Quasielastic Proton Spin-Transfer Observables at Intermediate Energies

by

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A thesis submitted in conformity with the requirements for the Degree of Doctor of Philosophy in the University of Toronto

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ABSTRACT

Three sets of new data for inclusive quasielastic polarized proton scattering are presented. These include two complete sets of data for $^{12}$C at two incident proton kinetic energies, 290 MeV and 420 MeV. The third is a partial set of spin observables for $^{16}$O at 420 MeV. The scattering angle at each energy is chosen for a central momentum-transfer of $1.9 \text{ fm}^{-1}$ in the nucleon-nucleon frame, which corresponds to an energy-transfer of 80 MeV, well above the giant resonances, yet well below the $\Delta$-isobar excitation.

A Fermi-gas model is used to calculate the quasielastic spin observables across the full width of the quasielastic peak, and the longitudinal-to-transverse ratio of the spin-isospin nuclear response functions is extracted from the data. Our results indicate that the inclusive interaction is dominated by the nucleon-nucleon scattering process, but no enhancement of the longitudinal nuclear response is seen, contrary to the suggestions of spin-isospin collectivity in the nuclear interaction. Comparison of the $^{12}$C and $^{16}$O data reveals a remarkable resemblance of the respective spin observables, in support of the peripheral nature of inclusive proton scattering, and indicates that the inclusive quasielastic interaction is insensitive to nuclear structure. Effects of medium modification to the free nucleon-nucleon process, however, are evident in the pronounced quenching of the polarization parameter relative to the free value.
To my parents
ACKNOWLEDGMENTS

I would like to express my deep gratitude to my supervisor, Prof. Tom Drake, whose insight led to the initiation of this project, and whose guidance, support and enthusiasm helped me to carry this work to its successful conclusion.

I would also like to thank our collaborators at TRIUMF. Their constant efforts in commissioning the longitudinally polarized proton beam and the focal-plane polarimeter made this experiment possible.

Finally, the financial assistance of the University of Toronto Open Doctoral Fellowship program is gratefully acknowledged.
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Chapter 1

INTRODUCTION

The measurement and calculation of proton spin-transfer observables for the continuum is a new area of research in nuclear physics. The precise measurement of these spin observables has only been recently made possible through the construction of focal-plane-polarimeters, and it was not until recently that the spin observables in the continuum region were being calculated.

For elastic and inelastic scattering at intermediate energies\(^1\) a number of different theoretical approaches has been used to compare with data. These vary in sophistication from the purely phenomenological fitting-of-parameters to the ambitious microscopic descriptions, using either the non-relativistic Schrödinger equation or the relativistic Dirac equation.\(^2\) The interest in the relativistic approach to nucleon-nucleus (N-A) scattering began with the observation that the typical 50 MeV nuclear potential is actually a remnant of relativistic potentials that are on the order of the nucleon mass.\cite{McN83, Tj85} It was therefore speculated that relativistic corrections to the N-A interaction could not be ignored. In fact, as it has already been shown that the relativistic approach is able to give a satisfactory description of the saturation property of nuclear matter \cite{An80} while the conventional Schrödinger approach fails to give quantitative results \cite{Da81}, it is not surprising that a lot of effort has gone into a search for the signature of relativistic effects in N-A scattering. Indeed, these studies gained impetus when the relativistic impulse-approximation was shown to surpass the standard non-relativistic impulse-approximation in describing some scatter-

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\(^1\) See, for example, \cite{AIP83, AIP80, AIP75, LA85} and references therein.

\(^2\) See, for example, \cite{Ar81, Br78, Cl83, Com80, Fra85, IIo81, Hor86a, Hu81, Hy85, Kel80, Lo81, McN83, Mu87, Na81, Pi85, Ray85, Sh83}.
However, from the studies of elastic scattering, it has been shown that spin observables are able to provide better sensitivity to the scattering amplitudes.\cite{Sh83, Cl83, Ray85, Gl79, Rah81} Recently, Horowitz et al.\cite{Hor86b, Hor88} suggested measuring the spin observables for quasielastic proton scattering. Indeed, they were able to predict, with a schematic model based on the relativistic impulse-approximation, that the polarization parameter for quasielastic scattering will be significantly quenched from the free N-N value.

