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Response of Laterally Loaded Rigid Monopiles and Poles in Multilayered Elastic Soil

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ABSTRACT: A new method of analysis of rigid monopiles and poles based on the principle of virtual work is developed. The analysis considers three-dimensional interaction between the rigid pile and surrounding soil, and quickly produces pile response maintaining accuracy comparable with that obtained from equivalent finite element analysis. Using this method, a systematic parametric study is performed to investigate the response of rigid piles in soil profiles where properties change either continuously or discretely with depth. Equations are developed based on the parametric study, which can be used to calculate pile head displacement and rotation. Numerical examples are provided that illustrate the use of the method.

Keywords: Rigid pile, monopile, pole, wind energy, variational calculus, analytical solution

INTRODUCTION

Monopiles are large diameter hollow cylindrical piles used as foundations for about 76% of wind turbines installed all over the world (EWEA 2014). In offshore regions, these piles are usually 4-6 m in diameter and installed into the sea bed to a depth of about 5-6 times the diameter (LeBlanc et al. 2010, Klinkvort and Hededal 2014). The monopiles are subjected to large lateral loads from wind, waves, and water currents. For these piles, the allowable lateral displacement at the pile head and rotation of the pile axis are important design criteria for serviceability limit states. Lateral pile displacements are usually calculated

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using the semi-empirical p - y method (Reese et al. 1974, 1975), which is included in the American Petroleum Institute (API 2000) and Det Norske Veritas (DNV 2014) codes of practice. However, finite element (FE) analysis results show that the p - y method underestimates monopile displacement (Abdel-Rahman and Achmus 2005; Lesny and Wiemann 2006). This is not surprising because the p - y method was developed from field lateral load tests performed on slender piles of 0.5-2.0 m diameter at a particular test site in Texas, U.S.A. (Reese et al. 1974, 1975), and may not work well for large-diameter piles installed in different soil profiles. Several researchers have performed FE analysis and centrifuge tests on monopiles in sandy and clayey soils to investigate their load-displacement behaviour (Klinkvort and Hededal 2012; Haiderali et al. 2013), and found that the monopiles undergo deformations typically by rigid-body rotation about a pivot point.

Available methods of analysis of laterally loaded rigid piles are mostly based on the theories of ultimate capacity (Broms 1964a,b; Zhang et al. 2005) that cannot be used to obtain the load-displacement response under working load conditions. Very few studies, e.g., FE analysis by Carter and Kulhawy (1992) and Higgins et al. (2013), and a simplified subgrade-reaction based analysis by Motta (2013), have focused on the load-displacement response of rigid piles under working loads. A systematic study on the deformation characteristics of rigid piles under working loads considering different possible soil conditions and pile geometry does not exist.

In this study, a new method for analysis of laterally-loaded rigid piles in multilayered, elastic soil is developed which can be used to calculate the load-displacement response of piles. In this method, rational soil and pile displacement fields are assumed maintaining compatibility and continuity, and the interaction between the pile and surrounding soil is taken into account by applying the principle of virtual work. The resulting equations governing the pile and soil displacements under equilibrium conditions are solved either

numerically or analytically. The analysis is based on the principles of continuum mechanics in which the soil is treated as a three-dimensional (3D) elastic continuum; yet solutions are obtained very quickly, with the simulation time comparable with that of a typical one-dimensional (1D) pile analysis performed using soil springs (e.g., the p - y method). Pile responses obtained from this analysis are compared with those of equivalent 3D FE analysis and field tests, and were found to be in good agreement. Results are obtained for piles embedded in one-, two-, three-, and four-layer soils. Results are also obtained for piles embedded in soil profiles with linearly varying soil modulus with depth. Based on the results, equations are developed and figures are produced which can be used to calculate pile head displacement and rotation with a hand-held calculator. Numerical examples are provided that illustrate the use of the equations.

ANALYSIS

Problem Description

Figure 1 shows a rigid pile with circular cross section of radius r_p and length L_p embedded in a layered elastic soil deposit consisting of n layers. The soil layers have infinite horizontal extent and have finite thickness in the vertical direction except the bottom (n^{th}) layer which extends to infinity downward. Any layer i has a thickness $H_i - H_{i-1}$ because the depth to the bottom of the i^{th} layer is denoted by H_i (with $H_0 = 0$ and $H_n = \infty$). Each layer is, homogeneous, isotropic, and elastic characterized by Lamé's constants λ_{si} and G_{si} . The pile head is at the ground surface and subjected to a horizontal force F_a and a moment M_a while the base is embedded in the n^{th} layer. There is no slippage or separation between the pile and the surrounding soil. A cylindrical (r - θ - z) coordinate system is chosen for analysis as shown in Figure 1.

Force Diagrams of Pile and Soil

In order to obtain the equilibrium equations describing pile and soil displacements, the force diagrams of the pile and soil are considered separately as shown in Figure 2. The interaction between soil and pile is captured by the distributed soil reaction force p acting along the pile shaft and by the concentrated shear force S_b at the pile base. The distributed reaction p is the resistance offered by the soil layers surrounding the pile against the lateral movement and rotation of pile, and the base shear force S_b is the resistance offered by the soil layers beneath the pile against its lateral movement.

Pile and Soil Displacement Fields

For a rigid pile with a point of rotation, it is reasonable to assume a linear displacement profile (Figure 3) as

$$w(z) = w_h - \Theta_h z \quad (1)$$

where w is the lateral pile displacement varying with depth z , w_h is the pile head displacement, and Θ_h is the clockwise rotation of the pile axis in the vertical plane in which F_a acts.

Assuming that the vertical soil displacement caused by lateral pile movement is negligible and that the horizontal soil displacements can be expressed as a product of three separable variables (Basu 2006), the following are obtained:

$$u_r = w(z) \phi_r(r) \cos \theta \quad (2a)$$

$$u_\theta = -w(z) \phi_\theta(r) \sin \theta \quad (2b)$$

where u_r and u_θ are the radial and tangential components of horizontal soil displacement, $\phi_r(r)$ and $\phi_\theta(r)$ are dimensionless displacement functions describing how the soil displacement decreases with increasing radial distance from the pile, and θ is the angle measured clockwise

from a vertical reference section ($r = r_0$) that contains the applied force vector F_a . In order to ensure perfect contact between the pile and soil, $\phi_r(r) = 1$ and $\phi_\theta(r) = 1$ are assumed at the pile-soil interface. At infinite radial distance, $\phi_r(r) = 0$ and $\phi_\theta(r) = 0$ are assumed, which ensures that soil displacements decrease monotonically with increase in radial distance from the pile.

