Canadian Geotechnical Journal
Revue canadienne de géotechnique

A semi-empirical elastic-thermoviscoplastic model for clay

Journal:	<i>Canadian Geotechnical Journal</i>
Manuscript ID	cgj-2015-0598.R1
Manuscript Type:	Article
Date Submitted by the Author:	13-Mar-2016
Complete List of Authors:	Kurz, David; University of Manitoba, Civil Engineering Sharma, Jitendra; York University, Lassonde School of Engineering Alfaro, Marolo; University of Manitoba, Civil Engineering Graham, James; University of Manitoba,
Keyword:	Clay, load duration, creep and viscosity, temperature, compression and shear

A semi-empirical elastic-thermoviscoplastic model for clay

***David Kurz¹, Jitendra Sharma², Marolo Alfaro³,
Jim Graham⁴***

***Revised manuscript submitted to the Canadian
Geotechnical Journal for publication as an Article***

Date submitted: March 2016

¹ Geotechnical Engineer, KGS Group, 865 Waverley Street, Winnipeg, MB, Canada, R3T 5P4.
E-mail: dkurz@ksgroup.com

² Professor and Chair, Civil Engineering, Lassonde School of Engineering, York University,
700 Keele St, Toronto ON M3J 1P3 Canada. E-mail jit.sharma@lassonde.york.ca

³ Professor, Civil Engineering Dept., Room E1-368A EITC, University of Manitoba, 15 Gillson
St., Winnipeg, Manitoba R3T 0V6, Canada. E-mail: marolo.alfaro@umanitoba.ca

⁴ *Corresponding author:* Professor Emeritus, Civil Engineering Dept., Room E1-368A EITC,
University of Manitoba, 15 Gillson St., Winnipeg, Manitoba R3T 0V6, Canada. E-mail:
james.graham@umanitoba.ca

Abstract

Clays exhibit creep in compression and shear. In 1-D compression, creep is commonly known as 'secondary compression' even though it is also a significant component of deformations resulting from shear straining. It reflects viscous behaviour in clays and therefore depends on load duration, stress level, the ratio of shear stress to compression stress, strain rate, and temperature.

Research described in the paper partitions strains into elastic (recoverable) and plastic (non-recoverable) components. The plastic component includes viscous strains defined by a creep rate coefficient ψ that varies with plasticity index and temperature, but not with stress level or overconsolidation ratio (OCR). Elastic-viscoplastic (EVP) models developed by Yin et al. (2002) and Kelln et al. (2008) have been modified so that $\psi = \psi(T)$ in a new elastic-thermoviscoplastic (ETVP) model.

The paper provides a sensitivity analysis of simulated results from undrained (CI $\bar{U}$) triaxial compression tests for normally consolidated and lightly overconsolidated clay. Axial strain rates range from 0.15%/day to 15%/day, and temperatures from 28°C to 100°C.

Keywords:

Clay, creep, viscosity, compression, shear, creep rate coefficient, strain rate, load duration, temperature.

Introduction

Creep, or more formally 'viscous behaviour', is an inherent property of inter-particle relationships in most natural clays, particularly if they contain bentonite, montmorillonite and organic material; or less significantly, chlorite-illites (Mitchell and Soga 2005). It is unfortunate that much of our understanding of clay behaviour comes from reconstituted kaolin, which shows little viscous behaviour (Graham 2006). Creep movements are seen in long-term settlements of foundations, in vertical and 'spreading' movements of embankments on soft ground, and in ongoing, often irregular, movements in natural and engineered slopes.

Viscous behaviour means that deformations depend on the duration of constant loading, the rate at which loading is applied, the soil temperature, or some combination of these processes. In clays, it also influences measured properties like preconsolidation pressure, yielding, and undrained shear strength (Graham et al. 1983; Graham and Yin 2001; Yin et al. 2002; Kelln et al. 2008). A successful constitutive model must therefore include relationships between stress, stress rate, strain, strain rate, and temperature. This implies that creep will be present during primary consolidation and secondary compression, and also in both compression and shear.

Origins of viscous straining

There are two processes that contribute to viscous straining and they can take place simultaneously. In one, geometric re-arrangement of soil particles involves statistical redistribution of more-stressed and less-stressed inter-particle contacts. As the former seek to relax their loading, particles move to accommodate the stress transfer, and the latter

accept additional loading. Redistribution can take place over long periods of time at essentially constant pore water pressure, not only in fine-grained soils like clay, but also in coarse-grained soils like scree, debris fields, or rock-fill in dams. In the second process, hydrated cations in diffuse double layers (DDLs) closely surrounding clay particles produce 'adsorbed' water with viscosity higher than that of 'free', or 'gravitational' water. The DDLs, only Ångstroms thick, vary with the mineralogy of the particles, dielectric constant, the chemistry (valency and concentration) of the pore fluid, temperature, etc, all of which can change due to engineering activity (Yong et al. 1992; Mitchell and Soga 2005).

Creep of soil particles is non-recoverable, that is, 'plastic', and involves local movement of water, even if the 'global' volume is constant. The combination of load transfer and high viscosity of adsorbed water means that the rate of creep movements will decay exponentially with time and will change with temperature.

Partitioning of stresses

A common way of thinking of viscous behaviour (Figure 1) is to consider a two-stage process starting with dissipation of excess pore water pressure (primary consolidation) produced by external loading. This is followed afterwards by ongoing settlements (secondary compression) whose rate decreases with time (Mesri and Choi 1985).

An alternative approach, which was initially controversial but is becoming more widely understood, divides settlements into 'instant' and 'delayed' components (Bjerrum 1967). The instant component is associated with largely reversible (elastic) deformations of the clay particles themselves, while the delayed component is a non-recoverable, time-dependent (viscoplastic) reorganization of the inter-particle micro-structure.

In a more general sense, total strains can be partitioned into elastic and viscoplastic components. That is, $\{\epsilon^{total}\} = \{\epsilon^{elastic}\} + \{\epsilon^{viscoplastic}\}$, (Yin et al. 2002, Kelln et al. 2008). Partitioning in this way applies to both compression and shear strains.

One outcome of this second approach is that preconsolidation pressures are not unique but vary with strain rate in constant-rate-of-strain (CRS) tests or the duration of loading in oedometer tests (Figure 2a) (Sällfors 1975). In the normally consolidated range of pressures, the compression index C_c , which is the slope of the Normal Consolidation Line (NCL), is constant, but the NCLs for different loading rates move to lower values of specific volume V with decreasing rates of loading. (Here, V is specific volume, that is, the volume occupied by unit volume of solids.) In the overconsolidated range, preconsolidation pressures and the unload-reload index C_r appear to vary with strain rate. It is now known that this apparent variation of C_r is because deformations in this range include both elastic recoverable strains and small viscoplastic non-recoverable strains that vary with strain rate, that is, with the duration of testing (Kelln et al. 2008).

Temperature effects

Effects of changing temperatures have received less attention than those of isothermal viscous effects, but must be considered in projects such as design of foundations for furnaces, transmission towers, nuclear waste containment, deep excavation and storage, thermal piles for accessing geothermal energy, and climate warming that produces thermal gradients in thawing permafrost under roadways.

Figure 2b shows results of CRS tests performed at different temperatures (Eriksson 1989). (No phase changes were involved in these tests.) Preconsolidation pressures and

unload-reload lines (URLs) again appear to vary with temperature, while normal consolidation lines (NCLs) are again parallel, with hotter specimens having smaller specific volumes. The similarities between Figure 2a and Figure 2b are striking. In particular, in the overconsolidated region in Figure 2b, values of C_r appear to vary with temperature even though the loading rate was constant. This implies that the creep rate coefficient $\psi = dV/d\{\ln(t)\}$, which is a material constant at one temperature, should change with changes in temperature. That is $\psi = \psi(T)$. Here, V is specific volume, t is time (that is, the duration of loading), and T is temperature.

A creep rate coefficient independent of pressure but varying with temperature

Data from constant-temperature oedometer tests are commonly interpreted in a way that has the coefficient of secondary compression $C_{\alpha e} = dV/d\{\log_{10}(t)\}$ varying with overconsolidation ratio. Secondary compression is slow in overconsolidated clay and faster in normally consolidated clay. If, as proposed earlier, viscoplastic compressions are related to particle reorganization and movement of viscous adsorbed water, it should be possible to identify a single material parameter - a creep rate coefficient ψ - that depends on the mineralogy of the clay, the chemistry of the pore fluid, and temperature. This means that compression behaviour can now be described by three parameters C_r , C_c , and $C_{\alpha e}$ if stresses are plotted in $\log_{10}$ -graphs; or κ , λ , and ψ in $\ln$ -graphs. Careful writing of the mathematics of the constitutive model can then simulate the apparently different values of C_r (or κ) in Figures 2a and 2b (Yin et al. 2002; Kelln et al. 2008).

Modeling

Two broad approaches are commonly used in constitutive modeling. The first is strongly based in physics and mathematics but is difficult to calibrate (Graham 2006). It usually involves many material parameters that are hard to isolate in laboratory testing (Zhou et al. 1998). As a result, this mechanics-based approach is more commonly used for sensitivity analysis than for simulating performance of actual projects.

The second approach is semi-empirical and relatively easy to calibrate using routine laboratory tests. It assigns simple mathematical expressions to laboratory data for recoverable deformations, yielding, plastic hardening, plastic shear strain increments (using a 'flow rule'), and a rupture envelope.

