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Study of black-hole as dissipative structure using negentropy

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Abstract: The area of the event horizon of a black-hole (A_{eh}) is so far linked only with its entropy (S_{BH}). In this theoretical investigation, it is shown that relating A_{eh} only to S_{BH} is inadequate; because A_{eh} is linked to the black-hole's *negentropy*, which encompasses its entropy. Increasing A_{eh} of black-holes that grow now follows from the *Negentropy Theorem (NET)* and also from the well-known *Area Theorem*. The decreasing A_{eh} of black-holes that decay follows from the *Converse to NET* and is not a violation of the *Area Theorem*. The *Corollary to NET* is proved for the case when two dissipative structures merge, which is the basis for the coalescence of black-holes. The *Converse of Corollary to NET* explains negentropy loss due to splitting of a dissipative structure. When applied to black-hole explosion, i.e. splitting in to infinite number of parts, *Converse of Corollary to NET* reduces to *Converse of NET*. The entropy-energy ratio of the exported Hawking radiance from black-holes contributes to the entropy increase of the universe. These aspects justify the consideration of black-holes as thermodynamic dissipative structures.

Keywords: black-hole; dissipative structure; event-horizon; Hawking radiance; negentropy

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1. Introduction

Non-equilibrium thermodynamics led to the understanding that the spontaneous formation, existence and growth of an unstable *dissipative structure* (a black-hole will be shown to be one of them), is the expected consequence of basic physical laws [1]. Mahulikar & Herwig [2] showed that as per the *Principle of Maximum Entropy Production* (PMEP) [3], a process can produce a *dissipative structure* (DS) if the flow of mass and / or energy through it produces net entropy (S) faster than flow without it. The *Isolated Embedding System* (IES), which is the actual subject of thermodynamic analysis, embeds the *dissipative structure* (an open system) and its mass and / or energy interactions with its surroundings [4]. A dissipative structure can spontaneously appear in general systems with energy flowing through it, by its ability to build its order by dissipating potentials in its surroundings [5]. As an illustration, when information is obtained about a physical system, then it is at the cost of an increase in the entropy of the system and its surroundings [6].

As illustrated in Fig. 1a, the creation (cr) path of DS can be selected (if available) because it generates entropy at a faster rate ($\dot{S}_{\text{IES,cr}} > \dot{S}_{\text{IES}}$). As per PMEP, if more entropy is generated ($\dot{S}_{\text{IES,dest}}$) by destroying (dest) the dissipative structure than the entropy generated due to its existence ($\dot{S}_{\text{IES,exist}}$), then the dissipative structure is destroyed if the destruction path is not blocked (ref. Fig. 1b). Ref. Table 1 for a list of *thermodynamic principles* that are the bases in this theoretical research investigation.

1.1 Black-hole thermodynamics

The laws of black hole mechanics are the ordinary laws of thermodynamics applied to a system containing a black hole [7]. *Black-hole Thermodynamics* describes black-holes in analogy to the Laws of Thermodynamics by relating [8]: mass to energy, area of the event horizon (A_{eh}) to entropy (S_{BH}), and surface gravity (κ) to temperature. For a Schwarzschild black-hole as per the 1st Law of Black-Hole Mechanics, $\delta M = \kappa \cdot \delta S_{\text{BH}} / (2\pi)$.

The *no-hair theorem* [9] postulates that all black-hole solutions can be characterized by just three externally observable classical parameters; viz. mass, angular momentum, and electric charge. A black-hole of given mass, angular momentum, and charge, can have several different internal configurations that are inaccessible to an external observer. They reflect the possible different initial configurations of matter that collapsed to form the black-hole. The black-hole's entropy (S_{BH}) can be regarded as a measure of the ignorance of an observer outside of the event horizon, about the black-hole's internal state or the pre-collapse configuration [10]. The logarithm of this number is S_{BH} [11], which is a measure of the amount of information about the initial state that was lost in forming the black-hole. Zhang *et al.* [12] interpreted S_{BH} as the uncertainty about the information of the black-hole forming matter's pre-collapsed, self-collapsed, and inter-collapsed configurations. Wald [13] reviewed black hole thermodynamics, which included classical black hole thermodynamics, Hawking radiation, the generalized 2nd law, and the issue of entropy bounds. A brief survey on the approaches for the calculation of S_{BH} was presented and some unresolved open issues were discussed.

The black-hole has a non-zero S_{BH} to satisfy the Entropy Principle (EP). The Bekenstein-Hawking entropy of a black-hole which exactly saturates the Bekenstein bound [14] is given as,

$$S_{\text{BH}} = [\pi c^3 \cdot k / (2G \cdot h)] \cdot A_{\text{eh}}. \quad (1)$$

In Eq. (1), c ($= 2.998 \times 10^8 \text{ m s}^{-1}$) is the speed of light in vacuum, G ($= 6.674 \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}$) is the universal gravitational constant, h ($= 6.626 \times 10^{-34} \text{ J s}$) is the Planck's constant, and k ($=$

$1.38065 \times 10^{-23} \text{ J K}^{-1}$) is the Boltzmann constant. The Schwarzschild radius (r_{SC}) of a black-hole is [15], $r_{\text{SC}} = (2G/c^2)M_{\text{BH}} [\approx 2950(M_{\text{BH}}/M_{\text{sun}})]$; where, M_{BH} and M_{sun} are the mass of black-hole and the sun, respectively. Therefore, the area of the event-horizon (projected as a circle) is,

$$A_{\text{eh}} = (4\pi G^2/c^4) \cdot M_{\text{BH}}^2 [\approx 1.1 \times 10^8 (M_{\text{BH}}/M_{\text{sun}})^2]; \quad (1.1)$$

hence, from Eq. (1),

$$S_{\text{BH}} = [2\pi^2 G \cdot k / (c \cdot h)] \cdot M_{\text{BH}}^2. \quad (1.2)$$

In this investigation, the entropy-energy ratio (denoted by, s^{en}) will be used. The specific entropy per unit mass (s) of mass is related to s^{en} of the mass as, $s^{\text{en}} = s/c^2$. The entropy-energy ratio of the black-hole is obtained as,

$$s_{\text{BH}}^{\text{en}} = [2\pi^2 G \cdot k / (c^3 \cdot h)] \cdot M_{\text{BH}} = [\pi^{3/2} k / (c \cdot h)] \cdot \sqrt{A_{\text{eh}}}. \quad (1.2.1)$$