Principle of Virtual Work

Applying a virtual displacement δw to the pile such that the virtual head displacement is δw_h and the virtual rotation of the pile axis is $\delta \Theta_h$, and invoking the principle of virtual work, the following equation is obtained:

$$\left[F_a - \int_0^{L_p} p(z) dz - S_b \right] \delta w_h + \left[M_a + \int_0^{L_p} p(z) z dz + S_b L_p \right] \delta \Theta_h = 0 \quad (3)$$

Because w_h and Θ_h are arbitrary, their first variations δw_h and $\delta \Theta_h$ are non-zero. Consequently, equation (3) is satisfied if and only if

$$F_a = \int_0^{L_p} p(z) dz + S_b \quad (4a)$$

$$M_a = - \left[\int_0^{L_p} p(z) z dz + S_b L_p \right] \quad (4b)$$

Equations (4a) and (4b) describe the equilibrium configuration of the pile (see Figure 2(b)).

Applying the principle of virtual work to the soil mass (see Figure 2(c)), the following equation is obtained:

$$\int_0^{L_p} p(z) \delta w dz + S_b \delta w_b - \int_0^\infty \int_0^{2\pi} \int_{r_p}^\infty \sigma_{lm} \delta \varepsilon_{lm} r dr d\theta dz - \int_{L_p}^\infty \int_0^{2\pi} \int_0^{r_p} \sigma_{lm} \delta \varepsilon_{lm} r dr d\theta dz = 0 \quad (5)$$

where $\delta \varepsilon_{lm}$ is the virtual strain tensor in the soil mass arising from the virtual pile displacement δw , and σ_{lm} is the soil stress tensor. The first two terms in the left hand side of

equation (5) represent the external virtual work. The third term represents the internal virtual work done by the entire soil mass except the soil column below the pile, and the fourth term represents the internal virtual work done by the soil column below the pile.

Using equations (2a)-(2b), the infinitesimal strain-displacement relationship, and the elastic stress-strain relationship, the term $\sigma_{lm}\delta\epsilon_{lm}$ (with summation implied) in equation (5) is expressed in terms of the functions w , ϕ_r , and ϕ_θ . This leads to an equation of the form $\int A(w)\delta w dz + \int B(\phi_r)\delta\phi_r dr + \int C(\phi_\theta)\delta\phi_\theta dr = 0$ in which δw , $\delta\phi_r$, and $\delta\phi_\theta$ are the first variations of the functions w , ϕ_r , and ϕ_θ , respectively, and $A(\cdot)$, $B(\cdot)$, and $C(\cdot)$ are differential operators. Considering the variations of w , ϕ_r , and ϕ_θ separately and equating the terms associated with δw , $\delta\phi_r$, and $\delta\phi_\theta$ individually to zero produce the governing differential equations $A(w) = 0$, $B(\phi_r) = 0$, and $C(\phi_\theta) = 0$ for w , ϕ_r , and ϕ_θ , respectively, along with appropriate boundary conditions.

Equations Describing Pile-Soil Interaction Forces and Displacements

After solving the differential equations of $w(z)$ (i.e., $A(w) = 0$) for each layer i using appropriate boundary conditions which ensure that the continuity of w across the boundaries of soil layers is maintained, that equilibrium is satisfied at the base of the pile, and that w becomes zero at infinite vertical distance down from the pile base, the following equations of soil reaction $p(z)$ and base shear force S_b are obtained:

$$p_i = k_i w_i \quad (6)$$

$$S_b = -2t_n \Theta_h + \sqrt{2k_n \zeta t_n} w_b \quad (7)$$

In equation (6), w_i and p_i represent $w(z)$ and $p(z)$ within the i^{th} layer. The parameters k_i and t_i in equations (6) and (7) are constants for any layer i , and are given by

$$k_i = \pi \left[(\lambda_{si} + 2G_{si}) \int_{r_p}^{\infty} r \left(\frac{d\phi_r}{dr} \right)^2 dr + G_{si} \int_{r_p}^{\infty} r \left(\frac{d\phi_{\theta}}{dr} \right)^2 dr + 2\lambda_{si} \int_{r_p}^{\infty} (\phi_r - \phi_{\theta}) \frac{d\phi_r}{dr} dr \right. \\ \left. + 2G_{si} \int_{r_p}^{\infty} (\phi_r - \phi_{\theta}) \frac{d\phi_{\theta}}{dr} dr + (\lambda_{si} + 3G_{si}) \int_{r_p}^{\infty} \frac{1}{r} (\phi_r - \phi_{\theta})^2 dr \right] \quad (8)$$

$$t_i = \begin{cases} \frac{\pi}{2} G_{si} \left[\int_{r_p}^{\infty} (\phi_r^2 + \phi_{\theta}^2) r dr \right]; & i = 1, 2, \dots, n \\ \frac{\pi}{2} G_{sn} \left[\int_{r_p}^{\infty} (\phi_r^2 + \phi_{\theta}^2) r dr + r_p^2 \right]; & i = n + 1 \end{cases} \quad (9)$$

and $\zeta = t_{n+1}/t_n$. Note that the n^{th} (bottom) layer is artificially split into two sub-layers above and below the pile base, and the sub-layer below the pile base is denoted by the subscript $n+1$ (i.e., $H_n = L_p$ and $H_{n+1} \rightarrow \infty$). The parameter k is analogous to the Winkler spring constant and represents the compressive resistance offered by the soil “springs” to the pile. The parameter t represents the shearing resistance of soil and can be interpreted as the horizontal shear force acting between the adjacent soil “springs”. In order to obtain k_i and t_i , numerical integrations are performed along the radial coordinate following equations (8) and (9), for which ϕ_r and ϕ_{θ} and their derivatives are required.

The differential equations of ϕ_r and ϕ_{θ} (i.e., $B(\phi_r) = 0$ and $C(\phi_{\theta}) = 0$), as obtained by separately considering the variations of ϕ_r , and ϕ_{θ} , are given by

$$\frac{d^2\phi_r}{dr^2} + \frac{1}{r} \frac{d\phi_r}{dr} - \left[\left(\frac{\gamma_1}{r} \right)^2 + \left(\frac{\gamma_2}{r_p} \right)^2 \right] \phi_r = \frac{\gamma_3}{r} \frac{d\phi_{\theta}}{dr} - \left(\frac{\gamma_1}{r} \right)^2 \phi_{\theta} \quad (10)$$

$$\frac{d^2\phi_{\theta}}{dr^2} + \frac{1}{r} \frac{d\phi_{\theta}}{dr} - \left[\left(\frac{\gamma_4}{r} \right)^2 + \left(\frac{\gamma_5}{r_p} \right)^2 \right] \phi_{\theta} = -\frac{\gamma_6}{r} \frac{d\phi_r}{dr} - \left(\frac{\gamma_4}{r} \right)^2 \phi_r \quad (11)$$

The corresponding boundary conditions are $\phi_r = 1$ and $\phi_{\theta} = 1$ at $r = r_p$, and $\phi_r = 0$ and $\phi_{\theta} = 0$ at $r = \infty$. Solutions of equations (10) and (11) are obtained using the finite difference method.

Because ϕ_r and ϕ_θ are interdependent and present in both the equations (10) and (11), an iterative algorithm was used to solve these two equations simultaneously.

