ELECTROMAGNETIC DECAY OF STATES OF HIGH SPIN
AND HIGH ISOSPIN IN FLUORINE-19

by

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ABSTRACT

The lowest $J^\pi = 11/2^+$ state and the lowest two $T = 3/2$ states in $^{19}$F have been located among a number of new resonances in the $^{15}$N($\alpha,\gamma$)$^{19}$F reaction. The spins and radiative widths of these three new levels have been established by measurements of angular distribution and yield of $\gamma$-radiation. Their isospin is inferred from their spins, excitation energies, radiative widths and decay schemes.

Measured transition matrix elements are compared with those calculated from shell-model wavefunctions which include all (sd)$^3$ configurations. Disagreements between experiment and theory indicate that the (sd)$^3$ description is inadequate for some of the positive parity states in $^{19}$F above about 4 MeV in excitation. Arguments are presented for the inclusion of (p)$^{-2}$(sd)$^5$ configurations to account for the properties of these states.

The new $11/2^+$ level and the $7/2^+$ level at 4.38 MeV (whose spin has been definitely established in the course of this work) are discussed also in terms of a simple adiabatic rotational model for $^{19}$F. This model is found to account reasonably well for the excitation energies and transition probabilities of the lowest levels of spins $1/2$ to $13/2$. 

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CHAPTER ONE

INTRODUCTION

1.1 General Introduction

This thesis in nuclear spectroscopy is concerned with an experimental study of certain of those excited states of the nucleus $^{19}$F accessible to study through the reaction $^{15}$N($\alpha$,γ)$^{19}$F. It has been found that by this reaction it is possible to excite and study many states, particularly those near 4 MeV and above, which are now crucially relevant to the considerable number and variety of theoretical studies of $^{19}$F(1-22). In the present study, the points of comparison between experimentally-observed excited states and their putative counterparts in theory are excitation energy, spin, parity, isospin, total and partial widths.

The primary (and perhaps rather wistful) aim in work of this kind is to shed more light on nuclear structure, perhaps by showing that experiment favours one model over another in certain respects. A less ambitious (and perhaps more realistic) aim is to amass data to test ever more all-embracing computer calculations of the nuclear many-body problem.
1.2 Reasons for Studying $^{19}\text{F}$

To early nuclear physicists the study of light nuclei, because of their small number of nucleons, probably seemed more likely to lead to an understanding of nuclear structure than the study of heavy nuclei. Or perhaps a more important factor was that because of their lower Coulomb barrier, light nuclei were more easily investigated by the low-voltage accelerators which were built in the early days of nuclear physics. Whether dictated by the accelerator technology or the rationale, the intensive study of light nuclei has yielded an impressive return in understanding nuclear structure and continues to be a fruitful field of research.

Of the light nuclei, $^{19}\text{F}$ has been of special importance to nuclear theory. It is, on the one hand, light enough for a full treatment in the independent-particle model yet, on the other hand it exhibits collective features\(^{(1-4)}\). This concurrence of collective and independent-particle traits in the one nucleus provided the stimulus for the development of the nuclear $\text{SU}_3$ model\(^{(11,23)}\). Recent calculations continue to explore the dual aspects of single-particle and collective motion in nuclei such as $^{19}\text{F}$ both by $\text{SU}_3^{(16,18)}$ and by projected Hartree-Fock methods\(^{(15,21,102)}\). To test their scope
and validity it is necessary to extend experimental spectroscopy to excitations well above 1.5 MeV - the recent limit of well-established information on spins, parities and transition probabilities in $^{19}F$.

In $^{19}F$, between the triplet of states at about 1.5 MeV excitation energy and the state at 6.07 MeV, at least 15 states were known at the time of starting this work ($^{24-29}$). However little or nothing was known about them other than their excitation energies. Only two or three had firm spin assignments and tentative parity assignments. This is in a region which is becoming increasingly important as nuclear model calculations for the low-lying bound states become more successful. At still higher excitation energies (i.e. above a rather arbitrary limit of 6 MeV), interest in the states themselves declines somewhat at present, except for states with special character such as high spin. However, higher states are often useful as intermediaries by which to excite and study the lower-lying levels through which they γ-decay. Most of them between 6.07 and 7.10 MeV have already been located and assigned spins and parities by Smotrich's $^{15}N(a,a)^{15}N$ work ($^{26,27}$). Prior to the present work however, there was no information on their radiative decay.
1.3 Radiative Capture

One reason for choosing to investigate the structure of a nuclear level by studying its radiative decay (as opposed to its particle decay, if this is energetically permitted) is that the electromagnetic interaction is quite well understood. Comparison of experiment with theory then tests the nuclear model rather than the theory of the interaction. Theoretical estimates of particle widths, on the other hand, involve the relatively poorly-understood strong nuclear forces. Of course for the most important levels — those which are bound or nearly bound — \( \gamma \)-emission is almost the only mode of decay.

By a radiative-capture nuclear reaction at an isolated resonance it is sometimes possible to form the nucleus of interest in a selected excited state whose radiative decay may then be studied. Alternatively, the nucleus of interest may be produced in a nuclear disintegration in which the final state consists of an outgoing particle plus the residual nucleus in an excited state whose radiative decay may then be observed. Often, however, the residual nuclei may be in any of several excited states and it may be necessary to observe the outgoing particle in coincidence with the \( \gamma \)-ray to obtain useful results. The radiative capture method is simpler though it is not always available, of course. Both methods may be used to infer spins of excited states by observing the angular distribution of the \( \gamma \)-radiation emitted.
1.4 The $^{15}_N(a,\gamma)^{19}_F$ Reaction

A table of nuclear reaction Q values indicates that conditions are very favourable for studying electromagnetic transitions in $^{19}_F$ by the $^{15}_N(a,\gamma)^{19}_F$ reaction at bombarding energies up to a few MeV.

1. The Q value of the reaction is 4.011 MeV$^{30}$. Apart from considerations of penetrability, at low and moderate bombarding energies one expects to observe $\alpha$-capture into states in $^{19}_F$ which are themselves of considerable interest.

2. $^{19}_F$ is bound against neutron emission up to 10.442 MeV and against proton emission up to 7.992 MeV$^{30}$. (Binding for nuclear fragments other than $\alpha$, $n$ and $p$ is still higher). Since these channels are closed at the bombarding energies being considered, it is more likely that sharp capture resonances will be found. Neutron production would have had the further technical disadvantage of contributing seriously to the background in the $\gamma$-ray spectrometers.

3. Since the first excited state of $^{15}_N$ is at 5.28 MeV, the $^{15}_N(a,a'\gamma)^{15}_N$ reaction cannot contribute spurious $\gamma$-rays since it cannot take
place at bombarding energies below 6.69 MeV.

As a means of studying $^{19}\text{F}$ by radiative capture
the $^{15}\text{N}(\alpha,\gamma)^{19}\text{F}$ reaction has to be compared with
$^{18}\text{O}(p,\gamma)^{19}\text{F}$ ($Q = 7.992 \text{ MeV}$). Some of the considerations
are as follows:

(i) From the point of view of simplicity in interpreting angular distribution measurements the
two reactions rate the same: both have channel spin 1/2.

(ii) The yield from $(\alpha,\gamma)$ reactions tends to be much lower than from $(p,\gamma)$ reactions at the same bombarding energies because of the large difference in $dE/dx$ between the two projectiles - about a factor of ten in Al at 2 MeV, for example$^{(31)}$.

(iii) The much higher $Q$ of the $(p,\gamma)$ reaction is somewhat of a disadvantage since the resonances may be more closely spaced and the states in $^{19}\text{F}$ above 8 MeV are not yet of great interest.

A search of the literature for references to the $^{15}\text{N}(\alpha,\gamma)^{19}\text{F}$ reaction shows that at the time of starting this work (January 1968) only three resonances were known$^{(24,27)}$:
these, occurring at 5.34, 5.47 and 5.50 MeV excitation energy in $^{19}$F, have been studied by Price (32) and by Tolbert (33-35). Since Price's work and before the start of the present work, many other levels in the 4.5 to 6.5 MeV region of excitation in $^{19}$F were discovered (25,26).

Since the lowest $T = 3/2$ state occurs at about $E_x = 7.5$ MeV in $^{19}$F (34), all these levels must have $T = 1/2$ and therefore their formation in the $^{15}$N($\alpha$,γ)$^{19}$F reaction is isospin-allowed. Further, since all the new levels reported by Smotrich (26) have $\Gamma > 1$ keV, it seemed that unless they had remarkably small $\Gamma_\gamma$'s, it should be possible to detect these, at least, as resonances in the $^{15}$N($\alpha$,γ)$^{19}$F reaction.

Though the formation of $T = 3/2$ states in $^{19}$F by $^{15}$N($\alpha$,γ)$^{19}$F is isospin-forbidden, it is well-known (37,38) that low-lying $T_\gamma = 3/2$ states in certain $T_x = 1/2$ nuclei may be excited as strong sharp resonances in ($p$,γ) reactions on $T = 0$ targets by exploiting slight violations of isospin conservation. Also, in an ($\alpha$,γ) reaction on a $T = 0$ target, it is possible to form the isospin-forbidden first $T = 1$ level in $^{20}$Ne by $^{16}$O($\alpha$,γ)$^{20}$Ne (39). In view of this it seemed that the isospin restriction against forming $T = 3/2$ states by $^{15}$N($\alpha$,γ) might similarly turn to advantage and that $T = 3/2$
states in $^{19}$F might appear as strong sharp resonances.

1.5 $^{19}$F Nuclear Data

The state of knowledge about $^{19}$F at the time of starting the experiments to be described here is summarized in Fig. 1.1, which is based mainly on information from Nuclear Data Sheets (1960). (Data for $E_x = 6.5$ to $7.4$ MeV and for $E_x > 7.7$ MeV have been left out as being irrelevant to the present work.) All the firm spin-parity assignments to levels with $E_x > 5.5$ MeV are from the $^{15}$N($\alpha,\gamma$)$^{15}$N work by Smotrich et al. (26, 27). The $T = 3/2$ suggestion for the level at $7.43$ MeV is due to Butler et al. (36), who found this level as a threshold in the $^{18}$O(d,ny)$^{19}$F reaction. Their isospin suggestion is based on the argument that (i) the level does not appear as a resonance in $^{15}$N($\alpha,\alpha$)$^{15}$N in which it is isospin-forbidden, (ii) it appears in $^{18}$O(d,ny)$^{19}$F in which it is isospin-allowed and (iii) calculation based on the $^{19}$O $-$ $^{19}$F and $^{17}$F $-$ $^{17}$O disintegration energies shows that the lowest $T = 3/2$ state in $^{19}$F should occur at about $7.5$ MeV.

In work after 1960, a number of new levels were discovered by $^{20}$Ne(t,α)$^{19}$F (25) and $^{19}$F(p,p')$^{19}$F (40) but nothing was known about their spins, parities or decay schemes at the time of starting this work. These levels are also shown in Fig. 1.1.
Fig. 1.1
CHAPTER TWO

ANGULAR DISTRIBUTION AND ABSOLUTE YIELD OF RADIATION AFTER RADIATIVE CAPTURE

2.1 Angular Distribution

$^{19}_F$ nuclei formed by capture of $\alpha$-particles in a parallel beam incident on $^{15}_N$ nuclei are restricted in their components $M$ of angular momentum along the beam direction to $M = \pm 1/2$. If the spin $J$ of the compound state is greater than $1/2$, the $^{19}_F$ nuclei thus formed are said to be aligned $[P(M) = P(-M) \neq \frac{1}{2J+1}]$ and the angular distribution of radiation from the state is, in general, anisotropic. Since the angular distribution is characteristic of the spins of the initial and final states of the radiating nucleus as well as the multipolarities of the radiation emitted, it may be possible to determine the nuclear spins by identifying the radiation pattern. In practice, since as many as three multipoles, each with its own distinctive patterns, may contribute in significant but unknown proportions, it is usually necessary to have a considerable amount of information about the energy levels from other experiments before the angular distribution can be interpreted unambiguously.

The theory of $\gamma$-radiation from a nucleus in a state of definite angular momentum and parity shows that
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The theory of $\gamma$-radiation from a nucleus in a state of definite angular momentum and parity shows that
the angular distribution may be expressed conveniently as a linear combination of even-order Legendre polynomials whose coefficients depend on the population parameters $P(M)$, the initial and final nuclear spins, and the $\gamma$-ray multipolarities and mixing ratios. The coefficients are obtainable from tables accompanying several different but equivalent formulations of the problem.

2.2 Absolute Yield and Electromagnetic Lifetime

Consider an $\alpha$-particle beam with variable mean energy $E_\alpha$ and finite energy spread which is small compared with the spacing between adjacent, sharp $(\alpha,\gamma)$ resonances in a target under bombardment by the beam. If the target is always thicker than the range of the $\alpha$-particles, then a graph of yield of primary photons per incident $\alpha$-particle versus $E_\alpha$ shows a series of ascending steps, each signifying the presence of a resonance. The height $H$ of a step is the number of primary photons per incident $\alpha$-particle for the corresponding resonance

$$H = \frac{\lambda^2}{2c} \omega_\gamma$$

where

$$\omega_\gamma = \frac{2J+1}{2} \frac{\Gamma_{\alpha \gamma}}{\Gamma}$$

for $\alpha$-particles incident on $^{15}N$. 

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\( \lambda \) = de Broglie wavelength in cm. for particles with the reduced mass and relative velocity of projectile and target.

\( \varepsilon \) = stopping power of the target material in eV - cm\(^2\) per disintegrable atom.

\( J \) = total angular momentum of the compound nucleus.

\( \Gamma_{\alpha} \) = entrance channel width in eV.

\( \Gamma_{\gamma} \) = sum of all open radiative exit channel widths in eV.

\( \Gamma \) = sum of all open channel widths in eV.

In the present series of experiments in which only the elastic \( \alpha \) and radiative channels are open, \( \Gamma = \Gamma_{\alpha} + \Gamma_{\gamma} \). For a given \( \alpha \)-particle orbital angular momentum, the approximation \( \Gamma_{\alpha} \gg \Gamma_{\gamma} \) is valid provided the \( \alpha \)-particle energy is high enough. The validity of the approximation is usually discussed in the light of the Wigner limits\(^{42,43} \) for \( \Gamma_{\alpha} \) and the Weisskopf estimates\(^{44} \) for \( \Gamma_{\gamma} \). The Wigner single-particle limit is calculated from

\[
\Gamma_{Wigner} = \frac{\rho}{A_{\alpha}^2} \frac{2\hbar^2}{\mu R^2}
\]

where \( \rho/A_{\alpha}^2 \) is the Coulomb penetrability function given by Sharp et al.\(^{43} \), \( \mu \) is the reduced mass, and \( R = 4.864 \text{ fm} \) is the interaction radius. If \( \Gamma_{\alpha} \gg \Gamma_{\gamma} \) then

\[
\omega_{\gamma} = \frac{2J+1}{2} \Gamma_{\gamma}
\]
and measurements of $H$ and $J$ lead to a value for $\Gamma_\gamma$, the electromagnetic width of the resonance level.
CHAPTER THREE

EXPERIMENT

3.1 Introduction

The experiments described in this thesis were carried out using electrostatic accelerators of the Van de Graaff type. One, of 3 MV, was located at the Ontario Cancer Institute, Toronto; the other, of 4 MV, is at the National Research Council, Ottawa. Both machines provide beams of $^4\text{He}^+$ of low enough energy spread and emittance for resonance capture and angular distribution studies. In addition, the Ottawa machine can also produce beams of $^4\text{He}^{++}$ at energies up to 7.5 MeV.

Price\(^{(32)}\) in 1955 and Tolbert\(^{(33-35)}\) in 1968 have described studies of $^{19}\text{F}$ using the $^{15}\text{N}(\alpha,\gamma)^{19}\text{F}$ reaction. Price used targets of tantalum nitride and observed the de-excitation $\gamma$-rays with NaI(Tl) scintillation spectrometers. Tolbert used targets of tantalum nitride also, beam currents of $<$10 $\mu$A, and a 15 cm$^3$ Ge(Li) semiconductor $\gamma$-ray spectrometer.

The studies reported here owe their success largely to two recent improvements in experimental technology. These are:

(1) The development of methods of making suitably thin nitrogen targets by A.M. Charlesworth at the
(ii) The commercial development of large volume, high resolution Ge(Li) γ-ray spectrometers.

3.2 Targets

The $^{15}\text{N}$ target material was contained in a thin layer of titanium nitride on a backing of tantalum 0.01" thick. The thickness of the TiN layer in the targets made for the experiments described here, varied from 12 to 25 μgm/cm$^2$. (The estimates are based on the mass of titanium used in the preparation of the targets and on the widths of peaks in the yield curve for $^{15}\text{N}(\alpha,\gamma)$ resonances some of which are known to be sharp).

Details of the method of making titanium nitride targets will be described by Charlesworth who developed the technique (45). Two highly desirable characteristics of nitrogen targets made by his method are:

(a) The thickness of nitrogen is sharply delimited to the thin layer of titanium nitride. This makes it possible to study separately resonances which are quite closely spaced.

(b) With direct water-cooling of the tantalum backing, the targets are extremely durable under bombardment with He$^+$ beams of ~100 μA. Several hours of useful target life were commonplace.
3.3 Gamma-ray Spectrometers.

Since their inception in the early 1960's, Ge(Li) semiconductor spectrometers and their associated low-noise electronic amplifiers have revolutionized \( \gamma \)-ray spectrometry. In resolving power they surpass NaI(Tl) spectrometers by more than an order of magnitude and in detection efficiency the 40 cm\(^3\) counter used in the present work is about the same as a 1.5" dia. by 1.5" NaI(Tl) spectrometer. Since the low-lying levels of \(^{19}\text{F}\) are quite closely spaced, high resolution spectrometers are indispensable to studies of the \( \gamma \)-radiation from this nucleus. The 40 cm\(^3\) Ge(Li) detector used for some of the experiments described here was made by the R.C.A. Victor Company, Montreal, Canada. Its resolution under good conditions was 3.4 keV (FWHM) for 1.33 and 1.17 MeV \( \gamma \)-rays. In addition, 35 cm\(^3\) and 30 cm\(^3\) Ge(Li) detectors of comparable performance were used. A 5" diameter by 4" thick NaI(Tl) scintillation spectrometer was also employed for some purposes.

The usual range of modern commercially-available pulse height amplifiers and analyzers was utilized.

3.4 Competing Reactions

No competing reactions involving the N, Ti or Ta in the target gave trouble. However carbon and to a
lesser extent boron, occurring as contaminants on the
target, contributed some background to the γ-ray spectra
through strong resonances in the \(^{13}\text{C}(\alpha,n)^{16}\text{O}\) and
\(^{10}\text{B}(\alpha,p\gamma)^{13}\text{C}\) reactions. The reactions \(^{18}\text{O}(\alpha,n\gamma)^{21}\text{Ne}\)
and \(^{17}\text{O}(\alpha,n\gamma)^{20}\text{Ne}\) also, together with others only
tentatively identified, sometimes contributed spurious
peaks, particularly at the bombarding energies above 4
MeV.

Neutrons from the \(^{13}\text{C}(\alpha,n)^{16}\text{O}\) and other reactions
interact by \((n,n')\) reactions in the Ge of the Ge(Li) spectrometers,
in the I of NaI(Tl) spectrometers and in Al, Cu, Zn, 
Fe and other material in the experimental area. The sub-
sequent γ-rays can produce spurious peaks, not all of which
have been identified, and obscure \(^{15}\text{N}(\alpha,\gamma)^{19}\text{F}\) resonances in
the excitation function. A comparison of Figs. 3.1 and 4.1
which show excitation curves before and after the difficulty
was overcome, gives some idea of the nature and magnitude
of the problem. The improvement was achieved* by careful
cold-trapping in the region of the target to minimize the
deposition of carbon-containing contaminants in the vacuum
system (39).

