# cjfr-2015-0345 entitled "Comparing the Efficacy of LP Models I and II for Spatial Strategic Forest Management"

<table>
<thead>
<tr>
<th><strong>Journal:</strong></th>
<th>Canadian Journal of Forest Research</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Manuscript ID</strong></td>
<td>cjfr-2016-0139.R2</td>
</tr>
<tr>
<td><strong>Manuscript Type:</strong></td>
<td>Article</td>
</tr>
<tr>
<td><strong>Date Submitted by the Author:</strong></td>
<td>10-Aug-2016</td>
</tr>
<tr>
<td><strong>Complete List of Authors:</strong></td>
<td>Martin, Andrew; Dalhousie University, Industrial Engineering Gunn, Eldon; Dalhousie University Richards, Evelyn; University of New Brunswick</td>
</tr>
<tr>
<td><strong>Keyword:</strong></td>
<td>forest management, strategic planning, linear programming, ecosystem based management, spatial constraints</td>
</tr>
</tbody>
</table>
Title: Comparing the Efficacy of LP Models I and II for Spatial Strategic Forest Management

Andrew B. Martin¹, Evelyn Richards²*, and Eldon Gunn³†

¹ Department of Industrial Engineering, Dalhousie University, andrew.b.martin@dal.ca, Halifax, Nova Scotia

² Faculty of Forestry and Environmental Management, University of New Brunswick, ewr@unb.ca, Fredericton, New Brunswick.

³ Department of Industrial Engineering, Dalhousie University, Halifax, Nova Scotia.

† deceased

* corresponding author:

Faculty of Forestry and Environmental Management
28 Dineen Drive
University of New Brunswick
Fredericton NB E3B 5A3
Abstract

Contemporary strategic forest management goals have become increasingly complex in spatial definition and scale. For example, the Canadian Council of Forest Ministers Criteria and Indicators (CCFM C&I) includes metrics that are expressed at multiple levels of spatial resolution, such as ecodistricts, watersheds, and vegetative communities. Supporting these criteria with aspatial models is sometimes difficult and results are often not transferable to the actual forest. We describe a spatial Model I stand and prescription based strategic forest planning model that includes spatial metrics in a realistic sized problem. We compare its formulation, capabilities, and computational efficiency with a Model II formulation using a case study on Nova Scotia's Crown Central forest. We demonstrate that the spatial Model I is better suited to support strategic forest management when spatial criteria are included.

Keywords

forest management, strategic planning, linear programming, ecosystem based management, spatial constraints
1. Introduction

Forest strategic planning addresses sustainability of forests with respect to environmental, economic, and social dimensions. In the Montreal Process, 12 member countries, accounting for 90% of the world's temperate and boreal forest, defined criteria and indicators for sustainable forest management (Montreal-Process, 1998). In Canada, the Canadian Council of Forest Ministers Criteria and Indicators (CCFM C&I) (CCFM, 2003) define the principles that forest management strategy must address to be sustainable. As one might expect, many of these have spatial specifications. For example, a goal such as forest contribution to water quality is measured at watershed scale, while ecosystem diversity is measured at ecodistrict scale. In provincial jurisdictions, management strategies are determined based upon what the C&I mean in local forest and policy contexts. Ecosystem Based Management (EBM) (Stewart & Neilly, 2008), (Pretzsch et al., 2008) is used as the framework around which strategy is constructed so that it is regionally relevant and nationally coherent.

Analyzing impacts of proposed plans on long term sustainability of large forests is a daunting task, and analysts need Decision Support Systems (DSS) to facilitate the process (Seely et al., 2004) These DSS are usually powered by optimization and/or simulation models of various types that project the impact of management actions on the forest over time. With many planning periods, stand types, and management options, solving optimization models of the full spectrum of key economic and environmental
metrics is challenging. Further, when objectives, constraints, or goals have spatial dimensions, the computational challenge increases significantly.

Mathematical optimization models that guarantee feasible and optimal solutions, such as Linear Programming (LP) models, are extremely useful for examining trade-offs between management choices. If solving to optimality is not attainable or necessary then approximate methods such as heuristics may be used to solve the model (Pukkala, 2013); see also the commercial Patchworks software package (Rouillard & Moore, 2008). Simulation systems do not optimize but can generate and assess proposed solutions. Nelson, for example, created a forest level spatial planning simulator (Nelson, 2003). Simulation is often combined with linear or mixed integer models to generate solutions (Gustafson et al., 2006).

Hierarchical Forest Management (HFM) (Bitran & Hax, 1977) is a system that organizes management into separate but linked strategic, tactical and operational planning phases. HFM systems typically remove spatial considerations from the strategic planning phase, and use an aspatial model to determine estimates of sustainable flows of timber over multiple forest rotations. These estimated flows are inputs to the subsequent tactical planning phase where, on a shorter time horizon and fewer planning periods, spatial goals as well as other tactical issues are addressed. This produces revised estimates of sustainable flows that are then input to the strategic planning process. This cycle is repeated until a defensible strategy is determined.
By using disaggregated models, HFM has the potential to produce suboptimal solutions, and there are uncertainties in predicting long term sustainability when spatial considerations are assessed at shorter time horizons (Weintraub & Cholaky, 1991). Gunn (2010) draws attention to the difficulties in interpreting outputs from strategic aspatial planning models as inputs to a tactical planning phase. The specific structure of an optimal solution from an aspatial strategic model in HFM may bear little relation to an optimal spatial solution, and Gunn concludes that a common practice of implementing details of the aspatial solution is “highly questionable”. Paradis et al. (2013) observed that when using HFM, systematic disconnectedness between planned and implemented forest management activities may result in a significant divergence between planned and actual forest condition over time, compromising credibility and performance of the forest management planning process. Gunn also suggested that a spatial Model I formulation can contain the elements for better HFM and strategic forest management planning (Gunn, 2010).

LP models provide an unambiguous guarantee of feasibility and optimality, and have been successfully applied to large scale problems in many fields. Two LP model forms, known as Model I and Model II, (Johnson & Scheurman, 1977), are well known formulations that have been widely used in strategic forest planning (Martell et al., 1998). Both models allow a manager to explore the effects of a suite of silvicultural activities on economic returns and future forest condition. However, they are quite different in structure. Model I is stand based, and in this sense is a spatial model. It retains stand spatial resolution in the landbase with decision variables that choose
prescriptions to be applied to specific stands. These prescriptions are a set of silvicultural interventions and their timing: each individual prescription spans the planning horizon. For a given set of stands, the number of decision variables grows proportionally with the number of valid stand-prescriptions pairs. Model II, on the other hand, is inherently aspatial. Stands that are similar in vegetation, age, and future development potential are aggregated into strata. Decision variables define the amount (area) and timing of silvicultural interventions on each strata. At the end of each planning period, strata are aggregated again via transition equations based on the development of the forest over that planning period. Model II has the ability to consider many prescription alternatives without having to define them explicitly. Its computation time is not sensitive to the number of stands, but to the number of strata and planning periods, and hence very large forest instances can be optimized.

