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Energy distribution of electrons under axial motion in a quadrupole Penning trap

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Abstract: We present results of the energy distribution function of trapped electrons through a measurement on their axial oscillation in a quadrupole Penning trap. Electrons emitted from a thoriated tungsten filament with a range of energies (3-5 eV) are trapped in a low magnetic field quadrupole Penning trap. Subsequent to storage the trapped electrons are detected through monitoring their axial oscillations by an electronic tank circuit that is weakly coupled to the trap, by resonant energy transfer to the electrons. The technique developed using LabView enables a direct measurement of the energy distribution function of the electrons in the trap. We obtain a normal distribution of energy, with a maximum that coincides with the potential applied on the electron filament, indicating that the energy distribution of electrons in the trap reflects the energy distribution of the thermionically emitted electrons from the filament.

Key words: Ion Traps, Penning Trap, Quadrupole Traps, Non-neutral Plasma, Electron energy distribution

1. Introduction

A quadrupole Penning trap[1-4] confines charged particles through combining a static electric quadrupole potential $\Phi$ with a uniform magnetic field $B$ along the symmetry axis of the trap. The quadrupole potential governing single particle motion is described by:

$$\Phi = \frac{U_0}{2a^2} (2z^2 - r^2)$$

where $U_0$ is the applied potential, $r$ and $z$ are the radial and axial co-ordinates respectively, and $a$ is the characteristic dimension of the trap.

Figure 1: Schematic of the Penning trap
Fig. 1 shows the electric and magnetic field lines in the trap. The degrees of freedom in the $x$ and $y$ directions are coupled (radial motion), resulting in coupled equations of motion of the ion. On the other hand, motion of the particle along the $z$ direction is uncoupled (axial motion). The solutions of the equations are obtained by a frame transformation, yielding the characteristic frequencies of motion for a single ion/electron:\[5,6\]:

$$\omega_z = \sqrt{\frac{2qU_0}{md^2}}$$

(2)

$$\omega_+ = \frac{\omega_c}{2} + \sqrt{\frac{\omega_c^2}{4} - \omega_0^2}$$

(3)

$$\omega_- = \frac{\omega_c}{2} - \sqrt{\frac{\omega_c^2}{4} - \omega_0^2}$$

(4)

where $q$ and $m$ are the charge and mass of the ion respectively, $\omega_z$ is the axial frequency of motion due to the harmonic potential, $\omega_c$ the cyclotron frequency, $\omega_+$ the modified cyclotron frequency, $\omega_-$ the magnetron frequency representing a drift in the radial plane around the centre of the trap, $d$ is the characteristic dimension of the trap given by $2d^2 = r_0^2 + 2z_0^2$; $r_0$ being the radius of the ring electrode and $2z_0$, the distance between the two end cap electrodes. Note that the axial oscillation has no dependence on the magnetic field. For stable solutions, we require $\omega_c \geq 2\omega_z$. In a real trap with a cloud of electrons, the radial and axial motion of the electrons are coupled due to coulomb interactions as well as perturbations in the trap potential that originate from imperfections in the trap geometry, leading to higher order frequencies in addition to the single particle frequencies described here.\[5\]

Penning traps are widely used as devices for confining non-neutral plasmas and exist in many geometries. An abundance of literature exists on Penning traps of different geometries and the relative suitability of these devices in confining non-neutral plasma \[7-12\]. The effects of individual and collective particle oscillations of the electron plasma have been investigated in confinements in Penning traps\[10-12\] and for other ions in Paul traps\[13,14\].

It is desirable in characterizing the trapped plasma in such traps, to measure the energy distribution of the plasma amongst the different degrees of freedom of motion of the particles, or the nature of the spatial distribution of the plasma within the trap. Measurements through laser fluorescence of trapped ions in a quadrupole Paul trap\[15,16\] reveal for ions under thermal equilibrium achieved from collisions with a buffer gas, a Gaussian form of spatial distribution of the ions from trap centre. Druyvesteyn et al.\[17\] have used Langmuir probes in the study of the Electron energy distribution functions. In their work measuring the electron energy distribution function in Tokamak edge plasma, Tsv K. Popov et al. use a Langmuir probe to obtain bi-Maxwellian distributions\[18,19\].