* I am indebted to Dr. R.S. Storey, National Research
Council of Canada, for drawing this technique to my
attention.
3.5 Experimental Layout

Fig. 3.2 is a scale drawing of the Ge(Li) counter, target and beam, in their relative positions as used in some of the experiments. Angular distributions of radiation were observed over the first quadrant. Provided the nuclear state being formed by α-capture has a definite parity, the angular distribution of the radiation is symmetric about 90° in centre-of-mass co-ordinates and, since the mass of the residual nucleus is very much greater than the mass of the emitted photon, the centre of mass is very close to the 19F nucleus.

Before starting a measurement of angular distribution, the following two adjustments were made:

(i) The beam spot on the target was arranged to lie at the centre of rotation of the Ge(Li) counter (to an accuracy of one or two percent of the distance between beam spot and counter) and in the plane of this rotation.

(ii) The beam spot's horizontal width was adjusted to be about 3% of the distance between beam spot and counter and its vertical dimension to be about 3 or 4 mm.

3.6 Experimental Procedures

Yield curves such as are shown in Figs. 3.1 and 4.1 were obtained by counting pulses within a certain range
of energy loss in the γ-ray detector for a fixed charge delivered to the target. The abscissa is an N.M.R. frequency proportional to the field strength in the accelerator's deflecting and analyzing magnet and therefore proportional to the momentum of the α-particles in the beam. The ordinate is in units which are arbitrary to the extent that the detector's position is not specified and no effort was made to suppress the emission of secondary electrons from the target. However, yield curves were used only to find resonances, choose bombarding energies and check target thickness and quality.

Angular distributions of γ-rays were determined by measuring their intensity in a Ge(Li) counter at some or all of the five angles θ = 0°, 30°, 45°, 60°, and 90° relative to their intensity in a monitor counter kept in a fixed position close to the target throughout the experiment. The effects of target deterioration and off-resonant bombardment (because of accidental changes in the beam energy) are thereby eliminated. Counts in total absorption, single and double escape peaks (or as many of these three as appeared in usable intensity and unobscured by background lines) were simply added to give a measure of intensity.

3.7 Measurements of Radiative Yield

$$\omega_\gamma = \frac{2J+1}{2} \frac{\Gamma_\alpha \Gamma_\gamma}{\Gamma}$$ (41)

The radiative yield was measured for each resonance studied by intercomparison with the radiative yield of the $E_\alpha = 1681$ keV resonance in the $^{15}\text{N}(\alpha,\gamma)^{19}\text{F}$ reaction. The same target, one
thick enough to give nearly the full thick-target yield, was bombarded at both resonances in succession while recording their $\gamma$-ray spectra. The effect of target deterioration was reduced by switching from one resonance to the other several times. In order to get an accurate relative measure of the charge delivered to the target at each resonance, secondary electron emission was minimized by surrounding the target with a suppressor grid at a high enough negative potential (−400 volts) that the indicated target current was independent of the suppressor’s potential. Details of the performance of the suppressor will be given by Charlesworth (45).

Since the number of $\alpha$-particles incident on the target is calculated from the measured charge of He$^+$ or He$^{++}$ delivered, there has to be a correction if some of the He$^+$ ions are stripped of their electron in passing through residual gas in the beam transport system. The correction was estimated using the data of Allison (46) and found to be negligible at the operating residual gas pressures ($\sim 10^{-5}$ mm.Hg). The largest effect amounted to 2% in the worst case which arose in using the N.R.C. machine (with beam path length ~10 metres) to intercompare the 1681 keV resonance on the He$^+$ beam with the 4467 keV resonance on the He$^{++}$ beam.

The number of primary photons per $\alpha$-particle on the target was determined for each resonance from the NaI(Tl) or
Ge(Li) spectra, with proper allowance for the decay scheme of the resonance and the angular distributions of its primary γ-rays. The radiative width of the resonance at $E_a$ was calculated from:

$$\frac{(\omega \gamma)_{E_a}}{(\omega \gamma)_{1681}} = \frac{(x^2 H)_{E_a}}{(x^2 H)_{1681}} \times \frac{\varepsilon_{1681}}{\varepsilon_{E_a}}$$

where $x$ is the reduced channel wavelength of the incident α-particle, $H$ is the "step" in a "thick target" yield and $\varepsilon$ is the specific energy loss of the incident α-particles in the target at the resonance energy indicated by the subscripts. In this equation the ratio of thick-target yields may be replaced without serious error by the ratio of yields from "semi-thick" targets, defined as several times thicker than the resonance width. This is because the yield then depends little on target thickness (47). All the targets used in the yield intercomparisons to be described here satisfied this requirement.

Values of $\varepsilon$ for titanium nitride (in ev·cm$^2$/molecule) were obtained from a semi-empirical formula given by Whaling (31) for protons together with a method of conversion to provide the relevant values for α-particles (formula B1 and table 2d in the reference).
3.8 Precautions

In general, peaks appearing in spectra observed on the Ge(Li) counter have been attributed to γ-rays from $^{19}F$ if they met the following conditions:

(i) The energy of the apparent γ-ray (with allowance for possible Doppler shift) must correspond to a transition between known energy levels in $^{19}F$. All primary γ-rays in the resonances discussed in detail here showed the full Doppler shift.

(ii) Total absorption, single and double escape peaks must appear in their proper relative intensities.

(iii) The intensity of the peak integrated over all angles must fit the overall decay scheme proposed for the resonance.

(iv) The relative intensity of the peak must show the appropriate angular distribution.

(v) The peak must not appear in a spectrum taken off-resonance. For each of the three resonances discussed in detail here, off-resonance spectra were observed by bombarding just below the resonance peak in the yield curve.

Except where otherwise noted, all γ-rays ascribed to transitions in $^{19}F$ in this thesis met all five conditions.
3.9 Corrections to Angular Distributions

An experimentally-observed angular distribution differs from the true angular distribution about the centre of mass of the \(^{19}\)F nucleus on account of the following effects:

(i) the finite solid angle subtended by the counter at the target. The formula given by Smith\(^{(49)}\) was used to correct for this attenuation of the distribution. Smith's formula applies to total absorption of \(\gamma\)-rays by a circular disc centred on the counter's axis and having the same area as the counter's cross-section. Since much of the sensitive volume of the Ge(Li) counter is annular in cross-section, this approximation may appear to underestimate the attenuation though only a full Monte Carlo calculation based on knowledge of the shape and dimensions of the sensitive volume could establish the true correction convincingly. One indication that the correction is adequate is that a measured angular distribution of pure quadrupole radiation is well fitted by a theoretical distribution which has no adjustable parameter. (This was observed in the study of another resonance to be described elsewhere. It was the angular distribution of the E2 primary transition from the \(7/2^-\) resonance
level \((E_a = 1.79 \text{ MeV})\) to the \(3/2^-\) level at 1.46 MeV). This justifies the method of calculating \(Q_4\) and a fortiori \(Q_2\), where \(Q_2\) and \(Q_4\) are the attenuation correction factors by which the theoretical coefficients of \(P_2\) and \(P_4\) must be multiplied.

(ii) different path lengths in the target backing for photons emitted at different angles. The absorption at different angles of passage through the tantalum backing, the cooling water and brass target holder was calculated and corrected for, using the data of Grodstein (50).

(iii) aberration. The correction for this effect is calculated in Appendix 1. It was applied to all angular distributions of \(\gamma\)-rays showing noticeable Doppler shift.

3.10 Testing Mixing Ratio and Spin Hypotheses

A theoretical angular distribution \(T_0\), being the true angular distribution modified by the effects of attenuation, may be defined as

\[
T_0 = \sum_{\sigma} a_\sigma (\delta_1, \delta_2, \ldots) Q_\kappa P_\kappa (\cos \theta)
\]

where \(a_o = Q_o = P_o (\cos \theta) = 1\) and \(\kappa\) is even.
The quantities $\delta$ are multipole mixing ratios defined as

$$\delta_L^\pi = \frac{\langle J_L^\pi | T_L^\pi | J_f \rangle}{\sqrt{2L+1}}$$

where $\tilde{L}$ is the lowest order of multipole occurring in the transition and $L$ is the order of the multipole which mixes with $\tilde{L}$ ($L = \tilde{L} + 1$ always in this thesis).

$\pi$ stands for the parity (electric or magnetic) of the radiation of multipole order $L$.

$T_{LM}^\pi$ is the multipole operator.

The above definition alone is insufficient to specify the sign of $\delta$ since there is no single conventional way of defining the phases of the reduced matrix elements. For consistency, in tables 6.5, 7.4 and 8.1 which summarize experimental results, the sign of $\delta$ follows the definition of Rose and Brink (60).

The distribution $T_\theta$ is normalized so as to minimize the quantity $\chi^2$ for various values assigned to the mixing ratio(s).

$$\chi^2 = \sum_\theta \frac{(\delta T_\theta - I_\theta)^2}{E_\theta^2}$$

where $\delta = \text{normalizing factor}$

$T_\theta = \text{theoretical intensity at angle } \theta$

$I_\theta = \text{experimental relative intensity at angle } \theta$

$E_\theta = \text{experimental error on } I_\theta$

and the sum is over the several angles $\theta$ of the distribution.

With certain assumptions, a probability can be calculated that,
given a true hypothesis, the dimensionless number $\chi^2$ will be exceeded because of random errors. A probability less than 1% will be assumed to rule out a hypothesis.

The width of the dip in a $\chi^2$ versus $\delta$ graph is a measure of the uncertainty in $\delta$. The method described by Thomas et al. (29) was used to estimate the error in $\delta$ by measuring this width in a consistent manner.

A measurement of $\Gamma_{\gamma}$ for a nuclear level and $\delta$ for its decay $\gamma$-ray leads to

$$\Gamma_{L\pi} = \frac{\delta^2}{1 + \delta^2} \Gamma_{\gamma}$$

(assuming only one $\delta$ and one decay branch) and hence to $\Gamma_{L\pi}$ in single particle units (see section 9.4). With some reservations about possible circularity of argument, the hypothesis leading up to this estimate of $\Gamma_{L\pi}$ may sometimes be accepted or rejected by comparing $\Gamma_{L\pi}$ with many other comparable measurements of this quantity. Skorka et al. (48), have prepared a compilation of measured transition strengths in light nuclei which is useful for this purpose. Later statements about the admissibility or inadmissibility of certain $(L,\pi)$ strengths are based on norms from this reference.
Fig. 3.1

Fig. 3.2

EXPERIMENTAL LAYOUT

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CHAPTER FOUR

NEW RESONANCES IN THE $^{15}_N(\alpha,\gamma)^{19}_F$ REACTION

Twelve new resonances have been discovered in the $^{15}_N(\alpha,\gamma)^{19}_F$ reaction between $E_\alpha = 600$ and 3150 keV, populating many levels in $^{19}_F$ which are of considerable interest. Prior to the present work, only the resonances at 1681, 1852 and 1888 keV studied by Price$^{(32)}$ and more recently by Tolbert et al.$^{(33-35)}$, were known. Both old and new resonances are shown on the yield curve, Fig. 4.1, which is a composite of partial curves obtained on different occasions.

Except for the one at 3150, all the resonances whose bombarding energies in keV are marked on Fig. 4.1 correspond to levels in $^{19}_F$ already known or suggested$^{(24-27,40)}$ at the time of first observing this yield curve. Those marked with full arrows have been further identified by observation of their radiative decay using a Ge(Li) spectrometer. A summary of the results of this study is presented in Fig. 4.2. In this diagram true $\gamma$-ray branching ratios and ratios observed at 55° to the beam direction are indicated by filled circles. Ratios referring to observation at 90° are shown as open circles. Only the resonance at 3150 keV will be described in detail in this thesis.
The branching ratios reported by Tolbert\(^{(34)}\) for the resonances at 1.68, 1.85 and 1.88 MeV agree substantially with those given here. However, in the present work, no decay (upper limit 2\%) was found to the 5/2\(^+\) state from the 5.34 MeV level. This puts in question Tolbert's assignment of positive parity to this state. Another minor discrepancy concerns the 3\% branch to the 5/2\(^+\) state in the decay of the 1.85 MeV resonance. This transition is not reported by Tolbert.

While using the He\(^{++}\) beam of the 4 MV accelerator at N.R.C. to investigate the region of the lowest \(T = 3/2\) state in \(^{19}\text{F}\), another two new resonances in \(^{15}\text{N}(\alpha,\gamma)^{19}\text{F}\) were discovered. These occur at \(E_\alpha = 4467\) and 4621 keV as strong, sharp and isolated resonances in an otherwise featureless background. See Fig. 7.2a. The properties of these resonances and the reasons for believing that they correspond to the two lowest \(T = 3/2\) states in \(^{19}\text{F}\) will be discussed in full detail later in this thesis.
CHAPTER FIVE

MEASUREMENT OF $\omega_\gamma$ FOR 1681 keV RESONANCE

The radiative yield of the 1681 keV resonance in the $^{15}\text{N}(\alpha,\gamma)^{19}\text{F}$ reaction was determined by comparison with $\omega_\gamma$ for the 1532 keV resonance in the $^{14}\text{N}(\alpha,\gamma)^{18}\text{F}$ reaction. Parker (52) has determined the radiative yield of this resonance in the $^{14}\text{N}(\alpha,\gamma)^{18}\text{F}$ reaction by an absolute method based on measuring the production of $^{18}\text{F}$ in the target by observing its residual $\beta$-activity. Since $^{19}\text{F}$ is stable, this method is not possible for $^{15}\text{N}(\alpha,\gamma)^{19}\text{F}$ resonances. However, since the 1681 keV and 1532 keV bombarding energies are quite close and since the two resonances are of comparable strength, their yields can be intercompared quite accurately. One important advantage of the intercomparison method is that it is not necessary to know the absolute concentration of nitrogen in the target. Only the $^{14}\text{N} : ^{15}\text{N}$ ratio need be known.

For the purposes of this intercomparison, titanium nitride targets were made using gas which was a mixture of $^{15}\text{N}(36 \pm 1\%)$ and $^{14}\text{N}(64 \pm 1\%)$. The isotopic composition of the nitrogen gas was determined by a mass spectrometer and it was assumed that this composition also existed in the nitride targets. Yield curves from the $^{14}\text{N}$ plus $^{15}\text{N}$ target
in the regions of the two resonant energies are shown in Fig. 5.1. Since both resonances have total widths less than 2 keV\(^{(24)}\), it is clear that the target thickness condition discussed in section 3.7 was satisfied.

An alternating sequence of bombardments was carried out, starting with the 1681 keV resonance and following with the 1532 keV resonance, such that there were in all 5 bombardments at 1681 keV and 4 at 1532 keV. The yields of primary γ-rays from the two resonances were intercompared by analysis of their spectra in a 5" NaI(T\(\text{I}\)) spectrometer set at 0° to the beam direction. The counts under the full energy peaks corresponding to the primary transitions in table 5.1 were determined. To calculate from thence the relative yield of primary photons, the following additional data were used:

(i) The branching ratios given in table 5.1
(ii) The angular distributions given in table 5.1
(iii) Angular distribution attenuation coefficients for the 5" NaI(T\(\text{I}\)) crystal calculated from the formula given by Smith\(^{(49)}\).
(iv) The relative efficiency of the NaI(T\(\text{I}\)) crystal interpolated and calculated from the data of Miller et al.\(^{(53)}\) and Miller and Snow.\(^{(54)}\)

The ratio of \(\gamma\)'s was found to be
This leads to $\omega \gamma = 1.75 \pm 0.20$ ev for the 1681 keV resonance in $^{15}\text{N}(\alpha,\gamma)^{19}\text{F}$ when Parker's (52) figure for the 1532 keV resonance in $^{14}\text{N}(\alpha,\gamma)^{18}\text{F}$ (i.e. 2.4$\pm$0.2 eV) is used*. Price's results (32,55) for the same two resonances are 3.3$\pm$0.7 eV (1681 keV) and 3$\pm$1 eV (1532 keV).

*N.B. Parker (52) and Price (24,32,55) quote results for the quantity $(2J + 1)\Gamma_\alpha \Gamma_\gamma /\Gamma$ which is larger by a factor of 2 than the quantity $\omega \gamma$ defined earlier in this thesis.
Table 5.1

Data concerning two resonances intercompared

<table>
<thead>
<tr>
<th>Resonance E keV</th>
<th>Energy Level (MeV)</th>
<th>Branching Ratio</th>
<th>Angular Distribution</th>
<th>Ref.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1681</td>
<td>0.0 unresolved</td>
<td>37</td>
<td>isotropic</td>
<td>34,56</td>
</tr>
<tr>
<td>1681</td>
<td>0.110</td>
<td>42</td>
<td>isotropic</td>
<td>34,56</td>
</tr>
<tr>
<td>1681</td>
<td>1.46</td>
<td>20</td>
<td>isotropic</td>
<td>34,56</td>
</tr>
<tr>
<td>1530</td>
<td>0.0</td>
<td>9</td>
<td>$1 - 1/4 P_2(\cos\theta)$</td>
<td>45</td>
</tr>
<tr>
<td>1530</td>
<td>1.045 unresolved</td>
<td>3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1530</td>
<td>1.082 unresolved</td>
<td>35</td>
<td>$1 + 1/2 P_2(\cos\theta)$</td>
<td>45</td>
</tr>
</tbody>
</table>
Yield Curves

5" NaI (Tl) \( E_\gamma = 3-6 \) MeV

TiN target #50

\[ ^{14}\text{N}(\alpha, \gamma)^{18}\text{F} \]

\( E_\alpha = 1532 \) keV resonance

\[ ^{15}\text{N}(\alpha, \gamma)^{19}\text{F} \]

\( E_\alpha = 1681 \) keV resonance

Fig. 5.1
6.1 Introduction

Angular distributions of the γ-rays from this resonance were measured with the 40 cm$^3$ Ge(Li) counter at the angles $\theta = 0^\circ$, $30^\circ$, $45^\circ$, $60^\circ$ and $90^\circ$ and with the front face of the Ge crystal 5.25 cm. from the source of radiation. The 30 cm$^3$ Ge(Li) detector, situated at 135$^\circ$ to the beam direction, was used as a monitor.

The γ-ray decay scheme finally established for the 3150 keV resonance (see Figs. 4.1 and 6.1a) is shown in part in Fig. 6.1b. In addition to the transitions shown in this figure, the Ge(Li) spectra (see Fig. 6.11) showed strong transitions from the resonance level to the ground state and first four excited states of $^{19}$F. Since the strength of the resonance (estimated initially from the size of the peak in the yield curve) requires dipole or quadrupole radiation*, the peak in the yield curves at 3150 keV was interpreted as being due to two closely spaced resonances, one with spin $\geq 9/2$ decaying to the $13/2^+$ and $9/2^+$ levels, the other with spin $\leq 5/2$ decaying to the $15/2^+$ and $11/2^+$ levels.

* If octupole were the lowest occurring multipole for any of the observed primary γ-rays, transition strengths $> 10^4$ W.u. would be implied. These are unphysical.
The two-resonance interpretation is supported by later work (57) which enabled a spin of 3/2 to be assigned to the low spin resonance and which resolved the single peak in the yield curve at 3150 keV into a doublet due to resonances at 3147 keV (J = 3/2) and 3149 keV.

The following analysis justifying an 11/2 spin assignment to the compound state formed at $E_a = 3149$ keV (with the decay scheme presented in Fig. 6.1b) first assumes that the spin assignments of 9/2 and 13/2 to the levels at 2.778 MeV and 4.648 MeV are now established (35,58). However, since the 13/2$^+$ level is newly-discovered (1968), an attempt was made to determine its spin independently, using the present experimental data, relying only on the 9/2$^+$ assignment for the 2.78 MeV level. Though the results of the analysis from this more conservative point of view are somewhat inconclusive, the details are given here since they do restrict the possible spins. In the following discourse, the two points of view will be referred to respectively as assumption A and assumption B.

