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Some Technical Issues and Horizon Problem

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Abstract

In this paper, we explore some technical issue in the study of spherical system coupled with isotropic matter distribution. We also study horizon problem under the influence of higher curvature quantities. We evaluated well-known equations of motion. We also join exterior region with an interior one by taking well-known matching conditions. We first find total matter content within the system and then calculate couple of horizons. We have developed technique for educating some dynamical variables which are involved in controlling the time duration of the emergence of horizons as well as singularities.

Keywords: Relativistic systems; Modified gravity.

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1 Introduction

Recent cosmic indications coming from reliable and authenticated sources, e.g., Supernovae type Ia, large scale structure, cosmic microwave background etc has opened a new window of research field. They evidenced that our

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cosmos, in the present epoch, is in the phase of accelerated expansion [1, 2, 3]. Relativistic astrophysicists suggested that an mysterious energy is responsible for this counteracting nature of gravity. They dubbed this as a dark energy. General relativity (GR), besides of its many achievements, cannot give a smooth way to solve this puzzle. In this setting, modified gravities occupied much attentions in the field of gravitational physics. The $f(R)$ gravity [4], which is one of the modified gravities, could be helpful to understand mysterious nature of the expanding universe. This theory is obtained by using a general Ricci scalar function instead of only Ricci scalar in the well-known Einstein-Hilbert (EH) action. The quadratic Ricci scalar corrections in EH give rise to field equation that could assist enough to understand the impact of dark energy (DE) on various cosmological issues. Such kind of corrections were proposed firstly by Starobinsky [5].

Gravitational implosion of spherical, planar and cylindrical systems occupy keen interest among researchers of relativity. Vaidya [6] and Santos [7] investigated collapsing models of dissipative stellar system. Afterwards, large number of people investigated influence of different fluid variables on the formation and their dynamics of imploding matter distributions. Bonnor and Santos [9] studied the role of locally non-anisotropic pressure gradient on the relativistic spheres. Tiwari and Ray [10] explored some solutions of stellar objects coupled relativistic matter distributions and found that all fluid dynamics can be written by means of electric charge. Herrera et al. [11] calculated some well-known equations of fluid velocity along the radial coordinate and investigated collapsing nature of stellar structures.

Di Prisco et al. [12] discussed some exact solutions of collapsing spherical system coupled with anisotropic as well as isotropic fluids by invoking some conditions on mass function. Usmani et al. [13] discussed curvature singularity emergence in the gravitational implosion of stellar structures. Herrera et al. [14] studied irregularity factors in the implosion of spherical cloud. Sharif and his team explored effects of dynamical variables in the collapse of spherical [15, 16], cylindrical [17, 18], planar [19] and axial [20] relativistic bodies in modified gravity. Pinheiro and Chan [21] analyzed impact of some physical factors on the dynamics of stellar collapsing system. Arbuzova et al. [22] explored spherically symmetric collapsing models and found that modified corrections in GR produce repulsive force among massive compact objects.

Roshan and Abbassi [23] did Jeans analysis to discuss the dynamics of stellar collapsing systems and concluded that higher degrees of freedom may

host galaxy configurations different from that of GR. Farinelli et al. [24] found numerical models of extended Lane-Emden equation and proposed that extra curvature corrections could lead to decrease instability of celestial structures. Kusaka [25] discussed dynamics of plane symmetric dust cloud. Chakrabarti and Banerjee [26] analyzed constant Ricci curvature gravitational implosion in metric $f(R)$ gravity and found BH depending upon the initial configurations of the relativistic geometry. Sheykhi et al. [27] explored cylindrical stellar system and found some generating solutions of the field equations.

The effects of extra curvature or dark source terms on the stability and evolution of stellar objects has gained much attractions. In this respect, a lot of work has been done about the study of R^2 [28]-[38], R^4 [39]-[41] and R^n [42]-[47] corrections on different aspects of the evolving universe. Furthermore, the study of inverse Ricci power [48]-[50] models has also gained attention. It was proposed that such model could help to unify late and early universe eras. The study of polynomial [51, 52] and three parametric forms [53]-[56] of cosmic models could also assist to unveil many unknown facts of the accelerating universe. Sharif and Yousaf [40] analyzed how pressure gradients affect the the collapsing processes of self-gravitating geometries. Yousaf and Bhatti [57] explored cylindrical imploding systems and found that αR^2 gravity favors the existence of more massive objects with smaller radii. The effects of modification of gravity have been analyzed under various assumptions by many authors [58, 59, 60], thereby showing the importance of extra curvature quantities in the investigation of stellar objects.

