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Pipe-soil interaction model for current-induced pipeline instability on a sloping sandy seabed *

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Abstract

As the offshore exploitation moving to deeper waters, ocean currents would become more prevailing hydrodynamics on pipelines, and meanwhile the sloping seabed is always encountered. The prediction of lateral soil resistance is vital in evaluating the pipeline on-bottom stability. Unlike the previous pipe-soil interaction models mainly for horizontal seabed conditions, a pipe-soil interaction model for current-induced downslope and upslope instabilities is proposed by using limit equilibrium approach. The Coulomb's theory of passive earth pressure for the sloping seabed is incorporated in the derivation. The model verification with the existing full scale tests shows a good agreement between the experimental results and the predicted ones. Parametric study indicates that the effect of slope angle on the pipeline lateral soil resistance is significant in the examined range of the slope angle from -15° to 15° . The critical pipeline embedment and the corresponding passive-pressure decreases approximately linearly with increasing slope angle.

Key words: Submarine pipeline; On-bottom stability; Sandy seabed; Analytical study; Pipe-soil interaction; Sloping seabed

Introduction

Lateral soil resistance is one of the fundamental issues in submarine pipeline on-bottom stability design for the hydrodynamic loading conditions in offshore environments (Wagner et al. 1989; Det Norske Veritas 2010). The behavior of the pipeline on-bottom instability in ocean environments is a complex phenomenon, involving significant flow-soil-structure interaction. Unlike the conventional foundations of structures, on-bottom pipelines can tolerate moderate movements across the seabed without exceeding a limit state, except where they are constrained by wellheads, other connections or obstructions on the seabed (Randolph and Gourvenec 2011). As the oil and gas exploitation moving into deeper waters, ocean current becomes one of the prevailing hydrodynamic loads on submarine pipelines. Besides the usual steady current, a turbidity current fast-moving down a slope can incise and erode continental margins and even cause serious damage to engineering structures. The interaction of internal waves with the seabed is another significant source of near bed currents (Boczar-Karakiewicz et al. 1991). It is noted that the submarine pipelines are more preferred to be laid directly on the seabed (seldom buried artificially) in deeper waters. Meanwhile, the submarine slopes are always encountered, e.g. at the continental slopes in South China Sea (Liu et al. 2002). As such, an improved understanding of the mechanism on current-induced instability of unburied pipelines on a sloping seabed would be beneficial to offshore engineering practices.

When ocean currents are in perpendicular to the axis of a horizontal pipeline

40 which is partially-embedded in the sloping seabed with certain slope angle (α), the
41 flow-induced pipeline on-bottom instability can be regarded as a plane strain problem
42 (see Fig. 1). There normally exists a balance between hydrodynamic loads (including
43 drag force, F_{Du} , and lift force, F_L), the submerged weight of the pipeline, W_s , and the
44 soil resistance, F_{Ru} . If the soil lateral resistance to the pipeline could not balance the
45 hydrodynamic loads and the submerged weight, the pipeline would break out from its
46 original locations, i.e. the lateral on-bottom instability occurs. Thus, an accurate
47 prediction of the ultimate lateral soil resistance is vital for properly evaluating the
48 on-bottom stability of the pipeline partially-embedded on a sloping seabed.

49

50 **Fig. 1. Illustration of the current-induced pipeline lateral instability on a sloping**
51 **seabed: (a) Downslope instability; (b) Upslope instability**

52

53 The conventional interfacial frictional theory (i.e. Coulomb friction model) was
54 ever suggested to predict the lateral soil resistance of the pipeline (Lyons 1973).
55 Previous pipe-soil interaction tests (Wagner et al. 1989; Brennodden 1989; Gao et al.
56 2007, 2011) showed that the loading history that increased the pipe penetration led to
57 a notable increase of the lateral on-bottom stability. The soil berm ahead of the pipe
58 provides passive resistance, which governs the lateral pipe-soil interaction force
59 (White and Cheuk 2008; Youssef et al. 2013). Hence, the soil resistance is far more
60 complex than the simple interfacial friction that calculated using the conventional
61 Coulomb friction model. A literature review on physical modeling of pipeline

on-bottom stability can be referenced in [Gao et al. \(2012\)](#). The existing test data indicated that the lateral resistance was significantly dependent on pipe penetration and soil strength.

An empirical pipe-soil model by [Wagner et al. \(1989\)](#) has been adopted for the dynamic lateral stability analysis in the current DNV Recommended Practice for on-bottom stability design of submarine pipelines ([Det Norske Veritas 2010](#)). Their model was based on the results of a series of pipe-soil interaction tests. The lateral resistance (F_R) was estimated by the model including the following two components, i.e. a sliding-resistance component (F_{Rf}) plus a passive-pressure component (F_{Rp}):

$$(1) \quad F_R = \underbrace{\mu_0 (W_s - F_L)}_{F_{Rf}} + \underbrace{\beta_0 \gamma' A_{0.5}}_{F_{Rp}} \quad (\text{for a horizontally flat sandy seabed})$$

where μ_0 is the sliding resistance coefficient, which was set as 0.60 for the pipe on sands; W_s is the submerged weight of the pipe per unit length (in kN/m); F_L is the hydrodynamic lift force on the pipe per unit length (in kN/m); γ' is the effective (buoyant) unit weight of the sand (in kN/m³); $A_{0.5}$ is a characteristic area which can be calculated from the initial estimated penetration, i.e. one half of the vertical cross sectional area of the soil displaced by the partially-embedded pipe (in m²); β_0 is a dimensionless empirical coefficient for the soil passive pressure, which is relative to the sand density and the loading history. For the simple monotonic lateral loading, the values of β_0 were recommended empirically with a wide range, from “38” for sands with $\gamma' < 8.6$ kN/m³ to “79” for sand with $\gamma' > 9.6$ kN/m³. It should be noticed that a direct sum in the scalar form of the sliding-resistance and the passive-pressure components (see [eq. \(1\)](#)) was not appropriate for describing the

