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Uncertainty analysis of a spatially-distributed hydrological model with rainfall multipliersArpana Rani Datta¹, and Tirupati Bolisetti^{1,*}

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Draft

Abstract

This paper has developed an input error model for accounting input uncertainty and applied the rainfall multiplier approaches to the calibration and uncertainty analysis of Soil and Water Assessment Tool (SWAT), a spatially-distributed hydrological model. The developed input error model has introduced the season-dependent rainfall multipliers to the Bayesian framework and reduced the dimension of the posterior probability density function. The method is applied to a watershed located in Southwestern Ontario, Canada. The results of the developed method are compared with two other methods. The SWAT model parameters and the input error model parameters are jointly inferred by a Markov chain Monte Carlo sampler. The results show the measured precipitation data overestimates the true precipitation values for the study area. The uncertainty in model prediction is underestimated for high flows and overestimated for low flows. There is no significant change in the estimation of parameter uncertainty and streamflow prediction uncertainty in the developed method from those in the other methods. The study emphasizes that the rainfall multiplier approaches are applicable to spatially-distributed hydrological modelling for accounting input uncertainty.

Keywords: Input uncertainty, rainfall multiplier, parameter uncertainty, predictive uncertainty, distributed hydrological modelling.

1 INTRODUCTION

The uncertainty in the hydrological modelling may arise from model inputs, parameters, structure and outputs (Vrugt 2004, Kavetski et al. 2006, Huard and Mailhot 2006, Ajami et al. 2007, Yang et al. 2008, Thyer et al. 2009, Koskela et al. 2012, Li et al. 2012). Among them, uncertainty in precipitation inputs plays a significant role because any hydrological model forced with inaccurate precipitation data cannot produce accurate model predictions (McMillan et al. 2011). Uncertainty in precipitation inputs may arise from data measurement errors and imperfect representation of precipitation data in the hydrological modelling (Huard and Mailhot 2006, McMillan et al. 2011). The precipitation measurement errors may occur at a station due to the effects of wind and evaporation during its measurement and/or instrumental error (Salamon and Feyen 2009). Even though the precipitation measurement is exact, there might be differences between the gauge reading and the input to the model due to rescaling of precipitation over space (Huard and Mailhot 2006, McMillan et al. 2011). These differences can be treated as the errors due to imperfect representation of precipitation. The interpolation techniques of precipitation measurements among the rain gauges are considered another source of input errors in hydrological modelling (Hwang 2005, McMillan et al. 2011).

The input data uncertainty may cause biases in model parameter estimation that consequently may result in increased uncertainty in model prediction. Therefore, in many recent studies, input uncertainty has been accounted for in the calibration process of hydrological models. Some examples are studies conducted by Thyer et al. (2009), Renard et al. (2010), McMillan et al. (2011), Koskela et al. (2012) and Li et al. (2012). Generally, two approaches are adopted to account for input uncertainty in hydrological model calibration. In one approach, the input data are corrected explicitly by introducing an appropriate input error model to the calibration

framework. The multiplicative input error model developed by Kavestki et al. (2006) (Bayesian Total Error Analysis, BATEA) and the additive input error model developed by Huard and Mailhot (2006) fall under this approach. In the second approach, the errors in model inputs, parameters, structure and outputs are lumped together and expressed as an additive error model. Some examples of this category are the approaches proposed by Sorooshian and Dracup (1980), Kuczera (1983), Duan et al. (1988), Bates and Campbell (2001), Engeland et al. (2005), Yang et al. (2007a, 2007b), Schaepli et al. (2007) and Laloy et al. (2010).

In BATEA, precipitation uncertainty is considered by introducing rainfall multipliers as latent variables to the hierarchical Bayesian system. Latent variables need to be estimated through the inference process. The temporal scale of the multipliers is either daily or storm-event basis. The rainfall multiplier approach has a problem of an increase in the number of latent variables with the increase in the length of calibration period. This implies an increase in the dimension of the posterior probability distribution and causes an increase in the computational effort. The input error model developed by Huard and Mailhot (2008) also needs extensive computational effort when the time scale of model simulation is reduced to daily. Moradkhani et al. (2005), Vrugt et al. (2005) and Salamon and Feyen (2009) proposed sequential data assimilation techniques where the state variables are recursively estimated/corrected each time an observation becomes available. These approaches also involve high computational requirement. Reichert and Mieleitner (2009) corrected the bias in model input explicitly by introducing stochastic, time-dependent model parameters. The approach is conceptually similar to the BATEA method. Vrugt et al. (2008) developed the Differential Evolution Adaptive Metropolis (DREAM) algorithm for parameter sampling and extended the rainfall multiplier approach by introducing uniform prior distributions for the rainfall multipliers. Ajami et al.

(2007) developed the Integrated Bayesian Uncertainty Estimator (IBUNE) framework and reduced the number of latent variables by sampling the rainfall multipliers from the same distribution for each rainfall observation. In IBUNE, the true rainfall is assumed to be corrupted each time by random rainfall multipliers sampled from the identical distribution with unknown mean and variance. Despite these advancements in the area of treatment of input uncertainty, there is a need for a computationally less intensive framework for calibration and uncertainty analysis of hydrological models. The computational cost of uncertainty analyzing methods can be reduced by reducing the dimension of the posterior probability density functions. The first objective of this paper is to develop a posterior probability density function that would be computationally less intensive for a hydrological model calibration considering input data uncertainty.