This paper studies effects of cosmic inflationary terms on some relativistic collapsing models of Lemaître-Tolman-Bondi (LTB) spherical geometry. We also evaluated time required for the appearance of black hole and cosmological horizon, represented respectively by BHH and CH. Our manuscript is organized as follows. Section TWO presents some fundamental contents describing LTB spacetime, fluid distribution, $f(R)$ model and joining conditions. We also check how inflationary terms modify collapsing nature of the relativistic sphere. In the THIRD section, we explore horizons and their dynamical evolution. In the FOURTH section, we outlook our remarks and results.

2 LTB Solutions in $f(R)$ Gravity

The modified EH action for $f(R)$ gravity is written as follows

$$S_{f(R)} = \frac{1}{2\kappa} \int d^4x f(R) \sqrt{-g}, \quad (1)$$

where $f(R)$ is a Ricci scalar function and κ is a coupling constant. On varying the above equation with respect to metric tensor, we get the following set of metric $f(R)$ field equations

$$R_{\alpha\beta} f_R - (\nabla_\alpha \nabla_\beta - g_{\alpha\beta} \square) f_R - \frac{1}{2} g_{\alpha\beta} f = \kappa T_{\alpha\beta}, \quad (2)$$

where ∇_β is an operator for covariant derivation, while $\square \equiv \nabla_\beta \nabla^\beta$. Here $T_{\alpha\beta}$ is a standard energy-momentum tensor. We assume that LTB spherical system is filled up with a perfect fluid configurations. Its mathematical form is

$$T_{\alpha\beta} = (P + \mu) U_\alpha U_\beta - P g_{\alpha\beta}, \quad (3)$$

where P is an isotropic pressure gradient and μ is the energy density of the fluid distribution. Here, we consider that motion of the spherical stellar system is supported with comoving coordinate frame. Under this context, the four-velocity, U_β is obeying the following relation

$$U_\beta = \delta_\beta^0.$$

Using Eqs.(11) and (3), we obtain

$$R_{\alpha\beta} = \frac{1}{f_R} \left[(\nabla_\alpha \nabla_\beta - g_{\alpha\beta} \square) f_R + \kappa \{ (P + \mu) U_\alpha U_\beta - P g_{\alpha\beta} \} + \frac{g_{\alpha\beta}}{2} f(R) \right]. \quad (4)$$

We consider a three dimensional boundary surface over a four dimensional spherical LTB relativistic system. We denote this boundary with Δ notation. The line element of interior region is given by

$$ds_-^2 = dt^2 - A^2(t, r) dr^2 - H^2(t, r) (d\theta^2 + \sin^2 \theta d\phi^2), \quad (5)$$

while exterior to Δ is

$$ds_+^2 = \left(1 - \frac{2M}{\mathbb{R}} \right) d\nu^2 + 2d\nu d\mathbb{R} - \mathbb{R}^2 (d\theta^2 + \sin^2 \theta d\phi^2), \quad (6)$$

where ν is a retardation time, while M is total black hole mass. The junction conditions occupy central importance in the matching of two distinct regions of the spacetimes. This approach assists to find some collapsing relativistic exact analytical solutions of the field equations. For the continuous matching of interior as well as exterior regions, we shall use following couple of junctions conditions:

- (i) Darmois [61] junction conditions
- (ii) Senovilla [62] junction conditions

These conditions collectively give the some important constraints that the system must satisfy over Δ . These are as follows

$$R|_{-}^{+} = 0, \quad f_{,RR}[\partial_{\nu}R]_{-}^{+} = 0, \quad f_{,RR} \neq 0, \quad (7)$$

$$M \stackrel{\Omega}{=} \frac{H}{2} \left(1 + \frac{Q^2}{H^2} + \dot{H}^2 - \frac{H'^2}{A^2} \right), \quad P \stackrel{\Omega}{=} \frac{-\alpha R^2}{2\kappa}. \quad (8)$$

