A Novel Method Of Computing The EM Response Of A Conductive Plate In A Conductive Medium

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by

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ABSTRACT

This thesis describes the development of a novel, simple, and robust integral equation technique to compute the electromagnetic response of a three dimensional plate-like conductor embedded in a conductive host. Thin sheet or plate-like conductors are mathematical models of many base metal mineral deposits, including mineralized veins, shears, and dipping strata. Solutions for the thin sheet models are usually formulated using the integral equation technique with the electric field as the variable. The appeal in this approach is its simplicity in derivation and the ease in interpretation of solutions. However, many of the approximate computer algorithms derived directly from the exact theory have suffered to some extent from the defect that the solutions fail to generate strong vortex and tend to become unstable when the host conductivity becomes too small. The determination of the scattered electric field within the plate requires the solution of an integral equation in which inductive and channeling terms couple together. The magnitudes of the terms vary with the selection of parameters. They can differ by many orders of magnitude particularly for very resistive hosts. The accuracy with which the terms can be derived numerically depends on the selection of appropriate basis functions. A small set of functions which can describe accurately both the current flows is difficult to find.

There are several powerful techniques to circumvent the problem, but these methods usually result in complicated and costly software. Our approach to resolve the problem is to deform the thin plate so as to create an equivalent network model for which the channeling and vortex currents can be described equally well by one simple set of unknowns. The new model uses a set of basis functions with a spatial
representation bearing a strong resemblance to a lattice composed of many thin conductive strips, called elements, arranged to look like a bottle-box with the top and base removed. It has finite length, width, and thickness, and consists of $m \times n$ basic square loops.

In order to check our formulation, the EM responses of the new model in a conductive whole space are compared with equivalent responses of a thin disk derived by West and Edwards (1985) for a wide range of induction and channeling response parameters. Other tests of the lattice plate have shown that the solutions are accurate, stable, and robust, and the algorithm is efficient. The lattice plate is used in forward modeling to study the EM responses of plate-like conductors in the earth and in the sea floor to various Controlled Source Electromagnetic sounding systems.
ACKNOWLEDGEMENTS

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Last but not the least, I acknowledge the generous financial assistance provided by the University of Toronto during the course of the study.
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LIST OF SYMBOLS

E  Electric field intensity
J  Electric current density
H  Magnetic field intensity
B  Magnetic induction
A  Magnetic vector potential
Z  Impedance or Scattering matrix
R  Real part of impedance matrix
X  Imaginary part of impedance matrix
J_n  Eigencurrents
G(r | r')  Green's tensor for electric field
g_s  Green's function from source to target
g_r  Green's function from target to receiver
f(r)  Functions
J_0  Zero order Bessel function
J_1  First order Bessel function
U  Electric scalar potential
M  Magnetic dipole moment
P  Electric dipole moment
sigma_o  Host conductivity
rho_o  Host resistivity
sigma_e  Host conductivity
rho_L  Layer resistivity
L  Layer thickness
S  Plate conductance
omega  Angular frequency
sigma_o  Host conductivity
Delta  Skin depth
theta  Propagation constant for medium
f  frequency
mu  Permeability of free space
epsilon  Permittivity of free space
lambda  Wave number
lambda_n  Eigenvalue
delta(r - r_o)  Dirac delta function
epsilon(r)  Equivalent source function
i_p  Impressed azimuthal current in the ring
m  Row index for the lattice plate
n  Column index for the lattice plate
delta  Length of the element
tau  Width of the element
alpha_p  Plate induction number
beta_p  Plate channeling number
gamma_h  Host induction number
a  Radius of the ring
w  Width of the ring
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Chapter 1

Introduction

1.1 Mineral prospecting and numerical methods

Many deposits and geological structures of economic and research interest, such as oil and gas, base metal minerals, faults and fracture zones, are buried beneath the earth’s surface and inaccessible to direct observation. The detection of these targets depends upon variations of those physical characteristics which differentiate them from the host media, and can be achieved by using geophysical methods. For example, the variations of elastic properties, density, magnetic permeability, and electrical conductivity can be detected by seismic, gravity, magnetic, and electrical techniques respectively. For mineral explorations, the most frequently applied geophysical prospecting technique is the electrical method, because of all the physical parameters of the mineral deposits, electrical conductivity has the largest range of variation, covering at least seven decades. Electrical prospecting methods detect the remote surface effects produced by electrical current flow in the ground, and include techniques such as self-potential, magnetotellurics, audio-frequency magnetic fields (AFMAG), resistivity, electromagnetic (EM), and induced polarization (IP). The most commonly used electrical technique in mineral exploration is the EM method, which involves the propagation of time-varying low-frequency electromagnetic fields, generated by either a natural or a man-made source. Most mineral deposits that have been explored and exploited are usually buried in the ground; however, in recent years, the targets of EM exploration have expanded to include polymetallic sulphide deposits buried beneath the sea floor (Francheteau et al. 1979).
(Rodger and Eastham 1982).

In contrast to the DE methods which compute solutions at every mesh point, IE methods require the solution of the unknown fields inside the conductor only. As a result, a considerable savings in computing time can be archived. The unknowns are usually expanded in terms of a suitable set of orthonormal basis functions and the coefficients of these functions are found either by equating the tangential electric fields or by variational methods. One disadvantage in the IE methods is that they usually require explicit and complicated Green's functions which may be difficult to compute. Engineering applications of the IE method include those of Preston and Reece (1982), Davey and Barnes (1983), McWhirter et al. (1982), and Simkin (1982) for eddy current problems. Since IE methods are most useful for computing the response of isolated conductors, they are often used to solve problems involving thin sheets or prisms, which are good approximations to conductive mineralized veins, shears, or dipping strata in the earth. The theories of thin plate and the related prismatic structure problems using the IE method have been thoroughly developed and have appeared continually in the geophysical literature. Existing solutions for prismatic structures include those for 2D prisms by Hohmann (1971) and Annan (1974), and those for 3D prisms in conductive host Hohmann (1975), Wannamaker et al. (1984), San Filipo and Hohmann (1985), and Newman and Hohmann (1988). Solutions for 3D thin sheets include those for thin plates in free space by Lamontagne and West (1971), Annan (1974) and those for thin sheets in conductive medium by Lajoie and West (1976), Hanneson and West (1984), and Walker (1988).

1.2 Numerical difficulty for thin sheets

One of the disturbing and widely recognized problems for three dimensional models using the IE method is the failure of many approximate computer algo-
vortex in the plate when the medium is resistive. For example, the prism model can produce accurate results if the contrast between the conductivity of the host and the plate conductance is no greater than 200 (Newman and Hohmann, 1988).

An approach adopted by Lajoie and West 1976 to circumvent the problem is to describe two current systems in a thin sheet separately using auxiliary divergence-free and curl-free potential. A similar method, used by Walker 1988, describes the galvanic terms by the curl-free electric scalar potential and the inductive terms by the divergence-free magnetic field. To resolve the difficulty in prisms, SanFilippo and Hohmann 1985 applied specially constructed concentric and divergence-free local basis functions, known as 'current tubes', to represent closed currents in a prism. Even though these remedies have succeeded admirably to improve the solution accuracy and stability, the resulting algorithms and software have become more costly and complicated than before. At the present, most numerical models are still quite computationally expensive, cumbersome, and inflexible for routine applications (West and Edwards, 1985). Quite obviously, there is a need for a new approach which not only provides accurate solutions for a wide range of host parameters, but also maintains the cost of computing at a reasonable level.

