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Unsteady flow over an expanding cylinder in a nanofluid containing gyrotactic microorganisms

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Abstract: In this paper, an analysis is made for the unsteady flow due to an expanding cylinder in a nanofluid which contains both nanoparticles and gyrotactic microorganisms with suction. The nonlinear system of partial differential equations is transformed to high order non-linear and ordinary differential equations by similarity transformations and then solved numerically using a shooting method with fourth-fifth-order Runge-Kutta integration technique. Besides, the influences of significant physical parameters on the distributions of the velocity, temperature, nanoparticle volume fraction, as well as the density of motile microorganisms are graphically presented and discussed in detail. It is found that dual solutions exist for both stretching and shrinking cases and the range of dual solutions increases with the strength of the expansion. The results also indicate that larger bioconvection Peclet number and smaller Schmidt number lead to higher the concentrate of microorganisms and thicker boundary layer thickness.

Keywords: Nanofluid, Gyrotactic microorganisms, Expanding cylinder, Stretching/Shrinking

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1 Introduction

Poor thermal conductivity of conventional fluids such as water, ethylene glycol, and engine oil, limit the performance enhancement and the compactness of many thermal applications. An innovative technique for improving heat transfer by using nanoparticles (diameter less than 50 nm) in a based fluid has been used extensively recently. It was first introduced by Choi [1] who proposed the term ‘nanofluid’ to describe the liquid suspension, which has the unique physical and chemical properties due to the ultrafine of nanoparticles [2]. After the pioneering work, nanofluids have attracted enormous interest from researchers because of its many significant applications such as medical applications, chemical engineering, transportations, electronics as well as manufacturing and food. A comprehensive literature on the topic of nanofluid can be found in the famous book by Das et al. [3] and Nield and Bejan [4], and in the review papers by Buongiorno [5], Kakaç and Pramuanjaroenkij [6], Wong and De Leon [7], Fan and Wang [8], Mahian et al. [9], Hussein et al. [10], and Mahdi et al. [11].

It should be mentioned that the nanoparticles in the base fluid are unstable, which may result in aggregating and decreasing of thermal conductivity of nanofluids. Introducing the upswimming microorganisms is a novel method presented by Kuznetsov [12] to solve unstable problem and avoid nanoparticles from agglomerating and aggregating. Thus, The nanofluid containing both nanoparticles and gyrotactic microorganisms has drawn considerable attention of several researchers such as Tham et al. [13], Zaimi et al. [14], Khan et al. [15, 16], Mutuku et al. [17], Xu and Pop [18] and many others. In a nanofluid containing nanoparticle and motile microorganisms, it should be also mentioned that the nanoparticles are not self-propelled and their motion is driven by Brownian diffusion and thermophoresis occurring in the nanofluids which explored by Buongiorno [5].

During the past few years, the problem of nanofluid over a cylinder have been explored by a great number of researchers due to its many relevant applications such as heat exchangers, nuclear reactor fuel rods and steel suspension bridge cables. It seems that Nazar et al. [19] have first studied the mixed convection flow past a circular cylinder embedded in a porous medium saturated by a nanofluid. They reported that heating the cylinder delays boundary layer separation and cooling the cylinder brings the boundary layer separation point nearer to the lower stagnation point. Later Grosan and Pop [20] numerically examined the steady axisymmetric mixed convection flow and heat transfer characteristics past a vertical cylinder immersed in a nanofluid. It was found that dual solutions exist when the cylinder surface is cooled. Ever since, much more attention has been drawn to this type of flow such as Tham et al. [21, 22], Rashad et al. [23, 25], Prasad et al. [25], El-Kabeir et al. [26], Rohni et al. [27], and Ashorynejad et al. [28].

