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Experimental and Numerical Studies on Pumping Viscoplastic Fluids

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Draft

Abstract

Experimental and numerical studies were performed to investigate the pumping withdrawal of a Bingham plastic fluid using a vertical circular pipe. A Laponite suspension with 3% mass concentration was used for the experimental study. Results are compared with the theoretical solution of a point sink. A virtual point sink can be identified to exist below the intake along its centerline. With the assumption of an axisymmetric flow condition, radial velocity is found to be the same within a conical zone, but varies with the axial angle outside this zone in spherical polar coordinates. It was found that the flow viscosity and yield stress do not change the location of the virtual sink but they reduce the horizontal velocity of the Laponite suspension. The extent of the sheared flow region was also studied and the deformation radius was found to be proportional to the $^{1/3}$ power of the pumping rate.

Keywords: Intake; Point sink; Pumping; Viscoplastic fluids; Velocity

INTRODUCTION

Pumping non-Newtonian fluids is important in many fields of engineering. Existing studies are mainly concentrated on the applications of pumps rather than the dynamics of the fluids. In biochemical engineering, researchers tested peristaltic pump or centrifugal pump numerically (Teran *et al.* 2008) and experimentally (Zhang *et al.* 2008), because blood and other types of fluids in the human body are all non-Newtonian. For transporting suspensions, centrifugal pump and progressive cavity pump (PCP) systems have been found to be suitable. Graham *et al.* (2009) experimentally investigated the performance of a centrifugal pump for lifting a power-law fluid and a Herschel-Bulkley fluid. The performance of PCP system was analyzed and modelled by Moreno and Romero (2007) and Gamboa *et al.* (2003). Numerical analysis was also presented by Li *et al.* (1999) for extrusion process of viscoelastic cementitious flows in a shallow flight screw extruder, which is similar to a PCP system.

Suction flow of a fluid is usually represented by a point sink. While many studies focused on Newtonian fluids (Xue and Yue 1998; Zhou and Graebel 1990; Robinson *et al.* 2010), only a few papers studied non-Newtonian fluids withdrawal. However, they were restricted in the investigation of surface/interface deformation and force balance (Zhou and Feng 2010; Blanchette and Zhang 2009; Berkenbusch *et al.* 2008; Jeong 2007). There have been no experimental studies to examine the flow field of a non-Newtonian flow withdrawing near an intake. Our experiments were designed to develop an understanding in this area.

This study is motivated by pumping oil sands mature fine tailings (MFT) from tailings ponds, which is a challenge due to the difficulty of shearing fluids containing high concentrations of fine solids. MFT can be treated as a non-Newtonian viscoplastic fluid, especially a Bingham

plastic fluid (Banas 1991). However, our attempts to understand the physical processes in MFT have been hampered by the opacity of MFT, not allowing observations of movements inside it. So in this study, a Laponite[®] dispersion is used as an artificial MFT material. Laponite is a rheological additive to make transparent clay, which also behaves like a viscoplastic fluid and can be made by mixing Laponite powder with tap water or demineralised water at different concentrations (Cai 2013). Pignon *et al.* (1996) reported the rheological properties of the Laponite suspension at different volume fractions.

This study is intended to investigate the flow field induced in a Bingham plastic fluid when the fluid is withdrawn using a vertical circular pipe intake in a large tank. Tracer particles were placed in the fluid and their movements were analyzed to study the velocity and the sheared zone of the fluid. Numerical simulations were also employed to study the effects of wall boundary and Laponite characteristics on the velocity field.

METHODOLOGY

Laboratory Experiments

To simulate the rheological behaviour of MFT, Laponite powder with a density of $\rho_L = 2600 \text{ kg/m}^3$ was used. The powder was mixed with demineralised water to form a gel. Rheological tests were conducted for a suspension of 3% mass concentration (i.e., volume fraction $\phi_v = 1.15\%$) using Brookfield rotational viscometer (DV-II+) at the room temperature with the pH level of the suspension at 7.

Figure 1 shows the variations of shear stress with shear rate of Laponite at different formation times, t_p . The apparent viscosity and yield stress of Laponite increases with formation time.

Rheological measurements of Pignon *et al.* (1996) for Laponite with $\phi_v = 0.8$ and 1.2 were also added in the figure for comparison. It should be noted that the Laponite suspension in Pignon *et al.* (1996) had the pH level of 9.5 and $t_p \geq 75$ days, whereas the Laponite suspension used in this study has the pH level of 7 and the formation time of 48 hrs. As can be seen from Figure 1, despite the difference in formation time and pH level, the viscosities and shear stresses of Laponite in the present study and Pignon *et al.* (1996) measurements follow similar rheological model.

