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Theory and calibration of the Pierre 2 stochastic rock fall dynamics simulation program

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Abstract

The use of computer models to determine rock fall hazards is increasingly common, with increasingly complex models being developed. In most practical applications, slopes potentially affected by rock falls are characterized in general terms only, thus a simpler model is desirable to reduce the parameter uncertainty. The model presented here utilizes a lumped-mass representation of the rocks. Key features are the stochastic roughness angle to represent contact geometry variability, hyperbolic restitution factors, and a stochastic shape factor, which have been developed considering impact mechanics theory. Together, these features can yield realistic results for linear and angular velocity, bounce height, run out distance, and normal restitution factors greater than one while still being easy to calibrate.

The model calibration has been carried out using detailed, full-scale experiments from a talus slope in France, a hard rock quarry in Austria and a weak bedrock and talus slope in Japan. An observed rock fall event in British Columbia was modeled as a pseudo-forward analysis to demonstrate the model validity. The usefulness of the model as a design tool has been demonstrated by using the simulation results as inputs for a hypothetical barrier design application, and calculating the reliability of the design values.

Key words: rock fall, dynamics, restitution, stochastic simulation

Introduction

Rock fall is the detachment, sliding, rolling and bouncing of rock fragments, primarily interacting with the slope substrate, with negligible dynamic interaction between the fragments (Hung et al. 2014). Rock fall is a common hazard below natural cliffs and talus slopes, and man-made environments such as quarries, road cuts and open pit mines. The total volume of material in these events is usually limited, but the rock fragments will usually attain extremely rapid velocities (e.g. Turner and Duffy, 2012). Thus, the affected areas are relatively small, but the intensity of the impact is high and there will likely be insufficient time for evasive action to be taken. Models of rock fall hazard have three components, the initiation (the probability of a failure of a given magnitude occurring), the reach (the probability of a block of given magnitude reaching a certain location) and the intensity (the jump height and kinetic energy at the location of risk) (Volkwein et al. 2011).

The potential source areas for rock falls are typically widely distributed. In a practical sense, this implies that it will not be feasible to gather very detailed site information for the entire area potentially affected by rock fall. However, it is important to be able to accurately represent the kinematics of this process, the velocities and bounce heights, to determine the hazard. The motion is primarily determined by the dynamic interactions of the falling rock with the slope surface, thus the impact process is a primary focus of rock fall modeling.

To simplify the complex impact processes, most rock fall models represent the loss of energy during an impact using coefficients of restitution. The loss is quantified by means of the ratio of rebound over incident velocity, momentum, or kinetic energy, with the ratio of velocities being the most common. These values are typically represented by constants (Lan et al. 2007) or by statistical distributions (Stevens, 1998) related to the material type.

A great deal of research has been done in the field of impact mechanics, and the physical processes during an impact have been well described (Goldsmith 1960, Stronge 2000). As shown by theoretical analyses derived in these references, the impact process is extremely sensitive to the precise geometry of the impact, as well as momentum magnitude and the elastic and plastic responses of the material forming the contact. The geometry and material properties for a falling rock and the contact area, such as the Young's modulus or strength properties, are variable and cannot be accurately and completely characterized in a practical application. This limits the usefulness of the established impact mechanics relationships for this application. However, this research does provide a stochastically-based conceptual framework for the modeling of rock fall impacts, which is detailed in the section on the restitution factors in this paper.

The current state of practice is to conduct computer modelling of the event trajectory, velocity and jump height to evaluate the reach probability as well as event intensities. The two general calculation approaches are a lumped mass model, or a rigid body model. A lumped mass model represents the boulder as a single, dimensionless point, with all of the properties of the boulder assigned to that point (Volkwein et al. 2011). A rigid body model explicitly accounts for the shape of the boulder, often idealizing the shape as a sphere, cube or ellipsoid. The final approach is a hybrid model, using a lumped mass model for the trajectory of the boulder through the air and calculating the rebound, rolling or sliding portions of the trajectories using a rigid body representation. All of the calculation approaches also utilize a coefficient or coefficients of restitution to model the impacts as well as friction coefficients for the rolling or sliding portions of the trajectory (Volkwein et al. 2011).

This paper describes the features of the Pierre 2 rock fall simulation program and the theory behind the various features. The authors' goal in deriving the model has been to make it as simple as possible, while still capturing the most important details of the physical processes at work. The model is able to predict apparent normal restitution values greater than one, as have been observed by other authors. Details of the calibration process to find model inputs for both firm talus and exposed rock slopes are presented. Model inputs correlated for general slope properties are presented. The results are also compared to design standards to demonstrate the applicability of the Pierre 2 program for design. Some specific results of this research have been reported previously by Gischig et al. (2015) and Preh et al. (2015). However, the present paper is intended to provide a complete coverage of the two-dimensional version of the model theory in full detail and to compare correlation results from the several test sites that have been analysed, ranging in terms of site characteristics (see Mitchell 2015). The goal is to present a model that is capable of making first order stochastic predictions of rock fall behaviour at different sites, depending on a generalized site description and a limited range of input parameters. To a large extent, this goal has been achieved. A new formal approach to calibration is also presented.

Model description

Particle definition

The assumptions of the model are first discussed in the context of a 2D analysis, with the extension of these ideas to 3D discussed later. The model considers the linear motion of the particle in the directions normal and tangential to the slope, as well as the particle's rotation. All particles are represented as equivalent spheres, with the diameter calculated from the particle mass input by the user. It is assumed that the impacts are collinear (the centre of mass of the particle is assumed coincident with the vector normal to the contact point), allowing the equations of normal, tangential and rotational motion to be

solved simultaneously (Goldsmith 1960). Random non-spherical shapes of real rocks and the non-collinear nature of the majority of real impacts are simulated by the introduction of stochastic impact “roughness” parameters, as described below.

Particle flight and collision

The flight of the fragment is described by a ballistic trajectory, neglecting air friction losses. The program calculates the position of the fragment (x , z) as well as the translational velocity (v_x , v_z) and rotational velocity (ω) along the trajectory with an iterative time stepping algorithm. With the assumption that losses are negligible, the only external force acting on the particle during flight is gravity (acting in the z direction) and the movement of the fragment can be described by classical ballistic equations.

When the program detects that the calculated position of the centre of gravity of the fragment crosses the slope surface, the program steps back one time step (so that the fragment will be a small distance above the slope) and begins the impact portion of the simulation. With short time steps, it is not necessary to calculate the precise intersection of the trajectory and the slope surface.