The most commonly used of these semi-empirical models is Modified Cam Clay (MCC), which partitions strains into elastic and plastic components without considering viscous or temperature effects (Wood 1990). The model is commonly referred to as a volumetric-hardening elastic-plastic (EP) model. It can often be used successfully in isothermal, saturated applications in which creep movements can be expected to be small, the soil chemistry is constant, and isotropic elasticity is a reasonable assumption.

Extra conditions can be added to the basic framework of MCC. For example, Yin et al. (2002) and Kelln et al. (2008) added effects of load duration and strain rate to produce related elastic viscoplastic (EVP) models that have provided successful simulations of settlements of large compacted fills. Graham et al. (2001) added the effects of temperature to the base MCC in a thermal elastic-plastic (TEP) model. Unfortunately, they associated heating and cooling with the elastic, unload-reload regions suggested by Figure 2b, and neglected the effects of temperature on viscosity. Following the insights provided by Crilly

(1996), this paper moves the effects of temperature into the viscoplastic component of straining to form an elastic-thermoviscoplastic (ETVP) model.

Terminology and Assumptions

The terminology used here is consistent with Wood (1990) and the EVP model developed by Kelln et al. (2008). This paper will deal only with triaxial compression tests in which mean effective stress is defined as $p' = p - u$, where $p = (\sigma_1 + \sigma_2 + \sigma_3)/3$ and σ_1 , σ_2 , and σ_3 are principal stresses in the axial and radial directions, and u is pore water pressure. Deviator stress is defined as $q = q' = \sigma_1 - \sigma_3$ and specific volume is denoted by V . The approach can of course be generalized to other stress paths and incorporated into finite element modeling.

Subscripts used in following sections include i , y , c , cs , m , and x to describe the states of stress as initial, yield, preconsolidation pressure, critical state, isotropic compression, and current state, respectively. A point on an NCL is defined by p'_m, V_m . Superscripts e , p , and vp indicate elastic (recoverable), plastic (non-recoverable), and viscoplastic (time- and temperature-dependent non-recoverable) states respectively. Time is denoted by t and dt ; temperature by T and dT .

The ETVP model uses the following assumptions.

1. Because the model is based on MCC, it assumes non-linear isotropic elasticity, with the bulk and shear moduli varying with p' and p'_m respectively. Volumetric strains and shear strains are denoted by ε_p and ε_q . This reflects decoupling of volumetric strains from q -stresses and shear strains from p' -stresses in isotropic elasticity. Elastic stress-strain behaviour is independent of both strain-rate and temperature, and has slope κ in $V, \ln(p')$ space.

2. Yield envelopes are elliptical, symmetrical about the p' -axis, and increase with p'_m .
3. An associated flow rule applies, so plastic potentials are also elliptical.
4. The plastic hardening law is a line with slope λ in $V, \ln(p')$ space corresponding to the Normal Consolidation Line NCL. Viscous behaviour produces what appears to be a series of lines that decrease in specific volume V as strain rate decreases and temperature increases (Figure 2), but these are due to additional components of viscoplastic compression.
5. Critical states are defined by $\partial p'/\partial \varepsilon_q = \partial q/\partial \varepsilon_q = \partial u/\partial \varepsilon_q = \partial V/\partial \varepsilon_q = 0$.
6. Large strain (critical state) failure is defined by the Mohr-Coulomb criterion where $(\sigma'_1/\sigma'_3)_{\max}$ is constant.
7. A constant isothermal creep rate coefficient with slope ψ denotes the rate of viscous compression. The creep rate coefficient does, however, increase with increasing temperature.

All parameters for MCC can be obtained from routine oedometer and triaxial compression tests.

The EVP Framework

Kelln (2007), following earlier work by Yin et al. (2002), proposed an elastic-viscoplastic (EVP) soil model that uses specific volume V to describe reorganization of clay particles, and elliptical plastic potentials to define the beginning of more compressible, non-recoverable behaviour at yielding ((Kelln et al. 2008). Elliptical plastic potentials and an associated flow rule allow comparisons with results from the non-viscous elastic-plastic MCC model for

normally consolidated (NC) and lightly overconsolidated (LOC) clays. Following paragraphs provide a brief overview of Kelln's EVP model, which is used in the new ETVP model.

In $V, \ln p'$ compression space, the specific volume of a specimen for a state of stress p'_m, V_m at time t can be written:

$$[1] \quad V_m = N - \lambda \ln p'_m - \psi \ln \left(\frac{t_0 + t}{t_0} \right)$$

where N is the intercept of the NCL at an assumed mean stress of $p' = 1$ kPa (Figure 3), and t is the duration of loading. Equation [1] is defined for $t > -t_0$, with $t_0 = 0$ being commonly taken at the end of primary consolidation (EOP) or the common loading interval of 24 hours. (The choice makes little difference in analyses.) In this form, t can be negative, which allows viscous deformations to be modelled during primary consolidation – the so-called Hypothesis B (Kelln et al. 2008).

The specific volume V_{ncl} on the isotropically consolidated NCL for the mean stress p'_m is calculated using the first two terms on the right side of Equation [1]. Viscoplastic compression, the third term, is plotted vertically in compression space. For $t = 0$, $V_{ncl} = V_m$ for a given stress state (p'_m, V_m) and an associated viscoplastic strain rate that produces decreases in V with increasing time. Using the third term, which defines creep, an incremental form of the delayed compression line can be related to time t in Equation [1] to obtain the viscoplastic volumetric strain rate in Equation [2].

$$[2] \quad \dot{\varepsilon}_p^{vp} = \frac{\delta \varepsilon_p^{vp}}{\delta t} = \left(\frac{\psi}{V_m t_0} \right) \exp \left(\frac{V_m - N}{\psi} \right) (p'_m)^{\frac{\lambda}{\psi}}$$

Equation [2] is only applicable to an isotropically consolidated state (p'_m, V_m) .

The mathematics of the formulation is not yet complete. The MCC framework assumes an associated flow rule for general p', q, V states. To satisfy this, the flow rule is defined using a scalar function S and plastic potential g , where:

$$[3a] \quad \dot{\varepsilon}_p^{vp} = S \frac{\partial g}{\partial p'} \text{ and}$$

$$[3b] \quad S = \left(\frac{\psi}{V_m t_0} \right) \exp \left(\frac{V_m - N}{\psi} \right) (p'_m)^{\frac{\lambda}{\psi}} \frac{1}{|\partial q / \partial p'|}$$

A general expression for the viscoplastic strain rate can then be written:

$$[4] \quad \dot{\varepsilon}_{ij}^{vp} = \left(\frac{\psi}{V_m t_0} \right) \exp \left(\frac{V_m - N}{\psi} \right) (p'_m)^{\frac{\lambda}{\psi}} \frac{1}{|\partial q / \partial p'|} \left(\frac{\partial g}{\partial \sigma'_{ij}} \right)$$

Yin et al. (2002) and Kelln et al. (2008) defined a viscoplastic limit line (vpl) as the boundary in p', V compression space at which viscoplastic straining is first encountered when p' increases from low values. To the left of the vpl in Figure 3 and Figure 4, deformations are purely elastic. To the right of the vpl, elastic and viscoplastic behaviour are both present. The vpl can be thought of as defining the p', V relationship at which void spaces in the soil have collapsed after very long loading durations. It can also be chosen so that the times needed to reach the vpl correspond with anticipated lifetimes of engineering structures. Doing so will improve computational efficiency. If the isotropic state is below or on the vpl, the stress state is inside the current yield locus, the stress-strain response is purely elastic and no viscoplastic strains will be experienced. Kelln et al. (2008) also redefined the third (creep) term in Equation [1] and introduced a method for establishing a suitable position for the vpl. Analyses are not sensitive to this choice because times are usually very long and viscoplastic volumetric strain rates are low.

Relating the vpl to the Normal Consolidation Line NCL and a given unload-reload line URL, leads to definitions for the viscoplastic volumetric strain rate $\dot{\varepsilon}_{ij}^{vp}$ and the scalar function S that defines the magnitude of shear strains:

$$[5] \quad \dot{\varepsilon}_{ij}^{vp} = \left(\frac{\psi}{V_m t_0} \right) \left(1 - \frac{N - \lambda \ln p'_m - V_m}{N - Z} \right)^2 \exp \left[\frac{N - \lambda \ln p'_m - V_m}{\left(\frac{N - \lambda \ln p'_m - V_m}{N - Z} - 1 \right) \psi} \right] \frac{1}{\left| \frac{\partial g}{\partial p'} \right|} \left(\frac{\partial g}{\partial \sigma'_{ij}} \right)$$

$$[6] \quad S = \left(\frac{\psi}{V_m t_0} \right) \left(1 - \frac{N - \lambda \ln p'_m - V_m}{N - Z} \right)^2 \exp \left[\frac{N - \lambda \ln p'_m - V_m}{\left(\frac{N - \lambda \ln p'_m - V_m}{N - Z} - 1 \right) \psi} \right] \frac{1}{\left| \frac{\partial g}{\partial p'} \right|}$$

Equations [5] and [6] do not operate if $V_{ncl} - V_m > N - Z$, where Z is the specific volume V on the vpl when $p' = 1$ kPa.

The analysis proceeds in a step-wise fashion, so $\delta q / \delta p'$ can be changed in triaxial tests to accommodate transition from consolidation to shear, from drained to undrained shearing, or to user-controlled changes in the effective stress path.