84 actual pipe-soil interactions. In the existing empirical lateral pipe-soil interaction
85 models (e.g. the aforementioned model (eq. (1)), and an energy-based pipe-soil
86 interaction model by Brennodden et al. (1989)), the ultimate lateral soil-resistance to
87 the partially-embedded pipeline has not been well understood.

88 Historically, plasticity theory has been used for calculating the lateral earth
89 pressure on conventional retaining walls, which is a central issue in the analysis of
90 retaining structures. In the plasticity analysis, a zone of soil is assumed to reach its
91 plastic equilibrium such that plastic collapse occurs. This plastic soil zone slips
92 relative to the rest of soil mass along the slip surface, where the peak soil strength is
93 assumed to be mobilized (Osman and Bolton 2004). The full range of soil strengths
94 can be expressed in terms of the variation of shearing resistance angle (ϕ) with
95 density and confining pressure (Bolton 1986). As is well-known, plasticity theory
96 can be employed for collapse load calculation, whereas elasticity theory is usually
97 used to predict strain or displacement. Limit equilibrium approach is efficient for
98 determination of passive pressure coefficients for retaining walls (Patki et al. 2015).
99 Numerical study by Potts and Fourie (1986) showed that the effect of Young's
100 modulus distribution on the overall stability of a conventional retaining wall
101 (characterized with passive or active pressure coefficients) appears to be negligible.

102 Force-resultant plasticity models for the combined vertical and horizontal
103 loading conditions have been successively developed and employed for simulating
104 the pipeline on-bottom responses (e.g., Zhang et al. 2002; Hodder and Cassidy 2010).
105 These numerical models were based on the plasticity theory and verified by series of

106 sideswipe tests of a partially embedded pipeline on calcareous sands. The behaviors
107 of the entire pipe foundation were encapsulated by relating the resultant forces to the
108 corresponding displacements of the pipeline.

109 The previous numerical and the empirical analyses were mainly for the
110 condition of horizontally flat seabed, which is typical for the shallow continental
111 shelf regions. As the offshore engineering practice moving to the deeper continental
112 slope regions, the influence of the seabed slope should be taken into consideration
113 for evaluating the ultimate lateral-resistance of the submarine pipelines. In the
114 existing theoretical investigations on the pipeline lateral stability, the influence of the
115 slope angle of the seabed has not been considered yet.

116 In this study, an improved analytical pipe-soil interaction model is developed on
117 the basis of the passive soil pressure theory to assess the lateral instability of
118 submarine pipelines on a sloping sandy seabed. The developed model is verified by
119 the existing experimental and numerical results. The effect of seabed slope angle on
120 the lateral on-bottom stability is further investigated.

121 **Critical Soil Resistance for a Partially-embedded Pipeline**

122 **Assumptions and application scopes**

123 For the pipeline-soil interaction system subject to ocean current loading, a proper
124 evaluation of the soil resistance is key to evaluate the pipeline on-bottom stability,
125 especially when a sloping seabed is encountered. If the hydrodynamic loads are large
126 enough to induce the pipeline instability, the consequence of the lateral pipeline
127 movement is to bring the neighboring soil of the sloping seabed from a quasi- K_0

state to a passive limiting equilibrium state. In this analytical investigation, in order to derive a reasonable analytical solution for evaluating the soil resistance to an unburied pipeline, the main assumptions and application scopes are discussed and listed as follows.

As the rigidity of a submarine pipeline is normally much larger than that of the soils, it would be reasonable to assume the pipeline as a rigid shallow foundation. In the offshore fields, the submarine pipeline diameter (D) normally ranges from several inches to around 40 inches (~ 1.0 m). The examined embedment-to-diameter ratio (e_0/D) is in the range of 0 to 0.5. Due to the constraints from the pipeline ends linking with the subsea well-heads and/or from the locking blocks, the anti-rolling condition is under consideration, i.e. the pipeline may move in parallel or normal to the seabed surface, but the free rolling is prohibited.

The hydrodynamics on the partially-embedded pipeline under the action of ocean currents include the drag force F_D (parallel to the seabed surface) and the lift force F_L (upward perpendicular to the seabed surface) (see Fig. 1), which can be calculated with the Morison equations (Morison et al. 1950), i.e.

$$(2a) \quad F_D = \frac{1}{2} C_D \rho_w D U^2$$

$$(2b) \quad F_L = \frac{1}{2} C_L \rho_w D U^2$$

where C_D and C_L are the drag and the lift force coefficient, respectively; ρ_w is the mass density of the water (in kg/m^3); D is the outer diameter of the submarine pipeline; U is the velocity of the ocean currents (in m/s). As recommended by Jones (1978), the effective hydrodynamic coefficients (C_D and C_L) for a pipeline resting on

the seabed ($e/D = 0$) can be determined with their correlations with the values of Reynolds number ($Re = UD/\nu$ is the ratio of inertia force to viscous force; ν is the kinematic viscosity of water (in m^2/s). $\nu \approx 1.5 \times 10^{-6} \text{ m}^2/\text{s}$ for water at 5°C). With Re increasing from 3.0×10^4 to 1.0×10^6 , both the drag coefficient C_D and the lift coefficient C_L decrease gradually to constant values with similar trends (also see [Gao et al. 2011](#)).

