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Simple criteria for ploughing and runout in post-failure evolution of submarine landslides

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ABSTRACT

The paper extends the shear band propagation analysis of slope failures to investigation of the ploughing and runout phenomena in submarine landslides. Ability to predict the two different modes of the post-failure landslide evolution is critical for determining the tsunami hazard and the type of the landslide impact on the offshore structures. The proposed analysis is based on the analogy between the ploughing and spreading failures. It uses the energy balance approach to develop the criterion for progressive shear band propagation driven by accumulation of the sliding material on top of the stable slope. This criterion is then combined with the kinematic passive block mechanism to produce analytical ploughing failure criteria formulated in terms of the critical rise in the seabed level. If the minimum rise of the seabed level at which the ploughing can take place is larger than the maximum possible free standing step in the seabed surface, the first passive failure block will start crumbling over the top of the stable zone causing the landslide to runout. Application of the derived criteria to the analysis of observed geomorphological features is demonstrated using example of a paleo-landslide complex in the Caspian Sea.

Keywords: submarine landslides, shear band propagation, ploughing, runout

INTRODUCTION

Evidence of historic submarine landslides shows that they can reach gigantic dimensions of tens to hundreds of kilometres in length, at very low slope inclinations of a few degrees (e.g., Hampton et al. 1996; Locat et al. 2002; Bryn et al. 2005; Masson et al. 2006). The hazard does not, however, end with the underwater slope failure. When a landslide runs out downslope, it can cause debris flow turning into a high-velocity turbidity current, demolishing offshore cables, pipelines and other infrastructure getting in its way (as in the 1929 Grand Banks landslide, Fine et al. 2005) and, possibly, generating a tsunami wave (as in 1998 Papua New Guinea tsunami, Tappin et al. 2001).

Understanding the mechanisms of the dynamic landslide evolution - from pre-failure conditioning, triggering, nucleation and propagation of localized shear zones to global failure and post-failure processes - is critical for reliable hazard assessment. Whereas the existing standard slope stability analysis approaches allow for both deterministic and probabilistic landslide hazard assessment of large seabed areas (e.g., Dimmock et al. 2012), they can only predict the slope failure in the steepest part of the slope. They cannot explain catastrophic and progressive dynamic propagation of a local shear zone into milder parts of the slope resulting in the observed post-failure geomorphology. In contrast, sophisticated finite element analysis can provide some insight into the basic landslide mechanisms, but the availability of high-quality data required for this type of analysis is limited. Enormous computational costs limit hazard assessment to deterministic analysis of small seafloor areas, with various stages of the landslide evolution treated as separate problems (e.g., Locat et al. 2002; Vanneste et al. 2013).

The recently developed shear band propagation (SBP) approach for submarine slope stability analysis helps to overcome these limitations, by explaining how a relatively short initial slip surface (localized shear zone of finite thickness, marked in red in Figure 1) can under specific conditions propagate dynamically tens of kilometres parallel to the slope surface through the quasi-stable part of the slope (orange in Figure 1), eventually causing a gigantic landslide and generating enormous tsunami waves. This approach has been proposed by Puzrin and Germanovich (2005) based on Fracture Mechanics principles developed by Palmer and Rice (1973), and, thanks to its evolutionary dynamic nature, the landslide geometry, volume and breakoff velocity could be determined to provide reliable input parameters for debris flow and tsunami wave propagation analysis (Puzrin et al. 2004; Puzrin et al. 2010; Trapper et al. 2016). A particular form of shear band propagation, which has received considerable attention (Locat et al. 2011; Locat et al. 2013; Quinn et al. 2011; Quinn et al. 2012; Dey et al. 2015), is a progressive spreading failure accompanied by an uphill growth of the shear band in sub aerial landslides driven by the removal of the downslope support. In the past few years a number of novel analytical, numerical and experimental tools have been developed and validated (Viesca and Rice 2012; Puzrin et al. 2015; Puzrin et al. 2016a,b; Germanovich et al. 2016; Zhang et al. 2015; Zhang et al. 2016), extending the SBP approach to account for actual non-linear slope geometries, different types of triggers (excess pore water pressures, seismic loading), dynamic propagation effects and retrogressive spreading above the failed slab in submarine landslides.

This paper takes the SBP approach further, towards a unified treatment of various geomorphological features, by exploring what happens at the bottom of the failed slab in the stable bottom zone of the slope (blue in Figure 1). Namely, what

makes some of the landslides to run out on top of the stable zone, while others to plough into it? Finding answer to this question is critical for determining the type of the landslide impact on the offshore structures.

The key to the solution of this problem is hidden in the observation that the ploughing has a direct analogy to the spreading (Puzrin et al. 2016a). Similar to spreading at the top of the slope, ploughing occurs at the bottom of the slope as a consequence of the slab failure, which is caused by the catastrophic (unstable) growth of the slip surface (Puzrin and Germanovich 2005). As described above, this slip surface nucleates in the unstable zone of the slope (red in Figure 1), propagates catastrophically (i.e., under existing external forces) through the quasi-stable part (orange in Figure 1) and arrests at the boundary between the quasi-stable and stable zones (blue in Figure 1). This arresting, however, does not necessarily define the final length of the slip surface. Indeed, as the slip surface grows downhill in the quasi-stable part, it causes additional loading of the sliding layer, which may result in lateral normal stresses approaching the passive earth pressure limit. This passive failure at the bottom of the slope (left side of the bottom diagram in Figure 1) may lead to the slab failure of the entire quasi-stable zone, followed by the heave h_p of the slab material at the start of the stable zone, which can drive the slip surface at the depth h_z further downhill into the stable zone.