All the above investigations deal with only the steady flow of the nanofluids along the cylinder. In reality the flow and heat transfer problems may be unsteady, due to motion of cylinder surface or the sudden changes of surface temperature. The steady (fully developed) flow is results of the development of viscous unsteady

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flow after a certain instant of time in most cases. However, very little work on unsteady nanofluids flow along the cylinder has been reported in the literatures including Zaimi et al. [29], Elbashbeshy et al. [30], Chamkha et al. [31], and Sarkar et al. [32]. It may be remarked that the innovative method on adding of gyrotactic microorganisms to avoid nanoparticles from agglomerating and aggregating, has not been considered in the problem of unsteady nanofluid flow over cylinder.

The major of the present paper is to investigate the unsteady flow due to an expanding cylinder in a nanofluid which containing both nanoparticles and gyrotactic microorganisms. It should be mentioned that the problems of unsteady flow in cylindrical coordinates are important issues in many biomicrosystems, although several aspects of nanofluid containing gyrotactic microorganisms have been presented. The present study is to extend the work investigated by Zaimi et al. [29] to the case of the unsteady flow due to an expanding cylinder in a nanofluid using Buongiorno's model. By means of similarity reductions, a set of four coupled nonlinear equations are solved numerically. Besides, the effects of the governing parameters on the velocity, temperature, nanoparticle volume fraction, as well as the density of motile microorganisms profiles are graphically presented and discussed. As far as we know, there seems to be no attempts in literature to consider this problem and we believe that the paper is original and the major results are new.

2 Flow analysis

We consider the unsteady laminar flow of a viscous and incompressible nanofluid containing both nanoparticles and gyrotactic microorganisms over an expanding stretching/shrinking cylinder. The physical model and coordinate system are shown in Fig. 1. The diameter of the cylinder is assumed as a function of time. It is assumed that the unsteady and incompressible nanofluid is without body force. Under these assumptions, the governing equations for the continuity, momentum, energy, nanoparticle volume fraction and microorganisms in the vectorial form [5, 12] are given in the following form:

$$\nabla \cdot \mathbf{V} = 0, \quad (1)$$

$$\frac{\partial \mathbf{V}}{\partial t} + (\mathbf{V} \cdot \nabla) \mathbf{V} = -\frac{1}{\rho} \nabla p + \nu \nabla^2 \mathbf{V}, \quad (2)$$

$$\frac{\partial T}{\partial t} + \mathbf{V} \cdot \nabla T = \alpha \nabla^2 T + \tau [D_B \nabla T \cdot \nabla C + (D_T/T_\infty) \nabla T \cdot \nabla T], \quad (3)$$

$$\frac{\partial C}{\partial t} + \mathbf{V} \cdot \nabla C = D_B \nabla^2 C + (D_T/T_\infty) \nabla^2 T, \quad (4)$$

$$\frac{\partial N}{\partial t} + \nabla \cdot \mathbf{j} = 0, \quad (5)$$

where $\mathbf{V}$, T , C , N denote the velocity vector of nanofluid flow, temperature, nanoparticle concentration and the concentration of microorganisms, respectively. p is the pressure, ν is the kinematic viscosity, k is the porosity

Fig. 1. Physical model and coordinate system

of the medium, ρ is the fluid density, α is the thermal diffusivity of the nanofluid, $\tau = (\rho c)_p/(\rho c)_f$ is ratio between the heat capacity of the nanoparticle and the heat capacity of fluid, D_B is the Brownian diffusion coefficient, D_T is the thermophoretic diffusion coefficient and j is the flux of microorganisms which is defined as

$$j = N\mathbf{V} + N\tilde{\mathbf{V}} - D_n\nabla N, \quad (6)$$

in which $\tilde{\mathbf{V}} = (bW_c/\Delta C)\nabla C$ is the velocity vector relating to the cell swimming in nanofluids with D_n is the diffusivity of microorganisms, b is the chemotaxis constant, $\Delta C = C_w - C_\infty$ and W_c is the maximum cell swimming speed.

