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Complete List of Authors:	Imran, Muhammad; National University of Sciences & Technology (NUST), School of Natural Sciences Baig, A. Q.; COMSATS Insitute of Information Technology, Attock, Pakistan, Mathematics Ali, Haidar; COMSATS Insitute of Information Technology, Attock, Pakistan, Mathematics
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On Topological Properties of Dominating David Derived Networks^{*}

Muhammad Imran^{*}, Abdul Qudair Baig¹, Haidar Ali²

^{*}Department of Mathematics,
School of Natural Sciences (SNS),
National University of Sciences and Technology (NUST),
Sector H-12, Islamabad, Pakistan
Tel: +92 333 4736997

Email: imrandhab@gmail.com

¹Department of Mathematics,
COMSATS Institute of Information Technology, Attock

²Department of Mathematics,
Government College University, Faisalabad
{aqbaig1, haidar3830}@gmail.com

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Abstract. Topological indices are numerical parameters of a graph which characterize its molecular topology and are usually graph invariant. In QSAR/QSPR study, physico-chemical properties and topological indices such as Randić, atom-bond connectivity (ABC) and geometric-arithmetic (GA) index are used to predict the bioactivity of chemical compounds. Graph theory has found a considerable use in this important area of research.

All the studied interconnection networks in this paper are constructed by the Star of David network. In this paper, we study the general Randić, first Zagreb, ABC , GA , ABC_4 and GA_5 indices for the first, second and third type of Dominating David Derived networks and give closed formulas of these indices for these networks. These results are useful in network science to understand the underlying topologies of these networks.

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1 Introduction and preliminary results

Graph theory has provided chemists with a variety of useful tools, such as topological indices. Molecules and molecular compounds are often modeled by a molecular graph. A molecular graph is a representation of the structural formula of a chemical compound in terms of graph theory, whose vertices correspond to the atoms of the compound and edges correspond to chemical bonds. *Cheminformatics* is new subject which is a combination of chemistry, mathematics and information science. It studies Quantitative structure-activity (QSAR) and structure-property (QSPR) relationships that are used to predict the biological activities and properties of the chemical compounds. In the QSAR /QSPR study, physico-chemical properties and topological indices such as Wiener index, Szeged index, Randić index, Zagreb indices and *ABC* index are used to predict bioactivity of the chemical compounds.

A graph can be recognized by a numeric number, a polynomial, a sequence of numbers or by a matrix. A *topological index* is a numeric quantity associated with a graph which characterize the topology of graph and is invariant under graph automorphism. There are some major classes of topological indices such as distance based topological indices, degree based topological indices and counting related polynomials and indices of graphs. Among these classes degree based topological indices are of great importance and play a vital role in chemical graph theory and particularly in theoretical chemistry. In more precise way, a topological index $Top(G)$ of a graph, is a number with the property that for every graph H isomorphic to G , we have $Top(H) = Top(G)$. The concept of topological indices came from the work done by Wiener¹ while he was working on the boiling point of paraffin (an important member of alkane family). He named this index as *path number*. Later on, the path number was renamed as *Wiener index*² and the whole theory of topological indices started.

The algorithm for constructing David Derived and Dominating David Derived network (of dimension n) is as follows: Consider a Star of David network $SD(n)$ ³ of dimension n . Split each edge of it into two by inserting a new vertex. Resulting graph is called David Derived network $DD(n)$ of dimension n . Consider a honeycomb network denoted by $HC(n)$ of dimension n . Split each edge of $HC(n)$ into two by inserting a new vertex. In each hexagon cell, connect the new vertices by an edge if they are at a distance of 4 units within the cell. Place vertices at new edge crossings. Remove the initial vertices and edges of $HC(n)$. Split each horizontal edge into two edges by inserting a new vertex. Resulting graph is called Dominating David Derived network $DDD(n)$ of dimension n .

The Dominating David Derived network of first type denoted by $D_1(n)$ of dimension n can be obtained by connecting vertices of degree two of $DDD(n)$ by an edge, which are not in the boundary. The Dominating David Derived network of second type denoted by $D_2(n)$ of dimension n can be obtained by subdividing once the new edge introduced in $D_1(n)$. The Dominating David Derived network of third type $D_3(n)$ of dimension n can be obtained from $D_1(n)$ by introducing parallel path of length 2 between the vertices of degree two which are not in the boundary⁴.

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In this article, G is considered to be a network with vertex set $V(G)$ and edge set $E(G)$, the $\deg(u)$ is the degree of vertex $u \in V(G)$ and $S_u = \sum_{v \in N_G(u)} \deg(v)$ where $N_G(u) = \{v \in V(G) \mid uv \in E(G)\}$. The notations used in this article are mainly taken from the books^{5,6}.

Let G be a connected graph. Then the Wiener index of G is defined as

$$W(G) = \frac{1}{2} \sum_{(u,v)} d(u,v) \quad (1)$$

where (u, v) is any ordered pair of vertices in G and $d(u, v)$ is $u - v$ geodesic.

The very first and oldest degree based topological index is *Randić index*⁷ denoted by $R_{-\frac{1}{2}}(G)$ and was introduced by Milan Randić and is defined as

$$R_{-\frac{1}{2}}(G) = \sum_{uv \in E(G)} \frac{1}{\sqrt{\deg(u)\deg(v)}} \quad (2)$$

The general Randić index $R_\alpha(G)$ is the sum of $(\deg(u)\deg(v))^\alpha$ over all the edges $e = uv \in E(G)$ and is defined as

$$R_\alpha(G) = \sum_{uv \in E(G)} (\deg(u)\deg(v))^\alpha \text{ for } \alpha = 1, \frac{1}{2}, -1, -\frac{1}{2} \quad (3)$$

An important topological index introduced by Ivan Gutman and Trinajstić is the *Zagreb index* denoted by $M_1(G)$ and is defined as

$$M_1(G) = \sum_{uv \in E(G)} (\deg(u) + \deg(v)) \quad (4)$$

One of the well-known degree based topological index is *atom-bond connectivity (ABC)* index introduced by Estrada *et al.*⁸ and defined as

$$ABC(G) = \sum_{uv \in E(G)} \sqrt{\frac{\deg(u) + \deg(v) - 2}{\deg(u)\deg(v)}} \quad (5)$$

Another well-known connectivity topological descriptor is *geometric-arithmetic (GA)* index which was introduced by Vukičević *et al.*⁹ and defined as

$$GA(G) = \sum_{uv \in E(G)} \frac{2\sqrt{\deg(u)\deg(v)}}{\deg(u) + \deg(v)} \quad (6)$$

The fourth version of atom-bond connectivity index denoted by ABC_4 and fifth version of geometric-arithmetic index denoted by GA_5 can be computed if we are able to find the edge partition of these interconnection networks based on sum of the degrees of end vertices of each edge in these graphs. The

fourth version of ABC index is introduced by Ghorbani *et al.*¹⁰ and defined as

$$ABC_4(G) = \sum_{uv \in E(G)} \sqrt{\frac{S_u + S_v - 2}{S_u S_v}} \quad (7)$$

Recently, the fifth version of GA index is proposed by Graovac *et al.*¹¹ and is defined as

$$GA_5(G) = \sum_{uv \in E(G)} \frac{2\sqrt{S_u S_v}}{S_u + S_v} \quad (8)$$

The general Randić index for $\alpha = 1$ is the second Zagreb index for any graph G .

2 Main results

In this paper, we study the general Randić, first Zagreb, ABC , GA , ABC_4 and GA_5 indices and give closed formulas of these indices for the first, second and third type of Dominating David Derived network. Hayat *et al.* studied recently the degree based topological indices for various networks like silicates, hexagonal, honeycomb and oxide¹². Nowadays there is an extensive research activity on ABC and GA indices and their variants. For further study of topological indices of various graph families and their invariants see,^{13–23}.

2.1 Results for Dominating David Derived networks of first type

In this section, we compute certain degree based topological indices of Dominating David Derived network of first type denoted by $D_1(n)$. We compute the general Randić $R_\alpha(G)$ for $\alpha = \{1, -1, \frac{1}{2}, -\frac{1}{2}\}$, ABC , GA , ABC_4 and GA_5 indices for first type of Dominating David Derived networks in this section.

In the following theorem, the general Randić index for Dominating David Derived network of first type is computed.

Theorem 2.1.1. *Let $G \cong D_1(n)$ be the Dominating David Derived network of first type, then its general Randić index is equal to*

$$R_\alpha(D_1(n)) = \begin{cases} 1089n^2 - 1357n + 501, & \alpha = 1; \\ (72\sqrt{3} + 171)n^2 + (56\sqrt{2} - 112\sqrt{3} + 4\sqrt{6} - 239)n - \\ 32\sqrt{2} + 48\sqrt{3} - 4\sqrt{6} + 95, & \alpha = \frac{1}{2}; \\ \frac{1}{4}(25n^2 - \frac{151}{9}n + \frac{41}{9}), & \alpha = -1; \\ (6\sqrt{3} + 12)n^2 + (7\sqrt{2} - \frac{28}{3}\sqrt{3} + \frac{2}{3}\sqrt{6} - \frac{46}{3})n - \\ 4\sqrt{2} + 4\sqrt{3} - \frac{2}{3}\sqrt{6} + \frac{20}{3}, & \alpha = -\frac{1}{2}. \end{cases}$$

Fig. 1. Dominating David Derived network of first type ($D_1(2)$)

(d_u, d_v) where $uv \in E(G)$	Number of edges
(2, 2)	$4n$
(2, 3)	$4n - 4$
(2, 4)	$28n - 16$
(3, 3)	$9n^2 - 13n + 5$
(3, 4)	$36n^2 - 56n + 24$
(4, 4)	$36n^2 - 52n + 20$

Table 1. Edge partition of Dominating David Derived network of first type ($D_1(n)$) based on degrees of end vertices of each edge.