In this work we present a technique, that relies on measuring the number of electrons at the centre of the trap, at different trapping potentials $U_0$\[20\]. This is carried out through monitoring the axial motion of the electrons. This measure of the number of electrons at the centre of the
trap as a function of the voltage $U_0$, results in a graph of the storage potential versus electron detection signal area. The area under the signal is proportional to the number of electrons\textsuperscript{[20, 21]}. This is similar to the I-V curves obtained using Langmuir probes\textsuperscript{[17]}. The detection of electrons is achieved by coupling the trap to a weakly excited tank circuit (detection circuit) and is described in earlier work\textsuperscript{[20]}. The typical storage time of electrons in the quadrupole Penning trap in our experiments being in the range of hundreds of milliseconds, is much greater than the time for the electrons entering the trap to achieve thermal equilibrium (about 10 micro seconds, as shown later).

**Theory:** The resistively heated thoriated tungsten filament emits electrons that enter the trap volume through a small orifice in one of the end caps. The filament bias is varied between 3-5 volts, resulting in electrons entering the trap having different energy distributions centered around the bias potential and trapped subsequently. Since the electrons possess energy of a few eV the electron temperature $T_e$ is much greater than that of the surrounding neutral gas molecules, the latter being at ambient temperature (300K). Thus the electron plasma constitutes a non-thermal plasma under Local Thermodynamic Equilibrium (LTE). Thermal equilibrium by electron-electron collisions is achieved over a time\textsuperscript{[22]}

$$\tau_{ee} = \frac{16 \pi \varepsilon_0^2 m_e^{1/2} \left( k_B T_e \right)^{3/2}}{n_e e^4 \ln \Lambda}$$

(1)

where, $\varepsilon_0$ is the Permittivity of free space, $m_e$, Mass of an Electron, $k_B$, Boltzmann constant, $T_e \approx 10^4 K$, the electron temperature, $e$, the charge of an electron, $n_e$, electron number density, $\ln \Lambda$ is the Coulomb logarithm [and is a weak function of temperature and density] wherein $\Lambda \approx 9 N_D$, $N_D$ being the Plasma parameter that describes the number of particles in the Debye sphere, and is given by

$$N_D = \frac{4}{3} \pi n_e \lambda_D^3$$

(2)

where, $\lambda_D = $ Debye radius,

$$\lambda_D = \sqrt{\frac{\varepsilon_0 k_B T_e}{n_e e^2}}$$

(3)

Number density $n_e$ is in our case\textsuperscript{[22]},

$$n_e = 1.4813 \times 10^{15}/m^3$$

This yields,

$$\lambda_D = 1.129 \times 10^{-4} m.$$

Thus, in our case we obtain, $\tau_{ee} \approx 10 \mu s$. This is much smaller than the time over which the measurements are carried out (few milliseconds) and hence we are probing the regime wherein the electron plasma has achieved LTE.

The distribution function may be obtained as follows: The number of electrons in the trap with energies up to $eV$ is given by
\[ N = e \int_0^{eV} f(E) \, dE, \]

where, \( f(E) \) is the distribution function of the electrons at an energy \( E \)

Similarly, the number of electrons from energy up to voltage \( V' = V - \Delta V \) is

\[ N' = e \int_0^{e(V-\Delta V)} f(E) \, dE \]

The reduction in number of trapped electrons, when \( V \) changes to \( V' \) is given by

\[ \Delta N = e \int_0^{eV} f(E) \, dE - e \int_0^{e(V-\Delta V)} f(E) \, dE \]

Taylor expanding around \( eV \), and considering only first order term, as \( \Delta E \to 0 \) yields

\[ \lim_{\Delta V \to 0} \frac{\Delta N}{\Delta V} = \frac{dN}{dV} = f(E) \]

A graph of area under the detection signal is plotted against the storage voltage. The area under the detection signal is proportional to the number of trapped electrons\(^{[20]} \). The derivative of the graph yields the desired distribution function.