1.3 A Novel Solution

The new approach to the problem is quite simple. From experience and intuition, we (Edwards and his EM group) learned that loop conductors in free space are very good in describing closed currents, and linear plate-like conductors in conductive medium are equally capable of describing channeling currents. By induction, one could imagine that a 'plate' endowed with the characteristics of both loop and thin strip conductors will likely describe both the inductive and channeling currents equally well. To find a plate which satisfies the above requirements is not as difficult as one might think. The easiest and safest way to do this without having
describe a small closed current, and collectively, it is capable of representing very complex vortex currents. If, however, the response of the plate are dominated by the channeling currents, the lattice can also be considered as consisting of many thin strips which are capable of describing many different kinds of channeling currents in the plate. Notice that a $1 \times 1$ lattice looks very similar to one of the concentric 'current tubes' (SanFilipo and Hohmann, 1985; Newman and Hohmann, 1988) used to constrain currents to flow within the plate in purely inductive situations.

![Network Conductivity Structure](image)

**Figure 1.2** A conductivity structure of a two-component mineral deposit which can be represented by the lattice plate.

Just like the existing thin sheets are sufficient mathematical models for certain
the plate in an infinite conductive host to a uniform external electric field. The second case deals with the galvanic and inductive responses of the plate in conductive whole space, excited by an electric dipole source and a magnetic dipole source respectively. The results of the second study are compared quantitatively with the results obtained by investigating the response of a circular thin-disk conductor by West and Edwards (1985). In addition, the efficiency of the new algorithm in terms of the CPU time is discussed.

In Chapter Four, the EM response of a vertical plate, embedded in a conductive half-space and then in a two-layered earth, to a fixed Slingram system is investigated. The results are presented in the form of Argand phase diagrams. The Slingram system, consisting of two co-planar, horizontal magnetic loops, is a widely used, land-based EM survey system. In order to obtain results useful for the interpretation of real EM sounding data collected on the ground, several theoretical profiles of a vertical plate in the earth to a moving Slingram array are computed.

In Chapter Five, the new plate is used in forward modeling of plate-like mineral deposits in crust below the sea water. In this chapter, the responses of a vertical plate embedded in the sea floor to two different EM systems are investigated. The first system is the horizontal, coaxial, magnetic dipole-dipole system, or HRHR whilst the second system is the horizontal in-line electric dipole-dipole system, or ERER. Cheesman et al. (1987) have shown that the HRHR and ERER are the only suitable systems for marine EM explorations. A number of theoretical EM response profiles over a vertical plate to the two systems are computed for the purpose of interpreting real EM data collected on the sea floor.
Consider an arbitrary system occupying a bounded region $V$, which has an input $f$, an output $u$, and a linear differential operator $\alpha$. The output $u$ satisfies the differential equation

$$\alpha(u) = f,$$  \hspace{1cm} (2.1.1)

and is subject to a boundary condition at the surface $S$ enclosing the region $V$. The Green's function in $V$ with a homogeneous boundary condition at $S$ satisfies the following equation

$$\alpha(G(r \mid r_0)) = \delta(r - r_0),$$  \hspace{1cm} (2.1.2)

where $r, r_0$ represent the coordinates of the observation point and the source point respectively, and $\delta(r - r_0)$ is the Dirac delta function representing the impulse. For certain classes of differential operators, the Green's function $G$ obeys the reciprocity condition, which can be stated as

$$G(r \mid r_0) = G(r_0 \mid r).$$  \hspace{1cm} (2.1.3)

The general solution of equation (2.1.1) can be expressed as a convolution integral of the impulse response or Green's function in $V$ and the system input $f$,

$$u = \int_V G(r \mid r_0) f(r_0) \, dV.$$  \hspace{1cm} (2.1.4)

In the EM problems of interest, the conductive target is usually occupied in a region $V'$ which is usually located in a much larger host medium $V$. The smaller system can be treated mathematically as an equivalent source; therefore, it is natural to describe the problem with two separate differential equations, one for the host, and the other for the target

$$\alpha(u(r)) = f(r) \quad r \in V, r \notin V'$$  \hspace{1cm} (2.1.5)

and

$$\alpha(u(r)) + \beta(u(r)) = f(r) \quad r \in V, r \in V',$$  \hspace{1cm} (2.1.6)
Fredholm integral equation of the second kind is derived directly from the Maxwell equations, in terms of the electric field. The following is a the derivation of the integral equation for a conductor of conductivity \( \sigma_1 \) in a conductive whole space of conductivity \( \sigma_o \).

Assuming \( e^{i\omega t} \) time dependence, the electric and magnetic fields in the conductor are related via Faraday’s and Ampere’s laws, i.e.

\[
\nabla \times \mathbf{E} = -i\omega \mu \mathbf{H},
\]

(2.2.1)

and

\[
\nabla \times \mathbf{H} = \sigma_1 \mathbf{E} + \mathbf{J}_e,
\]

(2.2.2)

where \( \mathbf{J}_e \) is the current in the conductive body impressed by an external source. All the boldface quantities are vectors. Since the ratio \( \omega \epsilon / \sigma \ll 1 \), the magnetic effects of displacement current can be neglected in comparison with those of the conduction current, a quasi-static approximation. The equivalent source in this problem is the scattered current density in the body, \( \mathbf{J}_s \), which is defined as

\[
\mathbf{J}_s = (\sigma_1 - \sigma_o)\mathbf{E}.
\]

(2.2.3)

Using (2.2.3), expression (2.2.2) can be rewritten as

\[
\nabla \times \mathbf{H} = \sigma_o \mathbf{E} + \mathbf{J}_e + \mathbf{J}_s.
\]

(2.2.4)

Taking the curl of (2.2.1) yields

\[
\nabla \times \nabla \times \mathbf{E} + \alpha^2 \mathbf{E} = -i\omega \mu (\mathbf{J}_e + \mathbf{J}_s),
\]

(2.2.5)

where \( \alpha^2 = i\omega \mu \sigma_o \). Moreover, \( \mathbf{E} \) can be separated into two independent parts,

\[
\mathbf{E} = \mathbf{E}_e + \mathbf{E}_s,
\]

(2.2.6)
2.3 Reduction of the integral equation

Since objective of this thesis is to develop a simple numerical algorithm to solve the electric field integral equation directly, the reduction procedure chosen should be as simple as possible. One of the simplest and widely used reduction procedures is the method of point matching or point collocation with sectional pulse functions (Harrington, 1968).

Rewriting (2.2.10) in terms of the anomalous current $J_s$, one obtains

$$E(r) - \int_{V'} G(r \mid r') \cdot J_s(r') \, dV' = E_e(r). \quad (2.3.1)$$

To reduce the integral equation into a discrete matrix form, the conductor has to be divided into a finite number of subsections. For a thin sheet, each subsection is a rectangular element with infinitesimal thickness, and for a prism, each subsection is a solid brick. The anomalous current can also be discretized in the same fashion. Let the anomalous current $J^s$ be expanded in terms of basis functions,

$$J_s = \sum_{n=1}^{N} j_n \delta_n \, u_n, \quad (2.3.2)$$

where $j_n$ and $u_n$ are the magnitude of current density and the unit vector in the $n^{th}$ subsection respectively, and

$$\delta_{mn} = \begin{cases} 1, & m = n \\ 0, & m \neq n \end{cases}.$$  

In mathematical parlance, the discrete function $\delta_{mn}$ is called the Kronecker delta. An inner product $<f, g>$ of two functions $f$ and $g$ can be written as

$$<f, g> = \int_{V'} fg \, dV'.$$  

For the EM problem treated here, a useful operator $L$ in the domain $V'$ can be defined as

$$L(f) = \int_{V'} G(r \mid r') \cdot f \, dV'.$$  

(2.3.4)
where $G_S$ and $G_I$ are Green's functions or interaction functions for the static and inductive terms.