Due to the spatial nature of its decision variables, Model I appears to be a natural choice when spatial considerations become important to land managers and simple wood supply models are no longer sufficient. However, historically, the large number of decision variables in Model I was viewed as an insurmountable computational hurdle, and aspatial Model II systems have been more popular. In the literature, Model II is widely viewed as the better LP formulation due to its reduced number of decision variables and consequent computational difficulty relative to Model I (Johnson & Scheurman, 1977), (McDill et al., 2016). On the other hand, anecdotal evidence has shown that Model II computation time increases rapidly when additional strata are added, and in particular when spatial resolution is added by defining strata that differentiate location. This has
frustrated users who are required to assess proposed plans relative to spatial outcomes. A closer look at the LP matrix structures indicates that the observed computational difficulties with Model II are likely due to the network of constraints required for each strata. Model I does not have this structure, and so, given the experiences of users, and observing the nature of the Model II constraints, we hypothesized that Model I would outperform Model II computationally, and would exhibit only a linear increase in computational burden as additional spatial constraints were added. We set out to learn more about the relative merits of these models in a systematic way.

We based our study on a forest in Nova Scotia, Canada, and EBM criteria of the Nova Scotia Department of Natural Resources. Their EBM process is consistent with forest management applied in many parts of the world (Montreal-Process, 1998). First, we created a stand based model and prescription system to optimize an objective function based on forest management priorities. These included meeting harvest regulation constraints, harvest sustainability, and spatial ecosystem targets. We then created a functionally equivalent Model II formulation with the same objective function and constraints. We compared the two systems based on their ease of use and computational tractability. We also tracked LP matrix composition changes as constraints were added, and took note of the relative ease with which solutions could be understood. Section 2 describes the structural and technical differences between models I and II in more detail. In section 3 we describe the case study forest and strategic planning priorities. Section 4 details the linear programming models used, and section 5 our study methods. Section 6
presents computational results, and finally our discussion of results and summary of the work in a larger context follow in sections 7 and 8.
2. Structure and Characteristics of Model I & Model II

Johnson & Scheurman (1977) were the first to publish a technical description of LP forest management models, and classified the two main variants as Model I and Model II. In practice, Model I has been employed by Curtis (1962), MaxMillion (Ware & Clutter, 1971), Timber RAM (Navon, 1971), JLP (Lappi, 1992), Nelson et al. (1991), Spectrum (Greer & Meneghin, 2002), and more recently in Heureka (Wikström et al., 2011) and in the updated version of JLP, J (Lappi & Lempinen, 2014). Model II is the basis of the Woodstock™ planning system (Feunekes & Cogswell, 2000) and also the USDA Forest Service Model, ForPlan (Kent et al., 1991). These systems are used widely and with considerable success in supporting sustainable harvest management. While either model can be used to support strategic planning, there are significant structural differences that impact their capabilities and solvability in particular circumstances.

Figures 1 and 2 show equations for generic Models I and II.

We refer to Model I as "spatial" because decisions are applied to specific locations in the forest, called stands. Stands are identical in all attributes that are tracked and used in the model. These attributes describe the stand forest type at varying levels of detail, its forest growth potential, and age. Non-biological attributes such as distance to riparian features, location in watershed, ecodistrict, ownership, and so forth are also identical in a stand. Stands do not change location over time, although their forest attributes change through growth and silvicultural interventions. Stands are commonly assumed to be
contiguous, although this has no bearing on the nature and performance of Model I. In fact, in our study, we included stands composed of non-adjacent polygons when all other relevant attributes are identical.

In Model II, forest stands are aggregated into strata that are defined in a similar way, but usually without any knowledge of spatial location of individual stands or other physical attributes such as area or edge length. What is "known" in Model II is the number of hectares in each strata at the beginning of each planning period. Hectares in each strata are transitioned to new strata at the end of each planning period as the forest is changed through growth, aging, and silviculture. Model II is often referred to as an aspatial model due to this lack of spatial definition, although spatial attributes can be incorporated by adding more strata to define location.

Both models incorporate a predefined set of silvicultural interventions or actions such as clearcut harvest, precommercial thinning, commercial thinning, planting, and shelterwood harvest. In Model I, prescriptions are composed of a series of interventions occurring over the span of the planning horizon, and each stand is assigned one prescription. For example, a prescription may be to "Clearcut at age 60, regenerate naturally and clearcut again at age 60". Model II applies actions at different levels (area) in each strata, and in each planning period. In Model II prescriptions are created dynamically in the model, and are controlled only by eligibility constraints on actions. In Model I, all prescriptions are pre-specified.
These differences are of course reflected in the decision variables (Figures 1 and 2), as well as some of the constraints. In Model I the variables are \( x_{ij} \): the number of hectares of stand \( i \) to manage under prescription \( j \). In Model II the variables are \( y_{abks} \): the number of hectares from strata \( s \) regenerated in period \( a \), and treated in period \( b \) using intervention \( k \). The number of decision variables in Model I is the number of valid stand - prescription pairs, which is usually much less than the number of stands times the number of prescriptions. In Model II, the number of variables increases with the number of strata, planning periods, and interventions that are included.

To manage transitions of forest at the end of each period in Model II, it is necessary to update the new area in each strata. This is done with constraints that transfer hectares from one strata to another, depending on their treatment in the current period. This makes for fewer decision variables, but stand identity is lost when actions are applied. The decision variables in Model II can be visualized as the arcs in an acyclic network (Figure 3), where actions applied in one planning period determine a transition of area to different strata in the subsequent period. It turns out that the constraints required to manage this network can impose a significant computational burden. Unfortunately for Model II users, when multiple and overlapping spatial goals are included, more strata and larger constraint networks are required. It has been observed by Model II users that models can “blow up” when too many strata are included, often as a result of attempts to include multiple levels of spatial resolution. Each additional spatial attribute with, say, \( n \) possibilities, increases the number of strata by \( n \). Then an additional arc is required for each of these \( n \) new strata, each eligible action on each strata, and for each period and
possible transition. This is what leads to substantial increases in model size which in turn leads to unacceptably long solution times.

