Design of Ultra-Wideband Reflectors

by

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Abstract
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It is well-known that reflectarrays typically have a narrow bandwidth which is commonly attributed to the resonant nature of the antenna elements used and the narrow bandwidth of the phase shifting networks used. Typical approaches to increase the bandwidth include the use of true-time-delay (TTD) devices, coupling multiple resonances together, stacking multiple layer of scatterers or the use of numerical optimization techniques. While these approaches have been shown to improve the bandwidth of reflectarrays, the upper bound remains at approximately 40% in fractional bandwidth. This thesis investigates how ultra-wideband reflectors can be designed using novel approaches going beyond the conventional approaches listed above. Specifically, we investigate two new methods to design reflectarrays. In the first method, the reflector is designed as a Transformation Optics (TO) device and in the second method, the reflector is designed as a metasurface. The TO method amounts to manipulating of electromagnetic waves using an appropriate set of material parameters such that the wave propagation inside the material behaves in a desired manner. We use TO as a means to design an ultra-wideband reflector and draw insights into the practical considerations associated with designing such a reflector. In the second method, we design a reflectarray as a metasurface. Traditionally, reflectarrays are considered to be an array of individual antenna elements with no surface properties typically defined. A metasurface often has homogenized surface properties such as surface impedance and admittance which are realized by sub-wavelength elements that make up the reflector. A novel method to design ultra-wideband reflector using a metasurface is derived from first principles and it is shown that the designed reflector exhibits excellent characteristics over a very wide band of frequencies. The advantages and disadvantages of the reflectors designed using each of these two methods are discussed. A comparison of our metasurface is made to a state-of-the-art wideband reflectarray.
Dedication

To my parents and Jessica,
for all their love and support
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## Contents

1 Introduction .................................................. 1
   1.1 Motivation and Thesis Goals .......................... 2
   1.2 Reflectarray as a Transformation Optics Device ...... 3
   1.3 Reflectarray as a Huygens' Surface ................. 4
   1.4 Thesis Outline ........................................... 5

2 Reflector Antenna and Reflectarray Background ......... 7
   2.1 Reflector Antenna Theory of Operation ............... 7
   2.2 Reflectarray Theory of Operation .................... 9
   2.3 Reflectarray Literature Review ....................... 11
      2.3.1 Phase Shift Mechanisms ......................... 12
      2.3.2 Bandwidth ........................................ 13
      2.3.3 Tunability ....................................... 16
      2.3.4 Passivity ........................................ 16
      2.3.5 Polarization ...................................... 16
   2.4 Summary ................................................. 17

3 Design of a Wideband Reflector Using Transformation Optics 18
   3.1 Transformation Optics Background ...................... 18
      3.1.1 Coordinate Transformations ...................... 21
   3.2 Transformation Optics in 3-D ............................ 22
   3.3 Transformation Optics in 2-D ............................ 23
      3.3.1 Conformal Coordinate Transformation .......... 25
      3.3.2 Graded-Index Approach ............................ 27
   3.4 Generation of 2-D Conformal Coordinate Transformation 27
      3.4.1 Schwarz-Christoffel Transformation .............. 29
   3.5 Reflector Design Using Schwarz-Christoffel Transformation 31
   3.6 Reflector Design Considerations and Permittivity Profile 32
   3.7 The Induced Relative Permittivity Profile ............ 37
   3.8 Simulated Performance of the TO Reflector .......... 37
      3.8.1 Simulation Setup .................................. 37
      3.8.2 Angle Scanning Performance ..................... 38
      3.8.3 Effect of Perturbing the Relative Permittivity Profile 39
      3.8.4 Directivity Patterns versus Frequency .......... 40
# 3.9 Practical Considerations

- 3.9.1 Design of a Proof-of-Concept TO Reflector
  - 3.10.1 Bandwidth Considerations
  - 3.10.2 Unit Cell Design and Simulation
  - 3.10.3 Simulation Setup and Fabricated TO Reflector
  - 3.10.4 Measurement Setup
  - 3.10.5 Measured Results

# 3.11 Conclusion

---

# 4 Ultra-Wideband Reflector Design Using an Impedance Surface

- 4.1 Introduction
- 4.2 Theory
  - 4.2.1 Problem Formulation
  - 4.2.2 Bandwidth Limitations
  - 4.2.3 Bessel Filters
  - 4.2.4 Transfer Function $\Gamma(s)$
  - 4.2.5 Dispersion Analysis
  - 4.2.6 Maximum Reflector Size and Selection of Filter Order
- 4.3 Selection of Circuit Topology
- 4.4 Logic Flow Diagram
- 4.5 Unit Cells
  - 4.5.1 Design of Unit Cells
  - 4.5.2 Inter-Cell and Intra-Cell Coupling Considerations
- 4.6 3-D Simulation Setup and Method of Moments
  - 4.6.1 Reflectarray Geometry
  - 4.6.2 Method of Moments Simulation Setup
  - 4.6.3 Experimental Setup
- 4.7 Results
  - 4.7.1 Far-Field Measurements
  - 4.7.2 Signal Fidelity of the Reflectarray
- 4.8 Other Filters Types
- 4.9 Conclusion

# 5 Comparison of TO Reflectors and Impedance Surface Reflectarrays

- 5.1 TO Reflector vs. Impedance Surface Reflectarray
  - 5.1.1 Reflector Topology
  - 5.1.2 Physical Profile
  - 5.1.3 Bandwidth
- 5.2 Comparison with State-of-the-Art Reflectarrays
  - 5.2.1 Qualitative Comparisons with Other Wideband Reflectarrays
List of Tables

4.1 A table of component values for circuits 1 – 3. ................................. 64
4.2 Percentage of area coverage for each cell configuration. ....................... 68
4.3 Simulated ranges for the geometric parameters defined in Fig. 4.15 in each cell configuration. 70
4.4 Table of simulated spillover and conductor efficiencies and the measured XPR in decibels over a 3 dB beamwidth of the reflectarray. ................................. 78

5.1 Summary of comparison between the TO reflector and the impedance surface reflectarray. 96
5.2 A comparison between the impedance surface reflectarray and the state-of-the-art in literature. ................................................................. 97
5.3 A comparison of our impedance surface reflectarray with other works in the literature. 98
List of Figures

1.1 A coordinate transformation between a virtual and physical space. ................. 4
1.2 Relationship between a reflectarray and a Huygens’ surface. .................... 5

2.1 A cross section of 3-D parabolic reflector and its delay profile. .................. 8
2.2 Illustrating the relationship between frequency (wave number $k$) and reflected phase on a 1-D reflectarray surface with phase wrapping. ................................. 9
2.3 Illustration of a generic reflectarray consist of an array of scatterers. .......... 11
2.4 Three approaches to phase shift: a guided-wave approach, an integrated approach and the use of material. ................................................................. 13
2.5 Three filters each representing the response of a unit cell on a reflectarray. ..... 14

3.1 An analogy between the refractive index profile and mechanical potential in classical Newtonian mechanics. ................................................................. 20
3.2 An example of conformal coordinate transformation. ................................ 26
3.3 Two examples of analytic conformal coordinate transformations. ............... 28
3.4 An illustration of a SC coordinate transformation from the $z$-plane to the $w$-plane. 30
3.5 Two examples of SC coordinate transformations. ....................................... 30
3.6 A SC transformation that maps a rectangular region to a closed polygon. .... 31
3.7 A parabolic reflector rotated about its apex. ........................................... 31
3.8 Three SC transformations that are mapped from the physical flat space to the virtual curved spaces shown. ................................................. 32
3.9 The design TO reflector: its coordinate transformation and relative permittivity profile. 33
3.10 Impact of the coordinate transformation size on permittivity profile. ........ 35
3.11 The maximum $\varepsilon_r$ along $y = 150$ mm line as a function of $y_{\text{shift}}$, the $y$ position of the apex of the parabolas in Figures 3.10(a) and (c). .............. 36
3.12 A SC transformation performed only in region B of Figure 3.9(a). .............. 36
3.13 Directivity patterns of the TO reflector and parabolic reflector for various scan angles. 38
3.14 Relative permittivity profile for various scan angles. ............................. 39
3.15 Actual scan angles vs. desired scan angle for the TO reflector. ................ 39
3.16 Directivity pattern of the TO reflector cover for different permittivity profiles at the center frequency of 5 GHz. ......................................................... 40
3.17 Directivity patterns for $\phi_o = 40^\circ$ for various frequencies for case B. ...... 41
3.18 Maximum directivity vs. frequency and actual beam angle vs. frequency. .... 41
3.19 A parabolic reflector and its corresponding TO reflector with $f/D = 0.2$. ...... 42
4.26 Measured and simulated directivities and gains as a function of frequency. 83
4.27 The normalized magnitude and phase of the transfer function of the reflectarray from 5
to 10 GHz. 83
4.28 Input signal and the radiated (reflected) E-field of the reflectarray. The signal fidelity of
these two signals is 0.951. 84

5.1 Two parabolic reflectors operating at two different frequencies with the same electrical size. 88
5.2 The unit cells of the state-of-the-art reflectarray in the current literature. 89
5.3 The lumped element model of the unit cells on the reflectarray. 89
5.4 A lumped element model consisting of only two capacitive layers. 90
5.5 A lumped element model consisting of only two layers. 90
5.6 Optimized phase of the reflection coefficient \( \Gamma(\omega) \) with respect to the target phase \(-\omega\tau\). 91
5.7 Far-field patterns of the impedance surface reflectarray and the ladder filter reflectarray
with optimized reflection phase for frequencies from 5 to 7 GHz. 92
5.8 Far-field patterns of the impedance surface reflectarray and the ladder filter reflectarray
with optimized reflection phase for frequencies from 8 to 10 GHz. 93

A.1 The layout of the impedance surface reflectarray. 103
B.1 The problem definition to extract the anisotropic sheet impedance of a metasurface. 104
Chapter 1

Introduction

Antennas are ubiquitous and important parts in modern wireless communication systems. They enable efficient transmission of information encoded in the radiated electromagnetic fields. Antennas are able to convert electromagnetic energy in a guided form to an unguided form that radiates out into open space, and vice versa. Antennas come in many different shapes and sizes. Their profiles have transformed from a typically protruded form factor visible from the outside to more integrated solutions where the antennas are more discreet, conforming to the device housing. In addition to the their physical requirements, more demands are being made to the their electrical requirements such as the ability to operate on multiple frequency bands while maintaining a reasonable radiation efficiency and a good impedance match. Modern smart phones have integrated antennas that operate over a number of different frequency bands for simultaneous operation of bluetooth, WiFi and LTE.

In many wireless applications, antennas are used to focus the emitted radiation as much as possible in a desired direction so that maximum signal power is available at the receiver for reliable communication. There is a limit in how much focusing can be achieved given a fixed aperture size which is proportional to the directivity and gain of an antenna. High directivity and gain requirements lead to antennas with a large aperture size. For example, in an inter-satellite point-to-point links, the distance could be hundreds of kilometers long, requiring a large antenna size which is problematic for such an application because the weight and size are limited.

Reflector antennas such as parabolic reflectors can provide large aperture sizes to produce high gain but they tend to be bulky and heavy. In addition, a mechanical apparatus is needed to point the reflector to a desired direction. Reflectarray antennas have been proposed as a superior alternative to mechanical reflectors with the benefits of potentially being much lighter and they can be made electronically reconfigurable to achieve dynamic beam-steering without mechanical movement. A typical reflectarray consists of a planar array of similar antenna elements to form an array. Their planar and low profile makes them more attractive compared to their curved reflector counterparts. Tunable elements such as varactor diodes and RF microelectromechanical systems (MEMS) can be incorporated into the reflectarray cell to make it dynamically reconfigurable to achieve capabilities such as dynamic beam steering and beamforming. These features are highly desirable in applications such as dynamic target tracking where no physical movement of the reflector is required. Reflectarrays are usually passive devices but they can incorporate active devices, such as amplifiers in their unit cells, to achieve an increased power gain. In such a design, the power emitted from the reflectarray surface can be more than the total
power incident power from a feed antenna.

The design of a reflectarray antenna amounts to producing a set of appropriately phased fields on its surface in such a way that a desired beam, such as a pencil beam, is achieved. Reflectarrays are typically fed using a feed antenna located in front of the reflector surface producing a set of incident fields which is then scattered by the reflector surface. Each antenna element on the reflectarray surface can be thought of as an independent antenna with its own scattering characteristics that can be tuned by changing some of its parameters, for example the shape or size of the element. The superposition of all the scattered fields from each element make up for the total radiated fields of the reflectarray.

Despite the good qualities of reflectarray antennas, the main disadvantage of modern reflectarrays is their limited bandwidth. The upper bound on the fractional bandwidth is typically quoted to about 20% [1]. Recent state-of-the-art reflectarrays have demonstrated a fractional bandwidth up to 40% [2] using stacked layers.

Recently, there has been some changes in the design methodology of reflectarray antennas. Although the end goal remains the same, which is controlling the reflected fields on the reflector surface, the new design methods allow novel reflectors to be designed that are conceptually more than just an array of antenna elements. Two novel methods to design reflectarrays are explored in this thesis. The first method is Transformation Optics (TO) and the second method is the design of a reflectarray using an impedance surface. There are many terminologies often used in literature to refer to the second method, namely Huygens’ surfaces, metasurfaces and miniaturized-element frequency-selective-surfaces (MEFSS) which are all refer to the same type of surface and used interchangeably throughout this thesis – a surface that in which macroscopic surface properties such as sheet impedance and sheet admittance can be defined. The electrical sizes of the elements used on such sheets are typically sub-wavelength.

This thesis describes novel methods to design ultra-wideband (UWB) reflector antennas that can operate over an exceptionally wide band of frequencies. At the time of writing, the achieved bandwidth is the state-of-the-art.

1.1 Motivation and Thesis Goals

There are strong motivations to have a wideband reflectarray as it is useful in a wide variety of applications such as communication, imaging, radar and remote-sensing. For example, in a wideband radar application, sharp temporal pulses are used in a target range detection application for producing a high spatial resolution. Such an application requires a wideband reflector so that all the frequencies contained in the pulse are radiated efficiently. Moreover, a satellite communication system typically operate on at least two bands, one for uplink and one for downlink, and potentially operating over other bands at the same time. Having a different reflector for each frequency is impractical and wasteful. Although multi-band reflectarrays have been proposed to address this problem, their designs tend to be complex, especially when more than two or three bands are required. Having a lightweight and low-profile reflector surface that can operate over all frequencies is ideal, particularly in an application such as satellite communication systems where space and weight are extremely scarce.

The main goal of this thesis is to investigate how to design wideband reflectarrays without using of true-time-delay (TTD) devices. Even though reflectarrays with TTD devices, such as transmission delay lines, is well-known to have a theoretically unlimited bandwidth (at least their delay bandwidth), their state-of-the-art bandwidth tend to hover at around only 20% [1] in fractional bandwidth due to
the resonant nature of the antenna elements used. Reflectarrays without TTD devices, typically with multiple layers of scatterers, have been shown in literature to have roughly the same fractional bandwidth. This thesis attempts to design wideband reflectors without using TTD device and experimentally verify that the proposed methods used in the thesis are able to produce a fractional bandwidth that far exceed the 20% limit.

This thesis attempts to provide answers to the following important questions.

- Since TO is known to manipulate the trajectory of electromagnetic waves, can it provide a platform to design wideband reflectors? If so, what are the advantages and disadvantages and design considerations for such a reflector?

- What happens to the reflectarray as its element size and spacing becomes small compared to the wavelength? Can we design some sort of homogenized surface with a macroscopic surface properties? If so, can we manipulate such surface properties to design wideband reflectors?

The answers to the above questions can provide insightful and useful feedback to address the bandwidth issue of reflectarray, aiding research to develop new methodologies that enhance their bandwidth.

The two main methods to be explored in this thesis for designing wideband reflectors are the TO method and the impedance surface method. We will see what type of wideband reflectors that can be designed using each method. In the TO method, a reflectarray is designed as a device that realizes a curved space by using an effective medium. In the impedance surface method, a reflectarray is designed as an impedance surface that realizes a Huygens' surface which supports the flow of arbitrary electrical currents. We will investigate the pros and cons of each approach as the two design methods are different, each yielding reflectors that maybe more suitable for some applications than others.

It should be noted here that the ideal reflectarray with an infinitely wide bandwidth is the one with true-time-delay (TTD) devices used as the phase shifting mechanism. Independent of the method in which the wide phase bandwidth is achieved, the end goal is always the same. That is, we would like the phase of the reflection coefficient on the reflectarray surface to approximate the response of TTD devices over as a wide bandwidth as possible. In the TO method, this is achieved using a bulk medium while in the impedance surface method, it is achieved using a surface with an appropriate surface impedance whose reflection coefficient approximates that of TTD devices over a wide bandwidth.

In the next sections, we provide a synopsis of the fundamentally different approaches involved in the two design methods, both of which can provide a platform to design novel reflectors.

1.2 Reflectarray as a Transformation Optics Device

Transformation optics (TO) [3] is a method to control propagation of electromagnetic waves using the fully tensorial material parameters, which are derived from a coordinate transformation. TO is derived using the form-invariant property of Maxwell’s equations that make coordinate transformation synonymous with material parameters for electromagnetic wave propagation. A flat space filled with an inhomogeneous tensorial material is the same electromagnetically as a corresponding curved space filled with a homogeneous material (or vacuum). TO amounts to producing a desired set of inhomogeneous material parameters such that the light wave in the medium behaves in some pre-determined manner. Figure 1.1 illustrates a coordinate transformation in 2-D, denoted by a complex map \( f \) between a virtual curved \( w \) space to a flat \( z' \) space which contains the inhomogeneous material.
Fundamentally, the principle of operation of TO is Fermat’s principle, which states the optical path length $s$ between two points $A$ and $B$

$$s = \int_{A}^{B} n\,dl$$

is an extremum. When the refractive index $n$ is a function of position, the shortest path is not a straight but a curved path. This principle made control of light waves possible using material parameters and it may cause optical illusions such as invisibility. One of the most well-known examples of light taking the shortest path is the light refraction at the interface between two mediums. An example of a light taking the longest path can be shown in the case for a lens [3]. Effectively, the optical path is different from the geometric path and the material parameters define a different measure of space for light. Regions with high refractive index appear to be larger than low index regions.

There are many interesting applications that are derived from TO, most notably the cloaking applications whereby an object can be concealed or transformed to appear electromagnetically the same as another object. A more detailed survey is presented in the Chapter 3. For the purposes of designing reflectarrays, one can envision an inhomogeneous planar reflector cover placed on top of a flat perfect electrical conductor (PEC). The reflector cover allows the incident fields to see the flat reflector as a curved surface, thereby mimicking a curved reflector. In effect, TO distorts space for light propagation by the use of materials.

### 1.3 Reflectarray as a Huygens’ Surface

To design reflectarrays as a Huygens’ surface, we view and design a reflectarray not as an array of antenna elements but a surface with miniaturized sub-wavelength elements. There is a point in which the size and the spacing of the antenna elements become sufficiently small compare to the wavelength, such that the overall array is not so much as an array of elements but more of a homogenized surface medium in which we can define a macroscopic parameter such as sheet impedance or sheet admittance associated with the surface. Traditional reflectarrays with element size and spacing comparable to the wavelength do not operate in this regime and hence, no macroscopic surface properties are typically or meaningfully defined. The link between reflectarrays and frequency-selective-surface (FSS) particularly the ones with sub-wavelength elements are becoming more synonymous. One of the first examples of reflectarray design using sub-wavelength elements for a reflectarray-like application is the mushroom structure by Sievenpiper [4]. Each sub-wavelength cell is locally modeled using a simple parallel LC resonator, through which the reflection coefficient is electronically tuned to achieve beam steering. In addition, using sub-wavelength elements has been recognized as the key to improve the bandwidth of
reflectarrays [5].

There are many terminologies that are synonymous with Huygens’ surface in literature. For example, impedance and admittance surfaces [6] and frequency-selective-surfaces [7] are a type of Huygens’ surface. An impedance surface is a surface that supports the flow of electric surface current density $J_s$ producing different $H$-fields on the two sides while $E$-fields stay the same. Conversely, an admittance surface is a surface that supports the flow of magnetic surface current density $M_s$ producing different $E$-fields on the two sides while $H$-fields stay the same. A Huygens’ surface refers to the superposition of impedance and admittance surfaces and it supports both $J_s$ and $M_s$ as illustrated in Figure 1.2. Impedance surfaces can be modeled as shunt lumped components and the admittance surface can be modeled as series lumped components [6].

The design of Huygens’ surfaces is becoming popular in recent years due to their ability, in principle, to produce arbitrary currents that can produce arbitrary scattered fields leading to a wide variety of applications such as novel antenna design and cloaking. The design of the Huygens’ surface amounts to designing the set of impedance and admittance surfaces that can generate the set of desired surface currents given a set of incident fields. Huygens’ surfaces are essentially the imaginary surfaces used in the equivalence principle [8], which states that the arbitrary fields can be maintained on the two sides of the surface by the introduction of an appropriate set of electric and magnetic surface currents. The presence of the surface currents ensures the satisfaction of boundary conditions on the surface. For the purpose of designing reflectarrays, we design an impedance surface, which is the reflectarray, that produces the set of desired fields. At each position on the reflectarray surface, the incident fields see a different impedance, which is designed with the help of filter theory, in such a way that the reflected fields is able to track linearly with both frequency and space to produce a wide band reflectarray.

![Figure 1.2: (a) Reflectarray consist of an array of elements shown as patch antennas. As the element size and spacing becomes small compared to sub-wavelength, the reflectarray effectively becomes a Huygens’ surface shown in (b) with macroscopic sheet impedance $\bar{Z}_s$ and admittance $\bar{Y}_s$ supporting the flow of electric and magnetic currents $J_s$ and $M_s$.](image)

### 1.4 Thesis Outline

This thesis begins in Chapter 2 by providing a brief review of the theory of operations of reflectors and reflectarrays in general with a literature survey of the existing reflectarrays that are relevant to this thesis. Chapter 3 introduces TO and the fundamentals behind its theory of operation without invoking much
differential geometry and tensor algebra, which are discussed only to the extend necessary to understand the concepts involved in the basics of the theory of operation of TO devices. Chapter 4 describes how reflectarrays can be designed using an impedance surface, which is in contrast to the tradition design method of reflectarrays by using an array of antennas with no surface properties typically defined. Chapter 5 compares the two different reflectarray design approaches described in the previous two chapters. The pros and cons of each approach are discussed. We will also compare our own reflectarray respect to the current state-of-the-art wideband reflectarray in literature. Chapter 6 concludes the thesis.
Chapter 2

Reflector Antenna and Reflectarray Background

2.1 Reflector Antenna Theory of Operation

We first provide a short review of the theory of operation of reflector antennas which we will simply referred to as reflectors, and illustrate their most important and relevant properties to this thesis. The materials presented here are essential for understanding how reflectors function, which is intrinsically related to the novel methods used to design wideband reflectors in Chapters 3 and 4.

Although many other types of reflectors exist [8] such as spherical reflectors and corner reflectors, here we focus on parabolic reflectors in particular as they are often used in high gain antenna applications and most relevant to this thesis. The primary purpose of a parabolic reflector, an example is illustrated in Figure 2.1(a), is to provide focusing of a non-directive beam into a more directive one by means of reflecting the incident fields so that they radiate coherently in the same direction. The coherence of the reflected field is critical to form a constructive interference that is responsible for producing a directive beam.

Figure 2.1(a) shows a cross section of 3-D parabolic reflector (a paraboloid) that is made by rotating a 2-D a parabola \( y = Ax^2 \), \( A > 0 \), about its axis. A feed antenna is placed at the focal point of the reflector which is typically located at a significant fraction of the reflector’s diameter. The feed antenna typically has a relatively non-directive pattern to illuminate most of the area of the parabolic reflector. The parabolic reflector shown in Figure 2.1(a) is in an offset configuration as only the top portion of the reflector is used. Offset configurations are commonly used to avoid the feed blockage as the reflected field would miss most of the feed antenna located at the focal point. Although a simple parabolic reflector system is illustrated, complex multi-reflector systems of different types can be built [8], such as the Cassegrain [9, 10] configuration, with the aim to further reduce the physical profile while still having a large aperture size.

The parabolic reflector has the following important properties.

- The distance traveled from the focal point to an imaginary plane via a point of reflection is a constant. This is illustrated by two paths \( FQP \) and \( FQ'P' \) having the same lengths in Figure 2.1(a), where \( Q \) and \( Q' \) are arbitrary points on the parabola. Due to the constant path length, the
electrical distance of such two paths are the same, achieving a coherence for the reflected fields.

- The parabola is the only shape that achieves this on-axis focusing and any off-axis rays incident on the parabola are not focused exactly [11].

The fundamental principle of operation of a parabolic reflector is to offer the appropriate amount of delay of the incident field such that there is no relative delay for the fields on an imaginary plane. The delay is achieved by using the geometry of the parabola that provides a different distance of compensation for each impinging ray on the reflector. It is important to realize that a parabolic reflector is a true-time-delay (TTD) device.

A red line denoting a planar device is drawn in Figure 2.1(a). One can imagine that if the planar device is able to compensate for the extra delay length highlighted in the blue path, then it would provide the same focusing effect as the original parabola reflector. This idea produces a planar reflectarray antenna where the surface performs the phasing of the incident fields to produce coherent reflected fields. Such an antenna is described in the next section. The length of the blue path is the distance compensation profile, a distance that needs to be compensated for when the planar device is used in placement of the parabolic reflector. Figure 2.1(b) illustrates such a distance delay profile as a function of $x'$, the coordinate in the planar surface as illustrated in Figure 2.1(a).

In lieu of what is coming in Chapters 3 and 4, there are other methods to produce a delay without using a curved geometry. In Chapter 3, the delay is produced by an inhomogeneous material, while in Chapter 4, the delay is produced by a surface with printed circuits that realizes delay cells. No matter how the delay is achieved, these methods approximate the ideal TTD response.

A wideband reflectarray has specific relationships between frequency in radians $\omega$, the wave number $k = \omega/c$, the required delay distance $d$ and the reflection coefficient $\Gamma = e^{-jkd}$. Consider a 1-D reflectarray where the distance of compensation $d(x)$ is shown in Figure 2.2(a). The ideal reflected phase at frequency $\omega_1$ is shown in (b), which is the electrical length of $d(x)$ at $\omega_1$. When the frequency is increased to $\omega_2 > \omega_1$, as shown in (c), then magnitude of the reflected phase $k_2d$ is greater than $\pi$ which it can be phase wrapped backed into the $(-\pi, \pi]$ range, or any other user-defined range for the principal argument of a complex number. For a wideband reflectarray, whatever the method is used to produce

![Figure 2.1: (a) A cross section of a 3-D parabolic reflector (a paraboloid) drawn to scale. The cross section is parabola in the form of $y = Ax^2$ for $A = 1$. A focal point is located at $f = 1/(4A)$. $x'$ is the coordinate in the red planar surface. (b) The 2-D distance compensation profile for the planar surface in red in (a).](image-url)
the reflected phase such as Transformation Optics or metasurface, it must track linearly with respect to both frequency and the required delay distance. It is important to keep this in mind when designing wideband reflectarrays.

\[
d(x) = \begin{cases} \pi & y = \pi \\ -\pi & y = -\pi \end{cases}
\]

Figure 2.2: Illustrating the relationship between frequency (wave number \(k\)) and reflected phase on a 1-D reflectarray surface. The wave numbers are related to frequency by \(k = \omega / c\). (a) the distance of compensation of the reflectarray. (b) The reflected phase at \(\omega_1\). (c) The reflected phase at \(\omega_2\) where \(\omega_2 > \omega_1\) so that \(|k_2d| > \pi\) is phase wrapped as shown. The range of the principal argument chosen here is \((-\pi, \pi]\).

### 2.2 Reflectarray Theory of Operation

The theory of operation of TTD reflectors discussed in the previous section was in the time domain. Here, we switch to a discussion in the frequency domain to review the theory of operation of a typical narrowband reflectarray. A reflectarray can be thought of as a device approximating the response of a TTD reflector, such as the parabolic reflector shown in Figure 2.1, at a single frequency or over very narrow bandwidth. The line between whether a reflectarray is considered to be TTD in nature blurs as its bandwidth increases from 0 Hz, in which case it is not considered to be a TTD reflector, to a sufficiently large bandwidth, after which the reflectarray may considered to be a TTD reflector. In the limit as the bandwidth approaches infinity, the reflectarray can be considered as a TTD reflector. For what follows, we review the theory of operation of a narrowband reflectarray in the frequency domain.

A reflectarray typically consists of an array of scatterers placed on a planar surface as shown in Figure 2.3. Each scatterer is typically arranged in a rectangular grid with uniform spacing, though nonuniform grids can also be used. The elements of the reflectarray are spatially excited with a feed antenna located at the focal point (F) of the reflectarray that emits a set of incident fields impinging on the reflectarray surface as shown.

The purpose of the reflectarray is to alter the incident fields in a desired manner such that a desired set of scattered fields is produced. As illustrated in Figure 2.3, the \(i^{th}\) incident ray from the feed antenna travels from the focal point \(F\) to a point on the reflectarray surface \(Q_i\), at which point the fields gets reflected back to open space. Three rays are drawn in Figure 2.3 to illustrate the path of the incident fields. At the point of reflection \(Q_i\), the incident fields experiences a reflection with phase \(\Gamma_i\) that controls the phasing of the scattered fields, although the control of the reflected amplitude is also possible. For example, to produce a pencil beam at a frequency \(f_o\), the reflected phase of each ray must constructive interfere on an imaginary plane whose normal is the desired direction of propagation as shown in Figure 2.3. Mathematically, this means the desired electrical length for each ray \(i\) must be...
a constant

\[ k_0(FQ_iP_i) + \Gamma_i = \text{const.} \pmod{2\pi}, \quad k_0 = \frac{2\pi f_o}{c}, \quad \forall i \]  

(2.1)
on the imaginary plane, where \((FQ_iP_i)\) denotes the physical distance of the path \(FQ_iP_i\) and \(\Gamma_i\) is a design parameter that can be used to achieve beam-steering and beamforming.