In any gravitational theory, it is very significant to realize the structure and its evolution in the solutions of spherical spacetimes, specifically, for black holes. In metric version of $f(R)$ gravity, the solutions of field equations are directly related to $f(R)$ model. It is seen from the literature that there exists some $f(R)$ which are unviable and not consistent with solar system and cosmological tests. Thus, it is very crucial to use an appropriate form of $f(R)$ model. Here, we are taking a well-known $f(R)$ model which is given as follows [5]

$$f(R) = R + \alpha R^2. \quad (9)$$

This model was firstly proposed by Starobinsky for the discussion of inflationary epochs of cosmos. Later, it was suggested that this model can be considered for dark matter [63] contents, for details see [64, 65]. In the above equation, α is a positive constant. Its positivity is required to make this model physically viable.

The trace of Eq.(11) provides

$$f_R R - 2f(R) + 3\Box f_R = \kappa T. \quad (10)$$

It is mentioned that there is a de-Sitter point that is related with a vacuum solution, i.e., ($T = 0$). Here the Ricci curvature scalar is constant. The quantity $f_R(R)$ is zero at this point, therefore

$$f_R R - 2f(R) = 0. \quad (11)$$

It is pertinent to mention that the model $f(R) = \alpha R^2$ obeys this constraint and provides the exact model of de-Sitter [66]. In $f(R) = R + \alpha R^2$, due to the Ricci scalar linear term, the inflationary cosmic expansion terminates whenever αR^2 reduces to become smaller as compared to this linear mathematical quantity. This is accompanied with a reheating phase in which the Ricci scalar perturbation yields the phenomenon of particle production. It is worthy to stress that the de-Sitter point could be used as a substitute for dark energy. However, models without introducing dark energy effects coming from cosmological constant have been studied by Cruz-Dombriz and Dobado [67].

Equations (4), (5) and (18) give the following set of Ricci tensor components

$$R_{00} = -\frac{\ddot{A}}{A} - 2\frac{\ddot{H}}{H} = \frac{1}{1+2\alpha R} \left[\kappa\mu + \frac{R}{2} + \frac{\alpha R^2}{2} - \left\{ \frac{2\alpha R''}{A^2} + \left(\frac{\dot{A}}{A} + 2\frac{\dot{H}}{H} \right) \right. \right. \\ \left. \left. \times 2\alpha\dot{R} + \left(\frac{A'}{A} - 2\frac{H'}{H} \right) \frac{2\alpha R'}{H^2} \right\} \right], \quad (12)$$

$$R_{01} = \frac{\dot{H}'}{H} - \frac{\dot{A}}{A} \frac{H'}{H} = \frac{\alpha}{(1+2\alpha R)} \left(\dot{R}' - \frac{\dot{A}}{A} R' \right), \quad (13)$$

$$R_{11} = A\ddot{A} + 2A\dot{A}\frac{\dot{H}}{H} - \frac{2}{H} \left(H'' - \frac{A'H'}{A} \right) = \frac{A^2}{(1+2\alpha R)} \left[\kappa P + 2\alpha\ddot{R} \right. \\ \left. - \frac{R}{2} - \frac{\alpha R^2}{2} - 4\frac{\alpha H'R'}{A^2 H} + 4\frac{\dot{H}\alpha\dot{R}}{H} \right], \quad (14)$$

$$R_{22} = H\ddot{H} + \dot{H}^2 + \frac{H\dot{A}\dot{H}}{A} - \frac{1}{A^2} \left(HH'' + H'^2 - \frac{HA'H'}{A} - A^2 \right) \\ = \frac{-H^2}{1+2\alpha R} \left[\frac{R}{2} + \frac{\alpha R^2}{2} - \kappa P + \left\{ -2\alpha\ddot{R} + \frac{2\alpha R''}{A^2} - \left(\frac{\dot{H}}{H} + \frac{\dot{A}}{A} \right) 2\alpha\dot{R} \right. \right. \\ \left. \left. + \left(\frac{H'}{H} - \frac{A'}{A} \right) \frac{2\alpha R'}{A^2} \right\} \right]. \quad (15)$$

The integration of Eq.(13) provides

$$A = \exp \int \frac{\dot{H}'(1+2\alpha R) + H\alpha\dot{R}}{H\alpha R' + H'(1+2\alpha R)} dt. \quad (16)$$

In view of present Ricci scalar invariant value, we obtain an explicit expression of A as

$$A = \exp \int \frac{\dot{H}'}{H'} dt. \quad (17)$$

Now, we restrict my analysis by assuming constant curvature condition in the inflationary $f(R)$ model. In this realm, the non-vanishing components of Ricci tensor are given by

$$-\frac{\ddot{A}}{A} - 2\frac{\ddot{H}}{H} = \frac{1}{1+2\alpha\tilde{R}} \left(\kappa\tilde{\mu} + \frac{(\tilde{R} + \alpha\tilde{R}^2)}{2} \right), \quad \frac{2\dot{H}'}{H} = \frac{2\dot{A}}{A} \frac{H'}{H}, \quad (18)$$

$$\frac{2}{A^2} \left(\frac{H''}{H} - \frac{A'H'}{AH} \right) - \frac{\ddot{A}}{A} - 2\frac{\dot{A}\dot{H}}{AH} = \frac{1}{1+2\alpha\tilde{R}} \left(-\kappa\tilde{P} + \frac{\tilde{R} + \alpha\tilde{R}^2}{2} \right), \quad (19)$$

$$\begin{aligned} & \frac{1}{A^2} \left\{ \frac{H''}{H} + \frac{H'}{H} \left(\frac{H'}{H} - \frac{A'}{A} \right) - \frac{A^2}{H^2} \right\} - \frac{\ddot{H}}{H} - \frac{\dot{H}}{H} \left(\frac{\dot{H}}{H} + \frac{\dot{A}}{A} \right) \\ &= \frac{1}{1+2\alpha\tilde{R}} \left(\frac{\tilde{R} + \alpha\tilde{R}^2}{2} - \kappa\tilde{P} \right), \end{aligned} \quad (20)$$

where $\tilde{X}$ indicates that quantity X is evaluated by assuming today Ricci curvature constraint.

Now, the solution of Eq.(17) yields

$$A = \frac{H'}{\zeta}, \quad (21)$$

where $\zeta = \zeta(r)$ is an integration function. It is seen that there exists an isotropic pressure in Eqs.(19) and (20). In order to eliminate P , we compare Eqs.(18) and (20), then manipulate resultant equation along with Eqs.(18) and (21), we obtain

$$\frac{\dot{H}^2}{H^2} - (\zeta^2 - 1) \frac{1}{H^2} + \frac{2\ddot{H}}{H} = -\frac{1}{2(1+2\alpha\tilde{R})} \left\{ \kappa(\tilde{\mu} - \tilde{P}) - \tilde{R} - \alpha\tilde{R}^2 \right\}. \quad (22)$$

This is a second order partial differential equation. The general solution of this is given as follows

$$\dot{H}^2 = \zeta^2 - 1 - \frac{1}{2(1+2\alpha\tilde{R})} \left\{ \kappa(\tilde{\mu} - \tilde{P}) - \tilde{R} - \alpha\tilde{R}^2 \right\} + 2\frac{m}{H}, \quad (23)$$

where $m = m(r)$ is an integration function. This function has a link with the LTB spherical mass function. Therefore, m is assumed to be non-zero and non-negative. By making use of Eqs.(18), (21) and (23), we obtain

$$m' = \frac{1}{2(1+2\alpha\tilde{R})} \left(\kappa\tilde{\mu} - \frac{1}{3}\kappa\tilde{P} \right) H^2 H'.$$

The solution of above equation gives

$$m(r) = \frac{1}{2(1+2\alpha\tilde{R})} \left(\kappa\tilde{\mu} - \frac{1}{3}\kappa\tilde{P} \right) \int_0^r H^2 H' dr + B(t), \quad (24)$$

where B is a temporal function of integration. The simultaneous manipulation of Eqs.(21) and (23) over the hyper surface Δ gives

$$M \triangleq \frac{Q^2}{2H} + m(r) + \frac{H^3}{1+2\alpha\tilde{R}} \left\{ \kappa(\tilde{P} - \tilde{\mu}) - \tilde{R} - \alpha\tilde{R}^2 \right\}. \quad (25)$$