From a different point of view, the utility of the nucleon as a spin-isospin sensitive probe of the nucleus has recently been emphasized, especially on the spin observables, which are relatively insensitive to nuclear distortion.\footnote{See, for example, \cite{Alb88, Bl82, Ca84, Cor81, Lo84, McC84, Mo82, Mo84, Pe81, Re86, SM82}.} In particular, an investigation of the spin-isospin ($\sigma \tau$) longitudinal ($\vec{\sigma} \cdot \vec{q}$) and transverse ($\vec{\sigma} \times \vec{q}$) interactions within the RPA framework predicted an enhancement of the longitudinal over the transverse response in nuclear matter.\cite{Alb80, Alb82} And it is the ability of a hadronic probe to provide both longitudinal ($\vec{\sigma} \cdot \vec{q}$) and transverse ($\vec{\sigma} \times \vec{q}$) coupling to the nucleus that makes proton quasielastic scattering particularly interesting in this area.\cite{Alb86, Alb88} However, for inclusive proton quasielastic scattering, the reaction is localized in the nuclear surface due to strong absorption. And to account for this effect, Alberico et al. have recently investigated the spin-isospin response with the inclusion of nuclear surface effects.\cite{Alb84, Alb88}
At the time of this work, experimental data on spin-transfer observables for quasielastic proton scattering was scarce, especially in the higher energy-loss (\(\omega\)) region for \(\omega \sim 60 - 100\) MeV. Here, we have measured two complete sets of data for \(^{12}\text{C}\) at two incident kinetic energies, 290 and 420 MeV, respectively; and a partial set of spin observables for \(^{16}\text{O}\) at 420 MeV. The scattering angle at each energy was chosen to correspond in the nucleon-nucleon frame to a central momentum transfer of \(q = 1.9\) fm\(^{-1}\), or \(\omega \sim 80\) MeV in energy-loss. Specifically, this is \(23.5^\circ \pm 0.3^\circ\) (lab) at 420 MeV, and \(29.5^\circ \pm 0.3^\circ\) (lab) at 290 MeV incident energy. This higher \(\omega\) value is in a region where the supposed two-body nature of quasielastic scattering is emphasized, well above the collective giant resonances, yet well below the threshold for \(\Delta\)-isobar excitation.

The experiment was conducted at TRIUMF using the Medium Resolution Spectrometer (MRS) and the Focal-Plane Polarimeter (FPP). In essence, a Lamb-shift ion-source (POLISIS) provided the polarized proton beam which was accelerated in the cyclotron to 290 and 420 MeV. After extraction from the cyclotron, the beam was steered into beam-line 4B. The beam at extraction was polarized in the vertical direction; however provision was made by a combination of dipole magnets and solenoids to rotate the polarization into the sideways or longitudinal directions. The degree of polarization of the incident beam was measured before impingement on the primary target. After scattering, the proton was momentum analyzed in the MRS magnetic spectrometer. The polarization of the scattered protons was determined by measuring the asymmetry for an auxiliary scattering using the focal-plane-polarimeter located beyond the MRS focal plane.

The experimental data were analyzed using procedures described in Chapter 5. To interpret our experimental results, we have used a Fermi-gas model to calculate the quasielastic observables across the full width of the quasielastic peak. Also, the longitudinal-to-transverse ratio of the spin-isospin nuclear response functions was extracted from the data.
Our results indicate that the inclusive quasielastic interaction is dominated by the nucleon-nucleon (N-N) scattering process, and that no enhancement of the longitudinal nuclear response is seen, contrary to the suggestions of spin-isospin collectivity in the nuclear interaction. A comparison of the $^{12}$C and $^{16}$O data reveals that the respective spin observables are almost identical, in support of the peripheral nature of inclusive proton scattering, and indicates that the inclusive quasielastic interaction is insensitive to nuclear structure. The effects of medium modification to the free nucleon-nucleon process, however, are seen in the pronounced quenching of the polarization parameter relative to the free value.