An important distinction between the models is the nature of their prescription sets. As mentioned earlier, Model I prescriptions are predefined by the user, and consist of a sequence of actions. These always include a 'do nothing' action, and hence one could visualize a prescription as a vector of actions, one for each planning period. The type and timing of silvicultural actions must be suitable for the stand type and consistent with forestry best management practices. Thus there will be a practical limit to the number of prescriptions that should be generated. However, care must be taken not to limit the model by too restrictive a prescription set. Thus, a disadvantage of Model I is that it is up to the user to determine the prescriptions it attempts to model (Davis et al., 2001), and generating a comprehensive prescription set can be arduous. On the other hand, explicitly specifying prescriptions is an advantage in that the process of generating prescriptions can be controlled to produce only those that satisfy certain forestry goals including local sustainability. This is not necessarily the case with Model II.

Model II creates prescriptions dynamically by applying actions in each period. If insufficient care is given to restricting eligibility of these actions, illogical sequences may be created. The user only has to define individual silvicultural interventions and the strata for which they are eligible, and is not forced to think about how they might be combined into prescriptions. Thus, Model II solutions can contain prescriptions such as “Shelterwood in periods 1 and 3, pre-commercial thin in period 5, commercial thin in
period 9, clearcut in period 12, and clearcut in period 20”. Each individual intervention may be reasonable, but the sequence lacks credibility with regard to good forestry practices. Further, since stand identity is not maintained in Model II, it is impossible to determine the prescription the model assigns to each stand, so it is not always possible for the user to know the percentage of their solution that was developed from these undesirable sequences.

Why do such inconsistencies arise in the optimal solution? The nature of optimization models is to aggressively seek opportunities to improve the objective function. Forest strategic planning models usually have an objective to maximize harvest value and/or volume. The LP solver will then compose any allowable sequence that increases the harvest. Since results of these sequences are not necessarily assigned to the same stand, these benefits (i.e. harvest volumes) may in fact be impossible to achieve. Thus, we can know that some of the objective function is due to these computer generated series of actions that do not represent good forestry practices, but it is near impossible to remove them all from the system.

Finally, Model I solutions are easier to interpret because allowable prescriptions are predefined and applied to particular stands. With Model II it may be quite difficult to determine the forestry decisions that create a given solution, and since there is no stand resolution it is impossible to determine the proposed sequence of interventions to verify that they make valid forestry sense.
3. Study Forest

The Crown Central forest has 379,000 ha, divided among 3 ownerships, 22 ecodistricts, 24 watersheds, and is spread over 5 counties: Halifax, Hants, Colchester, Cumberland, and Pictou. Figure 4 is a map of the forested area of the Crown Central Forest. Figures 5 & 6 show an example of how ecodistricts and watersheds overlap in this forest. The study area is in the Acadian ecozone (Webb & Marshall, 1999), a forest with high species richness, diversity in forest composition and structure, and number of natural disturbance types it experiences. The Nova Scotia Department of Natural Resources (NSDNR) planning system group defines 16 species associations (Table 1), according to vegetation type, previous management, and regeneration type. Most stands are under natural even aged management, but there are managed softwood plantations and uneven age managed stands as well. Crown land is either unlicensed or assigned to the Northern Pulp or Port Hawkesbury Paper licenses.

Softwood, especially Spruce and Fir, makes up the majority of harvests (NSDNR, 2013). Environmental policies on Crown land in Nova Scotia include maintaining representative species mixes and age class distributions for each natural disturbance regime (Neilly et al., 2007; NSDNR, 2008), limiting harvests in riparian buffer zones, reducing clearcuts to less than 50% of harvests by area (NSDNR, 2011), and selecting 12% of high conservation quality land for protection (NSENV, 2012). Their analysts have incorporated these in their current planning system. At the time of this study, they
were considering expanding the spatial environmental constraints and goals in their modelling. For example, continuous cover maintenance in watersheds was a new goal to be incorporated. Earlier attempts to include spatial environmental attributes using additional Model II strata definition were not successful due to computational difficulties. The analysts anticipated the need for extensive investigation of the impact of new constraints on their wood supply. This requires an interactive decision process, including sensitivity analysis of the importance of stochastic factors such as stand development uncertainties. Hence, they have a need for a system that could produce new solutions quickly, and generate many instances for analysis as scenarios are proposed and evaluated.
4. Study Strategic Planning Models

Two strategic planning LP models were used to investigate the capabilities of Model I and Model II. Both models needed to incorporate NSDNR goals for ecosystem based landscape management. We used their current system, Woodstock™, which incorporates a Model II type of optimization. We formulated a spatial Model I and developed a prescription system that matched the current aspatial model as closely as possible. Our Model I follows the structure of the theoretical, comprehensive Model I formulation presented in Gunn (2010).

Figure 7 delineates the sets, parameters and variables used for Model I. Note that sets are used to group stands by ownership, natural disturbance regime, ecodistrict, and watershed. Coefficients are used to calculate important features of a solution. For each stand and prescription combination, we have yield (timber flow) and standing timber inventory coefficients for various timber types. Membership indicators for cover types, development classes and seral classes are used to quantify forest composition. Parameter sets A, B, and W are desired targets for percentage composition of the forest in each development class, seral class, disturbance regime, and watershed, and over a number of planning periods. For example, a user might wish to maintain at least 10% of mature forest cover in each ecodistrict in each planning period and each natural disturbance regime. This would involve setting all $A_{dn}$, $B_{dn}$, and $W$ at $w$ equal to 0.10.
Decision variables $x_{ij}$ are the number of hectares of stand $i$ treated under prescription $j$. We generated only eligible variables: i.e. those $x_{ij}$ for which prescription $j$ makes sense for stand $i$. Thus, the number of variables in the model is dependent on the number of stands and the number of prescriptions eligible for each stand. Table 2 describes the types and number of options of prescriptions that were modelled. The number of eligible prescriptions per stand ranged from 1 to 41, with more than half of stands having 13 to 21 prescriptions, and more than a third having 1 to 2 prescriptions due to exclusion status. We used deviation variables ($J_{dent}$ and $G_{cent}$) to measure lack of conformance to each environmental forest composition target. As in the original NSDNR model, these are weighted in the objective function by an equivalency factor of 120 cubic metres of spruce-fir harvest per hectare of deviation.

$$\max \sum_{i \in I(u)} \sum_{j \in P_i} \sum_{t \in T} spbf_{ijt} * x_{ij} - 120 * \sum_{d \in D, e \in E} \sum_{n \in N, t \in T} J_{dent} - 120 * \sum_{c \in C, e \in E} \sum_{n \in N, t \in T} G_{cent}$$

(1)