For a 2-D uniformly spaced array, the array factor of the overall array is given by a superposition of the constructive and destructive interference from each scatterer. If the induced current on each scatterer is denoted by \(I_{mn} \in \mathbb{C}\), then the overall array factor is given by [8]

\[ AF(\theta, \phi) = \sum_n \sum_m I_{mn} e^{jk_0(m\Delta x \sin \theta \cos \phi + n\Delta y \sin \theta \sin \phi)}, \]  

(2.2)
where \(\Delta x, \Delta y\) are the spacings in the \(x\) and \(y\) directions respectively, the subscripts \(mn\) are the indexes of the \(mn\)th element in the array. The total radiated scattered fields of the reflectarray \(E_{ff}(\theta, \phi)\) is then the product of the array factor and the element factor \(EF(\theta, \phi)\) [8]

\[ E_{ff}(\theta, \phi) = EF(\theta, \phi) \times AF(\theta, \phi). \]  

(2.3)
This traditional array theory allows antennas to have a very large effective aperture producing a high gain without having a large bulky curved reflector. In addition, arrays can produce a versatile set of radiation patterns that otherwise would be difficult to achieve using a single antenna. For example, beam-steering can be achieved by changing the relative phase of the currents on each antenna element. The different phases on the elements collectively produce a constructive interference in a different direction, producing the main lobe. Various beam shapes can be engineered by controlling both the magnitude and the phase of the currents. Contour beams of a specific shape, e.g. the North American continent, can be specifically engineered. Moreover, a specific side lobe level can be achieved by tapering the current amplitudes across the array. This spatial window is analogous to the temporal windows used in signal processing [12] for Fourier analysis. The performance of the overall array can be made robust to component failure due to the large number of antennas present. The array factor pattern calculated using array theory in this manner assumes no coupling between the antenna elements, a condition that is violated in practice when the elements are in close vicinity of each other. In a practical design, the element coupling needs to be managed to ensure that the desired radiation characteristics of the array have been met. There exist simulation tools as commercial software packages with the ability to model infinite array in periodic boundary conditions to capture the effect of inter-element coupling, at least with identical adjacent elements.

Of particular interest in thesis is to design reflectarrays with as wide bandwidth as possible. Traditional array theory is insufficient for designing wideband reflectarrays because the direction of the main lobe is a function of frequency. As the frequency is changed, the direction in which the constructive interference occurs (main lobe) changes due to the changes in the electrical distance between the elements. This is known as beam squinting which is a necessary result of the traditional array theory. Beam squinting is an important issue to address because it has significant negative impact on the wireless system performance as the main lobe points away from the desired direction. Novel methods to design wideband reflectarrays must address beam squinting as part of the design problem, among other aspects such as designing wideband phase shift networks and wideband radiating elements.
Chapter 2. Reflector Antenna and Reflectarray Background

2.3 Reflectarray Literature Review

The first reflectarray was demonstrated by Berry in 1963 [13] consisted of an array of short-circuited waveguides, each of which has a different length to produce the required reflection phase. Although it functioned well and produced a directive beam, it was very bulky compared to modern standards due to the long waveguides. Reflectarrays did not gain much attraction until printed versions were demonstrated by Munson [14] and Huang [15] almost 20 years after their inception but they have come a long way since. Not only do they have a much slimmer profile now but they can have the ability to incorporate many different features integrated in the antenna. A typical modern reflectarray is relatively lightweight and can have tunable elements to achieve electronic beam-steering and beamforming. It can also be made active by incorporating RF amplifiers in their scatterers to enhance the radiated power.

We present a literature survey of reflectarray antennas, categorizing them from their most important aspects: phase shifting mechanisms, bandwidth, tunability or reconfigurability, passivity and polarization. The bandwidth of reflectarrays is of particular interest in this thesis and it deserves special attention. A definition for bandwidth is given here. Three of the most common ways to improve bandwidth are reviewed: the use of true-time-delay (TTD) devices, cascading multiple resonances on a single layer and stacking multiple layers of scatterers. In addition to these three methods, there are two other design methods that allow for enhanced reflectarray bandwidth. They are called Transformation Optics (TO) and Frequency selective surfaces (FSSs) which are the methods proposed in this thesis to design wideband reflectarrays. The literature review for wideband reflectarrays designed using TO and FSSs are given in their own respective Chapters 3 and 4.
2.3.1 Phase Shift Mechanisms

The main purpose of a reflectarray is to provide an appropriate amount of phase shift to an impinging incident field, though, amplitude control is also possible [16]. Here, we restrict the discussion to linear polarization as circular polarization has a different phase shift mechanism. There are three main methods to achieve the required phase shift on a reflectarray surface [17]: a guided-wave approach, an integrated approach and the use of material as illustrated in Figures 2.4 in (a), (b) and (c) respectively. In the guided-wave approach, the incident field impinges on the radiating structure, then it transitions to a guided-wave into a phase shift network, then it travels back out to the radiating structure which emits radiation out into open space. In this approach, the phase shift network is considered to be a separate network than the radiating structure and it is responsible for producing the required phase shift. The phase shift network is closely coupled to the radiating structure. In Figure 2.4(a), the guiding structure is shown as a transmission line that is responsible for coupling between the phase shifter and the radiating structure. The biggest advantage of the guided-wave approach is that it essentially outsources the analog signal processing to an external RF device which in theory can be designed independently. Practically, the external RF devices must fit within often limited space available on the reflectarray cell. For example, open-ended transmission line stubs [18, 19, 1, 20] can be directly attached (coupled) to patch antennas which are the radiating structure to provide a true-time delay response. Hybrid phase shifters with RF amplifiers [21] can be coupled to patch antennas to produce both phase shift and amplification simultaneously. A common method to couple the radiating structure to a phase shift network is to employ ground plane with a slot [22, 23] that also provides shielding of the phase shift network against the incident fields to limit the unwanted scattering of the phase shift network. One of the disadvantages of the guided-wave approach is that it often creates multi-layer structures which increases the fabrication complexity and spurious radiation maybe produced by the scattered fields of phase shift devices.

In the integrated approach in Figure 2.4(b), there is no separate phase shift network to produce phase shift. The scatterers that reside on the reflectarray are responsible for both radiation and for producing the required reflection phase simultaneously. In this method, scatterers of various shapes and sizes are used to tune the reflected phase. For example, varying the physical size of rectangular patches [24, 25] or circular patches [26] and dipoles [27] can tune their resonance frequency which in turn tunes the reflected phase. Other popular scatterers include metal rings [28, 29] or multiples thereof [28], loaded rings [30], ring slots [31] and rectangular slots [32]. In this integrated approach, the elements of the reflectarray can be made sub-wavelength, at which point the reflectarrays operate more like a miniaturized-element frequency-selective-surface (MEFSS) rather than an array of individual scatterers\footnote{It is clear that MEFSSs can be made transmissive but reflectarrays, by definition, can not.}. Some recent work on reflectarrays treats them as an impedance surfaces [4] which is a type of metasurface. Reflectarrays designed as a metasurface deserve their own special attention and a literature review is presented in Chapter 4.

The third mechanism to achieve the desired phase shifts is to use materials as shown in Figure 2.4(c). Although it has been known for a long time that wave propagation through materials incurs different phase shifts, a recently developed method in 2006 by Pendry et al [33, 34] allows for the computation of material parameters that directly corresponds to a geometry. This opened a whole new field known as Transformation Optics (TO) in which the distortion of space can be realized. TO is fundamentally based on Fermat’s principle or the principle of least time that a path taken between two points where light propagation uses the optical length as a measure of notion of space rather than the geometric length [3].
Many TO devices have been designed in the literature [35]. The design of TO reflectors is the topic of Chapter 3 and a full literature review is presented there.

![Phase Shifter Diagram](image)

Figure 2.4: Three approaches to provide a phase shifting mechanism. (a) A guided-wave approach where the incident wave is converted into a guided-wave that is fed into an external phase shift mechanism which coupled to the radiating structure. (b) An integrated approach where an array of scatterers provides both a mechanism for radiation and also provide the required phase shift simultaneously. (c) A block of inhomogeneous material is placed in front of a planar reflector to achieve phase shift.

### 2.3.2 Bandwidth

There are two types of bandwidth associated with a reflectarray because it acts as a filter both temporally and spatially. The fact that reflectarrays are temporal filters is well understood and it simply filters the spectrum of an input signal to produce an output spectrum just like any other temporal filter. However, reflectarrays are typically not thought of as a spatial filter although it is one nevertheless. It provides focusing of an non-directive beam to a more directive one. It acts as a spatial filter that takes a spatially wideband fields as the input and produces a spatially narrowband fields, a quasi plane-wave, as output. In general, any object can act as a spatial filter to a set of incident fields producing a set of output fields with a different spatial spectrum.

It is important to be aware of which type of bandwidth is under consideration because it produces different characteristics for reflectarrays and one needs to be clear which one, or both, that the reflectarray is being designed for. A temporally wideband reflectarray is an antenna that preserves the shape of the incident signal for as large a frequency range as possible, known as a high signal fidelity, but it may be spatially dispersive producing poor beam characteristics as the frequency is changed. Likewise, a spatially wideband reflectarray is an antenna that produces good beam characteristics for as a large frequency range as possible but it may be temporally dispersive, leading to poor signal fidelity. To illustrate the requirements to produce a spatially wideband reflectarray, Figure 2.5 shows three filters with transfer functions $H_i(s)$, $i = 1, 2, 3$, each representing the temporal response of single unit cell on a reflectarray surface. The input signals to the filters, which are illustrates as wave packets, are the incident fields emitted from the feed antenna and the output signals of the filters are the reflected fields due to the reflectarray cells. The incident signal arrives first at filter 1 compared to filter 2. Therefore, the transfer function $H_1(s)$ needs to provide a group delay of $\Delta x/c$ in order for the peaks of the envelope
to be aligned at the outputs, as shown at the output of filters 1 and 2. However, this group delay requirement does not constrain the transmission phase of the filters. At the output of filter 3, which has the same group delay as filter 1, the transmission phase is not coherent with the second filter and it can be arbitrary. Hence, if the reflected signals are not coherent at a particular frequency on the reflectarray surface, it would not able to produce desired beam patterns at that frequency even though the group delays of each unit cells are properly set at that frequency. Typically in the literature, reflectarrays that are labeled wideband are spatially wideband reflectarrays where beam patterns are plotted over a large frequency range. Their temporal dispersion is rarely characterized. A wideband reflectarray in the literature often does not explicitly specify which bandwidth is the reflectarray being designed for, although sometimes it is obvious by its design.

![Figure 2.5](image_url)

Figure 2.5: Three filters each representing the response of a unit cell on a reflectarray. Signals arrives first at the input of filters 1 and 3 compared to filter 2. Filters 1 and 3 has the same group delay in such a way that the output peaks of the envelops are all aligned among the 3 filters. However, filter 3 is dispersive and its transmission phase is arbitrary. Hence, its envelop peak is aligned with rest of the filters but its phase is not coherent.

An ideal wideband reflectarray is wideband in both the temporal and spatial sense, producing non-dispersive scattered fields and the main lobe remains directive and pointed in the same direction over a large frequency range. It must have wideband antenna elements and wideband phase shift networks simultaneously. In addition, the coupling mechanism between them needs to be wideband as well. In theory, true-time-delay (TTD) devices can be used as ideal phase shifters for delay compensation that can achieve ideal wideband reflectarrays. One of the earliest methods to realize a TTD reflectarray is by coupling an open-ended transmission line stub as delay elements to an array of patches [15]. It is reasoned that the incident fields are coupled to the transmission line, reflecting off its open end and re-radiating out into open space with the appropriate amount of delay determined by the length of the transmission line. This phase shift mechanism falls into the category of the guided-wave approach described in Section 2.3.1, where external delay devices are coupled to the main scatterers. In this case, they can all reside on the same plane, although other cases may not be possible to do so [21].
Despite some improvements in achieved bandwidth, this approach has some inherent limitations which are, mainly, the undesired scattering of the transmission lines, which produces spurious radiation that needs to be managed, and a limited space is available between the cells to place the long transmission lines. In addition, although wideband phase shifters are used, the radiating antennas are still resonant patches which limit the bandwidth of the reflectarray [1].

Bandwidth can be improved for single-layer reflectarrays using a multi-resonance approach. In essence, elements with different shapes and size but with similar resonant frequencies can be overlaid or placed together that collectively cover a wider frequency band than what can be covered using only one type of element. In this case, the phase shift mechanism falls into the integrated approach described in Section 2.3.1. For example, reflectarrays with different sized concentric rings [36, 28, 37, 38, 39], cross and rectangular loops [40], short dipoles [41, 42, 43] and cross-slot elements [44] have been demonstrated. This approach inherently requires that the elements of different shapes and sizes need to be physically compatible so that it can be realized on a planar surface without physically overlapping. Hence, concentric rings are among the most popular choice for element shapes for this approach.

Naturally, multi-layer structures have been proposed to improve the bandwidth of reflectarrays. Multiple layers of patches [45, 46, 47, 48, 49], patches with slots [32], loops [50] and rings slots [31] have been demonstrated. In essence, the improved bandwidth comes from the increased degrees of freedom associated with the stacked structure in the third dimension at the expense of design and fabrication complexity. Many multi-layer structures are designed with their unit cells acting as filters to achieve some desired filtering functions of incident fields. The current state-of-the-art wideband reflectarray design [2] known to the author at the time of this writing uses 3 substrate layers to achieve a beam fractional bandwidth of 40%. Recently, a generalized systematic approach is developed that can synthesize an arbitrary filter response in a reflectarray cell using $N$ layers [51]. The biggest advantage of the multi-layer approach is that it can, in theory, produce arbitrary high bandwidth for a reflectarray when a sufficient number of layers of scatterers is used, but it comes at the expense of increased design and fabrication complexity that may not be suitable in some applications such as sub-millimeter applications where the simplification of the manufacturing process is particularly important [41].

The design of wideband reflectors using TO amounts to having appropriate materials placed in front of a reflector such that the incident fields see a traditional curved reflector. Such a TO reflector, or TO devices in general, are wideband due to the spatial distortion for light is realized by the material corresponding to a coordinate transformation which is frequency-independent. Hence, in practice, the bandwidth of TO devices in general is only dependent on the frequency dependence of the material. The wideband reflectarrays have also been designed using non-resonant sub-wavelength elements. These elements that make up the reflectarray act like FSSs, rather than an array of individual antenna elements, with surface properties such as sheet impedances and sheet admittances. As a result, the surface is able to support the flow of arbitrary electric and magnetic surface currents as a result of the scattering of some incident fields. The ability to manipulate currents directly offers unprecedented control of the scattered fields, through which wideband reflectors can be designed. Literature reviews of the wideband reflectors designed using TO and FSSs are presented in their own respective Chapters 3 and 4.

\footnote{Subject to satisfaction of Maxwell's equations and boundary conditions}
2.3.3 Tunability

Reflectarrays can be made tunable or reconfigurable by embedding electrically tunable elements in their unit cells. Tunable capacitors are typically employed rather than tunable inductors as the tuning elements due to the superior availability of the former. Two of the most commonly used tunable capacitors are varactor diodes and RF micro-electromechanical (MEM) capacitors which allow for continuous tuning of capacitance controlled by an external DC bias voltage. The design of these tunable components is a subject of device physics and their tuning properties are either given by the specifications or measured directly by the user. Varactor diodes are cheaper to produce than RF MEM capacitors and more widely accessible as off-the-shelf components available for purchase. The theories of operations of these two devices are different, leading to different constraints for their usage. Varactors diodes are inherently non-linear devices which prohibits their use in high power applications whereas RF MEMs capacitors are much more linear in their device characteristics. Many reflectarrays have been designed using varactor diodes [52, 53, 54, 55] and RF MEM capacitors [56, 57] to achieve electronic beam-steering [4, 22].

Binary tunable elements such as PIN diodes [58, 54] and MEMs switches [59, 23] can be used to achieve reconfigurability. These devices act as RF switches, controlled by a bias voltage, to provide the control of their binary state which is useful in applications such as switchable beams [60] where discrete control is required. More exotic tuning mechanisms have also been employed in the literature. For example, the dielectric properties can be changed in a liquid crystal substrate [61, 62] and graphene can be used as conductors with tunable conductivity [63].

2.3.4 Passivity

Reflectarray are usually made up of passive components and most of the reflectarray do not incorporate active components. Here, active means the magnitude of the reflection coefficient on the reflectarray surface is greater than unity. Tunable elements such as the aforementioned ones are typically considered to be passive components because the active circuits that control them operate at DC and do not directly contribute to RF functions, though some reflectarrays in the category are labeled as active reflectarrays in the literature. RF amplifiers can be integrated in the reflectarray cells [64, 65, 66], making the reflectarray active. Most of the active reflectarrays employ the guided-wave approach to aid the placement of RF amplifiers.

An active reflectarray, despite the presence of amplifiers, still needs to phase the reflected fields which make them much more difficult to realize. There are some unique challenges such as stability that are not present in passive devices. Few works exist in literature on active reflectarrays, although one reflectarray is made both active and reconfigurable [17, 21].

2.3.5 Polarization

The polarization of a reflectarray depends on the polarization that the unit cells are intended to function. A reflectarray can have a very versatile set of polarizations. In theory, all polarizations states are possible, namely, single or multiple polarizations with linear, circular or elliptical polarization states operating over a single or multiple bands. It is also possible for the reflectarray to convert the state of the polarization [21], e.g. an incident field with horizontal linear polarization can be converted to a vertical linear polarization.
Conventional parabolic reflector produces relatively high cross-polarization ratios (XPR) [67, 68], particularly in offset cases and there is no inherent mechanism to control polarization state of the scattered fields. The XPR in a reflectarray can be managed by designing unit cells with low scattering in the cross-polarization component [69, 19] and can typically achieve a much better XPR than the conventional reflector [70, 69, 71].

Different polarization states can be specifically engineered in a reflectarray. For example, dual linear polarizations can be achieved on dual bands [72], or having each polarization produce a different contour beam [73]. A reflectarray can accept dual linear polarizations by attaching two open-ended transmission line stubs to an array of patches [19]. Circular polarizations (CP) can be achieved by using an array of patches [18] or ring slots [31], each of which is physically rotated to produce the scattered fields with different reflected phase.

2.4 Summary

In this chapter, we have reviewed the theory of operations for reflector antennas and reflectarrays. A survey of modern reflectarray is presented along with different methods to enhance the bandwidth of reflectarray antennas. In addition, the different phase shift mechanisms in the literature are identified in the three categories presented. The notion of bandwidth for reflectarrays are distinguished between spatially wideband and temporally wideband reflectarrays and they each give raise to different characteristics for the reflectarray. The distinction between these two concepts are rarely found in the literature and it is clarified in this chapter. In the next chapter, we introduce the fundamentals of TO and use it as a powerful method to design ultra-wideband reflectors.
Chapter 3

Design of a Wideband Reflector Using Transformation Optics

3.1 Transformation Optics Background

Transformation Optics (TO) is a method to control electromagnetic waves using fully tensorial material parameters which are derived from a set of coordinate transformations. The relationship between the coordinate transformation and the corresponding material parameters is derived from the set of Maxwell’s equations in general curvilinear coordinate systems, which requires some basic knowledge in differential geometry and tensor algebra. The form-invariant property of Maxwell’s equations has made coordinate transformation synonymous with an appropriate set of material parameters.

The notion of distance for light incorporates the medium in which it travels and therefore, it is a different notion than the geometric distance. A light path achieves an extremum for an optical length (1.1) as if it is performing an optimization algorithm over all possible paths from a point A to another point B and selecting the one that achieves the extremum for the total optical length. Feynman’s description of the Huygens’ principle offers an explanation of how this optimization phenomenon occurs [3]. In brief, the selected light path is the only one that had a constructive interference in the set of infinitely many possible paths where all but one is cancelled out.

Many interesting and novel devices have emerged from TO. Most predominately, TO devices have been used in cloaking applications [74, 75, 76, 77] where objects can be concealed under a “carpet” which is an electromagnetically undetectable region. Transformation media that rests above this region manipulate light in such a way that makes the curved carpet appear flat. Moreover, cylindrical regions can be made electromagnetic invisible [33, 34] by a radial coordinate transformation where the space in the radial direction is stretched. The resulting transformation media that surrounds the cylindrical region makes incident fields propagate around the region, whereby arbitrary objects can be concealed inside the cylindrical region. Many novel devices have been created using TO. For example, in light-guiding applications, wave propagation can be directed or confined [78, 79], bent or expanded [80, 81] and collimated [82] in unprecedented ways. In lensing applications, interesting devices such as an aberration-free lens [83], a flattened Luneberg lens [84, 85], and a compressed lens [86] are created. Superscatterers [87] and superabsorbers [88] can be engineered to alter the scattering characteristics of an object to make it electromagnetically appear arbitrarily large or small. TO is also a useful tool for designing antennas...
such as novel reflectors [89, 90], highly directive antennas [89, 91] and steerable antennas [92]. TO has found itself for uses in other branches in the scientific community such as manipulation of acoustic waves [93, 94, 95], matter waves [96] and providing earthquake protection using seismic metamaterials [97, 98]. Some recent reviews of the advances of TO in designing electromagnetic devices can be found in [35, 99, 100].

With a wide variety of applications for TO, the engineering of the artificial materials needed to realize TO devices is in the field of metamaterials. One of the biggest and earliest motivation for metamaterials is the design of negative-index metamaterials (NIMs) which is able to act as a perfect lens and form images of the source beyond the diffraction limit. NIMs or metamaterials in general have generated intense research efforts and it is an active area of research that is closely related to TO. It was pointed out that the perfect lens also employs a coordinate transformation [3, 101]. The negative index of refraction is analogous to a coordinate transformation with a negative-sloped partial derivative in one of the coordinates.

In TO, there are two types of coordinate transformations, one that is performed over the all of space or only over a finite subregion. Examples of the former are the well-know carpet cloaks where coordinate transformations are performed over open regions. The latter case is called embedded transformation because the coordinate transformation is performed over a closed, finite-area region of space, as shown in the lens-like example in Figure 1.1. For embedded transformations, there could be reflections at the device boundaries. Regions that are on the outside and inside the domain of coordinate transformation are independent with one another. Reflection properties on the boundaries of a TO device is important for its proper operation. For example, in TO reflector antennas [102, 103], waves travels in and out of the reflector cover could experience refraction which could jeopardize the quality of the main lobe of the reflector. A necessary condition is found in the literature [104, 80] for reflectionless boundaries that imposes a condition on the metric of the coordinate transformation along either side of a boundary. Although the condition is useful in predicting the reflection properties at the boundaries for some devices [104, 80], to the best of author’s knowledge, no rigorous analysis of the reflection properties of TO devices have been reported to date [35].

TO provides a method to design wide band electromagnetic devices in many different applications. Since the coordinate transformation is independent of frequency, the realized TO devices are inherently as wide band as the virtual devices. In fact, the bandwidth of the TO device is often limited by the bandwidth of the materials used to realize or approximate those required from the coordinate transformation. Ultra-wide band reflectors [102, 105] and lenses [106, 86] have been demonstrated in the literature. Light-guiding structures such as beam benders and splitters [80] can theoretically function at any frequency.

In this thesis, we show how UWB reflectors can be designed using TO that uses an embedded coordinate transformation. There are many design considerations that need to be carefully treated such as the generation of the coordinate transformation, the shape of the transformation region, realization of the effective permittivity profile, management of the reflection at the device boundary, etc.

First, we illustrate how light travels within an inhomogeneous material with only the refractive index $n$, a scalar quantity. This quantity provides a method to guide light in different directions that satisfies the Fermat’s principle. $n^2$ can be shown to be analogous to the Newtonian mechanical potential influencing the motion of a particle. The trajectory of light can be computed by solving the Euler-
Lagrange equation [3]
\[
\frac{d}{d\xi} \frac{\partial L}{\partial \dot{x}^i} = \frac{\partial L}{\partial x^i}, \quad L = L(x^i, \dot{x}^i, \xi), \quad \dot{x}^i \equiv \frac{dx^i}{d\xi},
\]
(3.1)

where \(x^i(\xi)\) denotes a coordinate \(x, y\) or \(z\), and \(\xi\) is a parameter that draws the \((x(\xi), y(\xi), z(\xi))\) trajectory. (3.1) can be used to derive the geodesic equation which is satisfied by the path of the shortest distance in general curvilinear coordinate systems [3]. Using variational calculus, solving for the Euler-Lagrange equation (3.1) subject to \(n\) to yield a trajectory \(r(\xi) = (x(\xi), y(\xi), z(\xi))\) that satisfies [3]
\[
\frac{d^2 r}{d\xi^2} = \nabla n^2 / 2,
\]
(3.2)

where \(r(\xi)\) is the position vector. Noting that the second derivative of the position vector is analogous to acceleration, (3.2) is the Newton’s second law \(F = ma\) for a unit mass particle with “time” denoted \(\xi\) under the influence of the potential
\[
U = -\frac{n^2}{2} + E,
\]
(3.3)

where \(E\) is an arbitrary constant. Regions of high refractive index are analogous to regions with low mechanical potential energy where particles tend to gravitate toward. As an example, an Eaton lens which is a spherical retroreflector has an refractive index profile
\[
n(r) = \sqrt{2/r - 1}, \quad r \leq 1,
\]
(3.4)

where \(r\) is the radial distance from the device center. Light rays bend back towards the direction from where it came as illustrated in Figure 3.1(a). In this elegant analogy, light acts as if it is moving ball traveling on a curved surface where its trajectory is determined shape of the curved surface as illustrated in Figure 3.1(b).

Figure 3.1: A diagram illustrates the regions with high refractive index is analogous to regions with low mechanical potential in classical Newtonian mechanics. (a) An illustration of light rays bending towards back to the direction of where they came from for a Eaton lens with refractive lens profile (3.4) which is shown as the shaded region. (b) An analogy in classical Newtonian mechanics where the trajectory of a moving ball is changed by a curved surface.
3.1.1 Coordinate Transformations

Consider a mapping between two coordinate systems in 3 dimensions $x^i = (x^1, x^2, x^3)$ and $x'^i = (x'^1, x'^2, x'^3)$ as

$$x'^i = x'^i(x^i), \quad \text{for } i = 1, 2, 3, \quad i' = 1', 2', 3', \quad (3.5)$$

and the inverse coordinate transformation as

$$x^i = x^i(x'^i), \quad \text{for } i = 1, 2, 3, \quad i' = 1', 2', 3', \quad (3.6)$$

where each coordinate is a function of all coordinates in the other coordinate system. (3.5) and (3.6) can be viewed as a set of mappings between the variables $x^i$ to $x'^i$ and vice versa. It is important to note that the coordinate transformation is a change of variables. Consider a coordinate transformation in 2-D expressed using complex notation as a mapping from the $z$-plane to $w$-plane, both of which are the complex planes, as $w = u(x,y) + jv(x,y)$, where $u$ and $v$ are arbitrary functions of $x$ and $y$. Therefore, $(u,v)$ is generally a curved coordinate system. Mathematical expressions such as the wave equation expressed in $(x,y)$ coordinate system are thus different from the expressions in $(u,v)$ coordinate system. This is what we mean by a coordinate transformation and it is not to be confused with the way that the function $w = u + jv$ is expressed which is still in the real part $u$ and imaginary part $v$ of a complex (Cartesian) representation of a complex number.

After a coordinate transformation, the unit vector which points in the direction of a coordinate line is generally different than in the original coordinate system and they are a function of position. Expressions for the vector component of a vector field is also different because the components are computed as the vector projection of the vector field onto the unit basis vectors. The most often encountered examples of a coordinate transformation is that of the transformation from the Cartesian coordinate system to the cylindrical or spherical coordinate systems. The expressions for the divergence, curl, Laplacian operators and differential length, surface and volume are all different than that of the Cartesian coordinate system. The quantities obtained from divergence, curl and Laplacian are coordinate-invariant quantities of a vector field independent of whether one chooses to cover the space with a coordinate system. Likewise, differential length, surface and volume elements are also coordinate-invariant quantities. To account for an arbitrary coordinate system in which the coordinate lines are curved, the coordinate-invariant quantities must take into the account for this “curvature” in order to “correct” for it, leading to expressions that contains this information. For example, the divergence operator in the spherical coordinate system is not just a superposition of the partial derivatives with respect to each coordinate like in the Cartesian coordinate system. Furthermore, since the unit basis vector itself is a function of position, the derivative of a vector has contributions not only from the components of the vector, like in Cartesian coordinate system, but also from the unit basis vectors itself. This forms the basis for the topic of differential geometry, a powerful mathematical tool needed to describe Maxwell’s equations in a general curvilinear coordinate system, from which the equivalence between a spatial media and a geometry is drawn. It is also the tool that underlies the theory of general relativity.
3.2 Transformation Optics in 3-D

Designing transformation optics devices in 3-D amounts producing a desired set of coordinate transformation between a virtual space and the physical space in which the TO devices reside. The fully tensorial material parameters $\varepsilon_r$ and $\mu_r$ are determined by the coordinate transformations between the two spaces. The coordinate transformation is generally determined by geometry of the devices in the two coordinate systems. For example, a bent rectangular waveguide can appear electromagnetically as a straight waveguide by filling the bent waveguide with an appropriate set of materials which is determined by a coordinate transformation between the bent waveguide and a straight waveguide.

For the purpose of design for our TO device, we regard the physical space as the flat space in which we allow the presence of an arbitrary material and the virtual space is curved and empty ($\varepsilon_r = \mu_r = 1$). In the general case for TO, these constraints on the properties of space and material do not have to be present – both spaces can be arbitrary curved and can be filled with arbitrary materials.

To derive the fully tensorial material parameters resulting from a coordinate transformation requires knowledge in differential geometry and tensor algebra. An introduction of the two topics in thesis would not provide their justice. Instead, we refer readers to [3, 107, 108, 109] for a thorough understanding of these topics. However, in this section we provide the results of the derivation and focus on the engineering considerations associated with realizing such artificial materials.