The second objective of this paper is to make the application of the rainfall multiplier approach feasible for spatially-distributed models. Due to the presence of large number of model parameters, the distributed hydrological models are challenging to calibrate by incorporating additional latent variables in the calibration process (Abbaspour 2008). For this reason, input uncertainty is commonly treated indirectly by aggregation with other sources of uncertainty in distributed hydrological modelling. The Generalized Likelihood Uncertainty Estimation (GLUE) (Beven and Binley 1992), an informal Bayesian approach, is commonly used for the uncertainty analysis in spatially-distributed hydrological modelling (Beven et al. 1995, Arabi et al. 2007, Blasone et al. 2008, Yang et al. 2008, Younger et al. 2009). However, the GLUE methodology has been criticized due to its subjectivity involved in choosing thresholds between behavioral and non-behavioral models (Blasone et al. 2008, Koskela et al. 2012). Li et al. (2012) developed a Bayesian model to estimate the parameters of a geomorphology-based distributed hydrological

model by incorporating rainfall input errors. They observed an improvement in the predictive distribution by jointly estimating input errors and hydrological model parameters in the calibration process. But the improvement was incremental. Yen et al. (2015) applied the IBUNE input error model to a spatially-distributed hydrological model called Soil and Water Assessment Tool (SWAT) (Arnold et al. 1998) to explore the effects of input uncertainty to model predictions. They did not observe any noticeable improvement in SWAT model simulation by the incorporation of latent variables in precipitation data. Therefore, further research needs to explore the application of rainfall multiplier approach to distributed hydrological model calibration considering input uncertainty. In this study, the developed input error model will be applied for the calibration and uncertainty analysis of a distributed hydrological model to investigate the effects of rainfall multiplier approach on parameter estimation and model prediction.

The application of season-dependent stochastic model parameters to account for input, structural and output errors has drawn the attention of recent researches in the hydrological modelling. For example, Yang et al. (2007a) used the variance and characteristic correlation time of the stochastic error model for dry season and wet season separately to represent the model input and structural errors implicitly. Schaeffli et al. (2007) divided the streamflow data into low flow and high flow for the calibration of a hydrological model to make the model parameter estimation more stable. Following the idea of season-dependent stochastic model parameters, the number of statistical distributions used for the inference of the rainfall multipliers for different rainfall observations is kept equal to the number of distinct seasons of the relevant watershed. The developed season-dependent input error model is conceptually similar to the approaches given by Kavetski et al. (2006), Vrugt et al. (2008) and Ajami et al. (2008), but differs in

handling the latent variables. The rainfall multipliers for the rainfall observations are inferred from a few distributions rather than from a large number of distributions. Therefore, the method will reduce the high-dimensionality of the posterior probability distribution for model parameters that occurs in the storm-event dependent approaches. The developed input error model will be described here as a season-dependent input error model.

The performance of the method is illustrated by developing a case study where the SWAT model (Arnold et al. 1998) is calibrated for a watershed located in Southwestern Ontario, Canada. The model is fed with the rainfall inputs obtained from a rain gauge located outside the watershed. Thus the developed input error model focuses on the errors that can be arisen from the rainfall measurement and its representation in hydrological modelling. For evaluating the season-dependent input error model, the uncertainty in model parameters and model prediction are compared with that estimated by two other calibration methods: the traditional calibration method based on assumption that precipitation input is error free and the IBUNE calibration method. The developed method is validated using a set of observed input and output data that were not used in calibration.

2 STUDY AREA, MODEL AND DATA

The SWAT model is widely used for streamflow simulation of agricultural watersheds. The Canard River watershed located in the Essex region, Southwestern Ontario, Canada covers an area of 348 km² and consists of relatively flat clay plain. The major land use of the watershed is agriculture which occupies 85% of the area. Therefore, SWAT model is selected for the evaluation of the new season-dependent input error model. The land elevation of the watershed ranges from 175 m to 197 m. There is one streamflow gauging station (Fig. 1) in the watershed.

The climate data, such as daily precipitation, temperature, humidity and wind speed for the Windsor Airport climatic station (Fig. 1) are obtained from the Environment Canada website (http://climate.weatheroffice.gc.ca/climateData/canada_e.html). The necessary Geographic Information System (GIS) data such as watershed boundary, DEM, land use and soil are obtained from the Essex Region Conservation Authority (ERCA). Based on the climatic normal record of Windsor Airport station for the period of 1971 to 2000, the annual average precipitation in the area is 920 mm with an average annual rainfall of 805 mm. Most of the snowfall occurs during the winter months of December - February. The winter temperature usually falls below 0°C while the average summer (June-August) temperature is around 20°C.

The ArcGIS interface of SWAT model called ArcSWAT divides the watershed into a number of sub-basins and extracts model input data from the map layers and other databases for each sub-basin. The land use and soil maps are overlaid on the Digital Elevation Model (DEM) map and each sub-basin is divided into a number of Hydrologic Response Units (HRUs). The SWAT model simulates water, sediment, nutrients and pesticides transport at a HRU level on daily basis. This paper considers the generation of runoff at HRU level and transportation of water from the HRUs to the watershed outlet. The movement of water at the HRU level for the study area simulated by SWAT is shown in Fig. 2. The figure is adapted from Neitsch et al. (2005). Infiltration and surface runoff from daily precipitation are calculated in SWAT by the SCS curve number method (SCS 1972). Lateral subsurface flow is estimated by using the kinematic storage model (Sloan and Moore 1984) and groundwater flow is estimated as return flow to stream from the shallow aquifer (Arnold et al. 1998). The Penman-Monteith method (Monteith 1965) is used to estimate the potential evapotranspiration for the watershed. The Muskingum method (Cunge 1969) is used for channel routing.