2. Experiment

In this section we describe a LabVIEW based voltage control system that is used for obtaining the distribution function. The detection schematic is shown in Fig. 2(a). After loading the trap with electrons of energy distributed around the filament bias voltage that is negative (varied from 3-5 V) and which determines the energy of the electrons, the loading is terminated by reducing the filament bias. The ring electrode is at a positive potential while the end caps are grounded. The potential in the trap is initially set at a higher value at about 10 V. Electrons that enter the trap are confined, partly owing to energy transfer from axial motion to the radial motion through coulomb interactions. The potential is then reduced to a value, \( V \), causing electrons whose energies are larger than \( V \), leave the trap. The time \( \tau \) over which this occurs, called the dwell time, is well within the storage time of the trap for electrons. The voltage is then ramped down. This results in varying the axial frequency of the trapped electrons, as in equation (2). At a certain ramp voltage the corresponding axial frequency matches the tuned detection circuit frequency resulting in energy transfer from the detection circuit to the electrons, and is registered as a voltage change in the demodulated DC signal from the detection circuit. The corresponding signal so obtained is recorded. Measurements are carried out for different \( V \). In our measurements, as the detection circuit was tuned to a frequency that corresponded to about 1 V, where resonant energy transfer took place from the circuit to the trapped electrons, we set the minimum \( V \) at 2V. The signal strength is thus recorded yielding the area under the signal.
Fig. 2(a): A schematic of the voltage ramp. Fig. 2(b): Schematic showing the variation in applied storage potential. The electron signal is obtained when the potential is ramped. Electrons are injected in the trap for 1 sec. The detection signal voltage variation is superimposed on the graph and the corresponding voltages on y-axis do not correspond to the variation in DC output of signal.

Figure 3: Electron plasma storage time measured when dwell time is varied. This is 80 ms for our trap and at a magnetic field of 0.05T.

Fig. 3 shows the detection signal fall off with dwell time τ, as the latter is varied, for a fixed storage potential. We define storage time, as the time for the signal strength measured initially and soon after electron loading is stopped, to fall off to 1/e of the initial value\(^\text{[23]}\). This storage time can vary depending on the external magnetic field, and is about 80ms for an external magnetic field of 0.05T. The energy distribution measurements in the trap are carried out for dwell times that are well within this storage time. All measurements here were carried out for a fixed external magnetic field of 0.05T. Pressure in all cases was in the range of \(8 \times 10^{-9}\) Torr.
3. Results and Discussion:

Figure 4(a) and 4(b) are respectively the graphs of area under the detection signal versus storage voltage and the corresponding derivatives of the signal area versus voltage. Figures 4(a) and 4(b) represent measurements carried out when electrons are injected from the filament at bias voltages of 5V and 3V respectively. The corresponding distribution function for both electron filament bias voltages shows that the maximum of the function is at voltages corresponding to the electron filament bias voltage. Thus, the distribution function of the electrons is a reflection of the total energy distribution, regardless of the fact that we monitor only the axial motion, in these measurements. This is a consequence of equipartition of energy into all degrees of motional freedom. The slope of the curves in figures 4(a) and 4(b) gives the corresponding distribution functions\cite{24}. The data fits well in each case, to a Gaussian distribution function.

Figure 5, is the distribution functions of the derivatives of signal area versus voltage for filament bias voltage around 5V and for different external magnetic fields. As can be seen from Fig. 5, there is no change in the shape of the energy distribution of trapped electrons with varying magnetic field, as expected, since the magnetic field has no role to play on the axial motion of the electrons. The storage time of the electrons in the trap, however, does depend on the external magnetic field and the nature of this dependence is currently under investigation.
4. Conclusion:

The trapped electrons that form a non neutral, non thermal plasma are in Boltzmann equilibrium for an electron gas under low pressures. We confirm in addition to this, that the energy distribution shows a peak that corresponds to the energy of the electrons determined by the filament bias voltage. By varying the filament bias we can see a clear shift in this peak in the distribution function. The electron detection signal that is obtained following a sequence of steps wherein the storage voltage is set to different values, varies as the storage voltage and this variation does depend on the range of electron energies determined by the filament bias voltage. Moreover, this variation has no dependence on the applied magnetic field. This work thus demonstrates the possibility of a direct measurement of the energy distribution function by monitoring the axial motion that is decoupled from the radial motion of the electrons, through electronic detection, under conditions of LTE of the trapped electrons.

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References: