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Complete List of Authors:	Mesri, Gholamreza; University of Illinois at Urbana-Champaign, Dept. of Civil and Environmental Engineering Kane, Thierno; University of Illinois at Urbana-Champaign, Civil and Environmental Engineering
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9 Discussion by

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11 Gholamreza Mesri¹ and Thierno Kane²

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13 The authors have done a valuable service illustrating the uncertainties associated with
14 defining effective normal stress for unsaturated soils (Konrad and Lebeau 2015). The difficulties
15 associated with definition of effective normal stress for unsaturated soils, however, has been only
16 part of the reason for shelving an expression for shear strength of unsaturated soils in terms of
17 effective normal stress. The other, more significant reason is the fundamental difference
18 between total normal stress and matric suction.

19 In soils, frictional shear strength results from interparticle shearing resistance through
20 interparticle contact. The increase in interparticle contact in response to increase in total normal
21 stress pushing particles together, is only limited by fabric changes that accompany normal stress
22 increase, through particle damage in granular soils and through decrease in edge-face interaction
23 in clay soils. In other words, there is a more or less direct relationship between shear strength
24 and total normal stress in unsaturated soils, expressed by means of a friction angle ϕ' (e.g., Fig.
25 19.9, Terzaghi et al. 1996).

26 On the other hand, there is a definite limit on the ability of matric suction pulling soil particles
27 together, and increasing interparticle contact and shearing resistance. In other words, the

¹ Ralph B. Peck Professor of Civil Engineering, University of Illinois at Urbana-Champaign, Urbana, IL 61801
E-mail for correspondence: gmesri@illinois.edu

² Graduate Student, University of Illinois at Urbana-Champaign, Urbana, IL 61801

relationship between shear strength of unsaturated soils and matric suction is highly non-linear and does reach a limit as suction increases (e.g., Fig. 19.10, Terzaghi et al. 1996).

The authors subject matric suction u_s to a reduction factor $S_{r,c}$; however, by employing an effective normal stress approach, they propose a linear relationship between shear strength and capillary stress, $S_{r,c} u_s$. Although the shear strength Eq. 1

$$(D1) \quad s = c' + \bar{\sigma}_n \tan \phi' + \chi u_s \tan \phi' ,$$

shear strength Eq. 7

$$(D2) \quad s = c' + \bar{\sigma}_n \tan \phi' + \beta u_s \tan \phi' ,$$

and shear strength Eq. 20

$$(D3) \quad s = c' + \bar{\sigma}_n \tan \phi' + S_{r,c} u_s \tan \phi'$$

where $\bar{\sigma}_n = \sigma_n - u_a$, u_s = matric suction, and u_a = pore air pressure, have been presented by the authors under, respectively, “effective stress formulation”, “independent stress formulation”, and “shear strength” formulation by the authors, the three formulations are mathematically similar, except that the Bishop χ factor in Eq. (D1) has been replaced by “effective stress resistance parameter” β in Eq. (D2), and by “degree of saturation ascribed to capillary forces” $S_{r,c}$ in Eq. (D3). It has been known that χ , β , and $S_{r,c}$ may depend on magnitude of suction and even total normal stress. Thus, the three expressions for shear strength are based on definitions of effective normal stress for unsaturated soils, respectively, as

$$(D4) \quad \bar{\sigma}'_n = \bar{\sigma}_n + \chi u_s$$

$$(D5) \quad \bar{\sigma}'_n = \bar{\sigma}_n + \beta u_s$$

$$(D6) \quad \bar{\sigma}'_n = \bar{\sigma}_n + S_{r,c} u_s$$

and shear strength of unsaturated soils

$$(D7) \quad s = c' + \bar{\sigma}'_n \tan \phi'$$

The alternative expression for shear strength of unsaturated soils separates the contribution of pushing soil particles together by total normal stress and contribution of pulling soil particles together by matric suction

$$(D8) \quad s = c' + \bar{\sigma}'_n \tan \phi' + b u_s^a$$

The existing shear strength data for unsaturated soils with suction control suggest a value of a near 0.5. Terzaghi et al. (1996) summarizes data on values of b; together with additional data this is shown in Fig. 1. Note that for any soil the value of b appears to depend on magnitude of suction and also total normal stress. Nevertheless, Eq. (D8) for shear strength of unsaturated soils may be preferred because it considers separately the contribution of total normal stress and suction, and together with mean values of empirical data on b provides a simple means of estimating shear strength of unsaturated soils.

