A Plausible Mechanism for Generating Negative Coincident-Loop Transient Electromagnetic Responses

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A plausible mechanism for generating negative coincident-loop transient electromagnetic responses

by

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Abstract

Coincident-loop transient electromagnetic measurements are normally positive at all delay times, but occasionally they become negative at late delay times. The most likely cause of these sign reversals are polarizable conductivity structures (i.e. structures whose conductivity varies in the sense $\partial\sigma/\partial s > 0$, where $s$ is the Laplace transform variable representing the frequency). However, structures such as half-spaces, buried conductors or buried conductive layers are not capable of explaining the negatives unless the polarizabilities are implausibly large.

The negatives can be explained with realistic polarizabilities if the structure is such that the following three ‘favourable coupling conditions’ are satisfied at the positions where negatives occur:

1. the transmitter couples well to the body,
2. at late delay times the electromagnetically induced current couples poorly to the receiver, and
3. the polarization couples well to the receiver.

Polarizable structures which satisfy the favourable coupling conditions are: a buried conductor in a very resistive host, a thin conductive overburden, two interacting conductors, an overburden which comes to an edge and a thin dipping dyke. The field examples of negative transients presented in this thesis are all associated with these structures and the polarizabilities required to model the negatives are small, so the favourable coupling conditions provide a plausible general explanation of the negative transients.
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Preface

The mineral exploration industry uses coincident-loop transient electromagnetic methods to explore for conductive ore deposits. Normally, the measured transients are positive and methods have been developed for interpreting this type of data. However, in some circumstances the transients mysteriously change sign to negative at late delay times, so the standard interpretation methods cannot be used. Phil Harman, David Isles (both of BHP-Utah Minerals International) and Bob Smith (of CRA Exploration), who visited Toronto in 1984, encouraged me to find a plausible cause of the negative transients and to develop methods for interpreting the data. I undertook this task with the whole-hearted support, encouragement and guidance of my supervisor, Prof. Gordon F. West. His initial suggestions provided a head-start which allowed me to perform an extensive investigation of this topic in a relatively short time. I am very grateful to him for the insight and advice given throughout this project.

When trying to explain a phenomenon, it is very important to compare theoretical predictions with real field occurrences. It would have been difficult to perform enough field work to provide a significant variety of examples, so I am very grateful for data generously supplied by the following geophysicists (and companies): Guido Staltari (formerly of Geophysical Exploration Consultants, now at Resource Consulting Services), Dick Irvine (BHP-Utah Minerals International), Peter Elliott (formerly Billiton Australia, now at Search Exploration Services), Peter Gidley (formerly at CSR Limited, now at Encom Technology), André Lebel (formerly BHP Minerals Exploration, now at Datascience), Mike Asten (BHP-Utah Minerals International), Miro Bosnar (Geonics Limited), Jock Buselli (CSIRO), Bruce Craven (Chevron Exploration Corporation, on behalf of the Teutonic Bore Joint Venture), Tom Kerr, Sandy Dodds (SADME), Van Reed (Zonge Engineering and Research
Organization (Australia)) and of course Bob Smith and Phil Harman.

The main body of the thesis describes the mechanism I have proposed as an explanation for the negative transients. To avoid distracting the reader from this theme, a great deal of the detail has been relegated to the appendices. Normally, the reference list comes between the conclusion and the appendices, but because of the quantity of the appended material, the reference list has been placed at the end of the thesis after the appendices where it is easier to find.

All the appendices contain original material, except the early sections of Appendix 5, which comprise an explanation of the Gaver-Stehfest algorithm based on previously published material. The latter sections of Appendix 5 describe an original unpublished technique, developed at the suggestion of Prof. R.N. Edwards, for increasing the efficiency of the Gaver-Stehfest algorithm.

Most of this work has already been published. One paper (Smith and West, 1988a) forms the basis of Chapters 1, 2, 4 and Appendix 1, while another paper (Smith and West, 1988b) has been rewritten as Chapter 3 and Appendix 3. Chapter 5 has recently been submitted for publication (Smith and West, 1988c) and a fourth paper (Smith, Walker, Polzer and West, 1988), describing the approximate convolution algorithm, has been included as Appendix 4.

The approximate convolution algorithm was conceived by Peter Walker (with help from Ben Polzer) in response to a challenge made by Prof. R. Nigel Edwards at a seminar given by Art Raiche to the Geophysics Laboratory of the University of Toronto. I am very grateful to Peter for allowing me to develop the algorithm for use in this thesis. The intuition thereby gained represents a watershed in understanding the problem of how currents flow in polarizable bodies.

Important points made throughout the thesis are illustrated with computer simulations and a number of the algorithms used are due to other workers. David Boerner wrote a routine for the numerical calculation of the Hankel transform based on the digital-linear-filter weights of Johansen and Sorenson (1979); a second routine for the same purpose, supplied by Prof. R.N. Edwards, is the method of Chave
(1983). The layered earth algorithm discussed in Appendix 2 was developed by Prof. R.N. Edwards; I am extremely grateful for the time he spent explaining the method, and his generosity in allowing me to use the algorithm. His ideas, encouragement and advice throughout this project have been an invaluable contribution. Professor Peter Weidelt very kindly gave me a copy of his program for calculating the electromagnetic response of a half-plane. The work of Chapter 5, an important part of the thesis, relies heavily on the results from this algorithm.

Computer drafting was used for most of the figures in this thesis. The X-Y plots were generated using the interactive plotting routine TPLOT (written by Dave Boerner), while the contour plots were produced with a program from the National Center for Atmospheric Research (U.S.A.). The computer drafted diagrams not suitable for publication were ably re-drafted by Khader Khan and Raul Cunha.

Producing the manuscript is one of the most difficult parts of the research project. The final version presented here was reached through many intermediate stages (monthly reports, conference abstracts, scientific articles, and thesis drafts). At each stage, comments and suggestions for improving the manuscript were solicited. I am grateful to the following for their help in this regard: Prof. G.F. West, Peter Walker, Ross Groom, Lorna Clark, Dave Boerner, Brian Spies, Marcus Flis, D. Guptasarma, Terry Lee, Prof. Keeva Vozoff and four anonymous reviewers.

Monetary support for this work was given by the Canadian Government via a Commonwealth Scholarship and the Canadian Society for Exploration Geophysicists via a CSEG Scholarship. Supplementary funding in the final stages was generously provided by Prof. G.F. West. Research costs such as computing, postage, stationery, photocopying, drafting, etc were covered by a NSERC operating grant to Prof. West. Financial assistance to attend the 1988 Australian Society for Exploration Geophysicists meeting in Adelaide was obtained from CRA (Bob Smith), BHP-Utah Minerals International (Mike Asten), and Resource Consulting Services (Guido Staltari).

Finally I would like to acknowledge my colleagues, friends and family for help,
support, encouragement and advice. The social environment at the Physics Department of the University of Toronto certainly made my stay an enjoyable experience.
Chapter 1

Introduction

1.0  Intention of the thesis

Coincident-loop transient electromagnetic (TEM) systems use a large horizontal receiver loop, coincident with the transmitter loop, to measure the voltage response associated with currents induced in the ground after a steady current in the transmitter is suddenly switched off. The conductivity structure of the ground is investigated by moving the transmitter/receiver loops along a profile and measuring the response at a number of different positions. Two examples of response profiles are shown on Fig. 1.1. The voltage response, normalized to unit transmitter current, is plotted on a log scale as a function of the transmitter/receiver position. The plotted curves represent the measured voltage at a number of specific delay times (channels). The channel numbers and delay times corresponding to each curve are used as labels on the left- and right-hand sides respectively. In general, a slow decay is indicative of a structure beneath the transmitter/receiver which is conductive and a rapid decay indicates resistive material.

The transient response at one or more positions can also be plotted as a function of the delay time. Figure 1.2 shows an example of three such ‘delay curves’, plotted in this case as a function of log time. The tick-marks on the time axis are positioned at the delay times which corresponding to each measurement channel.

Note that the voltage responses shown on Figs 1.1 and 1.2 are positive in all cases. In fact, Gubatynko and Tikshayev (1979), Weidelt (1982) and Guptasarma (1984a) have shown that the coincident-loop TEM response of any conductivity structure will always be positive if the conductivity and permeability are linear and independent of frequency. Because these requirements are normally satisfied in elec-
Figure 1.1  Two coincident-loop surveys collected along east-west profiles (After Buselli, 1980)

tromagnetic (EM) prospecting, the coincident-loop response is generally positive, as is seen here.

Occasionally however, the positive response changes sign to negative at late delay times; two situations where this occurs are shown on Fig. 1.3. In both these examples the negatives occur over a localized zone, although the region of occurrence can be quite extensive. A logarithmic scale cannot be used to display negative values, so a special bi-log plot has been used. On this plot values between 1 and -1 µV/A are plotted on a linear scale, values greater than 1 µV/A are plotted on a
logarithmic scale above the linear region and values less than \(-1\) \(\mu\text{V/A}\) are plotted on an inverted logarithmic scale below the linear region. This plotting format tends to over-emphasize the significance of small negatives such as those on Fig. 1.3.

As sign reversals are not expected under normal circumstances, an explanation is clearly required. At the time this thesis was commenced, a number of explanations had been proposed, but none were plausible or capable of explaining the wide variety of negative transients. This thesis describes an attempt to find an explanation for the negative transients which is both plausible and generally applicable.
Figure 1.3 Two examples of coincident-loop profiles showing negative transients, taken from Figures 3.9 and 4.5. The curve labels are the delay times (in milliseconds) on the left profile and the channel numbers on the right profile.

1.1 Coincident-loop TEM and related methods

The negative transients are mostly observed in Australia, where TEM systems utilizing a coincident-loop configuration (or something similar) are used extensively,
principally because of their ability to detect conductors buried below the highly-conductive non-uniform overburden existing over most of the continent (Spies, 1976, 1980a, 1980b; Buselli, 1980, 1982; McCracken et al, 1980; Doyle and Lindeman, 1985; McCracken et al, 1986a,b). The coincident-loop mode is used by the SIROTEM system (Buselli and O'Neill, 1977; Buselli, 1982); an earlier system, the Russian MPPO-1 (Velikin and Bulgakov, 1967; Spies, 1974) uses a single loop as the transmitter and receiver so the measured response is identical to the coincident-loop response. The EM37 (Geonics, 1982), Crone-PEM (Crone, 1982) and SIROTEM systems can all operate in a mode where a small receiver is placed at the centre of a large transmitter loop — the central-loop mode. If the ground is laterally uniform, the measured response of this mode is similar to that of the coincident-loop mode (Spies, 1980b) and negative transients are not expected under normal conditions (i.e. when the conductivity and permeability are linear and independent of frequency).

1.2 Problems experienced with TEM

The extensive use of the SIROTEM system in Australia has been successful, but it has also brought to light a number of problems:

(1) *Superparamagnetic effects.*

The secondary magnetic field is distorted if the near-surface material contains an ultra fine-grained mineral such as maghemite which is superparamagnetic (SPM). The distortion is strongest close to the transmitter, so the corresponding distortion of the TEM response will be greatest when the receiver is coincident with the transmitter. Because the distortion is much weaker away from the transmitter, a lateral displacement of the receiver loop from the transmitter loop by a distance of about 3 metres is sufficient to effectively eliminate SPM effects (Buselli, 1982). The response measured in this displaced-loop mode is similar to the coincident-loop response, so it is commonly deployed in regions where SPM effects would otherwise be significant.
(2) The loop effect.

In situations where the ground is laterally uniform, the response profile across a fixed transmitter loop measured with a small receiver is expected to have a vertical-field response which is greater inside the loop than outside. 'The loop effect' (Asten and Price, 1985) is an anomalous depression of the late-time response inside the loop compared with that outside.

(3) The negative transients.

The topic of this thesis, coincident-loop negatives, are normally observed in measurements which start out positive, but change sign to negative prior to becoming too small to measure. The first documented example of a coincident-loop negative transient was observed in 1974 (Spies, 1980c). Typically, the size of the negatives observed with 50 to 100m square loops is about 10μV/A, which is small compared with the early-time response. In rare circumstances a negative response is observed right from the earliest measurement time (G. Staltari, pers. comm.), and sometimes the response can change sign a second time (M. Bosnar, pers. comm.). The negatives are annoying because they obscure the response of deep conductive bodies.

The superparamagnetic effect has been adequately explained by Buselli (1982). The loop effect is related to the negative transients so it is dealt with briefly in Chapter 3. Possible explanations for the negative transients are reviewed briefly below.

1.3 Possible causes of the coincident-loop negatives

(1) Poor instrumentation.

The negatives could be an artifact of poor instrument design and engineering. Qian (1985) pointed out that a receiver filter with a high frequency cut-off which is too low might lead to the observation of spurious negative transients. It is unlikely for flaws to exist in the currently available commercial systems (MPPO-1, SIROTEM
and EM37), which are well designed and manufactured, and extremely unlikely for all systems to have identical flaws. When different systems are used to measure the response over a specific region, the observed negatives are similar (Smith and West, 1988a; Spies, 1980c), so poor instrumentation can be ruled out as a general explanation.

(2) Poor field procedures.

If sufficient care is not taken in the instrument set-up procedure, spurious negative transients may be generated. For example, they may occur in the following two situations:

(a) If the instrument is not insulated from the ground (e.g. if it is placed on wet soil) current induced in the ground may leak into the receiver instrumentation, producing negative transients.

(b) A transmitter/receiver wire which is not placed on the surface of the earth, but suspended by undergrowth will be deflected by the Earth's magnetic field when the transmitter current is flowing. After the current is switched off, the wire will return to the equilibrium position. The effect of this movement on the TEM response (termed 'the Ward effect') will generate small negative transients (Buselli, 1982).

Both of these explanations are very specific to particular circumstances and although they could cause negatives in isolated incidents, they will not explain all examples. The negatives are more likely caused by geological conditions than by instrument effects or field procedures, a conclusion supported by the fact that negatives are persistently observed (Spies, 1980c; R.J. Smith, pers. comm.) and are more prevalent in certain geological terrains than in others.

(3) Current/receiver coupling effects.

Using a semi-quantitative argument, McCracken et al (1981) proposed that negatives could be generated when current is induced in a non-polarizable dipping dyke or quarter-space. The induced current flow pattern would migrate underneath the receiver loop reversing the direction of the secondary magnetic field at the receiver and hence the sign of the TEM response. Since this mechanism was proposed, Gu-
batyenko and Tikshayev (1979) and Weidelt (1982) have shown that the response of this, or any other structure, cannot reverse sign when the conductivity and permeability of the ground are linear and independent of frequency. This is because a system with one means of energy storage (the magnetic field) and one means of energy loss (joule heating) necessarily produces a unipolar response (Guptasarma, 1984a). Thus, coupling effects are not able to generate negative transients.

(4) **Permittivity effects.**

The positivity constraint on the coincident-loop TEM response was derived assuming negligible electrical permittivity ($\varepsilon$). In EM prospecting this is normally an excellent assumption (Grant and West, 1965), but if the permittivity is non-negligible, the possibility of negative transients must be considered. A non-negligible permittivity can be caused by either dielectric polarization or electrochemical polarization; the later effect however is not normally accounted for with a non-negligible permittivity, but with an equivalent representation, a frequency-dependent complex conductivity. Electrochemical polarizations are discussed in (6) below, so the discussion here is restricted to dielectric polarization, an effect described with a frequency-independent relative permittivity.

Lee (1981a) found that a conductive half-space with a very large relative permittivity ($> 100$) has a response which is only marginally different from that of a conducting half-space with $\varepsilon = \varepsilon_0$. In fact, the only appreciable difference occurs at early time for a very resistive half-space; thus, it is extremely unlikely that dielectric polarization effects are the cause of the late-time negatives.

(5) **Frequency dependent permeability.**

Another possible explanation is that the magnetic permeability ($\mu$) of the ground is frequency dependent (dispersive). Superparamagnetic soils exhibit this behaviour (Buselli 1982), but the sense of the dispersion ($\partial \mu / \partial s < 0$, where $s$ is the Laplace transform variable representing the frequency) results in an increased coincident-loop response at late times, so a negative transient cannot be generated (Lee, 1984a,b). A permeability dispersion in the opposite sense may generate negatives,
but I am not aware of any such documented example in exploration geophysics. Thus, permeability dispersions are also unlikely as a general explanation for the negatives.

(6) Frequency dependent conductivity.

In a theoretical study, Bhattacharyya (1964) showed that negative transients can be generated when the conductivity is frequency dependent. The plausibility of this explanation is investigated in this section.

Rock conductivities are commonly weakly frequency dependent between 0.1 and 10 000Hz in the sense $\partial \sigma / \partial s > 0$; and rocks with a dispersion of this sense are termed ‘polarizable’. Qualitatively speaking the strength of the frequency dependence is the ‘polarizability’, a quantity which can be given a number of quantitative definitions and is commonly measured over the approximate frequency range 0.1 to 10Hz by frequency-domain induced polarization (IP) systems. Experience has shown that the functional form of the frequency dependence is most easily described with a Cole-Cole dispersion model (Pelton, 1977). Although it can be used to describe a variation in the conductivity, this model is most commonly used to describe variations in the resistivity, viz

$$\rho(s) = \frac{\rho_{dc}}{1 + (sT)^c[ (sT)^c(1 - m) + 1 ]}, \quad (1.1)$$

where $\rho_{dc}$ is the resistance at 0Hz, $T$ is the Cole-Cole time constant, and $c$ is the Cole-Cole dispersion factor. The Cole-Cole chargeability $m$ is one specific measure of the polarizability of the material, it describes the fractional decrease in the resistivity from dc to very high frequencies. In practice, the frequency-domain IP method measures over a restricted frequency range, so the polarizability is commonly quantified using the frequency effect (FE), which describes the following fractional change in resistivity

$$FE = \frac{\rho_{low} - \rho_{high}}{\rho_{norm}},$$

where $\rho_{low}$ and $\rho_{high}$ are the resistivities at the high and low limits of measure-
ment. The normalizing resistivity $\rho_{\text{norm}}$ can be either the low or high frequency value (glossary in Sumner, 1976), but I use $\rho_{\text{low}}$. The frequency effect is commonly multiplied by 100 to yield the percent frequency effect (PFE) and can be made somewhat independent of the choice of $\rho_{\text{low}}$ and $\rho_{\text{high}}$ by normalizing to the number of decades between these limits, yielding the PFE/decade. The specific descriptions of polarizability measured by time-domain IP systems is the 'chargeability', a quantity which is not directly comparable to the Cole-Cole chargeability, because the two measures are sensitive to different frequency ranges and the Cole-Cole chargeability is dimensionless, while the time-domain IP chargeability commonly has dimensions of time.

Polarizability data measured outside the IP frequency range are relatively rare, but laboratory and in-situ measurements of the resistivity over a broad range of frequencies do exist (Lee, 1975; Pelton, 1977; and Olhoeft, 1985). Example values of the polarizability over the EM and IP frequency ranges (10Hz to 10kHz and 0.1Hz to 10Hz respectively) obtained from these measurements are summarized on Table 1.1. The samples listed on the top half of the table are typical of 'average' crustal material, whereas those on the bottom half are representative of rarer economic ore deposits, which tend to be highly polarizable. In general, the PFEs/decade in the IP frequency range are neither larger nor smaller than those in the EM frequency range. The only correlation in the data is that most samples have a polarizability in one range which appears to be somewhat similar to that in the other range. In this thesis I assume that the polarizability in the EM frequency range is (a) roughly comparable to the IP polarizability and (b) will not commonly exceed a few percent per decade; both assumptions are generally consistent with the tabulated data.

A number of workers have tried to model negatives the size of those observed in the field with the following polarizable models: half-spaces, buried conductive layers and buried confined conductors (Morrison et al, 1969; Astrakhantsev et al, 1975; Lee, 1975, 1981b; Weidelt, 1982; Raiche, 1983; Lewis and Lee, 1984; Molchanov et al, 1984; Raiche et al, 1985; Flis et al, 1985; Thomas and Lee, 1988). If the
polarizability is realistic, the polarization response is either too small to generate negative transients or the negative transients generated are too small to measure, results consistent with the conclusions of Hohmann et al (1970) and Macnab (1980). Negatives of the required size can be generated by increasing the polarizability; however, it must be increased to implausibly large levels and it is highly unlikely that such polarizable structures will exist wherever negative transients are observed. Thus, polarizable structures alone cannot be invoked as a plausible explanation of the negative transients either.

<table>
<thead>
<tr>
<th>RESISTIVITY DISPERSION</th>
<th>PFE/decade</th>
</tr>
</thead>
<tbody>
<tr>
<td>SAMPLE</td>
<td>0.1-10Hz</td>
</tr>
<tr>
<td>Bingham Granite*</td>
<td>1.3</td>
</tr>
<tr>
<td>COFR1 7 Barren Rock (sat. KCl)†</td>
<td>0.1</td>
</tr>
<tr>
<td>387.1 5 DMD7 2 Carb. Silt/Sandstone (nat. water)†</td>
<td>3.0</td>
</tr>
<tr>
<td>2076 Tuff (nat. water)†</td>
<td>2.2</td>
</tr>
<tr>
<td>6-106WY Min. Sandstone (0.001mol/L KCl)†</td>
<td>3.6</td>
</tr>
<tr>
<td>AP1-26 Montmorillonite (0.1 mol/L KCl)†</td>
<td>-0.3</td>
</tr>
<tr>
<td>Shaly Sandstone‡</td>
<td>1.5</td>
</tr>
<tr>
<td>Lornex 4 (Dry Porphyry)*</td>
<td>3.3</td>
</tr>
<tr>
<td>Sudbury 4 (Pyrrhotite)*</td>
<td>5.3</td>
</tr>
<tr>
<td>Iron Mt. 5 (Magnetite)*</td>
<td>2.3</td>
</tr>
<tr>
<td>Labelle 1 (Graphite)*</td>
<td>10.9</td>
</tr>
<tr>
<td>Tyrone 3 (Wet Porphyry)*</td>
<td>13.4</td>
</tr>
<tr>
<td>Kidd Creek 1 (Massive Sulphide)*</td>
<td>18.4</td>
</tr>
<tr>
<td>80-027 Massive Amorphous Graphite†</td>
<td>34.0</td>
</tr>
<tr>
<td>Pyrite in Quartz Matrix (Model Cq)†</td>
<td>39.7</td>
</tr>
</tbody>
</table>

*Pelton (1977) †Oihocht (1985) ‡Lee (1975)

Table 1.1 Laboratory and in-situ PFE measurements

1.4 Brief outline of the thesis

It is here that an impasse is reached: all the mechanisms proposed for explaining negative transients are either irrelevant, not generally applicable, or implausi-
ble. I have undertaken a detailed study of the most likely explanation, frequency-dependent conductivities, in the hope that a means of generating negatives with more moderate polarizabilities can be found.

The thesis starts with a theoretical investigation, but in the latter chapters is guided by field examples, each chapter being an attempt to explain a different class of negative transient.
Chapter 2

The response of a simple polarizable conductor

2.0 Introduction

The response of a simple polarizable conductor model is studied to ascertain whether negative transients can be generated with realistically small polarizabilities.