The following chapters are organized as follows. In Chapter 2, we briefly review some of the theoretical framework and ideas leading towards our interpretation of quasielastic scattering. Details on the experiment are given in Chapters 3, 4 and 5: Chapter 3 on the experimental setup; Chapter 4 on data acquisition; and Chapter 5 on data analysis. In chapter 6, our experimental results are presented and compared with calculations obtained with a free Fermi-gas model. Chapter 7 is a summary of this work.
Chapter 2

THEORY

2.1 An overview

In the last few years, elastic and inelastic nucleon-nucleus (N-A) scattering at intermediate energies has been studied extensively.\footnote{See footnotes 1 and 2 on pg 1.} The majority of these studies was conducted in the multiple-scattering framework of Watson [Wat53] and Kerman-McManus-Thaler [Ker59], where the N-A interaction is taken to be a sum of two-body interactions between the projectile nucleon and the individual nucleons within the nucleus. Such approaches are considered to be most meaningful at intermediate energies between 200 to 500 MeV, where the nucleon-nucleon (N-N) interaction is the weakest [Fra85], [Lo81]. In these calculations the N-N interaction can be taken to be that in free-space; that is, the presence of neighbouring nucleons within the nucleus is ignored, in which case an effective free N-N t-matrix interaction can be used.[Ker59], [Lo81], [Fra85], [Com80] Alternatively, a microscopic calculation of the N-N potential can be obtained from a meson-exchange model, and then a Brueckner reaction g-matrix can be constructed self-consistently from the N-N potential.[Ge83] This g-matrix can then be used in place of the t-matrix to represent the N-N interaction as modified by the Fermi motion, Pauli-blocking and the dispersive effects in the single particle energies in the medium. This medium modification is seen to play a major role in describing inelastic scattering to discrete states especially in those cases where the transition density peaks below the nuclear surface.[Kel80] Indeed, the g-matrix approach is able to reproduce the experimental data surprisingly well for both elastic scattering and inelastic scattering to discrete states for a wide range of target masses out to large scatter-
ing angles. [Le88] Furthermore, it has been demonstrated that the total N-A reaction cross section can be accounted for on the basis of the N-N total cross section. [DG80], [Wu80] The success of the multiple-scattering approach to describe N-A scattering and the dominance of the N-N cross section to the total N-A reaction cross section therefore strongly suggest that the N-N scattering process dominates the N-A reaction mechanism, or equivalently, that the quasielastic scattering process accounts for most of the reaction cross section.

The quasielastic reaction can be identified in the energy spectrum as a broad peak in the continuum region. It is experimentally observed that the centroid of the quasielastic peak closely relates to the N-N two-body kinematics. Thus, in essence, the reaction corresponds to the knock-out of a target nucleon from the target nucleus, with the broadening of the peak generally taken to reflect the momentum distribution of the target nucleons. [Ch80], [Jac71], [Kr70] It is therefore expected that, for inclusive quasielastic scattering, the spin observables will be grossly described by the free N-N interaction. If such an approach is valid, then it is to be contrasted to the folding character of the N-A optical model potential, where the N-N interaction is folded with the ground state density of the target nucleus. That is, if the knock-on character dominates, then for quasielastic scattering, this folding procedure is not needed. Rather, the emphasis will be on the medium modification to the N-N interaction.

In this thesis, we take the simple view of comparing the quasielastic spin observables to the free nucleon-nucleon (N-N) observables, where we draw analogy to the impulse-approximation of N-A scattering in the multiple-scattering model. In the following sections, we start with a brief introduction to the t-matrix and g-matrix effective N-N interactions. We then examine the multiple-scattering theory of N-A scattering based on the N-N interaction, and derive the optical potential for elastic scattering. Subsequently, we discuss the reactive content of the first-order optical potential, then we describe the concept of
quasielastic scattering in the impulse-approximation.

In the last part of this chapter, we show how the N-N observables can be expressed in terms of the scattering amplitude in the density-matrix formalism. And, following the suggestions of Bleszynski-Bleszynski-Whitten [Bl82], we show how suitable combinations of the experimental quantities can be directly related to each individual component of a specific parametrization of the scattering amplitude.

2.2 The nucleon-nucleon interaction

We have argued that the knock-on interaction dominates the quasielastic scattering process. It is then possible to describe quasielastic scattering on the basis of the nucleon-nucleon interaction between the projectile and the target nucleons. In this section, we briefly summarize the conventional parametrization of the nucleon-nucleon (N-N) scattering amplitudes and the formalism with which they are extracted from the experimental N-N data. In the next section, we will then describe the multiple-scattering framework of nucleon-nucleus (N-A) scattering based on the two-body nucleon-nucleon (N-N) interaction.