For a spherical particle, the actual contact with the slope surface occurs not at the centre of gravity, but a distance of one half the particle diameter measured normal to the slope surface. This has no influence on the performance of the algorithm, except that the actual height of the calculated trajectory with respect to the slope surface is slightly higher.

The first stage of the impact model is to transform the velocity vector from the global x - z coordinate system to a local coordinate system with axes normal and tangential to the local slope.

Roughness Angle

Irregularities in the slope surface and the shape of the particle are both represented by a single stochastic “roughness angle”, designated θ . This roughness angle is added to the slope angle at the point of impact, shown in Figure 1. The local normal – tangential ($n - t$) coordinate system is rotated again by θ and the components of the velocity vector relative to this twice rotated coordinate system ($n' - t'$) are used for the subsequent calculations.

Initially, a uniform distribution of θ was examined, ranging from zero to some user input maximum angle. Somewhat surprisingly, the uniform distribution provided good results in early testing. However, a truncated normal distribution, greater than zero was introduced to improve the simulation of extreme values. The distribution is created using the Box-Mueller approximation for a normal distribution. The stochastic roughness, θ , is generated from a fixed mean and standard deviation, μ and σ respectively, to be

specified along with the user input scaling value, θ_{scale} . The implementation of this requires two successive samples, S_i and S_j , to be drawn from a uniform distribution in the range of $[0, 1]$.

$$\tan \theta = \theta_{scale} [\mu + \sigma(\cos 2\pi S_j) \sqrt{-2(\ln(S_i))}] \quad (\text{Eq. 1})$$

For numerical stability, values of θ that are negative are discarded and new random samples S_i and S_j are drawn until a positive value for θ is obtained. Not doing so could result in the trajectory being below the ground surface.

Observations of natural talus deposits show that larger boulders will tend to travel farther downslope than smaller ones, because larger boulders are able to override smaller perturbations on the slope surface. Thus, the surface roughness value should be scaled to the boulder size. A linear relationship was assumed to find the effective roughness angle, θ_{eff} , from a theoretical maximum roughness, b , as the boulder diameter approaches 0, decreasing at a rate of m multiplied by the boulder equivalent diameter D .

$$\theta_{eff} = \theta(b - mD) \quad (\text{Eq. 2})$$

Restitution Calculation

The commonly used “restitution coefficients” have been replaced with “restitution factors” for this model, because a coefficient implies a constant value, whereas our restitution factors vary from impact to impact, as described below. We also adopt the definition of the ratio of rebound over incident linear velocity for the restitution factors, as opposed to the ratio of rebound over incident kinetic energy.

Consider a rigid spherical boulder with mass m , diameter D and incident velocity vector $\mathbf{v}^{in}$, having a normal impact with a surface with a constant indentation resistance (strength-like parameter described by Pichler et al. 2005). It is assumed that fragmentation does not occur during the impact and the force of gravity applied on the boulder over the impact period is small and can be neglected. A conceptual force-time and force-penetration history can be derived from these assumptions. The surface applies a force to the boulder that is equal to the impact resistance multiplied by the contact area. The time integral of the resisting force and the contact duration gives the resisting impulse. The boulder's initial momentum is reduced by the resisting impulse. The maximum penetration occurs when the resisting impulse is equal to the initial momentum, i.e. the boulder's velocity is momentarily zero. Following this point, elastic strains stored in the surface are recovered and the surface exerts a rebound impulse on the boulder that is a function of the elastic strain recovered and the duration of the rebound phase. The force-penetration plot for this conceptual impact model is shown in Figure 2. The integral of the force-penetration curve allows for the identification of two regions. The area bounded by the upper curve and the rebound curve

represents the plastic work (permanent deformations) that occurred. The area bounded by the rebound curve and the vertical line from the maximum force point represents the elastic work recovered after the impact.

The concave shape of the force-penetration diagram is a reflection of the assumed constant yield strength of the impacted surface, so that the total force is a product of the contact area, which for a sphere, scales in proportion to D^2 . The plastic work done during the impact is equivalent to the change in kinetic energy, which is $1/2m\Delta v^2$. With the assumption that particle mass is constant with constant density, the mass is proportional to D^3 . Thus, the energy lost during an impact is a function proportional to D^3 . A larger-diameter particle will have a greater contact area for a given penetration depth relative to a smaller diameter particle. This concept is similar to the “bullet effect” proposed for analysis of rock fence failures by Buzzi et al. (2011). This is represented by having the incident kinetic energy distributed over the contact area, which is proportional to D^2 . As a result, the term “unit deformation energy”, E_d , used to represent the incident normal kinetic energy distributed over the contact area, has characteristic dimensions of $v_n^2 D$.

A hyperbolic relationship to describe the dependence of the restitution factors on impact energy was described in Gischig et al. (2015) and Preh et al. (2015). The hyperbola defining the normal and tangential restitution factors, k'_n and k'_t respectively, are scaled using the unit deformation energy, E_d , shown in Equations 2 and 3.

$$k'_n = \frac{E_{n,50}}{E_d + E_{n,50}} \quad (\text{Eq. 2})$$

$$k'_t = \frac{E_{t,50}}{E_d + E_{t,50}} \quad (\text{Eq. 3})$$

Here, $E_{n,0.5}$ and $E_{t,0.5}$ [m^3/s^2] are reference deformation energy values, i.e. the values at which k'_n and k'_t equal to 0.5. For a given impact velocity v_n , k'_n and k'_t are computed using $E_d^{\text{in}} = v_n^2 D$. The assumption here is that the non-frictional tangential momentum losses, characterized by k'_t , are also scaled by the value of the normal impact energy, to account for cratering and displacement of material from the impact site. Different relationships for the normal and tangential restitution are obtained by defining different reference deformation energies, i.e. where $E_{n,0.5} \neq E_{t,0.5}$ different values of k'_n and k'_t will be calculated from the impacts' v_n . The prime indicates the calculations are carried out relative to the slope rotated by the roughness angle. Note that the restitution factor approaches 1.0 when the deformation energy approaches zero and tends to zero at infinite values of energy.

It should be noted that the hyperbolic relationships can be scaled by any point on the relationship, not necessarily the 50 % momentum conservation point. The 95 % momentum conservation point has been

used to scale the tangential restitution relationship through out this paper. This is done for convenience so that the input reference energy is a smaller number.