2.1 The wire-loop circuit model

The simplest possible model is the filamental wire-loop circuit, commonly used as a crude approximation for confined sheet-like conductors imbedded in resistive bedrock (Ranasinghe, 1962; Grant and West, 1965; Vallée 1981; Barnett, 1984, Boyd and Wiles, 1984; and McCracken et al, 1986a,b). The circuit shown on Fig. 2.1, having a self inductance $L_1$ and a resistance $R_1$, is used to approximate a non-polarizable conductor. When this circuit is excited by a coincident-loop transmitter/receiver system the induced voltage is given by equation (A1.8) (Appendix 1)

$$v(t) = I_0 R_1 k_{01}^2 \mu_1^2 e^{-t/\tau_1},$$

where $I_0$ is the magnitude of the current $i_0$ flowing in the transmitter loop prior to switch off, $k_{01}$ is the coupling coefficient between the transmitter/receiver and the circuit, $\tau_1 = L_1 / R_1$ is the intrinsic time constant of the circuit, $\mu_1 = \sqrt{L_0 / L_1}$ and $L_0$ is the self-inductance of the transmitter. In what follows, this voltage is termed the intrinsic voltage response, and the current flowing in the non-polarizable circuit is called the intrinsic current. The intrinsic voltage is a simple exponential decay, so the response is positive at all delay times (as $R_1$ and $I_0$ are always positive), a result consistent with Weidelt (1982).
2.2 A dispersive circuit

A polarizable circuit can be modelled by replacing the resistance \( R_1 \) with a frequency-dependent Cole-Cole impedance \( Z_1(s) \), where

\[
Z_1(s) = \frac{R_1}{1 + (sT_1)^c} \left[(sT_1)^c(1 - m_1) + 1\right].
\]

The symbols are the same as those used in equation 1.1, except the subscript 1 has been added and \( R_1 \) is used as the the dc resistance of \( Z_1 \). In this analysis the mathematics has been simplified by setting \( c \), the Cole-Cole dispersion factor, to \( c = 1 \). The impedance is thus equivalent to a capacitor (impedance \( 1/sC \)) and a resistor \( (R_0) \) connected in series, both of which are connected in parallel to a resistor \( R_1 \) (see Fig. 2.2). The Cole-Cole chargeability \( m_1 \) and time constant \( T_1 \) in this case are given by

\[
m_1 = \frac{1}{1 + R_0/R_1}, \quad \text{and} \quad T_1 = C R_1/m_1.
\]

In Appendix 6, I show that requiring \( c = 1 \) is not overly restrictive. Figure 2.3,
Figure 2.2 The geometrical arrangement of the polarizable filamental-wire circuit model (c = 1 in the Cole-Cole impedance). The impedance of the capacitor is 1/sC, where s is the Laplace transform variable. The coincident-loop transmitter/receiver is also shown.

taken from that appendix, is a plot of the response for a broad range of c values. Each response is similar, so the response of a circuit in which c ≠ 1 can be mimicked by one in which c = 1. In each case, the values of T1 are the same, but the m1 and R1 are varied so that the values of Z1 and ∂Z/∂s at s = 1/T1 are identical. The similarity of the response shows that the negative transients are most strongly effected by the dispersion of Z1(s) over a small part of the spectrum around s = 1/T1. The c = 1 assumption can be made because the c = 1 model is quite adequate for representing most dispersions over such a small range of frequencies. Generalizing the mathematics of this chapter to c ≠ 1 is a complex algebraic task and merely results in the decay of the negative transients being smeared in comparison to the c = 1 decay. As such a modification is not difficult to imagine, the extra effort is better spent elsewhere. Also, the discrete, isolated, filamental loop is not likely
to be a viable quantitative model for the geometry of electromagnetically induced currents at either very early or very late times irrespective of the value of \( c \), so the generalization would be of little use.

\[
c = 0.1, 1.0, 0.1
\]

![Graph showing voltage responses]

**Figure 2.3** The voltage responses of a wire-loop circuit with a self-inductance of 0.0001H, a mutual inductance to the transmitter-receiver of 0.1H, a Cole-Cole time constant of \( 1.28 \times 10^{-3} \)s, and ten dispersive resistances with Cole-Cole dispersion parameters \( c = 0.1, 0.2, \ldots, 1.0 \). (After Fig. A6.2).

Because a variety of \( c, m, \) and \( R_1 \) values are capable of generating similar negatives, specifying \( c, m \) and \( R_1 \) does not result in a unique description of the dispersion required to generate a negative transient. A better quantitative descrip-
tion of the polarizability is to give the slope of the dispersion over the frequency range of interest; for this reason a percent variation between 10 and 10,000 Hz is used throughout the text of this thesis. The other parameters are of course still important, so they are given in the relevant figure captions whenever the Cole-Cole model is used.

2.3 The response of a dispersive conductor

The coincident-loop voltage response of the polarizable wire-loop circuit model is given by

\[
v(t) = -M_0 \frac{\partial i_1}{\partial t} = \frac{I_0 R_1 k_\infty^2 \mu^2}{\sqrt{q}} \left( e^{s'_1 t'} (s'_1 + s'_2 \lambda) - e^{s'_2 t'} (s'_2 + s'_2 \lambda) \right)
\]

(Appendix 1, equation (A1.11)), where primed quantities are dimensionless. The symbols used in equation (2.3) are defined by:

\[
t' = t/\tau_1 \\
s'_1, s'_2 = \frac{1}{2\lambda} \left[ -[1 + \lambda(1 - m)] \pm \sqrt{q} \right], \\
t'_1, t'_2 = \frac{1}{2} \left[ [1 + \lambda(1 - m)] \pm \sqrt{q} \right], \\
\lambda = T_1/\tau_1, \quad \lambda \geq 0 \\
q = [1 - \lambda(1 - m)]^2 - 4\lambda m, \quad \text{and} \\
m = m_1, \quad 0 \leq m \leq 1.
\]

The parameter \(\lambda\) is the ratio of the Cole-Cole time constant and the intrinsic time constant. The time constants of the two decay modes \(t'_1\) and \(t'_2\) are defined by \(t'_1 = -1/s'_1\). The value of \(q\), contoured on Fig. 2.4 for \(0 \leq m \leq 1\) and \(0 \leq \lambda \leq 8\), determines whether or not the two time constants \(t'_1\) and \(t'_2\) are purely real or complex.

For \(m\) large, \(q\) is negative, so \(t'_1\) and \(t'_2\) are a complex conjugate pair. The corresponding voltage response can be written as a single term which describes
an exponentially damped oscillation. As $m$ decreases, $q$ becomes less negative, so the imaginary part of the time constants will decrease and the voltage will oscillate more slowly. For further reductions in $m$, $q$ becomes positive, and the time constants become real but distinct quantities. The voltage response is thus comprised of two independent terms with different time constants.
For very small chargeabilities \( m \ll 1 \) and realistic time constants \( \lambda > 1 \), equation (2.3) can be written approximately as

\[
v(t) \approx I_0 R_1 k^2 q^2 \left[ e^{-t/\tau_1} - e^{-t/T_1} \left( \frac{m}{(\lambda - 1)^2} \right) \right].
\]

The first term has the time constant of a non-polarizable circuit, so it is independent of the dispersion and fundamental to the circuit. This 'fundamental response' is the same as the intrinsic response of a non-polarizable circuit for \( m \to 0 \); however, as \( m \) increases, the fundamental and intrinsic modes diverge, although they can usually be identified as being similar. These modes are given different names because they are associated with polarizable and non-polarizable structures respectively. The second term in equation (2.4) results in a very small negative voltage response and is associated with the polarization*. The total voltage can thus be written in the form

\[
v(t) = v^{\text{fund}}(t) + v^{\text{pol}}(t).
\]

Each of these terms have an associated current mode: the fundamental inductive current mode and the polarization current mode, the latter being generally much smaller. Seigel (1959) defined the polarization occurring in galvanic IP as being in the opposite direction to the dc charging current. In appendix 4, it is shown that the polarization current in EM is a sum of all the polarizations induced by the time varying fundamental inductive current. This sum, written mathematically as a convolution, results in a polarization current flowing in the opposite direction to the fundamental inductive current.

### 2.4 The occurrence and time of sign reversals

Setting \( v(t) = 0 \) in equation (2.3), yields an equation for the time of a sign reversal

* Wait (1983) has also shown that the decay associated with the intrinsic time constant has a sign opposite to the decay associated with the Cole-Cole time constant.
in the response

\[ e^{(s'_2 - s'_1)t'} = \frac{s'_1 + s'_2 \lambda}{s'_2 + s'_2 \lambda}. \]  \hspace{1cm} (2.5)

By investigating the conditions for which equation (2.5) is satisfied it is possible to determine the times at which sign reversals occur.

**Case 1: real roots** \((q > 0)\)

When \(q > 0\), the \(s'_1\) and \(s'_2\) are real and equation (2.5) can be solved to give the time of the sign reversal:

\[ t'_{\text{rev}} = \frac{1}{s'_1 - s'_2} \ln\left(\frac{s'_2 + s'_2 \lambda}{s'_1 + s'_2 \lambda}\right), \]

where I have assumed without loss of generality that \(s'_1 > s'_2\) (the \(s'_1 = s'_2\) case is not considered as it is the transition between case 1 and case 2). When the argument of the logarithm is less than one, no physically realizable sign reversals are obtained; for an argument greater than 1, there is only one sign reversal as the logarithm is single valued for real arguments.

**Case 2: complex roots** \((q < 0)\)

For \(q < 0\) the roots occur in complex conjugate pairs, and the voltage response oscillates in sign. This means that a negative transient can become positive again (i.e multiple reversals occur)\(^*\).

The times of the first and subsequent sign reversals can be found by writing the roots in polar form:

\[ s'_1 = re^{i\theta}, \]

\[ s'_2 = re^{-i\theta}, \]

where

\[ r = \frac{1}{2\lambda} \sqrt{[1 + \lambda(1 - m)]^2 + |q|}, \]

\(^*\) Multiple reversals are very occasionally observed in the field (M. Bosnar, pers. comm.); however, it is unlikely they are caused by a conductor with \(q < 0\), as this would require \(m \gg 1\) and/or \(\lambda \approx 1\), which is rarely true in practice.
and
\[ \theta = \pi + \tan^{-1}\left( \frac{-\sqrt{|q|}}{1 + \lambda(1 - m)} \right), \]

\((0 \leq \theta \leq \pi)\). The \(\pi\) has been inserted because the real part of the roots is always negative, so \(\theta\) is in the second quadrant.

Substituting \(s_1^r\) and \(s_2^r\) into equation (2.5) gives:
\[ e^{-2ris\sin\theta t'} = e^{i2\theta \frac{1 + 2r\lambda \cos \theta + r^2\lambda^2 \cos 2\theta + i(2r\lambda \sin \theta + r^2\lambda^2 \sin 2\theta)}{1 + 2r\lambda \cos \theta + r^2\lambda^2}}. \]

This can be simplified to
\[ e^{-2ris\sin\theta t'} = e^{i(2\theta + \Theta)}, \tag{2.6} \]

where
\[ \Theta = \tan^{-1}\left( \frac{2r\lambda \sin \theta + r^2\lambda^2 \sin 2\theta}{1 + 2r\lambda \cos \theta + r^2\lambda^2 \cos 2\theta} \right), \]

\((0 \leq \Theta \leq \pi)\). Taking the logarithm of both sides of equation (2.6) yields the scaled times of the sign reversals:
\[ t_{rev}' = \frac{2\theta + \Theta + 2\pi n}{-2r \sin \theta}, \quad n = 0, 1, 2... \tag{2.7} \]

These scaled times are a function of \(\theta, \Theta\), and \(r\), which in turn are functions of \(\lambda\) and \(m\). Sign reversals will always occur because a large enough positive value of \(n\) can always be found such that \(t_{rev}'\) is positive. The first sign reversal will correspond to the value of \(n\) which yields the smallest possible value of \(t_{rev}'\). Incrementing this value of \(n\) will yield subsequent sign reversals.

**Requirements for sign reversals**

Figure 2.5 shows the scaled time of the first sign reversal for \(0 < m < 1\) and \(0 < \lambda < 8\). The hatching denotes regions where no sign reversals occur as \(q > 0\) and the argument of the logarithm function is less than \(1\). The time that would elapse between any subsequent sign reversals is shown on Fig. 2.6.

For realistic dispersions (\(m \ll 1\) and \(\lambda \gg 1\)), \(q\) is generally positive and the single sign reversal occurs at large scaled time \((t/\tau_1)\). A negative can only be observed at a relatively early unscaled time if \(\tau_1\) is small, i.e., the larger positive
fundamental response will decay rapidly, allowing the smaller polarization response to dominate at late times. The resulting negatives will only be observed in field measurements if their magnitudes are large.
Figure 2.6 The scaled time between successive sign reversals for $0 < m < 1$ and $0 < \lambda < 8$.

2.5 Obtaining large negatives

The largest negative voltage $v_{neg}$ will occur when the time rate of change of voltage
is zero. Differentiating equation (2.3) and setting \( \partial v(t)/\partial t = 0 \) yields

\[
e^{(s'_{1} - s'_{1})t'} = \frac{s'_{1}^{2} + s'_{1}^{3} \lambda}{s'_{2}^{2} + s'_{2}^{3} \lambda}.
\]

When \( q > 0 \), only one maximum in the negative voltage is observed, and this occurs when

\[
t'_{\text{neg}} = \frac{\lambda}{\sqrt{q}} \ln \left[ \frac{s'_{1}^{2} + s'_{1}^{3} \lambda}{s'_{2}^{2} + s'_{2}^{3} \lambda} \right].
\]

(N.B. this only applies in regions where negatives are observed (i.e. \( q > 0, \lambda > 1 \).)

When \( q < 0 \) the stationary points occur for \( t' \) such that

\[
t'_{\text{neg}} = \frac{4\theta + \Theta \pm 2\pi n}{-2r \sin \theta}, \quad n = 0, 1, 2, \ldots
\]

As the voltage is a damped oscillation, the largest negative will be the first stationary point at which \( \partial^2 V/\partial t^2 \) is positive. For completeness \( t'_{\text{neg}} \) has been contoured as a function of \( \lambda \) and \( m \) on Fig. 2.7.

The maximum negative voltage response can be re-written in the form:

\[
v_{\text{neg}} = A_{0} f(m, \lambda, t'_{\text{neg}})
\]

where

\[
A_{0} = I_{0} R_{1} k_{01}^{2} \mu_{1}^{2} = \frac{I_{0} R_{1} k_{01}^{2} L_{0}}{L_{1}} = \frac{I_{0} k_{01}^{2} L_{0}}{r_{1}}, \quad \text{and}
\]

\[
f(m, \lambda, t') = \frac{1}{\sqrt{q}} \left( e^{s'_{1}t'} (s'_{1} + s'_{1}^{2} \lambda) - e^{s'_{2}t'} (s'_{2} + s'_{2}^{2} \lambda) \right).
\]

The parameter \( A_{0} \) is a scaling factor independent of time and a function only of the non-dispersive properties of the transmitter/receiver and the circuit. The time varying term, \( f(m, \lambda, t') \), is dependent on the dispersive properties of the conductor.

Rather than plot the unscaled maximum negative voltage, the dependence on the non-dispersive properties has been removed by normalizing to the scale factor, \( A_{0} \). As the resulting quantity varies over a large range of values, what is actually plotted on Fig. 2.8 is \( \log_{10}(-v_{\text{neg}}/A_{0}) \).

It can be seen from the figure that increasing the chargeability will increase the size of the negatives; however, because the chargeability is normally constrained
Figure 2.7 The value of $t'_{neg}$ for $0 < m < 1$ and $0 < \lambda < 8$. In region to the left, where $\lambda < 1$, no negatives occur, so $t'_{neg}$ is not defined.

to small values ($m \ll 1$), some other mechanism for increasing the negatives is required. The size of the negative is essentially unchanged as $\lambda$ increases, so the only way to increase its size is to increase the scale factor $A_0$. This can be done by:

i) *Increasing the transmitter current $I_0$. An increase of $I_0$ will increase the mea-
Figure 2.8 The magnitude of the largest negative voltage, plotted as $\log_{10}(-v_{neg}/A_0)$, where $A_0$ is the scale factor defined in the text.

sured voltage proportionally, but because the voltage response is usually normalized to unit transmitter current this will have no effect on the final result, except perhaps to increase the signal to noise ratio.

ii) *Increasing the resistance, decreasing the intrinsic time constant, or decreasing*
the self inductance of the conductor. All these actions are related, as decreasing the self inductance or increasing the resistance implicitly decreases the time constant (and vice-versa). A conductor with a smaller time constant decays more rapidly, allowing the negative transient to be observed at earlier times when it is larger. (N.B. this argument assumes $\lambda$ constant, so varying these parameters as specified also requires that $T_1$ vary proportionately).

iii) Increase the coupling of the transmitter to the conductor ($k_{01}$). This is the most effective way of increasing the size of the negatives because $v_{neg} \propto k_{01}^2$. Flis (pers. comm.) noted that the size of the negatives observed in field measurements also grow with greater coupling.

Each of the above actions will also increase the early-time response (as $v(0) = A_0(1 - m)$); thus, requiring a large $A_0$ is equivalent to specifying that the early-time response be large.

The conditions required to increase the scale factor will also result in a large fundamental inductive current flowing in the body at early time

$$i_1(0) = I_0 k_{01} \mu_1 = I_0 k_{01} \sqrt{L_0 \over L_1} = I_0 k_{01} \sqrt{L_0 R_1 \over \sqrt{\tau_1}}$$

(which is also independent of $\lambda$ and $m$). Workers in IP (Vacquier et al, 1957; Scott and West, 1969) have shown that polarization effects are proportional to the charging current (for current densities less than $1.0 \times 10^{-3} \text{A/m}^2$). In EM methods, the charging current is the fundamental inductive current (Appendix 4), so it is not unreasonable to expect a large fundamental current will result in a large polarization current and hence a large negative in situations when the body is polarizable.

2.6 Examples

Figure 2.9 shows a coincident-loop profile of the near-vertical wire-loop circuit shown schematically at the bottom of the figure. The two large positive lobes seen at
Figure 2.9 The response profile of the polarizable circuit shown schematically at the bottom of the figure. The parameters describing the circuit are: $\tau_1 = 0.13 \text{ ms}$, $R_1 = 0.1 \Omega$, $m = 0.017$, and $T_1 = 1.9 \text{ ms}$. The $i^{th}$ curve corresponds to the measured response in the $i^{th}$ standard SIROMEM time channel. The radius of the transmitter/receiver loop is 56m, giving the loop the same cross-sectional area as a 100x100m square loop. The plot is logarithmic except between 1 and -1, where it is linear.
early time occur at positions where the transmitter/receiver couples strongly to the circuit \((k_{01} \text{ large})\). At positions directly above the circuit, the transmitter/receiver coupling is weak \((k_{01} \text{ small})\), so the induced current and the observed response are small. At transmitter/receiver positions where the response is large, the strong coupling and small time constant results in a large rapidly decaying early-time current capable of charging the weakly polarizable circuit \((1.6\% \text{ variation in the resistance between 10 and 10\,000\,Hz})\). The polarization current is small, but the coupling of the polarization current to the receiver is identical to the coupling of the transmitter to the circuit, so the polarization response measured by the receiver will be large enough to dominate a positive response which has become small at late times. At positions where the coupling is less, the polarization response is too small to be measured.

An example of the currents induced in a single polarizable circuit \((\lambda = 6, m = 0.25; q > 0)\), shown on Fig. 2.10, illustrates the physical processes occurring in the conductor. The fundamental and intrinsic modes are similar, but not identical to each other (a large chargeability was selected so these differences would be apparent). The polarization current is significantly smaller than the fundamental current and has an opposite sign. The total current is the sum of the fundamental and intrinsic modes; it changes sign because the fundamental mode decays more rapidly than the polarization mode. The voltage measured in the receiver loop is related to the time derivative of the current so (a) the polarizable voltage will not change sign until the total current has reached its maximum negative value, and (b) the enhancement of the polarizable response in comparison to the non-polarizable response just prior to the sign reversal is due to a very rapid decrease in the total current at that time.

Negatives similar to those shown on Fig. 2.9 are not observed in the field because the small negative voltages are normally swamped by a larger positive background response associated with a half-space or a conductive overburden. Figure 2.11, taken from Flis (1987), is one such example. In this case, the near-vertical
Figure 2.10 The fundamental inductive and fundamental polarization current modes flowing in a polarizable circuit and the associated voltage response curves. These are compared with the intrinsic current mode and the associated intrinsic voltage response when the circuit is non-polarizable. For this figure $\lambda = 6, m = 0.25$.

Conductor is not thin, but thick, so it is expected that the early-time response profile will display a single broad maximum rather than two lobes; however, at later times two lobes should be seen. Relative to the background, the response on Fig. 2.11 shows a broad enhancement in the first channel, a hint of two positive lobes at 1.633ms, and two 'negative' lobes after 2.643ms. Without the large positive background response this data would display actual negative transients somewhat similar to those seen on Fig. 2.9.
Figure 2.11 SIROTEM response measured along line 131.133S at Blake South using 80m-square coincident transmitter and receiver loops. The delay time of each channel (rather than the channel number) is plotted on the right hand side (after Flis (1987)).
2.7 Summary

In order to observe measurable negatives at realistic times and plausible chargeabilities the early-time positive response must be large and decay away rapidly and the smaller polarization current must couple well to the receiver.

To generate a large early-time response requires that the coupling of the transmitter to the circuit be large. At late time, a rapidly decaying fundamental inductive current is small, so the fundamental magnetic field is weak and couples poorly to the receiver. The polarization current decays more slowly, and because it flows in the same circuit as the early-time fundamental current, it will couple well to a receiver which is coincident with the transmitter, allowing measurable negatives to be associated with moderate polarizabilities.

These provisions outline the conditions required for observing negative transients with realistic polarizabilities, the term used to describe them throughout this thesis is 'the favourable coupling conditions'.
Chapter 3

Negatives attributed to a thin surficial conductor

3.0 Introduction

The characteristics of the currents induced in a laterally extensive conducting structure will be different from those induced in a simple confined conductor. In this chapter, extensive conductors such as half-spaces and thin surficial overburdens are studied to determine the properties of the polarization current and reasonable conditions for generating negative transients with realistic polarizabilities. The discussion is illustrated with theoretical models and field data.

3.1 Induction in a polarizable half-space

In the wire-loop circuit analyzed in the previous chapter, the polarization and fundamental currents are both constrained to flow in the wire. For extended bodies such as half-spaces, overburdens etc., there is no such constraint, so the currents diffuse into the body.

The spatial pattern of the diffusion is studied for a 125Ωm half-space excited by a circular-loop of radius 50m in two cases: when the half-space is (i) non-polarizable, and (ii) polarizable. The flow patterns can be ascertained from contour plots of the electric field induced in the ground (Lewis and Lee, 1978; Nabighian, 1979; Hoversten and Morrison, 1982). For a circular loop, the electric field is cylindrically symmetric and need only be calculated outwards from the centre of the transmitter as a function of radius and depth. The values, calculated using the algorithm of Appendix 2, vary over a large range of values, so for ease of contouring the inverse
hyperbolic sinh (arcsinh)\(^*\) of the electric field (in nV/m) is plotted. Figure 3.1 shows the electric field induced in a non-polarizable ground at six delay times. At early time, the intrinsic current is largest in the vicinity of the transmitter loop, but as time progresses, its magnitude decreases substantially and the maximum of current density diffuses downward and outward, in this case moving off the plotted section by 0.1ms. At late delay times, the magnitude of the electric field decreases, and there is a gradual change in the shape of the contours. (In the last section, the contours are somewhat jagged because the algorithm is not very accurate for very small magnitudes.) The downward and outward diffusion is similar to the manner ‘smoke rings’ diffuse out from a smoker’s mouth (Nabighian, 1979).

Figure 3.2 shows electric field sections for the case when the half-space is polarizable. The field at 0.01ms is similar to the non-polarizable case, but by 0.1ms, the contours are beginning to be distorted by the negative polarization current. At 0.3ms, sign reversals have occurred in the region where the non-polarizable field is small and positive (i.e. \(r < 50\text{m}\)). After 1ms, the positive electric field is small enough that the negatives become dominant over a region covering almost the entire section.

Because the magnitude of the polarization is proportional to the magnitude of the charging current density (Vacquier et al., 1957; Scott and West, 1969), the greatest polarization current will be in the region where the fundamental inductive current is greatest. The maximum of current density associated with the fundamental current does migrate, but its magnitude decreases, so the largest value occurs at early-time (prior to 0.01ms) slightly below and inside the transmitter loop (Lewis and Lee, 1978). The polarization current is expected to be greatest in a similar position, but unlike the fundamental current, its maximum of current density does not appear to migrate! This behaviour is understandable because the dominant part of the polarization is accumulated while the strong early-time fundamental cur-

\(^*\) For small values of the argument, arcsinh is approximately linear (arcsinh\((x) = x\)), while for large values it is logarithmic (arcsinh\((x) = \text{sgn}(x) \ln(2|x|))\).
Figure 3.1  Contours of the electric field in the earth plotted on a section covering 50m of depth and 80m of radial distance from the centre of the transmitter loop. The conductivity of the ground is 125$m$ and it is excited by a circular transmitter loop of radius 50m. The arcsinh of the nanovoltage per metre is contoured, contours of 1, 4, 8, 12, 16 correspond to $1.17 \times 10^3$, $8 \times 10^4$, $4 \times 10^5$ nV/A. The plots labelled (a), (b), (c), (d), (e), and (f) represent the voltages at times of 0.01, 0.1, 0.3, 1.0, 3.0, 8.0ms respectively. In each case the contour interval is 0.1
Figure 3.2 The same as Fig. 3.1, except the conductivity is dispersive ($T = 0.1\text{ms}$, $m = .46$, $c = .55$). When the electric field is negative, the contours are dashed lines. The plots labelled (a), (b), (c), (d), (e), and (f) represent the electric field at times of 0.01, 0.1, 0.3, 1.0, 3.0, 8.0ms respectively. The contour intervals are 0.1, 0.1, 1.0, 0.4, 1/3, and 0.4. On the last graph the contour labels have been multiplied by 1000.
rent flows and the pattern of the polarization current does not change subsequently because the rate of decay of the polarization is independent of position*.