SWAT is a spatially distributed hydrological model. In this study, the 'aggregate parameter' concept developed by Yang et al. (2005) is implemented for the calibration of SWAT model. According to the concept, the model parameters are aggregated so that the model parameters needed to be calibrated are reduced to a few numbers. Four aggregate model parameters, such as, Curve Number (CN), available water holding capacity (AWC), the plant uptake compensation factor (EPCO) and soil evaporation compensation factor (ESCO) are the most sensitive parameters for the Canard River watershed modelling (Rahman, 2007) and are estimated through the calibration procedure. The parameter, CN controls the estimation of surface runoff while the parameter, AWC is very sensitive for estimating the soil storage and evapotranspiration. The parameter, EPCO is an indication of the changes in the depth distribution of soil layers used to meet water uptake demand of plant and the parameter, ESCO is an indication of the changes in the depth distribution of soil layers used to meet the soil evaporative demand. The aggregate parameters can be expressed as a__CN2.mgt, a__SOL_AWC ().sol, a__EPCO.bsn and a__ESCO.bsn, respectively. The variable 'a' indicates that the value is added to the existing values of those parameters. The symbol '()' indicates that the value will be changed to all of the layers of soil. The mgt, sol and bsn are the extension of data files that contain the calibration parameters.

For the calibration of SWAT model, the observed daily streamflow data for the period from 1990 to 1993 of the gauging station obtained from the Environment Canada website are used. The model is validated with the streamflow data for the period of 2000 to 2003. For stabilizing the initial state variables of the SWAT model, one year is considered as warm-up period during calibration and validation.

3 POSTERIOR PROBABILITY DENSITY FUNCTIONS

3.1 New calibration method

Any hydrological model can be represented by the following equation:

$$\tilde{\mathbf{q}} = \mathbf{g}(\mathbf{x}, \boldsymbol{\theta}) \quad \dots\dots\dots (1)$$

where $\mathbf{g}(\cdot)$ is the response function of the hydrologic system; $\tilde{\mathbf{q}} = \{\tilde{q}_1, \tilde{q}_2, \tilde{q}_3, \dots, \tilde{q}_n\}$ is a vector of model response, such as streamflow, of the hydrologic system at any time step; n is the number of time steps; $\mathbf{x} = \{x_1, x_2, x_3, \dots, x_n\}$ is a vector of true precipitation data to the hydrologic system and $\boldsymbol{\theta} = \{\theta_1, \theta_2, \theta_3, \dots, \theta_s\}$ is a vector containing the s model parameters to be estimated through the calibration process.

The season-dependent input error model assumes that the precipitation inputs are corrupted by the measurement and representation errors and is represented by the following equation:

$$\mathbf{x} = \mathbf{f}(\tilde{\mathbf{x}}, \mathbf{m}) \quad \dots\dots\dots (2)$$

$\mathbf{m} = \{m_1, m_2, m_3, \dots, m_i\}$ is a vector containing the i input error model parameters to estimate the true inputs $\mathbf{x}$, given $\tilde{\mathbf{x}} = \{\tilde{x}_1, \tilde{x}_2, \tilde{x}_3, \dots, \tilde{x}_n\}$, which is a vector of daily precipitation observation and i is the number of distinct seasons in the relevant watershed. Assuming that the precipitation input errors are multiplicative, the true precipitation at time step t can be expressed as follows:

$$x_t = m_{t \sim s} \tilde{x}_t \quad t = 1, 2, \dots, n; s = 1, 2, 3, \dots, i \quad \dots\dots\dots (3)$$

where, $m_{t \sim s}$ is the multiplicative error for the precipitation observation at time step t and in the s th season, i is the number of distinct seasons in a year considered during the calibration period and $\tilde{x}_t, x_t$ are the observed and true precipitations at time step t .

Now, considering errors in the modelling results that cannot be captured by input errors and implementing the season-dependent input error model, the following equation can be written:

$$\mathbf{q}_{obs} = \mathbf{h}(\mathbf{x}, \boldsymbol{\theta}, \mathbf{m}) + \mathbf{e} = \mathbf{q}_{com} + \mathbf{e} \dots\dots\dots (4)$$

where, $\mathbf{h}()$ is the selected hydrologic model for the watershed response, $\boldsymbol{\theta}$ is hydrological model parameter vector, $\mathbf{m}$ is the input error model parameter vector, $\mathbf{q}_{obs} = \{q_1, q_2, q_3, \dots, q_n\}^T$ is the observed response of the system, $\mathbf{q}_{com} = \{h_1, h_2, h_3, \dots, h_n\}^T$ is the computed response vector of the model and $\mathbf{e} = \{e_1, e_2, e_3, \dots, e_n\}^T$ is the error vector. The symbol T represents the transpose of the vector.