Defining a physical space in the primed coordinate system and a virtual space in the unprimed coordinate system as in (3.5), one can find the material parameters as a function of the partial derivatives of the coordinate mappings

$$\varepsilon_r = \mu_r = \frac{[\Lambda_{ij}']\times [\Lambda_{ij}']^T}{\det[\Lambda_{ij}']}, \quad \Lambda_{ij}' = \frac{\partial x^i}{\partial x'^j},$$

where $\varepsilon_r$ and $\mu_r$ are the required relative permittivity and permeability respectively, $[\Lambda_{ij}']$ denotes a matrix whose $(i, i')$-th entry is denoted by $\Lambda_{ij}'$, $T$ is the matrix transpose operator and $\det$ is the determinate operator of a matrix.

A media that is deduced from a coordinate transformation, such as (3.7), is called a transformation media. The E- and H-fields are the same in the two spaces. The media in the physical space simply moves the E- and H-fields into a different position according to the coordinate transformation. Transformation media are special media that achieve such a feat. Note that the transformation media is impedance-matched to vacuum since $\varepsilon_r = \mu_r$. It is clear that not all media are a transformation media. Light wave propagation in each of the two spaces is electromagnetically equivalent. By that, we mean that the optical length between any two points in one space is exactly the same as the optical length between the two corresponds points in the other space. Since the virtual coordinate system is arbitrary, TO in 3-D is a mechanism for arbitrary control of electromagnetic waves, bending, expanding, etc.

The realization of the material with arbitrary values, such as (3.7), falls into the field of metamaterials where the man-made materials are engineered to have a set of desired properties. In general, it is difficult to realize the required material described by (3.7) because the Jacobian matrix are a purely mathematical construct and is not subject to any physical constraints [99]. More specifically, the materials are difficult to realize because

- they are inhomogeneous in both permittivity and permeability,
- there is a need to produce a magnetic response ($\mu_r \neq 1$), and
the relative permittivity and permeability can be negative, zero, less than 1 or be arbitrarily large, all of which are not typically found in nature.

Any one of the above alone can present significant engineering challenges. In addition, metamaterials tends to use resonant structures like split-ring resonators [110, 111, 112] to create an artificial magnetic response for $\mu_r$ but they tend to be lossy [111, 113], making RF devices even more difficult to realize.

Having said that, however, metamaterial engineering has greatly helped the developments in TO devices as more exotic materials becomes possible as time progresses. Metamaterial research has advanced tremendously since its inception many decades ago and it is a topic worthy of its own thesis. Interesting properties of metamaterials includes but are not limited to, negative-index refraction [114, 115], negative group velocity [116], negative permittivity and permeability [117, 118], that allows the realization of novel applications such as cloaking [110, 119], super resolution imaging [120, 121, 122, 123], directive antennas [124, 125, 126], device miniaturization [127, 128, 129], etc. Tunable metamaterials [130, 131] can be realized by using varactors, PIN diodes, MEMs switches, microfluids, liquid crystals, etc [130]. Metamaterials allow design methods like TO to exercise its full potential to create even more novel devices for innovative applications.

TO devices are difficult to realize in 3-D due to the aforementioned challenges. However, a few 3-D TO devices have been realized [85, 77, 132, 133] but they are not true 3-D TO devices. The design method of these devices follow the same recipe but they are not true 3-D devices because no 3-D coordinate transformation is performed. These devices are designed by first generating a conformal coordinate transformation, which is described in detail in the upcoming sections, producing an inhomogeneous but dielectric-only profile in 2-D that is then rotated about an axis to produce the 3-D profile. Such devices have shown to function well and simple in the design principle but they can not manipulate light waves in an arbitrary manner.

### 3.3 Transformation Optics in 2-D

Transformation optics in 2-D deserves some special attention. The 2-D coordinate pair $(x, y)$ can be thought of as a single complex number where $x$ is the real part and $y$ is the imaginary part. The design of 2-D devices can leverage the power of complex analysis without necessarily the need to invoke differential geometry or tensor algebra like in 3-D cases. Partial derives can be expressed as complex derivatives which are well-defined. Furthermore, for the transverse-electric (TE) polarization, the polarization is completely decoupled from the rest of the polarizations. The Maxwell’s equations in source-free empty space are

$$\nabla \times \mathbf{E} = -j\omega\varepsilon_0\mathbf{H}, \quad \nabla \times \mathbf{H} = j\omega\varepsilon_0\mathbf{E},$$

(3.8)

where $\varepsilon_0$ and $\mu_0$ are the free space permittivity and permeability. For TE polarization in 2-D, $\mathbf{E} = E_\perp \hat{z}$, $\mathbf{H} = H_z \hat{x} + H_y \hat{y}$ and $\partial / \partial z = 0 \forall x, y$ which yields a scalar wave equation in the transverse ($z$ direction) electric field component $E_\perp$

$$\nabla^2 E_\perp + k^2 E_\perp = 0, \quad k^2 = \omega^2 \mu_0 \varepsilon_0 \mu_r \varepsilon_r.$$  

(3.9)

As alluded to earlier, complex analysis lends a powerful mathematical tool for analysis in 2 dimensions. We can write the wave equation in 2-D in complex notation as follows [3]. Expressing the
coordinate pair \((x, y)\) as a complex number \(z = x + jy\). Then \(z^* = x - jy\) and
\[
x = \frac{z + z^*}{2}, \quad y = \frac{z - z^*}{2j}.
\] (3.10)

The partial derivative with respect to \(x\) and \(y\) are, with the help of the chain rule of partial differentiation,
\[
\partial_x = \frac{\partial z}{\partial x} \partial_x + \frac{\partial z^*}{\partial x} \partial_{x^*}, \quad \partial_y = -j \frac{\partial z}{\partial y} \partial_x - j \frac{\partial z^*}{\partial y} \partial_{x^*} = \partial_x - \partial_{x^*}.
\] (3.11)

Hence, the \(\nabla^2\) operator becomes
\[
\nabla^2 = \partial_x^2 + \partial_y^2 = 4 \partial_{x^*} \partial_{x^*}.
\] (3.12)

The scalar wave equation (3.9) becomes
\[
(4 \partial_{x^*} \partial_{x^*} + k^2) E_\perp = 0, \quad k^2 = \omega^2 \mu_o \varepsilon_o \mu_r \varepsilon_r.
\] (3.13)

Consider a coordinate transformation from \(z = x + jy\) to \(w = u + jv\) where \(w\) is only a function of \(z\) and not \(z^*\), then
\[
w = f(z), \quad \partial_{z^*} w = 0.
\] (3.14)

Applying the chain rule to obtain
\[
\partial_z = \frac{dw}{dz} \partial_w, \quad \partial_{z^*} = \frac{dw^*}{dz^*} \partial_{w^*}.
\] (3.15)

Substituting (3.15) into (3.13), the scalar wave equation becomes
\[
\left(4 \left|\frac{dw}{dz}\right|^{2} \partial_w \partial_{w^*} + k^2 \right) E_\perp = 0,
\] (3.16)

Re-writing (3.16)
\[
\left(4 \partial_w \partial_{w^*} + \hat{k}^2 \right) E_\perp = 0, \quad \hat{k}^2 = k^2 \left|\frac{dz}{dw}\right|^{2}.
\] (3.17)

Note that the new wavenumber \(\hat{k}\) in \((u, v)\) coordinate system and the original wavenumber \(k\) are scaled by a factor that is determined by the coordinate transformation \(w = f(z) = u + jv\). More importantly, (3.17) is another wave equation expressed in a general curvilinear coordinates system \((u, v)\), an equation that governs the same electromagnetic wave propagation as the old coordinate system if
\[
\hat{k} = k \left|\frac{dz}{dw}\right|,
\] (3.18)
or equivalently
\[
\omega \sqrt{\mu_o \varepsilon_o \mu_r \varepsilon_r} = \omega \sqrt{\mu_o \varepsilon_o \mu_r \varepsilon_r} \left|\frac{dz}{dw}\right|.
\] (3.19)

It is easy to see that (3.19) is satisfied for
\[
\hat{\mu}_r = 1, \quad \sqrt{\hat{\varepsilon}_r} = \sqrt{\varepsilon_r} \left|\frac{dz}{dw}\right| \quad \text{or} \quad \hat{n} = n \left|\frac{dz}{dw}\right|,
\] (3.20)
where $\hat{n} = \sqrt{\hat{\varepsilon}_r}$ and $n = \sqrt{\varepsilon_r}$ are refractive indexes in the $w$ and $z$ spaces respectively. Hence, if the conditions (3.20) are satisfied, then the electromagnetic wave propagation is exactly the same in the two coordinate systems $w = u + jv$ and $z = x + jy$. The expression for $\hat{\varepsilon}_r$ in (3.20) can be explicitly expressed as the partial derivatives as

$$\hat{\varepsilon}_r = \left| \frac{dz}{dw} \right|^2 = \left( \frac{d\hat{u}}{dx} \right)^2 + \left( \frac{d\hat{u}}{dy} \right)^2 \right)^{-1}. \quad (3.21)$$

It is important to note that, for the TE polarization, no magnetic response is needed in the material because $\hat{\mu}_r$ is set 1 in (3.20) and only the permittivity needs to be engineered for this polarization. This is due to the conformal coordinate transformation used in (3.14) which is defined in the next section.

### 3.3.1 Conformal Coordinate Transformation

In this section, the notion of a conformal coordinate transformation in 2 dimensions is defined. Let $f$ be the mapping between two coordinate systems $z = x + jy$ and $w = u(x, y) + jv(x, y)$ on the complex plane. A conformal coordinate transformation $f_c: z \rightarrow w$, $z, w \in \mathbb{C}$ is closely related to the satisfaction of the well-known Cauchy–Riemann equations in complex analysis

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}, \quad \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}, \quad (3.22)$$

and the existence of the complex derivative $dw/dz$,

$$\frac{dw}{dz} = w'(z) = \lim_{\Delta z \rightarrow 0} \frac{w(z + \Delta z) - w(z)}{\Delta z}. \quad (3.23)$$

Their relationships are as follows:

- If the partial derivatives $\partial u/\partial x$, $\partial v/\partial x$, $\partial u/\partial y$ and $\partial v/\partial y$ are continuous in a neighborhood of $z = z_0$, then Cauchy-Riemann conditions are both necessary and sufficient conditions for the existence of $f'(z_0)$ [134].

- An equivalent statement to satisfaction of the Cauchy-Riemann conditions is that the mapping $f$ is independent of $z^*$, namely, $\partial w/\partial z^* = 0$ [3].

- If $w(z)$ is analytic in a domain $D$, then $w(z)$ is conformal every point in $D$ where $dw/dz \neq 0$ [134].

We provide an intuitive graphical interpretation of a conformal coordinate transformation. Consider a mapping $w = f_c(z) = e^z = u + jv$, which is shown in Figure 3.2. A rectangular grid in $z$ space is mapped to a set of curved grids in the $w$ space as shown which are the images of constant $x$ and $y$ lines. A few properties of a conformal coordinate transformation are immediately obvious from the graphical illustration:

- A conformal coordinate transformation preserves angles. A point is shown in each space and the angles formed between the two corresponding tangent vectors in red and blue are the same, so long as the $dw/dz \neq 0$. Note that the sense of the angle is also preserved.

- A conformal coordinate transformation stretches the space equally in all directions. The measure of length in all directions remains the same. Hence, a differential rectangular area in one space
is mapped to another differential rectangular area with the same aspect ratio but the size of the rectangle has changed, which is clearly shown in Figure 3.2.

These properties can be rigorously proven in an elementary complex analysis textbook which is omitted here. In the language of differential geometry, a conformal coordinate transformation is a coordinate transformation that yields a metric tensor $g_{ij}$ that is scaled by a constant at every point

$$g_{ij} = \Omega(x,y) g'_{ij}, \quad g_{ij} = \mathbf{g}_i \cdot \mathbf{g}_j, \quad \mathbf{g}_i = \frac{\partial \mathbf{r}}{\partial x^i},$$

(3.24)

where $\mathbf{r}$ is the position vector and $\mathbf{g}_i$ is the tangent vector along the $x^i$ coordinate line. Note that $|\mathbf{g}_i| \neq 1$. It is easily seen that a conformal coordinate transformation stretches the space in an isotropic manner.

$$z = x + jy \text{ space} \quad \quad \quad \quad \quad w = u + jv \text{ space}$$

Figure 3.2: An example of a conformal coordinate transformation from $z = x + jy$ to $w = f_c(z) = e^z = u + jv$. A conformal coordinate transformation preserves angles and stretches the space isotropically, independent of direction. Lines of constant $x$ and $y$ in $z$ space map to corresponding lines in $w$ space.

The coordinate transformation in (3.14) is actually a conformal coordinate transformation in which we drew an equivalence between a coordinate transformation and a set of materials shown in (3.19) and (3.20). A similar analysis can be performed on Maxwell’s equations using differential geometry and tensor algebra to produce the expression for the material parameters (3.7) in 3-D.

Instead of using complex analysis, the Cauchy-Riemann conditions (3.22) can be directly substituted into the 3-D relative permittivity and permeability (3.7) equations to yield

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \varepsilon_{r,zz}(x,y) \end{bmatrix}, \quad \varepsilon_{r,zz}(x,y) = \left[ \left( \frac{\partial u}{\partial x} \right)^2 + \left( \frac{\partial u}{\partial y} \right)^2 \right]^{-1},$$

(3.25)

which has the same expression for $\varepsilon_{r,zz}$ as in (3.21) and the two formulations are consistent. Note that for a TE polarization in 2-D, the two H-field components experience free space as $\mu_{r,xx} = \mu_{r,yy} = 1$ and the transverse E-field component $E_\perp$ experiences the relative permittivity $\varepsilon_{r,zz}$.

It is clear that not all coordinate transformations are conformal and not all media can correspond to a conformal transformation. The first fact is obvious from the requirements of a conformal coordinate transformation. We provide an example of the second fact. One can show that the only refractive index
profile which varies only in one direction \( y \) that corresponds to a conformal coordinate transformation is in the form [3]

\[
    n(y) = n_o e^{ay}, \quad n_o = \text{const.}, \quad a = \text{const}.
\]

(3.26)

All other \( n(y) \) functions do not correspond to a conformal coordinate transformation in 2-D.

In 2-D, there is a large variety of conformal coordinate transformations and it is only limited by the analytic property of a complex function. All regions in the domain of an analytic function \( w = f(z) \) where \( dw/dz \neq 0 \) is conformal. In 3-D, every conformal coordinate transformation is a Möbius transformation [135], which is a coordinate transformation composed of transitions, scales, rotations, reflections and inversions. Hence, conformal coordinate transformations in 3-D are quite limited in their types. Given that a 3-D conformal coordinate transformation, the material parameters can shown to be [107]

\[
    \varepsilon_r = \mu_r = \begin{pmatrix}
        f(x, y, z) & 0 & 0 \\
        0 & f(x, y, z) & 0 \\
        0 & 0 & f(x, y, z)
    \end{pmatrix}.
\]

(3.27)

The coordinate transformations from Cartesian to either cylindrical or spherical coordinate systems produce orthogonal coordinate lines, hence the angle is preserved, but the transformations are not conformal because the spaces described by cylindrical and spherical coordinate systems are stretched anisotropically.

### 3.3.2 Graded-Index Approach

The graded-index approach to design dielectric devices has been known for a long time. TO is closely related to the graded-index approach in 2-D where conformal coordinate transformation is used to produce dielectric-only or graded-index materials. TO provides an alternative approach to design graded-index materials particularly when the transformation of the device shape is needed. However, as mentioned before, not all refractive index profiles correspond to a conformal coordinate transformation. Hence, the refractive profile produced by a conformal coordinate transformation is a subset of all possible refractive index profiles. It is obvious that, in general, TO provides a method to design devices that can not be or difficult to be designed using graded-index approach in both 2-D and 3-D. For example, a beam splitter which is naturally designed by using a coordinate transformation that splits a coordinate plane into two pieces [104] but it would be difficult to design using the graded-index approach in which the gradient of the material needs to be carefully designed to guide the light to the desired directions.

Some useful devices can be designed using graded-index approach such as the Luneburg lens without the aid of TO. Certain refractive index profiles can be inferred for a desired set of light trajectories. To be clear, this is a design problem that solves for the refractive index \( n(r) \), where \( r \) is the position vector, when a family of desired path of light trajectories \( \mathbf{P}_i = (x_i(\xi), y_i(\xi), z_i(\xi)), \forall i \), are known. This problem is solved by Luneburg for spherically symmetric profiles \( n = n(r) \) whereby he derived the profile for the famous Luneburg lens [3].

### 3.4 Generation of 2-D Conformal Coordinate Transformation

The design of a conformal coordinate transformation depends on the geometry a curved virtual device and the geometry of the physical TO device. The generation of 2-D conformal maps is an involved topic
of its own. It can be achieved by using either analytic equations [136] or numerical quasi-conformal techniques [137] where rectangles map to rectangles with a slightly different aspect ratio.

Having a coordinate transformation that is as conformal as possible at all points is important because the material parameters may end up being too anisotropic. The advantage of having an analytic equation is that the produced coordinate transformations are perfectly conformal, whereas in quasi-conformal methods, the degrees of conformity at every point needs to be checked to ensure it is within acceptable limits.

Figure 3.3(a) shows an analytic conformal coordinate transformation from the upper-half of the complex plane to the region on the right half of the figure [136]. Such a transformation is potentially useful in cloaking applications where objects can be concealed beneath the wedge. Figure 3.3(b) shows another example of analytic conformal coordinate transformation but it is mapped from the strip shown [136]. Such a coordinate transformation could be useful in light-bending applications. A wide variety of conformal coordinate transformation can be found in literature with tabulated catalogs of many different types of the coordinates transformations [138, 139, 136, 137]. This is by no means an exhaustive list. However, it is difficult to design analytical expressions for an arbitrarily shaped geometries. A more versatile tool is needed to design conformal coordinate transformations that is ideally perfectly conformal. Schwarz-Christoffel transformation which is described in the next section provides a solution.

\[
a = 0.75 \\
t = \sqrt{\frac{z}{a^2z + 1}} \\
u = \frac{1 + at}{1 - at} \\
w = u \left(\frac{t - 1}{t + 1}\right)^a
\]

\[
p = 1.2 \\
s = e^z \\
t = \sqrt{\frac{s - 1}{s + p^2}} \\
w = p \text{arctanh} t - \text{arctan}(pt)
\]

Figure 3.3: Two examples [136] of analytic conformal coordinate transformation from \(z\)-plane to \(w\)-plane. (a) A mapping from the upper-half of the complex plane. This mapping could be used in applications where objects can be concealed underneath the wedge. (b) A mapping from the strip \(0 < \Im(z) < \pi\). This mapping could be used to bend light by 90°.
3.4.1 Schwarz-Christoffel Transformation

The Schwarz-Christoffel (SC) transformation is conformal coordinate transformation in the complex plane from a canonical region to the interior of a polygon. While many types of SC transformation exist, the standard SC transformation is a mapping from the upper-half of the complex plane to the interior of a polygon. A set of points on the real axis maps to the vertices of the polygon. The polygon under consideration is a generic polygon (a simply-connected region) which can be open or closed. In the case of an open polygon, zero or negative interior angles are allowed. We refer readers to references [134, 140, 139, 136] in order to obtain a full description of SC transformation. In this section, we illustrate the fundamental of how SC transformation works and some of the capabilities to illustrate its potential usefulness in designing 2-D TO devices.

We illustrate how the standard SC transformation can transformation points on the real axis to vertices of a polygon. Consider \([134, 140]\)
\[
dw = A(z - u_1)^{\alpha_1/\pi - 1}(z - u_2)^{\alpha_2/\pi - 1} \cdots (z - u_n)^{\alpha_n/\pi - 1} \, dz, \tag{3.28}
\]
where \(u_i, \alpha_i \in \mathbb{R}\). Taking the argument of both sides to obtain
\[
\text{arg} \, dw = \text{arg} \, A + \left(\frac{\alpha_1}{\pi} - 1\right) \text{arg}(z-u_1) + \left(\frac{\alpha_2}{\pi} - 1\right) \text{arg}(z-u_2) + \cdots + \left(\frac{\alpha_n}{\pi} - 1\right) \text{arg}(z-u_n) + \text{arg} \, dz. \tag{3.29}
\]
A point \(P\) on the real axis in the \(z\)-plane is marked in Figure 3.4 where it is located to the left of \(u_1\). As the point \(P\) moves to by \(dz = du\), \(\text{arg} \, dz = 0\) and each \((\alpha_i/\pi - 1) \, \text{arg} \, (z-u_i)\) term is \(\pi\), making \(\text{arg} \, dw\) a constant. Hence, for as long as \(P\) remains to the left of \(u_1\), \(\text{arg} \, dw\) is constant and it traces out a (red) line in the \(w\)-plane. As the point \(P\) moves across \(u_1\), the value of the term \((\alpha_1/\pi - 1) \, \text{arg} \, (z-u_1)\) changes from \(\alpha_1 - \pi\) to 0. Hence \(\text{arg} \, dw\) abruptly decreases by amount \(\alpha_1 - \pi\), or abruptly increases by amount \(\pi - \alpha_1\), changing the direction of the line segment traced out in the \(w\)-plane (blue line). Hence, \(u_1\) maps to the vertex \(w_1\) as shown. Likewise, \(u_2\) maps to \(w_2\) and the green lines maps to each other after \(P\) has across to the right side of \(u_2\) experiencing another abrupt increase in angle by \(\pi - \alpha_2\). At each vertex the interior angle is \(\alpha_i\) as shown.

The differential form \(3.28\) suggests that the mapping from the \(z\)-plane to \(w\)-plane is \([134]\)
\[
w = A \int^z (\xi - u_1)^{\alpha_1/\pi - 1}(\xi - u_2)^{\alpha_2/\pi - 1} \cdots (\xi - u_n)^{\alpha_n/\pi - 1} \, d\xi + B, \tag{3.30}
\]
where \(A, B \in \mathbb{C}\) are constants, \(u_i \in \mathbb{R}\) are the location of the preimage of the vertices of the polygon. The lower bound of the integral is not specified because it would be absorbed into \(A\) and \(B\). The proof that the upper-half of the complex plane is mapped to the interior of the polygon is omitted here. While we have illustrated a closed polygon, \(3.30\) is applicable to open polygons as well.

Other types of SC transformations allow for different canonical regions, such as the unit circle, infinite strips and rectangles to be mapped to a polygon. SC transformation also allows mappings to the exterior of polygons instead of their interior.

Figure 3.5(a) shows an example of a standard SC transformation that is a mapping from the upper-half of the complex plane to the open polygon shown. This open polygon is specified by three vertices, two of which are marked in the figure having interior angles of \(3\pi/2\) and \(\pi/2\). The third vertex is located at infinity and it has an interior angle of \(-\pi\), making the polygon an open polygon. Figure 3.5(b) shows
Chapter 3. Design of a Wideband Reflector Using Transformation Optics

Figure 3.4: An illustration of a SC coordinate transformation from the $z$-plane to the $w$-plane. Points $u_i$ on the real axis in $z$-plane are mapped to vertices $w_i$ of a polygon in the $w$-plane.

an example of a SC transformation that maps the unit disk to the interior of a rectangle (a polygon). Such a coordinate transformation is useful for designing devices that converts cylindrical to plane waves.

Figure 3.5: Two examples of SC coordinate transformations. (a) A SC transformation from the upper-half of the complex plane to this open polygon. The third vertex has interior angle of $-\pi$. (b) A SC transformation from the unit disk to this rectangle.

For the purpose of designing TO reflectors, we focus on the type of SC transformation that maps a rectangular region into an arbitrarily shaped closed polygon. This type of SC transformation is particularly useful because it allows the mapping of arbitrarily shaped devices to be approximated by a polygon with finite number of point in which a rectangle can map to, as illustrated in Figure 3.6. The inhomogeneous material within the rectangular region is given by (3.20) which is a function of the coordinate transformation between the rectangle and the polygon. The aspect of the rectangle $m$, which is called the conformal modulus associated with the polygon [139, 141], determined by the shape and the four chosen vertices of the polygon that maps to the four corners of the rectangle, as illustrated in Figure 3.6. In general, the conformal modulus is a property of a generalized quadrilateral illustrated in Figure 3.6, which is Jordan region with four distinct points on its boundary [142, 141]. Conformal coordinate transformations exist only between two generalized quadrilaterals with the same conformal modulus [139, 107]. It is clear that no conformal coordinate transformation can map a square to a rectangle.

Finding the locations of the prevertices that map to the polygon is a Schwarz-Christoffel parameter
3.5 Reflector Design Using Schwarz-Christoffel Transformation

There are several methods to design a reflector using conformal coordinate transformation. First, we define the geometry of the reflector in the virtual space. Figure 3.7 shows a set of incident fields impinging on a parabolic reflector that is rotated about its apex by an angle $\alpha_o$, producing a reflected wave at angle $\phi_o = 2\alpha_o$ with respect to the vertical axis. This method of rotation has advantages over other rotation methods because the reflector remains illuminated by the feed as the parabolic reflector is rotated. In addition, the rotation angle only needs to be half of a desired reflected angle $\phi_o$.

Figure 3.7: Rotation of a parabolic reflector about its apex by an angle $\alpha_o$ to produce reflected wave at angle $\phi_o = 2\alpha_o$ with respect to the vertical axis.
We use SC transformations to produce the rotated parabolic reflector described in Figure 3.7. Since SC transformations are mappings from polygons to polygons, the vertices of the polygons can be positioned such that arbitrarily shaped objects can be approximated using polygons. For example, a subset of the boundaries of a polygon can take the shape of a rotated parabolic reflector that is discretized spatially with a finite number of points, each of which is vertex of the polygon. Figures 3.8 shows some curved virtual spaces which are mapped from the physical flat space. In each coordinate transformation, a subset of the complex plane is mapped the polygon as shown. Blue lines highlight the boundaries of the SC transformation. Portions of the boundaries are marked by red circles are parabolic. (a) is a mapping from the upper half of that complex plane to this open polygon shown. (b) and (c) are mappings from a rectangular region to the closed polygons shown. The boundaries of (c) contains a section of the rotated parabolic described in Figure 3.7.

![Figure 3.8](image)

Figure 3.8: Three SC transformations that are mapped from the physical flat space to the virtual curved spaces shown here. Each of these is a polygon where a subset of its boundary, highlighted in red circles, makes up for a parabolic reflector. (a) is a mapping from the upper half of the complex plane to this open polygon, (b) and (c) are mappings from a rectangular region to the closed polygons shown. A section of the boundaries of the polygons is parabolic. (c) contains a section of the rotated parabolic reflector with $\alpha_o = 20^\circ$ described in Figure 3.7.

### 3.6 Reflector Design Considerations and Permittivity Profile

Figure 3.9(a) shows a SC coordinate transformation in the virtual space which is mapped from a rectangular region in the physical flat space in Figure (3.9)(b), which contains a set of inhomogeneous
materials. The entire space between the focal point and the rotated parabolic reflector is transformed. An open-ended waveguide (OEWG) is placed at the focal point. With the coordinate transformation known, the permittivity profile in the physical flat $z$ space is computed using (3.20) resulting in

$$\epsilon'_r = \left| \frac{dw}{dz} \right|^2. \quad (3.31)$$

The relative permittivity of the virtual space is unity, $\epsilon_r = 1$. The relative permittivity profile for a conformal coordinate transformation can easily be visualized given the set of curved coordinate lines in the virtual space. According to (3.31), regions with a large change in $w$ yields higher relative permittivity; larger cells yield higher relative permittivity. The SC transformation in Figure 3.9(a) is computed for $\alpha_o = 30^\circ$, for the maximum reflected beam angle of $\phi_{o,\text{max}} = 2\alpha_{o,\text{max}} = 60^\circ$ with respect to the vertical axis. The parabolic reflector has a diameter of $5\lambda$ at $5$ GHz with $f/D = 1.5$.

![Figure 3.9](image)

Figure 3.9: (a) is a SC transformation from a rectangular region in the physical flat space to this curved virtual space shown, whose boundaries contain a section of the rotated parabolic reflector described in Figure 3.7. $\alpha_o = 30^\circ$. (b) is the corresponding permittivity profile of the SC transformation in (a) computed by (3.31). The wavelength $\lambda = 60$ mm is at $5$ GHz.

Any polygon can be chosen in the virtual space so long as its boundary includes the desired parabolic section. However, there are some design considerations for the overall shape of the polygon and for the induced transformation media:

- The position of the focal point is a function of scan angle $\phi_o$. For each angle of rotation, there is a different coordinate transformation between the virtual and physical space since virtual space contains a different polygon at each angle. Hence, the focal point in the physical space moves according to the coordinate transformation. The amount of the movement depends on the coordinate transformation used which in turn depends on the shape of the overall polygon. When the focal point is far away from the reflector, as is the case in Figure 3.9(a), the amount of movement is minimal. The movement of the focal point may not be a concern for fixed designs, however, it
may need a consideration for reconfigurable designs that rotates the virtual parabolic reflector.

- The relative permittivity near the focal point is ideally unity, as in a practical scenario, a feed antenna is placed at the focal point. This means that the coordinate transformation does not stretch the spaces in the vicinity of the focal point.

- The light waves traveling in and out of the TO device need to have a smooth transition. This means that the relative permittivity needs to remain close to unity on the top and two side boundaries of the TO device. Otherwise the light waves would experience refraction and reflection which is expected to degrade the beam quality of the TO reflector.

- Since the relative permittivity profile is determined by a coordinate transformation, there could be regions of the relative permittivity profile that are less than unity. These regions create a problem particularly for wideband applications because the metamaterials associated with such properties tend be dispersive. Hence, the area of these regions needs to be kept in check.