The dual current modes induced in a polarizable ground contrast with the case of a non-polarizable layered earth where there is usually only one current mode (Hoversten and Morrison, 1982).

The coincident-loop voltage responses of the half-spaces of Figs 3.1 and 3.2 are shown on Fig. 3.3. The non-polarizable case (dotted line) is purely positive and decays as $t^{-5/2}$, while the polarizable case (solid line) displays a small negative transient at late time. The dispersion required to obtain this negative is unrealistically large (22% change in the resistivity between 10 and 10000Hz). Unfortunately, a half-space with a significantly weaker dispersion would have a polarization current too small to reverse the sign of the total response. Negatives will only be observed with more realistic levels of the polarizability if the positive fundamental response decays more rapidly than it does on Fig. 3.3.

3.2 The response of a thin conductive overburden

A thin conductive overburden is an extended structure with a voltage response which decays as $t^{-4}$ in the time ranges normally of interest, a decay rate more rapid than that of a half-space. Figure 3.4 shows the response of a 20Ωm overburden overlying a 6000Ωm half-space for the cases when the overburden is 10, 30, 100 and 300m thick. The early-time response is similar in each case, as it is dependent on the conductivity of the layer closest the transmitter. The voltage response at subsequent delay times is dependent on the thickness of the overburden; the thinner the layer, the earlier the response becomes small. When all the conductivities (including that of the basal half-space) are made weakly polarizable (5% change in the resistivity

* This assumes that the current induced by the decay of the polarization current’s magnetic field is insignificant, an assumption verified in Appendix 4.
between 10 and 10,000 Hz), negatives are only observed in the cases when the response decays rapidly and is small at relatively early-time (Fig. 3.5). Although the 600Ωm half-space has a small response at $10^{-3}$s, negatives are not seen because the early-time fundamental current is not as strong, and the rate of decay is not as rapid as it is for an overburden.

Figure 3.6 shows how the size of the early-time response depends on the coupling of the transmitter to a thin conductive layer (modelled here with an infinite thin sheet). The curves on this figure, and subsequent ones in this chapter, are
calculated using the algorithm described in Appendix 3. Each thin sheet has an identical conductivity-thickness product, but the depth of burial varies. The shallower the thin sheet, the larger the exciting field, so the early-time fundamental current and early-time response will be greater. A larger polarization current will be generated by a stronger fundamental current, which is one reason the observed negative transients are stronger.
Figure 3.5 The same as Fig. 3.4 except that all resistivities are dispersive, with $T = 1\text{ms}$, $m = 0.1$, $c = 0.3$.

The characteristics of the fundamental response in cases when negatives are seen are similar to those required to observe negatives with the confined conductor model of the previous previous chapter (i.e. a large early-time response which decays away rapidly).

The electric field sections of Fig. 3.7 show how the currents induced in the thinnest overburden model of Fig. 3.5 couple to the transmitter_receiver. The fundamental inductive current flows primarily within the top layer (c.f. Fig. 3.1), and its position of maximum current density migrates outwards very rapidly, moving off the plotted section by $0.1\text{ms}$. The early-time fundamental current is proximal
Figure 3.6 The coincident-loop response of a conductive thin sheet \( R_{dc} = 4 \text{S} \) excited by a circular-loop of radius 56m. The response has been plotted when the sheet is buried 0.1, 1, 3, 10, 30 and 100m below the transmitter/receiver loop. The Cole-Cole parameters describing the dispersion are \( T = 4 \text{ms}, m = 0.03 \) and \( c = 1 \).

To the receiver loop, resulting in strong coupling and a large response; however, the subsequent migration within the top layer decreases the coupling substantially, ensuring a very rapid decay. The position of maximum current density associated with the polarization current does not migrate a significant distance and couples well to the receiver, so by 1.0ms, the polarization response is dominant.

This example illustrates why the fundamental response decays quickly, but more importantly it shows how a very weak polarization current can generate a significant response — by coupling very strongly to the receiver.
Figure 3.7 Contours of the electric field in the earth plotted on a section covering 50m of depth and 80m of radial distance from the centre of the transmitter loop. The earth is comprised of a 10m thick 20Ωm polarizable overburden \((T = 1\text{ms}, m = 0.1, c = 0.3)\) over a 600Ωm non-polarizable half-space. The transmitter loop radius is 56m. The plots labelled (a), (b), (c), (d), (e), and (f) represent the voltages at times of 0.01, 0.1, 0.3, 1.0, 3.0, 7.0ms respectively. The contour intervals vary.
3.3 Field examples

Field examples of negatives associated with polarizable overburdens do exist, and two examples are discussed here.

Billiton Aust. and CRA Exploration Pty. Ltd. have provided data from an area near Cleve in South Australia. A resistivity/IP survey along line 15300E (Fig. 3.8) indicates a generally resistive background and a small shallow zone of material at 12200N which has a relatively low resistivity (less than 8Ωm) and a chargeability of approximately 30 ms (measured in the time domain over the period range 50 to 1550 ms). The position of this zone correlates with that of a saline creek (Elliott, 1987). Figure 3.9 is a SIROTEM profile along a similar line collected with a small receiver of dipole moment $10^4$ m$^2$ placed at the centre of a moving transmitter loop. Over the conductive overburden the response is large at early time, decays away rapidly and displays negative transients.

In this example, a fixed-transmitter moving-receiver survey was performed, allowing the characteristics of the current decay modes to be determined by locating the regions of maximum current density. A 100×100m transmitter loop was centred at 12200N, directly over the central-loop negatives. The vertical- and horizontal-field responses can be seen on Figs 3.10 and 3.11 respectively (the noise in the data is attributed to the receiver being blown about by strong winds during the survey). The zone of maximum current flow in a shallow thin-sheet is found where and when:

a) the vertical magnetic field changes most rapidly as a function of position (e.g. near a zero crossing), and

b) the horizontal magnetic field is a maximum positive or minimum negative (these positions are termed the 'shoulders').

The latter condition is a result of the horizontal magnetic field being equal to twice the surface current density (Appendix 3, equation (A3.10)). In practice, EM surveys measure the voltage response, which is related to the magnetic field by the time derivative operator. On Fig. 3.11, the shoulders in the voltage response
Figure 3.8 A dipole-dipole resistivity/IP pseudo-section along line 15350E, near Cleve in South Australia. The data were collected with a Huntex Mk IV 7.5 kW transmitter and a Huntex Mk IV receiver. The transmitted waveform has a period of 4 seconds, and the receiver integrated over channels 0 to 9. The dipole size is 50m (after Elliott, 1987). The apparent chargeability is actually in units of ms not mV/V.

associated with the fundamental inductive current migrate outwards so rapidly that after 1.2ms they are off the plotted profile and distant from the transmitter/receiver. However, the smaller shoulders associated with the polarization current do not move laterally, indicating that the polarization current remains close to the transmitter wire. The characteristics of the current are in qualitative accord with those of a uniform overburden model, so it is not surprising that negative transients are seen.

Whether the polarizability required to generate these negatives is plausibly small is determined by modelling the data quantitatively. The overburden is represented by a weakly-polarizable uniform thin sheet infinite in horizontal extent, and the 100 × 100m square-loop transmitter loop is approximated by a circular loop of
Figure 3.9 SIROTEM data collected with a 100m square-loop transmitter and an 'central-loop' vertical-dipole receiver (moment $10^4 m^2$), along line 15300E (the eastern edge of the transmitter loops are along line 15350E.) The delay time (in ms) for each SIROTEM channel is plotted on the right hand side (After Elliott, 1987).

equal area. The thin-sheet is assigned a dc resistance* ($R_{dc}$) of 0.45Ω and the dis-

* the resistance is equal to the resistivity divided by the sheet thickness
Figure 3.10 The voltage induced in a small vertical receiver dipole of moment $10^4 \text{m}^2$, as a function of distance across a fixed $100 \times 100\text{m}$ square transmitter loop centred at 12200N on line 15300E. Error bars have not been plotted, but the data is very noisy (after Elliott, 1987).

Persian is such that there is a 7% change in the resistance between 10 and 1000Hz. This polarizability cannot be compared with the time-domain chargeability as they
Figure 3.11 The voltage induced in a small horizontal receiver dipole of moment $10^4\,\text{m}^2$, as a function of distance across the same transmitter as used in Fig. 3.10. The sign convention chosen for this data results in the fundamental response to the left of the profile being positive. (After Elliott, 1987).

are different quantities and the latter measurement is sensitive to a different frequency range. The calculated vertical- and horizontal-field responses (Figs 3.12 and
Figure 3.12 The calculated voltage induced in a vertical receiver dipole (of moment $10^4 \text{m}^2$) plotted as a function of distance across the transmitter loop (in this case the loop is circular with a radius of 56m, which gives it a cross-sectional area equal to the 100x100m square loops used in Figs 3.9 to 3.11). The overburden is characterized by $R_{dc} = 0.45\Omega$, $T = 2\text{ms}$, $m = 0.09$ and $c = 0.6$. 

"FIXED LOOP—CALCULATED VERTICAL RESPONSE"

"TRANSIENT RESPONSE (\mu V/A)"

"Polarization Response"

"Fundamental Response"

"Station Location / Loop Radius"
Figure 3.13 The same model as Fig. 3.12, except the response is the voltage induced in a horizontal receiver dipole (of moment $10^4$ m$^2$). In this plot, 11 channels have been plotted whereas only 7 are plotted in the field data.
3.13) have a form similar to the field data. Any differences can be attributed to the field area being laterally heterogeneous, while the model is laterally uniform. The weak polarizability required to explain the field data suggests that the favourable coupling conditions, outlined in the previous chapter, are a plausible mechanism for explaining negative transients.

In the next example, from Willyama, N.S.W., the polarization current is only significant when the transmitter couples well to the polarizable zone.

Figure 3.14 is a resistivity/IP section from this region (data courtesy of Billiton Aust.). An interpretation of the resistivity section would indicate a thin shallow conductive zone of narrow lateral extent (between 1200W and 1400W) overlying a resistive substratum. Shallow drilling in the area shows that the conductive overburden is comprised of an alluvial clay in the top 2-4 metres and a weathered gneiss to 10 metres depth. Although the IP data does not give conclusive information about the polarizability in the TEM frequency range, the likelihood is that the region is only weakly polarizable. A 'displaced-loop' SIROTEM profile is available and shown on Fig. 3.15 (the 200×200m transmitter and receiver loops are displaced by only 15m, so the response will be very similar to the coincident-loop response). Negative transients are observed over the polarizable zone where the response is large at early time and decays away rapidly.

That the negatives are a result of the favourable coupling conditions can be seen from the fixed-loop vertical-field profile of Fig. 3.16. The placement of the transmitter loop at 1300W ensures a strong coupling to the conductive overburden and a significant early-time fundamental current. The large early-time positive response observed inside the loop is indicative of this large current. A rapid migration of the fundamental current away from the transmitter, indicated by the rapid movement of the zero crossing in the early channels, results in a correspondingly rapid decay of the positive response inside the loop and the rapidly decaying coincident-loop response. The existence of a fixed polarization current below the transmitter is implied by the presence of negative transients which remain inside the loop for
Figure 3.14 A dipole-dipole resistivity/IP pseudo-section along line 7300N, near the Willyama Block, N.S.W. The dipole size is 50m, and a Huntec Mk IV receiver was used to measure the response (after Elliott, 1987).

all delay times. (The importance of the coupling of the polarization current to the receiver can be seen on this profile. When the receiver dipole is inside the transmitter loop it couples well to the polarization so negative transients are observed; however, when the receiver is distant from the transmitter, it couples poorly and no significant polarization response is measured.)

Another fixed-loop survey (Fig. 3.17) was performed along the same line with the transmitter moved so its right edge is at 1500W. The coupling of the exciting magnetic field to the horizontal conductive and polarizable zone between 1200W
Figure 3.15 The SIROTEM response along the same line as Fig. 3.14, using 200×200m square transmitter and receiver loops displaced by 15m (after Elliott, 1987). Detail has been omitted when the response is between ±1μV/A.

and 1400W is sufficiently weak that a large fundamental inductive current is not generated in this zone. Consequently, the polarization current will be very small and the response will show no appreciable polarization effects. The most significant feature observed is the slow outward movement of the zero crossing between 1300W and 1100W, a well known effect exhibited by non-polarizable conductive zones (Smith and West, 1987). (Note that the anomalously low values at 1225W
Figure 3.16  The vertical component of the response along the same line as Fig. 3.14 using the roving vector receiver (dipole moment $10^4 m^2$) and a fixed, 200 x 200m transmitter loop centred on 1300W. The irregularity in the response between -1500E and -1750E is caused by the resistive structure at the left end of the resistivity/IP survey.
Figure 3.17 The same as Fig. 3.16, except the fixed transmitter loop is 600 × 300m with its right edge at 1500W.
could be caused by a local near-surface, conductive, but non-polarizable structure.)

The field examples of negative transients have confirmed the observations of theoretical modelling and provided some additional insights. Another field example, showing an effect related to the negative transients, is presented next.

3.4 The loop effect

The ‘loop effect’, mentioned in Chapter 1 and discussed briefly here, is a phenomenon difficult to explain by simple induction. It is sometimes observed in areas where the ground is essentially laterally uniform by surveys utilizing a large transmitter loop and measuring the vertical-field response with a small dipole receiver. The response inside the loop is depressed by an abnormal flexure of the response at the transmitter wire. An example of the loop effect, described by Asten and Price (1985), is shown on Fig. 3.18. The depression is similar in form to that shown on Figs 3.10, 3.12 and 3.14, except the polarization is much weaker and therefore not strong enough to reverse the sign of the total response.

The distortion of the data is easily explained by a slowly decaying polarization current flowing in a near-surface layer. For this example, the near surface layer is approximated by a thin-sheet overburden with a dc resistance of 0.16Ω that is weakly dispersive (the resistance varies by 1.2% between 10 and 10 000Hz). A circular loop of radius 56m is used to excite the thin-sheet model, and the calculated response profile is shown on Fig. 3.19. A sharp flexure in the response occurs near the transmitter wire, and the position and width of this distortion does not change with increasing delay time.

The field and calculated responses do have a slightly different shapes at early time and different fundamental response magnitudes at late time. The variation on the right hand side of the field case, likely due to a lateral heterogeneity, would be very difficult to model. The agreement in other regions could be improved by
using a model with a more complicated layered structure; however, the polarization response would be little changed as it depends principally on the properties of the overburden near the transmitter.

The loop effect distorts the coincident loop response and biases the resulting apparent resistivities of the ground. Unbiased estimates can be obtained using a scheme proposed by Asten and Price which corrects the downward bias inside the loop and the upward bias outside the loop by assuming that the magnitude of the bias is proportional to the primary field at each position. The polarization response is largest where the early-time fundamental response is greatest, which is approximately where the primary field is greatest, so their scheme will provide a correction accurate to first order. The polarization can be accounted for more exactly by
CALCULATED RESPONSE
FIXED LOOP – VERTICAL COMPONENT

Figure 3.19 The vertical component TEM profile across a fixed transmitter loop of radius 56m. The dashed line is the response when the overburden (resistance $R_{dc} = 0.16\Omega$) is non-polarizable, and the solid line is when it is weakly polarizable ($T = 4\text{ms}$, $m = 0.07$, $c = 0.1$). The delay time is plotted on the right hand side.

making all the conductivities in the model frequency dependent; however, because the most significant polarization will occur in the region nearest the transmitter, it would probably be sufficient to make only the near-surface material polarizable.

3.5 Dealing with negatives attributed to a near-surface polarization

When a negative transient is observed in the field, it is important to determine whether it is caused by a near-surface polarization current or a deeper polarizable structure. One method of doing this is to measure the vertical- and/or horizontal-field response along a profile line crossing the transmitter loop. A near-surface polarization is implicated if
1) The vertical-field response is depressed inside the loop, and

2) The horizontal-field response has late-time shoulders at the transmitter wire which have the opposite sign to the shoulders of the early-time response.

The distortion caused by the polarization current occurs close to the transmitter wire and is analogous to the distortion caused by the superparamagnetic effect. The SPM effect was virtually eliminated by displacing the receiver from the transmitter by approximately 3 metres. Because the polarization current affects a much broader zone a displacement of 3m will not be adequate to reduce its effect on coincident-loop data. However, placing the receiver a large distance outside the transmitter will decrease the polarization effects significantly. For example, a receiver displaced two loop radii outside the transmitter loops of Figs 3.10, 3.12 and 3.16 will not measure significant polarization effects. Early-time negative transients will be measured, but these are a result of normal inductive effects.

The polarization response is identified by comparing its spatial characteristics with those of the fundamental inductive current. Incorporating more spatial information into a survey simplifies identification of the polarization current and subsequent interpretation.

3.6 Summary

In the above synthetic and field examples, negative transients are generated with moderate polarizabilities when the following favourable coupling conditions are satisfied: (1) the transmitter couples well to the polarizable zone to ensure a large fundamental current, (2) the fundamental response decays away rapidly, and (3) the receiver couples well to the slowly decaying polarization current. The coupling of the polarization current to the receiver remains large at all times because the polarization current is stationary.

When these conditions are not satisfied, the effect of the polarization current will be diminished. This occurs when: the transmitter is moved (either laterally or
vertically), the fundamental decay rate is too slow (as it is for a half-space), and the receiver is displaced from the transmitter by more than two loop radii.

The polarization current distorts the response either strongly (causing negatives) or weakly (causing the loop effect), but it is possible to identify these distortions by measuring the vertical- and/or the horizontal-field response across the transmitter loop. The distortions could be removed or interpreted by modelling the polarization current. Because the polarization occurs near the transmitter, this could be done with a simple polarizable model positioned proximal to the transmitter, even if the actual conductivity structure elsewhere is more complex than the model.
Chapter 4

A negative attributed to interacting conductors

4.0 Introduction

A field example of a negative transient occurring between two nearby conductors is presented in this chapter. Prior to discussing the example, the conductors are approximated by two interacting wire-loop circuits and a model study is undertaken to determine the nature of the induced currents and the characteristics of the response.

4.1 Two interacting non-polarizable circuits

Initially, the case of two non-polarizable wire-loop circuits, such as those shown on Fig. 4.1, is studied. The coincident-loop response of this model is (Appendix 1, equation (A1.14))

\[
v(t) = \frac{I_0}{\sqrt{(\tau_1 - \tau_2)^2 + 4\tau_1 \tau_2 k_{12}^2}} \left[ e^{s \tau t}[\alpha s \xi + \beta s^2] - e^{s \tau t}[\alpha s \zeta + \beta s^2] \right],
\]

where

\[
\alpha = k_{01}^2 \tau_1 L_0 + k_{02}^2 \tau_2 L_0,
\]

\[
\beta = \tau_1 \tau_2 L_0 \left[ (1 - k_{12} k_{02} / k_{01}) k_{02}^2 + (1 - k_{12} k_{01} / k_{02}) k_{01}^2 \right],
\]

\[
s_i = -1/t_i, \quad i = \xi, \zeta.
\]

The \(k_{ij}\) are the coupling coefficients between circuits \(i\) and \(j\), and the \(t_i\) are time constants defined by

\[
t_{\xi, \zeta} = \frac{1}{2} \left[ \left( \tau_1 + \tau_2 \right) \pm \sqrt{(\tau_1 - \tau_2)^2 + 4\tau_1 \tau_2 k_{12}^2} \right].
\]
Figure 4.1 The geometric arrangement of the two interacting non-polarizable circuit model. The coincident-loop transmitter/receiver is also shown.

If there is no interaction, then $k_{12} = 0$ and each circuit $i$ has one current mode with a time constant equal to the intrinsic time constant of that circuit ($\tau_i = L_i/R_i$). When interaction takes place, an extra current mode is introduced into each circuit. The modes generally have different magnitudes, but the time constants of the two modes in one circuit ($t_1$ and $t_2$) are identical to the time constants of the two modes in the other circuit. For weak coupling ($k_{12}^2 \ll 1$), these time constants are similar to the intrinsic time constants of each circuit. Thus, in each circuit one mode has a time constant similar to the circuit's intrinsic time constant (the innate inductive mode), and the other mode has a time constant identical to the innate inductive mode of the other circuit (the transferred inductive mode). The magnitude of the innate inductive mode is generally larger than that of the transferred inductive
mode. As the coupling increases, the larger time constant increases and the other decreases, but their values remain identifiable with the intrinsic time constants of the uncoupled circuits (Vallée, 1981). For very strong coupling \((k_{12} \to 1)\), one mode has a time constant equal to the sum of the intrinsic time constants whereas the other tends to zero.

A synthetic example of a coincident-loop TEM profile over two coupled circuits is seen on Fig. 4.2. It is basically a four lobed response, being the superposition of the individual responses of the two circuits (both of which have a two lobed response). However, there are two important modifications produced by the interaction.

First, the additional decay mode in each circuit will have an effect on the amplitude and decay pattern. For example, the anomaly over circuit 2 (lobes C and D) is principally a short time constant decay, but at late times, a weaker decay with a long time constant is observed in lobe D. The time constant of the larger decay mode is similar to the intrinsic time constant of circuit 2, so this decay is associated with the innate inductive current mode. The weaker, late-time decay has a time constant equal to the innate time constant of circuit 1, associating the decay with the transferred inductive mode of circuit 2.

The second difference from a simple superposition is that the response is dependent on how the transmitter/receiver couples to the two buried circuits. North of circuit 2 (at lobe D) the response of the slowly decaying transferred inductive mode of circuit 2 adds to the response of the slowly decaying innate inductive mode of circuit 1, a situation which is termed ‘positive coupling’. In between the circuits, at lobe C, ‘negative coupling’ occurs at late time. In this case, the magnetic fields from the slowly decaying mode of circuit 1 has a component through the receiver which is in the opposite direction to that from the slowly decaying mode of circuit 2. Thus, the total secondary magnetic field will be very small, as will the corresponding voltage response.

In the position of negative coupling, the response is large at early time and
Figure 4.2 The response of the two interacting conductors shown schematically at the bottom of the figure. There is a region of rapid decay in the vicinity of lobe C. On this and subsequent plots the transmitter/receiver is circular with a radius of 56m. In this diagram $\tau_1 = 2\ ms$, $R_1 = 0.2\ \Omega$, $m_1 = 0$, $T_1 = 0\ s$, $\tau_2 = 0.13\ ms$, $R_2 = 0.1\ \Omega$, $m_2 = 0$, $T_2 = 0\ s$ and $k_{12} = 0.3$. 
decays away very rapidly, so negatives can be observed here when one or more of the circuits is weakly polarizable.