The general formula for obtaining the total mass of the spherical relativistic fluid distribution was proposed by Misner and Sharp [68]. This formula is given by

$$\mathcal{M}(t, r) = \frac{H}{2} (1 - g^{\mu\nu} H_{,\mu} H_{,\nu}),$$

where H is a parameter controlling the radial distribution of the spherically symmetric spacetime. For interior manifold (5), the mass function turns out to be

$$\mathcal{M}(t, r) = \frac{H}{2} \left\{ 1 + \dot{H}^2 - \frac{H'^2}{A^2} \right\}. \quad (26)$$

The LTB spherical mass function introduced by Misner-Sharp formula, $\mathcal{M}$ and that one we get from the integration function, m can be related as follows

$$\mathcal{M} = m(r) + \frac{H^3}{12(1+2\alpha\tilde{R})} \left\{ \kappa(\tilde{P} - \tilde{\mu}) - \tilde{R} - \alpha\tilde{R}^2 \right\}. \quad (27)$$

By making use of Eqs.(21) and (23), the exact analytical solutions of the metric $f(R)$ field equations are

$$A = \left(\frac{12m(1+2\alpha\tilde{R})}{\kappa(\tilde{P} - \tilde{\mu}) - \tilde{R} - \alpha\tilde{R}^2} \right)^{\frac{1}{3}} \left[\frac{1}{3} \sqrt{\frac{\kappa(\tilde{P} - \tilde{\mu}) - \tilde{R} - \alpha\tilde{R}^2}{12m'(\tilde{R} + \alpha\tilde{R}^2)}} \times \sinh^{\frac{2}{3}} \psi(t, r) \right]$$

$$+ \frac{t'_0}{3} \sqrt{\frac{3}{2(1+2\alpha\tilde{R})}} \left\{ \kappa(\tilde{P} - \tilde{\mu}) - \tilde{R} - \alpha\tilde{R}^2 \right\} \sinh^{\frac{-1}{3}} \psi(t, r) \cosh \psi(t, r) \right], \quad (28)$$

$$H = \left(\frac{12m(1+2\alpha\tilde{R})}{\kappa(\tilde{P} - \tilde{\mu}) - \tilde{R} - \alpha\tilde{R}^2} \right)^{\frac{1}{3}} \sinh^{\frac{2}{3}} \psi(t, r), \quad (29)$$

where

$$\psi = \frac{1}{2} \sqrt{\frac{3}{2(1+2\alpha\tilde{R})}} \left\{ \kappa(\tilde{P} - \tilde{\mu}) - \tilde{R} - \alpha\tilde{R}^2 \right\} \times [t_0(r) - t], \quad (30)$$

t_0 is function of r . It is worth mentioning that this function is very important in the in the analysis of the emergence of BH singularity. For the physically acceptable model, we require that $\frac{\kappa\tilde{P}}{1+2\alpha\tilde{R}} > \frac{\kappa\tilde{\mu}}{1+2\alpha\tilde{R}} + \tilde{R} + \alpha\tilde{R}^2$ along with $C = 1$.

The selection of $f(R)$ is very crucial. It has been observed by many relativistic astrophysicists that the MGs incorporated with inverse curvature powers is very helpful in the discussion of late universe evolution. It could serve as a purely geometric DE candidate. The motivation of including $\frac{1}{R}$ term in the EH action stem from the fact that it may helps to shed light over many cosmic puzzles in low curvature cosmic eras [69]. Despite from the proposal that such models could exhibit natural explanation for the accelerating cosmic phases [70, 71] yet it could not pass terrestrial and solar system tests. Furthermore, it suffers from some instability issues [72, 73]. However, the MG theory exhibiting quadratic curvature terms [74] or $\ln R$ terms [75] could helps simpler enough to understand inflationary mechanism of the cosmos. In order to see the effects of R^2 terms on the collapsing rate of the LTB spherical collapse. We perform the following analysis. The Newtonian potential $\chi = (1 - g_{00})^{\frac{1}{2}}$ for the spacetime outer to hypersurface Δ , can be casted as

$$\chi = \frac{H^2}{6(1+2\alpha\tilde{R})} \left\{ \kappa(\tilde{P} - \tilde{\mu}) - \tilde{R} - \alpha\tilde{R}^2 \right\} + \frac{2m}{\mathbb{R}}. \quad (31)$$

