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<th><strong>Journal:</strong></th>
<th><em>Canadian Geotechnical Journal</em></th>
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<tr>
<td><strong>Manuscript ID:</strong></td>
<td>cgj-2016-0015.R2</td>
</tr>
<tr>
<td><strong>Manuscript Type:</strong></td>
<td>Article</td>
</tr>
<tr>
<td><strong>Date Submitted by the Author:</strong></td>
<td>09-May-2016</td>
</tr>
<tr>
<td><strong>Complete List of Authors:</strong></td>
<td>Moon, Sung-Woo; National University of Singapore, Civil and Environmental Engineering Ku, Taeseo; National University of Singapore, Dept. of Civil &amp; Environmental Engineering</td>
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<td><strong>Keyword:</strong></td>
<td>Plasticity index, Void ratio, Unit weight, Shear wave velocity, Stress normalization</td>
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Development of global correlation models between in-situ stress-normalized shear wave velocity and soil unit weight for plastic soils

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ABSTRACT

Shear wave velocity \( V_s \) in geo-materials is strongly dependent on factors such as stress state, void ratio, and soil structure. Stress-dependency and void-ratio dependency can be represented by the equations:

\[
\frac{V_s}{\rho_g} = a \cdot (\sigma' \cdot e)^b
\]

and

\[
\frac{V_s}{\rho_g} = a \cdot (e)^b
\]

respectively. To consider the effect of soil disturbance and stress relief in geo-materials, shear wave velocity is often required to be normalized by adopting the site-specific model parameters \( \beta \) or \( b \).

Based on a special in-situ database compiled from 156 well-documented test sites that include various geo-materials, this study presents: 1) the apparent relationships of the model parameters \( \alpha \) and \( \beta \) for all soil and rock materials as well as \( a \) and \( b \) for all soil materials, 2) new global correlations between soil unit weight and two types of stress-normalized shear wave velocity \( V_{s1} \) and \( V_{sn} \), instead of the conventional \( V_s \)-soil unit weight relationship for clays, and 3) the best-fitted multi-regression models between soil unit weight and site-specifically normalized shear wave velocity as well as the plasticity index for plastic soils. Moreover, this study presents the importance of site-specific stress normalization \( V_{sn} \) in creating a better correlation model. The proposed relationships offer first order assessments of soil unit weight within the ranges of available data, which are also approximately guided by a hyperbolic unit weight model with depth.

Keywords: Shear wave velocity; Stress normalization; Plasticity index; Void ratio; Unit weight
1 Introduction

The initial shear modulus \( (G_0) \) at very small strains \( (\gamma_s < 10^{-6}) \) is a critical parameter of geomaterials that allows researchers to analyze geotechnical engineering problems under static and dynamic loading. Based on elastic theory using shear wave velocity\( (V_s) \), the initial shear modulus can be determined as: \( G_0 = \rho \cdot V_s^2 \), where \( \rho \) is the bulk soil density. The calculated value of \( G_0 \) is applied not only for predicting the dynamic soil responses of geo-materials, but also for designing and analyzing soil-structure interactions as well as conducting liquefaction potential assessments.

Several techniques have been employed in the laboratory and in the field to measure shear wave velocity. In-situ \( V_s \) measurements have been widely conducted using various invasive (e.g., downhole test, crosshole test) and non-invasive (e.g. surface wave survey, reflection and refraction test) field geophysics methods. In addition, laboratory testing methods such as the bender element test, torsional shear, and resonant column tests have been commonly used to obtain shear wave velocity based on retrieved samples (Clayton 2011; Kim et al. 2013; Stokoe et al. 2004).

Researchers need to carefully apply the in-situ \( V_s \) measurements to determine the reference \( G_0 \) values because of potential uncertainty/sensitivity and scattered outliers, which can exist due to site conditions and test methods. Nonetheless, as a design input parameter for geotechnical engineering problems, in-situ \( V_s \) is generally preferred to laboratory \( V_s \) because the in-situ \( V_s \) can be readily and economically determined with minimal soil disturbance. Moreover, site-specific \( G_0 \) values obtained from in-situ \( V_s \) measurements are more reliable when compared to those
obtained from laboratory testing because typically the lab $V_s$ represents lower $G_0$ values due to sample disturbance, the loss of aging effect and other factors (Ghionna and Jamiolkowski 1991; Ku and Mayne 2014; Stokoe and Santamarina 2000; Tatsuoka and Shibuya 1992).

In several studies, the in-situ $V_s$ measurements have been used to construct empirical correlations with the engineering properties and parameters of soils (e.g., soil unit weight, $\gamma_t$; peak friction angle, $\phi_p$; undrained shear strength, $s_u$; and lateral stress coefficient, $K_0$) (Ku and Mayne 2013, 2015; Levesques et al. 2007; Mayne 2007b; Uzielli et al. 2013). Using a special globally compiled database, this study describes interesting characteristics of $V_s$ in geomaterials related to stress- and void-dependent model parameters. Moreover, by considering the strong stress dependency of $V_s$, this paper also examines new global empirical relationships between soil unit weights and in-situ site-specific stress-normalized shear wave velocity (i.e., to develop more robust stress-independent correlation models), instead of conventional $V_s$–$\gamma_t$ models.

2 Previous Studies

2.1 Shear wave velocity – stress relationship

Previous studies have illustrated the influence of effective confining stress ($\sigma'_c$), void ratio ($e$), soil structure and fabric, and other factors on the magnitude of $V_s$ (Fumal and Tinsley 1985; Fumal 1978; Hardin and Black 1968; Roseler 1979; Santamarina et al. 2001; Sully and Campanella 1995; Sykora and Stokoe 1983; Yan and Byrne 1990). Among the various relationships between $V_s$ and suggested factors, the two simplest forms (1-1 and 1-2) for illustrating stress and void dependency of $V_s$ can be shown as follows:
\[ V_s = \alpha \cdot (\sigma'_c / 1 \text{kPa})^\beta \]  \hspace{1cm} (1-1)

\[ V_s = a \cdot (e)^b \]  \hspace{1cm} (1-2)