The dimensionless terms γ_1 , γ_2 , γ_3 , γ_4 , γ_5 , and γ_6 in equations (10) and (11) are given by

$$\gamma_1 = \sqrt{\frac{\sum_{i=1}^n (\lambda_{si} + 3G_{si}) \int_{H_{i-1}}^{H_i} (w_h - \Theta_h z)^2 dz + (\lambda_{sn} + 3G_{sn}) (w_h - \Theta_h L_p)^2 \sqrt{\frac{\zeta t_n}{2k_n}}}{\sum_{i=1}^n (\lambda_{si} + 2G_{si}) \int_{H_{i-1}}^{H_i} (w_h - \Theta_h z)^2 dz + (\lambda_{sn} + 2G_{sn}) (w_h - \Theta_h L_p)^2 \sqrt{\frac{\zeta t_n}{2k_n}}}} \quad (12)$$

$$\gamma_2 = r_p \sqrt{\frac{\sum_{i=1}^n G_{si} \int_{H_{i-1}}^{H_i} \Theta_h^2 dz + G_{sn} (w_h - \Theta_h L_p)^2 \sqrt{\frac{k_n}{8\zeta t_n}}}{\sum_{i=1}^n (\lambda_{si} + 2G_{si}) \int_{H_{i-1}}^{H_i} (w_h - \Theta_h z)^2 dz + (\lambda_{sn} + 2G_{sn}) (w_h - \Theta_h L_p)^2 \sqrt{\frac{\zeta t_n}{2k_n}}}} \quad (13)$$

$$\gamma_3 = \sqrt{\frac{\sum_{i=1}^n (\lambda_{si} + G_{si}) \int_{H_{i-1}}^{H_i} (w_h - \Theta_h z)^2 dz + (\lambda_{sn} + G_{sn}) (w_h - \Theta_h L_p)^2 \sqrt{\frac{\zeta t_n}{2k_n}}}{\sum_{i=1}^n (\lambda_{si} + 2G_{si}) \int_{H_{i-1}}^{H_i} (w_h - \Theta_h z)^2 dz + (\lambda_{sn} + 2G_{sn}) (w_h - \Theta_h L_p)^2 \sqrt{\frac{\zeta t_n}{2k_n}}}} \quad (14)$$

$$\gamma_4 = \sqrt{\frac{\sum_{i=1}^n (\lambda_{si} + 3G_{si}) \int_{H_{i-1}}^{H_i} (w_h - \Theta_h z)^2 dz + (\lambda_{sn} + 3G_{sn}) (w_h - \Theta_h L_p)^2 \sqrt{\frac{\zeta t_n}{2k_n}}}{\sum_{i=1}^n G_{si} \int_{H_{i-1}}^{H_i} (w_h - \Theta_h z)^2 dz + G_{sn} (w_h - \Theta_h L_p)^2 \sqrt{\frac{\zeta t_n}{2k_n}}}} \quad (15)$$

$$\gamma_5 = r_p \sqrt{\frac{\sum_{i=1}^n G_{si} \int_{H_{i-1}}^{H_i} \Theta_h^2 dz + G_{sn} (w_h - \Theta_h L_p)^2 \sqrt{\frac{k_n}{8\zeta t_n}}}{\sum_{i=1}^n G_{si} \int_{H_{i-1}}^{H_i} (w_h - \Theta_h z)^2 dz + G_{sn} (w_h - \Theta_h L_p)^2 \sqrt{\frac{\zeta t_n}{2k_n}}}} \quad (16)$$

$$\gamma_6 = \sqrt{\frac{\sum_{i=1}^n (\lambda_{si} + G_{si}) \int_{H_{i-1}}^{H_i} (w_h - \Theta_h z)^2 dz + (\lambda_{sn} + G_{sn}) (w_h - \Theta_h L_p)^2 \sqrt{\frac{\zeta t_n}{2k_n}}}{\sum_{i=1}^n G_{si} \int_{H_{i-1}}^{H_i} (w_h - \Theta_h z)^2 dz + G_{sn} (w_h - \Theta_h L_p)^2 \sqrt{\frac{\zeta t_n}{2k_n}}}} \quad (17)$$

Substituting equations (6) and (7) in equations (4a) and (4b) results in

$$F_a = \int_0^{L_p} k_i (w_h - \Theta_h z) dz - 2t_n \Theta_h + \sqrt{2k_n \zeta t_n} (w_h - \Theta_h L_p) \quad (18)$$

$$M_a = - \int_0^{L_p} k_i (w_h - \Theta_h z) dz - \left\{ -2t_n \Theta_h + \sqrt{2k_n \zeta t_n} (w_h - \Theta_h L_p) \right\} L_p \quad (19)$$

Equations (18) and (19) relate the head displacement and rotation of the rigid pile with applied force and moment. For a pile in a single, homogeneous soil layer with constants k and t , equations (18) and (19) simplify to

$$w_h = \frac{\left(\frac{kL_p^3}{3} + 2tL_p + \sqrt{2\zeta t} L_p^2 \right) F_a + \left(\frac{kL_p^2}{2} + 2t + \sqrt{2k\zeta t} L_p \right) M_a}{\left(kL_p + \sqrt{2k\zeta t} \right) \left(\frac{kL_p^3}{3} + 2tL_p + \sqrt{2k\zeta t} L_p^2 \right) - \left(\frac{kL_p^2}{2} + \sqrt{2k\zeta t} L_p \right) \left(\frac{kL_p^2}{2} + 2t + \sqrt{2k\zeta t} L_p \right)} \quad (20)$$

$$\Theta_h = \frac{\left(\frac{kL_p^2}{2} + \sqrt{2k\zeta t} L_p \right) F_a + \left(kL_p + \sqrt{2k\zeta t} \right) M_a}{\left(kL_p + \sqrt{2k\zeta t} \right) \left(\frac{kL_p^3}{3} + 2tL_p + \sqrt{2k\zeta t} L_p^2 \right) - \left(\frac{kL_p^2}{2} + \sqrt{2k\zeta t} L_p \right) \left(\frac{kL_p^2}{2} + 2t + \sqrt{2k\zeta t} L_p \right)} \quad (21)$$

Equivalent Soil Shear Modulus

Randolph (1981) found that soil Poisson's ratio ν_s has a minimal effect on lateral pile response and that its effect can be taken into account by using an equivalent shear modulus G_s^* given by

$$G_s^* = G_s (1 + 0.75\nu_s) \quad (22)$$

The advantage of equation (22) is that the pile response can be investigated in terms of a single soil parameter that takes into account the effect of both the elastic constants. Guo and Lee (2001) found that, for assumed soil displacement field similar to that described in equation (2a)-(2b), pile response is excessively stiff for soil Poisson's ratio ν_s close to 0.5. In order to avoid this excessive stiffness, Guo and Lee (2001) recommended setting $\nu_s = 0$ irrespective of its actual value (which is the same as setting $\lambda_s = 0$) and indirectly take into

account the effect of ν_s through equation (22). In this study, the recommendation of Guo and Lee (2001) was followed and it was found that the resulting pile response was quite accurate when compared with the results of equivalent 3D FE analysis.