In light of the considerations mentioned, we have placed the boundary of the closed polygon shown in Figure 3.9(a) that satisfies all of the above considerations. A SC toolbox [143] is used to compute a SC transformation that maps from a rectangular region, shown in Figure 3.9(b), to the designed closed polygon in Figure 3.9(a). Note that the aspect ratio of the rectangle is not chosen and it is the conformal modulus of the closed polygon. In addition, as the parabolic reflector is rotated in Figure 3.9(a) from \(\alpha_o = 0^\circ\) to \(\alpha_o = 30^\circ\), the focal point in the permittivity profile in Figure 3.9(b) moves by a vector \((-10.468, -57.904)\) mm. This movement of the focal point is caused by the different coordinate transformations used that maps the different polygons to its corresponding rectangular regions.

The chosen polygon has two features. First, it transforms all of the space between the focal point and the curved reflector, allowing the coordinate transformation to morph back to Cartesian-like grid near the vicinity of the focal point. This large space also restricts the amount of movement of the focal point in the physical space because the coordinate transformation near the focal point is more independent of the geometry of the curved reflector at the bottom boundary. Second, there are two extensions regions 1 and 2 that are added on the two sides of the reflector, which corresponds to regions A and C respectively in Figure 3.9(b). The purpose of these regions is to allow for the coordinate transformation to morph back to Cartesian-like grid along the top and two side boundaries of the TO reflector. The relative permittivity is close to unity near these boundaries which allows for light waves to smoothly travel in and out of the TO reflector with little refraction. In practice, the thickness of the permittivity profile can be truncated to a thickness \(T\) producing a thinner profile.

Since we want the relative permittivity to be as close to unity as possible along the top boundary of the TO reflector, a natural question to ask is whether it is better to have a smaller coordinate transformation region so that the permittivity profile does not need to be truncated as much, as illustrated in Figures 3.10(a) and (b), or a large coordinate transformation region and then truncate the permittivity profile, as illustrated in Figures 3.10(c) and (d). Figures 3.10(a) and (c) shows the same parabolic reflector and its extension regions but they are located at different vertical positions. The apexes of the parabolas are located at \((0, -0.5f)\) and \((0, -2f)\) respectively. A variable \(y_{\text{shift}}\) is defined as the amount of shift for the parabola in the vertical direction so a larger \(y_{\text{shift}}\) leads to a larger coordinate transformation region. We examine the maximum relative permittivity along the lines \(y = T = 150\) mm which are marked on Figures 3.10(b) and (d). Figure 3.11 shows the maximum relative permittivity along
these lines as a function of $y_{\text{shift}}$. As the size of the transformation region grows, the maximum relative permittivity decays rapidly but it settles to a minimum. This is the expected behavior as more space is added in front of the reflector, the finite coordinate transformation acts more like an open transformation as shown in Figure 3.8(a) and the added space further from the parabolic reflector have little impact on the grid lines near the reflector. Hence, Figure 3.11 shows that for the same truncation thickness $T$, the added space helps to reduce the relative permittivity closer to unity along the truncation boundary but only up to a point, beyond which the added space have little impact on the permittivity. In the finalized design shown in Figure 3.9(a), we have chosen $y_{\text{shift}} = f$ so that the maximum relative permittivity along the top a boundary at $y = 150$ mm is approximately 1.34.

![Figure 3.10: Impact of the coordinate transformation size on permittivity profile. (a) A coordinate transformation region with corresponding $\varepsilon_r$ map shown in (b). (c) A coordinate transformation region that contains the same parabolic section as in (a) but its apex is shifted down to $(0, -2f)$, making the transformation region much larger than (a). (d) The $\varepsilon_r$ profile corresponding to (c).](image)

We examine what happens when the coordinate transformation do not include the extension regions. When the coordinate transformation is performed only in region 2 as shown in Figure 3.12, the cell sizes along the left-boundary are larger than the rest of the cells and they correspond to a higher refractive
Figure 3.11: The maximum \( \varepsilon_r \) along \( y = 150 \text{ mm} \) line as a function of \( y_{\text{shift}} \), the \( y \) position of the apex of the parabolas in Figures 3.10(a) and (c).

index that would produce an impedance mismatch as waves propagates across this boundary. Note that the overall permittivity can be scaled by a constant such that the relative permittivity on this boundary is approximately unity so that no impedance mismatch occurs but it would make the rest of the permittivity profile to have a less-than-unity relative permittivity. This approach is not favored since such regions would be dispersive and degrade the wideband performance of the reflector. Therefore, the two extension regions are added to help mitigate this problem.

Figure 3.12: A SC transformation performed only in region B of Figure 3.9(a). \(|dw/dz|\) is large compared to unity on the left-most boundary creating a mismatch in the impedance.

Readers may have noticed that the size of the reflector in the physical size in Figure 3.9(b) is smaller than that of in (a). This is a result of the coordinate transformation and it is expected. Moreover,
since the material is a transformation media whose material parameters is induced by a coordinate transformation, the physical TO device is electromagnetically the same as the curved virtual device and hence, the electrical sizes of the reflector is the same in both spaces.

3.7 The Induced Relative Permittivity Profile

Figure 3.9(b) shows the relative permittivity profile that corresponds to the SC transformation shown in Figure 3.9(a). The curved reflector in Figure 3.9(a) is transformed into a flat reflector shown in Figure 3.9(b) as a straight PEC reflector. It is worth pointing out that a graded-index reflector can be constructed by having an appropriate set of refractive index, which is a linear function of the required delay, to make up an all-dielectric reflector. However, as discussed in Section 3.3.2, the TO method is a more general approach to produce the required material which can be difficult to deduce directly.

We truncate the permittivity profile in Figure 3.9(b) to a thickness $T = 2.5\lambda$ at 5 GHz to reduce its thickness, keeping only the significant region of the profile which is the reflector cover. This choice of $T$ is determined by the maximum angle of rotation for the virtual parabolic reflector at $\alpha_o = 30^\circ$ as this configuration needs the most amount of space for the relative permittivity decay down to unity at the top of the reflector cover. For the chosen $T$, the maximum relative permittivity is approximately 1.34 on along this boundary, inducing some refraction and reflection as wave impinges upon it. To help minimize the refraction, one can smoothly taper down the permittivity profile to that of free space. A new relative permittivity profile $\varepsilon_r^{\text{taper}}$ is created with linear taper as

$$\varepsilon_r^{\text{taper}}(x, y) = \begin{cases} 
1 & y > T + T_t \\
\kappa \varepsilon_r(x, y = T) + (1 - \kappa) & T \leq y \leq T + T_t \\
\varepsilon_r(x, y) & y < T 
\end{cases} \quad (3.32)$$

where $\varepsilon_r(x, y)$ is the relative permittivity profile induced by the coordinate transformation, $T_t = \lambda/3$ at 5 GHz is the thickness of the transition region and $\kappa = -\frac{1}{T_t}(y - T) + 1$ is weighting parameter that varies from 0 to 1 in the transition region. As a result, the relative permittivity profile gracefully transitions to unity on the top boundary. This linear taper can be applied to other boundaries of the reflector cover. The effects of the tapering will be shown in a later section.

3.8 Simulated Performance of the TO Reflector

3.8.1 Simulation Setup

Simulations are set up in COMSOL 4.3 with a probe-fed open-ended waveguide (OEWG) placed at the focal point as the fed antenna. All simulations in COMSOL are in 2-D. The OEWG is chosen as it has a relatively stable phase center. The aperture size of the OEWG is 60 mm which corresponds to a cut-off frequency of 2.5 GHz. Simulations have shown that the OEWG becomes multi-moded at frequencies beyond 7 GHz. The frequency range of interest is from 3 to 7 GHz, a fractional bandwidth of 80%.

The far-field patterns of the reflector cover are based on its scattered fields. 2-D directivity values are reported here includes the taper efficiency but not the spillover efficiency because realizing a high spillover efficiency with minimal blockage would require larger reflector sizes which would be computationally
challenging. Hence, the spillover efficiency is considered as a secondary issue compared to demonstrating the operation of the TO reflector cover.

### 3.8.2 Angle Scanning Performance

Figure 3.13 shows the directivity pattern of the reflector cover for scan angles $\phi_o \in \{0, 20, 40, 50, 60\}^\circ$. The solid curves are that of the reflector cover and the dashed curves are that of the parabolic reflector in virtual space. For now we have disabled the tapering in the permittivity profile and set $T = 2.5\lambda$ at 5 GHz and set regions of relative permittivity less than 1 have been forced to 1 to avoid the dispersion associated with the metamaterials needed to produce such relative permittivity. Here we show the scan angle to one side but the TO reflector can scan to the other side by mirroring the permittivity profile.

First, we examine the directivity patterns of the virtual parabolic reflector in the dashed curves in Figure 3.13. The quality of the beam patterns remains excellent even at 60° away from broadside. The side lobe levels are constant at about $-14\,\text{dB}$ at $\phi_o = 60^\circ$. This justifies our choice of the method of rotation of the parabolic reflector about its apex. One reason for the good angle scanning performance is that the tilt angle $\alpha_o$ of the virtual parabolic reflector only needs to be half of the reflected beam while at the same time the reflector is not laterally moving compared to some of the other means of rotation that have been considered [105].

Next, we examine the directivity patterns of the flat TO reflector cover shown in solid curves in Figure 3.13. The beam patterns remains directive up to 47° away from broadside despite some reflection and refraction at the top boundary of the reflector cover (tapering) and that the regions of relative permittivity less than 1 have been disturbed. At the extreme angle of $\phi_o = 60^\circ$, the pattern degrades but it still has a main lobe at approximately 57°.

![Figure 3.13: Directivity patterns of the reflector cover at 5 GHz (solid) and that of the curved parabolic reflector (dashed) for various scan angles $\phi_o \in \{0, 20, 40, 50, 60\}^\circ$. A radial line is drawn at each scan angle $\phi_o$.](image)

Figure 3.15 shows a plot of the actual angle of the reflected beam of the TO reflector cover and that of the parabolic reflector, along with their maximum directivity as a function of scan angle. The actual reflected beam angle is not exactly the same as that of the set scan angle $\phi_o$ because of the refraction along the top interface and the permittivity profile itself has been disturbed.

When the reflector cover scans to 47° away from broadside, its directivity drops by 2.5 dB relative to that of broadside and it is 2.9 dB lower compared to the directivity of the parabolic reflector scanned...
Figure 3.14: Relative permittivity profile for (a) $\phi_o = 0^\circ$, (b) $\phi_o = 20^\circ$, (c) $\phi_o = 40^\circ$, (d) $\phi_o = 50^\circ$. The permittivity profiles are truncated to a thickness of $T = 2.5\lambda$ at 5 GHz. The color scale is limited to $\varepsilon_r = 3$ for clarity.

to the same angle. We use a 3 dB criteria for the loss in directivity and we consider 47° away from broadside is the maximum angle of scan for the reflector cover.

Figure 3.15: Actual reflected beam angle and maximum directivity as a function of scan angle $\phi_o$ for the TO reflector cover (solid) and the parabolic reflector (dashed).

3.8.3 Effect of Perturbing the Relative Permittivity Profile

This section shows the effects of perturbing the relative permittivity profile on the directivity patterns. Figure 3.16 shows the directivity patterns when $\phi_o = 40^\circ$ for a few different scenarios of the relative permittivity profile labeled A through D. Case A is when the permittivity profile is left undisturbed and no tapering along its boundaries. Case B is the same as case A except that the regions of relative permittivity less than 1 have been forced to 1. Cases C and D are the same as that of A and B respectively except that we employ the tapering in the permittivity profile all of its boundaries according to the linear tapering described in (3.32). These boundaries are highlighted in green dashed lines in Figure 3.14(d).
The pattern for the parabolic reflector is also plotted in Figure 3.16 for comparison.

The directions of the main lobes for all cases are approximately at the desired angle of 40°. A radial line is drawn at this angle for reference. Cases A and C, with their full undisturbed permittivity profile, each have their main lobes pointed closer to the target angle and they are also narrower main lobes compared to that of cases B and D which have their relative permittivity profile disturbed by limiting the lower boundary for $\varepsilon_r$ to 1. This is an expected behavior because the full permittivity profile better approximates the virtual parabolic reflector. However, in the cases B and D, the angle of the main lobes does not appreciably deviate from the target angle but their side lobes are higher than that of cases A and C, leading to a lower directivity. Here, the accuracy of the main lobe and low side lobe levels are traded-off for much simpler material realization where no anisotropy and no magnetic responses are needed. It is important to point out that since no metamaterial are needed to realize materials with less than unity in relative permittivity, the bandwidth limitations that are typically associated with such a dispersive material are imposed on the TO reflector. Overall, the reflector cover mimics the curved parabolic reflector quite well.

![Figure 3.16: Directivity pattern of the TO reflector cover for different permittivity profiles at the center frequency of 5 GHz.](image)

### 3.8.4 Directivity Patterns versus Frequency

Figure 3.17 shows the directivity pattern for scan angle $\phi_o = 40^\circ$ over a range of frequencies. The solid curves are that of the reflector cover and the dashed curves are that of the parabolic reflector of the corresponding color. The relative permittivity profile used here is case B defined in the previous section. It is clearly shown that the beam patterns are good as the frequency is changed.

We characterize the directivity and the actual beam pointing angle in Figure 3.18 as a function of frequency. Solid and dashed curves are that of the reflector cover and that of the parabolic reflector respectively.

Figure 3.18(a) shows that the directivity on average degrades faster than that of the parabolic reflector. At broadside, the directivity difference over all frequencies between the reflector cover and that of the parabolic reflector is about 1 dB. However, this difference increases as the reflector cover is tuned to large scan angles. At $\phi_o = 50^\circ$, this average difference in directivity is about 2.5 dB. For a potentially tunable implementation of the reflector cover, this is an acceptable trade-off to scan to such a large angle. At $\phi_o = 60^\circ$, the loss in directivity for the reflector cover becomes very significant with an
average directivity about 5 dB less than the parabolic reflector. At such extreme scan angles, reflector cover does not have similar scattering properties to the parabolic reflector.

Figure 3.18(b) shows the actual angle of the main lobe as a function of frequency for various scan angles. It is evident here that there exists a consistent pointing error in the angle of the main lobe that is slightly less than the desired one. This is caused by the perturbed permittivity profile which is lower-bounded to 1. However, the direction of the main lobe is stable and it has little squinting from 3 to 7 GHz. A look-up table can be constructed to map the actual pointing angle to the set angle and the pointing error can be compensated with this look-up table.

Figure 3.17: Directivity patterns for \( \phi_o = 40^\circ \) for various frequencies when case B in Figure 3.16 is used. Solid and dashed curves are that of the reflector cover and the parabolic reflector respectively. Each color is a different frequency.

Figure 3.18: (a) Maximum directivity as a function of frequency for various scan angles. (b) Actual beam angles as a function of frequency. Solid and dashed curves are that of the reflector cover and parabolic reflector respectively.


3.9 Practical Considerations

A small $f/D$ ratio is often desired for a more compact reflector system. However, in the TO reflector cover, since the permittivity is induced by a coordinate transformation, some $f/D$ maybe prohibitively small because the relative permittivity in the vicinity of the feed differs too much from 1 to be ignored. In this case, the feed antenna would have be emerged in the transformation media as shown in Figure 3.19 for $f/D = 0.2$ as an example, potentially complicating the reflector system. Near the focal point denote by a black circle, the relative permittivity is 3.15.

![Figure 3.19: A parabolic reflector and its corresponding TO reflector with $f/D = 0.2$. Near the focal point in the relative permittivity profile denoted by a black circle, $\varepsilon_r = 3.15$. The any feed antenna would have to be emerged in the transformation media.](image)

In an actual realization of the TO reflector cover, the continuous permittivity profile needs to be realized somehow and it is difficult to build. The continuous profile can be spatially sampled for discretization. It is reasonable to expect that a larger spatial sampling period would produce poor reflector reflector performance. A trade-off is expected between the number of cells for the construction complexity versus the performance of the reflector. It is well-known that an array of sub-wavelength dipoles can produce an effective medium because they increase the polarization of the material. Fixed permittivity profiles have been realized for other TO devices in literature [84, 131] by using dipoles.

3.10 Design of a Proof-of-Concept TO Reflector

In this section, we design and build a TO reflector cover to measure its scattering characteristics, which will be compared to that of the simulated reflector discussed in Section 3.8.

We rotate the virtual parabolic in Figure 3.9(a) by an angle $\alpha_o = 15^\circ$ to produce a scan angle of $\phi_o = 30^\circ$. As before, the relative permittivity shown in Figure 3.20 is lower-bounded to 1 to avoid the use of potentially dispersive metamaterials. Only the regions of relative permittivity profile greater than 1 is realized. Here, we choose a smaller physical reflector than what was simulated before in Section 3.8.1 to aid the measurement of the TO reflector cover. The table in Figure 3.20 shows the parabolic-equivalent parameters for the TO reflector cover.

The TO reflector cover is in 2-D that can be realized by placing the device between two conducting plates known as a parallel-plate-waveguide (PPWG). The two plates are parallel to the $xy$-plane and are located at $z = 0$ and $z = d$ where $d$ is the distance of separation of the two plates. The polarization of interest is TE where the only the $z$-component of the E-field exist. All other E-field components are zero. The $E_\perp$ component is made to experience the permittivity profile in Figure 3.20.
Figure 3.20: Relative permittivity of the TO reflector cover. The profile is induced by a coordinate transformation but it is lower-bounded by 1. The table contains the parabolic equivalent parameters of the TO reflector cover.

### 3.10.1 Bandwidth Considerations

The continuous permittivity profile in Figure 3.20 is spatially discretized for ease of fabrication. It is realized by using an array of dipoles shown in Figure 3.21. Each dipole increases the local polarization and therefore the local permittivity of the material for $z$-polarization. Note that we refer to the dipoles as an array here which is not to be confused with the term “array” used when discussing reflectarrays. The purposes of the arrays are totally different. Here the dipoles array is producing an effective medium whereas the array of scatterers in a reflectarray is not.

Figure 3.21: Dipole array realizing an effective permittivity.

There are two major issues concerning the bandwidth of the TO reflector cover when designing for such a dipole array. First, any practical element used always has a frequency dependent response. In the case for the dipole array, it is desirable to minimize the element size and spacing in order for the dipoles response to be frequency independent as possible. The explanation is as follows. Each dipole can be modeled as a series LC resonator as shown in Figure 3.22(a). The inductance $L$ is proportional the length of the dipole and the capacitance $C$ depends on the capacitance formed between the two ends of the dipole and the top and bottom plates of the PPWG. It is desirable for the resonant frequency $1/\sqrt{LC}$ to be as high as possible so that the dipoles are made to be less frequency dependent over the same frequency range (i.e. the dipoles are far from self-resonance). The capacitance $C$ contributes to the local permittivity of the material. For an increase in the dipole length, both the $L$ and $C$ of the dipole increase. For a decrease in the dipole spacing, as illustrated in Figure 3.22(b) for the case when the spacing is halved, the effective inductance is decreases by a factor of two while the capacitance increases by the same factor. Hence, the resonant frequency stays constant. Figure 3.22(c) summarizes the effects of dipole length and spacing on $L$, $C$ and the resonant frequency $1/\sqrt{LC}$. In this table, ↑ denotes an increase of a parameter and ↓ denotes a decrease.
The second issue impacting on the bandwidth of the spatially discretized version of TO reflector stems fundamentally from the spatial discretization. Consider a spatially discretized version of a 1-D transmission using a lumped series inductor $L = L_o \Delta z$ and a lumped shunt capacitance $C = C_o \Delta z$, where $L_o$ and $C_o$ are the per unit length inductance and capacitance of the transmission line. In the limit as $\Delta z \to 0$, the lumped element unit cell exactly models the distributed transmission line. For a finite $\Delta z$, the LC cells would always resonant at some frequency, at which point the lumped element unit cell no longer model the transmission line. Our 2-D discretization suffers from the same spatial discretization effect. Hence, a high spatial sampling rate is again preferred. Typically, a rule of thumb to follow is that the cells are to be spaced at more than $\lambda/10$ at the highest frequency of interest [110].

### 3.10.2 Unit Cell Design and Simulation

The dipoles are simulated in Ansoft HFSS as shown in Figure 3.23. The dipoles are placed on a Rogers RT/Duroid 5880 substrate with $\varepsilon_r = 2.2$, a loss tangent $\tan \delta = 0.009$ and a thickness of 0.127 mm. Dipoles of various lengths $l_d \in [1, 8]$ mm are simulated in PPWG with thickness $d = 10$ mm so that its effective permittivity can be extracted based on the simulated two-port s-parameters [144] for normally-incident fields. PEC and PMC boundary conditions are setup in the simulation as shown in Figure 3.23.

The width of the simulation $\Delta x$ corresponds to the $x$ spatial sampling period of the dipole array and the reference planes for simulated s-parameters are de-embedded inward from the two wave-ports such that the distance between them is the $y$ spatial sampling period of the dipole array $\Delta y$. As a result, the extracted effective permittivity is the permittivity of the block of size $\Delta x \times \Delta y \times d$. The chosen spacings are $\Delta x = \lambda/10$ and $\Delta = \lambda/20$ where $\lambda = 60$ mm the wavelength at 5 GHz. The spacing in the $x$ direction is more coarse due to the width of the dipole at 4 mm. The wide width helps to reduce the inductance of the dipole.

Figure 3.24 shows the extracted relative permittivity of the dipoles of various dipole lengths as a function of frequency 1 to 10 GHz. $\Re(\varepsilon_r)$ is shown in solid curves and $\Im(\varepsilon_r)$ is shown in the dashed curved which are all nearly zero. For simplicity, we refer $\Re(\varepsilon_r)$ as just $\varepsilon_r$ since $\Im(\varepsilon_r)$ is not under
consideration during the design procedure. $\varepsilon_r$ depends slightly on frequency due to the series inductance of the dipole, which is more pronounced for longer dipole lengths. However, even in the worst case, the $\varepsilon_r$ variation is only about 20% for the 8 mm dipole. In addition, only a small region in the relative permittivity profile requires $\varepsilon_r > 3.5$ as shown in Figure 3.20. A look-up table is constructed so that a required $\varepsilon_r$ can be mapped to a dipole length by averaging $\varepsilon_r(\omega, l_d)$ with respect to frequency from 1 to 10 GHz.

![Figure 3.23: A HFSS simulation to extract effective permittivity of the dipole.](image)

![Figure 3.24: Extracted real (solid) and imaginary (dashed) part of $\varepsilon_r$ for various dipole length $l_d \in [1, 8]$ as a function of frequency.](image)

### 3.10.3 Simulation Setup and Fabricated TO Reflector

Figure 3.25 shows the simulation of the array of dipoles of the appropriate lengths in a PPWG. These dipoles are backed by the same substrate that is used in the HFSS simulations in Figure 3.23 when extracting the effective permittivity. The simulation is set up in SEMCAD X, a finite-difference-time-domain tool. A cylindrical current source is used for excitation to aid the experimental setup that also uses a cylindrical source. The scattered fields is computed by taking the difference between the total fields and the incident fields which are obtained by running a separate simulation with no scatterers present. In Figure 3.25, all metals are simulated as PECs and they are highlighted in red. The substrate
3.10.4 Measurement Setup

Figure 3.27 shows the measurement setup used to measure the scattered fields of the TO reflector cover. The PPWG consist of two aluminum sheets with perforated holes on the top plate (not shown in Figure 3.27). The perforated holes allow the E-field probe to be inserted into the PPWG for measurement without significantly disturbing the fields inside the waveguide. The holes are in a standard 60° staggered pattern with a diameter of 3.175 mm which is about λ/10 at the highest frequency of interest at
10 GHz. Simulations of the perforated PPWG are performed to ensure that the holes do not disturb wave propagation in the PPWG. Absorbers are placed on the outer rim of the PPWG. A vector network analyzer (VNA) is used to measured the transmission coefficient $s_{21}$ from the cylindrical source in Figure 3.27 to the E-field probe which can sample the E-field in the PPWG everywhere by moving it to any desired locations using the $xyz$-translator. A software is used to control the translator and record the measured $s_{21}$ values. To compute measured scattered fields of the TO reflector cover, a reference measurement is taken with an empty PPWG so the incident fields are known.

### 3.10.5 Measured Results

#### 2-D Near-Field Measurements of the TO Reflector Cover

The scattered near-field is measured in front of the TO reflector cover to qualitatively verify its operation. The area of scan is $285 \times 185$ mm which is highlighted in the blue rectangular area in Figure 3.27. Figure 3.28 shows the measured scattered near-fields of the TO reflector cover from 5 to 10 GHz. The restriction for the frequency to be above 5 GHz is due to the pyramidal absorbers as they do not function well below 5 GHz. It is clear in Figure 3.28 that the reflector redirects most of the energy to the left side, at an angle of approximately 24°. This angle is not exactly the same as the set scan angle of $\phi_o = 30^\circ$ because the relative permittivity profile is perturbed by lower-bounding it to 1.

Some fields can be observed to scatter in unwanted directions. In particular, there are fields that scatter to the opposite side of broadside which can be seen in Figure 3.28. These specular reflections lower the directivity of the patterns and contribute to higher side lobe levels in the far-field patterns, which is shown in the next section. The overall measured near-fields behave as expected.
Figure 3.28: Measured normalized scattered E-field of the TO reflector at various frequencies. Radial lines are drawn at 30° (red) and 24° (black) for reference.

Far-Field Measurement of the TO Reflector Cover

To measured the scattered far-field of the TO reflector cover, the scattered fields are measured along a circle, highlighted as the green circle in Figure 3.27 that encloses all sources and scatterers. The position of the center of the circle is immaterial so long as all sources and scatterers are enclosed. A shift of the circle center produces changes only in the phase of the far-field patterns. To obtain the measured far-field pattern of the TO reflector cover, the measured near-fields are projected onto a set of weighted Hankel functions of integer orders $n \in (-\infty, +\infty)$ [145], performing a cylindrical near-to-far-field transform. The scattered far-field patterns are computed using the same method.

The measured and simulated scattered patterns needs to be normalized appropriately for comparison. The simulated far-field patterns depend on the strength of the cylindrical source whereas the measured far-field patterns depend on the measured $s_{21}$ which includes, in part, the unknown vector effective length of the probe used. The impedance mismatch at the cylindrical source that feeds the PPWG also contributes to $s_{21}$.

A normalization procedure is used for both the measured and simulated E-fields is as follows. Since
the Green’s function in 2-D is the zero-th order Hankel function of the second kind \( H_0^{(2)} \), we normalize
the measured and simulated E-fields to this function as follows. In the empty PPWG, the near-fields
are sampled along a circle with a known distance to the cylindrical source. We define an error term
\[
e(f, \phi) = \frac{E_{m,s}(f, \phi)}{H_0^{(2)}(f, \phi)},
\]
where \( E_{m,s} \) is either the measured \( s_{21} \) or the simulated E-field sampled along a circle, both depend on
frequency \( f \) and angle \( \phi \). An average error term \( \hat{e}(f) \) is calculated by averaging \( e(f, \phi) \) for all \( \phi \). When
the simulated E-fields or the measured \( s_{21} \) are multiplied by their own respective error term \( \hat{e}(f) \), it
is as if an ideal cylindrical source is used in both simulation and measurement, thus, making a valid
comparison.

Figure 3.29 shows the set of measured and simulated far-field patterns of the scattered fields of the
TO reflector cover. Two sets of the simulated patterns are plotted which are the patterns of the array of
dipoles (solid blue) shown in Figure 3.25 and of the continuous permittivity profile in Figure 3.20. Note
that in all cases, the relative permittivity profile is lower-bounded to 1. The simulated patterns for the
continuous permittivity profile is obtained from COMSOL [146].

The measured patterns generally agree well with both sets of the simulated patterns. We can qualita-
tively estimate the effect of spatial discretization by comparing the two sets of simulated patterns where
one is spatially sampled by the use of dipoles while the other is the continuous permittivity profile. The
angle of the main lobes are pointed at 24° instead of the set scan angle of \( \phi_0 = 30° \). This offset in the
pointing angle is consistent across all frequencies and therefore it could be potentially corrected for, for
example, by setting the scan angle to a slightly larger value. More importantly, there is no significant
beam squinting from 5 to 10 GHz, a 67% fractional bandwidth and the shape of the main lobe roughly
remains the same across all frequencies measured. Note that the upper frequency limited is not strictly
bounded to 10 GHz as the dipoles still collectively represent an effective medium. For frequencies around
10 GHz, the relative permittivity would vary by more than 20% and it could cause the beam patterns
to degrade.

The peaks of the patterns between the measured and the simulated generally agree well but the
measured patterns agrees closer to the simulated case with the continuous permittivity profile. In
addition, the differences in the peaks becomes smaller for an increase in frequency. We attribute the
differences to the imperfect absorbers at the boundaries of the PPWG in the experimental setup as the
absorbers function better at high frequencies. The side lobe levels of the measured are generally on
the same order of magnitude compared to the simulated ones but they do not completely agree. The
side lobes are much weaker than the main lobe and they tend to be more sensitive to the quality of the
experimental setup, especially when small reflections off of the absorbers are present at low frequencies.
Simulations were performed with small reflections at the PPWG boundaries and it was qualitatively
found that the small reflections become exacerbated in the far-field patterns. Hence, we don’t expect a
perfect correlation between the simulated and measured patterns.

It is worth mentioning that the measurements of the scattered fields require two separate measure-
ments, one to determine the incident fields in an empty PPWG and the second to capture the total
fields. The measurement setup must remain still between the two measurements in order to accurately
capture the scattered fields. Moreover, the top PPWG has to be removed in order to insert the TO
reflector and then placed back into the same position as the reference measurement. Although great
 cares is exercised during the measurement process, it is reasonable to expect that the experimental setup introduces some errors in the measured fields that contributes to the differences between the measured and simulated far-field patterns. Despite the sources of potential measurement errors, the measured and the simulated far-field patterns agree well overall.