The objective function (Equation 1) is to maximize the harvested volume of the most important softwood species (Balsam Fir and Spruce), while minimizing deviations from two sets of desired forest composition targets. These environmental goals are to maintain at least a certain forest area in each development class (D) and in each seral class (C). Their constraints can be applied at different spatial scales, for example all forest, ecodistrict, ecosite, or watershed. They are also applied to each natural disturbance regime in constraints (6) and (7).
The model constraints include the usual logical and accounting constraints, constraints on timber volume and type, and environmental constraints.

\[ \sum_{j \in P_i} x_{ij} = area_i \quad i \in I \quad (2) \]

Constraints (2) state that the total area in each stand is assigned to prescriptions. Note that the 'do nothing' prescription is always available. There are three constraints on harvest and forest standing volume that are required for economic sustainability.

\[ \sum_{i \in I(u)} spbf_{ij} * x_{ij} \leq \sum_{i \in I(u)} spbf_{ij+1} * x_{ij} \quad u \in U, t \in T \quad (3) \]

Constraints (3) require non declining harvest of spruce and fir in each ownership and period.

\[ \sum_{i \in I(u)} other_{ij} * x_{ij} \leq 0.25 * \sum_{i \in I(u)} total_{ij} * x_{ij} \quad u \in U, t \in T \quad (4) \]

Low-value species, as defined by the NSDNR for Central Nova Scotia as intolerant hardwoods, beech, red oak, pine, eastern hemlock, and tamarack larch, are limited to less than 25% of total harvest in constraints (4). Natural stands in Acadian forests contain multiple species (see Table 1), some with no markets or only poor markets. In practice when clearcutting is used, these trees are harvested along with the more valuable species. This constraint steers the strategic planning process to produce a more valuable timber harvest.
\[ \sum_{i \in I(u)} \sum_{j \in P_i} spbf inv_{ijt} * x_{ij} \leq \sum_{i \in I(u)} \sum_{j \in P_i} spbf inv_{ijt+1} * x_{ij} \quad u \in U, t \in T \quad (5) \]

Constraints (5) require non-declining standing volume of spruce and fir for each ownership and for each planning period. These are intended to maintain sustainability of the most important economic species throughout the planning horizon. Note that standing volumes (constraint set 5, as well as 6, 7, and 8) are computed at the beginning of the period, as opposed to harvest volumes which are computed at the end of the period.

\[ \sum_{i \in I(n,e)} \sum_{j \in P_i} dev_{dijt} * x_{ij} + J_{dent} \geq A_{dn} * area_{en} \quad d \in D, e \in E, n \in N, t \geq 11, t \in T \quad (6) \]

\[ \sum_{i \in I(n,e)} \sum_{j \in P_i} ser_{cijt} * x_{ij} + G_{cent} \geq B_{cn} * area_{en} \quad c \in C, e \in E, n \in N, t \geq 11, t \in T \quad (7) \]

Constraint sets 6-8 define the environmental constraints. Coefficients \( dev_{dijt}, ser_{cijt} \) and \( cover_{ijt} \) are indicator data: they equal 1 if stand \( i \) under prescription \( j \) satisfies appropriate development class, seral stage or forest cover conditions in period \( t \), and 0 if it does not.

Constraints 6 state that in the last 19 periods the area of forest in each development class \( d \)
and natural disturbance regime (NSDNR, 2008), \( n \), and ecodistrict, \( e \), should be \( A_{de} \) percent of total area in that ecodistrict and natural disturbance regime. Deviations from these goals are recorded in the \( J_{dent} \) variables, which are penalized in the objective function at 120 \( m^3 \) per hectare. Constraints 7 are similar to 6 but applied to seral stage (Stewart & Neilly, 2008) instead of development class. The \( G_{cent} \) variables record deviations from these targets and are penalized in the same way as the \( J_{dent} \) variables. As is common in forest strategic planning, depending on the initial structure of the forest, the model may not be feasible in every period. Penalty weights and A and B parameter values were provided by the NSDNR. In this instance, the initial state of the forest was vastly different from these targets. Hence, the environmental constraints were applied after 11 planning periods, allowing a 'warm-up' time so that the forest can be managed to attain this structure over time.

\[
\sum_{i \in i(w), j \in P_i} \text{cover}_{ij} \times x_{ij} \geq \text{Wat}_w \times \text{area}_w \quad w \in W, t \geq 5, t \in T 
\]  

Constraints 8 are the sole case of an element being introduced to this study that was not in the original NSDNR Woodstock™ model. They state that in the last 25 periods at least \( \text{Wat}_w \) percent of the forest in each watershed, \( w \), must qualify as suitable watershed forest cover, i.e. not be in an establishment development class.

The Model II model, built using the Woodstock™ system, is analogously defined; copies of the files that describe its structure can be found in Martin (2013). Figure 2 shows a generic Model II formulation. Model II decision variables \( x_{iak} \) represent the area of stand \( i \) harvested
in period $a$ using intervention $k$. Variables $y_{abk}$ represent the area regenerated in period $a$, and then harvested again in period $b$, using intervention $k$. Parameters $c_{iak}$ and $d_{abk}$ represent the benefit accruing from harvesting stand $i$ in period $a$ using intervention $k$, and harvesting a hectare in period $a$ then again in period $b$ using intervention $k$, respectively. Network constraints (2) ensure the area harvested in period $b$ is regenerated and harvested again in period $f$ or allowed to remain unharvested and pass into $u_a$. The ending inventory constraints (3) ensure a certain harvestable area remains at the end of the planning horizon. Area accounting constraints (4) ensure the area harvested from each stand is equal to the area covered by that stand.

Identical yield data $y_{ijkl}$ was used in both models. It came from the NS Growth and Yield model for even-aged stands and from Permanent Sample Plot (PSP) data for uneven aged stands (O'Keefe & McGrath, 2006). The same silvicultural interventions were defined in both models: clearcut, pre-commercial thinning, commercial thinning, shelterwood harvest, selection harvest, and buffer harvest.

For all comparisons, 68,346 stands were modelled. These were derived from the NSDNR data of 176,480 forested contiguous stands by aggregating those that shared the same ecodistrict, natural disturbance regime, watershed, county, species association, forest state, stocking level, site-class, riparian status, exclusion status, ownership, and age.
5. **Study Methods**

We defined four scenarios (Table 3) that are increased in spatial resolution by successively adding ownership, ecodistrict, and watershed constraints. We solved Models I and II under each scenario twice. In phase 1, a restricted prescription set (Table 2) was used to calibrate the models, and to investigate relative performance on a test sized problem. The second phase expanded the prescription timing options so that Model I would be equal in complexity to the models used in practice by NSDNR. Phase 1 had 144 prescription types and phase 2 had 206. The study forest stands and data were used identically in all cases.