The 48-hour old sample, with a measured yield stress $\tau_0 = 12.7 \text{ N/m}^2$, is found to have the properties closest to MFT. According to our measurements, Bingham plastic model $\tau = \tau_0 + \mu_p \dot{\gamma}$ can be adapted to fit the rheological data of 3% (by weight) Laponite gel, where τ and $\dot{\gamma}$ refer to the shear stress and the shear rate of the suspension; τ_0 and μ_p are, respectively, the yield stress and the plastic viscosity in the Bingham plastic model. The best-fitting coefficients are $\tau_0 = 15.2 \text{ N/m}^2$ and $\mu_p = 0.0182 \text{ N}\cdot\text{s/m}^2$. It should be noted that the measured yield stress (τ_0) is usually smaller than the value that can fit for a real fluid (Wilkinson 1960). Details on the preparation procedures and the behaviours of the gel can be found in (Cai 2013).

As shown in Figure 2, a glass tank with a width (W) of 50 cm, depth (D) of 25 cm, and height (H) of 30 cm was used for all the experiments. A vertical PVC pipe was placed in the center of the tank and was attached to a PCP system. To investigate the impact of intake size on the pumping velocity field, two different pipe diameters (i.e., series A and B) were used as the intake for pumping; and for each size, the pumping discharge was varied. Table 1 shows a list of the experimental parameters, where the inner and outer diameters of the pipe are denoted as d_0 and

d_1 ; h_1 is the initial submergence of the intake in the gel, and h_2 is the distance from the intake entrance to the bottom of the tank; V_0 and Re_B are, respectively, the average velocity and the Reynolds number inside the pipe, wherein, Reynolds number was defined in a modified form $Re_B = \rho_g d_0 V_0 / \mu_p$ for Bingham plastic fluids. The density ρ_g of the 3% Laponite suspension was found to be 1002.93 kg/m³.

In each series, different pumping rates were set while the intake diameter was kept constant. The steady pumping flow rate (Q) was calculated by collecting the Laponite gel at the outlet of the pump several times during the process and averaging the measurements. In our experiments, all tests were running towards a steady state after a sharp flow rate rise in the beginning. To verify the flow regime, the Hedstrom number ($He = d_0^2 \rho_g \tau_0' / \mu_p^2$) was calculated and the critical Reynolds number, $(Re_B)_c$, was obtained using the correlation developed by Hanks (1963). This critical number marks the transition from laminar flow to turbulent flow for Bingham plastic fluids. Given $Re_B < (Re_B)_c$, all experiments listed in Table 1 were running within the laminar regime.

Black poppy seeds, 1.3 mm to 1.8 mm in diameter, were placed along the center plane of the tank as tracer particles in our experiments. Food colour was used to mark the center plane at the Laponite gel surface. It allows observation of the surface change during the experiment. After a period of 48 hours from Laponite formation, the experimental process was started and the gel in the tank was withdrawn from the intake. A CCD camera (PULNIX TM-1400CL), with a resolution of 1392×1040 pixels, was used to capture pictures at a rate of 30 frames per second (fps). The observation window size is about 22 cm in height and 16 cm in width providing an average resolution of 65 pixels/cm. At the exit of the pump, measuring cylinders were used to

collect the volume of the gel and the pumping discharge was calculated. Figure 3 shows a set of raw images of pumping Laponite from the 7.2 mm intake pipe at different times during the test (Case A1). The surface of the gel was initially flat and gradually curved towards the centerline of the intake. The pumping discharge was monitored and found to increase at the beginning but reached a stable state until the end of the experiment when the gel surface dropped down to the level of the intake entrance. In Figure 3a, a 3D Cartesian coordinate system shows its origin (o^*) located at the centerline of the entrance, the x -axis goes downwards and $z = 0$ is the center plane across the tank.