### 3.11 Conclusion

In this chapter, we have shown that materials can be used to control light propagation based on Fermat’s principle. A coordinate transformation induces a set of material parameters $\varepsilon_r$ and $\mu_r$ in such a way that the wave propagation in each of the two spaces are the same. In 2-D, a conformal coordinate transformation is able to produce isotropic dielectric-only TO devices which is useful for wide applications as demonstrated by the examples described in this chapter.
The designed proof-of-concept flat UWB reflector is able to mimic the reflection characteristic of a curved parabolic reflector. Simulations have shown that it has an excellent beam characteristic compared to that of an ideal parabolic reflector scanned to the same angle even at a scanning angle of 47° from broadside over a wide band of frequencies. In addition, we have generated many insights into the practical considerations in the design of TO reflectors in order to make them appealing in more realistic applications. The measured scattered near-field and far-field show that the TO reflector is able to redirect most of the incident power in the desired direction. The TO reflector exhibits good overall beam characteristics over the measured 67% fractional bandwidth with little observed beam squinting. In the experimental demonstration, the TO reflector redirected the incident beam at 30° and it was shown to have a good far-field patterns. Larger angles of scan is possible and can be achieved as long as the corresponding permittivity profile can be realized.

The TO reflector is relatively thick, which is a necessary condition for the incident field to experience an appropriate amount of delay while propagating in the transformation media. In addition, in order to leverage conformal coordinate transformations, the TO reflector is inherently in 2-D. In the next chapter, a novel design method is introduced to design UWB reflectors using a metasurface that addresses these issues.
Chapter 4

Ultra-Wideband Reflector Design
Using an Impedance Surface

4.1 Introduction

The previous chapter described the realization of an UWB reflector using Transformation Optics. Although the TTD TO reflector is shown to operate over an extremely wideband of frequencies, there are a few drawbacks that are inherent to the design. Perhaps most dominantly, the reflector are thick which is a necessary requirement in order for an incident wave to incur the appropriate amount of delay in the material. Moreover, the dielectric-only TO reflector are inherent to two dimensions in order to leverage of the conformal coordinate transformation, restricting its application to wave-front manipulation in only one plane such as a cylindrical wave-front collimation or redirection. In addition, the effective geometries of the wideband reflectors are restricted to those that have a relative permittivity greater than unity. Although it is possible to ignore the regions with less-than-unity relative permittivity in some cases by clever design of the transformation domain, like the case in the previous chapter, it is nonetheless a restriction imposed on the type of transformations that can take place for a wideband application. The reflectarray designed using the proposed method in this chapter addresses these issues, using a metasurface, while still having a wide bandwidth. The proposed method is applicable in full 3-D applications that require no bulky materials, producing reflectarrays with a low profile.

In this chapter, we look a reflectarray antenna as a filter both in the temporal sense and spatial sense simultaneously. A reflectarray is a temporal filter because it filters an incident signal just like a typical filter that changes the magnitude and phase of the input signal at every frequency. A reflectarray is also spatial filter because changes the spatial frequency content of the incident wave, typically from an undirective spatially broadband wave to, ideally, a single spatial harmonic otherwise known as a plane wave. Using Bessel filters, this chapter develops a framework in which wideband reflectarrays can be designed.

Traditionally, reflectarray antennas are thought of as an array of individual antenna element on a surface with element size and spacing comparable to the wavelength. Each antenna element is responsible for producing its own desired reflection phase. Coupling between the elements needs to be managed to ensure each element is producing the required reflection phase even in the presence of all the other elements. The reflectarray bandwidth is limited in part by the bandwidth of the antenna elements used
in the array and the bandwidth of the phase shifting mechanism used [24].

There are a number of ways to improve the bandwidth, for example, by using true-time-delay devices (TTD) [1, 20], cascading multiple layers of scatterers [2, 46, 147] and sub-wavelength elements or ring structures [148, 28, 36, 30, 149]. The classical implementation of a wideband reflectarray is to use true-time-delay (TTD) components that couple to the array of radiating elements. Short open-ended transmission line stubs can be attached to antenna elements to achieve a wideband response [1, 20] for the reflectarray. In theory, the achieved bandwidth can be as wide as the TTD components allow but it is often limited by practical considerations. For example, the required lengths of the stubs can be prohibitively long when large delays are required. In addition, there could be spurious radiation associated with the bent stubs degrading the cross-polarization ratio of the reflector [24]. TO reflectors designed in the previous chapter are also examples of TTD reflectors. Although no TTD devices are explicitly used, the TTD response comes from the wideband response of the engineered bulk material [102, 103, 105, 90]. Fundamentally, the TO reflector attempts to mimic an exact replication of the curved reflector, which essentially uses time delays to accomplish collimation. Multi-layered structures have been proposed to improve the bandwidth of reflectarrays. Two layers of patch antenna arrays have shown to significantly increase the range of reflection phase compared to a single layer [24]. A threelayer design have also been proposed [46]. Stacking multiple layers of scatterers is analogous to cascading multiple transfer functions together [147], creating more poles and zeros that provide additional degrees of freedom for enhanced phase control at the expense of design and fabrication complexity. More recently, sub-wavelength elements have been used to design reflectarrays with improved bandwidth performance. In particular, it was discovered that an array of sub-wavelength patch antennas with sub-wavelength spacing provides noticeably more bandwidth than the traditional array of resonant patch antennas with roughly $\lambda/2$ spacing [5, 148, 150, 151, 152, 148]. Multiple layers of sub-wavelength element arrays have also been proposed [47]. A surface of sub-wavelength elements is akin to creating a miniturized-element frequency-selective-surface (MEFSS) [7] in which macroscopic surface properties such as sheet impedance and admittance can be defined which supports the flow of electric and magnetic surface current densities. In a traditional reflectarray, no such macroscopic surface properties are usually defined. Reflectarrays have been designed as impedance and admittance surfaces [5, 148, 149, 4]. The combination of impedance and admittance surfaces are also known as Huygens’ surfaces or metasurfaces, and have been used in a wide variety of applications such as refraction [153, 6, 154], lensing [155, 156] and cloaking [6, 157]. The design of the very first reflectarray [13] was actually designed based on an effective surface impedance method realized by an array of short-circuited rectangular waveguides of different lengths. The impedance seen into the waveguides is a function of the angle of incidence and the waveguide length. Although sheet impedance concept is used to design the first reflectarray, the surface impedance is not homogenized as the size of the rectangular waveguides is comparable to the wavelength. In contrast, modern designs of Huygens’ surfaces used to realize reflectarrays usually have sub-wavelength element size and spacing.

Reflectarrays can be designed using numerical optimization algorithms. The two main evolutionary algorithms used for designing RF devices are the genetic algorithm [158] and the particle swarm optimization [159, 29]. Other evolutionary algorithms are also possible [160]. Various optimization goals such as the gain bandwidth [49] can be defined and optimized over. While good reflector performance can be achieved, these numerical optimization routines rarely offer insights into the theory of operational of the devices being designed and finding the optimal solution can take a long time as many iterations
are required to achieve convergence. Although useful, numerical optimization is not considered in this thesis.

In this chapter, we explore the type of reflectarrays that are designed using a metasurface. We do not consider reflectarrays designed with a) use of TTD devices b) numerical optimization of cells on the reflector surface and c) stacking multiple layers of scatterers to cascade transfer functions to create more poles. Instead, we first ask the question, what is required to design a wideband (theoretically unlimited bandwidth) reflectarray? Then we will see how an impedance surface can be engineered to maximize the bandwidth of reflectarrays.

The goals of this chapter are twofold. First a framework is developed to a design wideband reflectarray using Bessel filters to maximize its bandwidth. The theory of operation, the assumptions and the limitations of the theory are presented. Second, a proof-of-concept reflectarray is designed using the developed framework for maximum bandwidth. The designed reflectarray is then fabricated and its antenna parameters are measured and compared to the simulated ones.

4.2 Theory

4.2.1 Problem Formulation

We first describe the formulation of the problem and state assumptions made in the derivation of the theory. An array of sub-wavelength scatterers that make up a reflectarray is shown in Figure 4.1(a) with a plane wave impinging on them. The sub-wavelength cells collectively make up a metasurface, which we call a reflectarray interchangeably, and support the flow of macroscopic electric surface currents. The metasurface is assumed to have a macroscopic surface impedance tensor $Z_s$. The incident E-fields, H-fields and the wave vector are labeled as $E_i$, $H_i$ and $k_i$ respectively in Figure 4.1(a). The relationship between the surface currents and the incident and scattered fields is

$$
\begin{bmatrix}
E_x^i + E_x^s \\
E_y^i + E_y^s
\end{bmatrix} =
\begin{bmatrix}
Z_{xx} & Z_{xy} \\
Z_{yx} & Z_{yy}
\end{bmatrix}
\begin{bmatrix}
J_x \\
J_y
\end{bmatrix},
\tag{4.1}
$$

where $i$ and $s$ denotes the incident and scattered fields respectively. For our initial problem formulation, we make the following assumptions regarding the reflectarray:

1. A plane wave is impinging on each reflectarray cell. This condition is approximately satisfied if the reflectarray is sufficiently far away from the feed antenna. In this case, the spherical wave fronts becomes near planar locally for each cell.

2. For the purpose of developing a framework in which we can design wideband reflectarray, it is assumed that the E-field is normally incident on each reflectarray cell as shown in Figure 4.1(b). As will see later, we can correct for oblique incidence during the design process.

3. The impedance of the metasurface is designed only in the $y$-direction. For now, we will ignore the effects of $Z_{xy}$, $Z_{yx}$ and $Z_{yy}$ at the design stage but they will be included in the simulations to predict the far-field patterns of the reflectarray.

While a full analysis of a metasurface requires the manipulation the tensorial impedance in (4.1), our goal here is to first develop a framework in which we can use as a basis to design wideband unit cells
using the simplifications made above, some of which can be relaxed at a later stage. Applying the three assumptions made, the impedance of the highlighted cell in the red box in Figure 4.1(a) simplifies to Figure 4.1(c) where the impedance is modeled as an arbitrary load. With this simplified model in mind, we develop a method to engineer for this impedance to design wideband reflectarrays.

4.2.2 Bandwidth Limitations

We take a look at what is required for a planar reflectarray to have an unlimited bandwidth. Consider a prime focus reflectarray mimicking a parabolic reflector shown in Figure 4.2. In order for reflectarray and the parabolic reflector to be electromagnetically the same as seen by the incident wave, the reflectarray shown in red must compensate for a distance $d(x)$ as indicated, where $d(x)$ is the extra distance that the light must travel when the parabolic reflector is present compared to when the reflectarray is present. The reflection coefficient produced on the reflectarray surface is ideally

$$\Gamma_{\text{required}} = -e^{-j\omega d(x)/c}. \quad (4.2)$$

The corresponding impedance is the input impedance is

$$Z_{\text{required}} = jZ_o \tan \left( \frac{\omega d(x)}{2c} \right), \quad (4.3)$$

where $Z_o$ is the reference impedance for the reflection coefficient and $c$ is the speed of light. Note that this impedance is a transcendental $\tan()$ function with respect to $\omega$. A transcendental function is a function that can not be realized using a finite number of algebraic expressions resulting in an infinite number of poles and zeros. Hence, when designing or modeling an unit cell on a reflectarray surface using (finite) number of lumped elements, the bandwidth is inherently limited because there are only
a finite number of algebraic expressions in the resulting transfer function $\Gamma(s)$ having a finite number of poles and zeros. The design of the proposed reflectarray falls into this category and it becomes a question of how to maximize its bandwidth using unit cells with a finite number of lumped elements each providing one degree of freedom.

![Figure 4.2: Illustration of 1-D reflectarray compensating for a distance $d(x)$.](image)

### 4.2.3 Bessel Filters

A $N^{th}$-order Bessel filter has the form

$$H(s) = A \left( \sum_{i=0}^{N} a_i s^i \right)^{-1}, \quad (4.4)$$

where $a_i$ are the coefficients of the Bessel polynomial in the denominator of the transfer function and $A$ is a constant. (4.4) is used to approximate (4.2) for as wide frequency range as possible. For the time being, we analyze the frequency $s = j\omega$ to be the normalized frequency where the group delay is set to 1 second in (4.4). For an arbitrary delay, we simply map $j\omega \rightarrow j(\omega d/c) = j\omega(\tau_{g,s})$ where $d$ is the delay distance in meters, $c$ is the speed of light and $\tau_{g,s}$ is the set group delay in time. In addition, since $d$ and $\omega$ are interchangeable variables, all the properties of the Bessel filter described here with respect to $\omega$ also applies to $d$ or $d/c$.

The Bessel filter has the important property that its group delay is maximally flat about $\omega = 0$. This property is achieved by selecting for an appropriate set of $a_i$ coefficients such that the Taylor Series expansion of the group delay of (4.4) is zero for as many of the higher order terms as possible, starting from second derivative of $\angle H(\omega)$ with respect to $\omega$. The first derivative is set to the desired group delay $\tau_g = -\frac{\partial}{\partial \omega} (\angle H(\omega))$. For example, a 4th-order Bessel filter with its group delay normalized to one second is given by

$$H(s) = \frac{105}{s^4 + 10s^3 + 45s^2 + 105s + 105}, \quad (4.5)$$

and the Taylor Series expansion of its group delay $\tau_g$ about $\omega = 0$ is found to be

$$\tau_g = 1 - \frac{1}{11025} \omega^8 + \frac{1}{77175} \omega^{10} + O(\omega^{12}). \quad (4.6)$$

Figure 4.3 shows the normalized group delay profile where $\tau_g$ is set to 1 second, for the first 5 orders of the Bessel filter. In this figure, the abscissa can be $\omega$ or $d/c$ and the ordinate can be the group delay $-\frac{\partial \angle H}{\partial \omega}$ or the spatial group delay $-\frac{\partial \angle H}{\partial (d/c)}$. For now, consider this figure to be group delay versus $\omega$. Given a desired normalized group delay threshold $\tau_t = 1/\sqrt{2}$, a normalized maximum group delay
bandwidth can be defined $\omega_{n,\text{max}}$ which is shown in the figure to be 3.682 rad for the 4th order filter. It is clear that the group delay of the Bessel filter is maximally flat about $\omega = 0$ and higher orders produces higher $\omega_{n,\text{max}}$ for the same $\tau_t$. In addition, it is clear that a higher set group delay $\tau_{g,s}$ yields a lower group delay bandwidth as the frequency axis is mapped from $\omega$ to $\omega(\tau_{g,s})$.

![Figure 4.3: The normalized group delay ($\tau_{g,s} = 1$) of the Bessel filter up to order 5. A normalized maximum group delay bandwidth $\omega_{n,\text{max}}$ is defined here based on a threshold of $\tau_t = 1/\sqrt{2}$. Region A is the maximally flat and it is the least dispersive both in $\omega$ and $d$. Region B is the dispersive region.](image)

### 4.2.4 Transfer Function $\Gamma(s)$

The transfer function experienced by a normally incident field at each unit cell on the reflectarray surface is the reflection coefficient $\Gamma(s)$ of the cell. Here, the characteristics of $\Gamma(s)$ are analyzed for an arbitrary lossless passive LC (LPLC) network. The lossless property ensures that unit cells do not absorb energy of the incident fields and more importantly, it is an important condition for the proper operation of the framework.

Let $Z_{\text{in}}$ be the input impedance of a LPLC network

$$Z_{\text{in}} = \frac{P_M(s)}{Q_N(s)}, \quad (4.7)$$

where $P_M(s)$ and $Q_N(s)$ are $M^{\text{th}}$ and $N^{\text{th}}$ order polynomial respectively. Then either [161]

- $P_M(s)$ is an even polynomial and $Q_N(s)$ is an odd polynomial, or
- $P_M(s)$ is an odd polynomial and $Q_N(s)$ is an even polynomial.

Note that an even or odd polynomial is a polynomial with all of its powers being even or odd respectively.

Substituting (4.7) into the normally incident reflection coefficient on the surface of the reflector yields

$$\Gamma(s) = \frac{P_M/Q_N - Z_{\text{ref}}}{P_M/Q_N + Z_{\text{ref}}} = \frac{P_M - Z_{\text{ref}}Q_N}{P_M + Z_{\text{ref}}Q_N} = \frac{P_{\Gamma}(s)}{Q_{\Gamma}(s)}. \quad (4.8)$$
where $Z_{\text{ref}}$ is a reference impedance for the reflection coefficient. We would like the group of delay of (4.8) to be the same as the group delay of (4.5). Since the parity of $P_M(s)$ and $Q_N(s)$ are always the opposite, $P_\Gamma(s)$ and $Q_\Gamma(s)$ each has its even- and odd-powered terms interleaved. Moreover, the sign of terms in $P_\Gamma(s)$ differs from $Q_\Gamma(s)$ for every other term. As a result, the roots of $P_\Gamma(s)$ and $Q_\Gamma(s)$ are always at mirror-imaged locations about the $j \omega$ axis. Figure 4.4 illustrates the pole and zero locations of $\Gamma(s)$ for a 4$^{th}$ order transfer function. Furthermore, the group delay of $P_\Gamma(s)$ and $Q_\Gamma(s)$ are exactly the same.

$$\Gamma(s) = \frac{P_M(s) - Z_{\text{ref}}Q_N(s)}{P_M(s) + Z_{\text{ref}}Q_N(s)}$$

Figure 4.4: Locations of poles and zeros of $\Gamma(s)$ for 4$^{th}$ order transfer function on the complex plane.

Since the parities of $P_M(s)$ and $Q_N(s)$ are essential to the theory of operation, we provide a short proof. Consider the input impedance of a LPLC network

$$Z_{\text{in}} = \frac{M_1(\omega) + jN_1(\omega)}{M_2(\omega) + jN_2(\omega)},$$

(4.9)

where $M_i(\omega)$ are even polynomials and $N_i(\omega)$ are odd polynomials. The average dissipated power must be zero because the network is lossless

$$\frac{1}{2} \text{Re}(Z_{\text{in}}) |I|^2 = 0.$$  (4.10)

Since $|I| \neq 0$, then

$$\text{Re}(Z_{\text{in}}) = \text{Even Part}(Z_{\text{in}}) = \frac{M_1M_2 + N_1N_2}{M_2^2 + N_2^2} = 0.$$  (4.11)

Hence, the numerator must be zero

$$M_1(\omega)M_2(\omega) + N_1(\omega)N_2(\omega) = 0.$$  (4.12)

It follows that either

$$M_1 = N_2 = 0 \quad \text{or} \quad M_2 = N_1 = 0.$$  

Substituting either of the above conditions back into (4.9), the expression becomes either an even polynomial divided by an odd polynomial or vise versa.

We make a few interesting and important observation about $\Gamma(s)$ in (4.8):

- The coefficients of $Q_\Gamma(s)$ can always be set to that of the Bessel polynomial because both of
them are Hurwitz polynomials [162, 161]. Then, lumped-element circuits can be used to realize the transfer function $\Gamma(s)$. The achieved group delay bandwidth is the maximum group delay bandwidth without introducing ripples in the passband.

- Since the group delays of $P_{\Gamma}(s)$ and $Q_{\Gamma}(s)$ are the same, and that the total group of $\Gamma(s)$ is the superposition of the individual group delays, then the group delays of $P_{\Gamma}(s)$ and $Q_{\Gamma}(s)$ can be set to only half of the desired delay. As a result, the group delay bandwidth of the overall transfer function $\Gamma(s)$ is doubled compared to if only one Bessel polynomial (either $P_{\Gamma}(s)$ or $Q_{\Gamma}(s)$) is used.

- $|\Gamma(s)| = 1$ for all frequencies since the zeros are located at mirror-imaged locations about the $j\omega$-axis of the poles. This is due to the lossless nature of the one port network used.

- One has the freedom to choose an arbitrary LC network of a desired order to realize $\Gamma(s)$. This is an important feature as some circuits are more difficult to realize than others. For example, lumped-element circuits which contain components in the series branch are more difficult to realize than component in the shunt branch. A design example is described in a later section that will make clear and take advantage of this feature.

The design of the reflectarray amounts to producing a desired delay profile $d(x, y)$, for example by ray-tracing, to be realized by the unit cells on the reflector surface. The incident fields would experience the same delay as if the corresponding curved reflector is used.

Since any LPLC network can be used to realize a Bessel filter response in $\Gamma(s)$, the input impedance of different LPLC networks generally can be different. Here we analyze how they differ. Since the group delay of the $\Gamma(s)$ in (4.8) is the same independent of the choice of the LPLC circuit, then the phase of $\Gamma(s)$ can differ only by a constant for all frequency. Since the input impedance of any LPLC network at DC is either 0 or infinite, then phase of $\Gamma(s)$ between any two chosen LPLC networks can differ by either $0^\circ$ or $\pi$. For a difference of $0^\circ$, it is clear that the input impedance is exactly the same. For a difference of $\pi$, the input impedances are inverses of one another. This means that the $\Gamma(s)$ in (4.8) is realized by a family of different LPLC networks with the input impedances of any two members are exactly the same or the input impedance of one equals to the input admittance of the other.

Traditional reflectarrays often consist of resonant elements such as patch antennas that can be thought of having their own input impedance. However, this input impedance is not typically optimized to approximate (4.3). In the case of an array of sub-wavelength patches that make up the mushroom structure [4] as shown in Figure 4.5, which has been proposed for reflectarray-like applications, the lumped elements that model the structures are not optimized for bandwidth, even though it is a LPLC network that in theory could produce a reflection phase response of a second order Bessel filter. However, the produced inductance in this structure is practically difficult to be tuned as that would require the thickness of the substrate to be varied as a function of position. Only the capacitance can be easily varied from cell to cell by changing, for example, the patch size. In our developed framework, each lumped element is required to be independently controlled. A recent work [2] has shown that the reflection properties from multiple layers of capacitive patches are able to approximate a TTD response producing wideband reflectarrays. However, this approach requires many layers to be stacked on top of each other, significantly complicating the fabrication process. A detailed comparison of the reflectarrays designed using this approach and our own is provided in Chapter 5. Using our proposed design method, there is no need to use many layers as only one layer is sufficient to realize an even larger bandwidth as will be shown in the measurement data.
Chapter 4. Ultra-Wideband Reflector Design Using an Impedance Surface

\[ z_{in} = \frac{sL}{s^2LC + 1} \]

Figure 4.5: The mushroom structure proposed by Sievenpiper [4] that can also be used to realize a wideband reflectarray using the proposed design method because the lumped element model is a LPLC network. In theory, a second order Bessel filter can be realized using this structure given that the inductance can be changed on the reflectarray surface. (a) The unit cell of the mushroom structure. (b) The equivalent lumped-element model of (a).

4.2.5 Dispersion Analysis

This section provides a qualitative dispersive analysis of the reflectarray to aid a rapid assessment of whether the final reflectarray could potentially have poor beam characteristics or poor signal fidelity over the frequency range of operation.

The reflectarray designed using the developed framework has both a temporal and spatial response because the unit cells provide a Bessel filter phase response both temporally in \( \omega \) and spatially in the delay distance \( d \). The signal fidelity of the reflected wave depends on the temporal dispersion of the reflector surface, which maybe poor of the spectrum of incident signal cover portions in region B in Figure 4.3 where the fast roll-off occurs. The beam quality depends on the spatial dispersion of the reflector surface. Since \( d \) starts with zero and up to some maximum value, the reflectarray, in the spatial sense, necessarily operates in region A where it is the least dispersive and possibly in region B where it is more dispersive.

Whether a designed reflectarray has a low temporal dispersion and low spatial dispersion can be inferred at design stage by evaluating the region of operation of the reflectarray both temporally and spatially in Figure 4.3 as follows. Let \( \omega_{min}, \omega_{max} \) and \( d_{max} \) be the minimum and maximum frequency of interest and \( d_{max} \) the maximum compensation distance.

- For spatial dispersion, check whether \( \omega_{max}d_{max}/c \) is too far into region B in Figure 4.3. If so, the reflectarray may produce poor beam patterns. Use of higher order Bessel filters may be necessary.

- For temporal dispersion, check the dispersion in Figure 4.3 in the range \( \omega_{min}d_{max}/c \) to \( \omega_{max}d_{max}/c \). If this range do not cover much of the dispersive region in Figure 4.3, then the reflectarray have a low temporal dispersion and no further action is necessary. If this range do cover significant portions of the dispersive region in Figure 4.3, then a full signal fidelity analysis is necessary to estimate the temporal dispersion of the reflectarray by simulating or measuring the far-field patterns over a wideband of frequencies so that the transfer function of the reflectarray surface can be obtained.

Our proposed reflectarray, both the spatial response (beam patterns) and temporal response will be analyzed.
4.2.6 Maximum Reflector Size and Selection of Filter Order

The finite order of the Bessel filter establishes a constraint on the maximum electrical size that a reflector can have before the phase distribution on the reflector surface deviates too much from the ideal one. For simplicity, consider a prime focus reflectarray shown in Figure 4.6 in red and the curved parabolic reflector in black which the planar reflectarray is mimicking. $D$ is the diameter and $d_{\text{max}}$ is the maximum distance that the reflectarray needs to compensate in order to mimic the curved parabolic reflector. $f_D$ is the $f/D$ ratio of the parabolic reflector.

![Focal Point Diagram](image)

Figure 4.6: Two prime focused reflectors with diameter $D$. Red is the planar reflectarray and black is the curved parabolic reflector.

Let the equation of the parabolic reflector be

$$y = Ax^2, \quad A = \frac{1}{4f_D D}$$

and

$$d_{\text{max}} = \left(\frac{1}{2}\right) \left(2A \left(\frac{D}{2}\right)^2\right) = \frac{D}{16f_D}.$$  \hspace{1cm} (4.14)

The $1/2$ factor in front is because there are two polynomials in $\Gamma(s)$ that contributes to the total group delay, hence the group delay of each polynomials is set to $1/2$ of the total required delay. The relationship between $d_{\text{max}}$ and $f_{n,\text{max}} = \omega_{n,\text{max}}/(2\pi)$ is

$$\frac{f_{\text{max}}d_{\text{max}}}{c} = f_{n,\text{max}}$$  \hspace{1cm} (4.15)

where $f_{\text{max}}$ is the maximum frequency of interest and $\omega_{n,\text{max}}$ is the normalized maximum group delay as defined in Figure 4.3. Hence, using (4.15) and (4.14), it follows that the maximum diameter of the reflectarray is given by

$$D = 16f_D d_{\text{max}} = 16f_D c \frac{f_{n,\text{max}}}{f_{\text{max}}},$$

which leads to an electrical size given by

$$D_{\text{e,RA}} = \frac{D f_{\text{max}}}{c} = 16f_{n,\text{max}} f_D.$$  \hspace{1cm} (4.17)

Figure 4.7 shows $D_{\text{e,RA}}$ for the first 5 orders of the Bessel filter as a function of $f_D$. Using this figure, a desired Bessel filter order can be selected based on the maximum electrical size required for the reflectarray and the required $f/D$ ratio.
4.3 Selection of Circuit Topology

Any LPLC circuits can be used to realize the local transfer function $\Gamma(s)$. However, some circuits are more difficult to realize than others. Figures 4.9 to 4.11 show three 4th order circuits 1, 2 and 3 as examples that could be used. There are two aspects to be considered when making a circuit selection. The first aspect is the realizability of the component values, which must be within the bounds of practical means of realization such as using actual lumped elements or printed circuits. Second is the consideration of the circuit topology as components on the the series branches are generally more difficult to realize than the components on the shunt branches. We will elaborate on these two issues. First consider the $Q_{\Gamma(s)}$ for circuit 1,

$$Q_{\Gamma(s)} = L_1 L_2 C_1 C_2 s^4 + \frac{1}{Z_{\text{ref}}} L_1 L_2 C_1 s^3 + (L_1 C_1 + L_2 C_1 + L_2 C_2) s^2 + \frac{1}{Z_{\text{ref}}} L_2 s + 1. \quad (4.18)$$

The coefficients of the above polynomial are set to that of the Bessel filter of the same order,

$$\frac{1}{105} (\tau_{g,s} s)^4 + \frac{10}{105} (\tau_{g,s} s)^3 + \frac{45}{105} (\tau_{g,s} s)^2 + \tau_{g,s} s + 1, \quad (4.19)$$

where $\tau_{g,s}$ is the set group delay of the Bessel filter, the group delay at DC. The component values $L_1$, $L_2$, $C_1$ and $C_2$ can then be solved as a function of group delay $\tau_{g,s}$ and the reference impedance $Z_{\text{ref}}$. Table 4.1 shows the solved expressions for the components for all three circuits. Note that each circuit has a different required component values for the same $\tau_{g,s}$ and $Z_{\text{ref}}$. Hence, depending on the particular process used to realize the components, some circuits are more favorable than others.

Secondly, each circuit has a different topology. In circuit 1, all the components are located on the shunt branches which can be realized, for example, using off-the-shelf lumped elements or printed circuits. All components in theory can reside on the same layer. In circuit 2 and 3, however, there is a series inductor and a series resonance tank respectively. These components on the series branch are more difficult to realize than the components on the shunt branches in general. This is because the series branch components correspond to admittance surfaces as illustrated in Figures 4.8(a) and (b).
In Figure 4.8(a), the components on the series branch causes different voltages on both sides which is analogous to an admittance surface shown in Figure 4.8(b) with magnetic surface currents $M_s$, causing different tangential E-fields on either side of the surface. Although equivalent magnetic currents can be realized by using loaded resonant loops [31], they are much more difficult to synthesize, especially over a wide band of frequencies. In the special case when the series component is purely an inductor, it can be realized with the help of the free space inductance which can be accomplished by physically separating the layers. For completeness, the components on the shunt branch, as illustrated in Figure 4.8(c), analogous the impedance surface in Figure 4.8(d), which is much more easily realizable compared to the series case. Circuit 1 is chosen for realization.

Figure 4.8: Lumped components on the series and shunt branches corresponding to admittance and impedance surfaces. (a) Components on the series branch causing different voltages but identical currents on both sides. (b) An admittance surface that is equivalent to (a) supporting the flow of magnetic currents $M_s$. (c) Components on the shunt branch causing different currents but identical voltages on both sides. (d) An impedance surface that is equivalent to (c) supporting the flow of electric currents $J_s$.