Using the Bayesian theory, the posterior probability density function (*pdf*) conditioned on observed precipitation data, $\tilde{\mathbf{x}}$ and observed streamflow data, $\mathbf{q}_{obs}$ can be written as:

$$p(\boldsymbol{\theta}, \mathbf{m} | \tilde{\mathbf{x}}, \mathbf{q}_{obs}) \propto p(\boldsymbol{\theta}, \mathbf{m}) p(\mathbf{q}_{obs} | \boldsymbol{\theta}, \mathbf{m}, \tilde{\mathbf{x}}) \dots\dots\dots (5)$$

where $p(\boldsymbol{\theta}, \mathbf{m})$ is the prior *pdf* of hydrological model parameters, $\boldsymbol{\theta}$ and input error model parameters, $\mathbf{m}$. The prior *pdf* represents the prior knowledge about the hydrological model parameters and season-dependent input error model parameters. Following the method of Vrugt et al. (2008), the prior distributions of hydrological model parameters and input error model parameters are assumed to be uniform. Now applying the uniform prior distribution, the equation (5) can be written as:

$$p(\tilde{\boldsymbol{\theta}} | \tilde{\mathbf{x}}, \mathbf{q}_{obs}) \propto p(\tilde{\boldsymbol{\theta}}) p(\mathbf{q}_{obs} | \tilde{\boldsymbol{\theta}}, \tilde{\mathbf{x}}) \dots\dots\dots (6)$$

where $\tilde{\boldsymbol{\theta}} = [\boldsymbol{\theta}, \mathbf{m}]$.

To reveal the effects of input errors on parameter estimation and model prediction, a simple stochastic model is employed in this study for describing the residuals between observed and simulated responses. Thus, the posterior *pdf* can be written as:

$$p(\tilde{\theta}, \sigma^2 | \tilde{x}, q_{obs}) \propto p(\tilde{\theta}, \sigma^2) p(q_{obs} | \tilde{\theta}, \tilde{x}) \dots \dots \dots (7)$$

Where the output and model errors are assumed from a Gaussian distribution with zero mean and constant variance, σ^2 .

According to the Jeffry's rule, for the noninformative prior it can be written as (Box and Tiao 1992),

$$p(\tilde{\theta}, \sigma) \propto \frac{1}{\sigma} \dots \dots \dots (8)$$

Thus the posterior *pdf* becomes

$$p(\tilde{\theta}, \sigma^2 | \tilde{x}, q_{obs}) \propto \frac{1}{\sigma^2} (2\pi)^{-\frac{n}{2}} (\sigma^2)^{-\frac{n}{2}} \exp \left[-\frac{1}{2} \left\{ \sigma^2 \left(\sum_{t=1}^n e_t^2 \right) \right\} \right] \dots \dots \dots (9)$$

Integrating σ^2 out, the posterior *pdf* becomes

$$p(\tilde{\theta} | \tilde{x}, q_{obs}) \propto \mathcal{S}_{\mathcal{S}} (\tilde{\theta}, x, q_{obs})^{-\frac{n}{2}} \dots \dots \dots (10)$$

Where

$$\mathcal{S}_{\mathcal{S}} (\tilde{\theta}, x, q_{obs}) = \sum_{t=1}^n e_t^2 \dots \dots \dots (11)$$

The Markov chain Monte Carlo (MCMC) type numerical solution methods are used in this study to solve the posterior *pdf* [equation (10)] where the season-dependent input error model is incorporated. The results of the season-dependent input error model are verified with the results of the standard calibration method and the IBUNE calibration method. The posterior *pdfs* based on the assumptions of the two methods are described here briefly. The notations of

variables used for describing the posterior *pdf* of the two methods are the same as the notations used in formulating the season-dependent input error model.

3.2 Standard calibration method

The standard calibration method is the traditional calibration method and assumes that the precipitation input is error free. Therefore, no input error model is employed in Bayesian model for estimating the hydrological model parameters and the modelling errors are assumed from a Gaussian distribution with zero mean and constant variance, σ^2 . The mathematical form of the posterior distribution is similar to that of the standard least square regression method as shown below:

$$p(\theta|\tilde{x},q_{obs}) \propto \mathcal{S}(\theta,\tilde{x},q_{obs})^{-\frac{n}{2}} \dots\dots\dots (12)$$

where

$$\mathcal{S}(\theta,\tilde{x},q_{obs}) = \sum_{t=1}^n e_t^2 \dots\dots\dots (13)$$

The details of the above posterior *pdf* are available in Kavetski et al. (2006).

3.3 IBUNE calibration method

In IBUNE method, the rainfall multipliers are assumed to be random noises from a normal distribution with unknown mean and variance. Therefore, the mean and variance of rainfall multipliers are introduced to the Bayesian system as latent variables. This approach reduced the dimensional problem of rainfall multiplier model in BATEA. In BATEA framework, the dimension of the posterior distribution is equal to the number of hydrological model parameters and number of rainfall multipliers, while in the IBUNE calibration method, it is equal to the number of hydrological model parameters and two latent variables (Ajami et al. 2009). The IBUNE input error model is applicable for a relatively small variance of rainfall multipliers

(Ajami et al. 2009). The IBUNE calibration method is selected in the study to evaluate the developed season-dependent input error model. In IBUNE method, the input error model is expressed as:

$$x_t = M_t \tilde{x}_t \quad \text{..... (14)}$$

$$\text{where, } M_t \approx N(\mu_M, \sigma_M^2)$$

$\tilde{x}_t$, x_t are the observed and true precipitations at t th time step and M_t represents a random rainfall multiplier at t th time step. The rainfall multipliers are assumed to be normally distributed with mean equal to μ_M and variance equal to σ_M^2 .