6.2 Radiative Yield

Using the 40 cm$^3$ Ge(Li) counter at $\theta = 55^\circ$, the quantity $\omega_Y$ for the 3149 keV resonance was found to be $2.5 \pm 0.4$ eV by intercomparison with the yield from the 1681
keV resonance. In the spectrum from the 1681 keV resonance, all three peaks were utilized from each of the following transitions: R to g.s., R to 0.110 MeV and R to 1.46 MeV. In the spectrum from the 3149 keV resonance, the full energy peak corresponding to the transition R to 4.65 MeV and all three peaks from the transition R to 2.78 MeV were used. The same target was used throughout the following sequence of bombardments intended to average out the effect of target deterioration: E = 3149 keV (0.0165 coulomb), 1681 keV (0.0165 coulomb), 3149 keV (0.0165 coulomb), 1681 keV (0.0182 coulomb), 3149 keV (0.0180 coulomb). The relative numbers of photons from each primary γ-ray were calculated with the aid of efficiency curves prepared by Phyllis B. Dworkin for the 40 cm$^3$ Ge(Li) counter. These efficiency curves were also used in the determination of the branching ratios in Fig. 6.1b.

In order to have an estimate of $\Gamma_{\gamma}$ to assist in the following analysis, let it be assumed, tentatively, that $\Gamma_{\alpha} \geq \Gamma_{\gamma}$. This assumption will be discussed in section 6.6.

Then

$$\omega_{\gamma} \propto \frac{2J + 1}{2} \Gamma_{\gamma} = 2.5 \pm 0.4 \text{ eV}.$$  

6.3 3.72 MeV γ-Ray. (11/2 to 9/2 transition)

The angular distribution of this γ-ray is shown in Fig. 6.1c. Figs. 6.2a and 6.2b show that of the various spins postulated for the resonance level, only 11/2 (with $|\delta| = 0.02 \pm 0.02$,
leading to an E2* strength \( \geq 0.06^{+0.18}_{-0.06} \text{ W.u.} \) and 7/2 (with \( |\delta| = 0.11 \pm 0.04 \), leading to an E2 strength \( \geq 6 \pm 3 \text{ W.u.} \)) lead to acceptable fits of the measured angular distribution. Neither of the above E2 strengths is unlikely a priori. The 5/2 postulate which comes near to an acceptable fit (Fig. 6.2b) only with an E3 strength \( \geq 10^6 \text{ W.u.} \) is rejected along with 9/2 and 13/2. With assumption A, a spin of 11/2 is thus established for the resonance level. The small mixing ratio with a large error prevents any reliable conclusion from being drawn about the parity of the resonance level.

With assumption B, the spin of the resonance level must be either 11/2 or 7/2. In the 7/2 case, positive parity would be indicated by the quadrupole strength [since negative parity would imply that \( |M(M2)|^2 > 50 \text{ W.u.} \)]. A 7/2\( ^+ \) assignment for the spin-parity of the resonance level would reopen the question, apparently settled in section 6.1, of which of the two resonances is responsible for the transitions to the 5/2\( ^+ \) and 5/2\( ^- \) levels. (The presence of a second resonance would still have to be assumed to explain the transitions to the 1/2\( ^+ \), 1/2\( ^- \) and 3/2\( ^- \) levels). However it has now

* The calculation of E2 strengths at this stage is not intended to imply any prejudice in favour of positive parity for the resonance level. It is simply that the estimate of interest is the lowest quadrupole strength in W.u. implied in the mixing ratio.

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been shown\(^{(57)}\) that the transitions to the \(5/2^+\) and \(5/2^-\) levels are from the \(E_x = 3147\) keV resonance.

6.4 \(11/2\) to \(13/2\) to \(9/2\) Cascade

(i) Because of a near coincidence of energies, the spectral peaks corresponding to this \(\gamma\)-ray cascade were not completely resolved. Fig. 6.3 shows how the primary \(\gamma\)-ray, showing full Doppler shift, merges with the unshifted secondary \(\gamma\)-ray at forward angles of observation. This interpretation is consistent with the relatively long life-time \((2 \times 10^{-12}\) secs.) of the \(13/2\) level\(^{(58)}\), and the observation of full Doppler shift on the other primary \(\gamma\)-ray.

Attempts to extract the separate areas of the overlapping peaks were, on the whole, unsuccessful because of large errors arising from the lack of detailed knowledge of the line shapes. However, the separation and the resolution were good enough to allow separate areas to be obtained at \(\theta = 90^\circ\) and also at \(135^\circ\) (from the monitor Ge counter which was situated at this angle). The ratios of primary \(\gamma\)-ray intensity to secondary \(\gamma\)-ray intensity at \(90^\circ\) and \(135^\circ\) were thus obtained for comparison with the theoretical ratios. Because the energies of primary and secondary \(\gamma\)-rays are nearly the same, errors in intensity ratios originating from uncertainties in the relative efficiencies of the two different
counters are negligible.

Fig. 6.1d shows the angular distribution of the two cascade $\gamma$-rays treated as an unresolved doublet. With the secondary $\gamma$-ray assumed to be pure quadrupole and a multipole mixing ratio of $\delta = 0.03 \pm 0.03$ (see Fig. 6.4a) for the primary, a good fit is obtained to both the angular distribution and the intensity ratios (see table 6.1). Again the measured E2 strength of $3.0^{+9.0}_{-3.0}$ W.u. is too uncertain to permit any reliable inference regarding the parity of the resonance level.

Clearly the experimental data about this cascade are consistent with assumption A and with the assignment of $11/2$ for the spin of the resonance level.

<table>
<thead>
<tr>
<th>Table 6.1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ratio of Intensities of $11/2\rightarrow13/2$ and $13/2\rightarrow9/2$ Gamma–rays</td>
</tr>
<tr>
<td>Angle</td>
</tr>
<tr>
<td>---</td>
</tr>
<tr>
<td>$90^\circ$</td>
</tr>
<tr>
<td>$135^\circ$</td>
</tr>
</tbody>
</table>

(ii) Under assumption B, the experimental data for this cascade are now used to try to determine independently the spin of the 4.648 MeV level, and also to resolve the
11/2-7/2 ambiguity for the spin of the resonance level. The procedure is to try to fit both the ratio and the angular distribution data with the theoretical ratios and angular distributions corresponding to various postulated spin sequences. The results are shown in Figs. 6.4b to 6.8b and are summarized in Table 6.2.

Note that the calculated $\chi^2$ includes both the fit to the two ratios and the five angles of the angular distribution. Although minimum $\chi^2$ calculated in this way may reasonably be defined to correspond to optimum fit, difficulties arise in trying to assign absolute probabilities to the various fits. One difficulty is that the experimental ratio at 90° and relative intensity at 90° are not statistically independent: both derive from the moving counter. This tends to make the fit worse (i.e., make $\chi^2$ larger) than would be expected if the two numbers were statistically independent. A second difficulty has to do with the fact that the angular distribution data are fitted with the aid of a normalizing factor (this decreases $\chi^2$ and would be regarded as reducing by one the number, N, of statistical degrees of freedom if these were the only points being fitted) while the fit to the ratio data is not affected by this normalization. Since the
two effects tend to cancel and since, in any case, accurate absolute probabilities are not really at issue, let us neglect both effects on the number N, calculating N thus:

\[ N = (\text{no. of ang. distr. points}) + (\text{no. of ratios}) - (\text{no. of multipole mixing ratios}) \]

\[ = 5 + 2 - (1 \text{ or } 2) \]

Tables of \( \chi^2 \) were then used to find the numbers in column 4 of table 6.2.

From table 6.2, it is evident that the spin sequences numbered 1, 3 and 6 fit the data well while the others fit badly enough to be rejected. Further, the minimum E2 strengths of the primary \( \gamma \)-rays for the sequences 1, 3 and 6 (calculated from the corresponding \( \delta \)) are reasonable. The conclusion is that under assumption B and according to the data presented so far, the spins of the levels at 6.498 MeV and 4.648 MeV are restricted to, respectively, \( 11/2 \) and \( 13/2 \), \( 11/2 \) and \( 9/2 \), or \( 7/2 \) and \( 9/2 \).

6.5 \( 9/2 \) to \( 5/2 \) Transition

The identification of the \( \gamma \)-ray at 2.58 MeV with this transition, though not really in doubt, was verified by noting its varying spectral line shape at different angles of observation. This results from the "fast" and "slow" feeds to the \( 9/2^+ \) level through the \( 11/2 \rightarrow 9/2 \) transition (fast) and the \( 11/2 \rightarrow 13/2 \rightarrow 9/2 \)
Table 6.2

Analysis of partially resolved cascade through level at 4.648 MeV in an attempt to restrict spins of the latter level and of the resonance level.

<table>
<thead>
<tr>
<th>No.</th>
<th>Postulated Spin Sequence</th>
<th>Fig. No.</th>
<th>1% Limit $\chi^2$</th>
<th>Co-ordinates of Best Fit</th>
<th>Acceptable Yes/No</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$</td>
<td>\delta_1</td>
</tr>
<tr>
<td>1</td>
<td>11/2→13/2→9/2</td>
<td>6.4c</td>
<td>16.8</td>
<td>4.8</td>
<td>0.01 ± 0.04</td>
</tr>
<tr>
<td>2</td>
<td>11/2→11/2→9/2</td>
<td>6.6a</td>
<td>15.0</td>
<td>66</td>
<td>-</td>
</tr>
<tr>
<td>3</td>
<td>11/2→9/2→9/2</td>
<td>6.5</td>
<td>15.0</td>
<td>2.0</td>
<td>0.06 ± 0.02</td>
</tr>
<tr>
<td>4</td>
<td>11/2→7/2→9/2</td>
<td>6.4b</td>
<td>16.8</td>
<td>&gt;500</td>
<td>-</td>
</tr>
<tr>
<td>5</td>
<td>7/2→11/2→9/2</td>
<td>6.4b</td>
<td>16.8</td>
<td>&gt;200</td>
<td>-</td>
</tr>
<tr>
<td>6</td>
<td>7/2→9/2→9/2</td>
<td>6.7</td>
<td>15.0</td>
<td>4.0</td>
<td>0.03 ± 0.04</td>
</tr>
<tr>
<td>7</td>
<td>7/2→7/2→9/2</td>
<td>6.6b</td>
<td>15.0</td>
<td>74</td>
<td>-</td>
</tr>
<tr>
<td>8</td>
<td>7/2→5/2→9/2</td>
<td>6.4b</td>
<td>16.8</td>
<td>122</td>
<td>-</td>
</tr>
</tbody>
</table>
(slow) cascade, and also the fact that the lifetime of the 9/2^+ level is of the order of the slowing down time of the 19^F ions in the target (35,58).

The angular distribution of the 9/2 → 5/2 γ-ray is shown in Fig. 6.8a and again in Fig. 6.8b. Clearly it is quite well fitted (χ^2 = 4.2 from Fig. 6.4d) by the unbroken line which corresponds to spins of 11/2 and 13/2 for the resonance and 4.648 MeV levels, with the same choice of mixing ratios in the primary γ-rays as was required to fit the primary angular distributions. Thus, this angular distribution also is consistent with assumption A and with the 11/2 assignment for the spin of the resonance level.

The 9/2 → 5/2 angular distribution was used to try to resolve the remaining ambiguities, under assumption B, among the three possible spin sequences for the cascade, though the fit to the experimental points for the 9/2 to 5/2 transition is rather insensitive to the choice of mixing ratios in the upper γ-rays of the cascade. However, as is illustrated by Fig. 6.9, for the spin sequences under consideration the mixing ratios which best fit this angular distribution are partly inconsistent with those required to fit the primary angular distributions, except in the case of the 11/2 → 13/2 → 9/2 sequence. The results of fitting the
Table 6.3
Analysis of 9/2 to 5/2 angular distribution with a view to restricting spins of resonance level and 4.648 MeV level

<table>
<thead>
<tr>
<th>Spin Sequence</th>
<th>Fig. No.</th>
<th>1% Limit $\chi^2$</th>
<th>Lowest $\chi^2$ (occurs at $\delta_1=\delta_2=0$)</th>
<th>$\chi^2$ at $\delta_1$ and $\delta_2$ given in Table 6.2</th>
</tr>
</thead>
<tbody>
<tr>
<td>11/2, 13/2, 9/2 to 5/2</td>
<td>6.4d</td>
<td>11.3</td>
<td>4.2</td>
<td>4.2</td>
</tr>
<tr>
<td>11/2, 9/2, 9/2 to 5/2</td>
<td>6.9a</td>
<td>9.2</td>
<td>6.1</td>
<td>14±2</td>
</tr>
<tr>
<td>7/2, 9/2, 9/2 to 5/2</td>
<td>6.9b</td>
<td>9.2</td>
<td>9.7</td>
<td>17±3</td>
</tr>
</tbody>
</table>
9/2 to 5/2 angular distribution are summarized in table 6.3. Note that the 7/2, 9/2, 9/2 spin sequence fails to reach the 1% confidence limit for any choice of \( \delta_1 \) and \( \delta_2 \). Note also that only the 11/2, 13/2, 9/2 spin sequence gives a properly low \( \chi^2 \) when the mixing ratios are those observed in fitting the cascade \( \gamma \)-rays (table 6.2).

In an attempt to exploit these inconsistencies as a means of discriminating among the three spin sequences, Fig. 6.10 shows the results of fitting simultaneously the 9/2 \( \rightarrow \) 5/2 angular distribution and the cascade angular distribution. Table 6.4 lists the lowest overall \( \chi^2 \) and corresponding mixing ratios for the three spin sequences. Though it is clear that the 11/2 \( \rightarrow \) 13/2 \( \rightarrow \) 9/2 sequence fits best, the \( \chi^2 \) for the other two are not very high. Unfortunately, the statistical interdependence of the data now makes it even more difficult to assign probabilities to the various \( \chi^2 \) for, in addition to the difficulties already discussed in connection with the cascade data, there is a correlation of errors in the two separate angular distributions through the common monitor counter.

Finally, it should be noted that the dotted lines in Figs. 6.8a and 6.8b show that the larger \( \chi^2 \) associated with the 11/2, 9/2, 9/2 and 7/2, 9/2, 9/2 sequences is largely due
to the deviation of just one point. This fact, together with the rather equivocal $\chi^2$ values, forces the admission that, without invoking assumption A, the 11/2, 9/2, 9/2 and 7/2, 9/2, 9/2 spin sequences cannot be rigorously eliminated. However the sequence 11/2, 13/2, 9/2 is strongly favoured.

**TABLE 6.4**

**Analysis of simultaneous fits of cascade $\gamma$-rays and 9/2 to 5/2 $\gamma$-ray**

<table>
<thead>
<tr>
<th>Spin Sequence</th>
<th>Fig. Nos.</th>
<th>Overall $\chi^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>11/2, 13/2, 9/2, 5/2</td>
<td>6.4c and d</td>
<td>9.0</td>
</tr>
<tr>
<td>11/2, 9/2, 9/2, 5/2</td>
<td>6.10b</td>
<td>16</td>
</tr>
<tr>
<td>7/2, 9/2, 9/2, 5/2</td>
<td>6.10a</td>
<td>22</td>
</tr>
</tbody>
</table>

**6.6 Discussion and Summary**

Since the present work tends to support a $13/2^+$ assignment for the spin-parity of the level at 4.648 MeV, there is no reason to doubt this assignment of Jackson et al. (58). That is to say assumption A is valid and 11/2 is the spin of the new level at 6.498 MeV.

The parity of the level is not determined by the present work. However, a negative parity resonance level decaying only by El transitions would be quite different from
any of the others studied in $^{19}F$, and would be difficult to explain in terms of current ideas of the structure of $^{19}F$(13,21). On the other hand, several theories predict that the lowest $11/2^+$ level in $^{19}F$ should occur at about the excitation energy of this resonance level. For example, Elliott and Flowers (1) predict 6.7 MeV if the parameter $V_c$ in their $(2s - 1d)^3$ shell model calculation is chosen to give the observed excitation energy of the $13/2^+$ level ($V_c = 46$ MeV); Hamamoto and Arima (20) predict 6.65 MeV from projected Nilsson wavefunctions; and Benson and Flowers (21) predict 6.3 MeV in a restricted Hartree-Fock calculation with $K = 1/2$ a good quantum number. Further, as will be shown later, the decay scheme and transition probabilities of the new level are in accord with these theories. All this, of course, is no substitute for determining the parity experimentally; it is only suggestive. In the meantime, it is now assumed that the level at 6.498 MeV is the lowest $11/2^+$ state in $^{19}F$.

The assumption that for the resonance level $\Gamma_\alpha \geq \Gamma_\gamma$ will also have to be verified by experiment. Meanwhile it is plausible, for $\Gamma_\alpha$ (Wigner) for $l = 5$ $\alpha$-capture is 200 eV (for an interaction radius $R = 4.864$ fm) whereas $\Gamma_\gamma$ (Weisskopf) is 1.2 eV. However it is safer to write $\Gamma_\alpha \gg \Gamma_\gamma$ rather than $\Gamma_\alpha \gg \Gamma_\gamma$ since $\Gamma_\alpha$ may be much less than
If the state is well described as \((2s - 1d)^3\) in the shell model, \(\Gamma_\alpha\) (Wigner) if the state is well described as \((2s - 1d)^3\) \(0^{16}\) in the shell model. Note that this argument is only partly vitiated if the state has negative parity: \(\Gamma_\alpha\) (Wigner) is then only 12 eV, but the shell model description of the state would not then require that \(\Gamma_\alpha \ll \Gamma_\alpha\). Finally, the assumption that

\[
\frac{\Gamma_\alpha}{\Gamma_\gamma} = \frac{2 \omega_\gamma}{(2J+1)}
\]

leads to the reasonable transition strengths given in table 6.5.

**Table 6.5**

Summary of measured transition strengths. The \(7/2^+\) state is taken to be the level at \(E_x = 4.38\) MeV. The sign of \(\delta\) conforms to the sign convention of Rose and Brink (60).

| Transition | Branching Ratio | \(\delta\) | \(|M|^2\) W.u. |
|------------|----------------|-----------|----------------|
| \(11/2^+\) to \(13/2^+\) | 45% | 0.03±0.03 | 1.4±0.2 |
| to \(9/2^+\) | 55% | 0.02±0.02 | 0.20±0.03 |
| to \(7/2^+\) | <3% | - | - |

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$^{15}$N($\alpha$,γ)$^{19}$F YIELD CURVE

$3.0 < E_\gamma < 7.0$ MeV

35 cm$^3$ Ge(Li)

Fig. 6.1
$^{15}_N \xrightarrow{\alpha} 3.15 \text{ MeV}$

$\theta = 90^\circ$

$\theta = 45^\circ$

$\theta = 0^\circ$

Fig. 6.3
Fig. 6.6a

Fig. 6.6b

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$2.58 \text{ MeV } \gamma$-RAY

\begin{align*}
\text{RELATIVE INTENSITY} & \quad \text{THEORY} \\
\frac{1}{2}^+ & \quad \delta = 0.007 \\
\frac{3}{2}^- & \quad \delta = 0.017 \\
\frac{9}{2}^- & \quad \delta_2 = 0.728 \\
\frac{5}{2}^- & \quad \delta_2 = 0.638
\end{align*}

$\cos^2 \theta$

\text{Fig. 6.8a}

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\textbf{Fig. 6.10a}

\textbf{EQUI-$\chi^2$ CURVES}

\textbf{THEORY}

\begin{align*}
\text{ARCTAN} \delta_1 \text{ (DEGREES)} & & \text{ARCTAN} \delta_2 \text{ (DEGREES)} \\
\text{THEORY} & & \\
\end{align*}

\begin{align*}
\chi^2 & = 50 \\
\end{align*}

\begin{align*}
\text{TWO ANGULAR DISTRIBUTIONS} & & \text{TWO ANGULAR DISTRIBUTIONS} \\
\text{FITTED SIMULTANEOUSLY} & & \text{FITTED SIMULTANEOUSLY} \\
\end{align*}

\textbf{Fig. 6.10b}

\textbf{EQUI-$\chi^2$ CURVES}

\textbf{THEORY}

\begin{align*}
\text{ARCTAN} \delta_1 \text{ (DEGREES)} & & \text{ARCTAN} \delta_2 \text{ (DEGREES)} \\
\text{THEORY} & & \\
\end{align*}

\begin{align*}
\chi^2 & = 50 \\
\end{align*}

\textbf{TWO ANGULAR DISTRIBUTIONS SIMULTANEOUSLY FITTED.}

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\[ E_\alpha = 3150 \text{ keV Resonance Spectrum at } 135^\circ \]

**Fig. 6.11**

- **Counts**
  - 6000
  - 4000
  - 2000
  - 1000
  - 500
  - 300
  - 200
  - 100

- **Channel Number**
  - 2400
  - 2600
  - 2800
  - 3000
  - 3200

- **Factors**
  - x0.353
  - x1
  - x1
  - x2.08
  - x0.208
7.1 Introduction

A typical Ge(Li) spectrum obtained at the $E_\alpha = 4467$ keV resonance is shown in Fig. 7.1. Figs. 7.2b and 7.11 show the $\gamma$-ray decay scheme ascribed to this resonance (at $E_x = 7.537$ MeV in $^{19}$F). The results of angular distribution measurements at $\theta = 0^\circ$, $45^\circ$ and $90^\circ$ are presented in Figs. 7.3 to 7.10. The 30 cm$^3$ Ge(Li) counter was used for these measurements with its front face 4.5 cm from the source of radiation. The attenuation coefficients were calculated by averaging the absorption over the whole active volume of the detector (rather than over a disc at its centre) while taking into account the exponential absorption of the $\gamma$-rays in passing through the crystal (61).