Figure 4.9: Circuit 1. A 4th order circuit composed of only shunt elements.
Note that all the circuits presented are terminated in an open-end circuit boundary condition which ideally requires the realization of perfectly magnetic conductors (PMC) to back the unit cell. Although wideband PMCs can be realized and is a topic of active research area, its design can be quite involved. Moreover, we will show that by placing a perfectly electrical conductor (PEC) approximately λ/4 behind the metasurface at the center frequency would result in a reflectarray that produces good beam characteristics even at frequencies that are significantly different from the center frequency. Note that \(L_2\) in circuits described by Figures 4.9 to 4.11 could be realized by using the ground plane located at an appropriate distance away. However, since \(L_2\) needs to vary as a function of position on the reflectarray surface, the separation distance would also needs to be varied which would complicate the manufacturing process of the unit cells.

All of the circuits shown are of 4th order. Higher order circuits can be used to realize higher order Bessel filters to obtain a larger group delay bandwidth without necessarily increase the number of layers. For example, more series resonance tanks can be connected in parallel to circuit 1 to produce a higher order circuit while all the components can in theory still reside on a single layer.

<table>
<thead>
<tr>
<th>Circuit 1</th>
<th>Circuit 2</th>
<th>Circuit 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>(L_1)</td>
<td>(0.41) ((\tau_{g,s}Z_{\text{ref}}))</td>
<td>(0.29) ((\tau_{g,s}Z_{\text{ref}}))</td>
</tr>
<tr>
<td>(L_2)</td>
<td>(1.00) ((\tau_{g,s}Z_{\text{ref}}))</td>
<td>(0.71) ((\tau_{g,s}Z_{\text{ref}}))</td>
</tr>
<tr>
<td>(C_1)</td>
<td>(0.23) ((\tau_{g,s}/Z_{\text{ref}}))</td>
<td>(0.10) ((\tau_{g,s}/Z_{\text{ref}}))</td>
</tr>
<tr>
<td>(C_2)</td>
<td>(0.10) ((\tau_{g,s}/Z_{\text{ref}}))</td>
<td>(0.46) ((\tau_{g,s}/Z_{\text{ref}}))</td>
</tr>
</tbody>
</table>

It is worth noting that there is a direct method to synthesize the input impedance of a short-circuit
transmission line using lumped-elements, which is shown in Figure 4.12 [163], with components given by

\[ L_0 = \frac{l}{\alpha Y_o} \]  
\[ L_n = \frac{1}{2} L_0, \quad n = 1, 2, \ldots \]  
\[ C_n = \frac{2 l Y_o}{n^2 \pi^2 c}, \quad n = 1, 2, \ldots \]

where \( Y_o \) is the characteristics admittance of the transmission line, \( l \) is the physical length of the transmission line and \( c \) is the speed of light. In the limit where an infinite Bessel filter order is used, then letting the LPLC network in Figure 4.12 to realize a Bessel filter response in its reflection coefficient, we expect to obtain the same component values from the Bessel filter approach as we do from (4.20), (4.21) and (4.22). Note that the circuit in Figure 4.12 has the same topology as circuit 1 in Figure 4.9 where only shunt components are used.

Readers may be tempted to deduce that the direct method yields a set of component values in (4.20) to (4.22) that also can be used to realize a wideband TTD response in the reflection coefficient. This is true but only in the limit for a large number of components. In addition, this direct method may have slow convergence issues meaning a large number of components is required to produce a certain bandwidth. Furthermore, it should noted that for a finite number of components, which requires truncating the infinite series of components in Figure 4.12, the component values given by (4.20) to (4.22) can not be the optimal values to maximize the bandwidth for the reflection coefficient because these values differ from that obtained from the Bessel filter approach, which produces the optimized component values by definition.

![Figure 4.12: An infinite lumped-element circuit that realizes the input admittance of a short-circuit transmission [163].](image)

Correction For Oblique Angles of Incidence

In our developed framework, the incident angle to each unit cell is assumed to be at broadside. This condition is violated in practice because each unit cell sees a different incident angle at the different positions on the reflector surface. However, the effect of the non-broadside on the reflection phase can be mitigated by introducing a corrective factor \( \hat{c}_z(\theta_i) \) to the cell impedance \( Z_{in} \). \( \theta_i \) is the incident angle defined in Figure 4.1. It is determined as follows.

Let \( c_z(\omega, \theta_i, \theta_r) \) be the frequency and incident angle dependent factor to the cell impedance. \( c_z(\omega, \theta_i, \theta_r) \)
is found by setting the phase of the oblique incident reflection coefficient to the broadside one. For example, for a TM polarization

$$\angle (\Gamma_{\text{TM}}) = \angle \left( \frac{jXc_z - \eta_o \cos \theta_i}{jXc_z + \eta_o \cos \theta_r} \right) = \angle \left( \frac{jX - \eta_o}{jX + \eta_o} \right) ,$$  

(4.23)

where $X$ is the reactance of the impedance, $\eta_o$ is the free-space impedance and $\theta_i$ and $\theta_r$ are the incident and reflected angles defined in Figure 4.1. Solving for $c_z(\omega, \theta_i, \theta_r)$ yields

$$c_z(\omega, \theta_i, \theta_r) = \frac{1}{4X^2} (- (a + b)(\eta_o - X)(\eta_o + X)) + \frac{1}{4X^2} \left( \sqrt{16ab\eta_o^2X^2 + (a + b)^2(\eta_o^2 - X^2)^2} \right) ,$$  

(4.24)

where $a = \cos \theta_i$ and $b = \cos \theta_r$. A study of (4.24) shows that, for all $X$, it is bounded between a minimum $c_{z,\text{min}}$ and a maximum $c_{z,\text{max}}$ given by

$$c_{z,\text{min}} = c_z(\omega, \theta_i, \theta_r) \bigg|_{|X| \to 0} = \frac{2ab}{a + b} ,$$

$$c_{z,\text{max}} = c_z(\omega, \theta_i, \theta_r) \bigg|_{|X| \to \infty} = \frac{a + b}{2} .$$  

(4.25)

Figure 4.13 shows an example of $c_{z,\text{min}}$ and $c_{z,\text{max}}$ as a function of $\theta_i$, for $\theta_i \in [0, 60^\circ]$ and $\theta_r = 22^\circ$. Note that the reflected angle $\theta_r$ is a constant on the reflectarray surface. Since $c_z$ is tightly bounded, it is not sensitive to $X$ and therefore not sensitive to $\omega$. $c_z(\omega, \theta_i)$ can be averaged with respect to $\omega$ to obtain $\hat{c}_z(\theta_i)$ which can be applied to each component value in a cell where $L \to \hat{c}_z L$ and $C \to C/\hat{c}_z$.

A similar analysis can be performed for TE polarization and is omitted here. The corrective constant $\hat{c}_z(\theta_i)$ is applied to each location on the reflectarray surface.

Figure 4.13: The lower and upper bound of $c_z(\omega, \theta_i, \theta_r)$, $-\infty < X < \infty$, for a reflected angle of $\theta_r = 22^\circ$. 

4.4 Logic Flow Diagram

There are many parameters that influence the maximum electrical size that a reflectarray can have when it is designed using the proposed method. This maximum electrical size of the reflectarray stems the finite bandwidth of the Bessel filter used. A logic flow diagram is presented in this section that shows the relationship between these parameters and it can be used by a designer to determine the maximum electrical size of a reflectarray.

Figure 4.14 shows the logic flow diagram. A particular printed circuit have a maximum frequency of operation $f_{\text{max,LC}}$, beyond which the component values produced by the printed circuits are considered to be frequency dependent. For example, the length of the a printed inductor needs to be kept electrically short in order for its inductance to weakly depend on frequency. The printed circuits have a maximum inductance $L_{\text{max}}$ and capacitance $C_{\text{max}}$ that can be produced, which together with a chosen LPLC network determine a maximum delay $\tau_{\text{max}}$ that a reflectarray can compensate. Recall that different circuits require different maximums for the each of the component values as shown in Table 4.1. Given $\tau_{\text{max}}$ and a particular reflector geometry, the maximum physical size $D_{\text{max}}$ of the reflector can be calculated. $\tau_{\text{max}}$ with a chosen Bessel filter order $N$ determine a maximum frequency $f_{\text{max,cut}}$ before the reflected phase on the reflectarray surface deviates too much from the ideal one. This is due to the inevitable roll-off of the group delay of the Bessel filters. This frequency may or may not be above $f_{\text{max,LC}}$, hence the minimum of $f_{\text{max,LC}}$ and $f_{\text{max,cut}}$ is taken as the maximum frequency $f_{\text{max}}$ that the reflectarray can be operated at. At this frequency, the maximum electrical size of the reflectarray is evaluated $D_e = D_{\text{max}}/(c/f_{\text{max}})$. At this point, a designer needs to ensure that this maximum electrical size is greater than the required electrical size in a particular application.

![Logic Flow Diagram](image)

Figure 4.14: A Logic flow diagram that shows the dependence of the reflectarray’s geometric and electrical parameters.

4.5 Unit Cells

4.5.1 Design of Unit Cells

The design of the unit cells amounts to producing a sufficiently large range to cover the required inductance and capacitance. Since $\tau_g$ ranges from zero to a maximum value, so do the required component
values as shown in Table 4.1. We choose printed circuits to realize the components as they are easy to fabricate and they work better compared to off-the-shelf lumped elements at our design frequency range from 5 to 10 GHz. Inductors are realized using meander lines and capacitors are realized using interdigitated fingers. Printed circuits are commonly used to realize printed components, particularly in the realization of metasurfaces [154, 164, 153]. Mathematical models have been developed to accurately predict the inductance from grids and patches [165]. Figure 4.15 shows the printed circuits that are used to realize circuit 1 in Figure 4.9.

Before designing the printed circuits, the aspect ratio of the unit cell needs to be chosen so that the realized ranges of the printed components are able to satisfy the required ones. The required component values are a function of aspect ratio $\Delta y/\Delta x$ as follows

$$L_{i}^{\text{required}} \propto Z_{\text{ref}} \propto \frac{\Delta y}{\Delta x}, \quad C_{i}^{\text{required}} \propto \frac{1}{Z_{\text{ref}}} \propto \frac{\Delta x}{\Delta y}$$

(4.26)

where $\Delta x$ and $\Delta y$ are the horizontal and vertical dimensions of the unit cell defined in Figure 4.15. However, the dependence of the realized component values from the printed circuits roughly obey

$$L_{i}^{\text{printed circuit}} \propto \Delta y, \quad C_{i}^{\text{printed circuit}} \propto \Delta x.$$  

(4.27)

Hence, $\Delta x$ is determined when the required maximum inductance matches that produced by the printed circuit. Similarly, $\Delta y$ is determined when the required maximum capacitance matches that produced by the printed circuit. The initial sizing to guide the selection of aspect ratio is aided by the equations for inductance and capacitance in [165]. The final aspect ratio $\Delta y/\Delta x = 8 \text{ mm}/2 \text{ mm}$ is found by manual tuning.

Figures 4.15(a)–(e) shows the five half-circuits that realizes part of circuit 1 in Figure 4.9. Each half-circuit realizes a subset of the required range of the component values and they cover different portions of the reflectarray surface. Table 4.2 shows the percentage area covered by each cell configuration shown in Figure 4.15. In (a)–(e), the series $L_1$ and $C_1$ is on the left half of the unit cell while in (d) – (e), the parallel $L_2$ and $C_2$ is on the right half of the unit cell. Depending on the required component values for a particular cell, one of the half-circuits in (a) – (e) is chosen to be connected in parallel with one of the half-circuits in (d) – (e) to produce a full circuit shown in (f). In the simulations, the printed circuits are PECs and they all reside on two sides of a thin substrate, Rogers RT/duroid 5870 with a thickness of 0.127 mm. In Figure 4.15, the conductors are colored-coded as orange and blue each reside on the different sides of the substrate. The substrate needs to be as thin as possible because the series and parallel resonance tanks in circuit 1 have zero electrical distance between them. In theory, all the cells can reside on the same layer but it was easier to place $L_2$ on the other side of the substrate for space reasons.

<table>
<thead>
<tr>
<th>Cell Config.</th>
<th>Coverage</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fig. 4.15(a)</td>
<td>77.8%</td>
</tr>
<tr>
<td>Fig. 4.15(b)</td>
<td>2.5%</td>
</tr>
<tr>
<td>Fig. 4.15(c)</td>
<td>19.8</td>
</tr>
<tr>
<td>Fig. 4.15(d)</td>
<td>94.5%</td>
</tr>
<tr>
<td>Fig. 4.15(e)</td>
<td>5.5%</td>
</tr>
</tbody>
</table>
Figure 4.15: (a)–(e) half-circuits to realize part of the circuit shown in Figure 4.9. Capacitors are realized using interdigitated fingers and gaps. Inductors are realized using meander lines and metal strips. These partial circuits are then combined in parallel to realize the full circuit in Figure 4.9 as shown in (f). The aspect ratio of the cell is $\Delta x/\Delta y = 8\text{ mm}/2\text{ mm}$. The incident E-field is in indicated direction and is propagating in the $k_{\text{incident}}$ direction shown.

Each half-circuit in Figures 4.15(a)–(e) is simulated separately in HFSS in a parallel-plate waveguide to extract the component values. In each case, two geometric parameters are defined and they are swept in the simulations so that a look-up table is constructed to map the geometric parameters to a set of component values. The set of geometric parameters defined in Figure 4.15 are swept in the range shown in Table 4.3. The combined total time to simulate the database of cells is approximately 14 hours on a computer with Intel Core i7-4820k CPU at 3.7 GHz and a 64 GB of memory. The look-up table between the geometric parameters and component values are found by a brute-force search among all possible permutations of the component values within a pre-defined range which is known a priori. Once simulations of the cell database are completed, the generation of the look-up table took minutes to complete.
Table 4.3: Simulated ranges for the geometric parameters defined in Fig. 4.15 in each cell configuration.

<table>
<thead>
<tr>
<th>Cell Config.</th>
<th>Parameter</th>
<th>Range [mm]</th>
<th>Step [mm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fig. 4.15(a)</td>
<td>l\text{finger} \ A_{L1}</td>
<td>[0, 0.5] \ [0, 1.5]</td>
<td>0.025 \ 0.100 linear</td>
</tr>
<tr>
<td>Fig. 4.15(b)</td>
<td>l\text{finger} \ w_{L1}</td>
<td>[0, 0.5] \ [0.08, 3.82]</td>
<td>0.050 \ 0.170 linear</td>
</tr>
<tr>
<td>Fig. 4.15(c)</td>
<td>w_{C1} \ w_{L1}</td>
<td>[0.08, 3.80] \ [0.08, 3.82]</td>
<td>0.232 \ 0.170 linear</td>
</tr>
<tr>
<td>Fig. 4.15(d)</td>
<td>w_{C2} \ A_{L2}</td>
<td>[0.05, 1.80] \ [0.1, 3.82]</td>
<td>11 points per decade \ 0.100 linear</td>
</tr>
<tr>
<td>Fig. 4.15(e)</td>
<td>w_{C2} \ w_{L2}</td>
<td>[0.05, 1.80] \ [0.07, 3.80]</td>
<td>11 points per decade \ 0.266 linear</td>
</tr>
</tbody>
</table>

We show the magnitude of the reflection coefficient in the worst case for a unit cell near the reflectarray center in Figure 4.16. A copper material with a conductivity of $\sigma = 5.8 \times 10^7$ S/m is assumed which corresponds to a skin depth of 0.661 $\mu$m at 10 GHz. This skin depth corresponds to a sheet resistance of 0.026 $\Omega/\square$ which is used in the simulation across all frequencies. The cells in this region are expected to have the highest loss because the length of the meander lines that realize the inductors are the longest. The reflection coefficient is computed as

$$\Gamma = \frac{z_s - \eta_o}{z_s + \eta_o},$$  \hspace{1cm} (4.28)

where $z_s$ is the extracted sheet impedance from the HFSS simulations and $\eta_o$ is the free space impedance. It is expected that this ohmic loss contributes to the loss of aperture efficiency and therefore the gain of the reflectarray.

Figure 4.16: The magnitude of the reflection coefficient for a lossy unit cell near the reflectarray center. This cell is expected to have the worst case loss because the length of the meander line that realizes the inductors are the longest.
4.5.2 Inter-Cell and Intra-Cell Coupling Considerations

Each cell configuration in Figure 4.15 is simulated using periodic boundary conditions. The extracted impedances from these simulations are valid only when the adjacent cells that are physical identical. On the reflectarray surface, each cell is different from the adjacent cell as they need to realize different group delays. However, in practice, they are very close to each other due to the group delay profile being smooth and the cell spacing is small compared to the variations in the group delay profile. Hence, it is expected that the extracted sheet impedance from the simulations with periodic boundary conditions are valid on the reflectarray surface.

The cell database shown in Figure 4.15 are only half-circuits which are combined together to realize one completed unit cell. There is a potential for intra-cell coupling between the two halves of the circuit and this needed to be checked. Impedances from the simulated combined half-circuits are checked against that of the numerically combined circuits. For checking the intra-cell coupling, we choose the cells in Figure 4.15(a) and (d) because they are expected to produce the worst case coupling as they have the maximum component values which are needed to realize the relatively high delay values required near the reflectarray center. In addition, these two cells cover most of the area on the reflectarray surface as shown in Table 4.2. We define \( Z_{L_1,C_1} \) to be the sheet impedance obtained from the full wave printed circuit simulations of the \( L_1, C_1 \) cell in Figure 4.15(a). Likewise, we define \( Z_{L_2,C_2} \) to be the sheet impedance obtained from the full wave simulations of the \( L_2, C_2 \) cells in Figure 4.15(d). We compare the numerically combined reflection coefficient

\[
\Gamma_{\text{num}} = \frac{(Z_{L_1,C_1}|Z_{L_2,C_2}) - \eta_o}{(Z_{L_1,C_1}|Z_{L_2,C_2}) + \eta_o}, \quad (a||b) = \frac{ab}{a+b}
\]

(4.29)

to that obtained from the full wave simulation where both halves of the circuit are present. \( \eta_o \) is the free space impedance. Figure 4.17 shows the phase of the reflection coefficients computed using these two methods for an unit cell near the center of the reflectarray. The reflection phase appear to match quite well even in this worst case, indicating that there is low intra-cell coupling between the two halves of the circuit. For clarity, only one representative comparison is shown here but during the design stage, multiple cells were checked and they were found to have a better or an equal match between the numerically-combined and the full wave reflection phase than what is shown in Figure 4.17.

![Figure 4.17: Intra-cell coupling.](image-url)
Chapter 4. Ultra-Wideband Reflector Design Using an Impedance Surface

4.6 3-D Simulation Setup and Method of Moments

4.6.1 Reflectarray Geometry

Figure 4.18 shows the side view of the proposed reflectarray geometry in full 3-D. The reflectarray consists of an impedance surface shown in orange and a ground plane backing shown in red located distance $T = 10 \text{ mm}$ behind the impedance surface. The reflectarray is tilted $\theta_{\text{rot}} = 22^\circ$ with respect to the vertical axis. Over the entire surface of the reflectarray, it was checked that $c_z(\omega, \theta_i)$ depends weakly on frequency and $\hat{c}_z(\theta_i)$ ranges from 0.77 to 0.96. The elliptical reflectarray is offset in the E-plane and it mimics an offset parabolic reflector. The dominant incident E-field is in the $y$-direction as shown and it is predominately TM-polarized across the reflector surface. Since the unit cells are engineered to have a response in the $y$ direction, the co-pol of reflectarray is in the elevation plane. The geometric parameters of the reflectarray are defined in the figure and their values are defined in the accompanying table.

![Figure 4.18: Geometry of the 2-D reflectarray. $D_x$ is the length of the minor axis of the reflector oval into the page. The impedance surface (orange) and its ground plane (red) make up the reflectarray.](image)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D_y$</td>
<td>392.0 mm</td>
</tr>
<tr>
<td>$D_x$</td>
<td>364.0 mm</td>
</tr>
<tr>
<td>$H_f$</td>
<td>316.2 mm</td>
</tr>
<tr>
<td>$\Delta B$</td>
<td>43.20 mm</td>
</tr>
<tr>
<td>$\theta_{\text{rot}}$</td>
<td>$22^\circ$</td>
</tr>
<tr>
<td>$\theta_{\text{offset}}$</td>
<td>$18.85^\circ$</td>
</tr>
</tbody>
</table>

The delay profile $d(x, y)$ shown in Figure 4.19 is computed by ray-tracing. This distance profile is the distance that the reflectarray needs to compensate for in order to mimic the offset parabolic reflector. The outer rim of $d(x, y)$ highlighted in hatched lines is not realized because the delay distance approaches to zero in this region requiring small component values which are too small to be realized using printed circuits shown in Figure 4.15. Each component value is multiplicative constant of this delay profile according to the expressions in Table 4.1, and the correction factor $\hat{c}_z(\theta_i)$ defined earlier.

Figure 4.20 shows the corresponding component values for $L_1$, $L_2$, $C_1$ and $C_2$ corresponding to the distance profile in Figure 4.19 computed for circuit in Table 4.1. Figure 4.21 shows the impedance on the surface at the center frequency of 7.5 GHz. Note that this surface impedance is resonate on a elliptic-like contour which is required to produce the proper reflected phase. Figure 4.22 shows the ideal reflected phase distributions at various frequencies on the reflectarray surface. These reflected phases are simply the electrical lengths of the distance compensation profile in Figure 4.19. These ideal reflected phase shows that as frequency is increased, there are more phase wraps on the reflectarray surface.
Figure 4.19: Compensation distance profile of the reflectarray. Each component value is a multiplicative constant of this profile and a correction factor $\hat{c}_z$. The region in hatched lines are not realized because its corresponding component values are too small to be realized using printed circuits in Figure 4.15.

Figure 4.20: Required component values corresponding to the distance compensation profile in Figure 4.19. $\hat{c}_z(\theta_i)$ is not applied.
Chapter 4. Ultra-Wideband Reflector Design Using an Impedance Surface

Figure 4.21: Impedance on the metasurface at 7.5 GHz.

Figure 4.22: The ideal reflected phase distributions, namely, the electrical lengths of the distance compensation profile in Figure 4.19 at various frequencies.

4.6.2 Method of Moments Simulation Setup

A Method of Moments (MoM) simulator is created to calculate the far-field patterns of the scattered fields of reflectarray shown in Figure 4.18. The custom simulator is created because the simulation is a multi-scale problem where extremely fine features are present throughout the entire structure which is many wavelengths in size. A finite-element or finite-difference time-domain simulators would require prohibitive amounts of memory even with a modern heavy-duty workstation. The goal of the MoM code is to calculate for the unknown current densities $J_x$ and $J_y$ on the impedance surface and on the ground plane. The relationship between the incident fields, the excited currents and the anisotropic impedance surface is

$$
\begin{bmatrix}
E_x^i + E_x^s \\
E_y^i + E_y^s
\end{bmatrix} =
\begin{bmatrix}
Z_{xx} & Z_{xy} \\
Z_{yx} & Z_{yy}
\end{bmatrix}
\begin{bmatrix}
J_x \\
J_y
\end{bmatrix},
$$

(4.30)

where subscripts denote directions and superscript denote incident fields with $i$ and scattered fields with $s$ respectively. $Z_{ij}$ are the components of the anisotropic sheet impedance matrix of the impedance surface. Subscripts $i$ and $j$ denote either $x$ or $y$. In the designed unit cell, the engineered impedance is only in the $y$ direction. The values for $Z_{yy}$ in (4.30) for the input to the MoM code, here we call $Z'_{yy},$
is derived from the HFSS simulations of the database of the unit cells shown in Figure 4.15. This sheet impedance is extracted from the simulated $s$-parameters as [166]

$$Z'_{yy} = \frac{\eta_o}{2} \frac{1 + s_{11} + s_{21}}{1 - s_{11} - s_{21}}$$

(4.31)

where $\eta_o$ is the free space impedance. In (4.30), the rest of the sheet impedances in (4.30) $Z_{xx}$, $Z_{xy}$ and $Z_{yx}$ are extracted from HFSS based on the simulated mode-dependent $s$-parameters and they are input into the MoM simulations. A derivation of calculating these sheet impedances from the reflection and transmission coefficients is presented in Appendix B. It is clarified that there are two methods mentioned here to extract the $Z_{yy}$ for input to the MoM code. One method is through (4.31) while the other is through the mode-dependent $s$-parameters method shown in Appendix B. The $Z'_{yy}$ in (4.31) is chosen as the extraction method because it only requires $s$-parameters for the $y$-polarization, making the simulation time much shorter than that of the mode-dependent (Floquet) simulations. The simulation of the unit cells with the mode-dependent $s$-parameters is performed by quantizing the required delay profile into many different levels and each level corresponded to an elliptical shape on the reflectarray surface.

The expressions of the scattered fields due to the current densities are given by

$$E^s_x = -j\omega A_x - j \frac{1}{\omega \mu \varepsilon} \left( \frac{\partial^2 A_x}{\partial x^2} + \frac{\partial^2 A_y}{\partial x \partial y} \right),$$

(4.32)

$$E^s_y = -j\omega A_y - j \frac{1}{\omega \mu \varepsilon} \left( \frac{\partial^2 A_x}{\partial x \partial y} + \frac{\partial^2 A_y}{\partial y^2} \right),$$

(4.33)

where $A_i$ are the components of the magnetic vector potential. Note that $A_z(x, y) = 0$ on the impedance surface because there are no currents flowing in the $z$-direction. Furthermore, the expressions of $A_i$ are a function of the unknown current densities

$$A_x = \frac{\mu}{4\pi} \int_S J_x \frac{e^{-jk|\vec{r}' - \vec{r}|}}{|\vec{r}' - \vec{r}|^3} d\vec{r}',$$

(4.34)

$$A_y = \frac{\mu}{4\pi} \int_S J_y \frac{e^{-jk|\vec{r}' - \vec{r}|}}{|\vec{r}' - \vec{r}|^3} d\vec{r}',$$

(4.35)

where the integration is performed on the impedance surface $S$. Substituting (4.32), (4.33), (4.34), (4.35) into (4.30) to obtain

$$E^s_x = Z_{xx} J_x + Z_{xy} J_y + \left( \frac{j\omega \mu}{4\pi \varepsilon} + \frac{j}{4\pi \omega \varepsilon} \frac{\partial^2}{\partial x^2} \right) \int_S J_x \frac{e^{-jk|\vec{r}' - \vec{r}|}}{|\vec{r}' - \vec{r}|^3} d\vec{r}'$$

$$+ \frac{j}{4\pi \omega \varepsilon} \frac{\partial^2}{\partial x \partial y} \int_S J_y \frac{e^{-jk|\vec{r}' - \vec{r}|}}{|\vec{r}' - \vec{r}|^3} d\vec{r}',$$

(4.36)

$$E^s_y = Z_{yx} J_x + Z_{yy} J_y + \left( \frac{j\omega \mu}{4\pi \varepsilon} + \frac{j}{4\pi \omega \varepsilon} \frac{\partial^2}{\partial y^2} \right) \int_S J_x \frac{e^{-jk|\vec{r}' - \vec{r}|}}{|\vec{r}' - \vec{r}|^3} d\vec{r}'$$

$$+ \frac{j}{4\pi \omega \varepsilon} \frac{\partial^2}{\partial x \partial y} \int_S J_y \frac{e^{-jk|\vec{r}' - \vec{r}|}}{|\vec{r}' - \vec{r}|^3} d\vec{r}'.$$

(4.37)

These two equations are coupled integral equations which can be solved for the unknowns $J_x$ and $J_y$. 
using standard MoM techniques. The setup of the MoM matrices is common and is omitted here but it is worthwhile to mention some key points about the numerical techniques used in the code.

- Numerical integration is used everywhere except for integration over the region that contains the pole where $|\vec{r} - \vec{r}'| \approx 0$. The integration over the pole was handled in the limit near the pole analytically [167].

- A 7-point finite difference was used to approximate the second-order derivatives $\frac{\partial^2}{\partial x^2}, \frac{\partial^2}{\partial y^2}$ and $\frac{\partial^2}{\partial x \partial y}$.
  The method of undetermined coefficients was used to determine the weights of each point [168].

Having solved for the $J_x$ and $J_y$ everywhere, the far-fields patterns are then computed via the magnetic vector potential.

To feed the reflectarray in the simulations, two open-ended waveguides (OEWG) WR-137 and WR-112 are used to cover the wide frequency of interest from 5 to 10 GHz. OEWGs are chosen because an OEWG has a relatively stable phase center and its radiation pattern resembles closely to an ideal spherical source across a wide frequency range. The incident fields for the MoM simulator are obtained from HFSS simulations of two OEWGs. These simulations model the two real OEWGs used in the experiment.

4.6.3 Experimental Setup

The metal layout of the impedance surface is shown in Appendix A consisting of 7030 unit cells. Chemical etching is used to realize the metal patterning for the impedance surface on both sides of the substrate, 0.127 mm thick Rogers RT/duroid 5870. The impedance surface is paper-thin and it is backed by a copper ground plane located at $T = 10$ mm behind it, collectively make up the reflectarray, as shown in Figure 4.18. Figure 4.23 shows the experimental setup of the reflectarray on a near-field scanner that can measured its far-field patterns. In the figure, the outer rim of the impedance surface and of the copper ground plane are highlighted in yellow and red respectively. Foam spacers are used to ensure the separation distance $T = 10$ mm. An inset shows some unit cells viewed under the microscope. Due to the wide range of frequency, both the feed and the scan probe on the near-field scanner are wrapped accordingly to cover 5 to 10 GHz.

