COST EFFECTIVE DEPLOYMENT OF HETEROGENEOUS NETWORKS AND THE EFFECT OF PILOT CONTAMINATION

by

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A thesis submitted in conformity with the requirements for the degree of Master of Applied Science
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Abstract

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2016

Network densification by means of adding more antennas to the macro base station (MBS) or by placing single antenna small cell access points (SC AP) have great potential in meeting the growing user demand in wireless cellular networks. While more resources, properly used, is always better, these entail greater deployment cost. In this thesis we investigate how to achieve the highest sum rate in a two-tiered heterogeneous network (HetNet) combining MBSs and SCs under a cost constraint. We show that when ignoring the overhead of uplink training for channel estimation, it is more cost effective to deploy MBSs equipped with a large number of antennas; leading to a large-scale multiple input multiple output (LS-MIMO) system. On the other hand, with even a relatively low cost of training, it is beneficial to deploy SC APs. We then investigate the effect of training on LS-MIMO networks and develop a lower bound on the per user ergodic rate when uplink training, inter-cell interference, and pilot contamination are considered.
Acknowledgements

I would like to thank my supervisor, Professor Raviraj Adve, for his throughout support and guidance. His enthusiasm, energy, patience, and immense knowledge have helped and inspired every aspect of my research and academic life.

I would like to thank my family and friends for their love and support over my graduate years. A special thanks to Kianoush Hosseini for the helpful discussion and advice with regards to my research.
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Chapter 1

Introduction

1.1 Background and Motivation

Network densification implies significantly increasing the number of antenna elements in a wireless network to increase the overall capacity of the network. Due to the projected exponential growth in network capacity demand, network densification has currently become a widely researched topic. There are in general two approaches to realize the densification of the cellular network: small cell networks and large-scale multiple input multiple output (LS-MIMO) systems.

Small cell networks - one approach to densify the current network is to deploy a dense layer of small cells (SCs) comprising of many micro/femto cells on top of the existing macro base station (MBS) network. These SCs are characterized by low transmit power access points (APs), to help offload traffic from the current network. The sizes of these cells are small compared to the traditional macro cell network due to the low transmit power of the access points. Each SC is intended to serve very limited number of users, and it therefore can direct a large proportion of its transmit power and bandwidth to each intended user. In addition, the low transmit power and small cell size help reduce inter-cell interference, and promote frequent reuse of time and frequency resources. Due
to the reduced distance between AP and user, SCs also provide higher received signal power at the intended receiver. However, the benefits of small cell is subject to a few key technical challenges [2]. These challenges include implementing effective interference coordination for dense deployments, potential overhead with cell association and biasing, and to support for seamless handover when user is mobile.

Large scale MIMO - another popular approach of network densification is to install more antennas at the MBS of the macro network. This creates an LS-MIMO network when the number of antennas per BS gets large. The use of large antenna arrays enables the base station to focus emitted energy with great precision at small regions in space (the intended users) by appropriately adjusting the phase and gain of the signal emitted from each transmit antenna. This is known as coherent beamforming, which makes the signal wavefronts add up constructively at the locations of the intended users. The array gain from coherent beamforming mitigates inter-user interference and propagation losses, and improves the energy efficiency (EE) of the network [18][20]. Having more antennas than users also gives the transmitting base station the ability to spatially multiplex users on the same time and frequency resource and therefore improve the overall capacity of the system. However, there is a tradeoff involved here since multiplexing users reduces the spatial degrees of freedom (DoP) or diversity order per user.

To effectively perform coherent beamforming or spatial multiplexing of the channels, the transmitting MBS requires the knowledge of its channels through the acquisition of channel state information (CSI) [15]. For this reason, LS-MIMO works best under the time division duplexing (TDD) mode of operation, where the downlink channel is just the reciprocal of the uplink channel. The transmitting MBS is then able to estimate its downlink channel from uplink pilots transmitted by the users. Since there are fewer users than MBS antennas, this approach is more efficient than training in the downlink, followed by feedback.

A significant challenge in LS-MIMO systems is pilot contamination, which directly
limits the quality of channel estimation. Pilot contamination arises from the fact that the length of orthogonal pilot sequences must be greater or equal to the number of users, and, because the channel coherence time is finite, the same pilot sequence need to be reused in different cells when the number of users gets large.

Given that each of the two approaches to network densification has its own upsides and corresponding challenges, we would like to consider how to best combine the two approaches. We propose a network architecture where a layer of SC APs is overlaid on top of an LS-MIMO network. The proposed network architecture can be evaluated using different performance measures such as coverage, energy efficiency, spectral efficiency, sum rate, etc. In this thesis, we focus on using downlink sum rate as our performance measure. To keep the proposed architecture practical, we constrain the total number of antennas and the total cost of the system.

In the first part of this thesis we seek to accomplish the following task: for the proposed network architecture where we have a fixed total number of antennas and a total cost, how many antennas should be deployed as SC APs and the rest be allocated to the MBS such that the sum rate is maximized. In the second part of this thesis, we develop a lower bound on the per user ergodic rate in the presence of channel estimation error and pilot contamination in an LS-MIMO network.


d.2 Literature Review

Traditionally, base station locations are modelled by hexagonal lattice or square grid placements, and tractable analysis such as user signal-to-interference-plus-noise ratio (SINR) can be done for fixed user locations. However, as networks become more and more dense, it becomes intractable to do analysis on deterministic base station and user locations. Furthermore, a regular deployment does not capture the increasing irregularity in BS deployments or the expected inherent randomness in the deployments of SCs.
Currently, a widely adopted model in the literature is the Poisson Point Process (PPP) [3]. The PPP model assumes the placements of base stations to be truly random, in that BS locations are mutually independent. Further, the number of BSs in any area is a Poisson random variable with mean proportional to the area and the density of BSs. The PPP model includes undesirable situations where several base stations are placed close to each other. It has been shown that cellular networks modelled with a PPP yield a lower bound on network performance [11], and networks modelled with perfect hexagonal grids yield an upper bound. In [5] the authors have shown that when considering coverage probability, the performance of a modern real-world deployment fits in between that of a perfect hexagonal lattice and a PPP.

Although a PPP gives a lower bound on system performance, it captures system behavior as a function of system parameters [3]. Furthermore, it significantly simplifies the analysis and we adopt the PPP for the placements of MBSs, SC APs and users in our proposed architecture. In [1] the authors present closed form expressions for both the user coverage probability and downlink per user ergodic rate when the base station locations are modelled using a PPP. Their ergodic rate expression [Section IV, Theorem 3] for a typical user inside a network formed by single antenna base stations following a PPP can be directly extended to measure the performance of the small cell network in our proposed architecture.

In the LS-MIMO network, an important operation is pre-coding. Pre-coding is the operation where by the $K$-dimensional message-bearing symbol vector intended for $K$ users is mapped to an $N$ dimensional vector, where $N$ is the number of antennas on the MBS. The two most prominent forms of linear pre-coding for LS-MIMO system are: conjugate beamforming and zero-forcing beamforming (ZF-BF). In conjugate beamforming, also known as single user beamforming (SU-BF), the MBS communicates with only one user in each resource block, and pre-coding is done by multiplying the message-bearing signal by the conjugate of the estimated channel vector. Clearly, all spatial resources are
devoted to a single user, thereby maximizing diversity order.

In ZF-BF the transmitting MBS exploits space division multiplexing access (SDMA) where a length-$K$ signal vector is multiplied by the pseudo-inverse of the estimated channel matrix. If the CSI is known accurately, ZF-BF eliminates inter-user interference at the expense of noise enhancement. At high signal-to-noise ratios, this penalty is negligible. Importantly, ZF-BF lends itself to mathematical analysis [22]. Furthermore, it has been shown to capture system behavior [14].

In LS-MIMO networks, there is a fundamental trade-off between energy efficiency and spectral efficiency [18]. In [22] the authors have shown that ZF-BF gives high spectral efficiency and low energy efficiency, and the converse is true when conjugate beamforming is used. By communicating with $K$ users simultaneously, ZF-BF provides a multiplexing gain, at the expense of the diversity order per user. In this thesis, we adopt ZF-BF as our primary form of linear pre-coding for the LS-MIMO network.

There has been some related work analyzing the network architecture of combining LS-MIMO networks and small cells. In [6] the authors consider how to minimize the total power consumption in a single cell LS-MIMO augmented by small cells. They show that under spatial multiflow transmission (users can be served by multiple transmitters at the same time) the total power consumption can be greatly improved by combining LS-MIMO and small cells. The authors in [15] analyzed the combination of LS-MIMO and small cells by overlaying small cells onto multiple antenna MBSs placed on square grids. They show, via simulation, that by adopting the TDD protocol, both MBSs and SC APs could precode the downlink transmissions using their respective local received interference matrices and significantly increase the area throughput. However, the MBSs and SC APs are forced to be on the grid and the authors do not present tractable signal and interference distribution functions.

In [8], the authors model the base station locations using the PPP and analyze the uplink EE over different system parameters. They develop an iterative algorithm to
optimize the uplink EE over a set of parameters including base station density, number of transmit antennas, and number of users. They show that when user density is low (high) a single input multiple output (SIMO) system outperforms (is outperformed by) a MIMO system in terms of uplink EE. In a similar work [12] the authors compared the downlink performance of an LS-MIMO network and SC network with fixed total antenna density. They show that in terms of outage rate, a dense SC network performs better than an LS-MIMO network when the user density is moderate or large, and the LS-MIMO network outperforms the SC network when the user density is asymptotically small. In terms of EE they show that a SC network always outperforms an LS-MIMO network. However, the authors assumed the transmitting MBS for the LS-MIMO network to be using SU-BF and did not consider the SDMA case. Furthermore, they do not consider deployment costs of the two types of networks.