In cylindrical polar coordinates, the axis of the cylinder is taken along the z -axis, while the r -axis is taken in the radial direction. Based on the axisymmetric flow assumptions, the equations (1)-(5) can be written as

$$\frac{\partial}{\partial r}(rv_r) + \frac{\partial}{\partial z}(rv_z) = 0, \quad (7)$$

$$\frac{\partial v_r}{\partial t} + v_r \frac{\partial v_r}{\partial r} + v_z \frac{\partial v_r}{\partial z} = -\frac{1}{\rho} \frac{\partial p}{\partial r} + \nu \left(\frac{\partial^2 v_r}{\partial r^2} + \frac{1}{r} \frac{\partial v_r}{\partial r} + \frac{\partial^2 v_r}{\partial z^2} - \frac{v_r}{r^2} \right), \quad (8)$$

$$\frac{\partial v_z}{\partial t} + v_r \frac{\partial v_z}{\partial r} + v_z \frac{\partial v_z}{\partial z} = -\frac{1}{\rho} \frac{\partial p}{\partial z} + \nu \left(\frac{\partial^2 v_z}{\partial r^2} + \frac{1}{r} \frac{\partial v_z}{\partial r} + \frac{\partial^2 v_z}{\partial z^2} \right), \quad (9)$$

$$\begin{aligned} \frac{\partial T}{\partial t} + v_r \frac{\partial T}{\partial r} + v_z \frac{\partial T}{\partial z} = & \alpha \left(\frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial z^2} \right) + \tau \left(D_B \frac{\partial C}{\partial r} \frac{\partial T}{\partial r} + \frac{\partial C}{\partial z} \frac{\partial T}{\partial z} \right) \\ & + \frac{\tau D_T}{T_\infty} \left[\left(\frac{\partial T}{\partial r} \right)^2 + \left(\frac{\partial T}{\partial z} \right)^2 \right], \end{aligned} \quad (10)$$

$$\frac{\partial C}{\partial t} + v_r \frac{\partial C}{\partial r} + v_z \frac{\partial C}{\partial z} = D_B \left(\frac{\partial^2 C}{\partial r^2} + \frac{1}{r} \frac{\partial C}{\partial r} + \frac{\partial^2 C}{\partial z^2} \right) + \frac{D_T}{T_\infty} \left(\frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial z^2} \right), \quad (11)$$

$$\begin{aligned} \frac{\partial N}{\partial t} + v_r \frac{\partial N}{\partial r} + v_z \frac{\partial N}{\partial z} = & -\frac{bW_c}{\Delta C} \left[N \left(\frac{\partial^2 C}{\partial r^2} + \frac{1}{r} \frac{\partial C}{\partial r} + \frac{\partial^2 C}{\partial z^2} \right) + \frac{\partial C}{\partial r} \frac{\partial N}{\partial r} + \frac{\partial C}{\partial z} \frac{\partial N}{\partial z} \right] \\ & + D_n \left(\frac{\partial^2 N}{\partial r^2} + \frac{1}{r} \frac{\partial N}{\partial r} + \frac{\partial^2 N}{\partial z^2} \right). \end{aligned} \quad (12)$$

The boundary conditions for the physical problem are given by

$$v_r = -\frac{2\nu}{a_0\sqrt{1-\beta t}}, \quad v_z = \frac{4\nu z\varepsilon}{a_0^2(1-\beta t)}, \quad (13)$$

$$T = T_w, \quad D_B \frac{\partial C}{\partial r} + \frac{D_T}{T_\infty} \frac{\partial T}{\partial r} = 0, \quad N = N_w \text{ at } r = a(t), \quad (14)$$

$$v_z \rightarrow 0, \quad T \rightarrow T_\infty, \quad C \rightarrow C_\infty, \quad N \rightarrow 0 \text{ as } r \rightarrow \infty. \quad (15)$$