Kelln et al. (2009) used the model in a finite element analysis to simulate deformations under a well-instrumented highway embankment on soft organic silty clay at Limavady, Northern Ireland. The embankment was about 4 m high and settled 1.2 m in one year. Calibration of the model used data from high-quality commercial tests. Figure 5 compares field measurements for horizontal deformations under the toe of the embankment with values simulated using the MCC and EVP models. Traditional methods of calculating secondary compression and MCC modeling both have difficulty in calculating realistic lateral deformations. Traditional methods mostly consider only vertical compressions and ignore shear strains, while MCC assumes isotropic elasticity. Most clays are anisotropic ($K_0 \neq 1.0$)

and the resulting anisotropic elasticity leads, for example, to the non-elliptical yield loci observed in natural clays (Wood and Graham 1995).

Conventional drained triaxial compression tests are operated at a constant specified strain rate. The EVP model assumes that elastic strains instantly accompany changes in effective stress within the soil structure, while plastic strains develop with time (via reorganization of the soil particles). In general, a slower testing speed permits more plastic strains to develop and different stress-strain relationships are observed.

In lightly overconsolidated clay, the elastic-plastic MCC model, without viscosity, does not permit plastic strains to develop while effective stress states (p', q) are inside the yield locus. In contrast, elastic-viscoplastic (EVP and ETVP) models include viscoplastic straining immediately after loading is applied. Initially, viscoplastic strain rates are slow, but they accelerate as stresses approach the yield locus. Once the yield surface is reached, straining consists of elastic strains, stress-induced plastic strains, and time-dependent viscoplastic strains. Volumetric 'hardening' increases the size of yield surfaces.

In undrained lightly overconsolidated (LOC) undrained tests, the constant-volume requirement of the test can only be controlled by offsetting elastic and viscoplastic volumetric strains, with accompanying changes in pore water pressure and compensating changes in effective stress.

Without viscosity, and assuming isotropic elasticity, $\dot{\epsilon}_p^{tot} = 0 = \dot{\epsilon}_p^e + \dot{\epsilon}_p^p$ for effective stresses inside the current yield locus; changes in deviator stress are decoupled from changes in mean effective stress; and $\delta p' = \delta p - \delta u = 0$. This means that stress paths in q vs. p' plots are vertical. However, with viscosity, there are now viscoplastic volumetric strains as well as elastic strains, although they are typically small in overconsolidated clays. Their sum

must again be zero, so $\dot{\epsilon}_p^{tot} = 0 = \dot{\epsilon}_p^e + \dot{\epsilon}_p^{vp}$. In this case, stress paths in q vs. p' graphs are no longer vertical.

Incorporating temperature effects

Attempts have been made with varying degrees of success to model the effects of changes in temperature on the stress-strain behaviour of clay soils. Examples include Hueckel and Borsetto (1990), Baldi et al. (1991), Zhou et al. (1998), Cekeravac and Laloui (2004), Laloui and Cekerevac (2008), and Hueckel et al. (2009). Much of this work takes a mechanics-based approach, with its associated problems of large numbers of variables that are difficult to calibrate. These models mostly focus on q, p' stress space and pay less attention to $V, \ln(p')$ compression space. They introduce thermal coefficients to explain the physical effects of expansion and contraction of clay particles and water with changes in temperature. They also use temperature-dependent elastic parameters like the bulk modulus K to bridge the gap between known physics and the semi-empirical MCC framework.

Tanaka (1995), (later published as Graham et al. 2001), described a triaxial testing program on drained and undrained specimens of reconstituted illite. Figure 6a shows a large strain (critical state) envelope that is independent of temperature. In Figure 6b, despite experimental scatter, it is reasonable to draw parallel Critical State Lines (CSLs) like the NCLs observed by Campanella and Mitchell (1968) and by Eriksson (1989), see Figure 2b.

Tanaka used data from these and other tests, for example by Hueckel and Baldi (1990), to define a semi-empirical development of MCC that explained the effects of temperature on clay behaviour, particularly in compression space. The model in Figure 7 accommodated both constant-volume and drained heating, and simulated the complex pattern of volume

changes seen in drained tests with different overconsolidation ratios. To get agreement between measured and modelled results, the model assumed that the slopes κ of unload-reload lines varied with temperature in a way similar to those suggested by the low pressure sections of the test curves in Figure 2b. Graham et al. (2001) called it a thermoelastic-plastic (TEP) model.

To check the assumption that $\kappa = \kappa(T)$, Crilly (1996) ran a series of oedometer tests at different temperatures, but with constant load durations. The results showed the assumption was incorrect.

This disagreement between the assumptions in Tanaka's model and Crilly's experimental data forms part of the basis for this paper, which adds the effects of changes in temperature to the viscosity (creep) function used in the EVP model described by Kelln et al. (2008). The new model (Kurz 2014) attaches the effects of temperature to the viscoplastic component of the strain tensor and not to the elastic component. It is therefore an elastic-thermoviscoplastic (ETVP) model.

The ETVP and EVP models produce essentially identical results if the same value of the creep rate coefficient ψ is used in both. It can simulate results from tests at different temperatures and in principle, will allow simulations of non-isothermal behaviour.

A temperature-dependent viscosity function

Kelln (2007) introduced the viscoplastic volumetric strain rate function $\dot{\epsilon}_{ij}^{vp}$ (Equation [5]). It produces similar effects to those in Figure 2a, namely lower 'time-lines' in compression space that correspond to lower strain rates, and preconsolidation pressures that vary with strain rate. Kelln also suggested that his EVP model could be modified to accommodate

changes in temperature by defining the creep rate parameter ψ as a function of temperature, that is, $\psi = \psi(T)$. His suggestion addressed the dilemma discussed earlier in which Tanaka's TEP model (Figure 7) needed unload-reload coefficients that varied with temperature, whereas constant-duration loading tests by Crilly showed they were constant (Tanaka 1995; Crilly 1996; Graham et al. 2001). This paper is the first to develop the idea that viscoplastic volumetric strains now become thermoviscoplastic strains.

The ETVP model needs a defined relationship between ψ , (in $\ln$ time), and temperature. Published data are limited and in part contradictory, possibly because many clays have a substantial silt content. Towhata et al. (1993) concluded that secondary compression is initially accelerated by heating, meaning a faster decrease in voids ratio or specific volume, but it is followed by a similar rate of deformation to that observed at lower temperature. Batenipour et al. (2009) show a small difference in C_α , (in $\log$ time) for oedometer tests on silty clay performed at 3.0°C and 21.0°C. Leroueil and Marques (1996) cite evidence of an increase in strain rate, and therefore strains, with an increase in temperature. A potential relationship between creep and temperature is acknowledged in Burghinoli et al. (2000) and a linear relationship is presented in Houston et al. (1985). Fox and Edil (1996) show strong evidence for a C_α -temperature relationship for peat materials and proposed the following exponential relationship between temperature and C_α :

$$[7] \quad C_{\alpha 2} = C_{\alpha 1} \exp\{C_T(T_2 - T_1)\}$$

where $C_{\alpha 1}$ is the coefficient of secondary compression at temperature T_1 and $C_{\alpha 2}$ is the coefficient of secondary compression at temperature T_2 . The constant C_T is independent of voids ratio, stress level, and the change in temperature. We note that C_α is commonly measured only in the normally consolidated range of stresses and that most engineering is

done in the overconsolidated range where the creep compression rates are slower. The ETVP model overcomes this difficulty by assuming that C_α (or ψ) is a material constant that is independent of overconsolidation ratio but depends on temperature; and then arranging the mathematics appropriately, (Equation [1]).

Many 'active' clay particles are surrounded by diffuse double layers (DDLs) of water whose viscosity decreases with increasing temperature (Mitchell and Soga 2005). It seems reasonable, therefore, that a clay at higher temperature will experience higher creep rates and more creep straining than one at lower temperature. The picture becomes further complicated depending on whether the pore water can drain or is prevented from draining (Campanella and Mitchell 1968).

This project assumed two assumed relationships for $\psi = \psi(T)$, (Figure 8). (Experimental data appear to be scarce.) The simpler relationship is a linear increase in ψ between two selected temperatures T , one of which is the laboratory temperature at which ψ is measured. Houston et al. (1985) used 0°C as the other 'anchor' point for the relationship. Since frozen soils exhibit creep, the authors have chosen -273.15°C, or zero Kelvin, where all atomic motion ceases and no creep deformations are possible. Values of ψ associated with the first anchor point are discussed in following section.

The second relationship is the exponential relationship proposed by Fox and Edil (1996), but now with the creep parameter defined by ψ instead of C_α .

$$[8] \quad \psi_2 = \psi_1 \exp\{\Omega(T_2 - T_1)\}$$

where ψ_1 is again the creep rate coefficient at temperature T_1 , and ψ_2 is the creep rate coefficient at temperature T_2 . Different soils have different values of Ω . The required functional relationship is anchored to a measured value of ψ at laboratory temperature.

Implementing the $\psi = \psi(T)$ functions

To test the effects of temperature-dependent viscosity functions, Kurz (2014) developed a spreadsheet model based on (a) the MCC framework, (b) the viscoplastic formulation of Kelln et al. (2008), and (c) the linear and exponential relationships for $\psi = \psi(T)$ in Figure 8. Later paragraphs write the conditions used in our modeling in the form $(\psi:T)$, where ψ represents the value of the creep rate coefficient and T represents temperature. The spreadsheet uses selected values of $(\psi:T)$ to analyse drained, undrained, lateral expansion, or user-defined stress paths, starting from an initial stress state and moving through peak strength to large-strain strengths in triaxial loading. Stresses and strains are calculated incrementally in (q,p') stress space and $V, \ln(p')$ compression space.