For phase 1, we needed to modify the NSDNR Woodstock™ model to ensure that its prescription set was comparable to that of Model I (See Table 2). We severely restricted eligibility of the Woodstock™ system's actions, limiting second and third treatments depending on the initial intervention. For example, area that was clearcut as a first intervention could only be clearcut at a specified age for the second and third interventions or area that was commercially thinned as a first intervention would receive a commercial thin again after its initial final-felling. With the restricted model II, interventions could not be mixed freely, and we were able to create a problem that was nearly identical to our Model I instance. In addition to facilitating calibration, Phase 1 provided data for a small implementation that could be computed quickly and produced preliminary results on performance and feasibility of the models.
In the second phase, restrictions were removed from Model II. Commercial thinning, shelterwood, selection, and clearcut interventions could be combined within operability limits. Additional timing options were added to the Model I prescriptions so that the first entry did not determine future entries, and hence approximated Model II's expanded prescription set. Phase 2 models represented the current and anticipated level of use by NSDNR, and were verified using their historical data. This phase allowed us to gain insight into potential for implementing further spatial detail.

Due to licensing restrictions and for efficiency, Model I and Model II models were run on different computers. Model I was run on 64-bit Windows 7 with 8 Gb of Ram and a 2.53Ghz processor, and Model II was run on 64-bit Windows 7 with 8 Gb of Ram and a slightly faster 3.00Ghz processor. Both computers were networked and thus a fully controlled computational environment was not possible. Nevertheless, every attempt was made to keep the model runs as undisturbed as possible. Model I models were generated using AMPL (Fourer et al., 1993) and Model II models were generated within Woodstock™.
6. Results

All problem instances were solved to optimality using the concurrent optimizer in Gurobi 5.1.1 (Gurobi Optimization Inc., 2013).

Tables 4 and 5 show dimensions of Model I and Model II for each scenario in Phase 1 and Phase 2 respectively. Model I number of constraints (rows) and matrix density (Non-Zeros) change very little (.02 percent) as additional spatial considerations are added. Model II constraints increase from 260,296 to 722,002 in Phase 1 and from 322,852 to 1,156,015 in Phase 2, a total increase of 358%. The number of decision variables in Model II increases from 787,506 to 1,823,163 in Phase 1, and from 821,549 to 2,900,105 in Phase 2, 353%. Model II matrix Non-Zeros exhibits a similar, yet more dramatic pattern. It moves from 1,700,820 to 100,002,631 in Phase 1 and from 1,646,491 to 120,417,153 in Phase 2, a total increase of 7314%.

Model I has identical variables (columns) across scenarios. We calculated all harvest flows and environmental values indicators even if they were not constrained or penalized. This increased the size of the initial constraint set and hence the percentage increase in matrix density across scenarios is smaller than it could be.

Tables 6 and 7 summarize solution cpu time and quality for Phase 1 and Phase 2 scenarios respectively. Figures 8 and 9 present the results graphically. There was little
difference in model preparation times: all were in the order of 20 minutes. Hence, we omitted the fixed cost of pre-processing and model building in these results.

Computationally, Model I significantly outperformed Model II in scenarios 3 and 4 (Tables 6 and 7). In both phases, Model II CPU time increases dramatically as constraints are added to the model, while Model I increases are small. This is further illustrated in figures 8 and 9 that show the relatively large increase in solution time for Model II in both phases as spatial resolution increases. Computational results for scenarios 1 and 2 are anomalous: Model II outperforms Model I in these cases.

In Phase 1, Model I attained lesser objective values than Model II, at 98.43, 97.91, 95.46, and 95.43 percent of Model II objectives for scenarios 1, 2, 3, and 4 respectively. In Phase 2 however, Model I and Model II objective functions were nearly identical.

Recall that the objective function (Equation 1, section 4) is a sum of harvest volumes and penalties for not meeting environmental targets. Solution quality in this paper refers only to the numeric value of the mathematical objective function, which can be composed of any combination of harvest and penalties. As an example, Figure 8 shows the penalties per period for Model I, Phase 2, Scenario 3. The total of these penalties is 12,267 hectares, representing 1,472,040 cubic metres weighted at 120 cubic meters per hectare. These account for approximately 3.5 percent of the total objective function. Penalties in the equivalent Model II were almost exactly 5,000 hectares, corresponding to 600,000 m$^3$. 

7. Discussion

The strategic planning study we undertook is extensive in both scope and in problem size. It includes several important elements of sustainable spatial forest management, namely economic return from harvesting, and landscape level structure by ecodistrict, natural disturbance regime, ownership and watershed. The study was executed on a large forest of 379,000 ha divided between about 68,000 stands, and includes a comprehensive set of 206 prescriptions (Table 3).

The study results confirm our hypothesis that Model I would be computationally preferable to Model II. While both models can express spatial constraints and goals, Model I is more robust with respect to the number of such constraints. In absolute terms, Model II solution time increased to 5.7 hours on the full sized problem in Phase 2, scenario 4. This scenario was solved by Model I in 42 minutes. The results, though derived from a study of an Acadian forest in Nova Scotia, are of general interest to forest planners and researchers who use the Montreal Process, or a similar process for sustainable forest management planning.

There were two limitations in the study: a slight difference in the decision variables available to the models, and lack of definitive analysis of comparative running times for scenarios 1 and 2. The following two paragraphs explain these issues and comment on their importance to our conclusions.
In order to ensure that both models equally covered the solution space, we wanted to create functionally identical models I and II. We employed an iterative process to add prescriptions that were similar to Model II optimal choices until Model I and Model II objective function values were near identical. We were unable to replicate the full scope of the Model II decision variables. Table 6 shows that with Phase 1 restricted prescription sets, Model I achieves about 5% lower objective values than Model II. Thus, our ability to force the Model II system to duplicate a restricted set of prescriptions in Model I was close but not entirely successful. Objective function values for Phase 2 (Table 7) were near identical, within a fraction of one percent. This implies that we were able to better replicate Model II decision variables with the expanded Model I prescription set used in Phase 2. This approach does not guarantee that all interventions are modelled identically, explaining how Model I finds slightly higher objective values in phase 2, scenarios 2 - 4 Table 7. These small differences do not change our conclusions about the trends in model size and computational effectiveness as spatial constraints are added.