Traditionally, the nucleon-nucleon (N-N) interaction may be described either by microscopic potentials derived from a more fundamental understanding of the nuclear force such as the meson-exchange theory, or, by phenomenological effective interactions that are parametrized to fit the N-N data. Although idealistically the N-N interaction should be described by the interaction between the constituent quarks, such a quark model for the N-N interaction does not exist yet. Presently, the most comprehensive descriptions of the N-N interaction based on the meson-exchange model are given by the Paris collaboration (hence the Paris potential) [La80], and by the Bonn collaboration (hence the Bonn potential) [Hol75]. These bare potentials are, however, too strong to be used directly in actual
calculations. In practice, the effective interaction is usually identified in free space with the t-matrix $t$ through

$$ t|\vec{k} > = v\psi $$

where $|\vec{k} >$ is the two-body plane wavefunction for c.m. momentum $\vec{k}$, and $\psi$ is an eigenfunction of the Schrödinger equation with the the bare N-N potential $v$. In the limit of infinite nuclear matter, the medium-modified effective N-N interaction may be identified with the g-matrix $g$ through

$$ g|\vec{k} > = v\psi $$

where $\Psi$ is the true wavefunction in nuclear matter. We shall describe these in turn.

2.2.1 The t-matrix interaction

For the scattering of two nucleons in free space, the transition amplitude $t$ is given by (see, e.g. [Sa83])

$$ t = v + vG_0t $$

(2.1)

where $v$ is the free N-N interaction and $G_0$ the propagator in free space. It is clear from the definition of the transition amplitude

$$ t|\vec{k} > = v\psi $$

(2.2)

that the t-matrix can be interpreted as the effective free N-N interaction which operates on the initial plane wavefunction $|\vec{k} >$. The construction of the N-N effective interaction from the N-N data is described in detail by Love and Franey [Lo81], [Fra85]. Here, we sketch the principle in the procedure.

First of all, one may recall that the transition amplitude $t$ is simply related to the scattering amplitude $M$ by

$$ t = -\frac{4\pi(\hbar c)^2}{E_{cm}} M. $$

(2.3)
The scattering amplitude $M$ for N-N scattering can be parametrized by one of the many equivalent forms, for example,[Wo58]

$$M = A + B (\vec{\sigma}_1 \cdot \hat{n})(\vec{\sigma}_2 \cdot \hat{n}) + C (\vec{\sigma}_1 + \vec{\sigma}_2) \cdot \hat{n} + E (\vec{\sigma}_1 \cdot \vec{K})(\vec{\sigma}_2 \cdot \vec{K}) + F (\vec{\sigma}_1 \cdot \vec{P})(\vec{\sigma}_2 \cdot \vec{P})$$  (2.4)

where $\vec{\sigma}$ denotes the Pauli spin operator and the subscripts (1,2) denote particles 1 and 2, respectively. Henceforth, the c.m. right-handed rectilinear coordinate system is defined by

$$\vec{P} = \frac{\vec{k}_f + \vec{k}_i}{|\vec{k}_f + \vec{k}_i|}, \quad \vec{K} = \frac{\vec{k}_f - \vec{k}_i}{|\vec{k}_f - \vec{k}_i|}, \quad \hat{n} = \frac{\vec{k}_i \times \vec{k}_f}{|\vec{k}_i \times \vec{k}_f|},$$  (2.5)

where $\vec{k}_i$ and $\vec{k}_f$ are the initial and final c.m. momentum, respectively. The amplitudes $A$, $B$, $C$, $E$ and $F$ can be determined empirically from the N-N data.[Arn83]

The point to note here is that, while the empirical N-N data basically determine $t$, the N-N data include the exchange of the two nucleons implicitly. For the application to N-A scattering, it is desirable to construct an effective interaction which separates the direct and exchange contributions to the N-N scattering amplitude. The empirical N-N t-matrix is usually denoted by $t_{NN}^0$, while the “t-matrix interaction” is commonly denoted by $v_{12}$.

In practice, the interaction $v_{12}$ is assumed to have the local form

$$v_{12} = v^C(\vec{r}_{12}) + v^S(\vec{r}_{12})\vec{L} \cdot \vec{S} + v^T(\vec{r}_{12})S_{12}$$  (2.6)

where $\vec{r}_{12}$ is the relative distance between the two nucleons, while $\vec{L} \cdot \vec{S}$ and $S_{12}$ are the usual spin-orbit and tensor interactions, respectively. Guided by the one-boson exchange theory for the N-N interaction, the radial dependence of the central and the spin-orbit parts of $v_{12}$ are taken to be a sum of Yukawa terms with different range parameters, while that of the tensor part is taken to be $(r_{12})^2$ times a sum of Yukawa terms. For the central part $v^C$, the Yukawa term of the longest range is constrained to match the one-pion-exchange-potential, while that of the shorter range terms are adjusted to match the N-N data.
In the fitting procedure, the wavefunction for the incident and target nucleons is antisymmetrized with the operator $P_{12}$ which interchanges particles 1 and 2. Thus, the “t-matrix interaction” $v_{12}$ is obtained by matching