It is the same as the universal upper bound to the entropy-to-energy ratio of bounded systems [16]. From Eqs. (1), (1.2), and (1.2.1); $S_{\text{BH}} \propto A_{\text{eh}}$, $s_{\text{BH}}^{\text{en}} \propto (A_{\text{eh}})^{1/2}$, and $S_{\text{BH}} \propto M_{\text{BH}}^2$ i.e. $s_{\text{BH}}^{\text{en}} \propto M_{\text{BH}}$. Hence, for growing black-hole:

- (i) $\dot{A}_{\text{eh}} > 0$, $\dot{M}_{\text{BH}} > 0$, i.e. $\dot{s}_{\text{BH}}^{\text{en}} > 0$ [from Eq. (1.2.1)];
- (ii) entropy (S_{BH}) increases at a faster rate than its entropy-energy-ratio ($s_{\text{BH}}^{\text{en}}$).

Hawking [17] showed that black-holes emit thermal radiation in a black body spectrum at the Hawking temperature [18], $T_{\text{HR}} = [h/(2\pi k)] \cdot \kappa \approx 10^{-6} M_{\text{sun}}/M_{\text{BH}} \text{ K}$. Thus, large black-holes with high M_{BH} emit radiation at lower T_{HR} [19] and either grow fast or decay slowly. The cosmic blackbody background radiation pervades the entire Universe and falls into every astrophysical black-hole. Therefore, the growth or decay rate depends on the temperature of the Cosmic Microwave Background Radiation (CMBR), T_{CMBR} [20]. Wielgus *et al.* [15] analyzed the effect of a ‘firewall’ due to the gravitational blue-shift of the in-falling photons of CMBR and found it to be physically

unimportant. They found that the effects of CMBR interaction with the black-hole have little importance for the astrophysical processes, even in the past hot CMBR era.

Penrose & Floyd [21] first noticed that a black-hole has a natural tendency to increase its A_{eh} [i.e. S_{BH} , ref. Eq. (1)], when undergoing an *irreversible* transformation. The *irreducible (irr) mass* of a Kerr (rotating) black-hole, $M_{\text{BH,irr}} (\propto \sqrt{A_{\text{eh}}})$, represents energy that cannot be extracted by the Penrose processes [22]. Since, A_{eh} always increases in an irreversible process, $M_{\text{BH,irr}}$ of Kerr black-hole also increases. Carter [23] re-derived a result of Christodoulou [22] that $M_{\text{BH,irr}}$ is unchanged in a reversible transformation. Bekenstein [24] showed that the generalization of EP for black-hole is that S_{BH} plus the entropy outside of the black-hole increases. The relevant entropy outside the black-hole ($S_{\text{IES,BH}}$) is that of matter and radiation with which, it interacts. This generalized EP holds in statistically averaged form considering Hawking radiance from Kerr and Schwarzschild black-hole (stationary black-hole with only irreducible mass) [25]. Zurek [26] showed that the entropy of radiation irreversibly evaporated by a Schwarzschild black-hole into vacuum integrated over its entire lifetime, is $4/3$ times the initial entropy of the black hole.

Rabinowitz [27] noted the following two points that contributed to the existing popular belief that A_{eh} is only related to S_{BH} :

- (i) ‘no information’ on what is inside a black-hole can be made available to an outside observer by currently known means [modified by Rabinowitz [27] as ‘no direct knowledge’];
- (ii) A_{eh} increase in irreversible transformations ‘resembles’ S_{BH} increase in EP [8].

1.2 Objectives and scope

This investigation shows that a black-hole, which is basically unstable, not just has an entropy but more importantly a negentropy. New physical insight is gained by taking a re-look at the black-hole as a thermodynamic dissipative structure with a negentropy. Consequently, the Negentropy Theorem

(NET) [28] is now applied to growing black-holes, to explain their existence. The *Converse of* NET is applied to decaying black-holes to explain their destruction. The Corollary to NET is given for the merger of black-holes and its converse is given for the splitting of a dissipative structure.

2. Negentropy-based principles for black-hole

Figure 2 illustrates a black-hole as a dissipative structure, which has energy content, E_{BH} , and entropy-energy ratio, $s_{\text{BH}}^{\text{en}} = S_{\text{BH}}/E_{\text{BH}}$. From IES, mass can enter the black-hole at rate $\dot{m}_{\text{in}}$ with specific entropy per unit mass, $s_{\text{m},\text{in}}$. Similarly, energy (E) e.g. the omnipresent Cosmic Microwave Background Radiation (CMBR) [29] enters with entropy-energy ratio, $s_{\text{E},\text{in}}^{\text{en}}$. Hawking radiance leaves the black-hole at rate, $\dot{E}_{\text{HR},\text{out}}$, and exports higher entropy-energy ratio, $s_{\text{HR},\text{out}}^{\text{en}}$ to IES.

Schroedinger's *negentropy* (s_{n}) of a dissipative structure has been re-defined as [28], $s_{\text{n},\text{DS}} = s_{\text{DS}} - s_{\text{IES},\text{DS}}$. The s_{DS} is the specific entropy of the dissipative structure per unit mass and $s_{\text{IES},\text{DS}}$ is the specific entropy of IES per unit mass of the dissipative structure; where, $s_{\text{DS}} \ll s_{\text{IES},\text{DS}}$; i.e. $s_{\text{n},\text{DS}} \ll 0$. This re-definition, based on specific entropy per unit mass, is valid so long as the total mass is conserved in IES, i.e. mass-energy inter-conversion is unimportant. But for black-holes, irreversible transformation from reducible ($M_{\text{BH},\text{red}}$) to irreducible mass ($M_{\text{BH},\text{irr}}$) should be considered; where, $E_{\text{BH}} = M_{\text{BH}} \cdot c^2$, and $M_{\text{BH}} = M_{\text{BH},\text{red}} + M_{\text{BH},\text{irr}}$. Therefore, the negentropy ($s_{\text{n},\text{BH}}^{\text{en}}$) of black-hole is defined based on its entropy-energy ratio as,

$$s_{\text{n},\text{BH}}^{\text{en}} = s_{\text{BH}}^{\text{en}} - s_{\text{IES},\text{BH}}^{\text{en}} = s_{\text{BH}}^{\text{en}} \cdot [1 - (s_{\text{IES},\text{BH}}^{\text{en}}/s_{\text{BH}}^{\text{en}})]. \quad (2)$$