An image processing software, Davis 8, by LaVision[®], was used to calculate the velocity field for each experiment. It is designed to process Particle Image Velocimetry (PIV) images, but could be also applied in our case, because the idea of tracking particles works the same way as in a PIV setup. Compared with a typical PIV image, our particles are much bigger and the distribution density is relatively low, hence the interrogation window size was set to 256 by 256 pixels. The velocity fields were computed for all cases at different times. In processing the images, three regions were masked out for every image, as marked by the dotted lines in Figure 2: the rectangular area occupied by the intake pipe, the zone very close to the entrance and the lowest part due to the reflection from the bottom of the tank. The reason for removing the second region is that the speed of our camera is limited to 30 fps, only allowing for tracking movement below 1.2 m/s within an interrogation window (256 pixels). However, all mean velocities at the entrance are higher than 1.2 m/s as shown in Table 1. In addition, the tracking seeds were travelling so fast that very few could be captured near the entrance.

Numerical Simulation

In order to study the effects of tank size and Laponite characteristics on the velocity field, a solver package Ansys CFX (2009) with a homogenous multiphase laminar model was employed. Given the axisymmetry of the flow withdrawal, only one-quarter of the tank was numerically simulated. The simulations were performed on a computer work station that has two CPU each having eight processing core running at 3.00 GHz, and a random access memory of 16 GB. Considering the transient nature of the problem with an experimental duration and numerical time step of 0.1 second, the total computation time was between 36-73 hours for each simulation.

An opening boundary condition was employed at the top surface of the domain to control the air pressure and maintain hydrostatic pressure in Laponite suspension. This boundary condition allows the surface of the Laponite suspension to drop down. The outer wall of the domain was specified as free-slip boundary condition. Detailed mathematical models and the discretized form of boundary conditions were explained in Azimi *et al.* (2011). The initial velocity of the Laponite suspension is taken as zero, and the initial pressure is provided as hydrostatic pressure. A constant mass flow rate, measured at the pump intake, was used as the boundary condition for the domain outlet. The computational domain discretized into small cells with various sizes ranging from 0.5 mm to 5 mm. Mesh independence analysis was performed by systematically decreasing the mesh size to ensure that the numerical results are independent of the mesh resolution. Details of mesh independence analysis were explained in Azimi *et al.* (2011).

Three series of test were performed to investigate the effect of flow rate (Series C), dynamic viscosity and yield stress of the Laponite suspensions (Series D), and size and geometry of the tank (Series E). The modeling details are shown in Table 2.

RESULTS AND ANALYSIS

Figure 4 shows the measured velocity contour of Case B1. It was found that the flow reached the steady-state pumping rate after 63.8 seconds of the pumping commencement, and this steady-state pumping lasted for another 180 seconds. As can be seen from Figure 4c, the flow pattern appears to be different from the potential sink flow in a Newtonian fluid (Papanastasiou *et al.* 2000). The velocity contours are not spherical and the radial velocity decays at different rates along different angular directions. The horizontal and vertical components of the velocities are denoted as u_y and u_x , respectively. Figure 5 shows the variations of the horizontal velocity u_y at different y locations for Case A1. In this plot, each curve represents all horizontal velocity components along one vertical line in the center plane. These curves are bell-shaped close to the intake but flatten out away from it. The peak of each curve indicates where the maximum horizontal velocity component is located along x direction at different y location. It was found that the location of these velocity peaks do not change with y , and the maximum horizontal velocity along the y -axis is located at $1.4d_0$ below the intake level. Accordingly, a virtual sink (O) is assumed to be located along the centerline below the intake pipe and the distance away from the entrance is equal to $1.4d_0$ (see Fig. 2b). Solid curves show the horizontal velocity profiles from numerical simulation of Case A1. As can be seen, the model was able to predict the peak horizontal velocities and their location properly for $y < -5$ cm. Comparison between the CFD model results and laboratory measurements indicated that the numerical model over-predicts the peak horizontal velocity by 6.2% for $y = -3.09$ cm and it under-predicts the peak horizontal velocity by -5% and -4.2% for $y = -4.09$ cm and $y = -5.10$ cm, respectively. For $y > -5$ cm, the peak horizontal velocity smeared off and the CFD results under-predict the velocity profile with $\pm 9\%$ error.

In a spherically symmetric radial flow, the axisymmetric sink flow solution for the velocity in an infinite domain is

$$u_r = \frac{Q}{4\pi r^2} \quad (1)$$

where r is the radial distance from the point sink (Papanastasiou *et al.* 2000). The radial velocity increases dramatically as the fluid is being sucked into the point sink (O). If a spherical polar coordinate system is employed with its origin located at O (Fig. 2b), the radial velocity u_r in our experiments can be obtained at different angles. The sink flow solution has been used to model the selective withdrawal of a two-layer flow (Shammaa and Zhu 2010). Shammaa and Zhu (2010) experimentally showed that the flow noticed the pipe intake and the pipe wall at a distance of 3 and 1.5 intake diameters from the intake axis, respectively.