$$t_{NN}^0 = \frac{4\pi(hc)^2}{E_{cm}} M = \int d^3r_{12} e^{-i\mathbf{r}_{12} \cdot \mathbf{r}_{12}} v_{12}(1 - P_{12}) e^{i\mathbf{r}_{12} \cdot \mathbf{r}_{12}}. \quad (2.7)$$

2.2.2 The g-matrix interaction

The g-matrix interaction can be regarded as the medium-modified effective N-N interaction. Without going into details, let us illustrate by considering the case of infinite nuclear matter and assume that the correlation between the nucleons can be ignored. In this case, the medium modification to free scattering consists of Pauli-blocking below the Fermi level and the effects of a uniform potential $U$ among the nucleons. In nuclear matter, $U$ is a function of the nucleon momentum only.

The single-particle wavefunctions $\phi_i$ are the eigenfunctions of the single-particle Hamiltonian $H_i$ with eigenenergy $\epsilon_i$. Thus,

$$H_i \phi_i = (K_i + U_i) \phi_i = \epsilon_i \phi_i \quad (2.8)$$

where $K_i$ and $U_i$ are the single-particle kinetic and potential energy of nucleon $i$, respectively. A pair of colliding nucleons feels the overall single-particle potential $U$ as well as the free N-N interaction $v$. Thus, the total wavefunction $\Psi_{kl}$ for two colliding nucleons $k$ and $l$ is given by

$$(H_k + H_l + v) \Psi_{kl} = \epsilon_{kl} \Psi_{kl}. \quad (2.9)$$

To be specific, let us assume that all levels $i = 1, \ldots, k, l, m, n$ are occupied up to the level $n$ while levels $i = \alpha, \beta, \ldots$ are unoccupied. Consequently, the true wavefunction $\Psi_{kl}$
must be orthogonal to \( \phi_j (j = 1, \ldots, m, n; \ j \neq k, l) \). Thus, \( \Psi_{kl} \) can be written as

\[
\Psi_{kl} = \phi_k \phi_l + \sum_{p,q=a,b,...} a_{pq} \phi_p \phi_q,
\]

where

\[
a_{pq} = \frac{\langle \phi_p, \phi_q | v | \Psi_{kl} \rangle}{\epsilon_{kl} - \epsilon_p - \epsilon_q}.
\]

Hence, multiplying Eqn. (2.10) on the left by \( v \) and using the definition

\[
g|\phi_k, \phi_l > = v|\Psi_{kl} >,
\]

one obtains, in configuration space,

\[
< \phi_m, \phi_n | g | \phi_k, \phi_l > =< \phi_m, \phi_n | v | \phi_k, \phi_l >
\]

\[
+ \sum_{p,q=a,b,...} \frac{1}{\epsilon_{kl} - \epsilon_p - \epsilon_q} < \phi_m, \phi_n | v | \phi_p, \phi_q > < \phi_p, \phi_q | g | \phi_k, \phi_l >.
\]

This is the Bethe-Goldstone equation for the g-matrix of a pair of interacting nucleons in uncorrelated infinite nuclear matter.

von Geramb and collaborators [Ge83], [Ri84] have carried out the construction of the g-matrix in infinite nuclear matter using the Paris potential as input. Since the Fermi momentum \( p_F \) is related to the nuclear density \( \rho \) by

\[
\rho = \frac{2p_F^3}{3\pi^2 \hbar^3},
\]

it is apparent from Eqn. (2.13) that the g-matrix is density-dependent. The g-matrix is usually applied to the construction of the optical potential in nucleon-nucleus (N-A) scattering in the local-density approximation.[Br77] The resulting optical potential is therefore said to be density-dependent.
2.3 The multiple-scattering theory of nucleon-nucleus scattering

In the last section we have discussed the effective t-matrix and g-matrix nucleon-nucleon interactions. Here we will describe the multiple-scattering framework of nucleon-nucleus (N-A) scattering based on the two-body nucleon-nucleon (N-N) interaction. This approach to N-A scattering leads to the construction of an optical potential which in turn leads to the conclusion that the single knock-on interaction dominates the inclusive nucleon-nucleus (N-A) interaction.