Solution Algorithm

Pile head displacement w_h and rotation Θ_h can be calculated using equations (18)-(19) or (20)-(21). However, the soil parameters k_i and t_i must be known to calculate w_h and Θ_h . The parameters k_i and t_i depend on the functions ϕ_r and ϕ_θ which, in turn, depend on w_h and Θ_h through the six dimensionless constants γ_1 - γ_6 . Thus, an iterative algorithm is used to obtain solutions. Initial guesses on γ_1 - γ_6 are made (each of them was assigned a value 1.0 initially), which are used to determine ϕ_r and ϕ_θ by solving equations (10) and (11) iteratively using the finite difference method. From the calculated values of ϕ_r and ϕ_θ , k_i and t_i are calculated using equations (8) and (9). The calculated values of k_i and t_i are used to obtain w_h and Θ_h . Using the calculated pile displacement and rotation, γ_1 - γ_6 are calculated using equations (12)-(17). The calculated values of γ_1 - γ_6 are compared with the assumed initial values and if the differences are more than the tolerable limit of 0.001, the same set of calculations are repeated with the calculated values of γ_1 - γ_6 as the initial guesses. Iterations are continued until the values of γ_1 - γ_6 between successive iterations fall below 0.001. Figure 4 shows the solution flowchart.

VALIDATION AND RESULTS

Comparison with FE Analysis

The accuracy of the proposed analysis method is verified by comparing pile responses obtained from this analysis with those of equivalent 3D FE analysis performed using ABAQUS. Comparisons were made for two piles embedded in homogeneous and four-layer

soil profiles, respectively (Figure 5). In ABAQUS, the pile and soil were modeled as a single cylindrical part with appropriate partitioning to represent the pile and soil separately. This ensured that there was no slippage or separation between the soil and pile. The top soil surface was flush with the pile head and the bottom soil surface extended to a finite depth of 15 m below the pile base. The horizontal radial extent of the soil domain (i.e., the vertical curved boundary of the FE domain) for both the homogeneous and four-layer soil problems was at a distance of 80 m from the pile axis. Different boundary conditions were prescribed at the soil boundaries of the FE domain — all components of displacements were made zero along the bottom (horizontal) boundary and along the outer, curved (vertical) boundary. Eight-noded reduced integration (C3D8R) brick elements were used to model both the homogeneous and four-layer soil and pile domain. A rigid body constraint was applied to the entire pile section to ensure that the pile behaved as a rigid element. The soil material model was assumed to be homogeneous, isotropic, and linear elastic within each layer. Concentrated force and moment were applied to a reference point at the pile head, to which all the nodes of the pile were connected. A uniform discretization length of 2.5 m for the mesh was chosen with the total number of elements for the homogeneous and four-layer problems being 80650 and 84588, respectively. The optimal domains and meshes described above were obtained by performing convergence checks to ensure that there were no boundary effects.

Figure 5 shows the comparisons — the details of pile and soil properties are shown in Figure 5(a). Figure 5(a) also shows the normalized pile head displacement w_h/r_p profiles — a difference of 4.33% and 5.33% occurred in the pile head displacement for the cases of homogeneous and four-layer soil, respectively. Figures 5(b) and (c) show the normalized radial displacement u_r/r_p and octahedral shear strain ε_{oct} in soil at the ground surface (i.e., for $z = 0$) as functions of normalized radial distance in the direction of the applied load (i.e., for θ

= 0). Figure 5(d) shows the normalized soil reaction $p(z)$ as a function of normalized depth. The match between the results obtained from the present analysis and FE analysis are quite well.

Comparison with Field Test

Pile displacement obtained from the present analysis is compared with that obtained from a field pile load test reported by Yang and Liang (2006). The test was performed on a free-head drilled shaft with a diameter of 2.59 m and an embedment length of 20.12 m. The drilled shaft was embedded in rock (shale, sandstone and claystone) with an overlying sand layer of thickness 7.93 m. The average standard penetration test (SPT) $N = 10$ for the sand layer based on which the Young's modulus E_s of the sand layer is estimated to be 10 MPa following Budhu (2011). The Young's modulus of the rock varies with depth and was reported to be 44.56 MPa from a depth of 7.93 m to a depth of 15.7 m below the ground surface, 4.51 MPa from 15.7 m to 20.1 m below the ground surface, and 6.58 MPa from 20.1 m to great depth. The Poisson's ratio is assumed to be 0.25 for all the soil and rock layers. For an applied load $F_a = 400$ kN, the calculated head displacement is 2.6 mm while the measured head displacement in the field is 3.14 mm (the drilled shaft behaved rigidly in the field because of which this comparison is possible). Overall, the match in the head displacement is reasonably well and the difference in the results may be attributed to the uncertainty in the parameter estimation of the soil and rock properties.

Effect of Pile Diameter on Soil Reaction

Figure 6 shows the variation of soil reaction p with pile radius for the two pile cases described in Figure 5(a). To generate Figure 6, the radius is varied for both the piles while all other variables (i.e., soil layering and properties, pile length, and applied force and moment)

are kept the same as those described in Figure 5(a). It is evident that soil reaction is a function of pile diameter, and the results indicate that the p - y curves applicable for small diameter piles may not work well for large diameter monopiles.

PARAMETRIC STUDY

Pile Response in Homogeneous Soil

Figures 7(a) and (b) show the normalized head displacement $w_h G_s^* r_p / F_a$ and $w_h G_s^* r_p^2 / M_a$ and normalized rotation $\Theta_h G_s^* r_p^2 / F_a$ and $\Theta_h G_s^* r_p^3 / M_a$, caused by applied force and moment, respectively, as functions of the pile slenderness ratio L_p / r_p . Also plotted are the corresponding results of equivalent 3D FE analysis obtained using ABAQUS. Based on the plots obtained from the present analysis, fitted equations are developed, as shown below, that can be used to calculate head displacement and rotation of rigid piles in homogeneous soil:

$$w_h = 0.36 \left(\frac{F_a}{G_s^* r_p} \right) \left(\frac{L_p}{r_p} \right)^{-0.61} + 0.37 \left(\frac{M_a}{G_s^* r_p^2} \right) \left(\frac{L_p}{r_p} \right)^{-1.61} \quad (23)$$

$$\Theta_h = 0.42 \left(\frac{F_a}{G_s^* r_p^2} \right) \left(\frac{L_p}{r_p} \right)^{-1.61} + 0.46 \left(\frac{M_a}{G_s^* r_p^3} \right) \left(\frac{L_p}{r_p} \right)^{-2.42} \quad (24)$$

Pile Response in Two-layer Soil

Figure 8(a) shows the normalized pile head displacement caused by applied force F_a in two-layer soil. The normalization is performed with respect to the modified shear modulus G_{s1}^* of the top layer. Plots are generated for different thickness H_1 of the top layer. Similar plots of head displacement caused by applied moment and of head rotation caused by applied force and moment are generated for several different values of H_1 / L_p but not reported in this paper. Based on these plots, fitted equations are developed as shown below:

$$w_h = a_1 \left(\frac{F_a}{G_{s1}^* r_p} \right) \left(\frac{L_p}{r_p} \right)^{-a_2} + b_1 \left(\frac{M_a}{G_{s1}^* r_p^2} \right) \left(\frac{L_p}{r_p} \right)^{-b_2} \quad (25)$$

$$\Theta_h = c_1 \left(\frac{F_a}{G_{s1}^* r_p^2} \right) \left(\frac{L_p}{r_p} \right)^{-c_2} + d_1 \left(\frac{M_a}{G_{s1}^* r_p^3} \right) \left(\frac{L_p}{r_p} \right)^{-d_2} \quad (26)$$

where the regression coefficients a_1 , a_2 , b_1 , b_2 , c_1 , c_2 , d_1 , and d_2 are given in Tables 1-2.