4.7 Results

4.7.1 Far-Field Measurements

Figures 4.24 and 4.25 shows the simulated and measured co-polar gain patterns of the reflectarray from 5 to 10 GHz. The elevation and azimuthal planes are the $y'z'$-plane (co-polar) and $x'z'$-plane (cross-pol) defined in Figure 4.23 respectively. Two simulated cases are presented, a lossless case where the circuits 1 in Figure 4.9 is used as the unit cell and a lossy case where $L_1$ has a series resistor $R_s$ and $L_2$ has a parallel resistor $R_p$. Both $R_s$ and $R_p$ are obtained from HFSS simulations of the unit cells where printed circuits are realized by copper, which is modeled by a sheet resistance of 0.026 $\Omega//\square$ calculated from a skin-depth of 0.652 $\mu$m at 10 GHz. In addition, the substrate loss is also included in the simulations. As a result $R_s$ and $R_p$ include both the conductor loss and the dielectric loss due to the substrate and they are both a function of position on the reflectarray surface.
The measured and simulated patterns agree well. The shape of the main lobes match in beamwidth and the side lobe generally match as well. In addition, there is no significant beam squinting observed across all the frequencies measured. Note that the ground plane is placed $T = 10 \text{ mm}$ away which corresponds to a $\lambda/4$ distance at 7.5 GHz, the center frequency of operation. Despite this, the quality of the beam patterns remains excellent even for 5 and 10 GHz which are significantly different from the center frequency. Furthermore, the FF patterns were computed for $T \in [8, 12] \text{ mm}$ and it was found that the beam patterns still remains what is shown in Figure 4.24 and 4.25. This shows that the radiation patterns are not too sensitive to the ground plane distance.

Figure 4.26(a) shows the measured and simulated directivities of the reflectarray from 5 to 10 GHz. Again, the lossless and lossy cases are shown for the simulated curves. The measured directivity is only about 1 dB down from either of the two simulated cases. This confirms that the experimental setup is good overall. Little defocusing has occurred and the reflectarray is focusing the relatively un-directive beam from the feed.

Figure 4.26(b) shows the measured and simulated gains of the reflectarray. The both of the simulated curves match well to the measured one. The simulated curve with loss matches particular well to the measured gain, most frequency points are within about 1 dB. Note that in the simulated curves, losses from N-to-SMA adapters and waveguide-to-coaxial adapters in the experiment, though small, are not included. If these losses are taken out, then the true gain of the reflectarray would be slighted higher compared to what is shown in Figure 4.26(b).

Due to the un-directive feed antenna, the aperture efficiency of the reflectarray is dominated by the poor spillover efficiency. Table 4.4 shows the spillover efficiencies $\epsilon_s$ and conductor efficiency $\epsilon_c$. All efficiencies are computed from simulations. The conductor efficiency is the difference between the blue and the pink curve in Figure 4.26(b). In addition, the measured lowest cross polarization ratio over a 3 dB beamwidth is also shown. The cross polarization is defined as the co-pol E-field divided by the cross-pol E-field.
Table 4.4: Table of simulated spillover and conductor efficiencies and the measured XPR in decibels over a 3 dB beamwidth of the reflectarray.

<table>
<thead>
<tr>
<th>Freq. [GHz]</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\epsilon_s$</td>
<td>0.225</td>
<td>0.297</td>
<td>0.277</td>
<td>0.258</td>
<td>0.321</td>
<td>0.293</td>
</tr>
<tr>
<td>$\epsilon_c$</td>
<td>0.730</td>
<td>0.731</td>
<td>0.697</td>
<td>0.642</td>
<td>0.646</td>
<td>0.615</td>
</tr>
<tr>
<td>XPR [dB]</td>
<td>24.96</td>
<td>24.49</td>
<td>27.93</td>
<td>30.53</td>
<td>34.14</td>
<td>30.71</td>
</tr>
</tbody>
</table>

4.7.2 Signal Fidelity of the Reflectarray

In order to characterize the signal fidelity of the reflected signal of the reflectarray, we use a $\sin \theta$ as the feed pattern. Realistic radiation patterns of the OEWGs can not be used as they are temporally dispersive. The $\sin \theta$ pattern corresponds to the far-field pattern of a Hertzian dipole. Note that all far-field patterns of any antenna has a $j\omega$ term that corresponds to a temporal derivative of an input signal, for example, the excitation current. Here, since we wish to analyze the characteristics of the reflectarray, no $j\omega$ term is applied to the Hertizan dipole in the simulations so that the computed transfer function contains information only of the reflectarray surface itself and not of the feed. The reflected signal is taken at the maximum co-pol of the E-field of the main lobe of the reflectarray. By computing the spectrum of this radiated fields the transfer function is obtained and it only contains the temporal response of the reflectarray. Figure 4.27 shows the transfer function of the reflectarray. It has relatively flat magnitude response and a very linear phase response. Hence, we expect the radiated signal to faithfully represent the input signal.

We use a cosine modulated Gaussian pulse as the input signal to the Hertzian dipole to test the signal fidelity of the reflectarray and it is defined by

$$x(t) = \cos(2\pi f_o t) e^{-\frac{(t-t_0)^2}{\tau_p^2}}$$  \hspace{1cm} (4.38)

with $t_0 = 50$ ns, $\tau_p = 0.14$ ns and $f_o = 7.5$ GHz which is the center frequency. The spectrum of (4.38) is a Gaussian curve centered at $f_o$ and it tapers down to $-10$ dB from its peak at 5 and 10 GHz. The radiated fields due to this input signal is computed via a linear convolution between the input signal and the impulse response associated with the transfer function found. Figure 4.28 shows the input signal (4.38) and the radiated signal.

We define a measure for signal fidelity $F$ as a correlation between two signals as \cite{169}

$$F = \max_{\tau} \int_{-\infty}^{\infty} \hat{x}(t)\hat{y}(t+\tau)dt, \quad \hat{x}(t) = \frac{x(t)}{\left[\int_{-\infty}^{\infty} |x(t')|^2 dt'\right]^{1/2}}, \quad \hat{y}(t) = \frac{y(t)}{\left[\int_{-\infty}^{\infty} |y(t')|^2 dt'\right]^{1/2}},$$  \hspace{1cm} (4.39)

where $x(t)$ and $y(t)$ are the input and output signals and $\hat{x}(t)$ and $\hat{y}(t)$ have their energies normalized. If two signals have the same shape, then $F = 1$. The signal fidelity of the input and output signals shown in Figure 4.28 is computed to be 0.951 which is excellent. The reflectarray acts much like a filter with low temporal dispersion and it does not significantly distort the input signal over a wide band of frequencies.
4.8 Other Filters Types

Before concluding this chapter, it is worthwhile to mention some potential forms other filters for use in designing reflectarrays as oppose to the Bessel filter. The motivation for this analysis is that Bessel filters have their response center about $\omega = 0$ for the group delay which may not be suitable for applications with a high center frequency. It would be ideal to design a frequency-centered version of the Bessel filter (FCBF). The design of such a filter is non-trivial. A typical frequency transformation often used is

$$s \rightarrow \frac{\omega_o}{\text{BW}} \left( \frac{s}{\omega_o} + \frac{\omega_o}{s} \right),$$

which works well to transform low-pass to bandpass magnitude response do not work well here because the transformation itself introduces a group delay profile.

Generally, the desired frequency-centered transfer function filter must be in the form of

$$H(s, d) = \sum_i a_i(\omega_o, d)s^i$$

where each $a_i$ is an arbitrary function in $d$ and $\omega_o$ is the center frequency. The problem to find each function $a_i(\omega, d)$ such that

$$\angle H(\omega, d) \approx -\frac{\omega d}{c}$$

around $\omega = \omega_o$. One way to solve this problem is to take the 2-D Taylor series of $\angle H(\omega, d)$ about $\omega = \omega_o$ and design each $a_i(\omega, d)$ such that (4.42) is true. However the functions $a_i(\omega, d)$ are designed, they must obey all of the following conditions:

- $a_i(\omega_o, d)$ must be of one sign for all $i$ and for all $d$. Otherwise, it is not possible to realize the transfer function use passive circuits because some of the required poles would be on the right-half of the complex plane.
- The numerical values of $a_i(\omega, d)$ must be suitable for realization.

Note that in the case for the Bessel filter, $a_i(\omega_o, d) = (d/c)^i$, $\forall i$ and both of the above conditions are satisfied.

4.9 Conclusion

In this chapter, a novel method is derived and proposed to design wideband reflectarray as an impedance surface with unit cells that realizes a Bessel filter response. The reflectarray designed using the developed framework is a FSS with sub-wavelength elements, which collectively form surface properties such as surface impedance and surface currents. This is in contrary to the traditional method in which reflectarrays are designed using an array of individual antenna elements. The designed reflectarray is shown to fully capitalize on the maximum bandwidth provided by the Bessel filters. As a result, good beam characteristics are observed from 5 to 10 GHz, a 66.7\% fractional bandwidth with little beam squinting.

The design of process of the reflectarray is simple and straight forward. It amounts to producing a set of inductance and capacitance values to make up for an impedance surface that have a prescribed reflection coefficient on the reflector surface. Due to the finite order for the Bessel filter, there exist a
maximum size for the reflectarray that can be made before reflection phase deviates too much from the ideal distribution. However, in theory, an arbitrary high order Bessel filters can be used and any LPLC network can be used to realize it.

The ground plane placed at $\lambda/4$ at the center frequency produces good beam patterns even at 5 and 10 GHz. The simulated directivities and gains match well to that of the measured. Excellent cross polarization ratio (XPR) is measured over the main lobe of the reflectarray.
Figure 4.24: Measured and simulated gain patterns for the reflectarray in the azimuth and elevation planes from 5 to 7 GHz.
Figure 4.25: Measured and simulated gain patterns for the reflectarray in the azimuth and elevation planes from 8 to 10 GHz.
Figure 4.26: Measured and simulated directivities and gains as a function of frequency.

Figure 4.27: The normalized magnitude and phase of the transfer function of the reflectarray from 5 to 10 GHz.
Figure 4.28: Input signal and the radiated (reflected) E-field of the reflectarray. The signal fidelity of these two signals is 0.951.
Chapter 5

Comparison of TO Reflectors and Impedance Surface Reflectarrays

In this chapter, we make qualitative comparisons between the characteristics of the reflectors designed with the TO approach and the impedance surface approach, summarizing their advantages and disadvantages. We will also compare the state-of-the-art wideband reflectarray in the literature with respect to our own. From now on, we will refer to reflectarrays designed with the impedance surface approach as simply the impedance surface reflectarray. The principles of operation of the TO reflector and the impedance surface reflectarray are fundamentally different. The resulting reflectors each have their own unique attributes that may give rise to a preference towards some applications over others. This chapter attempts to identify the pros, cons and the limitations associated with each approach, so that the suitability of the two design methods given a particular application is more easily assessed.

Both design methods approximate a TTD reflector using fundamentally different approaches. In the TO reflector, the delay is realized by using an effective medium such that each incident ray experiences a different optical length in a homogeneous material to produce the required delay. In the impedance surface approach, the incident fields impinge on surface with an engineered impedance to produce scattered fields with the required delay. In the ideal case, the TO reflector would have its material parameters independent of frequency and the impedance surface reflector would use a Bessel filter with an infinite order to produce a perfect delay. We will adopt a terminology in referring to \( n \)-D reflectors with \( n \) denoting the number of dimensions in which the reflector operates in, not the dimension of the physical reflector.

5.1 TO Reflector vs. Impedance Surface Reflectarray

In this section, we compare the TO reflector designed in Chapter 3 and the impedance surface reflectarray designed in Chapter 4. Table 5.1 on page 96 summarizes the key aspects of the qualitative comparisons made and they are explained in detail in their respective sections.

It is emphasized here that even though the impedance surface reflectarray works in a fundamentally different way than the TO reflector, the end goal is still the same in terms of approximating a TTD response. The TO reflector, which is a distributed device, can be thought of as an multi-layer reflectarray with an infinite number of stacked layers. Stacking multiple layers has been shown in the literature to
enhance the bandwidth of reflectarrays and the infinite number of layers of scatterers used in the TO reflector can be considered as one end of the extreme case. The impedance surface uses only one layer in theory, and can be considered as the other end of the extreme case.

5.1.1 Reflector Topology

The types of reflectors that can be designed using TO with a conformal coordinate transformation are constrained to 2-D devices, or 3-D devices with no variation ($\partial/\partial z = 0$) in the third dimension. This is due to the lack of available conformal coordinate transformations in full 3-D, which exist but they are all Möbius transformations [135]. Intrinsically 2-D TO devices are useful in 3-D cylindrical applications, for example a cylindrical lens in which an incident wave is collimated, re-directed or refracted in one plane. 3-D dielectric-only TO devices can be made by rotating a 2-D permittivity profile about an axis. For example, a 3-D carpet cloak [133] can be realized.

For the impedance surface reflectarray, the theory of operation is developed only for a broadside incident wave, though the surface impedance is corrected for oblique angles of incidence using the corrective constant $\hat{c}_z(\theta_i)$ described in Section 4.3. Unlike TO reflectors, full 3-D reflectors can be made. It is shown to function well in Chapter 4 despite using the 1-D broadside incidence circuit model in Figure 4.9. Although TO is limited to intrinsically 2-D devices, it is a generic method to produce other types of 2-D RF devices that otherwise cannot be realized using the impedance surface method. The reason is that the impedance surface is more specifically designed for a reflector application whereas the TO method, being a more generic method, is applicable to lensing, beam-splitting and beam-bending devices.

5.1.2 Physical Profile

The physical profiles of reflectors designed using the two approaches are completely different. The TO method produces volumetric reflectors whereas the impedance surface method produces, in principle, surface reflectors although in practice a ground plane is needed at quarter wavelength away at the center frequency. Reflectors designed by the TO method tend to produce thick permittivity profiles which are dependent on the rate of decay of the permittivity profile. The thickness of TO reflectors can be reduced at an expense of an inferior beam quality.

Since TO reflectors are frequency independent, a measure of the electrical thickness of the reflector may not be so consequential. The physical thickness remains the same regardless the frequency of operation. If the proposed TO reflector is operated at frequencies 10 times higher than what is shown in Chapter 3, then the electrical thickness would be 10 times as thick. However, the thickness of the impedance surface reflectarray is dependent on the frequency of operation. The PEC ground plane is placed at $\lambda/4$ away at the center frequency of operation. Hence, for applications with a high center frequency, the physical thickness can be thin which can be more advantageous than TO reflectors. For example, in satellite applications operating at mm-wave frequencies where the physical weight and profile of the reflector is of great concern, the required thickness $\lambda/4$ is on the order of millimeters whereas the thickness of the TO reflectors would remain the same for the same beam steering angle as any other frequency. Interestingly, at low frequencies, this effect is reversed. Notwithstanding the fact that electrically small reflectors may not have sufficient gain, the physical size of the impedance surface reflectarray could be much thicker than that of the TO reflector. Practically, modern communication
systems tend to operate at higher frequencies in order to leverage a higher absolute bandwidth given the same fractional bandwidth for an increase in the information capacity, making the thin nature of the impedance surface approach potentially more appealing. In addition, as the physical size for the reflectors grow at a fixed frequency, the thickness of the TO reflector would need to grow accordingly to produce a larger delay needed but the thickness for the impedance surface reflector would remain at $\lambda/4$ at the center frequency.

### 5.1.3 Bandwidth

The maximum achievable bandwidth of the reflectors realized using the two design methods stems from different mechanisms. Ideally, the bandwidth is infinite for both cases.

The physical sizes of TO reflectors can scale with frequency to keep their electrical sizes the same. Their electrical properties, such as the undesired refraction at the top device boundary, would remain the same after it has been frequency scaled. The bandwidth of TO reflectors depends on how well the required material parameters are approximated in their realizations.

The bandwidth limitations of the impedance surface reflectarray are different and it depends on the maximum physical delay required, given a fixed order Bessel filter. Isolating the maximum frequency of operation $f_{\text{max}}$ in (4.15) yields

$$f_{\text{max}} = \frac{f_{n,\text{max}}(N)}{\tau_{\text{max}}},$$

where $f_{n,\text{max}}$ is the maximum normalized frequency which is a function of only the filter order $N$ as defined before, and $\tau_{\text{max}} = d_{\text{max}}/c$ is the maximum delay.

Figure 5.1 shows two parabolic reflectors (solid black) and their impedance surface reflectarrays (dashed red) operating at two different frequencies but with the same electrical size in their diameters. Even though the electrical sizes are the same, the required maximum delays $d_{\text{max},1}$ and $d_{\text{max},2}$ are different, leading to two different maximum frequencies, as per (5.1), for the impedance surface reflectarray. For the parabolic reflectors shown, we have $d_{\text{max},2} = 4d_{\text{max},1}$, yielding 4 times larger bandwidth for the smaller reflector. This is not to be confused with the maximum diameter size in wavelengths described in Section 4.2.6, where the impedance surface reflectarray has a maximum electrical size which is a function of only $f/D$ ratio and filter order $N$. Described here is the analysis of the bandwidth of an impedance surface reflectarray given a fixed electrical size. The bandwidth is greater for the physically smaller reflector due to the lower required delay compensation.

### 5.2 Comparison with State-of-the-Art Reflectarrays

In this section, we compare the impedance surface reflectarray in Chapter 4 with the state-of-the-art wideband reflectarrays in the literature. At the time of this writing, the most wideband and most relevant reflectarray is the one designed by Behdad [2] et al, where the reflectarray is realized by a MEFSS realizing a TTD response, similar to the one proposed in Chapter 4. However, the similarities end there.

The aim of this section is to qualitatively assess beam qualities over a wide bandwidth between the state-of-the-art wideband reflectarray and the impedance surface reflectarray. We provide simulations of the far-field patterns of the two reflectarrays for the same geometry ($f/D$ ratio, $D$ and offset angle) and frequency range to make the comparison as fair and meaningful as possible. For the remaining part
Chapter 5. Comparison of TO Reflectors and Impedance Surface Reflectarrays

Figure 5.1: Two parabolic reflectors operating at two different frequencies $2f_o$ and $f_o$ with the same electrical diameter size but with different physical sizes. The maximum distance of compensation of the larger reflector is four times bigger $d_{\text{max,2}} = 4d_{\text{max,1}}$ leading to four times greater bandwidth for the smaller reflector for the impedance surface reflectarray, as per (5.1), given the same Bessel filter order.

of this chapter, we will refer to the state-of-the-art reflectarray in [2] as the ladder filter reflectarray for reasons that will soon become clear.

Figure 5.2 shows an illustration of the unit cell of the ladder filter reflectarray [2]. In general, it can be consist of $N$ substrate layers but here we use 3 layers as per the example presented in their paper [2]. The dimensions of the patches are different for each position on the reflectarray producing the different required reflection coefficients. The corresponding lumped element model of this unit cell is shown in Figure 5.3, which resemble a filter in the ladder form [162]. The thickness of each of the substrate is modeled as a fixed series inductor $L_f$ and the patches are modeled as shunt capacitances $C_1$ and $C_2$. The middle substrate with no patches is modeled as a fixed shunt capacitor $C_f$ and a fixed series inductor $L_f$. We adopt the values of the fixed components from [2] and they depend on the thickness $h$ and dielectric constant $\varepsilon_r$ of the substrate,

$$L_f = \mu_0 h, \quad C_f = \varepsilon_0 \varepsilon_r h, \quad \varepsilon_r = 3.4, \quad h = 1.524 \text{mm.}$$

In Figure 5.3, the capacitance $C_1$ and $C_2$ are tuned via the physical size of the patches and tunes the input impedance $z_{\text{in}}$ looking into the unit cells. The reflection coefficient is defined as

$$\Gamma(\omega, C_1, C_2) = \frac{z_{\text{in}}(\omega, C_1, C_2) - \eta_0}{z_{\text{in}}(\omega, C_1, C_2) + \eta_0},$$

where $\eta_0$ is the impedance of free space and the capacitances $C_1$ and $C_2$ are defined in Figure 5.3. Defining $\tau(x, y)$ to be the time delay compensation profile, for an ideal TTD reflectarray, the reflection coefficient $\Gamma(\omega)$ is to satisfy the conditions

$$|\Gamma(\omega)| = 1 \quad \text{Arg}(\Gamma(\omega)) + 2\pi n = \omega \tau(x, y), \quad n \in \mathbb{Z}$$

at every required delay $\tau$ for all frequencies of interest. Arg denotes the principal argument of a complex number. The reference work [2] does not explicitly specify how this was achieved even though it was indeed achieved by tuning the dimensions of the patches. In this section, we use a brute-force method
to find the optimal $C_1$ and $C_2$ that satisfies (5.4). The cost function is defined as

$$f_{\text{cost}}(C_1, C_2, n) = \int_{f_1}^{f_2} |\text{Arg}(\Gamma(\omega, C_1, C_2)) + 2\pi n - \omega \tau(x, y)| d\omega,$$

which is evaluated for each $\tau$. $f_1$ and $f_2$ are the lower and upper bounds of the frequency range of interest. The cost function is to be minimized over the defined range as

$$\min_{C_1, C_2 \in R_{C_n} \in \mathbb{Z}} f_{\text{cost}}(C_1, C_2, n), \quad R_C = [0.1, 2000] \text{fF}. \quad (5.6)$$

The reason for choosing the maximum value of 2000 fF as the upper bound for the search range is that we want to cover a bit more than what can be typically produced by printed interdigitated capacitors to give the best chance for this approach to function properly. To the best of the author’s knowledge, with the current manufacturing tolerances, the printed interdigitated capacitors can typically produce a maximum capacitance around several hundreds of femto-farads.

Figure 5.2: The unit cells of the state-of-the-art reflectarray [2] in the current literature. We call this the ladder filter reflectarray. The fixed component values are $L_f = \mu_0 h$, $C_f = \varepsilon_0 \varepsilon_r h$, where $h = 1.524 \text{mm}$ is the substrate thickness and $\varepsilon_r = 3.4$ is the dielectric constant of the substrate. $C_1$ and $C_2$ are tuned by the dimensions of the patches and they can be optimized over.

Figure 5.3: The lumped element model of the unit cells on the reflectarray shown in Figure 5.2. $L_f = \mu_0 h$ and $C_f = \varepsilon_0 \varepsilon_r h$ are the fixed components with $h = 1.524 \text{mm}$ being the substrate thickness. $C_1$ and $C_2$ are optimized in the range $[0.1, 2000] \text{fF}$. 
Figure 5.4: A lumped element model that is the same as in Figure 5.3 except that the middle components $L_f$ and $C_f$ are removed, corresponding to the removal of the middle substrate in Figure 5.2.

Figure 5.5: A lumped element model consist of only one capacitive layer $C_1$ on top on a ground plane.

For each required $\tau$, the optimal capacitances $C_1^*$ and $C_2^*$ are found which minimize the defined cost function. Since all the component values in the lumped element model in Figure 5.3 are known, its input impedance can be calculated and treated as the input to the MoM simulator described in Section 4.6.2 to compute the far-field patterns of the ladder filter reflectarray. Before discussing the far-field patterns, two other lumped element models are described first, which are shown in Figures 5.4 and 5.5. These lumped element models have some components removed and they need a fewer number of layers of substrate and patches for realization. The aim here is to qualitatively deduce an approximate number of layers needed to produce far-field patterns that are comparable to that produced by the impedance surface reflectarray. In Figure 5.4, the middle components $L_f$ and $C_f$ are removed which corresponds to the removal of the middle substrate layer in Figure 5.2 producing a design with only two substrate layers. Likewise, the lumped element model shown in Figure 5.5 has all capacitive layers removed except for the one on top of the ground plane. The brute-force optimization procedure described previously is used to find the optimal capacitances $C_1^*$ and $C_2^*$ in all the lumped element models. The optimized phase of the reflection coefficient $\text{arg}(\Gamma(\omega))$ and the ideal reflection phase $-\omega \tau$ are shown in Figures 5.6(a), (b) and (c) corresponding to the 3 substrate layers design in Figure 5.3, the 2 substrate layers design in Figure 5.4 and the single substrate design in Figure 5.5 respectively. It is clear that when a fewer number of layers is used, the quality of the approximation of $\text{arg}(\Gamma(\omega))$ to the ideal reflection phase degrades as expected. In Figure 5.6, the ideal reflection $-\omega \tau$ is shown for several $\tau$ in the range from 0.1 ps to 180.1 ps in 30 ps steps.

With the optimized $C_1$ and $C_2$ found for each delay value in the range [0.1, 210.1] ps, the input impedance to each of the lumped element models are computed and input to the MoM simulator to calculate the far-field patterns. To make a fair comparison, the geometries of both the impedance surface reflectarray and the ladder filter reflectarray are the same and it is described in Figure 4.18. In the MoM simulation, an ideal Hertzian dipole is used as the feeding antenna for simplicity and there are no losses included in the simulations.

Figures 5.7 and 5.8 shows the far-field patterns for four cases. The first case is the impedance
Figure 5.6: Optimized phase of the reflection coefficient $\Gamma(\omega)$ with respect to the target phase $-\omega \tau$ for several delays $\tau = 0.1$ to 180.1 ps in 30 ps steps.

Surface reflectarray designed with Bessel filters as described in Chapter 4. The rest of the cases are the far-field patterns of the ladder filter reflectarray designed using the numerically optimized approach described in this section using 3, 2 and 1 substrate layers. The overall beam shapes of the ladder filter are comparable to that of the impedance surface reflectarray. However, there are a few differences. The impedance surface reflectarray performs better at the lower end of the frequency range. In particular, at 5 and 6 GHz, the impedance surface reflectarray as a higher directivity due its lower side lobe levels compared to the ladder filter reflectarray. This is the expected behavior as the impedance surface realizes a low-pass Bessel filter response. The worse-performing case is the ladder filter reflectarray with only one substrate layer as expected, particularly at the low-end of the frequency range. At the high end of the frequency range, the ladder filter with three substrate layers has less kinks in the shape of the main lobe compared to the impedance surface reflectarray. Two kinks are circled as examples in Figures 5.8(c) and (d). Qualitatively, the ladder filter reflectarray with two substrate layers has its far-field patterns the closest to the impedance surface reflectarray. The overall simulated patterns for both types of reflectarrays are excellent and no significant beam squinting is observed for the simulated frequency range from 5 to 10 GHz, a 67% fractional bandwidth.
Figure 5.7: Far-field patterns of the impedance surface reflectarray and the ladder filter reflectarray [2] with optimized reflection phase for frequencies from 5 to 7 GHz using 3, 2 and 1 capacitive layers, corresponding to the lumped element models in Figures 5.3, 5.4 and 5.5 respectively.
Figure 5.8: Far-field patterns of the impedance surface reflectarray and the ladder filter reflectarray with optimized reflection phase for frequencies from 8 to 10 GHz using 3, 2 and 1 capacitive layers, corresponding to the lumped element models in Figures 5.3, 5.4 and 5.5 respectively.
Here, we discuss the differences from a theoretical perspective. Despite the goal being the same for both the impedance surface reflectarray and the ladder filter reflectarray, which is to realize a TTD response over a wide bandwidth using sub-wavelength elements, the design process is entirely different. In the ladder filter reflectarray, a set of optimal lumped elements must be found in order for the reflection phase to approximate the ideal reflection phase. It is important for any numerical optimization routine used here to be fast and efficient as there could be a large set of parameters to be optimized over. The ladder filter reflectarray discussed in this section only has two capacitances to be optimized over but in general all other components, such as the series inductors can also be included in the domain of optimization. In contrast, in the design of the impedance surface reflectarray, since the optimization is performed analytically to find the optimal component values, closed-form expressions for the component values exist and they scaled multiples of the required delay. Moreover, the ladder filter reflectarray is a multi-layer structure that has implications on its manufacturing process. The multi-layer structure potentially requires bonding films that further complicates the manufacturing process and their frequency responses need to be taken into account [2]. In contrast, the impedance surface reflectarray has metal patterns only on two sides of the same substrate and no bonding films are needed. It is important to emphasize that the impedance surface reflectarray is an inherently single layer design where all the components can reside on the same layer in theory. However, the overall thickness of the ladder filter reflectarray is 0.16 \( \lambda \) which is electrically thinner than the 0.25 \( \lambda \) for the impedance surface reflectarray, evaluated at their own center frequencies. The quarter wavelength thickness for the impedance surface reflectarray arises from the need to provide an approximate open circuit termination required by the design method. In the ladder filter reflectarray, the termination is already a PEC by design. Further, an advantage of the ladder filter reflectarray is its ability to operate at an arbitrary center frequency. This is not the case for the impedance surface reflectarray, which by construction of its design method, the intrinsic center frequency is at DC due to the Taylor series expansion of the group delay about \( \omega = 0 \). However, we have shown that the impedance surface reflectarray still functions well over an useful frequency range despite that some of the available bandwidth, for example DC to 5 GHz, is potentially wasted as the electrical size of the reflector is too small. It is worth noting that the design method used in the ladder filter reflectarray could be used in lensing applications to synthesize a broadband transmittarray [170] and other filter responses such as Butterworth and Chebyshev types [51]. In the impedance surface reflectarray, the design method and the unit cells are specific to the reflectarrays.

Table 5.2 on page 97 summaries the most important properties of the impedance surface reflectarray and the ladder filter reflectarray discussed in this section.

### 5.2.1 Qualitative Comparisons with Other Wideband Reflectarrays

Table 5.3 shows some representatives of wideband reflectarrays in the current literature using patches, rings and delay lines. This is by no means an exhaustive list. A direct comparison between the fractional bandwidths of these reflectarrays is not possible nor meaningful because their bandwidths are not well-defined. Even if they are, it would be difficult to make a fair comparison as they have different electrical sizes, \( f/D \) ratios, focal lengths and offset angles. Here, we present their achieved fractional bandwidth based on what is presented in their papers and attempted to be clear, whenever possible, the definition used for the bandwidth. These fractional bandwidth numbers are not meant to be used in a quantitative comparison between the different works but rather a qualitative comparison to help readers to gauge the bandwidth achieved in this thesis compared to others currently in the literature. All of the presented
works have good beam characteristics with reasonable side lobe levels within their frequency bandwidth.