7.2 Radiative Yield

The quantity $\omega_Y$ for this resonance was determined by an intercomparison with the $E_\alpha = 1681$ keV resonance with the 30 cm$^3$ Ge(Li) detector set at $\theta = 55^\circ$ to avoid the need for corrections for angular distribution. (The angular distributions of the $\gamma$-rays from the $E_\alpha = 4467$ keV resonance are shown later to have no significant $P_4$.
(terms.) In the spectrum from the $E_\alpha = 4467$ keV resonance, all three peaks from the $\gamma$-rays corresponding to the transitions $R \to 0.198$ MeV and $R \to 1.55$ MeV were counted. In the case of the $E_\alpha = 1681$ keV resonance, all three peaks from the following $\gamma$-rays were counted: $R \to 0$ MeV, $R \to 0.110$ MeV and $R \to 1.46$ MeV. The same target — one 12 $\mu$gm/cm$^2$ or 8 keV thick at $E_\alpha = 4.4$ MeV — was used at both resonances. The sequence of bombardments was first 4467 keV (0.015 coulombs He$^{++}$), then 1681 keV (0.08 coulombs He$^+$) and finally 4467 keV again (0.015 coulombs He$^{++}$). No significant drop in $\gamma$-ray yield was noticed between the initial and final bombardments at 4467 keV. The conversion factors from counts to relative number of photons, required in this measurement of $\omega_\gamma$ and in determining the branching ratios shown in Fig. 7.2b and in Fig. 7.11 were obtained from detector efficiency calibrations carried out by Dixon and Storey\cite{61}.

The result of the intercomparison is

$$(\omega_\gamma)_{4467} \div (\omega_\gamma)_{1681} = 9.9 \pm 1.5,$$

whence $$(\omega_\gamma)_{4467} = 17 \pm 4 \text{ eV}.$$

For the purposes of calculating interim values of transition strengths, it has been assumed that $\Gamma_\alpha \gg \Gamma_\gamma$. The justification of this assumption is postponed to section 7.5.
Therefore, let it be assumed that

\[ \Gamma_\gamma = \frac{2}{2J+1} \omega \gamma \]

### 7.3 Spin of Resonance Level

Table 7.1 summarizes the results and conclusions from fitting (see Fig. 7.3 and 7.4) the observed angular distributions of the 5.98 MeV and 7.34 MeV \( \gamma \)-rays attributed to the transitions \( R \) to 1.55 MeV and \( R \) to 0.198 MeV respectively. Spins of 3/2 or 5/2 are possible for the resonance level but 7/2 is ruled out.

The experimental angular distribution of the 1.35 MeV \( \gamma \)-ray attributed to the transition from the 1.55 MeV level to the 0.198 MeV level, has been analyzed (see Figs. 7.5 and 7.6) with a view to deciding between 3/2 and 5/2 for the spin of the resonance level. If \( |\delta_1| \) for the unobserved \( R \) to 1.55 MeV transition is as given in Table 7.1, both spin postulates are consistent with the angular distribution of the 1.35 MeV \( \gamma \)-ray if its mixing ratio \( \delta_2 \) is unrestricted. Table 7.2 summarizes the situation. It is possible, however, to infer something about \( \delta_2 \) from other information about the 1.55 MeV level:

(i) Poletti, Becker and McDonald(62) in a recent attempted measurement of the lifetime of the 1.55 MeV level by the Doppler Shift Attenuation Method using \( ^{19}F(p,p')^{19}F^* \) found \( \tau_m < 1.7 \times 10^{-14} \) sec. However, using branching ratios
Table 7.1

Summary of results of analysis of Figs. 3 and 4—an attempt to determine the spin of the resonance level. 7/2 is eliminated; 5/2 seems the most reasonable, but 3/2 is not convincingly disposed of.

| Transition | Postulated Spin | Minimum $\chi^2$ | Admissible | $|\delta|$ for best fit | $|M(E2)|^2$ Enhancement W.u. | Admissible |
|------------|----------------|------------------|------------|------------------------|----------------------------|------------|
| R to 1.55 MeV |
| 7/2 | 670 | No | - | - |
| 5/2 | 0.11 | Yes | 0.02±0.02 | $(5_{-5}^{+20})\times10^{-2}$ | Yes |
| 3/2 | 0.11 | Yes | 0.77±0.07 | $72_{-9}^{+22}$ | Yes |
| 3/2 | 0.11 | Yes | 3.2±0.5 | 181±12 | Yes |
| R to 0.198 MeV |
| 5/2 | 0.11 | Yes | 0.11±0.02 | $(41_{-18}^{+20})\times10^{-2}$ | Yes |
| 3/2 | 0.11 | Yes | $0.47 \leq |\delta| \leq 1.33$ | $7 \leq |M|^2 \leq 30$ | Yes |
for the 1.55 MeV level found in their own work, they calculate the figure \((4.4^{+2.4}_{-2.0}) \times 10^{-15}\) sec. from the partial lifetimes given by Booth, Chasan and Wright\(^{(63)}\) from their resonance fluorescence studies. The limit found by Poletti et al.\(^{(62)}\) leads to a partial width \(\geq 3.5 \times 10^{-2}\) eV for the 1.55 MeV level with respect to decay to the 0.198 MeV; this is too small by a factor of about 4 if the figure derived from Booth et al.\(^{(63)}\) is adopted.

(ii) The wavefunctions of the three lowest positive parity states of \(^{19}\)F are known well enough to explain\(^{(1)}\), qualitatively at least, why the 3/2\(^+\) state at 1.55 MeV decays so strongly to the 5/2\(^+\) state at 0.198 MeV rather than to the 1/2\(^+\) g.s. In a recent calculation, Benson and Flowers\(^{(21)}\) quote transition strengths which imply that the ratio \(\Gamma(3/2\rightarrow 1/2):\Gamma(3/2\rightarrow 5/2)\) should be about 1% while the calculation of Elliott and Flowers\(^{(1)}\) gives 0.6%. The experimental figure is 2.4\%\(^{(62)}\). Moreover, the absolute width of the 3/2\(^+\) level for decay to the 5/2\(^+\) level calculated from the Benson and Flowers data is about 1.1 \times 10^{-1}\) eV which compares well with the experimental figure 1.5 \times 10^{-1}\) eV derived from \(\tau_m = 4.4 \times 10^{-15}\) sec. adopted by Poletti et al.\(^{(62)}\).

Another demonstration of the relevance of the shell-model wavefunctions for the low-lying states concerns the E2 strength of the transition connecting the 1/2\(^+\) and 3/2\(^+\) states. This has been measured by Coulomb excitation by Litherland et al\(^{(64)}\). Their result, 9 ± 3 W.u. is also in good agreement with
Table 7.2

Summary of results of analysis of the 1.55 MeV-to-0.198 MeV γ-ray angular distribution. The quantity $|M(E2)|^2$ is calculated from the lifetime of the 1.55 MeV level given by Poletti et al. (62) as a re-evaluation of the figure obtained by Booth et al. (63).

| Resonance Spin Postulate | $|\delta_1|$ for best fit R=1.55 MeV (from Table 7.1) | Fig.No. | Minimum $\chi^2$ | Admissible Yes/No | $|\delta_2|$ for best fit | $|M(E2)|^2$ Enhancement W.u. | Admissible Yes/No |
|-------------------------|--------------------------------------------------|--------|-----------------|----------------|------------------|-----------------------------|----------------|
| 5/2                     | 0.02                                             | 7.5    | 0.13            | Yes            | 0.06±0.02        | $50^{+80}_{-30}$            | Yes            |
| 5/2                     | 0.02                                             | 7.5    | 0.13            | Yes            | 7.4±0.7          | $\approx10^4$              | No             |
| 3/2                     | 0.77                                             | 7.6    | 9.0             | No             | -                | -                           | -              |
| 3/2                     | 3.2                                              | 7.6    | 0.3             | Yes            | 0.44±0.09        | $(1.4^{+2.8}_{-0.3})\times10^3$ | No             |
| 3/2                     | 3.2                                              | 7.6    | 0.3             | Yes            | 1.7±0.3          | $\approx10^4$              | No             |
the theoretical value, 7.2 W.u. (21, 51)

Having thus established some confidence in the essential correctness of the shell-model wavefunctions* for the low-lying positive parity states, one can argue that the calculated E2 - M1 mixing ratio for the 3/2\(^+\) to 5/2\(^+\) transition may not be too far wrong. The Benson and Flowers data for this transition give a mixing ratio
\[ |\delta_2| = 1.5 \times 10^{-2} \]
and an enhancement \[ |M(E2)|^2 = 2.3 \text{ W.u.} \]

Then Poletti et al. (62), quoting from a private communication from McGrory, give the results of another shell model calculation (19): \[ |\delta_2| = 1.6 \times 10^{-2} \] and \[ |M(E2)|^2 = 2.6 \text{ W.u.} \]

Finally, Dreizler (12), using an entirely different model of \(^{19}\text{F}\) (a 2s - 1d hole coupled to \(^{20}\text{Ne}\)), calculates
\[ |M(E2; 3/2 \text{ to } 5/2)|^2 = 2.4 \text{ W.u.} \]
and \[ |\delta_2| = 2.5 \times 10^{-2} \]

The inferences being drawn from the points (i) and (ii) above are that values of \(\delta_2 \gg 10^{-1}\) disagree with calculations from theoretical wavefunctions which are quite successful in other cases in which they have been tested and, if taken in conjunction with the measured partial width of the 1.55 MeV level, lead to unusually large E2 strengths.

Estimates of the E2 strength of the 3/2\(^+\) to 5/2\(^+\) \(\gamma\)-ray are given in table 7.2 calculated from the best fitting

* It is worth noting that the rotational model of Paul (3) gives substantially the same results as the shell model for the lifetime and branching of the 3/2\(^+\) state. This fact reduces the model dependence of the argument.
\( \delta_2 \)'s of its angular distribution and the partial width 
\((1.5^{+1.1}_{-0.5}) \times 10^{-1} \) eV derived from the work of Booth et al.\(^{(63)} \) (as readjusted by Poletti et al.\(^{(62)} \)). In selecting the best fits, appropriate values of the mixing ratios (\(|\delta_1|\)) of the unobserved primary \( \gamma \)-ray have been selected from table 7.1 although Fig. 7.6 shows that the fit is not very sensitive to this parameter.

Though the 5/2 postulate comes near to satisfying the criteria, even it seems to require quite a large E2 strength to be assumed for the 3/2\(^+\) to 5/2\(^+\) \( \gamma \)-ray. However the errors quoted for \( \delta_2 \) (which are reflected in the errors in \(|M(E2)|^2\)) do not allow for a possible systematic error due to the source of \( \gamma \)-radiation being eccentric with respect to the circle of rotation of the moving counter. Calculation shows that in the present case an eccentricity error of only 2\% could have shifted \( \delta_2 \) from zero to -0.06 (see Fig. 7.5). In the case of the 3/2 postulate, the same misalignment error would have shifted the lower limit on \(|M(E2)|^2\) from about 500 to 1200, which is the figure in table 7.2. Thus, whereas the rather large value for the 5/2 postulate can be explained by invoking a reasonable alignment error, the values for the 3/2 postulate are still much too large.

One piece of information which has been ignored in the above argument is the value for \( \delta_2 (3/2^+ \text{ to } 5/2^+) \) given by Giebbie.\(^{(65)} \) From polarization-direction correlation studies
using NaI(Tl) detectors and spectrum-stripping procedures, he reports \( \delta_2 = 0.23 \pm 0.02 \). In conjunction with the partial lifetime used already, this leads to \( 670^{+440}_{-200} \) W.u. for \( |M(E2)|^2 \). If the lifetime-limit of Poletti et al.\(^{(62)}\) is used \( |M(E2)|^2 \) is at least 135 W.u. Even Gebbie's own upper limit on the lifetime of the 1.55 MeV level gives \( |M(E2)|^2 > 17 \). These rather large numbers suggest that unless the nature of these low-lying positive parity levels is completely misunderstood and the lifetime estimates by Poletti et al. and Booth et al. are seriously wrong, Gebbie's value for \( \delta_2 \) is too large.

It is on these arguments that the 3/2 postulate in table 7.2 has been rejected, thus leaving 5/2 as the only spin postulate which satisfactorily explains all the data.

7.4 Spin of the 4.378 MeV Level

The level at 4.378 MeV, previously reported as 4.39 MeV\(^{(29,66)}\), is strongly fed at this resonance. In rough agreement with the observations of Thomas et al.\(^{(29)}\) and Olness and Wilkinson\(^{(66)}\), de-excitation \( \gamma \)-rays to the levels at 2.78 MeV (9%) and 0.198 MeV (91%) are seen. Since these \( \gamma \)-rays show greater than 95% of the full Doppler shift appropriate to the resonance bombarding energy, the lifetime of the 4.38 MeV level

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must be less than $2.5 \times 10^{-14}$ sec.* This information on its lifetime and decay scheme restricts the spin of the level to 5/2, 7/2 or 9/2.

Table 7.3 summarizes the results and conclusions from fitting (see figs. 7.7 to 7.9) the observed angular distributions of the 3.158 MeV and 4.180 MeV γ-rays attributed to the transitions R-to-4.378 MeV and 4.378 MeV-to-0.198 MeV respectively. Unfortunately the full-energy peak of the primary γ-ray overlaps the double-escape peak from the secondary γ-ray in the Ge(Li) spectra but adequate counting statistics were obtained from the remaining two peaks for each γ-ray. The overlapped peak was not used in the analysis. At $\theta = 90^\circ$, an unidentified background peak partially overlaps the double-escape peak of the primary γ-ray. This accounts for the somewhat larger error bar on the 90° point in fig. 7.9a than on the other points.

* This lifetime limit was calculated by a Doppler shift attenuation computer programme written by several members of the University of Toronto nuclear physics group. Although the 4.38 MeV level is unbound against α-emission, the Wigner limit on its α-width ($<10^{-8}$ eV for $\ell = 3$) indicates that since $\Gamma > 2.6 \times 10^{-2}$ eV, $\Gamma_{\gamma} = \Gamma_{\gamma}^\prime$ with negligible error. This is consistent with the estimate $\Gamma_{\gamma}/(\Gamma_{\alpha} + \Gamma_{\gamma}) = 0.84 \pm 0.24$ given by Thomas et al, (29) as a result of their $^{19}$F(p,p'γ)$^{19}$F investigation. The present experiment yields the result $\Gamma > \Gamma_{\gamma} > 0.85 \Gamma$. 

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Table 7.3

Results of testing various spin postulates for the 4.38 MeV level.
Spin of the resonance level is taken to be 5/2 and in calculating $|M(E2)|^2$, the lifetime limit for the 4.38 MeV level is taken as $<2.5 \times 10^{-14}$ sec.

| Transition | Spin Postulate | Fig.No. | Minimum $\chi^2$ | Admissible | $|\delta|$ for best fit | $|M(E2)|^2$ Enhancement W.u. | Admissible |
|------------|----------------|--------|------------------|------------|-------------------------|-----------------------------|------------|
| R to 4.38 MeV | 9/2 | 7.7a | 6.0, 3.1 | Yes | see text | see text | Yes |
| | 7/2 | 7.7a | 3.3 | Yes | $0.6^{+0.23}_{-0.18}$ | <100 | Yes |
| | 7/2 | 7.7a | <1 | Yes | >5.7 | >2000 | No |
| | 5/2 | 7.7a | 6.4 | Yes | $0.05<|\delta|<0.47$ | >0.02 | Yes |

| 4.38 MeV to 0.198 MeV | 5/2 | 7.7b, 7.8a | 19.7 | No | - | - | - |
| | 7/2 | 7.8a | 0.85 | Yes | $0.13\pm0.07$ | >0.04 | Yes |
| | 9/2 | 7.8a | 40 | No | - | - | - |
None of the three spin postulates fit the angular distribution of the primary $\gamma$-ray well (see fig. 7.7a) but $7/2$ fits in the 5 - 10% probability range. Fig. 7.9a shows that the difficulty is due to the $\theta = 90^\circ$ point which, as already mentioned, is less reliable than the other two. The $9/2$ postulate with a $|\delta_1|$ not significantly different from zero, and the $5/2$ postulate with a $|\delta_1|$ whose lower limit is only slightly different from zero, are admitted as slight possibilities and reconsidered in analyzing the angular distribution of the secondary $\gamma$-ray (see figs. 7.7b, 7.8 and 7.9b). In the case of the $9/2$ postulate, $\delta_1^2$ must be negligible since it implies octupole admixture in the 1.7 eV primary $\gamma$-ray; fig. 7.8a then rules out this postulate. $5/2$ is eliminated for all likely (as given by fig. 7.7a) values of $\delta_1$ by fig. 7.7b.

The conclusion drawn from the angular distributions considered in figs. 7.7 to 7.9 is that the spin of the 4.38 MeV level is $7/2$. Fig. 7.10 shows that this conclusion is in accord with the angular distribution of the weak 1.60 MeV $\gamma$-ray attributed to the 4.38 MeV-to-2.78 MeV transition.

7.5 Discussion of Resonance Level

The resonance at $E_\alpha = 4467$ keV was discovered in an attempt to excite the lowest $T = 3/2$ state in $^{19}$F, believed to lie between 7.4 and 7.6 MeV in excitation. (36) The
appearance of a strong, sharp and isolated resonance in 
the appropriate region of bombarding energy (see fig. 7.2a) 
where the resonances are generally quite broad,\(^{(26)}\) im-
mediately suggests its \(T = 3/2\) isospin-forbidden character 
for the following reasons:

(i) Since the steepness of the leading edge of the 
peak in the yield curve is entirely attributable 
to the beam energy-spread (a few keV) and target 
surface effects, the width of the resonance must 
be less than about 2 keV. The fact that it was 
not resolved in the \(^{15}\text{N}(\alpha,\alpha)^{15}\text{N}\) work of Smotrich 
et al.\(^{(26)}\) supports this contention. The limit 
on the reduced \(\alpha\)-particle width is therefore 
\(\Gamma_{\alpha}^2 < 1\%\) (assuming \(\ell = 3\) \(\alpha\)-capture), which is in 
keeping with the suggested isospin-forbidden 
character of the reaction.

(ii) The large value of \(\Gamma_\gamma\) indicated by the strength 
of the resonance (\(\omega_\gamma = 17\) eV) suggests that the 
excited configuration is rather a simple one, 
since it decays with near single-particle strength 
to the low-lying states. This also is a 
characteristic to be expected of the lowest \(T = 3/2\) 
state.

Of course the above two characteristics, even in combination, 
are not unique features of a \(T\)-forbidden analogue resonance.
A high spin (e.g. \( J = 13/2 \)) \( T = 1/2 \) resonance might be expected to display similar properties.