Note that in contrast to the catastrophic growth of the slip surface in the quasi-stable zone, which takes place under constant external forces, the growth in the stable zone does require work of additional external forces. And the increasing heave of the seafloor caused by the slab failure above the stable zone is what provides this additional loading. (In the case of spreading, additional unloading is provided by the drop of the seafloor level caused by the slab failure below the stable zone, illustrated

in the right side of the bottom diagram in Figure 1). This additional, heave-generated loading fully controls downhill growth of the slip surface, determining both its stable (*progressive*) nature and its length.

The strategy for the ploughing/runout problem solution sought in this paper can be briefly described as follows. The passive failure block mechanism is driven by the rise of the seabed level due to an upslope slab failure. For this mechanism to materialize, the slip surface should have the possibility to cut through the entire length of the next passive failure block, before the previous block starts crumbling and running out over the top of the stable zone. The criterion for progressive growth of the slip surface will be developed using the energy balance approach, and then applied to the passive block mechanism. This will produce analytical ploughing failure criteria in terms of the critical rise in the seabed level necessary to propagate a shear band progressively downhill through the entire length of the passive failure block. If the minimum rise of the seabed surface level at which the ploughing can take place is larger than the maximum step in the seabed surface which the soil can maintain unsupported, the first passive failure block will start crumbling over the top of the stable zone causing the landslide to run out. The derived criteria will be then applied to a paleo-landslide complex in the Caspian Sea in an attempt to understand the observed geomorphological features.

UNSTABLE GROWTH OF SLIP SURFACES IN THE QUASI-STABLE ZONE

Before investigating the ploughing/runout problem, we briefly introduce the framework of the SBP approach to catastrophic (unstable) growth of slip surfaces in quasi-stable zone leading to the slab failures (e.g., Puzrin and Germanovich 2005; Puzrin et al. 2015). In this approach, the slip surface is assumed to be parallel to the

slope surface (Figure 1). Note, that the slip surfaces in the submarine landslides differ from conventional shear bands in that they can have a finite, problem specific thickness d and not just a thickness of a few soil grain diameters. This has two important implications. The first one is that the thickness of the slip surface cannot be defined from laboratory shear tests and has to be determined from geotechnical and geophysical site investigations of existing landslides. The second problem is whether the classical SBP approaches, which assume zero thickness of the shear band and use the relative displacement δ between the boundaries of the band as a state parameter, can also be applied to the slip surfaces of finite thickness. In fact, this can be easily resolved by taking $\delta = \gamma d$, where γ is the shear strain within the slip surface.

The shear behavior of the soil on the slip surface is strain softening with the undrained shear strength dropping from peak τ_p to residual τ_r over the slip weakening displacement δ_r (Figure 2). Both strength values are assumed to be proportional to effective stresses and can be derived, together with the slip weakening displacement δ_r , from laboratory ring shear tests or in-situ vane shear tests performed on the soil of the slip surface at displacement rates close to those expected for the shear band propagation. These tests can provide the entire strain softening curve in Figure 2, allowing for the critical energy dissipation parameter $\bar{\delta}$ to be calculated directly from the area below the experimental curve. Excess pore water pressures are assumed to cause a flow normal to the slope surface.

Seismic slope stability analysis based on the shear band propagation has been performed in Puzrin et al. (2015) using pseudo-static approach with degradation of the shear strength as a result of the cyclic loading. However, in contrast to a typical slab failure in the quasi-stable zone of the slope, caused by the slip surface propagating catastrophically at the velocity close to that of the P-wave, ploughing is caused by

progressive propagation of the slip surface below the failed slab. This propagation takes place within the zone, which according to the shear band propagation approach is considered to be stable, and is driven by the rise of the seabed due to the above slab failure. This is a significantly slower process, which most likely takes place after the action of earthquake loads ends. Therefore, the work presented in this paper focuses on the analysis of conditions for the progressive slip surface propagation and slope failure in the stable zone, without the earthquake loading and inertia forces. If, however, the slope failure takes place short time after the earthquake, the post-earthquake value of the shear strength degradation index δ_d is not likely to increase considerably compared to its earthquake value.

The critical component of the SBP approach to slope stability is mapping the slope into unstable, quasi-stable and stable zones. This is achieved by defining the shear stress ratio, which for post-earthquake conditions is given by

$$r = \frac{\tau_g - \tau_r}{\tau_p - \tau_r} = \frac{s \frac{\tau_g}{\tau_p} - 1}{s - 1} \quad [1]$$

where

$$s = \frac{\tau_p}{\tau_r} \quad [2]$$

is the ratio between the peak and residual shear strength, referred here as sensitivity,

$$\tau_g = \gamma' h_z \sin \alpha \cos \alpha \quad [3]$$

is the gravitational shear stress,

$$\tau_p = \delta_d k \gamma' (1 - r_u) h_z \cos^2 \alpha \quad [4]$$

is the peak undrained shear strength, where $k = 0.2 - 0.3$ for normally consolidated sediments, δ_d - the shear strength degradation index and $r_u = u_e / \gamma' h_z$ is the normalized excess pore water pressure at the depth h_z . If the excess pore water pressure grows linearly with depth, $r_u = \text{const}$. Equation [1] can be then rewritten as:

$$r = \frac{s\chi \tan \alpha - 1}{s - 1}, \text{ where } \chi = \frac{1}{\delta_d k (1 - r_u)} \quad [5]$$