For the scattering of a nucleon from a nucleus of mass $A$, the total Hamiltonian $H$ can be written

$$ H = H_0 + V $$

(2.15)

where $H_0$ consists of the kinetic energy operator, $K$, of the projectile, and the Hamiltonian, $H_A$, of the target nucleus:

$$ H_0 = K + H_A. $$

(2.16)

In the framework of the multiple-scattering theory the interaction $V$ between the projectile nucleon and the target nucleus is written as a sum of two-body interactions $\nu_i$ between the projectile and the individual target nucleons $i = 1, \ldots, A$.\cite{Ker59} If the target wavefunction is properly antisymmetrized, the index $i$ may be dropped. Consequently, the scattering matrix $T$ can be written in the Lippmann-Schwinger form \cite{Me61}

$$ T = V + VGT = A(\nu + \nu GT) $$

(2.17)

where

$$ G = \frac{A}{E - H_0} $$

(2.18)

is the full Green function, $A$ is a projection operator for completely antisymmetrical nuclear states, and $E$ is the eigenenergy of the total Hamiltonian $H$. 

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The medium modified two-body interaction $\nu$ may be replaced with the in-medium two-body scattering matrix $\tau$ defined by

$$\tau = \nu + \nu G \tau . \quad (2.19)$$

Since

$$\nu GT = TG\nu \quad (2.20)$$

from Eqn. (2.17), and

$$\nu G \tau = \tau G \nu \quad (2.21)$$

from Eqn. (2.19), substituting

$$V = A \nu = A \left( \frac{1}{1 + \tau G} \right) \tau = A \tau \left( \frac{1}{1 + G \tau} \right) \quad (2.22)$$

into

$$T' = V \left( \frac{1}{1 - GV} \right) \quad (2.23)$$

yields

$$T = A \tau \left( \frac{1}{1 - (A - 1)G \tau} \right) . \quad (2.24)$$

Defining

$$T' = \left( \frac{A - 1}{A} \right) T , \quad (2.25)$$

Eqn. (2.24) can be recast in terms of $T'$ into the Lippmann-Schwinger form. Thus,

$$T' = U^{(0)} + U^{(0)}GT' , \quad (2.26)$$

where

$$U^{(0)} = (A - 1)\tau . \quad (2.27)$$

Eqn. (2.26) may be expressed in the iterated form, a Born series, as

$$T' = U^{(0)} + U^{(0)}GU^{(0)} + U^{(0)}GU^{(0)}GU^{(0)} + \ldots$$

$$= (A - 1)\tau + (A - 1)^2 \tau GT + (A - 1)^3 \tau GT GT + \ldots . \quad (2.28)$$
It is clear from this expression that the T-matrix is given by a sum of contributions from "single-scattering," "double-scattering," etc. This is the origin of the term "multiple-scattering" often used to describe this model.

Eqns. (2.26) and (2.27) show that the "effective interaction" $U^{(0)}$ is simply proportional to the two-body interaction $\tau$. It should be noted, however, that the two-body scattering matrix $\tau$ contains the effects of the nuclear medium since it is the solution of Eqn. (2.19) where $\nu$ is the in-medium interaction.

2.4 The optical potential

It must be noted that the matrix element of $T'$ in Eqn. (2.26) is to be taken between product wavefunctions $|\vec{k}, \phi_A\rangle$ of the projectile $|\vec{k}\rangle$ and target nucleus $|\phi_A\rangle$. In other words, Eqn. (2.26) is a many-body Lippmann-Schwinger equation.

In order to reduce Eqn. (2.26) to a one-body equation, one may consider the following. Suppose we introduce the projection operators $P$ and $Q$ such that $P + Q = 1$. We can therefore write Eqn. (2.26) as

$$PT' = PU^{(0)} + PU^{(0)}GPT' + PU^{(0)}GQT', \quad (2.29)$$

and similarly,

$$QT' = QU^{(0)} + QU^{(0)}GPT' + QU^{(0)}GQT'. \quad (2.30)$$

Substituting Eqn. (2.30) into Eqn. (2.29) iteratively, one can write

$$PT' = P\tilde{U} + P\tilde{U}GPT' \quad (2.31)$$

where

$$\tilde{U} = U^{(0)} + U^{(0)}GQU^{(0)} + U^{(0)}GQU^{(0)}GQU^{(0)} + \ldots \ . \quad (2.32)$$

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Here, we are particularly interested in the subspace where the initial and final states of the target nucleus are identical. In other words, we choose to separate the case of elastic scattering from inelastic scattering. This choice is also practical since the resulting optical potential (see below) can be determined from elastic scattering experiments.

