Supplementary Material

S1 Incorporating seasonality into mizer

To add seasonality into the existing mizer code, two steps are required - firstly to modify the resource spectrum, and secondly to alter the spawning function for the different species.

1. To add seasonal resource pulses, alter equation (3.15) of Scott et al. (2015) to Table 1, equation (19). The parameters $\omega$, $\nu_p$ and $t_p$ need to be added to the code, with values as in Table 2.

2. To add seasonal spawning to the model, the spawning timing strategy (1) for each species needs to be set, using values of $\nu_i$ and $t_i$ taken from Table 2. Once these are set up, the growth term for each species, given by equation (3.8) of Scott et al. (2015), needs to be updated to (10), with the egg production term correspondingly changed to (11).

Altering these functions will allow for seasonal reproduction within the mizer modelling framework. The R code for the model, simulations and figures are also available for download at the following link: https://figshare.com/s/e75f29d4cc9b94ae393b.

S2 Plotting the time-dependent function

Figure S1 shows the effect of increasing $\nu_i$ on the time-dependent function $s$ (1).

For $\nu_i = 0$, $s$ is constant over all time (solid line in Figure S1). As $\nu_i$ increases the Gaussian-shaped peak becomes steeper and narrower, but maintaining equivalent areas under the curves (1) for all values. Thus, as long as food supply is constant, an equal number of eggs are produced over a year by species in non-seasonal (i.e. $\nu_i = 0$) and seasonal ($\nu_i > 0$) systems.
Figure S1: The time-dependent function (1) for an individual, where $w$ is low enough that $\psi_i(w) = 0$ (so that seasonality is not damped). The function $s$ is plotted for $t_i = 0.5$ (so the peak of spawning is halfway through the year), and for a range of values of $\nu_i$.

S3 A derivation of the reproduction function

In previous work, the jump-growth equation for modelling the predation process was derived from a basic stochastic process involving one individual eating another and gaining weight (Datta et al. 2010). In a similar way, to model reproduction the individual level process is initially considered, summarised in Figure S2. Note that for this section we use the ln weight of organisms, which we refer to as $x$.

Figure S2: The change in abundances at different weights during a single reproductive event. A fish of weight $x_a$ loses a small amount of weight (to become weight $x_b$) while depositing a large number of eggs further down the spectrum at $x_c$.

From the individual level process shown in Figure S2 Capitan and Delius (2010) de-
The left hand side is the change over time in \( u(x,t) \), the number of organisms per unit volume of size \( x \). \( r(x,x',x'',t) \) is the rate at which individuals of weight \( x \) reduce to weight \( x' \) by producing offspring of weight \( x'' \). The three terms within the double integral can be intuitively understood from Figure S2 as a reproduction event reduces the number of individuals at one weight and increases numbers at two other weights. Hence the three terms are, in turn: (i) an organism of weight \( x \) spawns, and loses weight to become less than \( x \), (ii) an organism of higher weight than \( x \) spawns to lose weight to become weight \( x \), and (iii) weight \( x \) is the weight of the offspring, and receives numbers from a spawning individual (the preceding fraction calculates the number of offspring from the difference in weights before and after the spawning event). Note that the subscript \( b \) on the left hand side indicates that only the reproductive process is considered at present; reproduction is incorporated into the full dynamic model at the end of the section.

Using the model (S1) for reproduction would be computationally demanding, and require extremely fine discretisation of the weight range for numerical simulations, as noted for the jump-growth equation (Datta et al. 2010). Thus a simplified model is used here for the reproductive process; for the sake of argument, the continuous weight range is discretised into weight brackets, as in Datta et al. (2010), for a more intuitive grasp of the system. The following assumptions are now made. Firstly, the mass of eggs tends to vary little in many pelagic species (Ware 1975; Cury and Pauly 2000). Secondly, mature fish generally weigh much more than the eggs they produce (several orders of magnitude larger is commonplace); it is therefore reasonable to as-
sume that a parent fish has similar weights before and after a reproductive event. Thus the model of reproduction appears similar to the metabolic loss term of Capitan and Delius (2010); in a discretised system fish simply move to the weight bracket below in a reproductive event, while the appropriate amount of mass moves to the weight bin where offspring are born, and converted into the appropriate number of offspring (Figure S3). The log weight of newborns is labelled $x_0$. The equation for reproduction

\[ \frac{d}{dt} u_i = \sum_j \left( e^{x_j} - e^{x_j-1} \right) \Delta, \]