A similar question was asked in [10], where the authors compare the ergodic rate and area spectral efficiency (ASE) of three transmission techniques in a HetNet: SC APs, single user MIMO beamforming (SU-BF), and multi-user MIMO (MU-MIMO) through ZF-BF. They demonstrate, through ordering results, that the ergodic rate of SU-BF and SC APs is larger than in the MU-MIMO case. They also show that, keeping the number of antennas the same, using SC APs outperforms MU-MIMO in terms of ASE. Importantly, the analysis does not optimize the ratio of number of users to antennas, assuming the two to be equal, ignores the impact of training (the need for training adversely effects the SDMA case the most) and is entirely communication centric, i.e., it ignores issues such as cost (which, as we will see, impacts the SC APs the most).

As mentioned earlier, the performance of an LS-MIMO network is heavily affected by the quality of the CSI acquired during the uplink training phase. The biggest challenge for an LS-MIMO network operating in TDD protocol comes from pilot contamination, which results from users in different cells reusing the same pilot sequences during uplink training. In [17] the authors show in a multi-cell LS-MIMO network with universal pilot
reuse, the inter-cell interference power increases at the same rate as signal power with increasing number of transmit antennas under minimum mean-square error (MMSE) channel estimation and matched-filtering (MF) pre-coding. In [4], the authors show in a multi-cell LS-MIMO network with single user least-squares channel estimation and conjugate beamforming, pilot contamination can be avoided by sharing among all the MBSs the local message-bearing signals and slow fading coefficients from each cell.

In [16] the authors derive approximations of achievable uplink/downlink rates in a hexagonal deployment of LS-MIMO network operating in TDD protocol with universal frequency reuse. They account for both channel estimation error and pilot contamination in their model. They used MMSE estimation for uplink channel estimation and ZF-BF for downlink pre-coding. The authors in [7] considered the uplink of a multi-cell LS-MIMO network where the MBSs are distributed as a PPP. They incorporated pilot contamination in their system model with MMSE channel estimation and maximum ratio combining (MRC) for uplink signal detection. They have developed a lower bound on the uplink per user ergodic capacity in the presence of pilot contamination for fixed MBS and user locations. Both these results are useful as, in the latter part of this thesis, we investigate the effect of channel estimation error and pilot contamination under MMSE on the downlink per user ergodic rate of an LS-MIMO network. We develop a lower bound on the downlink per user ergodic rate in the presence of pilot contamination for an LS-MIMO network where the MBS and user locations are modelled by independent PPPs.

1.3 Thesis Contributions

In this thesis we make two contributions: in the first part of the thesis we develop the per user ergodic rate for a typical macro user (MUE) in an LS-MIMO network. We incorporate both hardware cost and training overhead into analyzing the sum rate of a
heterogeneous network combining LS-MIMO and SC APs. We show, through analysis, the optimum deployment of number of MBS, MBS antennas, and SC APs under hardware cost constraint and different training costs.

In the second part of the thesis we analyze the effect of pilot contamination on the per user ergodic rate in an LS-MIMO network. We show that under MMSE channel estimation the downlink ergodic rate depends on the relative locations of the MUEs. We then give a closed form lower bound of the per user ergodic rate.

1.4 Organization of Thesis

In Chapter 2 we describe in detail the system model and formulate the optimal deployment problem where the area sum rate is optimized over the number of MBS, MBS antenna, and SC AP. In Chapter 3 we first present the per user ergodic rate of a typical MUE/SUE in an LS-MIMO/SC AP network. Then we solve the optimal deployment problem presented in Chapter 2 and discuss the results. In Chapter 4, we incorporate MMSE channel estimation into the ergodic rate expression developed in Chapter 3. We conclude the thesis and discuss possible future works in Chapter 5.

1.5 Notation

We use lower case italics for scalars (e.g. $x$), lower case boldface italics for column vectors (e.g., $x$) and upper case boldface italics for matrices (e.g., $X$). $[X]^H$ denotes the matrix Hermitian and $|X|$ denotes the $L^2$ norm of matrix $X$. $I_N$ denotes the $N \times N$ identity matrix. $F_X(x)$ denotes the cumulative distribution function of random variable $X$. $\mathbb{E}[X]$ denotes the expectation of random variable $X$. $\mathcal{CN}(\mu, R)$ denotes the complex Gaussian random variable with mean $\mu$ and covariance matrix $R$. $\Gamma(a, b)$ denotes a gamma variable with shape parameter $a$ and scale parameter $b$. 
Chapter 2

System Model

We consider a two tier heterogeneous network deployed inside a large but finite circular area $\mathcal{A}$. The network comprises of a layer of SC APs overlaid on a tier of multi-antenna MBSs.

As mentioned in Chapter 1, a common model for the locations of MBSs and SC APs is independent PPPs. While a PPP is valid in infinite area networks, a PPP conditioned on a finite area leads to a uniform distribution within the area. The locations of the MBSs and SC APs each follows an individual homogeneous point process $\Phi_{\lambda_M}$ (with density $\lambda_M$ MBSs/km$^2$) and $\Phi_{\lambda_S}$ (with density $\lambda_S$ SC APs/km$^2$). Each MBS is equipped with $N$ transmit antennas and transmits with total power $P_M$. Each MBS transmits to $K$ pre-scheduled users (MUEs or macro-user equipments) in the downlink. Each SC AP is equipped with an single omnidirectional antenna, and transmits with power $P_S$. Each SC AP transmits in the downlink to a single pre-scheduled user (SUE or small cell user equipment).

Users (UEs) are distributed following a homogeneous point process $\Phi_{\lambda_{UE}}$ with density $\lambda_{UE}$ UEs/km$^2$. We assume that $\Phi_{\lambda_M}, \Phi_{\lambda_S}, \Phi_{\lambda_{UE}}$ are mutually independent spatial processes. Users are assumed to be associated with their closest transmitting MBS/SC AP, thereby partitioning the region $\mathcal{A}$ into Voronoi cells. We assume the UE intensity $\lambda_{UE}$
is high enough to ensure that the Voronoi regions formed by each MBS includes at least $K$ active MUEs; similarly, each SC AP Voronoi region is assumed to include at least one SUE.

We assume a Rayleigh fading channel between transmitters and receivers with average channel power affected by only distance attenuation. Each MBS/SC AP is assumed to have perfect knowledge of the instantaneous channel state information (CSI) of the users it is serving. In formulating the problem we will explicitly account for training required to acquire CSI. Later, in Chapter 4, we will investigate how the quality of the CSI affects the downlink rate of an LS-MIMO network adopting SDMA. Importantly, we assume the MBS and SC networks have orthogonal (non-overlapping) spectrum allocation.

### 2.1 Received Signal Model in Small Cell Network

We denote the small-scale Rayleigh fading component of the channel between the $i$th SC AP and the SUE it serves (the $i$th SUE) as $h_{ii}$, where $h_{ii} \sim \mathcal{CN}(0, 1)$. Similarly, the Rayleigh component of the interference channel between the $i$th SUE and the $j$th SC AP is denoted as $g_{ij}$; as with $h_{ii}$, $g_{ij} \sim \mathcal{CN}(0, 1)$. The received signal at SUE $i$ associated with SC AP $i$ can be written as

$$
g_{i}^{SUE} = \sqrt{P_{S}r_{ii}^{-\alpha}}h_{ii}s_{i} + \sum_{j \neq i} \sqrt{P_{S}r_{ij}^{-\alpha}}g_{ij}s_{j} + n_{i} \quad (2.1)
$$

The signal-to-interference-plus-noise ratio (SINR) at the $i$th SUE is given by

$$
\gamma_{i}^{S} = \frac{\rho_{S}|h_{ii}|^{2}r_{ii}^{-\alpha}}{\sum_{j \neq i} \rho_{S}|g_{ij}|^{2}r_{ij}^{-\alpha} + 1} \quad (2.2)
$$

where $\rho_{S} = P_{S}/\sigma^{2}$ denotes the ‘transmit’ signal-to-noise ratio (SNR) ($\sigma^{2}$ denotes noise power), and $\alpha$ is the path loss exponent; further, $r_{ii}$ denotes the distance between the $i$th SUE to its serving SC AP and the $r_{ij}$ the distance between the $i$th SUE and the
Chapter 2. System Model

The power of the unit-variance small scale components, \( |h_{ii}|^2 \) and \( |g_{ij}|^2 \) are exponentially distributed with unit mean.

Finally, the ergodic rate for the \( i \)th SUE is defined as

\[
R_S = \mathbb{E}_{\Phi, \mathbf{s}, \mathbf{h}, \mathbf{g}} [\log(1 + \gamma_i^S)] \tag{2.3}
\]

where the expectation is over the SC AP locations and the small-scale fading components.

2.2 Received Signal Model in an LS-MIMO Network

We denote the channel vector between the MBS in cell \( i \) and MUE \( k \) in cell \( j \) as \( \sqrt{\beta_{ijk}} \mathbf{h}_{ijk} \); here \( \mathbf{h}_{ijk} \in \mathbb{C}^{N \times 1} \) is the vector of small-scale Rayleigh fading components between the MUE and each antenna element on the MBS. Each component in \( \mathbf{h}_{ijk} \) is independently and identically distributed (i.i.d.) with \( \mathcal{CN}(0, 1) \). Further, \( \beta_{ijk} = r_{ijk}^{-\alpha} \) denotes the distance dependent path loss, and \( r_{ijk} \) is the distance between MUE \( k \) in cell \( j \) to the MBS in cell \( i \). The interference channel vector between MBS \( j \) and MUE \( k \) in cell \( i \) is denoted as \( \mathbf{g}_{jik} = \sqrt{\beta_{jik}} \mathbf{h}_{jik} \).