where the coefficients \( \alpha \) (m/s) and \( a \) (m/s) are material constants, and the exponents \( \beta \) and \( b \) represent the sensitivity of stress and the void dependent effect, respectively. As indicated in Eq (1-2), \( V_s \) decreases with an increasing void ratio that is independent of relative grain size, gradation and relative density (Fumal and Tinsley 1985; Fumal 1978; Hardin and Black 1968; Sykora 1987). Santamarina et al. (2001) described the \( \alpha \) and \( \beta \) as experimentally and site-specifically determined. The constant \( \alpha \) is not only inversely proportional to the exponent \( \beta \) (i.e., \( \beta \approx 0.36 - \alpha / 700 \)), but also estimated by contact behavior, packing type, material properties, and fabric changes. Theoretical \( \beta \) values have been recommended for various contact effects: 1) 0 for cemented soil; 2) 1/6 for Hertzian contacts (elastic spherical particles); 3) 1/4 for cone-to-plane contacts (rough or angular particles) and spherical particles with contact yield; and 4) 3/4 for contacts governed by Coulombian forces. In addition, particle characteristics such as particle shape and inter-particle friction have an influence on the fitting parameters. For instance, increasing particle irregularity leads to lower \( \alpha \) and higher \( \beta \) values (Cho et al. 2006; Otsubo et al. 2015). On the other hand, Ku et al. (2011) described that the coefficient \( \alpha \) and exponent \( \beta \), based on in-situ measurements, were remarkably dependent on site-specific conditions, which ranged widely for both constants.

Interestingly, the in-situ measurements of \( V_s \) have been normalized with respect to effective overburden pressure \( (\sigma'_\text{vo}) \) and a constant exponent of 0.25 at the reference stress of 1 atmospheric pressure (i.e., 101.325 kPa) for evaluating the liquefaction resistance of soils (Kayen...
et al. 2013). Conceptually, however, a site-specific exponent (n) depending on the actual
geostatic stress condition at each site can be employed for the normalization of \( V_s \) instead of the
constant exponent of 0.25. In this study, the stress-normalized shear wave velocity (\( V_{s1} \)) and the
site-specific stress-normalized shear wave velocity (\( V_{sn} \)) of various geo-materials are
respectively calculated from the following equations:

\[
V_{s1} = \frac{V_s (m/s)}{(\sigma'_{vo}/\sigma_{atm})^{0.25}} \tag{2-1}
\]

\[
V_{sn} = \frac{V_s (m/s)}{(\sigma'_{vo}/\sigma_{atm})^n} \tag{2-2}
\]

As noted in Eq. (2-1), the exponent 0.25 has been commonly determined as an empirical value
based on laboratory tests with clean silica sands (Stokoe et al. 1985; Yu and Richart 1984).
However, the exponent \( n \) in Eq. (2-2) can be estimated as various values because it is often
significantly dependent on site-specific conditions, as noted by Ku et al. (2011).

2.2 Correlations between Unit weight (\( \gamma_t \)) and Geotechnical and Geophysical Test
Results

A number of empirical correlations between unit weight and geotechnical and geophysical test
results have been developed by several studies, listed in Table 1. Marchetti and Crapps (1981)
and Schmertmann (1986) suggested a DMT based general chart and approximate diagram for
clays, silts, and sands. Since CPT based unit weight estimation using the Soil Behavioral Type
(SBT) classification system developed by Lunne et al. (1997) did not completely cover various
soil types, several CPT based empirical relations were developed using statistical regression
analyses (Mayne 2007b, 2014; Mayne and Peuchen 2013; Mayne et al. 2010; Robertson and
Cabal 2010; Singh and Chung 2013).

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Burns and Mayne (1996) suggested a fundamental relationship to estimate soil unit weight using $V_s$. This relationship strongly relies on not only void ratio and effective stress state but also on other elements such as soil structure, cementation, and aging, and it is based on a database accumulated from geotechnical and geophysical testing (Mayne 2001; Mayne 2007a). Table 1 also lists two compression wave velocity ($V_p$) based equations which are applicable for different soil types (Tezcan et al. 2009). Thus, equations listed in Table 1 offer possible linkages between CPT/DMT and soil unit weight, as well as between $V_s$ and soil unit weight.

Kim et al. (2001) performed free-free resonant column tests with reconstituted samples which are classified as SP-SM from the Hoengsung road construction site in Korea. They then described the typical variations in $V_s$ with density at different confining pressures. Based on the test results, it was reported that the $V_s$ varied significantly with both density and confinement. The $V_s$ was normalized by a confining stress that adopts an empirical exponent value, 0.25, which developed the correlation between $V_s$ and density while minimizing the effect of confinement. For example, Figure 1 illustrates the variation in stress-normalized $V_s$ with density.

Density can be evaluated using the developed relationship between stress-normalized $V_s$ and dry unit weight ($\gamma_d$). Although the relationship is limited only to the test site, the proposed methodology seems reasonable because (1) soil unit weight is directly related to void ratio and (2) $V_s$ strongly depends on confining stress and void ratio. Similarly, it indicates a feasibility of correlation between total unit weight and $V_s$. This study attempts to expand the site-specific approach and to eventually develop new global correlations based on the extensive collected in-situ database.
3 Compiled Database

In this study, we used an extensive database (Mayne et al. 2009) that includes data from 156 well-documented test sites consisting 61 clay sites, 6 fissured and calcareous sites, 8 silt sites, 35 sand sites, 32 rock sites, and 14 other sites (e.g., gravel, peat, diatom mudstone). The ranges of compiled engineering properties of soils such as plasticity index (PI), void ratio ($e$), and total unit weight ($\gamma_t$) are summarized in Table 2.

In general, the dynamic properties ($V_s$ and damping ratio) of soils and geomaterials are frequency dependent at lower shear strain levels (Bate et al. 2013; Qiu and Fox 2008; Santamarina et al. 2001). For instance, the Kramers – Kronig relationship shows that the inherent relationship between shear modulus and damping ratio is not independent with respect to frequency in viscoelastic materials (Kronig 1926). In laboratory testing, the $V_s$ values obtained from different sources (e.g., resonant column and bender element) that have the various ranges of frequency are slightly different and the effect of the frequency should be considered. However, in this study, the effect of the frequency is ignored because in-situ $V_s$ via field geophysics (e.g., very small strain level), can be generally assumed to be frequency independent, except for surface wave techniques (Foti et al. 2015).

3.1 Validation of database from analytical relationship

Mayne et al. (2009) described the trend for total unit weight ($\gamma_t$) in terms of shear wave velocity ($V_s$) and depth ($z$) for a wide range of non-cemented particulate geo-materials including clays, silts, sands, gravels, and mixed soils. Using statistical regression analyses in the database (total
number of data, \( N = 1018 \) except for non-compliant geo-materials such as calcareous sands, carbonates clays, and diatomaceous mudstone, the possible relationship was expressed:

\[
y_t(kN/m^3) = 8.63 \log(V_s (m/s)) - 1.18 \log(z (m)) - 0.53
\]  

(3)

Eq. (3) indicates that the magnitude of total unit weight depends on the effect of \( V_s \) and depth for non-cemented particulate soil materials. Thus, a measured \( V_s \) at a known depth makes it possible for us to profile the \( y_t \).