Both creep relationships are implemented in the same way. Starting from an initial known point $(\psi_1:T_1)$ from laboratory data, either a new temperature, T_2 , or a change in temperature, dT , can be specified. Both relationships use $\psi_0:T_0 = 0:0K$, noting also that the exponential relationship requires a value for Ω and cannot be precisely zero. The calculated creep parameter, ψ_2 is then used for calculating viscoplastic strains under specified triaxial test conditions.

In the results shown later, the temperature is kept constant during shearing. The simulations are therefore isothermal, although the effects of different temperatures on stress-strain-time behaviour can be examined. The model calculates incremental values of stresses and strains for both $(\psi_1:T_1)$ and $(\psi_2:T_2)$. Results are then calculated for a single set of test conditions using (a) the basic MCC model without temperature effects or viscous behaviour, $(\psi = 0)$; (b) Kelln's (2007) formulation of the EVP model without temperature

effects at $(\psi_1: T_1)$, and (c) the authors' ETVP model at $(\psi_2: T_2)$ as a result of a temperature change $dT = T_2 - T_1$.

A second ETVP model was developed to examine the effects of changes in temperature during triaxial consolidation and subsequent shearing. This non-isothermal model allows the initial temperature, T_1 to be changed to a final temperature T_2 , with ψ being changed in a series of increments until ψ_2 is reached. This model was developed for comparative purposes and was not used for the analyses shown in following sections. Details of both models and preparation of input data can be found in Kelln (2007) and Kurz (2014).

Limitations of the ETVP model

The ETVP model contains the assumptions and constraints listed earlier for MCC (Wood 1990), but it can also capture the effects of temperature-dependent clay viscosity that are observed when straining rates are varied, and when stresses are held constant. If the creep rate coefficient ψ is set to zero, the MCC model is recovered from both the EVP and ETVP models as a special case.

The new work described in this paper contains the following limitations.

- (1) The model focuses only on $\psi = \psi(T)$. That is, the purpose of the model is to gain an understanding of how temperature affects the creep component of plastic straining. All other parameters used by Kelln (2007) are unchanged.
- (2) Physics related to thermal expansion and contraction of the soil particles and water is not implemented (Laloui and Cekeravac 2008; Hueckel et al. 2009). Leaving the coefficients of thermal expansion and contraction out of the model isolated the impact of temperature on viscous strains. This was done to examine whether the ETVP 'semi-

empirical' model could replicate published data for tests at different temperatures. Some discussions suggest their inclusion in the ETVP model may be unnecessary.

- (3) Although the $\psi = \psi(T)$ relationships have been defined on the Kelvin-scale and sub-zero Celsius temperatures are achievable, the model does not implement phase changes. The volumetric expansion of water and the strength and viscous behaviour of ice are not included in the model at this stage.
- (4) Elasticity is assumed to be independent of temperature. As in MCC, elastic properties like bulk modulus K and shear modulus G , (or Young's modulus E and Poisson's ratio ν) of the microstructure of the clay, change with changes in mean stress and specific volume. They are not a direct function of temperature. Naming the model an elastic-thermoviscoplastic (ETVP) model reflects the assumption that thermal effects are included only in the viscoplastic component of strains.
- (5) The series of parallel 'time lines' in Yin's work, or the strain rate lines in Figure 2a all result from viscoplastic straining relative to a selected NCL. Similarly, in the work of Campanella and Mitchell (1968), Eriksson (1989), and Tanaka (1995), the parallel lines for different temperatures develop from thermoviscoplastic straining from an initial NCL at a chosen temperature. In the authors' modeling, the apparent changes in slope of unload-reload lines with temperature seen in Figure 2b result from changes in ψ . They are not the absolute changes in κ assumed in Tanaka's (1995) model (Figure 7).
- (6) The material parameter, Ω , is assumed to be constant, but with different values for various types of clays and different pore water chemistries. It will therefore affect the rate at which the creep rate coefficient ψ changes with temperature. To the author's knowledge, literature pertaining to this value is not available. This project used a

constant value of $\Omega = 0.015/^{\circ}\text{C}$, ($0.015/\text{K}$). This provides a near-zero value for ψ at zero Kelvin.

Sensitivity Analysis and Modeling Parameters

Factors such as test type and overconsolidation ratio impact soil behaviour considerably. To investigate the effects of viscosity and temperature, the ETVP model was used in a sensitivity analysis of undrained ($\text{CI}\bar{\text{U}}$) and drained (CID) triaxial compression tests on normally consolidated (NC), lightly overconsolidated (LOC), and heavily overconsolidated (HOC) clay.

Modeling assumed the same material properties used by Kelln (2007) in his EVP modeling, (Table 1). This allowed direct comparison with Kelln's results when the same creep coefficient ψ was used in both models. It also allowed comparison with MCC modeling using $\psi = 0$.

Analyses were run with three overconsolidation ratios; 1.00 for normally consolidated (NC) clay, 1.67 for lightly overconsolidated (LOC) clay, and 5.00 for heavily overconsolidated (HOC) clay. Isotropic preconsolidation pressures p'_c of 25.0 kPa and 50.0 kPa were examined. We acknowledge that these pressures are low.

Axial strain rates $d\varepsilon_1/dt$ were again chosen to be consistent with those used by Kelln (2007). Three values were chosen; 15.0%/day, 1.50%/day, and 0.15%/day. (For brevity, later figures will only show results for 15%/day and 0.15%/day.) While an axial strain rate of 15.0%/day can be incorporated into simulating potential results from drained tests, drainage will usually not be complete in actual tests at this shearing rate - excess pore water pressures will develop. The model does not accommodate this partial drainage condition.

The final variable needed in the sensitivity analysis was the creep rate coefficient $\psi = \psi(T)$. The linear and exponential relationships in Figure 8 were used. The linear relationship

was anchored at absolute zero (0.0K) and 28°C (Table 1). The exponential relationship was anchored to a typical laboratory value of $\Omega = 0.015/^{\circ}\text{C}$ (Equation [8]). The relationships in Figure 8 also need an anchor value of ψ_1 measured in a standard creep test at a known laboratory temperature T_1 , and labelled here as $(\psi_1:T_1)$. Our analyses used three values of ψ_1 that represent the common range of values for soil - 0.002, 0.006, 0.010 at a temperature, T_1 , of 28.0°C, (301.15K). (The intermediate value of $\psi = 0.006$ was measured by Kelln (2007) at his instrumented embankment in Northern Ireland.)

The temperatures used in the sensitivity analyses were 28°, 65°, and 100°C, with some additional analysis for 3°C to represent behaviour beneath one of our projects on embankment performance. The ‘anchor’ temperature, $T_1 = 28^{\circ}\text{C}$, is a little higher than typical laboratory temperatures. It was chosen to allow comparison with the thermoelastic-plastic (TEP) results from Tanaka (1995).

Representative results

Analyses using the parameters shown in the previous section produced over 540 graphs. Obviously, we can only include representative examples to indicate the effects of creep rate and temperature on clay behaviour. More detailed numerical information about peak strengths, large-strain strengths, and corresponding pore water pressures associated with the three chosen temperatures and three axial strain rates can be found in Kurz (2014).

Figures 9 to 13 deal with undrained (CI $\bar{U}$) tests. In these and other later figures, we have used the symbol u to represent *changes* in pore water pressure produced during shearing. This usage is common. We recognize, however, that most triaxial tests start with a ‘back pressure’ u_b , which is used to improve the measurement of pore water pressure changes,

whether increases or decreases, during shearing. Values of the changes u can then be applied to u_b and produce values for the pore water pressures that are actually measured. Figure 14 shows one example of drained (CID) results.

As an introductory example of results from the ETVP model, Figure 9 shows q, p' and q, ε_q responses for CI $\bar{U}$ NC test specimens consolidated to 50 kPa at the lowest (28°C) and highest (100°C) of our three chosen temperatures. The figure compares results from both linear and exponential creep relationships anchored at $(\psi_1: T_1) = (0.006: 28.0^\circ\text{C})$. It is assumed that no phase change has yet occurred at 100°C.

Although the ETVP simulations shown in Figure 9a all end at the same CSL, (Table 1), they indicate strain softening after peak undrained strength $s_u = q_{peak}/2$ is reached. Strain softening is seen in laboratory testing but is not reflected by MCC. The peak undrained strengths in this NC modeling increase with temperature. Large-strain strengths decrease with temperature. That is, higher temperatures produce higher peak strengths, more strain softening, and lower large-strain strengths than lower temperatures. The figure shows, as expected, a larger impact of higher temperature using the exponential relationship as compared with the linear relationship. All the remaining figures show results from only the exponential relationship anchored at $(\psi_1: T_1) = (0.006: 28.0^\circ\text{C})$.

As expected, using smaller (0.002) and larger (0.010) values for ψ_1 as the anchor creep rate coefficient at 28.0°C, produced smaller and larger differences respectively between the responses of the model at higher temperatures. Kelln (2007) showed that similar, but less marked, patterns of behaviour are produced by changes only in strain rate.