For the conventional thin sheet, the anomalous surface current density $\mathbf{J}_s$ is a continuous 2D quantity within the plate. For the lattice, $\mathbf{J}_s$ flows only in the network of elements, and in a particular element, it can flow in only one direction, depending on the orientation of the element. The electric field $E_m(x_m, y_m)$ inside the $m^{th}$ subsection is given by Ohm's law,

$$E_m = \frac{j_m}{S_m},$$  \hspace{1cm} (2.3.8)

where $S_m$ and $j_m$ are the conductivity-thickness product and surface current density for the $m^{th}$ subsection respectively. The equation (2.3.7) can be cast into a matrix equation,

$$Z \mathbf{J}_s = \mathbf{E}_e,$$  \hspace{1cm} (2.3.9)

where

$$Z_{mn} = \frac{\delta_{mn}}{S_m} - l_{mn}.$$  

$\mathbf{J}_s$ and $\mathbf{E}_e$ are the anomalous surface current density and impressed electric field vectors of dimension $N$ respectively. Compare equation (2.3.9) with the standard form of Ohm's law, it is clear that the matrix $Z$ has the dimension of resistivity; therefore, it is called the impedance matrix. The impedance or scattering matrix can be separated into real and imaginary parts,

$$Z = R + iX,$$  \hspace{1cm} (2.3.10)

where

$$R_{mn} = \frac{\delta_{mn}}{S_m} - \int_{L'} G_S(m \mid n) \, dL',$$

and

$$X_{mn} = \omega \mu \sigma \int_{S'} G_I(m \mid n) \, dS'.$$
modal solution for the anomalous current \( \mathbf{J}_s \) on the lattice can be calculated in the manner outlined below: assume \( \mathbf{J}_s \) to be the linear combination of the eigencurrents

\[
\mathbf{J}_s = \sum_{n=1}^{N} \alpha_n \mathbf{J}_n ,
\]

where the \( \alpha_n \) are coefficients to be determined, and the eigencurrents \( \mathbf{J}_n \) are defined by

\[
\mathbf{ZJ}_n = \lambda_n \mathbf{J}_n ,
\]

where \( \lambda_n \) is the eigenvalue. Substituting (2.4.1) into (2.3.11), and using the linearity of the impedance operator \( \mathbf{Z} \), one obtains

\[
\sum_{n=1}^{N} \alpha_n \mathbf{ZJ}_n - \mathbf{E}_e = 0 .
\]

Next, taking the inner product defined in (2.3.3) of the above equation with each \( \mathbf{J}_m \), gives a set of equations

\[
\sum_{n=1}^{N} \alpha_n < \mathbf{J}_m, \mathbf{ZJ}_n > - < \mathbf{J}_m, \mathbf{E}_e > = 0 .
\]

Using the orthogonality relationship of the eigencurrents, (2.4.4) is reduced to

\[
\alpha_n \lambda_n = < \mathbf{J}_n, \mathbf{E}_e > .
\]

Now, the modal solution of current \( \mathbf{J}_s \) is given by

\[
\mathbf{J}_s = \sum_{n=1}^{N} \frac{< \mathbf{J}_n, \mathbf{E}_e > \mathbf{J}_n}{\lambda_n} .
\]

Harrington and Mautz 1971 have shown that modal solutions converge as the number of eigencurrents is increased, and they are in good agreement with the solutions obtained using the direct matrix inversion method.
### Table 2.4.1b: Eigenvalues for Inductive Eigencurrents

<table>
<thead>
<tr>
<th>n</th>
<th>$\lambda_n$ (Ω · m)</th>
<th>max. current (A)</th>
</tr>
</thead>
<tbody>
<tr>
<td>13</td>
<td>$(2.0 \times 10^{-2}, 4.92 \times 10^{-3})$</td>
<td>0.261</td>
</tr>
<tr>
<td>14</td>
<td>$(2.0 \times 10^{-2}, 3.73 \times 10^{-3})$</td>
<td>0.266</td>
</tr>
<tr>
<td>15</td>
<td>$(2.0 \times 10^{-2}, 3.46 \times 10^{-3})$</td>
<td>0.351</td>
</tr>
<tr>
<td>16</td>
<td>$(2.0 \times 10^{-2}, 3.01 \times 10^{-3})$</td>
<td>0.238</td>
</tr>
<tr>
<td>17</td>
<td>$(2.0 \times 10^{-2}, 3.02 \times 10^{-3})$</td>
<td>0.253</td>
</tr>
<tr>
<td>18</td>
<td>$(2.0 \times 10^{-2}, 2.78 \times 10^{-3})$</td>
<td>0.329</td>
</tr>
<tr>
<td>19</td>
<td>$(2.0 \times 10^{-2}, 2.02 \times 10^{-3})$</td>
<td>0.252</td>
</tr>
<tr>
<td>20</td>
<td>$(2.0 \times 10^{-2}, 2.64 \times 10^{-3})$</td>
<td>0.242</td>
</tr>
<tr>
<td>21</td>
<td>$(2.0 \times 10^{-2}, 2.63 \times 10^{-3})$</td>
<td>0.329</td>
</tr>
<tr>
<td>22</td>
<td>$(2.0 \times 10^{-2}, 2.57 \times 10^{-3})$</td>
<td>0.259</td>
</tr>
<tr>
<td>23</td>
<td>$(2.0 \times 10^{-2}, 2.09 \times 10^{-3})$</td>
<td>0.232</td>
</tr>
<tr>
<td>24</td>
<td>$(2.0 \times 10^{-2}, 2.11 \times 10^{-3})$</td>
<td>0.256</td>
</tr>
</tbody>
</table>
Figure 2.4.1. (b) Eigencurrents
The complex eigenvalues, which have the same dimensions as the impedance matrix, i.e., resistivity Ω·m, are shown in Table 2.4.1a, and 2.4.1b, the former containing those for channeling eigencurrents, and the latter for inductive eigencurrents. In the table, the maximum current, which is the largest current of a set of eigencurrents, are also listed. For a discrete plate, the total number of eigencurrents is equal to the total number of elements, and for a 5 × 4 plate, the number is 49. Only 24 eigencurrents are chosen for display, and half of those are in the channeling mode, with large eigenvalues, and the other half are in the inductive mode, with relatively small eigenvalues. The fact that the eigencurrents can be clearly separated into channeling and inductive classes indicates that the algorithm and software for the new plate are working properly as expected. The first salient feature in the eigencurrents is that they are orthogonal to each other. The other feature is the amount of detail about the currents that the lattice can provide down to the scale of the basic square loop. The lattice can describe closed currents of the size of a basic loop on any parts of the plate, including the four corners, in a host that can be very resistive. In this particular example, the classification of eigencurrents seems possible by simply looking at the magnitude of their eigenvalues. However, the magnitude of eigenvalues is not a reliable indicator for the classification the eigencurrents, since it changes with the change of parameters.