Implementing the IBUNE input error model in the Bayesian theory, the posterior *pdf* becomes:

$$p(\tilde{\theta}, (\mu_M, \sigma_M^2) | \tilde{x}, q_{obs}) \propto \mathcal{S}(\theta, x, q_{obs})^{-\frac{n}{2}} \quad \text{..... (15)}$$

where

$$\mathcal{S}(\theta, x, q_{obs}) = \sum_{t=1}^n e_t^2 \quad \text{..... (16)}$$

The details of the above posterior *pdf* are available in Ajami et al. (2007).

4 COMPUTATIONAL FRAMEWORK AND EXPERIMENTAL DESIGN

For solving the posterior probability density function of model parameters and input error model parameters, the Shuffled Complex Evolution Metropolis (SCEM-UA) algorithm (Vrugt et al. 2003), a MCMC based calibration and optimization tool is used in the study. This algorithm was also implemented in IBUNE for parameter inferences. The SCEM-UA algorithm is based on the Metropolis-Hastings algorithm (Metropolis et al. 1953, Hastings 1970) and complex shuffling procedure for sampling the model parameters and finds the global optimum values of the parameters (Vrugt et al. 2003). The SWAT model is simulated under the GNU OCTAVE

environment using the text format input data files generated by the ArcSWAT interface for each of the HRU. Once the data files are extracted in text format by the ArcSWAT interface, it can be used outside the GIS environment for the SWAT model simulation. The computational flowchart used to calibrate the SWAT model with the SCEM-UA algorithm is shown in Fig. 3. The flowchart is similar to the SWAT calibration and uncertainty programs (SWAT-CUP) developed by Abbaspour (2008). For incorporating the season-dependent input error model in the calibration process, an algorithm is added to the SWAT-CUP flowchart (Fig. 3). The Markov chain Monte Carlo simulations are carried out with five parallel Markov chains for the inferences of SWAT model parameters and rainfall multipliers. The posterior *pdf* of parameters are analyzed using the samples after the chain has reached the stationary distribution. The convergence of Markov chain is checked using the convergence criteria of Gelman and Rubin (1992) known as the scale reduction factor ($\hat{R}$) [shown in equation (17)].

$$\hat{R} = \sqrt{\frac{n'-1}{n'} + \frac{m'+1}{m'.n'} \frac{V}{V'}} \dots\dots\dots(17)$$

where n' is the number of iterations within each sequence, V is the variance between the m' sequence means, and V' is the average of the m' within-sequence variances for the parameter under consideration. A value of $\hat{R}$ less than 1.2 for each of the parameters is considered as the convergence of the Markov chain (Gelman and Rubin 1992).

Five season-dependent rainfall multipliers are identified in the study area to account for the input errors in the measured precipitation data. The observed streamflow is the hydrologic response of the true precipitation input to the watershed. The season-dependent input error model parameters are selected on the basis of observed seasonal variation of streamflow in the watershed. The observed streamflow rises during the months of November-December and the

peak occurs during the months of February-March when the temperature is frequently above the freezing temperature. The streamflow starts to recess at the end of April and takes the lowest value in the months of July-August. In the period of July-August, the occurrence of the evapotranspiration is the highest in the study area. Therefore, the periods from January to April, May to June, July to August, September to October and November to December are identified as five distinct seasons in the watershed. The errors in the daily precipitation observations occurred during these seasons are accounted with five rainfall multipliers. These multipliers are defined in this paper as season-dependent rainfall multipliers and noted as the Jan_Apr_mult, May_Jun_mult, Jul_Aug_mult, Sep_Oct_mult and Nov_Dec_mult. The season-dependent rainfall multipliers are estimated in the calibration process along with the SWAT model parameters. The errors in precipitation measurement and its representation in modelling are estimated from the information of the observed streamflow and observed precipitation data. Five uniform distributions are assumed for sampling of the season-dependent precipitation multipliers. As mentioned earlier, the performance of the season-dependent input error model in calibration of SWAT model is evaluated by estimating the model parameters in three different calibration methods. One is based on the developed season-dependent input error model and others are based on the standard calibration method and IBUNE calibration method. These methods of calibration are summarized in Table 1. The SWAT model parameters and input error model parameters estimated by the calibration process are used to predict streamflow in the validation period. The results obtained from three calibration methods are described in terms of the performance of the methods for estimating the parameter uncertainty, input uncertainty and predictive uncertainty. The assumptions regarding the residual error models are verified in three approaches of model calibration.

To assess the predictive uncertainties produced by three calibration methods, one index called Average Relative Interval Length (ARIL) (Jin et al. 2010) and a tool called predictive QQ plots (Laio and Tamea 2007) are used in the study. The ARIL is an index used to measure the quality of data coverage by the predictive uncertainty. The difference between the upper limit and the lower limit of the confidence interval at any time step divided by the corresponding observed data is termed as the relative interval length and the average value of the time period is termed as ARIL (Jin et al. 2010). A smaller ARIL value and a larger percentage of data coverage indicate a better performance of the prediction method. The predictive QQ plot is a useful tool to verify the probabilistic forecasts of hydrological variables. The details of the construction of predictive QQ plot are described in Laio and Tamea (2007) and Thyer et al. (2009). The predictive QQ plot helps to quantify the reliability of the streamflow prediction. If the predictive distribution of streamflow (x_i) is consistent with observed data, the probability density function of x_i coincides with the true distribution of x_i . If z_i represents the value from the cumulative distribution function of the predictions in correspondence to the observed value of x_i , the distribution of z_i is uniform, $U [0, 1]$ (Laio and Tamea 2007). If the z -value curve is close to the bisector (the 1:1 line), the predictive distribution of x_i seems to be reliable, otherwise it indicates the biasness in prediction (Laio and Tamea, 2007). The deviation from the bisector can be quantified using the reliability index which is related to the area of z -value curve and bisector line (Renard et al. 2010). The value of reliability index close to 1 implies perfect reliability and the value of the index close to zero shows worst reliability of prediction (Renard et al. 2010).