\section*{4.2 Two interacting polarizable circuits}

The coincident-loop voltage response of two inductively-coupled polarizable circuits, such as those shown on Fig. 4.3, is

\[ v(t) = -M_{13} \frac{\partial i_1}{\partial t} - M_{23} \frac{\partial i_2}{\partial t}, \]  

(Appendix 1, equation (A1.19)), where

\[ \frac{\partial i_i}{\partial t} = H_i[\alpha_i \frac{\partial^3}{\partial t^3} Y(t) + \beta_i \frac{\partial^2}{\partial t^2} Y(t) + \gamma_i \frac{\partial}{\partial t} Y(t) + Y(t)], \quad i = 1, 2, \]

\[ = H_i[A_i e^{s_i t} \phi_{iI} + A_je^{s_j t} \phi_{jI} + A_K e^{s_K t} \phi_{iK} + A_L e^{s_L t} \phi_{iL}], \]

\[ \phi_{ij} = \alpha_i s_j^3 + \beta_i s_j^2 + \gamma_i s_j + s_j, \quad j \in \{I, J, K, L\} \]

\[ H_i = \frac{I_0 k_0 \tau_i \mu_i}{\tau_1 \tau_2 T_1 T_2 (1 - k_{12}^2)}, \]

the Cole-Cole time constants of the dispersive impedances of circuit 1 and 2 are $T_1$ and $T_2$ respectively, and the $\alpha_i$, $\beta_i$, $\gamma_i$ and $Y(t)$ are defined in Appendix 1. The $A_j$ are the coefficients obtained in decomposing $Y(s)$ (the Laplace transform of $Y(t)$) into partial fractions, and the $s_i$ are the roots of the denominator of $Y(s)$. The time constants of the four decay modes are given by $t_i = -1/s_i$.

In the case of small dispersion and moderate coupling, two of the modes in a given circuit will have time constants similar to the intrinsic time constant and the Cole-Cole time constant of that circuit, and these are termed the innate fundamental inductive and innate polarization modes of that circuit. The other two modes flowing in the circuit have time constants equal to those of the innate fundamental inductive and innate polarization modes of the other circuit, so these modes are termed the transferred fundamental inductive and transferred polarization modes.
4.3 Case A: A Polarizable circuit 2

Figure 4.4 shows an example where making circuit 2 polarizable has resulted in a negative transient in the region of negative coupling. In this case, the negative is associated with the innate polarization current of circuit 2. This current couples well to the receiver, so all the favourable coupling conditions are satisfied and the negative can be explained with a small dispersion (6.5% variation between 10 and 10 000Hz). This example was generated to model the field example of the following section.
Figure 4.4 The response of two interacting circuits with a polarizable circuit 2. In this diagram $\tau_1 = 2$ ms, $R_1 = 0.1 \Omega$, $m_1 = 0$, $T_1 = 0$ s, $\tau_2 = 0.25$ ms, $R_2 = 0.13 \Omega$, $m_2 = 0.07$, $T_2 = 1.9$ ms and $k_{12} = 0.7$. 
4.4 Field example

The data, collected near Dalgaranga, Western Australia, is supplied by CRA Exploration Pty. Ltd. of Adelaide, S.A. The SIROTEM system was used to measure the coincident-loop profile shown on Fig. 4.5 and a similar response was obtained using an EM37 system in the central-loop mode. No IP survey has yet been carried out along this line.

The channel 1 and 2 response indicates two discrete, steeply dipping conductors: one at 950N (conductor 1) is characterized by a slow decay, while another at 1300N (conductor 2) has a larger more rapidly decaying response. Measurements of the total static geomagnetic field reveal an anomaly which closely correlates with conductor 2 and suggests that conductor 2 is a relatively large permeable sheet-like body that dips steeply. Conductor 1 is probably smaller as the magnetic anomaly is weaker and narrower. The EM data from two adjacent parallel lines is similar, implying that the conductors strike parallel to one another in a direction normal to the profile line. In such a situation, it is likely the two conductors will have similar dips. Because the negatives seen on the southern flank of conductor 2 are similar to those seen in the position of negative coupling on Fig. 4.4, it would appear the synthetic model provides a plausible explanation for the negatives seen in the field data.

The amplitude of lobe D at late time is the portion of the synthetic data which is most unlike the field data. North of conductor 2, the lobe D response merges with the relatively weak response between 1450N and 1850N which is attributed to a structure other than the two conductors, either a conductive overburden, conductive host rock or a discrete conductor off the profile to the north. Deciding how to model the late time part of lobe D is critical in determining the polarizability of the circuit. If a large slowly decaying part is required, the model of Fig. 4.4, which has a 6.5% dispersion, would be adequate. However, if this part of the response can be
Figure 4.5 A SIROTEM profile from line 5050E, Dalgaranga, Western Australia. The transmitter/receiver consists of 100 m by 100 m coincident-loops.
Chapter 4  A negative attributed to interacting conductors

ascribed to another cause, a model with a smaller transferred fundamental inductive response, such as that shown on Fig. 4.6, would be more suitable. This latter example has a very small polarizability (1.6% variation in the resistance between 10 and 10 000 Hz). Between these two extreme examples lie a broad range of models which can be used to explain a range of late-time responses in lobe D.

Considering the simplicity of the circuit model (e.g. a fixed radius, circular-loop circuit, and the assumption that the Cole-Cole parameter, \( c \), be set to \( c = 1 \)), the agreement between the field and model data is remarkable, all the basic features being reproduced.

4.5  Case B: A Polarizable circuit 1

When circuit 1 is made polarizable instead of circuit 2, the favourable coupling conditions are no longer satisfied because the largest polarization current, the innate mode of circuit 1, couples poorly to the receiver in the region of negative coupling.

The transferred polarization current of circuit 2 is what will be observed in the region of negative coupling, and this is too small to measure if the dispersion is reasonable (<10% variation in the resistance between 10 and 10 000 Hz). A greater transferred polarization current can be generated by increasing \( k_{12} \), the coupling coefficient of the two circuits, but a significant increase is not possible in this example because the positions and relative sizes of the TEM and magnetic anomalies imply that \( k_{12} \) is roughly in the range \( 0.6 < k_{12} < 0.7 \). The only other way to increase the size of the transferred polarization current is to increases the polarizability of circuit 1. Unfortunately, this increases the innate polarization current of circuit 1, resulting in negative transients in lobes A and B. With delicate adjustments of the geometric, electromagnetic and dispersive parameters, the negatives in lobes A and B can be made small, while still allowing a negative to be observed in the region of negative coupling (Fig. 4.7).

Although this figure resembles the field data somewhat, there are a number of

...
Figure 4.6 The response of two interacting circuits with a polarizable circuit 2, a small chargeability, and late-time response which is almost non-existent at lobe D. In this diagram $\tau_1 = 2 \text{ ms}$, $R_1 = 0.2 \Omega$, $m_1 = 0$, $T_1 = 0 \text{ s}$, $\tau_2 = 0.13 \text{ ms}$, $R_2 = 0.06 \Omega$, $m_2 = 0.017$, $T_2 = 1.9 \text{ s}$ and $k_{12} = 0.6$. 
problems:

( i) The small late-time negatives at lobes A and B have not been eliminated completely.

( ii) The negative at lobe C is slightly too narrow and it decays too rapidly.

( iii) The slow part of the decay of lobe D is too large and is not monotonic in time. This contradicts the field data which is monotonic even if a superimposed response from an overburden or bedrock is subtracted out.

( iv) In the first and second channels, lobe C is larger than lobe D, the opposite of what is observed in the field data. Making circuit 2 dip to the south will correct this, but the fit to the late-time response becomes worse (Smith and West, 1988a).

Also, the required dispersion results in a 27% variation in the resistivity between 10 and 10000Hz. Although it is not impossible that conductor 1 could be this strongly polarizable, the weakly polarizable conductor 2, which satisfies the favourable coupling conditions, provides a better fit and a much more plausible explanation.

4.6 Including conductive overburden

A more complicated model would provide a better fit to the data north of conductor 2; incorporating a conductive overburden into the analysis is one method of doing this. The positive overburden response will mask the negative transients, requiring a larger polarizability to model the negatives. However, on the field data, the overburden response to the north is always less than the negatives after the third channel, so the effect of the overburden should really only shift the position of negative coupling at early time.

It is also possible that the polarization current causing the negatives is flowing in a horizontal surficial overburden layer. Measuring the vertical- and or the horizontal-field response across a fixed transmitter loop, as described in the previous chapter, would confirm whether this was the case. Even if it were found to
Figure 4.7 The response of two interacting circuits with a polarizable circuit 1. The response is roughly similar to the field data. In this diagram $\tau_1 = 2\text{ ms}$, $R_1 = 0.1\Omega$, $m_1 = 0.3$, $T_1 = 1.2\text{ ms}$, $\tau_2 = 0.27\text{ ms}$, $R_2 = 0.08\Omega$, $m_2 = 0$, $T_2 = 0\text{ s}$ and $k_{12} = 0.7$. 
be true, the general conclusions would still not be altered significantly: firstly, the region of negative coupling is still the only region where negatives are observed, and secondly the polarization still couples strongly to the receiver (more strongly than if the polarization current were in conductor 2).

4.7 Summary

The example of this chapter illustrates the importance of the polarization current coupling strongly to the receiver. In cases when the polarization current is distant, large polarizabilities are required to model the data; however, when the polarization is closer, negatives can be generated with moderate polarizabilities. The negatives are only seen in the region of negative coupling, showing that the large early-time response and rapid decay are also very important.

Only two interacting conductors were considered. When there are more than two conductors, inductive interaction will generate further regions of negative coupling and negatives could be observed in those regions if the magnetic field of the polarization current couples well to the receiver.
Chapter 5

Negatives attributed to edge effects

5.0 Introduction

When a coincident-loop system is over a conducting half-plane and near to the edge, the induced current pattern results in a fundamental inductive response which is large at early time and decays away rapidly. The work of Chapter 3 suggests that a polarization current induced in the half-plane will remain close to the transmitter/receiver so negative transients may be generated when the half-plane is weakly polarizable. This possibility is investigated, first with computer models and then with field examples.

5.1 The horizontal half-plane response

Non-polarizable half-plane

Peter Weidelt (1983) has written a computer program for calculating the coincident-loop response of a non-polarizable semi-infinite half-plane which is arbitrarily thin, yet has a finite conductance (conductivity-thickness product). The half-plane is assumed to be suspended in a perfectly insulating medium so it can be used to approximate a thin conductive overburden with an edge, overlying a resistive half-space. The computer algorithm is exact, but is subject to numerical quadrature errors and cannot be used to calculate the response too close to the edge. The latter problem can be avoided by burying the half-plane some distance below the surface of the earth.
Figure 5.1 is a SIROTEM coincident-loop profile of a non-polarizable half-plane whose conductance \((1/R_{dc})\) is 4.75S. The model, shown at the bottom of the figure, was selected to represent a field case of a conductive overburden with an edge at 9600E. Because the real overburden will be thick and the computer algorithm breaks down close to the edge, the half-plane is buried 50m below the surface of the earth.

When the transmitter is entirely over the half-plane it couples well to the structure. A large position-independent early-time response is obtained from the west end of the profile to about 9480E. Away from the edge (to the west of 8840E) the late-time response decays at the same rate as an infinite thin sheet (as \(t^{-4}\)). Closer to the edge, the response decays more rapidly. For example, at 9480E, the channel 7 response is significantly less than it is at other positions over the half-plane. As the delay time increases, this position of depressed response migrates, a consequence of the induced current migrating away from the edge of the sheet and under the receiver (Weidelt, 1983)*. When the current has migrated below the centre of the receiver loop, the secondary magnetic field couples poorly to the receiver and the measured response is small. This region of depressed response is termed the region of 'decreased coupling'.

**Polarizable half-plane**

The response of a polarizable overburden can be calculated from the non-polarizable response using the approximate convolution algorithm of Appendix 4. The results of Chapter 3 suggest that the polarization current will be greatest close to the transmitter loop and will not migrate a significant distance, so the coupling to a coincident receiver will be strong. Figure 5.2 is the response when the half-plane of Fig. 5.1 is weakly polarizable (2.7% variation between 10 and 10000Hz). The

---

* Currents induced in a finite plate by a uniform normal field migrate similarly away from the edges and towards the centre of the plate (Annan, 1974; Barnett, 1984; Dyck and West, 1984). These cases contrast with the half-space and overburden models of chapter 3, where the current expands outwards from the transmitter in all directions.
Model: Non-polarizable half-plane

Figure 5.1 The SIROTEM coincident-loop response of a half-plane with a conductance of 4.75S, that comes to an edge at 9600E, and is buried 50m below the surface of the earth. The transmitter-receiver loop is circular with a radius ($R$) of 90m. The model is shown at the bottom of the figure.

Negatives are observed, but only when the transmitter/receiver is over the half-plane as this is where the early-time response is large and the polarization is close to the receiver.

The larger negatives near the edge are not indicative of a larger polarization.
Model: Polarizable half-plane

Figure 5.2 The same as Fig. 5.1, except the half-plane is polarizable with the Cole-Cole parameters \( m = 0.03, T = 10\,\text{ms}, c = 1 \).

current, they are a consequence of the fundamental response decaying away more rapidly. For example, the earliest negative occurs in the channel 7 response at the position of decreased coupling (9480E), and the negatives are largest here because the polarization response is seen at an earlier time when it is larger. The region in which the response is negative broadens and appears to migrate to the west as
a function of increasing delay time, a result of the region of decreased coupling behaving in this manner.

The region of decreased coupling is where the favourable coupling conditions are best satisfied. Because measurable negatives can be generated when the polarizabilities are plausibly small, it is likely that response profiles similar to this synthetic example will be observed in field measurements.

5.2 Field examples

Teutonic Bore Joint Venture, Western Australia.

A field example, supplied to us courtesy of the Teutonic Bore Joint Venture (BP Minerals Australia, Mount Isa Mines, and Chevron Exploration Corporation) has been collected along line 65440N at Teutonic Bore, Western Australia (Fig. 5.3). This data is very similar to the synthetic data, so we ascribe it to a weakly polarizable conductive overburden which is truncated in the vicinity of 9600E. To the west, the size of the negative transients decrease more rapidly than they do in the model data, and this can either be attributed to a decrease in the polarizability of the overburden, and/or to a lateral variation in the conductivity structure. The most likely conductivity variation, a decrease in the conductance of the overburden to the west, will explain the early-time fundamental response decreasing to the west of 9080E. However, the polarization response actually starts to decrease at around 9320E, suggesting that the polarizability of the overburden is also decreasing to the west of this position.

Pincher Well, Western Australia

An EM37 profile, provided by BHP-Utah Minerals International, from line 18600N at Pincher Well, Western Australia is shown on Fig. 5.4.

The large amplitude early-time response is indicative of an overburden with an edge between 75300E and 75400E that is too shallow to be modelled quantitatively with the half-plane program. Rather than try and model the data exactly, the
Field data: Teutonic Bore

Figure 5.3 Coincident-loop SIROMEM collected along line 65440N at Teutonic Bore. The transmitter/receiver loops were 160×160m square (which has an area equal to a circular loop of 90m radius)

half-plane has been buried and I have attempted to reproduce the spatial form of the data with amplitudes which are smaller. Figure 5.5 shows the field response at station 76200E and the response of a 2.5S polarizable infinite thin sheet with a decay which is similar, but smaller by a factor of ten. This thin sheet is buried 120m below the earth and its conductance varies by 6.3% between 10 and 10 000Hz.

The response profile of a 2.5S polarizable half-plane, seen on Fig. 5.6, has a spatial form which is virtually identical to the field data. The polarizability of the half-plane is laterally invariant so it would appear that the polarizability of the overburden to the east of 75300E is also. Once again, the largest and earliest negatives are observed in positions where the favourable coupling conditions are
best satisfied.

In considering the polarizability which must be ascribed to the overburden, it should be noted that if the half-plane did not have to be buried so deeply, then the polarizability required to model the data would be less. A 4S infinite thin sheet buried 5m deep has a response (dot-dash-dot curve of Fig. 5.5) which matches the field data when the conductance only varies by 3.9% between 10 and 10000Hz.

Kangiara, N.S.W.

Figure 5.7 is a field example from line 6900N at Kangiara, N.S.W., supplied by BHP-Utah Minerals International. The early-time positive response is similar to the response near the edge of an overburden, but the negatives are more localized
Figure 5.5  The decay of the measured response at 76200E on line 18600N (the solid line with the greatest magnitude). The solid line, which is smaller than the measured response by a factor of ten, is the response of a 2.5S overburden \((m = 0.07, T = 10\text{ms}, c = 1)\) which is buried 120m below the transmitter/receiver. Also shown with the dash-dot-dot curve is the response of a 4S overburden \((m = 0.044, T = 12\text{ms}, c = 1)\) buried 5 metres deep. The overburden was modelled using an infinite thin-sheet, and the convolution approximation was used to calculate the polarization response. In all cases, the negative response is shown with a dotted line.

than in previous examples.

A half-plane which is weakly polarizable (2.8\% variation between 10 and 10\,000 Hz) has a similar response (Fig. 5.8), but the zone in which the negatives occur is much broader than it is in the field data. This could be due to: (1) a decrease in the polarizability of the overburden away from the edge, (2) a greater conductivity
Model: Polarizable half-plane

![Graph showing response profile](image)

Figure 5.6 The calculated response profile of a polarizable 2.5S overburden ($m = 0.07$, $T = 10\text{ms}$, $c = 1$) buried 120m. The edge of the overburden (shown at the bottom) is at 75350E. The response is calculated assuming the transmitter/receiver loops to be circular and coincident. The circular loop has a radius of 112.8m, which yields a transmitter moment equal to that of a square loop 160×160m.

at depth to the west (slowing the fundamental decay), or (3) a decrease in the conductance of the overburden to the west (decreasing the early-time current).

The computer program is not able to model any of these situations, but the latter one can be simulated by a half-plane dipping to the west because the decreased
Field data: New South Wales

Figure 5.7 Coincident-loop SIROTEM collected along line 6900N at Kangiara, N.S.W. The transmitter/receiver loops were 100 x 100m square.

coupling of the transmitter to the half-plane will result in a smaller early-time current and a smaller fundamental response (as is observed). A half-plane dipping
Figure 5.8 The SIROTEM coincident-loop response of a half-plane with a conductance of 4.0 S, that comes to an edge at 5275 E, and is buried 30 m below the surface of the earth. The transmitter-receiver loop is circular with a radius (R) of 56 m (which has an area equal to a 100 x 100 m square loop). The model, shown at the bottom of the figure, has the Cole-Cole parameters $m = 0.03$, $T = 4$ ms, $c = 1$.

at 10 degrees has the response shown on Fig. 5.9. If the model of Fig. 5.9 is made weakly polarizable (5.6% variation between 10 and 10000 Hz), then the region in which negatives occur (Fig. 5.10) is similar to that in the field data.

The data of Fig. 5.7 are more likely due to a lateral change in the conductivity
structure than to a dipping structure; however, the example does raise the possibility of negatives being associated with dipping sheet-like conductors.

Model: Non-polarizable dipping half-plane

Figure 5.9 The response for the same model as Fig. 5.8, except the half-plane is not polarizable and it dips to the west 10°.

5.3 The dipping half-plane response

The response of a 4.2S non-polarizable half-plane dipping at 55 degrees is plotted
Model: Polarizable dipping half-plane

Figure 5.10 The response for the same model as Fig. 5.9, except the half-plane is polarizable ($m = 0.06$, $T = 4$ms, $c = 1$).

on Fig. 5.11. At early time, the position of decreased coupling is near the edge at 0m, but by 5ms it has migrated to a position between -50m and -100m. Here, the early-time response is large, so the polarization current, which couples well to the receiver, will be large and hence negatives will be observed if the half-plane is made polarizable. Figure 5.12 shows this is the case when the conductance of the half-plane varies by only 2.1% between 10 and 10000Hz.
Model: Non-polarizable dipping half-plane

Figure 5.11 The response of a thin dipping dyke whose top edge is 30m below -25m and dips 55° to the north. The conductance of the half-plane is 4.2S, and a circular-loop radius 56m is used as the transmitter and receiver. The response at 1.2, 2.6 and 5.0ms is plotted with solid lines, and the other channels are plotted with dotted lines.
Figure 5.12  The response when the model of Fig. 5.11 is made polarizable ($m = 0.023$, $T = 5$ ms, $c = 1$).
5.4 Field example

A response profile, described by McCracken et al and shown on Fig. 5.13, was collected by CSIRO near Gani Village, India, over a 10m thick carbonaceous shale conductor which virtually outcrops in the vicinity of -25m to -50m. Although Fig. 5.12 is qualitatively similar to Fig. 5.13, the model amplitudes are too small provide a quantitative explanation of the field example. Larger amplitudes cannot be generated by increasing the conductance of the half-plane, as the resulting response will decay too slowly; however, decreasing the depth of burial would increase the amplitudes as required. A shallower half-plane would be consistent with the known geology of the area, but unfortunately the computer program is not able to calculate the response of such a structure. Even if it were, the field and model data would not agree exactly because the early-time response of Fig. 5.13 does not show the sharp local minimum in the response characteristic of infinitesimally thin sheet-like structures.

Although the half-plane is not able to model the field data exactly, the negatives are still observed close to the edge where the favourable coupling conditions appear to be satisfied. The modelling suggests that the polarizabilities required to explain the field data would not be significantly large.

5.5 Summary

A horizontal or dipping polarizable half-plane satisfies the favourable coupling conditions and negative transients are observed when the polarizabilities are moderate. The largest negatives occur close to the edge at early time; subsequent to this, the zone in which they are observed broadens and appears to migrate away from the edge, a direct consequence of the region of decreased coupling moving.

It is likely that a region of decreased coupling will also be observed near the edge of a quarter-space (McCracken et al (1981)). Whether negative transients
Field data: Gani village, India

Figure 5.13 Coincident-loop SIROTEM collected near Gani Village, India (After McCracken et al, 1981). The transmitter/receiver loops were 100×100m square (which has an area equal to a circular loop of 56m radius).
will be observed with moderate polarizabilities in this situation requires further investigation.
Chapter 6

Conclusions

The negative transients observed in coincident-loop TEM are a puzzling phenomenon. Although weak and relatively rare, the significant number of persistent, coherent examples requires that the phenomenon be explained. Empirical evidence indicates the negatives are not a consequence of instrument or operator error, but associated with the geologic structure. In theory, they will not occur if the properties of the ground are linear and independent of frequency, so structures with frequency dependent (dispersive) conductivities, familiar to workers in IP, would appear the obvious cause. However, the negatives cannot be quantitatively modelled with a dispersive half-space, a dispersive buried layer or a dispersive confined conductors in a conductive host medium unless the polarizabilities are implausibly large. The objective of this research has been to resolve this paradox.

In the field examples I considered, the negative transients can be explained with reasonable polarizabilities when the conductivity structure was chosen to model both the fundamental and polarization parts of the response. Each structure was noted to satisfy the following qualitative ‘favourable coupling conditions’ in positions where negatives occur:

(1) Strong coupling of the transmitter to the structure results in a large early-time fundamental response, inferred to be associated with a strong fundamental inductive current,

(2) The fundamental response decays away quickly, a consequence of the fundamental inductive current coupling poorly to the receiver at late time, and

(3) The polarization current is proximal to, and couples strongly with, the receiver. A large early-time fundamental inductive current (condition 1) will ensure that a significant (albeit small) polarization is induced. The small, slowly decaying
response associated with the polarization can only reverse the sign of the total response if the fundamental response becomes small at late time (i.e. condition 2 is satisfied). If the polarizability is moderate, the polarization response will be too small to overcome the fundamental response and generate a measurable negative transient unless condition 3 is satisfied.

Easing any of these requirements will reduce the size of the polarization response and hence the negatives, e.g.

- **Condition (1):** An example of the transmitter coupling being reduced is seen in the case history from the Willyama region of N.S.W. Here, a polarizable zone is excited by two different transmitter loops, one is directly over the zone, and the other is displaced laterally by 600m. In the former position, the transmitter couples well to the polarizable zone, so the polarization response can be measured (Fig. 3.16), but in the latter position the reduced coupling results in a polarization response too small to be identified (Fig 3.17).

- **Condition (1 and 3):** A shallow thin sheet satisfies the favourable coupling conditions, but making the sheet deeper (as in Fig. 3.6) will decrease the coupling of the transmitter/receiver to the body, resulting in a smaller early-time fundamental inductive current. Not only will the polarization current generated be smaller, it will couple less strongly to the receiver, so the negative transients are reduced.

- **Condition (2):** Increasing the late-time coupling of the fundamental inductive current to the receiver increases the late-time fundamental response. Examples of situations where this occurs are (i) when the conductivity-thickness of an overburden increases (Fig. 3.4), and (ii) when a transmitter, over the overburden but close to the edge, is moved and away from the edge (Fig. 5.1). The greater late-time fundamental response means that the polarization response will be too weak to reverse the sign of the total response, except perhaps at later times when the fundamental response is even smaller. If negatives are seen at later time, they will be smaller because the polarization response has
also decreased (Figs 5.2 and 3.5). When the decay of the fundamental response is very slow (as it is for a half-space), the polarization response will never be large enough to generate significant negative transients unless the polarizability is unrealistically large.

