CONTROLLING ELECTROMAGNETIC FIELDS USING PERIODIC STRUCTURES:
GRATINGS, METAMATERIALS, AND PHOTONIC CRYSTALS

by

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Abstract

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This thesis presents novel devices and techniques that enable new methods for enhancement, concentration, refraction, shaping, collimation, and directive beaming of electromagnetic fields. These unprecedented methods to control electromagnetic fields are achieved by exploring and harnessing the unique wave-interactions in periodic gratings, metamaterials, and photonic crystals, with emphasis on Epsilon-Near-Zero (ENZ) metamaterials and zero-index media. The presented solutions impact a wide variety of applications ranging from microwave to optical frequencies.

A discovery of dramatic radiation enhancement of an invisible array of sources next to a sub-wavelength periodic metal strip grating is reported, both theoretically and experimentally. The phenomenon is first systematically theorized by introducing the ‘spectral impulse response’ approach for the aperiodic excitation problem, followed by the ‘spectral array factor’ approach for designing the near-field of array of sources. Such radiation enhancement has applications in sensing, detection, and accurate measurement of distance.

The shaping and collimation of radiation of a simple dipole source near or buried inside a general anisotropic ENZ half-space is then systematically studied using the Lorentz reciprocity method. Various elliptic and hyperbolic anisotropic ENZ media are considered, showing how the air-side radiation can be enhanced and shaped using certain ENZs.

A novel device and technique is proposed for collecting, refracting and concentrating incident waves into an area of high power concentration, at extremely short distances. This flat low-profile light-concentrator comprises a hetero-junction of anisotropic ENZ metamaterials (hyperbolic or elliptic), and is realized with plasmonic layered media at optical frequencies. By
harnessing an extremely oblique refraction process in ENZs, the light-concentrator significantly outperforms the size requirements of existing thick high curvature lenses, useful in various applications (e.g. as microlenses). The hetero-junction can also serve as a thin beam-splitter and beam-shifter.

Lastly, the Dirac Leaky-Wave Antenna (DLWA) is introduced for reliable and continuous scanning of directive leaky-wave radiation from photonic crystals. The DLWA is based on a zero-index photonic crystal with a Dirac-type dispersion at its Γ-point. The DLWA solves the classic open broadside stopband in leaky-wave structures not only at microwaves, but for terahertz and optical frequencies, with feasible dimensions and low losses.
Dedication

This thesis is dedicated to the three women who have shaped my life: my grandmother, my mother, and my darling wife Parisa.
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Chapter 1

Introduction

1.1 Objective and Motivation

In electromagnetic and optical applications, the goal is to control electromagnetic fields in some form. For instance, it is desired to collimate and shape electromagnetic radiation into directive beams or increase the strength and efficiency of radiators in antenna engineering, or control transmission based on frequency in microwave filters. It is also desired for example to focus and concentrate waves using lenses for imaging in optics. There are many ways to control electromagnetic fields depending on the desired goal. A simple, popular, versatile and powerful method of achieving control of waves, is by using periodic structures.

The objective of this thesis is to devise novel devices and techniques using simple periodic structures, for new and/or improved methods of controlling electromagnetic fields, from radio waves to light. By control of electromagnetic fields, we are specifically interested in concentration, collimation, shaping, directive beaming, and radiation enhancement, as they encompass many of the important goals in numerous electromagnetic and optical applications. We aim to use simple periodic structures including metamaterials, photonic crystals, and gratings, in order to exploit new and unique features and capabilities from these structures, to achieve the desired control of electromagnetic fields.

These methods of control are used to address several important challenges in electromagnetics and optics. The results of this work may in fact impact a variety of applications including directive antennas, radars, sensing and detection, telecom and optical connections, solar cells, infrared surveillance and night vision, spectroscopy, terahertz and opto-electronics. Providing simple and effective novel solutions to some of the challenges in these applications, motivates this thesis.

1.2 Background

In the following, a brief literature review on periodic structures relevant to this thesis will be covered. The additional literature relevant to the specific topics/applications addressed in each
chapter (e.g. spectrum conversion, radiation above half-space, concentration, leaky waves, etc.), will be covered in the corresponding chapter.

It is assumed that the reader of this thesis is familiar with the basics of electromagnetics, including solutions to the Maxwell’s equations and the wave equation, periodic structures and Floquet/Bloch analysis, notions of propagation constant, dispersion and dispersion diagrams (relation between the propagation constant of the waves in the structure and frequency of operation), Bloch impedance, iso-frequency contours, as well as microwave networks and multi-port characterization, Green’s functions, plane-wave expansion, surface and leaky waves. For reading on such topics, the author’s personal experience has been to find references [1, 2, 3, 4, 5, 6, 7, 8] as excellent sources, covering both introductory and advanced explanations on such topics.

1.2.1 Periodic Structures

A periodic structure comprises a repeated pattern of a unit cell, in one, two or three dimensions. Periodic structures crop up in various areas of electromagnetics and optics. Metamaterials [9, 10, 11], photonic crystals and photonic bandgap structures [12], gratings [5], and frequency selective surfaces [13] are all examples of periodic structures. Due to their wide range, applications and variations, it is outside the scope of this thesis to provide a comprehensive study of all periodic structures in electromagnetics and optics. In this work, we are primarily interested in metamaterials, photonic crystals and grating structures, which encompass some of the more popular and more recently studied periodic structures in electromagnetics. The boundary between these categories are not always well defined. For example some experts may consider photonic crystals as part of metamaterials, or diffraction gratings, electromagnetic bandgap and photonic crystals as one category. The goal in this thesis is not to strictly categorize, but to embark on different periodic structures, to study, utilize and exploit new and unexplored useful behavior from them where possible, irrespective of their fundamental definitions.

In an infinitely periodic structure, the field solution repeats over every cell, multiplied by an extra complex-valued propagation factor, famously formalized in Floquet/Bloch theory. The Floquet/Bloch theory enables solving for the field over one unit cell, and automatically knowing the solution to all other points in space accordingly. Though realistic periodic structures are finite in practice, the analysis of an infinitely periodic structure provides valuable information regarding the behavior of the structure, and predicts the behavior of the finite structure that comprises many cells and/or is terminated in the appropriate Bloch terminations.

1.2.2 Periodic gratings

Periodic gratings [3, 5, 6] are probably one of the simplest of periodic structures in electromagnetics and optics. They are typically made from periodic arrangements such as simple strips, grooves or ridges, and are typically arranged in one or two dimensions. Thus, they normally form a two dimensional surface, which may or may not have a thickness. A classical example
of gratings are diffraction gratings in optics, where they are used to demonstrate interference patterns of monochromatic light, as well as polychromatic diffraction (rainbow patterns) of white light on a screen. The operation of a grating is governed by the phase matching of waves at the grating, leading to the famous grating equation, which states that the wavevector of the scattered waves differ from the incident wavevector by integer multiples of the grating wavenumber \( (2n\pi/\text{period}, n \text{ integer}) \), as a consequence of the Floquet theory. Depending on their periodicity with respect to the wavelength, gratings can be reflective, transmissive, or both, and polarizing. Gratings with a very sub-wavelength period are typically highly reflective to incident waves. For instance the deeply sub-wavelength mesh grid on the front door of a microwave oven prevents electromagnetic waves to leak from the oven and bounces them back inside. Larger periods (with respect to the operating wavelength) are used for generating interference and rainbow patterns in optics.

1.2.3 Metamaterials

Metamaterials (MTMs) are structures that can attain an effective electromagnetic response that is not typically attained in nature. Typical materials in nature normally have constitutive parameters \((\epsilon, \mu)\) larger than or equal to the free-space parameters \((\epsilon_0, \mu_0)\). This corresponds to the region indicated in green in Fig. 1.1, which includes materials such as normal dielectrics. Metamaterials however can attain effective material parameters beyond those of typical materials (such as negative or zero permittivity and/or permeability), corresponding to all regions outside the green region of Fig. 1.1.

A common behavior in normal materials and dielectrics for example is that light refracts ‘positively’ at their interface with air as shown in the top right inset of Fig. 1.1, due to the positive refractive index (PRI). With a metamaterial however it is possible to achieve simultaneous negative permittivity and permeability, which leads to negative refractive index (NRI) media. The interface of an NRI with air refracts waves ‘negatively’ with respect to the normal of the interface, as shown in the bottom left inset of Fig. 1.1, as a consequence of the difference between the wavevector and the Poynting vector inside the NRI.

The uncommon electromagnetic response of MTMs [14] has led to remarkable scientific findings such as the ‘Superlens’ for perfect imaging [15], electromagnetic ‘cloaking’ [16], negative index of refraction [17, 18], indefinite media [19], negative refraction [20] and near-field imaging [18, 21], to only name a few. The topic of metamaterials has grown and received significant attention in the past 15 years, especially in the scientific electromagnetic and optics communities. A great body of scientific papers and books (e.g. [9, 10, 11, 22]) have been written on the analysis, design, characteristics, and various forms of electromagnetic and optical metamaterials.

Aside from their scientific elegance and enabling new frontiers and interesting scientific properties, metamaterials have also gradually found practical applications. MTMs have shown to be beneficial for various existing applications in electromagnetics and optics. Metamaterials
and transmission-line metamaterials have been used in various applications of microwave and antenna engineering [9, 10]. Various applications have also been shown for higher frequencies such as in the terahertz (THz), infrared and optical frequencies using plasmonic metamaterials [11, 22]. MTMs have also shown superior performance over prior techniques for absorption, wave guidance and narrow-band emission of light for solar and/or thermal photovoltaic cell applications. Recently, various works have used metamaterials for the design of thermal absorbers [23, 24, 25, 26, 27, 28] and selective thermal emitters [29, 30, 31, 32, 33, 34] potentially for thermal photovoltaics (TPV), as well as Solar-TPV systems [35, 36].

Engineered periodic structures [17, 37], are the most popular method to realize metamaterials and their ‘effective’ electromagnetic response, over a range (or ranges) of frequencies. To date, many realizations of MTMs have been demonstrated. Split ring resonators, wire media, mesh grids, transmission line metamaterials, layered media, nano-rods and nano-spheres, are some examples of such realizations, each applicable to certain frequencies and applications [9, 10, 11, 22]. The period of a metamaterial structure is typically very small compared to the guided wavelength (sub-wavelength), such that each cell sees and acts on a small fraction of the wavelength (long wavelength limit), but the overall periodic structure attains a certain macroscopic or effective electromagnetic behavior. Coming up with an effective medium theory that fully captures the effective electromagnetic behavior of a metamaterial has been the subject of numerous research over the years.
1.2.4 Epsilon-Near-Zero and Zero-Index Metamaterials

The class of metamaterials which attain effective permittivity values close to zero are known as Epsilon-Near-Zero (ENZ) metamaterials [38, 39, 40]. More generally, metamaterials that attain an effective permittivity and/or permeability near zero are referred to as Zero-Index Metamaterials (ZIMs) [41, 42, 43]. In Fig. 1.1, ZIMs are shown as materials lying in the shaded purple region ($|\epsilon/\epsilon_0| < 1$ and/or $|\mu/\mu_0| < 1$), including the ENZ region $|\epsilon/\epsilon_0| < 1$. In this thesis, we shall be primarily using Epsilon-Near-Zero metamaterials and Zero-Index Metamaterials and media, due to some of their unique properties in enabling control over electromagnetic fields.

ENZs [38] and ZIMs offer unique and fascinating wave phenomena such as supercoupling and tunneling of waves through channels and bends [39, 43], tailoring the phase of the radiation patterns from arbitrary sources [40], directive emission of sources [44, 45], optical $\tilde{D}$ wires [46], and even a recent proposal for electric levitation of a dipole [47], to name a few.

A wave in an ENZ or a ZIM has an increasingly large effective wavelength ($\lambda \to \infty$) around the ENZ/ZIM frequencies. The propagating wave has a near-zero wavenumber ($\beta \to 0$) and thus a near-zero phase progression over a large distance ($\beta l \to 0$), and a superluminal phase velocity ($v_p > c$) around the ENZ/ZIM frequencies. In ZIMs [41, 42, 43], the potentially simultaneous near-zero permeability and permittivity provides an additional capability to tailor the wave impedance ($\sqrt{\mu/\epsilon}$). This facilitates impedance matching of waves (e.g. to free space $\sqrt{\mu_0/\epsilon_0}$), in addition to the fascinating behaviors resulting from the near-zero effective refractive index ($n_{eff} = \sqrt{\epsilon_{eff} \mu_{eff}/\epsilon_0 \mu_0} \to 0$). The directive emission (gain enhancement) of a radiator embedded inside bulk ZIMs (including ENZs), is primarily owed to Snell’s law. Air has a higher refractive index than the ZIM, hence the angle of refraction in air is smaller than in the ZIM, contrary to typical dielectrics. If a radiator is embedded inside the ZIM, the waves exiting the ZIM are collimated towards the broadside direction, causing a highly directive beam. Leaky waves [3, 48, 49] can also play a significant role in directive emission from finite ZIMs [50, 51].

The ENZ effect can be observed in various natural materials around their plasma frequency, such as in conducting oxides and noble metals. The plasma frequency of most materials is in the THz region and above, and thus the near zero effect is typically not noted in most materials, especially at microwave frequencies. They are also fixed at a certain frequency depending on the material, thus limiting their usability. The ENZ and ZIM effect can however be tailored at will for other frequencies using metamaterials and periodic structures. This can be achieved with different realizations depending on the desired frequency of operation. For example wire media and mesh grids [52, 44] have shown to exhibit near-zero plasma like behavior at microwave frequencies. The dispersion of near-cutoff modes of waveguides [53] has also been used to realize effective ENZ behavior.

For frequencies beyond microwaves, typically plasmonic metals are combined with dielectrics to obtain ENZ metamaterials. Although isotropic bulk ENZs are typically hard to design, anisotropic MTMs can be realized using sub-wavelength periodic bi-layers of two materials [54], at THz and optical frequencies. Such bi-layers of a plasmonic metal and a dielectric
can enable negative refraction in the optical and infra-red regimes [55, 56], but can also be tailored to achieve an effective near zero permittivity at least along one axis. The appropriate mixing of a negative permittivity material (e.g. metal at optical frequencies) and a regular positive permittivity dielectric, enables tailoring the near zero permittivity at will. Such layered ENZ media have enabled the realization of the hyperlens with hyperbolic dispersion MTMs [57, 58, 59, 60].

### 1.2.5 Photonic crystals

Photonic crystals [61, 62, 12] are another class of periodic structures which have been used for controlling and tailoring the flow of light in the past few decades. Much like electrons passing through a crystal that has a lattice of varying potentials, photons travel through photonic crystals with a periodic variation of material-parameters. Electromagnetic waves in a photonic crystal can experience bandgaps and passbands over ranges of frequencies corresponding to the allowable modes or states inside the photonic crystal, much like electrons inside crystals. A well known example of photonic crystals is a stack of layered dielectrics of two alternating permittivity values. Such structure can enable very different optical devices, such as anti-reflective coatings as well as mirrors. Optical photonic crystals using dielectrics have been realized in one two or three dimensions, with different and/or similar behaviors along the axes, thus making them spatially dispersive in general. The periods of photonic crystals are typically in the order of half to a full guided wavelength. This allows for resonance effects to establish inside the cell, and through coupling to adjacent cells and resonances, the desired overall behavior of the photonic crystal (partial or full reflection/transmission) is established, much like the operation of coupled resonator filters. Defects in photonic crystals can be used to further affect the flow of flight, enable localized modes of light [62], or enable surface waves at their terminated interface [12].

### 1.3 Thesis Outline

In Chapter 2 radiation of simple sources near sub-wavelength periodic metal-strip gratings are studied and analyzed. The study is aimed to harness the capability of gratings in converting the spectrum of field of the source for controlling electromagnetic fields. The chapter provides a theoretical framework using spectral techniques, exemplifying the evanescent-to-propagating wave conversion that takes place. The aperiodic Green’s function of the infinite strip gratings is constructed, by introducing the Spectral Impulse Response (SIR). All proposed results are validated against full wave electromagnetic simulations.

Chapter 3 exploits the studied evanescent-to-propagating wave conversion in gratings to theoretically and experimentally demonstrate radiation enhancement of ‘invisible’ sources using a simple sub-wavelength grating. The invisible source array is designed using a proposed ‘spectral array factor’ approach. The physical phenomenon is shown in simulations and measurements
at microwaves.

In Chapter 4, the radiation of simple dipole sources near the interface of anisotropic Epsilon-Near-Zero Metamaterials is analyzed using the Lorentz reciprocity theorem, and their possible radiation enhancement and pattern shaping effects are studied. ENZs with different dispersion, as well as practical anisotropic optical ENZs are studied for their effect on controlling the electromagnetic radiation.

In Chapter 5, hetero-junctions of anisotropic Epsilon-Near-Zero Metamaterials are used to realize low-profile and flat light-concentrators with very low focal distance. Atypical bending of light is demonstrated using such ENZs, and is used to realize the low-profile light concentrator, and the operation of the device is analyzed. Realizations in the optical regime are presented using periodic bi-layers of metal and dielectric. Important features and advantages of the proposed hetero-junction concentrator are shown via comparisons with lenses. The ENZ structure is also used to demonstrate beam splitting and beam shifting.

In Chapter 6, zero-index photonic crystals with a Dirac-type dispersion are used to introduce the Dirac Leaky Wave Antenna (DLWA) for reliable and continuous leaky-wave radiation through broadside. Various characteristics and features (including radiation at broadside), leaky-wave parameters and dispersion, modeling, scanning abilities, losses etc. of the antenna are demonstrated and discussed.

Chapter 7 briefly concludes the thesis, and outlines the contributions made during the course of this PhD thesis.
Chapter 2

Evanescent-to-Propagating Wave Conversion Using Sub-wavelength Metal Strip Gratings

2.1 Introduction

The near-field spectrum contains spatial information about the sub-wavelength variations of a source/object distribution. This spatial information in the near-field is carried by evanescent waves, which decay exponentially with distance. The sub-wavelength information is therefore typically lost in the far-zone, since only the propagating waves survive. For applications where small distances, compared to the operating wavelength, are of significance, such information cannot be resolved in the far field due to this limiting factor. This is the main cause of the diffraction limit. Sensing the evanescent near-field is the key for imaging beyond the diffraction limit as done in Near-field Scanning Optical Microscopy (NSOM) [63, 64, 65].

Several techniques have been reported in the literature, where sub-wavelength features of an object could be resolved in the far-zone [66, 67, 68, 69, 70]. For example, in a far-field optical super-lens setup [67], sub-wavelength-spaced nano-wires (object) were imaged in the far field, using a silver film and a diffraction grating placed near the object. In another experiment involving time reversal [68], independent MIMO (Multiple-Input Multiple Output) channels were realized in closely-spaced antenna elements. Although the elements were spaced at sub-wavelength distances, they were individually selected from the far-zone, by placing scatterers in the form of metallic brushes in the near-field zone of the antenna array. In a related study [69], it has been demonstrated that waves emanating from sources can be refocused with sub-wavelength resolution onto an image plane using phase-conjugating screens, provided that identical scatterers are placed both near the sources and the image plane.

The phenomenon, which is common among these applications, is spectrum conversion and specifically, the conversion of evanescent-to-propagating waves. In order to retain the sub-
wavelength resolution at the far-zone, the near-field spectrum is converted to a propagating spectrum, which can then reach the far-zone, in addition to the usual propagating spectrum.

Evanescent-to-propagating wave conversion is typically made possible by some form of near-field scattering close to a source, where obstacles or scatterers interact with the surrounding near-field, before the evanescent waves have decayed significantly. Converting these evanescent waves to propagating waves enables their detection in the far-zone [66]. The work in Ref. [68] used metallic brushes to attain random scattering in the near-field of a microwave antenna array, while Ref. [69] used point scatterers. Both Ref. [68] and Ref. [69] utilized such wave conversion in a random fashion. Aside from scattering, phase-matching of evanescent waves to propagating waves inside hyperbolic indefinite media [19] was used for partial wave focusing in Ref. [71] and in the development of the “hyperlens” [57, 58, 59].

In Ref. [67], evanescent waves were strengthened using a negative-index silver film, and then the controllable spectrum conversion capability of an optical grating was exploited to convert the evanescent waves into propagating waves. This conversion effect relies on the principle that an incident evanescent wave on a grating can phase match to other propagating or evanescent waves, provided that they differ in their wave-number by integer multiples of the grating wave-number. Wavenumber matching using gratings is also used to excite high wavenumber Surface Plasmon Polariton (SPP) waves by an incident plane-wave [72]. The periodic grating provides the additional wavenumber required for the low wavenumber incident plane-wave to phase match to the SPP.

It is the purpose of this chapter to rigorously analyze and study the phenomenon of the conversion of evanescent to propagating waves by ordered periodic structures, excited by finite aperiodic source(s). The developed understanding from this chapter will help to realize a novel phenomenon/application of controlling electromagnetic fields in the next chapter, involving radiation enhancement of an ‘invisible’ array of closely-spaced sources. The presented theory also sheds light on spectrum conversion of waves from finite sources in general. It may also be used for other conversions in spectrum, such as propagating-to-evanescent, evanescent-to-evanescent, and propagating-to-propagating spectrum conversion scenarios.

For this purpose, we analyze a commonly studied canonical metallic strip grating excited by point source(s). A spectral view of the scattering that takes place is depicted, to explicitly study the exact evanescent-to-propagating conversion that takes place. The spectrum conversion is first inspected in its fundamental form, for a single evanescent plane-wave incident on such grating. Then, the phenomenon is analyzed for a single finite source placed near the grating, and ultimately for arbitrary arrangements of sources, for both polarizations. Therefore, the theory involved in this work treats the problem of ‘aperiodic excitation’ of a ‘sub-wavelength periodic grating’.
Chapter 2. Evanesce.-to-Propag. Wave Conv. Using Sub-wavelength MSGs

2.1.1 Aperiodic excitation of a periodic structure

In many microwave and optical applications, a periodic structure is driven by finite source(s) that are placed in its vicinity. Some examples include sources above artificial periodic surfaces [73], excitation of leaky-wave antennas [74], directive beaming in the presence of partially reflection surfaces [51], and sources near metamaterial structures [75]. To solve for the fields, the break of the periodicity due to the aperiodic excitation complicates the standard periodic analysis that uses the Floquet modes. Ideally, it would be desirable to find the aperiodic Green’s function of the periodic structure in analytical closed form, which is equivalent to determining the field solution when a single point source (space impulse) drives the periodic domain. To date, numerical and semi-analytical methods have been developed for solving this class of problems [76, 77, 78, 79, 80, 81, 82]. Perhaps the most notable method is the Array Scanning Method (ASM) [76], which has been combined with the Method-of-Moments (MoM) [77, 78] and with FDTD [80, 81].

The solution to the problem in this chapter is carried out directly in the spectral domain through the introduction of the “Spectral Impulse Response” (SIR), which makes it particularly simple to monitor the evanescent-to-propagating wave conversion, and directly identify the far-zone field by asymptotic methods. The method separates the structure and source from each other, and makes it possible to find the solution to more complicated source arrangements without the need for re-solving the problem. The presented method essentially yields the spectral and spatial Green’s function of the metal strip grating, based on the plane-wave scattering by the grating [5]. It accounts for the complete fields emanating from the source, including the reactive near field. The final expressions appear as a natural expansion of Floquet modes, which offer physical insight to the diffraction process to complement other semi-analytical representations [73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 51, 83, 84, 85], rigorous [86], or fully numerical full-wave approaches such as FDTD, MoM, or FEM simulations.

2.1.2 Simplified Problem Space

Consider the problem space shown in Fig. 2.1. As shown, a canonical infinitely long diffraction grating is aligned in a vacuum along the $x$-axis in the $x-y$ plane. The grating is an array of metal strips, with period ‘$L$’, separated by a distance ‘$w$’, and the strip width is ‘$L-w$’. The strips are zero thickness Perfect Electric Conductors (PEC). Two polarizations are possible in the two-dimensional (2D) domain: Transverse Electric (TE) with three field components $\{H_x, E_y, H_z\}$, and Transverse Magnetic (TM) with three field components $\{E_x, H_y, E_z\}$. All fields are time-harmonic phasors of $e^{j\omega t}$. For the aperiodic analysis, a single frequency source excites the grating at an arbitrary distance $(0, z_0)$, making the overall problem aperiodic.
Chapter 2. Evanesc.-to-Propag. Wave Conv. Using Sub-wavelength MSGs

2.2 Spectrum conversion in an all periodic grating problem

We show here that indeed a single evanescent wave can be converted to a propagating plane-wave, in the infinite periodic metal strip grating, through periodic full-wave simulation. Consider the simulation setup of a single unit cell shown in Fig. 2.2, for a metal strip grating with period $L$ and an incident plane-wave, indicated by the propagation vector $\vec{k}_i = k_x \hat{x} + k_z \hat{z}$. The problem is open in the $z$ direction.

It is known that a single plane-wave incident on an infinite grating with an incident wavenumber $k_x$, diffracts into many higher-order modes. The higher-order mode wavenumbers ($k_{xn}$) are governed by the phase matching condition $k_{xn} = k_x + nk_L$ where $k_L = 2\pi/L$ is the grating wavenumber. In other words, a grating can provide shifts in the transverse wavenumber of the incident wave, by integer increments of $k_L$. For applications such as [67, 68, 69] where near-field data is to be transmitted to the far-field, the grating must provide a wavenumber shift larger than $2k_0$, such that the evanescent waves outside the propagating region only convert to propagating waves. This directly implies a sub-wavelength period and is the core reason we are interested in the study of sub-wavelength gratings for the purpose of evanescent-to-propagating
spectrum conversion.

For example, a sub-wavelength period size of $L = \lambda_0/5$ corresponds to a grating wavenumber of $k_L = 5\lambda_0$. With such a grating, we speculate that any waves within the spectrum range of $(4nk_0, 6nk_0)$ where $n$ is an integer, would be converted to the propagating region $(-k_0, +k_0)$, in the lower half space of the problem, and waves with any other incident wavenumbers would not diffract into a propagating wave.

We shine the grating with two different evanescent waves in the simulator. These incident evanescent waves may be due to any external source which is not of interest here. Comsol v4.2 is used with a background/scatter simulation approach, with the background field having the form

$$\vec{E}_b = \hat{y}e^{-jk_x x}e^{k_z z}$$

where $|k_x| > k_0$, and $k_x^2 + k_z^2 = k_0^2$, where the second exponent is chosen such that the wave is decaying along the $-z$ direction. The scattered field is calculated by the simulator, and the total field comprises the background and scattered field.

Fig. 2.3 (a) shows the magnitude of the total field plotted along the $x = 0$ path for the two incident cases, in the transmission region $z < 0$. For an incident wave with $k_x = 3.5k_0$, the dashed curve shows that the total field decays to zero in an exponential manner as we move away from the grating. This (as well as Fig. 2.3 (b)) shows that there are no propagating waves in the transmission region of the grating ($z < 0$) and the transmitted field consists solely of evanescent waves.

For the incident evanescent wave of $k_x = 5.5k_0$, the solid curve in Fig. 2.3 (a) (as well as Fig. 2.3 (c)) shows that the total field magnitude is now a non-zero constant for $z < 0$, implying a propagating wave with no decay when moving away from the grating. Hence an evanescent-to-propagating wave conversion is observed.

Fig. 2.3 (d) shows the absolute value of the real part of the electric field, for the case of $k_x = 5.5k_0$. It can be seen that the transmitted field has wavefronts corresponding to a single propagating wave. Since this wave is converted from an incident evanescent wave of $k_x = 5.5k_0$, it propagates with $k_x = 5.5k_0 - 5k_0 = 0.5k_0$, due to the phase matching condition.

A spectral view of the above two simulations is shown in Fig. 2.4, where only the window between $4k_0$ and $6k_0$ can be converted to the propagating region, due to the choice of the grating period. An incident wave at spectral location $3.5k_0$ does not diffract into the propagating region. However the wave at $5.5k_0$ does diffract into the propagating region having a transmitted wave number of $5.5k_0 - 5k_0 = 0.5k_0$. The amplitude of the diffracted wave is also multiplied by a complex weight named $B_1(5.5k_0)$, which will be determined later using the SIR theory. It must be noted that in both cases, the transmitted diffracted field contains an infinite number of higher-order harmonics but are not shown in Fig. 2.4. Here we show what only diffracts into the propagating region.
Chapter 2. Evanes. to Propag. Wave Conv. Using Sub-wavelength MSGs

Figure 2.3: An evanescent wave is incident on a grating from $z > 0$ (a) Magnitude of TE electric field, $|E_y|$, along the $z$-axis, in the transmission region of the grating unit cell ($z < 0$). (b) $|E_y| \approx 0$ in the unit cell for the case of no conversion $k_x = 3.5 k_0$. (c) $|E_y| = \text{constant} \neq 0$ in the unit cell for the case of conversion to a propagating wave, $k_x = 5.5 k_0$. (d) Absolute value of real part of $E_y$ for the case of $k_x = 5.5 k_0$ showing the excited propagating wave and its direction.

2.3 Plane-wave scattering from a metal strip grating

Consider Fig. 2.5 where a single plane-wave is incident on the grating. The incident wave, $E_{yi} \text{ (TE)}$ or $H_{yi} \text{ (TM)}$, has a transverse wavenumber $\beta$ and a longitudinal wavenumber $q = \ldots$
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Figure 2.4: Spectral representation of transmission through a grating with $k_L = 5k_0$. An incident wave of $k_x = 3.5k_0$ falls outside of any conversion band and does not convert to a propagating wave, but an incident wave of $k_x = 5.5k_0$, is converted to a propagating wave of $k_x = 0.5k_0$, via the $n = 1$ diffraction order, and weight $B_1(5.5k_0)$.

$\sqrt{k_0^2 - \beta^2}$, and an amplitude $A_0$. This is an all periodic problem of period $L$, where both the structure and the excitation are infinitely periodic.

Figure 2.5: Scattering due to an incident plane-wave on the grating.

$$TE : E_{yi}(x, z) = A_0 e^{-j\beta x + jqz} \tag{2.2}$$

$$TM : H_{yi}(x, z) = A_0 e^{-j\beta x + jqz} \tag{2.3}$$

In [5] the formulation is set up such that the total reflected field consists of a specular reflected wave $E_{yr}$ or $H_{yr}$ when the gaps are completely closed, as well as the reflected scattering $E_{ys1}$ and $H_{ys1}$ from the fields of the infinitely repeated periodic gaps. The scattered fields $E_{ys1}$ and $E_{ys2}$ are found to be equal at the grating plane, with opposing directions of propagation.
in the $+z$ and $-z$ directions.

\begin{align*}
\text{TE :} & & E_{yr}(x, z) &= -A_0 e^{-j\beta x - jqz} \\
& & E_{ys1}(x, z) &= \sum_{n=-\infty}^{\infty} B_{TE}^n(\beta) e^{-j\beta_n z - j\beta_n x} \quad (2.4) \\
\text{TM :} & & H_{yr}(x, z) &= A_0 e^{-j\beta x - jqz} \\
& & H_{ys1}(x, z) &= -\sum_{n=-\infty}^{\infty} B_{TM}^n(\beta) e^{-j\beta_n z - j\beta_n x} \quad (2.5) \\
\text{TE :} & & E_{ys2}(x, z) &= \sum_{n=-\infty}^{\infty} B_{TE}^n(\beta) e^{+j\beta_n z - j\beta_n x} \quad (2.6) \\
\text{TM :} & & H_{ys2}(x, z) &= -\sum_{n=-\infty}^{\infty} B_{TM}^n(\beta) e^{+j\beta_n z - j\beta_n x} \quad (2.7)
\end{align*}

The transmitted/reflected scattered fields are thus represented with a Fourier series of space harmonics according to Floquet’s theorem for the $x$–periodicity. Each term in the series, $B_n(\beta) e^{-j\beta_n z - j\beta_n x}$, is the $n^{th}$ order diffracted space harmonic term, with its transverse wavenumber $\beta_n = \beta + 2\pi n/L$, phase matched to the incident wave. These harmonics have an associated weight $B_n$, which is a function of the incident wavenumber $\beta$.