The validity of the hyperbolic restitution hypothesis is difficult to determine theoretically, especially its key assumption that both the normal and tangential restitution factor depend on the incident normal velocity. However, the calibrations presented here, as well as those by Gischig et al. (2015) and Preh et al. (2015), demonstrate its validity.

Goldsmith Impact Model

Goldsmith (1960, Pages 14 to 16) derived exact relationships for conservation of momentum using stereomechanical impact theory to describe the collinear impact of a perfect rigid sphere against a planar surface: A friction limit angle, γ , which represents the inclination of the incident impulse vector, is calculated using the following equation and used below:

$$\tan \gamma = \frac{2(k_t v_{t'}^{in} + R_{eff} k_t \omega^{in})}{7v_{n'}^{in}(1 + k_{n'})} \quad (\text{Eq. 4})$$

Goldsmith's theory takes k_t as equal to 1.0, assuming that all tangential energy losses are caused by friction only. In our model, we used k_t' values less than one. This accounts for non-frictional losses, such as those caused by indentation and cratering. Goldsmith's theory was developed for spherical particles. The effect of the non-spherical shape of real boulders is represented using the effective radius, R_{eff} , described in the Particle Shape Factor section following. It is also important to note that all these calculations are carried out in the plane normal and tangential to the slope modified by the roughness angle. The friction limit angle, γ , is compared to the contact friction angle between the fragment and slope, ϕ .

During an impact, slip will occur, which is sliding motion at the contact point. If the slip is fully contained within the impact period, the rotational and tangential velocities will be synchronized before the loss of contact between the particle and the substrate, i.e. the rebound $v_t^{re} = R\omega^{re}$. This scenario is referred to as a contained slip impact and occurs when the interface friction angle, ϕ , is greater than γ , obtained from Eq. 4.

The energy transferred during each contained slip impact is described by the following system of equations:

$$\begin{bmatrix} v_{t'}^{re} \\ v_{n'}^{re} \\ R\omega^{re} \end{bmatrix} = \begin{bmatrix} \frac{5}{7}k_{t'} & 0 & -\frac{2}{7}k_{t'} \\ 0 & -k_{n'} & 0 \\ -\frac{5}{7}k_{t'} & 0 & \frac{2}{7}k_{t'} \end{bmatrix} \times \begin{bmatrix} v_{t'}^{in} \\ v_{n'}^{in} \\ R_{eff}\omega^{in} \end{bmatrix} \quad (\text{Eq. 5})$$

If synchronization of the rotational and tangential velocities has not occurred during the impact, the situation is termed an uncontained slip impact and the following set of equations are used:

$$\begin{bmatrix} v_{t'}^{re} \\ v_{n'}^{re} \\ R\omega^{re} \end{bmatrix} = \begin{bmatrix} k_{t'} & -\tan\varphi(1+k_{n'}) & 0 \\ 0 & -k_{n'} & 0 \\ 0 & -\frac{5}{2}\tan\varphi(1+k_{n'}) & k_{t'} \end{bmatrix} \times \begin{bmatrix} v_{t'}^{in} \\ v_{n'}^{in} \\ R_{eff}\omega^{in} \end{bmatrix} \quad (\text{Eq. 6})$$

A detailed discussion of the differences between contained and uncontained slip impacts is given in Mitchell (2015). In practical terms, using the theory described herein results in a great majority of the impacts being of the contained type. The reason for this is that particle rotation in a developing trajectory tends to synchronize with the linear velocity, a phenomenon that is frequently observed in nature.

Particle Shape Factor

The program assumes the particle is a sphere, but for a natural rock the impact point will likely not be the equivalent spherical diameter distant from the centre of mass. To account for this difference, a shape factor, R_{eff} , is used to create a range of possible diameter modifications. At each impact, a uniformly distributed random number is multiplied by the range of the diameter modification. From examination of equations 5 and 6, it can be seen that increasing the radius will result in more of the momentum from ω^{in} being transferred to v_t^{re} and less momentum from v_t^{in} transferred to ω^{re} .

Representing the impacts as spheres impacting a plane surface in the model will result in the rotational velocity approaching 40% of the translational velocity. Mathematical verification of this is given by Chau et al. (2002). Experimental data from Ushiro et al. (2006) showed similar results to Chau et al. and the data from Ushiro et al. were used in the calibration of the shape factor, shown in the Ehime calibration section presented below.

This is a very simplified representation of the complexities resulting from the shape of real rocks, but the uniform distribution is presented here to demonstrate a simple method that manages to replicate the general relationship between translational and rotational kinetic energy. It is possible that there is some covariance between the roughness angle and the shape factor, where different shapes of rocks would be better represented with different roughness distributions as well. It has been assumed that the tests used in the calibration of this model have not had an extreme shape variety, so this idea has not been tested. It should be noted that the shape factor only influences the partition between rotational and linear energy during an impact. It does not account for the error caused by the assumption of collinearity implicit in the Goldsmith Equations, which is addressed by the roughness angle definition given above. The introduction of the shape factor in the calculations improves the calibration results somewhat, but does not have a very large influence on the results in term of linear velocity calculations.

Stopping Criterion

Models generally use an arbitrary minimum velocity to determine when the particle has stopped. The issue with this approach is that it does not consider larger rocks requiring a greater amount of rebound energy to keep them moving following an impact. As discussed in the introduction, the rock must have sufficient translational energy to overcome the weight of the rock to become airborne again, or sufficient angular momentum to overcome frictional forces over the contact area. As the model treats impacts as instantaneous, it is not possible to solve these force balances directly. However, the resisting forces acting over the contact area can all be related to the size of the block.

The rebound energy head (total kinetic energy divided by the fragment weight, units of m) is calculated for each impact and compared to a specified fraction of the particle radius, which has units of m. If the rebound energy head is less than the specified fraction of the particle radius, the simulation will stop, as it is implied that the particle does not have sufficient rebound energy to exit the impact crater. The fraction of the radius used for the stopping criteria was determined empirically by calibration (Mitchell 2015).

Extension to 3D model

This model has also been adapted for three dimensions, described by Gischig et al. (2015). The approach is very similar, but with a three dimensional flight trajectory and the tangential and angular velocities resolved in the longitudinal (downslope) and transverse (cross-slope) directions. The model demonstrated by Gischig et al. did not include the truncated normal distribution to describe the roughness, or the shape factor for the particles. These additional model features have since been implemented into the 3D model and calibrated, shown by Mitchell and Hungr (2015).