The above Morison equations with the modification of drag and lift coefficients by [Jones \(1978\)](#) may provide a convenient approach for the pipeline hydrodynamics calculation. Such a conventional calculation approach is semi-empirical, in which the force coefficients were determined from the tests. [Soedigdo et al. \(1999\)](#) proposed a more sophisticated analytical model (i.e. Wake II model) for predicting the near-wall pipeline hydrodynamics in waves, in which the wake velocity correction was derived based on a closed-form solution to the linearized Navier–Stokes model for oscillatory flow and the hydrodynamic forces coefficients were determined based on start-up effects. Note that in those models for hydrodynamic loads calculations, the penetration effect has not been taken into account. It was observed by [Jacobsen et al. \(1989\)](#) that while the pipeline partially penetrating into the seabed, the hydrodynamic loads are decreased gradually, noting that the lift coefficient is influenced slightly when the embedment-to-diameter ratio is less than 0.10. The recommended reduction factors due to pipeline penetration/embedment for the hydrodynamic loads can be referenced in [Det Norske Veritas \(2010\)](#).

For the current-induced pipeline on-bottom stability on the sloping seabed with

172 a slope angle (α), the following force equilibrium equations should be satisfied in
173 both directions of parallel (x) and perpendicular (y) to the seabed surface,
174 respectively (Fig. 1):

175 (3a)
$$F_R = F_D - W_s \sin \alpha \text{ (in } x \text{ direction)}$$

176 (3b)
$$F_C = W_s \cos \alpha - F_L \text{ (in } y \text{ direction)}$$

177 where F_C is the prop force of the seabed to the unburied pipeline, i.e. the net normal
178 load in between the pipeline and the underlying soil.

179 The sandy seabed is taken into account in this analytical investigation. Sand
180 sediments can be deposited at different rates, resulting in a range of initial densities
181 which influence subsequent behaviors (Potts and Zdravkovic 1999). As a shallow
182 foundation, the partially-embedded pipeline can be supposed as a retaining structure.
183 While losing lateral stability, the pipeline pushing the frontal sand ahead can be
184 regarded as a quasi-static process, where a fully drained condition is basically
185 satisfied in the shallow sand layer.

186 A two-dimensional (2-D) plane strain elasto-plastic Finite Element (FE) model
187 was recently proposed by Han (2012) to predict the pipeline-soil interaction behavior
188 on the sloping seabed. A series of FE analyses (Han, 2012) indicated that the plastic
189 failure zone developed in the proximity of the pipeline when losing lateral stability is
190 quite similar to that in the previous analyses on the retaining walls (Potts and
191 Zdravkovic 2001). The details for the typical numerical simulation can be seen in the
192 latter section for the model validation. This can also provide a reasonable
193 confirmation of the empirical pipe-soil interaction model based on the test

194 observations by [Wagner et al. \(1989\)](#), i.e. the total soil resistance includes the
195 sliding-friction and the passive-pressure components.

196 The submarine slopes are always encountered in the offshore pipeline
197 engineering, which are generally gentler than the typical slopes on land. In this study,
198 the influence of slope angle on the pipeline on-bottom instability is examined
199 analytically with the proposed model. Two typical on-bottom instabilities are
200 involved, i.e. (1) Type-I: downslope instability and (2) Type-II: upslope instability.
201 The effects of slope angle will be investigated in the later section.

202 Based on the aforementioned analyses and discussions, in the proposed
203 analytical model, the composite failure surface comprises a sliding-friction segment
204 and a passive-pressure segment. The passive pressure is to be calculated with the
205 well-known Coulomb's theory of passive earth pressure for the soil slopes at a
206 constant angle to the horizontal ([see Craig 2004; Chen and Liu 1990](#)). In this study,
207 the examined absolute values of the slope angle are in the range of $0\sim 15^{\circ}$, which
208 covers the common submarine in-situ conditions.

209 In this theoretical derivation, the plane-strain condition is under consideration,
210 i.e. the pipeline is aligned with the bathymetric contours of the sloping seabed, and
211 the current is flowing perpendicularly to the pipeline. For more general cases with
212 oblique flow and run-off elevation laying, the conditions would be three-dimensional
213 in nature and the axial flow-pipe-soil interaction effects would emerge, for which the
214 present theoretical solutions could not be extended directly and should be further
215 examined.

216 Derivation

217 As previously stated, the Coulomb's theory of passive earth pressure is incorporated
 218 in the present analytical model. The composite failure surface for the lateral pipe-soil
 219 interaction on a sloping seabed (see Fig. 2 and Fig. 3) includes a sliding-friction
 220 segment (denoted as "segment-DB") and a passive-pressure segment
 221 ("segment-BC"). Fig. 2 and Fig. 3 illustrate the geometry of failure mechanism and
 222 the force triangles for the downslope instability and those for the upslope instability,
 223 respectively. Along both segments (segment-DB and segment-BC), the shear
 224 strength of the soil is fully mobilized while the pipeline losing lateral stability.