Based on the shear stress ratio value, the slope can be divided into unstable ($1 \leq r$), quasi-stable ($1 < r \leq 0$) and stable ($r < 0$) zones (red, orange and blue, respectively, in Figure 1). For a curved slope, Puzrin et al. (2015) have demonstrated that the initial slip surface formed in the unstable zone ($r > 1$) will propagate catastrophically (i.e., under existing external forces) into the quasi-stable zone ($r > 0$) if its initial length L_i exceeds the critical value:

$$L_i \geq L_{cr} = \frac{1}{\bar{r}} \left(1 + \sqrt{\frac{E_l}{E_u}} \right) \sqrt{\frac{s\chi}{s-1} \frac{2E_u \bar{\delta}}{\gamma'}} \quad [6]$$

where

$$\bar{r} = \frac{s\chi \tan \bar{\alpha} - 1}{s - 1} \quad [7]$$

is the average shear stress ratio; $\bar{\alpha}$ is the average inclination of the slip surface;

$$\bar{\delta} = \frac{\int_0^{\delta_r} (\tau - \tau_r) d\delta}{\tau_p - \tau_r} \quad [8]$$

is the characteristic displacement; δ_r is the slip weakening displacement (Figure 2); E_l and E_u are deformation plane strain moduli in loading and unloading, respectively.

With the growth of the slip surface length L , the average gravitational shear stress ratio $\bar{r}$ in equation [7] decreases, so that the critical length L_{cr} in equation [6] increases and can, in principle, exceed the current length of the slip surface. Puzrin et al. (2015) have shown that if the slip surface starts propagating, it will always reach the boundaries of the quasi-stable zone, as long as the average shear stress ratio $\bar{r}$ remains positive (always true within the quasi-stable zone, where $r > 0$). This is, of course, assuming that excess pore water pressures and degradation index do not change during the slip surface growth.

In fact, the slip surface will grow even further, into the stable zone, where $r < 0$. By accounting for the dynamic effects of the slip surface growth, Germanovich et al. (2016) have shown that inertia and wave propagation could extend the final slip surface length beyond its static estimates, leading to the slip surface growth arrest criterion $\bar{r} = 0$. This is consistent with the upper bound static estimate [6], according to which, for $\bar{r} = 0$, the critical length for unstable slip surface growth becomes infinite. The important question, however, which the above analysis does not answer is whether the condition $\bar{r} = 0$ provides the ultimate limit of the slip surface growth and the corresponding landslide dimensions, or it is possible that after earthquake shaking has ceased, the shear band will propagate further into the downhill stable zone, potentially causing ploughing or runout.

PLOUGHING AND RUNOUT IN THE STABLE ZONE

Following Kvalstad et al. (2005), who proposed a block mechanism for the retrogressive spreading failure, in this work we propose a block mechanism for

ploughing into the stable zone with $r_2 < 0$ (Figure 3), driven by the accumulation of the material at the top of the semi-infinite slope α_2 . The source of this material is the slab failure of the upper steeper quasi-stable slope α_1 , with $r_1 > 0$. Because in rapid failure the undrained shearing of fine grained soils can be described by a Tresca type failure criterion, the limit analysis suggests that the failure planes should be inclined by 45° to a principal axis, assumed below to be parallel to the slope surface. In order to bring this passive failure mechanism into motion, the strength of soil at the bottom of the blocks is at its residual value, which is only possible if the slip surface propagates parallel to the slope and the length of this slip surface L_{sb} propagated into the intact layer of the original height h_z is larger than the length of the block L_2 (Figure 4):

$$L_{sb} \geq L_2 \quad [9]$$

If the inequality [9] becomes satisfied before the step in the seabed surface caused by the heave of the passive block 1 in Figure 3 reaches the maximum sustainable height, the block 2 will start moving parallel to the slope, causing the heave of the block 3 (Figure 4). This ploughing pattern will then repeat itself involving new passive failure blocks, as long as the failed slab keeps moving and provides material for the heave of the seabed surface (exact kinematics of the passive blocks in the failed part of the stable zone is beyond the scope of this work). If, however, the step caused by the heave of the passive block 1 in Figure 3 reaches the maximum sustainable free height before the inequality [9] is satisfied, block 1 will start crumbling on top of the stable slope α_2 , causing a runout.

It follows that inequality [9] is the key component of the criterion for the assessment of the probability of the ploughing. This criterion will be derived below,

after the second key component - the phenomenon of the progressive growth of slip surfaces - is investigated in the following section.

PROGRESSIVE GROWTH OF SLIP SURFACES IN THE STABLE ZONE

Progressive and catastrophic failure

While this issue is relatively well understood in Fracture Mechanics, in Soil Mechanics literature there is no clear distinction between catastrophic and progressive failure. Puzrin and Germanovich (2005) have demonstrated, that one of the advantages of the shear band propagation approach is that it allows for different types of failure to be clearly distinguished. In **progressive failure**, propagation of the shear band is stable in a sense that it requires work of additional external forces. In **catastrophic failure**, propagation of the shear band is unstable and takes place under existing external forces. In Puzrin and Germanovich (2005) study of the stability of an infinite slope, the shear band propagates catastrophically once its length exceeds the critical value.