Figure 2 presents a trend between total unit weight and void ratio for all geo-materials. We used this trend to validate the database by applying the analytical relationship between total unit weight, specific gravity and void ratio. Generally, the trend follows the calculated analytical relationship based on assumed specific gravity (\( G_s = 2.65 \)) and water unit weight (\( \gamma_w = 9.81 \) kN/m\(^3\)) corresponding to a wide range of void ratios \( (e = 0 - 5) \). However, some scattered data are observed in some sands and weathered rocks. This variance is due to a combination of various causes such as approximate \( G_s \) assumption, measurement sensitivity, and uncertainty related to saturated soil condition or other issues.

### 3.2 Validation of database from hyperbolic unit weight model

Generally, the engineering properties of soils that are used as input parameters to access and solve geotechnical design problems are obtained from: (1) direct measurements in the laboratory and field, and (2) theoretical and empirical approaches. The unit weight of soil (\( y_t \)) is a fundamental geotechnical parameter that enables the calculation of total vertical stress (\( \sigma_{v0} \)) and effective overburden stress (\( \sigma'_{v0} \)) in addition to other subsequent parameters.
Figure 3 shows a trend between in-situ total unit weight measurements and depth for clays and sands \((n = 939)\). The reported total unit weight values have ranges from 11.2 kN/m\(^3\) to 22.7 kN/m\(^3\) for clays and 14.9 kN/m\(^3\) to 21.2 kN/m\(^3\) for sands. The wide ranges of total unit weight at a shallow depth may have resulted from the degree of surface densification. They present a considerable scatter which is similar to the range of the total unit weight (clays: 11.2 kN/m\(^3\) - 23.1 kN/m\(^3\), sands: 13.2 kN/m\(^3\) - 23.2 kN/m\(^3\)) recommended by NAVFAC (1986). Considering the general guidelines by NAVFAC, the collected database for this study seems to have reasonable ranges. As shown in Figure 3, the ranges of total unit weight values slightly decrease from the surface to a 30m depth indicating a shallow zone, while there are consistent trends of total unit weight values increasing with depths below 30 m that can be explained by the effect of confining (or overburden) stress. Zekkos et al. (2006) proposed a hyperbolic relationship to represent the characteristic municipal solid waste (MSW) unit weight profile based on in-situ unit weight data and trends observed in large-scale laboratory tests. The hyperbolic MSW unit weight equation was suggested by a fitting process associated with near-surface in-place unit weight and depth. Based on the conceptual hyperbolic model, an apparent relationship between in-situ total unit weight and depth is investigated. The following equation illustrates the proposed hyperbolic model for total unit weight \((\gamma_t)\) of clays and sands as a function of depth \((z)\):

\[
\gamma_t = \gamma_{t,\text{max}} - \frac{\eta}{\left(1 + \frac{z}{\zeta}\right)^\tau}
\]

(5)

where \(\gamma_{t,\text{max}}\) = maximum in-situ total unit weight; \(z =\) depth; and \(\zeta, \eta\) and \(\tau\) = fitting parameters. While the first term in Eq. (5) represents the maximum boundary of the measured in-situ total unit weight...
unit weight for clays and sands, the second term indicates the effect of depth on the total unit weight below 30m which may describe the effect of confining stress. Because the in-situ total unit weight is generally obtained at a known depth in the field instead of a known confining stress, Eq. (5) is suitable to approximately estimate the total unit weight profile with depth. The model described by Eq. (5) is presented with the in-situ total unit weight measurements in Figure 3. The parameters, such as \( \zeta \), \( \eta \) and \( \tau \), are determined by the best-fitting model. In this study, the recommended fitting parameters for the two soil types are listed in Table 3. Interestingly, the fitting parameters \( \eta \) and \( \tau \) that are required to adjust the shape of the total unit weight versus depth curve are identical for clays and sands. The \( \gamma_{t,max} \) and \( \zeta \) parameter are determined based on the observed ranges of in-situ total unit weight measurements. In Figure 3, the hyperbolic models provide apparent lower bound limits with depth for total unit weight approximation. Overall, it is proven that the compiled database is within suitable ranges for this study to establish empirical correlations between soil unit weight and normalized shear wave velocity. In addition, the trend for the initially assumed \( \gamma_t \) of clays is proposed as the average of the lower limit boundary and the maximum boundary (Figure 3). Later in this paper, the proposed trend will be employed for estimating the total unit weight of clay based on normalized shear wave velocity.

4 Determination of Site Specific Parameters

4.1 Site specific example: Lilla Mellösa, Sweden

Several geotechnical and geophysical tests such as vane tests, SASW tests, and seismic cone tests in clay had been carried out at the Lilla Mellösa test field in Sweden (Cadling and Odenstad...
1950; Larsson 1977; Larsson and Eriksson 1989; Larsson and Mulabdic' 1991a, b; Svensson and Möller 2001; Wiesel 1975). The total database of \( N = 11 \) from these seismic cone tests performed by Larsson and Mulabdic' (1991b) is employed to develop the site-specific correlation. As mentioned earlier, since the exponent value in Eq. (2-2) is highly dependent on the site-specific conditions, the \( V_s \) expression should be examined site-specifically as an individual power function in terms of effective overburden stress and void ratio. Using the in-situ data, site-specific Eq. (6-1) and (6-2) are derived at this site, as shown in Figure 4. It is evident that \( V_s \) is related directly to the effective overburden stress and void ratio.

\[
V_s (m/s) = 2.82(\sigma'_{v0})^{0.910}, N = 11, R^2 = 0.99
\]  
(6-1)

\[
V_s (m/s) = 2055.4(e)^{-3.057}, N = 11, R^2 = 0.95
\]  
(6-2)