Here $a(t) = a_0\sqrt{1-\beta t}$ is the varying cylinder diameter, where β is a constant with $\beta > 0$ for a contracting cylinder and $\beta < 0$ for an expanding cylinder. To convert the governing equations into ordinary differential equations, using the similarity variables [33]

$$\eta = \frac{r^2}{a_0^2(1-\beta t)}, \quad v_r = -\frac{2\nu f(\eta)}{r}, \quad v_z = \frac{4\nu z\eta f'(\eta)}{r^2} \quad (16)$$

$$\theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}, \quad \phi(\eta) = \frac{C - C_\infty}{C_w - C_\infty}, \quad \omega(\eta) = \frac{N}{N_w}. \quad (17)$$

Since the equation (7) is automatically satisfied and the longitudinal pressure gradient is zero, Eqs. (8)-(12) are transformed into

$$\eta f''' + f'' + f f'' - f'^2 - \lambda(f' + \eta f'') = 0, \quad (18)$$

$$\eta \theta'' + \theta' + P_r(f\theta' - \lambda\eta\theta') + \eta(N_b\phi'\theta' + N_t\theta'^2) = 0, \quad (19)$$

$$\eta \phi'' + \phi' + L_e(f\phi' - \lambda\eta\phi') + \frac{N_t(\eta\theta'' + \theta')}{N_b} = 0, \quad (20)$$

$$\eta \omega'' + \omega' - P_e(\eta\omega\phi'' + \omega\phi' + \eta\omega'\phi') + S_c(f\omega' - \lambda\eta\omega') = 0, \quad (21)$$

and the boundary conditions become

$$f(1) = 1, \quad f'(1) = \varepsilon, \quad \theta(1) = 1, \quad \omega(1) = 1, \quad N_b \phi'(1) + N_t \theta'(1) = 0, \quad (22)$$

$$f'(\infty) = \theta(\infty) = \phi(\infty) = \omega(\infty) = 0 \quad (23)$$

where prime denotes differentiation with respect to η . In the above equations, $\lambda = a_0^2 \beta / 4\nu$ is the unsteadiness parameter for a contracting cylinder which displays the strength of the contraction. ε is the stretching/shrinking parameter and P_r is the Prandtl number. N_t and N_b are the thermophoresis and Brownian motion parameters respectively which depend on thermal diffusivity [34, 35], L_e is the Lewis number, P_e is the bioconvection Peclet number and S_c is the Schmidt number, which are given by

$$P_r = \frac{\nu}{\alpha}, \quad N_t = \frac{\tau D_T (T_w - T_\infty)}{\alpha T_\infty}, \quad (24)$$

$$N_b = \frac{\tau D_B (C_w - C_\infty)}{\alpha}, \quad L_e = \frac{\nu}{D_B}, \quad P_e = \frac{b w_c}{D_n}, \quad S_c = \frac{\nu}{D_n}. \quad (25)$$

The quantities of physical interest are the skin friction coefficient C_f , the local Nusselt number Nu_x , the local Sherwood number Sh_x and the local density of the motile microorganisms Nn_x , which are defined as [29]

$$C_f = \frac{\mu (\partial v_z / \partial r)_{r=a(t)}}{\rho v_{zw}^2 / 2}, \quad Nu_x = \frac{-a_0 k (1 - \beta t)^{1/2} (\partial T / \partial r)_{r=a(t)}}{2k (T_w - T_\infty)}, \quad (26)$$

$$Sh_x = \frac{-a_0 D_B (1 - \beta t)^{1/2} (\partial C / \partial r)_{r=a(t)}}{2D_B (C_w - C_\infty)}, \quad Nn_x = \frac{-a_0 D_n (1 - \beta t)^{1/2} (\partial N / \partial r)_{r=a(t)}}{2D_n N_w}. \quad (27)$$