The method to determine the characteristic of an eigencurrent by its eigenvalue is discussed in Harrington and Mautz (1971), who developed a comprehensive theory of characteristic modes for conductors by considering a weighted eigenvalue problem with real eigenvalues. The following is their approach to the eigenvalue problem:

\[(R + iX)J_n = \nu_n RJ_n ,\]  \hspace{1cm} (2.4.7)

The above is a weighted eigenvalue problem with weight operator \(R\), the real part
dipole source along the axis. The ring, along with a transmitter and a receiver, is shown in Figure 2.5.1. The parameters for the ring $a$ and $w$ are chosen such that the total surface area and the cross section of the ring equal to those of a square loop; i.e., the following relations must hold, $\pi a^2 = \delta^2$ and $2\pi aw = 4\delta r$. Due to the symmetry of the configuration, the impressed current $i \phi$ is independent of azimuth, and it is calculated in Appendix B,

$$i \phi = -(\frac{i\alpha}{1 + i\alpha})\left(\frac{Mwg_s}{2\pi f}\right).$$

(2.5.1)

where $M$ is the magnetic dipole moment of the source, $g_s$, and $\alpha$ are the frequency dependent Green’s function from source to target, and the induction parameter for
electric field \( E_s \) just outside the strip is

\[
E_s = -\frac{2i_x \delta}{4\pi \sigma \delta^3} = -\frac{i_x}{2\pi \sigma \delta^2}.
\]

(2.5.4)

Apply the boundary condition for the tangential electric fields, one obtains

\[
E_x - \frac{i_x}{2\pi \sigma \delta^2} = \frac{i_x}{\tau S}.
\]

(2.5.5)

and re-arrange the terms to give \( i_x \),

\[
i_x = E_x 2\pi \sigma \delta^2 \left( \frac{\beta}{1+\beta} \right),
\]

(2.5.6)

where

\[
\beta = \frac{\tau S}{2\pi \sigma \delta^2},
\]

which is called the channeling number for the thin strip. To generalize, the secondary channeling response of the thin strip in a conductive whole space at the receiver \( E_r \) can be written in a way similar to (2.5.2), if the source is an electric dipole,

\[
E_r = \frac{P \delta^3}{2\pi \sigma} g_s \left( \frac{\beta}{1+\beta} \right) g_r.
\]

(2.5.7)

where \( P \) is the the electric dipole moment for the source. Again, the channeling response of a thin strip is separated into three product terms. The first and third terms are Green's functions from the source to the conductor, and from the conductor to the receiver, and they are frequency dependent. The real function in the second term characterizes the channeling effect of the strip. As \( \beta \) increases from zero, the channeling effect increases quickly, and then approaches saturation, a state in which the channeling effect stays nearly constant and almost independent of \( \beta \).
the plate is more conductive than the host, then $E_i < E_e$. Since the host has finite conductivity, the primary electric field will cause currents to flow, which become more concentrated in the vicinity of the more conductive plate. These currents are called galvanic or channeled currents, which are denoted by $J_g$, as shown in Figure 3.1.1b. According to Ampere's Law, the anomalous galvanic current will induce the magnetic induction $B_g$ in the plate, which is shown in Figure 3.1.1b. Once again, the plate reacts to minimize the development of anomalous magnetic field, achieved through the induction of vortex currents $J_v$ in the plate as shown in Figure 3.1.1c.

If the host is relatively conductive, one would expect the induced currents in the plate are mostly galvanic. If the host is resistive, the currents in the plate will be mostly inductive. To demonstrate this assertion, a series of impressed current flow patterns were obtained for different host resistivity values. The numerical model consists of a 5 x 5 square plate with $\delta = 1m$ and $\tau = 0.5m$, excited by a uniform harmonic electric field $E_e$.

Similar to the characterization for the EM responses of a ring and a thin strip by induction and channeling parameters, as demonstrated in Section 2.5, the plate response can also be characterized by two dimensionless parameters: the plate channeling parameter $\beta_p = S/\sigma_o a$, and the plate induction parameter $\alpha_p = \omega \mu S a$, where $\omega$ is the angular frequency of the source, $S$ is the plate conductance, and $a$ is the characteristic size of the plate. The impressed currents $J_n$ ($n = 1, 60$), computed by solving the matrix equation (2.3.11), and their divergence are shown in Figure 3.1.2 for the case of $\beta_p = 50$ and $\alpha_p = 0.06$. Each arrow in the figure points the direction of current flow and the length of the arrow represents the magnitude of the current in each element. The dashed and solid circles indicate where the net currents enter or leave the plate respectively, while the size of the circles represent the magnitude of divergence at each node. The maximum value of divergence at the nodes in Figure 3.1.2 is 1.78 A/m. It is quite clear by inspection that the impressed
Figure 3.1.2 Impressed currents and current divergence in a plate in a conductive host of resistivity $\rho_o = 1 \ \Omega \cdot m$.

Figure 3.1.3 Impressed currents and current divergence in a plate in a conductive whole space with resistivity $\rho_o = 1000 \ \Omega \cdot m$.

currents are galvanic.

The impressed currents in the plate, with plate channeling number 0.05 while the plate induction number remains unchanged, are shown Figure 3.1.3, the currents near the corners of the plate begin to curl around, showing a pattern of two vortices.
terms in the scattering matrix is never divided by vanishingly small host conductivity. This is made possible through short range approximations to interactions between elements. The approximation also renders the separation of the galvanic terms and the inductive terms possible, thus decoupling the error in the inductive response from the error in the galvanic response.

3.2 Plate response under dipole sources

In order to check our formulation, the EM responses of the lattice plate in a conductive whole space under electric and magnetic dipole excitations are compared quantitatively here with equivalent responses of a circular thin disk derived by West and Edwards 1985. By equivalent responses we mean the EM responses of the lattice and the disk both of which contain the same amount of conductive material. The other reason to compare the responses is that a disk can describe concentric vortex currents naturally. In fact, the purpose of using a circular thin disk by West and Edwards is to investigate how an infinite conductive host medium influences the characteristic current channeling and local eddy-current induction responses.
secondary electric field $E^s_z$ at the source location due to the plate is

$$E^s_z = \frac{h}{4\pi\sigma_o} \sum_{i=1}^{N} \frac{r_i m_i}{R_i^5} (3 + 3\alpha R_i + \alpha^2 R_i^2) e^{-\alpha R_i}, \quad (3.2.2)$$

where $m_i = J_i \tau \delta$ is the current moment of the $i^{th}$ element, and the $J_i$ are obtained by solving the matrix equation (2.3.11).

![Diagram showing typical impressed channeling currents and the associated current divergence in the plate excited by an electric dipole.](image)

The electric field at the $i^{th}$ element due to an alternating magnetic dipole source of moment $M$ is (West and Edwards):

$$E_i(r_i) = -\frac{i\omega \mu M r_i}{4\pi R_i^3} (1 + \alpha R_i) e^{-\alpha R_i}, \quad (3.2.3)$$

and the vertical secondary magnetic field $H^s_z$ at the source location is

$$H^s_z = -\frac{1}{4\pi} \sum_{i=1}^{N} \frac{r_i m_i}{R_i^3} (1 + \alpha R_i) e^{-\alpha R_i}. \quad (3.2.4)$$

The impressed currents and the associated current divergence in the plate excited by a downward directed electric dipole are shown in Figure 3.2.2. Notice the
as predicted by equation 2.5.7. The effect of the host on the plate response is manifested by the phase rotation and amplitude attenuation, which are caused by the propagation loss of the fields in passing through the conductive host, as the host induction parameter $\gamma_h$ increases. Strong phase rotation occurs for $\gamma_h > 1$ and
plate's induction number $\alpha_p = \omega \mu S a$. In Figure 3.2.5, the Argand plot of $H_z^s$ as a function of $\gamma_h$ and $\alpha_p$. Again, the plot indicates two major effects: the inductive saturation of the plate and phase rotation and amplitude attenuation by the host. The free-space response is a semi-circle in the third quadrant for the same source-receiver configuration and it approaches saturation (inductive limit) for $\alpha_p > 100$, evidently predicted by equation 2.5.2. The free-space response arc is severely phase rotated for $\gamma_h > .3$ and strongly attenuated for $\gamma_h > 3$. The amplitude attenuation and phase rotation are seen as before, a consequence of the behaviour of the Green's functions which couple the source to the plate and the disk to the receiver (West and Edwards, 1985). They checked the assertion by plotting the product of the normalized Green's functions for source and receiver coupling to a point on the disk together with one of the disk's responses. The two curves keep in perfect step with one another, clearly validating the assumption.