A much more cogent argument in favour of \( T = 3/2 \) concerns the \( \gamma \)-ray decay scheme, which strongly suggests the analogy of the resonance level with the ground state of \(^{19}\text{O} \). Fig. 7.2b shows that the \( J = 5/2 \) resonance level at \( E_x = 7.537 \) MeV in \(^{19}\text{F} \) \( \beta^- \)-decays to the same three \( T = 1/2 \) states in \(^{19}\text{F} \) as are fed in the \( \beta^- \)-decay of the \( J = 5/2 \) ground state of \(^{19}\text{O} \). \(^{66,67}\) Since the \( \beta^- \)-decays are allowed Gamow-Teller transitions \( (\Delta T = 0) \) \( (\Delta J = 0, \pm 1; \Delta T = 1) \), the similarity between the M1 operator and the Gamow-Teller \( \beta^- \)decay operator suggests that there should be a resemblance between the two decay schemes if the resonance level is the analogue of the \(^{19}\text{O} \) ground state. The parallelism of the two decay schemes is displayed and discussed more fully in sections 9.7 and 9.8.

There is no direct evidence concerning the parity of the resonance level but, as was remarked in the case of the \( 11/2 \) resonance, decay solely by El transitions would be unusual and hard to explain. \(^{21}\) However if, on other grounds, the resonance level is established as being the analogue of the ground state of \(^{19}\text{O} \), there can be no doubt about its parity.

Because of the isospin-forbiddenness of the resonance reaction, it is unusually difficult to make a plausible assumption about the relative sizes of \( \Gamma_\alpha \) and \( \Gamma_\gamma \). Had the reaction

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been strictly isospin forbidden $\Gamma_\alpha$ would, of course, have been zero and the $T = 3/2$ state could not have been observed in the present experiment. A non-zero value for $\Gamma_\alpha$ implies some degree of isospin mixing with neighbouring states of the same spin and parity in the compound state (i.e. $T = 1/2$ admixture) induced by the Coulomb interaction and possibly by charge-dependent nuclear forces. ($T = 3/2$ admixtures in the $^{15}N$ ground state and $T = 1$ admixtures in the $\alpha$-particle ground state would also contribute but are expected to be much smaller because of the large energy separation of their lowest $T_>$ and $T_<$ states.) Since the $\alpha$-resolution in the present experiment is no better than two or three keV, any fine structure ($^{68,69}$) in the resonance arising from coupling between the $T_>$ and adjacent $T_<$ states is averaged over.

From the results reported here, all that can be said about the relative sizes of $\Gamma_\alpha$ and $\Gamma_\gamma$ is that the assumption $\Gamma_\alpha \gg \Gamma_\gamma = \frac{2}{2J+1} \omega_\gamma$ leads to transition probabilities in the single-particle range (see table 7.4) which are to be expected for the decay of a simple configuration. The contrary assumption, that $\Gamma_\gamma \gg \Gamma_\alpha$, would lead to much higher values which would be hard to understand.

Since the present work on this resonance was completed, the level at 7.537 MeV has been reported in two quite different experiments with results which neatly complement those described in this thesis:
(i) Lennon et al. report the discovery of the lowest $T = 3/2$ state in $^{19}$F in an $^{18}_0(\text{He},d)^{19}$F stripping reaction. The level, at $7.56 \pm 0.02$ MeV, is found to have a spectroscopic factor of 0.41 (with respect to $1d_{5/2}$) indicating its near single-particle character, while its $\ell = 2$ stripping pattern shows that it has $J = 3/2$ or $5/2$ and positive parity.

(ii) From a resonance in the $^{18}_0(p,\gamma)^{19}$F reaction Wright finds strong $\gamma$-ray transitions to the level at 7.537 MeV in $^{19}$F but detects no $\gamma$-rays which would correspond to its decay. This indicates that for the 7.537 MeV level $\Gamma_\alpha > 20 \Gamma_\gamma$, thus endorsing the assumption made in calculating the transition strengths in table 7.4 below.

7.6 Discussion of 4.378 MeV, $7/2$ Level

Positive parity has already been assigned to this level by Olness and Wilkinson as a consequence of the very low log ft (in the "super-allowed" range) measured for the transition to the 4.38 MeV state in the $\beta$-decay of $^{19}$O. However, although they and Thomas et al., who studied the 4.38 MeV level in the course of their $^{19}_F(p,p'\gamma)^{19}$F experiments, favour a $J^\pi = 7/2^+$ assignment, neither experiment excludes $3/2^+$ or $5/2^+$. The present results are definitive in this respect and,
in conjunction with other work identifying the lower states, (56) justifies the assertion that the level at 4.38 MeV is the lowest 7/2+ state in 19F. Until now, the level at 5.47 MeV (7/2+(35)) might have been in contention for this position since most calculations based on the various models suggest that the lowest 7/2+ state should occur at a somewhat higher energy than 4.38 MeV. Elliott and Flowers (1) indicate that it should occur between 4.4 and 5.1 MeV; Hamamoto and Arima (20) quote 4.81 MeV; and Benson and Flowers (21) (projected Hartree-Fock, K = 1/2+ band) predict 5.2 MeV. However, since the spins of all the intervening levels are now known, partly as a result of other 15N(α,γ)19F work (56), the level at 5.47 MeV must be the second 7/2+ state in 19F.
Table 7.4

Summary of transition strengths and decay schemes observed in the study of the $E_\alpha = 4467$ keV resonance. The sign of $\delta$ conforms to the sign convention of Rose and Brink.\(^{(60)}\)

| Transition $J^\pi$, $T$ to $J'^\pi$, $T'$ | Branching Ratio | $|M|^2$ W.u. |
|----------------------------------------|-----------------|-------------|
|                                        | $\delta$        | $M1$        | $E2$        |
| $5/2^+, 3/2$ to $7/2^+, 1/2$           | 28\%            | 2.6±0.6     | <100        |
| to $5/2^+$                             | 30\%            | 0.22±0.05   | ($41^{+20}_{-18}) \times 10^{-2}$ |
| to $3/2^+$                             | 42\%            | -0.56±0.14  | ($5^{+20}_{-5}) \times 10^{-2}$    |
| $7/2^+, 1/2$ to $9/2^+, 1/2$           | 9\%             | >2.7 $\times 10^{-2}$ | > 0.3 |
| to $5/2^+$                             | 92\%            | >1.7 $\times 10^{-2}$ |              |

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$E_x = 44.67$ keV Resonance Spectrum at $0^\circ$

Channel Number

Counts $\div 10^3$

Fig. 7.1

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**Fig. 7.2a**

RAWSON PROBE READING (GAUSS ÷ 2)

GAMMA-RAY YIELD $^{15}$N ($\alpha,\gamma$)$^{19}$F

$4.4 \text{ MeV} \leq E_{\gamma} \leq 8.0 \text{ MeV}$

$5''$ NaI (Tl)

**Fig. 7.2b**

MeV $J^\pi$

0.096 $3/2^+$

0.0 $5/2^+$

$0.16\%$

$60\%$

$40\%$

$\beta^-$

$19F$ $19O$

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The figure shows two graphs of relative intensity versus cosine squared of the angle (\(\cos^2 \theta\)) for different \(\gamma\)-rays.

1. **5.98 MeV \(\gamma\)-Ray:**
   - The graph is linear and decreasing.
   - The relative intensity is shown for different values of \(\cos^2 \theta\) from 0 to 1.
   - The theoretical curve is indicated with a straight line.
   - The figure is labeled as 'Theory' with a shift value of \(\delta = 0.02\).
   - The \(\frac{1}{2}^-\) state of \(^{15}\text{N} + \alpha\) is shown with a line segment of length 1.55.

2. **7.34 MeV \(\gamma\)-Ray:**
   - The graph is linear and increasing.
   - The relative intensity is shown for different values of \(\cos^2 \theta\) from 0 to 1.
   - The theoretical curve is indicated with a straight line.
   - The figure is labeled as 'Theory' with a shift value of \(\delta = 0.11\).
   - The \(\frac{1}{2}^-\) state of \(^{15}\text{N} + \alpha\) is shown with a line segment of length 0.198.

**Fig. 7.4**

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Figure 7.5

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Fig. 7.6

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Fig. 7.8b

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Fig. 7.9a

Fig. 7.9b

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Fig. 7.10

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$E_x = 4467 \text{ and } 4621 \text{ keV RESONANCES}$

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8.1 Introduction

A typical Ge(Li) spectrum obtained at the $E_\alpha = 4621$ keV resonance is shown in fig. 8.1. Figs. 7.2b and 7.11 show the $\gamma$-ray decay scheme ascribed to this resonance (at $E_x = 7.658$ MeV in $^{19}$F).

Observations were made only at the two angles $\theta = 0^\circ$ and $\theta = 90^\circ$ in this experiment using the 30 cm$^3$ Ge(Li) counter positioned as described in section 7.1. The relative number of counts in each spectral peak (relative to the monitor counter, that is) was determined at both angles and the quantity called "anisotropy $0^\circ:90^\circ$" in fig. 8.2 is the ratio of these two relative numbers.

8.2 Radiative Yield

To avoid the difficulty of switching between the doubly-charged and singly-charged He beams, the quantity $\omega \gamma$ for this resonance was obtained by intercomparison with the $E_\alpha = 4467$ keV resonance. The intercomparison is based on spectra obtained with a 5" dia. by 4" NaI(Tl) detector set at $\theta = 140^\circ$ during the angular distribution measurement at this resonance and at the 4467 keV resonance. Two beam spots, one for each resonance, on the same target strip were
used. Since the targets seem to be quite uniform across their surface and slow to deteriorate, the omission of the averaging effect of switching several times between the resonances is unlikely to be serious. At both resonances, the full-energy and single-escape peaks of the γ-ray corresponding to the transition R-to-1.55 MeV were well resolved in the NaI(Tl) spectra and these alone were used for the intercomparison. Branching ratios and angular distributions for these were obtained from the Ge(Li) measurements.

The result of the intercomparison is

\[(\omega_\gamma)_{4621} \div (\omega_\gamma)_{4467} = 0.19 \pm 0.04\]

whence

\[(\omega_\gamma)_{4621} = 3.3 \pm 1.0 \text{ eV}\]

Let it be assumed that \(\Gamma_\alpha \geq \Gamma_\gamma\); this will be justified in section 8.4. Therefore let it be assumed that

\[\Gamma_\gamma = \frac{2}{2J+1} \omega_\gamma\]

8.3 Spin of the Resonance Level

The strength of the resonance and its decay scheme indicate that it has \(J \leq 5/2\). Since the angular distributions of the γ-rays attributed to the transitions R-to-g.s. and R-to-1.55 MeV are clearly anisotropic (see figs. 8.2a and 8.2b), \(J = 1/2\) is excluded. The anisotropy of the ground state
transition (fig. 8.2a) alone suffices to eliminate the 5/2 postulate. From fig. 8.2a and without invoking arguments concerning octupole strengths, the probability of 5/2 can be shown as follows to be ~ 0.01% by considering the difference D between the experimental and theoretical anisotropies in relation to the experimental standard deviation σ:

Suppose first \( \arctan \delta = \phi = -35^\circ \)

Experimental anisotropy \( 0^\circ:90^\circ = 0.37 \pm 0.03^\circ \)

Theoretical anisotropy = 0.50 for \( J = 5/2 \) with \( \phi = -35^\circ \)

Difference \( D = \frac{2}{\sigma} \)

Probability of 5/2 being the correct spin = probability of \( D \) exceeding \( 4\sigma \)

\[ = 0.006\% \]

If now \( \delta \) is unrestricted:

\[ \text{Prob. of } 5/2 = \frac{1}{180} \int_{-90^\circ}^{90^\circ} \left[ \text{prob. of } D \text{ exceeding } x(\phi)\sigma \right] d\phi < 0.01\% \]

where \( x(\phi) \) is \( D/\sigma \) at \( \arctan \delta = \phi \).

However, if the possibility of octupole strength greater than 100 W.u. is disregarded, then \( \delta \leq 3 \times 10^{-3} \) and \( D \sim 50\sigma \). Then the probability of 5/2 being correct is negligible.

Inspection of fig. 8.2 shows that all four anisotropies are satisfactorily explained if the resonance has spin 3/2.

Table 8.1 lists the measured branching ratios, mixing ratios and transition strengths.

* The error includes errors in alignment.
(ii) Wright\(^{(71)}\) finds that this state (he quotes \(E_x = 7.658\) MeV) also is populated from the same resonance in \(^{18}_0(p,\gamma)^{19}F\) as populates the \(J = \frac{5}{2}, T = \frac{3}{2}\) state. From the fact that he detects no \(\gamma\)-rays corresponding to the radiative decay of either \(T = 3/2\) state, the assumption that \(\Gamma_\alpha > 20\Gamma_\gamma\) is justified for both.
Table 8.1
Summary of results from the study of the $E_a = 4621$ keV resonance.

Of the two values of $\delta$ for each transition, the larger one and the
values of $|M|^2$ which go with it, can probably be rejected on the
grounds that a significant collective enhancement is inconsistent
with the decay of a low-lying isobaric analogue state.

| Transition          | Branching Ratio | Fig.No. | $\delta$ for best fit | $|M(\text{M1})|^2$ W.u. | $|M(\text{E2})|^2$ W.u. |
|---------------------|-----------------|---------|-----------------------|--------------------------|--------------------------|
| R to g.s.           | 44%             | 8.3a    | $0.04 \pm 0.03$       | $0.07 \pm 0.02$          | $(2^{+4}_{-2}) \times 10^{-2}$ |
|                     |                 | 8.3a    | $1.60 \pm 0.08$       | $0.022 \pm 0.007$        | 8 ± 3                    |
| R to 1.55 MeV       | 39%             | 8.3b    | $0.02 \pm 0.04$       | $0.12 \pm 0.04$          | $<1.3 \times 10^{-1}$    |
|                     |                 | 8.3b    | $4.0 \pm 0.6$         | $0.034 \pm 0.015$        | 26±9                     |
| R to 0.198 MeV      | 17%             | 8.3c    | $0.14 \pm 0.08$       | $0.03 \pm 0.01$          | $(1.0^{+3.0}_{-0.5}) \times 10^{-1}$ |
|                     |                 | 8.3c    | $2.9 \pm 0.7$         | $(3 \pm 1) \times 10^{-3}$ | 4 ± 1                    |
| 1.55 MeV to 0.198 MeV |                 | 8.3d    | $\delta_2 > 0.22$     | -                        | -                        |
Fig. 8.2

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9.1 The Spherical Shell Model

It is well known that the shell model, in its extreme single-particle form, is less successful in explaining spins and parities of excited states than in explaining those properties in the ground state. Since, in the case of $^{19}$F, even the ground state spin is wrongly given by the extreme single-particle model, it is not surprising that one must turn to more sophisticated versions of nuclear shell theory to account for the properties of its excited states.

The first of the modern shell-model calculations treating nuclei of masses 18 and 19 was carried out by Elliott and Flowers. In their model for mass 19, they consider the states of three interacting nucleons in a spherically symmetric potential well provided by an $^{16}$O core which is otherwise inert. The inertness of the core is justified on the grounds that the nucleus $^{16}$O is a doubly closed shell and is known to have no excited states below about 6 MeV. In the absence of interactions among the three "loose" nucleons, the independent particle model of such a system in its ground state would have a harmonic
oscillator or other similarly-shaped potential well containing the loose particles in the highly degenerate levels of the 2s – 1d shell. In the model of Elliott and Flowers, this degeneracy is removed partly by the spin-orbit force and partly by postulating a residual interaction among the loose particles in the form of a central, attractive, two-body potential. Then, using the single-particle wavefunctions of the 2s – 1d shell as a basis, they diagonalize the complete Hamiltonian

$$H = H_0 + \xi \sum_i \hat{s}_i \hat{k}_i + \sum_{i>k} V_{ik}$$

to find the eigenvectors and energy eigenvalues of the low-lying states of positive parity.

There are two ways of constructing the required complete set of properly anti-symmetrized three-particle basis wave functions. One method, the L – S coupling scheme used by Elliott and Flowers, results in basis states which have L, S and T as good quantum numbers (in addition to others). In this scheme, a central residual interaction is diagonal whereas the spin-orbit potential is not. In the alternative $j-j$ coupling representation, chosen by Redlich in a similar treatment of the mass-19 nuclei, neither L nor S are good quantum numbers but J and T are. The spin-orbit term in the Hamiltonian is now diagonal while the particle-particle
central interaction term is not. Clearly, the combination of spin-orbit forces and central residual interactions in comparable strengths results in a coupling scheme which is intermediate between the L - S and j-j extremes. In both the Elliott and Flowers calculation and Redlich's calculation, the degree of intermediate coupling is an adjustable parameter chosen to fit the observed energy level schemes.

Many of the insights gained in this early work are still relevant. Besides the clear demonstration that neither L - S nor j-j coupling adequately describes the true situation, most revealing is the importance of the role played by the spatial symmetries of the basis wave functions. Because the inter-particle force is attractive and short-ranged, states with a large number of space-symmetric couplings between pairs of nucleons will lie lowest. This tendency results in T = 1/2 states lying generally lower than T = 3/2 states because of the greater spatial symmetry available to the former. Another general observation which emerges from the calculations - that T = 1/2 states are nearer to L - S coupled than j-j while the reverse is true of T = 3/2 states - is seen to be a consequence of the greater spatial symmetry of the T = 1/2 states leading to relatively larger expectation values of the particle-particle interaction.

Recent shell model calculations attempt a more rigorous solution of the problem. Whereas Elliott and Flowers derived
their two-body interaction matrix elements from a "conventional interaction" (with a Yukawa potential of adjustable strength $V_c$), Kuo$^{75}$ and Kallio and Kolltveit$^{76}$ relinquish adjustable parameters and calculate matrix elements from "correct" internucleon potentials which fit two-nucleon scattering data. Since these "realistic interactions" have a hard core, methods have to be devised to separate this out$^{77}$ in order to make the problem tractable.

9.2 Possible Spins and Isospins

Using the Pauli principle, simple enumeration of states according to their $m_j$ shows that $^{19}$F, with two neutrons and a proton active in the ($s$-$d$) shell, can have positive parity states with $J = 1/2, ..., 13/2$. By a similar argument, $^{19}$O and $^{19}$Na are restricted to $J^\pi = 1/2^+, ..., 11/2^+$. Sebe and Harvey$^{78}$, in a useful tabulation, show that $^{19}$F has two possible states with $J = 13/2, T = 1/2$; 5 possible states with $J = 11/2, T = 1/2$; and so on. Up to the present, most attention has been directed towards locating and studying the lowest-lying states of each possible $J$, denoted $(J)_{1}$. Since the states $(1/2^+)_{1}$, $(3/2^+)_{1}$, $(5/2^+)_{1}$, $(9/2^+)_{1}$ and $(13/2^+)_{1}$ were already known,$^{27,35,58}$ the identification of the $(7/2^+)_{1}$ and $(11/2^+)_{1}$ states in the work described here can be said to complete the first stage in a test of the shell model in $^{19}$F. Though considerable further work remains to be done on these states,
attention may now be focussed on the states \((1/2^+)_2\) \ldots \((13/2^+)_2\) and the question of the relevance of the \((2s - 1d)^3\) shell model in describing them. Some of these \((J^+)_2\) states are already identified and will be discussed later.

In the shell model of the positive parity states of \(^{19}\text{F}\), the three \((2s - 1d)\) particles are solely responsible for the isospin \(T\) (therefore \(T = 1/2\) or \(3/2\)) as well as the angular momentum \(J\), since the \(^{16}\text{O}\) core is coupled to \(J = T = 0\).

In the isospin formalism, the \(T = 1/2\) states in \(^{19}\text{F}\) and \(^{19}\text{Ne}\) form isospin doublets and the \(T = 3/2\) states in \(^{19}\text{O}\), \(^{19}\text{F}\), \(^{19}\text{Ne}\) and \(^{19}\text{Na}\) form isospin quartets. Though the states within a quartet are degenerate in the shell model, in fact this degeneracy is removed by the action of the Coulomb force (primarily). Thus the excitation energy of the lowest \(T = 3/2\) state in \(^{19}\text{F}\) may be estimated from the mass difference between the ground states of \(^{19}\text{O}\) \((T = 3/2)\) and \(^{19}\text{F}\) \((T = 1/2)\) and the difference in Coulomb energy between the two nuclei. That is,

\[
E_x^{^{19}\text{F}; T = 3/2} = (M_{c^2} - E_{\text{Coul}})^{^{19}\text{O}} - (M_{c^2} - E_{\text{Coul}})^{^{19}\text{F}}
\]

A calculation of this kind, using empirical data as far as possible, leads to \(E_x = 7.5\) MeV\(^{(36)}\) for the lowest \(T = 3/2\) state in \(^{19}\text{F}\)\(^*\). In view of the uncertainty in the Coulomb energies, this

---

\* The problem in doing this accurately is that whereas it is easy to get an estimate of the difference in Coulomb energies between the ground states of \(^{19}\text{O}\) and \(^{19}\text{F}\) (e.g. from the \(^{17}\text{F}\) to \(^{17}\text{O}\) \(\beta^+\) decay energy, corrected for the increased volume of mass 19 nuclei), what is required is the difference in Coulomb energies between the ground state of \(^{19}\text{O}\) and the lowest \(T = 3/2\) state in \(^{19}\text{F}\). Subtle differences of this kind are among those considered by Thomas.
is in good agreement with the measured $E_x = 7.538$ MeV (section 7).