It was observed that, for a given G_{s1}^* , pile head displacement is greater than the corresponding homogeneous-soil case (with $G_{s1}^* = G_s^*$) if the second layer is softer than the top layer. The reverse is also true if the second layer is stiffer than the top layer. For a stiffer second layer, the head displacement increases with increase in the thickness of the top layer, and the reverse is true if the second layer is softer. Clearly, soil layering has a significant effect on the pile response and, in order to investigate this aspect further, head-displacement ratio $w_{\text{two-layer}}/w_{\text{homogeneous}}$ is plotted as a function of normalized thickness of the top layer H_1/L_p (Figure 8(b)) where $w_{\text{two-layer}}$ is the head displacement in the two-layer soil and $w_{\text{homogeneous}}$ is the head displacement in a homogeneous soil with G_{s2}^* as the soil modulus. For $G_{s2}^*/G_{s1}^* < 1$, the head-displacement ratio becomes more or less constant beyond $H_1/L_p = 0.4$. For $G_{s2}^*/G_{s1}^* > 1$, however, the head-displacement ratio increases monotonically with increase in H_1/L_p .

Pile Response in Three-layer Soil

In order to further investigate the effect of soil layering on pile response, piles embedded in three-layer soil profiles are analyzed. Three different cases of layer thickness, L1, L2, and L3, are considered and, for each case, three different combinations, M1, M2, and M3, of soil modulus in the three layers are assumed. Thus, altogether nine cases are analyzed, as described in Figure 9. For layering case L1, the three soil layers divide the pile shaft into equal parts of length $L_p/3$ while, for cases L2 and L3, unequal thicknesses are

assumed (Figure 9). For case L2, $H_1 = L_p/6$ and $H_2 = L_p/2$, and for case L3, $H_1 = L_p/6$ and $H_2 = 5L_p/6$. For the modulus combinations M1, M2, and M3, the moduli G_{s1}^* , G_{s2}^* and G_{s3}^* of the three layers are so chosen that $(G_{s1}^* + G_{s2}^* + G_{s3}^*)/3 = G_s^*$ (Figure 9) and that their values increase in multiples of 3 from the minimum to the maximum (e.g., for case M2, G_{s2}^* is the minimum and G_{s3}^* is the maximum with $G_{s1}^*/G_{s2}^* = 3$ and $G_{s3}^*/G_{s1}^* = 3$).

Figure 9 shows the pile head displacement caused by an applied force at the head, normalized with respect to the average modulus G_s^* , for all the nine cases described above. It is evident from the plots that both the thickness and modulus of each layer influence the pile response. Unlike flexible long piles, layers close to the pile base may have a strong influence on the rigid pile response. Similar plots of head displacement caused by applied moment and of rotation caused by both applied force and moment were obtained although not shown in this paper, and using these plots, fitted equations of head displacement and rotation are obtained as

$$w_h = a_3 \left(\frac{F_a}{G_s^* r_p} \right) \left(\frac{L_p}{r_p} \right)^{-a_4} + b_3 \left(\frac{M_a}{G_s^* r_p^2} \right) \left(\frac{L_p}{r_p} \right)^{-b_4} \quad (27)$$

$$\Theta_h = c_3 \left(\frac{F_a}{G_s^* r_p^2} \right) \left(\frac{L_p}{r_p} \right)^{-c_4} + d_3 \left(\frac{M_a}{G_s^* r_p^3} \right) \left(\frac{L_p}{r_p} \right)^{-d_4} \quad (28)$$

where the regression coefficients a_3 , a_4 , b_3 , b_4 , c_3 , c_4 , d_3 , and d_4 are given in Tables 3-4.

Pile Response in Four-layer Soil

In order to study pile response in four-layer soil, only one case of layer thickness L1 with $H_1 = L_p/4$, $H_2 = L_p/2$, and $H_3 = 3L_p/4$ was considered (Figure 10) along with four combinations M1, M2, M3, and M4 of soil modulus. The moduli G_{s1}^* , G_{s2}^* , G_{s3}^* and G_{s4}^* of the four layers are so chosen that, for all the combinations M1-M4, the average is G_s^* . Figure 10 shows the normalized head rotation, caused by applied moment, for these cases as a

function of pile slenderness ratio. Similar plots of normalized head displacement caused by applied force and moment and of head rotation caused by applied force are also obtained but not presented here, and based on these plots, fitted equations are obtained as follows:

$$w_h = a_5 \left(\frac{F_a}{G_s^* r_p} \right) \left(\frac{L_p}{r_p} \right)^{-a_6} + b_5 \left(\frac{M_a}{G_s^* r_p^2} \right) \left(\frac{L_p}{r_p} \right)^{-b_6} \quad (29)$$

$$\Theta_h = c_5 \left(\frac{F_a}{G_s^* r_p^2} \right) \left(\frac{L_p}{r_p} \right)^{-c_6} + d_5 \left(\frac{M_a}{G_s^* r_p^3} \right) \left(\frac{L_p}{r_p} \right)^{-d_6} \quad (30)$$

where the regression coefficients a_5 , a_6 , b_5 , b_6 , c_5 , c_6 , d_5 , and d_6 are given in Tables 5-6.

Pile Response in Soil with Linearly Varying Modulus

Often it is found that soil properties vary gradually with depth and distinct layering may not always be present. For such cases, it is quite common to assume that soil modulus increases linearly with depth. In order to model such profiles, Randolph (1981) suggested that the rate of increase of soil shear modulus with depth, dG_s/dz , must be modified following equation (22) as

$$m^* = m(1 + 0.75\nu_s) \quad (31)$$

where $m = dG_s/dz$. In this study, equation (31) is used along with the assumption that $\lambda_s = 0$ (irrespective of the actual value of ν_s) just as was done for the cases with spatially constant soil modulus.