Proof. Let G be the Dominating David Derived network of first type. The Dominating David Derived network of first type ($D_1(n)$) has $20n - 10$ vertices of degree 2, $18n^2 - 26n + 10$ vertices of degree 3 and $27n^2 - 33n + 12$ vertices of degree 4. The edge set of $D_1(n)$ can be divided into six partitions based on the degree of end vertices. The first edge partition $E_1(D_1(n))$ contains $4n$ edges uv ; where $\deg(u) = \deg(v) = 2$. The second edge partition $E_2(D_1(n))$ contains $4n - 4$ edges uv , where $\deg(u) = 2$ and $\deg(v) = 3$. The third edge partition $E_3(D_1(n))$ contains $28n - 16$ edges uv ; where $\deg(u) = 2$ and $\deg(v) = 4$. The fourth edge partition $E_4(D_1(n))$ contains $9n^2 - 13n + 5$ edges uv ; where $\deg(u) = \deg(v) = 3$. The fifth edge partition $E_5(D_1(n))$ contains $36n^2 - 56n + 24$ edges uv ; where we have $\deg(u) = 3$ and $\deg(v) = 4$. The sixth edge partition $E_6(D_1(n))$ contains $36n^2 - 52n + 20$ edges uv ; where $\deg(u) = \deg(v) = 4$. Table 1 shows such an edge partition of $D_1(n)$. Thus from equation (3), it follows that

$$R_\alpha(G) = \sum_{uv \in E(G)} (\deg(u) \cdot \deg(v))^\alpha$$

For $\alpha = 1$

Now we apply the formula of general Randić index $R_\alpha(G)$ for $\alpha = 1$.

$$R_1(G) = \sum_{j=1}^6 \sum_{uv \in E_j(G)} \deg(u) \cdot \deg(v)$$

By using the edge partition given in Table 1, we get the following

$$R_1(G) = 4|E_1(D_1(n))| + 6|E_2(D_1(n))| + 8|E_3(D_1(n))| + 9|E_4(D_1(n))| + 12|E_5(D_1(n))| + 16|E_6(D_1(n))|$$

$$\implies R_1(G) = 1089n^2 - 1357n + 501$$

For $\alpha = \frac{1}{2}$

We apply the formula of $R_\alpha(G)$ for $\alpha = \frac{1}{2}$.

$$R_{\frac{1}{2}}(G) = \sum_{j=1}^6 \sum_{uv \in E_j(G)} \sqrt{\deg(u) \cdot \deg(v)}$$

By using the edge partition given in Table 1, we get

$$R_{\frac{1}{2}}(G) = 2|E_1(D_1(n))| + \sqrt{6}|E_2(D_1(n))| + 2\sqrt{2}|E_3(D_1(n))| + 3|E_4(D_1(n))| + 2\sqrt{3}|E_5(D_1(n))| + 4|E_6(D_1(n))|$$

$$\implies R_{\frac{1}{2}}(G) = (72\sqrt{3} + 171)n^2 + (56\sqrt{2} - 112\sqrt{3} + 4\sqrt{6} - 239)n - 32\sqrt{2} + 48\sqrt{3} - 4\sqrt{6} + 95$$

For $\alpha = -1$

We apply the formula of $R_\alpha(G)$ for $\alpha = -1$.

$$R_{-1}(G) = \sum_{j=1}^6 \sum_{uv \in E_j(G)} \frac{1}{\deg(u) \cdot \deg(v)}$$

$$R_{-1}(G) = \frac{1}{4}|E_1(D_1(n))| + \frac{1}{6}|E_2(D_1(n))| + \frac{1}{8}|E_3(D_1(n))| + \frac{1}{9}|E_4(D_1(n))| + \frac{1}{12}|E_5(D_1(n))| + \frac{1}{16}|E_6(D_1(n))|$$

$$\implies R_{-1}(G) = \frac{1}{4}(25n^2 - \frac{151}{9}n + \frac{41}{9})$$

For $\alpha = -\frac{1}{2}$

We apply the formula of $R_\alpha(G)$ for $\alpha = -\frac{1}{2}$.

$$R_{-\frac{1}{2}}(G) = \sum_{j=1}^6 \sum_{uv \in E_j(G)} \frac{1}{\sqrt{\deg(u) \cdot \deg(v)}}$$

$$R_{-\frac{1}{2}}(G) = \frac{1}{2}|E_1(D_1(n))| + \frac{1}{\sqrt{6}}|E_2(D_1(n))| + \frac{\sqrt{2}}{4}|E_3(D_1(n))| + \frac{1}{3}|E_4(D_1(n))| + \frac{\sqrt{3}}{6}|E_5(D_1(n))| + \frac{1}{4}|E_6(D_1(n))|$$

$$\implies R_{-\frac{1}{2}}(G) = (6\sqrt{3} + 12)n^2 + (7\sqrt{2} - \frac{28}{3}\sqrt{3} + \frac{2}{3}\sqrt{6} - \frac{46}{3})n - 4\sqrt{2} + 4\sqrt{3} - \frac{2}{3}\sqrt{6} + \frac{20}{3}$$

□

In the following theorem, we compute the first Zagreb index for Dominating David Derived network of first type denoted by $D_1(n)$.

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Theorem 2.1.2. For Dominating David Derived network of first type ($G \cong D_1(n)$), the first Zagreb index is equal to

$$M_1(D_1(n)) = 594n^2 - 682n + 242$$

Proof. Let G be the Dominating David Derived network of first type ($D_1(n)$). By using the edge partition from Table 1, the result directly follows. From the equation (4), we have

$$M_1(G) = \sum_{uv \in E(G)} (deg(u) + deg(v)) = \sum_{j=1}^6 \sum_{uv \in E_j(G)} (deg(u) + deg(v))$$

$$M_1(G) = 4|E_1(D_1(n))| + 5|E_2(D_1(n))| + 6|E_3(D_1(n))| + 6|E_4(D_1(n))| + 7|E_5(D_1(n))| + 8|E_6(D_1(n))|.$$

By doing some elementary calculation, we get

$$\implies M_1(G) = 594n^2 - 682n + 242$$

□

In the next theorem, we compute the ABC , GA , ABC_4 and GA_5 indices of Dominating David Derived network of first type denoted by $D_1(n)$.

Theorem 2.1.3. Let $G \cong D_1(n)$ be the Dominating David Derived network of first type for every positive integer $n \geq 2$; then we have

- $ABC(D_1(n)) = (6 + 9\sqrt{6} + 6\sqrt{15})n^2 + (\frac{54\sqrt{2}-39\sqrt{6}-28\sqrt{15}-26}{3})n + (\frac{10-30\sqrt{2}+12\sqrt{15}+15\sqrt{6}}{3})$.
- $GA(D_1(n)) = (\frac{315+144\sqrt{3}}{7})n^2 + (\frac{280\sqrt{2}-480\sqrt{3}+24\sqrt{6}-915}{15})n + (\frac{2625-1120\sqrt{2}+1440\sqrt{3}-168\sqrt{6}}{105})$.
- $ABC_4(D_1(n)) = (\frac{63\sqrt{20}+693\sqrt{2}+18\sqrt{3542}}{77})n^2 + (\frac{8}{9} - \frac{53}{3}\sqrt{2} + \frac{2}{3}\sqrt{6} + \frac{2}{7}\sqrt{26} + \frac{4}{13}\sqrt{26} + \frac{3}{13}\sqrt{39} + \frac{4}{\sqrt{7}} + \frac{5}{\sqrt{11}} + \frac{2}{3}\sqrt{10} + \frac{2}{11}\sqrt{110} + \frac{2}{7}\sqrt{42} + \frac{2}{21}\sqrt{357} + \frac{3}{11}\sqrt{374} + \frac{\sqrt{494}}{13} - \frac{7}{11}\sqrt{20} - \frac{34}{77}\sqrt{3542} + \frac{20}{\sqrt{182}})n + 10\sqrt{2} - \frac{2}{3}\sqrt{6} - \frac{3}{13}\sqrt{39} - \frac{2}{7}\sqrt{42} - \frac{4}{\sqrt{7}} - \frac{5}{\sqrt{11}} - \frac{4}{13}\sqrt{26} + \frac{2}{11}\sqrt{77} - \frac{2}{21}\sqrt{357} - \frac{2}{11}\sqrt{374} - \frac{\sqrt{494}}{13} + \frac{16}{77}\sqrt{3542} - \frac{20}{\sqrt{182}} - \frac{8}{9} + \frac{3}{11}\sqrt{20} - \frac{2}{7}\sqrt{26}$.
- $GA_5(D_1(n)) = (\frac{225+240\sqrt{14}+72\sqrt{154}}{25})n^2 + (3 + \frac{3}{2}\sqrt{7} + \frac{32}{27}\sqrt{11} + \frac{32}{29}\sqrt{13} - \frac{304}{15}\sqrt{14} + \frac{24}{23}\sqrt{14} + \frac{4}{5}\sqrt{21} + \frac{16}{19}\sqrt{21} + \frac{48}{19}\sqrt{22} + \frac{16}{21}\sqrt{26} + \frac{8}{13}\sqrt{42} + \frac{8}{17}\sqrt{66} + \frac{\sqrt{143}}{3} + \frac{8}{27}\sqrt{182} - \frac{136}{25}\sqrt{154})n + \frac{8}{3}\sqrt{2} - \frac{3}{2}\sqrt{7} - \frac{32}{27}\sqrt{11} - \frac{32}{29}\sqrt{13} + \frac{32}{3}\sqrt{14} - \frac{24}{23}\sqrt{14} - \frac{4}{5}\sqrt{21} - \frac{16}{19}\sqrt{21} - \frac{32}{19}\sqrt{22} - \frac{16}{21}\sqrt{26} + \frac{16}{23}\sqrt{33} - \frac{8}{13}\sqrt{42} - \frac{\sqrt{143}}{3} + \frac{64}{25}\sqrt{154} - \frac{8}{27}\sqrt{182} - 3$.