9.3 The Isobaric Mass Equation (I.M.E.)

$$M(A,T,T_z) = a(A,T) + b(A,T)T_z + c(A,T)T_z^2$$

is a relation among the masses of the members of an isobaric multiplet. It states that if the nuclear wavefunctions of all states of the multiplet are identical, the electrostatic energy of the nucleus changes quadratically with $T_z$. That is, changing the nuclear charge does not alter the nuclear structure. Wilkinson points out that even if the nuclear forces themselves are charge-dependent, a quadratic formula still holds provided the charge-dependence is small enough to be treated as a perturbation and provided it is of a two-body character. However, the coefficients $b$ and $c$ would then be different from their values calculated from Coulomb perturbation alone. Charge dependence of a three-body or higher rank will cause deviations from a quadratic formula.

Because of the possibility of checking the I.M.E., the exact excitation energies of states forming isospin quartets (or higher) are unusually important. In table 9.1, it is noteworthy that the accuracy of the excitation energies determined by γ-ray spectroscopy is better by an order of magnitude than can be achieved by charged particle spectroscopy. This is
significant because Hardy et al.,\cite{72} in discussing their extensive survey of $T_>$ states by two-nucleon transfer reactions using charged particle spectroscopy, remark that enough is now known about the validity of the I.M.E. to state that any deviations from its quadratic form are probably too small to be observed with their present experimental precision.

Table 9.1

Values of $E_x$ for two lowest $T = 3/2$ states in $^{19}F$ by various workers

<table>
<thead>
<tr>
<th>$J^\pi; T$</th>
<th>$21Ne(p,3He)^{19}F$</th>
<th>$18O(3He,d)^{19}F$</th>
<th>$18O(p,\gamma)^{19}F$</th>
<th>$15N(\alpha,\gamma)^{19}F$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$5/2^+; 3/2$</td>
<td>$-\quad 7.56 \pm 20$</td>
<td>$7.538$</td>
<td>$7.537 \pm 3$</td>
<td></td>
</tr>
<tr>
<td>$3/2^+; 3/2$</td>
<td>$7.660 \pm 35$</td>
<td>$-\quad 7.658$</td>
<td>$7.658 \pm 3$</td>
<td></td>
</tr>
</tbody>
</table>

a Ref.\cite{72,73} b Ref.\cite{70} c Ref.\cite{71}

Another example of the usefulness of the I.M.E. and the importance of precise excitation energies for the states of the lowest $T = 3/2$ quartet is the prediction by Hardy et al.\cite{72} of the mass of $^{19}Na$(g.s.) In the absence of information on the excitation energies of the lowest $T = 3/2$ states in $^{19}F$ and $^{19}Ne$, they had to use the observed values of $E_x$ for the second lowest
$T = 3/2$ states in these nuclei and in $^{19}O$. Then, assuming that the second excited state of $^{19}Na$ is at 0.095 MeV as in $^{19}O$, they calculate that $^{19}Na$(g.s.) is proton unstable by about 300 keV.\(^{(72)}\) This prediction has recently been verified by experiment.\(^{(81)}\)

### 9.4 $\gamma$-Ray Transition Strengths

In subsequent sections, the quantity

$$|M|_\pi,L|^2 = \frac{\Gamma_\pi,L}{\Gamma_\pi,L,W}$$

derived from the present experiments is compared with the same quantity calculated from model wave-functions. $\Gamma_\pi,L,W$ eV, the single-particle or Weisskopf estimate, is calculated from the expressions given by Wilkinson.\(^{(44)}\) The measured values of $\Gamma_\pi,L$ eV have already been given. $\Gamma_\pi,L$ eV can be calculated from the following expressions\(^{(83)}\) (if the wave-functions are known):

$$\Gamma_\pi,L = \frac{8\pi(L+1)}{L(2L+1)!} \left( \frac{E_\gamma}{\hbar c} \right)^{2L+1} B(\pi,L)$$

where $B(\pi,L)$ is the reduced transition probability, which is related to the reduced matrix element $\langle f | | \theta(\pi,L) | i \rangle$ by

$$B(\pi,L) = \frac{1}{(2J + 1)} |\langle f | \theta(\pi,L) | i \rangle|^2$$

where $\theta(\pi,L)$ is an electromagnetic transition operator.\(^{(83)}\)

The reduced matrix element results from the application of
the Wigner-Eckart theorem to the transition matrix element.

It seems preferable, when possible, to compare experiment with theory in respect to the quantity $|M|^2$ (or alternatively $B(\pi,L)$) rather than with respect to branching ratios, since energy-dependent factors are thereby removed from consideration. Excitation energies and transition probabilities then become separate and independent issues between theory and experiment.

9.5 $11/2^+$ State: Transition Strengths

Table 9.2 shows how theoretical transition strengths compare with experiment in the case of the $11/2^+$ level. The shell model calculations using first the Kuo matrix elements and then the Rosenfeld mixture (a "conventional force" of the Yukawa type with $V_C = 40$ MeV) were carried out by Wong (84) using the Rochester-Oak Ridge shell model computer programme. (85) The results for the Kallio and Kolltveit interaction are from Benson and Flowers. (21) In calculating the E2 transition strengths, Benson and Flowers used an effective charge of 0.25e for neutrons and 1.25e for protons (to take account of the distortion of the core by the orbits of the "loose" nucleons). Further, because of the sensitivity of the E2 matrix element to the tail of the wavefunction, they replaced their harmonic oscillator radial wavefunctions with more realistic ones calculated for a Woods-Saxon
Table 9.2

Comparison of some transition strengths measured in this work (section 6) with those calculated (i) from shell model wave-functions and (ii) according to a simple rotational model. The shell model wavefunctions are calculated for \((2s - 1d)^3\) in intermediate coupling, with the residual interactions of Rosenfeld, Kuo\(^{(75)}\) and Kallio and Kolltveit\(^{(76)}\).

| Transition | \(|M|_M^2\) W.u. | | \(|M|_{E2}^2\) W.u. |
|------------|-----------------|-----------------|-----------------|
|            | This Expt. |Ros.    | Kuo   | K.K.  | Rotor | This Expt. | K.K.  | Rotor |
| \(1\frac{1}{2}^+\) to \(1\frac{3}{2}^+\) | \(1.4 \pm 0.2\) | 0.60  | 0.18  | 0.84  | 1.5   | \(\frac{3}{-3}^{+9}\) | 2.0   | 0.34  |
| \(1\frac{1}{2}^+\) to \(9\frac{2}{2}^+\) | \((20\pm3)\times10^{-2}\) | 0.10  | 0.08  | 0.15  | 0.021 | \(\frac{6}{-6}^{+18}\) \times10^{-2} | 3\times10^{-2} | 0.40  |
potential.* For the Kallio and Kollnyeit interaction, theory and experiment are in particularly good agreement.

9.6 7/2 States: Decay Schemes

Only a lower limit on the radiative width of the 4.38 MeV state is available from the present experiment. This is \( \Gamma_\gamma > 2.6 \times 10^{-2} \) eV which is to be compared with a theoretical value of \( \Gamma \approx 3 \) eV. (21)

In the absence of experimental information on absolute partial widths of the \((7/2^+)\) state, a ratio of transition strengths for the two observed decay modes is presented in table 9.3 along with theoretical values for this ratio from the same sources referred to in the previous section. The experimental ratio is based on branching ratios of 14% to the 2.78 MeV level and 86% to the 0.198 MeV level. This is the result of averaging the ratios found in this experiment with those found by Olness and Wilkinson (66) and by Thomas et al. (29)

Table 9.3 also shows similar data for the decay of the second \(7/2^+\) state at 5.47 MeV. The experimental branching ratios for this state are given in fig. 4.2. These differ

* This improves the results because the harmonic oscillator \((2s - 1d)\) shell provides only a truncated basis. If a complete basis was used, the results of calculating observables would, of course, be independent of the choice of bases.
from those reported by Tolbert in that he does not report a branch to the 5/2\(^+\) state at 0.198 MeV.

Table 9.3

Ratios of transition strengths of the two decay modes of the 7/2\(^+\) levels. A comparison of experiment and theory.

See text and table 9.2 for further explanation.

<table>
<thead>
<tr>
<th>Ratio</th>
<th>This Expt.</th>
<th>THEORY</th>
</tr>
</thead>
<tbody>
<tr>
<td>[</td>
<td>M</td>
<td>^2 (7/2^+ to 9/2^+)]</td>
</tr>
<tr>
<td>[</td>
<td>M</td>
<td>^2 (7/2^+ to 5/2^+)]</td>
</tr>
<tr>
<td>[</td>
<td>M</td>
<td>^2 (7/2^- to 9/2^+)]</td>
</tr>
<tr>
<td>[</td>
<td>M</td>
<td>^2 (7/2^- to 5/2^+)]</td>
</tr>
</tbody>
</table>

Clearly experiment and theory would be in better accord in table 9.3 if the states (Kuo 7/2\(^+\))\(_1\) and (Kuo 7/2\(^+\))\(_2\) (in an obvious notation) had their subscripts interchanged. Since these states are separated by less than 0.5 MeV in the calculation (actually \(E_x = 5.20\) and 5.65 MeV) perhaps this discrepancy in itself has no serious implications for the theory. However, even with this interchange, the results for the decay of the 5.47 MeV state are still totally unsatisfactory.
Moreover, the \((2s - 1d)^3\) intermediate coupling shell model can offer no explanation at all for the strong El transition \((0.006\ \text{W.u.})\) from the 5.47 MeV level to the \(5/2^-\) state (see fig. 4.2).

9.7 \(T = 3/2\) States : Transition Strengths

Table 9.4 presents a similar juxtaposition of theory and experiment for the decay of the \(T = 3/2\) states. The states denoted \((1/2)^+_1\), \((3/2)^+_1\), \((5/2)^+_1\) and \((7/2)^+_1\) are taken to be the experimentally observed states at 0.0, 1.55, 0.198 and 4.38 MeV. The states denoted \((1/2)^+_2\), \((3/2)^-_2\), \((5/2)^-_2\) and \((7/2)^+_2\) are taken to be those observed at 5.34, 3.91, 4.57 and 5.47 MeV though these identifications are rather more controversial:

(i) The state at 5.34 MeV has been assigned \(J^\pi = 1/2^+\) by Tolbert\(^{(34)}\) though, as already noted in section 4, there is some doubt about its parity. However, it seems to be the only candidate for \((1/2^+)_2\) predicted by Elliott and Flowers to occur at about 3.7 MeV.

(ii) The weight of evidence now strongly favours positive parity for the \(J = 3/2\) state at 3.91 MeV, thus making it \((3/2^+)_2\). Firstly, the state at 4.01 MeV in \(^{19}\text{Ne}\), which is probably its analogue, exhibits a strong \(L = 0\) pattern in the \(^{21}\text{Ne}(p,t)^{19}\text{Ne}\) pick-up reaction\(^{(72)}\). Secondly, Wormald and Wright\(^{(71)}\) have been able to populate the 3.91 MeV state in \(^{19}\text{F}\) from a resonance in the \(^{18}\text{O}(p,\gamma)^{19}\text{F}\) reaction and, using a Ge(Li)
<table>
<thead>
<tr>
<th>TRANSITION</th>
<th>EXPERIMENT</th>
<th>THEORY</th>
</tr>
</thead>
<tbody>
<tr>
<td>(J,T) → (J,T)</td>
<td>log ft</td>
<td></td>
</tr>
<tr>
<td>(3/2, 3/2) → (3/2, 3/2)</td>
<td>4.45</td>
<td>0.56</td>
</tr>
<tr>
<td>- (3/2, 3/2)</td>
<td>&gt;5.5</td>
<td>&lt;0.06</td>
</tr>
<tr>
<td>- (5/2, 5/2)</td>
<td>5.41</td>
<td>0.22</td>
</tr>
<tr>
<td>- (7/2, 7/2)</td>
<td>&lt;0.11</td>
<td>4.7</td>
</tr>
<tr>
<td>- (9/2, 9/2)</td>
<td>3.54</td>
<td>2.6</td>
</tr>
<tr>
<td>- (7/2, 7/2)</td>
<td>&lt;0.32</td>
<td>3.4</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>TRANSITION</th>
<th>EXPERIMENT</th>
<th>THEORY</th>
</tr>
</thead>
<tbody>
<tr>
<td>(J,T) → (J,T)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(3/2, 3/2) → (1/2, 1/2)</td>
<td>*</td>
<td>0.07</td>
</tr>
<tr>
<td>- (1/2, 1/2)</td>
<td>*</td>
<td>&lt;0.5</td>
</tr>
<tr>
<td>- (3/2, 3/2)</td>
<td>*</td>
<td>0.12</td>
</tr>
<tr>
<td>- (5/2, 5/2)</td>
<td>*</td>
<td>&lt;0.04</td>
</tr>
<tr>
<td>- (7/2, 7/2)</td>
<td>*</td>
<td>0.03</td>
</tr>
<tr>
<td>- (5/2, 5/2)</td>
<td>*</td>
<td>&lt;0.11</td>
</tr>
</tbody>
</table>
detector, have shown that the 3.91 MeV level decays mainly by a ground state transition. Other recent Ge(Li) investigations of the decay of the 3.91 MeV state confirm that its main decay branch is to the ground state.\(^{(61, 86, 87)}\)

(iii) The level at 4.56 MeV (or 4.57 MeV) has been assumed, rather speculatively, to be \((5/2^+)\)\(^2\). Benson and Flowers\(^{(21)}\) quoting the experimental results of Allen and Lawson, suggest that the 4.56 MeV level may be \((5/2^+)\) and that it is a member of an unresolved doublet. The results of Lennon et al.\(^{(70)}\) also seem to imply that there are two closely-spaced levels in this region. The experimental decay scheme for the 4.56 MeV level given by Allen and Lawson (see Benson and Flowers\(^{(21)}\)) is consistent with a \(5/2^+\) assignment and agrees fairly well with one obtained by Wright.\(^{(71)}\)

(vi) The identification of the state \((7/2^+)\)\(^2\) with the 5.47 MeV level is based on the work of Tolbert et al.\(^{(35)}\) Assignments for all the lower neighbouring states have already been reported.\(^{(34, 56)}\)

The \(\beta\)-decay experimental data in the table is taken from the work of Olness and Wilkinson\(^{(66)}\) and from Johnson et al.\(^{(67)}\), together with a private communication from Olness\(^{(86)}\) regarding
the log ft limit to the state at 3.91 MeV, \((3/2^+)_2\).
The theoretical transition strengths in the table were calculated by Wong\(^{(84)}\) from wave-functions derived from a \((2s - 1d)^3\) intermediate coupling calculation with the same parameters as given in section 9.5.

A comparison of theory with experiment in table 9.4 reveals several points of disagreement and is, on the whole, rather disappointing. The worst discrepancies appear in the decay scheme of the \(J = 5/2, T = 3/2\) state. The theoretical decay strengths to the \((7/2^+)_1\) and \((7/2^+)_2\) states are badly wrong and, as in table 9.3, the situation would be better if the \((\text{Kuo} 7/2^+)\) \(_1\) and \((\text{Kuo} 7/2^+)\) \(_2\) states could be interchanged. To a lesser extent this is true also of the \((\text{Rosenfeld} 7/2^+)\) \(_1\) and \((\text{Rosenfeld} 7/2^+)\) \(_2\) states, though such an interchange would worsen the results in table 9.3. However, if these interchanges are made, theory and experiment (both \(\beta\)-decay and \(\gamma\)-decay) are in fair agreement on the relative disposition of transition strengths to the "subscript 1" states. The remarkably bad disagreement between theory and experiment for the decay to the \((3/2^+)\) \(_2\) state, both in \(\beta\)-decay and \(\gamma\)-decay, demands an explanation but this is postponed to section 11.
### 9.8 T = 3/2 States; β- and γ-decay Analogy

The similarity between the β- and γ-transitions which was referred to in section 7.5 as furnishing the best evidence of the analogy between the J = 5/2 resonance level and the ground state of $^{19}_{8}$O, is very plain in table 9.4. The β-decay transition strengths in this table have been calculated from (84, 88)

$$ft = \frac{6124}{1.41|<M_{GT}>|^2}$$

where $|<M_{GT}>|^2$ is the Gamow-Teller (GT) reduced transition probability. (The Fermi term is zero for a $\Delta T = 1$ transition.)

To demonstrate explicitly the connection between the β-decay and γ-decay results, consider first the reduced transition probability (but not reduced as to isospin) for a GT β-transition (88):

$$B(GT; J_iT_iM_i \rightarrow J_fT_fM_f-1) = \frac{8}{4\pi} \frac{1}{2J_i+1} |<J_fT_fM_f-1|| \sum_k t_-(k)\sigma_k|J_iT_iM_i>|^2$$

where g is the GT coupling constant and $t_-$ is the operator which, acting only on the isospin part of the wave-function, transforms a neutron into a proton in the same orbit (i.e. $<p|t_-|n> = 1$). This is to be compared with the reduced transition probability for an M1 transition

$$B(M1; J_i \rightarrow J_f) = \frac{1}{2J_i+1} <J_f||\hat{\delta}(M1)||J_i>$$
where $\hat{\delta}(M1)$ is the operator.

$$\hat{\delta}(M1) = \frac{1}{2} \sum_{k} \left( (1-t_{zk}) \hat{t}_{k} + \left[ (1-t_{zk}) \mu_{p} + (1+t_{zk}) \mu_{n} \right] \hat{\sigma}_{k} \right)$$

where $\mu_{p} = 2.79$ and $\mu_{n} = -1.91$ (in nuclear magnetons).

This operator consists of an isoscalar part and an isovector part, only the latter of which can contribute to $\Delta T = 1$ transitions.

$$\hat{\delta}(M1, isovector part) = \sum_{k} t_{zk} \left\{ (\mu_{n} - \mu_{p}) \hat{\sigma}_{k} - \hat{t}_{k} \right\}$$

Thus if the matrix element of the orbital term of the $M1$ operator is much smaller than the matrix element of the spin term, the $\beta$ and $\gamma$-transition strengths will be closely related. However, even without this inequality, the similarity of the operators indicates a connection between the two decay modes. This connection has been made in the following relation quoted by Hanna for $\Delta T = 1$ $M1$ $\gamma$-transitions and the analogous $\beta$-transitions:

$$\Lambda(M1) = 11.1 \left( \frac{T_{f} \gamma \Gamma_{0} \Gamma_{13} \Gamma_{14}}{T_{f} \beta \Gamma_{0} \Gamma_{13} \Gamma_{14}} \right)^{2} \left( 1 + 0.11 \frac{<f| \sum_{k} \frac{\hat{t}_{k} \Gamma_{k}}{\hat{t}_{k}} |i>}{<f| \sum_{k} \frac{\hat{\sigma}_{k} \Gamma_{k}}{\hat{\sigma}_{k}} |i>} \right) \Lambda(GT)$$

$$= 7.4 \left( 1 + 0.11 \frac{<f| \sum_{k} \frac{\hat{t}_{k} \Gamma_{k}}{\hat{t}_{k}} |i>}{<f| \sum_{k} \frac{\hat{\sigma}_{k} \Gamma_{k}}{\hat{\sigma}_{k}} |i>} \right)^{2} \Lambda(GT)$$ in this case.
A(Ml) and A(GT) are proportional to reduced transition probabilities and are related to quantities already defined:

\[ A(Ml) = 7.64 \left| \mathbf{M} \right|_{Ml}^2 \quad \text{(see section 9.4)} \]

\[ A(GT) = 4390/f \]

If the orbital matrix element is small compared with the spin matrix element, then

\[ A(Ml) \approx 7.4 \ A(GT) \]

This relation is compared with the results of the full intermediate coupling calculation and with experiment in table 9.5 from which it is evident that it fails drastically, particularly for the analogue to anti-analogue transition. Hanna\(^{(89)}\) also gives the following alternative form of the relation between A(Ml) and A(GT):

\[ A(Ml) = 5.9 \left\{ 1 + 0.12 \frac{\langle f| \sum_k \hat{J}_k \hat{t}_k \mid i \rangle^2}{\langle f| \sum_k \hat{s}_k \hat{t}_k \mid i \rangle} \right\} \ A(GT) \]

where \[ \hat{J}_k = \hat{r}_k + \hat{s}_k \]

Warburton and Weneser\(^{(103)}\) point out that for single particle transitions of the type \( j \rightarrow j \pm 1 \) (spin flip) the \( j \) matrix element vanishes and \( A(Ml) = 5.9 \ A(GT) \). Examination of the wave-functions given in table 9.6 show that only the transition to the \((7/2^+)\)_1 state can be described approximately in simple, single-particle, spectroscopic terms. Since it is of the type
there is no reason to expect the j matrix element to vanish. The wave-functions of the \( (5/2^+) \) and \( (3/2^+) \) are too complicated to allow one to predict by inspection whether or not the \( \hat{t} \) or \( \hat{j} \) transition matrix elements will be small. The results of the detailed calculation and of experiment indicate that the \( \hat{\lambda} \) matrix element is certainly not negligible for two of the transitions.