Using the applying the similarity transformation (16) and (17), we obtain

$$C_f \frac{z \varepsilon^2}{a(t)} = f''(1), \quad Nu_x = -\theta'(1), \quad (28)$$

$$Sh_x = -\phi'(1), \quad Nn_x = -\omega'(1). \quad (29)$$

3 Result and discussions

Numerical solutions to the nonlinear ordinary differential equations (18)-(21) with the boundary conditions (22)-(23) are obtained by using a shooting method with fourth-fifth-order Runge-Kutta integration technique. The appropriate finite value of boundary layer thickness η_∞ ($\eta_\infty = 15$) is chosen to acquire sufficiently accurate (error less than 10^{-5}) results. Then a Newton-Raphson iteration is applied to assure the iteration convergence required to satisfy the boundary conditions at infinity. Dual solutions are obtained by setting different initial guesses of $f''(1)$, $-\theta'(1)$, $-\phi'(1)$ and $-\omega'(1)$ where all the velocity, temperature, nanoparticle concentration and the concentration of microorganisms profiles satisfy the infinity boundary conditions (23) asymptotically but with different shapes. For the accuracy verification of the applied numerical method, comparisons of the skin friction coefficient $f''(1)$ for $\varepsilon = -1$ are made with the available results of Wan Zaimi et al. [36]. The results are presented in Table 1 and the excellent agreement is noted. Moreover, some typical results of $f''(1)$, $-\theta'(1)$ and $-\omega'(1)$ of the present study are shown in Table 2. It is note that the values of $-\phi'(1)$ can be obtained from the boundary condition (22).

Table 1. Comparison with previously published data for the values of $f''(1)$ when $\varepsilon = -1$ (shrinking case)

λ	Wan Zaimi et al.		Present work	
	First sol	Second sol	First sol	Second sol
-4	4.7879	-34.6866	4.78798	-34.68664
-3.5	4.2610	-25.3598	4.26102	-25.35983
-3	3.7255	-17.5991	3.72552	-17.59912
-2.5	3.1761	-11.3068	3.17615	-11.30678
-2	2.6012	-6.3803	2.60122	-6.38004

Table 2. Numerical values of $f''(1)$, $-\theta'(1)$ and $-\omega'(1)$ for various values of ε and λ when $Pr = 6.2$, $N_t = 3.1$, $N_b = 3.1$, $L_e = 2$, $P_e = 1$, $S_c = 1$

ε	λ	$f''(1)$		$-\theta'(1)$		$-\omega'(1)$	
		First sol	Second sol	First sol	Second sol	First sol	Second sol
-1	-3	3.72552	-17.59912	17.31518	17.04931	-12.52810	-12.89707
	-2	2.60122	-6.38004	12.97483	12.77171	-9.33659	-9.58686
	-1	1.00006	0.08412	8.431026	8.377436	-6.14246	-6.19341
-0.5	-3	1.93373	-20.05711	17.42158	17.15534	-12.55719	-12.93786
	-2	1.40729	-8.30500	13.12004	12.91153	-9.37511	-9.64797
	-1	0.83514	-1.63337	8.69476	8.56770	-6.18298	-6.32863
0	-3	6.95988	-22.54364	17.52511	17.25976	-12.58648	-12.97636
	-2	2.15681	-10.23227	13.25785	13.04863	-9.41433	-9.70278
	-1	1.93952	-2.98030	8.90958	8.77248	6.24103	-6.42112
0.5	-3	-2.06235	-25.07129	17.62606	17.36248	-12.61585	-13.01300
	-2	-1.58293	-12.19980	13.38965	13.18228	-9.45371	-9.75295
	-1	-1.12342	-4.35223	9.10275	8.96811	-6.30034	-6.49620
1	-3	-4.24278	-27.64898	17.72467	17.46346	-12.64523	-13.04806
	-2	-3.31709	-14.22902	13.51640	13.31225	-9.49299	-9.79964
	-1	-2.45157	-5.82261	9.28127	9.15232	-6.35882	-6.56178