We use ZF-BF on the downlink to spatially multiplex the \( K \) MUEs for each MBS. Since each MBS is assumed to have perfect knowledge of its local channel vectors for each of its \( K \) MUEs, we can construct the \( N \times K \) ZF beamforming matrix as

\[
\mathbf{W}_i = \left[ \mathbf{G}_i (\mathbf{G}_i^H \mathbf{G}_i)^{-1} \right]_{1:K} = [\mathbf{w}_{1i}, ..., \mathbf{w}_{Ki}]
\]

where \( \mathbf{G}_i = [\mathbf{g}_{i1}, ..., \mathbf{g}_{iK}] \in \mathbb{C}^{N \times K} \) and \( \mathbf{g}_{iik} = \sqrt{\beta_{iik}} \mathbf{h}_{iik} \) is the channel vector between MBS \( i \) and its own MUE \( k \). Each \( \mathbf{w}_{ki} \) denotes the ZF pre-coding vector associated with MUE \( k \), and they are normalized to have unit magnitude to ensure equal power transmitted. The expression for the BF matrix does not show the normalization. The received signal at MUE \( k \) in cell \( i \) can be written as
\[ y_{ik}^{MUE} = \sqrt{\frac{P_M}{K}} g_{iik}^H w_{ki} s_{ki} + \sum_{j \neq i} \sum_{k=1}^{K} \sqrt{\frac{P_M}{K}} g_{jik}^H w_{kj} s_{kj} + n_{ik} \]  

(2.4)

where \( s_{ik} \) denotes the information symbol intended for MUE \( k \) in cell \( i \), and \( n_{ik} \) denotes the circularly symmetric complex additive white Gaussian noise with variance \( \sigma^2 \). We assume that \( \mathbb{E}[|s_{ik}|^2] = 1 \quad \forall \ i, k \). The SINR of MUE \( k \) in cell \( i \) is given by

\[ \gamma_{ik}^M = \frac{\rho_M |g_{iik}^H w_{ki}|^2}{\sum_{j \neq i} \sum_{k' = 1}^{K} \rho_M |g_{jik}^H w_{k'j}|^2 + 1} \]  

(2.5)

where \( \rho_M = P_M/(K\sigma^2) \) denotes the SNR of the MUE. The ergodic rate for MUE \( k \) in cell \( i \) is defined as

\[ \hat{R}_M = \mathbb{E}_{\Phi_M, \{g\}}[\log(1 + \gamma_{ik}^M)] \]  

(2.6)

### 2.3 Distributions of Signal and Interference Powers in an LS-MIMO network

From the channel model described above, it follows that \( g_{iik} \sim \mathcal{CN}(0, \beta_{iik} I_N) \) and \( g_{ijk} \sim \mathcal{CN}(0, \beta_{ijk} I_N) \). We note that \( g_{iik} \) and \( g_{ijk} \) are random isotropic vectors in a \( N \)-dimensional vector space whose powers are a sum of \( N \) independent exponentially distributed random variables, and distributed as \( g_{iik}^H g_{iik} \sim \Gamma(N, \beta_{iik}) \) and \( g_{ijk}^H g_{ijk} \sim \Gamma(N, \beta_{ijk}) \) [14] [Section IV-A]. Based on the ZF orthogonality property, the pre-coding vector associated with user \( k \) is orthogonal to the subspace spanned by the channel vectors between MBS \( i \) and the other \( K - 1 \) users, i.e. \( w_{ki} \perp \text{span}\{g_{iik'}\}_{k' \neq k} \). As a consequence, the received signal power \( |g_{iik}^H w_{ki}|^2 \) is the power of an isotropic \( N \)-dimensional random vector projected on a \( N - K + 1 \) dimensional beamforming space. The statistical distribution for the received signal power at user \( k \) in cell \( i \) is therefore [14]

\[ |g_{iik}^H w_{ki}|^2 \sim \Gamma(N - K + 1, \beta_{iik}) \]  

(2.7)
Additionally, the pre-coding vector for each MBS is independent of the interference channels to other MBSs. From the perspective of user $k$, an interfering beam lies within an one-dimensional vector space. Therefore, $|g_{jik}^H w_{kj}|^2$ is the power of the interfering channel $g_{jik}$ projected onto a one-dimensional beamforming space. The statistical distribution for the interference power from beamforming vector $k'$ from cell $j$ to the user $k$ in cell $i$ is

$$|g_{jik}^H w_{k'j}|^2 \sim \Gamma(1, \beta_{jik})$$

Furthermore, we assume the interference power from different beamforming vectors from cell $j$ to user $k$ in cell $i$ is independent from each other, and the aggregate interference power from cell $j$ to user $k$ in cell $i$ is well approximated by

$$\sum_{k'=1}^{K} |g_{jik}^H w_{k'j}|^2 \sim \Gamma(K, \beta_{jik}) \quad (2.8)$$
Chapter 3

Area Sum Rate Optimization

In this chapter we first formulate the problem of optimal deployment strategy. Then we present the ergodic rate per user both for the typical SUE and MUE. Lastly, we present the numerical solutions to the proposed optimal deployment problem under different system parameters and discuss the implications.

3.1 Problem Formulation

We are now ready to formulate the problem of an optimal deployment strategy within the area $A$. A deployment strategy entails choosing a specific number of SC APs, MBSs, and antennas per MBS to be deployed in $A$. The performance of a deployment is measured by the sum rate inside $A$ the deployment provides. For our deployment strategy, we will consider the following

- Per user ergodic rate provided by the SC and MBS network
- Relative hardware cost of MBS, MBS antenna element, and SC AP and a constraint on the total cost
- Percentage of total spectrum allocated to each of the two tiers
• Uplink training required for the LS-MIMO network

Hardware Cost

To model the economics of a deployable heterogeneous network, we constrain the total hardware cost to limit the number of MBS/SC APs deployed. Let $C_T$ denote the total allowed hardware cost for the entire network; we have the following constraint

$$C_T \geq n_S C_S + n_M (C_{M0} + NC_{M1}). \quad (3.1)$$

where $n_S$ ($n_M$) denotes the number of SC APs (MBSs) deployed within $\mathcal{A}$. Note $n_S = \lambda_S |\mathcal{A}|$ and $n_M = \lambda_M |\mathcal{A}|$. $C_S$ denotes the hardware cost of a single SC AP, $C_{M0}$ the hardware cost of a MBS tower, and $C_{M1}$ is the hardware cost of a single MBS antenna element. Essentially, $C_{M0}$ denotes the minimum cost of deploying a MBS while $C_{M1}$ the incremental cost of adding an antenna to the MBS. In practice, one would expect $C_{M1} \ll C_S \ll C_{M0}$.

Spectrum Allocation

As mentioned, we adopt orthogonal spectrum allocation across the two tiers, thus eliminating inter-tier interference. The amount of spectrum allocated to each tier is in proportion to the number of active users in each tier. A total available spectrum of $W$ is divided into MBS and SC components and allocated to each tier in proportion to the number of users served by each tier. Since each SC AP brings into the network one SUE, the total number of SUEs in the network is $n_S$. Each MBS however, brings $K$ MUEs into the network, and the total number of MUEs in the network is $n_M K$. We therefore allocate $[n_S/(n_S + n_M K)W]$ Hz of spectrum to the small cell tier and $[n_M K/(n_S + n_M K)W]$ to the LS-MIMO tier. \footnote{It may be possible to further improve the performance by optimizing this fraction. We do not consider this issue in this thesis.}
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Training Penalty

When training the downlink channel in a TDD LS-MIMO network, each MUE transmits, on the uplink, a series of pilot symbols for the MBS to process. Since the MBS must reserve at least the equivalent of one time slot for each MUE, the minimum training period is $K$ symbols, the total number of MUEs the MBS serves. To improve on the quality of channel estimation, the MBS may require each MUE train for more than one symbol. In general, therefore, this training overhead scales linearly with $K$. Denoting as $D_p$ the fraction of time each MUE spends in training, the true rate achieved by assigning a cost of training, $\kappa_t$, to penalize the ergodic rate in our LS-MIMO network. We have

$$\kappa_t = (1 - D_p K).$$ \hfill (3.2)

Note that the overhead for training scales with $K$ for each user since no user can communicate data while the others are training; $\kappa_t$ acts as a penalty factor, and, for a positive communication, we must have $\kappa_t \in [0, 1]$. This essentially limits the number of users $K$ the MBS can serve. Now taking into account the penalty training cost, the effective MUE ergodic rate $R_M$ in an LS-MIMO network per user is

$$R_M = \kappa_t \hat{R}_M,$$ \hfill (3.3)

where $\hat{R}_M$ is given in (2.6).

Sum Rate Optimization Problem

Finally, the sum rate $R_A$ in $\mathcal{A}$, in bits/s, is calculated as the product of the bandwidth, the number of users and ergodic rate per user over the two tiers, i.e.

$$R_A(n_S, n_M, N) = W \left( \frac{(n_M K)}{n_S + n_M K} (n_M K) R_M + \frac{n_S}{n_S + n_M K} n_S R_S \right)$$

$$= W \left( \frac{(n_M K)^2}{n_S + n_M K} R_M + \frac{n_S^2}{n_S + n_M K} R_S \right).$$ \hfill (3.4)
where, $R_M$ and $\hat{R}_M$ (in (3.3) and (2.6) and $R_S$ in (2.3)) are the average rates per MUE and SUE respectively.