5 EVALUATION OF SEASON-DEPENDENT INPUT ERROR MODEL

5.1 Assessment of parameter uncertainty and input uncertainty

The SWAT model parameters and the input error model parameters are inferred jointly from their uniform prior distributions. The maximum and minimum ranges of prior distributions of the parameters are given in Table 2. In this study, the effects of narrow and wide ranges of the values of SWAT model parameters on model calibration are performed and the limits of uniform distributions are set for model calibration and uncertainty analysis. The ranges of the season-dependent input error model parameters are set on the basis of the study carried out by Vrugt et al. (2008). The ranges of the IBUNE input error model parameters are selected from the values recommended by Ajami et al. (2009). The uncertainty in SWAT model parameters are estimated using 10,000 samples after the convergence of Markov chain. The marginal posterior probability distributions of the SWAT model parameters are presented in Fig. 4. The posterior probability distributions for two model parameters, $a_{\text{SOL_AWC}}$ () and $a_{\text{ESCO.bsn}}$ are changed from that of Standard method when the input error models are incorporated in the calibration process. The posterior distributions of $a_{\text{CN2.mgt}}$ and $a_{\text{EPCO.bsn}}$ remain almost uniform in any of the methods. The wider posterior distributions of the parameters $a_{\text{CN2.mgt}}$ and $a_{\text{EPCO.bsn}}$ give an indication of the presence of local optima in model parameter estimation. This may arise from the existence of model structural uncertainty and other uncertainties that are not considered in the calibration process. No significant change is observed in posterior distributions of SWAT model parameters obtained from the developed method and the IBUNE method.

The rainfall multiplier approaches used to account for input uncertainty can sometime compensate for the model structural uncertainty. This can be tested by the correlation of input

error model parameters with the hydrological model parameters. In the season-dependent rainfall multiplier approach, the correlation between the rainfall multiplier during May to June with the parameter $a_ESCO.bsn$ is -0.50 and the correlation between the rainfall multiplier during July to August with the parameter $a_SOL_AWC ().sol$ is -0.29. In the IBUNE method, the correlations of the mean of rainfall multiplier with the parameters $a_ESCO.bsn$ and $a_SOL_AWC ().sol$ are -0.31 and 0.54, respectively. These correlations indicate that the input error model parameters are interacting with the hydrological model parameters such as soil evaporation compensation factor (ESCO) and available water holding capacity (AWC). The parameter, ESCO affects the estimation of soil evaporative demand and the parameter, AWC affects the estimation of soil storage and evapotranspiration. The interactions between the input error model parameters and the hydrological model parameters can affect the estimation of model parameters and therefore the water balance in the hydrologic system can be changed. This finding is similar to other studies such as, Koskela et al. (2012), Thyer et al. (2009), Ajami et al. (2007), etc.

The marginal posterior probability distributions of season-dependent input error model parameters (Fig. 5) are generated by the 10,000 samples after the stationary distribution is achieved. The figure shows that the distribution of each season-dependent rainfall multiplier is not spread over the entire prior range. The prior and posterior values of the season-dependent rainfall multipliers shown in Table 3 also represent narrow width of the posterior distributions in comparison with prior distributions. Thus it can be stated that the multipliers are well-defined from the streamflow data used in the calibration of SWAT model. The mean value of the season-dependent rainfall multipliers is different from one. This indicates an existence of input errors in observed precipitation data. From Table 3, it can be estimated that mean precipitation errors are

approximately 15% in January-April months, 7% in May-June months and 32% in July to August months. The rainfall multipliers show an underestimation of true precipitation in January to April and an overestimation of true precipitation in July-August. The overall mean of the season-dependent precipitation multipliers is 0.97. In this case study, the rain gauge is located outside the watershed and it is expected that the spatial representation of the measured rainfall can be reduced over the entire watershed. Therefore, it is necessary to check if the developed input error model can account for the rainfall measurement and rainfall representation errors. Further research needs in this direction by using other sources of rainfall information. It is noted that like other rainfall multiplier approaches, the developed input error model cannot quantify the errors in the zero-depth precipitation measurements. This is the limitation of the rainfall multiplier based calibration framework.

The marginal posterior distributions of mean and variance of rainfall multipliers in the IBUNE method are shown in Fig. 6. The estimated posterior median, mode, 5th and 95th percentiles of IBUNE rainfall multipliers are 0.96, 0.97, 0.94 and 0.98 respectively. This indicates that the IBUNE rainfall multipliers are well-defined from the data used in the calibration process. While using the IBUNE input error model in the calibration process, the mean of the rainfall multipliers is observed to be 0.96, on average. This value is very close to the overall mean of season-dependent precipitation multipliers. The IBUNE input error model indicates that the measured precipitation data overestimates the true precipitation values. The posterior distribution of the variance of rainfall multipliers shows a wider range. The estimated posterior median, mode, 5th and 95th percentiles of variance of rainfall multipliers are 1×10^{-3} , 9×10^{-4} , 9×10^{-5} and 4×10^{-3} respectively. This is an indication of that the observed streamflow data cannot provide enough information regarding the random errors of precipitation measurement. A

comparison of the new method and IBUNE method for the estimation of precipitation uncertainty at the optimal input error model parameters is shown in Fig. 7. The figure shows the comparison between observed and estimated precipitation and between observed and simulated streamflow for one year daily precipitation and streamflow data from February/1992 to January/1993. The figure reveals that except one peak flow in July/1992, the streamflow simulated by the new method is consistent with that of IBUNE method.