Figs. 2-4 display the variation of skin friction coefficients $f''(1)$, local Nusselt number $-\theta'(1)$ and local density of motile microorganisms $-\omega'(1)$ with ε for different values of λ . It is observed that there are dual solutions (first solution and second solution) when $\varepsilon > \varepsilon_c$, and one solution when $\varepsilon = \varepsilon_c$, but no solution is found in the range of $\varepsilon < \varepsilon_c$. The range of the dual solution increases as the strength of

Fig. 2. Variation of the skin friction coefficient $f''(1)$ with ε for different values of λ when $Pr = 6.2$, $N_t = 3.1$, $N_b = 3.1$, $L_e = 2$, $P_e = 1$, $S_c = 1$

Fig. 3. Variation of the local Nusselt number $-\theta'(1)$ with ε for different values of λ when $Pr = 6.2$, $N_t = 3.1$, $N_b = 3.1$, $L_e = 2$, $P_e = 1$, $S_c = 1$

Fig. 4. Variation of the local density of the motile microorganisms $-\omega'(1)$ with ε for different values of λ when $Pr = 6.2$, $N_t = 3.1$, $N_b = 3.1$, $L_e = 2$, $P_e = 1$, $S_c = 1$

expansion $|\lambda|$ increases. It is also seen from Figs. 2 and 4 that $f''(1)$ and $-\omega'(1)$ decrease with an increase in ε for the two solutions. In contrast, $-\theta'(1)$ increases with ε which indicates that the stretching enhances the heat transfer at the surface, but the shrinking inhibits the effect of heat transfer at the surface. It is interesting to note from Fig. 2 that the value of $f''(1)$ always equals to zero when $\varepsilon = 0$ which is consistent with the result reported by Bachok et al. [37]. As shown in Fig. 3, the local Nusselt number is consistently higher with higher value of $|\lambda|$. It is worth noting that the local Nusselt number is always positive, i.e. the heat is transferred from the hot surface to the cold fluid, which is consistent with physical fact. It is shown that local density of the motile microorganisms decreases dramatically with $|\lambda|$ in Fig. 4, while the effect of strength of expansion $|\lambda|$ on local Sherwood number is not remarkable.

The influences of λ on velocity, temperature, nanoparticle concentration and the concentration of microorganisms profiles are presented in Figs. 5-9. It is clear that the far field boundary conditions (23) are satisfied asymptotically, which shows the validity of the results. All of the boundary layer thicknesses of first solution are smaller than that of second solution for a particular value of the unsteady parameter λ . As shown in Fig. 5, the velocity gradient at the surface is positive for the first solution and negative for the second solution for the shrinking case ($\varepsilon = -0.5$), which is accordant with the results presented in Fig. 2. We expect the first solution to be the physically realizable solution. For the first solution, the velocity profile increases as $|\lambda|$ increases, and the velocity boundary layer thickness decreases with $|\lambda|$. Moreover, the velocity gradient at the surface increases with increasing $|\lambda|$, and in consequent increase the skin friction coefficient. For the stretching case ($\varepsilon = 2$) as shown in Fig. 6, the velocity profile and boundary layer thickness of first solution decrease with increasing $|\lambda|$, i.e. the velocity gradient at the surface is decreased. This phenomenon indicates that the effect of strength of expansion on the first solution of velocity profile for shrinking and stretching case is opposite. These results are also agreed with the result showed in Fig. 2. It is observed from the profiles presented in Figs. 7-9 that the

Fig. 5. Velocity profiles $f'(\eta)$ for different values of λ when $Pr = 6.2$, $N_t = 3.1$, $N_b = 3.1$, $L_e = 2$, $P_e = 1$, $S_c = 1$, $\varepsilon = -0.5$ (shrinking case)

Fig. 6. Velocity profiles $f'(\eta)$ for different values of λ when $Pr = 6.2$, $N_t = 3.1$, $N_b = 3.1$, $L_e = 2$, $P_e = 1$, $S_c = 1$, $\varepsilon = 2$ (stretching case)