To find the weights $B_n$, [5] approximates the tangential electric field, with a function $f(x')$ using the edge condition, across the gaps in a single unit cell, at the grating plane. The TE/TM weights of the diffracted field are

\begin{align*}
B_{TE}^n(\beta) &= \frac{1}{L} \int_{-w/2}^{w/2} f^{TE}(x') e^{+j\beta_n x'} dx' \quad (2.8) \\
B_{TM}^n(\beta) &= \frac{\omega \epsilon}{L d_n} \int_{-w/2}^{w/2} f^{TM}(x') e^{+j\beta_n x'} dx' \quad (2.9)
\end{align*}

In the TE case, across a unit cell at $z = 0$ (grating), the tangential electric field $E_y$ on the strips will be zero and the tangential field across the gaps would have a profile with a maximum in the gap center and vanishing at the strips. For the TM case, the tangential electric field $E_x$ will exhibit a singularity at the strip edge and attain a minimum at the gap center. Explicitly, the two functions describing the tangential TE and TM electric fields across the gap are:

\begin{equation}
 f^{TE}(x') = C^{TE}(\beta) \sqrt{(w/2)^2 - x'^2} e^{-j\beta x'} \quad (2.10)
\end{equation}
\[ f^{TM}(x') = \frac{C^{TM}(\beta)e^{-j\beta x'}}{\sqrt{(w/2)^2 - x'^2}} \]  

The unknown constants \( C^{TE} \) and \( C^{TM} \) in the two functions (2.10) and (2.11) are then found by applying the boundary condition of continuity of the tangential fields across the gap, as was done in [5]. However it was noted that those final results had errors that ultimately did not match simulation results. We carefully carry out the analysis of [5] with the aid of [87] as detailed in Appendix A, and with some corrections to [5], find that the TM and TE constants are

\[
C^{TE}(\beta) = -A_0jq\left(\pi \right)\left[\sum_{n=-\infty}^{\infty} \frac{q_n}{L} \left(\frac{w}{2}\right)^2 \left(\frac{nJ_1(n\pi w/L)}{n\pi w/L}\right)^2\right]^{-1} 
\]

\[
C^{TM}(\beta) = A_0J_0(\frac{\beta w}{2}) \left[-j\omega \epsilon \sum_{n=-\infty}^{\infty} \frac{\pi}{jq_nL} j_0^2(\frac{n\pi w}{L})\right]^{-1} 
\]

where \( J_0 \) and \( J_1 \) are the zero-th and first order Bessel functions of the first kind. It can also be seen that the \( C \) coefficients (2.12) and (2.13), and hence the \( B_n \) coefficients are directly proportional to \( A_0 \), the amplitude of the incident wave. Therefore scaling the input by an amplitude \( \alpha \), scales all output terms in (2.6 ) and (2.7) by the same factor \( \alpha \).

In the analysis of [5], the incident wave is a propagating wave with wavenumbers \( \beta = k_0 \sin \theta \), and \( q = k_0 \cos \theta \), hence \(|\beta| < k_0 \), and \( q \) is real. But in fact this solution can be extended for incident evanescent waves, where an evanescent slow wave with a lateral wavenumber \(|\beta| > k_0 \), is incident on the grating and consequently \( q \) is imaginary. This extension of the solution to include incident evanescent waves is an important generalization of the plane-wave solution, which is necessary to solve for our aperiodic problem of interest. It must be noted that for evanescent waves with a high lateral wavenumber, higher accuracy modal functions (2.10) and (2.11) may be needed to account for more rapid variations of the fields across the gap.

2.4 Spectral impulse response of metal strip grating based on the plane-wave solution

We now utilize the discussed solution of scattering due to a single plane-wave incident on the grating. The idea is to apply this solution to all constituent plane-waves in the source field spectrum. The total transmitted field is then found by summing the contribution of scattering due to all incident waves. This is valid by virtue of the superposition principle.

We shall carry out the above analysis in the spectral domain. For any complex function \( U(x, z) \) in our spatial domain, the spectrum \( \tilde{U}(k_x, z) \) is determined using the Fourier Transform
(F.T.) in Cartesian coordinates to be

$$\tilde{U}(k_x, z) = \int_{-\infty}^{+\infty} U(x, z) e^{jk_xx} dx = e^{-jk_z z} \int_{-\infty}^{+\infty} U(x, 0) e^{jk_xx} dx$$  \hspace{1cm} (2.14)$$

$$k_z = \sqrt{k_0^2 - k_x^2}, \text{ and } k_x \text{ is the spatial frequency variable in the } x-\text{direction. Let us now convert the previous plane-wave solution into the spectral domain. Applying F.T. (2.14) to } E_{yi} (2.2) \text{ and } H_{yi} (2.3) \text{ for } A_0 = 1, \text{ the spectrum of a single incident plane-wave on the grating is}$$

$$\tilde{E}_{yi}(k_x) = \tilde{H}_{yi}(k_x) = \int_{-\infty}^{+\infty} E_{yi}(x, 0^+) e^{jk_xx} dx = \int_{-\infty}^{+\infty} e^{-j\beta x} e^{jk_xx} dx = \delta(k_x - \beta)$$ \hspace{1cm} (2.15)$$

which is simply a shifted impulse in the spectral domain ($k_x$ domain). The spectrum of the transmitted field directly below the grating is the F.T. of $E_{ys}$ (2.6) and $H_{ys}$ (2.7) at $z = 0^-$, given by

$$SIR^{TE} \big|_\beta (k_x) = \int_{-\infty}^{+\infty} E_{ys2}(x, 0^-) e^{jk_xx} dx = \sum_{n=-\infty}^{\infty} B_{TE}^n(\beta) \delta(k_x - \beta_n) \hspace{1cm} (2.16)$$

$$SIR^{TM} \big|_\beta (k_x) = \int_{-\infty}^{+\infty} H_{ys2}(x, 0^-) e^{jk_xx} dx = \sum_{n=-\infty}^{\infty} B_{TM}^n(\beta) \delta(k_x - \beta_n) \hspace{1cm} (2.17)$$

which is an infinite sum of shifted impulses of order n in the $k_x$ domain, weighted by $B_n$. Consider now the grating as a ‘system’, with the incident wave spectrum $\tilde{E}_{yi}(k_x)$ or $\tilde{H}_{yi}(k_x)$ in (2.15) as the ‘input’ spectrum to this system, and the transmitted wave spectrum (2.16) or (2.17) as the ‘output’ from the system. The input is a spectral unit impulse (a plane-wave in real space). For any shifted version of this unit impulse, the functions (2.16) and (2.17) describe the system’s output. This statement suggests that, in the spectral domain, these functions describe the ‘general impulse response’ for the grating system, i.e. they describe the response of the system due to any shifted spectral impulse $\delta(k_x - \beta)$. Hence we name (2.16) and (2.17) as the shift variant ‘Spectral Impulse Response’ (SIR) of the grating, for the TE and TM polarizations respectively.

The input/output relations for two functions characterizing the grating system behavior, i.e. the SIR and the Green’s function, are depicted in Fig. 2.6 (a) and (b) respectively. These functions characterize the same system, but with two different inputs. The SIR describes the response of the system, when it is driven with a ‘spectral impulse’, at an arbitrary spectral position $\beta$. In the space domain, the SIR is the plane-wave solution, i.e. the response due to a spatial exponential input. On the other hand, the Green’s function describes the response of the system due to a ‘spatial impulse’ positioned at an arbitrary location $x_0$. In the spectral domain,
the spectral Green’s function describes the response of the system to a spectral exponential input, where the effect of \( x_0 \) is now in the phase of the exponential. There is a close connection between the two functions, and as we shall see, the Green’s function for the transmitted field can be recovered from the SIR/plane-wave solution.

The SIR of (2.16) and (2.17) are general impulse response functions, and the response is \( k_x \) variant. This is particularly useful as it describes the output spectrum of a grating for every arbitrary shifted version of the input impulse. As the input impulse is shifted in the \( k_x \) space, the output is not a mere shift by the same amount in the \( k_x \) space, but rather a unique set of complex \( B_n \) factors scale the output components, depending on the input \( k_x \). For every impulse in the \( k_x \) space, there exists a unique response defined by (2.16) and (2.17) that specifies what the output will be.

Once the SIR is determined, we can then find the transmitted spectrum or ‘output’ of the grating due to the field of any source, be it an aperiodic or a periodic excitation impinging on the grating, directly in the spectral domain. This is simply achieved by a superposition integral on the spectrum of the field due to these source(s), and the shift-variant SIRs (2.16) or (2.17), in the spectral domain. Since we can decompose the input spectrum into spectral impulses, which is always possible due to the F.T., we can always determine the scattered spectrum. We shall see that the far-zone characteristics of the structure for an aperiodic excitation are then derived from the asymptotic approximation of the spectrum in the far-zone.

The total transmitted field due to an arbitrary input spectrum to this system will contain scattered waves due to every \( k_x \) in the incident spectrum, and these scattered waves are governed by the shift-variant SIR. Consider an arbitrary TE field spectrum input to the grating, \( \tilde{E}_{\text{in}}(k_x) \), which is a function of \( k_x \). This function can be re-written as an integral of weighted impulses,

\[
\tilde{E}_{\text{in}}(k_x) = \int_{-\infty}^{+\infty} \tilde{E}_{\text{in}}(\beta)\delta(k_x - \beta)d\beta \quad (2.18)
\]

The term in the integrand now is a shifted spectral impulse \( \delta(k_x - \beta) \), multiplied by \( \tilde{E}_{\text{in}}(\beta) \).
The SIR\(^{TE}\) (2.16), describes the output due to such a shifted impulse, and is also directly proportional to the input amplitude as found in the plane-wave solution. Therefore the total output of the system can be written as

\[
\hat{E}_{\text{out}}(k_x) = \int_{-\infty}^{+\infty} \hat{E}_{\text{in}}(\beta) \sum_{n=-\infty}^{\infty} B_n(\beta) \delta(k_x - \beta_n) d\beta
\]

which is the superposition of scattered waves due to all incident \(k_x\) wavenumbers in \(\hat{E}_{\text{in}}\). Rearranging and switching the order of integration and summation yields,

\[
\hat{E}_{\text{out}}(k_x) = \sum_{n=-\infty}^{\infty} \int_{-\infty}^{+\infty} \hat{E}_{\text{in}}(\beta) B_n(\beta) \delta(k_x - \beta - \frac{2\pi n}{L}) d\beta
\]

By means of the sifting theorem, the transmitted spectrum is

\[
\hat{E}_{\text{out}}(k_x) = \sum_{n=-\infty}^{\infty} \hat{E}_{\text{in}}(k_x - \frac{2\pi n}{L}) B_n(k_x - \frac{2\pi n}{L})
\]

This is the transmitted or output spectrum of the grating due to an arbitrary input spectrum \(\hat{E}_{\text{in}}(k_x)\). This demonstrates that the output is an infinite summation of weighted copies of the input spectrum. In fact (2.21) provides a clear spectral view of the diffraction process of any field distribution impinging on the grating. It is an important result which clearly explains the relation between the input and output spectrum of the grating, and as we shall see, it will help us identify what portions of the original evanescent spectrum, and with what weights, will convert into the propagating region. The SIR approach may be utilized to solve the field for the aperiodic excitation of a periodic structure based on a general plane-wave solution.

### 2.5 Aperiodic excitation of a periodic diffraction grating

In this section the SIR approach is used to find the aperiodic response of the periodic grating in various scenarios.

#### 2.5.1 Aperiodic Green’s function of the strip grating

If the input spectrum \(\hat{E}_{\text{yin}}(k_x)\) or \(\hat{H}_{\text{yin}}(k_x)\) (for TM) is the spectrum due to a point source excitation, we essentially determine the aperiodic Green’s function of the structure in the spectral domain, i.e. \(\hat{G}(k_x)\). In our domain of interest, a possible input spectrum to the grating system, due to a point source placed at \((x_0, z_0)\), i.e. \(\delta(x - x_0)\delta(z - z_0)\), is:

\[
\hat{E}_{\text{yin}}(k_x) = \hat{H}_{\text{yin}}(k_x) = e^{-jk_x x_0} e^{-jk_z z_0}
\]

Using (2.22) in (2.21) we have
\[
\tilde{G}(k_x|x_0, z_0) = \sum_{n=-\infty}^{\infty} e^{-j(k_x - \frac{2\pi n}{L})x_0} e^{-j\sqrt{k_0^2 - (k_x - \frac{2\pi n}{L})^2}z_0} B_n(k_x - \frac{2\pi n}{L})
\] (2.23)

where \(\tilde{G}(k_x|x_0, z_0)\) stands for the spectral domain representation of the Green’s function for TE or TM polarizations. To find the actual Green’s function in space, an inverse Fourier Transform is required to be applied to (2.23), giving:

\[
G(x, z|x_0, z_0) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \tilde{G}(k_x|x_0, z_0) e^{-jk_xx} e^{-jk_zz} dk_x
\] (2.24)

Incidentally, if this source is at the origin \((0, 0)\), then \(\tilde{E}_{yin} = \tilde{H}_{yin} = 1\), and

\[
\tilde{G}(k_x) = \sum_{n=-\infty}^{\infty} B_n(k_x - \frac{2\pi n}{L})
\] (2.25)

therefore

\[
G(x, 0^-) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \tilde{G}(k_x) e^{-jk_xx} dk_x = \frac{1}{2\pi} \int_{-\infty}^{\infty} \sum_{n=-\infty}^{\infty} B_n(k_x - \frac{2\pi n}{L}) e^{-jk_xx} dk_x
\] (2.26)

By a change of variables we obtain

\[
G(x, 0^-) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \sum_{n=-\infty}^{\infty} B_n(k_x) e^{-j(k_x + \frac{2\pi n}{L})x} dk_x
\] (2.27)

which is equivalent to integrating the original plane-wave solution, (2.6) or (2.7), over the entire spectrum of the transverse wavenumbers. This is an intuitive result as one would expect that a single impulse source at the origin would excite all plane-waves in the spectrum with equal amplitudes, and it confirms that the original plane-wave scattering solution (2.6) and (2.7) can be linked to the aperiodic excitation problem through the plane-wave expansion.

### 2.5.2 Response to a current source excitation

In (2.21), \(\tilde{E}_{in}(k_x)\) or \(\tilde{H}_{in}(k_x)\) can be the spectrum of the field impinging on the grating, due to one or many periodic or aperiodic sources, placed at arbitrary locations from the grating, and of any type (e.g. current or voltage source). As long as we know the total spectrum of the waves incident on the grating due to the sources, we can find the total transmitted spectrum via (2.21). For instance, by the plane-wave expansion of fields [2], here we find the spectrum of current sources adjacent to the grating, for TE and TM field polarizations, and use the SIR to determine the scattered spectrum.

For the TE case, consider an out of plane line current placed at \((0, z_0)\) above the grating
$J_y = I \delta(x) \delta(z - z_0)$ as depicted in Fig. 2.7 (a). By plane-wave decomposition and applying the FT (2.2), the input spectrum due to this source at $z = 0^+$ is [2]:

$$\tilde{E}_{y\text{in}}(k_x) = -\frac{k_0 \eta I e^{-j\sqrt{k_0^2-k_x^2}z_0}}{2}$$

(2.28)

Using (2.28) in (2.21), we find that the transmitted spectrum directly below the grating at $z = 0^-$ is:

$$\tilde{E}_{y\text{t}}(k_x) = -\frac{k_0 \eta I}{2} \sum_{n=-\infty}^{\infty} \frac{e^{-j\sqrt{k_0^2-(k_x-\frac{2\pi n}{L})^2}z_0}}{\sqrt{k_0^2-(k_x-\frac{2\pi n}{L})^2}} B_{n}\text{TE}(k_x - \frac{2\pi n}{L})$$

(2.29)

An $x-$directed electric line dipole [2], $J_x = p \delta(x) \delta(z - z_0)$, on the other hand gives rise to TM fields as shown in Fig. 2.7 (b). If such distribution is placed at $(0, z_0)$ above the grating at $(0, z_0)$, the incident spectrum onto the grating would be,

$$\tilde{H}_{y\text{in}}(k_x) = -pe^{-j\sqrt{k_0^2-k_x^2}z_0}$$

(2.30)

By substituting (2.30) into the TM equivalent of (2.21), the transmitted TM spectrum due to the $x-$directed current element, directly below the grating at $z = 0^-$ is:

$$\tilde{H}_{y\text{t}}(k_x) = -\frac{p}{2} \sum_{n=-\infty}^{\infty} e^{-j\sqrt{k_0^2-(k_x-\frac{2\pi n}{L})^2}z_0} B_{n}\text{TM}(k_x - \frac{2\pi n}{L})$$

(2.31)

Equations (2.29) and (2.31) respectively describe the TE and TM transmitted field spectrum due to a corresponding electric line source and electric line dipole, located at $(0, z_0)$.

### 2.5.3 Far-field

The antenna characteristics of actual sources, such as the discussed current sources, placed in the vicinity of the grating, are of particular interest especially in far-field applications such as [67] and [68], where near-field data is to be transmitted to the far-zone. This requires the knowledge of the field behavior in the far-zone. By knowing the spectrum of the transmitted field, the Green’s function can be asymptotically approximated, and no inverse Fourier Transform is required. The far-zone field and radiation characteristics can be obtained through the method of stationary phase: The far-field at coordinates $(r, \theta)$, is related to the spectrum (Fourier Transform of tangential field at $z = 0$) by asymptotic approximation, and letting $k_x = k_0 \sin \theta$, where $\theta$ is the far angle measured from the $z-$axis. In a two dimensional space we have [88]:

$$\left\{ \begin{array}{l}
E_y(r, \theta) \\
H_y(r, \theta)
\end{array} \right\} \approx -\frac{e^{-j(kr-\frac{\pi}{4})}}{\sqrt{r \lambda}} \cos \theta \int_{-\infty}^{\infty} \left\{ \begin{array}{l}
E_y(r, \theta) \\
H_y(r, \theta)
\end{array} \right\} e^{jk_x x} \, dx$$

(2.32)

Therefore we can directly write the TE and TM far-zone electric and magnetic fields for the
Figure 2.7: Current source near grating. (a) A line source gives rise to TE polarization. The electric field $E_y$ (red vector) attains a maximum at the gap center and reduces to zero at the metal strips (its envelope shown in dashed red), as it is parallel to the neighboring metal strip boundaries. (b) An $x$–directed line dipole gives rise to TM polarization. The electric field (green vector) across the gaps are normal to the metal strip boundary condition (its envelope shown in dashed green).

current source excitations, by applying (2.32) to (2.29) and (2.31).

$$E_y(r, \theta) \approx \frac{k_0 \eta I}{2\sqrt{r \lambda}} e^{-j(kr - \frac{\pi}{4})} \cos \theta \sum_{n=-\infty}^{\infty} e^{-j\sqrt{k_0^2 - (k_0 \sin \theta - \frac{2\pi n}{L})^2}z_0} B_n^T E(k_0 \sin \theta - \frac{2\pi n}{L})$$  \hspace{1cm} (2.33)

$$H_y(r, \theta) \approx -\frac{pe^{-j(kr - \frac{\pi}{4})}}{2\sqrt{r \lambda}} \cos \theta \sum_{n=-\infty}^{\infty} e^{-j\sqrt{k_0^2 - (k_0 \sin \theta - \frac{2\pi n}{L})^2}z_0} B_n^T M(k_0 \sin \theta - \frac{2\pi n}{L})$$  \hspace{1cm} (2.34)

The need to find the output spectrum due to an arbitrary excitation, and arriving at (2.21) can now be better understood. Not only does expression (2.21) give us a clear spectral view of the diffraction/conversion process, it can be readily used to find the far-field, which is of interest in this work, and in general where evanescent-to-propagating wave conversion is of interest.

### 2.5.4 Reflected field

In the reflection region, $z > 0$, the field solution will have additional terms. The reflected field for a single plane-wave incidence was shown in section 2.3. The additional terms for the reflection region include the effect of the incident source waves $E_{yi}$, and their reflection wave $E_{yr}$ if the
gaps were closed. It was also found from the plane-wave analysis that \( E_{ys2}(x,0^-) = E_{ys1}(x,0^+) \), i.e. the scattered field due to the infinitely periodic gap openings, is equal on the two sides of the grating, and with opposite directions of flow in the +z and −z directions. Therefore, for any point \( z > 0 \), we have to add the solution of a source near the infinite PEC plate solution. For the far-zone, this part of the solution can also be approximated to yield simpler expressions and this can be done directly in the space domain. Alternatively, and to be consistent with our spectral representation, the reflected spectrum can be found. For instance, for the line current giving rise to TE field, the spectrum of the reflected waves that exist in \( z > z_0 \), above the source is:

\[
\tilde{E}_{yt}(k_x) = \tilde{E}_{yt}(k_x) - \frac{k_0 \eta I}{2} \frac{e^{+j\sqrt{k_0^2-k_z^2}z_0}}{\sqrt{k_0^2-k_z^2}} + \frac{k_0 \eta I}{2} \frac{e^{-j\sqrt{k_0^2-k_z^2}z_0}}{\sqrt{k_0^2-k_z^2}}
\]  

(2.35)

where \( \tilde{E}_{yt}(k_x) \) is defined in (2.29), and is essentially the spectrum of the scattering due to infinitely periodic gap fields, the second term is the spectrum of the +z directed waves from the source translated to the \( z = 0 \) plane (i.e. multiplied by \( e^{+jk_zz_0} \)), and the third term is the reflection of the input spectrum (2.28) if the gaps were closed.

### 2.6 Validation of the field solution

The transmitted field found using the SIR method is validated by comparing against Comsol’s 2D full wave simulator, at three field zones (\( z = 0 \), near field, and far-zone). Fig. 2.8 shows results for a very dense grating, with dimensions \( L = \lambda_0/20 \) (\( \lambda_0 = 30 \text{cm} \)) and \( w = L/2 \). For the plots of \( z = 0^- \) (a) and \( z = -\lambda_0/20 \) (b), the source is placed in the \( x-z \) plane at a very close distance of \( z_0 = \lambda_0/20 \) and runs infinitely along the \( y- \) axis. Such an arrangement is compatible with applications [68] and [69]. The field is found using inverse Fourier Transform on the spectrum (2.29) and (2.31). We observe close agreement between theory and simulation for both the magnitude and phase of the complex field.

Fig. 2.8 (c) shows the far-zone field due to a current source placed at \( z_0 = \lambda_0/4 \) away from the same dense grating. Such a scenario is usually approximated as a solid infinite ground plane with no transmission for the TE polarization, in which case image theory applies. Although this is largely valid given that the transmitted field in \( z < 0 \) is small compared to the field in \( z > 0 \), some field will always leak through the gaps into the transmission region (\( 90^\circ < \theta < 270^\circ \)). Fig. 2.8 shows that the SIR method can correctly predict both the extremely weak transmitted TE far-field Fig. 2.8 (c-left), as well as the strong transmitted TM field Fig. 2.8 (c-right), leaking through the grating. It should be noted that the corresponding full-wave simulations used a large domain with over 100 grating periods terminated into absorbing walls to mimic a grating with infinite extent. This, along with the need for a very fine mesh to capture the details of diffraction, requires a long CPU time to achieve reasonable accuracy.

It can be seen in the TE case that image theory does not hold entirely, although the grating
gaps are very small compared to the wavelength. The image theory approximation is mostly valid for normally incident propagating waves on the grating, as they reflect almost completely off of the sub-wavelength grating. The high grazing propagating waves, as well as the evanescent portion of the spectrum, do not bounce off the mesh reflector in the same manner. Rather, they seep through the grating as their lateral wave number is in the order of the grating wavenumber. For the case of the $x-$directed line dipole excitation giving rise to TM fields, it is also seen that the overall pattern is equal on either side of the grating as shown in Fig. 2.8 (c-right) and the transmission occurs with little reflection.

2.7 Evanescent-to-propagating wave conversion using SIR theory

We are now in the position to use the developed SIR theory to investigate evanescent-to-propagating conversion in finite sources, such as a single current source excitation as well as extended source excitations.

2.7.1 Single current source excitation

To study the effect of spectrum conversion in a single source excitation setting, we find the radiation of the current sources discussed, placed both near and far from a grating of $k_L = 3.517k_0$. Fig. 2.9 (a) shows the TM incident spectrum on the grating, where a line dipole excites the domain, for two source-grating separation distances of $z_0 = \frac{\lambda_0}{20}$ (black curve), and $z_0 = \frac{\lambda_0}{4}$ (dashed red curve). In the case of $z_0 = \frac{\lambda_0}{20}$, it can be seen that the evanescent region of the incident spectrum extends well beyond the propagation region and several parts of the spectrum fall within the conversion bands of the grating. However if the source is then taken back at a distance of $\frac{\lambda_0}{4}$, the incident spectrum is such that most of the evanescent region has decayed to zero and almost no significant portion of the spectrum falls even in the first diffraction order. The weight of each diffraction order that is multiplied by the corresponding portion of the input spectrum is shown in Fig. 2.9 (b), i.e. the $B_{\lambda}^{TM}$ coefficients in the TM equivalent of (2.21). These weights are dependent on the grating geometry and are independent of the source location. Fig. 2.9 (c) shows the radiated field in the far-zone for the two source distances of interest. In addition Fig. 2.9 (c) shows the radiation field for cases where the grating gaps are completely closed, i.e. an infinite PEC ground plate, for the same source distances.

Similarly in the TE case, a line current is placed at these two distances from the grating, and the incident spectrum is shown in Fig. 2.10 (a). The corresponding diffraction weights ($B_{\lambda}^{TE}$) are shown in Fig. 2.10 (b). The far-zone radiation patterns are shown in Fig. 2.10 (c), and compared to the case of free space.
Figure 2.8: Verification of TE complex field values (a) at the grating location \( z = 0 \), and (b) at a near-field plane of \( z = -\lambda_0/20 \), (c) TE/TM far-field (dB magnitude). The TE fields are excited with a line current \( J_y \), and the TM fields are excited with a line dipole \( J_x \), placed at \((0, z_0)\).

### 2.7.2 Enhanced transmission through a sub-wavelength grating

Let us inspect the transmission region of Fig. 2.9 (c), when the grating is close and far from the source. Intuitively we expect that when a grating is in front of the source, the transmitted...
Figure 2.9: An $x-$directed line dipole source in the vicinity of the grating for TM polarization
(a) Incident spectrum at the grating plane for a source at two distances of $z_0 = \lambda_0/20$ and $z_0 = \lambda_0/4$, (b) The $n$-th order diffraction weights ($B_n$’s) for spectrum conversion into the propagating region for $|n| < 2$. (c) Far-field radiation pattern (linear scale) showing exceeding transmission compared to free space and solid PEC, as predicted by SIR (solid curves) and validated with Comsol (dashed curves of corresponding color). Inset shows the grating and source relating to the transmission and reflection regions, $L = \lambda_0/3.517$, $w = L/5$.

field through the grating would be weaker than the free space case, as one would think of the grating as a blockage against radiation into the transmission region. In fact, it seems logical
Figure 2.10: A \( y \)-directed line source in the vicinity of the grating for TE polarization. (a) Incident spectrum at the grating plane for a source at two distances of \( z_0 = \lambda_0/20 \) and \( z_0 = \lambda_0/4 \), (b) The \( n \)-th order diffraction weights (\( B_n \)'s) for spectrum conversion into the propagating region for \(|n| < 2\). (c) TE Far-field radiation pattern (linear scale), with almost zero transmission. Inset shows the grating and source relating to the transmission and reflection regions, \( L = \lambda_0/3.517 \), \( w = L/5 \).

to expect that the field magnitude in the transmission region of the grating, to be somewhere between the free space case, and the case of a solid plate, which is zero. In Fig. 2.9 (c) we can
see that indeed the magnitude of the far-field for the case of \( z_0 = \lambda_0/4 \) (red curve) is overall weaker than the free space case (blue curve) as expected, and therefore the grating is indeed acting as a blockage against transmitted radiation.

However, an interesting observation can be made here which is in some respect counter intuitive. For the source-grating distance of \( z_0 = \lambda_0/20 \) (black curve), we see that the transmitted far-field magnitude is actually stronger, than the free space case (blue curve) at every angle in the transmission region. This counter-intuitive result shows an enhanced transmission beyond the free-space transmission, for the case of a source close to the sub-wavelength grating. In other words not only the grating does not act as a blockage for the transmitted fields, but in fact it enhances them at the far zone.

The explanation for this result is as follows. The intuitive expectation is actually only valid for propagating waves, and when no significant spectrum conversion occurs. This is the case for \( z_0 = \lambda_0/4 \), where most of the waves reaching the grating are propagating as shown in Fig. 2.9 (a) with the red curve. Therefore what transfers to the transmission region is weaker than the free space case radiation, since the \( B_0 \) coefficient for the zeroth order is always less than unity as shown in Fig. 2.9 (b). However, for the case of \( z_0 = \lambda_0/20 \), aside from the original propagating waves, evanescent waves have diffracted and added to the propagating spectrum, and hence the overall radiation field is stronger than the free space case. This is reminiscent of “extra-ordinary” transmission [89] through a sub-wavelength grating enabled by the evanescent-to-propagating wave conversion properties of the grating.

It must be noted that for the TE case of Fig. 2.10 (c) we do not observe an extra-ordinary transmission beyond the free space transmission. This is primarily due to the weakening effect of the grating for this polarization. In other words, the fact that the electric field cannot be tangential to the metal boundary, allows for very weak fields to develop across the gaps, yielding significantly smaller coefficients \( B_n \), as seen in Fig. 2.10 (b). Additionally, the denominator in the spectrum (2.28), weakens the incident field at higher spatial frequencies, which can also be seen in Fig. 2.28.

### 2.7.3 Radiated power

The radiation patterns of Fig. 2.9 (c) also show a stronger overall magnitude, in both the reflection and transmission regions, for the case of an electric line dipole placed at a distance of \( z_0 = \lambda_0/20 \) from the grating. This leads us to examine the total radiated power more closely, for the line sources at an arbitrary chosen frequency of 1 GHz. The total radiated power by a source arrangement, per unit length in the \( y \) direction, calculated using the far-zone electric or magnetic fields integrated over an enclosing cylindrical surface of radius \( r_{obs} \) around the antenna arrangement is

\[
    P_{rad} = \frac{1}{2\eta} \int_0^{2\pi} |E(r_{obs}, \theta)|^2 r \, d\theta = \frac{\eta}{2} \int_0^{2\pi} |H(r_{obs}, \theta)|^2 r \, d\theta
\]

(2.36)
As a benchmark, the total radiated power in free space per unit length in the $y$-direction, for the electric line dipole of strength $p = 1 \text{[A.m/m]}$, can be found analytically by integration of the far-field [2]:

$$P_{rad} = \frac{\eta k_0 p^2}{16} = 493.82 \text{ W/m} \quad (2.37)$$

Similarly in the TE case, the radiated power per unit length in the $y$-direction, for the electric line current $I = 1 \text{[A]}$ radiating in free space, is found analytically by integrating the far electric field:

$$P_{rad} = \frac{\eta k_0 I^2}{8} = 987.64 \text{ W/m} \quad (2.38)$$

The total radiated power, for all other cases of interest in Fig. 2.9 and Fig. 2.10, where a source is placed near a grating at different distances as well as close to an infinite PEC ground plane, are found by numerically evaluating (2.36) with the far-field data, and are tabulated in Table 2.1.

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Power (W/m) from line dipole (TM)</th>
<th>Power (W/m) from Line source (TE)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Source in free space</td>
<td>494</td>
<td>988</td>
</tr>
<tr>
<td>Grating at $z_0 = \lambda_0/20$</td>
<td>1019</td>
<td>112</td>
</tr>
<tr>
<td>Grating at $z_0 = \lambda_0/4$</td>
<td>540</td>
<td>1307</td>
</tr>
<tr>
<td>Infinite PEC at $z_0 = \lambda_0/20$</td>
<td>72</td>
<td>96</td>
</tr>
<tr>
<td>Infinite PEC at $z_0 = \lambda_0/4$</td>
<td>894</td>
<td>1299</td>
</tr>
</tbody>
</table>

* A grating with $L = \lambda_0/3.517$, $w=L/2$.

* radiated power only in the reflection region, (half space $z>0$).

Table 2.1: Total radiated power from a single source at 1 GHz

As one would expect, when the line source (TE) or line dipole (TM) is very near the PEC ground plane at $z_0 = \lambda_0/20$, it almost does not radiate at all, i.e. the current source is shorted out because it is parallel to the ground plane in both polarizations. But again an interesting observation from the grating results shows that the line dipole source (for TM pol.) radiates very strongly, when a grating is instead placed at the same close distance. This total radiated power, for the combination of reflection and transmission regions, is more than that of the total radiated power when the source lies in free space, or if a solid infinite PEC plane is used. This is in agreement with our previous observation and confirms that indeed higher amount of power is radiated in the far-zone altogether. This is suggestive once more that some reactive near-field around the antenna has diffracted and converted into real radiated power, i.e. the radiation resistance of the overall radiator increases.
2.7.4 Response to arbitrary source arrangements

In the most general case, we are interested in finding the response to an arbitrary source excitation, and to see if spectrum conversion can be utilized and exemplified in more involved source arrangements. Suppose that the excitation of the problem is an arbitrary arrangement of sources shown in Fig. 2.11. Such sources may fit entirely into a unit cell, may extend beyond a unit cell, or fall on the boundary. In the proposed SIR method, irrespective of how the source arrangement is situated with respect to the grating, and the grating periodicity, the output spectrum of the diffracted fields is determined by means of (2.21), as long as the spectrum of the entire source arrangement is found at the grating plane (through plane-wave decomposition or other means), and used as the input spectrum to the grating system. This is an important benefit of the SIR method compared to numerical and even semi-analytical techniques such as the ASM [76]. Such scenarios of extended source arrangements may be solved using other techniques [76, 77, 78, 79, 80, 81] by breaking up the problem into sub-problems, each having different portions of the extended source arrangement, solving each sub-problem individually, and then combining the results. This is a more laborious solution and requires solving multiple aperiodic problems, whereas the SIR technique yields the total diffracted field instantly.

![Figure 2.11: An arbitrary aperiodic source excitation that extends beyond a unit cell of the periodic grating.](image)

Typically the unit cells of 1, 2 and 3 are solved separately, each with the corresponding portion of the total current. However, the SIR solution directly yields the total output spectrum by finding the free space input spectrum.