The parameters under the system designer’s control are $n_s$, $n_M$, $N$ and $K$. However, as shown in [13], in an LS-MIMO system using ZF-BF, the sum rate is maximized when $K \simeq 0.6N$. We therefore reduce the parameter space by setting $K = 0.6N$ and optimize the remaining three variables. We define the optimal deployment strategy in $A$ as the following sum rate optimization problem

$$
\max_{n_S, n_M, N} R_A(n_S, n_M, N),
$$

such that

$$
n_S C_S + n_M (C_{M0} + NC_{M1}) \leq C_T,
$$

$$
0 \leq \kappa_t \leq 1,
$$

$$
n_S \geq 0, n_M \geq 0, N \geq 0.
$$

\section{3.2 Ergodic Rate Per User}

\subsection{3.2.1 Ergodic Rate Per MUE}

The ergodic rate expression (as defined in (2.6)) for a typical MUE in our proposed LS-MIMO network is presented as the following proposition. We give a detailed proof of how the expression is derived.

\textit{Proposition}: The ergodic rate for a typical MUE at the center of region $A$ is well-approximated by the expression in (3.6).

$$
\hat{R}_M = \int_0^\infty e^{-\pi \lambda_M r^2} \int_0^\infty e^{-z} \frac{e^{-z}}{z} \exp\left( -\lambda_M \int_0^{2\pi} \int_0^\infty \left[ 1 - (1 + \rho_M z r_j^{-\alpha}) \right]^{-K} dr_j d\theta \right)
\cdot \left[ 1 - (1 + \rho_M z r^{-\alpha})^{-(N-K+1)} \right] dz 2\pi \lambda_M dr
$$

\textit{Proof}. We denote as $i$ the MBS nearest to the typical MUE $k$, and let $r$ be the dis-
distance from MBS \( i \) to the typical MUE \( k \) (i.e. \( r_{ik} = r \)). Since each MUE is associated with its closest MBS, all the interfering MBS cannot be closer than \( r \). The probability density function (pdf) of \( r \) can be derived using the simple fact that the null probability of a 2-D Poisson process in an area \( A \) is \( \exp(-\lambda MA) \). We have 

\[
P[r > R] = P[\text{No MBS closer than } R] = e^{-\lambda M \pi R^2}.
\]

Therefore, the cdf is 

\[
P[r \leq R] = F_r(R) = 1 - e^{-\lambda M \pi R^2}.
\]

and the pdf can be found as 

\[
f_r(r) = \frac{dF_r(r)}{dr} = e^{-\lambda M \pi r^2} \frac{2\pi \lambda M}{r}.
\]

(3.7)

The ergodic rate of the typical MUE \( k \) is given by 

\[
\hat{R}_M = \mathbb{E}[\log(1 + \gamma_{ik}^M)].
\]

we can express it as:

\[
\hat{R}_M = \mathbb{E}_{\Phi, \{g\}}[\log(1 + \gamma_{ik}^M)]
\]

\[
= \mathbb{E}_r[\mathbb{E}_{\Phi \setminus \{i, g\}}[\log(1 + \gamma_{ik}^M)|r]]
\]

\[
\overset{(a)}{=} \int_0^\infty e^{-\pi \lambda M r^2} \mathbb{E}_{\Phi \setminus \{i, g\}}[\log(1 + \gamma_{ik}^M)|r] 2\pi \lambda M r dr
\]

where in \((a)\) we took the conditional expectation of MBS \( i \) being at a distance \( r \) from the typical MUE \( k \), and used the pdf given in (3.7).

\[
\hat{R}_M = \int_0^\infty e^{-\pi \lambda M r^2} \mathbb{E}_{\Phi \setminus \{i, g\}} \left[ \int_0^\infty \frac{e^{-t}}{t} (1 - e^{-t\gamma_{ik}^M}) dt \right] 2\pi \lambda M rdr
\]

Further, since \( \log(1 + x) = \int_0^\infty \frac{e^{-t}}{t} (1 - e^{-xt}) dt \). Using the SINR expression given in (2.5), we have:

\[
\hat{R}_M = \int_0^\infty e^{-\pi \lambda M r^2} \mathbb{E}_{\Phi \setminus \{i, g\}} \left[ \int_0^\infty \frac{e^{-t}}{t} \left(1 - \exp\left(-t \frac{\rho_M |g_{ik}^H w_{ki}|^2}{\sum_{j' \neq i} \sum_{k'=1}^K \rho_M |g_{j'k}^H w_{k'j}|^2 + 1}\right)\right) dt \right] 2\pi \lambda M rdr
\]

Now, let

\[
t = z \left( \sum_{j \neq i} \sum_{k'=1}^K \rho_M |g_{jik}^H w_{k'j}|^2 + 1 \right) \Rightarrow \quad dt = \left( \sum_{j \neq i} \sum_{k'=1}^K \rho_M |g_{jik}^H w_{k'j}|^2 + 1 \right) dz
\]
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substituting \( z \) for \( t \), we have

\[
\hat{R}_M = \int_0^\infty e^{-\pi \lambda_M r^2} \mathbb{E}_{\Phi_M \setminus \{i\}} \left[ \int_0^\infty \frac{\exp(-z(\sum_{j \neq i} \sum_{k'=1}^K \rho_M |g_{jik}^H w_{kj}|^2 + 1))}{z} \right. \\
\left. \cdot \left( 1 - \exp(-z\rho_M |g_{iik}^H w_{ki}|^2) \right) dz \right] 2\pi \lambda_M dr
\]

Since \( \{g_{iik}\} \) and \( \{g_{jik}\} \) are independent distributions, we can separate out the expectation

\[
\hat{R}_M = \int_0^\infty e^{-\pi \lambda_M r^2} \int_0^\infty \frac{e^{-z}}{z} \mathbb{E}_{\Phi_M \setminus \{i\}} \left[ \exp(-z(\sum_{j \neq i} \sum_{k'=1}^K \rho_M |g_{jik}^H w_{kj}'|^2)) \right] \\
\cdot \left( 1 - \mathbb{E}_{\{g\}} \left[ \exp(-z\rho_M |g_{iik}^H w_{ki}'|^2) \right] r \right) dz 2\pi \lambda_M dr
\]

The locations of the interfering MBSs follow independent and identical distributions, and so we can rewrite the sum in the exponent as a product. Further simplification is to by separate the expectations \( \mathbb{E}_{\Phi_M} \) and \( \mathbb{E}_{\{g\}} \) since these are two independent processes, resulting in

\[
\hat{R}_M = \int_0^\infty e^{-\pi \lambda_M r^2} \int_0^\infty e^{-z} \mathbb{E}_{\Phi_M \setminus \{i\}} \left[ \prod_{j \in \Phi_M \setminus \{i\}} \mathbb{E}_{\{g\}} \left[ \exp(-z\sum_{k'=1}^K \rho_M |g_{jik}^H w_{kj}'|^2) \right] \right] \\
\cdot \left[ 1 - \mathbb{E}_{\{g\}} \left[ \exp(-z\rho_M |g_{iik}^H w_{ki}'|^2) \right] r \right] dz 2\pi \lambda_M dr
\]

Using the Gamma distributions in (2.7) and (2.8) and applying the MGF of the Gamma distribution, we have

\[
\mathbb{E}_{\{g\}} \left[ \exp(-z\rho_M |g_{iik}^H w_{ki}'|^2) \right] = (1 + \rho_M z r^{-\alpha})^{-(N-K+1)}
\]
\[
\mathbb{E}_{\{g\}} \left[ \exp(-z\sum_{k'=1}^K \rho_M |g_{jik}^H w_{kj}'|^2) \right] = (1 + \rho_M z r^{-\alpha})^{-K}
\]

Now let \( r_j = r_{jik} \) denote the distance from the origin (where our typical MUE \( k \) is)
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to the interfering MBS \( j \), we have

\[
\hat{R}_M = \int_0^\infty e^{-\pi \lambda_M r^2} \int_0^\infty \frac{e^{-z}}{z} \mathbb{E}_{M \setminus i} \left[ \prod_{j \in \Phi_M \setminus i} (1 + \rho_M z r_j^{-\alpha})^{-K} \right] \\
\cdot \left[ 1 - (1 + \rho_M z r_j^{-\alpha})^{-(N-K+1)} \right] dz 2\pi \lambda_M dr
\]

Finally, the last step follows from the probability generating functional (PGFL) of the PPP, which states for some function \( f(x) \) that \( \mathbb{E}_{\Phi} [\prod_{x \in \Phi} f(x)] = \exp(-\lambda \int_{\mathbb{R}^2} (1 - f(x))dx) \).

We have

\[
\mathbb{E}_{\Phi_M \setminus i} \left[ \prod_{j \in \Phi_M \setminus i} (1 + \rho_M z r_j^{-\alpha})^{-K} \right] = \exp \left( -\lambda_M \int_0^{2\pi} \int_r^\infty [1 - (1 + \rho_M z r_j^{-\alpha})]^{-K} dr_j d\theta \right)
\]

We now have (3.6), i.e.

\[
\hat{R}_M = \int_0^\infty e^{-\pi \lambda_M r^2} \int_0^\infty \frac{e^{-z}}{z} \exp \left( -\lambda_M \int_0^{2\pi} \int_r^\infty [1 - (1 + \rho_M z r_j^{-\alpha})]^{-K} dr_j d\theta \right) \\
\cdot \left[ 1 - (1 + \rho_M z r_j^{-\alpha})^{-(N-K+1)} \right] dz 2\pi \lambda_M dr
\]

Although this ergodic rate per MUE is for MBS locations following a PPP on an infinite area, we can easily convert it to our finite circular area \( \mathcal{A} \) by setting the upper limit of the outermost integral to the radius of \( \mathcal{A} \). The expression in (3.6) can be easily evaluated using hypergeometric functions [21].