The comparison between the galvanic responses of the lattice plate and of the circular thin disk is quite good in unsaturated regions, where the $\gamma_h$ is small, in the sense the skin depth $\Delta$ is several orders of magnitude larger than the size of an element. The skin depth and the host induction parameter are related in the following manner: $\Delta = h \sqrt{2/\gamma_h}$. As the host induction number becomes large, for example, $\gamma_h = 7$, the square loop size is only one third of skin depth. Therefore, the distance between elements has become comparatively large, so that correct description of the field by pulse basis functions becomes difficult. As a result, the comparison is poor in saturated regions. If the loop size is reduced to one sixth of skin depth by using a $10 \times 10$ plate, whose responses are indicated by open circles, then the comparison becomes much better.

The next set of tests address the question of uniform convergence in terms of the plate responses as a function of the number of elements in the plate. The convergence test is conducted in the following manner: the secondary electric and magnetic field are computed for a particular host induction number while the plate channel-
induction numbers are 1, 3, 10, 30, 100, and 200. The host induction number is set very close to the free space limit, thus making the skin depth very large. The inductive responses of the plate show a definite trend of uniform convergence as the number of elements increases.

In Figure 3.2.8, the results of the self-consistent convergence tests are displayed. The relative errors of typical galvanic and inductive responses with respect to the 10 × 10 responses are plotted versus the number of elements in the plate. The purposes of the display are first to estimate the rate of convergence for the responses of the plate, which is approximately 1/N, and second to shown the robustness of the
Figure 3.2.9 The CPU time for typical response as a function of N

give very accurate results.

3.3 Summary

So far, we have been able to show that the new algorithm for modeling the EM response of a conductive plate in a conductive medium is quite robust, and should be able to run on a PC to give approximate but nevertheless physically correct results using a small number of unknowns. The smooth transition of the scattering current in the plate from channeling mode to inductive mode as the resistivity of the host medium increases confirms that the solutions of the plate can indeed model the galvanic and inductive currents equally well. The robustness of the numerical
Chapter 4

HLEM responses of the plate in the earth

One of the most popular EM exploration techniques on land is the moving source horizontal loop method, also known as the Slingram, meaning two carried loops in Swedish. For the Slingram system, both transmitter and receiver are moved, a fixed spacing between them being maintained by a cable. The coils, 0.5 to 1 m in diameter, are coplanar and almost always oriented to detect the vertical component. The readings taken in the vicinity of a conductor are the in-phase quadrature components of the secondary field, generally in percent of the primary field. The traverses made by a Slingram are perpendicular to the strike of conductor where possible, and the readings are symmetric, one of the advantages of the system, and plotted for the mid-point the system. Due to the portability of the system, the transmitter power is limited to a few watts so that the maximum depth for a good conductor is often considered to be half of the coil spacing, in the range of few hundred meters. For more description on the Slingram system, please refer to Telford et al. 1976. In this chapter, the Horizontal Loop EM (HLEM) response of the plate in a conductive half-space will be investigated.

4.1 Half-space and overburden effects

Although the computation of EM responses of the lattice plate in a conductive whole space is useful for the understanding some of its important properties and the effect of the host, it is not directly applicable to real geophysical problems because conductors to be modeled are buried in the earth which is better approximated by a conductive half-space. In this section, the EM responses of the plate to the Slingram system for a wide range of host induction numbers are computed in order
distance from the source, $\lambda$ is the radial wavenumber, and $J_1$ is the first order Bessel function. Once the external electric field at each element is calculated, the matrix equation in (2.3.11) can then be solved for the scattered currents in the plate. The secondary magnetic field $H_2^s$ at the receiver location is calculated using the Reciprocity Theorem, described in Jordan and Balmain 1968 (p.481). The Hankel transforms in the above equation are evaluated using the fast algorithm developed by Anderson 1982. Typical impressed currents and current divergence
host medium is conductive, a current circulation is also induced in the host medium flowing in horizontal circles about the transmitter. The more conductive plate attracts some of this external current, creating an additional scattering current in the plate and correspondingly increasing the anomaly intensity. The anomaly
Figure 4.1.5 The Slingram system along with a plate in a half-space with an overburden conductive layer of thickness 1m. The resistivity of the half-space is $\rho_0 = 3 \ \Omega \cdot m$. The plate is $5 \times 5$, and the dimensions of an element in the plate are $\delta = 1m$ and $\tau = 5/12m$. The source is a harmonic magnetic dipole with frequency $f = 1000Hz$. The loop separation is 10m. The distance from the top of the plate to the bottom of the overburden is 5m.

The EM responses of the plate in a two-layered earth to the Slingram system are also computed for the investigation of effect of the conductive overburden. The system configuration is shown in Figure 4.1.5. A $5 \times 5$ plate is buried in a conductive half-space with host induction number $\gamma_h = 0.04$. The overburden induction
Figure 4.1.7 The HLEM sounding system for the investigation of the effect of different source and receiver locations.

is given by (Kaufman and Keller 1983):

\[
E_\phi = \frac{i \omega \mu M}{4\pi} \int_0^\infty \frac{2\lambda R}{\lambda R + m_1} J_1(\lambda r) \, d\lambda ,
\]

(4.1.2)

where

\[ R = \coth[m_1 T + \coth^{-1}(m_1/m)] , \]

\[ m_1 = \sqrt{\lambda^2 + i \omega \mu \sigma_1} , \text{ and } m = \sqrt{\lambda^2 + i \omega \mu \sigma_0} . \]

The response ratios at the receiver
Figure 4.1.8 The anomalous magnetic fields $H'_x/H''_x$ of a vertical plate in a conductive half-space to the HLEM system as a function of receiver locations for various plate induction parameter values.

to push the field to its inductive saturation limit. The saturation occurs when $\alpha_p$ approaches 100.
separation is the ratio $H_z^s/H_z^p$, where $H_z^s$ is the secondary vertical magnetic field due to the plate and $H_z^p$ is the primary vertical magnetic field due to the presence of the half-space. Since the response is symmetrical with respect to the plate, the real part of the response is plotted only for positive $X$ locations and the imaginary part of the response is plotted only for negative $X$ positions. The first set of response curves for host induction numbers are displayed in Figure 4.2.2, while the plate induction number is fixed at 377. The other relevant parameters are:

$$l = 1\, \text{m} \quad d = 0.4\, \text{m}.$$ 

![Figure 4.2.2](image)

Figure 4.2.2 HLEM response of a $3 \times 3$ plate in a half-space as a function of host induction numbers to the HLEM system shown in Figure 4.1.1.
a two-dimensional body excited by a two-dimensional source, but host and plate
induction parameters for this experiment are in the region of the Argand diagram
(Figure 4.1.3) where the current channeling effect dominates, so it is expected that
the Slingram response of the plate increases with decreasing resistivity of the host.