The choice of \( P \) just mentioned can be written as

\[
P = \int d^3 \vec{k}' |\vec{k}', \phi_A \rangle <\vec{k}', \phi_A| .
\] (2.33)

From this definition, it follows immediately that

\[
PG = GP .
\] (2.34)

Since \( Q = 1 - P \), it also follows that

\[
QG = GQ .
\] (2.35)

Consequently, Eqn. (2.32) can be written

\[
\hat{U} = U^{(0)} + U^{(0)} Q [G + GQU^{(0)} QG + \cdots ]QU^{(0)}
\] (2.36)

and, using the operator identity

\[
\frac{1}{A} - \frac{1}{B} = \frac{1}{A} \frac{(B - A)}{B} \frac{1}{B}
\] (2.37)

or, writing \( A = B - C \),

\[
\frac{1}{A} = \frac{1}{B} \frac{1}{C} \frac{1}{B} + \frac{1}{B} \frac{1}{C} \frac{1}{B} + \cdots ,
\] (2.38)

one obtains

\[
\hat{U} = U^{(0)} + U^{(0)} Q \left[ \frac{A}{Q(E - H_0 - U^{(0)}) Q} \right] Q U^{(0)} .
\] (2.39)

Also, Eqn. (2.31) can be written as

\[
PT'P = P \hat{U} P + P \hat{U} PGPT'P .
\] (2.40)
Now, if one defines

$$\langle \vec{k}' | T_{el} | \vec{k} \rangle \equiv \langle \vec{k}', \phi_A | T' | \vec{k}, \phi_A \rangle , \tag{2.41}$$

$$\langle \vec{k}' | U_{el}^{opt} | \vec{k} \rangle \equiv \langle \vec{k}', \phi_A | U | \vec{k}, \phi_A \rangle , \tag{2.42}$$

and

$$\langle \vec{k}' | \tilde{G} | \vec{k} \rangle \equiv \langle \vec{k}', \phi_A | G | \vec{k}, \phi_A \rangle , \tag{2.43}$$

Eqn. (2.40) can be written as

$$T_{el} = U_{el}^{opt} + U_{el}^{opt} \tilde{G} T_{el} , \tag{2.44}$$

which is now a one-body Lippmann-Schwinger equation. Historically, $U_{el}^{opt}$ is known as the “optical potential” for elastic scattering.

In passing, we point out that in the first-order Born approximation to Eqn. (2.32), i.e.

$$\tilde{U} \sim U^{(0)} \propto \sum_i A \tau_i , \tag{2.45}$$

if one chooses a local interaction such that

$$\tau_i \equiv \tau(|\vec{r}_0 - \vec{r}_i|) , \tag{2.46}$$

where $\vec{r}_0$ and $\vec{r}_i$ denote the position of the projectile and the target nucleons, respectively, the elastic optical potential can be written as

$$U_{el}^{opt} = \langle \phi_A | \sum_i A \tau(|\vec{r}_0 - \vec{r}_i|)|\phi_A \rangle$$

$$= \int d^3 \vec{r}' \langle \phi_A | \sum_i A \tau(|\vec{r}_0 - \vec{r}_i|) \delta(|\vec{r}_i - \vec{r}'|)|\phi_A \rangle$$

$$= \int d^3 \vec{r}' \tau(|\vec{r}_0 - \vec{r}'|) \langle \phi_A | \sum_i \delta(|\vec{r}_i - \vec{r}'|)|\phi_A \rangle$$

$$= \int d^3 \vec{r}' \tau(|\vec{r}_0 - \vec{r}'|) \rho_A(\vec{r}') , \tag{2.47}$$
where \( \rho_A \) is by definition the nuclear ground state density. Thus, the first-order optical potential for elastic scattering may be written in terms of the folding of a local two-body scattering matrix with the nuclear ground state density. This is the origin of the so-called "folding models" for the optical potential when only the "single-scattering" term is kept in the multiple-scattering approximation to nucleon-nucleus scattering.