### 4.2 Site specific relationship (\( \alpha \) and \( \beta, a \) and \( b \))

As illustrated in Eq. (1-1) and (1-2) as well as in the section 4.1, the relationships of \( V_s - \sigma'_{v0} \) and \( V_s - e \) can be determined at each site. Effective confining pressure (\( \sigma'_{c} \)) in Eq. (1-1) has been acknowledged as 1) the mean normal stress (\( \sigma'_m \)); 2) the average stress (\( \sigma'_{avg} \)); and 3) the individual stress, which is considered to be a factor affecting the \( V_s \) in references (Kang et al. 2014b; Sully and Campanella 1995; Yan and Byrne 1990). In this study, the effective vertical stresses are used to investigate the relationship between alpha-beta fitting, because it is very difficult to measure actual in-situ horizontal effective stresses. Thus, the \( V_s - \sigma'_{v0} \) equation in this study should be cautiously compared to some other data even though the results are not significantly different from adopting other confining stresses (e.g., \( \sigma'_m, \sigma'_{avg} \)). After careful
examination of all individual relationships from the in-situ database, the fitted data of ‘α and β’ and ‘a and b’ are presented with trend lines in Figure 5 (e.g., one data point per each site). In Figure 5(a), the fitted α and β values of this study establish a log-linear relationship, Eq. (7), displaying an array of β values ranging mostly from 0 (dense sands) to 1 (soft clays) at corresponding α values. Several negative β values are plotted together to represent the corresponding unusual in-situ \( V_s \) profiles which decrease slightly with depth. Compared to other studies based on laboratory tests reported by Cha et al. (2014), Kang et al. (2014a), and Santamarina et al. (2001), the β values get larger until α = 40 but they have similar values at α = 40 – 100 due to a difference database being employed for the best-fitting (e.g., in-situ \( V_s - \sigma'_v \) model in this study). On the other hand, Figure 5(b) shows the best-fitted linear relationship between coefficient a and b for both clays and all soil materials (Eq. (8-1) and (8-2)). The sites that have incomplete datasets (e.g., missing void ratio) and statistically poor void ratio-dependent trends are not included. Nevertheless, it is noted in Figure 5(b) that the coefficient a is also inversely proportional to the exponent b term that represents the sensitivity of void ratio dependent behavior of \( V_s \) in soils:

\[
\beta = -0.168 \ln (\alpha) + 0.953, \quad \text{for all geo-materials} \quad (7)
\]

\[
b = -0.846 \ln (\alpha) + 3.534, \quad \text{for clays} \quad (8-1)
\]

\[
b = -0.796 \ln (\alpha) + 3.253, \quad \text{for soil materials (except for rock)} \quad (8-2)
\]
5 Simple Correlations for Total Unit Weight ($\gamma_t$) of Soil

5.1 Unit weight – Normalized $V_s$

In this study, the soil unit weight is empirically correlated with normalized in-situ shear wave velocity, so that the effect of confining stress can be minimized. Data for clay sites that are plotted in terms of normalized $V_s$ and $\gamma_t$ are shown in Figure 6. The following simple empirical equations are adopted as an initial approach for soil unit weight assessment using $V_s$. The equations use linear regression analysis for clays in terms of (a) $V_{s1}$ and (b) $V_{sn}$.

$$\gamma_t = 4.75 \ln (V_{s1}) - 6.73, \ N = 67, \ R^2 = 0.676, \ S.E.Y. = 1.480$$  \hspace{1cm} (9)

$$\gamma_t = 4.76 \ln (V_{sn}) - 7.24, \ N = 49, \ R^2 = 0.731, \ S.E.Y. = 1.356$$  \hspace{1cm} (10)

For the regressions, a representative mean value at each site is applied to both indicate the equivalent site weights and to avoid any unwanted sensitive errors. Figure 6(b) shows that regression analysis using $V_{sn}$ provides a greater $R^2$ of 0.731, when compared with the $R^2$ of 0.676 using $V_{s1}$ in Figure 6(a). It describes that an improved correlation coefficient can be achieved by adjusting the exponents of stress-normalized $V_s$ (Eq. 2). A site-specific exponent value (i.e., ‘$n$’ in Eq. 2-2) is preferable to merely adopting an approximate value of 0.25 (Eq. 2-1) to normalize $V_s$. However, the preference of $V_{s1}$ versus $V_{sn}$ is determined by examining the stress-dependency of $V_s$ on each site. $V_{s1}$ values might be still useful in some sites where site-specific exponent values cannot be provided for the calculation of site-specific $V_{sn}$ (e.g., sites which have low $R^2$ values indicating unclear stress-dependency).
5.2 Unit weight – Plasticity index (PI)

The plasticity index (PI), the difference between the liquid and plastic limits, is employed to classify soil types by measuring the range in water contents over which the soil remains in plastic states. Figure 7 shows the strong dependence of $\gamma_t$ upon the PI based on the special database (i.e. clay and clay tills), including both the PI and $\gamma_t$. In Figure 7, the initial trend of the total unit weight decreases with the plasticity index of the clay and clay tills, and the scatter may be due to the effect of effective overburden pressure ($\sigma'_v$). The general trend shown in Figure 7 is expressed ($N = 50; R^2 = 706; \text{S.E.Y.} = 0.075$) as follows:

$$\gamma_t (kN/m^3) = 28.88 \cdot PI^{-0.159}$$  \hspace{1cm} (11)

Based on the extensive database of 50 test sites, this newly developed expression presents a trend similar to the general observation (e.g., $\gamma_t$ decreases with increasing PI) that was reported by Mayne and Peuchen (2013).

6 Normalized $V_s$ – PI – Unit Weight Relationships

6.1 Normalized $V_s$ – Plasticity index (PI) or Void ratio ($e$)

Empirical relationships between the PI and the normalized undrained shear strength ratio ($s_u/\sigma'_v$) have been studied for several decades. Linear relationships between the PI and $s_u/\sigma'_v$ for normally and over-consolidated clays were suggested by Skempton (1957), Chandler (1988), and Worth and Houlsby (1985), and another empirical relationship for non-swelling clay was investigated by Dolinar (2009). On the other hand, Levesques et al. (2007) developed an
empirical relationship between $s_u$ and $V_s$ for intact clays based on the regression analysis of in-situ test results obtained from three sites: Saguenay Fjord, Eastern Canada, and the North Sea. Similarly, several empirical correlations between $s_u$ and $V_s$ have also been examined by various researchers (Dickenson 1994; Kulkarni et al. 2010; Long et al. 2013; Yun et al. 2006). Consequently, these linked empirical relationships between three terms such as $V_s$, $s_u$, and the PI, make it possible for $V_s$ to be applied towards an assessment of PI characteristics for clays.