(S2)

By taking the limit $\Delta \to 0$ the continuum limit of (S2) is derived. This follows steps taken in previous work (Datta et al. 2010; Capitan and Delius 2010). Thus the number $u_i(t)$ becomes a number density per unit volume $u(x,t)$, such that $u(x_i,t) = u_i(t) / \Delta$, and $r_i(t)$ becomes $r(x_i,t)$, such that $r(x_i,t) = r_i(t) \Delta$. The sum in (S2) is replaced by an integral. Rewriting the fraction in (S2) as

\[ \frac{e^{x_i}}{e^{x_0}} - \frac{e^{x_i-1}}{e^{x_0}} = \frac{e^{x_i}(1 - e^{-\Delta})}{e^{x_0}} \]

(S3)

where the first order approximation for $1 - e^{-\Delta}$ is taken, then taking $\Delta \to 0$ changes the sum $\sum \Delta$ to an integral $\int dx$. Thus the reproductive process in the continuum limit
is given by
\[
\begin{align*}
\left( \frac{\partial u(x, t)}{\partial t} \right)_b &= \frac{\partial}{\partial x} (r(x, t)u(x, t)) \quad \text{for } x \neq x_0 \\
\left( \frac{\partial u(x_0, t)}{\partial t} \right)_b &= \int e^{x-x_0} r(x, t)u(x, t) \, dx
\end{align*}
\]
(S4)
where time-dependence has been re-introduced to the reproduction and abundance functions.

It is noted that the assumptions used to derive (S4) only hold as long as the loss in weight during reproductive events is small. Thus, if spawning individuals are close in mass to that of newborns, or many offspring are produced over a short period of time, then the approximation to (S1) becomes worse. This is analogous to the McKendrick-von Foerster equation being a suitable approximation to the jump-growth equation only when prey are generally much smaller than predators (Datta et al. 2010).

Reproduction is included in the McKendrick-von Foerster equation with diffusion by altering the the first-order growth term, which moves biomass up the spectrum. As reproduction involves individuals losing weight, biomass is shifted the opposite way; the overall flux of biomass is the difference of the two rates. Thus, the McKendrick-von Foerster equation with diffusion becomes
\[
\frac{\partial u(x)}{\partial t} = -\mu u - \frac{\partial}{\partial x} \left( (g - r)u \right) + \frac{1}{2} \frac{\partial}{\partial x} \left( e^{-x} \frac{\partial}{\partial x} (du) \right)
\]
(S5)
where \( r(x, t) \) is the reproduction rate and \( g - r \) is the overall growth rate. Removing the final term in (S5) gives the equivalent form of the McKendrick-von Foerster equation (without diffusion). The abundance of offspring is similar to the boundary condition of Blanchard et al. (2011), although reproduction has explicit time-dependence here.

In this paper a simple income breeding method is used to model the reproduction process, using a time-dependent fraction of mass assimilated from feeding (12). However, the inclusion of reproduction into (S5) can generally be applied to any marine growth
model, as long as offspring are assumed to be orders of magnitude smaller than the spawner. Importantly, capital as well as income breeding methods can be used, enabling organisms to potentially lose mass during spawning. However, if an organism drops in weight significantly in a reproduction event (by producing many eggs over a short period of time) the assumption of small weight changes no longer applies, and an alternative model for reproduction is required.

Realistically a number of different factors affect reproductive output (Lambert and Du-til [2000]), and a simple model is employed to allow greater tractability. Organisms do not reproduce below the maturity weight of fish $w_{i}^*$ (for species $i$). The reproduction function is specified as (11), with both size- and time-dependence. Only female fish are assumed to contribute biomass to reproduction, hence the $1/2$ factor in population level egg production (13). Note that for simplicity the stages between spawning (i.e. incubation, hatching and yolk resorption) are not modelled here (see Duarte and Alcaraz [1989] Huss et al. [2012]), and offspring are assumed to immediately feed in a size-based way.

References