3.2.2 Ergodic Rate Per SUE

In this section we present the ergodic rate (as defined in (2.3)) per SUE in our proposed small cell network. Similarly, the analysis is based on a typical SUE at the center of the circle. Let \( r \) denote the distance from the typical SUE to its closest serving SC AP and
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\( \mu_S = \frac{1}{P_S} \) as the inverse of the SC AP transmit power. The ergodic rate per SUE can be expressed as

\[
R_S = \mathbb{E}_{\Phi_S,\{h,g\}}[\log(1 + \gamma_i^S)]
\]

\[
= \mathbb{E}_r[\mathbb{E}_{\Phi_S,\{h,g\}}[\log(1 + \gamma_i^S)|r]]
\]

\[
= \int_0^\infty e^{-\pi \lambda_S r^2} \mathbb{E}_{\Phi_S,\{h,g\}} \left[ \log \left( 1 + \frac{\rho_S |h|^2 r^{-\alpha}}{\sum_{j \neq i} \rho_S |g_j|^2 r_j^{-\alpha} + 1} \right) \right] 2 \pi \lambda_S r dr
\]

\[
= \int_0^\infty e^{-\pi \lambda_S r^2} \int_0^\infty \mathbb{P} \left[ \log \left( 1 + \frac{\rho_S |h|^2 r^{-\alpha}}{\sum_{j \neq i} \rho_S |g_j|^2 r_j^{-\alpha} + 1} \right) > t \right] dt 2 \pi \lambda_S r dr
\]

\[
= \int_0^\infty e^{-\pi \lambda_S r^2} \int_0^\infty \mathbb{P} \left[ |h|^2 > r^\alpha \left( \sum_{j \neq i} |g_j|^2 r_j^{-\alpha} + \frac{1}{\rho_S} \right) (e^t - 1) \right] dt 2 \pi \lambda_S r dr
\]

\[
= \int_0^\infty e^{-\pi \lambda_S r^2} \int_0^\infty \mathbb{E}_{\Phi_S,\{h,g\}} \left[ e^{-r^\alpha \left( \sum_{j \neq i} |g_j|^2 r_j^{-\alpha} + \frac{1}{\rho_S} \right) (e^t - 1)} \right] dt 2 \pi \lambda_S r dr
\]

\[
= \int_0^\infty e^{-\pi \lambda_S r^2} \int_0^\infty e^{-\frac{1}{\rho_S} r^\alpha (e^t - 1)} \mathbb{E}_{\Phi_S,\{h,g\}} \left[ e^{-r^\alpha \left( \sum_{j \neq i} |g_j|^2 r_j^{-\alpha} (e^t - 1) \right)} \right] dt 2 \pi \lambda_S r dr
\]

Noting the interference channel power \(|g_j|^2\) is exponentially distributed with unit mean and applying the PGFL of PPP, the ergodic rate per SUE can be simplified to ([1], [Section IV-A, Theorem 3])

\[
R_S = \int_0^\infty e^{-\pi \lambda_S r^2} \int_0^\infty e^{-\frac{1}{\rho_S} r^\alpha (e^t - 1)} \exp \left( -\pi \lambda_S r^2 (e^t - 1)^\frac{2}{\alpha} \int_{(e^t - 1)^\frac{2}{\alpha}}^\infty \frac{1}{1 + u^2} du \right) dt 2 \pi \lambda_S r dr
\]

(3.9)

Again, this expression can be easily evaluated using hypergeometric functions.

### 3.3 Numerical Results

Given the complex relationship between the optimization variables and the objective function, the proposed optimization problem (3.5) does not fit into any framework that allows for computationally efficient solutions for the global optimum. We can easily
find locally optimal solutions using standard techniques such as sequential quadratic programming [9].

This approach proves to be fairly robust and leads to the same locally optimal solutions when exhaustive search is used. In this section we illustrate the results from exhaustive search using 3D plots of the objective function, and this leads to a far richer understanding of the problem at hand.

### 3.3.1 Load Factor

First, we would like to see how the number of MUEs $K$ and the number of MBS transmit antennas $N$ affect the per MBS sum rate. The per MBS sum rate is calculated by multiplying the MUE ergodic rate (3.6) by the number of MUEs $K$. We define the ratio between $K$ to $N$ using a load factor $L_f$. We first show in Figure 3.1 that the per MUE ergodic rate decreases with increasing $L_f$. This is not surprising since as the number of MUEs increases, each user receives less power and diversity order. In this figure, the total number of MBS antennas $N$ is set to 100 and the total bandwidth $W$ to $10^7$ Hz.

---

2Implemented as *fmincon* in MATLAB.
Figure 3.2: Per-MBS sum rate as a function of the load factor $L_f = K/N$.

We compared in the figure the MUE ergodic rate expression we derived in (3.6) and the result from Monte Carlo simulation. As is clear from the figure, the analysis captures the behavior of the MUE ergodic rate expression.

While increasing $L_f$ decreases the MUE ergodic rate, but we are also introducing more MUEs into the network and this affects the per MBS sum rate. As mentioned before, it has been shown in [13] that, to maximize the sum rate to the $K$ users, we need $K/N \approx 0.6$. In Figure 3.2, we confirmed this result. The figure plotted the sum rate per MBS for different $L_f$ while fixing $N = 100$ and $W = 10^7$, and confirms that, indeed, the per-MBS rate is maximized when $K/N \approx 0.6$.

In the following section, therefore, we use the expression in (3.6) for the MUE ergodic rate. Further, since our goal is to maximize the sum rate, we fix $K = 0.6N$ in (3.5).
Table 3.1: Parameters used in the simulations

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Path loss exponent</td>
<td>$\alpha$</td>
<td>3.75</td>
</tr>
<tr>
<td>MBS transmit power</td>
<td>$P_M$</td>
<td>43 dBm</td>
</tr>
<tr>
<td>SC AP transmit power</td>
<td>$P_S$</td>
<td>23 dBm</td>
</tr>
<tr>
<td>Noise power</td>
<td>$\sigma^2$</td>
<td>-174 dBm</td>
</tr>
<tr>
<td>MBS load factor</td>
<td>$L_f$</td>
<td>0.6</td>
</tr>
<tr>
<td>Bandwidth</td>
<td>$W$</td>
<td>$10^7$ Hz</td>
</tr>
<tr>
<td>MBS tower hardware cost</td>
<td>$C_{M0}$</td>
<td>1 unit</td>
</tr>
<tr>
<td>MBS antenna hardware cost</td>
<td>$C_{M1}$</td>
<td>0.01 unit</td>
</tr>
<tr>
<td>SC AP hardware cost</td>
<td>$C_S$</td>
<td>1/12 unit</td>
</tr>
<tr>
<td>Total hardware cost</td>
<td>$C_T$</td>
<td>30 units</td>
</tr>
<tr>
<td>Radius of network area $\mathcal{A}$</td>
<td></td>
<td>2500m</td>
</tr>
</tbody>
</table>

### 3.3.2 Results From Exhaustive Search

In this section, we present the numerical solutions to the optimization problem (3.5) when exhaustive search is used. The values of the system parameters used to solve (3.5) are listed Table 3.1. Note that we have set $C_S/C_{M0} = 1/12$, i.e., a SC AP is assumed to cost 1/12th a MBS [19]. A single antenna costs only 1% of the MBS cost ($C_{M1} = 0.01C_{M0}$).

The exhaustive search is done by first generating all feasible triples of $(N, n_M, n_S)$ that satisfy the overall cost constraint $C_T$, then calculating the sum rate in (3.5) for each of the triplets. It is important to note that the objective function in (3.5) is an increasing function in $N, n_M,$ and $n_S$, so the generated triplets are the ones meet the cost constraint with equality.

In analyzing the multiuser MIMO case in (3.6) and in Figure 3.2, the inherent assumption is that the MBS is able to adaptively beamform to $K$ users simultaneously. If this were not possible, a MIMO MBS would have to use single user BF, i.e., $K = 1$. To understand the role of adaptive BF (via ZF BF) we first numerically solve (3.5) for the case when SU-BF is used (i.e., $K = 1$). We then numerically solve (3.5) for different $D_P$ when ZF-BF is used.

Figure 3.3 plots the user sum rate in $\mathcal{A}$ with respect to different $n_M$ and $n_S$ when SU-
BF is used. From [10] we know that SU-BF provides a higher ergodic rate than ZF-BF, but we can see here that SU-BF gives a low overall sum rate due to the lower allocated bandwidth. The optimal rate is achieved at $n_M = 0$, $N = 0$, and $n_S = 360$ (deploy SC APs only). Essentially, if the MIMO MBSs can service one user only, the relatively low rate achieved makes the cost associated with a MBS not worth it. A greater rate is achieved by the far larger number ($12 \times$) of SC APs that can be deployed for the same cost.