Table 9.5
Comparison of \( \beta \) and \( \gamma \)-decay strengths in theory and experiment. According to an approximate theory in which the orbital term is neglected in the M1 matrix element, \( \Lambda(\text{M}1) \div \Lambda(\text{GT}) \approx 6 \) or 7.

<table>
<thead>
<tr>
<th>Transition to</th>
<th>( \Lambda(\text{M}1) \div \Lambda(\text{GT}) )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Theory</td>
</tr>
<tr>
<td></td>
<td>Kuo</td>
</tr>
<tr>
<td>( (3/2)_1 )</td>
<td>19.3</td>
</tr>
<tr>
<td>( (5/2)_1 )</td>
<td>341</td>
</tr>
<tr>
<td>( (7/2)_1 )</td>
<td>3.08</td>
</tr>
</tbody>
</table>

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Table 9.6

Percentage analysis of $^{19}$F wavefunctions into $jj$ coupled configuration from Wong's data for the Rosenfeld interaction.\(^{(84)}\) The numbers are similar but not identical to those given by Elliott and Flowers.\(^{(1)}\) Note the close similarity between the configurations of the $(7/2^+)_1$ and $T = 3/2, J = 5/2$ states which undoubtedly accounts for the large matrix element connecting them.

<table>
<thead>
<tr>
<th>$T$</th>
<th>$J$</th>
<th>$d^3$</th>
<th>$d^2s$</th>
<th>$ds^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$(5/2)^3$</td>
<td>$(5/2)^2(3/2)$</td>
<td>$(5/2)(3/2)^2$</td>
</tr>
<tr>
<td>1/2</td>
<td>$(3/2^+)_1$</td>
<td>21</td>
<td>37</td>
<td>2</td>
</tr>
<tr>
<td>1/2</td>
<td>$(5/2^+)_1$</td>
<td>47</td>
<td>11</td>
<td>7</td>
</tr>
<tr>
<td>1/2</td>
<td>$(7/2^+)_1$</td>
<td>83</td>
<td>6</td>
<td>4</td>
</tr>
<tr>
<td>3/2</td>
<td>$5/2^+$</td>
<td>85</td>
<td>1</td>
<td>3</td>
</tr>
</tbody>
</table>
CHAPTER TEN

DISCUSSION OF RESULTS : II ROTATIONAL AND UNIFIED MODELS

10.1 Symmetric Core Rotational Model

Despite its success, the shell model of Elliott and Flowers\(^{(1)}\), with its mixing of configurations and coupling schemes, inevitably blurs the simple physical picture of single-particle orbits. It is natural, therefore, to enquire whether a more realistic choice of bases might restore clarity to the picture. The discovery of rotational states in light nuclei\(^{(90,91)}\) led Paul\(^{(3)}\) to try to account for the positive parity states in \(^{19}\text{F}\) by a model which incorporates both rotational and "extreme" single-particle bases.

Paul and other authors (Rakavy\(^{(4)}\), Bhatt\(^{(9)}\)) use a symmetric core rotational model in which the intrinsic states are given by the single-particle Nilsson model. As is proper before postulating rotational degrees of freedom, Rakavy discusses the grounds for believing \(^{19}\text{F}\) to be deformed and shows by a simple calculation that deformation in the nuclei following \(^{19}\text{O}\) increases very rapidly, reaching a flat maximum in the \(A = 20 - 25\) region. Using a value
for the moment of inertia which is intermediate between values used to explain rotational spectra in $^{18}_0$ and $^{20}_{Ne}$, Paul found that a $K = 1/2^+$ rotational band based on the ground state gave the positive parity levels in correct order but with spacing much too large. However, when the $K = 3/2^+$ band was admixed with the $K = 1/2^+$, according to the provisions of the model (the effect of the rotation-particle coupling term), the positions of the low-lying levels then known were well reproduced.

The intrinsic states in the model are assumed to be determined by the state of the last odd nucleon. For the $K = 1/2$ band, the intrinsic state is constructed by filling as completely as allowed by the Pauli principle the lowest Nilsson orbits. In the lowest configuration, the last odd particle, a proton, goes into an $\Omega = 1/2$ orbit. The intrinsic state for the $K = 3/2^+$ band is formed by promoting the last odd particle from the $\Omega = 1/2$ orbit to the $\Omega = 3/2$ orbit. For the deformation assumed by Paul, the energy required to do this is about 2.8 MeV (and about 2.4 MeV in the calculation by Rakavy) but when the $K = 1/2^+$ and $K = 3/2^+$ bands are mixed, the $K = 3/2^+$ band head is pushed up to about 4 MeV.

Both Rakavy and Bhatt are careful to point out that only the two simplest intrinsic excitations have been
taken into account in these calculations and above about 4 MeV (the energy required to separate the paired nucleons in the $\Omega = 1/2$ orbit), states built on more complex intrinsic excitations may appear. Such states will, of course, mix with the states which are purer in $K$. However, with the restriction to low excitation energies, Paul's calculation provided a valid and important insight into nuclear structure which is missing in the Elliott and Flowers formulation.

10.2 Adiabatic Rotational Model Compared with Experiment

Now that all* the states in $^{19}\text{F}$ which would form the members of a possible $K = 1/2^+$ rotational band have been identified, it is interesting to see how well they fit the simple adiabatic symmetric-core rotational model without band mixing. There are several reasons for trying this: for instance:

(i) It has been suggested by Newton, Clegg and Salmon that $K = 3/2$ admixture in the $K = 1/2$ band is weaker than Paul postulated.

(ii) Projected Hartree-Fock calculations indicate that the states $(1/2^+)$ to $(13/2^+)$ in $^{19}\text{F}$ are quite well described as forming a pure $K = 1/2$ band.  

* i.e. all the states of the ground state band concealed in the $(2s - 1d)^3$ configuration-mixing calculations.
(iii) The appeal of the model lies in its simplicity. The large amount of band mixing used by Paul detracts from this, K ceases to be a good quantum number and yet all but the simplest $K = 1/2$ and $3/2$ intrinsic configurations are still omitted from the account. And, furthermore, the $11/2^+$ and $13/2^+$ states are still much too high.\(^{(21)}\)

Table 10.1 shows the results of a least-squares fit of the observed energy levels to the expression.

$$E = A + \frac{\hbar^2}{2I} \left\{ J(J + 1) + a(-1)^J + \frac{1}{2}(J + 1/2) \right\}$$

where $I$ is the moment of inertia of the rotor about its minor axis and "a" is a decoupling parameter depending on the intrinsic state. The quantities $\frac{\hbar^2}{2I}$ and "a" were varied freely to achieve the best fit which was found to occur at

$$\frac{\hbar^2}{2I} = 0.14 \text{ MeV} \quad a = 2.2$$

(The quantity $A$ was chosen to reduce the energy to zero for the $J = 1/2$ state i.e. $E_{1/2} = 0$). The value of "a" is well within the range 1 to 3 given by the Nilsson model for the $\Omega = 1/2$ orbit\(^{(4)}\). However, the value of the parameter $\frac{\hbar^2}{2I}$ is considerably smaller than the value chosen by Paul (0.3 MeV) as being intermediate between values used for $^{18}_0$ and $^{20}_\text{Ne}$. Bhatt\(^{(9)}\) justifies a somewhat smaller value for an odd $A$ nucleus than for neighbouring even nuclei on the grounds that the last odd particle tends to polarize and deform the core still further. However, even with the present unrestricted choice of $\frac{\hbar^2}{2I}$, the fit is not very good.
Table 10.1

Theoretical energy spectrum provided by the adiabatic rotational model for an odd A nucleus (pure \( K = 1/2 \) band) compared to experiment for the nucleus \(^{19}\text{F} \). The moment of inertia and decoupling parameters have been varied to obtain a least-squares fit.\(^{93}\)

<table>
<thead>
<tr>
<th>( J^\pi )</th>
<th>( E_x ) MeV</th>
<th>Expt.</th>
<th>Theory</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 1/2^+ )</td>
<td>0.00</td>
<td>0.00</td>
<td></td>
</tr>
<tr>
<td>( 3/2^+ )</td>
<td>1.55</td>
<td>1.34</td>
<td></td>
</tr>
<tr>
<td>( 5/2^+ )</td>
<td>0.20</td>
<td>0.50</td>
<td></td>
</tr>
<tr>
<td>( 7/2^+ )</td>
<td>4.38</td>
<td>3.61</td>
<td></td>
</tr>
<tr>
<td>( 9/2^+ )</td>
<td>2.78</td>
<td>2.11</td>
<td></td>
</tr>
<tr>
<td>( 11/2^+ )</td>
<td>6.50</td>
<td>7.00</td>
<td></td>
</tr>
<tr>
<td>( 13/2^+ )</td>
<td>4.65</td>
<td>4.82</td>
<td></td>
</tr>
</tbody>
</table>

M1 transition strengths within the \( K = 1/2 \) band have been calculated in this model from the expression\(^{94,95}\)

\[
B(\text{M1}) = \frac{3}{4\pi} \left( J_\downarrow 1/2 0 | J_f \downarrow 1/2 \right)^2 G_0^2/4 \left[ 1 + (-1)^{I^+ + 1/2} \frac{1}{b_0} \right] (\text{n.m.})^2
\]

where \( I_\uparrow \) is the larger of \( J_\downarrow \) and \( J_f \).
$G_o$ is the difference between the gyromagnetic ratio of the core and a gyromagnetic ratio associated with the decoupled particle(s) and $b_o$ is an intrinsic function (akin to "a"). In the Nilsson model, $G_o$ and $b_o$ are related to quantities which may be determined by experiment. Thus

$$G_o = 3\mu - a(g_\perp - g_R) - 1/2 g_s + g_\perp - 2g_R$$

$$b_o = -1/2G_o \left[ 3\mu + a(g_\perp - g_R) + 1/2 g_s - g_\perp - g_R \right]$$

where $\mu = 2.63$ n.m. is the magnetic moment of $^{19}$F (g.s.),

$a = 2.2$ to fit the energy spectrum.

$g_R$ = gyromagnetic ratio of the rotor core.

For heuristic purposes, it is permissible to use the values of $g_\perp$ and $g_s$ appropriate to a free proton and $g_R = \frac{Z-1}{A-1}$, corresponding to a flow of uniformly charged nuclear matter. This leads to $b_o = -1.30$ and $G_o = 4.00$ n.m. and to the M1 transition strengths quoted for the rotational model in tables 9.2 and 9.3. It is interesting to note, as did Paul, that this simple rotational model is as successful as the shell model in explaining the rather puzzling branching ratios of the $3/2^+$ level. Up-to-date experimental values for the branching ratios and lifetime of the $3/2^+$ level and the parameters given already for the theory have been used to illustrate this in table 10.2
Table 10.2

Comparison of Rotation Model Theory with experiment for the decay of the $3/2^+$ level. The lifetime and branching ratios for the $3/2^+$ level are those quoted by Poletti et al. (62)

<table>
<thead>
<tr>
<th>Transition</th>
<th>Expt.</th>
<th>Theory</th>
</tr>
</thead>
<tbody>
<tr>
<td>$3/2^+ \rightarrow 5/2^+$</td>
<td>2.7</td>
<td>1.65</td>
</tr>
<tr>
<td>$3/2^+ \rightarrow 1/2^+$</td>
<td>0.04</td>
<td>0.016</td>
</tr>
</tbody>
</table>

On the whole the agreement with experiment as regards the M1 transitions is not too bad. The theoretical results are particularly sensitive to the value adopted for $b_0$ since the branching $|J\rangle$ to $|J + 1\rangle$ and $|J\rangle$ to $|J - 1\rangle$ depends on $(1 + b_0)$ and $(1 - b_0)$ where $b_0 \sim -1$.

To a good enough approximation, E2 transition strengths in the $K = 1/2$ band are proportional to squares of Clebsch-Gordan coefficients. (97) Thus $|M|_M^2$ (E2; $i \rightarrow f$) = $34.4 (J_i 2 1/2 0 \mid J_f 1/2)^2$ where the constant of proportionality has been obtained from the known strength of the $5/2^+ \rightarrow 1/2^+$ transition. (98) This formula has been used to calculate the rotational model predictions for the E2 strength in the two decay branches of the $11/2^+$ level. See table 9.2. The same formula, normalized in the same way, gives reasonably good estimates (within a factor of two) for the pure E2 transitions.
within the band, viz. \(13/2^+\) to \(9/2^+\) and \(9/2^+\) to \(5/2^+\).\(^{99}\)

On the whole, the agreement between theory and experiment in respect of the energy spectrum and transition strengths is quite good, considering the naivety of the theory. If \(b_0\) were taken as an adjustable parameter the agreement for the M1 transition strengths could be improved since all err in the same direction: the \(|J>\) to \(|J-1>\) transition is much too weak. A smaller value of \(g_R\) than \((Z-1)/(A-1)\) improves the agreement and can be justified.\(^{100}\)

10.3 Unified Models

Unification of the rotational model with the independent particle model has already been tacitly introduced in the previous section by using Nilsson's results for a deformed potential well to describe the intrinsic state of the odd proton coupled to the core. A more modern but similar approach is to construct a single-particle intrinsic state according to Nilsson and project out states which are eigenstates of \(J^2\) and \(J_z\). Redlich\(^{101}\) and Hamamoto and Arima\(^{20}\) have used this method and the latter have shown that the wave-functions for the states \((1/2^+)_{1}, \ldots, (13/2^+)_{1}\) in \(^{19}F\), obtained by projection from the lowest Nilsson state, have good overlaps with the wave functions obtained from a full intermediate coupling calculation. Only the \(7/2^+\) and \(11/2^+\) states require some admixture of an excited intrinsic configuration to obtain
a really good overlap. Such good agreement between the full shell-model and a much simpler model, besides revealing something about the physical structure of the shell-model states, suggests that good wave functions may be obtained from the simpler model for nuclei with too many extra-core particles to be treated in the full shell-model.

The projection method can be generalized by constructing self-consistently, according to the Hartree-Fock method, a deformed three-particle intrinsic state with sharp K and projecting out angular momentum eigenstates. All configurations having the same K are now mixed into the intrinsic state by the residual interaction, and deformations other than the Nilsson Y\textsubscript{20} may be admitted. Benson and Flowers have refined this general procedure by postponing the Hartree-Fock minimization until after projection. By minimizing for each angular momentum eigenstate separately they allow for centrifugal stretching of the intrinsic configuration.

Of course, as long as the deformed intrinsic states are confined to superpositions of (2s - 1d) spherical basis functions, such calculations as have been outlined above cannot improve on the results of the full shell-model calculation.
CHAPTER ELEVEN

CONCLUSIONS

11.1 General

In this thesis the possibilities of the $^{15}\text{N}(\alpha,\gamma)^{19}\text{F}$ reaction for studying $^{19}\text{F}$ in an interesting region of excitation energy have been amply demonstrated and three of the particularly important states have been studied in detail. The points of agreement between the experimental results for these states and shell and rotational model calculations have already been discussed; they further substantiate the well-known validity of these models in describing the positive parity states of $^{19}\text{F}$.

Of considerably greater significance are the points at which the current theories fail conspicuously to account for observed properties. In particular, the total failure of the $(2s - 1d)^3$ intermediate coupling calculation to explain the absence or weakness of the decay of the $J^\pi = 5/2^+, T = 3/2$ level to the states $(3/2^+)_2$ and $(7/2^+)_2$ (see table 9.4) must be significant. The situation may be summarized in the statement that whereas in the theory both $(J^+)_1$ and $(J^+)_2$ states are about equally favoured in the decay scheme of the $J = 5/2, T = 3/2$ state, in experiment the $(J^+)_2$ states are heavily discriminated against in favour of the $(J^+)_1$ states. Indeed, as far as the $\gamma$-ray

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transition strengths are concerned, theory and experiment are in much better accord if the theoretical strengths to the \((J^+)_{1}\) and \((J^+)_{2}\) states are summed in pairs and compared with the experimental strengths to the \((J^+)_{1}\) states. Evidently there is a great difference in structure, for which the theory cannot account, between the \((J^+)_{1}\) states and the \((J^+)_{2}\) states.

11.2 El Transitions

An interesting difference between the \((J^+)_{1}\) levels and the \((J^+)_{2}\) levels emerges if one compares their El decay strengths in relation to their M1 strengths. Table 11.1, in which this comparison is made, shows that El decay branches are about 5 times more prominent for \((J^+)_{2}\) states than for \((J^+)_{1}\). The most outstanding in this regard is the \((3/2^+)^{2}\) state at 3.91 MeV which is also the most important of the \((J^+)_{2}\) states to understand because it is the lowest-lying and because it is the most anomalous (in the context of table 9.4). More generally, the results strengthen the impression that the \((J^+)_{1}\) and \((J^+)_{2}\) states belong to two different categories as far as competition between El and M1 transitions is concerned.
This table demonstrates the greater relative importance of El transitions for the $(J^+)_2$ states than for the $(J^+)_1$ states. Instead of absolute El strengths, which are not available for all of the states, the quantity

$$\frac{\text{El strength}}{\text{Ml strength}} = \left( \sum_{\text{El}} \frac{\text{El branching ratio}}{(\Delta E)^3} \right) \div \left( \sum_{\text{Ml}} \frac{\text{Ml branching ratio}}{(\Delta E)^3} \right)$$

has been calculated and appears in column 3. The summations are over all El transitions in column 1 and over all Ml transitions observed in the work cited.

<table>
<thead>
<tr>
<th>El Transition</th>
<th>Branching ratio assumed</th>
<th>El Strength x $10^2$</th>
<th>Ref.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(11/2^+)_1$ to $(9/2^-)_1$</td>
<td>&lt; 4%</td>
<td>&lt; 3</td>
<td>This work</td>
</tr>
<tr>
<td>$(7/2^+)_1$ to $(5/2^-)_1$</td>
<td>&lt; 4%</td>
<td>&lt; 4</td>
<td>&quot;</td>
</tr>
<tr>
<td>$(3/2^+)_1$ to $(1/2^-)_1$</td>
<td>5%</td>
<td>5</td>
<td>(62)</td>
</tr>
<tr>
<td>$(7/2^+)_2$ to $(5/2^-)_1$</td>
<td>30%</td>
<td>13</td>
<td>(34)</td>
</tr>
<tr>
<td>$(5/2^+)_2$ to $(3/2^-)_1$</td>
<td>8%</td>
<td>18</td>
<td>(71)</td>
</tr>
<tr>
<td>$(3/2^+)_2$ to $(1/2^-,3/2^-,5/2^-)_1$</td>
<td>see ref.</td>
<td>25</td>
<td>(71)</td>
</tr>
</tbody>
</table>
The shell-model-dependent El selection rule enunciated by Harvey (13) and later by Benson and Flowers (21) should be well exemplified in $^{19}$F. This is because the rule's requirements for El inhibition are satisfied by both the $(J^+)_{1}$ states and the low-lying negative parity states: it has been shown in this thesis and elsewhere that the $(J^+)_{1}$ states are well described by $(s - d)^3$ configurations and it is well-known that the low-lying negative parity states may be described as $p_{\frac{1}{2}}$ proton hole states in $^{20}$Ne (12, 21, 82). Benson and Flowers have shown that negative parity states can be constructed in this way with good $T(=1/2)$ and low centre-of-mass "spuriosity". To the extent that the positive and negative parity states may be thus described, El transitions connecting them are forbidden. The upper half of table 11.1 indicates that in fact the El inhibition is effective.