It is simpler to calculate Newtonian force, $\mathbb{F}$, from the above equation

$$\mathbb{F} = -\frac{2m}{\mathbb{R}^2} + \frac{\mathbb{R}}{3(1+2\alpha\tilde{R})} \left\{ \kappa(\tilde{P} - \tilde{\mu}) - \tilde{R} - \alpha\tilde{R}^2 \right\}. \quad (32)$$

Now, we shall check the behavior of $\mathbb{F}$ in the subsequent analysis of the LTB isotropic gravitational collapse over the boundary surface. It can be seen from the above relation that the value of the Newtonian force is directly controlled by couple of parameters, over the hypersurface. These parameters are m and $\mathbb{R}$. We now study the influence of Newtonian force by assuming some special cases of these parameters.

- It is been observed that impact of $\mathbb{F}$ will be zero, if during evolution, parameters m and $\mathbb{R}$ of the LTB collapse achieve the following values.

$$m = \frac{1+2\alpha\tilde{R}}{6\sqrt{\{\kappa(\tilde{P}-\tilde{\mu})-\tilde{R}-\alpha\tilde{R}^2\}}} \text{ and } \mathbb{R} = \frac{1+2\alpha\tilde{R}}{\sqrt{\{\kappa(\tilde{P}-\tilde{\mu})-\tilde{R}-\alpha\tilde{R}^2\}}}.$$

- The $\mathbb{F}$ exhibits repulsive effects in the dynamics of the collapsing LTB star if $\mathbb{F} > 0$. If the parameters have values $m > \frac{1+2\alpha\tilde{R}}{6\sqrt{\{\kappa(\tilde{P}-\tilde{\mu})-\tilde{R}-\alpha\tilde{R}^2\}}}$ and $\mathbb{R} > \frac{1+2\alpha\tilde{R}}{\sqrt{\{\kappa(\tilde{P}-\tilde{\mu})-\tilde{R}-\alpha\tilde{R}^2\}}}$, then $\mathbb{F} > 0$. For this purpose, $\tilde{R} < \kappa(\tilde{P} - \tilde{\mu}) - \alpha\tilde{R}^2$. This depicts the importance of quadratic correction terms in determining the LTB collapse rate.
- If parameters m $\mathbb{R}$ have effects opposites to the above mentioned values then, the systems undergoes the quick collapse. This is impossible as it is indicated that in our analysis, one needs the positivity of $\kappa(\tilde{P} - \tilde{\mu}) - \tilde{R} - \alpha\tilde{R}^2$.

3 Apparent Horizons

Here, we calculate apparent horizon surface of the spherical stellar object. The most generic formulate for its evaluation over a fixed hypersurface (with normals nulls directed outward) is

$$g^{\mu\nu} H_{,\mu} H_{,\nu} = \dot{H}^2 - \frac{H'^2}{A^2} = 0. \quad (33)$$

In this context, Eqs.(21) and (23) give

$$\left\{ \kappa(\tilde{P} - \tilde{\mu}) - \tilde{R} - \alpha\tilde{R}^2 \right\} H^3 - 12m(1 + 2\alpha\tilde{R}) + 6H(1 + 2\alpha\tilde{R}) = 0. \quad (34)$$

For the specific Ricci scalar value, i.e., $\tilde{R} = \kappa(\tilde{P} - \tilde{\mu}) - \alpha\tilde{R}^2$, we have a Schwarzschild horizon, i.e., $H = 2m$. Equation (34) has the following non-negative and non-zero roots.