As shown in Figure 8, the $V_{s1}$ and $V_{sn}$ are employed for examining the relationship with PI and $e$, respectively. The available set of in-situ measurements provides a total dataset of $N = 50$ and $N = 36$ for clay and clay tills [Figure 8(a, c)] and $N = 62$ and $N = 46$ for all clays [Figure 8(b, d)]. Some missing data were due to a lack of information on the measured PI and $e$ with respect to $V_{s1}$ and $V_{sn}$. From Figure 8(a, c), both $V_{s1}$ ($R^2=0.635$, S.E.Y. = 0.236) and $V_{sn}$ ($R^2=0.642$, S.E.Y. =0.251) are shown to exhibit comparable trends to the PI. Similarly, from Figure 8(b, d), both $V_{s1}$ ($R^2=0.626$, S.E.Y. =0.261) and $V_{sn}$ ($R^2=0.704$, S.E.Y. = 0.241) exhibit equivalent trends with void ratio $e$, although $V_{sn}$ provides better estimates. The empirical equations using normalized $V_s$ are expressed in terms of PI or void ratio ($e$):

$$V_{s1} = 623.54(PI)^{-0.427} \quad (12-1)$$

$$V_{sn} = 689.64(PI)^{-0.441} \quad (12-2)$$

$$V_{s1} = 172.10(e)^{-0.524} \quad (13-1)$$

$$V_{sn} = 187.96(e)^{-0.558} \quad (13-2)$$
From Figure 8, one can note that the normalized $V_s$ trends with PI or $e$ are expressed as logarithmic equations resulting from regression analyses. Thus, it is postulated that the normalized $V_s$ can be correlated with the PI as well as the unit weight.

6.2 Unit weight multi-regressions based on normalized $V_s$ and plasticity index (PI)

In terms of soil unit weight, higher order regressions are investigated using normalized $V_s$ and PI in an attempt to improve the fitting and statistics, including an increase in the coefficient of determination ($R^2$) and a decrease in the standard error of the dependent variable (S.E.Y.). Significant statistical trends relating total unit weight as a function of both the plasticity index (PI) and normalized shear velocities ($V_{s1}$ and $V_{sn}$) are observed in Figure 9 and are expressed as:

$$\gamma_t = 11.27(V_{s1})^{0.147}(PI)^{-0.096} \quad (14)$$

$$\gamma_t = 7.91(V_{sn})^{0.194}(PI)^{-0.068} \quad (15)$$

Eq. (14 and 15) present improved $\gamma_t$ relationships for clay and clay tills from the results considered, as illustrated by Figure 9. Also, the proposed expression ($N = 39; \ R^2 = 0.768; \ S.E.Y. = 0.069$) using $V_{sn}$, Eq. (15), is comparable with Eq. (14) ($N = 53; \ R^2 = 0.726; \ S.E.Y. = 0.074$) using $V_{s1}$. Based on the observed statistic information from the log-log regression, the regression model using $V_{sn}$ can be considered to be slightly more robust. Both normalized shear wave velocities are proven to have strong relationships with soil unit weight in spite of some scattered data.

Through multi-regression analysis, the total unit weight, $\gamma_t$ for clay and clay tills, are plotted versus: (1) $V_{s1}$, and (2) $V_{sn}$ in Figure 10. This figure also shows the calculated $\gamma_t$ lines.
corresponding to three different PIs based on the Eqs. (14) and (15). It is indicated that $\gamma_t$ increases with increasing normalized $V_s$ (i.e., $V_{s1}$ and $V_{sn}$) and decreases with increasing PI values, which exhibit three sets of data (i.e., PI = 5, 50, and 200). With respect to the multi-regression lines for PI = 50 covering a wide range of normalized $V_s$, it is observed that $\gamma_t$ ranging from 15.0 kN/m³ to 20.0 kN/m³ corresponds to (1) $V_{s1}$ ranging from 90 m/s to 637 m/s and (2) $V_{sn}$ ranging from 107 m/s to 470 m/s. This confirms the construction of apparently reasonable relationships between $\gamma_t$ and normalized $V_s$ for PI = 50. Also, the relationships are applied for the other two ranges of PI, such as 5 and 200, which follow the form derived for PI = 50. These three lines are incorporated to examine the sensitivity of the $\gamma_t$ relationship for different values of PI and normalized $V_s$. The relationship between $V_{sn}$ and $\gamma_t$, Figure 10(b), shows a similar trend to that between $V_{s1}$ and $\gamma_t$, Figure 10(a). The regression using $V_{sn}$ shows more narrow ranges from PI = 5 to 200 indicating less sensitivity to PI. It appears that the general trend of collected in-situ data is more smoothly captured with $V_{sn}$ because $\gamma_t$ corresponding to $V_{sn}$ shows well-fitted narrow ranges indicating less uncertainty (more reasonable sensitivity) for a different PI as well as slightly improved statistics. The correlation models proposed in this study (e.g., relationships between $\gamma_t$ and stress-normalized $V_s$) are summarized in Table 4.

7 Application of the Proposed Models in Practice

Based on the proposed hyperbolic model and correlation models the following six steps are recommended for characterizing the total unit weight of the clay versus depth profile.

Step 1: Assumption of average $\gamma_t$ trend based on Figure 3 and Eq. (5) with parameters at Table 3, and calculation of $\sigma_{\psi 0}'$ with depths
Step 2a: Determination of $V_{s1}$ using a constant exponent of 0.25

Step 2b: Determination of $V_{sn}$ using a site-specific parameter ($\beta$) from the $V_s - \sigma'_v$ relationship as illustrated in section 4

Step 3: Estimation of $\gamma_t$ with depths using Eq (14) and/or Eq (15)

Step 4: Re-calculation of $\sigma'_v$ and $\beta$ using $\gamma_t$ with depths obtained from step 3

Step 5: Re-estimation of $\gamma_t$ with depths using Eq (14) and/or Eq (15)

Step 6: Iteration of Step 4 and Step 5 until convergence

The suitable number of iterations is determined based on the initial total unit weight ($\gamma_t$) assumed at step 1, but it was observed that two to three iterations (i.e., convergence tolerance: 1%) are usually enough to produce a quite reasonable estimate. In fact, the initially assumed $\gamma_t$ value is independent of the calculated final $\gamma_t$ values, but a poor initial assumption will require more iterations.

8 Case Studies

To describe the applications of the $\gamma_t$ expressions derived from the combination of PI and normalized $V_{s1}$ and $V_{sn}$, we discuss the results predicted from the well-known Bothkennar clay in the United Kingdom and the Fucino clay in Italy. A variety of site investigation programs have been performed over several decades for Bothkennar clay (Hight et al. 2003) and for Fucino
clay (Soccodato 2003). Figure 11(a, b) and Figure 12(a, b) present selected engineering properties of soils, such as PI and \( V_s \) for Bothkennar clay and Fucino clay, respectively. Initially, \( y_t \) based on the average trend in Figure 3 is assumed, and it is updated through three iterations for each clay.