In Figure 4.2.3, the EM responses of the plate as a function of plate induction
parameter $\alpha_p$ are displayed. The host induction parameter $\gamma_h = 0.25$, and other pa-
rameters are unchanged and the amplitude of real and imaginary responses increases
with increasing plate induction number, but approaching saturation for $\alpha_p > 100$.
But the amplitude enhancement by unit increase of plate conductance is less than
the enhancement by unit increase of host induction number. In other words, the
EM response of the plate is more sensitive to unit change in host induction number
than to unit change in plate induction number in channeling regime.

Finally, we computed a set of theoretical profiles, displayed in Figure 4.2.4, as
a function of host resistivities and frequencies for two different configurations. It is
clear that the responses increase with decreasing host resistivity. However, there is
an optimum frequency which gives the maximum channeling response of the plate,
as demonstrated by the plot in bottom-right corner of Figure 4.1.4. The purpose of
computing the set of profiles is to show the capability of the new model to simulate
the EM response of a plate-like mineral targets in the earth. The CPU time for
computing a single profile is only 40 seconds on a MicroVax. Again, the program
assumes no plate symmetry, and no interpolations done in calculating the Green's
functions at scattered sources.
Chapter 5

EM response of the plate in the sea floor

In recent years, frequency and time domain controlled source electromagnetic (CSEM) sounding methods have been developed to explore the electrical conductivity structure of the crust beneath the sea water. The applications of the EM sounding methods in marine environments include the delineation of massive sulphide deposits of very high conductivity found along Mid-Ocean ridges, the detection of subsea permafrost and hydrothermal vents, and the estimation of porosity and water saturation of the sea floor. A CSEM system for use on the sea floor normally consists of a source transmitter energized at one or more frequencies. Receivers placed on the sea floor at various distances measure the EM fields which diffuse outwards from the source through both the sea water and the crust. Since the sea water is normally more conductive than the underlying crustal material, it is more difficult for EM fields to diffuse through the sea water than the crust. Thus, signals observed at large distance must have diffused through the earth. The characteristics of these signals as a function of frequency and range are diagnostic of the conductivity structure of the earth. To assist geological interpretation of the sounding data, the frequency domain response of a sea-earth double half-space model along the response of a vertical plate buried beneath the sea floor with horizontal magnetic and electric dipole sources will be computed.

5.1 HRHR response of a plate in the sea floor

The discovery of massive sulphide deposits and high-temperature hydrothermal vents near Mid-Ocean Ridges has spurred the interest in the development of towed sea floor electromagnetic mapping tools. Sea floor conductivity mapping using the
Figure 5.1.1 Schematic illustration of the HRHR system on the sea floor.

\[ E_z = \frac{i\omega \mu M \sigma_o y}{4\pi \sigma_1 \rho} \int_0^\infty \frac{\lambda^2}{\theta_o} (1 + R_{TM}) e^{-\theta_o z} J_1(\lambda \rho) d\lambda, \] (5.1.2)

and

\[ B_\rho(z = 0) = \frac{M}{4\pi} \left\{ -i\omega \mu \rho^2 \sigma_o \sigma_1 \int_0^\infty \frac{1}{\theta_1 \sigma_o + \theta_o \sigma_1} J_1(\lambda \rho) d\lambda \right. \\
- \int_0^\infty \frac{\rho^3 \lambda \theta_o \theta_1}{\theta_1 + \theta_o} J_1(\lambda \rho) d\lambda \right. \\
+ \int_0^\infty \frac{\rho^3 \theta_o \theta_1}{\theta_1 + \theta_o} J_1(\lambda \rho) d\lambda \} , \] (5.1.3)

where

\[ \theta_o = \sqrt{\lambda^2 + i\omega \mu \sigma_o} \quad \theta_1 = \sqrt{\lambda^2 + i\omega \mu \sigma_1} \]

\[ R_{TM} = \frac{\theta_o \sigma_1 - \theta_1 \sigma_o}{\theta_o \sigma_1 + \theta_1 \sigma_o} \quad R_{TE} = \frac{\theta_o - \theta_1}{\theta_o + \theta_1} , \]
Figure 5.1.3 Normalized primary EM response of the double half-space to the fixed HRHR system located on the crustal surface as a function of frequency for a range of $\sigma_0/\sigma_1$ values.

of sea floor and sea water conductivities respectively.

In Figure 5.1.4 the normalized total responses to the HRHR system as a function of position are displayed, simulating some actual conditions in a typical sea floor EM survey. Two characteristic minima on either side of the center position in the total response profile is the result of the massive conductive plate shielding the propagation of magnetic field through the crust. This indicates that conductors buried in the crust are easily detectable by the towed HRHR system operated in the marine environment. The amplitude of the response increases with decreasing
Figure 5.1.5 Normalized total EM response of the double half-space and the vertical plate embedded in the crust to the moving HRHR system located on the crustal surface as a function of moving array location for a range of frequency values. The ratio $\sigma_0/\sigma_1$ is fixed at 30. The parameters are the same as in the previous figures except the depth of the plate, which is 20m.

5.2 ERER response of a plate in the sea floor

Edwards (1988) investigated the ERER response of a vertical two-dimensional (2-D) structure in the sea floor. Even though the 2-D model assumption can greatly simplifies the mathematics and the numerical computations, 2-D models usually fail to represent all the physics involved. For example, the EM fields associated with the poloidal mode of induction, which are present in the complete 3-D solution,
where \( z \) is positive downward. The primary electric field required on the sea floor is given by:

\[
E_x(z = 0) = \frac{2 \cos 2\phi l d l}{4\pi \rho^3 \sigma_1} \left\{ \int_0^\infty \rho^2 \left[ \frac{\theta_0 \theta_1 \sigma_1}{\theta_1 \sigma_o + \theta_o \sigma_1} - \frac{i \omega \mu \sigma_1}{\theta_1 + \theta_o} \right] J_1(\lambda \rho) d\lambda \right\}
- \frac{2Idl}{4\pi \rho^3 \sigma_1} \int_0^\infty \rho^2 \left[ \frac{i \omega \mu \sigma_1}{\theta_1 + \theta_o} \sin^2 \phi \right] \lambda \rho J_0(\lambda \rho) d\lambda
- \frac{2Idl}{4\pi \rho^3 \sigma_1} \int_0^\infty \rho^2 \left[ \frac{\theta_0 \theta_1 \sigma_1}{\theta_1 \sigma_o + \theta_o \sigma_1} \cos^2 \phi \right] \lambda \rho J_0(\lambda \rho) d\lambda .
\]

(5.2.3)

Figure 5.2.2 Typical impressed currents and the associated current divergence in a plate embedded in the sea floor to the ERER system shown in Figure 5.2.1. The parameters are: source frequency \( f = 1000 \) Hz, seawater conductivity \( \sigma_o = 3.0 \) S/m, and sea floor conductivity \( \sigma_1 = 0.1 \) S/m.

In Figure 5.2.2, typical impressed currents and the associated current divergence in a plate embedded in the sea floor to the ERER system are displayed. The relevant geometric parameters are \( a = 10m \) and \( b = 1m \). The pattern of current flow is a mixture of toroidal and poloidal modes.