Table 7 shows that with a more realistic prescription set Model I achieves almost identical objective values as Model II and solves these models up to 8x faster. We note the anomaly in relative computational performance of Model I in scenarios 1 and 2. As we see in Tables 6 and 7, and graphically in Figures 8 and 9, in scenarios 1 and 2 Model II actually outperforms Model I. We suggest that this may be explained by the the fact that the initial Model I matrix is already quite dense. In Table 5 we see that the Model I
matrix size increases only marginally from scenario 1-4 in both phases. This is partly
due to the fact that when defining the number of stands and prescriptions available to
each stand, most of the matrix is determined at the basecase level. Additionally, all
scenario models had the same inventory constraints and variables. So, for example, all of
the scenario 1 models had watershed inventory variables. These variables and
constraints only tracked quantities, hence did not constrain the solution. For this reason
they would have likely been removed during the Gurobi presolve, potentially skewing
Model I solution times to be slightly higher. The effect is not obvious in Scenario 3 as
the performance of Model I overwhelms that of Model II. In scenario 4 they are
formulated equally.

We have stated earlier that it is easier to understand prescription based solutions, and
that explicitly defining prescriptions can assure users that only best forestry practices are
modelled. This is not the case with Model II, where sequences of actions are less easy to
control. Model II can create a sequence of treatments that are less than acceptable. An
example of one of these 'inconsistent prescriptions' from our case study is: if a stand was
14 periods old in period 1, it could receive a shelterwood first entry immediately and
second entry in period 3, then be placed on a commercial thinning regime starting in
period 16. This odd sequence of events was produced by a computer "puzzle solver". It
is not the sort of silviculture that meets good forestry practice standards. And so, subject
to the caveat that a comprehensive set of prescriptions suitable for the landscape is
generated, Model I in our opinion is a better choice. Generating a complete set of
prescriptions is a non-trivial task and does require additional effort required by users.
Some prescription generation systems exist, for example in the Heureka system (Wikström et al., 2011).

Structure of the optimal solution can be quite different given even small changes to the linear programming model. In our models, deviation variables are used to ensure feasible solutions for constraints (6) and (7). The objective function has two terms: the sum of harvest volumes and penalties due to non-compliance to environmental constraints. In this limited study, Model I tended to produce solutions with higher penalties. For example, violations to the ecosystem constraints for the Model I phase 2 scenario 3 model (Table 8) sum to 12,267 ha of violation. For all phases and scenarios, penalties were about 2.5 times higher in Model I models than Model II models. We did not focus on this aspect of solution quality. In practice, users would address this aspect of the solution by employing additional constraints to limit penalties or to spread penalties over planning periods.

A natural question to ask is `Will Model I continue to perform well on larger forests and with increased complexity in constraint structure?' The example in this paper, with 68,346 stands and three spatial constraint sets, exhibited a modest increase in computation time with added constraints as exhibited visually in figures 8 and 9. Model I size is directly proportional to the number of valid stand-prescription pairs. Therefore, we hypothesize that this trend of gradual increase in solution time will remain as larger problem instances (forests) and additional spatial constraints are included. However, increased size of problem instance and complexity of model may lead to other
complications in computing, such as memory management issues. That said, the problems solved in this study were done on very modest computers, and without effort to optimize code and data transfer. The next obvious step is to expand the study to larger forests, and to include other important spatial considerations. One obvious candidate is to include mill demands, prices, and costs of transportation to mills so that profit maximization can be done. These supply chain factors are enormously important in managing forests sustainably. In Martin (2013), other work demonstrates that the inclusion of basic economics as suggested in Gunn (2010) is entirely feasible within the Model I framework.

This paper should not be viewed as a criticism of the Woodstock system per se, other than the limitations inherent in its Model II structure. We mentioned the system specifically because it is the software the NSDNR uses. Further, we did not have access to source code, and that impacted our ability to identically reproduce their model in a Model I form. The main point of our paper is the Model I versus Model II modelling framework comparison, not the environment in which they are implemented.

8. Summary and Conclusions

This paper has shown that Model I is a promising framework in which to model forest management strategy with spatial constraints. With respect to computation time, it outperformed Model II conclusively, obtaining comparable optimal solutions in the order of 10 percent of Model II cpu time while incorporating multiple overlapping
spatial goals, including environmental objectives. Model II cpu time increased, as spatial constraints were added, to unacceptable levels. This empirical evidence of relative model performance and model matrix changes is supported by the technical exposition of the difference in model structure. Hence, we are comfortable in stating that these results are generalizable, i.e. not due to case-specific factors.

This is to our knowledge the first published comparison of these models on modern spatial strategic forest management planning problems, and our results provide new and important guidance to users and researchers about expected relative performance of these two formulations. Model II may outperform Model I in some situations, particularly when there are no spatial constraints. However, we have refuted the common expectation that Model II reduces the size and difficulty of linear programming harvest scheduling models relative to Model I in all situations.

Modern forest management must address the cumulative effects of silvicultural interventions over large landscape mosaics and long timeframes. Increasingly, stakeholders have collaborated and developed international standards for sustainable forest management and conservation. The Montreal Process criteria and indicators (Montreal-Process, 1998) for sustainable forest management framework is a globally adopted strategy. Credible forest management processes must incorporate these “essential components of strategic forest management” (Montreal-Process, 1998) in locally meaningful ways. This, in most if not all jurisdictions, means that the forest must be managed and assessed by its current and future condition at several scales: in
watersheds, ecodistricts, riparian zones, and ownership at a minimum. Optimization models like Model I that support this spatial resolution can more accurately assess and compare proposed policies and strategies, since important spatially referenced ecosystem based goals and constraints can be explicitly included. Spatial considerations have seldom been incorporated directly in strategic planning models. They may be dealt with exogenously, for example by delineating habitat and protected areas prior to optimization (Nalli et al., 1996; Naesset, 1997). Model I systems such as Heureka (Wikström et al., 2011) and Simo (Rasinmäki et al., 2009) have been used in Europe, but the predominant LP form for strategic planning has been Model II.

Strategic planning as the first phase in a HFM management process is commonly used to determine a first estimate of sustainable cutting levels, or Annual Allowable Cut (AAC). This aspatial AAC is input to a subsequent tactical planning process that assesses spatial feasibility over a shorter planning horizon as an AAC target. Increased spatial resolution in the strategic planning model will improve this estimate, strengthening the linkage between strategic and tactical planning phases of HFM.

In addition to the inherent likelihood of sub-optimal solutions from systems of disaggregated models, there remains uncertainty about long term ecosystem sustainability when spatial considerations are assessed on short time horizons and, sometimes, on a subset of the forest. Other concerns about the divergence between planned and implemented forest operations have been noted. Spatial strategic planning models can reduce the gap between strategic planning model outputs and tactical or
implemented plans, and hence contribute to improving HFM processes. Paradis et al. (2013) noted that adding increased detail in forest product demands in strategic level models increased coherence between long and short term harvest planning solutions significantly. In the same way, increasing spatial detail in strategic planning models reduces the gap between strategic and tactical management solutions by better forecasting timber harvests that depend on location relative to demand points, ecosystems, watersheds, and communities.