Data

A total of four sites were used in this study, all of which have been described previously, summarized in Table 1 with references to the original research. Specific details of the sites and experimental programs or rock falls observed are described following.

Insert Table 1 here.

The initial calibration of the 2D model was done using data from detailed observations of multiple rock fall parameters during a full-size rock fall experiment performed by the IRSTEA Institute (formerly CEMAGREF) of Grenoble near Vaujany in the French Alps. Details of the experiments were described by Dorren et al. (2005). The initial 2D calibration was done concurrently with the 3D calibration

presented by Gishig et al. (2015). Further details about the test site and procedure and site photos are available in Gishig et al (2015).

The total runout distances and the velocity and jump height of each block, as recorded by cameras at two locations, designated Screens 1 and 2. The translational kinetic energy was obtained from the measured block mass and velocity were also calculated (Berger and Dorren 2006).

A second parametric study was done using data from the Preg Quarry site in Austria. Four slope sections along one bench in the quarry were used in this testing. The sections ranged from slightly convex to slightly concave. The surfaces involved were rough, steep faces created by uncontrolled blasting of strong rock and flat, horizontal quarry floors of highly compacted rock debris. The experimental protocol and additional details on the site were presented by Preh et al (2015).

An excavator was used to move the boulders gently over the edge of the slope. At the end of each experiment, the stopping point of the boulders was measured. The impact distances were estimated by video analyses. After 5 throws, the boulders at the bench were removed by a wheel-loader in order to prevent boulder interaction during the following throws. Rock fall trajectories were filmed using two digital cameras, installed at the top and the toe of the slope (Preh et al. 2015). The key information used for this study are the first impact location, where the boulders first impacted the quarry floor, and the total runout distance, determined by video analysis and ground survey.

The third calibration dataset is from the Ehime prefecture in Japan. The experimental procedure and test site for their 2003 rock fall study were described by Ushiro et al. (2006). The runout path consisted of rock with thin colluvial cover and fine-grained talus. Both of these surfaces were similar to the Vaujany site. In the testing both cast concrete blocks and natural rocks were launched down a relatively open slope using a hydraulic excavator. The trajectories of the blocks were tracked using high speed cameras and the blocks were instrumented with triaxial accelerometers and angular rate sensors. This allowed for the tracking of the position, translational velocity and rotational velocity of all the test blocks (Ushiro et al. 2006).

The fourth dataset was from Tornado Mountain in British Columbia, which was described in detail by Wyllie (2014). In 2004, two separate natural rock falls originating from a 50 m high, steep rock face travelled distances of approximately 340 m vertically and 610 m horizontally and came to a stop on a horizontal bench excavated for the railway. The runout path was along a uniform talus slope, similar in description to the Vaujany site. There was no evidence of recent rock falls on the talus slope prior to the 2004 event. The impact points on the slope were identified from a field survey and the co-ordinates were recorded with a GPS (global positioning system) unit and a laser rangefinder. Broken trees were noted at

certain locations along the trajectories and it was assumed that the trees broke at the elevation that they were impacted. Those elevations were used to reconstruct the trajectory of the rocks between the impact points before and after, by fitting a ballistic parabola to the three points, assuming breaking the tree did not affect the trajectory. For the simulations we compared the final stopping positions of the two blocks and jump heights where available.

Model calibration

Vaujany

The Vaujany dataset was used to perform an initial calibration for a ground surface characteristic of relatively fine-grained talus slopes. In order to observe the effects of various values of the model inputs on the calculated outputs, a parametric study was conducted. This was achieved by varying the reference normal unit impact energy and roughness angle. The losses due to the tangential coefficient of restitution are, in general, much smaller than those due to the normal coefficient of restitution. The observed values also generally fall in a smaller range (Turner and Duffy 2012), which is why variability in the reference tangential unit impact energy was not considered in the initial parametric study. A tangential reference value of 95 % energy conservation, $E_{t,95} = 50 \text{ m}^3/\text{s}^2$ was used for all trials. Variation in the contact friction angle was also not considered and the value of 30° reported by Bourrier et al. (2012) was used.

An initial range of normal reference values for 50 % energy conservation, $E_{n,50} = 1$ to $50 \text{ m}^3/\text{s}^2$ and a roughness scale, $\theta_{\text{scale}} = 0.1$ to 1.0 was used to establish the response of the model over a wide range of inputs. For simplicity a uniform distribution was used to represent the roughness angle distribution, so that only θ_{scale} is needed to describe the distribution. The initial run looked at finding the minimum root mean squared error (RMSE) for the runout distance distribution to establish an optimum zone to focus on for more detailed analysis. Using two variables for the analysis also allows for the results to be represented as a surface plot, with the lower areas indicating a better fit.

The best results for the runout distribution were found at the lower ranges of the normal reference impact energy and at high roughness. A more refined parametric search was conducted for $E_{n,50}$ from 5 to $20 \text{ m}^3/\text{s}^2$ and θ_{scale} from 0.7 to 1.2. In this range, the RMSE for the distributions of total runout distance, jump height and translational velocity at Screens 1 and 2 were calculated for all parameter sets tested. Since design of protective structures will be governed by the maximum, or some upper-range value, the difference between the simulated maximum jump height and maximum velocity and the observed maxima were calculated. While comparison of maxima is not preferred statistically, because the simulations used the same number of rocks as the actual tests the comparison is valid. Examples of some

of the plots for the RMSE and the absolute value of the difference between the simulated and observed maxima are shown in Figure 3.

From a visual inspection of the data, some general observations can be made:

- The plots show a banded behavior, indicating there is not a unique best fit for any output using a two parameter set;
- Increasing roughness tends to decrease the runout distance, mean jump height and mean velocity;
- Increasing normal reference value tends to increase the runout distance and maximum jump height, with a negligible effect on the velocity; and
- The model is most sensitive to the roughness.

There is not one clear area where the model performs best for all outputs, so a compromise was made to try to capture the general behavior of the system while also capturing the maximum values. An $E_{n,50}$ of $15 \text{ m}^3/\text{s}^2$ with $\theta_{\text{scale}} = 0.9$ on the forest slope section and $\theta_{\text{scale}} = 0.5$ on the road were selected. Using these inputs the model would give acceptable predictions of the jump height and velocity distributions, including the maxima, but it would tend to over-predict the runout. This parametric study did not provide the final calibration of the model, however; it narrowed the range of parameters moving forward into more detailed calibration.