We make some qualitative comparisons between our impedance surface reflectarray and other works shown in the table. Most of other works use multiple layers of scatterers possibly with sub-wavelength non-resonant elements to improve the bandwidth. Although these methods are effective, the multi-layer approach inherently complicates the fabrication process as bonding agents may need to be used, as demonstrated in [2]. The use of concentric rings on one layer is akin to cascading multiple resonances close together and it eliminates the need to have multiple layers of scatterers but the achievable improvements in bandwidth are often limited as there are only a limited number of concentric rings that can be placed within a single unit cell. Although this comparison is qualitative, it is evident that our impedance surface reflectarray has advantages over other reflectarrays in terms of the achieved bandwidth and the number of layers needed to achieve the same bandwidth. However, the impedance surface tends to be thicker than the other works due to the ground plane needed at $\lambda/4$ away at the center frequency, a condition that is not needed for the other reflectarrays. Hence, a trade-off is expected between the profile thickness and bandwidth.
## Chapter 5. Comparison of TO Reflectors and Impedance Surface Reflectarrays

<table>
<thead>
<tr>
<th>TO Reflector</th>
<th>Impedance Surface Reflectarray</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Reflector Topology</strong></td>
<td><strong>Impedance Surface Reflectarray</strong></td>
</tr>
<tr>
<td>• TO reflectors are mostly limited to 2-D reflectors due to the wide variety of available conformal coordinate transformation available in 2-D. “3-D” conformal TO devices can be realized by rotating a 2-D profile along an axis.</td>
<td>• Full 3-D reflectors can be designed.</td>
</tr>
<tr>
<td><strong>Physical Profile</strong></td>
<td><strong>Physical Profile</strong></td>
</tr>
<tr>
<td>• The physical profile is frequency independent.</td>
<td>• The thickness is at λ/4 thickness at center frequency, leading to thin physical profile at high frequencies. Hence, it is more suitable for higher frequency of operation within the frequency range provided by the chosen Bessel filter.</td>
</tr>
<tr>
<td>• The electrical thickness is less meaningful due to lack of frequency concept.</td>
<td>• Profile remains at λ/4 thickness for larger reflector at a fixed frequency.</td>
</tr>
<tr>
<td>• There is a tradeoff between thickness and beam quality.</td>
<td>• There is a tradeoff between thickness and beam quality.</td>
</tr>
<tr>
<td>• Thicker profile for larger reflector at a fixed frequency.</td>
<td>• Thicker profile for larger reflector at a fixed frequency.</td>
</tr>
<tr>
<td><strong>Bandwidth</strong></td>
<td><strong>Bandwidth</strong></td>
</tr>
<tr>
<td>• The reflector bandwidth depends on the bandwidth in which the realized material approximates the required one.</td>
<td>• The reflector bandwidth depends on the bandwidth in which the realized material approximates the required one.</td>
</tr>
<tr>
<td>• For constant f/D ratio and electrical size, different frequency of operation leads the same TO reflector with a different physical size.</td>
<td>• For constant f/D ratio and electrical size, different frequency of operation leads the same TO reflector with a different physical size.</td>
</tr>
<tr>
<td><strong>Maximum Reflector Size</strong></td>
<td><strong>Maximum Reflector Size</strong></td>
</tr>
<tr>
<td>• No theoretical limit on the electrical or physical size of the reflector.</td>
<td>• No theoretical limit on the electrical or physical size of the reflector.</td>
</tr>
<tr>
<td>• For a fixed chosen Bessel filter order, the maximum bandwidth and the maximum electrical size are fixed.</td>
<td>• For a fixed chosen Bessel filter order, the maximum bandwidth and the maximum electrical size are fixed.</td>
</tr>
<tr>
<td><strong>Advantages</strong></td>
<td><strong>Advantages</strong></td>
</tr>
<tr>
<td>• It has very wide bandwidth.</td>
<td>• The reflector is thinner and lightweight. It can potentially be rolled-up for transportation or storage.</td>
</tr>
<tr>
<td>• No theoretical limit on bandwidth or electrical size of the reflector.</td>
<td>• It can be easily manufactured by printing metal patterns. No stacking is necessary.</td>
</tr>
<tr>
<td>• It can be made lightweight due to the planar printed circuits used.</td>
<td>• It can be easily manufactured by printing metal patterns. No stacking is necessary.</td>
</tr>
<tr>
<td>• It can be easily manufactured by printing metal patterns but they need to be stacked together.</td>
<td>• It can be easily manufactured by printing metal patterns. No stacking is necessary.</td>
</tr>
<tr>
<td><strong>Disadvantages</strong></td>
<td><strong>Disadvantages</strong></td>
</tr>
<tr>
<td>• Profile tends to be bulky and thick.</td>
<td>• Finite electrical diameter for a fixed Bessel filter order.</td>
</tr>
</tbody>
</table>

Table 5.1: Summary of comparison between the TO reflector and the impedance surface reflectarray.
<table>
<thead>
<tr>
<th></th>
<th>Impedance Surface Reflectarray</th>
<th>Ladder Filter Reflectarray</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Reflector type</strong></td>
<td>• TTD</td>
<td>• TTD</td>
</tr>
<tr>
<td><strong>Design principle</strong></td>
<td>• Bessel filter is capitalized in a special way.</td>
<td>• An optimization method can be used to obtain an approximate TTD response from the unit cells.</td>
</tr>
<tr>
<td></td>
<td>• Normal incidence only.</td>
<td>• Normal incidence only.</td>
</tr>
<tr>
<td><strong>Cell size and spacing</strong></td>
<td>• Sub-wavelength.</td>
<td>• Sub-wavelength.</td>
</tr>
<tr>
<td></td>
<td>• Macroscopic surface properties such as surface impedance and surface currents are defined and solved for on the reflectarray surface.</td>
<td>• Macroscopic surface properties such as surface impedance and surface currents are defined on the surface.</td>
</tr>
<tr>
<td><strong>Generality</strong></td>
<td>• The design method and unit cell structure is specific to reflectarrays.</td>
<td>• The design method and unit cell structure can be applied to other filter types such as Butterworth and Chebyshev, and lensing applications.</td>
</tr>
<tr>
<td><strong>Profile</strong></td>
<td>• Inherently single layer design for any filter order.</td>
<td>• Multi-layer design.</td>
</tr>
<tr>
<td></td>
<td>• Needs a PMC termination that increases the profile thickness to λ/4 at center frequency.</td>
<td>• It has as thinner profile at 0.16 λ at the center frequency due to the PEC termination. No PMC is required.</td>
</tr>
<tr>
<td><strong>Fabrication process</strong></td>
<td>• Relatively easy.</td>
<td>• Relatively more difficult. Bonding agents such as bonding films are needed between layers.</td>
</tr>
<tr>
<td><strong>Center frequency</strong></td>
<td>• Intrinsically at DC.</td>
<td>• No theoretical restriction.</td>
</tr>
</tbody>
</table>

Table 5.2: A comparison between the impedance surface reflectarray and the state-of-the-art ladder filter reflectarray [2].
### Table 5.3: A comparison of our impedance surface reflectarray with other works in the literature.

<table>
<thead>
<tr>
<th>Paper</th>
<th>Delay Mechanism</th>
<th>Layers</th>
<th>Thickness</th>
<th>Bandwidth Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>[2]</td>
<td>Sub-wavelength patches</td>
<td>3 substrate layers 2 metal layers 1 ground plane</td>
<td>0.16 λ</td>
<td>Designed to work from 8 to 12 GHz (40%)</td>
</tr>
<tr>
<td>[149]</td>
<td>Sub-wavelength loops</td>
<td>1 metal layer 1 ground plane</td>
<td>0.11 λ</td>
<td>Designed to work from 9.5 to 12 GHz (23.3%)</td>
</tr>
<tr>
<td>[20]</td>
<td>Aperture coupled delay lines</td>
<td>3 metal layers 1 ground plane</td>
<td>0.35 λ</td>
<td>9.1 to 11.75 GHz (25.4%) in which the measured gain is within 3 dB compared to that of a reflectarray with an ideal TTD response</td>
</tr>
<tr>
<td>[1]</td>
<td>Aperture coupled delay lines</td>
<td>3 metal layers 1 ground plane</td>
<td>0.11 λ</td>
<td>8.5 to 11.6 GHz (30%) in which the measured gain is within 3 dB compared to that of a reflectarray with an ideal TTD response</td>
</tr>
<tr>
<td>[28]</td>
<td>Cascade of resonances using rings</td>
<td>1 metal layer 1 ground plane</td>
<td>0.11 λ</td>
<td>21 to 23.5 GHz (11.2%) bandwidth in which the gain stays within 3 dB of the peak</td>
</tr>
<tr>
<td>Our Impedance surface RA</td>
<td>Metasurface</td>
<td>2 metal layers 1 ground plane</td>
<td>0.25 λ</td>
<td>Designed to work for 5 to 10 GHz (66.7%)</td>
</tr>
</tbody>
</table>
Chapter 6

Conclusion

The goal of this thesis was to investigate how to improve the narrow bandwidth typically associated with reflectarrays. In this thesis, we designed reflectors with an ultra-wide bandwidth without explicitly using TTD devices. Two main approaches investigated are the TO approach and the impedance surface approach. Reflectors designed using both approaches are of TTD nature which is responsible for their wideband characteristics, though the mechanism to achieve a TTD response was complete different between the two approaches. The unique aspects of both gave approaches arise to their own set of pros and cons which were discussed in details in the previous chapter.

In Chapter 3, the objective was to investigate whether TO can be used effectively as a means to design wideband reflectors and to investigate the practical considerations for designing such a reflector. It was found that wideband reflectors can be designed, using conformal coordinate transformations, to produce reflectors with UWB characteristics. Schwarz-Christoffel mappings are ideal to produce the needed conformal coordinate transformation because they allowed for perfectly isotropic materials to be used in the TO reflectors without the need to monitor the degree of anisotropy potentially introduced by the coordinate transformation. The conformal property of the coordinate transformation was critical to produce the isotropic materials that were easily realized over a wide band of frequencies. Although the TO reflector exhibited a UWB response, it had a relatively thick profile, on the order of a few wavelengths at the center frequency of operation. It was reasoned that this is a necessary condition for the TO reflector to have a wide bandwidth. In addition, the TO reflector had many layers of scatterers that needed to be held together to make up for the dielectric volume. Moreover, it was found that most of the 3-D TO devices in literature are not a direct result of a 3-D conformal coordinate transformation, as they would be all Möbius transformations, but as a result of rotating a 2-D profile about an axis to produce the 3-D profile. The lack of available conformal coordinate transformations in 3-D means that the materials needed in an intrinsically 3-D TO device must be anisotropic. This poses an even greater challenge to engineer materials for 3-D applications because the materials responses not only change as a function of position but also of orientation. This constrains TO reflectors and TO devices in general to intrinsically 2-D devices. Although TO reflectors are ultra-wideband, these drawbacks provided strong motivations to design reflectors in a different approach that overcomes these issues.

With the aforementioned issues in mind, a new method to design reflectarrays was developed in Chapter 4. The design method was developed from first principles. The new method was able capitalize on the maximal bandwidth provided by Bessel filters in which the cells of reflectarrays are able to realized.
Instead of engineering for bulk material in the TO reflector, this new method called for engineering specific surface properties, namely a surface impedance profile, which was realized by a metasurface using printed components. The reflectarray designed as a result of this method brought great improvements to the previously considered TO approach in many aspects, namely, full 3-D reflectors can now be realized and the resulting reflectarrays are much thinner profile at \( \lambda/4 \) at the center frequency. It was shown that the impedance surface reflectarray exhibited a wideband behavior both spatially and temporally. The temporal characteristics was characterized by comparing the pulse shapes of the incident and the reflected pulses and a simulated signal fidelity of 0.98 was achieved.

Through these investigations, we have accomplished the original goals and objectives of this thesis:

1. We have identified TO as viable approach to design reflectors with ultra-wide bandwidth. We have investigated the practical details of how reflectors can be designed with TO and generated insight into what is involved in designing such reflectors. The advantages and drawbacks of this approach are clearly outlined.

2. We have developed a novel method for designing reflectarrays as an impedance surface that addresses the drawback associated with the TO reflector. The impedance surface was able to leverage the optimized response of the Bessel filter from which the reflectarray cells are designed. The novel method provided an even more practical approach to design UWB reflectors.

3. We have systematically compared each of the above approaches against each other and summarized the findings in Table 5.1. We have also compared our own impedance surface reflectarray with the current state-of-the-art ladder filter reflectarray. It was found that although good beam characteristics can be produced by the ladder filter reflectarray, it needed multiple layers of scatterers to produce beam patterns that are similar to that of the impedance surface reflectarray, which is inherently a single layer design.

With practical considerations in mind, such as the ease of fabrication, low-profile and the ability to produce 3-D reflectors, this thesis recommends reflectors designed the impedance surface method over that of the TO reflector and over that of the state-of-the-art ladder filter reflectarray. The impedance surface is less bulky than the TO reflector, it is simple to design and to fabricate and it is inherently a single layer design.

6.1 Future Work

The TO reflector in Chapter 3 presents interesting challenges for future work. The current physical form is rather bulky and is likely to limit its usefulness in some practical applications. In the presented TO reflector, designers had little intrinsic control over the physical shape of the TO reflector other than simply truncating the permittivity profile. This is due to the requirement that the conformal modulus of the TO reflector and that of the polygon in virtual space must be the same in order for a conformal coordinate transformation to be applied. As an example to help reduce the profile, a more compact non-conformal coordinate transformation can be used to produce a thinner TO reflector. The non-conformal coordinate transformation induce anisotropic materials with magnetic response that need to be realized. Putting aside the obvious challenges associated with engineering such metamaterials, especially over a wide frequency range, a rigorous study is needed to determine the reflection properties...
Chapter 6. Conclusion

at the boundaries of embedded TO devices. Such a study would help to determine the conditions that need to be imposed on a coordinate transformation on the device boundary for a reflectionless or a low reflection boundary. Currently, best to author's knowledge, only a heuristic measure exists, which is based on a metric-matching condition [35], to predict the reflection properties at the boundaries of embedded TO devices. On the practical side, the TO reflector shown in Chapter 3 had excellent angular beam scanning characteristics, a useful feature that could be realized by having electrically tunable dielectrics in the cells of the reflector. This can be accomplished by using electronically tunable elements such as varactor diodes. Having both the wideband and tunable features would make the reflector extremely attractive and potentially useful in many applications. However, it is expected to be challenging to realize a tunable version of the TO reflector as there are a large number of cells that need to be controlled. The aforementioned future works are merely examples and they are by no means a completed list. These challenges make the future research of reflector design using TO interesting.

In this thesis, a reflectarray was also realized by designing the reflectarray as an impedance surface. Although it was shown to have an excellent wideband characteristics, it had some drawbacks that were outlined in Table 5.1 that can benefit from improvements in the future. The unit cells of the impedance surface reflectarray realized Bessel filters whose responses are centered at DC. It would be ideal for the unit cells to be able to realize a bandpass version of the Bessel filter; that is, a filter with a maximally flat group delay response centered at a particular frequency. A bandpass version could potentially yield more bandwidth compared to the current low-pass version as all of their passband is utilized. Although the bandpass versions of the Bessel filter exist as coupled-resonator filters which are described in common filter textbooks [171], they are in the transmit mode where the transfer function is defined from an input voltage to an output voltage on the other side of the filter circuit. This is incompatible with reflectarray mode which has the reflection coefficient $\Gamma(s)$ as the transfer function. In addition, the coupled-resonators filters have series inductances in their circuits which are typically realized by a spatial separation between multiple layers of scatterers. This presents interesting future research in how to best utilize this filter circuit topology in a reflectarray mode operation. The second issue concerns the method of termination for the impedance surface reflectarray. Currently, the design method requires a perfect open or PMC as the termination which was approximated by having a ground plane located at $\lambda/4$ away at the center frequency of operation. Instead of using the ground plane, wideband artificial high impedance surfaces can be used to back the reflector and it would be located, in theory, in the same plane as the impedance surface. Though, in practice the artificial impedance surface would be slightly offset and it would not be in contact with the impedance surface to ensure their proper functions. As a result, the reflector would have a paper-thin, light-weight and have an ultra-low profile so that it could be easily stored or transported.

6.2 Closing Remarks

The practicality of TO devices largely depends on the practicality of engineering metamaterials needed to realize the material parameters that are induced by the coordinate transformations. Recall that a coordinate transformation in general yields arbitrary material parameters, $\varepsilon_r, \mu_r$, which are difficult to realize as they are fully tensorial and have a magnetic response. In addition, using TO to design wideband devices adds additional engineering challenges because the materials need to hold their electrical properties over a wide frequency range. Many of the TO devices currently in literature reside
only in simulations as the required materials are often difficult to realize for an experimental validation. This thesis has paved the way to design wideband TO reflectors by using a specific type of coordinate transformation, for at least for one polarization, it overcomes some of the aforementioned challenges. TO in general is a still relatively new area of research and it will undoubtedly experience a great deal of research growth. The metamaterials of the future would be able to capitalize the full potential of anisotropic materials, allowing the designers to leverage the full power of TO. The design of reflectors using TO is still at its infancy but as more exotic metamaterials becomes available, the practicality of TO reflectors and TO devices in general will become more apparent in realistic applications.

The design of reflectors using metasurfaces is also at the beginning of its development. There is a shift in perspective in what can be considered as a reflectarray. Traditionally, they are considered to be an array of individual antenna elements each responsible for producing an appropriate reflection phase. The element size and spacing are comparable to the wavelength. More modern reflectarrays can be made out of MEFSSs where the element size and spacing are sub-wavelength, forming homogenized surface properties such as surface impedance and admittances, which are absent in the design process of traditional reflectarrays. The design of MEFSSs offers a promising outlook to design the next generation of reflectarrays because they are currently the closest RF devices that can support the flow of arbitrary electric and magnetic currents. These currents are the source currents that are able to produce an arbitrary set of fields, paving the way to design novel reflectors such as the one shown in the thesis, among many other interest devices.

6.3 Contributions

• Journal Publications

• Other Journal Publications

• Conferences Publications
Appendix A

Impedance Surface Reflectarray Mask

Figure A.1: The layout of the impedance surface reflectarray. Pink is the top metal layer and yellow is the bottom metal layer. An inset shows the zoomed in version. There are 7030 unit cells in total. The metal pattern is accomplished by a chemical etching process. The square at the top right is not part of the impedance surface and it is for alignment and orientation purposes.
Appendix B

Extraction of Anisotropic Sheet Impedance

In this section, we present a derivation of extracting fully anisotropic sheet impedances of a metasurface from its mode-dependent $s$-parameters. Figure B.1 defines the setup of the problem. In this section, the subscripts $x$ or $y$ denote the respective components of the vector field quantities, subscripts 1 or 2 denote the region in which the fields reside and superscripts $+$ or $-$ denote incident and reflected fields respectively.

![Diagram of the problem definition to extract the anisotropic sheet impedance of a metasurface.](image)

Figure B.1: The problem definition to extract the anisotropic sheet impedance of a metasurface.

The relationship between the incident and reflected E-fields as a function of the mode-dependent reflection coefficients is

\[
\begin{bmatrix}
E_{x,-,1}^- \\
E_{x,-,1}^-
\end{bmatrix} = \begin{bmatrix}
\Gamma_{xx} & \Gamma_{xy} \\
\Gamma_{yx} & \Gamma_{yy}
\end{bmatrix} \begin{bmatrix}
E_{x,+,-,1}^+ \\
E_{y,+,-,1}^-
\end{bmatrix}.
\]

(B.1)

These reflection coefficients are for the fundamental (propagating) Floquet mode only. The relationship between the incident and the transmitted E-fields as a function of the mode-dependent transmission coefficients is

\[
\begin{bmatrix}
E_{x,-,2}^- \\
E_{y,-,2}^-
\end{bmatrix} = \begin{bmatrix}
T_{xx} & T_{xy} \\
T_{yx} & T_{yy}
\end{bmatrix} \begin{bmatrix}
E_{x,+,-,1}^+ \\
E_{y,+,-,1}^-
\end{bmatrix}.
\]

(B.2)

Again, these transmission coefficients are for the fundamental (propagating) Floquet mode only. We
would like to express
\[
\begin{bmatrix}
E_{\text{tot}}^x \\
E_{\text{tot}}^y
\end{bmatrix}
= \overline{\mathbf{Z}}_s
\begin{bmatrix}
J_x \\
J_y
\end{bmatrix},
\overline{\mathbf{Z}}_s = \begin{bmatrix}
Z_{xx} & Z_{xy} \\
Z_{yx} & Z_{yy}
\end{bmatrix}
\]
(B.3)

using (B.1) and (B.2) and then solve for \( \overline{\mathbf{Z}}_s \). \( E_{\text{tot}}^x \) and \( E_{\text{tot}}^y \) are the \( x \) and \( y \) components of the total E-field on the metasurface. We express the E-fields using the H-fields as
\[
\begin{bmatrix}
E_{-,1}^x \\
E_{-,1}^y
\end{bmatrix} = \eta_0 \begin{bmatrix}
H_{-,1}^x \\
-H_{-,1}^y
\end{bmatrix},
\begin{bmatrix}
E_{-,2}^x \\
E_{-,2}^y
\end{bmatrix} = \eta_0 \begin{bmatrix}
-H_{-,2}^y \\
H_{-,2}^x
\end{bmatrix},
\begin{bmatrix}
E_{+,1}^x \\
E_{+,1}^y
\end{bmatrix} = \eta_0 \begin{bmatrix}
H_{+,1}^x \\
H_{+,1}^y
\end{bmatrix}
\]
(B.4)

We substitute (B.4) into (B.1) and (B.2) to obtain the relations between the incident and reflected H-fields,
\[
\begin{bmatrix}
H_{-,1}^y \\
H_{-,1}^x
\end{bmatrix} = \begin{bmatrix}
-\Gamma_{xx} & \Gamma_{xy} \\
\Gamma_{yx} & -\Gamma_{yy}
\end{bmatrix} \begin{bmatrix}
H_{+,1}^y \\
H_{+,1}^x
\end{bmatrix},
\]
(B.5)

and
\[
\begin{bmatrix}
H_{-,2}^y \\
H_{-,2}^x
\end{bmatrix} = \begin{bmatrix}
T_{xx} & -T_{xy} \\
-T_{yx} & T_{yy}
\end{bmatrix} \begin{bmatrix}
H_{+,2}^y \\
H_{+,2}^x
\end{bmatrix},
\]
(B.6)

where the negative signs are absorbed into the matrix as shown. We also note the relationship between the incident E-fields and H-fields,
\[
\begin{bmatrix}
H_{+,1}^y \\
H_{+,1}^x
\end{bmatrix} = \frac{1}{\eta_0} \begin{bmatrix}
-E_{+,1}^x \\
E_{+,1}^y
\end{bmatrix}
\]
(B.7)

We express the surface currents as the difference in the H-fields of both regions as
\[
\begin{bmatrix}
J_x \\
J_y
\end{bmatrix} = \begin{bmatrix}
H_{\text{tot}}^{y,1} - H_{\text{tot}}^{y,2} \\
-H_{\text{tot}}^{x,2} + H_{\text{tot}}^{x,1}
\end{bmatrix},
\]
(B.8)

where the superscript tot denotes the total field in their respective regions. We express the total H-fields as
\[
\begin{bmatrix}
H_{\text{tot}}^{y,2} \\
H_{\text{tot}}^{x,2}
\end{bmatrix} = \begin{bmatrix}
H_{-,2}^y \\
H_{-,2}^x
\end{bmatrix} = \begin{bmatrix}
T_{xx} & -T_{xy} \\
-T_{yx} & T_{yy}
\end{bmatrix} \begin{bmatrix}
H_{+,1}^y \\
H_{+,1}^x
\end{bmatrix}
\]
(B.9)

and
\[
\begin{bmatrix}
H_{\text{tot}}^{y,1} \\
H_{\text{tot}}^{x,1}
\end{bmatrix} = \begin{bmatrix}
H_{+,1}^y \\
H_{+,1}^x
\end{bmatrix} + \begin{bmatrix}
H_{-,1}^y \\
-H_{-,1}^x
\end{bmatrix} = \left( \begin{bmatrix}
1 & 0 \\
0 & 1
\end{bmatrix} + \begin{bmatrix}
-\Gamma_{xx} & \Gamma_{xy} \\
\Gamma_{yx} & -\Gamma_{yy}
\end{bmatrix} \right) \begin{bmatrix}
H_{+,1}^y \\
H_{+,1}^x
\end{bmatrix}
\]
(B.10)

We substitute (B.9), (B.10) and (B.7) into (B.8) to obtain
\[
\begin{bmatrix}
J_x \\
J_y
\end{bmatrix} = \left( \begin{bmatrix}
T_{xx} & -T_{xy} \\
T_{yx} & -T_{yy}
\end{bmatrix} + \begin{bmatrix}
-1 & 0 \\
0 & 1
\end{bmatrix} + \begin{bmatrix}
-\Gamma_{xx} & -\Gamma_{xy} \\
-\Gamma_{yx} & \Gamma_{yy}
\end{bmatrix} \right) \begin{bmatrix}
H_{+,1}^y \\
H_{+,1}^x
\end{bmatrix}
\]
(B.11)

\[
\begin{bmatrix}
J_x \\
J_y
\end{bmatrix} = \begin{bmatrix}
-1 + \Gamma_{xx} + T_{xx} & -\Gamma_{xy} - T_{xy} \\
\Gamma_{yx} + T_{yx} & 1 - \Gamma_{yy} - T_{yy}
\end{bmatrix} \begin{bmatrix}
-\frac{E_{+,1}^x}{\eta_0} \\
E_{+,1}^y
\end{bmatrix}
\]
(B.12)
We express the total E-fields as

\[
\begin{bmatrix}
E_{x,\text{tot}} \\
E_{y,\text{tot}}
\end{bmatrix} = \frac{1}{2} \left( \begin{bmatrix}
E_{x,1}^+ \\
E_{y,1}^+
\end{bmatrix} + \begin{bmatrix}
E_{x,1}^- \\
E_{y,1}^-
\end{bmatrix} \right),
\]

(B.13)

and substituting (B.1) and (B.2) into (B.14) to obtain

\[
\begin{bmatrix}
E_{x,\text{tot}} \\
E_{y,\text{tot}}
\end{bmatrix} = \frac{1}{2} \left( \begin{bmatrix}
1 & 0 \\
0 & 1
\end{bmatrix} + \begin{bmatrix}
\Gamma_{xx} & \Gamma_{xy} \\
\Gamma_{yx} & \Gamma_{yy}
\end{bmatrix} + \begin{bmatrix}
T_{xx} & T_{xy} \\
T_{yx} & T_{yy}
\end{bmatrix} \right) \begin{bmatrix}
E_{x,1}^+ \\
E_{y,1}^+
\end{bmatrix}.
\]

(B.14)

The \(\frac{1}{2}\) factor in front of (B.14) is due to the averaging of the fields on both regions. Substituting (B.14) and (B.12) into (B.3) to obtain

\[
\frac{1}{2} \left( \begin{bmatrix}
1 & 0 \\
0 & 1
\end{bmatrix} + \begin{bmatrix}
\Gamma_{xx} & \Gamma_{xy} \\
\Gamma_{yx} & \Gamma_{yy}
\end{bmatrix} + \begin{bmatrix}
T_{xx} & T_{xy} \\
T_{yx} & T_{yy}
\end{bmatrix} \right) \begin{bmatrix}
E_{x,1}^+ \\
E_{y,1}^+
\end{bmatrix} = \begin{bmatrix}
\eta_o \\
\eta_o
\end{bmatrix}
\]

(B.15)

and

\[
\frac{\eta_o}{2} \begin{bmatrix}
1 + \Gamma_{xx} + T_{xx} & \Gamma_{xy} + T_{xy} \\
\Gamma_{yx} + T_{yx} & 1 + \Gamma_{yy} + T_{yy}
\end{bmatrix} \begin{bmatrix}
E_{x,1}^+ \\
E_{y,1}^+
\end{bmatrix} = \overline{Z}_s \begin{bmatrix}
1 - \Gamma_{xx} - T_{xx} & -\Gamma_{xy} - T_{xy} \\
-\Gamma_{yx} - T_{yx} & 1 - \Gamma_{yy} - T_{yy}
\end{bmatrix} \begin{bmatrix}
E_{x,1}^+ \\
E_{y,1}^+
\end{bmatrix}
\]

(B.16)

We cancel the incident fields on both sides of (B.16) and isolating for \(\overline{Z}_s\) to obtain the final expression for the tensorial sheet impedance \(\overline{Z}_s\),

\[
\overline{Z}_s = \frac{\eta_o}{2} \begin{bmatrix}
1 + \Gamma_{xx} + T_{xx} & \Gamma_{xy} + T_{xy} \\
\Gamma_{yx} + T_{yx} & 1 + \Gamma_{yy} + T_{yy}
\end{bmatrix} \begin{bmatrix}
1 - \Gamma_{xx} - T_{xx} & -\Gamma_{xy} - T_{xy} \\
-\Gamma_{yx} - T_{yx} & 1 - \Gamma_{yy} - T_{yy}
\end{bmatrix}^{-1}.
\]

(B.17)

For completeness, the mapping between the reflection and transmission coefficients shown here to the mode-dependent simulated s-parameters are

\[
\begin{bmatrix}
\Gamma_{xx} & \Gamma_{xy} \\
\Gamma_{yx} & \Gamma_{yy}
\end{bmatrix} = \begin{bmatrix}
s(1 : x, 1 : x) & s(1 : x, 1 : y) \\
s(1 : y, 1 : x) & s(1 : y, 1 : y)
\end{bmatrix},
\]

(B.18)

and

\[
\begin{bmatrix}
T_{xx} & T_{xy} \\
T_{yx} & T_{yy}
\end{bmatrix} = \begin{bmatrix}
s(2 : x, 1 : x) & s(2 : x, 1 : y) \\
s(2 : y, 1 : x) & s(2 : y, 1 : y)
\end{bmatrix}.
\]

(B.19)

Here, we adopted the convention used in HFSS of expressing the mode-dependent s-parameters, \(s(p_1 : m_1, p_2 : m_2)\) where \(p_i\) is the port number and \(m_i\) is the mode number. In our case, \(m_i\) is either \(x\) or \(y\) corresponding to TE and TM polarizations respectively.