Proof. By finding the edge partition given in Table 1 and then by using the definition, we get the required result. From equation (5), it follows that

$$ABC(G) = \sum_{uv \in E(G)} \sqrt{\frac{deg(u) + deg(v) - 2}{deg(u) \cdot deg(v)}} = \sum_{j=1}^6 \sum_{uv \in E_j(G)} \sqrt{\frac{deg(u) + deg(v) - 2}{deg(u) \cdot deg(v)}}$$

$$ABC(D_1(n)) = \frac{1}{\sqrt{2}}|E_1(D_1(n))| + \frac{1}{\sqrt{2}}|E_2(D_1(n))| + \frac{1}{\sqrt{2}}|E_3(D_1(n))| + \frac{2}{3}|E_4(D_1(n))| + \frac{\sqrt{15}}{6}|E_5(D_1(n))| + \frac{\sqrt{6}}{4}|E_6(D_1(n))|.$$

By doing the some calculation, we get

$$\Rightarrow ABC(D_1(n)) = (6 + 9\sqrt{6} + 6\sqrt{15})n^2 + \left(\frac{54\sqrt{2}-39\sqrt{6}-28\sqrt{15}-26}{3}\right)n + \left(\frac{10-30\sqrt{2}+12\sqrt{15}+15\sqrt{6}}{3}\right).$$

From equation(6), we get

$$GA(D_1(n)) = \sum_{uv \in E(G)} \frac{2\sqrt{\deg(u) \cdot \deg(v)}}{\deg(u) + \deg(v)} = \sum_{j=1}^6 \sum_{uv \in E_j(G)} \frac{2\sqrt{\deg(u) \cdot \deg(v)}}{\deg(u) + \deg(v)}$$

By doing some calculation, we get

$$\begin{aligned} GA(D_1(n)) &= |E_1(D_1(n))| + 2\frac{\sqrt{6}}{5}|E_2(D_1(n))| + 2\frac{\sqrt{2}}{3}|E_3(D_1(n))| + |E_4(D_1(n))| + 4\frac{\sqrt{3}}{7}|E_5(D_1(n))| + \\ &|E_6(D_1(n))| \\ \Rightarrow GA(D_1(n)) &= \left(\frac{315+144\sqrt{3}}{7}\right)n^2 + \left(\frac{280\sqrt{2}-480\sqrt{3}+24\sqrt{6}-915}{15}\right)n + \left(\frac{2625-1120\sqrt{2}+1440\sqrt{3}-168\sqrt{6}}{105}\right). \end{aligned}$$

If we consider edge partitions based on degree sum of neighbors of end vertices; then the edge set $E(D_1(n))$ can be divided into twenty edge partition $E_j(D_1(n))$, $7 \leq j \leq 26$, where the edge partition $E_7(D_1(n))$ contains $4n$ edges uv with $S_u = S_v = 6$; the edge partition $E_8(D_1(n))$ contains $4n$ edges uv with $S_u = 6$ and $S_v = 11$; the edge partition $E_9(D_1(n))$ contains 4 edges uv with $S_u = 6$ and $S_v = 12$; the edge partition $E_{10}(D_1(n))$ contains $4n - 4$ edges uv with $S_u = 6$ and $S_v = 14$; the edge partition $E_{11}(D_1(n))$ contains $4n - 4$ edges uv with $S_u = 7$ and $S_v = 9$; the edge partition $E_{12}(D_1(n))$ contains $4n - 4$ edges uv with $S_u = 7$ and $S_v = 12$; the edge partition $E_{13}(D_1(n))$ contains $12n - 8$ edges uv with $S_u = 8$ and $S_v = 11$; the edge partition $E_{14}(D_1(n))$ contains $4n - 4$ edges uv with $S_u = 8$ and $S_v = 13$; the edge partition $E_{15}(D_1(n))$ contains $2n - 2$ edges uv with $S_u = S_v = 9$; the edge partition $E_{16}(D_1(n))$ contains $4n - 4$ edges uv with $S_u = 9$ and $S_v = 14$; the edge partition $E_{17}(D_1(n))$ contains $9n^2 - 7n + 3$ edges uv with $S_u = S_v = 11$; the edge partition $E_{18}(D_1(n))$ contains 4 edges uv with $S_u = 11$ and $S_v = 12$; the edge partition $E_{19}(D_1(n))$ contains $4n - 4$ edges uv with $S_u = 11$ and $S_v = 13$; the edge partition $E_{20}(D_1(n))$ contains $36n^2 - 68n + 32$ edges uv with $S_u = 11$ and $S_v = 14$; the edge partition $E_{21}(D_1(n))$ contains $4n - 4$ edges uv with $S_u = 11$ and $S_v = 16$; the edge partition $E_{22}(D_1(n))$ contains $4n - 4$ edges uv with $S_u = 12$ and $S_v = 14$; the edge partition $E_{23}(D_1(n))$ contains $4n - 4$ edges uv with $S_u = 13$ and $S_v = 14$; the edge partition $E_{24}(D_1(n))$ contains $4n - 4$ edges uv with $S_u = 13$ and $S_v = 16$; the edge partition $E_{25}(D_1(n))$ contains $4n - 4$ edges uv with $S_u = S_v = 14$ and the edge partition $E_{26}(D_1(n))$ contains $36n^2 - 76n + 40$ edges uv with $S_u = 14$ and $S_v = 16$.

From equation (7), we get

$$ABC_4(G) = \sum_{uv \in E(G)} \sqrt{\frac{S_u + S_v - 2}{S_u S_v}} = \sum_{j=7}^{26} \sum_{uv \in E_j(G)} \sqrt{\frac{S_u + S_v - 2}{S_u S_v}}.$$

By using the edge partition given in Table 2, we get the following

$$\begin{aligned} ABC_4(D_1(n)) &= \frac{\sqrt{10}}{6}|E_7(D_1(n))| + \frac{\sqrt{110}}{22}|E_8(D_1(n))| + \frac{\sqrt{2}}{3}|E_9(D_1(n))| + \frac{\sqrt{42}}{14}|E_{10}(D_1(n))| + \frac{\sqrt{2}}{3}|E_{11}(D_1(n))| + \\ &\frac{\sqrt{357}}{42}|E_{12}(D_1(n))| + \frac{\sqrt{374}}{44}|E_{13}(D_1(n))| + \frac{\sqrt{494}}{52}|E_{14}(D_1(n))| + \frac{4}{9}|E_{15}(D_1(n))| + \frac{\sqrt{6}}{6}|E_{16}(D_1(n))| + \end{aligned}$$

(S_u, S_v) where $uv \in E(G)$	Number of edges	(S_u, S_v) where $uv \in E(G)$	Number of edges
(6, 6)	$4n$	(11, 11)	$9n^2 - 7n + 3$
(6, 11)	$4n$	(11, 12)	4
(6, 12)	4	(11, 13)	$4n - 4$
(6, 14)	$4n - 4$	(11, 14)	$36n^2 - 68n + 32$
(7, 9)	$4n - 4$	(11, 16)	$4n - 4$
(7, 12)	$4n - 4$	(12, 14)	$4n - 4$
(8, 11)	$12n - 8$	(13, 14)	$4n - 4$
(8, 13)	$4n - 4$	(13, 16)	$4n - 4$
(9, 9)	$2n - 2$	(14, 14)	$4n - 4$
(9, 14)	$4n - 4$	(14, 16)	$36n^2 - 76n + 40$

Table 2. Edge partition of Dominating David Derived network of first type ($D_1(n)$) based on sum of degrees of end vertices of each edge.

$$2\frac{\sqrt{5}}{11}|E_{17}(D_1(n))| + \frac{\sqrt{77}}{22}|E_{18}(D_1(n))| + \frac{\sqrt{26}}{13}|E_{19}(D_1(n))| + \frac{\sqrt{23}}{\sqrt{154}}|E_{20}(D_1(n))| + 5\frac{\sqrt{11}}{44}|E_{21}(D_1(n))| + \frac{\sqrt{7}}{7}|E_{22}(D_1(n))| + \frac{5}{\sqrt{182}}|E_{23}(D_1(n))| + 3\frac{\sqrt{39}}{52}|E_{24}(D_1(n))| + \frac{\sqrt{26}}{14}|E_{25}(D_1(n))| + \frac{\sqrt{2}}{4}|E_{26}(D_1(n))|.$$

After some calculation, we get the following

$$\Rightarrow ABC_4(D_1(n)) = \left(\frac{63\sqrt{20}+693\sqrt{2}+18\sqrt{3542}}{77}\right)n^2 + \left(\frac{8}{9} - \frac{53}{3}\sqrt{2} + \frac{2}{3}\sqrt{6} + \frac{2}{7}\sqrt{26} + \frac{4}{13}\sqrt{26} + \frac{3}{13}\sqrt{39} + \frac{4}{\sqrt{7}} + \frac{5}{\sqrt{11}} + \frac{2}{3}\sqrt{10} + \frac{2}{11}\sqrt{110} + \frac{2}{7}\sqrt{42} + \frac{2}{21}\sqrt{357} + \frac{3}{11}\sqrt{374} + \frac{\sqrt{494}}{13} - \frac{7}{11}\sqrt{20} - \frac{34}{77}\sqrt{3542} + \frac{20}{\sqrt{182}}\right)n + 10\sqrt{2} - \frac{2}{3}\sqrt{6} - \frac{3}{13}\sqrt{39} - \frac{2}{7}\sqrt{42} - \frac{4}{\sqrt{7}} - \frac{5}{\sqrt{11}} - \frac{4}{13}\sqrt{26} + \frac{2}{11}\sqrt{77} - \frac{2}{21}\sqrt{357} - \frac{2}{11}\sqrt{374} - \frac{\sqrt{494}}{13} + \frac{16}{77}\sqrt{3542} - \frac{20}{\sqrt{182}} - \frac{8}{9} + \frac{3}{11}\sqrt{20} - \frac{2}{7}\sqrt{26}$$

From equation(8), we get

$$GA_5(G) = \sum_{uv \in E(G)} \frac{2\sqrt{S_u S_v}}{S_u + S_v} = \sum_{j=7}^{26} \sum_{uv \in E_j(G)} \frac{2\sqrt{S_u S_v}}{S_u + S_v}.$$