The maintenance of the low $s_{\text{BH}}^{\text{en}}$ is because, $s_{\text{E},\text{in}}^{\text{en}} \ll s_{\text{E},\text{out}}^{\text{en}}$; hence, $s_{\text{E},\text{in}}^{\text{en}} \leftarrow s_{\text{BH}}^{\text{en}} \ll s_{\text{E},\text{out}}^{\text{en}}$, and $s_{\text{BH}}^{\text{en}} \ll s_{\text{IES},\text{BH}}^{\text{en}} \rightarrow s_{\text{E},\text{out}}^{\text{en}}$. Therefore, in terms of the energy exchanged, Eq. (2) is re-written as,

$$s_{\text{n},\text{BH}}^{\text{en}} \sim s_{\text{BH}}^{\text{en}} \cdot [1 - (s_{\text{E},\text{out}}^{\text{en}}/s_{\text{E},\text{in}}^{\text{en}})] \sim -(s_{\text{E},\text{out}}^{\text{en}} - s_{\text{E},\text{in}}^{\text{en}}). \quad (2.1)$$

This equation gives, $s_{n,BH}^{en}$ in terms of the:

- i) entropy-energy ratio of black-hole (s_{BH}^{en});
- ii) ratio of the entropy-energy ratios of energy emitted and entering the black-hole,

$$s_{ratio}^{en} = (s_{E,out}^{en}/s_{E,in}^{en}), \quad (2.2)$$

which is a measure of the negentropy build-up of the black-hole, as per Eq. (2.1).

Using Eq. (2.1), the negentropy of a black-hole ($S_{n,BH}$) with energy content, E_{BH} , is obtained as,

$$S_{n,BH} = s_{n,BH}^{en} \cdot E_{BH} = S_{BH} - s_{IES,BH}^{en} \cdot E_{BH} \sim S_{BH} \cdot (1 - s_{ratio}^{en}). \quad (2.3)$$

The $S_{IES,BH}$ ($= s_{IES,BH}^{en} \cdot E_{BH}$) is the entropy of IES with the same energy content as the black-hole (E_{BH}). The black-hole's *Isolated Embedding System* (IES) comprises of Hawking radiance; hence, $s_{IES,BH}^{en} \sim s_{E,out}^{en} = s_{HR,out}^{en}$. From Eq. (2), the negentropy, $s_{n,BH}^{en}$, is the negative contrast between the black-hole's entropy-energy ratio (s_{BH}^{en}) and the entropy-energy ratio of the exported Hawking radiance to IES [$s_{HR,out}^{en}$ ($\sim s_{IES,BH}^{en}$)]. The high value of the exported $s_{HR,out}^{en}$ to the surroundings pays the black-hole's negentropy debt for its existence, i.e. to maintain its relatively low s_{BH}^{en} . Refer Table 2 for an exhaustive list of mathematical relations in Black-hole Thermodynamics that are used and / or derived here.

2.1 Negentropy Theorem (NET) for black-hole growth

The NET for the existence of a dissipative structure is the anti-symmetric form of EP, which is mathematically proved [28]. As per NET, '*For a dissipative structure to exist and to be sustained in its surroundings, its negentropy ($|S_{n,DS}|$) must increase*'; i.e. its negentropy generation rate, $\dot{S}_{n,DS} < 0 \Rightarrow |\dot{S}_{n,DS}| > 0$. The NET for a black-hole is proved by differentiating Eq. (2.3) w.r.t. time as, $\dot{S}_{n,BH} = \dot{S}_{BH} - \dot{S}_{IES,BH}$. Since, $\dot{S}_{BH} \ll \dot{S}_{IES,BH}$, " $\dot{S}_{IES,BH} > 0$ " due to EP, results in the validity of NET when the black-hole exists (and grows), i.e.

$$|\dot{S}_{n,BH}| > 0. \quad (3)$$

When a black-hole exists and grows, i.e. $\dot{E}_{BH} > 0$, its $s_{IES,BH}^{en}$, $S_{n,BH}$, S_{BH} , and A_{eh} , will increase, which is its negentropy build-up. Thus, for a growing black-hole, validity of the *Area Theorem* (2nd Law of Black-Hole Mechanics) is an outcome of NET.

An important reason for the *information loss* in black hole radiation is attributed to the pure thermal spectrum of the Hawking radiation [30]. This investigation considers this pure thermal spectrum (i.e. heat) for the estimation of thermodynamic parameters of the Hawking radiation. The Hawking temperature ($T_{HR} \propto M_{BH}^{-1}$) is given as,

$$T_{HR} = [c^3 \cdot h / (16\pi^2 G \cdot k)] / M_{BH} [\approx 6.17 \times 10^{-8} (M_{sun}/M_{BH}) \text{ K}] = c \cdot h / (8\pi^{3/2} k \cdot \sqrt{A_{eh}}). \quad (4)$$

Hence, the rate at which the Hawking radiance is lost from the surface of the event horizon is,

$$\dot{E}_{HR,out} = \sigma \cdot T_{HR}^4 \cdot (4A_{eh}) = \sigma \cdot c^8 \cdot (h/k)^4 / (4096\pi^7 G^2 \cdot M_{BH}^2) = \sigma \cdot (c \cdot h/k)^4 / (1024\pi^6 A_{eh}); \quad (4.1)$$

where, σ ($= 5.6704 \times 10^{-8} \text{ W m}^{-2} \text{ K}^{-4}$) is the Stefan-Boltzmann constant. Since, $\dot{E}_{HR,out} \propto M_{BH}^{-2}$, a large astrophysical black-hole can grow faster, as it loses less amount of energy as Hawking radiance (also inferred from Rabinowitz [31]). The entropy-energy ratio of the exported Hawking radiance from the black-hole is,