In contrast, **ploughing** is an example of progressive failure, driven by the additional loading caused by the heave of the seafloor at the start of the stable zone due to the slab failure of the quasi-stable zone. This loading drives the progressive downhill growth of the slip surface into the stable zone (with $r < 0$). In this case, the length of the slip surface will be uniquely determined by the driving force at the start of the stable zone. Solution of this problem is presented below.

Problem formulation

Consider the problem of the progressive growth of the slip surface at the depth h_z parallel to the slope surface (Figure 5). The normal horizontal stress σ_x will be taken as an average value of this stress over the thickness h_z of the layer:

$$\sigma_x(x) = \frac{1}{h_z} \int_0^{h_z} \sigma_x(x, z) dz \quad [10]$$

Let us assume that in an intact slope this average normal stress would be equal to $\sigma_x = p_{x0}$.

In the further derivations we are going to adopt for the sliding layer the beam approximation. It follows that the principal axes can be taken parallel to the slope, so that

$$\sigma_x(x) = \sigma_\alpha(x) \cos^2 \alpha_2 \quad p_{x0} = p_0 \cos^2 \alpha_2 \quad [11]$$

where $\sigma_\alpha(x)$ is the principal average stress acting on a plane perpendicular to the slope surface subjecting a soil element to uniaxial compression; p_0 is the initial value of this stress. This assumption has been shown to provide realistic results for the problems of shallow discontinuities ($L \gg h$) propagating parallel to the surface of a mildly sloped half space (e.g., Palmer and Rice 1973; Zlatin and Khrapkov 1986; Viesca and Rice 2012), because in the sliding body the work of the sub-horizontal shear stresses on the corresponding shear strains is small compared to that of the sub-horizontal normal stresses on the linear strains.

At the bottom of the quasi-stable slab, soil failed in the passive mode and the seafloor experienced the heave h_p . Therefore, the horizontal earth pressure acting over the height $h_z + h_p$ is the horizontal component p_{xp} of the passive pressure p_p ,

which is the principal average stress acting on the plane perpendicular to the slope surface (beam approximation):

$$p_{xp} = p_p \cos^2 \alpha_2 \quad [12]$$

This pressure serves as the driving force for the propagation of the slip surface into the stable zone ($r_2 < 0$), where the residual shear resistance τ_r is higher than the gravitational shear stress τ_g . This propagation will cause the normal stresses in the stable zone to increase above their initial value:

$$\sigma_x(x) = p_{xp} r_p - (\tau_r - \tau_g) \frac{x}{h_z} \quad [13]$$

where

$$\tau_g = \gamma' h_z \sin \alpha_2 \cos \alpha_2; \quad r_p = \frac{h_z + h_p}{h_z} \quad [14]$$

is the gravitational shear stress, while in the far field along the slope, the average stress asymptotically returns to the initial value $\sigma_x = p_{x0}$. We shall be looking for the maximum distance L , which the shear band can propagate into the stable zone for the given value of r_p .

Energy balance approach - assumptions

The elasto-plastic relationship between the average horizontal normal stress σ_x in the layer along the shear band and the average horizontal linear strain ϵ_x is given by the linear elastic - perfectly plastic model:

$$\sigma_x = E_l \epsilon_x + p_{x0}, \quad \text{for } p_{x0} \leq \sigma_x \leq p_{xp} \quad [15]$$

where p_{xp} is the average value of passive lateral pressure in the layer, representing the yield stress, while E_l is the plane strain modulus in loading, which can be

calculated from the stress-strain behavior of the soil samples from the stable zone, subjected to undrained triaxial compression tests with confining stress kept equal to the overburden pressure. Fitting the resulting stress-strain curves with a linear-elastic perfectly plastic relationship, will provide the values of triaxial strength and stiffness, which can then be converted into the plane strain values of p_{xp} and E_L .

In this section we derive the propagation criterion for the slip surface in the problem in Figure 5 directly from the energy balance, following Palmer and Rice (1973). The main assumptions made in this analysis are that under condition of $L \gg h$, and in the absence of dilation in the shear band, most of the energy transfer during slip surface propagation will take place due to:

- external work made by gravitational forces on downslope movements of the layer above the slip surface;
- internal work made by the normal stress acting parallel to the slope surface on deformations of the layer caused by changes in these stresses;
- plastic work dissipated on the slip surface.

We shall consider the case, where the end zone of the slip surface is small compared to the thickness of the sliding layer. Displacements below the slip surface are neglected, therefore average displacements $u_x(x)$ in the soil layer parallel to the slope surface will be equal to the relative slide δ on the slip surface and given by integration of the average strain ϵ_x :

$$\delta(x) = \frac{u_x(x)}{\cos\alpha_2} = \frac{1}{\cos\alpha_2} \int_x^L \epsilon_x(x) dx \quad [16]$$

so that at the tip of the slip surface $x = L$ the relative slide is zero.