By using the edge partition given in Table 2, we get the following

$$GA_5(D_1(n)) = |E_7(D_1(n))| + 2\frac{\sqrt{66}}{17}|E_8(D_1(n))| + 2\frac{\sqrt{2}}{3}|E_9(D_1(n))| + \frac{\sqrt{21}}{5}|E_{10}(D_1(n))| + 3\frac{\sqrt{7}}{8}|E_{11}(D_1(n))| + 4\frac{\sqrt{21}}{19}|E_{12}(D_1(n))| + 4\frac{\sqrt{22}}{19}|E_{13}(D_1(n))| + 4\frac{\sqrt{26}}{21}|E_{14}(D_1(n))| + |E_{15}(D_1(n))| + 6\frac{\sqrt{14}}{23}|E_{16}(D_1(n))| + |E_{17}(D_1(n))| + 4\frac{\sqrt{33}}{23}|E_{18}(D_1(n))| + \frac{\sqrt{143}}{12}|E_{19}(D_1(n))| + 2\frac{\sqrt{154}}{25}|E_{20}(D_1(n))| + 8\frac{\sqrt{11}}{27}|E_{21}(D_1(n))| + 2\frac{\sqrt{42}}{13}|E_{22}(D_1(n))| + 2\frac{\sqrt{182}}{27}|E_{23}(D_1(n))| + 8\frac{\sqrt{13}}{29}|E_{24}(D_1(n))| + |E_{25}(D_1(n))| + 4\frac{\sqrt{14}}{15}|E_{26}(D_1(n))|.$$

After some calculations, we have

$$\Rightarrow GA_5(D_1(n)) = \left(\frac{225+240\sqrt{14}+72\sqrt{154}}{25}\right)n^2 + \left(3 + \frac{3}{2}\sqrt{7} + \frac{32}{27}\sqrt{11} + \frac{32}{29}\sqrt{13} - \frac{304}{15}\sqrt{14} + \frac{24}{23}\sqrt{14} + \frac{4}{5}\sqrt{21} + \frac{16}{19}\sqrt{21} + \frac{48}{19}\sqrt{22} + \frac{16}{21}\sqrt{26} + \frac{8}{13}\sqrt{42} + \frac{8}{17}\sqrt{66} + \frac{\sqrt{143}}{3} + \frac{8}{27}\sqrt{182} - \frac{136}{25}\sqrt{154}\right)n + \frac{8}{3}\sqrt{2} - \frac{3}{2}\sqrt{7} - \frac{32}{27}\sqrt{11} - \frac{32}{29}\sqrt{13} + \frac{32}{3}\sqrt{14} - \frac{24}{23}\sqrt{14} - \frac{4}{5}\sqrt{21} - \frac{16}{19}\sqrt{21} - \frac{32}{19}\sqrt{22} - \frac{16}{21}\sqrt{26} + \frac{16}{23}\sqrt{33} - \frac{8}{13}\sqrt{42} - \frac{\sqrt{143}}{3} + \frac{64}{25}\sqrt{154} - \frac{8}{27}\sqrt{182} - 3.$$

□

2.2 Results for Dominating David Derived networks of second type

In this section, we calculate some degree based topological indices of Dominating David Derived network of second type denoted by $D_2(n)$ of dimension n . We compute the general Randić index $R_\alpha(G)$ with $\alpha = \{1, -1, \frac{1}{2}, -\frac{1}{2}\}$, ABC , GA , ABC_4 and GA_5 for $D_2(n)$.

Theorem 2.2.1. *Let $G \cong D_2(n)$ denotes the Dominating David Derived network of second type, then its general Randić index is equal to*

$$R_\alpha(D_2(n)) = \begin{cases} 1116n^2 - 1396n + 516, & \alpha = 1; \\ (72\sqrt{3} + 18\sqrt{6} + 144)n^2 + (56\sqrt{2} - 112\sqrt{3} - 22\sqrt{6} - 200)n - 32\sqrt{2} + 48\sqrt{3} + 6\sqrt{6} + 80, & \alpha = \frac{1}{2} \\ \frac{108n^2 - 85n + 27}{12}, & \alpha = -1; \\ (6\sqrt{3} + 3\sqrt{6} + 9)n^2 + (7\sqrt{2} - \frac{28}{\sqrt{3}} - \frac{22}{\sqrt{6}} - 11)n + 5 - 4\sqrt{2} + 4\sqrt{3} + \sqrt{6}, & \alpha = -\frac{1}{2}. \end{cases}$$

Fig. 2. Dominating David Derived network of second type ($D_2(4)$)

(d_u, d_v) where $uv \in E(G)$	Number of edges
(2, 2)	$4n$
(2, 3)	$18n^2 - 22n + 6$
(2, 4)	$28n - 16$
(3, 4)	$36n^2 - 56n + 24$
(4, 4)	$36n^2 - 52n + 20$

Table 3. Edge partition of Dominating David Derived network of second type ($D_2(n)$) based on degrees of end vertices of each edge.

Proof. Let G be the Dominating David Derived network of second type. The second Dominating David Derived network denoted by $D_2(n)$ has $9n^2 + 7n - 5$ vertices of degree 2, $18n^2 - 26n + 10$ vertices of

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degree 3 and $27n^2 - 33n + 12$ vertices of degree 4. The edge set of $D_2(n)$ is divided into five partitions based on the degree of end vertices. The first edge partition $E_1(D_2(n))$ contains $4n$ edges uv ; where $\deg(u) = \deg(v) = 2$. The second edge partition $E_2(D_2(n))$ contains $18n^2 - 22n + 6$ edges uv ; where $\deg(u) = 2$ and $\deg(v) = 3$. The third edge partition $E_3(D_2(n))$ contains $28n - 16$ edges uv ; where $\deg(u) = 2$ and $\deg(v) = 4$. The fourth edge partition $E_4(D_2(n))$ contains $36n^2 - 56n + 24$ edges uv ; where $\deg(u) = 3$ and $\deg(v) = 4$. The fifth edge partition $E_5(D_2(n))$ contains $36n^2 - 52n + 20$ edges uv ; where $\deg(u) = \deg(v) = 4$. Table 3 shows such an edge partition of Dominating David Derived network of second type ($D_2(n)$). Thus from equation (3), it follows that

$$R_\alpha(G) = \sum_{uv \in E(G)} (\deg(u) \cdot \deg(v))^\alpha$$

For $\alpha = 1$

Now we apply the formula of general Randić index $R_\alpha(G)$ for $\alpha = 1$.

$$R_1(G) = \sum_{j=1}^5 \sum_{uv \in E_j(G)} \deg(u) \cdot \deg(v)$$

By using the edge partition given in Table 3, we get

$$R_1(G) = 4|E_1(D_2(n))| + 6|E_2(D_2(n))| + 8|E_3(D_2(n))| + 12|E_4(D_2(n))| + 16|E_5(D_2(n))|.$$

$$\implies R_1(G) = 1116n^2 - 1396n + 516.$$

For $\alpha = \frac{1}{2}$

We apply the formula of $R_\alpha(G)$ for $\alpha = \frac{1}{2}$.

$$R_{\frac{1}{2}}(G) = \sum_{j=1}^5 \sum_{uv \in E_j(G)} \sqrt{\deg(u) \cdot \deg(v)}$$

By using the edge partition given in Table 3, we get

$$R_{\frac{1}{2}}(D_2(n)) = 2|E_1(D_2(n))| + \sqrt{6}|E_2(D_2(n))| + 2\sqrt{2}|E_3(D_2(n))| + 2\sqrt{3}|E_4(D_2(n))| + 4|E_5(D_2(n))|.$$

$$\implies R_{\frac{1}{2}}(D_2(n)) = (72\sqrt{3} + 18\sqrt{6} + 144)n^2 + (56\sqrt{2} - 112\sqrt{3} - 22\sqrt{6} - 200)n - 32\sqrt{2} + 48\sqrt{3} + 6\sqrt{6} + 80$$

For $\alpha = -1$

We apply the formula of $R_\alpha(G)$ for $\alpha = -1$.

$$R_{-1}(G) = \sum_{j=1}^5 \sum_{uv \in E_j(G)} \frac{1}{\deg(u) \cdot \deg(v)}$$

$$R_{-1}(D_2(n)) = \frac{1}{4}|E_1(D_2(n))| + \frac{1}{6}|E_2(D_2(n))| + \frac{1}{8}|E_3(D_2(n))| + \frac{1}{12}|E_4(D_2(n))| + \frac{1}{16}|E_5(D_2(n))|$$

$$\implies R_{-1}(D_2(n)) = \frac{108n^2 - 85n + 27}{12}$$

For $\alpha = -\frac{1}{2}$

We apply the formula of $R_\alpha(G)$ for $\alpha = -\frac{1}{2}$.

$$R_{-\frac{1}{2}}(G) = \sum_{j=1}^5 \sum_{uv \in E_j(G)} \frac{1}{\sqrt{\deg(u) \cdot \deg(v)}}$$

$$R_{-\frac{1}{2}}(D_2(n)) = \frac{1}{2}|E_1(D_2(n))| + \frac{1}{\sqrt{6}}|E_2(D_2(n))| + \frac{\sqrt{2}}{4}|E_3(D_2(n))| + \frac{\sqrt{3}}{6}|E_4(D_2(n))| + \frac{1}{4}|E_5(D_2(n))|$$

$$\implies R_{-\frac{1}{2}}(D_2(n)) = (6\sqrt{3} + 3\sqrt{6} + 9)n^2 + (7\sqrt{2} - \frac{28}{\sqrt{3}} - \frac{22}{\sqrt{6}} - 11)n + 5 - 4\sqrt{2} + 4\sqrt{3} + \sqrt{6}$$

□

In the following theorem, we compute the first Zagreb index of Dominating David Derived network of second type denoted by $D_2(n)$.

Theorem 2.2.2. For the Dominating David Derived network of second type $G \cong D_2(n)$, the first Zagreb index is equal to

$$M_1(D_2(n)) = 630n^2 - 734n + 262$$

Proof. Let $D_2(n)$ denotes the second Dominating David Derived network of second type. By using the edge partition from Table 3, the result follows. From equation (4), we have

$$M_1(G) = \sum_{uv \in E(G)} (\deg(u) + \deg(v)) = \sum_{j=1}^5 \sum_{uv \in E_j(G)} (\deg(u) + \deg(v))$$

$$M_1(D_2(n)) = 4|E_1(D_2(n))| + 5|E_2(D_2(n))| + 6|E_3(D_2(n))| + 7|E_4(D_2(n))| + 8|E_5(D_2(n))|$$

By doing the some calculation, we get the following

$$\implies M_1(D_2(n)) = 630n^2 - 734n + 262$$

□

Next, we compute the ABC , GA , ABC_4 and GA_5 indices of Dominating David Derived network of second type denoted by $D_2(n)$.