225
 226 **Fig. 2. Downslope instability of a submarine pipeline: (a) Geometry of failure**
 227 **mechanism; (b) Triangle of the forces on the wedge-ABD (shaded area in Fig**
 228 **2(a))**

229 **Fig. 3. Upslope instability of a submarine pipeline: (a) Geometry of failure**
 230 **mechanism; (b) Triangle of the forces on the wedge-ABD (shaded area in Fig**
 231 **3(a))**

232
 233 Based on the Coulomb's theory of passive earth pressure, the shearing
 234 resistance on the segment-BC and the weight of the wedge-ABC would be balanced
 235 by the thrust force (E_1) on a virtual retaining wall-AB. The length of the virtual
 236 retaining wall-AB has the same value with the pipeline embedment (e_0). As
 237 illustrated in Fig. 2(a) and Fig. 3(a), the retaining wall-AB is supposed perpendicular

to the seabed surface, and the sliding-friction segment-DB is parallel to the seabed surface (i.e. perpendicular to the wall-AB).

Choosing the wedge-ABD (the shaded areas in Fig. 2(a) and Fig. 3(a)) as the analysis object, the main forces acting on the wedge-ABD at failure for these two types of instabilities include: (1) The passive earth pressure on the virtual retaining wall-AB, the total force of which, as stated above, is denoted as the thrust force E_1 ; (2) The sliding-friction force (E_2) on the segment-DB, with an inclination angle (φ) to the normal; (3) The submerged weight of the wedge-ABD; and (4) The total pipe-soil interfacial force (P). The details of the calculation for these forces are as follows.

The passive earth pressure E_1 can be calculated with Coulomb's theory of passive earth pressure for the soil surface slopes (see Craig 2004):

$$(4) \quad E_1 = \frac{1}{2} \gamma' (e_0 \cos \alpha)^2 K_p$$

where " $e_0 \cos \alpha$ " is the vertical component of the length of the wall-AB (see Fig. 2(a) or Fig. 3(a)); K_p is the passive pressure coefficient for the sloping soil with a constant slope angle (α):

$$(5) \quad K_p = \left[\frac{\cos(\varphi + \alpha') / \cos(\alpha')}{\sqrt{\cos(\varphi' - \alpha') - \sqrt{\sin(\varphi + \varphi') \sin(\varphi + \alpha) / \cos(\alpha - \alpha')}}} \right]^2$$

in which, the internal friction angle of the sand (φ) is the drained (effective stress) shear strength parameter for the sand; α' is the angle between the virtual retaining wall-AB and the vertical; φ' is the mobilized friction angle at the wall-AB. As for a sloping seabed with slope angle α , the virtual retaining wall-AB is supposed to be

inclined with an inclination angle α' . Both angles (α and α') are included in the expression of K_p by eq. (5). Considering the examined values of α' range from $-15^\circ \sim 15^\circ$, the values of α' can be regarded as the same with the slope angle α for the purpose of simplification in the derivation. The friction angle along the retaining wall (φ') is always partially mobilized, whose values in the passive case are usually less than $\varphi/3$ (Craig 2004). As such, choosing the value of φ' as nil would be conservative for evaluating the lateral soil resistance to the partially-embedded pipeline. Submitting $\alpha'=\alpha$ and $\varphi' \approx 0$ into eq. (5), then

$$(6) \quad K_p = \left[\frac{\cos(\varphi + \alpha) / \cos(\alpha)}{\sqrt{\cos(\alpha)} - \sqrt{\sin(\varphi) \sin(\varphi + \alpha)}} \right]^2$$

Fig. 4 gives the variation of values of the passive pressure coefficient (K_p) with the slope angle (α) for certain values of the internal friction angle of the sand ($\varphi = 25^\circ, 30^\circ, 35^\circ, 40^\circ$ and 45°). Note that the values of α are positive for the upslope instability, whereas they are negative for the downslope instability. When $\alpha = 0$ (meanwhile $\varphi' = 0$), the passive pressure coefficient (K_p) in the Coulomb theory (eq. (6)) is identical to that of the Rankine theory for the case of a vertical wall and a horizontal soil surface, i.e. $K_p = (1 + \sin \varphi) / (1 - \sin \varphi)$. As shown in Fig. 4, for a certain value of φ , the values of K_p increase gradually with increasing slope angle α (from -15° to 15°). Meanwhile, if the values of α is fixed, the K_p increases gradually with the increase of φ .

Fig. 4. Variation of the passive pressure coefficient (K_p) with the slope angle

280 (α)

281 The submerged weight of the wedge-ABD (i.e. the shaded areas in Fig. 2(a) and
282 Fig. 3(a)) can be calculated with

$$283 \quad (7) \quad W_b = \frac{\gamma'}{8} \left[4e_0^2 \frac{1 + \cos \theta_0}{\sin \theta_0} - D^2 (\theta_0 - \sin \theta_0) \right]$$