5.2 Assessment of predictive uncertainty

In the Standard calibration method, the modelling errors are captured by the parameter uncertainty only, while in new and IBUNE methods, the modelling errors are represented by both parameter uncertainty and input uncertainty. The rainfall multipliers may reduce the model structural uncertainty by interacting with the hydrological model parameters. This may result in reduction in modelling errors in the rainfall multipliers based calibration approaches. This can be observed from Fig. 8 where the probability density functions of Daily Root Mean Square Error (DRMSE) for three calibration methods are shown. The figure shows that consideration of input uncertainty has resulted in reduction in modelling errors. This finding is similar to the study of Ajami et al. (2007). In the developed method, four hydrological model parameters and five rainfall multipliers are estimated jointly in the calibration process, while in the IBUNE method, four hydrological model parameters and two input error model parameters are estimated. The presence of more number of parameters may be responsible for allowing some extra degrees of freedom during the calibration process of new method and may result in marginal improvement of the value of DRMSE in the developed method over the IBUNE method.

The simulated streamflow with 95% confidence interval and the uncertainty in streamflow prediction due to parameter uncertainty in the calibration period are estimated (Fig.

9). In the developed method, at some time steps, the streamflow predictive uncertainty is quantified solely by the model parameter uncertainty. This indicates a better performance of the estimated model parameters in the new method. Quantitatively, the percentages of observed streamflow data covered by predictive uncertainty due to parameter uncertainty are 14.4%, 12.7% and 9.4% in new, IBUNE and Standard methods, respectively. The percentage of observed streamflow data covered by total 95% predictive interval is 95.2% in all of the calibration methods. The assumption of Gaussian error models may lead to a wider uncertainty bounds in any of the calibration methods. The value of ARIL in the calibration period is 71.7 in new method while it is 75.3 in the Standard and IBUNE methods. The 95% uncertainty bounds in any of the calibration methods show that the uncertainty in streamflow simulation is overestimated for low flows and underestimated for high flows. The streamflow observations that fall outside the uncertainty bounds are the high flows. This is an indication of the presence of model structural uncertainty in the calibration process.

For quantifying the uncertainty in streamflow prediction during model validation period, the uncertainty in input data and SWAT model parameters estimated by the calibration process, are propagated through the model simulation. Therefore, the SWAT model parameters and the input error model parameters used in validation are selected from their posterior distributions. The streamflow prediction with 95% confidence interval and the predictive uncertainty due to parameter uncertainty are estimated for the validation period (Fig. 10). The figure shows that the predictive uncertainty due to total uncertainty in the developed method is consistent with that of other two methods. The prediction uncertainty due to parameter, input and other sources of uncertainty is termed here as total uncertainty. Quantitatively, the percentages of observed streamflow data covered by predictive uncertainty due to parameter uncertainty are 8.9%, 8.0%

and 6.2% while the percentages of observed streamflow data covered by total 95% predictive interval are 95.0%, 95.4% and 94.4% in new, IBUNE and Standard methods, respectively. The values of ARIL are 151.5, 158.2 and 158.4 in new, IBUNE and Standard methods, respectively. During the validation period, the value of Nash-Sutcliffe coefficient of efficiency (NSE) (Nash and Sutcliffe, 1970) for monthly streamflow simulation using the parameters obtained at the highest posterior probability density 0.80 in the new method and 0.75 in IBUNE and Standard methods. The NSE is a normalized statistics that compares the variance of model residuals to the variance of measured data.

For assessing the consistency of total predictive uncertainty with the observed streamflow, the predictive QQ plots are used in the study. The predictive QQ plots in different calibration methods are shown in Fig. 11 for the model parameters obtained at the maximum posterior density. The figure shows that the observed z-values are higher or lower than the theoretical quantiles. This indicates that the estimated predictive distributions more frequently underestimate or overestimate the predictive uncertainty. The higher streamflows are frequently underestimated and lower streamflows are frequently overestimated, because the observed z-values are clustered near 1 or 0. This observation is consistent with the findings from the Figs 9 and 10. In quantitative terms, the values of reliability index are 0.69 in both Standard and IBUNE methods during calibration and validation. In the developed method, the values of reliability index are 0.67 in calibration and 0.71 in validation. In the model calibration process, some more degrees of freedom are introduced in the developed method than the other methods. However, the results obtained from the developed method are comparable to those of other methods in terms of the assessment of parameter uncertainty, input uncertainty and streamflow prediction uncertainty. There is no inconsistency in the results when the rainfall multiplier

approaches are applied to the SWAT model in comparison with Standard method. This indicates the applicability of the rainfall multiplier approach to the distributed hydrological models.