The graphs in Figures 9 to 13 also compare end-of-test behaviour with the 'steady-state' behaviour predicted by MCC at large strains. When viscous behaviour is added in EVP or

ETVP simulations, ongoing viscous straining means that the ratios $\partial p'/\partial \varepsilon_q$, $\partial q/\partial \varepsilon_q$, $\partial u/\partial \varepsilon_q$, and $\partial V/\partial \varepsilon_q$ do not reach the zero values required by MCC at critical state; see for example Figure 9b.

In Figures 10 to 13, deviations of $\partial p'/\partial \varepsilon_q$, $\partial q/\partial \varepsilon_q$, $\partial u/\partial \varepsilon_q$, and $\partial V/\partial \varepsilon_q$ from zero were larger when the strain rate of the tests was slower and the temperature higher. That is, longer test durations allow the addition of larger thermoviscoplastic volumetric strains in undrained tests. These must be offset by larger elastic volumetric strains to keep the total volume constant. To do this, pore water pressures must increase, p' must decrease, and therefore deviator stresses must also decrease if large-strain strengths are to remain on the critical state line, CSL. The result is strain softening behaviour.

Another feature may influence the results. At large axial strains, the measured variables change slowly and numerical procedures need to divide small values in the equations by other smaller values. As a result, numerical instabilities sometimes developed. Kurz (2014) tabulated the ratios of $\partial p'/\partial \varepsilon_q$, $\partial q/\partial \varepsilon_q$, $\partial u/\partial \varepsilon_q$, and $\partial V/\partial \varepsilon_q$ when modeling was terminated, usually at shear strains of 12 – 17% in undrained tests, and larger shear strains in drained tests.

Figures 10 to 14 show additional information for Normally Consolidated (NC) and Lightly Overconsolidated (LOC) specimens for axial strain rates of 15% per day and 0.15%/day. Compared with faster strain rates, slower strain rates allow larger plastic volumetric strains to develop. Increasing temperatures amplify this effect.

Modeling Normally Consolidated CI $\bar{U}$ Behaviour.

Graphs (a) and (d) in Figure 10 repeat the results from the exponential $\psi = \psi(T)$ modeling in Figure 9, but now also include results for 65°C.. The remaining graphs (b), (c), and (e), show

additional relationships between specific volume V and mean effective stresses p' and p'_m ; viscoplastic volumetric strain rate $\dot{\epsilon}_p^{vp}$ and p'_m ; and changes in pore water pressure u with shear strain ϵ_q . The values of p'_m represent the viscoplastic hardening and elastic softening that take place during CI $\bar{U}$ tests.

The graphs in Figure 10 provide insights into EVP behaviour. In graph (b), the horizontal line that extends leftward from the isotropic consolidation pressure of 50 kPa on the NCL towards the intersection of the unload-reload line URL_f, and the CSL represents the constant-volume stress path of CI $\bar{U}$ tests. As the tests proceed toward peak failure, the volumetric viscoplastic strain rate $\dot{\epsilon}_p^{vp}$ in (c) increases and so do compensating pore water pressures in (e) and the values of p'_m in (b) that define the size of the plastic potential. After peak shear strength is reached, $\dot{\epsilon}_p^{vp}$ and p'_m decrease toward end-of-test (large strain) values, and pore water pressures increase. The zero-conditions for Critical States are not reached, but large-strain strengths all lie close to the CSL in (a) with slightly different values of q'_f , p'_f , p'_{mf} , and u_f .

Comparing the NC modeling in Figures 10 and 11, the faster straining rate in Figure 10 produces higher peak deviator stresses that increase with increasing temperature. However, due to differences in strain softening related to the changes in p'_m in Figures 10c, higher temperatures produce lower stresses at the End-of-Test (EOT), (Figures 10c, 10d). In contrast, the slower straining rate in Figure 11 produces lower peak deviator stresses that decrease with increasing temperatures, as well as considerably lower deviator stresses at EOT (Figures 11c, 11d). The only difference between the two figures is strain rate.

Since $CI\bar{U}$ tests do not actually experience volume changes, pore water pressures must increase to offset viscoplastic volumetric compressions. By definition, this results in a slower decrease in mean effective stress p' in Figure 10 than in Figure 11. Compared with the faster Figure 10a, the slower strain rate in Figure 11a produces more viscoplastic volumetric strains. The differences in the effective stress paths are considerable. These differences are reflected in the changes u in pore water pressure shown in Figures 10e and 11e, where higher effective stresses result in lower pore water pressures and vice versa. The ETVP (and EVP) models generate strain softening in NC tests due to the development of thermoviscoplastic strains with increasing axial strains.

Viscoplastic volumetric strain rates $\dot{\epsilon}_p^{vp}$ correspond to the increases and decreases in plastic potentials, indicated by p'_m in graphs labelled (b) and (c) in the figures. As the graphs approach peak deviator stress, there are increases in p'_m and the rate of viscoplastic straining in Figure 10b, and decreases in Figure 11b. After peak deviator stress, as the modeling moves towards EOT, the rate of viscoplastic straining and the plastic potential decrease. The initial decreases observed in viscoplastic strain rate in the slower NC $CI\bar{U}$ test (Figure 11c) are believed to be due to viscoplastic strains (and pore water pressure changes) developing early in the test simulation. The less sloping lines at higher temperatures (compare Figures 10c and 11c) imply that changes in the rate of viscoplastic straining decrease as temperatures increase.

Modeling Overconsolidated $CI\bar{U}$ Behaviour.

With some important exceptions, the LOC $CI\bar{U}$ tests (Figures 12 and 13) present broadly similar responses of the model to temperature-dependent viscosity as those in NC tests. Of course, details of stress, strain, and pore water pressure relationships depend on

overconsolidation ratio (OCR) and are different. The faster strain rate in Figure 12a, (15%/day), produces an apparently similar elastic response for all temperatures within the yield ellipse. Because strains, and therefore test durations, are small in the overconsolidated section, only small viscoplastic volumetric strains will occur, although viscoplastic volumetric strain rates increase as the yield locus is approached. Even though the initial part of the p',q relationship is largely elastic, it also includes small amounts of viscoplastic straining. Temperature-dependency is only apparent after peak strengths have been passed and the differences are small.

In contrast, with the slower strain rate in Figure 13, (0.15%/day) and consequently longer test durations, the stress path inside the initial yield ellipse begins to deviate from a purely elastic response once the viscoplastic strain rate (Figure 13c) becomes large enough to produce larger amounts of viscoplastic volumetric compression. The compression is countered with an increase in pore water pressure (Figure 13e) and a decrease in mean effective stress (Figure 13a). If the creep rate coefficient is sufficiently high, peak strength will be reached before the stress path can reach the original yield ellipse. Here, peak strengths and post-peak strengths in ETVP modeling can be noticeably smaller than in MCC modelling.

Similar responses are observed in HOC CI $\bar{U}$ tests. We have not, however, provided accompanying figures, partly for brevity and partly because the base MCC formulation does not model HOC clay particularly well. As before, faster strain rates produce higher peak deviator stresses at higher temperatures compared with slower strain rates at the same temperatures. The difference lies with the response of the pore water pressure. For incremental stress ratios $\partial q/\partial p'$ below the CSL, (that is, when $\eta = q/p' < M$), the behaviour is

primarily elastic volumetric compression, and hence increasing pore water pressures. (Viscoplastic volumetric strain rates are low when the overconsolidation ratio OCR is high.) Above the critical state line, with $\eta > M$, the behaviour transitions to plastic volumetric expansion and leads eventually to decreases in pore water pressure. The peak of the pore water pressure graph does not coincide with the first crossing of the critical state line ($\eta = M$) but instead represents a gradual transition from compressive to expansive behaviour. Again, this is what is seen in laboratory tests but is missing from MCC modeling.

Another reason for not including details of HOC results is that some simulations produced sudden drops in strength and pore water pressure after peak strength was reached. The discontinuities seemed to be associated with the numerical procedures used in the modeling of slow axial strain rates and large values of $\psi(T)$.

Modeling Drained (CID) Behaviour.

We have chosen to show results for only one LOC CID test with the slower of our two strain rates, 0.15%/day (Figure 14). As with CI $\bar{U}$ tests, this slower strain rate allows larger viscoplastic strains to develop than the faster strain rate of 15.0%/day, which in any case, is unreasonably fast for CID tests. Increasing temperature amplifies this effect. Our work with the ETVP model has not yet been coupled with a pore water pressure dissipation model - it simply assumes that tests are run sufficiently slowly that no excess pore water pressures develop.

As seen in the laboratory data in Figure 2b, Figure 14b shows isotropic compression lines moving to lower specific volumes. While the stress path in Figure 14a is the same for all temperatures, elastic strains and viscoplastic strains will both develop from the beginning of the test. When the stress path in Figure 14a reaches the elliptical yield envelope, there are sudden changes in the $\dot{\epsilon}_p^{vp}$ versus p'_m curves in Figures 14c, which then show a decreasing

viscoplastic strain rate; the q, ε_q stress-strain curve in Figure 14d; and a marked increase in volumetric compression in Figure 14e. Kelln et al. (2008) explain that the transition to a predominantly plastic response is caused by a change in viscoplastic strain rate and not simply because of a sudden transition to a yielded condition. These features of behaviour are all affected by temperature.

As with the HOC CI $\bar{U}$ tests, convergence difficulties were experienced with the HOC CID tests and critical (p', q, V) states were usually not achieved. Attention to the numerical procedures used in the analysis may improve simulations close to critical states.