The normalized primary amplitude response of the double half-space to the ERER system is shown in Figure 5.2.3. The response in the low frequency regime is indicative of the conductivity of the sea water, while the position and amplitude
below:

\[ E_x^s \approx \left( \frac{\sigma_o + \sigma_1}{2\sigma_1} \right) \left( \frac{\beta}{1 + \beta} \right) F(x_t, b + \frac{\delta}{2}) F(x_r, b + \frac{\delta}{2}) \]  

(5.2.4)

where

\[ \beta = \frac{2\tau S}{\pi \sigma_1 \delta \sqrt{\delta^2 + \tau^2}}. \]

The input and output Green's function is given by

\[ F(x, z) = \frac{\delta r^2}{2} \int_0^\infty \frac{2\lambda^2 \theta_o \sigma_1}{\theta_o \sigma_1 + \theta_1 \sigma_o} e^{-\theta_1 z} J_1(\lambda x) d\lambda, \]

where \( r = \sqrt{x^2 + z^2} \). Expression (5.2.4) consists of four terms, representing the correction for the attenuation in the measured electric field caused by the contrast in conductivity between the sea water and the sea floor, the saturation of the impressed secondary current in the strip due to free charge buildup on the strip, the propagation of the source field from the transmitter to the plate and the propagation of the secondary field from the plate to the receiver. The last two terms are highly frequency dependent.

In Figure 5.2.4, the normalized amplitude curves of \( F(x_t, b + \frac{\delta}{2}) F(x_r, b + \frac{\delta}{2}) \) of a single element for several frequencies as a function of array position are shown. The amplitude of the curves increases first with increasing frequency, reaching a maximum at about 3000 Hz, and then decreases with increasing frequency.

The normalized total EM response profile over a conductive plate within the crustal half-space is shown in Figure 5.2.5. The curves are amplitude of the responses as a function of array position for a range of frequencies. The variations of the anomaly are quite small, suggesting that small targets of high conductivity can be detected if they are buried not deeper than a few hundreds meters in the crust. The amplitude of the anomaly increases with increasing frequency and then decreases, indicating there exists an optimum frequency.
Figure 5.2.5 The normalized total electric field response of the double half-space and the vertical plate to a moving ERER system. The electric field is plotted as a function of moving array location relative to the buried plate for a range of frequencies. The parameters are $a = 100\, \text{m}$, $b = 20\, \text{m}$, $\sigma_0 = 3\, \text{S/m}$, $\sigma_1 = 0.1\, \text{S/m}$. The plate is $50\, \text{m} \times 50\, \text{m}$. 
consideration of spatial distribution of the basis functions to meet both the needs of inductive and channeling terms, we have also deliberately separated the two terms via first order approximations in the scattering matrix, so that the errors in one term will not mix with those in the other. With these two measures, the solutions of the lattice plate has been shown to be quite robust even in extremely resistive hosts.

Overall, a number of results for the new plate have been established. The eigen-analysis of the scattering matrix show that the eigencurrents consist two distinct classes: inductive and channeling. Modal solution of currents requires the explicit knowledge of eigencurrents, and with the ability to classify them, the number of eigencurrents needed for a satisfactory solution may be reduced, and so the cost of computing, if a priori knowledge of the solution, either purely inductive or purely channeling, or a mixture of both modes, is made available. The display of two dozen eigencurrents indicates the natural ability of the lattice to describe both the inductive and channeling currents to a high degree of accuracy limited only by the number of elements in the lattice, with no more mathematical complexity than that of the sectional pulse basis functions and point matching. No only the formulation of the new model is very simple and straightforward, the analysis of its EM response is also very easy, and it requires no more than two dimensionless parameters to describe the scattering by the plate. The inductive response of the lattice is analysed using the small circular ring approximation, and it consists of three parts: the input and output coupling functions between the source and the plate, the plate and the receiver, and the scattering by the plate which can be described by the plate induction number $\alpha$. The channeling response is studied in a similar fashion using a thin strip approximation, and again the scattering of channeling currents by the plate can be explained by the plate channeling number $\beta$. These dimensionless parameters are found to be very convenient to describe the plate response in the development of the new plate.
Chapter 6 Conclusions

Itatively with similar results published in the literature, for example, those derived by Haneson and West 1984, but no attempt is made to compare them quantitatively. The effect of a conductive overburden is also investigated, and its effect, as we conclude, is to reduce the effectiveness of EM prospecting, due to the propagation loss of the coupling functions in passing through the conductive layer. Convergence test for Slingram responses is conducted, and the results indicate the solutions do converge uniformly. Slingram response profiles are quite easy to compute for a $5 \times 5$ plate, but the cost for computing Green’s functions accounts for most of the CPU time, which could become prohibitive for plate with large $N$. The amplitude of the profiles increases with decreasing host resistivity, same phenomenon discussed demonstrated by Hohmann 1975, and there exists an optimum frequency at which the plate response is maximized.

Man has always been looking for new resources to meet our ever-increasing need for consumption, and large-scale exploration and exploitation of such resources like oil and gas have already expanded into the sea. So there is no doubt in the future that some of the rich mineral deposits in the sea floor will be mined for commercial use. In Chapter Five, some theoretical profiles of a three-dimensional vertical lattice plate in the sea floor to the HRHR and ERER systems, the only two instruments prove to be useful in the sea floor EM exploration, are computed. The profiles have the similar characteristics to those computed by Edwards 1988, and Edwards and Cheesman 1987 using a two-dimensional plate in the sea floor. Results indicate that highly conductive targets buried a few hundred meters deep can be detected electromagnetically using the HRHR or ERER system. The amplitude of the HRHR profile increases with decreasing frequency, and for the ERER, there exists an optimum frequency at which the response is maximum.

As there is no question that there are merits in the new plate, one cannot deny that there are some demerits as well. One of the major defects in the plate is that in a purely channeling situation, the plate response may be in error by as
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Appendix A

Interaction functions for the lattice

Calculation of electric fields of a thin sheet of current in a conductive whole space, can be done by integrating the fields of infinitely many elementary current dipoles distributed uniformly over the sheet. The electromagnetic fields of a current dipole are given in many standard textbooks dealing with the subject of electromagnetic radiation, for example, Kaufman and Keller (1983). If an electric dipole with time variation $e^{i\omega t}$ and dipole moment $P = I\delta l \hat{z}$ is placed at the origin of the cylindrical coordinate system $(\rho, \phi, z)$, shown in Figure A.1, then the azimuthal magnetic induction $B_\phi$ is given by

$$B_\phi = \frac{\mu I\delta l}{4\pi R^2} \sin \theta (1 + \alpha R) \exp(-\alpha R), \quad (A.1)$$

where $\alpha^2 = i\omega \mu \sigma_0$, $\sin \theta = \rho/R$, and $R^2 = \rho^2 + z^2$. The corresponding electric fields of the dipole can be calculated using Ampere’s law, that is

$$\nabla \times \mathbf{B} = \mu \sigma_0 \mathbf{E}. \quad (A.2)$$

Separating (A.2) into its component yields

$$\mu \sigma_0 E_\rho = -\frac{\partial B_\phi}{\partial z}, \quad (A.3)$$

and

$$\mu \sigma_0 E_z = \frac{1}{\rho} \frac{\partial}{\partial \rho} (\rho B_\phi). \quad (A.4)$$

Substitute the expression for $B_\phi$ into the above equations to obtain the electric fields

$$E_\rho = \frac{\mu I\delta l}{4\pi \sigma_0 R^3} \sin \theta \cos \theta (3 + 3\alpha R + \alpha^2 R^2) \exp(-\alpha R), \quad (A.5)$$
Figure A.1 Illustrations of an elementary current dipole in the cylindrical coordinates and a thin sheet in the Cartesian coordinates. The thin sheet shown represents a single element in the lattice plate. The thin sheet has length $2b$ in the direction of current flow, and width $2a$.