Case (a): If $3m < \zeta_1$, where $\zeta_1 = \sqrt{\frac{2(1+2\alpha\tilde{R})}{3\{\kappa(\tilde{P}-\tilde{\mu})-\tilde{R}-\alpha\tilde{R}^2\}}}$, then I have the following cosmological and BH horizons

$$H_c = \frac{2\sqrt{2(1+2\alpha\tilde{R})}}{\left\{\kappa(\tilde{P}-\tilde{\mu})-\tilde{R}-\alpha\tilde{R}^2\right\}^{\frac{1}{2}}} \cos \frac{\varphi}{3}, \quad (35)$$

$$H_{bh} = \frac{-2\sqrt{2(1+2\alpha\tilde{R})}}{\left\{\kappa(\tilde{P}-\tilde{\mu})-\tilde{R}-\alpha\tilde{R}^2\right\}^{\frac{1}{2}}} \left(\cos \frac{\varphi}{3} - \sqrt{3} \sin \frac{\varphi}{3}\right), \quad (36)$$

where subscripts c indicates CH and bh describes BHH, while

$$\cos \varphi = \frac{-3m}{\sqrt{2(1+2\alpha\tilde{R})}} \sqrt{\kappa(\tilde{P}-\tilde{\mu})-\tilde{R}-\alpha\tilde{R}^2}.$$

On making null m integration function in Eq.(34), I have the following values of LTB horizons

$$H_c = \frac{\sqrt{6(1+2\alpha\tilde{R})}}{\left\{\kappa(-\tilde{\mu}+\tilde{P})-\tilde{R}-\alpha\tilde{R}^2\right\}^{\frac{1}{2}}}, \quad H_{bh} = 0. \quad (37)$$

Case (b): If $3m = \zeta_1$, then the following configurations of horizon can be evaluated as

$$H_c = H_{bh} = \sqrt{\frac{2(1+2\alpha\tilde{R})}{\left\{\kappa(\tilde{P}-\tilde{\mu})-\tilde{R}-\alpha\tilde{R}^2\right\}}}. \quad (38)$$

The inequality relating couple of cosmological and BH horizons is given by

$$0 \leq H_{bh} \leq \frac{\sqrt{2(1+2\alpha\tilde{R})}}{\left\{\kappa(\tilde{P}-\tilde{\mu})-\tilde{R}-\alpha\tilde{R}^2\right\}^{\frac{1}{2}}} \leq H_c \leq \frac{2\sqrt{2(1+2\alpha\tilde{R})}}{\left\{\kappa(\tilde{P}-\tilde{\mu})-\tilde{R}-\alpha\tilde{R}^2\right\}^{\frac{1}{2}}}. \quad (39)$$

The maximum BHH area is $H^2 = \frac{(1+2\alpha\tilde{R})}{\kappa(\tilde{P}-\tilde{\mu})-\tilde{R}-\alpha\tilde{R}^2}$ and the maximum CH proper area belongs to the open interval whose end points are $\frac{\kappa(1+2\alpha\tilde{R})}{\kappa(\tilde{P}-\tilde{\mu})-\tilde{R}-\alpha\tilde{R}^2}$ and $\frac{3\kappa(1+2\alpha\tilde{R})}{\kappa(\tilde{P}-\tilde{\mu})-\tilde{R}-\alpha\tilde{R}^2}$.

Case (c): When $3m > \zeta_1$, then Eq.(34) will have no positive roots, This scenario indicates that LTB spherically symmetric objects do not have any apparent horizon.

Now, we are interested to calculate the time duration by the imploding spherical system to build its apparent horizon. For this purpose, we solve Eqs.(29) and (34). Thus, we have

$$t_n = t_0 - \frac{2\sqrt{2(1+2\alpha\tilde{R})}}{\sqrt{3\{\kappa(\tilde{P}-\tilde{\mu})-\tilde{R}-\alpha\tilde{R}^2\}}} \sinh^{-1} \sqrt{\frac{H_n}{2m} - 1}, \quad (n = 1, 2). \quad (40)$$

In the limit $\tilde{R} \rightarrow \kappa(\tilde{P}-\tilde{\mu})-\alpha\tilde{R}^2$, Eq.(40) provides

$$\frac{H_n}{2m} = \cosh^2 \psi_n, \quad (41)$$

where

$$\psi = \frac{1}{2} \sqrt{\frac{3}{2} \{\kappa(\tilde{P}-\tilde{\mu})\}} \times [t_0(r) - t_n].$$

Equations (39) and (40) indicate that $B_c \geq H_{bh}$ along with $t_{bh} \geq t_c$. The condition $t_{bh} \geq t_c$ comes due to the reason that CH always appear first to BHH.