- **Condition (3):** Decreasing the coupling of the polarization current to the receiver will decrease the size of the polarization response. In the case of an overburden, the largest polarization current is induced close to the transmitter. Moving a small dipole receiver away from the transmitter decreases the coupling to this current, so the polarization response is reduced (see Fig. 3.10 and 3.12).

- **Condition (3):** The two interacting circuit model of Chapter 4 also illustrates that the coupling of the polarization current to the receiver must be large. A polarization current in the more proximal circuit 2 can generate observable negative transients with realistic polarizabilities, but because circuit 1 is more distant, negatives only occur if the polarizability is large.

Although additional explanations of negative transients may be found, the favourable coupling conditions do provide a plausible qualitative explanation which appears generally applicable. In the examples modelled, the polarizabilities are sufficiently small that it is likely the causative body has the required dispersion.

In the vicinity of a negative transient, the conductivity structure will be dispersive. In some of the field examples, IP surveys have been carried out across the anomalous features to try and measure these dispersions. No significant anomaly was detected in association with the negatives, which is not surprising for the following reasons:

1. The magnitude of the dispersions required to explain the negatives are small (the favourable coupling conditions being satisfied).

2. The IP method measures average or bulk properties, so the dispersion is diluted in magnitude by the surrounding non-dispersive material (Seigel, 1959; Guptasarma, 1984b). Dispersions which are already small will yield an insignificant
IP response when further diluted.

(3) The IP and TEM methods are sensitive to different frequency/time ranges; the dispersions in the TEM frequency range (which cause the negatives) cannot therefore be detected using the IP method.

The only way of confirming that the modelled polarizabilities are correct is to measure the polarizability of the causative body over the TEM frequency range. Such a measurement is beyond the scope of this thesis.

Given that the coincident-loop negatives are observed quite often, it is surprising that polarization effects have not been observed in other EM systems. One reason is that although the polarization effects measured by the coincident-loop TEM method are small, they do manifest themselves as negative transients which are easy to distinguish from the positive non-polarizable response. The polarization response measured by other EM systems will also be small, but much more difficult to identify. Also, transient EM systems, such as SIROTEM, are more liable to observe a strong polarization response because they are designed to measure the response at late times when relatively strong polarization effects are expected. Finally, the coincident-loop method uses a transmitter/receiver configuration which is conducive to the favourable coupling conditions being satisfied. For example, when an overburden layer is polarizable, the polarization current is generated close to the transmitter, the magnetic field of which couples strongly to a receiver positioned near to, or coincident with, the transmitter. Other systems rarely measure the response close to the transmitter, so the third favourable coupling condition will not be satisfied.

Because the favourable coupling conditions are satisfied so strongly for a polarizable overburden layer, most negatives will be associated with this type of conductive structure (as are the majority of the examples presented in this thesis). Thus, if a negative is observed in the field, it is important to determine whether a polarizable overburden is the causative body. One method of doing this is to measure the vertical- and/or horizontal-field response along a profile across the transmitter
loop. A near-surface polarization is implied if the vertical-field response is depressed inside the loop and shoulders are observed in the horizontal-field response at the transmitter wire. When the polarization response is not large enough to give rise to negative transients, the distortion of the vertical-field response in the vicinity of the transmitter wire is known as 'the loop effect' (Asten and Price, 1985). Asten and Price found it is possible to correct for this distortion by utilizing measurements taken inside and outside the transmitter loop. The additional spatial information is useful because the polarization and fundamental responses have different spatial forms. For this reason, it is recommended that as much spatial information as possible be utilized when the polarization response must be identified or quantified.

The fact that weak polarization effects can be seen in TEM measurements, raises the possibility of using TEM methods to measure the polarizability of a body. This would be particularly useful when the polarization effects decay too rapidly to be observed by the IP method. The TEM polarization effects are generally very small and will be difficult to identify and interpret unless methods exist for calculating the time-domain response of polarizable structures. For models where the response is calculated in the frequency domain and then converted to the time domain, this is not difficult; however, when the response is calculated directly in the time-domain, it is normally for a non-polarizable body. The time-domain response of a polarizable body can be calculated from this non-polarizable response using the approximate convolution algorithm discussed in Appendix 4.

The first three appendices to this thesis describe other techniques for interpreting data subject to polarization effects. The algorithms, either developed or enhanced in the course of this research, can be used to model two polarizable confined conductors (Appendix 1), a polarizable layered earth (Appendix 2), and a polarizable infinite thin sheet overburden (Appendix 3). Other appendices contain a description of an enhancement to the Gaver-Stehfest algorithm, used extensively in this research (Appendix 5), and a justification of the assumption that the Cole-Cole dispersion parameter \( c \) can be set to \( c = 1 \) (Appendix 6). Readers wanting more
details are invited to read further.
Appendix 1

The inductively-coupled two-circuit response

A1.0 Introduction

A single wire-loop circuit is commonly used to approximate a non-polarizable conductor (Grant and West, 1965; Barnett, 1984; Boyd and Wiles, 1984; McCracken et al, 1986a,b), and less commonly a polarizable conductor (Weidelt, 1982; Wait, 1983). In this appendix, the inductively-coupled two-circuit model (Ranasinghe, 1962; Grant and West, 1965; Vallée, 1981) is extended to approximate two polarizable conductors. Each circuit has a frequency-independent self inductance and a resistance with a frequency dependence described by the Cole-Cole impedance model. The Cole-Cole dispersion factor is set to $c = 1$, a requirement which is not overly restrictive (see Appendix 6). Conduction in the host medium and the overburden are neglected.

The concept of mutual inductance is used to account for the inductive coupling between the conductors. Circuits $i$ and $j$ have a mutual inductance $M_{ij}$ defined by

$$\Phi_{ij} = M_{ij}i_j,$$

where $\Phi_{ij}$ is the magnetic flux linking circuit $i$ associated with the current $i_j$ flowing in circuit $j$. In the case of two circular wire-loop circuits whose normals intersect, the mutual inductance is calculated using equation (1) and (5) from section 8.06 of Smythe (1950). The inductive interaction occurs when the currents flowing in the circuits vary as a function of time, this generates a time varying flux through each circuit, which induces a voltage

$$v(t) = -\frac{\partial \Phi}{\partial t},$$
and a corresponding current.

Initially, the response of two inductively coupled polarizable circuits is derived in the frequency domain. This solution is then used to calculate the time-domain response of: (i) a non-polarizable circuit, (ii) a polarizable circuit, (iii) two inductively coupled non-polarizable circuits, and (iv) two inductively coupled polarizable circuits.

A1.1 The frequency domain response of two circuits

Kirchhoff’s voltage law for each circuit yields

\[ i_1 * z_1 + L_1 \frac{di_1}{dt} + M_{12} \frac{di_2}{dt} = -M_{01} \frac{di_0}{dt}, \]  
\[ (A1.1) \]

\[ i_2 * z_2 + L_2 \frac{di_2}{dt} + M_{12} \frac{di_1}{dt} = -M_{02} \frac{di_0}{dt}, \]  
\[ (A1.2) \]

where \( z_1 \) and \( z_2 \) are the impedances of circuits 1 and 2, \( * \) denotes the convolution operator, the subscript 0 is used to denote quantities associated with the transmitter, and the geometric arrangement of the circuits is shown on Fig. 4.3. The forcing functions on the right hand side of equations (A1.1) and (A1.2) are the voltages induced in circuit \( i \) by the time rate of change of \( \Phi_{i0} \).

In time-domain systems, the current in the transmitter is normally switched off suddenly. The time derivative of the current \( i_0 \) is thus a negative impulse function and the Laplace transform of \( \partial i_0 / \partial t \) is \( -I_0 \), where \( I_0 \) is the magnitude of the current prior to switch-off. Taking the Laplace transform of equations (A1.1) and (A1.2) gives

\[ I_1(s) \left[ \frac{R_1}{1 + sT_1} [1 + sT_1(1 - m_1)] + sL_1 \right] + sM_{12}I_2(s) = M_{01}I_0 \]  
\[ (A1.3) \]

\[ I_2(s) \left[ \frac{R_2}{1 + sT_2} [1 + sT_2(1 - m_2)] + sL_2 \right] + sM_{12}I_1(s) = M_{02}I_0, \]  
\[ (A1.4) \]

where the Cole-Cole impedances have been substituted for the frequency-domain expressions for \( z_1 \) and \( z_2 \), and \( I_1(s) \) and \( I_2(s) \) are the Laplace transforms of \( i_1(t) \)
Appendix 1  The inductively-coupled two-circuit response

and \(i_2(t)\) respectively. These equations can be solved for the \(I_i(s)\) to give:

\[
I_1(s) = k_{01} \tau_1 \mu_1 I_0 \frac{\alpha_1 s^3 + \beta_1 s^2 + \gamma_1 s + 1}{a_4 s^4 + a_3 s^3 + a_2 s^2 + a_1 s + 1}, \tag{A1.5}
\]

\[
I_2(s) = k_{02} \tau_2 \mu_2 I_0 \frac{\alpha_2 s^3 + \beta_2 s^2 + \gamma_2 s + 1}{a_4 s^4 + a_3 s^3 + a_2 s^2 + a_1 s + 1}, \tag{A1.6}
\]

where

\[
\tau_i = L_i / R_i,
\]

\[
\mu_i = \sqrt{\frac{L_0}{L_i}},
\]

\[
k_{ij} = \frac{M_{ij}}{\sqrt{L_i L_j}},
\]

\[
a_4 = \tau_1 \tau_2 T_1 T_2 (1 - k_{12}^2),
\]

\[
a_3 = [\tau_1 \tau_2 (T_1 + T_2) (1 - k_{12}^2) + T_1 T_2 (\tau_1 (1 - m_1) + \tau_2 (1 - m_2))],
\]

\[
a_2 = [\tau_1 \tau_2 (1 - k_{12}^2) + \tau_1 \{T_2 (1 - m_2) + T_1 \}
\]

\[
+ \tau_2 \{T_1 (1 - m_1) + T_2 \} + T_1 T_2 (1 - m_1) (1 - m_2)],
\]

\[
a_1 = [\tau_1 + \tau_2 + T_1 (1 - m_1) + T_2 (1 - m_2)],
\]

and

\[
\alpha_1 = T_1 T_2 \tau_2 [1 - \frac{k_{12} k_{02}}{k_{01}}],
\]

\[
\beta_1 = \tau_2 (T_1 + T_2) [1 - \frac{k_{12} k_{02}}{k_{01}}] + T_1 T_2 (1 - m_2),
\]

\[
\gamma_1 = \tau_2 [1 - \frac{k_{12} k_{02}}{k_{01}}] + T_1 + T_2 (1 - m_2),
\]

\[
\alpha_2 = T_1 T_2 \tau_1 [1 - \frac{k_{12} k_{01}}{k_{02}}],
\]

\[
\beta_2 = \tau_1 (T_1 + T_2) [1 - \frac{k_{12} k_{01}}{k_{02}}] + T_1 T_2 (1 - m_1),
\]

\[
\gamma_2 = \tau_1 [1 - \frac{k_{12} k_{01}}{k_{02}}] + T_2 + T_1 (1 - m_1).
\]

The parameter \(k_{ij}\) is the coupling coefficient of circuit \(i\) to circuit \(j\) and \(\tau_i\) is the intrinsic time constant of the current induced in an isolated non-polarizable circuit of resistance \(R_i\) and self inductance \(L_i\).
Equations (A1.5) and (A1.6) represent the frequency-domain solution for the current induced in each circuit. Each equation is a ratio of polynomials in the Laplace transform variable \( s \). If the roots of the denominator are denoted by \( s_i \), then these equations can be written as a sum of terms with simples poles \( s_i \). When converted to the time domain, each term will be an exponential decay with time constant \(-1/s_i\). For the \( s_i \) with an imaginary part equal to zero, the associated decays will be simple exponentials, whereas those with a non-zero imaginary part have decays which are exponentially damped oscillations. In the following sections, the solution is converted to the time-domain for the four different cases considered. These are treated separately because the nature of the roots, and the methods by which the roots are obtained, are different in each case.

A1.2 Case 1: A single non-polarizable circuit

When circuit 1 is non-polarizable and circuit 2 is ignored (Fig. 2.1), \( T_1 = T_2 = \tau_2 = k_{12} = k_{02} = 0 \), so equation (A1.5) reduces to

\[
I_1(s) = k_{01} \mu_1 \frac{I_0}{s + 1/\tau_1},
\]

which has the inverse Laplace transform

\[
i_1(t) = I_0 k_{01} \mu_1 e^{-t/\tau_1}.
\]  
(A1.7)

For coincident transmitter and receiver loops, the voltage induced in the receiver is

\[
v(t) = -M_{01} \frac{\partial i_1}{\partial t} = I_0 R_1 k_{01}^2 \mu_1^2 e^{-t/\tau_1},
\]  
(A1.8)

in agreement with previous solutions (Grant and West, 1965).

The root of the denominator is real \((-1/\tau_1\)\), so the associated voltage decay is a positive, simple exponential.
A1.3 Case 2: A polarizable single circuit

If circuit 2 is ignored and circuit 1 is ascribed a polarizability (Fig. 2.2), then \( T_2 = \tau_2 = k_{12} = k_{02} = 0 \), and \( I_1(s) \) reduces to

\[
I_1(s) = \frac{I_0 k_{01} \mu_1 (1 + s T_1)}{T_1 (s - s_1)(s - s_2)},
\]

(A1.9)

where \( s_1 \) and \( s_2 \) (defined below) are the roots of the denominator. The time constants of the characteristic modes of the current decay are defined by \( t_i = -1/s_i \), where

\[
s_1, s_2 = \frac{1}{2T_1} \left[ -[1 + \lambda(1-m)] \pm \sqrt{q} \right],
\]

\[
t_1, t_2 = \frac{T_1}{2} \left[ [1 + \lambda(1-m)] \pm \sqrt{q} \right],
\]

\[
\lambda = T_1/\tau_1, \quad \lambda \ge 0
\]

\[
q = [1 - \lambda(1-m)]^2 - 4\lambda m, \quad \text{and}
\]

\[
m = m_1.
\]

The coincident-loop voltage response can be obtained by inverse Laplace transforming equation (A1.9) using equations (29.3.12) and (29.3.13) of Abramowitz and Stegun (1965):

\[
v(t) = \frac{I_0 R_1 k_{01} \mu_1^2}{\sqrt{q}} \left( e^{s_1 t} (s_1 + s_1^2 \lambda) - e^{s_2 t} (s_2 + s_2^2 \lambda) \right),
\]

(A1.11)

where \( s'_i = s_i \tau_1 \), and a dimensionless time \( t' = t/\tau_1 \) is now used.

Depending on the sign of \( q \), the \( s_i \) can either be real or complex, so the associated voltage decay terms will either be simple or oscillatory exponentials.

A1.4 Case 3: Two interacting non-polarizable circuits

For the non-polarizable two-circuit case (Fig. 4.1), \( T_1 = T_2 = 0 \) and equations (A1.5) and (A1.6) reduce to (Vallée, 1981):

\[
I_1(s) = \frac{I_0 k_{01} \tau_1 \mu_1}{\tau_1 \tau_2 (1 - k_{12}^2)} \left[ \frac{1}{(s - s_1)(s - s_2)} \right],
\]
\[ I_2(s) = \frac{I_0 k_{02} \tau_2 \mu_2}{\tau_1 \tau_2 (1 - k_{12}^2)} \left[ \frac{1 + s \gamma_2}{(s - s_\xi)(s - s_\zeta)} \right]. \]

The \( s_\xi \) and \( s_\zeta \) are the roots of the denominator, defined by \( s_i = -1/t_i \), where the time constants \( t_\xi \) and \( t_\zeta \) are

\[ t_\xi, t_\zeta = \frac{1}{2} \left[ (\tau_1 + \tau_2) \pm \sqrt{(\tau_1 - \tau_2)^2 + 4 \tau_1 \tau_2 k_{12}^2} \right]. \]

Because the intrinsic time constants \( \tau_1, \tau_2 \) are always positive and \( k_{12}^2 < 1 \) (Lorrain and Corson (1970), page 350), the time constants \( t_\xi \) and \( t_\zeta \) are real and positive.

Taking the inverse Laplace transform using equations (29.3.12) and (29.3.13) of Abramowitz and Stegun (1965) gives

\[ i_1(t) = \frac{I_0 k_{01} \tau_1 \mu_1}{\sqrt{(\tau_1 - \tau_2)^2 + 2 \tau_1 \tau_2 k_{12}^2}} \left[ e^{\xi t} - e^{\zeta t} + \gamma_1 (s_\xi e^{\xi t} - s_\zeta e^{\zeta t}) \right], \quad (A1.12) \]

\[ i_2(t) = \frac{I_0 k_{02} \tau_2 \mu_2}{\sqrt{(\tau_1 - \tau_2)^2 + 2 \tau_1 \tau_2 k_{12}^2}} \left[ e^{\xi t} - e^{\zeta t} + \gamma_2 (s_\xi e^{\xi t} - s_\zeta e^{\zeta t}) \right]. \quad (A1.13) \]

The voltage response is

\[ v(t) = -M_{13} \frac{\partial i_1}{\partial t} - M_{23} \frac{\partial i_2}{\partial t}. \]

Substituting for the time derivatives of \( i_1 \) and \( i_2 \) and collecting like exponentials yields

\[ v(t) = \frac{I_0}{\sqrt{(\tau_1 - \tau_2)^2 + 4 \tau_1 \tau_2 k_{12}^2}} \left[ e^{\xi t} \left[ \alpha s_\xi + \beta s_\xi^2 \right] - e^{\zeta t} \left[ \alpha s_\zeta + \beta s_\zeta^2 \right] \right], \quad (A1.14) \]

where for coincident loops (i.e. loop 3 is identical to loop 0), \( \alpha \) and \( \beta \) are defined by:

\[
\alpha = k_{01}^2 \tau_1 L_0 + k_{02}^2 \tau_2 L_0, \\
\beta = \tau_1 \tau_2 L_0 \left[ (1 - k_{12} k_{02} / k_{01}) k_{01}^2 + (1 - k_{12} k_{01} / k_{02}) k_{02}^2 \right].
\]

The \( t_\xi \) and \( t_\zeta \) are real and positive, so each voltage decay term in equation (A1.14) will be a simple exponential.
A1.5 Case 4: Two interacting polarizable circuits

The response in the case of two inductively coupled polarizable circuits (Fig. 4.3) can be calculated by inverse Laplace transforming equations (A1.5) and (A1.6). Define a function $Y(s)$ by:

$$Y(s) = \frac{1}{s^4 + \hat{a}_3 s^3 + \hat{a}_2 s^2 + \hat{a}_1 s + 1/a_4}$$

or

$$Y(s) = \frac{1}{G(s)},$$

where $G(s) = s^4 + \hat{a}_3 s^3 + \hat{a}_2 s^2 + \hat{a}_1 s + 1/a_4$, and $\hat{a}_i = a_i/a_4$ ($a_4$ is assumed non-zero). From the fundamental theorem of algebra

$$G(s) = (s - s_I)(s - s_J)(s - s_K)(s - s_L), \quad (A1.15)$$

where the $s_i$ are the roots of $G(s)$, found using Abramowitz and Stegun (1965) equation (3.8.3). Provided no two roots are the same, $Y(s)$ can be written in the form

$$Y(s) = \frac{A_I}{(s - s_I)} + \frac{A_J}{(s - s_J)} + \frac{A_K}{(s - s_K)} + \frac{A_L}{(s - s_L)} \quad (A1.16)$$

(Kreyszig (1979), page 229), where

$$A_i = \left. \frac{1}{\partial G(s_i)/\partial s} \right|_{s_i} = \frac{1}{4s_i^3 + 3\hat{a}_3 s_i^2 + 2\hat{a}_2 s_i + \hat{a}_1}. \quad (A1.17)$$

(If $a_4$ is zero, which occurs when only one of the circuits is polarizable, $G(s)$ reduces to $G(s) = s^3 + \hat{a}_2 s^2 + \hat{a}_1 s + 1/a_3$, where $\hat{a}_i = a_i/a_3$. In such a situation, there are three roots, so equation (A1.16) will contain three terms. This can be accounted for by setting $A_L = 0$.)

Using the definition of $Y(s)$, equations (A1.5) and (A1.6) become

$$I_i(s) = H_i[\alpha_i s^3 Y(s) + \beta_i s^2 Y(s) + \gamma_i s Y(s) + Y(s)],$$
where
\[ H_i = \frac{I_0 k_i \tau_i \mu_i}{a_4}, \]
and \( i = 1, 2 \) respectively. (The denominator of \( H_i \) is \( a_3 \) when \( a_4 = 0 \).) The inverse Laplace transform of \( Y(s) \) is
\[ Y(t) = A_I e^{s_1 t} + A_J e^{s_2 t} + A_K e^{s_K t} + A_L e^{s_L t}, \]
so the inverse Laplace transform of the current is
\[
\begin{align*}
i_i(t) &= H_i [\alpha_i \frac{\partial^3}{\partial t^3} Y(t) + \beta_i \frac{\partial^2}{\partial t^2} Y(t) + \gamma_i \frac{\partial}{\partial t} Y(t) + Y(t)] \\
&= H_i [A_I e^{s_1 t} \phi_1 + A_J e^{s_2 t} \phi_J + A_K e^{s_K t} \phi_K + A_L e^{s_L t} \phi_L], \quad (A1.18)
\end{align*}
\]
where
\[ \phi_{ij} = \alpha_i s_j^3 + \beta_i s_j^2 + \gamma_i s_j + 1. \]

The voltage induced in the receiver is
\[ v(t) = -M_{13} \frac{\partial i_1}{\partial t} - M_{23} \frac{\partial i_2}{\partial t}, \quad (A1.19) \]
where the formula for \( \partial i_i / \partial t \) is the same as that for \( i_i \), except the \( \phi_{ij} \) become
\[ \phi_{ij} = \alpha_i s_j^4 + \beta_i s_j^3 + \gamma_i s_j^2 + s_j. \]

The roots of the denominator can either be real or complex, depending on the relative values of the coupling coefficient, the intrinsic time constants and the Cole-Cole parameters, so the voltage decay terms can either be simple or oscillatory exponentials.

### A1.6 Conclusion

Formulae for calculating the time-domain response of two inductively coupled wire-loop circuits have been derived in three limiting cases and one more general case. The relevant physics is discussed in the body of the thesis where each model is introduced.
Appendix 2
The polarizable layered earth response

A2.0 Introduction


A2.1 The time-domain large-loop voltage response

Consider the large circular horizontal transmitter loop shown on Fig. A2.1. The loop has a radius $b$ and is a height $h$ above the surface of the layered earth. In this analysis, the co-ordinate system is a right-hand circular cylindrical system with $z$ positive down, the origin being on the surface of the earth directly below the centre of the loop. The layered earth model is depicted on Fig. A2.2.