The energy balance criterion for the slip surface propagation investigates the changes produced in the body by slip surface propagating by increment ΔL (Figure 5). As a result of this incremental propagation the sliding layer experiences deformation at the tip of the slip surface leading to the displacement parallel to the slope:

$$\Delta\delta = \varepsilon_\alpha(L)\Delta l \quad [17]$$

where $\varepsilon_\alpha(x)$ is the principal average strain acting on a plane perpendicular to the slope surface (beam approximation):

$$\varepsilon_\alpha(x) = \frac{\varepsilon_x(x)}{\cos^2\alpha_2}, \quad \text{for } \tan^2\alpha \ll 1 \quad [18]$$

The incremental length of the slip surface:

$$\Delta l = \frac{\Delta L}{\cos\alpha_2} \quad [19]$$

Substituting equations [18] and [19] into [17] gives

$$\Delta\delta = \frac{\Delta u}{\cos^3\alpha_2} \quad [20]$$

where

$$\Delta u = \varepsilon_x(L)\Delta L \quad [21]$$

Horizontal and vertical components of the downslope displacement due to the incremental slip surface propagation are obtained using equation [16]:

$$u_x = \Delta\delta \cos\alpha_2 = \frac{\Delta u}{\cos^2\alpha_2} \quad u_z = \Delta\delta \sin\alpha_2 = \frac{\Delta u}{\cos^2\alpha_2} \tan\alpha_2 \quad [22]$$

Energy balance calculations

Having made all the necessary assumptions we may proceed to derivation of the energy balance criterion for the slip surface growth. This criterion requires that the energy surplus produced in the body by incremental propagation of the slip surface ΔL should exceed the work required for this incremental propagation. Mathematically this can be expressed as the following inequality:

$$\Delta W_e - \Delta W_i - \Delta D_L \geq \Delta D_\omega \quad [23]$$

where

$$\begin{aligned} \Delta W_e &= \gamma' h_z L u_z + p_p (h_z + h_p) \cos\alpha_2 \Delta\delta \\ &= \left(\tau_g L + p_{xp} (h_z + h_p) \right) \frac{\Delta u}{\cos^4\alpha_2} \end{aligned} \quad [24]$$

is the increment of the external work made in our case by gravitational forces and passive pressure on downslope movements of the layer above the slip surface;

$$\Delta W_i = h_z \Delta L \int_0^{\varepsilon_\alpha(L)} \sigma_\alpha(\varepsilon_\alpha) d\varepsilon_\alpha = \frac{h_z \Delta L}{\cos^4\alpha_2} \int_0^{\varepsilon_x(L)} \sigma_x(\varepsilon_x) d\varepsilon_x \quad [25]$$

is the increment of the internal work made by the normal stress acting parallel to the slope surface on deformations of the layer caused by changes in these stresses;

$$\Delta D_L = \tau_r l \Delta \delta = \tau_r L \frac{\Delta u}{\cos^4 \alpha_2} \quad [26]$$

is the increment of the plastic work dissipated on the slip surface, which is required to overcome the residual shear resistance along the slip surface;

$$\Delta D_\omega \approx \Delta l \int_0^{\delta_r} (\tau(\delta) - \tau_r) d\delta = \frac{\Delta L}{\cos \alpha_2} (\tau_p - \tau_r) \bar{\delta} \quad [27]$$

is the increment of the plastic work dissipated in the slip surface during its propagation, which is required to overcome the shear resistance in excess of residual in the end zones of the slip surface.

Expressions [24]-[27] after being substituted into inequality [23] yield the sufficient condition for the slip surface propagation for a general form of constitutive law:

$$\begin{aligned} & \left((\tau_g - \tau_r)L + p_{xp}(h_z + h_p) \right) \frac{\varepsilon_x(L)\Delta L}{\cos^4 \alpha_2} - \frac{h_z \Delta L}{\cos^4 \alpha_2} \int_0^{\varepsilon_x(L)} \sigma_x(\varepsilon_x) d\varepsilon_x \quad [28] \\ & \geq \frac{\Delta L}{\cos \alpha_2} (\tau_p - \tau_r) \bar{\delta} \end{aligned}$$

Energy balance criterion for the progressive slip surface propagation

Noting that the first term in brackets on the left side of the inequality [28] is proportional to the right side of the equilibrium equation [13], this term can be expressed via the normal stress $\sigma_x(L)$, and, therefore, by using the constitutive law

[15], via the linear strain $\varepsilon_x(L)$. Next, substituting the constitutive law [15] into the integral on the left side of the inequality [28] and integrating, the integral can also be expressed via the strain $\varepsilon_x(L)$. After adding resulting terms and dividing both sides by $\frac{\Delta L}{\cos \alpha_2}$, inequality [28] reduces to

$$\frac{E_l h_z}{2} (\varepsilon_x(L))^2 \geq (\tau_p - \tau_r) \bar{\delta} \quad [29]$$

and can be further transformed into

$$L \leq L_{sb} = \frac{p_0 - p_p r_p}{\tau_p - \tau_r} \frac{h_z}{r_2} \cos^2 \alpha_2 + \frac{1}{r_2} \sqrt{\frac{2E_l h_z \bar{\delta} \cos^3 \alpha_2}{\tau_p - \tau_r}}; \quad r_p = \frac{h_z + h_p}{h_z} \quad [30]$$

where the shear stress ratio

$$r_2 = \frac{\tau_g - \tau_r}{\tau_p - \tau_r} = \frac{s\chi \tan \alpha_2 - 1}{s - 1} < 0; \quad \chi = \frac{1}{\delta_d k(1 - r_u)} \quad [31]$$

for post-earthquake conditions.