Moreover, in the SIR formulation, the excitation is decoupled from the problem solution and can be dealt with independently of the geometry of the grating. In other words, given a fixed grating, the response to any source may be found through (2.21) without having to re-solve the grating itself. When re-iterating the design to find the optimum excitation, say for a desired far-zone behavior, the same diffraction weights can be used for different arrangements of source distribution, without having to solve the full problem every time, in contrast to other techniques.

The spectrum at a distance from the source has an exponential decay profile as seen in Fig. 2.9 (a) or Fig. 2.10 (a). This ensures rapid convergence of (2.21) as the higher spectral components exhibit more decay. Knowing the spectral bandwidth of the incident spectrum,
one can decide where to truncate the series. As for the numerical efficiency of the SIR method, an analysis such as [90] is required to investigate its performance compared to other techniques such as the ASM.

A further demonstration of evanescent-to-propagating conversion in the case of extended sources, is shown in the next chapter, which leads to the idea of radiation enhancement of invisible sources.
Chapter 3

Radiation Enhancement of Invisible Sources

3.1 Introduction

The spectrum conversion of fields emanated from sources using a simple grating, as studied in the previous chapter, can be used for novel ways of radiation enhancement of electromagnetic sources. In this chapter, we demonstrate theoretically and experimentally, that the radiation of a specially designed ‘invisible source’ can be significantly enhanced, by placing an appropriate sub-wavelength metal strip grating close to it. This novel ‘radiation enhancement’ is made possible by a controllable and significant amount of evanescent-to-propagating wave conversion and without any resonances, for both polarizations.

The ‘invisible’ source is itself a novel, highly reactive source that hardly radiates in free space. The source is designed such that it has almost all of its energy allocated to the evanescent spectrum, or the invisible region, and the propagating (visible) region of the source field is significantly attenuated by design. Thus when the source is in free space, the very weak radiation from it is undetectable or ‘invisible’ to a far-field observer, especially in the presence of an ambient electromagnetic field/noise. It is then shown, that a particular metal strip grating can in fact cause this source to radiate strongly into the far-zone, and thus become ‘visible’. Based on the observed ‘radiation enhancement’ and devised arrangement, we propose applications in sensing, detection, and remote measurement of sub-wavelength distances.

The highly reactive ‘invisible’ source is designed using an array of simple point-radiators with specific excitation weights and is excited by a common passive feed network, reminiscent of an antenna array. The array however operates opposite to classic antenna arrays as it is non-radiating and invisible. To design such a reactive invisible array, we propose a spectral method to directly design the full spectrum of the near-field of a general array. Our approach extends the existing array factor concept [91], which is inherently limited to only the far-zone. In the proposed approach, it is found that the spectrum of field is essentially a multiplication
of a ‘spectral array factor’ and a ‘spectral element factor’. Thus the problem boils down to tailoring the polynomial of the ‘spectral array factor’ to have a desired behavior. This approach allows the capability to tailor the entire visible and invisible regions of the field (and not just designing the far-field using the array factor, as done in antenna arrays). For our scenario of interest, we design the spectral array factor for a low visible and a strong portion of invisible region. The method however is quite general and can be applied for other desired spectral responses. Such invisible sources may find applications in sensors and detectors, especially when combined with gratings as will be discussed in this chapter, as well as wireless power transfer, and imaging beyond the diffraction limit, due to their strong and tailored evanescent spectrum.

### 3.2 Converting an invisible array to a highly radiating one in TM polarization

Consider an array of five closely spaced TM line dipole sources as shown in Fig. 3.1 (a). Based on the coordinate system used, propagation is considered only in two dimensions in the \(x - z\) plane, and for Transverse Magnetic (TM) polarization, the available field components are \(\{E_x, H_y, E_z\}\). \(x\)-directed current elements are considered as sources of the TM polarized field as in the previous chapter.

The array is properly weighted (as will be described in the next section) to have a free space spectrum shown in Fig. 3.2 along the array axis (along \(z = 0\) in Fig. 3.1 (a)). It can be seen in this spectrum that there are significant lobes in the evanescent region, while the propagating region \((-k_0 < k_x < k_0)\) is significantly small in magnitude. Such a spectrum implies that when this antenna array is in free space, it radiates very weak electromagnetic fields into the far-zone, due to the weak propagating spectrum. The large lobes in the evanescent region also imply that the array is highly reactive, meaning significant reactive power is stored in the vicinity of the antenna arrangement, and a low amount of real power is radiated. Incidentally, such large lobes in the evanescent region are also encountered in super-directive arrays [92], where considerable reactive power is stored in the vicinity of the antenna array and is detrimental to their operation.

![Figure 3.1](image)

Figure 3.1: (a) An array of five x-directed current elements, closely spaced with separation ‘\(d\)’ is designed to radiate weak TM fields in free space (b) Same array placed close to a grating at distance \(z_0\).
Consider now that a grating with period ‘L’ and a gap size ‘w’ is placed in the close vicinity of this reactive array, at a distance \( z_0 \), as depicted in Fig. 3.1 (b). The grating is aligned along the x-axis, and the array is at \( z = z_0 \). We investigate the radiation characteristics of this arrangement using the SIR technique, and also monitor the spectrum conversion process. It is also interesting to note that this arrangement of five x-directed current elements extends beyond the center unit cell of the grating, and each element is at a different location within their respective unit cell, as the separation of the source elements ‘\( d \)’, is different than the grating period ‘\( L \)’. However, the SIR approach directly yields the field solution by just knowing the incident spectrum.

To calculate the output scattered field in the transmission region, we again utilize the results of the SIR method in equation (2.31). For the incident spectrum on the grating, instead of (2.30), we now need the FT of the array’s field in free space a distance \( z_0 \) away, which is a superposition of the spectrum due to each shifted current dipoles, and explicitly it is:

\[
\tilde{H}_{\text{gin}}(k_x) = \sum_{m=1}^{5} \frac{p_m e^{-j k_z (m-3) d} e^{-j k_z z_0}}{2} \quad (3.1)
\]

where \( k_z \) is the lateral wave-number and the spatial frequency for the Fourier transform of the field \( \tilde{H}(k_x, z) = \int_{-\infty}^{\infty} H(x, z) e^{j k_x x} dx \). All fields and currents are again time harmonic phasors of \( e^{+j \omega t} \).

The magnitude of this input spectrum, for a grating-array distance of \( z_0 = \lambda_0 / 20 \), is shown in Fig. 3.3 (a). This spectrum is essentially the result of the free space propagation/attenuation of the field of the array in Fig. 3.1 (a). In the spectrum of (3.1), the term \( e^{-j k_z z_0} \) creates a phase variation for propagating waves, whereas it provides a decay factor for evanescent waves as the exponent is real and negative. Moreover, evanescent waves that are at a higher spatial frequency \( (k_x) \), have a larger \( k_z \), and would therefore experience a greater decay than evanescent waves with lower \( k_x \). Therefore, as we see in Fig. 3.3 (a), the spectrum only contains two major lobes in the evanescent region, whereas the original spectrum along the array axis had repeated lobes over the spectrum, as depicted in Fig. 3.2. These other higher frequency lobes that were present in Fig. 3.2, have now decayed greatly. On the other hand, the distance \( z_0 \) is still small enough, such that the two major lobes experienced little decay and hence the propagating region is still much weaker than these two remaining evanescent lobes.

We choose the grating dimension as \( L = \lambda_0 / 3.517 \), i.e. a grating wavenumber of \( k_L = 3.517 k_0 \). With this dimension, the regions of evanescent waves that diffract into the propagating region are between \((2.517 n k_0, 4.517 n k_0)\), \( n \) is a non-zero integer. The first few regions are indicated in Fig. 3.3 (a) by dashed red lines, and the propagating region is indicated with a dashed green line.
Chapter 3. Radiation Enhancement of Invisible Sources

3.2 Diffraction orders

Fig. 3.3 (b) shows the magnitude of the conversion weights $B_n(k_x - 2\pi n/L)$, for the first few diffraction orders $-3 < n < 3$. As we are primarily interested in what diffracts into the propagating region, the figures are showing the weights over the range $-(2.517 nk_0, 4.517 nk_0)$. It can be seen that the weights of $B_n$ are mainly on the order of unity throughout the propagating region. This is in accordance with the earlier result, and our expectation of diffraction of TM fields through the grating. For this polarization, the field seeps through the grating with little attenuation.

Fig. 3.3 (c) shows the magnitude of the contribution to the output propagating spectrum, due to each diffraction order, i.e. the $n$th order term in the summation (2.21). These distributions are again shown for $-k_0 < k_x < k_0$. They are the $B_n$s coefficients multiplied with the corresponding parts of the incident spectrum between $(2.517 nk_0, 4.517 nk_0)$ in Fig. 3.3 (a), and end up in the propagating region. The magnitude of the different contributions, indicate that the $n = \pm 1$ diffraction orders are the most prominent, whereas the $n = 0$ and $n = \pm 2$ diffracted orders will make little contribution. The point to be made here is that the small contributions of $n \in \{0, \pm 2\}$ orders are not due to the grating attenuation, as the corresponding $B_n$s are all on the same order. But rather it is due to the weak incident spectrum. For $n = 0$, we designed the input propagating spectrum to be weak, and for $|n| > 1$ the attenuation of evanescent waves at higher spatial frequencies, significantly weakens the corresponding portion of the input spectrum.

Fig. 3.3 (d) shows the final output spectrum, i.e. the spectrum of the transmitted field directly below the grating at $z = 0^-$. This plot is the result of adding the diffraction orders $-3 < n < 3$. For validation, the same arrangement of sources and grating as in Fig. 3.1 (b),
Figure 3.3: (a) Spectrum at distance \( z_0 \) away from the array in free space, corresponding to the incident spectrum at \( z = 0^+ \) in Fig. 3.1 (b) used for SIR. (b) The n-th order diffraction weights \( (B_n)'s \) for spectrum conversion into the propagating region for \( |n| < 2 \) (c) contributions of n-th order diffraction into the output propagating region (d) Overall final output spectrum directly through the grating at \( z = 0^-, \) compared with the spectrum of fields sampled at \( z = -\lambda_0/10000 \) from Comsol. \( k_L = 3.517k_0, \) \( L = \lambda_0/3.517, \) \( w = L/5, \) \( z_0 = \lambda_0/20, \) \( d = \lambda_0/8, \) \( [p_{1-5}] = [0.4534, -1.476, 2.0713, -1.476, 0.4534](A.m/m). \)

is simulated in Comsol 2D and results are compared in Fig. 3.3 (d). It can be seen that the calculated transmitted spectrum is in close agreement with the simulation results.

### 3.2.2 Antenna characteristics

Fig. 3.4 shows the far-field pattern of the radiated magnetic field. The radiation pattern of the array without the grating is shown with a blue dashed curve. The black and dashed red curves
both show the radiation pattern for the array in the presence of the grating, from SIR theory and Comsol simulations respectively. We can see that in the latter cases the far-zone magnetic field attains a much larger absolute value. For example at broadside $\theta = 180^\circ$, the far-zone magnetic field $|H(r, \theta)|$ is orders of magnitude stronger when the grating is present, compared to the case where the array radiates in free space.

From the SIR formulation of the far-field, we are directly identifying the radiating modes. For $|k_x| < k_0$, the zero-th order mode in the summation is the direct conversion from the incident propagating spectrum, with the conversion factor $B_0$. The other terms in the summation, represent radiating modes which are a result of conversion from evanescent modes in the incident spectrum, to propagating modes, with conversion factor $B_n$ ($n$ non-zero). In this TM scenario, what causes the overall change in the far-field pattern is the strong contribution of the first diffracted order evanescent fields. The grating itself does little in terms of attenuating the shape of the diffracted distribution over $k_x$, as the $B_n$ coefficient distribution is found to be in the order of unity over this range. The key functionality of the grating is exploited here, which is adding up various portions of the input spectrum into the propagating region.

For the invisible array in free space of Fig. 3.1 (a), the total power per unit length radiated by the system is 0.0651 W/m. But when the same antenna arrangement is placed near the grating as in Fig. 3.1 (b), numerical evaluation of the power from the far-field data of Fig. 3.4, yields a radiated power of 1000 W/m. This value is a factor of 15000 larger than the total power radiated in free space, equivalent to a 42 dB increase in radiation power.

![Radiation Pattern Diagram](image1)

**Figure 3.4:** (a) Far-field radiation pattern (dB magnitude of $H_y$) of antenna array placed near the grating calculated using the SIR method, and validated with Comsol simulations. The pattern is much stronger than the pattern of the array in free space with no grating, and total radiated power is 42 dB higher. (b) Phase of radiated $H_y$ field in the presence of the grating predicted by the SIR method and validated with Comsol simulation.
3.3 Converting an invisible array to a highly radiating one in TE polarization

Consider now a 2D TE domain with a metal strip grating as shown in Fig. 3.5. Based on the coordinate system used, propagation is considered only in two dimensions in the $x-z$ plane, and for Transverse Electric (TE) polarization, the available field components are \{$H_x, E_y, H_z$\}. Line currents flowing out of the page are considered as sources of the TE polarized field. A similar grating is used as before, with a period $L$, and a gap size $w$.

First let us consider only one of the line currents shown at a distance $z_0 = \lambda_0/40$ away from the metal strip grating. A 2D full-wave simulation shows that the field strength at a transmission point $(0, -\lambda_0/2)$, is weaker (almost half) than the field strength at that point if the grating was removed (free space). This is intuitive as the grating is highly reflective (attenuating for transmission) under the TE polarization. The transmission vanishes in the limiting case when the gaps are completely closed, i.e., in the case of a solid perfect electric conductor (PEC) and all fields reflect. Moreover under such circumstances, for a line current parallel and close to the infinite PEC, the reflected field strength will be very small, as the PEC essentially shorts out any parallel currents close to the surface through image theory. The attenuating effect of the grating in TE polarization is due to the boundary condition at the metal edges of each gap of the grating, requiring the out-of-plane electric field to vanish at those locations and hence reaching a small value in the middle of the gap. This is while for the TM case, the field can be much more easily sustained across the gap and hence the grating is not attenuating.

Thus for the array arrangement shown in Fig. 3.5, since we are adding more sources, one expects that the total transmitted field through the sub-wavelength grating should be lower than if all these sources would radiate in free space collectively, by argument of superposition. The reflected field strength from such a sub-wavelength grating is also expected to be in the order of the fields reflected by a solid PEC boundary. However, we obtain a peculiar result with the same number of sources placed at the same distance from the grating, but excited with the appropriate current values discussed earlier for the TM polarization. The source-grating arrangement would then result in transmitted/reflected fields that are significantly stronger than the free space radiation of the same current arrangement without the grating, similar to what was seen in the TM polarization.

3.4 An ‘invisible’ radiating antenna array in the far zone

We stated earlier that an array of certain current values yields an 'invisible' array. Here we describe in detail what such arrays exactly mean and how their complex current values are achieved. Let us first examine the previously stated array of line sources with inter-element spacing $d = \lambda_0/8$, and currents $I_{1-5} = \{0.2189, -0.7126, 1.0000, -0.7126, 0.2189\}$, using existing
antenna array theory. It is well known that the far-zone radiated field of an array is [91]:

\[
\text{Array Field} = \text{Array Factor} \times \text{Element Factor}
\]  

(3.2)

The normalized radiation pattern of this linear array in free space is shown in Fig. 3.6 (a), which is essentially the plot of the normalized array factor (AF) as the elements are isotropic, with \( AF = \sum_{m=1}^{5} |I_m| e^{j(m-1)\psi} \), \( \psi = k_0 d \sin \theta + \pi \) and \( \theta \) is measured from the \( z \)-axis. Using the Schelkunoff method [92, 91] and treating the AF as a polynomial of a complex variable \( u = e^{j\psi} \), Fig. 3.6 (b) shows the Schelkunoff circle in the complex \( u \)-plane, the locations of zeros of the AF on the unit circle, the un-normalized AF for \( 0 \leq \theta \leq 2\pi \) in dB format, and the visible/invisible regions for this array.

The visible region of Fig. 3.6 (b) is what gives rise to the radiation pattern Fig. 3.6 (a), and due to the element spacing \( d = \lambda/8 \), the visible region only subtends an arc of \( 2k_0 d = \pi/2 \) radians on the unit circle of Fig. 3.6 (b). The un-normalized array factor pattern in Fig. 3.6 (b) shows that the amplitude of the AF is significantly lower than unity (0 dB) throughout the visible region. This implies that the radiated field from this 5-element array is actually very weak over all angles, considering that the 0 dB unit circle corresponds to the AF of just the center element \( I_3 = 1 \)(A). This weak radiation can be understood by the fact that the currents \( I_1 \) to \( I_5 \) tend to cancel out each other’s fields in the far-zone, especially as their distance is very small compared to the wavelength. The corresponding AF polynomial, \( I_5/I_3 \prod_{m=1}^{4} (u - u_m) \), attains small amplitudes in the visible region due to small values of \( |u - u_m| \) i.e. the short distance of every point \( u \) in the visible region, to its nearby zeros \( u_{1-4} \).

The remainder of the circle is the ‘invisible’ region. But, what is missing from this picture, is the behavior of the array in the invisible region. However, the AF polynomial and the Schelkunoff circle representation do not directly characterize the array in the invisible region, as they are inherently formulated for far-field radiation, and the invisible region in the AF formulation has no physical correspondence. Therefore, in order to investigate the array response in both near and far-field, we shall analyze the spectrum of field at or near the array, instead of the array factor polynomials.
3.4.1 Designing the spectrum of the ‘invisible’ array

In order to fully characterize the behavior of this weakly radiating array of current sources, it is imperative to inspect the near-field, and in particular the spectrum of the field near the array. In 2D free space (no grating present), the corresponding spectrum of fields at a distance $z_0$ away from the array is given by

$$
\tilde{E}_{\text{yin}}(k_x) = -\frac{k_0 \eta}{2} \frac{e^{-j\sqrt{k_0^2 - k_x^2}z_0}}{\sqrt{k_0^2 - k_x^2}} \sum_{m=1}^{5} I_m e^{-jk_x(m-3)d}.
$$

where $k_x$ is the lateral wave-number and the spatial frequency for the Fourier transform of the field $\tilde{E}(k_x, z) = \int_{-\infty}^{\infty} E(x, z)e^{jk_x x} dx$, and $\eta$ is the intrinsic wave impedance.

Fig. 3.7 (a) now shows the magnitude of the spectrum for the array of currents of Fig. 3.6. The far-zone or the visible region of the AF discussed in Fig. 3.6 (b) maps on to the propagating portion of the spectrum ($|k_x| \leq k_0$) between the dashed green lines. The far-field has a direct relation with the propagating region of the spectrum by means of the far-zone asymptotic approximation (see Chapter 2).

Similar to what was seen in the TM polarization, in Fig. 3.7 (a) it is seen that there are two significantly large lobes in the evanescent region of the spectrum ($|k_x| \geq k_0$), and the propagating region is significantly weaker in amplitude. The weak propagating region again implies that when this array resides in free space, it actually radiates very weak electromagnetic
fields into the far zone, i.e. a small amount of real power is radiated by the entire arrangement. The large lobes in the evanescent region imply that the array is highly reactive, meaning significant reactive power is stored in the vicinity of the source arrangement. Thus this array is termed invisible, as it has its energy stored in the invisible region of the Schelkunoff circle (evanescent region of the near-field spectrum). Moreover, it is also invisible to a far-field observer especially in presence of electromagnetic noise, as it is an extremely weak radiator.

Figure 3.7: (a) Spectrum of the proposed invisible array at a distance of \( z_0 = \lambda_0/40 \), bearing currents \( I_{1-5} = \{0.2189, -0.7126, 1.0000, -0.7126, 0.2189\} \) (A), and element spacing \( d = \lambda_0/8 \). (b) Magnitude of the spectral array factor and the element spectrum.

The nature of the spectrum and the presence of the large evanescent lobes can be better understood from the interpretation of the spectrum in Eq. 3.3 as a multiplication of two terms: the spectrum of an array of isotropic radiators which is the summation term \( \sum_{m=1}^{5} I_m/I_3e^{-jk_0(m-3)d} \), and an element spectrum which is \( -k_0\eta I_3e^{-j\sqrt{k_0^2-k_z^2}z_0}/(2\sqrt{k_0^2-k_z^2}) \) for the center line current in TE polarization. The magnitudes of these two components, namely the ‘spectral array factor’ as well as the element spectrum are shown in Fig. 3.7 (b). The element spectrum (blue curve) has a significant magnitude over the propagating region. This
is because a single radiator radiates well into the far-zone (i.e. it is visible). On the other hand, the spectral array factor (black curve) has a very low (close to zero) magnitude over the propagating region.

Therefore the overall spectrum in Fig. 3.7 (a), which is the multiplication of the two curves, has a close to zero magnitude over the propagating region. However in the evanescent region the effect is reversed. The spectral array factor grows beyond its last root upto $k_x = \pm 4k_0$ and repeats periodically over $k_x$, while the element spectrum decays with larger $|k_x|$. Therefore in the evanescent region, the decaying effect of the element spectrum multiplied by the enhancing effect of the spectral array factor results in only two major lobes in the overall spectrum of Fig. 3.7 (a), before the exponential decay of the element spectrum has completely reduced the amplitude. The currents of our array are in fact optimized to yield the carefully designed spectrum of Fig. 3.7 (a). By placing nulls in or close to the visible region, we make sure that the propagating region as well as the low $k_x$ evanescent region, attain a relatively small amplitude. Consequently this leads to the large lobes in the evanescent portion of the spectrum.

With this perspective, the design of the overall array spectrum boils down to the design of the spectral array factor by choosing the appropriate array weights (currents), either analytically or through optimization. To make the overall array invisible, the objective is then to minimize the magnitude of the polynomial of the spectral array factor over the entire propagating region $-k_0 \leq k_x \leq k_0$ by placing the roots of the polynomial in the correct $k_x$ locations. This optimization procedure was done here using a simple computer code, however an analytical approach could be to apply a Chebyshev type polynomial with equal ripple to the spectral array factor polynomial, over the propagating region.

It should be noted that this is not the typical perspective of standard antenna array theory where all effort is focused on the design of the visible region (e.g. Ref. [91]). In essence by working in the spectral domain, we are extending the element/array-factor multiplication approach to the near-field spectrum. Therefore, in general for an array of radiators, the spectrum of the field is:

$$\text{Array Spectrum} = \text{Spectral Array Factor} \times \text{Spectral Element Factor} \quad (3.4)$$

at some plane parallel to the array and extending to infinity. For far-field, asymptotic approximations on this spectrum yields the array field directly in real-space for the far-zone as in (3.2). Thus, the existing array factor [91] concept is a subset of the more general spectral equation (3.4), for the full-field.

It should be made clear that the multiplication effect of two components in (3.4) is valid for the ‘spectrum’ of the near-field (i.e. in $k_x$ space). The near field in real-space can be obtained either from the inverse Fourier transform of the array spectrum in (3.4), or as a convolution of the inverse Fourier transforms of the two spectra. Due to asymptotic approximations and the one-to-one correspondence between the actual far-field and the propagating region of the spectrum, a similar multiplication is consequently valid for the actual far-field in (3.2).
In this spectral approach, we are not designing the far-field array factor for the visible region only, but rather the full spectrum for both the visible and invisible regions simultaneously. This is made possible by analyzing the spectrum and recognizing the multiplication of a spectral array factor with an element spectrum, and designing the array weights accordingly, to achieve a certain behavior for the spectral array factor polynomial. This spectral array factor concept can also be extended to 2D arrays. Various polynomial types and array synthesis/analysis techniques as done in [91] may also be applied for the spectral array factor concept, which is left for future work. It may also be interesting and of practical use to analyze existing antenna arrays, for their full spectral array factor to characterize their near-field behavior and their reactive nature.

3.4.2 Spectrum conversion with a grating

If a grating were situated at a distance $z_0$ from the array as in Fig. 3.5, then Eq. 3.3 would describe the spectrum of the field that is incident on the grating. The transmitted spectrum of fields directly below the grating was found in the previous chapter to be:

$$\tilde{E}_{y_{out}}(k_x) = \sum_{n=-\infty}^{+\infty} \tilde{E}_{y_{in}}(k_x - \frac{2\pi n}{L})B_n(k_x - \frac{2\pi n}{L})$$  (3.5)

which stated that the transmitted spectrum is a summation of weighted copies of the incident spectrum, shifted by the grating wave-number $2\pi/L$. The weights, $B_n(k_x)$, are determined by the geometry of the grating, as discussed in the previous chapter. Hence by placing an appropriate grating at this location we can convert the strong evanescent lobes into propagating waves. The condition is to use a sub-wavelength grating with the right period. For Fig. 3.7 (a), we use a grating of $L = \lambda_0/3.5$, resulting in first order conversion into the propagating region with a wavenumber shift of $3.5k_0$. With the incident spectrum shown in Fig. 3.7 (a), the $n = \pm 1$ orders contribute the most to the transmitted propagating spectrum, whereas the other orders have little contribution due to the low strength of the incident spectrum. The weak incident propagating region multiplied by the small coefficient $B_0$ results in very low zeroth order contribution. For $|n| > 1$ orders the decaying magnitude of the incident spectrum for higher $k_x$, as seen in Fig. 3.7 (a), results in little contribution to the transmitted spectrum.

3.5 Experimental Setup

We devised an experiment to test the proposed radiation enhancement phenomenon, in 2D TE polarization. The overall experimental setup is shown in Fig. 3.8. The operating frequency of the experiment is at 1 GHz, equivalent to a free space wavelength of $\lambda_0 = 30$ cm. The main components of the experiment are a parallel plate waveguide, the array, grating, and the feeding network.
To enforce the 2D TE polarization, we use a Parallel Plate Waveguide (PPW) operating in the TEM mode, as depicted in Fig. 3.9. The electric field is $y$–directed normal to the top and bottom metal walls, and consequently the magnetic field is always parallel to the x-z plane. Through image theory, the top and bottom plates ensure that any vertical current segment and electric field, and horizontal magnetic field are repeated indefinitely in the $y$–direction, and therefore this closely mimics the 2D TE polarization assumed in Fig. 3.5.

The height of the guide is chosen to be $\lambda_0/20 = 1.5cm$ to keep other possible guided modes well below cut-off. In fact the cut-off frequency of the next propagating mode for this guide is at 9.99GHz. For the two top and bottom metals of the guide, two FR4 single-sided substrate boards are used. The metalized (Copper) side is facing inside the guide to meet the metal boundary condition. As shown in Fig. 3.9 absorbers are placed in the surrounding edges of the guide to avoid any reflections and the buildup of resonant modes inside the PPW.

In 2D TE polarization, a line current is used as the source of the TE field. In our experiment we use a shunt standing probe equal to the height of the guide carrying the RF current. An array of five such probes is placed inside the guide with a spacing of $d = \lambda_0/8 = 3.75cm$. The inner conductor of each probe runs to a hole in the second plate, where the two conductor system of probe and plate is terminated to a 50$\Omega$ load (terminations in Fig. 3.8). The metal strip grating is held vertically in the guide, using a foam layer with relative permittivity close to 1 around the operating frequency.

We enforce the desired current ratios by sending the correct ratios of powers incident on the 5 probes. A power divider was implemented to split a common input power unequally into five output branches, with a prescribed phase on each output port. This 1 to 5 way power splitter is designed in microstrip and fabricated on an FR4 substrate at 1GHz. The design is
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Figure 3.9: Array of 5 probes next to a grating exciting a parallel plate waveguide. Absorbers are placed around the edge of the guide to avoid reflections.

shown in Fig. 3.8 and is inspired from standard unequal power divider circuits, but with no resistors for simplicity. The downside of using no isolating resistors is not necessarily achieving simultaneous matching and isolation at the output ports. However this is not essential as we are interested in the transmission from the common input to the output ports. Meandered lines are added for proper phasing of each branch.

3.6 Results

The 3D simulation results of the 5 element reactive array of probes exciting the PPW are shown in Fig. 3.10, where the correct ratios of incident powers on the ports are assumed by the simulator. The figure shows the magnitude of the electric field ($E_y$) throughout the guide, over a plane parallel and half way in between the guide walls. The results clearly show that when only the array is in the guide Fig. 3.10(a), it hardly radiates any field in the guide (dark blue indicates zero field). The transmission is even smaller in Fig. 3.10(b), when the same array is placed next to the attenuating metal strip grating at $z_0 = \lambda_0/4$. But when the same array is placed closer to the grating, the radiated field starts to increase as in Fig. 3.10(c), and is significantly enhanced at least by an order of magnitude for $z_0 = \lambda_0/40$ as shown in Fig. 3.10(d).

Another important observation is in the reflection region. The grating is reflective as shown in Fig. 3.10(b), and the field strength is similar to what a solid PEC plate would reflect, as the openings are sub-wavelength. But in Fig. 3.10(d), we see that the field in the reflection region
is also significantly enhanced compared to the case of a solid PEC plate.

![Electric field magnitude](image)

**Figure 3.10**: Electric field magnitude $|E_y|$ (V/m) inside the PPW. Weak radiation from (a) array inside the guide (b) array plus a grating at $z_0 = \lambda_0/4$, and (c) array plus a grating at $z_0 = \lambda_0/8$. (d) Strong radiation from array and a grating at $z_0 = \lambda_0/40$.

In our experiment, we measure the relative field strength established inside the guide for these cases of interest. The common port of the feed network is fed from port 1 of a Vector Network Analyzer (VNA). A probe is connected to the second port of the VNA placed at a point $(0, -\lambda_0/2)$ inside the PPW, i.e. an observation point at the broadside of the array in the transmission region, and 2-port S-parameters are measured. The distance of the point from the array was chosen based on available space in the small prototype made. The results of the measurements are shown in Fig. 3.11 and Fig. 3.12.

Fig. 3.11 shows the transmission coefficient $S_{21}$, between the input common port to the feed network, and the probe placed at broadside inside the guide. It can be observed that when the array radiates on its own inside the guide, there is a weak transmission level of $-68$ dB sensed at 1GHz. We then place the grating inside the guide at $z_0 = \lambda_0/40$ and re-measure $S_{21}$ for the same observation point. In this case a transmission of $-49$dB is measured, which is about 19dB
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Figure 3.11: Raw measured transmitted field strength (in dB) at an observation point at broadside, showing extraordinary transmission for the case of the grating close to the invisible array of current sources.

The transmitted field sensed at the location of the probe is greatly enhanced in the presence of the grating near the weak radiator. This is in accordance with the full-wave simulation results which show a greater than 20 dB improvement (an order of magnitude increase in radiation intensity). The slight difference between the simulation and measurement results can be attributed to the imperfections in fabrication of the guide and connections of the grating to the top and bottom walls of the guide, as well as differences in the response of the feed network from the ideal case due to fabrication tolerances.

Fig. 3.11 also shows a third trace, which is the measured transmission when the same grating is now placed at a distance of \( \frac{\lambda_0}{4} \) away from the source. Though it may not be very clear from the figure, the sensed field in this case is actually slightly weaker than the case of the source with no grating. It is consequently significantly weaker than the case where the grating is placed at \( z_0 = \frac{\lambda_0}{40} \). Therefore, the grating is reflective as commonly expected for larger source grating distances, where most of the incident spectrum is composed of propagating waves, and confirms Fig. 3.10(b).

Fig. 3.12 shows the reflection coefficient seen at the common port 1 for the three cases of interest. It can be seen that the \( S_{11} \) in the case with no grating is higher than the case of the
grating at $z_0 = \lambda_0/40$, as in the latter case there is more power radiated. In other words, the whole system is better matched in the latter case.

![Reflection Coefficient Graph]

Figure 3.12: Measured reflection coefficient at the common input port 1 of the feed, for three cases of interest.

Fig. 3.13 shows the magnitude (a) and phase (b) of the transmission coefficient $S_{21}$, between the common port of the feed network and the output termination of each probe in the array. Each transmission is taken as a measure of the currents established on the ports, and confirms the correct excitation of the current amplitudes needed to realize the spectrum shown in Fig. 3.7 (a), at 1GHz. Although closely spaced sources may experience mutual coupling between the ports, this measurement validates the correct operation of the array and its weights in the presence of any coupling.

In the transmission response of Fig. 3.11 there is a frequency dependence. One determining factor for this dependence is the frequency response of the feed network as seen in Fig. 3.13. One can potentially analyze the frequency behavior of Fig. 3.11, by finding the spectrum based on the currents at each frequency, as well as accounting for change in the apparent sizes compared to the wavelength, as the frequency varies (e.g. grating period). Such analysis can potentially be used to extract other information such as measuring an unknown grating period illuminated by the source. But here we are only interested in the operation point of 1GHz, at which all dimensions such as the array element spacing, grating period size and distance from source,
Figure 3.13: Measured through transmission $S_{11}$ between the feed common port and the terminating end of array elements (a) magnitude and (b) phase.

were designed to yield the prescribed spectrum conversion. Therefore we limit the discussion and verification to our frequency of interest, to reliably validate that the enhanced transmission is only due to the speculated evanescent-to-propagating wave conversion, and not due to other effects. It is worth noting however, that the radiation enhancement effect here is not due to
any resonances, and measurements also indicate that it does not appear to be a very narrow-band effect. This may be considered as an additional benefit of this radiation enhancement technique, enabling wide-band operation. For instance our proposed radiation enhancement may have applications in scenarios similar to the Purcell effect, but not requiring narrow-band high-Q resonant cavity effects.