Fig. 7. Temperature profiles $\theta(\eta)$ for different values of λ when $Pr = 6.2$, $N_t = 3.1$, $N_b = 3.1$, $L_e = 2$, $P_e = 1$, $S_c = 1$, $\varepsilon = -0.5$

Fig. 8. Nanoparticle valume fraction profiles $\phi(\eta)$ for different values of λ when $Pr = 6.2$, $N_t = 3.1$, $N_b = 3.1$, $L_e = 2$, $P_e = 1$, $S_c = 1$, $\varepsilon = -0.5$

Fig. 9. Density of motile microorganisms profiles $\omega(\eta)$ for different values of λ when $Pr = 6.2$, $N_t = 3.1$, $N_b = 3.1$, $L_e = 2$, $P_e = 1$, $S_c = 1$, $\varepsilon = -0.5$

boundary layer thicknesses of temperature, nanoparticle concentration and concentration of microorganisms for the shrinking case decrease with an increase in $|\lambda|$ for the two solutions. Different from the temperature which is monotone decreasing with η , the nanoparticle concentration and concentration of microorganisms increase rapidly to a maximum value when η is small and then decrease to the far field conditions. It is noteworthy that the difference between the first and second solutions is small, which is also consistent with the results presented in Figs. 3 and 4. In addition, for the stretching case, temperature, nanoparticle concentration and concentration of microorganisms profiles are toward the same trend with the shrinking case, so that we do not present the corresponding graphs in this section.

Figs. 10-15 show the effect of the Brownian motion parameter N_b and the thermophoresis parameter N_t on the temperature, nanoparticle concentration and concentration of microorganisms profiles for the shrinking case. As shown in these figures, the first solutions approach to the far field conditions are faster than the second solutions, i.e. the boundary layer thicknesses of the first solutions are smaller than that of the second solutions. It is noted from Fig. 10 that the temperature and the thermal boundary layer thickness increase as N_b increases. In contrast, the nanoparticle concentration and concentration of microorganisms decrease with increasing N_b as illustrated in Figs. 11 and 12. The thermophoresis parameter N_t exhibits an opposite effect on these profiles compared with Brownian motion parameter N_b as shown in Figs. 13-15, which is in agreement with the physical fact.

Fig. 10. Temperature profiles $\theta(\eta)$ for different values of N_b when $\lambda = -2$, $Pr = 6.2$, $N_t = 3.1$, $L_e = 2$, $P_e = 1$, $S_c = 1$, $\varepsilon = -0.5$

Fig. 11. Nanoparticle volume fraction profiles $\phi(\eta)$ for different values of N_b when $\lambda = -2$, $Pr = 6.2$, $N_t = 3.1$, $L_e = 2$, $P_e = 1$, $S_c = 1$, $\varepsilon = -0.5$

Fig. 12. Density of motile microorganisms profiles $\omega(\eta)$ for different values of N_b when $\lambda = -2$ $Pr = 6.2$, $N_t = 3.1$, $L_e = 2$, $P_e = 1$, $S_c = 1$, $\varepsilon = -0.5$

Fig. 13. Temperature profiles $\theta(\eta)$ for different values of N_t when $\lambda = -2$ $Pr = 6.2$, $N_b = 3.1$, $L_e = 2$, $P_e = 1$, $S_c = 1$, $\varepsilon = -0.5$

Fig. 14. Nanoparticle volume fraction profiles $\phi(\eta)$ for different values of N_t when $\lambda = -2$ $Pr = 6.2$, $N_b = 3.1$, $L_e = 2$, $P_e = 1$, $S_c = 1$, $\varepsilon = -0.5$

Fig. 15. Density of motile microorganisms profiles $\omega(\eta)$ for different values of N_t when $\lambda = -2$ $Pr = 6.2$, $N_b = 3.1$, $L_e = 2$, $P_e = 1$, $S_c = 1$, $\varepsilon = -0.5$

It is noticed that bioconvection Peclet number P_e and Schmidt number S_c have only effects on the concentration of microorganisms profile $\omega(\eta)$ from the governing equations. As shown in Fig. 16, the increasing P_e results in the enhancement of $\omega(\eta)$. Moreover, a maximum value of $\omega(\eta)$ increases with P_e and appearance of the maximum value is delayed as P_e increases. The influence of S_c on $\omega(\eta)$ is opposite to that of P_e as plotted in Fig. 17.