Figure 6 shows the variations of the radial velocity of Laponite flow u_r in radial distance from the intake pipe for Cases A1-A3 and B1-B2 within a conical region $0 \leq \theta \leq 15^\circ$. Radial velocity measurements were compared with the CFD model results and analytical solutions of the potential flow theory. The analytical solutions were truncated for $r < 1.5d_i$ since the solution is not accurate in this region due to the pipe wall effect (Shammaa and Zhu 2010). Experimental results showed that the radial velocity u_r does not change with the angle θ and the velocity results are smaller than the axisymmetric point sink solution within a conical region $0 \leq \theta \leq 15^\circ$. Beyond the conical zone, radial velocity u_r does not vary or decay at the same rate along different angular directions. These velocity variations can justify a kidney shaped sheared zone when the flow is in steady-state condition (see Figure 4). As can be seen from Figures 6a, 6c, and 6e, predictions of the analytical solution of the potential theory became more accurate as the initial intake velocity became smaller.

Numerical results for the region that the intake pipe has no impact on the flow field ($r > 1.5d_i$) were compared with the analytical solutions and laboratory measurements for Cases A1 and B1. As can be seen in Figures 6a and 6b, numerical simulations performed better than the analytical solutions. The prediction errors of numerical simulations and analytical solutions were on average -6% and $+9\%$, respectively.

For a steady-state fluid in spherical coordinates, the continuity equation can be written as:

$$\frac{1}{r^2} \frac{\partial}{\partial r} (r^2 u_r) + \frac{1}{r \sin \theta} \frac{\partial}{\partial \theta} (u_\theta \sin \theta) + \frac{1}{r \sin \theta} \frac{\partial u_\phi}{\partial \phi} = 0 \quad (2)$$

If there is only radial movement and an axisymmetric flow is assumed along x -axis, Eq. 2 can be simplified into $\frac{1}{r^2} \frac{\partial}{\partial r} (r^2 u_r) = 0$, thus

$$u_r = \frac{f_1(\theta)}{r^2} \quad (3)$$

where f_1 is a function of θ only. Equation 3 can be further rewritten in the form of Eq. 1:

$$u_r = f(\theta) \frac{Q}{4\pi r^2} \quad (4)$$

where $f(\theta)$ is a function of θ , and at a fixed angle, it becomes a constant value. Therefore, the length scale of $\sqrt{Q/u_r}$ should increase linearly with r . The relationship between the length scale of $\sqrt{Q/u_r}$ and radial distance r is plotted for different angles in Figure 7. In this plot, all the radial velocities are plotted in terms of the value of θ . A best-fit dashed line is also added to indicate the average of different values of $f(\theta)$. For the two pipe sizes, all the data points follow the same curve and Eq. 4 can be used to predict the velocity. Due to experimental limitation, the

results presented in Figure 7 was only valid for a short radial distance ranges of $3 \text{ cm} < r < 6 \text{ cm}$ whereas, the numerical simulation can provide the results for a wider range.

Figure 8 shows the results of the numerical simulation for Case C1 (which is the same as the experimental Case B1). As can be seen, variation of $\sqrt{Q/u_r}$ with r is not linear, and its slope (a function of $f(\theta)$, see Eq. 4) changes with r . For a relatively short length ($3 \text{ cm} < r < 6 \text{ cm}$), this slope can be approximated as constant. The value of $f(\theta)$ is linear for $r < 3 \text{ cm}$. For $r > 6 \text{ cm}$, $f(\theta)$ increases for $0 < \theta < 90^\circ$ and almost constant for $90^\circ < \theta < 120^\circ$. Experimental data points of Case B1 were added for validation of the CFD model. The minimum and maximum errors of prediction were related to $\theta = 90^\circ$ and $\theta = 120^\circ$ with the values of 4.5% and 10.8%, respectively.