3.7 Additional remarks

The radiation enhancement presented in theory and in experiments is made possible by a controllable and significant amount of evanescent-to-propagating wave conversion. Therefore through the experiment, we report the first explicit physical demonstration of controllable evanescent-to-propagating wave conversion at microwave frequencies. The experiment is carried out for the more challenging case of TE polarization, but our method is generally applicable to both polarizations. The TE case radiation enhancement results are even more remarkable, because a sub-wavelength metal grating is highly attenuating/opaque under TE polarization, thus typically causing 'radiation reduction' even in the presence of evanescent-to-propagating wave conversion. The results indicate that a non-resonant sub-wavelength metal grating (opaque screen) enhances the radiation of such invisible sources, rather than attenuating it, for both polarizations.

This overall ‘radiation enhancement’ of ‘invisible’ sources is shown in the absence of any resonances for both TM and TE polarizations. The unique combination of the specialized ‘invisible’ source next to the sub-wavelength metal grating of the appropriate period, can render the grating transparent and also achieve strong radiation through the grating in the far field. This is akin to extra-ordinary transmission (ET) through an opaque screen, except that it is observed for transmission of the fields from a close-by source, rather than for incident plane waves as done in previous ET work [89, 93]. The mechanism responsible for this enhanced transmission is not due to Surface Plasmon Polaritons (SPPs) [89] or Spoof Surface Plasmons (SSP) [93], but rather due to evanescent-to-propagating wave conversion. It also does not use or require the structure to support Fabry-Perot or waveguide mode type resonances in order to build up the incident evanescent waves, as was required in Ref. [70], nor does it use a negative index silver film as in Ref. [67] for growing the evanescent waves. Instead, we utilize a zero thickness sub-wavelength metal grating which does not support any resonant modes. Only by phase matching of the propagating and evanescent modes at the interface of the grating (spectrum conversion), the ‘invisible’ source is made to radiate strongly into the far-zone and become visible.

The radiation enhancement discussed here converts evanescent waves of the source into propagating waves using the metal strip grating. Thus the overall radiating mechanism is larger. It should be noted however that this radiation enhancement arrangement is not the same as a feed antenna shining a larger reflector/transmit-array. In a reflector situation, the large aperture
is in the far-zone of the feed, and thus the reflector/transmitter alters/tailors the “propagating” waves emanating from the source feed. In the radiation enhancement discussed here, the larger grating exclusively converts the “evanescent” waves of the source feed (the invisible array) into propagating waves. Thus the grating is placed in the near-field of the source feed. Moreover, in a reflector/transmitter, the numerical aperture of the radiator is increased in order to increase the directivity by collimating the waves. The radiation enhancement via spectrum conversion discussed here is used to “strengthen” the level of radiation of the invisible source, and not to increase the directivity (although the numerical aperture is also increased). This radiation enhancement mechanism can be used along with tailoring the converted propagating waves such that directivity is also increased.

3.8 Applications

The demonstrated concept of dramatic radiation enhancement of invisible/reactive sources via evanescent-to-propagating wave conversion, can lead to various novel applications such as detection, sensing, and measurement of sub-wavelength distances.

The weak field due to the designed array in free space may be undetectable or ‘invisible’ to a far-field observer in the presence of an ambient electromagnetic field/noise, depending on their relative strengths. By placing an appropriate grating in front of the source arrangement, a much stronger field reaches the observation point in the far-field, which may be designed to be higher than the noise floor, and hence the presence of the array is sensed or detected in the observation location.

When an object equipped with the appropriate grating passes over such an invisible array, the array radiates strongly, and this increased radiation may be readily sensed at some far observation point, indicating the presence of the object at that location. This scenario can have particular applications for traffic control (such as railways etc.) where the passing of vehicles over checkpoints needs to be detected. This can be achieved through the appropriate choice of the operating frequency and the invisible-array/grating combination. One benefit here is that power is radiated only during the short periods when the vehicle passes over the checkpoint and does not radiate power at all other times, suggesting a very energy efficient non-contact method. The passing of the vehicle may also be sensed locally at the checkpoint via measurement of the reflection coefficient seen at the input of the feed of the invisible-source/array as in Fig. 3.12. The feed/array design would be tailored to exhibit large swings in the reflection coefficient between the cases with and without grating, owing to the change in the overall radiation resistance of the array.

Another interesting application of the proposed arrangement of a specialized reactive source and a sub-wavelength grating is the remote accurate measurement of sub-wavelength distances. For the range of source-grating distances where evanescent-to-propagating wave conversion occurs, we note that at a far observation point from the invisible-array/grating arrangement,
the sensed field strength has a one-to-one mapping with the sub-wavelength distance between the grating and the invisible array. For the example presented, this one-to-one correspondence is shown in the full-wave simulation results in Fig. 3.14. The far-field is stronger the closer the grating is to the invisible array, because the various evanescent waves reaching the grating (with strength $e^{-|k_z|z_0}$, $k_z = \sqrt{k_0^2 - k_x^2}$) become stronger at shorter distances from the grating ($z_0$), and therefore convert to stronger propagating waves. Moreover, since $|k_z|$ is larger than $k_0$ for these evanescent waves, the conversion process is sensitive to sub-wavelength distances and magnifies small variations. Small changes in the distance, translate into large changes in the field strength which may be easily measured by a remote observer.

![Figure 3.14: Dependence of far-zone field strength on source-grating distance when evanescent-to-propagating wave conversion occurs.](image)

An additional application based on the observations made in this experiment is a method for measuring an unknown grating period. By shifting the source evanescent region in $k_x$ space and or changing the frequency, and sensing the far-field, one can find regions of spectrum which there is maximum radiation (i.e. the grating has converted to propagating waves), which can then be used to find the unknown grating wavenumber and period.

A final application noted here is for the invisible array itself, in the area of wireless power transfer. The design approach taken here to tailor the entire spectrum of the array, can be used to design efficient send/receive mechanisms in short distance wireless power transfer (near-field coupling) systems, utilizing electric and/or magnetic coupling approaches. The spectral array factor approach can be used to tailor the full spectrum of the field of the transmitter and the receiver. For instance, their field spectrum can be designed to have no propagating region (such that the system does not lose power through radiation), but has a strong invisible/reactive re-
region. Moreover, both arrays are designed to operate in the same certain region of the evanescent spectrum, such that their spectra is well matched and power couples efficiently from one array to the other.
Chapter 4

Radiation of Dipoles Close to Epsilon-Near-Zero Media

4.1 Introduction

Radiation of simple sources at or near the interface of two media (e.g. dielectric half-space) has been the subject of numerous studies over the past century. In this chapter, radiation of a dipole at or below the interface of an (an)isotropic Epsilon-Near-Zero (ENZ) media is investigated, akin to the classic problem of a dipole above a dielectric half-space. To this end, the radiation patterns of dipoles at the interface of air and a general anisotropic medium (or immersed inside the medium) are derived using the Lorentz reciprocity method. By using an ENZ half-space, air takes on the role of the denser medium. Thus, shaped radiation patterns are obtained in air, which were only previously attainable inside the dielectric half-space. Following the early work of R. E. Collin on anisotropic artificial dielectrics, practical anisotropic ENZs can be realized by simply stacking sub-wavelength periodic bi-layers of metal and dielectric at optical frequencies. It is shown that when such a realistic anisotropic ENZ has a low longitudinal permittivity, the desired shaped radiation patterns are achieved in air. In such cases the radiation is also much stronger in air than in the ENZ media, as air is the denser medium. Moreover, the subtle differences of the dipolar patterns when the anisotropic ENZ dispersion is either elliptic or hyperbolic are investigated.

4.2 Radiation of sources at or near interface of two half-space media

The radiation of antennas at the interface of media has been the subject of numerous studies to date [94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106]. The scenario of interest is a classic problem in electromagnetics dating back to the work of Sommerfeld in 1909, investigating the radiation of a source above a lossy half-space [94]. For instance in [105] the radiation of dipoles
placed on an air-dielectric interface was studied and it was found that the radiation mainly occurs inside the dielectric with interesting radiation pattern shapes. The radiation pattern in the air-side primarily had a single lobe, and more importantly, it was much weaker than the radiation in the dielectric, with an approximate power ratio of $1 : \varepsilon^{3/2}$ \cite{102}. Ref. \cite{96} studied the problem of a dipole at the interface of an anisotropic plasma interface. Ref. \cite{104} investigated the radiation patterns of both horizontal and vertical interfacial dipoles, deducing the location of the nulls and power ratios in either half-spaces. Other effects such as subsurface peaking was also explored by the same authors in \cite{107}. Such studies have been intended for various applications such as Ground Penetrating Radar \cite{108, 109}, antennas for communication above earth or under water \cite{95, 106}, antennas on semiconductors \cite{102} or above dielectrics for imaging \cite{105}, to only name a few.

Different techniques have been used thus far for analyzing this problem, mainly developing the Green’s function and using asymptotic approximations to find the far-field radiation patterns inside the air or the dielectric regions. A great body of literature to date has been dedicated to solving the Sommerfeld type integrals that arise in these problems, (e.g. see \cite{103} for a review of various works). The poor convergence of Sommerfeld type integrals has been an important reason for devising various efficient techniques for solving these types of problems as done in \cite{110} and using integral equations solved with the Method of Moments \cite{111, 112}, and exact solutions such as \cite{113}. Finite Difference Time Domain (FDTD) methods have also been used to analyze radiation patterns of such scenarios \cite{109, 114}, analyzing both the near-field \cite{114} and the far-field \cite{109, 114}, pointing out some ripple effects on the patterns obtained due to finite observation distances. Effects of lateral waves were also explored in works such as \cite{109, 115, 116}. Other time domain techniques have also been used for solving the problem of a source above a lossy half-space as in \cite{117}. Furthermore, a few studies have investigated radiation from anisotropic media \cite{96, 97, 118, 119} mainly through developing the Green’s function of their scenario of interest.

In almost all the work to date such as \cite{102, 104, 105}, the study has been on dipoles at the interfaces of dielectrics, which have permittivity greater than vacuum, $\varepsilon_r > 1$. In this thesis however, we aim to systematically study the radiation pattern of a dipole at the interface of an air-metamaterial (MTM), in which the metamaterial is a homogenized medium with an effective permittivity lower than free space, $\varepsilon < \varepsilon_0$. Such materials would be classified as Epsilon-Near-Zero (ENZ) media. One motivation here is to obtain the interesting dielectric-side radiation patterns of \cite{102, 104, 105} in air. The argument for using ENZ is simple. By using an ENZ instead of the dielectric, air plays the role of the dielectric in \cite{105}. Therefore those patterns obtained inside the dielectric in \cite{102, 104, 105}, should be attainable now in the air side, as air now acts as the higher permittivity medium compared to the MTM. Aside from the shape, the intensity of the radiation is also stronger in air, rather than in the ENZ. This is for instance very desirable in telecommunication applications. We further generalize the problem to that of a dipole above an ‘anisotropic medium’, with potentially low value(s) in the permittivity
Since most ENZ media are realized with layered or wire medium type structures, the resulting effective medium is inherently anisotropic and typically similar to a uniaxial crystal with a well defined optical axis.

A simple approach for determining the radiation pattern is using the Lorentz Reciprocity Theorem, which has usually been used for finding the radiation pattern of dipoles on isotropic dielectrics [105]. In this thesis we utilize the reciprocity method for systematically studying the dipole radiation above an anisotropic half-space, which is potentially an ENZ medium. We expand the theory to solve for dipoles immersed inside an ENZ medium. Realizations of ENZs are usually anisotropic, that is the near zero permittivity is achieved only along one axis, e.g. using layered media. Based on the pioneering work of R.E. Collin on artificial dielectrics [54, 4], the ENZs in this work are realized by interleaving layers of metal and dielectric with a sub-wavelength period. In Ref. [54] Collin showed that such a periodic structure can be homogenized into an effective medium with an anisotropic (uniaxial) permittivity tensor and derived simple expressions which have been rediscovered and used extensively to date. In this work, the ENZ realizations are tailored for optical frequencies where it can enable various applications for better light emission, such as shaping the radiation of optical antennas or enhancing the radiation of fluorescent molecules. Both elliptic and hyperbolic anisotropic ENZ media are considered and the subtle differences between the corresponding far-field patterns are highlighted.

A related scenario to our problem of interest is the work of [44], which utilizes a source immersed in a low permittivity MTM to achieve highly directive emission at microwaves. The structure was realized using a mesh grid, operating just above the plasma frequency resulting in \( 0 < \epsilon_r < 1 \). In [44] however the source was fully immersed in the MTM. The propagating waves from the source inside the ENZ reach the interface and refract close to normal in air due to Snell’s law. Therefore a highly directive beam is emitted in air. The work in [44] was demonstrated at microwaves, but such a scenario has potential at optical frequencies. Our work extends to the case of fully immersed sources such as the scenario in [44] and shows potential realizations at optical frequencies that enhances the radiation of the source in air. In this effort, the distinct difference between the patterns from interfacial and immersed dipoles is investigated.

4.3 Theory

Consider Fig. 4.1, where a dipole is radiating at the interface of air and a medium with an arbitrary permittivity tensor. We utilize the Lorentz reciprocity method as done in [105], showing that such analysis is applicable to general anisotropic media as well. According to the reciprocity method, in order to find the radiated field \( E_1 \) due to the dipole current \( I_1 \), one can find the field \( E_2 \) due to the far zone dipole current \( I_2 \). As long as the two currents are equal, so will be the tangential components of the field.
We are therefore solving the reciprocal problem, that is finding $E_2$ which is the total field at the interface when $I_2$ is radiating. The field from $I_2$ is a spherical wave of the form $e^{-jkr}/4\pi r$. For a source $I_2$ in the far-zone, the wave from $I_2$ incident on the interface can be approximated as a plane wave $E_i(\theta)$. As shown in Fig. 4.2 (a), under plane-wave illumination the field at the interface is equal to the field just below the interface. The total field just below the interface is equal to $\tau(\theta)E_i(\theta)$, i.e. the incident field multiplied by the transmission coefficient going from air to the MTM. The problem therefore reduces to finding the Fresnel transmission coefficient $\tau(\theta)$ of an obliquely incident plane wave at the interface of a general anisotropic medium, under both polarizations. This is arguably an easier problem to solve than other techniques, hence the reason the reciprocity method is a powerful and simple method especially for finding far-zone radiated fields. Once $E_2(\theta)$ is found from this reciprocal scenario, we have essentially determined the desired radiation pattern of $I_1$ in transmit mode. We only need to multiply by the free space angular pattern of the radiating element $I_1$ (if any) to find out the overall transmit mode pattern we originally desired.
4.3.1 Plane-wave incidence

The Fresnel reflection and transmission coefficients for an interface between air and a uniaxial crystal is found by enforcing the boundary condition for the continuity of the tangential fields at the interface. One can formulate the expressions based only on the incident angle from air, $\theta$. For example in the $x-z$ plane of incidence, the reflection coefficient for the TM (Transverse Magnetic) or $p$-polarization is

$$r_{TM} = \frac{\cos \theta - \sqrt{(\mu_r/\epsilon_{xx} - \sin^2 \theta/\epsilon_{zz}\epsilon_{xx})}}{\cos \theta + \sqrt{(\mu_r/\epsilon_{xx} - \sin^2 \theta/\epsilon_{zz}\epsilon_{xx})}}$$ (4.1)

and for the TE (Transverse Electric) or $s$-polarization is

$$r_{TE} = \frac{\cos \theta - \sqrt{\mu_r\epsilon_{yy} - \sin^2 \theta}}{\cos \theta + \sqrt{\mu_r\epsilon_{yy} - \sin^2 \theta}}$$ (4.2)

The expressions presented in this form are applicable to any half-space that is an anisotropic medium, with arbitrary permittivity along its different axes.

The main cases of interest in this work are anisotropic ENZs, i.e. media where the permittivity is close to zero, at least along one axis. For TE polarization, the out-of-plane permittivity is only relevant and can be close to zero. In TM polarization, two permittivity values (longitudinal and transverse) are relevant. Either of these two permittivity values can be close to zero, and either can be positive or negative, in general. Therefore there are eight dispersion cases for this polarization, with $\{\epsilon_{xx} \rightarrow 0^\pm, \epsilon_{zz} = \pm 1\}$ or $\{\epsilon_{xx} \rightarrow \pm 1, \epsilon_{zz} \rightarrow 0^\pm\}$. As will be explained later, in this work we are primarily interested in the scenarios with low longitudinal permittivity ($\epsilon_{zz} \rightarrow 0^\pm$). Three of these scenarios lead to propagation inside the ENZ which will be used here.

Fig. 4.3 (a) shows the magnitude of the TM reflection coefficient at the interface of an anisotropic ENZ with $\epsilon_{zz} = 0.1$, $\epsilon_{xx} = 1$ (black curve). It also compares it to the reflection from an isotropic ENZ with $\epsilon_x = 0.1$ (red curve). The figure shows that the reflection is lower in the anisotropic case, for all angles below $15^\circ$, compared to the isotropic ENZ of the same low permittivity. It also shows that the two media only accept plane waves that are incident up to a critical angle equal to $\sin^{-1}\sqrt{\epsilon_{zz}}$. Beyond the critical angle the reflection coefficient going from air (dense medium) to the ENZ is 1, as the incident wave experiences Total Internal Reflection (TIR) back into air. Hence for any angle beyond the critical angle up to $90^\circ$ the transmitted wave into the ENZ is an evanescent wave (only showing up to $30^\circ$ for illustration purposes). There is also no Brewster’s angle (angle at which reflection is zero) for the anisotropic case, while in the isotropic ENZ case there exists a zero reflection angle of incidence close to the critical angle under the TM ($p$-polarization). For comparison, reflection from a typical dielectric such as glass (blue curve) is also shown. It can be seen that the reflection from glass is higher from the anisotropic case at normal incidence, up to $13^\circ$ off-normal, after which the reflection from
Figure 4.3: Magnitude of the TM reflection coefficient at the interface of an anisotropic ENZ and air, for wave incident from air (black curve), using an (a) anisotropic ENZ with $\epsilon_{xx} = 1$, $\epsilon_{zz} = +0.1$ (b) anisotropic ENZ with $\epsilon_{xx} = 1$, $\epsilon_{zz} = -0.1$ (c) ENZ with $\epsilon_{xx} = -1$, $\epsilon_{zz} = 0.1$. In each plot, the reflection is also compared with a corresponding isotropic ENZ case of $\epsilon_{xx} = \epsilon_{zz}$ (red curve), as well as that of glass (blue curve).

glass is lower.

Fig. 4.4 (a) shows the corresponding iso-frequency contour and refraction at the interface of air and the anisotropic ENZ with \{\(\epsilon_{xx} = 1, \epsilon_{zz} = 0.1\}\}. Such a medium has an elliptic iso-frequency curve as shown in the figure. An incident wave from air, phase matches at the interface to another wave with equal lateral wave-number $k_x$ in the ENZ. The wave-vector in the ENZ is the vector joining the origin to the corresponding point on the elliptical iso-frequency contour. The direction of power flow (Poynting vector) is normal to the iso-frequency contour at any given point as indicated in Fig. 4.4 (a). Although the power flows in the direction of the wave-vector (and hence phase velocity) in air, the power flow inside the ENZ is at an angle with respect to the phase velocity due to the anisotropy.
If the signs of the two in-plane permittivity values agree, the iso-frequency contour is elliptic and if they are of opposite signs, the iso-frequency contour is hyperbolic [56] giving rise to a hyperbolic metamaterial (as in the hyperlens [57, 58, 59]). Two additional hyperbolic cases with low longitudinal permittivity \( \{\epsilon_{xx} \to \pm 1, \epsilon_{zz} \to 0^\mp\} \) are shown in Fig. 4.4 (b) and (c), showing refraction in each case. In the case of Fig. 4.4 (b), \( \{\epsilon_{xx} = +1, \epsilon_{zz} = -0.1\} \), the medium is an indefinite medium. The magnitude of the reflection coefficient from this medium as a function of the incident angle is shown in Fig. 4.3 (b). We see that there is no critical angle in this case for all incident angles from air such that the wave never experiences TIR back into air. The amount of reflection coefficient increases gradually with the incident angle in an almost linear trend. Reflection from a typical dielectric such as glass (blue curve) is also shown which has a Brewster’s angle at 56.3° in this polarization. It can be seen that the reflection from glass is higher from this hyperbolic anisotropic case at normal incidence, up to 17° off-normal, after which the reflection from glass is lower.

In the case of Fig. 4.4 (c), \( \{\epsilon_{xx} = -1, \epsilon_{zz} = +0.1\} \), we have another hyperbolic ENZ with low longitudinal permittivity where the medium acts somewhat strangely in terms of Total Internal Reflection. What is surprising in this case is that for all incident angles from broadside up to a critical angle \( \theta_c' = \sin^{-1}\sqrt{\epsilon_{zz}} \), the wave experiences TIR back into air because there is no allowed propagation in the ENZ. For all incident angles beyond that critical angle the wave phase matches to a propagating wave in the ENZ. This type of operation is quite the opposite of typical TIR in dielectrics (and even the elliptic ENZ), where in fact the TIR occurs for angles beyond the critical angle. This is also evident when inspecting the reflection coefficient in Fig. 4.3 (c). We see that the reflection magnitude for this ENZ (black curve) is 1 for all angles up to \( \theta_c' \) (i.e. there is TIR back into air for angles close to broadside), while there is transmission into the ENZ for all angles above the critical angle with gradual increase in the reflection coefficient magnitude.

### 4.3.2 Interfacial dipoles

Using the expressions obtained thus far we can find the radiation patterns of a horizontal dipole placed at the interface of the two media. Depending on the orientation of the dipole relative to the anisotropic MTM, different polarization planes are realized. Here we are primarily interested in the principal planes which are the planes containing the principal axes of the MTM. Moreover, we are interested in the cases where the dipole is oriented along one of these major axes. Fig. 4.5 shows the four primary polarization planes for the horizontal dipole above a fixed anisotropic MTM.

Given the chosen geometry of Fig. 4.1 and the previously discussed reciprocity method, the radiation pattern for an interfacial \( x \)-directed dipole in the \( x-z \) plane is found to be

\[
S_{E\text{-plane}}(\theta) = \left[ \frac{\cos \theta \sqrt{\mu_r - \sin^2 \theta / \epsilon_{zz}}}{\cos \theta \sqrt{\epsilon_{xx} + \sqrt{\mu_r - \sin^2 \theta / \epsilon_{zz}}} \times \left[\frac{\cos \theta \sqrt{\mu_r - \sin^2 \theta / \epsilon_{zz}}}{\cos \theta \sqrt{\epsilon_{xx} + \sqrt{\mu_r - \sin^2 \theta / \epsilon_{zz}}} \right]^2} \right.
\]

(4.3)
Figure 4.4: Iso-frequency contour showing wave-vectors and Poynting vector for the interface of air and half-space ENZ with (a) elliptic \( \epsilon_{xx} = +1, \epsilon_{xx} = +0.1 \) (b) hyperbolic \( \epsilon_{xx} = +1, \epsilon_{xx} = -0.1 \) and (c) hyperbolic \( \epsilon_{xx} = -1, \epsilon_{xx} = +0.1 \) characteristic.

for the E-plane (E field in the \( x - z \) plane). The additional \( \cos \theta \) term in the numerator is due to the element pattern of a horizontal dipole in free space, in this plane. The H-plane of such scenario is the \( y - z \) plane and the pattern is:

\[
S_{H-plane}(\theta) = \left[ \frac{\mu_r \cos \theta}{\mu_r \cos \theta + \sqrt{\mu_r \epsilon_{xx} - \sin^2 \theta}} \right]^2. \tag{4.4}
\]

For a \( y \)--directed dipole (i.e. current out of \( x - z \) plane), the \( x - z \) plane is the H-plane and the radiation pattern (\( E_y \) only) is

\[
S_{H-plane}(\theta) = \left[ \frac{\mu_r \cos \theta}{\mu_r \cos \theta + \sqrt{\mu_r \epsilon_{yy} - \sin^2 \theta}} \right]^2. \tag{4.5}
\]

whereas the \( y - z \) plane is the E-plane and the radiation pattern is

\[
S_{E-plane}(\theta) = \left[ \frac{\cos \theta \sqrt{\mu_r - \sin^2 \theta / \epsilon_{yy}}}{\cos \theta \sqrt{\epsilon_{yy} + \sqrt{\mu_r - \sin^2 \theta / \epsilon_{yy}}}} \right]^2. \tag{4.6}
\]

This formulation now allows for the relative permittivity \( \epsilon_{xx}, \epsilon_{yy} \) or \( \epsilon_{zz} \) to be of different values and potentially less than 1. A similar analysis may be applied to media with an anisotropic permeability tensor.

The theory assumes that the homogeneous MTM medium is an infinite half space with no bounds and reflections. Such a scenario may be attainable in practice by using a large enough
4.3.3 Immersed dipole in an ENZ

We can also extend the theory to account for the source buried below the interface of the MTM at a distance $z_0$ below the surface. Revisiting the reciprocity solution, in the reciprocal problem, the transmitted wave $\tau(\theta)E_i(\theta)$ now travels an extra longitudinal distance of $z_0$ before reaching the source plane, as depicted in Fig. 4.2 (b). Hence this wave needs to be multiplied by an $e^{-jk_zz_0}$ propagation factor. Moreover, this propagation occurs inside the anisotropic MTM. For instance in the $x−z$ plane, the propagation inside an anisotropic crystal for the TM case is described by

$$\frac{k_z^2}{\epsilon_{zz}} + \frac{k_z^2}{\epsilon_{xx}} = k_0^2$$

(4.7)

and for the TE case it is governed by

$$k_z^2 + k_z^2 = \epsilon_{yy}k_0^2$$

(4.8)

The transmitted wave just below the interface phase matches such that it has a transverse wave-number component equal to that of the incident wave, $k_z = k_0\sin\theta$. Therefore the longitudinal component of the wavenumber is found from (4.7) or (4.8). The overall pattern of the immersed dipole can be therefore approximated as

$$S_{E−plane}(\theta)|_{z_0} = e^{-jk_zz_0}S_{E−plane}(\theta)|_0$$

(4.9)

in the $x−z$ plane for the $x$-directed dipole and

$$S_{H−plane}(\theta)|_{z_0} = e^{-jk_zz_0}S_{H−plane}(\theta)|_0$$

(4.10)
in the $x-z$ plane for the $y$-directed dipole.

A note of interest is that this additional exponential phase term can become an attenuation factor. In fact, for all angles of incidence from air above the critical angle, the wave phase matches to an evanescent wave in the ENZ (due to TIR in air) which is characterized by an exponentially decaying factor, $e^{-|k_z|z_0}$. As we shall see, in the transmit mode where the dipole is radiating from within the ENZ, a sufficiently distant source from the interface can lead to directive single lobe radiation explaining the observations reported in [44].

4.4 Radiation Patterns

4.4.1 Dipole on an isotropic ENZ

Using the derived radiation patterns for general anisotropic half-space, we can inspect radiation from both isotropic and anisotropic ENZ half-spaces. As a first example we inspect the radiation pattern of a dipole at the interface of an isotropic ENZ as shown in Fig. 4.6. The ENZ is chosen to have a relative permittivity of $\epsilon_{xx} = \epsilon_{yy} = \epsilon_{zz} = 0.1$. It can be seen that these radiation patterns closely resemble the radiation patterns that are typically attained inside dielectrics reported in various works such as [102, 104, 105, 106]. However, these radiation patterns behave oppositely to the dielectric half-space scenario, as air is now the denser medium compared to the ENZ. This means that a critical angle occurs in air relative to the ENZ. We are primarily interested in the radiation pattern in the air-side.

The E-plane pattern has three lobes, a broadside lobe and two side-lobes beyond the critical angle of air. Another important consequence of air being the denser medium is that the amount of radiated power is also much stronger in the air-side. This is the reverse of the case of the dielectric half-space, where most power radiates into the dielectric side. The H-plane pattern also exhibits the “pointed ears” radiation patterns that are typically attained in the H-plane pattern of the dielectric half-space (e.g. see [102, 104, 105, 106]). The radiation in the ENZ side is significantly weaker than in air as seen in the H-plane pattern.

4.4.2 Interfacial dipole on an anisotropic ENZ

As stated earlier, ENZs are usually anisotropic in practice. The derived radiation patterns can handle such anisotropy for different orientations of the dipole.

In H-plane, only the out-of-plane permittivity is relevant. Fig. 4.7 shows the H-plane radiation patterns for four values of $0 < \epsilon_{yy} < 1$, for a $y$-directed dipole. It can be seen that the angles at which the two peaks occur in the pattern (which is determined by the critical angle in air), separate further for larger permittivity values.

In the E-plane the anisotropy affects the patterns with two permittivity values (transverse and longitudinal), as apparent in the pattern expressions (4.3) and (4.6). The four cases of low transverse permittivity, i.e. $\{\epsilon_{xx} \to \pm 0$ and $\epsilon_{zz} = \pm 1\}$, do not lead to similarly shaped E-plane radiation patterns but they rather yield a single lobe. The requirement is then to use an ENZ
with a low longitudinal permittivity, in order to achieve the dielectric-side E-plane radiation patterns of [102, 104, 105, 106] (three lobes), in air. Therefore for the scope of this thesis, we primarily investigate low transverse permittivity cases.

Fig. 4.8 (solid blue curves) shows the E-plane radiation patterns for four values of the longitudinal permittivity $0 < \epsilon_{zz} < 1$, while the transverse permittivity is $\epsilon_{xx} = \epsilon_{yy} = 1$, for a $x-$directed dipole. Each plot also contains a second trace (dashed red) showing the E-plane radiation pattern if the ENZ were isotropic with the corresponding relative permittivity $\epsilon_r = \epsilon_{zz}$.

We can see from these results that similarly shaped radiation patterns can be attained in air with the anisotropic case, despite the large anisotropy between the transverse and longitudinal components. This is particularly applicable to practical ENZ scenarios that exhibit large anisotropy, as shown later.

The E-plane radiation patterns for the hyperbolic ENZ case of Fig. 4.4 (b) are shown in Fig. 4.9, for varying values of $-1 < \epsilon_{zz} < 0$, while $\epsilon_{xx} = 1$. The patterns in this case do not have three lobes and nulls as in Fig. 4.8, rather two merged lobes (without separating nulls) exist in the pattern and there is no main broadside lobe. The lack of nulls (and hence lack of distinct lobes) is due to the absence of a critical angle and no TIR into air for this type of hyperbolic ENZ (in order for the incident and reflected waves to cancel at the interface in the reciprocal problem). The two lobes merge further together into a single lobe as $\epsilon_{zz} \to -1$.

The E-plane radiation patterns for the hyperbolic ENZ case of Fig. 4.4 (c) are shown in Fig. 4.10, for varying values of $0 < \epsilon_{zz} < 1$, while $\epsilon_{xx} = -1$. A narrow broadside lobe and two prominent side-lobes exist in the pattern for $\epsilon_{zz} = 0.01$ of Fig. 4.10 (a), with distinct separating nulls due to TIR. The two side-lobes reduce in strength relative to the main lobe as $\epsilon_{zz} \to +1$, and the broadside lobe becomes dominant. The two side-lobes diminish more abruptly in this case compared to the corresponding isotropic ENZ case of $\epsilon_r = \epsilon_{zz}$ (dashed red curve).
4.4.3 Dipole immersed in an anisotropic ENZ

Fig. 4.11 shows the evolution of the radiation pattern in air as an immersed dipole is moved towards the interface (in the case of an elliptical ENZ). Fig. 4.11 (a) shows the H-plane pattern of an in-plane dipole, while Fig. 4.11 (b) shows the case of E-plane pattern for an out of plane dipole. For the H-plane pattern, it can be seen that when the dipole is fully immersed, a single directive lobe is primarily noticeable in the radiation pattern. As the dipole is moved closer to the interface, the two side-lobes start to emerge. In the case of an interfacial dipole the pattern has two prominent side-lobes and a smaller broadside lobe. The emergence of the side-lobes for interfacial dipoles, as well as those close to the interface, can be attributed to the evanescent near-field waves of the dipole. In such cases, the evanescent waves of the dipole can couple to propagating waves beyond the critical angle of air. This is while such evanescent waves become significantly attenuated for larger distances from the interface. Hence in the case of $z_0 = -\lambda_0/2$, there is almost no radiation beyond the critical angle of air with no noticeable side lobes. A similar scenario exists for the E-plane pattern. The pattern for the case of $z_0 = -\lambda_0/2$ has little radiation strength beyond the critical angle with an almost flat-top type radiation pattern. The pattern widens as the dipole moves closer to the interface, such that the familiar “pointed ears” radiation patterns of the interfacial case develops. The cases of $z_0 = -\lambda_0/2$ for the two planes essentially recover and explain the scenario of directive emission using ENZs as proposed by [44]. Only the propagating waves of the immersed source reach the surface, all refracting close to normal due to the Snell’s law, resulting in a directive broadside lobe.