$$s_{HR,out,BH}^{en} = \left(\frac{4}{3} a \cdot T_{HR}^3 \right) / (a \cdot T_{HR}^4) = 4/(3T_{HR}) = [64\pi^2 G \cdot k / (3c^3 \cdot h)] \cdot M_{BH} = [32\pi^{3/2} k / (3c \cdot h)] \cdot \sqrt{A_{eh}}; \quad (5)$$

where, the radiation constant, $a = 8\pi^5 k^4 / [15(c \cdot h)^3] = 7.565 \times 10^{-16} \text{ J m}^{-3} \text{ K}^{-4}$. From Eqs. (4.1) and (5),

$$\begin{aligned} \dot{S}_{HR,out} &= \dot{E}_{HR,out} \cdot s_{HR,out}^{en} = 4A_{eh} \cdot \sigma \cdot T_{HR}^3 = \sigma \cdot (c/\pi)^5 \cdot (h/k)^3 / (192G \cdot M_{BH}) \\ &= \sigma \cdot (c \cdot h/k)^3 / (96\pi^{4.5} \sqrt{A_{eh}}). \end{aligned} \quad (5.1)$$

Since, $s_{\text{HR,out_BH}}^{\text{en}} \propto M_{\text{BH}}$ (or $\sqrt{A_{\text{eh}}}$), a larger black-hole gives more garbled information; but the amount of entropy given out is less because, $\dot{s}_{\text{HR,out}} \propto (1/M_{\text{BH}})$ (or $1/\sqrt{A_{\text{eh}}}$).

If it is considered that the omnipresent CMBR enters the black-hole, then $s_{\text{E,in}}^{\text{en}} = 4/(3T_{\text{CMBR}})$, and from Eqs. (2.2) and (5),

$$s_{\text{ratio}}^{\text{en}} = (T_{\text{CMBR}}/T_{\text{HR}}) = [16\pi^2 G \cdot k \cdot T_{\text{CMBR}} / (c^3 \cdot h)] M_{\text{BH}}. \quad (5.2)$$

Since, $T_{\text{CMBR}} = \text{const.}$ ($= 2.73 \text{ K}$, current value [32]), the negentropy of the black-hole is determined by $s_{\text{ratio}}^{\text{en}} (\propto M_{\text{BH}})$ [ref. Eq. (2.3)]. From Eqs. (2), (2.3), and (5.2), negentropy build-up (growth) of black-hole occurs for, $T_{\text{CMBR}} > T_{\text{HR}}$. With increasing age of the universe, T_{CMBR} reduces, hence:

- (i) for the same negentropy build-up, T_{HR} must also reduce;
- (ii) from Eq. (4), black-holes with increasing size can exist and grow.

From Eqs. (1.2.1), (2), and (5), the negentropy of the black-hole based on its entropy-energy ratio is,

$$s_{\text{n_BH}}^{\text{en}} = -[58\pi^2 G \cdot k / (3c^3 \cdot h)] \cdot M_{\text{BH}} = -29[\pi^{1.5} k / (3c \cdot h)] \cdot \sqrt{A_{\text{eh}}}. \quad (5.3)$$

From Eq. (2.3), the negentropy of the black-hole of energy content, $E_{\text{BH}} = M_{\text{BH}} \cdot c^2$ is,

$$S_{\text{n_BH}}(M_{\text{BH}}) = -[58\pi^2 G \cdot k / (3c \cdot h)] \cdot M_{\text{BH}}^2 = -29[\pi c^3 \cdot k / (6G \cdot h)] \cdot A_{\text{eh}}; \quad (5.4)$$

i.e. $|S_{\text{n_BH}}(M_{\text{BH}})| = f(A_{\text{eh}})$. As per NET [Eq. (3)], $|\dot{S}_{\text{n_BH}}| = [116\pi^2 G \cdot k / (3c \cdot h)] \cdot M_{\text{BH}} \cdot \dot{M}_{\text{BH}} = 29[\pi c^3 \cdot k / (6G \cdot h)] \cdot \dot{A}_{\text{eh}}$; and for the case of growing black-hole, $\dot{M}_{\text{BH}} > 0$ and $\dot{A}_{\text{eh}} > 0$, hence, $|\dot{S}_{\text{n_BH}}| > 0$.

From Eqs. (1.2.1) and (5), $\dot{s}_{\text{HR,out_BH}}^{\text{en}} = (32/3)\dot{s}_{\text{BH}}^{\text{en}}$; from Eqs. (1.2.1) and (5.3), $|\dot{s}_{\text{n_BH}}^{\text{en}}| = (29/3)\dot{s}_{\text{BH}}^{\text{en}}$; and from Eqs. (2.3) and (5.4), $|\dot{S}_{\text{n_BH}}| = (29/3)\dot{S}_{\text{BH}}$. Thus, $|S_{\text{n_BH}}|$ increases with A_{eh} and exceeds increase in S_{BH} by a factor of 9.67; therefore, increasing A_{eh} is a much stronger measure of black-hole's increasing negentropy, i.e. $s_{\text{n_BH}}^{\text{en}}$ and $S_{\text{n_BH}}$. The exported high entropy-energy ratio of

the Hawking radiance from black-hole contributes to the entropy build-up of IES, justifying the role of the black-hole as a thermodynamic dissipative structure. The relatively lower entropy-energy ratio of black-hole ($s_{\text{BH}}^{\text{en}}$) remains with the black-hole until it explodes (also inferred from Bekenstein [33]). Thus, increase in $S_{\text{BH}}(A_{\text{eh}})$ and $s_{\text{BH}}^{\text{en}}(A_{\text{eh}})$ is due to the growth of black-hole, because $S_{\text{n,BH}}(A_{\text{eh}})$ and $s_{\text{n,BH}}^{\text{en}}(A_{\text{eh}})$ increase more sharply than $S_{\text{BH}}(A_{\text{eh}})$ and $s_{\text{BH}}^{\text{en}}(A_{\text{eh}})$, respectively. Therefore, A_{eh} is related not just to $S_{\text{BH}}(A_{\text{eh}})$ or $s_{\text{BH}}^{\text{en}}(A_{\text{eh}})$; but more importantly, A_{eh} correlates with the negentropies, $|S_{\text{n,BH}}(A_{\text{eh}})|$ and $|s_{\text{n,BH}}^{\text{en}}(A_{\text{eh}})|$. Mahulikar & Herwig [34] thermodynamically analyzed a black-hole fed by CMBR and showed that its negentropy, $s_{\text{n,BH}}^{\text{en}}$, more strongly correlates with A_{eh} than its $s_{\text{BH}}^{\text{en}}$.