Inequality [30] provides an estimate for the maximum length L_{sb} , which the progressively propagating slip surface can reach for a given value of the height ratio r_p . If this length satisfies inequality [9]: $L_{sb} \geq L_2$, progressive slip surface growth will lead to the ploughing, which can take place even if fin horizontal slopes with $\alpha_2 = 0$.

PLOUGHING AND RUNOUT CRITERIA

Estimate of the progressive slip surface length

Equation [30] uses values of initial and passive effective earth pressures, acting parallel to the slope surface, which are assumed to be principal stresses following the beam approximation. Initial effective earth pressure (average over depth h_z) can be expressed via the normal effective stress σ'_n acting at the depth $z = h_z$ perpendicular to the sliding surface:

$$p_0 = K_0 \frac{\sigma'_n(z = h_z)}{2} = \frac{1}{2k} K_0 \tau_p(z = h_z) \quad [32]$$

where from equation [4]:

$$\sigma'_n(z = h_z) = \gamma' \delta_a (1 - r_u) h_z \cos^2 \alpha_2 = \frac{\tau_p(z = h_z)}{k} \quad [33]$$

and the K_0 is the at rest earth pressure coefficient parallel to the slope.

The passive effective pressure at failure can also be estimated as a function of the normal effective stress σ'_n acting perpendicular to the sliding surface. From equations [4], [33] and the Tresca's failure criterion $p_p = \sigma_z + 2\tau_p$ it follows that the average passive pressure over depth $(h_z + h_p)$ can also be expressed via $\tau_p(z = h_z)$:

$$\begin{aligned} p_p &= \frac{1}{2} \sigma'_n(z = [h_z + h_p]) + \tau_p(z = [h_z + h_p]) \\ &= \tau_p(z = h_z) r_p \left(\frac{1}{2k} + 1 \right) \end{aligned} \quad [34]$$

where $r_p = \frac{h_z + h_p}{h_z}$.

Substituting expressions [32] and [34] into [30] we obtain the estimate for the maximum length L_{sb} :

$$L_{sb} = \frac{1/k [K_0 - r_p^2] - 2r_p^2}{1 - 1/s} \frac{h_z}{2r_2} \cos^2 \alpha_2 + \frac{1}{r_2} \sqrt{\frac{sk_E h_z \bar{\delta} \cos^3 \alpha_2}{s - 1}} \quad [35]$$

where

$$k_E = \frac{2E_l}{\tau_p}; \quad r_p = \frac{h_z + h_p}{h_z}; \quad r_2 = \frac{s\chi \tan \alpha_2 - 1}{s - 1} \quad [36]$$

Estimate of the length of the passive ploughing block

The length of the passive failure block L_2 depends on the kinematics of the failure mechanism. From the observations it seems that

$$L_2 \leq 2h_z \cos^2 \alpha_2 \quad [37]$$

is a reasonable first approximation for the length of the passive failure block, consistent with the limit analysis using the Tresca type failure criterion, which suggests that the failure planes should be inclined by 45° to principal axes.

Critical depth of the ploughing failure surface

The criterion for the ploughing can be formulated in terms of the critical depth of the failure surface h_{cr} at which the failure can take place. Indeed, substituting expressions [35]-[37] into [9] and resolving the resulting inequality with respect to depth h_z gives the two necessary conditions for the ploughing to occur:

$$\begin{cases} a_p > 0 \\ h_z \geq h_{cr} = \frac{sk_E \bar{\delta}}{(s-1)a_p^2 \cos \alpha_2} \end{cases} \quad [38]$$

where

$$a_p = 2r_2 - \frac{1/k [K_0 - r_p^2] - 2r_p^2}{2(1 - 1/s)} \quad [39]$$

If either one of the conditions [38] is not satisfied, ploughing is impossible.

Critical heave in the seabed level

An alternative way to formulate the criterion for the ploughing is in terms of the minimum rise of the seabed surface level h_{pcr} at which the failure surface can develop at the depth h_z . Indeed, substituting expressions [35]-[37] into [9] and resolving the resulting inequality with respect to the rise h_p in the seabed surface gives the following necessary conditions for the ploughing:

$$b_p < 1, \text{ or } \begin{cases} b_p \geq 1 \\ h_p \geq h_{pcr} = h_z (\sqrt{b_p} - 1) \end{cases} \quad [40]$$

where

$$b_p = \frac{-4(1 - 1/s)r_2 + K_0/k + 2\sqrt{(s-1)k_E \bar{\delta}/(sh_z \cos \alpha_2)}}{1/k + 2} \quad [41]$$

is the progressive propagation parameter. If either one of the conditions in the [40] is not satisfied, ploughing is impossible.

Conditions for the runout

If the minimum heave of the seabed surface level h_{pcr} at which the failure surface can develop at the depth h_z is larger than the maximum height of the step in the seafloor h_{max} , that can stand free (unsupported), the ploughing will not take place and the sliding layer will start crumbling on top of the stable zone, resulting in a runout. The two necessary conditions for the runout follow from equations [40]:

$$\begin{cases} b_p \geq 1 \\ h_{pcr} = h_z (\sqrt{b_p} - 1) \geq k_2 h_z \end{cases} \quad [42]$$

where

$$k_2 = h_{max}/h_z \quad [43]$$

with the maximum possible free standing step in the seabed surface estimated as

$$h_{max} \approx 4\tau_p^{av}/\gamma' \quad [44]$$

using the average undrained peak shear strength in the heaved soil layer τ_p^{av} .