Figure 11(c) and Figure 12(c) compare the measured \( y_t \) profiles with the estimated \( y_t \) profiles from Eqs. (14) and (15) derived in this study for Bothkennar clay and for Fucino clay. In addition, the predicted \( y_t \) profiles are compared with four empirical correlations using 1) PI from Mayne and Peuchen (2013); 2) \( V_s \) from Burns and Mayne (1996) and Mayne (2001); and 3) \( V_{s1} \) from Mayne (2007a), as listed in Table 1. In order to assess the relative differences between measured \( y_t \) profiles and predicted \( y_t \) profiles, each residual (i.e., defined as \( R = y_t(\text{measured}) - y_t(\text{predicted}) \)) was also compared in Figure 11(d) and Figure 12(d). Figure 11(c, d) and Figure 12(c, d) show that the predicted \( y_t \) profiles from the equations using \( V_{s1} \) and \( V_{sn} \) derived in this study provide more reasonable agreements with the measured \( y_t \) profiles than the estimated \( y_t \) profiles from the existing correlations derived in previous studies. For Bothkennar clay, the equations derived in this study appear to underestimate the \( y_t \) at depths below 14m (Figure 11[c, d]). This can be attributed to the existence of occasional shell fragments and silt/fine sand laminae at those regions, which causes increased density (Hight et al. 1992). For Fucino clay, the predicted \( y_t \) profiles from the equation using \( V_{sn} \) appear to match the measured \( y_t \) profiles in the depth range of 15m and 33m (Figure 12[c, d]).
9 Summary and Conclusion

In this study, the global correlation models between soil unit weight and normalized shear wave velocities ($V_{s1}$ and $V_{sn}$) were examined via a special in-situ database compiled from 156 well-documented geotechnical test sites including 61 clay sites, 6 fissured and calcareous sites, 8 silt sites, 35 sand sites, 32 rock sites, and 14 other sites.

Since it was found that the degree of stress and void ratio dependency of $V_s$ in geomaterials in situ is considerably site-specific, the parameters $\alpha$ and $\beta$ for stress-dependent model and $a$ and $b$ for void ratio-dependent model were determined site-specifically in the database: 

$$V_s = \alpha \cdot (\sigma'_c)^{\beta} \quad \text{and} \quad V_s = a \cdot (e)^b.$$ 

This study then described apparent inverse relationships between the model parameters.

It was also noted that $V_{sn}$ can be utilized to directly correlate with soil unit weight by statistical analyses in the database. Beyond various prior empirical approaches, global stress-independent expressions for soil unit weight were newly derived using multiple regression studies which considered $V_{s1}$ and $V_{sn}$ with PI. Based this suggested approach, some recommendations for engineering application within the compiled database are as follows:

1. For sands and clays, a good first-order evaluation of total unit weight ($\gamma_t$), specifically for a lower boundary, can be implemented directly from the hyperbolic relationship between total unit weight ($\gamma_t$) and depth ($z$) based on the comprehensive in-situ database.

2. For clay and clay tills, using the shear wave velocity normalized by site-specific coefficients, $V_{sn}$, rather than the shear wave velocity normalized by a traditional exponent
value of 0.25, $V_{s1}$, is recommended if possible, for total unit weight ($\gamma_t$), as the results seem more dependable.

The effect of normalized $V_s$ on engineering properties of soil via in-situ database is implicitly taken into account in terms of (1) PI, (2) void ratio, and (3) unit weight. Further investigations associated with other influencing factors such as age, cementation, and the complex interrelationships of different $V_s$ modes can be expected to be conducted with a larger collected database.

Acknowledgements

The Authors appreciates the financial support from the Singapore Ministry of Education (MOE). We also sincerely thank Professor Paul W. Mayne, Georgia Tech, for sharing in-situ data.

References


Sykora, D.W. 1987. Creation of a data base of seismic shear wave velocities for correlation analysis. US Army Engineer Waterways Experiment Station.

Sykora, D.W., and Stokoe, K.H., II. 1983. Correlations of In Situ Measurements in Sands With Shear Wave Velocity. The University of Texas at Austin.


Table 1. Empirical correlations between unit weight and geotechnical and geophysical testing results (revised after Singh and Chung (2013))

Table 2. Details of collected database: soil type, number of site and data, and range of soil properties (PI, $e$, $\gamma_t$, $V_s$, $V_{sn}$) used for correlation (data from Mayne et al. 2009)

Table 3. Hyperbolic fitting parameters for each soil type

Table 4. Developed relationship between $\gamma_t$ and normalized $V_s$
Table 1. Empirical correlations between unit weight and geotechnical and geophysical testing results (revised after Singh and Chung (2013))