As per the generalized EP, the net entropy generation rate ($\dot{S}_{\text{gen-BH}}$), which is always positive, is given as,

$$\dot{S}_{\text{gen-BH}} = (\dot{S}_{\text{HR,out}} - \dot{S}_{\text{in}}) + \dot{S}_{\text{BH}} > 0. \quad (6)$$

For a growing black-hole, $\dot{S}_{\text{BH}} > 0$, and $(\dot{S}_{\text{HR,out}} - \dot{S}_{\text{in}}) < 0$, and vice versa for a decaying black-hole. In Eq. (6):

- (i) $(\dot{S}_{\text{HR,out}} - \dot{S}_{\text{in}})$, is the net rate of entropy exported to the universe;
- (ii) $\dot{S}_{\text{HR,out}} = \dot{E}_{\text{HR,out}} \cdot s_{\text{HR,out}}^{\text{en}} = 4A_{\text{eh}} \cdot (\sigma \cdot T_{\text{HR}}^4) \cdot (1/T_{\text{HR}}) = 4A_{\text{eh}} \cdot \sigma \cdot T_{\text{HR}}^3$, is the rate at which, entropy is exported as Hawking radiance;
- (iii) $\dot{S}_{\text{in}} = \dot{E}_{\text{in,tot}} \cdot s_{\text{in}}^{\text{en}}$, is the rate at which, entropy enters the black-hole. The total rate of energy entering the black-hole is given as, $\dot{E}_{\text{in,tot}} = \dot{E}_{\text{in}} + \dot{m}_{\text{in}} \cdot c^2$; with entropy-energy ratio, $s_{\text{in}}^{\text{en}} = (\dot{E}_{\text{in}} \cdot s_{\text{E,in}}^{\text{en}} + \dot{m}_{\text{in}} \cdot s_{\text{m,in}}) / \dot{E}_{\text{in,tot}}$.
- (iv) From Eq. (1),

$$\dot{S}_{\text{BH}} = [\pi c^3 \cdot k / (2G \cdot h)] \cdot \dot{A}_{\text{eh}}, \quad (6.1)$$

is the rate of entropy change of the black-hole.

The rate of energy accumulation in the black-hole is given as, $\dot{E}_{\text{BH}} = \dot{E}_{\text{in,tot}} - \dot{E}_{\text{HR,out}}$.

Differentiating Eq. (1.1), the rate of energy accumulation in the black-hole is obtained as,

$$\dot{E}_{\text{BH}} = [c^6/(8\pi G^2 \cdot M_{\text{BH}})] \cdot \dot{A}_{\text{eh}} = [c^4/(4\pi^{1/2} G)] \cdot (\dot{A}_{\text{eh}}/\sqrt{A_{\text{eh}}}). \quad (6.2)$$

From Eqs. (6.1) and (6.2),

$$(\dot{E}_{\text{BH}}/\dot{S}_{\text{BH}}) = (c \cdot h) / [2\pi^{3/2} k \cdot \sqrt{A_{\text{eh}})]; \quad (6.3)$$

therefore, the growth of a black-hole ($\dot{E}_{\text{BH}} > 0$ i.e. $\dot{A}_{\text{eh}} > 0$) also results in, $\dot{S}_{\text{BH}} > 0$. Hence, for a growing black-hole, $\dot{S}_{\text{BH}}$ increases and the net entropy exported to the universe, $(\dot{S}_{\text{HR,out}} - \dot{S}_{\text{in}})$ decreases. The list of increasing and decreasing parameters as a black-hole grows is summarized in Table 3 (vice versa holds for a decaying black-hole). From Eq. (6.3), as a black-hole grows, its entropy increase rate ($\dot{S}_{\text{BH}}$) increases at a much faster rate than its energy increase rate ($\dot{E}_{\text{BH}}$). Black-holes that exist satisfy NET given by the inequality, $s_{\text{HR,out}}^{\text{en}} \gg s_{\text{in}}^{\text{en}}$; i.e. the exported Hawking radiance has higher entropy-energy ratio.

2.2 Converse of Negentropy Theorem (NET) for black-hole decay

Hawking & Penrose [35] explained that the decrease in A_{eh} for decaying black-holes is caused by ‘a violation of the weak energy condition, which arises from the indeterminacy of particle number in curved space time’. Later, Hawking [18] explained, $A_{\text{eh}} \rightarrow 0$, of decaying black-holes based on the concept of ‘negative energy flow down the black-hole’.

It is shown here that the *Converse of* NET directly explains the decrease in A_{eh} : ‘If a dissipative structure decays then, $-S_{\text{n,DS}} (= |S_{\text{n,DS}}|)$ reduces, i.e.

$$|S_{\text{n,DS}}| \rightarrow 0'. \quad (7)$$

Thus, decrease in A_{ch} of decaying black-holes is an outcome of the *Converse of NET* and is not in violation of the *Area Theorem* (which is applicable for growing black-holes). For decaying black-holes that evaporate and shrink, their mass-energy merges with the surroundings. From Eq. (5.4), the *Converse of NET* given by Eq. (7) reduces to, $|S_{\text{n,BH}}| = [58\pi^2 G \cdot k / (3c \cdot h)] \cdot M_{\text{BH}}^2 (\rightarrow 0) = 29[\pi c^3 \cdot k / (6G \cdot h)] \cdot A_{\text{ch}} (\rightarrow 0)$; i.e. $M_{\text{BH}} \rightarrow 0$ and $A_{\text{ch}} \rightarrow 0$. Decaying black-holes are unable to pay adequate negentropy debt to their surroundings, due to the decreasing specific entropy of Hawking radiance, $s_{\text{HR,out}}^{\text{en}}$ ($s_{\text{HR,out}}^{\text{en}} \leq s_{\text{in}}^{\text{en}}$), although, $\dot{S}_{\text{HR,out}} > \dot{S}_{\text{in}}$. Based on the *Converse of NET* and relating A_{ch} to the black-hole's negentropy [Eq. (2.3)], $A_{\text{ch}} \rightarrow 0$ is also the same as, $|S_{\text{n,BH}}| \rightarrow 0$; i.e. decreasing A_{ch} of decaying black-holes decreases their negentropy. Explosion of black-holes is determined by PMEP; since the inequality, $\dot{S}_{\text{IES,BH_dest}} > \dot{S}_{\text{gen-BH_exist}}$ [ref. Eq. (7)] is satisfied, black-hole's decay path is followed (Fig. 1b).