The two figures also show a secondary dashed curve which is the result of full-wave simu-
Figure 4.8: E-plane radiation pattern in air for a dipole above an anisotropic ENZ (solid blue curve) with $\epsilon_{xx} = \epsilon_{yy} = 1$, (a) $\epsilon_{zz} = 0.01$ (b) $\epsilon_{zz} = 0.2$ (c) $\epsilon_{zz} = 0.5$ (d) $\epsilon_{zz} = 0.7$, and (dashed red curve) for the corresponding isotropic ENZ $\epsilon_r = \epsilon_{zz}$.

The expressions and pattern results presented so far are applicable to general anisotropic media using only the permittivity tensor, independent of the realization of the MTM. Depending on...
the realization of the MTM, the permittivity tensor may be effectively related to the geometry and material parameters of the underlying unit cells of the actual MTM. The interest in this work has primarily been on ENZs, which may be realized with unit cells such as bi-layers, mesh grids, or wire media depending on the frequency of operation, typically showing some sort of anisotropy. Here we utilize the bi-layer media at optical frequencies.

R. E. Collin showed in [54] that periodic sub-wavelength layers of two dielectrics can be effectively homogenized as one dielectric with an anisotropic uniaxial permittivity tensor, in the investigations related to artificial dielectrics [54, 4]. Such artificial dielectrics were the fore-runners to what is now known as a type of metamaterials. To date this stacked bi-layer medium has been used in many studies, especially at optical frequencies in the past decade [57, 58, 59, 55, 38, 120, 121, 122, 123, 124, 125], primarily due to their simple nature and ease of fabrication. The bi-layer concept has been particularly useful for the realization of Epsilon-Near-Zero MTMs [38] where the desired “close to zero permittivity” is typically achieved by interleaving sub-wavelength layers of a material with positive permittivity and a material with negative permittivity, such that the effective medium permittivity is zero. This was the key for realizing the highly anisotropic ENZs of the hyperlens [57, 58, 59], where the effective medium has a hyperbolic dispersion characteristic. Ref. [126] utilized such layered MTMs to propose extreme boundary conditions such as perfect electric and magnetic conductors at optical frequencies and determined the radiation pattern of a dipole near a layered structure.
Figure 4.10: E-plane radiation pattern in air for a dipole at the interface of an anisotropic ENZ (solid blue curve) and air, for an anisotropic ENZ with $\epsilon_{xx} = -1$, (a) $\epsilon_{zz} = 0.01$ (b) $\epsilon_{zz} = 0.2$ (c) $\epsilon_{zz} = 0.5$ (d) $\epsilon_{zz} = 0.7$, and (dashed red curve) for the corresponding isotropic ENZ $\epsilon_r = \epsilon_{zz}$.

that is operating in the theoretical extreme limit $\{\epsilon_{\text{tangential}} \to \infty, \epsilon_{\text{normal}} \to 0\}$.

The desired anisotropic low permittivity MTMs in this work can also be realized with sub-wavelength layers of a metal (negative real permittivity) and a dielectric at the optical frequency of interest. Two orientations of the layered structure are possible as shown in Fig. 4.12. Primarily, horizontal stacks as in Fig. 4.12 (b) have been used in the past [57, 58, 59] as transverse zero permittivity was required. Here we utilize vertical layers as shown in Fig. 4.12 (a), as they offer an advantageous capability for our purposes of realizing close to zero longitudinal permittivity and the air-interface refractions shown in Fig. 4.4. The effective permittivity along the two principal axes of such the structure in Fig. 4.12 (a) can be found using the following first order effective medium theory (EMT) formulas [54]:

\[
\begin{align*}
\epsilon_{xx} &= \epsilon_{\text{opt. axis}} = \frac{\epsilon_m \epsilon_d}{(1 - p)\epsilon_m + (p)\epsilon_d} \\
\epsilon_{zz} &= \epsilon_{\perp\text{opt. axis}} = p\epsilon_m + (1 - p)\epsilon_d
\end{align*}
\]  

(4.11)  

(4.12)

where $\epsilon_m$ and $\epsilon_d$ are the permittivity values of the metal and dielectric layers respectively and 'p' is the filling ratio of the metal layer (thickness of the metal layer divided by the sum of the thickness of metal and dielectric layers). A second order effective medium approximation was also presented in [54], however we use the first order formulas to obtain initial values. The
Figure 4.11: Effect of interfacial versus immersed source in an ENZ MTM (a) E-plane pattern of an in-plane dipole (b) H-plane pattern of an out-of-plane dipole. Each graph shows the theoretical far-field pattern using the presented theory (solid curves) and the pattern from full-wave simulation (dashed curves) at a finite observation distance.

period used here is deeply sub-wavelength ($L = \lambda_0/22$) and therefore the second order effects were found not to cause noticeable difference in the effective permittivity values.

We utilize the case of vertical layers to realize zero longitudinal ($z-$axis) permittivity, mainly due to the fact that close to zero response can be easily achieved along the direction normal to the optical axis with readily available optical materials. A trend of the variation of the complex permittivity as a function of the filling ratio of the metal is shown in Fig. 4.13, utilizing the first order EMT formulas as in [54]. The operation region of interest has been magnified in Fig. 4.13 (b). The variation is dependent on the choice of the two materials. Here we have used Ag ($\epsilon_m = -2.4012 + 0.2488j$) [59] and PMMA (Polymethyl methacrylate - $\epsilon_d = 2.301$) at $\lambda_0 = 365nm$. 
Chapter 4. Radiation of Dipoles Close to Epsilon-Near-Zero Media

Figure 4.12: TM light incident on stacked periodic layers realizing an anisotropic medium with two different principal axes having its optical axis oriented along (a) $x$–axis and (b) $z$–axis.

One drawback with using the EMT formulas of [54] is the lack of incorporating additional modes which can arise from rapid field variations compared to the scale of the layers, especially in the optical regime where the metal layer has negative permittivity [120, 121, 122, 123, 124, 125]. For example, surface plasmon polaritons can exist when a metal layer with negative permittivity is next to a dielectric [121], creating a non-local response. Such non-local effects are known to give rise to spatial dispersion and have sparked research into providing corrections to the simple EMT model [120, 121, 122, 123, 125]. Depending on how sub-wavelength the period is, especially relative to the plasma wavelength, the effective permittivity values obtained from EMT can be significantly different and also dependent on the direction of propagation. This can be better seen by referring to the accurate dispersion equations governing the layered media obtained using the transfer-matrix method for photonic crystals (e.g. see [120, 121, 125]). However, the EMT formulas hold for a variety of angles when the period is sufficiently sub-wavelength (especially when well below the plasma wavelength of the metal layer) and are a good approximate first step when designing layered structures. A benefit of the general pattern expressions presented earlier is that they can be used along with more refined effective permittivity expressions that incorporate nonlocal effects such as [121, 122, 123, 125] to arrive at more accurate radiation pattern expressions, if required.

4.5.1 Radiation from a finite slab

The bi-layer structure of Ag and PMMA is tailored at $\lambda_0 = 365\,nm$ with a filling ratio of $p = 0.43$, and period $L = \lambda_0/22$, which leads to an effective permittivity of $\epsilon_{xx} = 13.28 + 3.46j$ and $\epsilon_{yy} = \epsilon_{zz} = 0.28 + 0.11j$ using the EMT expressions. Fig. 4.14 and 4.15 show the radiation of a horizontal dipole placed at the interface of a finite slab made of such layered structure.
Figure 4.13: (a) Variation of complex permittivity as a function of metal filling ratio “p” according to EMT expression, using bi-layers of Ag and PMMA at $\lambda_0 = 365nm$. (b) Operation region of interest yielding close to zero anisotropic permittivity with low losses.

2D full-wave simulation results presented here are for a 'finite' slab ($6\lambda_0 \times 6\lambda_0 \times 2\lambda_0$) as it is a more practical case to both simulate and potentially fabricate instead of the infinite half-space case.

The inset in Fig. 4.14 (a) shows the vertical stacked layers as well as the orientation of the dipole normal to the layers at the interface. The far-field radiation pattern in Fig. 4.14 (b) is an E-plane pattern. Three radiation patterns are shown in the figure. The blue curve marks the theoretical patterns for the dipole at the infinite half-space using the theory presented earlier, with the permittivity values obtained from the EMT expressions. The black curve shows the radiation pattern of the actual finite slab made of layers of Ag and PMMA obtained from full-wave simulations using the COMSOL 4.3a software package. The dashed red curve shows the pattern from full-wave simulation of a finite slab of the same size, filled with a homogeneous material having permittivity values from the EMT expressions. For the latter two cases, the dipole is placed at a slight distance above the interface in air ($z_0 = 0.009\lambda_0$) due to numerical issues with simulating a fully interfacial case. The discrepancy between the black and red curves is most likely due to a combination of simulation inaccuracies and non-local/spatial dispersion effects. Inaccuracies in simulation of the layered slab (black curve) arise from simulating a large domain with extremely fine plasmonic features, which poses particular challenges as also reported in [121], requiring dense meshing of a large domain. Aside from simulation inaccuracies, a contribution to this discrepancy may be due to non-local effects such that the EMT homogenized slab does not fully capture the complete behavior of the actual layered slab. It should be noted that the slab is illuminated with an adjacent source that has a wide range of spatial frequency components, including propagating and evanescent waves, exemplifying the potential influence of non-local effects and spatial dispersion (e.g. the excitation of TM SPPs).
Figure 4.14: (a) Power density color map of an interfacial horizontal dipole on a slab made of stacked bi-layer of Ag and PMMA with $p = 0.43$ at $\lambda_0 = 365\,nm$. Inset shows the layered slab, dipole orientation normal to the layers, and pattern plane (b) E-plane radiation pattern for three cases: infinite half-space from theory (blue curve), an EMT homogenized finite slab (red curve) from full-wave simulations, and the vertically layered finite slab (black curve) from full-wave simulations. Blue and red curves result from using an MTM with $\epsilon_{xx} = 13.28 + 3.46j$ and $\epsilon_{zz} = 0.28 + 0.11j$.

Fig. 4.15 shows the scenario of the interfacial dipole parallel to the layers as shown in the inset of power intensity plot of Fig. 4.15 (a). Three H-plane radiation patterns are now presented
in Fig. 4.15 (b). The patterns resemble the H-plane patterns of dielectric with two pointed peaks, with some rounding of the peaks due to losses. This time we see better agreement between the three cases, which shows that the EMT expressions provide a more reliable description of the behavior of the layered slab in the TE polarization than the TM polarization where nonlocal effects are stronger.

The above analysis treats a single frequency operation where the effective permittivity of the slab made of layers, closely resembles the assumed permittivity values of the homogenous slab. As the frequency of operation varies, the effective permittivity of the layered slab will change, particularly due to the dispersion of the plasmonic metal layer (silver). Thus, a certain shaped radiation pattern of interest may be achieved over a narrow range of frequencies, corresponding to the frequency bandwidth over which the required permittivity values are achieved. In other words, the bandwidth of operation will be affected by the bandwidth of the low longitudinal permittivity, as well as other band-limiting factors such as that of the source itself. In this work however, we show that the shaped radiation patterns of interest are indeed achieved at least at one frequency. The investigation of the frequency response of the layered slab, and its consequences on the air-side radiation patterns are left for future work.

From these results, it can be seen that shaped radiation patterns can be achieved in air, by placing the radiator on top of a finite slab of an ENZ realized using the stacked bi-layer structure of [54] at optical frequencies. Such a scenario can enhance the radiated power and tailor the radiation pattern of an optical radiator, e.g. an optical antenna or a florescent molecule for better radiation into far-zone in air. Realizations of the ENZ concept using wire media and mesh grids [44] are also possible for microwave applications.

The studied scenario of a dipole above an ENZ half-space, and the previously known scenario of a dipole close to an artificial magnetic conductor (AMC), can both increase the air-side (dipole side) radiation strength. The two scenarios are not the same however by virtue of field considerations as well as the process of shaping of the air pattern and polarization considerations, as discussed for the ENZ. It is important to note however that up to now, AMC boundaries have only been attainable with periodic structures at microwaves, e.g. using mushroom type structures, for increasing the radiation of a radiator with a substrate backing. Thus, the presented scenario of a dipole above a layered ENZ slab, opens the doors to such radiation increase for higher frequencies especially at optical frequencies, which was not previously attainable. It does so by additionally providing mechanisms for shaping of the pattern for different polarizations.

4.6 Chapter Conclusions

The radiation of a source at the interface of, or immersed in an anisotropic Epsilon-Near-Zero (ENZ) Metamaterial is systematically studied. To this end, the radiation patterns of a dipole at or below the interface of air and a general anisotropic MTM half-space are derived using
Figure 4.15: (a) Power density color map of an interfacial horizontal dipole on a slab made of stacked bi-layer of Ag and PMMA with $p = 0.43$ at $\lambda_0 = 365\, \text{nm}$. Inset shows the layered slab, dipole orientation parallel to the layers, and pattern plane (b) H-plane radiation pattern for three cases: infinite half-space from theory (blue curve), an EMT homogenized finite slab (red curve) from full-wave simulations, and the vertically layered finite slab (black curve) from full-wave simulations. Blue and red curves result from using an MTM with $\epsilon_{yy} = 0.28 + 0.11j$.

The Lorentz Reciprocity method. It is observed that shaped radiation patterns, which were previously only attained inside dielectrics of high permittivity, are achieved in air by using an ENZ half-space. The intensity of radiation is also much stronger in the air-side, due to role
reversal of air as the denser medium. Isotropic ENZs as well as anisotropic ENZs with low longitudinal permittivity were studied for their effect on the radiation pattern in the relevant polarization planes.

In the H-plane, two pointed peaks were observed in the air-side radiation pattern, similar to those obtained in the H-plane patterns in dielectrics. In the E-plane, a dipole on either an isotropic ENZ, an anisotropic elliptic ENZ, or an anisotropic hyperbolic ENZ has two clear nulls in the air-side radiation pattern, as long as the ENZ has low and positive longitudinal permittivity. The nulls give rise to a broadside lobe and two side-lobes in air, resembling the E-plane radiation patterns in dielectrics. These pattern features were explained via the reciprocal problem and studying the iso-frequency contours and reflection properties of the ENZ interface under plane-wave incidence. It was seen that as long as there is positive low longitudinal permittivity (e.g. Fig. 4.4 (a) and Fig. 4.4 (c)), a critical angle exists in air such that Total Internal Reflection (TIR) occurs back into air for some range of incident angles. The incident field and the totally reflected field cancel out at the interface for an incident angle corresponding to the angle of the null in the pattern of the dipole. The elliptic ENZ with low positive longitudinal permittivity (Fig. 4.4 (a)) exhibits TIR back into air above a critical angle. The hyperbolic ENZ with low positive longitudinal permittivity (Fig. 4.4 (c)) showed a peculiar case of TIR below the critical angle, which is completely opposite to that of regular dielectrics. It was also shown that a dipole on a hyperbolic ENZ with low and negative longitudinal permittivity (Fig. 4.4 (b)) has no nulls in the E-plane and only two merged side-lobes. This is because with such an ENZ there is no critical angle in air and TIR does not occur back into air for any incident angle.

The effect of varying the permittivity was shown to affect the critical angle and therefore the patterns in both planes. The effect of immersing the source inside the ENZ was also shown to increase the directivity of the radiation and dampening of off-broadside radiation, both in the H-plane as well as in the E-plane for the isotropic and elliptic ENZ with low longitudinal permittivity. This was due to the occurrence of TIR for all angles above the critical angle in the reciprocal problem, attenuating the waves that reach the source plane in the ENZ.

Following the pioneering work of R. E. Collin, sub-wavelength periodic alternating layers of metals and dielectrics were used for the realization of an anisotropic elliptic ENZ at optical frequencies. It was observed that radiation patterns from the interface of a finite slab of vertically layered medium with air, provides similarly shaped radiation patterns previously only attainable in dielectrics. The presented scenarios have applications in enhancing and shaping the radiation patterns of optical radiators such as optical antennas and fluorescent molecules.
Chapter 5

Light Concentration Using Hetero-junctions of Anisotropic Low-Permittivity Metamaterials

5.1 Introduction

In many electromagnetic and optical applications, it is desired to collect light incident over a wide area and focus/concentrate it into a smaller area. Although the most notable application is imaging, other applications include non-imaging and illumination optics [127], concentrated photovoltaics (CPV) [127, 128, 129], and electromagnetic (EM) concentrators [130, 131, 132, 133] to name a few. In all these scenarios, the collected electromagnetic waves are refracted and guided to create an area of high power concentration. Various solutions have been proposed so far, such as using traditional lenses [127, 128], Fresnel lenses [127, 129] and Transformation Optics (TO) devices [16, 130, 131] depending on the concentration application.

Traditional lenses formed from dielectrics (e.g. glass), refract light at their spherical or parabolic surfaces to focus a normally incident plane wave at their focal point [6]. Lenses however have various limitations and constraints. For example ideal lens performance is hindered by the diffraction-limit, and spherical aberrations and coma cause spreading of the focal point under plane-wave illumination. Modern developments in electromagnetics and optics have helped to overcome some of the limitations of lenses, such as metamaterials [15, 134, 135, 136, 137], transformation optics [136], metasurfaces [138, 139] and Graded-Index Lenses [6, 137], to name a few. Graded-Index lenses (GRIN) can achieve focusing without curvature and with a fixed height, creating appropriate phase delays for different regions of the incident wavefront using a gradual variation of the lateral refractive index of the lens [6, 137]. Metamaterial based lenses can capture/enhance/recover the evanescent waves near a source object, and thus enable imaging below the diffraction limit, most notably done in the superlens, hyperlens, and the far-field optical superlens [15, 134]. Focusing via prescribed flow of light [136], as well as using only
a surface [138, 139] has also been demonstrated. Another limitation of lenses is in achieving short focal distances, typically requiring very high curvature, thickness, and/or permittivity values. A potential solution for light concentration at short distances is using the growing field of metamaterials, in particular Epsilon-Near-Zero MTMs and zero-index media.

In this chapter, we propose a flat low-profile device based on a hetero-junction of anisotropic ENZ metamaterials, that leads to focusing and power concentration of the incoming light. The anisotropic MTMs have low longitudinal permittivity and are matched in the transverse direction for minimal reflections. The proposed device is low profile and flat and concentrates the field much like a lens does into a spot. In contrast to other designs, where the field intensity is increased inside a transformed medium [130, 131, 132, 133], this design focuses the power into a region outside in the host medium (e.g. air). It is shown that such a device can in fact outperform thick lenses with very low \( f/D \) ratios, to create areas of high power concentration very close to the exit facet of the device, i.e. at short focal distances. It also outperforms the recently reported flat and thin lenses realized using arrays of optical antennas [138, 139] in terms of their \( f/D \) ratios. The proposed focusing hetero-junction device is an alternative method for extremely close power concentration under normal incidence.

It is demonstrated how to logically construct such a focusing device out of ENZ hetero-junctions, by exploring the unique refraction properties of anisotropic ENZs based on ray optics and corresponding dispersion diagram analysis. Unlike Transformation Optics [16] which typically results in very complex media that are hard to realize in practice, this approach yields simple uniaxial structures that are easier to realize in the optical regime.

### 5.2 Theory

Our uniaxial MTM media of interest are assumed to be non-magnetic with \( \mu_r=1 \) and have a permittivity tensor

\[
\epsilon = \begin{pmatrix}
\epsilon_{xx} & 0 & 0 \\
0 & \epsilon_{yy} & 0 \\
0 & 0 & \epsilon_{zz}
\end{pmatrix} \epsilon_0
\]  

(5.1)

For an anisotropic uniaxial medium \( (\epsilon_{yy} = \{\epsilon_{xx} \text{ or } \epsilon_{zz}\}, \epsilon_{xx} \neq \epsilon_{zz}) \), TM (Transverse Magnetic) polarized light with an in-plane electric field in the \( x-z \) plane experiences different permittivity values along the \( z \) and \( x \) axes.

#### 5.2.1 Bending light away from normal

Consider a TM plane wave that is incident at a slightly off-normal angle on the interface of air and an arbitrary medium, as shown in the \( x-z \) plane of incidence in Fig. 5.1 (a). If the medium is a typical dielectric such as glass, according to Snell’s law, the wave refracts such that it always makes a smaller angle with the normal as shown in Fig. 5.1 (b), since glass is a dense medium \( (\epsilon > \epsilon_0) \) compared to air. But if the medium has low permittivity \( (\epsilon < \epsilon_0) \), the off-normal
incident light refracts differently, such that it makes a larger angle away from the normal to the interface. In the case of a zero permittivity MTM ($\epsilon \rightarrow 0$), the refracted wave flows almost parallel to the interface of the MTM and air, as shown in Fig. 5.1 (a). This seemingly simple refraction is not encountered in nature and is not typically noted. It is however of significant importance and will be used for the operation of the light concentrator. It should be noted that such refraction is achieved with a homogenous ENZ slab, and does not utilize a varying refractive index as in anomalous refraction in PCs or graded-index lenses [6].

![Figure 5.1](image)

Figure 5.1: (a) Off-normal light incident on the interface of air and a half-space of a material filled with an anisotropic MTM with low longitudinal and matched transverse permittivity values \( \{\epsilon_{zz} \rightarrow 0, \epsilon_{xx} = 1\} \) causing refraction away from normal. Both positive (blue) and negative (red) refraction is possible depending on the choice of MTM. (b) Interface with a regular isotropic dielectric $\epsilon_r > 1$ which always positively refracts light closer to the normal. (c) A rotated MTM with an inclined interface and (d) a rotated MTM cleaved surface along $x$-axis, to refract the $z$-directed incident light away from the $z$-axis.

An isotropic zero permittivity medium is typically not realizable in nature and it is also mismatched to air (impedance=$\sqrt{\mu/\epsilon}$). The latter issue causes reflections for the incident light at the interface of the MTM reducing the transmitted power. We find however that in order to achieve the desired refraction, the MTM does not need to be isotropic. The condition for refracting the wave away from the normal is to have a close to zero longitudinal permittivity (e.g.
The permittivity in the transverse direction does not need to be low and can in fact be used to match the MTM to air for normally incident light. Deriving the reflection coefficient at the interface by applying the boundary condition for the continuity of the tangential electric and magnetic fields, it is found that for an arbitrary angle of incidence \( \phi \) the reflection coefficient is

\[
r = \frac{\cos \phi \sqrt{\epsilon_{xx}} - \sqrt{1 - \sin^2 \phi / \epsilon_{zz}}}{\cos \phi \sqrt{\epsilon_{xx}} + \sqrt{1 - \sin^2 \phi / \epsilon_{zz}}}
\]

This shows that for normal incidence \( \phi = 0^\circ \), \( \epsilon_{xx} = 1 \) results in zero reflections.

Therefore, we arrive at an anisotropic MTM with low longitudinal \( \epsilon_{zz} \to 0^\pm \) and matched transverse \( \epsilon_{xx} = 1 \) permittivity values as the optimum solution for our refraction of interest. Incidentally this is the opposite of the operation of the hyperlens [57] where the transverse permittivity was close to zero. As noted, there are two possible solutions depending on the sign of \( \epsilon_{zz} \). Fig. 5.1 (a) shows the power flow (Poynting vector) for the two possibilities corresponding to positive (blue) and negative (red) refraction. From a dispersion standpoint, this means that the MTM either follows elliptic or hyperbolic dispersion respectively.

The same anisotropic medium can be used to refract a completely \(-z\) directed incident light away from the \(z\)-axis. This is done by rotating the crystal by a slight angle \( \alpha \) in the \(x-z\) plane as shown in Fig. 5.1 (c), where the rotation causes an inclined surface. The permittivity tensor of such rotated crystal can be found by a coordinate transformation and using a rotation matrix \( R(\alpha) \), where

\[
R(\alpha) = \begin{pmatrix}
\cos \alpha & 0 & -\sin \alpha \\
0 & 0 & 0 \\
\sin \alpha & 0 & \cos \alpha
\end{pmatrix}
\]

The final permittivity matrix of the rotated slab \( \tilde{\epsilon} \) in the \(x-z\) coordinates is \( \tilde{\epsilon} = R(\alpha) \tilde{\epsilon} R(-\alpha) \). The same rotated crystal may also be cleaved along the \(x\)-axis. The flat surface will still bend the normal incident light with a large angle away from the normal, making it propagate along the interface as shown in Fig. 5.1 (d).

We can therefore bend the direction of power flow of normal (as well as off-normal) incident light farther away from the normal to the interface. This refraction with minimal reflections is not possible with other choices of values for the permittivity tensor (e.g. it is not encountered in nature with typical dielectrics) and is a direct consequence of using an anisotropic low permittivity medium. This refracted wave is a free propagating wave in the lower half-space and is not a "bound surface wave", thus can be further guided if needed. Our intention is not to couple to surface waves bounded to the interface as they do not carry real power when leaving the interface.

The refraction capability suggests that one can create a longer transverse path length for the refracted wave and a much shorter longitudinal path. It therefore suggests that one can use a thin material, as the longitudinal component of the ray path is much shorter than the transverse. This has a direct consequence for realizing low profile focusing devices as shown in
this work. Another consequence of this result is that it can potentially allow thinner absorbers (say for thermo-photovoltaic or TPV applications), by increasing the chance of absorption of the incoming power in the transverse direction despite its small thickness.

5.3 Light Concentration using MTM Hetero-junctions

Fig. 5.2 (a) proposes the MTM based focusing device that collects light over a large area and concentrates it into a hot spot. The device comprises a hetero-junction of two anisotropic low permittivity MTMs \( \{\epsilon'_{z'} \to 0, \epsilon'_{x'} = 1\} \) placed side-by-side. Each MTM is rotated by a small angle (\( \pm \alpha \)) giving rise to oppositely rotated principal axes on the two sides of the junction. The MTMs are finite slabs and their interfaces are cleaved along the \( x \)-axis.

According to the operation discussed in Fig. 5.1 (d), the normally incident light bends away from the normal on each side of the hetero-junction. The MTMs are rotated such that the waves are refracted accordingly on each side such that power is forced to flow towards the center of the structure. The light is then transmitted through to create a spot of high field concentration.
on the other side of the structure in air. Hence the overall hetero-junction operates similarly to
a lens under normal incidence and focuses light into a spot. The device shown in Fig. 5.2 (a)
uses negative refraction with $\epsilon_z' \rightarrow 0^-$. A similar device using positive refraction with $\epsilon_z' \rightarrow 0^+$
can also be envisioned as shown in Fig. 5.2 (b). As shown the MTMs in Fig. 5.2 (b) must be
rotated opposite to that of Fig. 5.2 (a), such the refraction again forces the power towards the
center.

Fig. 5.2 (c) shows full-wave simulation results of the field magnitude for a device with
parameters $\{\epsilon_z' = -0.1, \epsilon_x' = 1\}$, i.e. the case of Fig. 5.2 (a) having MTMs with hyperbolic
dispersion. The simulation results of a device realized with elliptical dispersion MTMs $\{\epsilon_z' = +0.1, \epsilon_x' = 1\}$ is shown in Fig. 5.2(d). The tilt angle of the MTMs for both devices is $\alpha = \pm 10^\circ$.
It can be clearly seen that an area of high field concentration or hot spot is indeed achieved
in both cases close to the bottom interface. It should be noted that the top interface of the
devices does not necessarily need to be cleaved and can be left at the corresponding slanted
angles as was done in Fig. 5.1 (c). However, a cleaved surface along the $x$-axis allows realizing
a more flat and low profile design.

## 5.4 Operation of the Light Concentrator

The exact operation of the negative refracting device in Fig. 5.2 (a) can be best understood using
the iso-frequency contours shown in Fig. 5.3 (a). The wave-vector/phase velocity and the power
flow are traced out throughout the structure and outside. The diagram is constructed based on
the phase matching boundary condition at each interface, i.e. the tangential wave-vector must
be continuous. The direction of the Poynting vector (power flow) as well as the group velocity
is normal to the iso-frequency contour. The propagation in the anisotropic medium prior to
rotation is governed by

$$\frac{k_x'^2}{\epsilon_z'} + \frac{k_z'^2}{\epsilon_x'} = k_0^2$$

(5.4)

The figure shows the propagation of a TM wave incident on the MTM on the right side of the
hetero-junction. The normal incident wave $\vec{k}_i$ impinging on the air-MTM interface (1), phase
matches to a wave-vector $\vec{k}_r$ in the right-side MTM that also has a zero tangential wave-vector.
Because of the crystal rotation, the iso-frequency contour of the right-side MTM is tilted by the
angle $\alpha$ causing $\vec{k}_r$ to be larger than $\vec{k}_i$. The normal to the iso-frequency contour at this point
defines the direction of the Poynting vector $\vec{S}_r$, which as mentioned before is bent along the
interface, causing power to flow away from the normal and to the left. Due to the choice of the
permittivity tensor, the dispersion in both MTMs is hyperbolic leading to negative refraction
at interface (1) and is consistent with the designated power flow. The negatively refracted wave
in the right-side MTM travels towards the center along $\vec{S}_r$, while its phase changes along the
wave-vector $\vec{k}_r$. At the interface (2), $\vec{k}_r$ is completely tangential to the interface and therefore
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Figure 5.3: (a) Diagram showing wave-vectors and refraction at different boundaries of the hetero-junction, iso-frequency contour inside hyperbolic media, as well as predicted directions of Poynting vector. full-wave simulations of the power flow (Poynting vector) for (b) partial illumination and (c) full illumination of the device.

phase matches a wave $\vec{k}_l$ in the left side MTM as shown. In order to maintain causality, the power now must flow away from interface (2) and therefore we need to choose the appropriate branch of the hyperbola in the left-side MTM. The direction of power flow changes to $\vec{S}_l$, due to the $-\alpha$ tilt of the optical axis in the left side MTM and the normal to the contour at that point. The wave in the left-side MTM finally experiences another negative refraction at the MTM-air interface (3). Again the tangential wave-vector at the interface is continuous, leading to the output wave $\vec{k}_o$. The output power flow in air $\vec{S}_o$ is now in the $+x$ direction opposite to the flow in the MTM and is directed towards the center.

Fig. 5.3 (b) shows the power flow (Poynting vector) for a Gaussian beam incident on the right side of the device, using full-wave simulation. The simulated power flow confirms the predicted behavior from theory in Fig. 5.3 (a). The figure confirms that the power incident on
the right-side MTM is refracted towards the middle, then refracted into the left-side MTM and is then negatively refracted into air towards the center at the bottom side.

If the power was only incident on the left-side MTM, a similar behavior could be traced out for the transmitted and internal waves, only mirrored about the \( z \)-axis. In the final case where the incident wave illuminates both sides of the device, the resulting transmitted wave is therefore the superposition of the output from both sides. The two transmitted power flows concentrate towards the middle, creating a high concentration focal spot along the \( z \)-axis on the other side of the structure. Such focusing action is shown in the simulation results of the power flow in Fig. 5.3 (c).

## 5.5 Bi-layered Negatively Refracting Focusing Device

The desired anisotropic low permittivity MTMs can be realized with layers of metal and dielectric at optical frequencies. As also discussed in the previous chapter, it has been long known that sub-wavelength periodic bi-layers of materials can be effectively treated as a homogeneous medium with an effective uniaxial birefringent permittivity tensor [54]. This has been used in various studies [55] and is particularly useful for the realization of Epsilon-Near-Zero MTMs [38] where the desired “close to zero permittivity” is typically achieved by interleaving sub-wavelength layers of a material with positive permittivity and a material with negative permittivity, such that the effective medium is zero [57, 58, 59, 60].

As noted in the previous chapter, the effective permittivity values along the optical axis and normal to the optical axis of a layered structure with a sub-wavelength period, using the first order effective medium theory (EMT) formulas are [54, 60]:

\[
\epsilon_{\text{opt. axis}} = \frac{\epsilon_m \epsilon_d}{(1 - p) \epsilon_m + p \epsilon_d} \\
\epsilon_{\perp \text{opt. axis}} = p \epsilon_m + (1 - p) \epsilon_d
\]

where \( \epsilon_m \) and \( \epsilon_d \) are the permittivity values of the metal and dielectric layers respectively, 'p' is the filling ratio of the metal layer (thickness of the metal layer divided by the sum of the thickness of metal and dielectric layers) and the optical axis is normal to the layers.

Depending on the operating wavelength and desired level of loss, one may choose a variety of materials for realizing the required anisotropy. For instance for the low loss focusing device presented here we utilize bi-layers of silver (Ag) having \( \epsilon_m = -18 + 0.5j \) [57] for the metal layer (loss and dispersion accounted for) and glass \( \epsilon_d = 2.2 \) for the dielectric layer, at a free-space operating wavelength of \( \lambda_0 = 633 \text{ nm} \).

The full-wave simulation results of an actual focusing device realized with the Ag/glass bi-layer implementation are shown in Fig. 5.4 (a) and (b). The power density in Fig. 5.4 (a) is again normalized to the power density of the normally incident plane wave illuminating the structure. It can be clearly seen that a distinct bright focus point is achieved, with significant
field enhancement at that point, similar to the results of the ideal case of the negatively refracting focusing device in Fig. 5.2 (c). This demonstrates a focusing device with realistic materials and having losses included, operating at $\lambda_0 = 633$ nm. The power flow through this layered focusing device is also shown in Fig. 5.4 (b). It can be clearly seen that the focusing device collects and bends the incident normal power at the top surface and towards the middle via refraction, resulting in a high concentration of transmitted power at the middle of the structure. The MTMs of the focusing device are implemented with the Ag-Glass bi-layer combination with a period of $L = \lambda_0/10$, filling ratio of $p = 0.126$ and tilt angle of $\alpha = \pm 10^\circ$. Although this device has an effective transverse permittivity of $\epsilon_{xx} \approx 2.5$ due to the choice of materials, it is still far better matched to air than the corresponding isotropic low-permittivity medium ($\epsilon_{xx} = \epsilon_{zz} \rightarrow 0$), according to the expression for reflection “$r$” presented earlier. Note that the amount of the reflected power is related to $|r|^2$.