<table>
<thead>
<tr>
<th>Empirical relationships</th>
<th>Based</th>
<th>References</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma_t (kN/m^3) = f(I_D, E_D)$</td>
<td>DMT</td>
<td>Marchetti &amp; Crapps (1981)</td>
</tr>
<tr>
<td>$\gamma_t / \gamma_w = 1.12(E_D/P_\sigma)^{0.1}(I_D)^{-0.05}$</td>
<td>DMT</td>
<td>Schmertmann (1986)</td>
</tr>
<tr>
<td>$\gamma_t = \text{function of SBT classification system}$</td>
<td>CPT</td>
<td>Lunne et al. (1997)</td>
</tr>
<tr>
<td>$\gamma_d = 1.89 \log(q_t) + 11.8$ where, $q_t = (q_t/P_\sigma)/(\sigma_{vo}'/P_\sigma)^{0.5}$</td>
<td>CPT</td>
<td>Mayne (2007b)</td>
</tr>
<tr>
<td>$\gamma_t / \gamma_w = [0.27 \log(R_f) + 0.36 \log(q_t/P_\sigma) + 1.236]G_s/2.65$</td>
<td>CPT</td>
<td>Robertson &amp; Cabal (2010)</td>
</tr>
<tr>
<td>$\gamma_t / \gamma_w = 0.27 \log(R_f) + 0.36 \log(q_t/P_\sigma) + 1.236$ where, $R_f = (f_s/q_t)100%$ (friction ratio)</td>
<td>CPT</td>
<td>Mayne (2010)</td>
</tr>
<tr>
<td>$\gamma_t (kN/m^3) = 11.46 + 0.33 \log(z)$ where, $z$= depth (m)</td>
<td>CPT</td>
<td>Singh &amp; Chung (2013)</td>
</tr>
<tr>
<td>$\gamma_t (kN/m^3) = 3.10 \log(f_s/kPa) + 0.70 \log(q_t/kPa)$</td>
<td>CPT</td>
<td>Mayne &amp; Peuchen (2013)</td>
</tr>
<tr>
<td>$\gamma_t (kN/m^3) = 1.95 \gamma_w (kN/m^3) [(\sigma_{vo}'/P_\sigma)(f_s/P_\sigma)]^{0.86}$</td>
<td>CPT</td>
<td>Mayne (2014)</td>
</tr>
<tr>
<td>$\gamma_t (kN/m^3) = 20.159 - 0.1632(l_c^4/q_{t1})^{0.01}$ where, $l_c$ = soil behavioral type index</td>
<td>CPT</td>
<td></td>
</tr>
<tr>
<td>$\gamma_t (kN/m^3) = 30.4 \cdot PL^{-0.174}$</td>
<td>CPT</td>
<td></td>
</tr>
<tr>
<td>$\gamma_t (kN/m^3) = 0.636(q_t kPa)^{0.72}(10 + mq_k N/m^3/8)$ where, $m_q$= cone resistance - depth ratio</td>
<td>CPT</td>
<td></td>
</tr>
<tr>
<td>$\gamma_t = 26 - 14/(1 + [0.5 \cdot \log(f_s) + 1])^2$</td>
<td>CPT</td>
<td></td>
</tr>
<tr>
<td>$\gamma_t \approx 12 + 1.5 \cdot \ln(f_s + 1)$</td>
<td>CPT</td>
<td></td>
</tr>
<tr>
<td>$\gamma_t (kN/m^3) = 6.87(V_s m/s)^{2.27} / (\sigma_{vo}' kPa)^{0.57}$</td>
<td>$V_s$</td>
<td>Burns &amp; Mayne (1996)</td>
</tr>
<tr>
<td>$\gamma_t (kN/m^3) = 8.32 \log(V_s m/s) - 1.61 \log(z \text{ meters})$ where, $V_s$= shear wave velocity</td>
<td>$V_s$</td>
<td>Mayne (2001)</td>
</tr>
<tr>
<td>$\gamma_t (kN/m^3) = 4.17 \ln(V_{s1} m/s) - 4.03$ where, $V_{s1} = V_s(m/s)/(\sigma_{vo}'/P_\sigma)^{0.25}$</td>
<td>$V_{s1}$</td>
<td>Mayne (2007a)</td>
</tr>
<tr>
<td>$\gamma_d (kN/m^3) = 0.0629 V_{sn}(m/s) + 8.75$ where, $V_{sn} = V_s(m/s)/(\sigma_{vo}'/P_\sigma)^n$</td>
<td>$V_{sn}$</td>
<td>Kim et al. (2001)</td>
</tr>
<tr>
<td>$\gamma_t (kN/m^3) = 3.2(V_p m/s)^{0.25}$ where, $V_p$= compression wave velocity</td>
<td>$V_p$</td>
<td>Tezcan et al. (2009)</td>
</tr>
<tr>
<td>$\gamma_t (kN/m^3) = \gamma_0 (kN/m^3) + 0.002 V_p (m/s)$ where, $\gamma_0$= reference unit weight</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Where, $P_a$ = atmospheric pressure and $z$ = depth (m), $G_t$ = specific Gravity, $q_t$ = tip resistance, $q_d$ = stress-normalized tip resistance, $f_s$ = sleeve friction, $\gamma_w$ = unit weight of water, $I_D$ = material index, $E_D$ = dilatometer modulus.
Table 2. Details of collected database: soil type, number of site and data, and range of soil properties ($\pi$, $e$, $\gamma_t$, $V_{s1}$, $V_{sn}$) used for correlation (data from Mayne et al. 2009)

<table>
<thead>
<tr>
<th>Soil Type</th>
<th>No. of Site</th>
<th>No. of Data</th>
<th>PI</th>
<th>$e$</th>
<th>$\gamma_t$ (kN/m$^3$)</th>
<th>$V_{s1}$ (m/s)</th>
<th>$V_{sn}$ (m/s)</th>
<th>Symbol</th>
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<tbody>
<tr>
<td>Intact Clay</td>
<td>61</td>
<td>698</td>
<td>0-250</td>
<td>0.40-6.75</td>
<td>11.2-22.7</td>
<td>35-406</td>
<td>39-438</td>
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<tr>
<td>Fissured Clay</td>
<td>3</td>
<td>21</td>
<td>12-55</td>
<td>0.43-0.84</td>
<td>18.8-21.3</td>
<td>178-313</td>
<td>187-306</td>
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<tr>
<td>Calcareous Clay</td>
<td>3</td>
<td>18</td>
<td>0-11</td>
<td>0.95-1.38</td>
<td>16.2-19.7</td>
<td>186-400</td>
<td>182-535</td>
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<tr>
<td>Silts</td>
<td>8</td>
<td>32</td>
<td>0-15</td>
<td>0.64-1.43</td>
<td>16.7-20.2</td>
<td>122-319</td>
<td>142-215</td>
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<tr>
<td>Sands</td>
<td>35</td>
<td>200</td>
<td>0-11</td>
<td>0.43-2.15</td>
<td>14.9-22.2</td>
<td>106-621</td>
<td>88-728</td>
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<tr>
<td>Peat</td>
<td>3</td>
<td>3</td>
<td>-</td>
<td>-</td>
<td>10.4-11.8</td>
<td>50-149</td>
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<tr>
<td>Gravels</td>
<td>7</td>
<td>43</td>
<td>-</td>
<td>0.27-0.70</td>
<td>19.6-22.5</td>
<td>120-366</td>
<td>263-280</td>
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<tr>
<td>Clay Till</td>
<td>3</td>
<td>16</td>
<td>0-11</td>
<td>0.19-0.56</td>
<td>20.1-24.0</td>
<td>188-611</td>
<td>242-645</td>
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<td>Intact Rock</td>
<td>19</td>
<td>131</td>
<td>-</td>
<td>0.03-0.71</td>
<td>19.2-26.0</td>
<td>-</td>
<td>-</td>
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<tr>
<td>Weathered Rock</td>
<td>13</td>
<td>51</td>
<td>-</td>
<td>0.03-1.13</td>
<td>16.7-26.0</td>
<td>418-606</td>
<td>343-402</td>
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<tr>
<td>Diatom Mudstone</td>
<td>1</td>
<td>8</td>
<td>-</td>
<td>-</td>
<td>12.9-14.4</td>
<td>140-309</td>
<td>154-232</td>
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</table>
Table 3. Hyperbolic fitting parameters for each soil type