Figure 9 shows a comparison between the value of $f(\theta)$ from experimental results (Case B1) and from numerical simulations (Cases C1 and C2) for $3 \text{ cm} < r < 6 \text{ cm}$ by linearly fitting of the curves in Figure 8. A cosine function was used to predict $f(\theta)$ at different angles. If $f(\theta) = C_1 \cos 2\theta + C_2$ is used, then $C_1 = -0.39$ and $C_2 = 1.0$ is found to be the set of best-fitting coefficients within $0^\circ < \theta < 150^\circ$ when compared with the experimental data in Figure 9. In this figure, the error bars are generated by fitting each data points individually for all five tests and thereby they demonstrate the range of $f(\theta)$ for best fitting all data points.

$$f(\theta) = -0.39 \cos 2\theta + 1.0 \quad (5)$$

As can be seen from Figure 9, Eq. 5 provides a reasonable estimation of $f(\theta)$ at $\theta = 90^\circ$ but it overestimates $f(\theta)$ for $\theta < 90^\circ$ and underestimates $f(\theta)$ for $\theta > 90^\circ$. The reason for the discrepancy between Eq. 5 and computed $f(\theta)$ is likely due to the curvature of the correlation curves between $\sqrt{Q/u_r}$ and r .

DISCUSSION

Velocity Field

The velocity field obtained using Eq. 4 can be integrated to compare with the pumping rate. By the definition of flow rate, at a distance of r from the virtual sink, it can be calculated using the integration as shown below,

$$Q^* = 2\pi r^2 \int_0^\pi u_r \sin \theta d\theta \quad (6)$$

Substituting u_r using Eq. 4 and Eq. 6, it unveils the comparison of the integrated discharge and the measured value as $Q^*/Q=1.13$. It shows the estimation of discharge using $f(\theta)$ is 13% higher than our measurement. The difference likely comes from a number of sources: the flow was confined behind the pipe wall and very small amount of gel was withdrawn from the zone $\theta > 150^\circ$, where $f(\theta)$ is not suitable to describe the flow; u_r changed not only with θ but r ; and there were experimental errors in flow rate measurement.

In a 3D axisymmetric domain of infinite size, the stream function of a potential sink is (Childress 2009)

$$\psi(\theta) = -\frac{Q}{4\pi}(1 - \cos \theta) \quad (7)$$

To investigate the boundary effect of the tank bottom, an image sink O' is placed at a distance of $2a$ away from the sink O , wherein $a = h_2 - 1.4d_0$. According to Eq. 7, the stream function of the flow induced by O and its image O' can be calculated. The resulting stream function becomes

$$\psi(\theta) = -\frac{Q}{4\pi} \left[2 - \cos \theta - \frac{r \cos \theta - 2a}{(r^2 - 4ar \cos \theta + 4a^2)^{1/2}} \right] \quad (8)$$

From Eq. 8, the radial velocity can be derived as

$$u_r = \frac{1}{r^2 \sin \theta} \frac{\partial \psi}{\partial \theta} = -\frac{Q}{4\pi r^2} \left[1 + \frac{r^3 - 2ar^2 \cos \theta}{(r^2 - 4ar \cos \theta + 4a^2)^{3/2}} \right] \quad (9)$$

Assume $r = a/k$, Eq. 9 becomes

$$u_r = -\frac{Q}{4\pi r^2} \left[1 + \frac{1 - 2k \cos \theta}{(1 - 4k \cos \theta + 4k^2)^{3/2}} \right] \text{ or } \frac{u_r}{-\frac{Q}{4\pi r^2}} = 1 + \frac{1 - 2k \cos \theta}{(1 - 4k \cos \theta + 4k^2)^{3/2}} \quad (10)$$

According to Eq. 10 at $r = a/2$, $a/4$ and $a/8$, the variations of $u_r / (-Q/4\pi r^2)$ with θ are plotted in Figure 10. It demonstrates that the impact of the image sink on the radial velocity is limited when it is close to the virtual sink. Figure 10 shows that the velocity profile is more affected by the virtual sink when the virtual sink is further away from the nozzle. To examine the effect of angle θ , Eq. 9 is derived by θ :

$$\frac{du_r}{d\theta} = -\frac{Q}{4\pi r^2} \cdot \frac{4ar^2 \sin \theta (2a^2 - r^2 + ar \cos \theta)}{(r^2 - 4ar \cos \theta + 4a^2)^{5/2}} \leq 0 \quad (11)$$

Eq. 11 indicates the magnitude of radial velocity u_r increases with angle θ , given its direction is opposite to r . So the minimum value of u_r occurs when $\theta=0^\circ$, and Eq. 9 yields

$$u_r = -\frac{Q}{4\pi r^2} \left[1 - \left(\frac{2a}{r} - 1 \right)^{-2} \right] \quad (12)$$

which shows the highest change of u_r due to the image sink or the tank bottom. For instance, at a radial distance of $r = a/2$, $u_r = -8/9 \times Q/(4\pi r^2)$, which means the radial velocity is reduced by $1/9$ when an image sink is present. In our experiments, Figure 7 only includes data up to $r = 6$ cm, which is in the range from $0.52a$ to $0.56a$ in each test. Accordingly the reduced rate of u_r is from 12.5% to 14.8% in the case of potential flow. However, in a viscoplastic fluid, the sheared region is limited within the zone close to the intake, and the flow will not be affected by the bottom much. Nevertheless, if the boundary effect has to be estimated, it should be less than 14.8% in our case.