and

$$E^y_s = -\frac{\partial U}{\partial y} = \frac{j^z_s}{4\pi\sigma_o} \sum_{l=1}^{2} (-1)^l \int_{-a}^{a} \frac{ydz'}{[(x - (-1)^l b)^2 + y^2 + z'^2]^{3/2}},$$ \hspace{1cm} (A.11)

where $j^z_s$ is the anomalous surface current density in the $x$ direction, and the primed quantities refer to the coordinate system of the plate. The above results can be summarized in vector notation,

$$E_s = \int_{L'} G_S \cdot J_s dL',$$ \hspace{1cm} (A.12)

where $L'$ denote the line integral, and $G_S$ is the vector interaction functions for the
The integrals in A.15 and A.16 are difficult to evaluate analytically, except at the center of the plate, where

\[ E_f^r(0,0,0) = \frac{j\omega}{4\pi\sigma_0} \left[ 4a \ln \left( \frac{c+b}{c-b} \right) + 2b \ln \left( \frac{c+a}{c-a} \right) \right], \quad (A.18) \]

and

\[ E_f^\theta(0,0,0) = 0, \quad (A.19) \]

where \( c = \sqrt{a^2 + b^2} \). Numerical quadrature methods are used to compute the inductive terms at other locations on the \( xy \) plane.
where $J_1$ is the first order Bessel function. The radial magnetic field at offset $z$ is

$$
B_r(z) = B_r(0) \cdot e^{-\theta |z|},
$$

where $\theta^2 = \lambda^2 + i\omega \mu \sigma$. Apply one of the Maxwell's equations, i.e,

$$
\nabla \times E = -\frac{\partial B}{\partial t}
$$
or

$$
\frac{dE_0}{dz} = i\omega B_r.
$$

and integrate (B.6) with respect to $dz$, one obtains

$$
E_0(z) = -\frac{i\omega \mu j_0 a}{2\theta} e^{-\theta |z|} J_1(\lambda a) \Delta z.
$$

To calculate $E_o$ for the ring of finite width $w$, we have to integrate the expression (B.7),

$$
E_o(z) = -\frac{i\omega \mu j_0 a}{2} \int_{-w/2}^{w/2} e^{-\theta |z|} J_1(\lambda a) dz.
$$

Take the inverse Hankel Transform of (B.8), we have

$$
E_o(z) = -\frac{i\omega \mu j_0 a}{2} \int_{-\infty}^{\infty} \frac{\lambda}{\theta} \int_{-w/2}^{w/2} e^{-\theta |z|} dz J_1^2(\lambda a) d\lambda,
$$

$$
= -\frac{i\omega \mu j_0 a}{2} f(w/a)
$$

where $f(w/a)$ is called the geometric factor of the ring, which depends only on the dimensions of the ring if it is in free space. The external electric field $E_e$ due to the magnetic source is

$$
E_e(z) = -\frac{i\omega \mu M a}{4\pi R^3} e^{-\theta_o R (1 + \theta_o R)}
$$

$$
= -\frac{i\omega \mu M a}{4\pi} g_s
$$

where $g_s$ is the Green's function from the source to the ring, $\theta_o^2 = i\omega \mu \sigma$, $R^2 = z^2 + a^2$, and $M$ is the dipole moment of the source. Apply the boundary conditions for the
Appendix C

The EM fields in a conductive half-space of a VMD

In this section, the electromagnetic field of a Vertical Magnetic Dipole (VMD) source located on the surface of a uniform conducting half-space will be derived in detail. The general layered earth problem can be found in Kaufman and Keller (1983).

The most convenient coordinate system in which the above problem can be solved in is a cylindrical coordinate system with the origin on the surface of the half-space. The $z$-axis is perpendicular to the surface and positive downwards. The vertical magnetic dipole, shown in Figure B.1, has moment $M$ and is situated at height $h$ above the surface.

The conductivity and magnetic permeability of the half-space are represented by the symbols $\sigma$ and $\mu$ respectively. The value $\mu$ is assumed to have the same value as for the free space, that is, $\mu_0 = 4\pi \times 10^{-7}$ H/m. Assuming the electromagnetic fields have a time-dependence $e^{i\omega t}$, Maxwell's equations can be written as:

\[
\nabla \times \mathbf{E} = -i \omega \mu \mathbf{H}, \quad \nabla \cdot \mathbf{E} = 0 \quad (C.1)
\]
\[
\nabla \times \mathbf{H} = \sigma \mathbf{E}, \quad \nabla \cdot \mathbf{H} = 0
\]

where $\mathbf{E}$ and $\mathbf{H}$ are complex electric and magnetic fields. The constitutive equation for current, $\mathbf{J} = \sigma \mathbf{E}$, is assumed, and the displacement currents are ignored.

Using the magnetic vector potential, $\mathbf{A}$, which is defined as:

\[
\mathbf{E} = i \omega \mu \nabla \times \mathbf{A}, \quad (C.2)
\]

the curl of $\mathbf{H}$ can be written as

\[
\nabla \times \mathbf{H} = i \sigma \mu \omega \nabla \times \mathbf{A}
\]
and the electromagnetic field components are given by:

\[
\mathbf{E} = i \omega \mu \nabla \times \mathbf{A} \\
\mathbf{H} = \alpha^2 \mathbf{A} + \nabla \nabla \cdot \mathbf{A}
\] (C.6)

where \( \alpha^2 = i \sigma \mu \omega \). Because \( \mathbf{E} \) has only a tangential component, the first equation in B.6 implies that the vector potential has only a vertical component,

\[
\mathbf{A} = (0, 0, A_z)
\]

Equation (C.5) is valid everywhere except at the boundary of the two half-spaces. The boundary conditions, expressed in terms the the vector potential, can be written as:

\[
A_{1z} = A_{2z} \quad \text{and} \quad \frac{\partial A_{1z}}{\partial z} = \frac{\partial A_{2z}}{\partial z} \quad \text{at} \quad z = 0
\] (C.7)

where \( A_{1z} \) and \( A_{2z} \) are the \( z \) components of vector potential in the upper and lower half-spaces respectively. Taking the Hankel Transform of (C.5), one obtains

\[
\frac{d^2 \overline{A_z}}{dz^2} - \lambda^2 \overline{A_z} = \alpha^2 \overline{A_z}
\]

or

\[
\frac{d^2 \overline{A_z}}{dz^2} = m^2 \overline{A_z}
\] (C.8)

where \( m = \sqrt{\lambda^2 + \alpha^2} \). The general solution to (C.8) is:

\[
\overline{A_z} = \overline{C}_\lambda e^{-mz} + \overline{D}_\lambda e^{mz}
\] (C.9)

Taking the inverse Hankel Transform of (C.9), the general solution to (C.5) can be obtained,

\[
A_z = \frac{M}{4\pi} \int_0^\infty (\overline{C}_\lambda e^{-mz} + \overline{D}_\lambda e^{mz}) J_0(\lambda r) d\lambda
\] (C.10)

Because the EM field decreases with increasing \( z \) in a conducting medium, the vector potential in the lower half-space takes the form:

\[
A_{2z} = \frac{M}{4\pi} \int_0^\infty \overline{C}_\lambda e^{-mz} J_0(\lambda r) d\lambda
\] (C.11)