Because of the stochastic roughness element in the model there will be some variation from run to run. The model was run a number of times with the same input parameters. It was found that using the aggregate of 5 runs (each of the 100 blocks thrown 5 times in the simulation) provided repeatability of the mean and maximum values compared in this study. All of the subsequent results are the averages of 5 combined model runs.

From the observation that the model is most sensitive to roughness, further investigation was made into the roughness angle distribution. The effect of using the truncated normal distribution (Eq. 1) described previously was tested. The objective of using a truncated normal distribution was to have more impacts with a low roughness to result in higher translational velocities, with the occasional high roughness value to improve the fit of the runout distribution and to better simulate extreme values.

To test the truncated normal distribution a number of different mean (μ) and standard deviation (σ) values were used with all other model values held constant. Values of μ ranging from 0.3 to 0.5 were used with σ ranging from 0.15 to 0.45. Increasing the input μ will tend to decrease the runout distance, average

jump height and average velocity. It was found that increasing σ tends to reduce the runout and increase the variation in the jump height and velocity distributions, which also results in a slight reduction in the mean values. It was determined that the agreement between runout, jump height and velocity were best where μ is equal to σ , detailed in Mitchell (2015). A μ and σ of 0.5 were chosen, which could then be scaled by the roughness scaling factor, θ_{scale} , input by the user. This roughness distribution and the natural rock shape factor from the Ehime calibration, discussed following, were used as the optimal values for the Vaujany calibration, summarized in Table 4.

Comparisons between the observed and simulated runout distributions using these inputs are shown in Figure 4 and the comparisons between the observed and simulated jump height and velocity distributions at both Screen locations are shown in Figure 5.

Preg Quarry

The Preg Quarry data was used to develop an initial set of parameters for rough exposed rock slopes, and to examine the effects of boulder size on the roughness calculation. The process of conducting an initial parametric study was repeated for two of the four slope sections in the Preg quarry in Austria with the goal of establishing parameters for exposed rock faces. The truncated normal distribution was used for the roughness angle distribution. An initial range of $E_{n,50}$ from 5 to 25 m^3/s^2 and θ_{scale} from 0.1 to 1.0 was used to establish the response of the model over a wide range of inputs. The distribution of the first impact locations was used to calibrate the quarry face parameters, with the distribution of runout distances used to calibrate the quarry floor parameters, again using the RMSE as the objective parameter. Preg Slopes 1 and 2 were selected for this study.

The normal unit deformation energy was set as being the same for both the quarry face and quarry floor, meaning the θ_{scale} value is the only difference between the two parameter sets. The locations of the minima on the parametric plots do not match for the two slopes, however the inputs $E_{n,50} = 10$ to 20 and quarry face $\theta_{\text{scale}} = 0.4$ to 0.8 give results with similarly low errors for the distributions of first impact locations, shown in Figure 6. In this case the model produces reasonable total runout distributions with a quarry floor roughness between 0.3 and 0.5 over the range of normal unit deformation energies tested. In order to further constrain the range for the quarry face θ_{scale} , the difference between the maximum observed and simulated first impact locations were also examined. This constrained the quarry face roughness to a range of 0.5 to 0.8.

As with the Vaujany calibration, a number of runs were necessary to get repeatable results. For example, on Preg Slope 1, with the same input parameters the maximum simulated runout ranged from 9.97 m to

20.3 m and the mean runout ranged from 5.09 m to 6.33 m for ten individual simulations of 25 blocks each. Using the aggregate of each of the 25 blocks thrown ten times in the simulation reduced the variation between the means to the order of 0.1 m, and the variation between predicted maxima was reduced to the metre scale. All of the subsequent results are the averages of ten combined model runs.

With no size scaling of the roughness, the smaller blocks tend to have a further runout. To address this issue, the linear roughness scaling (Eq. 2) was introduced. To calibrate the relationship, the simulated and observed runout versus boulder mass were compared. As the previous calibration of the Vaujany model was achieved without the roughness scaling, the relationship was set so that it would average to 1 (no net change on average) for the Vaujany boulders. The values for slope and intercept for the roughness relationship were chosen with that constraint, which resulting in the following relationship.

$$\theta_{eff} = \theta(1.36 - 0.3D) \quad (\text{Eq. 7})$$

With a large enough boulder, this equation could give a negative roughness value. To deal with this, a minimum value for θ_{eff} was set to 0.3. A plot of the mass versus runout, with and without roughness reduction, is shown in Figure 7. Without the roughness reduction the model tended to over-predict the runout for small boulders and under-predict the runout for larger boulders. As desired, introducing the roughness reduction reduced the average runout for the smaller boulders and increased the average runout for the larger boulders.

After determining the initial range of inputs from the parametric study and calibrating the roughness reduction relationship, the calibration was refined using a trial and error process, detailed by Preh et al. (2015). The optimal values determined from the refined calibration summarized in Table 4. As there was not data on the rotational motion from these tests the particle shape factor was not considered in this calibration.

The duration of each test was compared to the simulated duration to calibrate the constant term in the stopping criterion. Values of 0.1, 0.25, 0.5 and 0.75 were tested. The experimental values for the runout duration were plotted against the runout distance. A linear trend line was fit to the data and the 95 % prediction interval was calculated. The simulated results were plotted and the criterion that resulted in the best correspondence to the 95 % prediction interval was selected. The effect of the stopping criteria on the total runout distance was found to be negligible for the range of values tested, indicating the part of the trajectory being eliminated was in fact very small bounces at the end of the trajectory. An example of the comparison plot is shown in Figure 8.

Ehime

The main objectives of the Ehime calibration were to see if the Vaujany parameters would work for a similar site, and to calibrate the shape factor, as the Ehime data set allowed for a comparison of the simulated to observed rotational velocity. The calibrated parameters from the Vaujany simulation were used as a first estimation for Ehime as both sites featured dense talus slopes and thin soil cover. The optimum parameters for Vaujany combined with the size dependent roughness scaling resulted in a good fit for the translational velocity profiles, however, there was a significant over prediction of the rotational velocity. The results presented by Ushiro et al. (2006) gave plots of the translational kinetic energy, E_k , versus the angular kinetic energy E_ω . The model could not predict the observed scatter without the addition of the shape factor described previously. The ratio of E_ω / E_k plots as a linear relationship with the slope determined by the particle radius if the only random element is the roughness angle, as shown in Figure 9a. If the random variation of the radius is included in the model, the simulation can have the same amount of variation as was observed, shown in Figure 9b.