284 in which, $\theta_0 (= \angle AOD)$, see Fig. 2(a) or Fig 3(a)) is a half of the angle of the
285 pipeline penetration:

$$286 \quad (8) \quad \theta_0 = \arccos \left(1 - 2 \frac{e_0}{D} \right)$$

287 It should be noticed that the pipe-soil interface is the circular arc-AD (Fig. 2(a)
288 and Fig. 3(a)). For a better description for the loading angle of the total pipe-soil
289 interfacial force (P), the circular arc-AD is simplified as the straight line segment
290 AD', i.e. the diagonal-line for the secant and the tangent lines from point-A of the
291 pipe-soil contacting circular arc-AD. This simplification treatment was approved
292 appropriate by a series of calculation trials. The angle $\angle D'AB$ (termed as “ β ”) is
293 the intersection angle between the virtual retaining wall-AB and the line segment
294 AD'. If the value of θ_0 is given, the value of β can be calculated with

$$295 \quad (9) \quad \beta = \frac{\pi}{2} - \frac{3}{4} \theta_0$$

296 Once the geometry of the proposed model is provided as described above, the
297 total pipe-soil interfacial force (P) can thereby be derived following the analysis on
298 the forces on the wedge-ABD (Fig. 2 and Fig 3). By using the “law of sines” to the
299 triangle of forces ($\triangle LMN$):

$$(10) \quad \frac{P}{\sin(\angle MNL)} = \frac{F_{MN}}{\sin(\angle MLN)}$$

in which, $\angle MNL = \pi/2 + \omega + \varphi$; $\angle MLN (= \pi/2 - (\beta - \delta) - \varphi) = 3\theta_0/4 + \delta - \varphi$;

F_{MN} is the resultant force of E_1 and W_b : $F_{MN} = (E_1 \cos \varphi' + W_b \sin \alpha) / \cos \omega$. Thus,

the total pipe-soil interfacial force P can be obtained:

$$(11) \quad P = \frac{\cos(\varphi + \omega)}{\cos(\omega) \sin(3\theta_0/4 + \delta - \varphi)} (E_1 \cos \varphi' + W_b \sin \alpha)$$

where δ is the inclination angle to the normal for P . Note that the signals of δ are

positive for the clockwise of the P in the case of downslope instability (Fig. 2(a))

and for the anti-clockwise of the P in case of upslope instability (Fig. 3(a)),

respectively. ω is the intersection angle between the direction of F_{MN} to the

seabed surface (Fig. 2(b) and Fig 3(b)), which can be calculated by

$$(12) \quad \omega = \arctan \left(\frac{E_1 \sin \varphi' - W_b \cos \alpha}{E_1 \cos \varphi' + W_b \sin \alpha} \right)$$

When the friction angle along the retaining wall-AB approaching zero, i.e. the thrust

force E_1 is acting approximately normally to the retaining wall, eq. (12) can then be

expressed as

$$(12') \quad \omega \approx \arctan \left(\frac{-W_b \cos \alpha}{E_1 + W_b \sin \alpha} \right) \quad (\text{for } \varphi' \approx 0)$$

Once the total pipe-soil interfacial force (P) is predicted by eq. (11), the critical

(maximum) lateral soil resistance (F_R) and the corresponding prop force (F_C) for

the pipeline instability on the sloping seabed can be further obtained:

$$(13a) \quad F_R = P \cos(\beta - \delta)$$

$$(13b) \quad F_C = P \sin(\beta - \delta)$$

The force equilibrium conditions (eqs. (3a) and (3b)) are utilized to identify the unique failure surface by solving these equation group. Submitting eqs. (13a) and (13b) into the force equilibrium equations eqs. (3a) and (3b), then

$$(14) \quad \tan(\beta - \delta) = \frac{W_s \cos \alpha - F_L}{F_D - W_s \sin \alpha}$$

Furthermore, submitting eq. (9) into eq. (14), the geometry relationship between the pipeline penetration and the direction for total pipe-soil interfacial force can be established:

$$(15) \quad \frac{3}{4}\theta_0 + \delta = \arctan\left(\frac{F_D - W_s \sin \alpha}{W_s \cos \alpha - F_L}\right)$$

If the values of the following parameters for the soil and the pipeline are known, i.e. α , φ , D , γ' , W_s and U , then the two unknown values of θ_0 and δ can be determined by eq. (15) together with one of the two eqs. (3a) and (3b). When the value of θ_0 is obtained, the pipeline embedment (e_0) can be further calculated by eq. (8). In the engineering practice, this calculated value of e_0 could be treated as the critical (minimum) pipeline embedment for on-bottom stability (termed as “ e_{cr} ”).

Similar to the above ‘scene representation’, if the value of the pipeline embedment (e_0) is given (W_s is not known in advance), the values of W_s together with δ can also be determined by solving the same equation group, i.e. eqs. (15) and (3a) or (3b).

Note that the signals of δ can be either positive or negative. Nevertheless, the absolute values of the pipe-soil interfacial friction angle ($|\delta|$) should be no larger than its critical value (δ_{crit}), i.e. $|\delta| \leq \delta_{crit}$; Otherwise, the partially-embedded

pipeline would breakout from its in-place location through the pipe-soil interfacial slippage. In accordance with classical plasticity theory, the critical pipe-soil interfacial friction angle can be evaluated with

$$\delta_{\text{crit}} = \arctan \left(\frac{\sin \varphi \cos \nu}{1 - \sin \varphi \sin \nu} \right) \quad (16)$$

in which, ν is the angle of soil dilation. Eq. (16) is a direct consequence of the assumption of coincidence of stress and the plastic strain increment directions, and that the soil is plastic immediately adjacent to the wall (pipe-soil interface) (Potts and Fourie 1986; Lee and Herington 1972).