Theorem 2.2.3. Let $G \cong D_2(n)$ be the Dominating David Derived network of second type, then we have

• $ABC(G) = (9\sqrt{2} + 9\sqrt{6} + 6\sqrt{15})n^2 + (5\sqrt{2} - 13\sqrt{6} - \frac{28}{3}\sqrt{15})n - 5\sqrt{2} + 5\sqrt{6} + 4\sqrt{15}$, for every positive integer $n \geq 1$.

• $GA(G) = (\frac{1260+720\sqrt{3}+252\sqrt{6}}{35})n^2 + (\frac{56}{3}\sqrt{2} - 32\sqrt{3} - \frac{44}{5}\sqrt{6} - 48)n + 20 - \frac{32}{3}\sqrt{2} + \frac{96}{7}\sqrt{3} + \frac{12}{5}\sqrt{6}$, for every

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positive integer $n \geq 1$.

• $ABC_4(G) = (\frac{315\sqrt{2}+21\sqrt{210}+18\sqrt{770}}{35})n^2 + (2 - 19\sqrt{2} + \frac{2}{3}\sqrt{10} + \frac{5}{\sqrt{11}} + \frac{2}{7}\sqrt{26} + \frac{2}{7}\sqrt{35} + \frac{2}{7}\sqrt{42} + \frac{4}{\sqrt{7}} + \frac{2}{11}\sqrt{110} + \frac{\sqrt{182}}{7} + \frac{20}{\sqrt{182}} - \sqrt{210} + \frac{351}{\sqrt{13}} + \frac{2}{21}\sqrt{357} + \frac{3}{11}\sqrt{374} + \frac{\sqrt{494}}{13} - \frac{36}{35}\sqrt{770} - \frac{4}{55}\sqrt{2090} + \frac{2}{65}\sqrt{2730} + \frac{2}{77}\sqrt{3542})n + \frac{34}{3}\sqrt{2} - \frac{2}{7}\sqrt{26} - \frac{2}{7}\sqrt{35} - \frac{2}{7}\sqrt{42} - \frac{4}{\sqrt{7}} + \frac{2}{11}\sqrt{77} - \frac{5}{\sqrt{11}} - \frac{20}{\sqrt{182}} - \frac{\sqrt{182}}{7} + \frac{7}{15}\sqrt{210} - \frac{\sqrt{351}}{13} - \frac{2}{21}\sqrt{357} - \frac{2}{11}\sqrt{374} - \frac{\sqrt{494}}{13} + \frac{18}{35}\sqrt{770} - \frac{2}{55}\sqrt{2090} - \frac{2}{65}\sqrt{2730} - \frac{2}{77}\sqrt{3542} - 2$, for every positive integer $n \geq 2$.

• $GA_5(G) = (\frac{96\sqrt{14}+45\sqrt{15}+60\sqrt{35}}{10})n^2 + (8 + \frac{16}{7}\sqrt{3} + \frac{16}{11}\sqrt{7} + \frac{32}{27}\sqrt{11} + \frac{32}{29}\sqrt{13} + \frac{16}{15}\sqrt{14} - \frac{96}{13}\sqrt{14} - \frac{15}{2}\sqrt{15} + \frac{4}{5}\sqrt{21} + \frac{16}{19}\sqrt{21} + \frac{48}{19}\sqrt{22} + \frac{16}{21}\sqrt{26} - 12\sqrt{35} + \frac{8}{13}\sqrt{42} + \frac{8}{17}\sqrt{66} + \frac{8}{11}\sqrt{110} + \frac{8}{23}\sqrt{130} + \frac{8}{25}\sqrt{154} + \frac{8}{27}\sqrt{182})n + \frac{8}{3}\sqrt{2} - \frac{16}{7}\sqrt{3} - \frac{16}{11}\sqrt{7} - \frac{32}{27}\sqrt{11} - \frac{32}{29}\sqrt{13} + \frac{32}{3}\sqrt{14} - \frac{16}{15}\sqrt{14} + \frac{7}{2}\sqrt{15} - \frac{4}{5}\sqrt{21} - \frac{16}{19}\sqrt{21} - \frac{32}{19}\sqrt{22} - \frac{16}{21}\sqrt{26} + \frac{16}{23}\sqrt{33} + 6\sqrt{35} - \frac{8}{13}\sqrt{42} - \frac{4}{11}\sqrt{110} - \frac{8}{23}\sqrt{130} - \frac{8}{25}\sqrt{154} - \frac{8}{27}\sqrt{182} - 4$, for every positive integer $n \geq 2$.

(S_u, S_v) where $uv \in E(G)$	Number of edges	(S_u, S_v) where $uv \in E(G)$	Number of edges
(6, 6)	$4n$	(10, 11)	$8n - 4$
(6, 8)	$4n - 4$	(10, 13)	$4n - 4$
(6, 10)	$18n^2 - 30n + 14$	(10, 14)	$36n^2 - 72n + 36$
(6, 11)	$4n$	(11, 12)	4
(6, 12)	4	(11, 14)	$4n - 4$
(6, 14)	$4n - 4$	(11, 16)	$4n - 4$
(7, 8)	$4n - 4$	(12, 14)	$4n - 4$
(7, 12)	$4n - 4$	(13, 14)	$4n - 4$
(8, 11)	$12n - 8$	(13, 16)	$4n - 4$
(8, 13)	$4n - 4$	(14, 14)	$4n - 4$
(8, 14)	$4n - 4$	(14, 16)	$36n^2 - 76n + 40$

Table 4. Edge partition of Dominating David Derived network of second type ($D_2(n)$) based on sum of degrees of end vertices of each edge.

Proof. By using the edge partition given in Table 3, we get the result. From equation (5), it follows that

$$ABC(G) = \sum_{uv \in E(G)} \sqrt{\frac{\deg(u) + \deg(v) - 2}{\deg(u) \cdot \deg(v)}} = \sum_{j=1}^5 \sum_{uv \in E_j(G)} \sqrt{\frac{\deg(u) + \deg(v) - 2}{\deg(u) \cdot \deg(v)}}.$$

$$ABC(D_2(n)) = \frac{1}{\sqrt{2}}|E_1(D_2(n))| + \frac{1}{\sqrt{2}}|E_2(D_2(n))| + \frac{1}{\sqrt{2}}|E_3(D_2(n))| + \frac{\sqrt{15}}{6}|E_4(D_2(n))| + \frac{\sqrt{6}}{4}|E_5(D_2(n))|.$$

By doing the some calculation, we get

$$\Rightarrow ABC(D_2(n)) = (9\sqrt{2} + 9\sqrt{6} + 6\sqrt{15})n^2 + (5\sqrt{2} - 13\sqrt{6} - \frac{28}{3}\sqrt{15})n - 5\sqrt{2} + 5\sqrt{6} + 4\sqrt{15}$$

From equation(6), we get

$$GA(D_2(n)) = \sum_{uv \in E(G)} \frac{2\sqrt{\deg(u) \cdot \deg(v)}}{\deg(u) + \deg(v)} = \sum_{j=1}^5 \sum_{uv \in E_j(G)} \frac{2\sqrt{\deg(u) \cdot \deg(v)}}{\deg(u) + \deg(v)}.$$

By using the edge partition given in Table 3, we get

$$GA(D_2(n)) = |E_1(D_2(n))| + 2\frac{\sqrt{6}}{5}|E_2(D_2(n))| + 2\frac{\sqrt{2}}{6}|E_3(D_2(n))| + 4\frac{\sqrt{3}}{7}|E_4(D_2(n))| + |E_5(D_2(n))|.$$

By doing the some calculation, we get

$$\implies GA(D_2(n)) = \left(\frac{1260+720\sqrt{3}+252\sqrt{6}}{35}\right)n^2 + \left(\frac{56}{3}\sqrt{2} - 32\sqrt{3} - \frac{44}{5}\sqrt{6} - 48\right)n + 20 - \frac{32}{3}\sqrt{2} + \frac{96}{7}\sqrt{3} + \frac{12}{5}\sqrt{6}.$$

If we consider edge partitions based on degree sum of neighbors of end vertices, then the edge set $E(D_2(n))$ can be divided into twenty two edge partition $E_j(D_2(n))$, $6 \leq j \leq 27$; where the edge partition $E_6(D_2(n))$ contains $4n$ edges uv with $S_u = S_v = 6$. The edge partition $E_7(D_2(n))$ contains $4n - 4$ edges uv with $S_u = 6$ and $S_v = 8$, the edge partition $E_8(D_2(n))$ contains $18n^2 - 30n + 14$ edges uv with $S_u = 6$ and $S_v = 10$, the edge partition $E_9(D_2(n))$ contains $4n$ edges uv with $S_u = 6$ and $S_v = 11$; the edge partition $E_{10}(D_2(n))$ contains 4 edges uv with $S_u = 6$ and $S_v = 12$; the edge partition $E_{11}(D_2(n))$ contains $4n - 4$ edges uv with $S_u = 6$ and $S_v = 14$; the edge partition $E_{12}(D_2(n))$ contains $4n - 4$ edges uv with $S_u = 7$ and $S_v = 8$; the edge partition $E_{13}(D_2(n))$ contains $4n - 4$ edges uv with $S_u = 7$ and $S_v = 12$; the edge partition $E_{14}(D_2(n))$ contains $12n - 8$ edges uv with $S_u = 8$ and $S_v = 11$; the edge partition $E_{15}(D_2(n))$ contains $4n - 4$ edges uv with $S_u = 8$ and $S_v = 13$, the edge partition $E_{16}(D_2(n))$ contains $4n - 4$ edges uv with $S_u = 8$ and $S_v = 14$, the edge partition $E_{17}(D_2(n))$ contains $8n - 4$ edges uv with $S_u = 10$ and $S_v = 11$; the edge partition $E_{18}(D_2(n))$ contains $4n - 4$ edges uv with $S_u = 10$ and $S_v = 13$; the edge partition $E_{19}(D_2(n))$ contains $36n^2 - 72n + 36$ edges uv with $S_u = 10$ and $S_v = 14$, the edge partition $E_{20}(D_2(n))$ contains 4 edges uv with $S_u = 11$ and $S_v = 12$, the edge partition $E_{21}(D_2(n))$ contains $4n - 4$ edges uv with $S_u = 11$ and $S_v = 14$; the edge partition $E_{22}(D_2(n))$ contains $4n - 4$ edges uv with $S_u = 11$ and $S_v = 16$; the edge partition $E_{23}(D_2(n))$ contains $4n - 4$ edges uv with $S_u = 12$ and $S_v = 14$; the edge partition $E_{24}(D_2(n))$ contains $4n - 4$ edges uv with $S_u = 13$ and $S_v = 14$; the edge partition $E_{25}(D_2(n))$ contains $4n - 4$ edges uv with $S_u = 13$ and $S_v = 16$; the edge partition $E_{26}(D_2(n))$ contains $4n - 4$ edges uv with $S_u = S_v = 14$ and the edge partition $E_{27}(D_2(n))$ contains $36n^2 - 76n + 40$ edges uv with $S_u = 14$ and $S_v = 16$.