This work has shown that a Model I framework is a more suitable modelling framework for representing spatial strategic forest management than the more commonly used Model II. Future work will investigate performance on larger forests and determining best structures and modelling for larger problem instances. Additional spatial considerations such as supply chain modelling will also be investigated.
Acknowledgements

We acknowledge the Nova Scotia Department of Natural Resources (NSDNR) for their indispensable support for this research. This case-study was made possible through collaboration with the NSDNR; they provided the stand inventory, yield data, a copy of their Woodstock™ model, and numerous discussions during model development. It remains that all analysis and conclusions in this work are the sole responsibility of the authors.

Funding for this project was provided by the National Sciences and Engineering Research Council, The VCO network, Dalhousie University, and the University of New Brunswick.
References


Hardwood - Intolerant hardwood
Hardwood - Mixed intolerant/tolerant hardwood
Hardwood - Tolerant hardwood
Mixedwood - Intolerant hardwood leading softwood
Mixedwood - Softwood leading intolerant hardwood
Mixedwood - Tolerant hardwood leading
Softwood - Red and black spruce dominant
Softwood - White Spruce dominant
Softwood - Balsam Fir dominant
Softwood - Spruce and Balsam Fir dominant
Softwood - Hemlock, Pine, and Spruce dominant
Plantation - White Spruce
Plantation - Red Spruce
Plantation - Pine
Plantation - Exotic species

Table 1: Species Associations in the Crown Central Forest, Nova Scotia, Canada
<table>
<thead>
<tr>
<th>Prescription type</th>
<th>Age range</th>
<th>Number of options Phase 1</th>
<th>Phase 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Clearcut</td>
<td>55+</td>
<td>80</td>
<td>80</td>
</tr>
<tr>
<td>Shelterwood</td>
<td>60 &amp; 70</td>
<td>1</td>
<td>18</td>
</tr>
<tr>
<td>Thinning</td>
<td>15 - 95</td>
<td>45</td>
<td>96</td>
</tr>
<tr>
<td>Selection</td>
<td>80</td>
<td>6</td>
<td>11</td>
</tr>
<tr>
<td>Bu er</td>
<td>60</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 2: Phase 2 Model I Prescriptions
## Constraints

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Description</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Base</td>
<td>no</td>
<td>no</td>
<td>no</td>
<td>no</td>
<td>no</td>
<td>no</td>
</tr>
<tr>
<td>2</td>
<td>Timber Constraints</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
<td>no</td>
<td>no</td>
<td>no</td>
</tr>
<tr>
<td>3</td>
<td>Ecodistrict Constraints</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
<td>no</td>
</tr>
<tr>
<td>4</td>
<td>Watershed Constraints</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
</tr>
</tbody>
</table>

Table 3: Scenario Definitions
<table>
<thead>
<tr>
<th>Scenario</th>
<th>Rows</th>
<th>Columns</th>
<th>Non-Zeroes</th>
<th>Rows</th>
<th>Columns</th>
<th>Non-Zeroes</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>100,679</td>
<td>665,381</td>
<td>60,435,407</td>
<td>260,296</td>
<td>787,506</td>
<td>1,700,820</td>
</tr>
<tr>
<td>2</td>
<td>100,910</td>
<td>665,381</td>
<td>60,435,959</td>
<td>262,723</td>
<td>788,952</td>
<td>8,927,070</td>
</tr>
<tr>
<td>3</td>
<td>106,190</td>
<td>665,381</td>
<td>60,446,519</td>
<td>511,513</td>
<td>1,360,729</td>
<td>60,268,649</td>
</tr>
<tr>
<td>4</td>
<td>106,790</td>
<td>665,381</td>
<td>60,447,119</td>
<td>722,002</td>
<td>1,823,163</td>
<td>100,002,631</td>
</tr>
</tbody>
</table>

| Ratio of scenario 4 to 1 | 1.06 | 1.00 | 1.00 | 2.77 | 2.32 | 58.80 |

Table 4: Phase 1: Model I and Model II Matrix Sizes
<table>
<thead>
<tr>
<th>Scenario</th>
<th>Rows</th>
<th>Columns</th>
<th>Non-Zeros</th>
<th>Rows</th>
<th>Columns</th>
<th>Non-Zeros</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>100,679</td>
<td>768,427</td>
<td>71,227,495</td>
<td>322,852</td>
<td>821,549</td>
<td>1,646,491</td>
</tr>
<tr>
<td>2</td>
<td>100,910</td>
<td>768,427</td>
<td>71,228,047</td>
<td>325,279</td>
<td>822,995</td>
<td>6,999,298</td>
</tr>
<tr>
<td>3</td>
<td>106,190</td>
<td>768,427</td>
<td>71,238,607</td>
<td>738,450</td>
<td>1,855,004</td>
<td>59,330,090</td>
</tr>
<tr>
<td>4</td>
<td>106,790</td>
<td>768,427</td>
<td>71,239,207</td>
<td>1,156,015</td>
<td>2,900,105</td>
<td>120,417,153</td>
</tr>
</tbody>
</table>

| Ratio scenario 4 to 1 | 1.06 | 1.00 | 1.00 | 3.58 | 3.53 | 73.14 |

Table 5: Phase 2: Model I and Model II Matrix Sizes
<table>
<thead>
<tr>
<th>Scenario</th>
<th>Model I CPU</th>
<th>Model I Objective $10^8$ m$^3$</th>
<th>Model II CPU</th>
<th>Model II Objective $10^8$ m$^3$</th>
<th>Percent of Model II</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>417.81</td>
<td>4.715</td>
<td>4.41</td>
<td>4.79</td>
<td>9474.15</td>
</tr>
<tr>
<td>2</td>
<td>1623.81</td>
<td>3.788</td>
<td>603.44</td>
<td>3.869</td>
<td>269.09</td>
</tr>
<tr>
<td>3</td>
<td>1615.75</td>
<td>3.574</td>
<td>9754.32</td>
<td>3.744</td>
<td>16.56</td>
</tr>
<tr>
<td>4</td>
<td>2488.03</td>
<td>3.573</td>
<td>12,032.67</td>
<td>3.744</td>
<td>20.68</td>
</tr>
</tbody>
</table>