Normalized head displacement $w_h m^* r_p^2 / F_a$ and $w_h m^* r_p^3 / M_a$ and rotation $\Theta_h m^* r_p^3 / F_a$ and $\Theta_h m^* r_p^4 / M_a$ caused respectively by the applied force and moment are obtained as functions of pile slenderness ratio for different values of $G_{s0}^* / (m^* L_p)$ where G_{s0}^* is the equivalent shear modulus at the soil surface. Figure 11 shows sample plots of normalized pile head displacement caused by the applied force. A comparison with the FE results of

Higgins and Basu (2011) for the case with $G_{s0}^* = 0$ show a good match. The corresponding fitted equations are given by

$$w_h = a_7 \left(\frac{F_a}{m^* r_p^2} \right) \left(\frac{L_p}{r_p} \right)^{-a_8} + b_7 \left(\frac{M_a}{m^* r_p^3} \right) \left(\frac{L_p}{r_p} \right)^{-b_8} \quad (32)$$

$$\Theta_h = c_7 \left(\frac{F_a}{m^* r_p^3} \right) \left(\frac{L_p}{r_p} \right)^{-c_8} + d_7 \left(\frac{M_a}{m^* r_p^4} \right) \left(\frac{L_p}{r_p} \right)^{-d_8} \quad (33)$$

where the regression coefficients a_7 , a_8 , b_7 , b_8 , c_7 , c_8 , d_7 , and d_8 are given in Tables 7-8.

Sometimes, soil profiles consist of a desiccated top crust overlying the main soil deposit, and such profiles are often modeled with a spatially constant modulus for the crust (i.e., the top layer) and with a linearly increasing modulus with depth for the main deposit (i.e., the second layer). Pile responses are obtained for several such soil profiles as well — Figure 12 shows the plots of normalized head rotation caused by applied force for $H_1/L_p = 0.1$. Similar plots were obtained for different values of H_1/L_p based on which fitted equations are developed as shown below

$$w_h = a_9 \left(\frac{F_a}{m^* r_p^2} \right) \left(\frac{L_p}{r_p} \right)^{-a_{10}} + b_9 \left(\frac{M_a}{m^* r_p^3} \right) \left(\frac{L_p}{r_p} \right)^{-b_{10}} \quad (34)$$

$$\Theta_h = c_9 \left(\frac{F_a}{m^* r_p^3} \right) \left(\frac{L_p}{r_p} \right)^{-c_{10}} + d_9 \left(\frac{M_a}{m^* r_p^4} \right) \left(\frac{L_p}{r_p} \right)^{-d_{10}} \quad (35)$$

where the regression coefficients a_9 , a_{10} , b_9 , b_{10} , c_9 , c_{10} , d_9 , and d_{10} are given in Tables 9-10.

DESIGN EXAMPLES

Soil with Linearly Varying Modulus

An example is presented for a rigid pile with diameter 0.92 m and embedment depth of 6 m, in a clay layer subjected to a lateral force of 450 kN and a moment of 900 kNm at the head. The undrained shear strength (s_u) of clay is assumed to vary linearly over the pile

length from 130 kPa at ground surface to 410 kPa at a depth of 6 m. The Young's modulus E_s of clay can be estimated from the relationship $E_s = 500s_u$ (Selvadurai 1979). Thus, E_s varies from 65 MPa at the ground surface to 205 MPa at a depth of 6 m. The Poisson's ratio ν_s of clay is assumed to be 0.45. The shear modulus G_{s0} at the ground surface is 22.41 MPa (calculated using the equation $G_s = 0.5E_s/(1 + \nu_s)$), while at 6 m depth, $G_s = 70.7$ MPa. Therefore, $G_{s0}^* = 22.41 \times (1 + 0.75 \times 0.45) = 29.97$ MPa, and $m = dG_s/dz = (70.7 - 29.97)/6.0 = 6.78$ MPa/m from which $m^* = 6.78 \times (1 + 0.75 \times 0.45) = 9.08$ MPa/m is obtained. The ratio $G_{s0}^*/(m^*L_p) = 0.55$. Using the regression coefficients obtained from Tables 7 and 8 corresponding to $G_{s0}^*/(m^*L_p) \approx 0.5$ and using equations (32) and (33), the head displacement and rotation for $F_a = 450$ kN and $M_a = 900$ kNm are calculated to be 3.1 mm and 5.5×10^{-4} radians, respectively. The pile head displacement and rotation obtained directly from the simulations are 2.6 mm and 4.8×10^{-4} radians, respectively, and are in good agreement with the results obtained from the fitted equations.

Two-layer Soil

An example of a monopile with $r_p = 3$ m and $L_p = 30$ m and subjected to $F_a = 3$ MN and $M_a = 40$ MNm is assumed to be embedded in a two-layer soil profile. The top layer consists of a 10 m thick ($H_1 = 10$ m) loose sand with Young's modulus $E_{s1} = 10$ MPa and Poisson's ratio $\nu_{s1} = 0.3$. The second layer consists of stiff clay with undrained shear strength (s_u) 100 kPa for which Young's modulus (E_{s2}) is estimated to be 50 MPa as per Selvadurai (1979). The Poisson's ratio ν_{s2} of stiff clay is assumed to be 0.45. Therefore, the shear modulus for the loose sand layer $G_{s1} = 0.5E_{s1}/(1 + \nu_{s1}) = 0.5 \times 10/(1 + 0.3) = 3.84$ MPa and the corresponding equivalent shear modulus $G_{s1}^* = G_{s1}(1 + 0.75\nu_{s1}) = 3.84 \times (1 + 0.75 \times 0.3) = 4.7$ MPa. For the stiff clay layer the shear modulus $G_{s2} = 0.5E_{s2}/(1 + \nu_{s2}) = 0.5 \times 50/(1$

+ 0.45) = 17.24 MPa and the corresponding equivalent shear modulus $G_{s2}^* = G_{s2}(1 + 0.75 \nu_{s2}) = 17.24 \times (1 + 0.75 \times 0.45) = 23.06$ MPa. The ratio $G_{s2}^* / G_{s1}^* = 4.91$. Therefore, using the regression coefficients corresponding to $G_{s2}^* / G_{s1}^* \approx 5$ and $H_1/L_p = 10/30 \approx 0.3$ from Tables 1 and 2 and using equations (25) and (26), the pile head displacement and rotation are obtained as 11.2 mm and 5.5×10^{-4} radian, respectively. The pile head displacement and rotation obtained directly from the simulations are 12.34 mm and 5.53×10^{-4} radians, respectively.

CONCLUSIONS

A continuum-based method of analysis for laterally loaded rigid piles with circular cross sections embedded in multi-layered elastic soil was presented. Differential equations for the pile and soil displacements were obtained using the principle of virtual work. The equations were solved using an iterative numerical scheme to obtain the pile head displacement and rotation. The advantage of the method is that it produces results fast with accuracy comparable with those of finite element analysis. The analysis requires simple inputs of pile geometry, soil layering and properties in a text file, and produces pile head displacement and rotation in a matter of seconds.

The responses studied are pile displacement and rotation, displacements and strains in soil, and the soil reaction on pile. It was observed that both displacements and strains in soil decrease with increase in radial distance from pile. The soil reaction was found to be a function of pile diameter.