Deformed Area

Understanding the extent of the deformed area is important as it indicates the sheared zone in the fluid which contributes to the pumping production. In a 3D domain, if the following assumptions are applied: (a) the Laponite gel is incompressible, (b) the pump is running steady-state, (c) the flow is laminar, (d) the flow is axisymmetric with respect to x -axis, and (e) there is radial motion only, the rate of deformation tensor can be simplified as Tadmor and Gogos (2006):

$$\dot{\gamma} = \begin{bmatrix} \dot{\gamma}_{rr} & \dot{\gamma}_{r\theta} & \dot{\gamma}_{r\phi} \\ \dot{\gamma}_{\theta r} & \dot{\gamma}_{\theta\theta} & \dot{\gamma}_{\theta\phi} \\ \dot{\gamma}_{\phi r} & \dot{\gamma}_{\phi\theta} & \dot{\gamma}_{\phi\phi} \end{bmatrix} = \begin{bmatrix} 2\frac{\partial u_r}{\partial r} & \frac{1}{r}\frac{\partial u_r}{\partial \theta} & 0 \\ \frac{1}{r}\frac{\partial u_r}{\partial \theta} & 2\frac{u_r}{r} & 0 \\ 0 & 0 & 2\frac{u_r}{r} \end{bmatrix} \quad (13)$$

The Bingham fluid constitutive laws are

$$\tau_{ij} = (\mu_p + \frac{\tau_0}{|\dot{\gamma}|}) \dot{\gamma}_{ij} \Leftrightarrow |\tau| > \tau_0 \quad (14a)$$

$$|\dot{\gamma}| = 0 \Leftrightarrow |\tau| \leq \tau_0 \quad (14b)$$

where $|\tau|$ and $|\dot{\gamma}|$ are, respectively, the second principal invariant of the stress tensor and the rate

of deformation tensor. They are defined as $|\tau| = (\frac{1}{2} \sum_{i,j=1}^3 \tau_{ij}^2)^{\frac{1}{2}}$ and $|\dot{\gamma}| = (\frac{1}{2} \sum_{i,j=1}^3 \dot{\gamma}_{ij}^2)^{\frac{1}{2}}$ (Childress

2009). From Eq. 13, we obtain

$$|\dot{\gamma}| = \sqrt{2 \left(\frac{\partial u_r}{\partial r} \right)^2 + 4 \frac{u_r^2}{r^2} + \frac{1}{r^2} \left(\frac{\partial u_r}{\partial \theta} \right)^2} \quad (15)$$

In the fluid domain, the area where $|\tau| \leq \tau_0$ is under a zero rate-of-strain condition, hence it moves like a rigid body. Fluid in other regions is moving like viscous liquid and the components of stress tensor can be calculated using Eqs. 14a and 15.

In the regions with low shear stress the fluid move like rigid body, commonly referred as “rigid” regions or plug regions (Widjaja *et al.* 2003). In contrast, the rest are sheared by relatively high stresses and they are called as “flow” regions. The radius of these flow regions depends on the pumping discharge, and a high discharge yields a large deformation radius. In our tests, the maximum shear rate occurs at $\theta = 90^\circ$ and the minimum is at $\theta = 0^\circ$. The magnitude of $|\dot{\gamma}|$ is proportional to the withdrawal discharge Q . Using Eq. 4 to Eq. 15, we have

$$|\dot{\gamma}| = f'(\theta) \frac{Q}{4\pi r^3} \quad (16)$$

If the deformed area is defined as the region where $|\dot{\gamma}| \geq 0.1s^{-1}$, according to Eq. 16, the deformation radius increases with the cube root of the withdrawal flow rate: $\tilde{r} \propto Q^{1/3}$.