From equation (7), we get

$$ABC_4(G) = \sum_{uv \in E(G)} \sqrt{\frac{S_u + S_v - 2}{S_u S_v}} = \sum_{j=6}^{27} \sum_{uv \in E_j(G)} \sqrt{\frac{S_u + S_v - 2}{S_u S_v}}.$$

By using the edge partition given in Table 4, we get the following

$$\begin{aligned} ABC_4(D_2(n)) = & \frac{\sqrt{10}}{6}|E_6(D_2(n))| + \frac{1}{2}|E_7(D_2(n))| + \frac{\sqrt{210}}{30}|E_8(D_2(n))| + \frac{\sqrt{110}}{22}|E_9(D_2(n))| + \frac{\sqrt{2}}{3}|E_{10}(D_2(n))| + \\ & \frac{\sqrt{42}}{14}|E_{11}(D_2(n))| + \frac{\sqrt{182}}{28}|E_{12}(D_2(n))| + \frac{\sqrt{357}}{42}|E_{13}(D_2(n))| + \frac{\sqrt{374}}{44}|E_{14}(D_2(n))| + \frac{\sqrt{494}}{52}|E_{15}(D_2(n))| + \\ & \frac{\sqrt{35}}{14}|E_{16}(D_2(n))| + \sqrt{\frac{19}{110}}|E_{17}(D_2(n))| + \sqrt{\frac{21}{130}}|E_{18}(D_2(n))| + \frac{\sqrt{770}}{70}|E_{19}(D_2(n))| + \frac{\sqrt{77}}{22}|E_{20}(D_2(n))| + \end{aligned}$$

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$$\sqrt{\frac{23}{154}}|E_{21}(D_2(n))| + 5\sqrt{\frac{11}{44}}|E_{22}(D_2(n))| + \frac{\sqrt{7}}{7}|E_{23}(D_2(n))| + \frac{5}{\sqrt{182}}|E_{24}(D_2(n))| + 3\sqrt{\frac{39}{52}}|E_{25}(D_2(n))| + \frac{\sqrt{26}}{14}|E_{26}(D_2(n))| + \frac{\sqrt{2}}{4}|E_{27}(D_2(n))|.$$

After some calculation, we get the following

$$\Rightarrow ABC_4(D_2(n)) = \left(\frac{315\sqrt{2}+21\sqrt{210}+18\sqrt{770}}{35}\right)n^2 + (2 - 19\sqrt{2} + \frac{2}{3}\sqrt{10} + \frac{5}{\sqrt{11}} + \frac{2}{7}\sqrt{26} + \frac{2}{7}\sqrt{35} + \frac{2}{7}\sqrt{42} + \frac{4}{\sqrt{7}} + \frac{2}{11}\sqrt{110} + \frac{\sqrt{182}}{7} + \frac{20}{\sqrt{182}} - \sqrt{210} + \frac{351}{\sqrt{13}} + \frac{2}{21}\sqrt{357} + \frac{3}{11}\sqrt{374} + \frac{\sqrt{494}}{13} - \frac{36}{35}\sqrt{770} - \frac{4}{55}\sqrt{2090} + \frac{2}{65}\sqrt{2730} + \frac{2}{77}\sqrt{3542})n + \frac{34}{3}\sqrt{2} - \frac{2}{7}\sqrt{26} - \frac{2}{7}\sqrt{35} - \frac{2}{7}\sqrt{42} - \frac{4}{\sqrt{7}} + \frac{2}{11}\sqrt{77} - \frac{5}{\sqrt{11}} - \frac{20}{\sqrt{182}} - \frac{\sqrt{182}}{7} + \frac{7}{15}\sqrt{210} - \frac{\sqrt{351}}{13} - \frac{2}{21}\sqrt{357} - \frac{2}{11}\sqrt{374} - \frac{\sqrt{494}}{13} + \frac{18}{35}\sqrt{770} - \frac{2}{55}\sqrt{2090} - \frac{2}{65}\sqrt{2730} - \frac{2}{77}\sqrt{3542} - 2.$$

From equation (8), we get

$$GA_5(G) = \sum_{uv \in E(G)} \frac{2\sqrt{S_u S_v}}{S_u + S_v} = \sum_{j=6}^{27} \sum_{uv \in E_j(G)} \frac{2\sqrt{S_u S_v}}{S_u + S_v}.$$

By using the edge partition given in Table 4, we get the following

$$GA_5(D_2(n)) = |E_6(D_2(n))| + 4\sqrt{\frac{3}{7}}|E_7(D_2(n))| + \frac{\sqrt{15}}{4}|E_8(D_2(n))| + 2\sqrt{\frac{66}{17}}|E_9(D_2(n))| + 2\sqrt{\frac{2}{3}}|E_{10}(D_2(n))| + \frac{\sqrt{21}}{5}|E_{11}(D_2(n))| + 4\sqrt{\frac{14}{15}}|E_{12}(D_2(n))| + 4\sqrt{\frac{21}{19}}|E_{13}(D_2(n))| + 4\sqrt{\frac{22}{19}}|E_{14}(D_2(n))| + 4\sqrt{\frac{26}{21}}|E_{15}(D_2(n))| + 4\sqrt{\frac{7}{11}}|E_{16}(D_2(n))| + 2\sqrt{\frac{110}{21}}|E_{17}(D_2(n))| + 2\sqrt{\frac{130}{23}}|E_{18}(D_2(n))| + \frac{\sqrt{35}}{6}|E_{19}(D_2(n))| + 4\sqrt{\frac{33}{23}}|E_{20}(D_2(n))| + 2\sqrt{\frac{154}{25}}|E_{21}(D_2(n))| + 8\sqrt{\frac{11}{27}}|E_{22}(D_2(n))| + 2\sqrt{\frac{42}{13}}|E_{23}(D_2(n))| + 2\sqrt{\frac{182}{27}}|E_{24}(D_2(n))| + 8\sqrt{\frac{13}{29}}|E_{25}(D_2(n))| + |E_{26}(D_2(n))| + 4\sqrt{\frac{14}{15}}|E_{27}(D_2(n))|.$$

After calculation, we get

$$\Rightarrow GA_5(D_2(n)) = \left(\frac{96\sqrt{14}+45\sqrt{15}+60\sqrt{35}}{10}\right)n^2 + (8 + \frac{16}{7}\sqrt{3} + \frac{16}{11}\sqrt{7} + \frac{32}{27}\sqrt{11} + \frac{32}{29}\sqrt{13} + \frac{16}{15}\sqrt{14} - \frac{96}{13}\sqrt{14} - \frac{15}{2}\sqrt{15} + \frac{4}{5}\sqrt{21} + \frac{16}{19}\sqrt{21} + \frac{48}{19}\sqrt{22} + \frac{16}{21}\sqrt{26} - 12\sqrt{35} + \frac{8}{13}\sqrt{42} + \frac{8}{17}\sqrt{66} + \frac{8}{11}\sqrt{110} + \frac{8}{23}\sqrt{130} + \frac{8}{25}\sqrt{154} + \frac{8}{27}\sqrt{182})n + \frac{8}{3}\sqrt{2} - \frac{16}{7}\sqrt{3} - \frac{16}{11}\sqrt{7} - \frac{32}{27}\sqrt{11} - \frac{32}{29}\sqrt{13} + \frac{32}{3}\sqrt{14} - \frac{16}{15}\sqrt{14} + \frac{7}{2}\sqrt{15} - \frac{4}{5}\sqrt{21} - \frac{16}{19}\sqrt{21} - \frac{32}{19}\sqrt{22} - \frac{16}{21}\sqrt{26} + \frac{16}{23}\sqrt{33} + 6\sqrt{35} - \frac{8}{13}\sqrt{42} - \frac{4}{11}\sqrt{110} - \frac{8}{23}\sqrt{130} - \frac{8}{25}\sqrt{154} - \frac{8}{27}\sqrt{182} - 4.$$

□

2.3 Results for Dominating David Derived networks of third type

We calculate certain degree based topological indices of Dominating David Derived network of third type denoted by $D_3(n)$ (of dimension n) in this section. We compute the general Randić index $R_\alpha(G)$ with $\alpha = \{1, -1, \frac{1}{2}, -\frac{1}{2}\}$, ABC , GA , ABC_4 and GA_5 in the following theorems for $D_3(n)$.

The general Randić index for $D_3(n)$ is computed in the following theorem.

Theorem 2.3.1. *Let $G \cong D_3(n)$ denotes the Dominating David Derived network of third type, then its general Randić index is equal to*

$$R_{\alpha}(D_3(n)) = \begin{cases} 1440n^2 - 1872n + 704, & \alpha = 1; \\ (288 + 72\sqrt{2})n^2 - (40\sqrt{2} + 424)n + 176, & \alpha = \frac{1}{2} \\ \frac{36n^2 - 33n + 11}{4}, & \alpha = -1; \\ (18 + 9\sqrt{2})n^2 - (25 + 5\sqrt{2})n + 44, & \alpha = -\frac{1}{2}. \end{cases}$$

Fig. 3. Dominating David Derived network of third type type ($D_3(2)$)

(d_u, d_v) where $uv \in E(G)$	Number of edges
(2, 2)	$4n$
(2, 4)	$36n^2 - 20n$
(4, 4)	$72n^2 - 108n + 44$

Table 5. Edge partition of Dominating David Derived network of third type ($D_3(n)$) based on degrees of end vertices of each edge.

Proof. Let $D_3(n)$ be the Dominating David Derived network of third type. The Dominating David Derived network of third type denoted by $D_3(n)$ has $18n^2 - 6n$ vertices of degree 2 and $45n^2 - 59n + 22$ vertices of degree 4. The edge set of $D_3(n)$ is divided into three partitions based on the degree of end vertices. The first edge partition $E_1(D_3(n))$ contains $4n$ edges uv ; where $deg(u) = deg(v) = 2$. The second edge partition $E_2(D_3(n))$ contains $36n^2 - 20n$ edges uv ; where $deg(u) = 2$ and $deg(v) = 4$. The third edge partition $E_3(D_3(n))$ contains $72n^2 - 108n + 44$ edges uv ; where $deg(u) = deg(v) = 4$. Table 5 shows such an edge partition of $D_3(n)$. Thus from equation (3), it follows that

$$R_{\alpha}(G) = \sum_{uv \in E(G)} (deg(u) \cdot deg(v))^{\alpha}$$

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For $\alpha = 1$

Now we apply the formula of $R_\alpha(G)$ for $\alpha = 1$.

$$R_1(G) = \sum_{j=1}^3 \sum_{uv \in E_j(G)} \deg(u) \cdot \deg(v)$$

By using the edge partition given in Table 5, we get

$$R_1(G) = 4|E_1(D_3(n))| + 8|E_2(D_3(n))| + 16|E_3(D_3(n))|$$

$$\implies R_1(G) = 1440n^2 - 1872n + 704$$

For $\alpha = \frac{1}{2}$

We apply the formula of $R_\alpha(G)$ for $\alpha = \frac{1}{2}$.

$$R_{\frac{1}{2}}(G) = \sum_{j=1}^3 \sum_{uv \in E_j(G)} \sqrt{\deg(u) \cdot \deg(v)}$$

By using the edge partition given in Table 5, we get

$$R_{\frac{1}{2}}(G) = 2|E_1(D_3(n))| + 2\sqrt{2}|E_2(D_3(n))| + 4|E_3(D_3(n))|$$

$$\implies R_{\frac{1}{2}}(G) = (288 + 72\sqrt{2})n^2 - (40\sqrt{2} + 424)n + 176$$

For $\alpha = -1$

We apply the formula of $R_\alpha(G)$ for $\alpha = -1$.

$$R_{-1}(G) = \sum_{j=1}^3 \sum_{uv \in E_j(G)} \frac{1}{\deg(u) \cdot \deg(v)}$$

$$R_{-1}(G) = \frac{1}{4}|E_1(D_3(n))| + \frac{1}{8}|E_2(D_3(n))| + \frac{1}{16}|E_3(D_3(n))|$$

$$\implies R_{-1}(G) = \frac{36n^2 - 33n + 11}{4}$$

For $\alpha = -\frac{1}{2}$

We apply the formula of $R_\alpha(G)$ for $\alpha = -\frac{1}{2}$.

$$R_{-\frac{1}{2}}(G) = \sum_{j=1}^3 \sum_{uv \in E_j(G)} \frac{1}{\sqrt{\deg(u) \cdot \deg(v)}}$$

$$R_{-\frac{1}{2}}(G) = \frac{1}{2}|E_1(D_3(n))| + \frac{\sqrt{2}}{4}|E_2(D_3(n))| + \frac{1}{4}|E_3(D_3(n))|$$

$$\implies R_{-\frac{1}{2}}(G) = (18 + 9\sqrt{2})n^2 - (25 + 5\sqrt{2})n + 44$$

□

In the following theorem, we compute the first Zagreb index of Dominating David Derived network of third type denoted by $D_3(n)$.

Theorem 2.3.2. *For Dominating David Derived network of third type denoted by $G \cong D_3(n)$, the first Zagreb index is equal to*

$$M_1(D_3(n)) = 792n^2 - 968n + 352.$$

Proof. Let $D_3(n)$ denotes the Dominating David Derived network of third type. By using the edge partition from Table 5, the result follows. From equation(4) we have

$$M_1(G) = \sum_{uv \in E(G)} (\deg(u) + \deg(v)) = \sum_{j=1}^3 \sum_{uv \in E_j(G)} (\deg(u) + \deg(v))$$

$$M_1(G) = 4|E_1(D_3(n))| + 6|E_2(D_3(n))| + 8|E_3(D_3(n))|$$

By doing some calculation, we get

$$\implies M_1(G) = 792n^2 - 968n + 352$$

□

Next, we compute the ABC , GA , ABC_4 and GA_5 indices for Dominating David Derived network of third type denoted by $D_3(n)$.

Theorem 2.3.3. *Let $G \cong D_3(n)$ be the Dominating David Derived network of third type, then we have*

- $ABC(G) = (18\sqrt{2} + 18\sqrt{6})n^2 - (8\sqrt{2} + 27\sqrt{6})n + 11\sqrt{6}$, for every positive integer $n \geq 1$.
- $GA(G) = (72 + 24\sqrt{2})n^2 - (\frac{40}{3}\sqrt{2} + 104)n + 44$, for every positive integer $n \geq 1$.
- $ABC_4(G) = (\frac{32\sqrt{3}+9\sqrt{30}+6\sqrt{78}}{4})n^2 + (\frac{7}{3}\sqrt{2} - 11\sqrt{3} + \frac{2}{3}\sqrt{10} + \frac{2}{5}\sqrt{15} + \frac{6}{5}\sqrt{20} + \frac{2}{3}\sqrt{22} + \frac{2}{7}\sqrt{26} - \frac{19}{4}\sqrt{30} + \frac{2}{7}\sqrt{35} + \frac{2}{7}\sqrt{42} + \frac{8}{\sqrt{7}} - \frac{5}{2}\sqrt{78})n + \frac{7}{3}\sqrt{2} + 4\sqrt{3} - \frac{2}{5}\sqrt{15} - \frac{6}{5}\sqrt{20} - \frac{2}{7}\sqrt{26} + \frac{5}{2}\sqrt{30} - \frac{2}{7}\sqrt{35} - \frac{2}{7}\sqrt{42} + \frac{2}{3}\sqrt{78}$, for every positive integer $n \geq 2$.
- $GA_5(G) = (\frac{1260+720\sqrt{3}+504\sqrt{6}}{35})n^2 + (\frac{8}{3}\sqrt{2} - \frac{240}{17}\sqrt{3} + \frac{16}{3}\sqrt{5} - \frac{88}{5}\sqrt{6} + \frac{16}{11}\sqrt{7} + \frac{16}{13}\sqrt{10} + \frac{16}{15}\sqrt{14} + \frac{4}{5}\sqrt{21} + \frac{16}{13}\sqrt{42} - 60)n + \frac{8}{3}\sqrt{2} + \frac{64}{7}\sqrt{3} - \frac{16}{3}\sqrt{5} + \frac{32}{5}\sqrt{6} - \frac{16}{11}\sqrt{7} - \frac{16}{13}\sqrt{10} + \frac{16}{15}\sqrt{14} - \frac{4}{5}\sqrt{21} - \frac{16}{13}\sqrt{42} + 36$, for every positive integer $n \geq 2$.

(S_u, S_v) where $uv \in E(G)$	Number of edges	(S_u, S_v) where $uv \in E(G)$	Number of edges
(6, 6)	$4n$	(12, 12)	$8n$
(6, 12)	$4n + 4$	(12, 14)	$8n - 8$
(6, 14)	$4n - 4$	(12, 16)	$36n^2 - 60n + 16$
(8, 10)	$12n - 12$	(14, 14)	$4n - 4$
(8, 12)	$36n^2 - 44n + 16$	(14, 16)	$4n + 4$
(8, 14)	$4n - 4$	(16, 16)	$36n^2 - 76n + 40$
(10, 16)	$4n - 4$		

Table 6. Edge partition of Dominating David Derived network of third type($D_3(n)$) based on sum of degrees of end vertices of each edge.

Proof. By using the edge partition given in Table 5, we get the result. From equation (5), it follows that

$$ABC(G) = \sum_{uv \in E(G)} \sqrt{\frac{\deg(u) + \deg(v) - 2}{\deg(u) \cdot \deg(v)}} = \sum_{j=1}^3 \sum_{uv \in E_j(G)} \sqrt{\frac{\deg(u) + \deg(v) - 2}{\deg(u) \cdot \deg(v)}}.$$

$$ABC(D_3(n)) = \frac{1}{\sqrt{2}}|E_1(D_3(n))| + \frac{1}{\sqrt{2}}|E_2(D_3(n))| + \frac{\sqrt{6}}{4}|E_3(D_3(n))|.$$

By doing some calculation, we get

$$\Rightarrow ABC(G) = (18\sqrt{2} + 18\sqrt{6})n^2 - (8\sqrt{2} + 27\sqrt{6})n + 11\sqrt{6}$$

From equation (6), we get

$$GA(G) = \sum_{uv \in E(G)} \frac{2\sqrt{\deg(u) \cdot \deg(v)}}{\deg(u) + \deg(v)} = \sum_{j=1}^3 \sum_{uv \in E_j(G)} \frac{2\sqrt{\deg(u) \cdot \deg(v)}}{\deg(u) + \deg(v)}.$$

By doing some calculation, we get

$$GA(D_3(n)) = |E_1(D_3(n))| + 2\frac{\sqrt{2}}{3}|E_2(D_3(n))| + |E_3(D_3(n))|.$$

By doing some calculation, we get

$$\Rightarrow GA = (72 + 24\sqrt{2})n^2 - (\frac{40}{3}\sqrt{2} + 104)n + 44.$$

If we consider edge partitions based on degree sum of neighbors of end vertices, then the edge set $E(D_3(n))$ can be divided into thirteen edge partition $E_j(D_3(n))$, $4 \leq j \leq 16$; where the edge partition $E_4(D_3(n))$ contains $4n$ edges uv with $S_u = S_v = 6$; the edge partition $E_5(D_3(n))$ contains $4n + 4$ edges uv with $S_u = 6$ and $S_v = 12$; the edge partition $E_6(D_3(n))$ contains $4n - 4$ edges uv with $S_u = 6$ and $S_v = 14$; the edge partition $E_7(D_3(n))$ contains $12n - 12$ edges uv with $S_u = 8$ and $S_v = 10$, the edge partition $E_8(D_3(n))$ contains $36n^2 - 44n + 16$ edges uv with $S_u = 8$ and $S_v = 12$, the edge partition $E_9(D_3(n))$ contains $4n - 4$ edges uv with $S_u = 8$ and $S_v = 14$; the edge partition $E_{10}(D_3(n))$ contains

$4n - 4$ edges uv with $S_u = 10$ and $S_v = 16$; the edge partition $E_{11}(D_3(n))$ contains $8n$ edges uv with $S_u = S_v = 12$; the edge partition $E_{12}(D_3(n))$ contains $8n - 8$ edges uv with $S_u = 12$ and $S_v = 14$; the edge partition $E_{13}(D_3(n))$ contains $36n^2 - 60n + 16$ edges uv with $S_u = 12$ and $S_v = 16$; the edge partition $E_{14}(D_3(n))$ contains $4n - 4$ edges uv with $S_u = S_v = 14$; the edge partition $E_{15}(D_3(n))$ contains $4n + 4$ edges uv with $S_u = 14$ and $S_v = 16$; and the edge partition $E_{16}(D_3(n))$ contains $36n^2 - 76n + 40$ edges uv with $S_u = S_v = 16$.