2.3 Corollary to Negentropy Theorem (NET) for merger of black-holes

For studying the merger of two black-holes (i.e. inter-collapse), *Corollary to NET* is formulated as: 'When two black-holes with energy contents, E_{BH1} , E_{BH2} , merge in to one with energy content, $E_{\text{BH,tot}} = E_{\text{BH1}} + E_{\text{BH2}}$, then its negentropy after merger $|S_{\text{n,BH_tot}}|$ exceeds the sum of the unmerged individual negentropies ($|S_{\text{n,BH1}}|$, $|S_{\text{n,BH2}}|$)'. Mathematically,

$$|S_{\text{n,BH_tot}}| > |S_{\text{n,BH1}}| + |S_{\text{n,BH2}}|; \quad (8)$$

where, each of the three negentropies, $|S_{\text{n,BH1}}|$, $|S_{\text{n,BH2}}|$, and $|S_{\text{n,BH_tot}}|$ increase as per NET [inequality (3)]. From Eq. (2.3), $|S_{\text{n,BH1}}| = S_{\text{IES,BH1}} - S_{\text{BH1}}$, $|S_{\text{n,BH2}}| = S_{\text{IES,BH2}} - S_{\text{BH2}}$, and $|S_{\text{n,BH_tot}}| = S_{\text{IES,BH_tot}} - S_{\text{BH,tot}}$. In these three equations, the following 3 inequalities must be satisfied:

- (i) $S_{\text{BH_tot}} > S_{\text{BH1}}$, $S_{\text{BH_tot}} > S_{\text{BH2}}$ (entropy of the merged black-hole exceeds the entropies of the two individual black-holes);

(ii) $S_{\text{IES,BH}_{\text{tot}}} > S_{\text{IES,BH1}}, S_{\text{IES,BH}_{\text{tot}}} > S_{\text{IES,BH2}}$ (entropy of the IES of the merged black-hole exceeds the entropies of the IES of the two individual black-holes);

(iii) $S_{\text{IES,BH1}} \gg S_{\text{BH1}}, S_{\text{IES,BH2}} \gg S_{\text{BH2}}, S_{\text{IES,BH}_{\text{tot}}} \gg S_{\text{BH}_{\text{tot}}}$ (entropies of the IES of black-holes are much larger than the respective black-hole entropies).

Using the definitions of negentropies in inequality (8),

$$\Delta S_{\text{IES,BH}} > \Delta S_{\text{BH}}; \quad (8.1)$$

where, $\Delta S_{\text{IES,BH}} = S_{\text{IES,BH}_{\text{tot}}} - (S_{\text{IES,BH1}} + S_{\text{IES,BH2}})$, and $\Delta S_{\text{BH}} = S_{\text{BH}_{\text{tot}}} - (S_{\text{BH1}} + S_{\text{BH2}})$. The inequality in Eq. (8.1) holds, because after merger, the increase in entropy of IES of black-hole exceeds that of the increase in entropy of the black-hole. Also, from Eq. (5.4), $S_{\text{n,BH}}(N \cdot M_{\text{BH}}) / [N \cdot S_{\text{n,BH}}(M_{\text{BH}})] = N$; hence, the negentropy of one black-hole of mass, $N \cdot M_{\text{BH}}$, exceeds the negentropy of N black-holes each of mass M_{BH} . Hence, the *Corollary to NET* [inequality (8)] is proved. Thus, *Corollary to NET* prefers the existence of one large black-hole over several smaller black-holes of the same total mass as the merged black-hole.

Merger of black-holes is possible if the entropy generated by their coalescence exceeds that of individual black-holes, as per PMEP. If two Schwarzschild black-holes of irreducible masses, $M_{\text{BH1}}, M_{\text{BH2}}$, merge then [36],

$$A_{\text{eh,tot}} > A_{\text{eh1}} + A_{\text{eh2}}. \quad (8.2)$$

This inequality directly follows from the inequality given by Eq. (8), because using the definition of $S_{\text{n,BH}}$ given by Eq. (5.4), Eq. (8) reduces to,

$$M_{\text{BH,tot}}^2 > M_{\text{BH1}}^2 + M_{\text{BH2}}^2; \quad (8.2.1)$$

which holds. Therefore, using Eq. (1.1) for the relation between A_{eh} and M_{BH} , inequality given by Eq. (8.2) is obtained and thus proved. The extra missing term ($\propto M_{\text{BH1}} \cdot M_{\text{BH2}}$) on the right-hand side

of Eq. (8.2.1) measures the correlation generated by gravitational interactions. This correlation constitutes the actual information describing dynamics of the coalescence due to gravitational force. Before coalescence, the information on the correlation can be gained by an external observer; but after the merger, the correlation is covered by the resulting event horizon ($A_{\text{eh,tot}}$). Therefore, the external observer cannot obtain the information on the correlation, which results in increase in entropy of the new merged black hole ($S_{\text{BH_tot}}$). This increase in $S_{\text{BH_tot}}$ after merger holds because gravitational wave radiations alone cannot carry all the information about the gravitational interactions during the collapse.

Bardeen *et al.* [8] used Eq. (1) to correlate A_{eh} and S_{BH} , but noted that their 2^{nd} Law of Black-Hole Mechanics actually missed a link with EP. The EP permits decrease of entropy of one of the two systems by transfer to the other, so long as the total entropy increases. But one cannot transfer A_{eh} from one black-hole to the other, since black-holes cannot bifurcate [37]. This gap in correlating Eq. (8.2) with EP is filled by correlating A_{eh} to $S_{\text{n,BH}}$ [Eq. (2.3)] and not with S_{BH} . Inequality (8.2) follows from inequality (8) for the negentropies, but individually, A_{eh1} and A_{eh2} , and $|S_{\text{n,BH1}}|$ and $|S_{\text{n,BH2}}|$, respectively, cannot decrease for existing black-holes, as per NET [Eq. (3)].