Discussion

Figure 15, Figure 16, and Table 2 summarize the effects of the chosen temperatures and strain rates on deviator stresses q_p and changes in pore water pressure u_p at peak failure. The figures also show values from MCC simulations. Temperature and strain rate effects are not of course included in MCC.

While our paper has dealt only with isotropically consolidated triaxial compression tests at various constant pressures, the ETVP model can, in principle, simulate anisotropic consolidation, step-wise changes in stress path, and the effects of heating or cooling. Further studies will be undertaken and reported later.

It is sometimes thought that the effects of temperature on creep in clay are small (for example, Mesri 1973). We disagree, although it is reasonable to say they are commonly smaller than those associated only with creep. Table 2 shows the effects of temperature-influenced creep rates on undrained shear strengths and pore water pressure changes in NC clay (Figure 15) and LOC clay (Figure 16). The table shows simulated ETVP behaviour for a

creep rate coefficient $\psi_1 = 0.006$ at 28°C , and an axial strain rate of $15\%/ \text{day}$, which can be considered typical values for laboratory tests. It then compares results for peak deviator stress and changes in pore water pressure with comparable results from a combination of temperature and strain rate that produced the maximum effects from thermoviscosity, 100°C and $0.15\%/ \text{day}$. This would correspond, for example, to rapid loading of a thick, slow draining layer of plastic clay subjected to considerable heating, but without phase changes. The results in Table 2 appear substantial.

Earlier discussion of Figures 2b and 7 said the TEP model described by Tanaka (1995) assumed the slope κ of unload-reload lines varied with temperature; that is, $\kappa = \kappa(T)$ (Graham et al. 2001). This was done to account for volumetric compressions seen by Hueckel and Baldi (1990) during isotropic consolidation of NC and LOC specimens, and the volumetric expansions seen in HOC specimens. Tanaka and his co-workers were correct that the behaviour needed to be modelled, but incorrect in associating it with elastic deformations and the κ -lines. As mentioned earlier, Crilly (1996) showed that κ was independent of temperature. This agrees with how we currently understand the mechanisms of intra- and inter-particle deformations.

The apparent dependency of κ on temperature in Figure 2b should be associated with a combination of temperature-independent elasticity and temperature-dependent viscous effects (Kurz 2014). Figure 17 shows simulated isotropic consolidation curves at temperatures of 28.0°C , 65.0°C , and 100.0°C for normally consolidated (NC) specimens and overconsolidated specimens with OCRs of 1.20, 1.67, and 2.00. The creep rate coefficient ψ is again defined by an exponential relationship for $\psi = \psi(T)$, with $(\psi_1 : T_1) = (0.006 : 28.0^\circ\text{C})$.

Loading rates are constant and the purely elastic stiffnesses are independent of temperature but vary with pressure. Drawing lines through the points of maximum curvature for each OCR for a given temperature suggests that slopes of the initial loading lines, which include thermoviscoplastic behaviour, vary with temperature and Tanaka's assumption can be justified in a qualitative sense. The behaviour that Tanaka and others observed is more correctly handled by the combined effects of temperature and viscoplasticity in the ETVP model.

Figure 17 also shows that the ETVP model includes preconsolidation pressures that decrease with increasing temperature, and parallel NCLs that move to lower values of specific volume with temperature increase. This was shown in the laboratory data by Eriksson (1989) in Figure 2b, and by Campanella and Mitchell (1968) and Houston et al. (1985), among others.

Hueckel and Baldi (1990) reported results from a series of drained tests in which temperatures were increased on specimens with different (constant) isotropic consolidation pressures and overconsolidation ratios (OCRs). The specimens had previously reached equilibrium at room temperature. Heating a normally consolidated specimen produced compressive volumetric strains that increased with increasing temperature. In overconsolidated specimens, volumetric strains increased with increasing temperature but decreased with increasing OCR. They became expansive at high OCRs (see also Towhata et al. 1993).

The now-questionable assumption that $\kappa = \kappa(T)$ allowed Tanaka's TEP model to simulate volumetric strains under these test conditions (Graham et al. 2001). Figure 18a shows a broadly similar pattern of behaviour to the Hueckel and Baldi (1990) data, with volumetric

strains increasing with temperature, and becoming expansive with increasing OCR.

Kurz (2014) examined this question with his ETVP model using a creep rate coefficient ψ that was once more defined by $(\psi_1 : T_1) = (0.006 : 28.0^\circ\text{C})$. The modeling shown in Figure 18b also captures the small initial compressions that are seen in tests on heavily overconsolidated specimens. The patterns in Figure 18b are again broadly similar to the Hueckel and Baldi (1990) data with respect to the effects of overconsolidation ratio. The magnitudes of the volumetric strains are different in the two graphs in Figure 18, with the ETVP relationships in Figure 18b trending more in the direction of compression. The results depend on the material properties selected for the modeling. We note that neither the TEP or ETVP modeling includes volumetric expansion of soil particles and water, but both models produce the families of parallel NCLs seen in Figure 2a for changes in strain rate and in Figure 2b for changes in temperature. The behaviour in each case is related to defining the creep rate coefficient ψ as a material constant. The curvatures of the volume-change relationships in Figure 18 are not totally consistent with values published for example by Hueckel and Baldi (1990) and Cekerevac and Laloui (2004). These two issues require further study.

The model can simulate non-isothermal behaviour but this has not yet been explored in detail, for example in simulating heating tests reported by Graham et al. (2001) and Hueckel et al. (2009), among others.

In its current form, the ETVP model has only been validated qualitatively against published data. Quantitative comparisons were not possible, largely because in our opinion, publications rarely include enough detailed information about how specimens were prepared and tests were conducted. A separate laboratory program focused on triaxial

testing at various temperatures and axial strain rates was not part of this program, nor was validation of the ETVP model in an instrumented field application. Nevertheless, the semi-empirical ETVP model can qualitatively simulate laboratory tests with different loading rates and different temperatures. In principle, it can be generalized into a finite element load-deformation model that includes pore water pressure dissipation.

The relationship of the creep rate coefficient ψ with temperature remains unclear. A laboratory program designed to measure ψ at various elevated (and potentially sub-zero) temperatures would be useful. Coupled with thermal modeling of temperatures, including phase-change relationships, the model would help in examining permafrost degradation associated with climate change and its impact on deformations of infrastructure in cold regions.

Conclusions

The project examined relationships between time, temperature, and load-deformation behaviour in clay soils subject to mechanical loading. This has been done in a sensitivity analysis using arbitrarily selected, but realistic, properties for elastic, normally consolidated, unloading-reloading, creep, yielding, and failure behaviour of clay. For a particular problem, these properties can be readily measured in commonly-used standard laboratory tests.

The research combined thermoplasticity with viscoplasticity into an elastic thermo-viscoplastic (ETVP) model for isotropically consolidated triaxial shear tests. It was developed from an earlier viscoplastic strain rate formulation with the addition of a relationship between the creep rate coefficient and temperature. This relationship needs additional investigation. Modeling of normally consolidated shearing shows strain softening

after peak deviator stress has been reached. This is seen in laboratory testing but not in Modified Cam Clay modeling.

The ETVP model is compatible with MCC modeling if ψ is set to zero, and with EVP modeling if the same value of ψ is chosen in both. It can be developed for more general stress states and used in finite element modeling.

Acknowledgements

Dr. Curtis Kelln provided helpful technical advice for developing his EVP model into the ETVP model described in the paper.

References

- Baldi, G., Hueckel, T., Peano, A. and Pellegrini, R. 1991. Developments in modelling of thermohydro-mechanical behaviour of Boom clay and clay-based buffer materials. ISBN: 92-826-0582-2.
- Batenipour, H, Kurz, D, Alfaro, MC and Graham, J. 2009. 'Effect of Temperature on the 1-D Behaviour of Plastic Clay', 62nd Canadian Geotechnical Conference and 10th Joint CGS/IAH-CNC Groundwater Conference., Halifax NS.
- Bjerrum, L. 1967. Engineering geology of Norwegian normally consolidated marine clays as related to settlements of buildings, 7th Rankine Lecture. *Géotechnique*, **17**(2): 81-118.
- Burghinoli, A., Desideri, A., and Miliziano, S. 2000. A laboratory study on the thermomechanical behaviour of clayey soils. *Canadian Geotechnical Journal*, **37**: 764-780.
- Campanella, R.G., and Mitchell, J.K. 1968. Influence of temperature variations on soil behaviour. *Journal of the Soil Mechanics and Foundations Division. ASCE*, **94**(3): 709-734.

- Cekerevac, C. and Laloui, L. 2004. Experimental study of the thermal effects on the mechanical behaviour of a clay. *International Journal for Numerical and Analytical Methods in Geomechanics* **28**: 209-228.
- Crilly, T.N. 1996. Unload-reload tests on saturated illite specimens at elevated temperatures. M.Sc. thesis, University of Manitoba, Winnipeg, Man.
- Eriksson, L.G. 1989. Temperature effects on consolidation properties of sulphide clays. In *Proceedings of the 12th International Conference on Soil Mechanics and Foundation Engineering*, Rio de Janeiro, A.A. Balkema, Rotterdam, Netherlands. Vol. 3, pp. 2087-2090.
- Fox, P.J. and Edil, T.B. 1996. Effects of stress and temperature on secondary compression of peat. *Canadian Geotechnical Journal*, **33**: 405-415.
- Graham, J. 2006. The 2003 R. M. Hardy Lecture: Soil parameters for numerical analysis in clay. *Canadian Geotechnical Journal* **43**: 187-2009.
- Graham, J., Crooks, J.H.A., and Bell, A.L. 1983. Time effects on the stress-strain behaviour of natural soft clays. *Géotechnique*, **33**(3): 327-340.
- Graham, J., Tanaka, N., Crilly, T. and Alfaro, M. 2001. Modified Cam-Clay modelling of temperature effects in clays. *Canadian Geotechnical Journal* **38**, 608-621. doi: 10.1139/t00-125.
- Graham, J. and Yin, J.-H. 2001. On the time-dependent stress-strain behaviour of soft soils. Invited Keynote Address, Conference on Settlement on Soft Soils, Hong Kong, December 2001. C.F. Lee et al. (eds.) A.A. Balkema Publishers, Lisse, 13-24.
- Houston S.L., Houston, W.N. and Williams, N.D. 1985. Thermo-mechanical behavior of seafloor sediments. *Journal of Geotechnical Engineering, ASCE*, **111**: 1249-1263.