Figure 3.3 shows that for MBSs to be worth their cost, they must use adaptive BF, here implemented as ZF-BF. Figure 3.4 plots the user sum rate in $\mathcal{A}$ with respect to different $n_M$ and $N$ when ZF-BF is used. We begin with the ideal case where the CSI is acquired for free ($D_P = 0$). The optimal rate is achieved at $n_M = 3$, $N = 900$, and $n_S = 0$ where $R_A = 26.190$ Gbits/s. Note that since $D_P = 0$ the number of antennas $N$ is only constrained by $C_{M1}$, and can essentially grow arbitrarily large. The optimization shows that with perfect CSI available without cost, it is optimal to deploy a few massive MIMO MBSs; in this case, the additional rate achieved by adding a SC AP, is not worth the
cost (relative to the cost of an additional antenna since each SC AP represents the same cost as about 8 antennas). Furthermore, since $K \simeq 0.6N$, this represents an increase of approximately 5 users served.

The previous example started with an unrealistic assumption of no training cost and ended with the likely unrealistic solution of $N = 900$. The next example sets $D_P = 0.5\%$. Figure 3.5 shows the user sum rate in the region with respect to different $n_M$ and $N$ for $D_P = 0.5\%$. The optimal rate is achieved at $n_M = 14, N = 114$, and $n_S = 0$ where $R_A = 11.21 \text{ Gbits/s}$. Notice a close second optimal $R_A = 9.92 \text{ Gbits/s}$ is achieved when the deployment strategy is shifted to $n_M = 0, N = 0$, and $n_S = 360$.

Interestingly, both these ‘solutions’ occur at extremes, i.e., either the solution is to deploy MBSs or SC APs, but not both. Figure 3.6 shows a slice of Figure 3.5 at number of MBS is equal to 9, and we can clearly see that taking away SC APs and adding more MBS antennas is unrewarding in terms of sum rate until at the point where there’s no SC APs left. Again, the relatively low cost of an antenna (and of training) suggests that a large number of antennas is more useful than a few SC APs. Note, however, that even
the modest cost of training of 0.5% of a channel coherence period per user has reduced the optimal number of antennas from 900 to 114.

Figure 3.7 shows the user sum rate in $\mathcal{A}$ with respect to different $n_M$ and $n_S$ for $D_P = 0.75\%$. The optimal rate is achieved at $n_M = 0$, $N = 0$, and $n_S = 360$ where $R_A = 9.92$ Gbits/s. Note the second optimal $R_A = 9.06$ Gbits/s is achieved when the deployment strategy is switched to $n_M = 16$, $N = 87$, and $n_S = 0$. Interestingly a small change in the cost of training (0.75% of the channel coherence time) moves the optimal point to the use of SC APs exclusively. Figures 3.5 and 3.7, taken together, illustrate the importance of the training and how they affect aggregate system behavior.

One could add a constraint on the number of antennas per MBS. In the next example we cap the number of antennas to 40. Figure 3.8 shows the sum rate for different number of MBS with this additional constraint on the number of antennas. Here $D_P = 0.75\%$. We can see that in this case the optimal deployment strategy is again to deploy only SC APs.

Furthermore, we note that MBSs are better suited at serving mobile users than SC
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Figure 3.6: A slice of Figure 3.5 at number of MBS is equal to 9

Figure 3.7: User sum rate for $D_P = 0.75\%$
APs. In a practical heterogeneous network, a minimum number of MBSs are deployed to ensure widespread coverage. In this final example we set the minimum number of MBSs to be three and investigate how the sum rate changes with varying number of SC APs and MBS antennas. Figure 3.9 shows the sum rate for a minimum number of three MBSs and $D_P = 0.75\%$. We see in this case it is still more cost effective to deploy a maximum number of SC APs.
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Figure 3.8: Sum rate when $N$ is capped at 40

Figure 3.9: Sum rate when the minimum number of MBS is 3
3.4 Discussion

In this chapter we addressed a question of growing importance in the implementation of multi-tier networks: we ask what should be the mix of macro MBSs and SC APs. In taking a new look at this question, we take into account issues of the required training and the relative cost of MBS and SC APs. Our results lead to some interesting results - for MIMO MBSs to be cost-effective, it must be able to serve multiple users simultaneously and also train the users efficiently. We see for a fixed load factor $L_f$, the MBS is able to bring into the network $L_f$ users by adding one antenna at a very low cost. It is therefore logical that for a fixed hardware cost and no training cost, the sum rate peaks at few deployments of MBS each equipped with large number of antennas. However, the cost of training cannot be ignored in real LS-MIMO networks and it limits the number of MUEs each MBS serves. Training cost essentially brings down the cost effectiveness of LS-MIMO and even with a relatively low training period of $0.75\%$ of the channel coherence, the optimization results in a single tier deployment of SC APs.
Chapter 4

Effect of Pilot Contamination

In this chapter, we abandon the perfect CSI assumption for LS-MIMO and add MMSE channel estimation and pilot contamination to our channel model. We then analyze how pilot contamination affect the downlink rate in the LS-MIMO network under ZF-BF.

4.1 Channel Estimation

As was mentioned our system model, to exploit channel reciprocity we adopt the TDD scheme for our LS-MIMO network. We assume the uplink channel to be equal to the downlink channel during the coherence period $T_c$. To estimate the local channel, the $K$ local MUEs first transmit synchronously on the uplink to their serving MBS a set of mutually orthogonal pilot sequences which the MBS then uses to compute estimates of the local channels. We denote the set of $K$ pilot sequences as $\{\psi_1, ..., \psi_K\}$ and each $\psi_k$ is a length-$\tau$ column vector, where $K \leq \tau \leq T_c$. Since $T_c$ is of finite length, the same set of $K$ pilot sequences needs to be reused in each cell.

The $i$-th MBS receives the following $N \times \tau$ matrix

$$Y_{ii}^{tr} = \sum_{j=1}^{J} \sum_{k=1}^{K} \sqrt{p_{tr} \tau} g_{ijk} \psi_k^T + n_{jk}^{tr}, \quad (4.1)$$

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where \( n_{jk} \sim \mathcal{CN}(0, I_N) \) and \( p_{tr} \) is the pilot transmit power. Since the pilot sequences are mutually orthogonal (i.e. \( \psi_k^T \psi_{k'} = \delta_{kk'} \)), the MBS can single out the observation for the local channel with each MUE \( k \) simply by \( Y_{ii}^{tr} \psi_k \) and we get

\[
y_{iik}^{tr} = g_{iik} + \sum_{j \neq i} g_{ijk} + \frac{1}{\sqrt{p_{tr}}} n_{jk}^{tr}
\]  

(4.2)

The MMSE estimate of the channel to MUE \( k \) by MBS \( i \) is

\[
\hat{g}_{iik} = r_{iik}^{-\alpha} \left( 1 + p_{tr} \tau \sum_j r_{ijk}^{-\alpha} I_N \right)^{-1} y_{iik}^{tr}
\]

\[
= \frac{r_{iik}^{-\alpha} p_{tr} \tau}{1 + p_{tr} \tau \sum_j r_{ijk}^{-\alpha}} \left( \sum_j g_{ijk} + \frac{1}{\sqrt{p_{tr}}} n_{jk}^{tr} \right)
\]

(4.3)

Notice the MMSE estimate \( \hat{g}_{iik} \) is simply a scaled version of \( y_{iik}^{tr} \). Also, from the previous chapter, \( r_{ijk} \) denotes the distance between MBS \( i \) to the \( k \)th MUE in cell \( j \). It can be shown the distribution of \( \hat{g}_{iik} \) is \([16]\)

\[
\hat{g}_{iik} \sim \mathcal{CN} \left( 0, \frac{r_{iik}^{-2\alpha}}{p_{tr} \tau} + \sum_j r_{ijk}^{-\alpha} I_N \right)
\]

(4.4)

Next, using the orthogonality property of MMSE we can decompose the true channel \( g_{iik} \) into \( g_{iik} = \hat{g}_{iik} + \tilde{g}_{iik} \), where \( \tilde{g}_{iik} \) is the uncorrelated estimation error. Additionally, since the estimation and error vectors are jointly Gaussian, orthogonality implies independence; we therefore have

\[
\tilde{g}_{iik} \sim \mathcal{CN} \left( 0, \frac{r_{iik}^{-2\alpha}}{p_{tr} \tau} + \sum_j r_{ijk}^{-\alpha} I_N \right)
\]

\[
\sim \mathcal{CN} \left( 0, r_{iik}^{-\alpha} \left( \frac{1}{p_{tr} \tau} + \sum_{j \neq i} r_{ijk}^{-\alpha} \right) I_N \right)
\]

(4.5)
4.2 Downlink Analysis

In this section, we investigate how the MMSE estimation error introduces intra-cell interference and affects the downlink SINR at the user. The ZF-BF pre-coding matrix is constructed from the estimated channel vectors as

$$\hat{\mathbf{W}}_i = \left[ \hat{\mathbf{G}}_i (\hat{\mathbf{G}}_i^H \hat{\mathbf{G}}_i)^{-1} \right]_{1:K} = [\hat{\mathbf{w}}_i, \ldots, \hat{\mathbf{w}}_K]$$

where $$\hat{\mathbf{G}}_i = [\hat{\mathbf{g}}_{i1}, \ldots, \hat{\mathbf{g}}_{iK}] \in \mathbb{C}^{N \times K}$$. The intra-cell interference comes from the fact that due to estimation errors, the pre-coder weights $$\hat{\mathbf{w}}_{ki}$$, are not orthogonal to the channels of all other users $$k' \neq k$$. The received SINR under MMSE channel estimation for MUE $$k$$ in cell $$i$$ is given by

$$\gamma_{ik}^{\text{MMSE}} = \frac{\rho_M |\hat{\mathbf{g}}_{ik}^H \hat{\mathbf{w}}_{ki}|^2}{\sum_{k' \neq k} \rho_M |\hat{\mathbf{g}}_{ik}^H \hat{\mathbf{w}}_{k'i}|^2 + \sum_{j \neq i} \sum_{k' = 1}^{K} \rho_M |\hat{\mathbf{g}}_{jk}^H \hat{\mathbf{w}}_{k'j}|^2 + 1}$$

(4.6)

Note, the first summation in the denominator corresponds to the intra-cell interference power from other MUEs ($$k' \neq k$$) to the typical MUE. The second term in the numerator corresponds to the contribution of channel estimation error to the signal power.