For the peak shear strength increasing linearly with depth:

$$\tau_p(z) = \tau_{p0} + \tau_{pk}z \quad [45]$$

the maximum possible free standing step h_{max} in the seabed surface can be estimated from condition of the equilibrium of the passive failure block:

$$\frac{1}{2}\gamma' h_{max}^2 \frac{\sqrt{2}}{2} = \int_0^{h_{max}} (\tau_{p0} + \tau_{pk}z) d\sqrt{2}z \quad [46]$$

as

$$h_{max} = \frac{4\tau_{p0}}{\gamma' - 2\tau_{pk}} \quad [47]$$

This results in the following runout condition:

$$b_p = \frac{-4(1 - 1/s)r_2 + K_0/k + 2\sqrt{(s-1)k_E \bar{\delta}/(sh_z \cos \alpha_2)}}{1/k + 2} \quad [48]$$

$$\geq \left(1 + \frac{4\tau_{p0}}{\gamma' h_z - 2\tau_{pk} h_z}\right)^2$$

EXAMPLE

This section provides an example of application of the ploughing/runout criteria to a shallow paleo-landslide from the Azeri-Chirag-Gunashli (ACG) field complex in the Azerbaijani sector of the south Caspian Sea, 130km to the south-east of Baku in water depths of 95 to 435m. Throughout the Pleistocene, mass transport events have repeatedly occurred producing significant seabed topography visible today (Figure 6). The most recent sediments comprise soft clay draped over an older ‘mega’ landslide

(MM700). A complex of landslides that occurred about 5ka BP within this drape (MM900) has been described by Gray et al. (2015). At the top of the failure scarp, MM900 does not exhibit any traces of the spreading failure (Figure 6), while at the bottom of the failure scarp there is a clearly defined zone of ploughing and runout. The lack of the spreading failure in the MM900 cross-section in Figure 6 has been explained by Puzrin et al. (2016b) using SBP spreading failure criteria with parameters from Table 1 given in Gray et al. (2015). The maximum thickness of the sediments involved in the MM900 mass movement was $h_z = 25$ m. It has been shown that, on one hand, for the spreading failure not to occur for MM900 slab failures with $h_z \leq 25$ m and, on the other hand, for it to occur in the same cross-section in the much deeper and older landslide MM700, the characteristic displacement has to stay within a rather narrow interval of $\bar{\delta} = 0.10 - 0.16$ m.

It would be interesting to explore, whether the proposed in this paper ploughing and runout criteria would be capable to confirm the observed presence of both of these phenomena for the same set of parameters, which allowed for explaining the lack of spreading. Substituting soil properties from Table 1 together with geometric parameters from Table 2 (after Gray et al. 2015) into expressions [31], [40] and [41] provides confirmation that after an earthquake the bottom of the MM900 failure scarp in Figure 6 is indeed stable ($r_2 = -0.61 < 0$), and that $b_p \geq 1$ for the whole range of parameters. This justifies the use of the above ploughing criteria to calculate the critical heave of the seabed $h_{p_{cr}}$ for the limiting values of $\bar{\delta} = 0.10$ m and $\bar{\delta} = 0.16$ m for the whole range of MM900 slab thicknesses $0 < h_z \leq 25$ m (Figure 7).

As is seen in Figure 7, the required heave of the seafloor to drive progressive growth of the slip surface and cause ploughing is not particularly large and reaches the maximum values of 0.68 m and 1.09 m, for $\bar{\delta} = 0.10$ m and $\bar{\delta} = 0.16$ m, respectively. If the soil could sustain this height free standing, the ploughing would be the only observed mode of the post failure evolution at the bottom of the slab failure. However, apparently, the normally to lightly over-consolidated fine-grained soils in the top meter of the drape are not capable of sustaining such a height, and start crumbling on the top of the stable zone when this height is exceeded, not being able to propagate the slip surface through the whole length of the passive failure block. In such a case, the runout becomes the dominant failure mode. For example, if the maximum sustainable height were $h_{max} = 1.0$ m, from Figure 7 it follows that in the case of $\bar{\delta} = 0.16$ m, the slab failures with depths of $6.0 \text{ m} < h_z < 17.4 \text{ m}$ could cause a runout. Outside of this range of the slab thickness, ploughing is still possible, thus confirming the observed presence of both phenomena in the MM900 event.

SUMMARY AND CONCLUSIONS

The paper extends the shear band propagation approach, previously applied to slab and spreading failures, to investigation of additional post-failure evolution modes in submarine landslides, such as ploughing and runout. By exploring what happens at the bottom of the failed slab after the catastrophic propagation of the slip surface has arrested in the stable zone of the slope, it provides simple energy balance criteria explaining why some of the landslides tend to runout on top of the stable zone, while others plough into it. Ability to distinguish between these two types of the post-failure landslide evolution is critical for determining the tsunami hazard and the type of the landslide impact on the offshore structures.