Three components of the critical soil resistance

As aforementioned, in the pipe-soil interaction model (Wagner et al. 1989), the lateral resistance F_R to the submarine pipeline on a horizontal sandy seabed ($\alpha = 0$) was evaluated by the form of eq. (1). As discussed in the introduction, their model is essentially empirical, with high uncertainty in the empirical coefficient β_0 for evaluating the passive pressure. Unlike the previous model, the present pipe-soil interaction model for a sloping sandy seabed may provide an explicit expression of the three components of the critical lateral soil resistance (Figs. 3(b) and 4(b)):

$$F_R = \underbrace{0.5\gamma'(e_0 \cos \alpha)^2 \cos(\varphi') K_p}_{F_{Rp}} + \underbrace{E_2 \sin \varphi}_{F_{Rf}} + \underbrace{W_b \sin \alpha}_{F_{Rw}} \quad (17)$$

in which F_{Rp} , F_{Rf} and F_{Rw} are the passive-pressure, the sliding-friction, and the additional submerged weight (from the wedge-ABD) components, respectively; K_p and W_b can be calculate by eq. (6) and eq. (7), respectively; the total sliding-friction E_2 along the bottom of the wedge-ABD (Figs. 2(a) and 3(a)) can be calculated in

362 accordance with the law of sines for the forces of triangle (ΔLMN ; see Fig. 2(b) and
 363 3(b)):

$$364 \quad \frac{F_{MN}}{\sin \angle MLN} = \frac{E_2}{\sin \angle LMN}, \text{ i.e. } \frac{(E_1 \cos \varphi' + W_b \sin \alpha) / \cos \omega}{\sin(\pi/2 + \delta - \beta - \varphi)} = \frac{E_2}{\sin(\beta - \delta - \omega)}.$$

365 Thus, the total sliding-friction E_2 can be expressed as

$$366 \quad (18) \quad E_2 = \frac{\sin(\beta - \delta - \omega)}{\cos(\omega) \cos(\beta - \delta + \varphi)} (E_1 \cos \varphi' + W_b \sin \alpha)$$

367 In the following sections, the verification and mechanism analysis will be made
 368 on the pipe-soil interaction, in which the force components of the critical soil
 369 resistance will be presented in detail.

370 Verification of the Proposed Model

371 The proposed pipe-soil interaction model is verified with the existing results of a
 372 series of full scale tests by Wagner et al. (1989). Table 1 gives the detailed
 373 comparisons between the existing test results and the predictions with the present
 374 model for pipe-soil interactions on flat sand-beds.

375 Table 1 lists the results of 10 series of pipe-soil interaction tests on a loose
 376 medium/coarse sand, and 5 series of tests on dense medium/coarse sand for the
 377 comparison with the predicted values. In the reference (Wagner et al. 1989), the
 378 information on the internal friction angle (φ) was not provided, but values of the
 379 relative density for the test sands were given. As listed in Table 1, the values of φ
 380 are evaluated by considering the concept of relative dilatancy index (Bolton 1986),
 381 i.e. for a plane strain problem:

$$\varphi \approx \varphi_{\text{crit}} + 5I_R$$

where φ_{crit} is the critical state angle of shearing resistance of sands (the recommended $\varphi_{\text{crit}} = 35^\circ$ for quartz sands); I_R is the relative dilatancy index: $I_R = D_r(10 - \ln p') - 1$, in which D_r is the relative density of sands, p' is the mean effective stress (in kPa). In addition, those pipe-soil interaction tests mainly involved monotonic and cyclic loadings. Note that in their cyclic loading tests, the oscillations were applied in advance, which were only to obtain the additional pipe penetration. In the table, e_{cr}/D refers to the ratio of the total embedment (including initial embedment and additional penetration) to the pipe diameter. The breakout loads was measured to obtain the values of F_R ($=F_D$ for the case of horizontal seabed). The values of " $W_s - F_L$ " are the net vertical prop loads between the pipe and the underlying sand.

As aforementioned, if the parameters for the sand and the pipeline (i.e. φ , D , γ' , W_s , F_D and F_L) are given, the critical value of θ_0 for the pipeline losing on-bottom stability can be determined by eq. (15) and one of the two eqs. (3a) and (3b). When the value of θ_0 is obtained, the corresponding critical pipeline embedment ratio (e_{cr}/D) can be calculated by eq. (8). With present model, the passive-pressure and sliding-friction components (F_{Rp} and F_{Rf}) of the total lateral soil resistance (F_R) can be easily identified and calculated by eq. (17). The predicted values of F_{Rp} and F_{Rf} are also listed in the right two columns in Table 1.

Fig 5 gives the comparison of the predicted critical pipeline embedment-to-diameter ratio with the experimental results. The comparison

404 indicates that the predictions by the present model and the measured values by
 405 [Wagner et al. \(1989\)](#) are generally in good agreement. As shown in [Fig. 5](#), there
 406 exists some scattering in the data for the conditions of shallow embedment or light
 407 submerged weight of pipelines (see [Table 1](#)), where the passive-pressure
 408 component is less dominant compared to the contributions from the sliding-friction
 409 mechanism. Except for those shallow embedments, the predictions are in general
 410 larger than the experimental results ([Fig. 5](#)), which may be attributed to that the
 411 effect of soil heave was not taken into account in the present model. This may imply
 412 the proposed model would be somewhat conservative for predicting the soil lateral
 413 resistance.