Fig. 16. Density of motile microorganisms profiles $\omega(\eta)$ for different values of P_e when $\lambda = -2$ $Pr = 6.2$, $N_t = 3.1$, $N_b = 3.1$, $L_e = 2$, $S_c = 1$, $\varepsilon = -0.5$

Fig. 17. Density of motile microorganisms profiles $\omega(\eta)$ for different values of S_c when $\lambda = -2$ $Pr = 6.2$, $N_t = 3.1$, $N_b = 3.1$, $L_e = 2$, $P_e = 1$, $\varepsilon = -0.5$

4 Conclusions

In the present study, the problem of the unsteady flow over an expanding cylinder in a nanofluid containing gyrotactic microorganisms has been investigated. Different from the previous works, here we first build the unsteady nanofluid model containing gyrotactic microorganisms in cylindrical coordinates. The major conclusions are listed below:

- Dual solutions exist for both stretching ($\varepsilon > 0$) and shrinking cases ($\varepsilon_c < \varepsilon < 0$) with curves which bifurcate at the critical values ε_c .
- The range of the dual solutions increases with the strength of the expansion.
- A more rapidly expanding cylinder can generate higher skin friction coefficients and local Nusselt number, but lead to smaller local Sherwood number and local density of the motile microorganisms at the surface.
- Increase in the strengths of Brownian motion effect results in the increase of local Sherwood number and local density of the motile microorganisms, but the opposite effect is reported for the thermophoresis parameter.
- The concentrate of microorganisms is found to increase with higher bioconvection Peclet number and lower Schmidt number.

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List of symbol

$a(t)$	radius of cylinder
a_0	positive constant
b	chemotaxis constant
C	nanoparticle concentration
C_f	skin friction coefficient
C_w	wall nanoparticle concentration
C_∞	ambient nanoparticle concentration
D_B	Brownian diffusion coefficient
D_n	diffusivity of microorganisms
D_T	thermophoretic diffusion coefficient
f	dimensionless stream function
j	flux of microorganisms
k	thermal conductivity
L_e	Lewis number
N	concentration of microorganisms
N_B	Brownian motion parameter
Nn_x	local density number of the motile microorganisms
N_t	thermophoresis parameter
Nu_x	local Nusselt number
N_w	wall concentration of microorganisms
N_∞	ambient concentration of microorganisms
p	pressure
Pe	bioconvection Peclet number
Pr	Prandtl number
Sc	Schmidt number
Sh_x	local Sherwood number
t	time
T	temperature inside the boundary layer
T_w	wall temperature
T_∞	ambient temperature
v_r	velocity components along the r-axes
v_z	velocity components along the z-axes
$v_z w$	wall velocity components along the z-axes
V	velocity vector of nanofluid flow
W_c	maximum cell swimming speed
r, z	coordinates in the radial and axial directions, respectively
α	thermal diffusivity
β	strength of expansion/contraction
ΔC	characteristic nanoparticle volume fraction
ε	stretching/shrinking parameter
η	similarity variable
θ	dimensionless temperature
λ	unsteady parameter
μ	dynamic viscosity
ν	kinematic viscosity
ρ	fluid density
$(\rho c)_f$	heat capacity of the fluid
$(\rho c)_p$	heat capacity of the nanoparticle
τ	ratio between the heat capacity of nanoparticle and fluid
ϕ	dimensionless nanoparticle concentration
ω	dimensionless concentration of microorganisms
∇^2	Laplacian operator

Subscripts

f	fluid
p	nanoparticle
w	wall condition
∞	ambient condition

Superscript

$'$	differentiation with respect to η
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