Table 6: Phase 1 Comparisons

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Model I CPU</th>
<th>Model I Objective $10^8$ m$^3$</th>
<th>Model II CPU</th>
<th>Model II Objective $10^8$ m$^3$</th>
<th>Percent of Model II</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>512.14</td>
<td>5.296</td>
<td>5.27</td>
<td>5.38</td>
<td>9718.03</td>
</tr>
<tr>
<td>2</td>
<td>1528.38</td>
<td>4.294</td>
<td>269.44</td>
<td>4.191</td>
<td>567.24</td>
</tr>
<tr>
<td>3</td>
<td>1798.42</td>
<td>4.126</td>
<td>7170.34</td>
<td>4.119</td>
<td>25.08</td>
</tr>
<tr>
<td>4</td>
<td>2541.74</td>
<td>4.126</td>
<td>20,480.18</td>
<td>4.119</td>
<td>12.41</td>
</tr>
</tbody>
</table>

Table 7: Phase 2 Comparisons
<table>
<thead>
<tr>
<th>Period*</th>
<th>Violation (ha)</th>
</tr>
</thead>
<tbody>
<tr>
<td>11</td>
<td>1586.5</td>
</tr>
<tr>
<td>12</td>
<td>954.4</td>
</tr>
<tr>
<td>13</td>
<td>671.6</td>
</tr>
<tr>
<td>14</td>
<td>372.3</td>
</tr>
<tr>
<td>15</td>
<td>314.9</td>
</tr>
<tr>
<td>16</td>
<td>82.4</td>
</tr>
<tr>
<td>17</td>
<td>60.1</td>
</tr>
<tr>
<td>18</td>
<td>22.7</td>
</tr>
<tr>
<td>19</td>
<td>4.4</td>
</tr>
<tr>
<td>20</td>
<td>4.6</td>
</tr>
<tr>
<td>21</td>
<td>38.0</td>
</tr>
<tr>
<td>22</td>
<td>5.7</td>
</tr>
<tr>
<td>23</td>
<td>20.6</td>
</tr>
<tr>
<td>24</td>
<td>59.9</td>
</tr>
<tr>
<td>25</td>
<td>84.8</td>
</tr>
<tr>
<td>26</td>
<td>169.9</td>
</tr>
<tr>
<td>27</td>
<td>356.9</td>
</tr>
<tr>
<td>28</td>
<td>999.6</td>
</tr>
<tr>
<td>29</td>
<td>2417.9</td>
</tr>
<tr>
<td>30</td>
<td>4039.9</td>
</tr>
</tbody>
</table>

Table 8: Per Period Violations on Model I Phase 2 Scenario 3 Model
*The ecosystem constraints these violations are associated with are not applied prior to period 11.
Sets

\[ I \] Stands
\[ P \] Prescriptions
\[ Y \] Yields
\[ T \] Periods
\[ W \] Watersheds

Parameters

\[ \text{area}_i \quad i \in I \] area of stand \( i \)
\[ y_{ijkt} \quad i \in I, j \in P, k \in Y, t \in T \] yield of type \( k \) in \( t \) from \( i \) under \( j \)
\[ c_{ij} \quad i \in I, j \in P \] benefit of applying \( j \) to \( i \)

Variables

\[ x_{ij} \quad i \in I, j \in P \] Area of \( i \) assigned to \( j \)
\[ Q_{kt} \quad k \in Y, t \in T \] Yield of type \( k \) to be achieved in \( t \)

Objective

\[
\max \sum_{i \in I, j \in P} c_{ij} x_{ij}
\]

Constraints

\[
\sum_{j \in P} x_{ij} = \text{area}_i \quad i \in I \quad \text{area accounting constraints}
\]
\[
\sum_{i \in I, j \in P} y_{ijkt} x_{ij} \geq Q_{kt} \quad k \in Y, t \in T \quad \text{yield constraints}
\]

---

**Figure 1.** Generic Model I LP Formulation
Sets
\( I \) Stands
\( K \) Interventions
\( T \) Periods
\( S \) Spatial Strata

Parameters
\( area_i \) \( i \in I \) area of stand \( i \)
\( U \) ending inventory condition
\( c_{iak} \) \( i \in I, a \in T, k \in K \) benefit of harvesting \( i \) in \( a \) using \( k \)
\( d_{abk} \) \( a \in T, b \in T, k \in K \) benefit of harvesting area regenerated in \( a \) in \( b \) using \( k \)
\( \zeta \) \( a \in T \) ending inventory condition for stands harvested in \( a \)

Variables
\( y_{abk} \) \( a \in T, b \in T, k \in K \) Area regenerated in \( a \) and harvested in \( b \) using \( k \)
\( x_{iak} \) \( i \in I, a \in T, k \in K \) Area of \( i \) harvested in \( a \) using \( k \)
\( u_a \) \( a \in T \) area regenerated in \( a \), not harvested again

Objective
\[
\max \sum_{i \in I, a \in T, k \in K} c_{iak} x_{iak} + \sum_{a \in T, b \in T, k \in K} d_{abk} y_{abk}
\]

Constraints
\[
\sum_{i \in I, k \in K} x_{iak} + \sum_{b \in T, k \in K} y_{abk} - \sum_{j \in T, k \in K} y_{ajk} - u_a = 0 \quad a \in T \quad \text{transfer constraints}
\]
\[
\sum_{a \in T} \zeta a \cdot u_a \geq 0 \quad a \in T \quad \text{ending inventory constraints}
\]
\[
\sum_{a \in T, k \in K} x_{iak} = area_i \quad i \in I \quad \text{area accounting constraints}
\]

Figure 2. Generic Model II LP Formulation
Figure 3. Model II Acyclic Network (Source: Gunn, 2010)
**Figure 4.** The Crown Central Forest. Located in Nova Scotia, Canada
**Figure 5.** Section of central Nova Scotia showing ecodistricts (Source: NSDNR, 2007)

**Figure 6.** Section of central Nova Scotia showing watersheds (Source: NSENV, 2011)
Figure 7. Model I LP Formulation: Sets, Parameters and Variables
Figure 8. Phase 1 Model I and Model II solution times
Figure 9. Phase 2 Model I and Model II solution times
Figure Captions

Figure 1. Generic Model I LP Formulation
Figure 2. Generic Model II LP Formulation
Figure 3. Model II Acyclic Network (Source: Gunn, 2010)
Figure 4. The Crown Central Forest. Located in Nova Scotia, Canada
Figure 5. Section of central Nova Scotia showing ecodistricts (Source: NSDNR, 2007)
Figure 6. Section of central Nova Scotia showing watersheds (Source: NSENV, 2011)
Figure 7. Model I LP Formulation: Sets, Parameters and Variables
Figure 8. Phase 1 Model I and Model II solution times
Figure 9. Phase 2 Model I and Model II solution times