2.4 Converse of corollary to Negentropy Theorem (NET)

The *Converse of the Corollary to NET* is given as: ‘When a finite dissipative structure (DS_{tot}) splits in to a large number of parts, N , ($DS1$, $DS2$, ..., DSN), then the sum of their split negentropies ($S_{\text{n,DS1}} + S_{\text{n,DS2}} + \dots + S_{\text{n,DSN}}$) is lower than the earlier un-split negentropy ($S_{\text{n,DS_tot}}$)’; i.e.

$$|S_{\text{n,DS1}}| + |S_{\text{n,DS2}}| + \dots + |S_{\text{n,DSN}}| < |S_{\text{n,DS_tot}}|. \quad (8.3)$$

The proof of inequality (8.3) is similar to that of inequality (8) up to inequality (8.1); which now reduces to, $\Delta S_{\text{DS}} < \Delta S_{\text{IES,DS}}$. Validity of inequality (8.3) is based on the increase in the entropy of the dissipative structure being lower than that of the increase in the entropy of IES, after splitting.

In the limit, $N \rightarrow \infty$, the negentropy of individual infinitesimal split dissipative structures tends to zero, which is applicable for black-hole explosion. In this limit, *Converse of Corollary to NET* [Eq. (8.3)] reduces to *Converse of NET* [Eq. (7)]. Black-hole of initial mass, $M_{\text{BH,tot}}$, explodes in to infinite number of parts ($N \rightarrow \infty$), i.e. $M_{\text{BH}1}, M_{\text{BH}2}, \dots, M_{\text{BHN}}$; where, $M_{\text{BH,tot}} = \sum_{i=1}^N M_{\text{BH},i}$. Equation (8.3) is valid, because using Eq. (5.4), $\sum_{i=1}^N M_{\text{BH},i}^2 < (\sum_{i=1}^N M_{\text{BH},i})^2 = M_{\text{BH,tot}}^2$, which holds. Based on the *Converse to the Corollary of NET*, the negentropy and gravitation of an exploding black-hole would be destroyed.

The four thermodynamic theorems derived in this investigation, i.e. (i) Negentropy Theorem (NET) for Black-Hole Growth, (ii) Converse of NET for Black-Hole Decay, (iii) Corollary to NET for Merger of Black-Holes, (iv) Converse of Corollary to NET, are listed in Table 1b.

3. Conclusions and summary

- i) Complete understanding of a black-hole as a thermodynamic dissipative structure is provided by the following parameters: entropy (S_{BH}), entropy-energy ratio ($s_{\text{BH}}^{\text{en}}$), and negentropies ($s_{\text{n_BH}}^{\text{en}}$, $S_{\text{n,BH}}$).
- ii) Relating the event horizon area of a black-hole (A_{eh}) to its entropy (S_{BH}) alone is incomplete. The A_{eh} is more importantly linked with its negentropy ($S_{\text{n,BH}}$) and the entropy-energy ratio of the exported Hawking radiance.
- iii) The Hawking radiance from growing black-holes at low Hawking temperature has high entropy-energy ratio ($s_{\text{HR,out}}^{\text{en}}$), i.e. more garbled information release. It is the payment of the black-hole's negentropy debt to its surrounding and contributes to the negentropy build-up of the black-hole. The high value of $s_{\text{HR,out}}^{\text{en}}$ justifies the consideration of a black-hole as a thermodynamic dissipative structure.

- iv) The relatively much lower entropy-energy ratio of the black-hole ($s_{\text{BH}}^{\text{en}} \ll s_{\text{HR,out}}^{\text{en}}$) remains with the black-hole, until it completely evaporates and exports Hawking radiance to the universe.
- v) The increasing A_{ch} of black-holes that grow as per the *Area Theorem* is an outcome of the *Negentropy Theorem* (NET). The decreasing A_{ch} of decaying black-holes is due to the *Converse of NET* and is not a violation of the *Area Theorem*.
- vi) The inequality based on A_{ch} before and after the merger of black-holes is explained by the *Corollary to NET*. This corollary prefers the existence of one large black-hole over several smaller ones of the same total mass.
- vii) The missing link in correlating the 2^{nd} Law of Black-Hole Mechanics with the 2^{nd} Law of Thermodynamics (EP), is filled by linking A_{ch} directly to $S_{\text{n,BH}}$ (and not S_{BH}). This is because, $S_{\text{n,BH}}$ of an existing black-hole cannot decrease as per NET (but the entropy of an individual system can decrease, so long as the total entropy increases).
- viii) The *Converse to the Corollary of NET* is the basis for the splitting of a dissipative structure and explains the resulting negentropy loss. When applied to black-hole explosion, i.e. splitting in to infinite number of parts, *Converse of Corollary to NET* reduces to *Converse of NET*.
- ix) A consolidated list of theorems and mathematical relations for black-hole thermodynamics are in Tables 1 and 2, respectively.

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(a) Creation of Dissipative Structure (DS)

(b) Existence / Destruction of Dissipative Structure (DS)

Fig 1. Creation, existence, and destruction of Dissipative Structure (DS) in chaos.

Fig 2. Black-hole as a thermodynamic dissipative structure.

Table 1. List of thermodynamic theorems used and / or derived in this investigation

(a) Existing Principles

<i>Thermodynamic Principle</i>	<i>Statement</i>
<i>Entropy Principle (EP) / 2nd Law of Thermodynamics</i>	The entropy of an isolated system never decreases, because isolated systems spontaneously evolve towards global thermodynamic equilibrium - the state of maximum entropy.
<i>2nd Law of Black-Hole Thermodynamics</i>	The event horizon area of a black-hole is, assuming the weak energy condition, a non-decreasing function of time: $\dot{A}_{\text{eh}} > 0$
<i>Principle of Maximum Entropy Production (PMEP)</i>	A system will select the path or assemblage of paths out of available paths that minimizes the potential or maximizes the entropy at the fastest rate for given constraint/s.