Benson and Flowers also point out that breakdown of the El rule may occur for positive parity states above about 4 MeV through admixtures of states derived from $5p = 2h$ configurations. This would explain the experimental results in the lower half of table 11.1.

11.3 Deformed States in the Nilsson Model (94)

Consideration of low-lying deformed states with $5p - 2h$ configurations is prompted by the El results in table 11.1 interpreted in the light of Benson and Flowers' remarks,
together with a suggestion by Zuker\(^{(104)}\) regarding the anomalously weak transition to the \((3/2^+)\)_2 state in table 9.4.

The energy of the lowest 5p - 2h state in \(^{19}\text{F}\) may be estimated from the following data interpreted in the light of the Nilsson model:

(i) \(0.110\ \text{MeV}\) is required to excite the lowest negative parity level in \(^{19}\text{F}\) by raising a \(p_{1/2}\) proton to the \(d_{5/2}\) level\(^{(13)}\) thus filling the \(\Omega = 1/2\) orbital (Nilsson \#6).

(ii) \(4.97\ \text{MeV}\) is required to excite the lowest negative parity level in \(^{20}\text{Ne}\) by raising a \(p_{1/2}\) nucleon to the \(\Omega = 3/2\) orbital (Nilsson \#7).

The lowest 5p - 2h state in \(^{19}\text{F}\), with the structure \((p_{1/2})^{-2}(d_{5/2})^5\), may therefore be expected at about 5.1 MeV. This is an upper limit because less energy than 4.97 MeV may be required to promote the second \(p_{1/2}\) nucleon than the first (the \(^{20}\text{Ne}\) case) since the \(p_{1/2}\) subshell is then broken. It is difficult to estimate from nuclear masses and energy-level diagrams what this difference may amount to because the nuclear deformation changes rapidly with the addition or subtraction of a nucleon for nuclei in this region. However, since the separation energy for a nucleon from \(^{16}\text{O}\) is much larger than from \(^{15}\text{N}\) or \(^{15}\text{O}\), it would not be surprising if the minimum energy to excite a 5p - 2h configuration were thereby reduced by an MeV, i.e. to
about 4 MeV.

Whereas the spherical shell model provides no guidance or insight concerning possible differences in structure between \( (J^+)_1 \) states and \( (J^+)_2 \) states, the Nilsson model suggests that these form two separate rotational bands and explains quite plausibly why \( (J^+)_2 \) states such as the \( (3/2^+) \) at 3.91 MeV might be admixed with 5p - 2h configurations while \( (J^+) \) states such as the \( (11/2^+) \) at 6.50 MeV are not. It is evident from these two examples that it is not simply a matter of excitation energy: rather it is a matter of intrinsic excitation. In the \( K = 3/2^+ \) band, of which the 3.91 MeV level is taken to be the lowest member, there are two vacancies in the lowest Nilsson orbit \( (#6) \) which may be readily filled by a pair of nucleons from the core. On the other hand, in the \( K = 1/2^+ \) band comprising all the \( (J^+) \) states, there is only one vacancy in \( #6 \) orbit and one nucleon of any pair excited from the core must go into the higher \( #7 \) orbital. However, in the Nilsson model of the low-lying \( T = 3/2^+ \) states, there are two vacancies in the \( #6 \) orbit; this explanation therefore implies that there may be 5p - 2h admixture in these states also, although no El transitions were observed from the lowest two (see fig. 7.11). Though this objection may indicate that the limit of such qualitative arguments has been reached, an answer along the
following lines may be possible:

(1) It is assumed that the important $5p - 2h$ configurations are those which take advantage of the strong binding of the Nilsson orbit $#6$ when filled and coupled to $J = 0, T = 0$. It is this strong $\alpha$-particle-like binding which, in the Nilsson model, forms the intrinsic structure of the low-lying positive parity levels in $^{20}\text{Ne}$ and negative parity levels in $^{19}\text{F}$. The intrinsic state function of the lowest $T = 3/2$ levels is therefore of the form

\[
\begin{array}{c}
\text{\includegraphics[width=0.5\textwidth]{figure1.png}}
\end{array}
\]

The point to be noted is that since the $5p - 2h$ term has two protons excited from the core, $E1$ transitions are forbidden by the rule discussed in the previous section.

On the other hand, the intrinsic state function of the lowest $K = 3/2^+ T = 1/2$ level is

\[
\begin{array}{c}
\text{\includegraphics[width=0.8\textwidth]{figure2.png}}
\end{array}
\]

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In this case, the EL inhibition rule does not apply to the 4th term and EL transitions may occur.

(ii) According to Zamick's rule governing p-h configurations, the amplitude of 5p - 2h terms in the T = 3/2 state should be smaller than in the T = 1/2 state, since in the former case T_p = 1/2 and T_h = 1 must couple in the "stretch" configuration which is energetically unfavourable.

11.4 "Intruder" States

If the two holes couple to zero spin, the weak coupling model of Arima et al. suggests that the well-defined K = 3/2 ground state band in ^{21}_{\text{Ne}} (96,107) may be replicated in ^{19}_{\text{F}} as 5p - 2h states. In this model, the wave-functions of the 5p - 2h states in ^{19}_{\text{F}} would be products in which the wave-functions of the 5 particles are the same as the wave-functions of the ^{21}_{\text{Ne}} ground state band, and the two-hole wave-function is the same as that of the ^{14}_{\text{N}} ground state. In fact, if the 3.91 MeV (3/2^+) level in ^{19}_{\text{F}} is taken to correspond to the 3/2^+ ground state of ^{21}_{\text{Ne}}, the model gives the excitation energies of the (5/2^+) and (7/2^+) states in ^{19}_{\text{F}} rather well: 5/2^+ occurs at 4.3 MeV (c.f. 4.56 MeV) and 7/2^+ at 5.4 MeV (c.f. 5.47 MeV).
Since the \((2s - 1d)^3\) intermediate coupling calculation places the \((3/2^+)\_2\ level in \(19^F\) at 5.5 MeV (with the Kallio and Koltveit interaction\(^{21}\)) or 5.25 MeV (with the Rosenfeld interaction\(^{84}\)), or 6.45 MeV (with the Kuo interaction\(^{84}\)), it is now extremely plausible to describe the anomalous \((3/2^+)\_2\ level at 3.91 MeV as an "intruder" 5p - 2h level, possibly admixed with and pushed down by the \((2s - 1d)^3(3/2^+)\_2\ level. It is reasonable then to identify the experimentally observed \((3/2^+)\_3\ level at 5.49 MeV\(^{26}\) (see Fig. 4.2) with the \((3/2^+)\_2\ level given by the \((2s - 1d)^3\ intermediate coupling calculation. The presence of p - h positive parity "intruder" states in addition to those given by the \((2s - 1d)^2\ intermediate coupling calculation is, of course, well known in \(18^O\) and \(18^F\)(106,108-111).

Unfortunately, if the transition strengths predicted by the \((2s - 1d)^3\ shell model to the \((J^+)\_2\ states are now sought in transitions to the experimentally-observed \((J^+)\_3\ states, one finds that the experimental upper limits on the transition strengths are generally too high to permit any conclusions about the presence or absence of the transition in question. For example, the limit on the transition strength to the \((3/2^+)\_3\ level at 5.49 MeV is \(|M|^2 < 0.32. The trouble is that the \((\Delta E)^3\ factor and the higher background for low energy \(\gamma\)-rays force up the limit of observation for transitions to states as high as 5.49 MeV.

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11.5 Further Work

The success of Middleton et al. (108,110) in highlighting the p - h states in $^{18}_0$ and $^{18}_F$ by direct particle-transfer reactions leads to the hope that reactions like $^{14}_N(7Li,d)^{19}_F$ ($Q = 6$ MeV) or $^{14}_N(6Li,p)^{19}_F$ ($Q = 11$ MeV) might do the same for $^{19}_F$. Both of these 5-particle-transfer reactions have been seen on $^{12}_C$ (113,114) and, since the $^{14}_N(7Li,t)^{18}_F$ reaction has recently been studied by Middleton et al. (110), the gas "targetry" presents no problems.

Finally, it will be interesting to see the results of new shell-model calculations on $^{19}_F$, taking into account the 5p - 2h space. Some such calculations are now in progress, stimulated in part by the experimental results presented here (84,104,112).
APPENDIX 1

Aberration: the effect of a moving source on angular distribution observed in the laboratory.

Consider the radiation in a pencil, solid angle \( d\Omega' \), in the frame \( S' \) moving with the source velocity \( v \). In the laboratory frame, this radiation will be in a pencil whose solid angle \( d\Omega \) is given by (116)

\[
\frac{d\Omega}{d\Omega'} = \frac{(1-v/c \cos \theta)^2}{1-v^2/c^2} = 1-2v/c \cos \theta
\]

Let angular distribution in \( S \) frame be \( W(\theta) \) and in \( S' \) frame \( W(\theta') \). Then

\[
W(\theta')d\Omega' = W(\theta) d\Omega
\]

\[
W(\theta') = (1-2v/c) W(\theta)
\]

\( W(\theta) \) is what is observed in the laboratory; \( W(\theta') \) is the desired angular distribution.
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FIGURE CAPTIONS

Fig. 1.1 Published experimental information (c. January 1968) on the $^{19}$F level scheme. References to states of present interest are as follows:

- T = Thomas et al. Ref. 29
- A = Allen et al. Ref. 28
- O = Olness and Wilkinson Ref. 66
- S = Silbert and Jarmie Ref. 25
- B = Butler et al. Ref. 36
- Price
- Smotrich et al. Ref. 26

Fig. 3.1 An early 5" NaI(Tl) γ-ray yield curve showing the prominence of resonances in the contaminant $^{13}$C(α,n)$^{16}$O reaction and how they may obscure the $^{15}$N(α,γ)$^{19}$F resonances of interest. Full arrows indicate definite assignment to $^{15}$N(α,γ)$^{19}$F. Titanium nitride target was 14 keV thick at $E_α = 1681$ keV.

Fig. 3.2 Scale drawing of 40 cm$^3$ Ge(Li) counter, target holder and chamber used for the study of the 11/2$^+$ state in $^{19}$F.

Fig. 4.1 Composite drawing of more recent γ-ray yield curves taken using the liquid-nitrogen-cooled target-shroud. A 5" NaI(Tl) detector was used for the yield curves except for
Figure Captions continued

the resonance at 3150 keV for which a 40 cm$^3$ Ge(Li) detector was used. Confirmed $^{15}_\alpha(N,\gamma)^{19}_F$ resonances are indicated by full arrows. The resonances at 2610 and 2640 keV can easily be resolved with a thinner target (see fig. 3.1). Two of the strongest $^{14}_\alpha(N,\gamma)^{18}_F$ resonances also show up even though the nitrogen was 99% pure $^{15}_N$. Titanium nitride target thickness was 29 keV at $E_\alpha = 1681$ keV.

Fig. 4.2 Decay schemes for resonances in $^{15}_N(N,\gamma)^{19}_F$ reaction. True branching ratios and ratios observed at $\theta = 55^\circ$ are indicated by filled circles. Open circles indicate that the ratios refer to observation at $90^\circ$.

Fig. 5.1 Yield curves in the regions of the two resonances being compared as to their strengths. The titanium nitride target had 36% $^{15}_N$ and 64% $^{14}_N$.

Fig. 6.1 (a) Yield curve taken with Ge(Li) detector in the region of the 3150 keV resonance.
(b) Decay scheme and branching ratios of $11/2^+$ level together with upper limits on other dipole or quadrupole transitions which were not observed.
(c) Angular distribution of $\gamma$-ray attributed to $11/2^+$ to $9/2^+$ transition. $\delta$ for the theoretical curve is from
Figure Captions continued

fig. 6.2a.
(d) Angular distribution of sum of primary and secondary \( \gamma \)-rays treated as unresolved and attributed to \( 11/2^+ \) to \( 13/2^+ \) to \( 9/2^+ \) cascade. \( \delta \) for the theoretical curve is from fig. 6.4a.

Fig. 6.2 Spin postulates from \( 5/2 \) and \( 13/2 \) for the resonance level are here tested (according to the \( \chi^2 \) criterion) against the experimental angular distribution shown in fig. 6.1c.

Fig. 6.3 This shows the primary \( \gamma \)-ray from the resonance level Doppler-shifting into the secondary \( \gamma \)-ray from the long-lived \( 13/2^+ \) level. The energies of the two \( \gamma \)-rays are 1.850 MeV and 1.870 MeV respectively and these are full-energy peaks.

Fig. 6.4 (a) \( \chi^2 \) test of theory against the experimental angular distribution in fig. 6.1d.
(b) and (c) \( \chi^2 \) tests of various theories against the experimental angular distribution in fig. 6.1d and the experimental ratios in table 6.1.
(d) \( \chi^2 \) test of theory against the experimental angular distribution in fig. 6.8.

Fig. 6.5 Equi-\( \chi^2 \) curves - a test of theory against the experimental angular distribution in fig. 6.1d and the experimental ratios in table 6.1. The lower figure
Figure Captions continued

shows the region of best fit in more detail.

Fig. 6.6 (a) and (b) Equi-$\chi^2$ curves - tests of theory against the experimental angular distribution in fig. 6.1d and the experimental ratios in table 6.1.

Fig. 6.7 Equi-$\chi^2$ curves - a test of theory against the experimental angular distribution in fig. 6.1d and the experimental ratios in table 6.1. The lower figure shows the region of best fit in more detail.

Fig. 6.8 Angular distribution of $9/2$ to $5/2$ $\gamma$-ray. The values of the mixing ratios used in the theoretical curves are from figs. 6.4d and 6.10. Note that irrespective of where the experimental points lie, the three theoretical curves shown would be indistinguishable within the experimental errors.

Fig. 6.9 Equi-$\chi^2$ curves - tests of theory against the experimental angular distribution in fig. 6.8. Since $\delta_1$ and $\delta_2$ have to be consistent with the values given in table 6.2, they have been varied over a restricted range only.

Fig. 6.10 $\chi^2$ tests of simultaneous fits of the angular distributions in figs. 6.1d and 6.8 together with the ratios in table 6.1, for the spin assignments shown in this figure. It was hoped that, since different values of the $\delta$'s are required to fit the two dis-
tributions separately, the $\chi^2$ for a fit to both would be high enough to exclude one or both of the spin sequences shown here.

Fig. 6.11 Spectrum taken on monitor Ge(Li) counter at 135° to beam direction. In the labelling of the peaks, °, ' and " denote full-energy, single-escape and double-escape respectively; $R_L$ and $R_H$ denote low and high spin resonances at $E_\alpha = 3147$ and 3149 keV respectively, which were excited simultaneously in this work. Note that the peaks $R_H \rightarrow 4.65$ and $4.65 \rightarrow 2.78$, which are shown also in fig. 6.3, are well separated at 135°. The unlabelled peaks are believed to have nothing to do with the two resonances excited in $^{15}N(\alpha,\gamma)^{19}F$: all of the strong ones appear off-resonance also.

Fig. 7.1 Ge(Li) spectrum from 30 cm$^3$ counter. The unlabelled peaks are believed to have nothing to do with the resonance in $^{15}N(\alpha,\gamma)^{19}F$: all of the strong ones appear off-resonance also.

Fig. 7.2 (a) The two resonances at $E_\alpha = 4467$ and 4621 keV, attributed to radiative capture into the two lowest $T = 3/2$ states in $^{19}F$, are the only prominences in the excitation curve over quite a range of bombarding energy. The titanium nitride target thickness was about 12 $\mu$g/cm$^2$. 

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(b) $\gamma$-decay schemes found for lowest two $T = 3/2$ states in $^{19}$F and $\beta$-decay scheme for $^{19}_0$.\(^{(66,67)}\)

Fig. 7.3 $\chi^2$ tests of theory (for various spin-postulates for the resonance level) against the experimental angular distributions in fig. 7.4.

Fig. 7.4 Experimental angular distributions of two of the primary $\gamma$-rays, fitted with optimum theoretical distributions determined from fig. 7.3.

Fig. 7.5 Experimental angular distribution of secondary $3/2^+ \rightarrow 5/2^+ \gamma$-ray and $\chi^2$ test of $5/2$ spin-postulate for the resonance level. The mixing ratio $\delta_1 = 0.021$ has been chosen to fit the angular distribution of the primary $\gamma$-ray to the 1.55 MeV level (see figs. 7.3 and 7.4).

Fig. 7.6 $\chi^2$ test of $3/2$ spin-postulate for the resonance level against the experimental angular distribution in fig. 7.5. The lower graph of $\chi^2$ versus $\delta_2$ shows two sections of the upper two-$\delta$ plot along the abscissae marked "best $\delta_1$". Of the two values of "best $\delta_1$" which fit the angular distribution of the primary $\gamma$-ray, only $|\delta_1| = 3/2$ also fits the angular distribution of the secondary $3/2^+ \rightarrow 5/2^+ \gamma$-ray.
Figure Captions continued

Fig. 7.7 (a) $\chi^2$ tests of various spin-postulates for the 4.38 MeV level against the experimental angular distribution in fig. 7.9a.

(b) $\chi^2$ test of $5/2$ spin-postulate for the 4.38 MeV level against the experimental angular distribution in fig. 7.9b. The range of variation of $\delta_1$ is restricted to that which approximately fits the angular distribution of the primary $\gamma$-ray (see Fig. 7.7a).

Fig. 7.8 (a) $\chi^2$ tests of various spin-postulates for the 4.38 MeV level against the experimental angular distribution in fig. 7.9b. In this test $\delta_1 = 0$ because (i) for $J = 5/2$, it gives the best fit in fig. 7.7b; (ii) for $J = 7/2$, it fits quite well in both figs. 7.7a and 7.8b; and (iii) for $J = 9/2$, appreciable octupole admixture in the primary $\gamma$-ray is ruled out.

(b) $\chi^2$ test of $7/2$ spin-postulate for the 4.38 MeV level against the angular distribution in fig. 7.9b. The range of variation of $\delta_1$ is restricted to that which approximately fits the angular distribution of the primary $\gamma$-ray (see fig. 7.7a).

Fig. 7.9 (a) Experimental angular distribution of primary $\gamma$-ray to 4.38 MeV level. $\delta$ for the theoretical curve is from fig. 7.7a.
Figure Caption continued

(b) Experimental angular distribution of secondary γ-ray from 4.38 MeV level to the 0.198 MeV level. δ's for the theoretical curves are from figs. 7.7b and 7.8a.

Fig. 7.10 Experimental angular distribution for weak 7/2 → 9/2 secondary γ-ray and χ² test of the theory.

Fig. 7.11 Decay scheme and branching ratios for the two lowest T = 3/2 levels and for the 7/2⁺ level at 4.378 MeV.

Fig. 8.1 Ge(Li) spectrum from 30 cm³ counter. The unlabelled peaks are believed to have nothing to do with the resonance in $^{15}$N(α,γ)$^{19}$F: all of the strong ones appear off-resonance also.

Fig. 8.2 The experimental information in these diagrams is contained in the shaded bar across each. The height of the bar represents
(counts/monitor count at 0°) / (counts/monitor count at 90°) and the width of the bar represents the experimental error in this ratio. The theoretical value of the ratio is graphed as a function of mixing ratio.
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