414 An alternative approach is performed by finite element analysis (FEA) to study
 415 the soil-structure interaction ([Potts and Fourie 1986](#)). As stated in the previous
 416 section, a 2-D plane strain elasto-plastic FE model proposed by [Han \(2012\)](#) was
 417 employed for predicting the pipeline-soil interaction behavior on the sloping seabed.
 418 [Fig. 6](#) shows the FE results of the case study for the plastic zones around
 419 partially-embedded pipelines while losing lateral instability on a sloping sand-bed
 420 ($D=0.5\text{m}$, $e_0/D=0.2$, $W_s=1.568\text{ kN/m}$, $\mu=0.3$, $\varphi=30^\circ$). As illustrated in [Fig. 6](#),
 421 for both the downslope instability ($\alpha=-10^\circ$) and the upslope instability ($\alpha=10^\circ$),
 422 the plastic yielding zones that developed in the proximity of the partially-embedded
 423 pipeline hold typical characteristics of retaining structures. It was observed that the
 424 plastic yielding zones were close to the pipeline bottom and protruded gradually to
 425 the soil surface. The passive failure was clearly identified by the plastic strain

development in these plots. Such observations (Figs. 6(a) and 6(b)) in the numerical modeling facilitate the construction of the failure modes (Figs. 2(a) and 3(a)) in the present analyses.

Table 1. Test results by Wagner et al. (1989) and predictions with the present model for pipe-soil interactions on flat sand-beds.

Fig. 5. Comparison of the predicted critical pipeline embedment (e_{cr}/D) with the experimental results

Fig. 6. FE results of plastic zones around partially-embedded pipelines while losing lateral instability on a sloping sand-bed ($D=0.5\text{m}$, $e_0/D=0.2$, $W_s=1.568\text{ kN/m}$, $\varphi=30^\circ$): (a) Downslope instability ($\alpha=-10^\circ$); (b) Upslope instability ($\alpha=10^\circ$)

It should be noticed that the instability of a submarine pipeline under the action of waves or currents is frequently accompanied by local scour or liquefaction of the soil (Gao et al. 2002; Teh et al. 2003; Gao et al. 2007). As previously pointed by Palmer (1996), the sediment transport of the seabed surface layer can be significant under the extreme conditions in the offshore fields. There exists a non-linear relationship between the non-dimensional critical flow velocity (Shields number) and the particle diameter of the sediments (Chien and Wan, 1999). Therefore, in the pipe-soil interaction analysis, the seabed mobility should be well evaluated simultaneously. When the seabed mobility is not predominant, the proposed pipe-soil model can be employed for a satisfactory prediction of the soil resistance.

Effects of Slope Angle

As aforementioned, the sloping seabed is encountered more frequently in deeper waters. The seabed in the South China Sea holds rich varieties of its topographic feature including the vast continental shelf, the continental slope and deep sea basin. The seabed slope angle changes much at various locations, e.g., the measured slope angle generally reaches up to 6.7-17.6 degree at the western continental slope of South China Sea (Liu et al. 2002). To investigate the influence of slope angle on the pipeline lateral instability on a sloping seabed, a case study is performed by using the proposed pipe-soil interaction model.

Table 2 gives the input parameters of the pipeline, the sand and the ocean current. The examined slope angle (α) is in the range of $-15^0 \sim 15^0$. Given the value of ϕ and the α range, the variation of passive pressure coefficients can be calculated by eq. (6). As aforementioned, if the values of the parameters listed in Table 2 are known, the values of the critical pipeline embedment (e_{cr}) could be predicted using the proposed model.

The predicted results are shown in Figs. 7(a) and 7(b). It is indicated in Fig. 7(a) that the values of e_{cr} (and e_{cr}/D) decreases approximately linearly with the increase in slope angle (α from -15^0 to 15^0). Fig. 7(b) illustrates the variations of the total soil resistance (F_R) and its three components (F_{Rp} , F_{Rf} and F_{Rw}) with the slope angle. It could be found in this figure that, the sliding-friction component F_{Rf} and the submerged weight component F_{Rw} change slightly with the variation of the slope angle. Nevertheless, the passive-pressure component F_{Rp} decreases approximately

linearly with increasing the slope angle, which is accompanied by the significant decrease in the critical embedment. This implies that to keep the submarine pipeline stable under the action of a downslope current, a larger value of pipe embedment (e_{cr}) is needed to avoid the occurrence of downslope instability, where a higher passive-pressure (F_{Rp}) could be mobilized to obtain the required soil resistance.

475

Table 2. Input data for case study of the slope angle effect on pipeline lateral instability

Fig. 7. Effects of the slope angle on the pipeline instability: (a) Variation of critical pipeline embedment with slope angle; (b) Variations of the total soil resistance and its three components with slope angle

481

482 Conclusions

As the offshore exploitation shifting from shallow to deep waters, the ocean current would exert the prevailing hydrodynamics on the submarine pipeline. Meanwhile, the sloping seabed would be encountered frequently, especially at the continental slopes. In this study, the ocean current-induced on-bottom stability of a submarine pipeline laid on a sloping sandy seabed is investigated analytically. The main conclusions drawn from this analysis are as follows:

1. Unlike the previous pipe-soil interaction models for the horizontal seabed conditions, a pipe-soil interaction model is proposed for evaluating the lateral soil resistance to a partially-embedded pipeline on a sloping sandy seabed. The

mechanics for the two types of the current-induced pipeline instability are analyzed, i.e. the downslope instability and the upslope instability.

2. By using limit equilibrium approach, the analytical expression of the total lateral soil resistance are derived, which is composed of the sliding-friction component, the passive-pressure component, and the component of submerged weight of the carried soil wedge. The Coulomb's theory of passive earth pressure for the sloping soil is incorporated in the derivation. The model verification with the existing full scale tests shows a good agreement between the experimental results and the predictions.