(b) Theorems derived

<i>Negentropy Theorem (NET) for Black-Hole Growth</i>	For a Dissipative Structure (DS) to exist (to be sustained) in its surroundings, its negentropy must increase: $\dot{S}_{\text{n,DS}} < 0 \Rightarrow \dot{S}_{\text{n,DS}} > 0$.
<i>Converse of NET for Black-Hole Decay</i>	If a dissipative structure decays then its negentropy reduces and tends to zero.
<i>Corollary to NET for Merger of Two Black-Holes</i>	When two black-holes coalesce in to one, which continues to exist after merger, the negentropy after merger exceeds the sum of the unmerged individual negentropies: $ S_{\text{n,BH tot}} > S_{\text{n,BH1}} + S_{\text{n,BH2}} $
<i>Converse of Corollary to NET</i>	When a finite dissipative structure splits in to N number of parts, then the sum of their split negentropies is lower than the initial un-split negentropy: $ S_{\text{n,DS1}} + S_{\text{n,DS2}} + \dots + S_{\text{n,DSN}} < S_{\text{n,DS tot}} $

Table 2. List of mathematical relations for black-hole thermodynamics

<i>Description</i>	<i>Mathematical Relation</i>
Schwarzschild radius	$r_{\text{SC}} = (2G/c^2)M_{\text{BH}}$
Event-horizon area	$A_{\text{eh}} = (4\pi G^2/c^4)M_{\text{BH}}^2$
Hawking temperature	$T_{\text{HR}} = [c^3 \cdot h / (16\pi^2 G \cdot k)] / M_{\text{BH}} = [c \cdot h / (8\pi^{3/2} k)] / \sqrt{A_{\text{eh}}}$
Rate of energy accumulation in black-hole	$\dot{E}_{\text{BH}} = [c^6 / (8\pi G^2)] (\dot{A}_{\text{eh}} / M_{\text{BH}}) = [c^4 / (4\pi^{1/2} G)] \cdot (\dot{A}_{\text{eh}} / \sqrt{A_{\text{eh}}})$
Rate at which, Hawking radiance is exported from black-hole	$\dot{E}_{\text{HR,out}} = 4\sigma A_{\text{eh}} \cdot T_{\text{HR}}^4$
Bekenstein-Hawking entropy of Black-Hole (BH)	$S_{\text{BH}} = [\pi c^3 \cdot k / (2G \cdot h)] A_{\text{eh}} = [2\pi^2 G \cdot k / (c \cdot h)] M_{\text{BH}}^2$
Entropy-energy ratio of black-hole	$S_{\text{BH}}^{\text{en}} \{ = [2\pi^2 \cdot G \cdot k / (c^3 \cdot h)] M_{\text{BH}} \} = [\pi^{3/2} k / (c \cdot h)] \sqrt{A_{\text{eh}}}$
Entropy-energy ratio of exported Hawking radiance from black-hole	$S_{\text{HR,out,BH}}^{\text{en}} = 4 / (3T_{\text{HR}}) = (32/3) S_{\text{BH}}^{\text{en}} = [64\pi^2 G \cdot k / (3c^3 \cdot h)] \cdot M_{\text{BH}} = [32\pi^{3/2} k / (3c \cdot h)] \cdot \sqrt{A_{\text{eh}}}$
Entropy generation rate by black-hole	$\dot{S}_{\text{gen-BH}} = (\dot{S}_{\text{HR,out}} - \dot{S}_{\text{in}}) + \dot{S}_{\text{BH}} > 0$
Rate at which, entropy is exported as Hawking radiance	$\dot{S}_{\text{HR,out}} = 4\sigma A_{\text{eh}} \cdot T_{\text{HR}}^3 = \sigma \cdot (c/\pi)^5 \cdot (h/k)^3 / (192G \cdot M_{\text{BH}}) = \sigma \cdot (c \cdot h/k)^3 / (96\pi^{4.5} \sqrt{A_{\text{eh}}})$
Re-definition of Schroedinger's <i>negentropy</i> of Dissipative Structure (DS)	$s_{\text{n,DS}} = s_{\text{DS}} - s_{\text{IES,DS}}$
Ratio of the entropy-energy ratios of energy emitted and entering the black-hole (measure of negentropy build-up of the black-hole)	$s_{\text{ratio}}^{\text{en}} = s_{\text{E,out}}^{\text{en}} / s_{\text{E,in}}^{\text{en}} (= T_{\text{CMBR}} / T_{\text{HR}}, \text{ for black-hole fed only by CMBR})$
Negentropy of black-hole based on its entropy-energy ratio	$s_{\text{n,BH}}^{\text{en}} = s_{\text{BH}}^{\text{en}} - s_{\text{IES,BH}}^{\text{en}} = -[58\pi^2 G \cdot k / (3c^3 \cdot h)] M_{\text{BH}} = -29[\pi^{1.5} k / (3c \cdot h)] \sqrt{A_{\text{eh}}} s_{\text{n,BH}}^{\text{en}} \sim s_{\text{BH}}^{\text{en}} \cdot (1 - s_{\text{ratio}}^{\text{en}})$
Negentropy of black-hole with net energy content, E_{BH}	$S_{\text{n,BH}} = s_{\text{n,BH}}^{\text{en}} \cdot E_{\text{BH}} = S_{\text{BH}} - s_{\text{IES,BH}}^{\text{en}} \cdot E_{\text{BH}} = -[58\pi^2 G \cdot k / (3c \cdot h)] M_{\text{BH}}^2 = -29[\pi c^3 \cdot k / (6G \cdot h)] A_{\text{eh}} \sim S_{\text{BH}} \cdot (1 - s_{\text{ratio}}^{\text{en}})$
2 nd law of black-hole mechanics	$A_{\text{eh,tot}} > A_{\text{eh1}} + A_{\text{eh2}}$

Table 3. Trends of thermodynamic parameters of existing (growing) black-hole

<i>Increasing Parameters</i>	<i>Decreasing Parameters</i>
$A_{\text{eh}}, E_{\text{BH}}, M_{\text{BH}}, s_{\text{BH}}^{\text{en}}, S_{\text{BH}}, s_{\text{HR,out}}^{\text{en}}, \dot{S}_{\text{gen-BH,exist}}$	$S_{\text{HR,out}}, (S_{\text{HR,out}} - S_{\text{in}})$