Effects of Laponite characteristics and the tank size

It is important to study the pumping capability of Laponite suspension at different conditions. Figure 11a shows the simulated Laponite velocity field at the vicinity of the intake. Numerical results were extracted from vertical profiles at different distances from the axis of the intake pipe. Vertical profiles were located from 30 mm to 60 mm from the axis of the intake pipe to show the variations of the axial velocities. Comparison between the numerical results (Figure 11a) and experimental observations (Figure 5) indicates that the model can predict the peak horizontal velocity at different y locations within an average of 12% error. The thick horizontal line shows the location of the virtual sink which is close to $1.4d_o$, consistent with the lab experiments.

Figure 11b shows the effects of Laponite characteristics on the flow velocity field. Two tests were run to investigate the sensitivity of plastic viscosity (Cases D1 and D2). In both cases the yield stress was kept constant and the dynamic viscosity was varied. The dynamic viscosity and yield stress in Cases D3 and D4 were increased by 1.5 and 2 times of the experimental run, respectively. Figure 11b, shows the effect of dynamic viscosity and yield stress on the horizontal Laponite velocity at $y = -30$ mm. It was found that by increasing the dynamic viscosity by four times the peak horizontal velocity of Laponite decreased by 4.4% and the location of the peak Laponite velocities are in consistent with the experimental observation. The peak horizontal velocities for Cases D3 and D4 decreased by 3.6% and 9.7%, respectively. It was found that the effect of dynamic viscosity becomes negligible far from the intake (i.e., $y = -80$ mm) but an increase in yield stress reduces the Laponite velocity by 50%.

Effect of the bottom wall on the flow structure was investigated by reducing the distance between the intake pipe and the bottom wall, h_2 . In experimental Case B1 this distance was 0.119 m and this distance was reduced to 0.059 m and 0.029 m in Cases E1 and E2, respectively, while the flow rate was kept constant as 95 mL/s. Figure 12 shows the contour plots of the Laponite velocity field at the vicinity of the outlet. The Laponite velocity range in contour plots was selected between 0.1-0.3 m/s for better visualization. The yellow dashed curves show the trajectory of the Laponite flow and the yellow color in contour plots represents the Laponite velocity of around 0.22 m/s. As can be seen from Figure 12, the Laponite velocity field did not alter by the bottom wall for $h_2 = 0.119$ and 0.059. The flow structure begins to be affected by the bottom wall boundary when the distance reduced to 0.029 m which is comparable to $4d_o$. As can be seen from Figure 12a, the Laponite velocity reduced at $\theta = 0^\circ$ and increased at $\theta > 90^\circ$. The tank dimensions were increased and decreased by a factor of two in Cases E3 and E4, respectively. No noticeable effects were found due to the effect of wall boundaries on the velocity field of Laponite suspension.

SUMMARY AND CONCLUSIONS

This paper presents laboratory and numerical studies on pumping a Bingham plastic fluid using a vertical circular pipe. The experimental results show that the intake pipe can be simplified as a virtual point sink located below the intake entrance. The horizontal velocity profiles close to the intake appear to be symmetric to a horizontal line but far from the intake the horizontal velocity profiles appear to be asymmetric. It was found that the location of the virtual sink is $1.4d_o$ below the intake pipe. A parameter $f(\theta)$ was identified to predict radial velocities at different angles. It was found that $f(\theta)$ can be modeled using a cosine function. Integration of radial velocity over a sphere with a radius of r showed that the estimation of discharge using $f(\theta)$ resulted in 13%

overestimation of the measured flow rate. In spherical polar coordinates, a conical zone is found to be within 15° under the intake. In this region, the magnitudes of radial velocity at different directions are almost the same; while in the outer region from 15° to 150° , the radial velocity varies at different angles. In the flowing domain, the radius of the deformed area is proportional to the $^{1/3}$ power of the pumping flow rate.

Effects of Laponite volume fraction ϕ_v on the horizontal velocity profiles were numerically studied by changing the viscosity and yield stress of the 3% Laponite. By increasing the dynamic viscosity by four times the peak horizontal velocity of Laponite at $y=-30$ mm decreased by 4.4%. The peak horizontal velocity at $y=-30$ mm decreased by 9.7% when both dynamic viscosity and yield stress doubled. Effects of bottom boundary on the radial velocity distribution were numerically investigated by systematically decreasing the distance between the intake pipe and the bottom wall h_2 . The flow structure begins to be affected by the bottom wall boundary when the distance reduced to 0.029 m ($\approx 4d_o$).

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Table 1: Experimental parameters in a tank with width (W) of 50 cm, depth (D) of 25 cm, and height (H) of 30 cm.