To our knowledge this is the first time that a flat low-profile concentrator of light has been devised with the use of anisotropic MTMs. Also, in contrast to other designs where the field intensity is increased inside a medium [130, 132, 133], this design focuses the power in the host medium (e.g. air) similar to a lens. This is particularly useful for solar-concentration applications, where light with high concentration needs to shine the photovoltaic cell, thus may be used as a non-imaging concentrator in focusing power for CPV applications [127].

In general, the proposed light collecting hetero-junction structure may be used in two different types of applications. The first set of applications is when it operates as a focusing mechanism to transmit and concentrate the power into a hot spot in air as shown in Fig. 5.2. This hetero-junction operates as a compact and low profile focusing device with small dimensions on the order of several wavelengths at the operating frequency. Various scenarios such as CPV applications or microlenses can benefit from this configuration. Such scenarios would require maximum transmission of field to the hot spot and therefore the device should be designed for low losses. Although we suggest here a layered realization with Ag and glass, other metal/dielectric combinations may also offer advantageous low loss operation, as well as the more recently explored low loss plasmonic metamaterials [140], depending on the wavelength of operation. For example, a very low-loss all-dielectric version of this hetero-junction concentrator, using a zero-index Dirac-type photonic crystals (described in the next chapter), is presented in Appendix B.

In a second type of application, the same structure can be designed to be highly lossy, in order to be used as a thin absorber (or equivalently a narrow band and narrow angle emitter) applicable to solar and TPV applications, if tuned to the desired wavelength. In this scenario, the transmission of the field is not of importance and the effort is to make a long transverse path for the rays inside the structure in order to increase the chance of absorption. It can create an area of high concentration in the middle of the structure such that this power is absorbed in order to heat up the structure itself and/or a secondary structure in contact with the hetero-junction. Even a single rotated slab of the proposed device may also be used as the
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5.5.1 Short focal distance

The proposed focusing device can achieve a focal point very close to the exit facet of the device, closer than bulky lenses with extremely low $f/D$ ratios. Typically, short focal distances are achieved with a thick lens having a high radius of curvature, or even elliptical shapes. Not only elliptical designs are harder to make but such large designs may not be always feasible, especially if the lens is a micro-lens with a small size in the order of several wavelengths and/or in applications where a low profile is desired.
The close focusing ability of the device can be better seen in the comparison study summarized in Fig. 5.5. We compare the locations of the focal points for a plano-convex, a convex-convex lens and a hyper-hemispherical lens all made with glass $\varepsilon_r = 2.5$, with the proposed ideal hetero-junction focusing device with parameters $\{\varepsilon'_x = -0.3, \varepsilon'_x = 1, \alpha = \pm 10^2\}$. The aperture size and thickness (bounding dimensions) of the first two lenses are kept equal to the focusing device $(8\lambda_0 \times 2\lambda_0)$, such that the performance of the three scenarios are evaluated under the same size requirements. In order to achieve the closest focus, the two lenses must have the maximum radius of curvature that fits this bounding dimension. Therefore the lenses have zero thickness on the sides to a maximum thickness of $2\lambda_0$ in the middle. All scenarios are illuminated with a Gaussian beam of waist size $\omega_0 = 6\lambda_0$, and the power is normalized to the peak power of the incident beam in free space.

The simulation results in Fig. 5.5 (b) and (c) demonstrate that the distance of the peak of the focal area from the exit face is $3.55\mu m = 5.53\lambda_0$ for the convex lens and $4\mu m = 6.3\lambda_0$ for the plano-convex lens, respectively. However, the proposed hetero-junction focusing device in Fig. 5.5 (a) develops a focus at $0.388\mu m = 0.61\lambda_0$, a very short focal distance given the allowed dimensions. This shows a considerable improvement over traditional lenses of the same size in achieving close by focal points using the proposed structure, for the same aperture and thickness dimensions.

Fig. 5.5 (d) shows a thick hyper-hemispherical glass lens of the same aperture size $(2R = 8\lambda_0)$ and a thickness of $R(1 + \frac{1}{\sqrt{\varepsilon_r}}) = 4.13\mu m = 6.53\lambda_0$, according to [141]. Such a thick lens provides a focal distance of $0.9\mu m = 1.42\lambda_0$, which is still longer than the focal length achieved with the proposed device. A hyper-hemispherical lens with the same aperture size and made of quartz ($\varepsilon_r = 3.8$) has a focal distance roughly equal to the proposed focusing device but is considerably thicker at $6.05\lambda_0$. This demonstrates that a high permittivity lens of very thick size with hyper-hemispherical (or elliptical) design is required to achieve the same focal distance, showing the limitation of traditional lenses in achieving focal spots very close to the lens. Another point of interest is that without curvature, a simple dielectric cannot provide focusing whereas here the proposed design provides a flat structure without curvature that yields better results than lenses.

From the color plots of Fig. 5.5(a), (b) and (c) it is evident that the proposed device develops a focal spot that is much closer and also stronger than the focus of the traditional lens of the same dimensions. Of course an actual lossy implementation of the device can reduce the strength of the fields at the focal spot slightly depending on the materials and implementation. This is mainly due to the losses that the ray paths exhibit by traveling through the lossy device. However, even a lossy device outperforms the corresponding lens by developing a very close-by focus that is actually still stronger than the lens’s focal strength. For instance, consider an actual lossy focusing device made with bilayers of Ag/glass similar to Fig. 5.4 and with dimensions $(8\lambda_0 \times 1.5\lambda_0)$. If tuned for a short focal distance, the peak power of the focus is about 1.25 of the peak power of the lossless convex-convex lens of the same bounding dimensions. This lossy
Figure 5.5: Power density plot (normalized to incident power density) showing focal spot (a) focusing device with \( \{ \epsilon_z' = -0.3, \epsilon_x' = 1, \alpha = 10^\circ \} \) (b) a convex lens and (c) plano-convex lens, all with bounding dimensions \( 8\lambda_0 \times 2\lambda_0 \) and (d) a hyper-hemispherical lens with bounding dimension \( 8\lambda_0 \times 6.53\lambda_0 \). Lenses made of glass with \( \epsilon_r = 2.5 \). The incident Gaussian beam has a waist size of \( \omega_0 = 6\lambda_0 \).

This result even significantly outperforms (by an order of magnitude) the recently demonstrated extreme short focusing using transmitarrays of optical antennas [138, 139], that reported a best
There are several parameters that determine the operation of the device, primarily the effective permittivity \( \{\epsilon_z', \epsilon_x'\} \), the tilt angle \( \alpha \) and the thickness. These parameters determine the location of the focal spot with respect to the exit facet as well as the strength of the focus. For an ideal device with parameters \( \{\epsilon_z' = -0.14, \epsilon_x' = 1.068, \alpha = 10^\circ\} \) and a fixed aperture size of \( 8\lambda_0 \), Fig. 5.6 shows the change in the focal spot location as a function of the device thickness, as well as the strength of the focus sensed. The results show that thinner devices (even thinner than one wavelength) are also possible yielding light concentration at extremely short distances. Very thin devices though come at the expense of a reduced focal strength. Given the observed trends for focal strength and location, one may conclude that a good range for the thickness is \( \lambda_0/2 - 3\lambda_0/2 \), which provides 5 to 11 times power intensification of the incident power density. For instance, a thin device with thickness of only \( \lambda_0 \) yields a short focal distance of \( 0.6\lambda_0 \) and a focal strength intensification of 10 times (an order of magnitude) higher than the incident power density. Such device outperforms Fresnel lenses in terms of size, as they are normally in the order of many wavelengths in thickness [139]. The \( f/D \) ratio for the range of devices in Fig. 5.6 (a) is between 0.02 – 0.2, for instance \( f/D = 0.075 \) for the \( \lambda_0 \) thick case.

Fig. 5.6 (b) shows a half wavelength thick lossy focusing device realized with Ag/glass, which is still able to achieve concentration at a distance of about \( \lambda_0/4 \). Note that the low effective longitudinal permittivity and the unique refraction is enabling this very thin design, as the refracted waves have a very short longitudinal path inside the structure as discussed earlier. With a layered realization, the permittivity of the focusing device is adjustable by tuning of the filling ratio using the same materials, e.g. Ag and glass used here. This is while a lens permittivity can only be changed by using a different material to increase the refractive index. In addition to these extremely thin designs, the device may also be scaled to electrically large aperture size lenses by adjusting its thickness accordingly.

The lensing effect occurs for any wavelength in the theoretical device as long as the effective longitudinal permittivity, \( \epsilon_z' \), is within an acceptable limit. Therefore the principle of operation is potentially wideband. However, depending on the implementation (e.g. whether bi-layered, nano-wire media, nano-composite, etc.) the bandwidth limits vary accordingly. The bandwidth of the bi-layered implementation primarily depends on the choice of the metal and its frequency dispersion characteristics. As the frequency changes, the negative permittivity of the metal varies according to its dispersion, hence the effective \( \epsilon_z' \) varies. Some of the overall effects of varying the operating frequency are changes in the focus intensity and a slight reduction of the focal distance from the exit facet of the device. For the specific Ag/glass example presented in Fig. 5.4, simulation results indicate that the lens has an operating line-width of 200nm ranging from 590 nm to 790 nm, using a Drude model for the silver layer. This is based on a stringent condition requiring that a distinct focus with at least 2x power intensification is achieved at the band edges. The power density (normalized to the incident power density in free space) along
Figure 5.6: (a) Effect of device thickness on focal distance and the strength of the focal spot (normalized to the incident power density). (b) Power density normalized to incident power density for a lossy focusing device that is \( \lambda_0/2 \) in height and \( 8\lambda_0 \) wide, realized with bi-layers of Ag/glass.

the center of the device and just below the exit face of the device (i.e. \( x = 0, -2 < z/\lambda_0 < 1 \)) is shown Fig. 5.7 for various wavelengths across this range.

5.5.2 Other materials and design wavelengths

The plasmonic bi-layer realization can be implemented with various metal combinations at different optical and infrared (IR) frequencies. Table 5.1 lists the various metals and wavelengths considered and the corresponding design dimensions, as well as the strength of the hot-spot. It can be seen that silver still provides the highest concentration strength among all metals due to its dispersive nature and its lower losses. The Figure of Merit (FoM) = \( \text{real}(\epsilon_m)/\text{imag}(\epsilon_m) \) plays an important role in the performance of the metal layer and the overall ENZ effect. The
optical parameters were obtained from [142]. It can be seen that the maximum intensification varies for different designs at different wavelengths, mainly owing to the FoM of silver. A high strength of focus is achieved with 8× intensification at 1087.8 nm operating wavelength. This shows that even with the effect of material losses present, the lens provides a strong focus, considering the ideal intensification level of 10×, which is achieved in the hetero-junction made of homogenous lossless slabs of anisotropic ENZ as in Fig. 5.2.

A design is also noted for the Far IR range at an operating wavelength of 11.84μm. This design uses SiC as the plasmonic metal layer along with a typical dielectric such as glass. The dimensions of layers in this design are much larger than the other designs. Therefore fabrication of this device is much simpler. Fabricating this device may be a good starting point for realizing prototypes of a plasmonic multilayer light concentrator and to prove the concept.

In many microlens applications, power is received by a sensor placed at the focal length of the lens, with the sensor potentially having a much smaller width than the lens itself. By computing the total power available at the sensor with and without the lens, one can obtain a good measure of improvements achieved. In our scenario, the sensor is assumed to be 1/8 of the width of the lens, and the lens is the realistic lossy hetero-junction at 633nm. This analysis thus includes both losses due to Fresnel reflections from the lens, as well as material losses in the layers of the device. We compute in simulation, the total power traveling in the \( z \) direction as intercepted by a sensor placed at the focus of the hetero-junction, compared to the
Table 5.1: Various Metals at different wavelengths in the design of the plasmonic ENZ concentrator

<table>
<thead>
<tr>
<th>Wavelength (μm)</th>
<th>Metal</th>
<th>Dielectric</th>
<th>Metal thickness (nm)</th>
<th>Dielectric thickness (nm)</th>
<th>Spot Strength</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.633</td>
<td>Ag</td>
<td>Glass (2.2)</td>
<td>14</td>
<td>92</td>
<td>6×</td>
</tr>
<tr>
<td>1.0878</td>
<td>Ag</td>
<td>Glass (2.2)</td>
<td>6.5</td>
<td>175</td>
<td>8×</td>
</tr>
<tr>
<td>1.2157</td>
<td>Ag</td>
<td>Glass (2.2)</td>
<td>6.3</td>
<td>196.5</td>
<td>6×</td>
</tr>
<tr>
<td>2.07</td>
<td>Ag</td>
<td>Glass (2.2)</td>
<td>5</td>
<td>340</td>
<td>5×</td>
</tr>
<tr>
<td>1.2157</td>
<td>Au</td>
<td>Glass (2.2)</td>
<td>9.3</td>
<td>194</td>
<td>&lt; 4×</td>
</tr>
<tr>
<td>0.89209</td>
<td>Au</td>
<td>Glass (2.2)</td>
<td>12</td>
<td>135</td>
<td>&gt; 4×</td>
</tr>
<tr>
<td>1.1271</td>
<td>Al</td>
<td>Glass (2.2)</td>
<td>182</td>
<td>6.5</td>
<td>&lt; 2.5×</td>
</tr>
<tr>
<td>0.89209</td>
<td>Cu</td>
<td>Glass (2.2)</td>
<td>10</td>
<td>139</td>
<td>3.1×</td>
</tr>
<tr>
<td>11.84</td>
<td>SiC</td>
<td>Glass (2.2)</td>
<td>160</td>
<td>1020</td>
<td>5×</td>
</tr>
</tbody>
</table>

intercepted power by the same sensor if the hetero-junction layer were not present. Our devices suggest that a minimum of twice the power is available to the sensor in the presence of the hetero-junction, i.e. we make use of the power incident on the entire lens area. Performance at operating wavelength of 1μm to 1.5μm is even higher as the concentration ability of the device is higher, as was shown in Table 5.1.

A three-dimensional design of the plasmonic ENZ concentrator for red light is presented in Figure 5.8, along with simulations results showing focusing of the incident light into an area of higher power density.

5.6 Lensmakers’ equation based on Geometric Optics

In this section, we explicitly analyze the refraction of light in the hyperbolic hetero-junction lens of 5.2 (a). We derive simple first-order formulas for its operation and design using geometric optics and dispersion relations, leading to an equivalent lensmaker’s equation for the device. Subsequently, the analysis is used to go beyond the limits of light focusing, opening the door to realizing thin and flat near-field beam-splitters and shifters, using the hetero-junction structure.

The geometry of the hetero-junction is again shown in Figure 5.9, along with the refraction that takes place in the device. If we assume that the refraction angles $\theta_{S,r}, \theta_{S,l}$ and $\theta_{o}$ are known, we can find the distance $f$ by noting:

\[
\begin{align*}
    l_o \sin \theta_{o} &= l_t \sin \theta_{S,l} \implies l_o = \frac{l_t \sin \theta_{S,l}}{\sin \theta_{S,o}} \\
    l_t \cos \theta_{S,l} + l_r \cos \theta_{S,r} &= h \implies l_t = \frac{h - l_r \cos \theta_{S,r}}{\cos \theta_{S,l}} \\
    m &= l_r \sin \theta_{S,r} \implies l_r = \frac{m}{\sin \theta_{S,r}}
\end{align*}
\]

where $m$ is assumed to be the distance form the axis at which the ray is incident on the top surface. Therefore $f$ is:
Figure 5.8: A 3D realization of the plasmonic ENZ concentrator. (a-c) Different cuts of the normalized power density (normalized to peak of power density at focus) showing confined concentration at the focal region directly below the device. Red light is illuminated from the top.

\[ f = l_o \cos \theta_o = l_i \frac{\sin \theta_{S,l}}{\sin \theta_{S,o}} \cos \theta_o = \]

\[ \frac{h - l_r \cos \theta_{S,r} \cdot \sin \theta_{S,l}}{\cos \theta_{S,l}} \cdot \frac{\sin \theta_{S,l}}{\sin \theta_{S,o}} \cos \theta_o = \]

\[ \left( h - \frac{m}{\sin \theta_{S,r}} \cos \theta_{S,r} \right) \frac{1}{\cos \theta_{S,l}} \cdot \frac{\sin \theta_{S,l}}{\sin \theta_{S,o}} \cos \theta_o \] \hspace{1cm} (5.8)

which leads to

\[ f = \frac{\tan \theta_{S,l}}{\tan \theta_o} \left( h - \frac{m}{\tan \theta_{S,r}} \right) \] \hspace{1cm} (5.9)

Equation (5.9) can be considered as the lensmaker’s equation describing the focal distance
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Figure 5.9: (a) The three-step refraction process of focusing and the focal distance of the hetero-junction lens using geometric optics. (b) The two-step refraction process.

Equation (5.9) shows that there is aberration in the focal distance, analogous to spherical aberration in spherical lenses. That is, depending on the ray distance ‘m’ from the main axis, the exiting rays converge at \( f \propto -m \) with a linear trend. Rays farther from the main axis converge at shorter \( f \), while rays at the axis converge at the longest distance \( f_{\text{max}} \), which is attained for \( m = 0 \):
\[ f_{\text{max}} = \frac{\tan \theta_{S,l}}{\tan \theta_{o}} h. \] (5.10)

Such aberrations may potentially be reduced by shaping of the input and exit surfaces of the lens, as can be done in dielectric lenses, to refine the spot size[6]. Equation (5.10) is also valid for the most part when \( \theta_{S,r} \rightarrow 90^\circ \), i.e. when the light bends inside the ENZ extremely away from the normal to the top interface (1). \( \tan \theta_{S,r} \) is large and consequently \( \frac{m}{\tan \theta_{S,r}} \) is a small number compared to \( h \).

For a wide device that is fully illuminated by a plane-wave, a focus develops starting from \( f = 0 \) upto the maximum distance \( f_{\text{max}} \). If the device width is finite or if a wide device is under-illuminated, then the focus spread will be between \( f_{\text{min}} \) and \( f_{\text{max}} \), where \( f_{\text{min}} \) is determined by the maximum illumination distance from the axis and \( f_{\text{max}} \). For instance, for a Gaussian beam of waist size \( \omega_0 \) incident on the lens and having the same axis as the lens, then
\[ f_{\text{min}} = \frac{\tan \theta_{S,l}}{\tan \theta_{o}} \left( h - \frac{\omega_0}{\tan \theta_{S,r}} \right). \]

The range of validity of equation (5.9) is for \( m < h \tan \theta_{S,r} \). Rays incident beyond this distance do not undergo the three-step refraction process shown in Figure 5.9 (a), but rather a two step refraction as in Figure 5.9 (b). For instance a ray incident on the right side MTM with \( m > h \tan \theta_{S,r} \), will refract once at the top surface, travel only in the right side MTM towards the middle, and exit with another refraction on the bottom surface of the right side MTM but closer to the main axis. Such a mechanism does not contribute to focusing on the main axis via constructive interference with its symmetric counterpart that is incident at \(-m\). However, this result can be used for a different operation of the device as a beam splitter, as will be shown in a later section.

### 5.7 Calculation of the refraction angles

The analysis thus requires solving for the three unknown angles \( \theta_{S,r}, \theta_{S,l} \) and \( \theta_{o} \) in (5.9). These unknowns can be derived from the dispersion of the ENZ hetero-junction and following the path of the light as it refracts in the device. The unknown angles are found to be related only to the permittivity of the ENZ slabs and axis rotation angles \( \pm \alpha \). The refraction, phase matching, and axes of the ENZ slabs are depicted in Figure 5.10.

#### 5.7.1 Right-side MTM

The incident wave is normal to the interface (1) in Figure 5.10. Phase matching at interface (1) requires continuity of tangential wave-vector at that interface, thus the wave-vector in the right-side MTM \( \vec{k}_r \) will also be normal to the interface. Moreover, the tip of \( \vec{k}_r \) will also lie on the iso-frequency contour of the anisotropic medium. The principal axes of the right-side MTM are denoted with \( x' \) and \( z' \) axes, which are rotated by angle \(-\alpha\) with respect to \( x \) and \( z \). The dispersion equation of an anisotropic medium with principal axes \( x' \) and \( z' \) dictates
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\[ \frac{k_{rx}^2}{\epsilon_{z'}} + \frac{k_{rz}^2}{\epsilon_{x'}} = k_0^2 \]  

(5.11)

The angle between the principal axis \( z' \) and \( k_r \) is

\[ \theta_{k,r}' = \tan^{-1} \left( \frac{k_{rx}'}{k_{rz}'} \right) = \alpha \]  

(5.12)

Therefore the components of \( \vec{k}_r \) along the principal \( x' - z' \) axes of the crystal are

\[
\begin{align*}
    k_{rx'} &= k_r \sin \alpha \\
    k_{rz'} &= k_r \cos \alpha
\end{align*}
\]  

(5.13)

substituting these in (5.11) we find:

\[ k_r = \frac{k_0}{\sqrt{\frac{\sin^2 \alpha}{\epsilon_{z'}} + \frac{\cos^2 \alpha}{\epsilon_{x'}}}} \]  

(5.14)

which means \( k_r \) is a function of the known material parameters and the crystal rotation angle \( \alpha \). The angle of refraction, or the angle that the Poynting vector makes with the principal axis \( z' \) in the right-side MTM is therefore

\[ \theta_{S,r}' = \tan^{-1} \left( \frac{k_{rx'}}{k_{rz'}} \right) = \tan^{-1} \left( \frac{\epsilon_{x'}}{\epsilon_{z'}} \tan \alpha \right) \]  

(5.15)

and hence the second unknown refraction angle in (5.9) is

Figure 5.10: The three step refraction, phase matching, and local coordinate systems.
\[ \theta_{S,r} = |\theta'_{S,r}| + \alpha \] (5.16)

For example for the case of \( \epsilon_{r_x} = 1, \epsilon_{r_z} = -0.1 \) and \( \alpha = 10^\circ \), we have \( \theta_{S,r} = 70^\circ \).

### 5.7.2 Left-side MTM

The left-side MTM is rotated by angle \( \alpha \), and the principal axes of the crystal are denoted the \( x'' \) and \( z'' \) axes. The dispersion of the left-side crystal in its principal axes leads to

\[ \frac{k^2_{l x''}}{\epsilon_{z''}} + \frac{k^2_{l z''}}{\epsilon_{x''}} = k^2_0 \] (5.17)

In the left-side slab, the wave-vector \( \vec{k}_l \) has an angle \( \theta''_{k,l} \) with the principal axis \( z'' \), such that the components along the principal axes are:

\[
\begin{cases}
  k_{l x''} = k_l \sin \theta''_{k,l} \\
  k_{l z''} = k_l \cos \theta''_{k,l}
\end{cases}
\] (5.18)

Phase matching of the wave-vector at interface (2) requires continuity of the tangential wave-vector at the interface, \( k_{rz} = k_{lz} \). Due to the opposite rotation of the left-side crystal we have

\[ k_l \cos (\alpha + \theta''_{k,l}) = k_{lz} = k_{rz} \] (5.19)

Based on (5.19) and (5.17), we need to solve for the two unknowns \( k_l \) and \( \theta''_{k,l} \) with two equations:

\[
\begin{cases}
  k_l \cos (\alpha + \theta''_{k,l}) = k_{lz} \\
  \frac{k_l^2 \sin^2 \theta''_{k,l}}{\epsilon_{z''}} + \frac{k_l^2 \cos^2 \theta''_{k,l}}{\epsilon_{x''}} = k^2_0
\end{cases}
\] (5.20)

Therefore the following equation needs to be solved to find the angle of the wave-vector in the left-side MTM, \( \theta''_{k,l} \)

\[ \frac{k_0}{\sqrt{\frac{\sin^2 \theta''_{k,l}}{\epsilon_{z''}} + \frac{\cos^2 \theta''_{k,l}}{\epsilon_{x''}}}} \cos (\alpha + \theta''_{k,l}) = k_{rz} \] (5.21)

The angle of refraction, or the angle that the Poynting vector makes with the principal axis \( z'' \) of the left-side MTM is therefore

\[ \theta''_{S,l} = \tan^{-1} \left( \frac{k_{lz''}}{k_{lx''}} \right) = \tan^{-1} \left( \frac{\epsilon_{x''}}{\epsilon_{z''}} \tan \theta''_{k,l} \right) \] (5.22)

and hence the refraction angle in (5.9) is
\[ \theta_{S,l} = |\theta''_{S,l}| - \alpha \]  

Phase matching of wave-vector at interface (3) dictates \( k_{ox} = k_{lx} \), thus the last unknown refraction angle in (5.9), i.e. the refraction in the output of the lens is

\[ \theta_o = \sin^{-1} \left( \frac{k_{ox}}{k_o} \right) = \sin^{-1} \left( \frac{k_{lx}}{k_o} \right) \]  

### 5.8 Beam-splitter

It was noted earlier that for \( m > h \tan \theta_{S,r} \), the rays travel on the same side of the device and exit from the other face of the device, closer to the main axis. By reciprocity, light can travel in the same path in the opposite direction. This result suggests that we can use the hetero-junction as a beam splitter and beam shifter, using the reciprocal path of light. Consider Figure 5.11 (a), where the anisotropic ENZ hetero-junction is illuminated from the back, clearly showing a beam-splitting operation. A Gaussian beam is incident from the bottom of the figure on the device, with its axis normal to the device and falling right at the junction at \( x = 0 \). The figure shows the power density over the simulation domain normalized to the peak of the incident power density of the Gaussian wave that has a waste size of \( \omega_0 = 4\lambda_0 \). The simulation domain is \( 60\lambda_0 \times 60\lambda_0 \), and the bottom face of the hetero-junction is placed on the \( z = 0 \) axis, symmetrically about the origin. The ENZ hetero-junction used here has parameters \( \{\epsilon_{x'} = \epsilon_{x''} = 1, \epsilon_{z'} = \epsilon_{z''} = -0.08, \alpha_r = -\alpha_l = -10^\circ, h/\lambda_0 = 2\} \).

When illuminated from the back, each part of the power incident on either side of the axis bends obliquely and refracts away from the axis, and travels farther away from the axis in the same side, and then exits from the top face with another negative refraction, following the reciprocal path of interest. It can be clearly seen from the figure that a single beam is split equally into two areas of high concentration at a far distance from the device, and away from each other. The beam splitting action is however achieved with the very thin and flat hetero-junction. Figure 5.11 (b) shows cuts of the transmitted power density away from the device at \( z/\lambda_0 = 50 \), as well as at the back of the device at \( z/\lambda_0 = -5 \). The transmitted power density shows two equal strength concentrated beams around \( x/\lambda_0 \approx \pm 10 \). The sampled field at the back of the device represents the reference input wave, and it can be seen that it is unaffected by the device as there are minimal reflections. Fig. 5.11 (c) shows another scenario with unequal power splitting, by shining the same hetero-junction with the Gaussian beam axis at \( x/\lambda_0 = 1 \). In the extreme case where the beam is shining only a single rotated ENZ slab, the slab operates as a beam shifter as shown in Fig. 5.12.

A refocusing of the beam can be observed at \( z/\lambda_0 = 54 \) in Fig. 5.11 and Fig. 5.12. This is due to the slightly different refraction angles of the off-normal incident waves inside the ENZ, dictated by the rotated hyperbolic dispersion. The incident Gaussian beam contains more than just the normal plane-wave component. Thus its constituent waves take on slightly different
Figure 5.11: The anisotropic ENZ hetero-junction as a near-field beam-splitter, when illuminated from the back. Equal power division of a Gaussian beam with $\omega_0/\lambda_0 = 4$ normally incident with axis on $x = 0$: (a) 2D power density plot and (b) at the input and output. (c) Unequal power division by shifting the axis of the incident beam to $x/\lambda_0 = 1$.
Figure 5.12: An anisotropic ENZ slab as a beam-shifter. (a) 2D power density plot and (b) Input and output power density.

Gaussian beam is decomposed into three rays. Each colored arrow represents a plane-wave component with a different angle of incidence on the slab, at the same frequency (orange being the normally incident wave as in all previous figures). The Gaussian beam waist is on the slab, and so the incident rays are converging together at the interface.

As noted, the off-normal incident (red and magenta) waves will have a refraction angle slightly different from each other, and from that of the normally incident (orange) wave. Each ray travels a slightly different path/distance in the MTM as shown. The magenta ray refracts with a slightly smaller angle than the red ray, and thus exits the other face of the MTM slab at a closer location than the red ray. Moreover, these rays also exit the other face with their original angle of incidence (as required from phase matching). This causes the exiting rays to point towards a focal point at some distance away from the exit face.

This is the reason for the re-focusing seen in Fig. 5.11 and Fig. 5.12. Thus, the waist of the incident Gaussian beam is not only shifted laterally (or splitted and shifted in case of the hetero-junction), but is also refocused at some distance away from the device, before diverging again. This enables an area of high concentration to develop in the near/intermediate region of the device.

5.9 Chapter Conclusions

A flat low-profile focusing device is presented, using a hetero-junction of anisotropic low permittivity metamaterials (MTMs). These MTMs have the unique ability to refract the wave
obliquely away from the normal and guide them along the interface, allowing for a thin structure. The proposed device can focus an incoming plane-wave or a Gaussian-wave into an area of high power concentration, at a distance much closer than lenses with low $f/D$ ratios for the same size requirements, as well as the recently reported arrays of optical antennas and Fresnel lenses. The proposed hetero-junctions are realizable at optical frequencies using periodic bilayers of metal and dielectric. The proposed hetero-junction maybe used in the low-loss focusing mode for applications requiring light concentration such as concentrated photovoltaics, non-imaging optics, micro and nano Fresnel lenses, photodetectors, etc., as well as in the high-loss mode for realizing thin absorbers/emitters for solar and/or thermo photovoltaics. The same hetero-junction was shown to also operate beyond the regime of concentration when illuminated from the back. This opens a door to realizing near-field beam-splitter and beam-shifters, using a very thin and low profile device, with potential applications in various areas such as optoelectronics and fiber optics.
Chapter 6

Dirac Leaky-Wave Antennas

6.1 Introduction

Leaky-wave radiation involves gradual leakage of electromagnetic energy over a structure, most commonly utilized in the context of Leaky-Wave Antennas (LWAs). This simple gradual leakage of energy enables radiation of highly directive and narrow beams from the antenna. LWAs are also capable of scanning the beam direction by simply changing the frequency of operation. LWAs have been traditionally used and extensively studied at microwaves [3, 48, 49]. But these structures offer capabilities that also make them excellent candidates for new frontiers in the terahertz (THz), infrared (IR) and even optical applications [143].

The radiation of a unidirectional leaky wave towards broadside has been a challenging and elusive concept. Continuous leaky structures such as slot waveguides are only capable of forward scanning above the cutoff frequency (broadside) of the leaky mode [3]. Periodically perturbed waveguides, such as periodically loaded dielectric waveguides [144, 143] are capable of both forward and backward scanning away from broadside, but typically have an open stopband around broadside due to mode coupling [3, 145]. Hence, such structures so far could never achieve true broadside radiation and could only radiate ‘close’ to broadside [3]. Consequently, these structures are typically mismatched at their input terminals around broadside frequencies. By center feeding a leaky structure, it is possible to produce a single broadside beam from the two opposing beams leaking from the two sides of the feed [146]. However, this beam quickly splits into two halves as the frequency changes, preventing a scanning operation. Moreover, a typical side-fed (unilateral) excitation of these LWAs (and other typical leaky structures) can never achieve true broadside radiation.

It was demonstrated at microwaves that the stopband can be closed in transmission-line metamaterial LWAs, by “balancing” the shunt and series transmission-line impedances [147, 37, 148, 149]. Such antennae can be simply identified as transmission-lines with sub-wavelength unit cells having both series and shunt lossy resonators. When the two resonant frequencies are made equal, a closed stopband situation arises. Additionally, the existence of both series and shunt radiation resistances that have a specific ratio in the unit cell is a necessary condition
to achieve actual broadside radiation even under the balanced condition [148]. Elimination of
the open stopband in one-dimensional periodic printed leaky-wave antennas at microwaves was
demonstrated in [150], using a stub-matching technique in the unit cell. These concepts have so
far been demonstrated only at microwaves. Primarily this is because they are either applicable
to periodic structures with a very sub-wavelength unit-cell period (\(\ll \lambda_g\)) such as TEM and
quasi-TEM mode (mainly micro-strip type) LWAs, or require situations where it is possible to
realize printed-circuit or stub elements, or more complicated unit-cells with additional mecha-
nisms [150, 149], and/or situations where it is appropriate to apply transmission-line modeling
as in [147, 37, 148, 149] to determine the balanced condition and close the stopband.