Now in the first term of the signal power we have an $$N$$-dimensional vector $$\hat{\mathbf{g}}_{ik}$$ with distribution given in (4.4) projected onto the $$N - K + 1$$ subspace spanned by $$\{\hat{\mathbf{g}}_{ik'}\}_{k' \neq k}$$, and as we have shown in Chapter 2, the signal power follows a Gamma distribution

$$|\hat{\mathbf{g}}_{ik}^H \hat{\mathbf{w}}_{ki}|^2 \sim \Gamma\left( N - K + 1, \frac{r_{ik}^{-2\alpha}}{p_{tr}^{-1} + \sum_j r_{ijk}^{-\alpha}} \right)$$

(4.7)

Note, that if the uplink pilot training energy $$p_{tr}$$ grows large and the distances $$r_{ijk}$$ to the pilot contaminating users from other cells ($$j \neq i$$) grow large, we recover the case with perfect knowledge of CSI in (2.7). The presence of the $$\sum r_{ijk}^{-\alpha}$$ terms complicates
the analysis.

Additionally, the second term in the signal power and the intra-cell interference power terms have the same distribution because the estimation error vector $\tilde{g}_{ik}$ is independent of the pre-coding vector $\hat{w}_{k'i}$. Therefore, $|\tilde{g}_{ik}^H\hat{w}_{k'i}|$ is the power of projecting the estimation error vector onto a one-dimensional subspace. Using the distribution of the estimation error vector in (4.5) we have

$$|\tilde{g}_{ik}^H\hat{w}_{k'i}|^2 \sim \Gamma \left( 1, \frac{1}{p_{ir}} + \frac{1}{p_{ir}'r_{ijk}^{-\alpha}} + \sum_j r_{ijk}^{-\alpha} \right), \quad \forall k' \quad (4.8)$$

Furthermore, we assume each of the $\hat{w}_{k'i}$ is independent from each other, and the aggregate intra-cell interference power is the sum of $K - 1$ independent Gamma distributions given by

$$\sum_{k' \neq k} |\tilde{g}_{ik}^H\hat{w}_{k'i}|^2 \sim \Gamma \left( K - 1, \frac{1}{p_{ir}} + \frac{1}{p_{ir}'r_{ijk}^{-\alpha}} + \sum_j r_{ijk}^{-\alpha} \right) \quad (4.9)$$

The inter-cell interference power $|g_{jik}^H\hat{w}_{kj}|^2$ distribution is the same as before since the inter-cell interference is not affected by local channel estimation error.

### 4.2.1 Ergodic Rate

Following a similar approach as in Chapter 3, we present here the lower bound on the downlink ergodic rate for a typical MUE $k$ in an LS-MIMO system with ZF-BF and MMSE channel estimation.

We let $r$ denote the distance between the typical MUE $k$ located at the origin to its serving MBS $i$, we have
Using the SINR expression given in (4.6), we have:

\[
R_{M}^{MMSE} = \mathbb{E}_{\Phi_{,i,\{g\}}} \left[ \log(1 + \gamma_{ik}^{MMSE}) \right] \\
= \mathbb{E}_{r} \left[ \mathbb{E}_{\Phi M \setminus i,\{g\}} \left[ \log(1 + \gamma_{ik}^{MMSE}) | r \right] \right] \\
= \int_{0}^{\infty} \mathbb{E}_{\Phi M \setminus i,\{g\}} \left[ \log(1 + \gamma_{ik}^{MMSE}) | r \right] f_{r}(r) dr \\
= \int_{0}^{\infty} e^{-\pi \lambda r^2} \mathbb{E}_{\Phi M \setminus i,\{g\}} \left[ \log(1 + \gamma_{ik}^{MMSE}) | r \right] 2\pi \lambda M r dr \\
= \int_{0}^{\infty} e^{-\pi \lambda r^2} \mathbb{E}_{\Phi M \setminus i,\{g\}} \left[ \int_{0}^{\infty} e^{-t} \left( 1 - e^{-t\gamma_{ik}^{MMSE}} \right) dt \right] r \left( 2\pi \lambda r \right) dr
\]

From (4.6), the signal power has two terms. We make the first simplifying approximation by neglecting the second term \( |\hat{\mathbf{g}}^{H}_{ik} \hat{\mathbf{w}}_{ki}|^2 \) in the signal term of the SINR. It is a reasonable approximation as we find through simulation that \( |\hat{\mathbf{g}}^{H}_{ik} \hat{\mathbf{w}}_{ki}|^2 \gg |\tilde{\mathbf{g}}^{H}_{ik} \tilde{\mathbf{w}}_{ki}|^2 \), i.e., for reasonable estimation accuracy, the estimation error does not contribute much to the signal term.

Now let

\[ t = z \left( \sum_{k' \neq k} \rho_{M} |\hat{\mathbf{g}}^{H}_{ik} \hat{\mathbf{w}}_{k'i}|^2 + \sum_{j \neq i} \sum_{k' = 1}^{K} \rho_{M} |\hat{\mathbf{g}}^{H}_{jik} \hat{\mathbf{w}}_{k'j}|^2 + 1 \right) \]

\[ dt = \left( \sum_{k' \neq k} \rho_{M} |\hat{\mathbf{g}}^{H}_{ik} \hat{\mathbf{w}}_{k'i}|^2 + \sum_{j \neq i} \sum_{k' = 1}^{K} \rho_{M} |\hat{\mathbf{g}}^{H}_{jik} \hat{\mathbf{w}}_{k'j}|^2 + 1 \right) dz \]
substituting $z$ for $t$, we have

\[ R_{MMSE}^{M} \approx \int_{0}^{\infty} \int_{0}^{\infty} e^{-\pi \lambda M r^2} \mathbb{E}_{\Phi \setminus \{i\}} \left[ \int_{0}^{\infty} e^{-z} \left( \sum_{k' \neq k} \rho_M |\hat{g}_{ik}^H \hat{w}_{k'i}|^2 + \sum_{j \neq i} \rho_M |g_{ij}^H \hat{w}_{ij}|^2 + 1 \right) \right] \cdot \left( 1 - \exp(-z \rho_M |\hat{g}_{ik}^H \hat{w}_{ki}|^2) \right) dz \right] \frac{2 \pi \lambda}{r} \mathbb{d}r \]

Since \( \{\hat{g}_{ik}\}, \{\tilde{g}_{ik}\}, \{g_{jk}\}, \text{ and } \Phi_M \) are independent distributions, we can separate out the expectations

\[ R_{MMSE}^{M} = \int_{0}^{\infty} e^{-\pi \lambda M r^2} \mathbb{E}_{\Phi \setminus \{i\}} \left[ \prod_{j \in \Phi \setminus i} \mathbb{E}_{\{g\}} \left[ \exp(-z \sum_{k' \neq k} \rho_M |\hat{g}_{ik}^H \hat{w}_{k'i}|^2) \right] \right] \mathbb{E}_{\{g\}} \left[ \exp(-z \rho_M |\hat{g}_{ik}^H \hat{w}_{ki}|^2) \right] \mathbb{E}_{\{g\}} \left[ \exp(-z \rho_M |g_{ij}^H \hat{w}_{ij}|^2) \right] \right] \frac{dz}{2 \pi \lambda} \mathbb{d}r \]

Noting the Gamma distributions in (4.7), (4.9), and (2.8) and applying the MGF of Gamma distribution, we have

\[ \mathbb{E}_{\{g\}} \left[ \exp(-z \rho_M |\hat{g}_{ik}^H \hat{w}_{ki}|^2) \right] = \left( 1 + \rho_M z \left( \frac{1}{\rho_M \tau} + \sum_{j} r_{ij}^{-\alpha} \right) \right)^{-(N-K+1)} \]

\[ \mathbb{E}_{\{g\}} \left[ \exp(-z \sum_{k' \neq k} \rho_M |g_{ij}^H \hat{w}_{k'i}|^2) \right] = \left( 1 + \rho_M z \left( \frac{1}{\rho_M \tau} + \sum_{j \neq i} r_{ij}^{-\alpha} \right) \right)^{-K+1} \]

\[ \mathbb{E}_{\{g\}} \left[ \exp(-z \sum_{k' = 1}^{K} \rho_M |g_{ij}^H \hat{w}_{k'j}|^2) \right] = (1 + \rho_M z r_{ij}^{-\alpha})^{-K} \]

Now to simplify the notation, let \( r_j = r_{ijk} \) denote the distance from the origin (where our typical MUE \( k \) is) to the interfering MBS \( j \) and let \( r_k = r_{ijk} \) denote the distance from the pilot contaminating user \( k \) in cell \( j \) to our MBS of interest \( i \). We have
\[ R_M^{MMSE} \approx \int_0^\infty e^{-\pi \lambda_M r^2} \int_0^\infty e^{-z} \frac{e^{-z}}{z} \left[ 1 - \left( 1 + \rho_M z \left( \frac{r^{-2\alpha}}{1 + \frac{1}{\rho_M} + \sum_j r_j^{-\alpha}} + \sum_j r_j^{-\alpha} \right) \right)^{-(N-K+1)} \right] \]

\[ \cdot \left[ 1 + \rho_M z \left( \frac{1}{\rho_M} + \sum_j r_j^{-\alpha} \right)^{K+1} \right] \mathbb{E}_{\Phi_{M\setminus i}} \left[ \prod_{j \in \Phi_{M\setminus i}} (1 + \rho_M z r_j^{-\alpha})^{-K} \right] dz 2\pi \lambda_M dr \]

Notice this expression depends on the distance \( r_k \) from the pilot contaminating MUEs in other cells to the serving MBS, and it gives the downlink ergodic rate for the typical MUE for a fixed set of pilot contaminating MUE locations. Therefore, to take out the dependency on \( r_k \), we need to take the expectation over the pilot contaminating MUEs locations. Since the same set of \( K \) pilot sequences is used in each cell, there is one MUE from each cell other than \( i \) that transmits the same pilot sequence as the typical MUE. Therefore, the distribution of the pilot contaminating MUEs follow the same PPP \( \Phi_{\lambda_M \setminus i} \) as the interfering MBSs.