A parametric study was performed for rigid piles in homogeneous, two-layer, three-layer, and four-layer soil with spatially constant soil modulus in each layer. Additionally, pile response in soil with linearly increasing modulus with depth was investigated. Dimensionless head displacement and rotation caused by applied horizontal force and moment at the pile head were plotted as functions of pile slenderness ratio, and these plots

were found to follow a power law with different regression coefficients. Based on these plots, fitted equations were obtained, which can be used to calculate the pile head displacement and rotation as functions of the applied forces and moments and of the different soil layering and properties. These equations and plots can be used in design. Numerical examples are worked out, which illustrate the use of the method.

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Figure Captions

Figure 1. A laterally loaded rigid pile embedded in a multilayered soil

Figure 2. (a) A rigid pile-soil system, (b) forces acting on pile, and (c) forces acting on soil

Figure 3. Assumed rigid-pile displacement profile

Figure 4. Solution flowchart

Figure 5. Comparison between the results obtained from present analysis and FE analysis for rigid piles in homogeneous and four-layer soil subjected to a lateral force and a moment at the head: (a) normalized pile displacement profile, (b) normalized radial soil displacement at the surface in the direction of applied force, (c) octahedral shear strain in soil at the surface in the vertical plane in which applied force acts, and (d) soil reaction acting on the pile shaft

Figure 6. Soil reaction versus pile radius at different depths along the shaft of rigid piles in homogeneous and four-layer soil

Figure 7. Normalized (a) head displacement and (b) rotation versus slenderness ratio of rigid piles in homogeneous soil subjected to a lateral force and a moment at the head

Figure 8. (a) Normalized head displacement versus slenderness ratio, and (b) head-displacement ratio versus normalized thickness of top layer for rigid piles in two-layer soil subjected to a lateral force at the head

Figure 9. Normalized head displacement versus slenderness ratio of rigid piles in three-layer soil subjected to a lateral force at the head

Figure 10. Normalized head rotation versus slenderness ratio of rigid piles in four-layer soil subjected to a moment at the head

Figure 11. Normalized head displacement caused by applied horizontal force at the head versus slenderness ratio of rigid piles in soil profiles with linearly increasing modulus with depth

Figure 12. Normalized head rotation caused by applied horizontal force at the head versus slenderness ratio of rigid piles in soil profiles with spatially constant modulus in the top layer overlying a second layer in which the modulus increases linearly with depth.

Table 1. Regression coefficients for head displacement of rigid piles in two-layer soil

G_{s2}^*/G_{s1}^*	H_1/L_p	a_1	a_2	b_1	b_2
0.10	0.10	1.62	0.78	0.91	1.55
	0.30	0.65	0.68	0.61	1.50
	0.50	0.42	0.58	0.59	1.47
0.25	0.10	1.11	0.75	0.74	1.58
	0.30	0.64	0.71	0.47	1.51
	0.50	0.47	0.65	0.45	1.54
0.50	0.10	0.72	0.70	0.59	1.57
	0.30	0.49	0.63	0.40	1.48
	0.50	0.47	0.68	0.41	1.53
0.75	0.10	0.54	0.69	0.39	1.56
	0.30	0.48	0.68	0.35	1.55
	0.50	0.45	0.67	0.33	1.55
1.00	-	0.36	0.61	0.37	1.60
1.25	0.10	0.35	0.67	0.26	1.54
	0.30	0.37	0.65	0.29	1.55
	0.50	0.40	0.66	0.35	1.55
1.50	0.10	0.30	0.66	0.24	1.57
	0.30	0.34	0.65	0.28	1.56
	0.50	0.42	0.71	0.32	1.56
1.75	0.10	0.26	0.65	0.20	1.53
	0.30	0.31	0.64	0.24	1.54
	0.50	0.37	0.65	0.32	1.55
2.00	0.10	0.23	0.66	0.21	1.54
	0.30	0.29	0.64	0.25	1.52
	0.50	0.31	0.61	0.31	1.56
5.00	0.10	0.10	0.64	0.08	1.57
	0.30	0.14	0.60	0.12	1.49
	0.50	0.21	0.58	0.21	1.53
10.0	0.10	0.05	0.63	0.04	1.52
	0.30	0.07	0.55	0.07	1.49
	0.50	0.13	0.55	0.13	1.49

Table 2. Regression coefficients for head rotation of rigid piles in two-layer soil

G_{s2}^*/G_{s1}^*	H_1/L_p	c_1	c_2	d_1	d_2
0.10	0.10	1.24	1.60	2.27	2.45
	0.30	0.78	1.55	1.87	2.43
	0.50	0.71	1.51	1.45	2.38
0.25	0.10	0.91	1.59	1.38	2.49
	0.30	0.63	1.60	1.08	2.46
	0.50	0.54	1.54	1.02	2.45
0.50	0.10	0.65	1.60	0.86	2.48
	0.30	0.50	1.57	0.69	2.44
	0.50	0.46	1.56	0.68	2.45
0.75	0.10	0.48	1.58	0.64	2.48
	0.30	0.44	1.57	0.61	2.49
	0.50	0.42	1.56	0.58	2.47
1.00	-	0.42	1.61	0.46	2.42
1.25	0.10	0.33	1.57	0.42	2.47
	0.30	0.36	1.56	0.44	2.46
	0.50	0.37	1.58	0.45	2.47
1.50	0.10	0.27	1.55	0.36	2.46
	0.30	0.33	1.57	0.41	2.49
	0.50	0.37	1.60	0.40	2.47
1.75	0.10	0.25	1.56	0.31	2.45
	0.30	0.29	1.55	0.36	2.47
	0.50	0.32	1.56	0.41	2.49
2.00	0.10	0.21	1.54	0.28	2.46
	0.30	0.27	1.55	0.32	2.45
	0.50	0.31	1.56	0.36	2.47
5.00	0.10	0.10	1.58	0.12	2.47
	0.30	0.14	1.51	0.18	2.47
	0.50	0.21	1.53	0.23	2.45
10.0	0.10	0.05	1.55	0.06	2.45
	0.30	0.08	1.51	0.09	2.44
	0.50	0.14	1.50	0.14	2.43

Table 3. Regression coefficients for head displacement of rigid piles in three-layer soil

Layering case	Modulus combination	a_3	a_4	b_3	b_4
L1	M1	0.73	0.58	0.69	1.52
	M2	0.62	0.67	0.54	1.59
	M3	0.24	0.62	0.28	1.49
L2	M1	0.48	0.58	0.46	1.51
	M2	0.51	0.60	0.48	1.54
	M3	0.38	0.70	0.33	1.51
L3	M1	0.74	0.65	0.65	1.58
	M2	1.04	0.74	0.90	1.67
	M3	0.34	0.66	0.34	1.53

Table 4. Regression coefficients for head rotation of rigid piles in three-layer soil