The geometry of the particles can be used to obtain a first estimate for the shape factors. Theoretically, the spheres do not require a shape factor. The shortest and longest distances from the centroid to the surface of a cube divided by the radius of a sphere with an equivalent volume are 0.81 and 1.4, respectively. The moment of inertia of a cube is within 10% of the moment of inertia of a sphere with an equivalent volume, thus the effect of representing a cube as a sphere is negligible. Due to the variable geometry of the natural rocks it was not attempted to theoretically estimate shape factors, they were determined by model calibration only.

Calibration of the shape factor showed that it was necessary to include a relatively small shape factor to replicate the experimental results for the sphere dataset. It is believed that this is required due to the modification of the spheres during the trajectory due to fragmentation as well as other, small magnitude processes occurring at the impact location not explicitly dealt with by the model. The shape factor for the cubes was slightly less than the theoretical range. This difference between the range of shape factors based on the geometry only and the calibrated values is not surprising given the simplifications to the impact process that have been made. The natural rocks required the highest shape factor to replicate their observed behavior.

The optimal results were found using a minimum value of 1.0 and a maximum shape factor of 1.2 for the spheres, a minimum of 0.85 and a maximum of 1.3 for the cubes and a minimum of 1.0 and a maximum of 1.5 for the natural rocks. Plots of the translational velocity and angular velocity versus elevation for the natural rock dataset are shown in Figure 10.

Pseudo-forward analysis

Tornado Mountain

Using the Vaujany/Ehime parameters directly resulted in some of the simulated blocks having a runout equal to or slightly greater than the observed runout. Decreasing the roughness from 0.7 to 0.6 increases the probability of the simulated blocks matching the observed runouts and gives a good match to the observed jump heights, shown in Figure 11.

The masses of the two boulders, designated Boulder A and Boulder B used in the simulation are the final mass (Wyllie 2014). The initial mass of the boulders was greater, as there was evidence of fragmentation along the trajectory. If the initial mass is used, the roughness will be reduced due to the larger diameter, which could result in a greater runout distance. Significant amounts of fragmentation were not noted for the Vaujany or Ehime tests, so this may explain the necessity to reduce the roughness to achieve the same results.

Relationship Between Incidence Angle and Normal Restitution

It has been noted that low angle impacts can result in normal restitution values of greater than 1. A linear relationship describing the relation between the incidence angle, α^{in} and the normal restitution, k_n , was presented by Spadari et al. (2011).

$$k_n = -0.04\alpha^{in} + 1.8 + \Delta, \text{ with } \Delta \in \{-0.5, 0.5\} \quad (\text{Eq. 8})$$

The random number, Δ , ranging from -0.5 to 0.5 represents the 90% prediction interval from the experiments that were used to derive this relationship. This relationship was developed specifically for incidence angles less than 30° .

The correlation between the incidence angle and the normal restitution has also been described using a power law (Wyllie 2014).

$$k_n = 19.5(\alpha^{in})^{-1.03} \quad (\text{Eq. 9})$$

The restitution factor evaluations carried out by the above studies relate to “apparent” restitution factors. With reference to Figure 1, “apparent” normal restitution factor is the ratio between observed rebound and incident velocity, normal to the local slope surface, unperturbed by “roughness”. Roughness is a random parameter that is “invisible” in surface models or surveyed ground profiles. In contrast, the “true” restitution factor, as used in the Goldsmith Equations (Eq. 4-6), is the ratio between rebound and incident velocity normal to the slope angle modified by roughness. The difference between apparent and true

restitution factors explains the apparent paradox of factors exceeding 1, which would suggest that the law of conservation energy is being violated (see Appendix).

Our model does not explicitly take α^{in} into the restitution calculation, however, it can produce calculated relation between this parameter and the apparent restitution factors as output, based on the more fundamental input parameters described earlier. Details of this calculation are given in the Appendix.

The data from Wyllie for Tornado Mountain as well as the general points from Spadari et al. were plotted along with the simulated values for k_n and α^{in} , shown in Figure 12. Although Spadari et al. and Wyllie chose different forms of the relationship between incidence angle and the normal restitution, the observed points that were used to derive their relationships are very similar. The scatter of the points from each dataset and the simulated values were plotted without attempting to fit a trend line. Evidently, the stochastic algorithm described in this paper simulates both the trend and variability shown by the observational data. This demonstrates how, although the stochastic roughness angle is a simplified representation of the actual impact processes, it can replicate the key behavior of rock fall impacts.

Discussion

One of the motivations of making the model as simple as possible is to develop an engineering design tool that will provide reliable results when limited site information is available. A hypothetical design for placing flexible barriers at the Vaujany Screen 1 and Screen 2 locations was conducted following the ETAG 027 design guideline.

The design kinetic energy, determined from the velocity and block mass and the barrier height were determined from the Pierre 2 analyses following the procedure outlined in Peila and Ronco (2009). The inputs to determine the design velocity, v_p , are the 95th percentile of the computed velocities, v_t and a safety factor, γ_F . The design block mass, m_p , is determined from the rock density, γ , the design block volume, vol_b and a safety factor, γ_M . The design interception height, h_i , is determined from the 95th percentile of the forecast bounce height, h_p , plus an additional clearance, f , of not less than $\frac{1}{2}$ of the average block diameter.

The computed velocity was calculated as the 95th percentile value of a Weibull distribution fit to the velocity distributions. A safety factor of $\gamma_F = 1.16$ was used, corresponding to the case with the velocity determined by computer simulation and the slope characterized by a precise topographic survey. The maximum block volume, 1.26 m^3 and rock density, 2800 kg/m^3 , were given by Berger and Dorren (2006), which were used in the calculation of the block mass, $\gamma \text{ vol}_b$. A safety factor of $\gamma_M = 1.05$, corresponding to a high level of confidence in the rock size, was selected. The following table presents

the results of the calculations to determine the maximum energy levels (MEL) and serviceability energy levels (SEL). The 95th percentile of the bounce height was also calculated from Weibull distributions fit to the simulation data. The mean block diameter was 1.2 m, so the minimum clearance height is 0.6m.