To date however, neither a dielectric leaky-wave antenna has been proposed that has a closed
broadside stopband; nor has a leaky-wave antenna been demonstrated based on a photonic
crystal that is devoid of a Γ-point bandgap, and satisfies the appropriate radiation mechanisms
(similar to [148]), to enable true unilateral broadside radiation and continuous beam scanning.
Such structures are extremely useful for reliable leaky-wave operation, especially at higher
frequencies beyond microwaves. Some dielectric based and optical LWAs have been reported
thus far in the literature [144, 151, 152, 143, 153] which, however, have not demonstrated
continuous scanning of a true leaky-wave beam through broadside.

Recently [43] reported a remarkable discovery of an effective zero-index behavior in photonic
crystals (PCs). This zero-index PC is a typical two-dimensional (2D) photonic crystal operating
in a regime where two inverted Dirac dispersion cones meet “at the Γ-point” of the PC, due to
an accidental degeneracy of modes. Reference [43] showed that this leads to an effective zero-
index behavior around the Γ-point, and verified this by effective medium theory beyond the long
wavelength limit [154]. Based on the work of [43], [45] recently demonstrated the all-dielectric
zero-index photonic crystal for directive emission of point sources at infra-red frequencies.

In this Chapter, the Dirac type PCs of [43] and [45] are utilized to demonstrate that these
structures can be made to radiate in order to realize directive Dirac Leaky-Wave Antennas
(DLWAs). The proposed class of DLWAs has an important advantage over common LWAs as
they are capable of radiating through broadside as the directive beam is scanned by varying
the frequency. The DLWA has a closed stopband, matched response, non-fluctuating leakage
constant, and real non-zero Bloch impedance at and around broadside. This is achieved by
utilizing the Dirac-type dispersion of the PC at the Γ-point, as well as appropriate shunt
and series radiation mechanisms in the unit cell. The DLWA is also capable to steer or scan
the directive beam over a wide range of angles off-broadside (in both forward and backward
directions) simply by changing the frequency, with a matched response. These DLWAs can
open doors for continuous and reliable leaky-wave operation at microwave to THz and optical
wavelengths.
6.2 Exploiting the Dirac Dispersion of PCs for Leaky-Wave Radiation

Dirac cones in dispersion diagrams and band diagrams of crystals have long been known, for instance in graphene, and are mainly observed at the band edges of the crystal [43, 45]. The 2D Dirac PC of [43] however, is a unique PC which attains such Dirac cones at the Γ-point of the PC, and thus offers some unique features.

As shown in the dispersion diagram of Fig. 6.1 (a), there are two inverted dispersion cones (Dirac cones) touching at a single Dirac point \( f_D \). Moreover, the PC is designed such that this Dirac point occurs at the Γ-point \( (k_x = k_y = 0) \). The dispersion bands on either Dirac cones have a linear change in frequency when traversing the Γ−X path on the Brillouin zone near the Dirac point. The continuous and linear dispersion curves are actually the characteristic feature of the Dirac point [43]. Each band is shown in [43] to comprise an electric monopole and a transverse magnetic dipole. At frequencies just about the Dirac point, the medium behaves as an effective zero-index medium in an almost isotropic manner as shown in Fig. 6.1 (b). Such effective zero-index behavior was verified in [43] using effective medium theory beyond the long wavelength limit [154]. Therefore the PC supports a propagation that has a gradual change in the transverse wave-number as the frequency changes, including the Γ-point. This Dirac-type dispersion behavior also implies that the photonic crystal has a closed bandgap at the Γ-point at a particular frequency \( f_D \), as a result of the two inverted and touching dispersion cones. The dispersion diagrams and mode chart results of Fig. 6.1 are generated for a Dirac PC similar to the examples of [43] and [45] for clarity.

It was found in [43] that the Dirac point can occur at the Γ-point as a result of an “accidental degeneracy” of modes in the PC. Accidental degeneracy of eigenmodes can occur both in dielectric resonators [155] and in dielectric PCs [43]. In accidental degeneracy, multiple orthogonal eigenmodes of a structure become degenerate (resonate at the same frequency), but this degeneracy is “accidental” as it is not due to symmetry of the structure or lattice. Such a multi-resonant effect typically occurs at a certain set of dimensions given the boundary conditions. Accidental degeneracy of three-dimensional (3D) resonant modes of a typical cylindrical (puck-shaped) and half-cut dielectric resonator was previously demonstrated in [155] to realize quadruple-mode and dual-mode dielectric resonators. At the degenerate point, a single physical cylinder was shown to sustain four orthogonal eigenmodes at the same frequency. This allowed the development of ‘single cavity four-pole’ filters, offering a significant size reduction [155].

The accidental degeneracy of Bloch eigenmodes in the photonic crystal of [43] can be simply seen with the trend in the resonance frequencies of the allowable Bloch eigenmodes with respect to a change in a critical geometrical dimension. This is best exemplified through the mode chart as shown in Fig. 6.1 (c) for the unit cell of the 2D Dirac PC of [43] and [45], similarly to what was previously shown for modes of a dielectric resonator in a metallic cavity [155]. The PC is composed of periodic posts or rods with a square cross section of side ‘w’ and high permittivity.
Figure 6.1: Exploiting a photonic crystal with a Dirac-type dispersion for leaky wave radiation. (a) Two inverted Dirac dispersion cones touching at the Γ-point of the Brillouin zone, showing a closed band-gap in a 2D Dirac-type PC. (b) Iso-frequency contours above the Γ-point showing an iso-tropic zero index behavior in the PC. (c) Mode chart of three eigenmodes at the Γ-point around the accidental degeneracy dimensions for the 2D Dirac PC. Inset shows the periodic arrangement of the PC. (d) The Dirac photonic crystal with an interface to air for leaky-wave radiation.

(e.g. Si, $\epsilon_r = 13.7$), arranged in a lattice with period ‘a’ inside a lower permittivity host medium (e.g. SiO$_2$, $\epsilon_r = 2.1$) as depicted in the inset of Fig. 6.1 (c). At a specific fill factor of the dense dielectric ($w/a$), three TM$_n$ Bloch modes (a pair of symmetrically degenerate modes and a third mode) are accidentally degenerate and resonate at the same Dirac frequency ($f_D$), for the Γ-point boundary conditions applied to the unit cell. Due to their orthogonality, the modes do not couple and can co-exist at the same frequency at the degenerate point.

The central idea here is that at the interface of such a PC with air, like the one shown in
Fig. 6.1 (d), a leaky mode may be supported of which its propagation constant is dependent on frequency. More importantly, the propagation can exist even at the Γ-point with no frequency gap. Note that a bandgap at the Γ-point of a leaky photonic crystal is equivalent to the open broadside stopband in 1D LWAs. Therefore we can achieve backward and forward, as well as broadside (at the Dirac point) leaky radiation from the interface of a PC with such Dirac dispersion behavior, resulting in the Dirac Leaky Wave Antenna (DLWA). The resulting DLWA can also be matched at the Γ-point as there is no stopband and it also has both of the necessary shunt and series radiation resistances in the lossy resonator model of the unit cell, as will be discussed later.

6.3 The Leaky Dirac Photonic Crystal

In order for the PC to couple to propagating waves in air, we need to create an interface between the PC and air. Depending on the scenario, one could envision different mechanisms for creating a leaky interface.

Let us consider the unit cell of the top perforated leaky Dirac PC for 2D propagation in Fig. 6.2 (a) and for 1D propagation in Fig. 6.2 (b). The unit cells are made of a high permittivity dielectric rod inside a low permittivity host medium in both cases. To enforce the electric field to lie along the $z$-axis, the rod and host are placed between two parallel perfect electric conductors (PEC).

The 2D PC case therefore resembles a dielectric filled, Parallel-Plate Waveguide (PPW). As the 2D PC is infinitely periodic in the $x- y$ plane, the opposing sides of the cell are covered with periodic boundaries in order to enforce a Floquet periodicity of $(k_x, k_y)$ between them. The 1D PC is similarly periodic in the $x$–direction, but in the transverse direction ($y$–direction) the cell is terminated with PEC walls. This turns the unit cell into a section of a dielectric-loaded rectangular waveguide. The top PEC plate is then perforated to allow the leakage of fields outside the guides.

The perforations in the top metal allow for coupling of the mode(s) inside the parallel plate walls to spatial modes outside. The similarly sized slots are designed such that the modes also have a finite (due to radiation) and comparable quality factors $Q$, which is a representation of the radiation of the modes and the leakage constant. The design procedure of the cell is explained in the next section. The perforations are placed such that the overall configurations still support the original triple (2D) and double (1D) modes of the guide. The perforations can be in the form of different shapes such as circular or square, placed at appropriate edges of the unit cell. It should be noted that by creating the top perforations in the guide, the original Dirac point of the un-perforated PC breaks up and shifts in frequency, i.e. the accidental degeneracy of the new crystal differs from the original PC in a non-perforated waveguide (a similar remark applies to the 2D case). Therefore, we adjust the filling ratio of the dielectric to achieve another degenerate point, and adjust the period in order to account for the frequency
Figure 6.2: A leaky Dirac photonic crystal. Unit Cell of the (a) 2D Dirac PC and (b) 1D Dirac PC with top perforations realized with slots in the metal waveguide. In (b) the periodicity is in the $x-$direction, and PEC walls terminate the cell in the $y-$direction (as opposed to being periodic as in (a)).

...shift, using the open eigenmode simulations described herein.

In the 2D-leaky case, simulations reveal that the three TM$_n$ Bloch modes of the Dirac PC (as discussed earlier and reported in Fig. 6.1 (c)) are established between the parallel plates. These three modes can be made degenerate (with appropriate adjustment of the dimensions) at the $\Gamma$-point in order to establish the touching Dirac dispersion cones for 2D propagation. Two of these three modes are actually dual degenerate (as shown in Fig. 6.1 (c)) due to symmetry.

In the 1D-leaky case, eigenmode simulation shows that only two Bloch eigenmodes are established inside the waveguide section given the periodic boundary condition established between the two ends of the guide, around the frequency of interest. The two modes can again be made degenerate at the $\Gamma$-point of the Brioullin zone with the appropriate dimensions of the post. These are two of the original three TM$_n$ Bloch modes of the Dirac PC. In the 1D leaky-
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In the 1D case, we are only dealing with 1D propagation, and only two TM\(n\) Bloch modes are required to become degenerate in order to achieve a Dirac-type dispersion behavior. One of the symmetric dual-degenerate modes from the 2D propagation case is not used in the 1D propagation.

The two Bloch modes of the 1D-leaky case are shown in Fig. 6.3 (a), where they are both degenerate at the Γ-point of the Brioullin zone, i.e. at \(k_x = 0\), which leads to a 1D Dirac-type dispersion for the PC. These two Bloch eigenmodes are orthogonal by definition for the given boundary conditions. Any field that exists outside the guide must also meet these boundary conditions at the unit-cell walls. If the eigenmode is radiating, then propagating waves outside the guide in air would propagate away from the top surface of the guide. This is shown graphically in Fig. 6.3 (b) with the magnitude of the real part of the electric field outside the unit cell. The figure depicts propagating waves emanating away from the top surface for the two radiating eigenmodes.

Figure 6.3: Accidental Degeneracy of two eigenmodes in the leaky Dirac PC at the Γ-point. (a) Normalized E-field magnitude in the \(x - y\) plane for the two eigenmodes of interest at the Γ-point frequency of the 1D Dirac PC. (b) Normalized \(|\text{Real}\{E\}|\) in the \(x - z\) plane for the two leaky eigenmodes at the Γ-point frequency.

6.3.1 Design of the 1D Leaky Dirac PC Unit-cell

We begin from the unperforated (non-leaky) 2D Dirac PC of Fig. 6.1 as a starting point in the design towards the 1D leaky Dirac PC. Around the Dirac frequency, the Dirac PC will have an effective zero index. Therefore in the 1D PC cell of Fig. 6.2 (b), if the metallic walls of
the waveguide touch the side-walls of the unit-cell (the dashed lines in Fig. 6.4 (c)), we are faced with a zero-index filled waveguide which will be shorted. Therefore, one needs Perfect Magnetic Conductors (PMCs) on the two sidewalls of the unit cell. This was done in [43] by using mushroom type PMC structures. An alternative simple approach is implemented here. The walls of the guide are simply extended by about $\lambda_0/(4n_{host})$ on either side of the guide, to convert the electric short into an electric open at the two sides of the PC. Therefore an open boundary condition at the two sides of the unit cell is created at the Dirac frequency allowing the two modes of interest to exist. This non-leaky 1D PC with extended sidewalls can have a closed bandgap when the Dirac-type dispersion is established due to the accidental degeneracy of its two Γ-point eigenmodes.

We then introduce perforations to the unit cell to allow for leakage, leading to the 1D leaky PC unit-cell of Fig. 6.2 (b). Similar sized slots were used for this purpose to make the $Q$ of both eigenmodes finite (both radiating) and of comparable magnitude to enable true broadside radiation as well as proper matching [148].

This however perturbs the PC and thus may separate the frequency of the Γ-point eigenmodes from each other, so much so as to open a stopband in the leaky PC. To account for this impact of the slot perforations and to minimize their effects as much as possible, we then perform eigenmode simulations to tune the “1D leaky PC unit-cell”. In this step we adjust the leaky unit-cell accordingly to again establish the Dirac dispersion in the 1D leaky PC cell. By proper adjustment of the post dimension and period, we make the Γ-point ($k_x = 0$) eigenmodes of the leaky unit-cell degenerate again. This was possible as these two eigenmodes are orthogonal and can coexist at the same frequency at the Γ-point. Thus we arrive at a 1D leaky Dirac PC unit-cell.

A similar strategy can be taken for tuning the three Γ-point ($k_x = 0, k_y = 0$) eigenmodes of the 2D leaky PC unit-cell of Fig. 6.2 (a) to become accidental degenerate, in order to establish the Dirac dispersion for a 2D leaky periodic structure, i.e. arriving at a 2D leaky Dirac PC unit-cell.

The eigenmode simulations of the leaky PC unit-cell are carried out using the Finite Element Method eigenmode solver in Ansoft HFSS v15. Absorbing Perfectly Matched Layer (PML) boundaries are placed in the transverse plane around the cell (at a distance away from the structure) to mimic the radiation condition. Periodic boundaries are placed in the direction of periodicity, covering the entire area and extended to also cover the relevant sides of the PMLs. Γ-point condition is applied to these periodic boundaries for the tuning stage.

Hence the final stage of the design of the 1D DLWA involved tuning the filling ratio and period of the leaky PC unit-cell to achieve degeneracy of the two eigenmodes at the Γ-point with finite $Q$s. This step helps to re-establish the Dirac dispersion in the Leaky PC cell and achieve the necessary radiation conditions as in [148]. We then directly cascaded N of such cells into an N-cell DLWA. The reported results for the DLWAs are using this design approach. As the reported results indicate, we readily reach a very well performing N-cell antenna, without
6.4 The Dirac Leaky-Wave Antenna

The proposed 1D Dirac-type Leaky-Wave antenna is shown in Fig. 6.4 with the 3D (a), top (b), and front cross-section (c) views. This antenna is made directly from cascading the leaky Dirac PC unit-cell that was described previously and shown in Figure 6.2 (b). The design of the antenna therefore mainly simplifies to designing the appropriately tuned leaky unit-cell as explained in the Methods section. The cross section of the antenna (in the \( y-z \) plane) is simply the metallic rectangular waveguide filled with the host dielectric and loaded with one high permittivity rod. The length of the antenna (in the \( x \)-direction) has enough cells to radiate a desired amount of input power (typically > 90% in LWAs). The structure is fed from one of its ends and can be either terminated at the other end with a matched load (as done here) or left open.

The structure of the DLWA in Fig. 6.4 has a period of \( a = 12.4 \mu m \) (in the \( x \)-direction), a substrate height of 3.9\( \mu m \) (in the \( z \)-direction), and a waveguide width of 21.28\( \mu m \) (in the \( y \)-direction). The dielectric rods are at the center of each cell and have a square cross section with dimensions 5.222\( \mu m \times 5.222 \mu m \) (in the \( x-y \) plane). The perforations in the top metal of the guide are slots with dimensions 8\( \mu m \times 1.5 \mu m \) (parallel to the \( x \)-axis) and 8\( \mu m \times 3 \mu m \) (parallel to the \( y \)-axis). The 19-cell antenna has a total length of 235.6\( \mu m \) in the \( x \)-direction. The leaky region is connected to an incoming air-filled metallic waveguide of equal cross section. Note that the incoming air-filled waveguide has only its fundamental \( TE_{10} \) mode above cutoff. The second cut-off frequency is at 14THz, away from the design frequency.

The low dielectric constant of the host material is \( \epsilon_r = 2.79 \), and the high dielectric constant of the rods is \( \epsilon_r = 12.11 \), similar to [143]. The choice of these PC materials can be altered based on the frequency of operation (microwaves, THz or optical) and availability of material/technology, losses, etc. As long as there is enough contrast between the permittivity of the rods and the host to sustain the Bloch modes of interest with no immediate spurious modes in their close vicinity in terms of frequency, the choice of materials is flexible. For instance [45] demonstrated experimentally that Si and SiO\(_2\) may be used at \( \lambda_0 = 1390nm \) for the rods and the host, respectively.

The S-parameters of the DLWA are reported in Fig. 6.5 around the Γ-point, showing a remarkable performance in the context of existing LWA literature. We can see that the antenna is perfectly matched (\(|S_{11}| < -20dB\)) for frequencies at and close to the Γ-point of the PC, i.e. all of the incident power is accepted. The low transmission to the port 2 (\(|S_{21}| < -10dB\)) also indicates that very little portion of the input power remains at the end of the antenna (almost all of the incident power is radiated out). The simulated structure is assumed to be lossless (aside from radiation), although losses can be accounted for as discussed later.

This dipping trend of \(|S_{11}|\) of the DLWA is a behavior completely opposite to that of typical
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Figure 6.4: A one dimensional Dirac Leaky-Wave Antenna or DLWA. (a) 3D and (b) top views of the 19-cell DLWA made from cascading the unit-cells of the leaky Dirac PC. Unilateral excitation of the antenna from port 1. Residual power is collected at port 2 which is matched to the leaky structure. Port 2 can be left open in long antenna where all power is radiated. (c) Cross section of the antenna showing placement of the high permittivity rods and sidewall extension.

periodic LWAs with a broadside stopband. Periodic LWAs typically have a rise in $|S_{11}|$ near the broadside frequency (referred to as increased VSWR) [3, 150], which becomes more serious with increasing the antenna length (the closer they approach to the infinitely periodic case). This is definitely not the case here and hence this is one of the indications that the DLWA is markedly different.

6.5 Radiation Characteristics

The radiation pattern of the DLWA is reported in Fig. 6.6 (a), for the principal $\phi = 0^\circ$ plane, showing a directive beam pointing exactly at broadside. A gain of 14.1dB is attained exactly at broadside for this 19-cell example. Note that all reported gain values are essentially ‘realized gain’ values, as the antenna is fully matched, as was shown in Fig. 6.5.

As the frequency is changed about the broadside frequency, the beam direction varies as
Figure 6.5: Perfect matching of the DLWA around broadside. S-parameters of the DLWA for three antenna lengths close to broadside (solid lines are $S_{11}$, dashed lines are $S_{21}$). Perfect matching ($|S_{11}| \to 0$) shows a closed bandgap at the $\Gamma$-point, as well as close to zero transmission ($|S_{21}| \to 0$) shows complete radiation of power for all lengths.

shown in Fig. 6.6 (b). The beam is highly directive with negligible variation in directivity around broadside, which can in fact be further mitigated in refined designs by adjusting the $Q$ of the $\Gamma$-point eigenmode that contributes less to the broadside radiation. This shows that the antenna can sweep a beam from the forward direction through broadside, and to the backward direction, and vice versa by changing the frequency, without any detriment to any of the antenna performance metrics or gaps in radiation.

The sidelobe levels are low and in fact there is only one main beam with ripples on the pattern due to the finite length of the antenna simulated (an infinite structure would not have any ripples). The three-dimensional gain plots of the DLWA are shown in Fig. 6.7 which demonstrate the directive fan-beam nature of the pattern as expected from a 1D leaky structure. The results pertaining to longer antennae confirm that the DLWA indeed has a closed broadside stopband, as the antenna becomes longer. The S-parameters in Fig. 6.5 show that all power is again accepted (dipping $S_{11}$) and more power is radiated (lower $S_{21}$) as the antenna length is increased. The gain of the broadside beam also increases to 16.4dB, and the pattern ripples are almost completely removed with no sidelobes, as the length of the DLWA is increased from 19 cells to 57 cells, as seen in Fig. 6.6 (a). This further confirms broadside radiation is in fact occurring due to a true leaky wave and is not due to finite-LWA effects.
Figure 6.6: Directive broadside radiation of the DLWA. 2D Gain plot of the broadside beam in the $\phi = 0^\circ$ plane for three different antenna lengths (a). Figure shows broadside radiation is in fact occurring due to a true leaky wave and is not due to finite antenna effects, at 9.892 THz. As the length of antenna is increased, the gain increases, and the pattern ripples are removed. (b) Radiation around broadside for the 19-cell DLWA, in the $\phi = 0^\circ$ plane.
Figure 6.7: 3D Radiation pattern of the DLWA. Gain of the 19–cell DLWA (in a 40dB scale range) showing a narrow fan beam pointing right at broadside, at 9.892 THz.

It is emphasized that the resulting DLWA does not experience mode coupling/cancellation at the Γ-point, contrary to typical LWAs. Rather, it utilizes the orthogonal degenerate Bloch TM eigenmodes of the PC and directly establishes fast waves at its interface with air. The accidental degeneracy guarantees a closed bandgap in the dispersion diagram of the resulting photonic crystal. We make the connection here that a Dirac-type dispersion behavior is equivalent to the closed stopband condition in metamaterial transmission-lines [37], and along with the shunt+series radiative losses (see Supplementary Methods) leads to the true broadside leaky-wave radiation from the PC.

6.5.1 Zero-index behavior of the DLWA

The broadside radiation can also be interpreted from a zero-index perspective. In fact, our DLWA is in effect a waveguide filled with an effective zero-index medium, such that the phase progression of fields along the surface and at the edges of each unit cell is zero, leading to in-phase radiating elements and broadside radiation. These two viewpoints are equivalent and converge to the same effect.

The original work of [43] showed that the Dirac PC is an effective zero-index medium, even though it is a photonic crystal. First [43] showed that the Dirac cones can occur at the Γ-point (rather than at zone edges as had been typically seen prior to that work). Hence a zero-index
behavior is possible when an effective medium can be applied. The latter condition is based on choosing dense enough materials to push the degeneracy point to as low a frequency as possible (to make a small unit cell with respect to the free space wavelength) such that the effective medium approach for these cells beyond the long wavelength limit [154] is acceptable. Reference [43] shows that the effective medium [154] does indeed describe the behavior of this PC around the Dirac point to be that of a zero-index medium with a finite impedance and non-zero group velocity. Similar results can be obtained for the leaky PC. A similar effective zero-index behavior viewpoint is reasonable for the DLWA. It is also worth noting that even if an effective medium theory could not be applied, the DLWA would still be of closed stopband since the two modes are still degenerate and orthogonal at the Γ-point forming the Dirac cones as in [43].

Reference [43] identifies each of the Dirac cone modes of the Γ−X direction of propagation (each mode that constitutes a branch of the Dirac cone dispersion) with an electric monopole and a transverse magnetic dipole within the unit cell. We can identify exactly the same electric monopole and transverse magnetic dipole of [43], for the 1D unit-cell as shown in Fig. 6.8. Results are shown according to the Supplementary material of [43] and it can be seen to match identically. Note that the third longitudinal magnetic monopole of [43] is not of relevance here in the 1D situation. We utilize similar materials and dimensions, such that our unit-cell period is \( a = 0.413\lambda_0 \) for the examples shown which is in fact slightly smaller than the case in [43], which was about \( a = 0.541\lambda_0 \).

### 6.6 Leaky-Wave parameters

The extracted dispersion diagram of the antenna is shown in Fig. 6.9 for frequencies close to the Γ-point. It can be seen that there is no stopband and the dispersion curve continuously links the backward region to the forward region without any gap or change in the linear trend. Moreover, the group velocity is not zero at broadside (i.e. the tangent to the curve does not have a zero slope) confirming that broadside radiation is indeed possible.

The zero-index behavior is consistent with a zero phase progression of the field along the length of the antenna, leading to broadside radiation when excited unilaterally and with no reflected waves. This zero phase progression is demonstrated in Fig. 6.10, which shows the plot of the phase of the electric field sampled at the center of the slots, for the frequency corresponding to the Γ-point of the DLWA with 57 cells. The exponential decay of the field magnitude due to the unilateral leaky operation can also be confirmed in the figure.

The attenuation (leakage) constant \( \alpha \) and phase constant \( \beta \) were obtained using S-parameter retrieval from a driven \( N \)-cell simulations. It is found that at \( f_D = 9.892\text{THz} \), \( \beta = 0 \) which is the Γ-point or the Dirac degeneracy frequency. The leakage constant shown in Fig. 6.9 (b) is found to be \( \alpha/k_0 = 0.0267 \) at \( f_D \). The figure also depicts how the leakage constant does not vary close to the Γ-point, contrary to typical LWAs. Various effects such as an open stopband
Figure 6.8: Eigenmode of the DLWA unit-cell with no slots (periodically loaded waveguide), at \( f = f_D - |\Delta f| \), for BC corresponding to slightly to the right of the \( \Gamma \)-point (\( \beta = 0.001\pi/a \)). (a) Real\( \{E_z\} \) and Imag\( \{\vec{H}\} \) (b) Imag\( \{E_z\} \) and Real\( \{\vec{H}\} \). Another identical eigenmode with reverse sign of field components also exists at \( f = f_D + |\Delta f| \) for the same BC, which is not shown here for brevity. These two modes meet at the same frequency \( f_D \) at the \( \Gamma \)-point. Periodicity is in the \( x \)-direction. PEC walls terminate the cell in the \( y \)-direction (as opposed to being periodic as in a 2D crystal). Electric field is out of plane \( (E_z) \).

or not supporting the appropriate radiation mechanisms typically manifest themselves with abrupt variations in the leakage constant around broadside in LWAs [148].

The size of the perforations provides a control mechanism to tailor the leakage factor. Adjustment of the perforation size is done with the appropriate minor adjustments of the filling ratio and/or period to ensure that the degeneracy point (closed stopband) is maintained at the desired design frequency for broadside radiation.

The behavior of the 1D leaky Dirac PC unit-cell is modeled with a lossy series resonator and a lossy shunt resonator around the \( \Gamma \)-point. The extracted resonances \( (f_{se} \approx f_{sh} \approx 9.892\text{THz}) \) and the fitted circuit-model parameters \( (L_{se} = 1 \times 10^{-11}\text{H}, C_{se} = 2.5871 \times 10^{-17}\text{F}, R_{se} = 8.113\Omega, L_{sh} = 2.25 \times 10^{-13}\text{H}, C_{sh} = 1.15 \times 10^{-15}\text{F}, G_{sh} = 5.738 \times 10^{-4}\text{S}) \) of the unit cell show that the stopband of such a structure is indeed closed. Equally important, there exists both a series resistance and a shunt conductance (meaning both a series radiation resistance and a shunt radiation conductance) in the circuit model of the unit cell and in an appropriate ratio to ensure matching at broadside. This condition guarantees that the Bloch impedance of the periodic structure can be made real and finite at the \( \Gamma \)-point, with little variation over the frequency. This in fact can be seen as a necessity for correct broadside radiation of a general periodically leaky unit cell (regardless of its dimensions with respect to the wavelength) and encompasses the condition of [148] for leaky metamaterial transmission-lines with sub-wavelength unit-cells.

The Bloch impedance \( (Z_B) \) close to broadside is shown in Fig. 6.11 (a). The shunt and series radiation element values are such that the Bloch impedance of this periodic structure
around the Γ-point is 119.4Ω (also equal to $\sqrt{R_{se}/G_{sh}}$). This is very close to the characteristic impedance of the incoming waveguide (121Ω), therefore allowing a perfect matching at broadside. The Bloch impedance is also fairly constant around the broadside frequency, enabling the consistent matching of the antenna as the beam is scanned from backward to broadside to forward direction.
Figure 6.10: Aperture field along the length of the antenna ($x$-direction) sampled at the center of the slots, for broadside radiation at 9.892 THz. Logarithm of the normalized $|E_x|$ (black curve) showing leaky-mode exponential decay due to radiation. Phase progression of $E_x$ (blue curve) showing a zero slope, i.e. $\beta = 0$, which is consistent with the ZIM behavior.

The 19-cell example radiates about 93% of the incident power as typically required in LWA literature [3, 48, 49]. This is shown in Fig. 6.11 (b), which is the plot of the quantity $1 - |S_{11}|^2 - |S_{21}|^2$. Longer antennae provide higher amounts of radiated power as also seen in Figure 6.11 (b). The 38 and 59 cell DLWAs radiate almost 100% of the incident power. These longer antennae were simulated mainly for validation purposes of the closed stopband and to show broadside radiation is not due to reflection from the end or other finite effects. Thus the 19-cell design may be considered sufficient for all practical purposes. The design of the leakage constant was done such that the DLWA had a reasonable length to radiate this much power and to be manageable for simulation purposes. Higher and lower leakage constants are also possible as mentioned earlier.

### 6.6.1 Extraction of Leaky-Wave parameters around broadside

The two-port S-parameters of the 19-cell antenna are found using full-wave simulations in Ansoft HFSS and then converted into an $ABCD$ transmission matrix. Then the $ABCD$ matrix of one cell is found:

$$ABCD_1 = 19^{\sqrt{ABCD_{19}}}$$

(6.1)

Where $ABCD_1$ and $ABCD_{19}$ are matrices of a single unit cell and the 19-cell structure, respectively. All 'extracted' parameters, such as the phase ($\beta$) and leakage ($\alpha$) constants, Bloch impedance ($Z_B$), and circuit model parameters are then found from the $ABCD_1$ matrix.
Figure 6.11: Bloch impedance and radiation performance for different lengths of the DLWA. (a) Real (solid) and Imaginary (dashed) components of the Bloch Impedance of the DLWA around the Γ-point frequency extracted from the 19—cell simulation. (b) Amount of radiated power from the DLWA around broadside.

Similarly for the 38 and 57 cell examples, $ABCD_1 = 3\sqrt{ABCD_{38}}$ and $ABCD_1 = 3\sqrt{ABCD_{57}}$ were used respectively. This multi-cell simulation approach was done to extract the full periodic effects of the PC and have as much resemblance to an infinitely periodic structure. A single cell simulation using driven ports does not account for the full coupling of the Bloch modes.
between adjacent cells in an infinite structure. In this method we are capturing both periodic effects as well as any input/output coupling effects on the simulations.

### 6.6.2 Two-resonator Circuit model for the Leaky Dirac PC

We apply a circuit model to the leaky PC unit cell, showing that:

1- The unit cell of the leaky Dirac PC can be modeled with a circuit having both a lossy series and a lossy shunt resonator.

2- Important implications arise from such a simple model regarding the PC behavior, such as confirming that the stopband is closed and a real and constant Bloch impedance can be achieved near the broadside frequency, enabling matched operation.

We first postulate and rationalize, without a formal proof, that the $\mathbf{ABCD}$ matrix of a single unit-cell of the leaky Dirac PC (extracted from the full-wave simulations) can indeed be represented by the $\mathbf{ABCD}$ matrix of a two resonator circuit around the frequency of degeneracy. Note that a one-to-one structure-to-circuit-component correspondence may not be easily seen with this structure, contrary to what is possible in transmission-line metamaterial unit cells which support quasi-TEM modes (e.g. microstrip MTMs [37, 148]). However, we can still model the unit cell’s two-port behavior with a simple circuit to explain its dispersion and matching characteristics.

The 1D PC has two resonant Bloch eigenmodes around the degeneracy point. A single resonance can always be modeled with an inductor/capacitor or LC resonator. The two Bloch eigenmodes are also orthogonal by definition (they can even occur at the same frequency as in the accidental degeneracy dimensions). Therefore any circuit model of the unit cell where both resonances occur must decouple these two resonances. This can be achieved with a separate series and shunt resonator (similar to the modeling in [1]), such that the two resonator circuit elements do not combine into one resonance. Moreover, as each of these eigenmodes has a finite $Q$ (due to radiation losses), they are therefore lossy resonators, i.e. resistor/LC ($RLC$) or conductance/LC ($GLC$) circuits.

The $\mathbf{ABCD}$ matrix of a circuit having both a series impedance ($Z_{se}$) and a shunt admittance ($Y_{sh}$) is:

$$\mathbf{ABCD}_{se \times sh} = \mathbf{ABCD}_{se} \times \mathbf{ABCD}_{sh} = \begin{bmatrix} 1 + Z_{se}Y_{sh} & Z_{se} \\ Y_{sh} & 1 \end{bmatrix}$$  \hspace{1cm} (6.2)

where

$$Z_{se} = R_{se} + j\omega L_{se} + \frac{1}{j\omega C_{se}}$$  \hspace{1cm} (6.3)

$$Y_{sh} = G_{sh} + j\omega C_{sh} + \frac{1}{j\omega L_{sh}}$$  \hspace{1cm} (6.4)

This means that inspecting the $B$ and $C$ elements of the $\mathbf{ABCD}$ matrix of such circuit,
independently reveals the series impedance and shunt admittance respectively.

\[ Z_{se} \text{ and } Y_{sh} \text{ represent two resonators, therefore each has a resonant frequency } f_{se} \text{ and } f_{sh} \text{ respectively:} \]

\[
f_{se} = \frac{1}{2\pi \sqrt{L_{se}C_{se}}} \]

\[
f_{sh} = \frac{1}{2\pi \sqrt{L_{sh}C_{sh}}} \tag{6.5}
\]

At each of these two resonances, the impedance/admittance is real such that \( Z_{se} = R_{se} \) and \( Y_{sh} = G_{sh} \).