For the prescribed source, there is no $z$ or $r$ components of the electric field, so the total field can be written

$$\mathbf{E} = E_\phi(r, z) \hat{\phi},$$

where $\hat{\phi}$ denotes the unit vector in the $\phi$ direction. When displacement currents are ignored and an implicit $e^{i\omega t}$ time dependence is assumed, the electric field satisfies

$$(\nabla \times \nabla \times \mathbf{E})_\phi = -i\omega \mu \sigma E_\phi,$$
where \( \mu \) is the permeability, assumed to be equal to \( \mu_0 \), the free-space value. This partial differential equation, written

\[
\frac{\partial^2 E_\phi}{\partial r^2} + \frac{1}{r} \frac{\partial E_\phi}{\partial r} - \frac{E_\phi}{r^2} + \frac{\partial^2 E_\phi}{\partial z^2} = i\omega \mu_0 \sigma E_\phi
\]

(A2.1)

in the circular cylindrical co-ordinate system, can be solved in the first-order Hankel-transform domain. The relevant transform pair is defined by

\[
\widehat{f}(\lambda, z) = \int_0^\infty r f(r, z) J_1(\lambda r) \, dr
\]

(A2.2)

\[
f(r, z) = \int_0^\infty \lambda \, \widehat{f}(\lambda, z) J_1(\lambda r) \, d\lambda,
\]

(A2.3)

where \( J_1(u) \) is the first-order Bessel function of the first kind with argument \( u \).
Substituting equation (A2.1) into (A2.2) gives
\[
\frac{d^2 \hat{E}_\phi(\lambda, z)}{dz^2} - \lambda^2 \hat{E}_\phi(\lambda, z) = i\omega \mu_0 \sigma \hat{E}_\phi(\lambda, z),
\]
a second-order ordinary differential equation in \( z \). In air, where \( \sigma = 0 \), the solution to this equation can be written
\[
\hat{E}_\phi(\lambda, z) = \hat{A}_{air}(\lambda)e^{-\lambda z} + \hat{B}_{air}(\lambda)e^{\lambda z}. \tag{A2.4}
\]
In the \( i \)th layer of the earth, where the conductivity \( \sigma_i \) is constant, the solution is
\[
\hat{E}_\phi(\lambda, z) = \hat{A}_i(\lambda) \cosh(\theta_i z) + \hat{B}_i(\lambda) \sinh(\theta_i z), \tag{A2.5}
\]
\( \theta_i \) being the propagation constant defined by \( \theta_i = \sqrt{\lambda^2 + i\omega \mu_0 \sigma_i} \).

The radial component of Faraday's law of induction is
\[
i\omega \hat{B}_r = \frac{d \hat{E}_\phi}{dz},
\]
so the magnetic field is
\[ \hat{B}_r(\lambda, z) = \hat{C}_{air}(\lambda)e^{-\lambda z} + \hat{D}_{air}(\lambda)e^{\lambda z} \]  
\[ (A2.6) \]
in the air, and
\[ \hat{B}_r(\lambda, z) = \hat{C}_i(\lambda) \cosh(\theta_iz) + \hat{D}_i(\lambda) \sinh(\theta_iz) \]  
\[ (A2.7) \]
in the ith layer. Note that \( \lambda, \theta_i \) and \( i\omega \) have been absorbed into the arbitrary constants \( \hat{C} \) and \( \hat{D} \) which are implicitly assumed to be frequency dependent.

These equations comprise the general solution to the differential equation, the particular solution is obtained by incorporating the effect of the source field from the transmitter. In the \( z = -h \) plane, the magnetic field of the transmitter is vertical, except at the transmitter wire where it is purely in the radial direction. The field at this position can be obtained from Ampere’s circuital law which states: the line integral of the tangential magnetic field intensity around a closed path is equal to the enclosed current. In this case, the path of integration is chosen to be around the outside of the transmitter wire in the counter-clockwise direction. The cross-section of the transmitter wire is rectangular with a vertical thickness \( a \) and a horizontal width \( \delta r \). The source current density flowing in the transmitter is defined by
\[ j_s(\omega) = \frac{I(\omega)}{\delta r}, \]  
\[ (A2.8) \]
where \( I(\omega) \) is the current flowing in the \(+\hat{r}\) direction. When the thickness \( a \) is infinitesimally small, the magnetic flux density on the upper side of the transmitter \( (z = -h^-) \) is
\[ B_r(r, -h^-) = \begin{cases} 0, & 0 < r < b; \\ -\mu_0 j_s(\omega)/2, & b < r < b + \delta r; \\ 0, & b + \delta r < r < \infty, \end{cases} \]
whilst on the lower side of the transmitter \( (z = -h^+) \) it is
\[ B_r(r, -h^+) = \begin{cases} 0, & 0 < r < b; \\ \mu_0 j_s(\omega)/2, & b < r < b + \delta r; \\ 0, & b + \delta r < r < \infty. \end{cases} \]
The Hankel transform of the flux density on the lower side of the loop is thus

$$
\hat{B}_r(\lambda, -h^+) = \frac{b\mu_0 I(\omega)}{2} J_1(\lambda b),
$$

(42.9)
in the limit $\delta r \to 0$. The field from the transmitter can be continued downwards by determining the values of $\hat{C}_{air}$ and $\hat{D}_{air}$ and utilizing equation (A2.6). Matching equation (A2.9) with (A2.6) at $z = -h^+$ and ensuring that there are no unbound solutions for $z \geq -h^+$ yields a continued magnetic flux density of the form

$$
\hat{B}_r(\lambda, z) = \frac{b\mu_0 I(\omega)}{2} J_1(\lambda b) e^{-\lambda(z+h)}.
$$

The complete solution for $\hat{B}_r(\lambda, z)$ between the layered earth and the transmitter is this particular solution plus a secondary field

$$
\hat{B}_r(\lambda, z) = \frac{b\mu_0 I(\omega)}{2} J_1(\lambda b) e^{-\lambda h} \left[ e^{-\lambda z} + \hat{R}(\lambda) e^{\lambda z} \right],
$$

(A2.10)

where the reflection coefficient $\hat{R}(\lambda)$ is a function of $\lambda$ yet to be determined. From Faraday's law the associated electric field $\hat{E}_\phi$ is of the form

$$
\hat{E}_\phi(\lambda, z) = -\frac{i\omega b\mu_0 I(\omega)}{2\lambda} J_1(\lambda b) e^{-\lambda h} \left[ e^{-\lambda z} - \hat{R}(\lambda) e^{\lambda z} \right].
$$

(A2.11)

The only task remaining is to obtain the reflection coefficient. The ratio $\hat{Q}$, defined by

$$
\hat{Q} = \frac{-\hat{E}_\phi}{i\omega \hat{B}_r},
$$

(A2.12)

allows this to be done. Because $\hat{E}_\phi$ and $\hat{B}_r$ are continuous throughout the depth section and across layer boundaries $\hat{Q}$ is also continuous. The reflection coefficient will be calculated from the value of $\hat{Q}$ at the surface ($\hat{Q}_s$) which is obtained by calculating the value of $\hat{Q}$ at the top of the basal half-space ($\hat{Q}_1$) and continuing it up through the stack of layers, ensuring continuity at each layer boundary.

In the basal half-space (layer 1), the solution for the electric field $\hat{E}_\phi$ must be a decaying exponential

$$
\hat{E}_\phi \propto e^{-\theta_1 z}.
$$
The corresponding \( \hat{B}_r \) is calculated from Faraday's law
\[
\hat{B}_r \propto \frac{-\theta_1}{i\omega} e^{-\theta_1 z},
\]
so the ratio \( \hat{Q}_1 \) is
\[
\hat{Q}_1 = 1/\theta_1.
\]
Now consider the \( i \)th layer, where it is assumed the value of \( \hat{Q}_{i-1} \) on the bottom boundary is already known. The value on the top boundary \( \hat{Q}_i \) can be obtained by continuing \( \hat{Q} \) up through the layer. Without any loss of generality set \( z = 0 \) at the bottom of the layer, so \( z = -d_i \) at the top of the layer. From equation (A2.5), \( \hat{E}_\phi \) is proportional to
\[
\hat{E}_\phi \propto \cosh(\theta_i z) + \hat{G} \sinh(\theta_i z),
\]
where \( \hat{G} \) is the only arbitrary constant required. The ratio of \( \hat{E}_\phi \) and the corresponding value of \( \hat{B}_r \) gives
\[
\hat{Q} = \frac{1}{\theta_i} \left[ \frac{1 + \hat{G} \tanh(\theta_i z)}{\tanh(\theta_i z) + \hat{G}} \right].
\]
This is true for all \( z \) in the \( i \)th layer. Substituting \( z = 0 \) and solving for \( \hat{G} \) in terms of known quantities gives
\[
\hat{G} = \frac{1}{\theta_i \hat{Q}_{i-1}}.
\]
The ratio \( \hat{Q}_i \) can thus be obtained by substituting \( z = -d_i \)
\[
\hat{Q}_i = \frac{1}{\theta_i} \left[ \frac{\theta_i \hat{Q}_{i-1} + \tanh(\theta_i d_i)}{1 + \theta_i \hat{Q}_{i-1} \tanh(\theta_i d_i)} \right].
\]
When \( d_i \to \infty \) the \( i \)th layer becomes a half-space, and \( \hat{Q}_i = 1/\theta_i \) as expected.

The value at the upper interface of the layered earth can be obtained by using equation (A2.14) as a recursion relation, calculating \( \hat{Q}_2, \hat{Q}_3, \ldots \hat{Q}_s \) from the known value, \( \hat{Q}_1 \). Continuity of \( \hat{Q} \) requires that the values in the layered-earth/air interface are the same, i.e.,
\[
\hat{Q}_s = \hat{Q}_{air} \big|_{z=0}.
\]
The latter can be obtained by setting $z = 0$ in equations (A2.10) and (A2.11) and taking the ratio to yield

$$
\hat{Q}_{air} = \frac{1}{\lambda} \left( \frac{1 - \hat{R}}{1 + \hat{R}} \right).
$$

Using equation (A2.15) and solving for $\hat{R}$ gives

$$
\hat{R}(\lambda) = \left( \frac{1 - \lambda \hat{Q}_s}{1 + \lambda \hat{Q}_s} \right),
$$

which completes the solution.

The electric field on the surface $z = 0$ for a transmitter of radius $b$ at a height $h$ above the earth (equation (A2.11)) can now be written

$$
\hat{E}_\phi^s = -\frac{i \omega \mu_0 I(\omega)}{2 \lambda} J_1(\lambda b) e^{-\lambda h} \left[ \frac{2 \lambda \hat{Q}_s}{1 + \lambda \hat{Q}_s} \right].
$$

(A2.16)

The voltage ($v = \int E \cdot dl$) induced in a coincident circular receiver loop (radius $b$) is

$$
v(\omega) = -\frac{i \omega \mu_0 b I(\omega)}{2} \int_0^\infty \frac{2 \lambda e^{-\lambda h}}{\lambda + 1/\hat{Q}_s} J_1(\lambda b) J_1(\lambda b) d\lambda,
$$

provided $h$ is small. For a step-off in current ($I(\omega) = -1/i\omega$), the time-domain voltage response is

$$
v(t) = \mu_0 b^2 I \pi \int_0^\infty 2 \lambda e^{-\lambda h} \mathcal{L}^{-1} \left\{ \frac{1}{\lambda + 1/\hat{Q}_s(s)} \right\} J_1^2(\lambda b) d\lambda,
$$

where $\mathcal{L}^{-1}$ denotes the inverse Laplace transform of the quantity in braces, and $s$ is the inverse Laplace transform variable ($s = i\omega$). The polarizability of the layers is accounted for by making the $\sigma_i$ frequency dependent, and using these to calculate $\hat{Q}_s(s)$ prior to inverse Laplace transformation. In the computer program written, the Gaver-Stehfest algorithm, discussed in greater detail in Appendix 5, is used to calculate the inverse Laplace transform and Chave's (1983) routine is used to perform the Hankel transform.
A2.2 The electric field in the ground

The electric field sections depicting the 'smoke rings' in Chapter 3 are contour plots of the electric field as a function of radial distance and depth, calculated by continuing the field at the surface (equation A2.16) down through the stack of layers, a process requiring a recursion relation similar to equation (A2.14).

Consider the ith layer. Without loss of generality, set the depth at the top of the layer to \( z = 0 \), and the depth at the bottom to \( z = d_i \). The ratio of the electric field (equation (A2.5)) at the top of the layer (\( \hat{E}_\phi^i \)) to that at the bottom (\( \hat{E}_\phi^{i-1} \)) gives

\[
\hat{E}_\phi^{i-1} = \hat{E}_\phi^i (\cosh(\theta_i d_i) + \hat{G} \sinh(\theta_i d_i)),
\]

where \( \hat{G} \) is an arbitrary constant to be determined. The value of \( \hat{Q} \) at the base of the ith layer is

\[
\hat{Q}_{i-1} = \frac{-\hat{E}_\phi^{i-1}}{i \omega \hat{B}_r^{i-1}} = \frac{-1}{\theta_i} \left[ \frac{\cosh(\theta_i d_i) + \hat{G} \sinh(\theta_i d_i)}{\sinh(\theta_i d_i) + \hat{G} \cosh(\theta_i d_i)} \right].
\]

Because \( \hat{E}_\phi^i \) is known, all the \( \hat{Q}_i \) are known (in particular \( \hat{Q}_{i-1} \)) so the value of \( \hat{G} \) can be obtained by a simple rearrangement

\[
\hat{G} = -\left[ \frac{1 + \theta_i \hat{Q}_{i-1} \tanh(\theta_i d_i)}{\theta_i \hat{Q}_{i-1} + \tanh(\theta_i d_i)} \right].
\]

Substituting this expression into equation (A2.17) and using the identity \( \cosh^2 \alpha - \sinh^2 \alpha = 1 \) gives

\[
\hat{E}_\phi^{i-1} = \hat{E}_\phi^i \text{sech} (\theta_i d_i) \left[ \frac{\theta_i \hat{Q}_{i-1}}{\theta_i \hat{Q}_{i-1} + \tanh(\theta_i d_i)} \right],
\]

the required recurrence relation for \( \hat{E}_\phi \) at each interface.

At an arbitrary depth \( l \), between layer boundaries, the electric field is obtained by placing a fictitious boundary at \( z = l \). The complete procedure involves calculating the values of \( \hat{Q}_i \) (including the value of \( \hat{Q}_l \)) recursively upward through the
stack, evaluating \( \hat{E}^s_\phi \), and then downward continuing this using

\[
\hat{E}^l_\phi = \hat{E}^s_\phi \prod_{N \geq i > l} \text{sech}(\theta_i d_i) \left[ \frac{\theta_i \hat{Q}_{i-1}}{\theta_i \hat{Q}_{i-1} + \tanh(\theta_i d_i)} \right].
\]

If the electric field is required at a depth \( l \) below the basal interface, a fictitious boundary could be inserted, or the electric field at the basal interface \( \hat{E}^1_\phi \) could be downward continued using

\[
\hat{E}^l_\phi = \hat{E}^1_\phi e^{-\theta_1 (l-z_1)}.
\]

### A2.3 The continuity of \( E_\phi \) and its spatial derivatives

The contour routine used to obtain the electric field sections assumes that \( E_\phi \) and its spatial derivatives are continuous across layer boundaries. \( E_\phi \) is tangential to the boundary, so this will be continuous. The non-zero spatial derivatives of \( E_\phi \) can be obtained from Faraday’s law:

\[
\frac{\partial E_\phi}{\partial z} = -\mu_0 \frac{\partial H_r}{\partial t}, \quad (A2.18)
\]

\[
\frac{\partial E_\phi}{\partial r} = \frac{\partial B_z}{\partial t} - \frac{E_\phi}{r}. \quad (A2.19)
\]

As each of the terms on the right hand side of equations (A2.18) and (A2.19) are continuous across interfaces (for \( t > 0 \)), so are the spatial derivatives on the left hand side.

### A2.4 Conclusion

Using the equations outlined above, programs have been written to calculate (i) the coincident-loop response of polarizable layered earths, and (ii) electric field contour sections (‘smoke ring diagrams’). Results from these programs are presented and discussed in Chapter 3.
Appendix 3

The polarizable overburden response

A3.0 Introduction

The response of an overburden can be calculated by assuming the overburden is a conductive thin sheet, a common assumption in geophysics. Situations in which this approximation has been made are reviewed briefly in a recent paper by Smith and West (1987).

The time-domain response of a non-polarizable uniform thin sheet can be calculated relatively easily using the well known retreating image solution (Grant and West, 1965). The polarizable thin-sheet response is more difficult to calculate because the response must first be calculated in the frequency domain using a frequency-dependent resistance and then inverted to the time domain.

The nomenclature and derivations used in this appendix follow those of Smith and West (1987).

A3.1 Basic theory

The boundary condition satisfied by the magnetic field on the upper surface of a uniform thin sheet is

\[
R \frac{\partial H_z^s}{\partial z} = \frac{\mu_0}{2} \left. \frac{\partial (H_z^p + H_z^s)}{\partial t} \right|_{z=0^+}
\]  

(Price, 1949; Grant and West, 1965), where \( R \) is the resistance of the sheet, \( H_z^p \) and \( H_z^s \) are the \( z \) components of the primary and secondary fields respectively, \( \mu_0 \) is the permeability of free space, \( t \) is time, and the sheet lies in the \( z = 0 \) plane.
This equation can be solved in the Laplace and two-dimensional (2D) Fourier transform domains. The former transform pair is defined by

\[ \mathcal{F}(s) = \int_0^\infty F(t)e^{-st}dt, \]

and

\[ F(t) = \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} \mathcal{F}(s)e^{st}ds \]

where \( s \) is the frequency and \( c \) is a real number greater than the real part of any poles of \( \mathcal{F}(s) \), while the latter transform pair is

\[ \mathcal{F}(p, q) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x, y)e^{-ipx-iqy}dx\,dy, \]

and

\[ f(x, y) = \frac{1}{(2\pi)^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \mathcal{F}(p, q)e^{ipx+iqy}dp\,dq, \]

where \( p \) and \( q \) are the Fourier-wavenumber variables in the \( x \) and \( y \) directions respectively. The Laplace transform of the \( \partial/\partial t \) operator is \( s \), and the 2D Fourier transform of the \( \partial/\partial z \) operator is \( -\sqrt{p^2 + q^2} \) (because \( H \) satisfies Laplace's equation). If the primary field \( H_z^p \) is switched off suddenly at time \( t = 0 \), the Laplace and Fourier transforms of equation (A3.1) yields

\[ \mathcal{F}^{s}(p, q, s) = \frac{H_z^p(p, q)}{s + \frac{2R}{\mu_0} \sqrt{p^2 + q^2}}, \quad (A3.2) \]

where \( H_z^p(p, q) \) is the Fourier transform of the primary field prior to \( t = 0 \).

If the sheet is non-polarizable, \( R \) is independent of frequency and the inverse Laplace transform of (A3.2) is

\[ H_z^s(p, q, t) = H_z^p(p, q)e^{-\frac{2R}{\mu_0} \sqrt{p^2 + q^2}}. \]

However, for a polarizable sheet \( R \) is frequency dependent, so

\[ H_z^s(p, q, t) = H_z^p(p, q) \mathcal{L}^{-1}\left(\frac{1}{s + \frac{2R(s)}{\mu_0} \sqrt{p^2 + q^2}}\right), \quad (A3.3) \]

where \( \mathcal{L}^{-1} \) denotes the inverse Laplace transform operator.
A3.2 The vertical-field response of a large circular loop

Up to this stage, the transmitter can be an arbitrary shape; however, the computations can be simplified by assuming the loop is circular (e.g. as done by Raiche and Spies, 1981; Knight and Raiche 1982). The cylindrical symmetry of the primary field can be exploited to give

$$\bar{H}_z^p(p, q) = \hat{H}_z^p(\lambda)$$

(Sneddon, 1951, page 62), where $\hat{H}(\lambda)$ is the zeroth-order Hankel transform of $H(x, y)$ and $\lambda = \sqrt{p^2 + q^2}$. This transform pair is defined by

$$\hat{f}(\lambda) = \int_0^{\infty} r f(r) J_0(\lambda r) \, dr,$$

and

$$f(r) = \int_0^{\infty} \lambda \hat{f}(\lambda) J_0(\lambda r) \, d\lambda,$$

where $J_0(u)$ is the zeroth-order Bessel function of the first kind with argument $u$ and $r = \sqrt{x^2 + y^2}$. Thus, (A3.3) can be written

$$\hat{H}_z^s(\lambda, t) = \hat{H}_z^p(\lambda) \mathcal{L}^{-1}\left(\frac{1}{s + \frac{2R(s)}{\mu_0} \lambda}\right). \quad (A3.4)$$

The primary field of a circular loop of radius $a$ has a zeroth-order Hankel transform

$$\hat{H}_z^p(\lambda) = \frac{Ia}{2} J_1(\lambda a) e^{-\lambda |z|} \quad (A3.5)$$

(Wait, 1982, page 103), where $|z|$ is the vertical distance of the horizontal transmitter loop from the sheet and $J_1$ is the first-order Bessel function. Inverting equation (A3.4) to the space domain via a Hankel transform will yield an expression for the vertical component of the secondary field as measured by a unit-dipole receiver placed a distance $r$ from the centre of the loop:

$$H_z^s(r, t) = \int_0^{\infty} \frac{Ia}{2} J_1(\lambda a) e^{-\lambda |z|} \mathcal{L}^{-1}\left(\frac{1}{s + \frac{2R(s)}{\mu_0} \lambda}\right) J_0(\lambda r) \lambda \, d\lambda. \quad (A3.6)$$
The inverse Laplace transform in equation (A3.6) is calculated via the Gaver-Stehfest inverse Laplace transform algorithm (Appendix 5) and the Hankel transform is done using a fast Hankel transform which utilizes lagged convolution (e.g. Anderson, 1982) and uses the digital filter weights of Johansen and Sørensen (1979). The accuracy of both the digital filter, and the Gaver-Stehfest algorithm are discussed in section A3.5.

A3.3  The horizontal-field response of a large circular loop

Equation (A3.6) is the explicit expression for the vertical field, but we also wish to calculate the horizontal-field response and the surface current density. These can be obtained via the scalar magnetic potential \( \phi^s \) which has a Hankel transform

\[
\hat{\phi}^s(\lambda) = \frac{\hat{H}^s_z(\lambda)}{\lambda},
\]  

(Smith and West, 1987). The radial component of the secondary field is related to \( \hat{\phi}^s \) via

\[
H^s_r(r) = -\frac{\partial \phi^s(r)}{\partial r}.
\]

Substituting equation (A3.6) and (A3.7) into (A3.8) gives

\[
H^s_r(r) = \int_0^\infty \frac{Ia}{2} J_1(\lambda a) e^{-\lambda |z|} \mathcal{L}^{-1}\left(\frac{1}{s + \frac{2R(s)}{\mu_0} \lambda}\right) J_1(\lambda r) \lambda \, d\lambda,
\]

where the relation \( J'_0(u) = -J_1(u) \) has been used. Equations (A3.6) and (A3.9) are similar, except that the zeroth-order Hankel transform is used to calculate \( H^s_z(r) \) and the first-order transform (Appendix 2) is used to calculate \( H^s_r(r) \).

The surface current density is given by

\[
K^s_\phi(r) = 2H^s_r(r)
\]

(Smith and West, 1987), which can be calculated from equation (A3.9).
A3.4 The voltage response

The voltage induced in a small receiver is the dipole moment multiplied by the time derivative of the magnetic flux density linking the receiver. The time derivative is obtained by multiplying the term to be inverse Laplace transformed (in either equation (A3.6) or (A3.9)) by $s$, the Laplace transform of $\partial / \partial t$.

Equations (3.6) and (3.9) give the secondary fields as a function of radial distance. The coincident-loop response can be obtained by integrating the voltage response over the area of the receiver loop.

A3.5 Numerical checks

Some of the results obtained using the digital-linear-filter method of performing the fast Hankel transform are compared with the results calculated using Chave's (1983) routine which uses the gaussian quadrature rules of Patterson (1973) and Pade approximants. On Fig. A3.1, where the vertical component of the voltage response is plotted, the two methods agree very closely, except at early time and small radial distances where the digital-linear-filter results commonly display minor numerical noise (Chave, 1983). Although the adaptive routine is more accurate, the linear filter method is significantly faster, and accurate enough for the purposes of this research.

The accuracy of the Gaver-Stehfest algorithm in a number of different situations is discussed in Appendix 5. In this case, using 18 terms gave results which were not significantly different from those obtained using 16 terms (Fig. A3.2), so the algorithm was concluded to give useful results.

A3.6 Conclusion

In this appendix, I have outlined a method for calculating the response of a uniform thin sheet excited by a large circular loop. Coincident- or fixed-loop responses
Figure A3.1 The same as Fig. 3.12, except that two methods of Hankel transformation are used. The dashed line uses a digital linear filter, and the solid line an adaptive integration scheme of Chave (1983). Numerical noise occurs with the digital filter method at small radii and early time.
Figure A3.2 The same as Fig. 3.13, except that the number of terms used in the Gaver-Stehfest Laplace transform algorithm is varied. The differences are small, and they are most visible in zones where the EM response is changing very rapidly with position.
can be calculated using this method when the overburden is either polarizable or non-polarizable.
Appendix 4

The approximate convolution algorithm

A4.0 Introduction

Ordinarily, the TEM response of polarizable bodies is obtained by calculating the frequency domain response at many frequencies and transforming it to the time domain via Fourier, Laplace or Hankel transforms, a computationally laborious task.