Layering case	Modulus combination	c_3	c_4	d_3	d_4
L1	M1	0.67	1.52	0.74	2.47
	M2	0.54	1.60	0.56	2.52
	M3	0.35	1.55	0.78	2.45
L2	M1	0.49	1.54	0.53	2.45
	M2	0.48	1.54	0.54	2.48
	M3	0.42	1.58	0.89	2.45
L3	M1	0.61	1.57	0.66	2.49
	M2	0.75	1.63	0.77	2.56
	M3	0.42	1.59	0.82	2.50

Table 5. Regression coefficients for head displacement of rigid piles in four-layer soil

Layering case	Modulus combination	a_5	a_6	b_5	b_6
L1	M1	0.62	0.59	0.58	1.52
	M2	0.58	0.66	0.51	1.58
	M3	0.48	0.74	0.32	1.51
	M4	0.27	0.62	0.37	1.62

Table 6. Regression coefficients for head rotation of rigid piles in four-layer soil

Layering thickness	Modulus combination	c_5	c_6	d_5	d_6
L1	M1	0.57	1.52	0.66	2.47
	M2	0.51	1.59	0.54	2.49
	M3	0.37	1.62	0.41	2.51
	M4	0.39	1.55	0.77	2.46

Table 7. Regression coefficients for head displacement of rigid piles in soil with modulus increasing linearly with depth

$G_{s0}^*/(m^*L_p)$	a_7	a_8	b_7	b_8
0.00	0.98	1.53	1.02	2.50
0.01	0.84	1.58	0.78	2.50
0.05	0.7	1.52	0.76	2.53
0.10	0.65	1.57	0.75	2.51
0.25	0.63	1.52	0.59	2.52
0.50	0.55	1.57	0.55	2.55
0.75	0.50	1.61	0.41	2.48
1.00	0.42	1.58	0.40	2.42

Table 8. Regression coefficients for head rotation of rigid piles in soil with modulus increasing linearly with depth

$G_{s0}^*/(m^*L_p)$	c_7	c_8	d_7	d_8
0.00	1.50	2.63	1.13	3.43
0.01	0.78	2.50	0.86	3.44
0.05	0.74	2.50	0.84	3.46
0.10	0.72	2.53	0.59	3.37
0.25	0.52	2.46	0.59	3.40
0.50	0.44	2.50	0.64	3.48
0.75	0.42	2.43	0.57	3.50
1.00	0.40	2.53	0.48	3.47

Table 9. Regression coefficients for head displacement of rigid piles in a soil profile with spatially constant modulus in the top layer that overlies a layer with linearly increasing modulus with depth

H_1/L_p	$G_{s1}^*/(m^*L_p)$	a_9	a_{10}	b_9	b_{10}
0.10	0.01	1.00	1.57	0.95	2.50
	0.05	0.95	1.56	0.92	2.51
	0.10	0.91	1.57	0.88	2.52
	0.25	0.80	1.59	0.75	2.54
	0.50	0.66	1.59	0.62	2.52
	0.75	0.57	1.60	0.54	2.54
	1.00	0.49	1.59	0.46	2.53
0.25	0.01	1.75	1.54	1.69	2.48
	0.05	1.59	1.53	1.54	2.48
	0.10	1.46	1.55	1.42	2.49
	0.25	1.18	1.56	1.14	2.51
	0.50	0.91	1.59	0.87	2.53
	0.75	0.74	1.60	0.69	2.54
	1.00	0.63	1.61	0.58	2.54
0.50	0.01	5.56	1.46	5.79	2.45
	0.05	4.43	1.51	4.37	2.48
	0.10	3.56	1.54	3.46	2.51
	0.25	2.31	1.59	2.20	2.55
	0.50	1.47	1.61	1.37	2.56
	0.75	1.07	1.62	0.99	2.57
	1.00	0.84	1.62	0.78	2.57

Table 10. Regression coefficients for head rotation of rigid piles in a soil profile with spatially constant modulus in the top layer that overlies a layer with linearly increasing modulus with depth

H_1/L_p	$G_{s1}^*/(m^*L_p)$	c_9	c_{10}	d_9	d_{10}
0.10	0.01	0.90	2.49	0.98	3.43
	0.05	0.88	2.49	0.99	3.45
	0.10	0.84	2.50	0.94	3.45
	0.25	0.68	2.51	0.65	3.39
	0.50	0.62	2.52	0.70	3.47
	0.75	0.51	2.51	0.57	3.46
	1.00	0.44	2.52	0.52	3.46
0.25	0.01	1.60	2.47	1.71	3.42
	0.05	1.50	2.48	1.62	3.43
	0.10	1.37	2.48	1.49	3.44
	0.25	1.10	2.50	1.20	3.45
	0.50	0.83	2.52	0.92	3.46
	0.75	0.67	2.52	0.76	3.47
	1.00	0.56	2.53	0.60	3.44
0.50	0.01	5.61	2.44	5.29	3.37
	0.05	4.15	2.49	4.29	3.43
	0.10	3.29	2.50	3.45	3.46
	0.25	2.02	2.52	2.15	3.48
	0.50	1.23	2.53	1.35	3.48
	0.75	0.91	2.54	1.02	3.49
	1.00	0.72	2.54	0.82	3.48

Figure 1. A laterally loaded rigid pile embedded in a multilayered soil

Figure 2. (a) A rigid pile-soil system, (b) forces acting on pile, and (c) forces acting on soil

Figure 3. Assumed rigid-pile displacement profile

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Figure 4. Solution flowchart

Figure 5. Comparison between the results obtained from present analysis and FE analysis for rigid piles in homogeneous and four-layer soil subjected to a lateral force and a moment at the head: (a) normalized pile displacement profile, (b) normalized radial soil displacement at the surface in the direction of applied force, (c) octahedral shear strain in soil at the surface in the vertical plane in which applied force acts, and (d) soil reaction acting on the pile shaft

Figure 6. Soil reaction versus pile radius at different depths along the shaft of rigid piles in homogeneous and four-layer soil

Figure 7(a).

Figure 7(b).

Figure 7. Normalized (a) head displacement and (b) rotation versus slenderness ratio of rigid piles in homogeneous soil subjected to a lateral force and a moment at the head

Figure 8(a).

Figure 8(b).

Figure 8. (a) Normalized head displacement versus slenderness ratio, and (b) head-displacement ratio versus normalized thickness of top layer for rigid piles in two-layer soil subjected to a lateral force at the head

Figure 9. Normalized head displacement versus slenderness ratio of rigid piles in three-layer soil subjected to a lateral force at the head

Figure 10. Normalized head rotation versus slenderness ratio of rigid piles in four-layer soil subjected to a moment at the head

Figure 11. Normalized head displacement caused by applied horizontal force at the head versus slenderness ratio of rigid piles in soil profiles with linearly increasing modulus with depth

Figure 12. Normalized head rotation caused by applied horizontal force at the head versus slenderness ratio of rigid piles in soil profiles with spatially constant modulus in the top layer overlying a second layer in which the modulus increases linearly with depth