To keep the expression tractable, we will make another approximation by taking the expectation over the pilot contaminating MUE locations in the denominator. Note \( \sum_j r_j^{-\alpha} = r + \sum_{j \neq i} r_j^{-\alpha} \), and we approximate \( \sum_{j \neq i} r_j^{-\alpha} \) by \( \mathbb{E}_{\Phi_{M\setminus i}} \left[ \sum_{j \neq i} r_j^{-\alpha} \right] \). By Jensen’s inequality we have the following lower bound

\[ R_M^{MMSE} \geq \int_0^\infty e^{-\pi \lambda_M r^2} \int_0^\infty e^{-z} \frac{e^{-z}}{z} \left[ 1 - \left( 1 + \rho_M z \left( \frac{r^{-2\alpha}}{1 + \frac{1}{\rho_M} + r^{-\alpha} + \zeta(r)} \right) \right)^{-(N-K+1)} \right] \]

\[ \cdot \left[ 1 + \rho_M z \left( \frac{1}{\rho_M} + r^{-\alpha} + \zeta(r) \right)^{K+1} \right] \mathbb{E}_{\Phi_{M\setminus i}} \left[ \prod_{j \in \Phi_{M\setminus i}} (1 + \rho_M z r_j^{-\alpha})^{-K} \right] dz 2\pi \lambda_M dr \]

For \( \alpha = 3.75 \), we have \( \zeta(r) = \mathbb{E}_{\Phi_{M\setminus i}} \left[ \sum_{j \neq i} r_j^{-\alpha} \right] = \frac{3.59 \lambda_M (e^{-\pi \lambda_M r^2} - 2.72 (\lambda_M r)^2 \theta (1 + \pi \lambda_M r^2))}{r^4} \).

The expectation is taken over \( r_k \geq r \) as we assume the pilot contaminating MUEs are further from the serving MBS than the typical MUE.

Finally, the last step follows from the probability generating functional (PGFL) of the
Figure 4.1: MUE ergodic rate as a function of uplink training power $p_{tr}$.

PPP, which states for some function $f(x)$ that $\mathbb{E}_\Phi[\prod_{x \in \Phi} f(x)] = \exp(-\lambda \int_{\mathbb{R}^2} (1-f(x))dx)$. We have

$$\mathbb{E}_{\Phi \setminus i} \left[ \prod_{j \in \Phi \setminus i} (1 + \rho_M z r_j^{-\alpha})^{-K} \right] = \exp \left(-\lambda_M \int_0^{2\pi} \int_0^\infty \left[1 - (1 + \rho_M z r_j^{-\alpha})\right]^{-K} dr_j d\theta \right)$$

We have the following lower bound on the ergodic rate for a typical MUE in an LS-MIMO network with ZF-BF and MMSE channel estimation

$$R_M^{MMSE} \geq \int_0^\infty e^{-\pi \lambda_M r^2} \int_0^\infty \frac{e^{-z}}{z} \exp \left(-\lambda_M \int_0^{2\pi} \int_r^\infty \left[1 - (1 + \rho_M z r_j^{-\alpha})\right]^{-K} dr_j d\theta \right) \cdot \left[1 - \left(1 + \rho_M z \left(\frac{1}{p_{tr}^\alpha} + r^{-\alpha} + \zeta(r)\right)\right)^{-K} \left(\frac{1}{p_{tr}^\alpha} + r^{-\alpha} + \zeta(r)\right)^{-K+1} \right] \frac{1}{2\pi \lambda} dr$$

Figure 4.1 shows a comparison between the results using (4.11) and Monte Carlo
Figure 4.2: User sum rate (MMSE channel estimation) with ZF-BF for $D_P = 0.75\%$ and $p_{tr} = 23$ dBm.

Simulation with the same system parameters listed in Table 3.1. The ergodic rate is plotted against the uplink training power $p_{tr}$ with $\tau = K$, and we can see the overall ergodic rate is lower than in the case of perfect CSI. Comparing (4.11) with simulation, we see it indeed gives a lower bound on the ergodic rate, and the bound becomes tight as $p_{tr}$ increases. Note as $p_{tr}$ increases, the rate first shows a steep increase and then plateaus at high $p_{tr}$. At high $p_{tr}$ the contribution of $\frac{1}{p_{tr}}$ becomes insignificant in the denominators of (4.11) and pilot contamination becomes the limiting factor.

Keeping the uplink training power constant at 23 dBm, we make the sum rate comparison between the two cases where we assume perfect CSI and MMSE channel estimation. Figure 3.7 from Chapter 3 shows the user sum rate in $\mathcal{A}$ for $D_P = 0.75\%$ and Figure 4.2 shows a similar plot, the only difference is that the MMSE ergodic rate approximation (4.11) is used instead of the perfect CSI ergodic rate (3.6). We can see that in Figure 4.2 the intensities at the regions of the LS-MIMO network (nonzero MBS with large number of antennas $N$) are significantly lower than those in Figure 3.7. This can be interpreted
as the effect of channel estimation and pilot contamination significantly lower the sum rate for the LS-MIMO network.

### 4.3 Load Factor

In chapter 3 we have confirmed that, under perfect CSI assumption, a load factor $L_f \approx 0.6$ maximizes the per cell sum rate. However, it is unclear whether it is the same case when channel estimation is considered and whether the uplink training power will affect the optimal load factor.

Figure 4.3 plots the per MBS sum rate against different $L_f$ for four low values of uplink training power $p_{tr}$. The per MBS sum rate is calculated by multiplying the per MUE ergodic rate (4.11) with the number of MUEs $K$ and a total bandwidth $W = 10^7$ Hz. We can see that the optimal $L_f$ which maximizes the sum rate shifts from 0.3 (when $p_{tr} = -2$ dBm) to 0.5 (when $p_{tr} = 4$ dBm). The result indicates that at low $p_{tr}$, it is beneficial to serve a smaller number of MUEs.
Figure 4.4: MBS sum rate as a function of load factor $L_f$ for high $p_{tr}$.

Figure 4.4 plots the per MBS sum rate against different $L_f$ for four values of high $p_{tr}$. We can see that at high $p_{tr}$, the optimal $L_f$ that maximizes the sum rate settles at 0.5. This optimal $L_f$ is not far from the perfect CSI case of 0.6, and since we have used (4.11) as an approximation for the per MUE ergodic rate calculation, the true optimal $L_f$ is likely to lie between 0.5 and 0.6.
Chapter 5

Conclusion

In this thesis we investigated the problem of maximizing the area sum rate of a HetNet combining LS-MIMO and SC networks under a hardware cost constraint. We developed the per user downlink ergodic rate in an LS-MIMO network performing adaptive beam-forming via ZF-BF. Using the per user ergodic rate we developed, we then confirmed a previously discovered result that a load factor of 0.6 maximizes the per MBS sum rate. We showed under perfect CSI assumption, it is more cost effective to deploy a few MBSs equipped with large number of antennas. We then showed that the benefit of LS-MIMO is very dependent on the efficiency of uplink training for channel estimation. The overhead of training arises from the fact that a fraction of the total available resources (channel coherence period) is dedicated to users transmitting pilots, and the overhead linearly scales with the number of users. We showed that SC network proves to be more cost effective than LS-MIMO when even a small fraction of channel coherence period used per user for training is considered.

We then investigated how the per user ergodic rate was affected when the perfect CSI assumption is lifted. We applied MMSE for channel estimation and considered uplink training with reused pilots in each cell. We developed a lower bound on the per user ergodic rate and showed that pilot contamination degrades the ergodic rate and its severity is dependent on the relative distances of the pilot contaminating users to the
serving MBS. Furthermore, using the lower bound we developed, we showed that the optimal load factor which maximizes the per MBS sum rate shifts from 0.1 to 0.5 as the uplink training power increases.

The work of this thesis could be extended in the following ways:

- As was mentioned earlier, there’s a fundamental trade-off between energy efficiency and spectral efficiency in LS-MIMO networks. Having looked at the area sum rate, it will be interesting to see how the results will change when energy efficiency is used as an alternative performance measure.

- In the optimal deployment problem we have assumed the LS-MIMO network to be operating under a fixed load factor and users stationary. It may be useful to analyze the coverage of the network with added user mobility and sparse user density.

- We have assumed orthogonal spectrum allocation to the optimal deployment problem. It will be interesting to see how inter-tier interference will affect the results in a co-channel deployment of the LS-MIMO and SC AP networks.
Bibliography


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