5.3 Residual errors

The assumptions of any statistical error model needs to be tested during the calibration of a hydrological model. While developing the posterior probability density functions, the residuals were assumed to be independent, Gaussian with zero mean and constant variance. The QQ plot is used to verify the type of distribution of the residual errors while the Autocorrelation Function (ACF) is used to test the correlation of the residual errors. The QQ plot of standardized residuals and the ACF plot of the residuals in different calibration methods are shown in Fig. 12. The residuals are calculated as the difference between the observed and simulated streamflow and are standardized by the standard deviation estimated by the different calibration methods. The QQ plot shows that the residuals in any of the calibration methods are far from the 'Theoretical' line.. If the assumption of normality is perfect, the QQ plot will follow the 'Theoretical' line. The QQ plot shows that the probability distribution of the residuals is peaked in any calibration method. The slope of the QQ plot is steeper than the theoretical line indicating that the high streamflows are underestimated in any of the calibration methods. The similar observations were stated in the previous sections.

The ACF plots show that there exist correlations in model residuals in any of the calibration methods, even though the value of ACF is lower in the developed method than the two other methods. The residuals are observed to vary systematically with the simulated streamflow in any of the calibration methods. This indicates that the variance of the residuals is not constant. Therefore, the assumptions of errors to be independent and Gaussian with constant variance are not satisfied in any of the calibration methods. In some studies, the statistical

properties of the residuals are not always consistent with the assumptions. Thyer et al. (2009) applied BATEA for quantifying parameter uncertainty and predictive uncertainty and reported imperfect match of the residuals to the theoretical normal distribution due to the presence of many near-zero runoffs. In that study, the autocorrelation of residuals was improved in BATEA in comparison with other methods though the independence assumption of the residuals was not fully satisfied. Li et al. (2012) developed a Bayesian model by incorporating input uncertainty to calibrate a distributed hydrological model. In that study, the first-order autoregressive model was considered to represent the temporal dependence of streamflow and the residuals were described by the Gaussian noise with zero mean and unknown variance. However, the residuals were observed to be deviated from the normal distribution for large absolute values and significantly correlated at lag one. The distributional properties of error models need to be satisfied for being confident in the uncertainty estimation. This paper is an initial step to evaluate the performance of the rainfall multiplier approaches to the spatially-distributed hydrological model and a simple error model is used while developing the posterior probability density functions. In future studies, the correlation, non-constant variance and non-normality of the modelling errors would be incorporated in the Bayesian framework for model calibration considering input uncertainty.

6 CONCLUSIONS

In this paper, a posterior probability density function considering season-dependent rainfall multipliers has been developed for the calibration and uncertainty analysis of a hydrological model. The developed method has reduced the number of the latent variables in the Bayesian framework and thus reduced the dimension of posterior probability density function. The method has been applied to the uncertainty analysis of SWAT, a distributed hydrological model. The performance of the developed method is evaluated by assessing parameter uncertainty, input

uncertainty and model prediction uncertainty. The results of the developed method are compared with those of IBUNE method and Standard calibration method.

In general, the inclusion of rainfall multipliers in the calibration process has shown a marginal improvement in the assessment of parameter uncertainty. The rainfall multiplier approaches show that the measured precipitation data overestimates the true precipitation values for the study area. The rainfall multipliers are observed to be well-identified in their posterior distributions obtained from the calibration process. The uncertainty in model prediction is observed to be underestimated for high flows and overestimated for low flows due to the presence of model structural uncertainty.

The results of the predictive QQ plots, the values of ARIL and the values of DRMSE reveal that there is no significant improvement in the developed method over the IBUNE and Standard methods for the calibration and uncertainty analysis of SWAT model. However, the study concludes that the rainfall multiplier approaches are applicable to spatially-distributed hydrological modelling for accounting input uncertainty. The rainfall multipliers can be inferred either from the same statistical distribution for each rainfall observation or from a few statistical distributions. The rainfall multiplier approaches are limited to quantify the errors in the measurements of nonzero precipitation. In future studies, to be more confident in the uncertainty estimation, the correlation, non-constant variance and non-normality of model residuals would be considered in the calibration framework.

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Table 1 List of methods used for SWAT model calibration.

Calibration method	Input error model implemented in the calibration process	Notation used for the method
1	No input error model	Standard
2	Season-dependent input error model	New
3	IBUNE input error model	IBUNE

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Table 2 The maximum and minimum ranges of prior distribution of SWAT model parameters and input error model parameters.

Parameters	Upper bound	Lower bound
SWAT model parameters		
a__CN2.mgt	5.00	-5.00
a__SOL_AWC ().sol	0.05	-0.05
a__EPCO.bsn	0.05	-0.05
a__ESCO.bsn	0.05	-0.05
Season-dependent input error model parameters		
Jan_Apr_mult	0.25	1.50
May_Jun_mult	0.25	1.50
Jul_Aug_mult	0.25	1.50
Sep_Oct_mult	0.25	1.50
Nov_Dec_mult	0.25	1.50
IBUNE input error model parameters		
μ_M	0.80	1.20
σ_M^2	1×10^{-5}	1×10^{-2}

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Table 3 Prior and posterior values of season-dependent rainfall multipliers

Season-dependent rainfall multipliers	Prior range	Posterior values			
		Median	Mode	5th percentile	95th percentile
Jan_Apr_mult	(0.25,1.5)	1.16	1.16	1.14	1.22
May_Jun_mult	(0.25,1.5)	0.93	0.91	0.86	1.01
Jul_Aug_mult	(0.25,1.5)	0.67	0.67	0.62	0.73
Sep_Oct_mult	(0.25,1.5)	1.07	1.06	1.01	1.14
Nov_Dec_mult	(0.25,1.5)	1.01	1.05	0.95	1.08

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457x269mm (96 x 96 DPI)

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284x239mm (96 x 96 DPI)

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