We then inspect the elements of the \( ABCD_1 \) matrix obtained from full-wave simulations. Fig. 6.12 and Fig. 6.13 show the complex values of the inverse of the B and C matrix elements of \( ABCD_1 \) over frequency (blue curves), which are used for model extraction. These curves suggest that each of the elements correspond to a resonance. Moreover, the A and D elements are not shown but are found to be \( A = D \sim 1 + 0j \). These results confirm that the assumed series/shunt model for the unit-cell can capture the two-port behavior of the cell.

![Figure 6.12: Lossy (radiating) series resonator of the unit cell. (blue) Plot of the inverse of the B element of \( ABCD_1 \) near the \( \Gamma \)-point, found from a 19-cell simulation, and (red) the curve fit of the \( Z_{se}^{-1} \) circuit model.](image)

The frequencies where zero imaginary of \( B \) and \( C \) occur are taken as the resonant frequencies \( (f_{se} \text{ and } f_{sh}) \), and the real component of \( B \) and \( C \) are equated to \( R_{se} \) and \( G_{sh} \). We then use curve fitting to find the L and C components in each case, by fitting the \( L_{sh} \) and \( L_{se} \) values, and keeping \( C_{sh} \) and \( C_{se} \) as functions of their \( Ls \) according to \( (6.5) \).
The extracted resonances \( f_{se} \sim f_{sh} \sim 9.892\text{THz} \) and the fitted model parameters \( L_{se} = 1 \times 10^{-11}\text{H}, C_{se} = 2.5871 \times 10^{-17}\text{F}, R_{se} = 8.113\Omega, L_{sh} = 2.25 \times 10^{-13}\text{H}, C_{sh} = 1.15 \times 10^{-15}\text{F}, G_{sh} = 5.738 \times 10^{-4}\text{S} \) show that the stopband of this structure is indeed closed (balanced condition). Equally important, there exits both series resistance and shunt conductance (meaning both a series radiation resistance and a shunt radiation conductance) in the circuit model of the unit cell at an appropriate ratio to ensure broadside matching and radiation. This condition guarantees that the Bloch impedance of the periodic structure can be made real and finite at the Γ-point (in order to properly match with a real characteristic impedance), with little variation over the frequency. This in fact can be seen as a necessity for correct broadside radiation of a general periodically leaky unit cell (regardless of its dimensions with respect to the wavelength) and encompasses the condition of [148] for leaky metamaterial transmission lines with sub-wavelength unit-cells.

### 6.7 Scanning ability of the DLWA

Changing the frequency of operation is the most common method in LWAs to vary the angle at which the beam is pointing at [3, 48, 49]. Some techniques have also been proposed that enable fixed frequency scanning as in [156, 157] at microwaves. The continuous scanning or steering ability of the DLWA is depicted in Fig. 6.14 for the leaky beam in the \( \phi = 0^\circ \) plane. It can be seen that the DLWA has a wide scan range over the simulated bandwidth from 61° to
−37°, when changing the frequency from 8.3THz to 11.12THz. The antenna is matched with |S_{11}| < −13.5dB over the entire range of frequencies. The gain does not vary over this frequency range and the antenna keeps its performance with about 14dB to 16dB change in peak gain.

In the forward region of scanning, the transverse plane of the beam has two additional nulls in the pattern, therefore increasing the peak directivity of the beam slightly (by about 1dB). The sidelobe levels are about −10dB lower than each peak gain over the entire frequency range for the 19-cell example. These sidelobes are arising from pattern ripples due to finite size, and are greatly reduced by using longer antennae as was shown in Fig. 6.6 (a).

Figure 6.14: Scanning ability of the DLWA. Changing the beam direction from backward to forward direction, through broadside by varying the frequency. The scan range is from +61° (at 8.3THz) to −37° (at 11.12THz). No degradation in pattern shape or beam, and with less than 2dB gain variation over the wide scan range.

The limits of the scan range are determined by the existence of other spurious modes, which can be pushed farther with refined designs. For a general leaky-wave antenna, the half-power beamwidth is given by \( \Delta \theta \propto \frac{\lambda_0}{L \cos \theta_m} \), where \( L \) is the total length of the antenna, \( \theta_m \) is the angle the beam is pointing to, and \( \lambda_0 \) is the operating wavelength at which the beam angle is \( \theta_m \) [48]. Therefore we observe slight widening of the beamwidth towards the +60° scan direction (8.3THz), due to the shortening of the electrical length of the antenna at lower frequencies. A longer antenna with more cells has a larger electrical length at lower frequencies, and thus a narrower beamwidth is achieved at the lower frequencies of the scan range (the +60° direction).

The antenna can also be made periodic in the transverse direction (i.e. using more cells in the \( y \)-direction). This would utilize the 2D leaky PC in Fig. 6.2 (a), and again can be
Chapter 6. Dirac Leaky-Wave Antennas

terminated with extended PEC walls. Additionally, the proposed DLWA can be made to leak not only from the top/bottom as previously shown, but also leak efficiently from the sides by opening the sidewalls (instead of the closed sidewalls of the waveguide situation). It is interesting to note that an open end of a PPW does not radiate significantly, however the open end of the PPW filled with the Dirac PC can leak power quite efficiently. 6.15 shows 3D directivity plots for a hybrid, side-and top- leaky structure, realized with 4 cells in the transverse direction, and 20 cells in the x-direction. It can be seen that the fan beam now has peaks at $\phi = 0^\circ$ (due to top leakage) as well as $\phi = \pm 90^\circ$ due to the side leakage. Lastly, it must be noted that a 2D leaky Dirac PC can be used to realize sharp pencil beams radiating right at broadside, by exciting an $N \times N$ cell array of the DLWA, either from the center, side, or corner.

6.8 Losses

At microwave frequencies, metallic waveguides have minimal conductor losses compared to any other printed-circuit technology such as printed TEM structures. Therefore the DLWA has better loss performance compared to microstrip or planar type implementations as it is based on a rectangular waveguide. Moreover, it does not use sub-wavelength traces and elements which cause significant loss. Hence, the losses at microwaves are of little concern for this design, and the antenna provides a great deal of flexibility for designing various leakage constants for the desired amount of gain, at various microwave frequencies. DLWAs realized at microwaves not only provide an alternative to existing printed solutions with lower losses, they enable high power handling capabilities due to their waveguide nature which may not be attainable with existing printed microwave LWAs, for continuous beam scanning. Additionally, as DLWAs are based on resonant cells, their cells are larger than existing metamaterial solutions with respect to the operating wavelength. Thus fewer cells may be required to achieve the same numerical aperture and directivity, compared to existing LWA solutions with very sub-wavelength unit cells, which can ease fabrication.

At optical frequencies, metal losses are of higher concern, as well as dispersive effects of metals causing them to behave as non-metals. A dispersive model (such as Drude model) for the actual metal must be used in order to characterize the losses at optical frequencies. At low THz however, it is generally adequate to model the metal losses with the skin effect. For the 19-cell example shown here using the finite conductivity boundary condition in Ansoft HFSS and Aluminum having a DC conductivity of $3.8 \times 10^7$ S/m, and Copper with DC conductivity of $5.8 \times 10^8$ S/m, the antenna still radiates with a sharp pattern and peak gains of 8.6dB and 9.3dB at broadside, respectively. The antenna efficiency is about 41% for the case with Aluminum walls, and 46% for the case with Copper walls. Reference [158] shows that the skin effect model used along with the ‘finite boundary condition’ in Ansoft’s HFSS simulator may even overestimate the losses in a metal rectangular waveguide in the low THz frequencies. Therefore our simulations are a safe estimate of the deteriorating effects due to metal losses. This suggests
Figure 6.15: Leakage from sides and top of a DLWA. 3D Directivity plots for a hybrid top and side leaky DLWA in a 40dB scale range, showing a 16dB peak gain. Design for broadside frequency of 1 THz.

that a lossy implementation of the DLWA with metals at THz frequencies can yield perfectly usable gain levels and performance.

As metal losses are higher for IR and optical frequencies, it is more desirable to design the antenna with a metal-free slab of the Dirac PC and utilize leaky-wave radiation from its interfaces with air, similar to what was shown in Fig. 6.1 (d). This can be done in the form of a dipole at the surface or line-source exciting the PC, coupling of an external Laser source or a Gaussian beam from one side, evanescent wave coupling, etc. A common scenario would be to use a waveguide coupling. Fig. 6.16 shows the directivity plots of an all-dielectric DLWA made from a 4 × 15 cells slab of an all-dielectric PC operating in the telecom regime. The antenna
is excited with an incoming dielectric slab waveguide, as shown in the inset of Fig. 6.16 and it is shown that leaky radiation from the sides is also possible. The antenna is excited with a dielectric slab waveguide of a similar cross-section, as shown.

![Figure 6.16: Radiation from a Dirac PC slab. Directivity plots for a 4 × 15 cell all-dielectric DLWA in the φ = 90° plane. Inset shows the DLWA and the incoming dielectric waveguide.](image)

### 6.9 Chapter Conclusions

Leaky-Wave Antennas (LWAs) enable directive and scannable radiation patterns which are highly desirable attributes at terahertz, infrared, and optical frequencies. However, a LWA is generally incapable of continuous beam-scanning through broadside, due to an open stopband in its dispersion characteristic. This issue has yet to be addressed at frequencies beyond microwaves, mainly as existing microwave solutions (e.g. transmission-line metamaterials) are unavailable at these higher frequencies. The concept of the Dirac Leaky-Wave antenna is introduced which operates based on leaky-wave radiation from the interface of a photonic crystal with a Dirac-type dispersion behavior. The DLWA has a closed stopband as well as appropriate radiation mechanisms such that it is capable of true matched broadside radiation. The DLWA has a perfect matching, non-fluctuating leakage constant, and a real Bloch impedance at and around the Γ-point. The 1D structure can be made to radiate from different sides in order to create narrow and directive fan beams. The 2D version of the DLWA can also yield sharp
pencil beams aiming directly at broadside. With changing the frequency, the DLWA is capable of continuously scanning the beam off broadside at both forward and backward directions over a wide range of angles and without breaking up into two separate beams. A simple design procedure was also presented which only required the design and optimization of the unit cell (as opposed to optimizing the whole antenna). DLWAs can find various applications from microwave all the way up to optical frequencies to achieve highly directive leaky-wave radiation, including radar, THz and optical beam-shaping, spectroscopy, free-space multiplexed optical interconnects, efficiency enhancements in opto-electronic devices, etc. These DLWAs can be designed at microwave as well as terahertz to optical frequencies, with feasible dimensions and low losses.
Chapter 7

Summary and Conclusions

In this thesis, simple periodic structures were studied and utilized to enable new or improved methods of controlling electromagnetic fields. Different types of periodic structures, including periodic gratings, metamaterials, and photonic crystals were utilized. Emphasis was placed on low-permittivity and low-index media to harness their unique and unexplored behaviors for controlling the flow of electromagnetic fields. Unprecedented and improved levels of control including radiation enhancement, pattern shaping, concentration and focusing, beam splitting/shifting, and directive leaky-wave beam scanning was demonstrated. New and revised theory and analysis was also presented along the way, and various applications of the achieved levels of control were discussed.

Chapter 2 provided a theoretical framework using spectral techniques for the analysis of simple aperiodic dipole sources near periodic metal strip gratings, to rigorously analyze the evanescent-to-propagating wave conversion that takes place. By introducing the spectral impulse response (SIR) of the grating, a theory was provided which helped to capture all diffracted fields and clearly highlight the spectrum conversion phenomenon, and construct the Green’s function of the infinite strip grating. The method solves the well-known problem of the aperiodic excitation of a periodic grating. In this method, the plane-wave solution of the grating is converted to the spectral domain to determine the grating’s SIR. The spectrum of the fields of an arbitrary source in free space is also found separately, and the total response is calculated through a superposition integral. The SIR method allows identifying the various diffraction orders and harmonics, and provides insight and clear explanation of the evanescent-to-propagating wave conversion that takes place in sub-wavelength gratings, and can also be used for studying other spectrum conversions such as propagating-to-evanescent conversions. All proposed results were validated against full wave electromagnetic simulations. The theory was also used to highlight and explain ‘extraordinary transmission’ through a sub-wavelength metal strip grating when excited by a current source. The proposed SIR may be used to study various conversion processes in gratings, such as propagating-to-evanescent, propagating-to-propagating, and evanescent-to-evanescent spectrum conversion, in addition to the demonstrated evanescent-to-propagating conversion process. Such schemes may be utilized in various scenarios, such as to
devise techniques for creating tight and sub-wavelength spots for focusing beyond the diffraction limit and near-field focusing.

In Chapter 3, this evanescent-to-propagating wave conversion was exploited for radiation enhancement of specialized sources using a simple grating. We experimentally demonstrated dramatically increased radiation from an ‘invisible’ source placed next to a sub-wavelength metal strip grating. The ‘invisible’ source is a novel, non-radiating (highly reactive) array made of radiating antennas, excited by a common feed. The metal grating used is sub-wavelength and non-resonant which typically attenuates the overall radiation of a nearby source, especially in the Transverse Electric polarization. However, we showed that such a grating screen with proper dimensions, placed next to the ‘invisible’ source, can in fact significantly enhance the radiated field strength, far beyond the free space radiation of this ‘invisible’ radiator, by an order of magnitude. This radiation enhancement is facilitated through the conversion of evanescent waves of the specially designed reactive source into propagating waves, and its level is inversely related to the source-grating distance. The physical phenomenon was shown in simulations and measurements at microwaves. This novel radiation enhancement effect was shown to have potential applications in various areas such as proximity sensing, detection and measurement of distance. The introduction of the Spectral Array Factor in chapter 3 also enabled investigation and design of the full spectrum of the field near an arbitrary arrangement of sources (or their near-field), and extends the existing far-zone array factor concept in antenna theory. For instance, the spectral array factor polynomial of several radiating sources was tailored to make the overall array be effectively non-radiating and invisible. The spectral array factor can be applied to the near-field of any antenna array, or array of sources.

In Chapter 4, the radiation of simple dipole sources near anisotropic Epsilon-Near-Zero Metamaterials and the radiation enhancement and shaping achieved were studied. We investigated radiation of a dipole at or below the interface of (an)isotropic Epsilon-Near-Zero (ENZ) media, akin to the classic problem of a dipole above a dielectric half-space. To this end, the radiation patterns of dipoles at the interface of air and a general anisotropic medium (or immersed inside the medium) were derived using the Lorentz reciprocity method. By using an ENZ half-space, air takes on the role of the denser medium. Thus we obtain shaped radiation patterns in air which were only previously attainable inside the dielectric half-space. We then followed the early work of R. E. Collin on anisotropic artificial dielectrics which readily enabled the implementation of practical anisotropic ENZs by simply stacking sub-wavelength periodic bi-layers of metal and dielectric at optical frequencies. We showed that when such a realistic anisotropic ENZ has a low longitudinal permittivity, the desired shaped radiation patterns are achieved in air. In such cases the radiation is also much stronger in air than in the ENZ media, as air is the denser medium. Moreover, we investigated the subtle differences of the dipolar patterns when the anisotropic ENZ dispersion is either elliptic or hyperbolic. The ENZ realizations were tailored for optical frequencies where it can enable various applications for better light emission, such as shaping the radiation of optical antennas or enhancing the radiation of
fluorescent molecules.

In Chapter 5, hetero-junctions of anisotropic Epsilon-Near-Zero metamaterials were used to realize low-profile and flat light-concentrators with very short focal distances, or low $f/D$. The flat low-profile device is formed from joining two cleaved finite slabs of anisotropic low (near zero) permittivity MTMs with rotated optical axes. The MTMs have near-zero longitudinal permittivity while matched in the transverse direction. Such MTMs were shown to provide a unique ability to bend the Transverse Magnetic or p-polarized light far away from the normal and along the interface, contrary to conventional dielectrics, and with minimal reflections; hence allowing for a low profile design. Realizations in the optical regime were presented using periodic bi-layers of metal and dielectric. The proposed hetero-junction focusing device concentrates the normally incident plane-wave and/or beam into a corresponding focal region similar to a lens, but via an unconventional series of multiple refractions. The hetero-junction is capable of creating a hot-spot very close to the device, much closer than dielectric lenses and it significantly outperforms the size requirements of thick high curvature lenses with low $f/D$ ratios. The same hetero-structure was shown to also provide beam-splitting and beam-shifting properties, when illuminated from the back. Such lateral splitting and shifting of the beam is done inside a very thin and flat hetero-junction structure. The presented light concentration, as well as beam manipulating devices in chapter 5 are a direct consequence of harnessing the unique extremely oblique refraction using anisotropic ENZs. The proposed designs can find applications in various scenarios including solar and thermo-photovoltaics, photo-detectors, concentrated photovoltaics, non-imaging optics, micro and nano Fresnel lenses, and optoelectronic devices.

In Chapter 6, leaky wave radiation from a class of photonic crystals were used to introduce the Dirac Leaky Wave Antennas. Leaky-wave antennas (LWAs) enable directive and scannable radiation patterns which are highly desirable attributes for terahertz, infrared, and optical frequencies, but currently continuously scanning LWAs do not exist at these frequencies. We reported leaky-wave radiation from the interface of a photonic crystal (PC) with a zero-index Dirac-type dispersion, and air. The resulting Dirac Leaky-Wave Antenna (DLWA) can radiate at broadside even with unilateral excitation, chiefly owing to the closed $\Gamma$-point bandgap of the Dirac PC. Moreover, the DLWA can continuously scan a directive beam over a wide range of angles by varying the frequency. These DLWAs can be designed at microwave as well as terahertz to optical frequencies, with feasible dimensions and low losses, for reliable and continuous leaky-wave radiation and scanning through broadside. The proposed designs not only provide a novel solution for continuous scanning at high frequencies, but can also be used at lower frequencies, e.g. at microwaves, where currently transmission-line metamaterial LWAs exist. One benefit of realizing DLWAs at these frequencies are higher power handling capabilities of the DLWA due to its waveguide nature, especially in high power applications where printed LWAs cannot handle the power requirements. Even in low power applications, if losses are of concern, the DLWA has much lower losses than printed structures, due to its waveguide nature. In addition, the DLWA has a simpler structure without the need for vias or finely printed inter-digital
capacitors, and the unit cell is bigger than its MTM counterparts, as it is based on a photonic crystal. Consequently one would need to fabricate fewer cells for a given physical length of an antenna, compared to other MTM techniques where many sub-wavelength cells are typically required.

7.1 Contributions

The research in this thesis has lead to the following scientific contributions [159, 160, 161, 162, 163, 164, 165, 166, 167]:

7.1.1 Journal papers


7.1.2 Conference papers


Appendix A

Derivation of the TE and TM gap fields of the Metal Strip Grating

A.1 TE polarization

Following [5] and having written the boundary condition at the metal strip grating, one obtains

\[
\int_{-w/2}^{w/2} \frac{-jq_n}{L} e^{-j\beta_n(x-x')} f(x')dx' = -2qA_0e^{-j\beta x} \tag{A.1}
\]

As stated in [5], according to the edge condition the gap field in the TE polarization is described by the function

\[
f(x') = C\sqrt{\left(\frac{w}{2}\right)^2 - x'^2}e^{-j\beta x'} \tag{A.2}
\]

where \(C\) is a complex constant. Therefore, in solving (A.1) using this assumed gap field, we aim to solve for the unknown constant \(C\), which is independent of \(x\) or \(x'\), and must only be dependent on the incident wavenumber \(\beta\) (or \(q\)), strength of the incident wave \(A_0\), and size of the gap \(w\).

To solve for \(C\), we multiply both sides of (A.1) by \(f^*(x)\), and integrate over the aperture to obtain

\[
\int_{-w/2}^{w/2} dx \int_{-w/2}^{w/2} \sum_{n=-\infty}^{n=+\infty} \frac{-jq_n}{L} e^{-j\beta_n(x-x')} f^*(x)f(x')dx' = -2qA_0 \int_{-w/2}^{w/2} f^*(x)e^{-j\beta x}dx \tag{A.3}
\]

The left hands side of (A.3) can be rearranged as

\[
LHS = \sum_{n=-\infty}^{n=+\infty} \frac{-jq_n}{L} \int_{-w/2}^{w/2} dx \int_{-w/2}^{w/2} e^{-j\beta_n(x-x')} f^*(x)f(x')dx' \tag{A.4}
\]
Appendix A. Derivation of the TE and TM gap fields of the Metal Strip Grating

By substituting \( f^*(x) = C^* \sqrt{(w/2)^2 - x^2} e^{j \beta x} \) and \( f(x') = C \sqrt{(w/2)^2 - x'^2} e^{-j \beta x'} \), and separating the integrals inside the summation we obtain

\[
LHS = C^* C \sum_{n=-\infty}^{n=+\infty} \frac{-jq_n}{L} \times \int_{-w/2}^{w/2} e^{-j \frac{2\pi n x}{L}} \sqrt{(w/2)^2 - x^2} dx \times \int_{-w/2}^{w/2} e^{+j \frac{2\pi n x'}{L}} \sqrt{(w/2)^2 - x'^2} dx' \tag{A.5}
\]

The argument of the summation in (A.5) contains two similar looking integrals multiplied by each other. These integrals are not trivial to solve analytically, but can be treated using the Poisson representation of the Bessel function.

We borrow the following identity from Ref. [87], page 953, which states:

\[
J_v(z) = \frac{(\frac{z}{2})^v}{\Gamma(v + \frac{1}{2}) \Gamma(\frac{1}{2})} \int_{-1}^{1} e^{izt} (1 - t^2)^{v-\frac{1}{2}} dt \tag{A.6}
\]

for \( \text{Real}(v) > -\frac{1}{2} \). \( J_v(z) \) is the Bessel function of the first kind of order \( v \), \( \Gamma(v) \) is the Gamma or general complex Factorial function (specifically \( \Gamma(\frac{1}{2}) = \sqrt{\pi} \)), \( i = j = \sqrt{-1} \), and \( z \) is an independent complex variable and argument of the Bessel function.

If we let \( v = 1 \) in (A.6), \( \Gamma(v + \frac{1}{2}) = \sqrt{\pi}/2 \), and substitute \( t = \frac{2}{w} x \) and \( dt = \frac{2}{w} dx \), we have:

\[
J_1(z) = \frac{z}{\sqrt{\pi} \sqrt{\pi}} \left( \frac{2}{w} \right)^2 \int_{-w/2}^{w/2} e^{\frac{2\pi i x}{w}} \sqrt{(w/2)^2 - x^2} dx \tag{A.7}
\]

Thus the two integrals in (A.5) become:

\[
\int_{-w/2}^{w/2} e^{-j \frac{2\pi n x}{L}} \sqrt{(w/2)^2 - x^2} dx = \pi \left( \frac{w}{2} \right)^2 \frac{J_1 \left( -n\pi w/L \right)}{-n\pi w/L} \tag{A.8}
\]

and

\[
\int_{-w/2}^{w/2} e^{+j \frac{2\pi n x}{L}} \sqrt{(w/2)^2 - x'^2} dx' = \pi \left( \frac{w}{2} \right)^2 \frac{J_1 \left( n\pi w/L \right)}{n\pi w/L} \tag{A.9}
\]

Therefore (A.5) becomes

\[
LHS = C^* C \sum_{n=-\infty}^{n=+\infty} \frac{-jq_n}{L} \times \pi \left( \frac{w}{2} \right)^2 \frac{J_1 \left( -n\pi w/L \right)}{-n\pi w/L} \times \pi \left( \frac{w}{2} \right)^2 \frac{J_1 \left( n\pi w/L \right)}{n\pi w/L} =
\]

\[
C^* C \sum_{n=-\infty}^{n=+\infty} \frac{jq_n}{L} \left( \frac{w}{2} \right)^4 \left( \frac{\pi J_1 \left( n\pi w/L \right)}{n\pi w/L} \right)^2 \tag{A.10}
\]

Additionally, the right hand side of (A.3) is
Appendix A. Derivation of the TE and TM gap fields of the Metal Strip Grating

\[ RHS = -2qA_0 \int_{-w/2}^{w/2} f^*(x)e^{-j\beta x}dx = \]
\[ = -2qA_0 \int_{-w/2}^{w/2} C^* \sqrt{(w/2)^2 - x'^2}e^{j\beta x'}dx' \]
\[ = -2qA_0C^* \int_{-w/2}^{w/2} \sqrt{(w/2)^2 - x'^2}dx' \quad (A.11) \]

substituting \( x' = \frac{w}{2}z \), \( dx' = \frac{w}{2}dz \) and factorization we have

\[ RHS = -2qA_0\left(\frac{w}{2}\right)^2C^* \int_{-1}^{1} \sqrt{1-z^2}dz = \]
\[ = -2qA_0\left(\frac{w}{2}\right)^2C^* \frac{1}{2} \left[ z\sqrt{1-z^2} + \sin^{-1}z \right]_{-1}^{1} = \]
\[ = -A_0q\left(\frac{w}{2}\right)^2\pi C^* \quad (A.12) \]

Equating (A.10) and (A.12), we find the final expression for the unknown constant of the gap field in TE polarization (\( C^{TE} \)) as:

\[ C^{TE} = -jqA_0 \frac{\pi}{2} \left\{ \sum_{n=-\infty}^{n=+\infty} \frac{q_n}{L} \left( \frac{w}{2} \right)^2 \left( \frac{\pi J_1(n\pi w/L)}{n\pi w/L} \right)^2 \right\}^{-1} \quad (A.13) \]
A.2 TM polarization

Applying the boundary conditions in TM polarization at the grating plane leads to

\[
A_0 = -j\omega \epsilon \int_{-w/2}^{w/2} \sum_{n=-\infty}^{n=+\infty} f(x') \frac{e^{-j\beta(x-x')}}{j\eta_n L} \, dx'
\]  
(A.14)

Following [5], according to the edge condition the tangential electric field \( E_x \) in the TM polarization is described by the function

\[
f(x') = \frac{Ce^{-j\beta x'}}{\sqrt{(w/2)^2 - x'^2}}
\]  
(A.15)

where \( C \) is a complex constant. Therefore, in solving (A.14) using this assumed gap field, we aim to solve for the unknown constant \( C \), which is independent of \( x \) or \( x' \), and must only be dependent on the incident wavenumber \( \beta \) (or \( q \)), strength of the incident wave \( A_0 \), and size of the gap \( w \).

To solve for \( C \), we multiply both sides of (A.1) by \( f^*(x) \), and integrate over the aperture to obtain

\[
\int_{-w/2}^{w/2} A_0 f^*(x) \, dx = -j\omega \epsilon \int_{-w/2}^{w/2} dx \int_{-w/2}^{w/2} \sum_{n=-\infty}^{n=+\infty} f^*(x) f(x') \frac{e^{-j\beta(x-x')}}{j\eta_n L} \, dx' \]  
(A.16)

Substituting \( f^*(x) \) in left hand side of (A.16) and rearranging we have

\[
LHS = A_0 \int_{-w/2}^{w/2} \frac{C^* e^{j\beta x}}{\sqrt{(w/2)^2 - x'^2}} \, dx
\]  
(A.17)

This integral is again not trivial to solve analytically, but can be treated using the Poisson representation of the Bessel function. If we let \( v = 0 \) in (A.6), \( \Gamma(\frac{1}{2}) = \sqrt{\pi} \), and substitute \( x = \frac{w}{2} t \) and \( dx = \frac{w}{2} dt \), we have:

\[
J_0(z) = \frac{1}{\sqrt{\pi} \sqrt{\pi}} \int_{-w/2}^{w/2} e^{j \frac{2z x}{w}} \, dx
\]  
(A.18)

Re-arranging and changing variable \( z = \frac{\beta w}{2} \) we get

\[
\int_{-w/2}^{w/2} \frac{e^{j\beta x}}{\sqrt{(w/2)^2 - x'^2}} \, dx = \pi J_0(\frac{\beta w}{2})
\]  
(A.19)

thus (A.17) is

\[
LHS = A_0 C^* \pi J_0(\frac{\beta w}{2})
\]  
(A.20)

Substituting \( f^*(x) \) and \( f(x') \) in right hand side of (A.16) and rearranging we have
Appendix A. Derivation of the TE and TM gap fields of the Metal Strip Grating

\[ \text{RHS} = -j\omega \sum_{n=-\infty}^{n=+\infty} \left\{ \frac{1}{jq_n L} \times \int_{-w/2}^{w/2} \frac{C^* e^{-j\beta_n x} e^{+j\beta x}}{\sqrt{(w/2)^2 - x^2}} dx \times \int_{-w/2}^{w/2} \frac{C e^{j\beta_n x'} e^{-j\beta x'}}{\sqrt{(w/2)^2 - x'^2}} dx' \right\} \]

\[ = -j\omega CC^* \sum_{n=-\infty}^{n=+\infty} \left\{ \frac{1}{jq_n L} \times \int_{-w/2}^{w/2} \frac{e^{-j2\pi n x/L}}{\sqrt{(w/2)^2 - x^2}} dx \times \int_{-w/2}^{w/2} \frac{e^{+j2\pi n x'/L}}{\sqrt{(w/2)^2 - x'^2}} dx' \right\} \tag{A.21} \]

and once again with the aid of (A.18)

\[ \text{RHS} = -j\omega CC^* \sum_{n=-\infty}^{n=+\infty} \left\{ \frac{1}{jq_n L} \times \pi J_0(-n\pi w/L) \times \pi J_0(n\pi w/L) \right\} \tag{A.22} \]

Equating (A.20) and (A.22) we find that

\[ C^{TM} = A_0 J_0 \left( \frac{\beta_w}{2} \right) \left[ -\omega \sum_{n=-\infty}^{n=+\infty} \frac{\pi}{jq_n L} J_0^2(\frac{n\pi w}{L}) \right]^{-1} \tag{A.23} \]
Appendix B

All-Dielectric Light Concentrator
Using The Dirac Photonic Crystal

In this design we do not use metals at all. The lens is based on the all-dielectric photonic crystal with a Dirac dispersion at its \( \Gamma \)-point [43, 45] which was used in Chapter 6. Fig B.1 shows the (a) 2D and (b) 3D implementation of the lens. Si (dark color) rods of relative permittivity 13.7 with square cross section are embedded inside a host SiO\(_2\) medium (light color) with relative permittivity 2.25. In this particular design tailored for \( \lambda_0 = 1.39\mu m \), the Si rod cross sections are squares of 260nm by 260nm, arranged with a period of 600nm. The design is particularly flexible and scales well with frequency without dispersion effects. It is always matched to air, because as discussed in Chapter 6 it has both an electric and magnetic response which simultaneously allows for matching to air to minimize reflected power. The PC has an effective zero-index behavior as shown in [43, 45], and thus we have utilized it here to again bend the light obliquely away from the normal to the top interfaces, and guide it towards the center of the device for focusing.

The simulation results of the 2D lens are shown in Fig. B.2, where (a) shows the power density and (b) shows the power flow vector, for \( \lambda_0 = 1.39\mu m \) light incident from the top. The plots show how the power is accepted from the top facet and flows towards the middle, and concentrated on the other side of the lens in a hot region.

The 3D simulation results of the power density of this all-dielectric concentrator are shown in Fig. B.3, confirming how a close-by hotspot exists just below the concentrator at \( \lambda_0 = 1390\text{nm} \). The dimensions of the device are 11.2\( \mu \text{m} \) in width, 2.4\( \mu \text{m} \) (4 periods) high, and the rods have a depth of 2.78\( \mu \text{m} \). Lower depth of rods as low as 2\( \mu \text{m} \) to 1.8\( \mu \text{m} \) are also possible. Generally, enough depth must be given to the rods to enable the TM Bloch modes of interest to establish inside the rods.

This shows that a very low-loss all-dielectric zero-index concentrator can be realized using the concepts presented. This may be very suitable for certain applications, and may provide easier methods of fabrication compared to the plasmonic ENZ concentrators in Chapter 5.
Figure B.1: A zero-index all-dielectric Dirac PC concentrator operating at $1.39\mu m$ (a) 2D TE implementation and (b) 3D implementation. The device is $11.2\mu m$ wide and $2.78\mu m$ in $x$-direction and $2.4\mu m$ in $z$-direction, with a $5^\circ$ tilt angle with respect to the $y$–axis on each side.

Figure B.2: 2D TE simulation results of the all-dielectric concentrator at $1.39\mu m$ (a) power density normalized to the incident power density and (b) power flow vectors. Electric field is out of plane (TE polarized).
Figure B.3: 3D simulation results of the power density normalized to the maximum spot intensity, in the all-dielectric concentrator at 1.39μm. The incident light has electric field polarized along the rods’ axes (x-direction).
Bibliography


Bibliography


[160] ——, “Dirac leaky-wave antennas for continuous beam scanning from photonic crystals,” *Nature Communications*, vol. 6, Jan. 2015.