From equation (7), we get

$$ABC_4(G) = \sum_{uv \in E(G)} \sqrt{\frac{S_u + S_v - 2}{S_u S_v}} = \sum_{j=4}^{16} \sum_{uv \in E_j(G)} \sqrt{\frac{S_u + S_v - 2}{S_u S_v}}.$$

By using the edge partition given in Table 6, we get the following

$$\begin{aligned} ABC_4(D_3(n)) = & \frac{\sqrt{10}}{6} |E_4(D_3(n))| + \frac{\sqrt{2}}{3} |E_5(D_3(n))| + \frac{\sqrt{42}}{14} |E_6(D_3(n))| + \frac{\sqrt{5}}{5} |E_7(D_3(n))| + \frac{\sqrt{3}}{4} |E_8(D_3(n))| + \\ & \frac{\sqrt{35}}{14} |E_9(D_3(n))| + \frac{\sqrt{15}}{10} |E_{10}(D_3(n))| + \frac{\sqrt{22}}{12} |E_{11}(D_3(n))| + \frac{\sqrt{7}}{7} |E_{12}(D_3(n))| + \frac{\sqrt{78}}{24} |E_{13}(D_3(n))| + \frac{\sqrt{26}}{14} |E_{14}(D_3(n))| + \\ & \frac{\sqrt{2}}{4} |E_{15}(D_3(n))| + \frac{\sqrt{30}}{16} |E_{16}(D_3(n))|. \end{aligned}$$

After some calculation, we get

$$\begin{aligned} \Rightarrow ABC_4(D_3(n)) = & \left(\frac{32\sqrt{3} + 9\sqrt{30} + 6\sqrt{78}}{4} \right) n^2 + \left(\frac{7}{3}\sqrt{2} - 11\sqrt{3} + \frac{2}{3}\sqrt{10} + \frac{2}{5}\sqrt{15} + \frac{6}{5}\sqrt{20} + \frac{2}{3}\sqrt{22} + \frac{2}{7}\sqrt{26} - \right. \\ & \left. \frac{19}{4}\sqrt{30} + \frac{2}{7}\sqrt{35} + \frac{2}{7}\sqrt{42} + \frac{8}{\sqrt{7}} - \frac{5}{2}\sqrt{78} \right) n + \frac{7}{3}\sqrt{2} + 4\sqrt{3} - \frac{2}{5}\sqrt{15} - \frac{6}{5}\sqrt{20} - \frac{2}{7}\sqrt{26} + \frac{5}{2}\sqrt{30} - \frac{2}{7}\sqrt{35} - \frac{2}{7}\sqrt{42} + \frac{2}{3}\sqrt{78}. \end{aligned}$$

From equation (8), we get

$$GA_5(G) = \sum_{uv \in E(G)} \frac{2\sqrt{S_u S_v}}{S_u + S_v} = \sum_{j=4}^{16} \sum_{uv \in E_j(G)} \frac{2\sqrt{S_u S_v}}{S_u + S_v}.$$

By using the edge partition given in Table 6, we have

$$\begin{aligned} GA_5(D_3(n)) = & |E_4(D_3(n))| + 2\frac{\sqrt{2}}{3} |E_5(D_3(n))| + \frac{\sqrt{21}}{5} |E_6(D_3(n))| + 4\frac{\sqrt{5}}{9} |E_7(D_3(n))| + 2\frac{\sqrt{6}}{5} |E_8(D_3(n))| + \\ & 4\frac{\sqrt{7}}{11} |E_9(D_3(n))| + 4\frac{\sqrt{10}}{13} |E_{10}(D_3(n))| + |E_{11}(D_3(n))| + 2\frac{\sqrt{42}}{13} |E_{12}(D_3(n))| + 4\frac{\sqrt{3}}{7} |E_{13}(D_3(n))| + |E_{14}(D_3(n))| + \\ & 4\frac{\sqrt{14}}{15} |E_{15}(D_3(n))| + |E_{16}(D_3(n))|. \end{aligned}$$

After calculation, we get

$$\begin{aligned} \Rightarrow GA_5(D_3(n)) = & \left(\frac{1260 + 720\sqrt{3} + 504\sqrt{6}}{35} \right) n^2 + \left(\frac{8}{3}\sqrt{2} - \frac{240}{17}\sqrt{3} + \frac{16}{3}\sqrt{5} - \frac{88}{5}\sqrt{6} + \frac{16}{11}\sqrt{7} + \frac{16}{13}\sqrt{10} + \frac{16}{15}\sqrt{14} + \right. \\ & \left. \frac{4}{5}\sqrt{21} + \frac{16}{13}\sqrt{42} - 60 \right) n + \frac{8}{3}\sqrt{2} + \frac{64}{7}\sqrt{3} - \frac{16}{3}\sqrt{5} + \frac{32}{5}\sqrt{6} - \frac{16}{11}\sqrt{7} - \frac{16}{13}\sqrt{10} + \frac{16}{15}\sqrt{14} - \frac{4}{5}\sqrt{21} - \frac{16}{13}\sqrt{42} + 36. \end{aligned}$$

□

3 Conclusion

In this paper, certain degree based topological indices, namely general Randić index, atom-bond connectivity index (ABC), geometric-arithmetic index (GA) and first Zagreb index for Dominating David Derived networks of first, second and third type were studied for the first time and analytical closed formulas for these networks were determined which will help the people working in network science to understand and explore the underlying topologies of these networks.

In future, we are interested to design some new architectures/networks and then study their topological indices which will be quite helpful to understand their underlying topologies.

References

1. Wiener H. *J. Am. Chem. Soc.* **1947**, 69, 17.
2. Deza, M.; Fowler, P. W.; Rassat, A.; Rogers, K. M. *J. Chem. Inf. Comput. Sci.* **2000**, 40, 550.
3. Star of David [online] available, http://en.wikipedia.org/wiki/Star_of_David.
4. Simonraj, F.; George, A. *GRAPH-HOC* **2012**, 4, 11.
5. Diudea, M. V.; Gutman, I.; Lorentz, J. *Molecular Topology*; Nova Science Publishers: Hauppauge, NY, 2001.
6. Gutman, I.; Polansky, O. E. Mathematical concepts in organic chemistry, *Springer-Verlag, New York*, **1986**.
7. Randić, M. *J. Amer. Chem. Soc.* **1975**, 97, 6609.
8. Estrada, E.; Torres, L.; Rodríguez, L.; Gutman, I. *Indian J. Chem.* **1998**, 37A, 849.
9. Vukičević, D.; Furtula, B. *J. Math. Chem.* **2009**, 46, 1369.
10. Ghorbani, M.; Hosseinzadeh, M. A. *Optoelectron. Adv. Mater. Rapid Commun.* **2010**, 4, 1419.
11. Graovac, A.; Ghorbani, M.; Hosseinzadeh, M. A. *J. Math. Nanosci.* **2011**, 1, 33.
12. Hayat, S.; Imran, M. *Appl. Math. Comput.* **2014**, 240, 213.
13. Bača, M.; Horváthová, J.; Mokrišová, M.; Suhányiová, A. *Appl. Math. Comput.* **2015**, 251, 154.
14. Baig, A. Q.; Imran, M.; Ali, H. *Optoelectron. Adv. Mater. Rapid Commun.* **2015**, 9, 248.
15. Baig, A. Q.; Imran, M.; Ali, H. *Canad. J. Chem.* **2015**, 93, 730.
16. Hayat, S.; Imran, M. *J. Comput. Theor. Nanosci.* **2015**, 12, 70.
17. Hayat, S.; Imran, M. *J. Comput. Theor. Nanosci.* **2015**, 12, 533.
18. Hayat, S.; Imran, M. *Appl. Math. Comput.* **2014**, 240, 213.
19. Iranmanesh, A.; Zeraatkar, M. *Optoelectron. Adv. Mater. Rapid Commun.* **2010**, 4, 1852.
20. Lin, W.; Chen, J.; Chen, Q.; Gao, T.; Lin, X.; Cai, B. *MATCH Commun. Math. Comput. Chem.* **2014**, 72, 699.
21. Manuel, P. D.; Abd-El-Barr, M. I.; Rajasingh, I.; Rajan, B. *J. Discrete Algorithms* **2008**, 6, 11.
22. Palacios, J.L. *MATCH Commun. Math. Comput. Chem.* **2014**, 72, 709.
23. Simonraj, F.; George, A. *Int. J. Future Comput. Commun.*, **2013**, 2, 90.

Figure Captions

Fig. 1. Dominating David Derived network of first type ($D_1(2)$)

Fig. 2. Dominating David Derived network of second type ($D_2(4)$)

Fig. 3. Dominating David Derived network of third type type ($D_3(2)$)

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