- Hueckel, T., and Baldi, G. 1990. Thermoplasticity of saturated clays: experimental constitutive study. *Journal of Geotechnical Engineering, ASCE*, **116**: 1778–1796.
- Hueckel, T., and Borsetto, M. 1990. Thermoplasticity of saturated soils and shales: constitutive equations. *Journal of Geotechnical Engineering, ASCE*, **116**: 1765–1777.
- Hueckel, T; Francois, B; Laloui, L. 2009. Explaining thermal failure in saturated clays, *Géotechnique*, **59**: 197-212. doi:10.1680/geot.2009.59.3.197.
- Kelln, C.G. 2007. An elastic-viscoplastic constitutive model for soil. Ph.D. thesis. Queen's University Belfast, Belfast, Northern Ireland, UK.
- Kelln, C., Sharma, J., Hughes, D, and Graham, J. 2008. An improved elastic-viscoplastic soil model. *Canadian Geotechnical Journal*, **45**, 1356-1376. doi:10.1139/T08-057.
- Kelln, C., Sharma, J., Hughes, D. and Graham, J. 2009. Finite element analysis of an embankment on soft estuarine deposit using an elastic-viscoplastic soil model. *Can. Geotech. J.* **46**: 357-368.
- Kurz, D.R. 2014. Understanding the effects of temperature on the behaviour of clay. PhD thesis, University of Manitoba, Winnipeg, Canada.
- Laloui, L., and Cekervac, C. 2008. Non-isothermal plasticity model for cyclic behaviour of soils. *International Journal for Numerical and Analytical Methods in Geomechanics* **32**: 437-460. doi: 10.1002/nag.629
- Leroueil, S. and Marques, M.E.S. 1996. Importance of strain rate and temperature effects in geotechnical engineering, in TC Sheahan, VN Kaliakin (eds.), *Measurement and modeling time dependent soil behaviour*, Geotechnical Special Publication, ASCE, Reston, Va.
- Mesri, G. 1973. Coefficient of secondary compression. *Journal of the Soil Mechanics and Foundations Division. ASCE*, **99**: 123-137.

- Mesri, G., and Choi, Y.K. 1985. The uniqueness of end-of-primary (EOP) void ratio effective stress relationship. In Proceedings of the 11th International Conference on Soil Mechanics and Foundation Engineering, San Francisco, USA. pp.587-590.
- Mitchell, J.K. and Soga, K. 2005. Fundamentals of Soil Behavior, 3rd Edition. John Wiley and Sons, New York, NY.
- Sällfors, G. 1975. Preconsolidation pressure of soft, high-plastic clays. PhD thesis, Chalmers University of Technology, Göteborg, Sweden.
- Tanaka, N. 1995. Thermal elastic plastic behaviour and modelling of saturated clays. Ph.D. thesis, University of Manitoba, Winnipeg, Canada.
- Tidförs, M., and Sällfors, G. 1989. Temperature effect on preconsolidation pressure. *Geotechnical Testing Journal*, **12**: 93-97.
- Towhata, I., Kuntiwattanakul, P., Seko I., and Ohishi, K. 1993. Volume change of clays induced by heating as observed in consolidation tests. *Soils and Foundations*, **33**(4): 170-183.
- Wood, D.M. 1990. Soil behaviour and critical state soil mechanics. Cambridge University Press, Cambridge, England.
- Wood, D.M. and Graham, J. 1995. State boundary surface and anisotropy. Proc. 5th Int. Conf. on Numerical Models in Geomechanics NUMOG V. Davos, Switzerland, Sept. 1995, 77-82.
- Yin, J.-H., Zhu, J.-G., and Graham, J. 2002. A new elastic viscoplastic model for time-dependent behaviour of normally and overconsolidated clays: theory and verification. *Canadian Geotechnical Journal* **39**, 157-173.
- Yong, R.N., Mohamed, A.M.O., Warkentin, B.P. 1992. Principles of contaminant transport in soils. Elsevier, Amsterdam.

Zhou, Y., Rajapakase, R.K.N.D., and Graham, J. 1998. Coupled heat-moisture-air transfer in deformable unsaturated media. *ASCE Journal of Engineering Mechanics*, **124**, 1090-1099.

Draft

List of Figures

Figure 1. Definition of 'instant' and 'delayed' compression compared with 'primary' and 'secondary' compression (Bjerrum 1967, with permission).

Figure 2. Differences in compression behaviour with (a) changes in strain rate (and Sällfors 1975, with permission), and (b) changes in temperature (Eriksson 1989, with permission).

Figure 3. Compression space $V, \ln(p')$ showing a point a_0 proceeding to a_1 after time t (Kelln et al. 2008)

Figure 4. Admissible development of viscoplastic volumetric strains with constant stress for a stress state which resides outside a current yield locus with size defined by p'_o that does not fall on the isotropic normal compression line p'_n . The locus of p'_o in the V, p' compression plane forms the viscoplastic limit line (vpl) parallel with the isotropic normal compression line (Kelln et al. 2008).

Figure 5. Computed and measured horizontal displacements beneath toe of an embankment on soft organic silty clay at Limavady, Northern Ireland (Kelln et al. 2009).

Figure 6. Peak strength (in this case, critical state) data from triaxial compression tests on normally consolidated illite at 28°C, 65°C, and 100°C: (a) strength envelope, (b) normal compression lines (Graham et al. 2001).

Figure 7. Representation of $V, \log p'$ compression space in Tanaka's thermoelastic-plastic (TEP) model, assuming $\kappa = \kappa(T)$: (Graham et al. 2001).

Figure 8. Linear and exponential relationships for the creep rate coefficient ψ versus temperature anchored at a temperature of 28°C, (301.1K) for given values of ψ .

Figure 9. ETVP modeling of a CI $\bar{U}$ test for NC soil at a strain rate of 15.0%/day and anchored at $(\psi_1 : T_1) = (0.006 : 28.0^\circ\text{C})$: (a) stress space (q, p') ; (b) deviator stress versus shear strain (q, ϵ_q) .

Figure 10. ETVP modeling of a CI $\bar{U}$ test for NC soil at a strain rate of 15.0%/day and $(\psi_1 : T_1) = (0.006 : 28.0^\circ\text{C})$: (a) q, p' ; (b) V, p' ; (c) $\dot{\epsilon}_p^{vp}, p'_m$; (d) q, ϵ_q ; (e) u, ϵ_q : $\dot{\epsilon}_p^{vp}$ is viscoplastic volumetric strain rate.

Figure 11. ETVP modeling of a CI $\bar{U}$ test for NC soil at a strain rate of 0.15%/day and $(\psi_1 : T_1) = (0.006 : 28.0^\circ\text{C})$: (a) q, p' ; (b) V, p' ; (c) $\dot{\epsilon}_p^{vp}, p'_m$; (d) q, ϵ_q ; (e) u, ϵ_q : $\dot{\epsilon}_p^{vp}$ is viscoplastic volumetric strain rate.

Figure 12. ETVP modeling of a CI $\bar{U}$ test for LOC soil at a strain rate of 15.0%/day and $(\psi_1 : T_1) = (0.006 : 28.0^\circ\text{C})$: (a) q, p' ; (b) V, p' ; (c) $\dot{\epsilon}_p^{vp}, p'_m$; (d) q, ϵ_q ; (e) u, ϵ_q : $\dot{\epsilon}_p^{vp}$ is viscoplastic volumetric strain rate.

Figure 13. ETVP modeling of a CI $\bar{U}$ test for LOC soil at a strain rate of 0.15%/day and $(\psi_1 : T_1) = (0.006 : 28.0^\circ\text{C})$: (a) q, p' ; (b) V, p' ; (c) $\dot{\epsilon}_p^{vp}, p'_m$; (d) q, ϵ_q ; (e) u, ϵ_q : $\dot{\epsilon}_p^{vp}$ is viscoplastic volumetric strain rate.

Figure 14. ETVP modeling of a CID test for LOC soil at a strain rate of 0.15%/day and $(\psi_1 : T_1) = (0.006:28.0^\circ\text{C})$: (a) q, p' ; (b) V, p' ; (c) $\dot{\epsilon}_p^{vp}, p'_m$; (d) q, ϵ_q ; (e) $V, \epsilon_q : \dot{\epsilon}_p^{vp}$ is viscoplastic volumetric strain rate.

Figure 15. Simulated results from CI $\bar{U}$ -NC test at different temperatures; consolidation pressure $p'_i = p'_c = 50$ kPa. (a) peak deviator stress q_p , (b) increase in pore water pressure change u at q_p .