<table>
<thead>
<tr>
<th>Soil type</th>
<th>$\gamma_{t,\text{max}}$ (kN/m$^3$)</th>
<th>$\zeta$ (m)</th>
<th>$\eta$ (kN/m$^3$)</th>
<th>$\tau$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Clay</td>
<td>22.65</td>
<td>85</td>
<td>11.5</td>
<td>3.0</td>
</tr>
<tr>
<td>Sand</td>
<td>21.15</td>
<td>85</td>
<td>7.5</td>
<td>3.0</td>
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</table>
Table 4. Developed relationship between $\gamma_t$ and normalized $V_s$

<table>
<thead>
<tr>
<th>Relationship between $\gamma_t$ and $V_s$</th>
<th>Soil Type</th>
<th>Best-fit relationship</th>
<th>No. Data</th>
<th>$R^2$</th>
<th>S.E.Y.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma_t - V_{s1}$</td>
<td>All clays</td>
<td>$\gamma_t = 4.75 \ln(V_{s1}) - 6.73$</td>
<td>67</td>
<td>0.676</td>
<td>1.480</td>
</tr>
<tr>
<td>$\gamma_t - V_{sn}$</td>
<td>All clays</td>
<td>$\gamma_t = 4.76 \ln(V_{sn}) - 7.24$</td>
<td>49</td>
<td>0.731</td>
<td>1.356</td>
</tr>
<tr>
<td>$\gamma_t - V_{s1}$</td>
<td>Clay and clay tills</td>
<td>$\gamma_t = 11.27(V_{s1})^{0.147}(PI)^{-0.096}$</td>
<td>53</td>
<td>0.726</td>
<td>0.074</td>
</tr>
<tr>
<td>$\gamma_t - V_{sn}$</td>
<td>Clay and clay tills</td>
<td>$\gamma_t = 7.91(V_{sn})^{0.194}(PI)^{-0.068}$</td>
<td>39</td>
<td>0.768</td>
<td>0.069</td>
</tr>
</tbody>
</table>
Figure Captions

Figure 1 Normalized shear wave velocity – dry density – confining pressure relationship (revised from Kim et al. (2001))

Figure 2 Trend between total unit weight ($\gamma_t$) and void ratio ($e$) for all geo-materials ($G_s$: specific gravity, $N = 932$) (data obtained from Mayne et al. (2009)).

Figure 3 Trend between total unit weight ($\gamma_t$) and depth ($z$) from in-situ tests for clays and sands ($N = 939$).

Figure 4 Site specific shear wave velocity trend at Lilla Mellösa in Sweden with (a) $\sigma_{v0}'$ and (b) void ratio ($e$).

Figure 5 Relationship for best-fit corresponding to geo-materials between (a) $\alpha$ and $\beta$, (b) $a$ and $b$.

Figure 6 Total unit weight trends with normalized shear wave velocity (a) $V_{s1}$, (b) $V_{sn}$.

Figure 7 Total unit weight trend with PI for clay and clay tills.

Figure 8 Apparent trends of normalized shear wave velocity with PI and void ratio for geo-materials: (a) $V_{s1}$ vs $PI$, (b) $V_{s1}$ vs $e$, (c) $V_{sn}$ vs $PI$, (d) $V_{sn}$ vs $e$.

Figure 9 Reference total unit weight versus empirically derived total unit weight using multiple regression as a function of PI and normalized shear wave velocity (a) $V_{s1}$, (b) $V_{sn}$.

Figure 10 Total unit weight versus normalized shear wave velocity with prediction lines from multiple regression analysis: (a) $V_{s1}$ and (b) $V_{sn}$.

Figure 11 Case study for Bothkennar clay (data from Hight et al. (2003)): (a) PI; (b) $V_s$; (c) $\gamma_t$ prediction; (d) residual.

Figure 12 Case study for Fucino clay (data from Soccodato (2003)): (a) PI; (b) $V_s$; (c) $\gamma_t$ prediction; (d) residual.
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Figure 4 Site specific shear wave velocity trend at Lilla Mellösa in Sweden with (a) $\sigma'_0$ and (b) void ratio ($e$).
Figure 5: Relationship for best-fit corresponding to geo-materials between (a) $\alpha$ and $\beta$, (b) $a$ and $b$. 

- **All Soil & Rock Materials:**
  - $\beta = -0.168 \ln(\alpha) + 0.9528$; $N = 93, r^2 = 0.924$
- **Clays:**
  - $b = -0.846 \ln(a) + 3.5335$; $N = 31, r^2 = 0.829$
- **All Soil Materials (Except for Rock):**
  - $b = -0.796 \ln(a) + 3.253$; $N = 38, r^2 = 0.701$
- **No Dependency**
Figure 6 Total unit weight trends with normalized shear wave velocity (a) $V_{s1}$, (b) $V_{sn}$.
Figure 7 Total unit weight trend with PI for clay and clay tills.
Figure 8 Apparent trends of normalized shear wave velocity with PI and void ratio for geomaterials: (a) $V_{s1}$ vs $PI$, (b) $V_{s1}$ vs $e$, (c) $V_{sn}$ vs $PI$, (d) $V_{sn}$ vs $e$. 

Clay and Clay tills:
- $V_{s1}(m/s) = 623.54 (PI)^{-0.427}$
  - $N = 50, r^2 = 0.635, S.E.Y. = 0.236$
- $V_{sn}(m/s) = 689.64 (PI)^{-0.441}$
  - $N = 36, r^2 = 0.642, S.E.Y. = 0.251$

All Clays:
- $V_{s1}(m/s) = 172.1 (e)^{-0.524}$
  - $N = 62, r^2 = 0.626, S.E.Y. = 0.261$
- $V_{sn}(m/s) = 187.96 (e)^{-0.558}$
  - $N = 46, r^2 = 0.704, S.E.Y. = 0.241
Figure 9 Reference total unit weight versus empirically derived total unit weight using multiple regression as a function of PI and normalized shear wave velocity (a) $V_{s1}$, (b) $V_{sn}$.
Figure 10 Total unit weight versus normalized shear wave velocity with prediction lines from multiple regression analysis: (a) $V_{s1}$ and (b) $V_{sn}$.

\[
\gamma_t (kN/m^3) = 11.27(V_{s1})^{0.147}(PI)^{0.096} \text{ with } V_{s1} \text{ (m/s)}
\]

\[
\gamma_t (kN/m^3) = 7.91(V_{sn})^{0.194}(PI)^{0.068} \text{ with } V_{sn} \text{ (m/s)}
\]
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