Insert Table 2 here.

These design values were then compared to the distributions of the experimental data. The goal was to look at the reliability of the predictions, i.e., what is the probability that the actual factor of safety is less than 1.0 (Duncan 2000). Both the SEL and MEL exceed the maximum observed kinetic energies at Screen 1 and Screen 2, which were 786 kJ and 958 kJ, respectively. Using a Weibull distribution fit to the observed kinetic energy distribution, the probability of the design values being exceeded was calculated. The minimum interception height was less than the observed maximum height for both Screen locations. However, the maximum simulated bounce height did exceed the observed value for both Screen locations. The probability of exceeding the calculated h_i and the simulated maximum, $h_{\max}$, are shown in the following table.

Insert Table 3 here.

The results obtained for the prediction of the SEL or MEL show using the factors of safety given in the ETAG 027 guideline will result in a low probability of failure. Using the minimum clearance specified by the guideline results in an appreciable probability of exceedance. However, a more appropriate level of safety can be used if the clearance height, f , is set so h_i is equal to the maximum height for the simulation.

Model limitations

The loss of energy due to fragmentation can be a significant portion of the loss of energy during impacts. This is especially true for rocks with planes of weakness impacting hard rock slopes (Giacomini et al, 2009). This process can also lead to relatively small fragments impacting barriers, resulting in high localized stresses on a barrier, the “bullet effect” described by Spadari et al. (2012). The model in its current version cannot address fragmentation.

Models that rely on a spherical representation of particles tend to over-predict runout (Leine et al. 2014). Much of the recent development in the field of rock fall modeling has attempted to address the limitations of a spherical representation by using more complex geometric shapes and contact relationships (c.f. Andrew et al. 2012; Leine et al. 2014). The disadvantages of this approach are two-fold; the first is the greatly increased computational cost, which is less of a deterrent with modern computers. The second disadvantage is the increased data required to characterize the rocks and the slope in the models. The

characterization of sites being analyzed in engineering practice is notoriously poor. This uncertainty in the input parameters results in a great deal of parameter uncertainty when using these complex models.

The present model relies heavily on the hypothesis that the stochastic roughness angle can mimic the effects of real, non-collinear impacts of rough, irregular shaped rocks in a model that uses an idealized representation of smooth spheres with collinear impacts. This is clearly a major simplification of the actual physical processes occurring during an impact. However, the simplified model reduces the parameter uncertainty and makes the model easier to apply to a forward analysis. No attempt has been made to theoretically demonstrate the validity of representing impacts using a single, stochastic variable to have the net effect of the non-collinear contact processes. The model calibration is used to test this hypothesis.

The shape factor described here also does not account for the non-collinearity of the contact points for real, irregular rock, or the gyroscopic forces resulting from the rotation of irregular rocks. The fact that tabular or wheel shaped boulders can be stabilized by rotation and achieve unusually long runouts has been shown previously (Evans and Hungr 1993). It has also been demonstrated that this process is not fully random, with a certain critical velocity and resonance in the motion of the center of mass needed to be achieved for stabilization (Leine et al. 2014). It has been demonstrated that despite these limitations, the model provides a reasonable approximation of these processes for cases where limited input information is available. It is also assumed that the roughness and the shape factor are independent stochastic variables. There is insufficient detailed information on the rock shapes in the calibration data present here to make a meaningful test of this assumption. In the future, it should be possible to approximate the unique extreme behaviour of tabular or elongated particles by providing them with a special set of roughness parameters. However, additional calibration will be required.

Conclusions

The calibrated parameter sets for firm talus slopes and exposed rock faces are summarized in Table 4.

Insert Table 4 here.

The results of the calibration demonstrate that using this simple model can reproduce all the main characteristics of the kinematic behavior of rock falls on both talus and exposed rock slopes at several sites, using a small selection of input parameters, as listed in Table 3. The initial calibration of the model using the roughness angle as the only stochastic element within the point mass model demonstrated that it could be used to accurately predict ranges of runout distances, velocities and jump heights simultaneously. The combination of a stochastic contact roughness and particle shape factor in the model

can be used to produce a realistic range of translational and angular velocities, as well as achieving an improved fit for the distributions of runout distances, velocities and jump heights. These concepts can also be effectively applied to a 3D model space, demonstrated by Gischig et al. (2015) and Mitchell and Hungr (2015). The model can also produce a distribution of apparent normal restitution factors greater than one, consistent with field observations.

It has been demonstrated that the model is capable of making realistic predictions of both average rock fall behaviour, variability and extremes over several different sites, characterized by general slope descriptions. The model is also able to predict normal restitution factors greater than one consistent with observations made by other authors.

A hypothetical design calculation for the Vaujany test site was conducted and compared to the experimental data using reliability engineering principles. From this comparison it was found the model's results for translational velocities would provide design kinetic energy values with a probability of exceedance of less than 1 in 1000 for all cases considered. To achieve a similar level of safety, the maximum values predicted for the model should be used instead of the 95th percentile plus the minimum clearance value specified in ETAG 027.

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Appendix – Apparent Restitution Values

As discussed in the theoretical development of the restitution factors, the observed normal restitution values are dependent on the incidence angle of the impact. The research of Spadari et al. (2011) and Wyllie (2014) have shown the normal restitution factor can be greater than 1. This is represented in the model with the rotation of the incident and rebound vectors using the roughness angle. As the values of k_n' and k_t' are calculated in the n' - t' plane, the values for k_n and k_t , which are relative to n - t plane, will be different. The rotation of the local coordinate system results in the incident normal velocity having an apparent effect on the tangential rebound velocity, and the incident tangential velocity having an apparent effect on the normal rebound velocity. The rotation of the incident and rebound vectors can be shown in the rotation matrix $\mathbf{R}_\theta$.

$$\mathbf{R}_\theta = \begin{bmatrix} \cos \theta & \sin \theta & 0 \\ -\sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Since the output from PIERRE is given relative n - t plane, but the calculations are performed in the n' - t' plane, both sides of the modified Goldsmith system of equations, designated $\mathbf{G}$, must be multiplied by the rotation matrix $\mathbf{R}_\theta$, shown in the following equation.

$$\mathbf{R}_\theta \mathbf{v}^{re} = \mathbf{G} \mathbf{R}_\theta \mathbf{v}^{in}$$