The proposed analysis is based on the direct analogy between the ploughing and the spreading failures, for which the SBP approach has been already successfully applied. Indeed, similar to spreading, ploughing occurs after arrest of the *catastrophic* propagation of the sub-horizontal slip surface in the stable zone. In contrast to spreading, however, ploughing results not from the active, but rather from the passive failure in the sliding layer, downhill from the quasi-stable zone. Further downhill propagation of the slip surface into the stable zone is only possible if the slab material accumulates at the top of the stable slope. This propagation will be *progressive*, with the length of the propagated slip surface controlled by the height of the accumulated material. At the same time, this increase in pressures will cause development of inclined shear bands forming the passive failure block mechanism in the sliding body. However, for this mechanism to get activated, the sub-horizontal slip surface should have the possibility to cut through the entire length of the next passive failure block, before the previous block starts crumbling and running out over the top of the stable zone.

The study uses the energy balance approach to develop the criterion for progressive growth of the slip surface, which is then combined with the passive block mechanism to produce analytical criteria for ploughing. The criteria are formulated in terms of the critical heave in the seabed level, which is necessary to propagate the slip surface progressively downhill through the entire length of the passive failure block. If the minimum heave of the seabed surface level, at which the ploughing can take place, is larger than the maximum step in the seabed surface, which the soil can maintain unsupported, the first passive failure block will start crumbling over the top of the stable zone causing the landslide to runout.

Application of the derived criteria to analysis of observed geomorphological features is demonstrated using example of the MM900 paleo-landslide complex in the Caspian Sea. This complex has an interesting feature: while no traces of spreading failure could be found at above the failed slabs, below them both ploughing and runout could be observed. The proposed ploughing/runout criteria allowed for these observations to be reconciled.

Needless to say that in spite of their mechanical consistency and successful application to the particular example, the proposed analytical criteria represent a significant simplification of the problem and have to be further validated and refined using sophisticated numerical and experimental tools. The main purpose of this paper has been to attempt the next step towards a unified treatment of various observed geomorphological features within the SBP approach to slope stability, with an ultimate goal of developing a consistent mechanical framework for simulating and better understanding of the entire process of dynamical submarine landslide evolution.

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LIST OF SYMBOLS

b_p progressive propagation parameter

ΔD_L increment of the plastic work dissipated on the shear band

ΔD_ω increment of the plastic work dissipated in the end zone shear band

E_l	plane strain deformation modulus in loading
E_u	plane strain deformation modulus in unloading
h_z	depth of the slip surface
h_{cr}	critical depth of the failure surface at which the sliding can take place
h_p	heave of the seafloor level
h_{pcr}	critical heave of the seafloor level
k	undrained shear strength coefficient
k_E	normalized plane strain deformation modulus in loading
K_0	earth pressure coefficient at rest
L	length of the slip surface
L_i	initial length of the slip surface
L_{cr}	critical length of the slip surface
L_{sb}	the maximum length to which the slip surface can grow for a given heave in the seafloor surface
L_2	the length of the passive failure block
ΔL	the increment of the length of the slip surface
p_p	average passive failure pressure in the sliding layer
p_0	average initial lateral pressure in the sliding layer
p_{x0}	horizontal component of the average initial lateral pressure
r	shear stress ratio
r_2	shear stress ratio of the downslope stable zone
r_p	normalized heave of the seafloor level
r_u	normalized excess pore water pressure
$\bar{r}$	average shear stress ratio
s	sensitivity ratio between peak and residual shear strength

u_e	excess pore water pressure
ΔW_e	increment of the external work
ΔW_i	increment of the internal work
α	slope angle
α_2	slope angle of the downslope stable zone
γ	shear strain within the slip surface
γ'	submerged unit weight of the soil
δ	relative displacement between the boundaries of the slip surface
δ_r	slip weakening length / displacement
$\bar{\delta}$	characteristic displacement
δ_d	seismic degradation index
$\Delta\delta$	increment of the relative displacement due to incremental slip surface propagation
ε_x	average horizontal strain in the sliding layer
ε_α	average principal strain in the sliding layer
σ_x	average horizontal normal stress in the sliding layer
σ_α	average principal normal stress in the sliding layer
σ'_n	normal effective stress on the sliding surface
τ_g	gravitational shear stress
τ_p	undrained peak shear resistance on the slip surface
τ_r	undrained residual shear resistance on the slip surface
χ	coefficient defined in equation [5]

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FIGURE CAPTIONS

- Figure 1. Dynamic evolution of submarine landslides.
- Figure 2. Strain softening post-failure behavior on the slip surface.
- Figure 3. Passive failure and heave driving the slip surface into the stable zone.
- Figure 4. Activation of the passive block mechanism in ploughing.
- Figure 5. Progressive growth of the slip surface: problem formulation.
- Figure 6. A section of the Caspian Sea Azeri basin, after Gray et al. (2015), published with permission of OTC: (a) Bathymetry of the studied seafloor section and its location (inset in the top right corner) within the ACG oil field; (b) 3DHR (top) and chirp sub-bottom profiler (bottom) seismic sections of the MM900 landslide.
- Figure 7. Ploughing/runout criteria for the MM900 landslide as a function of its thickness.

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Draft

Table 1. Parameters of the MM900 sediments (after Gray et al. 2015)

Parameter	k_E	γ'	s	K_0	k	χ	δ_d	r_u	$\bar{\delta}$
Units	-	kN/m ³	-	-	-	-	-	-	m
Value	254	7.5	2.4	0.8	0.21	4.76	1.0	0	0.10-0.16

Table 2. Parameters of the stable zone of the MM900 landslides

Parameter	h_z	α_2	r_2
Units	m	(°)	-
Value	< 25	0.731	-0.61