Case No.	d_0 (mm)	d_l (mm)	He	$(Re_B)_c$	Re_B	Q (mL/s)	V_0 (m/s)	h_l (cm)	h_2 (cm)
A1	7.2	13.8	1986	2300	824	84	2.08	8.5	12.5
A2					1342	137	3.39	8.5	12.5
A3					1837	188	4.64	8.6	12.4
B1	10.1	17.1	3922	2900	661	95	1.19	9.1	11.9
B2					1067	153	1.92	9.1	11.9

Table 2: Numerical experiments and mesh characteristics.

Case No.	Domain size*			Flow rate (mL/s)	μ (Pa.s)	τ_o (Pa)	Cell size (mm)		Number of elements	Number of nodes	Δt (sec)	t (sec)
	L (m)	W (m)	h_2 (m)				max	min				
C1 ⁺	0.25	0.125	0.119	95	0.0182	15.2	3	1	3053726	526888	0.2	66
C2				190	0.0182	15.2					0.1	30
D1	0.25	0.125	0.119	95	0.0091	15.2	3	1	3053726	526888	0.1	66
D2					0.0364	15.2						
D3					0.0273	22.8						
D4					0.0364	30.4						
E1	0.25	0.125	0.029	95	0.0182	15.2	3	1	2446429	422951	0.2	66
E2			0.059						2140659	370664		
E3	0.5	0.25	0.119	95	0.0182	15.2	5	2	2649242	457605	0.4	120
E4	0.125	0.0625					1	0.5	2568985	445725	0.1	30

* Due to symmetry of the test, only a quarter of the experimental tank was numerically modeled.

⁺ Numerical Case C1 is identical to the experimental Case B1.

Figure Captions:

Figure 1: Variations of shear stress with shear rate for Laponite suspension with $\phi_v = 1.15$ and the effect of formation time t_p on Laponite suspension and development of yield stress.

Figure 2: Schematic of laboratory setup, coordinate system and the observation window.

Figure 3: Raw images taken from Case A1 with Cartesian and spherical Polar coordinate systems. Black dots in images are poppy seeds used as tracer particles. a) Before pumping at $t = 0$ sec, b) 23 seconds after pumping commencement, c) 46 seconds after pumping commencement.

Figure 4: Contour plots of velocity field for Case B1 at different time from the beginning of pumping. The contour plots of the velocity are in the unit of cm/s. a) 4 seconds after pumping commencement, b) 35.7 seconds after pumping commencement, c) 63.8 seconds after pumping commencement ($t = 63.8$ s indicates a steady pumping condition).

Figure 5: Horizontal velocity components along the x -axis at different y locations for Case A1. Data points are experimental measurements and solid thin curves are the results of numerical simulation. The dashed line shows the location of the maximum horizontal velocity.

Figure 6: Variations of the radial velocity in the conical region ($2 \text{ cm} < r < 5 \text{ cm}$) of all five Cases (A1-A3 and B1-B2). The dashed curve shows the theoretical solution of a potential flow with point sink. The solid curve shows the result of the numerical simulation.

Figure 7: Variations of $(Q/u_r)^{1/2}$ at different angles for experimental tests. The dash line indicates the best fitting for each angle.

Figure 8: Experimental data (Points) and numerical results (Lines) show the relationship of $(Q/u_r)^{1/2}$ with r for Laponite suspension at different angles. Data extracted from Case B1 with $Q=95 \text{ mL/s}$ and $d_0=10.1 \text{ mm}$.

Figure 9: Variation of $f(\theta)$ with θ for different flow rates. $f(\theta)$ values were estimated from 3 cm $< r < 6$ cm. The dashed curve is the proposed equation for prediction of $f(\theta)$ for different angles.

Figure 10: Variations of the normalized radial velocity with θ when an image sink added in case of withdrawing a Newtonian Fluid.

Figure 11: Numerical results of the horizontal velocity of Laponite along the x -axis. a)

Horizontal velocity of Laponite at different horizontal distance from the pipe intake for Case C1.

Solid line shows $1.4d_o$ distance from the nozzle. b) Effect of dynamic viscosity and yield stress on horizontal velocity of Laponite at $y = -3$ cm (Cases D1-D4).

Figure 12: Contour plot of Laponite velocity shows the effect of h_2 on the flow development

near the intake $Q=95$ mL/s. a) $h_2= 2.9$ cm [Case E1], b) $h_2= 5.9$ cm [Case E2], c) $h_2 = 11.9$ cm [Case C1].

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