Finally, to isolate the rebound velocities vector, both sides must be multiplied by the inverse of $\mathbf{R}_\theta$. Carrying through this calculation using the matrix for a contained slip impact, where the 3×3 matrix is the result of $\mathbf{R}_\theta^{-1} \mathbf{G} \mathbf{R}_\theta$, the following matrix is found.

$$\begin{bmatrix} v_t^{re} \\ v_n^{re} \\ R\omega^{re} \end{bmatrix} = \begin{bmatrix} \frac{5}{7}k_t \cos^2 \theta - k_n \sin^2 \theta & \frac{5}{7}k_t \cos \theta \sin \theta + k_n \sin \theta \cos \theta & -\frac{2}{7}k_t \cos \theta \\ \frac{5}{7}k_t \sin \theta \cos \theta + k_n \sin \theta \cos \theta & \frac{5}{7}k_t \sin^2 \theta - k_n \cos^2 \theta & -\frac{2}{7}k_t \sin \theta \\ -\frac{5}{7}k_t \cos \theta & -\frac{5}{7}k_t \sin \theta & \frac{2}{7}k_t \end{bmatrix} \times \begin{bmatrix} v_t^{in} \\ v_n^{in} \\ R_{eff}\omega^{in} \end{bmatrix}$$

Rotation of the impact plane by the roughness angle allows the program to calculate normal restitution values greater than 1, while not violating the law of conservation of energy.

Table 1: Summary of test sites used in model calibration and validation

Site	General Description	Slope height and angle	Blocks thrown/observed	Reference
Vaujany	Open talus slope, vegetated, but without trees. Particle sizes gravel to small boulder.	243 m elev. 32° average slope	100 natural boulders, 1500 to 3500 kg	Dorren et al. 2005
Preg	Hard rock quarry, rough blasted walls	16.1 to 17.4 m elev. 53 to 71° average slope	100 natural boulders, 140 to 8130 kg	Preh et al. 2015
Ehime	Thin soil over bedrock and talus slope below, sparse vegetation. Particle sizes gravel to small boulder.	26 m elev. 44° average upper slope (thin soil) 16 m elev. 35 ° average talus slope	4 concrete spheres, 194 kg, 7 concrete cubes 520 kg, 12 natural boulders, 120 kg to 2060 kg	Ushiro et al. 2006
Tornado Mountain	Forested talus slope, conifers up to 500 mm diameter. Particle sizes gravel to small boulder.	340 m elev. 26° average talus slope	2 natural boulders, 3750 and 5600 kg	Wyllie, 2014

Table 2: Calculation table for the SEL, MEL and interception height for hypothetical barriers at Vaujany Screen 1 and 2.

	$v_p = v_t \gamma_F$	$m_p = \gamma \text{ vol}_b \gamma_M$	$\text{SEL} = 0.5m_p v_p^2$	$\text{MEL} = 1.3(0.5m_p v_p^2)$	$h_i \geq h_p + f$
	(m/s)	(kg)	(kJ)	(kJ)	(m)
Screen 1	24.0	3700	1070	1390	4.02
Screen 2	24.9	3700	1150	1490	4.16

Table 3: Hypothetical barrier failure probability for the design scenarios

	$P(E_K > \text{SEL})$	$P(E_K > \text{MEL})$	$P(h > h_i)$	$P(h > h_{\max})$
Screen 1	0.072 %	0.004 %	2.407 %	0.006 %
Screen 2	0.099 %	0.006 %	6.185 %	0.389 %

Table 4: Calibrated parameter sets

Parameter	General Slope Description ²		
	Fine to medium, firm talus, residual soil, light vegetation	Rough rock face, strong to very strong, uncontrolled blasting	Quarry floor, compact rock debris
Unit energy for normal restitution, $E_{0.5,n}$	15	5	5
Unit energy for tangential restitution, $E_{0.95,t}$	50	50	50
Roughness scale value, θ_{scale}	0.6 – 0.7	0.65	0.3
Shape factor for natural rocks, R_{eff}	1.0 – 1.5	NC ¹	NC ¹
Data sets used for calibration	Vaujany ² , Ehime, Tornado Mountain	Preg Quarry ²	Preg Quarry ²

¹ NC – Not calibrated² Photos of the three surfaces can be seen in Gishig et al. (2015) and Preh et al. (2015).

Figure 1: Modification of the local slope angle by the roughness angle.

Figure 2: Conceptual force-penetration diagram, loading path indicated by the arrows. The shaded area indicates the total plastic work done during the impact.

Figure 3: Parametric surface plots obtained by varying the slope roughness and normal unit deformation energy for: a) RMSE of the cumulative runout distribution, b) the absolute value of the residual of the maximum runout, c) RMSE of the cumulative jump height distribution, d) the absolute value of the residual of the maximum jump height at Screen 1, e) RMSE of the cumulative velocity distribution at Screen 1, and f) the absolute value of the maximum residual velocity at Screen 1.

Figure 4: Observed versus simulated runout distribution for the calibrated Vaujany model.

Figure 5: Comparison of observed and simulated jump height and translational velocity at the Vaujany Screen 1 and Screen 2 locations with the calibrated model inputs.

Figure 6: Parametric surface plots of the RMSE for the distribution of first impact locations obtained by varying the slope roughness and normal unit deformation energy for on the quarry face section: a) Preg Slope 1, and b) Preg Slope 2.

Figure 7: Comparison of runout versus mass, a) without roughness reduction, and b) with roughness reduction for Preg Quarry Section 1.

Figure 8: Calibrated stopping criterion with plot of simulated and observed runout distance versus duration for Preg Slope 3.

Figure 9: Comparison of rebound translational kinetic energy to angular kinetic energy from Ehime cube dataset, with the dashed lines indicating the mean and upper and lower bounds from Ushiro et al., 2006, a) without the inclusion of a shape factor, and b) with a shape factor uniformly distributed between 0.85 and 1.3.

Figure 10: a) Plot of observed and simulated incident velocities vs elevation for the Ehime natural rock dataset, and b) plot of observed versus simulated angular velocities versus elevation.

Figure 11: Comparison of simulated and observed jump heights versus the runout distance, and the observed runout distance, indicated by the dashed line, for Boulders A and B from Tornado Mountain.

Figure 12: Comparison of PIERRE 2 calculated normal restitution factors relative to the impact incidence angle for the Tornado Mountain simulation compared to incidence angle and normal restitution values presented by Spadari et al. (2011) and Wyllie (2014).