For some bodies however, it is possible to obtain the response of a non-polarizable body directly in the time domain; the solutions for a filamental wire loop circuit and a uniform thin sheet overburden are described in Grant and West (1965), and the solution for a thin plate is described in Annan (1974) and Dyck et al, (1980). Generally, the response of a weakly polarizable body and the response of the same body when it is non-polarizable are similar. Thus, it should be possible to write the response of a polarizable body as the response of a non-polarizable body plus a small perturbation. The non-polarizable body considered is the same as the polarizable body, except its conductivity is equal to the conductivity of the polarizable body in the high frequency limit. The response of such a body is called the fundamental response, whilst the small perturbation is termed the polarization response. An approximation to the decay of the polarization response can be calculated by convolving the fundamental response with the impulse response of the polarization, a technique requiring no frequency domain calculations.

The solution obtained by the approximate convolution algorithm is found to be similar to the exact solution in a number of representative cases.
A4.1 The decomposition

Kamenetskii and Timofeev (1984) have shown that it is possible to separate the polarization response from the fundamental response when currents are induced in a half-space and measurements are taken close to the transmitter loop at late delay times. In this appendix, the fundamental response and the polarization response are assumed to be separated, and it is shown that the latter can be calculated from the former using an approximate convolution method.

Let the total magnetic field, electric field and current density associated with the currents induced in a polarizable body be denoted by \( H^{tot}, E^{tot}, J^{tot} \). (Normally these fields are referred to as the secondary fields as they are induced by the primary field from the transmitter). The conductivity of the body can be written as a high frequency limit \( \sigma_\infty \) plus a small frequency dependent part which describes the polarization

\[
\tilde{\sigma}(s) = \sigma_\infty + \Delta \tilde{\sigma}(s), \quad (A4.1)
\]

where \( s \) is the Laplace transform variable \( (s = i\omega) \), and \( \tilde{\cdot} \) denotes a Laplace transformed quantity. In galvanic IP, the conductivity is normally described as a perturbation from the low frequency (dc) limit; however, the high frequency limit is used here, because it makes the EM analysis simpler. Bhattacharyya (1964), in deriving the EM response of a polarizable half-space, also described the variation of the conductivity in this way. The fundamental magnetic field and electric field and current density \( (H^{fund}, E^{fund} \text{ and } J^{fund}) \) are defined as those associated with a non-polarizable body identical to the polarizable body except the conductivity is frequency independent and equal to \( \sigma_\infty \). In most geological materials, the variation of the conductivity from \( \sigma_\infty \) is only very slight over the EM frequency range, so the fields and currents associated with the polarization will be relatively small. These quantities \( H^{pol}, E^{pol} \text{ and } J^{pol} \) are defined by the following relations

\[
H^{tot} = H^{fund} + H^{pol}, \quad (A4.2)
\]

\[
E^{tot} = E^{fund} + E^{pol}, \quad (A4.3)
\]
and

\[ J^{tot} = J^{fund} + J^{pol}. \]  \hspace{1cm} (A4.4)

(An implicit spatial dependence is assumed for \( \mathbf{H}, \mathbf{E}, \) and \( J; \) \( \sigma \) is assumed constant within the body, and zero outside the body.)

The total current density flowing in the body can be obtained from Ohm’s law

\[ \tilde{J}^{tot}(s) = \tilde{\sigma}(s) \tilde{E}^{tot}(s), \]  \hspace{1cm} (A4.5)

whilst the definition of the fundamental current density implies that

\[ \tilde{J}^{fund}(s) = \sigma_{\infty} \tilde{E}^{fund}(s). \]  \hspace{1cm} (A4.6)

Thus, the resulting polarization current is given by

\[ \tilde{J}^{pol} = (\sigma_{\infty} + \Delta \tilde{\sigma}(s)) \tilde{E}^{pol} + \Delta \tilde{\sigma}(s) \tilde{E}^{fund}. \]  \hspace{1cm} (A4.7)

Standard electromagnetic methods don’t normally measure the currents, but rather the magnetic field, which is related to the current via Ampere’s law

\[ \nabla \times \tilde{H}^{tot}(s) = \tilde{J}^{tot}(s). \]  \hspace{1cm} (A4.8)

With the use of equations (A4.2), (A4.3), (A4.4), (A4.5) and (A4.6) this becomes

\[ [\nabla \times \tilde{H}^{fund}(s)] + \nabla \times \tilde{H}^{pol}(s) = [\tilde{J}^{fund}] + \frac{\Delta \tilde{\sigma}(s)}{\sigma_{\infty}} \tilde{J}^{fund}(s) + (\sigma_{\infty} + \Delta \tilde{\sigma}(s)) \tilde{E}^{pol}. \]  \hspace{1cm} (A4.9)

The fundamental fields obey Ampere’s law, so the terms in square brackets cancel. The approximate convolution solution can be obtained by assuming that the effect of the \( (\sigma_{\infty} + \Delta \sigma)E^{pol} \) term, is negligible compared with the effect of the \( \Delta \tilde{\sigma}(s)/\sigma_{\infty} J^{fund} \) term. This assumption implies that the fundamental current energizes the polarization magnetic field, and the EM interaction of the polarization current with itself and the fundamental current is negligible. The approximation appears to be a good one for most geologic materials, and this is demonstrated in section A4.6.
Given the approximation, equation (A4.6) becomes
\[ \nabla \times \tilde{H}^{\text{pol}}(s) \approx \frac{\Delta \tilde{\sigma}(s)}{\sigma_\infty} \nabla \times \tilde{H}^{\text{fund}}(s). \]

Hence,
\[ \tilde{H}^{\text{pol}}(s) \approx \frac{\Delta \tilde{\sigma}(s)}{\sigma_\infty} \tilde{H}^{\text{fund}}(s) + \nabla \tilde{\psi}(s), \tag{A4.10} \]
where \( \tilde{\psi} \) is a scalar potential. Taking the divergence of equation (A4.10), and using Gauss's law for a region of uniform magnetic permeability (\( \nabla \cdot \mathbf{H} = 0 \)), \( \tilde{\psi} \) can be shown to satisfy \( \nabla^2 \tilde{\psi} = 0 \). As there are no sources of scalar potential, Liouville’s theorem infers that \( \tilde{\psi} = 0 \). Thus, \( \nabla \tilde{\psi} = 0 \), and
\[ \tilde{H}^{\text{pol}}(s) \approx \frac{\Delta \tilde{\sigma}(s)}{\sigma_\infty} \tilde{H}^{\text{fund}}(s), \]
which becomes
\[ \mathbf{H}^{\text{pol}}(t) \approx \frac{\Delta \sigma(t)}{\sigma_\infty} \ast \mathbf{H}^{\text{fund}}(t) \tag{A4.11} \]
in the time domain (the \( \ast \) is the convolution operator). The function \( \frac{\Delta \sigma(t)}{\sigma_\infty} \) is defined as the impulse response of the polarization. Equation (A4.11) shows that an approximation to the magnetic field associated with the polarization current can be computed by convolving the fundamental magnetic field with the impulse response of the polarization. This method of calculating the polarization response is called the ‘approximate convolution algorithm’. Using empirical arguments, Sidorov and Yakhin (1979) obtained a similar equation for the electric field; however, they avoided an evaluation of the convolution integral by making further approximations.

The approximate convolution algorithm can also be used to compute the IP and MIP response. As a simple example, a constant amplitude voltage source \( v^{t_{\text{tot}}}(t) \) is imposed between two input electrodes. In the frequency domain, the total current flowing into the ground is
\[ ^* \tilde{I}^{t_{\text{tot}}}(s) = a \sigma(s) \tilde{V}^{t_{\text{tot}}}(s), \]
where \( a \) is the distance between the electrodes, and \( \sigma(s) \) is defined as the effective conductivity of the ground. When the ground is frequency dependent, the effective
conductivity can be written \(\tilde{\sigma}_e(s) = \sigma_\infty + \Delta \tilde{\sigma}(s)\). The total current is thus
\[
\tilde{I}^{\text{tot}}(s) = a [\sigma_\infty + \Delta \tilde{\sigma}(s)] \tilde{V}^{\text{tot}}(s).
\]
The imposed voltage \(\tilde{V}^{\text{tot}}(s)\) is independent of the conductivity, so
\[
\tilde{I}^{\text{fund}}(s) = a \sigma_\infty \tilde{V}^{\text{tot}}(s),
\]
and hence the polarization current becomes
\[
\tilde{I}^{\text{pol}}(s) = \tilde{I}^{\text{tot}}(s) - \tilde{I}^{\text{fund}}(s),
\]
\[
= \frac{\Delta \tilde{\sigma}(s)}{\sigma_\infty} \tilde{I}^{\text{fund}}(s).
\]
This equation is a restatement of Seigel's fundamental law of IP (Seigel, 1959), except a frequency dependence has been incorporated. In the time domain, the polarization current becomes
\[
i^{\text{pol}}(t) = \frac{\Delta \sigma(t)}{\sigma_\infty} * i^{\text{fund}}(t), \tag{A4.12}
\]
which is a convolution as before.

The convolution in equations (A4.11) and (A4.12) reflects the fact that the previous history of the fundamental response effects the present state of the polarization. Specifically, the existence of the fundamental response implies the existence of a fundamental current which polarizes or charges the body. The polarization generated decays at a rate determined by the frequency dependence of the conductivity, and the polarization current at any time is a superposition of all the polarizations induced by the previous existence of the fundamental current.

### A4.2 The form of the frequency dependence of the polarization

Experience with the IP method suggests most polarizable bodies can be characterized with a conductivity whose frequency dependence has the form of a Cole-Cole dispersion (Pelton et al, 1978):
\[
\tilde{\sigma}(s) = \sigma_\infty - \frac{\sigma_\infty m}{1 + (1 - m)(sT)^\tau}, \tag{A4.13}
\]
where $m$ is the chargeability, $c$ is the Cole-Cole exponent, and $T$ is the Cole-Cole time constant. From equation (A4.1) it can be seen that
\[
\frac{\Delta \tilde{\sigma}(s)}{\sigma_\infty} = \frac{-m}{1 + (1 - m)(sT)^c}.
\] (A4.14)

The chargeability is almost universally positive (as $\partial \sigma/\partial \omega > 0$), which implies the sign of the polarization response will be opposite to that of the fundamental response. For example, if $I_{\text{fund}}$ in equation (A4.12) is positive, $I_{\text{pol}}$ is negative, and the total current is less than the fundamental current, a fact consistent with routine IP measurements. Similarly, the magnetic field polarization response in equation (A4.11) is negative relative to the fundamental response, so the total response of a polarizable body will be less than that of a non-polarizable body. In coincident-loop TEM a sufficiently large polarization response will change the sign of the total response to negative.

### A4.3 Calculation of the voltage response

The approximate convolution algorithm can be used for time-domain IP and MIP; however, the example given here is the one for which the algorithm was developed — calculating the coincident-loop TEM response of polarizable bodies. Coincident-loop systems measure the voltage induced in a receiver loop when the current flowing in a coincident transmitter loop is switched off suddenly (ideally as a step).

For a horizontal loop receiver, the part of the voltage associated with the polarization current is
\[
v_{\text{pol}}(t) = -\frac{\partial}{\partial t} \int_A \mu H_{z}^{\text{pol}} dA,
\]
where $A$ is the area of the transmitter/receiver loop and the $z$ axis is vertically directed. Substituting the convolution for $H_{z}^{\text{pol}}$ and exchanging the order of integration yields
\[
v_{\text{pol}}(t) \approx -\frac{\partial}{\partial t} \int_{-\infty}^{\infty} \Delta \sigma(t - t') \frac{u(t - t')}{\sigma_\infty} \left[ \int_A \mu H_{z}^{\text{fund}}(t') u(t') dA \right] dt',
\]
where $u(t)$ is the unit step-on function. The surface integral in square brackets is $\phi_{\text{fund}}^\text{pol}(t')$, the magnetic flux through the receiver.

Changing the order of differentiation and integration gives

$$v_{\text{pol}}(t) \approx -\frac{\Delta \sigma(0)}{\sigma_\infty} \phi_{\text{fund}}^\text{pol}(t) - \int_0^t \frac{\partial}{\partial t} \left( \frac{\Delta \sigma(t - t')}{\sigma_\infty} \right) \phi_{\text{fund}}^\text{pol}(t') \, dt',$$  \hspace{1cm} (A4.15)

or, equivalently,

$$v_{\text{pol}}(t) \approx -\frac{\Delta \sigma(t)}{\sigma_\infty} \phi_{\text{fund}}^\text{pol}(0) - \int_0^t \frac{\Delta \sigma(t')}{\sigma_\infty} \frac{\partial}{\partial t} \left( \phi_{\text{fund}}^\text{pol}(t - t') \right) \, dt',$$  \hspace{1cm} (A4.16)

since convolution is commutative. In both these equations, the polarization voltage is obtained by summing two terms. In the case of equation (A4.16), these two terms are similar in magnitude, opposite in sign, and are much larger than their sum. Thus, any small numerical error in one of the terms will make a significant, although erroneous, contribution to the answer. The errors introduced by numerical integration of the convolution term are too large to give accurate answers unless double precision adaptive integration is used. The integral in equation (A4.15) can be evaluated using simpler quadrature schemes (e.g., the rectangle rule) because the two terms on the right are not similar in magnitude and opposite in sign.

One of the following sets of quantities must be known if $v_{\text{pol}}(t)$ is to be calculated, viz,

(i) the impulse response of the polarization and the time derivative of the magnetic flux (for when equation (A4.16) is used), or

(ii) the magnetic flux and the time derivative of the impulse response of the polarization (for when equation (A4.15) is used).

Methods for calculating these quantities are outlined in the following two sections.

A4.4 The impulse response of the polarization

The impulse response of the polarization is obtained by inverse Laplace transformation of equation (A4.14). Analytic forms exist for certain values of $c$, for example,
when \( c = 1 \) the inverse Laplace transform of (A4.13) is

\[
\frac{\Delta \sigma(t)}{\sigma_\infty} = -mb e^{-bt} u(t), \tag{A4.17}
\]

where \( b \) (the inverse of the polarization time constant) is defined by \( b = 1/[(1 - m)T^c] \). For the case when \( c = 1/2 \) the inverse Laplace transform is (equation 29.3.37, Abramowitz and Stegun (1965))

\[
\frac{\Delta \sigma(t)}{\sigma_\infty} = -mb\left[\frac{1}{\sqrt{\pi t}} - be^{b^2t} \text{erfc} (b \sqrt{t})\right] u(t). \tag{A4.18}
\]

The time derivative of equation (A4.17) can be found by simple differentiation. However, for the case when \( c = 1/2 \) equation (A4.18) can not be differentiated simply, so equation (A4.16) must be used to calculate the polarization voltage.

Analytic expressions for the inverse Laplace transform do not exist when \( c \) is neither 1 or 1/2; however, the transform can be calculated for arbitrary \( c \) using infinite series (Pelton et al, 1978), asymptotic expansions (Lee, 1981c), digital linear filters (Guptasarma, 1982), or the Gaver-Stehfest inverse Laplace transform algorithm (Appendix 5). For the calculations of Chapter 5, the last technique was used because it provides a very rapid and easily programmable means of inverting equation (A4.14) to the time domain.

### A4.5 The magnetic flux and its time derivative

In this section, the magnetic flux and its time derivative are calculated for a filamental wire-loop circuit, and an infinite uniform thin-sheet.

For a filamental wire circuit of resistance \( R_\infty \) and self inductance \( L \), \( \phi^{und}(t) \) is given by

\[
\phi^{und}(t) = u(t) I_T e^{M^2 S_\infty} e^{-t/\tau} \tag{A4.19}
\]

(Grant and West, 1965), where \( I_T \) is the current flowing in the transmitter prior to switch off, \( M \) is the mutual inductance of the transmitter/receiver and the wire-loop circuit, \( \tau = L/R_\infty \), and \( S_\infty = 1/R_\infty \). The time derivative of \( \phi^{und}(t) \) is found by differentiating (A4.19).
A uniform thin sheet of conductance $S_\infty$ excited by a current switched off at $t = 0$ is considered next. The magnetic field associated with the secondary current induced in this sheet is equivalent to the magnetic field of an image of the transmitter current which retreats away from the sheet at a velocity $2/(S_\infty \mu_0)$, where $\mu_0$ is the permeability of free space (Grant and West, 1965). The initial position of the image current is that of the transmitter reflected in a mirror plane coincident with the thin sheet. The flow direction of the image current is chosen to preserve the component of the primary field normal to the sheet at time $t = 0$. The retreating image magnetic flux $\phi^{fund}(t)$ linking the receiver is given by

$$\phi^{fund}(t) = I^T_z \Gamma_{fund}(t),$$

where $\Gamma_{fund}$ is the image-receiver inductance. For coincident transmitters and receivers the inductance $\Gamma_{fund}$ can be written

$$\Gamma_{fund}(t) = \frac{2\mu_0}{k} r [(1 - \frac{1}{4}k^2) K(k) - E(k)],$$

(Smythe (1950) equation (1) section 8.06), where $r$ is the radius of the circular transmitter (and image), and $K(k)$ and $E(k)$ are the elliptic integral functions. The argument of the elliptic integral functions $k$ is defined by

$$k = \frac{1}{\sqrt{1 + \frac{z^2}{4r^2}}},$$

where $z$ is the distance of the image from the receiver. Using the image solution

$$z = 2h + \frac{2t}{S_\infty \mu_0},$$

where $h$ is the initial height of the transmitter above the overburden.

The time derivative of the flux is

$$\frac{\partial \phi^{fund}}{\partial t} = I^T_z \frac{\partial \Gamma_{fund}}{\partial k} \frac{\partial k}{\partial z} \frac{\partial z}{\partial t}$$

$$= \frac{I^T_z \sigma r k^2}{\sigma r} \left[ \frac{k}{2} + \frac{1}{k} \right] K - E/k + E' - (1 - k^2/2) K',$$
where $K' = \frac{\partial K}{\partial k}$ and $E' = \frac{\partial E}{\partial k}$. Formulae for the elliptic integral functions, given in Dwight (1957) (equations (773.2) and (774.2)), are infinite series in the variable $m$, defined by
\[
m = \frac{1 - \sqrt{1 - k^2}}{1 + \sqrt{1 - k^2}}.
\]
(A4.20)

The derivatives of the elliptic functions
\[
K' = \frac{\partial K}{\partial m} \frac{\partial m}{\partial k},
\]
and
\[
E' = \frac{\partial E}{\partial m} \frac{\partial m}{\partial k},
\]
can be calculated by (a) differentiating equation (A4.20) with respect to $k$
\[
\frac{\partial m}{\partial k} = \frac{2k}{\sqrt{1 - k^2}(1 + \sqrt{1 - k^2})^2},
\]
and (b) differentiating the infinite series with respect to $m$
\[
\frac{\partial K}{\partial m} = \frac{\pi}{2} \left[1 + \frac{1}{2^2} m^2 + \frac{1^2 3^2}{2^2 4^2} m^4 + \ldots\right] + \frac{\pi}{2} (1 + m) \left[\frac{1}{2^2} m + \frac{1^2 3^2}{2^2 4^2} m^3 + \frac{1^2 3^2 5^2}{2^2 4^2 6^2} m^5 + \ldots\right],
\]
and
\[
\frac{\partial E}{\partial m} = \frac{\pi}{2} \frac{-1}{(1 + m)^2} \left[1 + \frac{1}{2^2} m^2 + \frac{1^2}{2^2 4^2} m^4 + \frac{1^2 3^2}{2^2 4^2 6^2} m^6 + \ldots\right] + \frac{\pi}{2} \frac{1}{(1 + m)^3} \left[\frac{1}{2} m + \frac{1^2}{2^2 4^2} m^3 + \frac{1^2 3^2}{2^2 4^2 6^2} m^5 + \ldots\right].
\]

These series are valid for $k^2 < 1$ which is always true because $t > 0$.

### A4.6 Results

The algorithm is applied to two types of conductor: (a) a confined conductor (the filamental wire loop circuit), and (b) an extended conductor (the uniform thin sheet overburden). This is done to show: (i) that the approximations made in deriving the
solution are valid, and (ii) that the approximate convolution algorithm is capable of calculating the response for two very different classes of bodies.

In each case, the coincident-loop response is calculated, and the Cole-Cole dispersion model is used to describe the frequency dependence of the conductivity. For both the conductor models the response is calculated for $c = 1$ (with equation...
(A4.15)), $c = 1/2$ (with equation (A4.16)) and $m = 0.01, 0.05, 0.3$. These results are plotted on Figs A4.1 to A4.4 as dashed lines, the solid lines are the exact solution obtained by inverse Laplace transforming the frequency-domain solution. In all cases, there is good agreement between the exact and approximate results.

![FILAMENTAL LOOP CIRCUIT, c = 1/2](image)

**Figure A4.2** The voltage response of a polarizable filamental wire loop circuit calculated using the approximate convolution method (dashed line), and the exact method (solid line). In this case $c = 1/2$, $m = 0.0, 0.01, 0.05, 0.3$, $M = 1 \text{H}$, $S_{\infty} = 1 \text{S}$, $\tau = 0.0005 \text{s}$, $T = 5\text{ms}$. The geometrical arrangement is as in Fig. A4.1 The scale between -1 and 1 is linear, while outside this range it is logarithmic.

In the overburden case, the algorithm for calculating $\phi^{fund}(t)$ is very slowly convergent when $t$ is small and the transmitter loop is very close to the overburden.
Figure A4.3 The voltage response of a polarizable overburden calculated using the convolution approximate method (dashed line), and the exact method (solid line). In this case $c = 1$, $m = 0.0, 0.01, 0.05, 0.3$, $S_\infty = 3.5\, S$ and $T = 1.3\, ms$. The transmitter/receiver loop is circular with a radius of 56m, and is a negligible height (5m) above the overburden. The scale between -1 and 1 is linear, while outside this range it is logarithmic.

This difficulty is overcome by raising the transmitter a small height (5m) above the overburden. This makes the algorithm much quicker, and does not alter the response significantly.

The approximate convolution algorithm was derived by assuming that the EM effect of the polarization current is negligible compared with that of the fundamental current. This appears to be the case, even when the chargeability is as large as 0.3,
and is likely because EM induction effects are not proportional to the size of the current, but to its time rate of change. In most geological materials, a polarization current will decay relatively slowly, so the resulting EM induction will be small, even when the polarization current is large compared with the fundamental current (i.e. at late times or when $m$ is large).

The slight discrepancy between the times at which the sign reversals occur in
Fig. 4.4 may be because the rates of decay of the fundamental and polarization decays are similar. Plotting the decays as a function of log time makes the magnitude of the discrepancy appear more significant than it would otherwise appear.

A4.7 Conclusions

The TEM response of a polarizable body is the response of a non-polarizable body (the fundamental decay) plus a small perturbation (the polarization decay). An approximation for the polarization decay can be calculated by convolving the fundamental decay with the impulse response of the polarization.

In principle, the polarization current contributes along with the fundamental inductive current to the creation of the secondary magnetic field which controls the induction process. In this analysis, the effect of the polarization current has been ignored. This is a good approximation because the polarization current is generally smaller and has a significantly slower decay rate than the fundamental inductive current.

The approximate convolution algorithm has been tested on extensive and confined conductors and is found to work well. It could also be used for more complicated conductivity structures (e.g. a conductor buried in a host medium), provided the fundamental current flowing through the conductor is known. Hence, existing time-domain algorithms which do not account for polarization effects can readily be converted to ones that do.

Scale modelling is an established method of obtaining the EM response of bodies which cannot be approximated by simple mathematical models. It is very difficult to simulate a polarizable body with a scale model, so the approximate convolution algorithm could be used for obtaining the response of these bodies. The non-polarizable fundamental response of the scale model would have to be measured at early time to ensure that the convolution can be computed accurately.

The relative efficiency of the algorithm depends on the type of conductor for
which the response is calculated. Generally, the convolution algorithm is more rapid when the non-polarizable response can be calculated directly and efficiently in the time-domain. For example, in the case of an overburden, the convolution algorithm takes less than a cpu second to calculate the EM response at one delay time, whereas the exact method, which uses a Hankel transform, an inverse Laplace transform and a numerical integration of the magnetic flux linking a large receiver loop takes about a minute of cpu time (on a microVAX).