Now, we shall find time duration in the formation of LTB singularity and its CH. Equations (35) and (36) yield

$$\frac{d}{dm} \left(\frac{H_c}{2m} \right) = \frac{1}{m} \left(-\sin \frac{\varphi}{3} \sin^{-1} \varphi + 3 \cos \frac{\varphi}{3} \cos^{-1} \varphi \right) < 0, \quad (42)$$

$$\frac{d}{dm} \left(\frac{H_{bh}}{2m} \right) = \frac{1}{m} \left(-\sin \frac{(\varphi+4\pi)}{3} \sin^{-1} \varphi + 3 \cos \frac{(\varphi+4\pi)}{3} \cos^{-1} \varphi \right) > 0. \quad (43)$$

If $t_0 - t_n = T_n$, then from Eq.(41), I have

$$\frac{dT_n}{d\left(\frac{H_n}{2m}\right)} = \frac{1}{2 \cosh \psi_n \sinh \psi_n} \times \frac{2\sqrt{2(1+2\alpha\tilde{R})}}{\sqrt{3\{\kappa(\tilde{P}-\tilde{\mu})-\tilde{R}-\alpha\tilde{R}^2\}}}. \quad (44)$$

One can get the variations of time duration with respect to temporal coordinate by using the chain rule. In this way, Eqs.(42) and (44) give

$$\frac{dT_c}{dm} = \frac{\sqrt{2(1+2\alpha\tilde{R})}}{m\sqrt{3\left\{\kappa(\tilde{P}-\tilde{\mu})-\tilde{R}-\alpha\tilde{R}^2\right\}}\sinh\psi_1\cosh\psi_1} \times \left(-\sin\frac{\varphi}{3}\sin^{-1}\varphi+3\cos\frac{\varphi}{3}\cos^{-1}\varphi\right) < 0. \quad (45)$$

Thus T_c is a monotonically decreasing m function. This indicates if there is an increment in the mass function, then there will correspondingly decrease in time gap between the LTB spherical singularity and CH emergence. Thus this time interval is inversely related to the amount matter content in the collapsing star. Similarly, using Eqs.(43) and (45), we obtain

$$\frac{dT_{bh}}{dm} = \frac{\sqrt{2(1+2\alpha\tilde{R})}}{m\sqrt{3\left\{\kappa(-\tilde{\mu}+\tilde{P})-\tilde{R}-\alpha\tilde{R}^2\right\}}\sinh\psi_1\cosh\psi_1} \left\{-\sin\frac{(\varphi+4\pi)}{3}\right. \\ \left.\times\sin^{-1}\varphi+3\cos\frac{(\varphi+4\pi)}{3}\cos^{-1}\varphi\right\} > 0, \quad (46)$$

This indicates T_{bh} to be a monotonically increasing function of m . This suggests the direct correspondence between time interval for the formation of singularity and BHH and mass function.

4 Outlook

The study of gravitational collapse under various scenarios has always been one of the fascinating phenomenon in the field of relativistic astrophysics. In this work, we have discussed the role of inflationary model in the dynamics of the collapsing LTB spherical system. In this realm, we take LTB relativistic sphere equipped with non-radiating perfect fluid configurations in the context of cosmic inflationary model. In order to join two distinct metric over the boundary, we use criteria proposed by Senovilla and Darmois. After that, we have explored some exact analytical solutions in the context of today Ricci curvature invariant, i.e., $\tilde{R}$. We analyzed rate of implosion with the help of two parameters m and $\mathbb{R}$. We calculated BHH and CH and then

discussed their formation during collapsing processes. The main results are summarized as follows:

- Due to the repulsive behavior of inflationary dark source terms, we found relatively stable system as compared to GR. This can be easily analyzed from Eq. (32). This result supports the analysis of [24, 64].
- As the system undergoes in the collapse, it is seen that CH is first to appear than that of BH. It is worthy to stress that dark source terms of $f(R)$ model tends to lessen down the BHH as well as CH formation.
- Equations (45) and (46) indicate that horizons are also first to form than BH singularity. This sparks that there will be covered singularity in the LTB spherical perfect fluid collapse. This validates cosmic censorship conjecture in quadratic Ricci scalar gravity.
- Constant α quadratic curvature corrections induce same effects as that of the cosmological constant, Λ . This can be seen from Eqs. (31), (32), (45) and (46). This result is consistent with [64].

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