3. Parametric study indicates that the effect of slope angle on the pipeline lateral soil resistance is significant in the examined range of the slope angle from -15° to 15° . The critical pipeline embedment and the corresponding passive-pressure decreases approximately linearly with increasing slope angle.

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Table Captions:

Table 1. Test results by [Wagner et al. \(1989\)](#) and predictions with the present model

for pipe-soil interactions on flat sand-beds.

Table 2. Input data for case study of the slope angle effect on pipeline lateral

instability

Draft

Figures Captions:

Fig. 1. Illustration of the current-induced pipeline lateral instability on a sloping seabed: (a) Downslope instability; (b) Upslope instability

Fig. 2. Downslope instability of a submarine pipeline: (a) Geometry of failure mechanism; (b) Triangle of the forces on the wedge-ABD (shaded area in Fig 2(a))

Fig. 3. Upslope instability of a submarine pipeline: (a) Geometry of failure mechanism; (b) Triangle of the forces on the wedge-ABD (shaded area in Fig 3(a))

Fig. 4. Variation of values of the passive pressure coefficient (K_p) with the slope angle (α)

Fig. 5. Comparison of the predicted critical pipeline embedment (e_{cr}/D) with the experimental results

Fig. 6. FE results of plastic zones around partially-embedded pipelines while losing lateral instability on a sloping sand-bed ($D=0.5\text{m}$, $e_0/D=0.2$, $W_s=1.568\text{ kN/m}$, $\phi=30^\circ$): (a) Downslope instability ($\alpha=-10^\circ$); (b) Upslope instability ($\alpha=10^\circ$)

Fig. 7. Effects of the slope angle on the pipeline instability: (a) Variation of critical pipeline embedment with slope angle; (b) Variations of the total soil resistance and its three components with slope angle

Fig 1

343x303mm (96 x 96 DPI)

Fig 2

455x175mm (96 x 96 DPI)

635x212mm (96 x 96 DPI)

Fig 4

289x202mm (150 x 150 DPI)

Fig 5

289x202mm (150 x 150 DPI)

Fig 6

288x110mm (96 x 96 DPI)

Draft

Fig 7

678x258mm (150 x 150 DPI)

Table 1. Test results by Wagner et al. (1989) and predictions with the present model for pipe-soil interactions on flat sand-beds.

Test No.	ϕ ($^{\circ}$)	γ' (kN/m)	D (m)	W_s (kN/m)	Test Results			Predictions with present model		
					e_{cr}/D	$W_s - F_L$ (kN/m)	F_R (kN/m)	e_{cr}/D	F_{Rp} (kN/m)	F_{Rf} (kN/m)
LMS-1	35	8.6	1.0	3.0	0.08	1.60	1.67	0.12	0.26	1.41
LMS-2	35	8.6	0.5	0.8	0.07	0.50	0.44	0.14	0.07	0.37
LMS-3	35	8.6	1.0	2.0	0.05	1.25	1.00	0.08	0.09	0.91
LMS-4	35	8.6	1.0	1.0	0.03	0.74	0.54	0.03	0.02	0.52
LMS-5	35	8.6	1.0	3.0	0.17	1.39	1.98	0.25	0.85	1.13
LMS-6	35	8.6	1.0	3.0	0.17	1.26	2.12	0.26	0.96	1.16
LMS-7	35	8.6	0.5	0.8	0.09	0.51	0.48	0.17	0.09	0.39
LMS-8	35	8.6	1.0	2.0	0.07	1.15	1.07	0.12	0.20	0.87
LMS-9	35	8.6	1.0	3.0	0.26	1.46	2.16	0.27	0.97	1.19
LMS-10	35	8.6	1.0	1.0	0.10	0.72	0.81	0.11	0.23	0.58
DMS-1	40	9.6	1.0	3.0	0.05	1.84	1.57	0.03	0.04	1.53
DMS-2	40	9.6	1.0	2.0	0.03	1.30	1.16	0.05	0.05	1.11
DMS-3	40	9.6	0.5	0.8	0.07	0.52	0.44	0.03	0.02	0.42
DMS-4	40	9.6	1.0	3.0	0.06	1.65	1.58	0.09	0.14	1.44
DMS-5	40	9.6	1.0	3.0	0.14	1.59	1.79	0.15	0.36	1.43

Note: "LMS" and "DMS" refer to the Loose Medium/coarse Sand ($D_r \approx 0.3$) and the Dense

Medium/coarse Sand ($D_r \approx 0.7$) respectively in the tests by Wagner et al. (1989).

Table 2. Input data for case study of the slope angle effect on pipeline lateral instability

Input parameters	Values	Note
Flow velocity of the ocean current U (m/s)	1.5	
Pipeline diameter D (m)	0.5	
Reynolds number Re	0.5×10^6	
Drag force coefficient C_D	0.65	(Jones,1978)
Lift force coefficient C_L	0.86	(Jones,1978)
Drag force on the pipeline F_D (kN/m)	0.366	eq. (2a)
Lift force on the pipeline F_L (kN/m)	0.484	eq. (2b)
Submerged weight of the pipeline W_s (kN/m)	0.75	
Effective unit weight of the sands γ' (kN/m ³)	9.6	
Internal friction angle of the sands ϕ (°)	35°	
Examined range of slope angle α (°)	-15°~15°	
Variation of passive pressure coefficients K_p	3.25~4.33	Fig. 4