A further advantage of the algorithm is that it is relatively simple to implement on a computer.

The approximate convolution algorithm is important, not only because it provides a simple method of calculating the response in the time domain, but also because it illustrates the physics of the polarization process. It can be inferred that a polarization current arises as a result of a fundamental current flowing inside the body. The mechanism by which the fundamental current is introduced into the body (i.e. via grounded electrodes or inductive processes) is immaterial. The resulting polarization current is essentially a small negative image of the fundamental current which exists everywhere that the fundamental current has flowed, and has a magnitude which decays with a time constant equal to the polarization time constant.
Appendix 5

The Gaver-Stehfest algorithm

A5.0 Introduction

The Gaver-Stehfest algorithm (Stehfest, 1970a,b) is an approximate method for inverting a numerical function from the Laplace transform domain $\tilde{F}(s)$ to the time domain $f(t)$. A single realization of $f(t)$ is obtained by evaluating the function $\tilde{F}(s)$ at $n$ values of the argument $s$. The required values of $s$ are real and complex arithmetic is not needed in the computer program. The number of evaluations ($n$) is relatively small ($n \approx 16$); however, if a large number of realizations ($k$) are required, then the number of function evaluations ($m$) can become large ($m = kn$). An appropriate choice of the times $t$ at which $f(t)$ is required, can approximately halve the number of function evaluations.

The use of real arithmetic and the small number of function evaluations, makes the Gaver-Stehfest algorithm easy to implement and an efficient method of calculating the inverse Laplace transform. For these reasons, it is frequently used in geophysical contexts (Knight and Raiche, 1982; Villinger, 1985; Zemanian, 1985; Edwards and Cheesman, 1987; and Edwards, 1988). Many other techniques for performing inverse Laplace transforms exist; a paper by Davies and Martin (1979) contains a comprehensive review of these methods and a description of their particular advantages.

A5.1 The Gaver-Stehfest algorithm

The algorithm does not attempt to calculate the inverse Laplace transform at a particular time exactly, but obtains an approximate value ($f_a^n$), which is an average
over a certain time range i.e.,

\[ f_a^n = \int_0^\infty \delta_n(a, \tau) f(\tau) d\tau, \tag{45.1} \]

where \( \delta_n \) is the weighting function

\[ \delta_n(a, \tau) = \frac{(2n)!a}{n!(n-1)!} (1 - e^{-a\tau})^n e^{-na\tau} \tag{45.2} \]

(Gaver, 1966) which has the following properties:

1. \( \int_0^\infty \delta_n(a, \tau) d\tau = 1 \),
2. The modal \( \tau \) of \( \delta_n(a, \tau) = \ln 2/a \),
3. The variance of the distribution is \( \text{var}(\tau) = 1/a^2 \sum_{i=0}^n 1/(n + i)^2 \)

(Stehfest, 1970a). These properties are similar to the properties of a dirac delta function centred at \( \tau = \ln 2/a \), hence

\[ \delta_n(a, \tau) \approx \delta(\ln 2/a - \tau). \tag{45.3} \]

The value \( f_a^n \) is thus an approximation to \( f(t) \) at time \( t = \ln 2/a \).

Expanding equation (A5.2) using the binomial theorem,

\[ \delta_n(a, \tau) = \frac{(2n)!a}{n!(n-1)!} \sum_{k=0}^n (-1)^k \binom{n}{k} e^{-a(k+n)\tau}, \tag{45.4} \]

and substituting this into equation (A5.1) gives

\[ f_a^n = \frac{(2n)!a}{n!(n-1)!} \sum_{k=0}^n (-1)^k \binom{n}{k} \int_0^\infty f(\tau) e^{-a(k+n)\tau} d\tau. \]

Using the definition of the Laplace transform, this can be written

\[ f_a^n = \frac{(2n)!a}{n!(n-1)!} \sum_{k=0}^n (-1)^k \binom{n}{k} \bar{F}(a(k+n)), \tag{45.5} \]

where \( a(k+n) \) is the value of the Laplace transform variable. Thus, \( f_a^n \) can be found from \( n \) evaluations of \( \bar{F}(s) \). Gaver has shown that \( f_a^n \) has an asymptotic expansion

\[ f_a^n \approx f(\ln 2/a) + \alpha_1/n + \alpha_2/n^2 + \alpha_3/n^3 + ... \tag{45.6} \]
so \( f_a^n \) converges to the required value \( f(\ln 2/a) \) as \( 1/n \). Stehfest (1970a) proposed a method of speeding the rate of convergence by summing \( f_a^1, f_a^2, \ldots, f_a^{n/2} \), in such a manner that the inverse powers of \( n \) in the asymptotic expansion are eliminated. This is done by defining the coefficients of the sum \( x_i(k) \) such that

\[
\sum_{i=1}^{k} x_i(k) \frac{1}{(n/2 + 1 - i)j} = \delta_{j0}, \quad \text{for} \quad j = 0, 1, \ldots, k - 1, \quad k \leq n/2,
\]

where \( \delta_{j0} \) is the kronecker delta. Solving these \( k \) equations for the \( x_i \) yields

\[
x_i(k) = \frac{(-1)^{i-1}}{k!} \binom{k}{i} i(n/2 + 1 - i)^{k-1}.
\]

Multiplying these coefficients by the \( f_a^{n/2}, \ldots, f_a^1 \) gives

\[
\sum_{i=1}^{k} x_i(k) f_a^{n/2+1-i} = f(\ln 2/a) + (-1)^{k+1} \alpha_k \frac{(n/2 - k)!}{(n/2)!} + o\left(\left(\frac{n/2 - k}{n/2}\right)\right). \quad (A5.7)
\]

If \( k = n/2 \), then the error is minimized, so

\[
f(\ln 2/a) \approx \sum_{i=1}^{n/2} x_i(n/2) f_a^{n/2+1-i}.
\]

Substituting for the \( x_i \), setting \( a = \ln 2/t \), and utilizing equation (A5.5) gives an approximate value of \( f(t) \) at time \( t \)

\[
f_a(t) \approx \frac{\ln 2}{t} \sum_{i=1}^{n} V_i \tilde{F}(\frac{\ln 2}{t}i),
\]

where

\[
V_i = (-1)^{n/2+i} \sum_{j=\frac{i+1}{2}} \frac{j^{n/2}(2j)!}{(n/2 - j)!j!(j-1)!(i-j)!(2j-i)!}.
\]

This is the Gaver-Stehfest algorithm. Choosing \( n \) even will ensure that the terms on the right-hand-side of equation (A5.7) do not reduce the accuracy of the algorithm. The approximation can be improved by increasing \( n \) to decrease the spread of \( \delta_n(a, \tau) \); however, if \( n \) becomes too great, the absolute values of the \( V_i \) become so large that significant round-off errors are introduced. The optimal value of \( n \) is approximately equal to the number of decimal digits used by the computer to
represent floating point numbers (Stehfest, 1970a). The calculations of this thesis were performed using double precision arithmetic on a VAX, so \( n \approx 16 \).

Some insight into the Gaver-Stehfest algorithm can be obtained by taking the Laplace transform of \( f_a \). Substituting equation (A5.3) into (A5.1) gives

\[
 f_a(t) \approx \int_0^\infty \delta(t - \tau)f(\tau)d\tau,
\]

which has a Laplace transform

\[
 \tilde{F}_a \approx \mathcal{L}(\delta(t)) \tilde{F}(s), \tag{A5.8}
\]

where \( \mathcal{L} \) denotes the Laplace transform operator. The Gaver-Stehfest algorithm does not use the exact Dirac delta function, but the weighting function in equation (A5.4) which has a Laplace transform

\[
 \tilde{\delta}_n(a, s) = \frac{(2n)!a}{n!(n - 1)!} \sum_{k=0}^{n} (-1)^k \binom{n}{k} \frac{1}{s + a(k + n)}.
\]

Thus, equation (A5.8) can be written

\[
 \tilde{F}_a \approx \frac{\tilde{F}(s)(2n)!a}{n!(n - 1)!} \sum_{k=0}^{n} (-1)^k \binom{n}{k} \frac{1}{s + a(k + n)}.
\]

Hence, the spectrum of \( \tilde{F}_a \) is comprised of a weighted sum of the spectra of \( n \) decaying exponentials with time constants \( 1/(ia) \), where \( i = n, 2n \). The Gaver-Stehfest algorithm is thus related to YVESFT (Holladay, 1981), which writes \( \tilde{F}(s) \) as a sum of exponential spectra with time constants which are chosen somewhat arbitrarily, and inverts these spectra to the time domain analytically.

### A5.2 An efficient Gaver-Stehfest algorithm

The value of \( f_a(t) \) at time \( t = t_1 \) is calculated by evaluating \( \tilde{F}(s) \) at \( s = s_i \) where

\[
 s_i = \frac{i \ln 2}{t_1}, \quad i = 1, n, \tag{A5.9}
\]
and \( n \) is an even number. If the inverse Laplace transform is then calculated at time \( 2t_1 \), then the function evaluations are required at \( s = s_j \), where

\[
s_j = \frac{j \ln 2}{2t_1}, \quad j = 1, n. \tag{A5.10}
\]

The \( j = 2, 4, 6, \ldots, n \) evaluations in equation (A5.10), correspond to the \( i = 1, 2, 3, \ldots, n/2 \) evaluations in equation (A5.9), so these need not be computed a second time. The only additional evaluations required are for \( j = n/2 + 1, \ldots, n \). If \( f_a \) is evaluated at \( k \) times which comprise a geometric progression, \( t_1, 2t_1, \ldots, 2^{k-1}t_1 \), then only \( n + (k - 1)n/2 \) evaluations of \( \tilde{F}(s) \) are required. This is approximately half the \( kn \) evaluations than would be required for \( k \) arbitrarily spaced times.

Because the geometrically spaced times are equally spaced in log time, the response at a different set of times, such as that used by the SIROTEM system, can be calculated by: (a) calculating the response at times which are geometrically spaced, and (b) interpolating between these times using a quintic spline (e.g. that given in Boerner and West, 1984). A spline can be applied more accurately to slowly varying functions, so it is applied to the 'bilogarithm' of the voltage defined by

\[
bilog(v) = \begin{cases} 
\log_{10}(v), & v > 1; \\
v, & 1 > v > -1; \\
-\log_{10}(|v|), & v < -1.
\end{cases}
\]

Inaccurate results are obtained if the times comprising the geometric progression, between which the spline interpolates, are widely spaced. The above binary progression has about 3.5 points per decade, which is a relatively sparse sampling. Greater sampling can be obtained if two interlaced sequences of binary progressions are used. Figure A5.1 shows two sequences chosen to cover the times that the SIROTEM response is required.

### A5.3 Testing the Gaver-Stehfest algorithm

The Gaver-Stehfest algorithm is tested on two functions with known analytic transforms. One function has a transform with a time dependence which is similar to the
response of a plate-like conductor (exponential decay), and the other is the response of a thin conductive overburden (power law decay).

A5.4 The exponential decay

Plate-like conductors have a voltage decay which is essentially of the form \( v(t) \sim Ae^{-t/\tau} \) (Annan, 1974), where \( A \) is the constant of proportionality, and \( \tau \) is the time constant of the decay. The Laplace transform of the voltage decay is

\[
\tilde{V}(s) = \frac{A}{s + 1/\tau}.
\]

This function is inverse Laplace transformed for 5 different values of \( \tau \), using the Gaver-Stehfest algorithm (GSA) and the efficient Gaver-Stehfest algorithm (EGSA).

Figure A5.2 shows the results when 14 terms are used in the Gaver-Stehfest algorithm. The voltage response obtained using the EGSA (dotted lines) and the GSA (dashed lines) both agree with the exact solution (solid lines), except at small voltages. The EGSA and the GSA results differ by small amounts, likely due to
14 term Gaver–Stehfest

$\tau=0.035\text{ms}$

$\tau=0.1\text{ms}$

$\tau=1.0\text{ms}$

$\tau=3.5\text{ms}$

![Graph showing exponential decay for different time constants](image)

Figure A5.2. The exponential decay for 5 values of the time constant ($\tau = 0.035\text{ms}, 0.1\text{ms}, 0.35\text{ms}, 1.0\text{ms}$ and $3.5\text{ms}$). The voltages are calculated using the efficient Gaver-Stehfest algorithm (dotted lines), the Gaver-Stehfest algorithm (dashed lines), and the analytic solution (solid lines). In this case 14 terms are used in the Gaver-Stehfest algorithm. The constant $A$ is arbitrarily set to $10^4$.

interpolation errors. The differences between the exact response and the two Gaver-Stehfest responses are attributed to deficiencies in the Gaver-Stehfest algorithm. Increasing the number of terms in the algorithm to 16 (Fig. A5.3) decreases these errors substantially. Further increases are probably unnecessary, as the results obtained using 18 terms not significantly better (Fig. A5.4).

These voltage response curves are purely positive. The algorithm is found to have a similar accuracy when the voltage changes sign to negative; however, the
negatives do occur when the Gaver-Stehfest algorithm is least accurate (at small values of the voltage), so caution should be exercised.

A5.5 The $t^{-4}$ decay

A thin conductive overburden overlying a more resistive half-space can be approximated by a thin sheet with a conductivity-thickness product $\sigma d$. If the current flowing in a transmitter dipole $m$ positioned at $(x_0, y_0, z_0)$ is switched-off suddenly, then the secondary magnetic field $H^s(x, y, z)$ can be calculated using the Maxwell
18 term Gaver–Stehfest

Figure A5.4 The same as Fig. A5.2, except 18 terms are used in the Gaver-Stehfest algorithm.

retreating image solution

\[ H^s = \frac{1}{4\pi} \nabla \left( \mathbf{m} \cdot \nabla_0 \left[ (x - x_0)^2 + (y - y_0)^2 + (z + z_0 + \frac{2t}{\mu \sigma d})^2 \right]^{-1/2} \right) \]  \hspace{1cm} (A5.11)

(Grant and West (1965), equation (17.24)). For simplicity, define the origin such that \( x = y = x_0 = y_0 = 0 \), and assume that \( \mathbf{m} \) is a vertical dipole of magnitude \( m \)
positioned at \( z_0 = h \). Hence, \( \mathbf{m} = (0, 0, m) \), and equation (A5.11) reduces to

\[
H_z^s = \frac{m}{4\pi} \left[ \frac{\partial^2}{\partial z \partial z_0} \left( z + z_0 + \frac{2t}{\mu \sigma d} \right)^{-1} \right]
\]

\[
= \frac{m}{4\pi} \frac{2}{\left( z + h + \frac{2t}{\mu \sigma d} \right)^3}.
\]

The voltage induced in a receiver dipole is

\[
v(t) = -A \frac{\partial H_z^s}{\partial t},
\]

where \( A \) is the effective area of the receiver dipole. If the receiver is coincident with the transmitter, then

\[
v(t) = \frac{6mA/4\pi}{\left( \frac{2}{\mu \sigma d} \right)^3 \left[ h \mu \sigma d + t \right]^4},
\]

(A5.12)

the Laplace transform of which is

\[
\tilde{V}(s) = \frac{6mA}{(2h)^3 4\pi} e^{\tau s} E_4(\tau s).
\]

(A5.13)

(Abramowitz and Stegun equation (29.3.128)), where \( \tau = h \mu \sigma d \), and \( E_4(z) \) is the exponential integral of order 4 defined by

\[
E_4(z) = \int_1^\infty \frac{e^{-zt}}{t^4} dt \quad \text{Real}(z) > 0.
\]

The time domain voltage response, calculated by applying the GSA and the EGSA to equation (A5.13), can be checked against the analytic inverse in equation (A5.12). Figure A5.5 shows the results obtained for five values of the time constant \( \tau \), when 14 terms are used. The results are inaccurate at low voltages and small time constants, but the accuracy improves as the time constant increases. The inaccuracies seen at large voltages for the \( \tau = 3.5 \text{ms} \) case are due to the exponential integral not being accurate for large values of the argument (i.e. large \( \tau \) and/or \( s \)). The problem is therefore not associated with the Gaver-Stehfest algorithm, but with calculating \( \tilde{V}(s) \) using equation (A5.13).

When 16 terms are used (Fig. A5.6), the discrepancies at low voltages decrease. Because no significant improvement is obtained using 18 terms (Fig. A5.7), the 16
Figure A5.5 The $t^{-4}$ decay for 5 values of the time constant ($h\mu_\sigma d = 0.035\text{ms}, 0.1\text{ms}, 0.35\text{ms}, 1.0\text{ms}$ and 3.5ms). The voltages are calculated using the efficient Gaver-Stehfest algorithm (dotted lines), the Gaver-Stehfest algorithm (dashed lines), and the analytic solution (solid lines). In this case 14 terms are used in the Gaver-Stehfest algorithm, $h = 10$ and $A = 4\pi/6m$.

term results are probably sufficient. As the number of terms increases, the errors in the $\tau = 3.5\text{ms}$ case get worse. This is because $\tilde{V}(s)$ must be calculated at increasingly large $s$ values.

A5.6 Conclusions

For the cases considered, the Gaver-Stehfest algorithm appears adequate for inver-
Figure A5.6 The same as Fig. A5.5, except 16 terms are used in the Gaver-Stehfest algorithm.

...ing frequency domain data to the time domain. There is no definitive method of ensuring that the results are without error; however, the following strategy would be sufficient for the examples presented:

1. Calculate the response using the optimal number of terms \( n \), and then \( n + 2 \) terms.

2. If the differences between the two responses are small, then the method is concluded to be adequate.

3. If the errors are significant, the method is inadequate. It may be possible to
Figure A5.7 The same as Fig. A5.5, except 18 terms are used in the Gaver-Stehfest algorithm.

decrease the error by further incrementing \( n \) by 2; however this may result in larger round of errors as \( n \) becomes too large.

Increasing the number of terms and using quadruple precision will improve the accuracy, but the computing costs increase significantly.
Appendix 6

The \( c=1 \) assumption in the Cole-Cole model

A6.0 Introduction

In the model of Appendix 1, the algebra is simplified substantially by setting the Cole-Cole exponent to \( c = 1 \). In this appendix, I show that this is not an overly restrictive requirement, as the response of a conductor with \( c \neq 1 \) can be modelled by a conductor with \( c = 1 \).

A6.1 Constant slope

Table A6.1 shows 10 sets of Cole-Cole exponents, chargeabilities and dc resistances.

<table>
<thead>
<tr>
<th>( c )</th>
<th>( m )</th>
<th>( R_1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>0.44</td>
<td>1.22</td>
</tr>
<tr>
<td>0.2</td>
<td>0.25</td>
<td>1.085</td>
</tr>
<tr>
<td>0.3</td>
<td>0.17</td>
<td>1.041</td>
</tr>
<tr>
<td>0.4</td>
<td>0.13</td>
<td>1.018</td>
</tr>
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<td>0.5</td>
<td>0.108</td>
<td>1.004</td>
</tr>
<tr>
<td>0.6</td>
<td>0.091</td>
<td>0.995</td>
</tr>
<tr>
<td>0.7</td>
<td>0.079</td>
<td>0.988</td>
</tr>
<tr>
<td>0.8</td>
<td>0.069</td>
<td>0.984</td>
</tr>
<tr>
<td>0.9</td>
<td>0.062</td>
<td>0.980</td>
</tr>
<tr>
<td>1.0</td>
<td>0.056</td>
<td>0.977</td>
</tr>
</tbody>
</table>

Table A6.1

These sets of parameters, along with the Cole-Cole time constant \( T = 1.28 \times 10^{-3} \) are used to characterize 10 polarizable wire-loop circuits. The corresponding voltage responses are calculated using the Gaver-Stehfest algorithm, and the decay
Figure A6.2 The voltage responses of a wire-loop circuit with a self-inductance of 0.0001H, a mutual inductance to the transmitter-receiver of 0.1H, a Cole-Cole time constant of $1.28 \times 10^{-3}$s, and dispersive resistances characterized by the Cole-Cole parameters given in Table A6.1.

curves, shown on Fig. A6.1, all display negative transients. The times that the sign reversals occur and the magnitudes and time constants of the negative transients are very similar in each case. The only differences are the shape of the negative decay curves: the responses associated with the smaller $c$ values decay more quickly at first, but are sustained for longer, whilst those associated with the larger $c$ values are more exponential in nature. As these variations are quite subtle and occur at low signal levels, it would be difficult to distinguish between these curves if the data
Figure A6.2 The dispersion in the resistance for the suite of Cole-Cole parameters given in Table A6.1

were corrupted by noise.

The reason for the similarity in the decay curves can be understood from Fig. A6.2, where the resistances for each c value have been plotted as a function of frequency between 10 and $10^5$ Hz. The spectra of the resistances are similar; in fact, each set of parameters was chosen so that the slope of the curve about the characteristic frequency $(1/T) =$ would be the same. This suggests that the slope of the dispersion is what determines the properties of the negative transient. The charge-ability, and the Cole-Cole exponent do effect the decay curve, but only because adjusting these parameters adjusts the slope.
Restricting the choice of \( c \) to \( c = 1 \) in Appendix 1 does not limit the ability of the wire-loop circuit to model negatives. In fact, if a small difference in the shape of the decay curve can be tolerated, the value of \( c \) might have been set to any value, and the negatives could still have been modelled. The only limitation is that the corresponding values of \( m \) and \( R_1 \) are affected; the values used under a \( c = 1 \) approximation will be less than those used when \( c \neq 1 \).

The difficulty of uniquely resolving Cole-Cole parameters from data is clearly illustrated by this example.

### A6.2 Constant limits

Table A6.2 shows another suite of Cole-Cole parameters.

<table>
<thead>
<tr>
<th>( c )</th>
<th>( m )</th>
<th>( R_1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>0.45</td>
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<tr>
<td>0.2</td>
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<td>0.3</td>
<td>0.19</td>
<td>1.05</td>
</tr>
<tr>
<td>0.4</td>
<td>0.15</td>
<td>1.03</td>
</tr>
<tr>
<td>0.5</td>
<td>0.135</td>
<td>1.02</td>
</tr>
<tr>
<td>0.6</td>
<td>0.123</td>
<td>1.013</td>
</tr>
<tr>
<td>0.7</td>
<td>0.116</td>
<td>1.009</td>
</tr>
<tr>
<td>0.8</td>
<td>0.111</td>
<td>1.006</td>
</tr>
<tr>
<td>0.9</td>
<td>0.107</td>
<td>1.004</td>
</tr>
<tr>
<td>1.0</td>
<td>0.105</td>
<td>1.0027</td>
</tr>
</tbody>
</table>

Table A6.2

Figure A6.3 is a plot of the resistance spectra for these parameters. The values of the resistance at the extremes of the frequency range are constrained to be the same, which is similar to constraining the average slope over the whole frequency range. Note however that the slopes at the characteristic frequency are different. Although the disparity in the corresponding response curves is small (Fig. A6.4), it is greater than the disparity on Fig. A6.1, so the parameter which has the strongest effect on the shape of the decay curves is not the ‘average slope’, but the slope at
Figure A6.3 The dispersion in the resistance for the suite of Cole-Cole parameters given in Table A6.2

the characteristic frequency.

A6.3 Conclusions

A negative transient associated with a resistivity dispersion described by an arbitrary $c$ value can be modelled using a dispersion with another $c$ value (say $c = 1$), provided that the slopes of the dispersions at the characteristic frequency are the same. If negative transients are observed in the field, and they are modelled using a Cole-Cole impedance, the results should be interpreted with care, since the
The voltage responses of a wire-loop circuit with a self-inductance of 0.0001H, a mutual inductance to the transmitter-receiver of 0.1H, a Cole-Cole time constant of $1.28 \times 10^{-3}$s, and dispersive resistances characterized by the Cole-Cole parameters given in Table A6.2.

parameter values are not unique.

The best way of characterizing a dispersion is to specify the slope at the characteristic frequency. Quoting the dispersions as a percent variation over the EM frequency range (e.g. 7% variation in the conductivity between 10 and 10 000Hz) is equivalent to describing the 'average slope', and although this is not as good as specifying the slope at the characteristic frequency, it is better than merely stating the chargeability.
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