Because the application of Eq. (D3) to compute shear strength of an unsaturated soil requires the water retention data for the soil, and we could not identify an empirical correlation for $S_{r,c}$ in terms of soil composition and structure, we decided to compare shear strength by Eqs. (D3) and (D8) for the soils and example total normal stress and matric suction conditions considered by the authors. Table D1 shows that there is reasonably good agreement between values of shear strength according to Eqs. (D3) and (D8). We are aware that Eq. (D8) may overestimate shear strength for values of suction less than

$$(D9) \quad u_{ss} = \left(\frac{b}{\tan \phi'} \right)^2$$

For values u_s less than u_{ss}

$$(D10) \quad s = c' + \bar{\sigma} \tan \phi' + u_s \tan \phi'$$

Note that for (b, ϕ') combinations on Fig. D1 from $(2, 18^\circ)$ to $(8, 40^\circ)$, the values of u_{ss} are in the range of 38 kPa to 91 kPa; less than one atmosphere at which cavitation in a saturated soil may take place. For Brown sand, with a suction equal to 8.8 kPa, Eq. (D10) results in shear strength equal to 15 kPa which is close to the 13 kPa according to Eq. (D3). The authors may offer explanation in terms of $S_{r,c}$ for the difference in shear strength by Eqs. (D3) and (D8) for Madrid clayey sand and reconstituted silty clay.

The authors' concept of $S_{r,c}$ appears to be based on the assumption that drying of soils involving the removal of adsorbed water does not contribute to capillary forces and thus interparticle shearing resistance. According to the authors for Madrid clayey sand beyond suction of 761 kPa water is held by adsorptive forces. Yet Fig. 5 of Escario et al. (1989) shows that shear strength continues to increase beyond 761 kPa. An alternative explanation of shear strength-matric suction relationship for unsaturated soils is that as a soil dries and capillary force increases, its fraction normal to interparticle contact decreases.

In summary, Eqs. (D8) and (D10), together with mean empirical correlation for b in terms of ϕ' in Fig. D1, provide a simple means of calculating shear strength for unsaturated soils. This approach does not require water retention data for the soil.

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Table D1. Shear Strength of Unsaturated Soils Based on Eqs. (D3) and (D8).

Soils	$\bar{\sigma}_n$ ¹ (kPa)	u_s (kPa)	ϕ' (deg)	Equation (D3) ²		Equation (D8) ³	
				$S_{r,c}$	s (kPa)	b	s (kPa)
Indian Head Glacial Till	25	500	23	0.46	108	4.0	104
Madrid Clayey Sand	600	1175	39/41	0.13	654	8.0	805
Brown Sand	13.7	8.8	34	0.63	13	7.5	31.5
Indian Head Glacial Till	75	300	23	0.66	115	4.0	105
Reconstituted Silty Clay	870	851	32	0.70	916	7.0	748

¹ $\bar{\sigma}_n = \sigma_n - u_a$

² In Eq. (D3), $c' = 0$ is used by the authors for all soils, and for Madrid clayey sand $\phi' = 41^\circ$ is used by the authors, which is the average of the slopes from s vs. $\bar{\sigma}_n$ for six values of suction.

³ In Eq. (D8), $c' = 4.0$ kPa from Vanapalli et al. (1996) is used for Indian Head Glacial Till; and $c' = 45$ kPa and $\phi' = 39^\circ$ from Escario et al. (1989) is used for Madrid clayey sand, which is the friction angle of saturated soil.

Fig. D1. Values of parameter b from shear strength measurements with suction control on undisturbed and on compacted unsaturated soils (Modified after Terzaghi et al. 1996).