Figure 16. Simulated results from CI $\bar{U}$ -LOC test at different temperatures, preconsolidation pressure $p'_i = 50$ kPa; consolidation pressure $p'_i = 30$ kPa, (OCR = 1.67). (a) peak deviator stress q_p , (b) increase in pore water pressure change u at q_p .

Figure 17. ETVP modeling of compression of overconsolidated specimens with constant κ and temperatures of 28°C, 65°C, and 100°C. The creep rate coefficient ψ is defined by an exponential relationship for $\psi = \psi(T)$, with $(\psi_1 : T_1) = (0.006:28.0^\circ\text{C})$. Preconsolidation pressures, unload-reload lines and normal consolidation lines all appear to vary with temperature.

Figure 18. Temperature T vs. volume strain ϵ_v during heating at constant isotropic pressure p'_i for various overconsolidation ratios (OCRs). (a) from the TEP model by Graham et al. (2001), (b) from the ETVP model with the creep rate coefficient ψ defined by an exponential relationship for $\psi = \psi(T)$, with $(\psi_1 : T_1) = (0.006:28.0^\circ\text{C})$.

Figure 1. Definition of ‘instant’ and ‘delayed’ compression compared with ‘primary’ and ‘secondary’ compression (Bjerrum 1967, with permission).

Figure 2. Differences in compression behaviour with (a) changes in strain rate (Sällfors 1975, with permission), and (b) changes in temperature (Eriksson 1989, with permission).

Figure 3. Compression space $V/\ln(p')$ showing a point a_0 proceeding to a_1 after time t (Kelln et al. 2008).

Figure 4. Admissible development of viscoplastic volumetric strains with constant stress for a stress state which resides outside a current yield locus with size defined by p'_o that does not fall on the isotropic normal compression line p'_n . The locus of p'_o in the V, p' compression plane forms the viscoplastic limit line (vpl) parallel with the isotropic normal compression line (Kelln et al. 2008).

Figure 5. Computed and measured horizontal displacements beneath toe of an embankment on soft organic silty clay at Limavady, Northern Ireland (Kelln et al. 2009).

Figure 6. Peak strength (in this case, critical state) data from triaxial compression tests on normally consolidated illite at 28°C, 65°C, and 100°C: (a) strength envelope, (b) normal compression lines (Graham et al. 2001).

Figure 7. Representation of $V, \log p'$ compression space in Tanaka's thermoelastic-plastic (TEP) model, assuming $\kappa = \kappa(T)$: (Graham et al. 2001).

Figure 8. Linear and exponential relationships for the creep rate coefficient ψ versus temperature anchored at a temperature of 28°C, (301.1K) for given values of ψ .

Figure 9. ETVP modeling of a CIU test for NC soil at a strain rate of 15.0%/day and anchored at $(\psi_1:T_1) = (0.006:28.0^\circ\text{C})$: (a) stress space (q,p'); (b) deviator stress versus shear strain (q,ϵ_q).

Figure 10. ETVP modeling of a CIU test for NC soil at a strain rate of 15.0%/day and $(\psi_1 : T_1) = (0.006 : 28.0^\circ\text{C})$: (a) q, p' ; (b) V, p' ; (c) $\dot{\epsilon}_p^{vp}, p'_m$; (d) q, ϵ_q ; (e) u, ϵ_q ; $\dot{\epsilon}_p^{vp}$ is viscoplastic volumetric strain rate.

Figure 11. ETVP modeling of a CIU test for NC soil at a strain rate of 0.15%/day and $(\psi_1 : T_1) = (0.006 : 28.0^\circ\text{C})$: (a) q, p' ; (b) V, p' ; (c) $\dot{\epsilon}_p^{vp}, p'_m$; (d) q, ϵ_q ; (e) u, ϵ_q ; $\dot{\epsilon}_p^{vp}$ is viscoplastic volumetric strain rate.

Figure 12. ETVP modeling of a CIU test for LOC soil at a strain rate of 15.0%/day and $(\psi_1 : T_1) = (0.006 : 28.0^\circ\text{C})$: (a) q, p' , (b) V, p' , (c) $\dot{\epsilon}_p^{vp}, p'_m$, (d) q, ϵ_q , (e) u, ϵ_q ; $\dot{\epsilon}_p^{vp}$ is viscoplastic volumetric strain rate.

Figure 13. ETVP modeling of a CIU test for LOC soil at a strain rate of 0.15%/day and $(\psi_1:T_1) = (0.006:28.0^\circ\text{C})$: (a) q, p' ; (b) V, p' ; (c) $\dot{\epsilon}_p^{vp}, p'_m$; (d) q, ϵ_q ; (e) u, ϵ_q ; $\dot{\epsilon}_p^{vp}$ is viscoplastic volumetric strain rate.

Figure 14. ETVP modeling of a CID test for LOC soil at a strain rate of 0.15%/day and $(\psi_1 : T_1) = (0.006 : 28.0^\circ\text{C})$: (a) q, p' ; (b) V, p' ; (c) $\dot{\epsilon}_p^{vp}, p'_m$; (d) q, ϵ_q ; (e) V, ϵ_q : $\dot{\epsilon}_p^{vp}$ is viscoplastic volumetric strain rate.

Figure 15. Simulated results from CIU-NC test at different temperatures; consolidation pressure $p'_i = p'_c = 50$ kPa. (a) peak deviator stress q_p , (b) increase in pore water pressure change u at q_p .

Figure 16. Simulated results from CIU-LOC test at different temperatures; preconsolidation pressure $p'_i = 50$ kPa; consolidation pressure $p'_i = 30$ kPa, (OCR = 1.67). (a) peak deviator stress q_p , (b) increase in pore water pressure change u at q_p .

Figure 17. ETVP modeling of compression of overconsolidated specimens with constant κ and temperatures of 28°C, 65°C, and 100°C, with $(\psi_1 : T_1) = (0.006 : 28.0^\circ\text{C})$. Preconsolidation pressures, unload-reload lines and normal consolidation lines all 'appear' to vary with temperature.

Figure 18. Temperature T vs. volume strain ε_v during heating at constant isotropic pressure p'_i for various overconsolidation ratios (OCRs). (a) from the TEP model by Graham et al. (2001), (b) from the ETVP model with the creep rate coefficient ψ defined by an exponential relationship for $\psi = \psi(T)$, with $(\psi_1: T_1) = (0.006: 28.0^{\circ}\text{C})$.

List of Tables

Table 1. Soil parameters used to simulate CI $\bar{U}$ and CID triaxial compression tests.

Table 2. Comparison of peak deviator stress $q_p = 2 \times s_u$, change in pore water pressure u_p , and pore water pressure parameter A_p in simulations of CI $\bar{U}$ tests for NC and LOC specimens with ‘low’ and ‘high’ potentials for viscous behaviour. ‘Low’ is represented by $\psi = 0.006$, $T = 28^\circ\text{C}$, $\dot{\epsilon}_1 = 15\%/ \text{day}$, and ‘high’ by $\psi = 0.006$, $T = 100^\circ\text{C}$, $\dot{\epsilon}_1 = 0.15\%/ \text{day}$.

Table 1. Soil parameters used to simulate CI $\bar{U}$ and CID triaxial compression tests.

Soil Parameter	Symbol	Value
Slope of the Critical State Line (CSL)	M	1.00
Slope of the Normal Consolidation Line (NCL)	λ	0.25
Slope of the unload-reload line	κ	0.05
Creep rate coefficient ¹	ψ_1	0.006
Initial temperature ¹	T_1	28°C
Material constant for exponential $\psi = \psi(T)$	Ω	0.015/°C
Origin of the normal consolidation line	N	3.00
Origin of the viscoplastic line (vpl)	Z	1.00
Curve-fitting parameter	t_0	1.00 day
Poisson’s ratio	ν'	0.30

¹ Values correspond to the anchor point for the function $\psi = \psi(T)$.
Values of $\psi_1 = 0.002$ and 0.10 were also used in sensitivity analyses.

Table 2. Comparison of peak deviator stress $q_p = 2 \times s_u$, change in pore water pressure u_p , and pore water pressure parameter A_p in simulations of CI $\bar{U}$ tests for NC and LOC specimens with 'low' and 'high' potentials for viscous behaviour. 'Low' is represented by $\psi = 0.006$, $T = 28^\circ\text{C}$, $\dot{\epsilon}_1 = 15\%/ \text{day}$, and 'high' by $\psi = 0.006$, $T = 100^\circ\text{C}$, $\dot{\epsilon}_1 = 0.15\%/ \text{day}$.

Conditions for simulation:	28°C, 15%/day	100°C, 0.15%/day	Percent change	MCC
<u>Normally consolidated (NC)¹</u>				
Peak deviator stress q_p - kPa	29.1	21.5	-26.1	28.7
Pore water pressure u_p - kPa	27.9	31.2	+11.8	30.8
Pore pressure parameter A_p	0.96	1.45	+51.0	1.07
<u>Overconsolidated (LOC)²</u>				
Peak deviator stress q_p - kPa	26.3	19.5	-25.9	25.9
Pore water pressure u_p - kPa	10.0	13.1	+31.0	12.7
Pore pressure parameter A_p	0.38	0.67	+76.3	0.49

¹ Consolidation pressure $p'_i = p'_c = 50$ kPa

² Preconsolidation pressure $p'_c = 50$ kPa, consolidation pressure $p'_i = 30$ kPa, (OCR = 1.67).