2.5 The reactive content of the first-order optical potential

In the multiple-scattering theory we have just examined, the first-order approximation to \( T \) is given by (c.f. Eqn. (2.17))

\[
T \sim \sum_i^A \nu_i .
\]  

(2.48)

Obviously, the knock-out of a single target nucleon is included in the reactive content of this approximation. However, it is not immediately apparent whether this single-particle knock-out exhausts the reactive content implied by Eqn. (2.48). This subject has been studied by a number of authors, see, for example, [Ta75], [Kol79], [Er76]. In particular, the result of Koltun and Schreider [Kol79] shows that in the first-order approximation given by Eqn. (2.48), the reaction cross section can be written as a sum of exclusive cross sections corresponding to the knock-out of one, two, ... up to \( A \) target nucleons.

In practice, the two-body scattering matrix \( \tau \) may be approximated by the g-matrix described in section 2.2.2 or, in the so-called "impulse approximation," by the free N-N t-matrix.\(^5\) The result of [Kol79] shows that, at incident energies above several hundred MeV, where the impulse-approximation is expected to be meaningful, the inclusive reaction cross section is largely dominated by single-nucleon knock-out. (The same conclusion has also been reached by Tandy, Redish and Bollé [Ta75].) Indeed, this result has been

\(^5\) In fact, most calculations on spin observables are carried out in this approximation.
experimentally confirmed by showing that the measured p-A total reaction cross sections are essentially described by the experimental p-N total cross sections integrated over the target nucleons.[Wu80], [DG80], [Ale80]

That the reaction cross section is dominated by single-nucleon scattering has already been suggested by the observation that the centroid of the broad continuum in proton-nucleus scattering follows the N-N kinematics [Co72], [Wal66] and this is the origin of the term “quasielastic” scattering. The quasielastic peak is generally regarded to represent N-N knock-on scattering in the nuclear medium, with the broadening of the peak taken to reflect the Fermi momentum distribution of the target nucleons. In fact, most studies of the quasielastic excitation in the past have been centred on the possibility of extracting the target nucleon momentum distribution.[Ch80], [Jac71], [Kr70], [Wh74]

The fact that quasielastic scattering dominates the reaction mechanism has stimulated a series of studies which were aimed to explore the so called “y-scaling” property of nucleon-nucleus scattering.6) The central theme there is that scaling is expected if the single knock-on interaction dominates. And, if scaling is observed, the momentum distribution can be extracted from the scaling parameter. (See also the review by Sick [Si88].) The results of these studies indicate that, although other processes such as final state interaction may contribute to produce the observed results especially at backward angles, the inclusive cross sections nevertheless support the single-scattering mechanism.[Fr77b], [Fr78b], [Kom79], [Gr83] It is therefore indicative that the proton-nucleus interaction is limited to the nuclear surface – as strong absorption dominates inside the nucleus, penetration of the proton beyond the nuclear surface would substantially increase the total reaction cross section beyond that predicted by the impulse approximation. As will be seen later, our new data

6) For example, [Ale77], [Am76], [Fr77a], [Fr77b], [Fr76], [Fr78a], [Fr78b], [Fr79], [Fr81], [Gr83], [Kom79].
on the quasielastic proton spin observables also support this view.

2.6 The proton spin-transfer observables

The proton is a spin-$\frac{1}{2}$ particle; therefore its spin orientation is described by a vector polarization. Physically, the spin-transfer parameters relate the final polarization to the initial polarization of a particle scattered from an unpolarized target. These parameters can be conveniently defined in terms of the scattering amplitude $M$.

In principle, we are interested in the expectation value of the final polarization. The expectation value of any operator $\mathcal{O}$ is by definition

$$\langle \mathcal{O} \rangle \equiv \frac{\langle \chi|\mathcal{O}|\chi \rangle}{\langle \chi|\chi \rangle} = \frac{\sum_{i,j} \chi_j^i \mathcal{O}_{ji} \chi^i}{\sum_i \chi_i^i \chi_i}.$$  

(2.49)

For spin-$\frac{1}{2}$ protons, the spin wavefunction $\chi$ can be represented by a Pauli spinor

$$\chi = \begin{pmatrix} a \\ b \end{pmatrix}. \quad \quad (2.50)$$

Defining the density matrix

$$\rho = \chi \chi^\dagger, \quad \quad (2.51)$$

the expectation value can be written as:

$$\langle \mathcal{O} \rangle = \frac{Tr\{\rho \mathcal{O}\}}{Tr\{\rho\}}. \quad \quad (2.52)$$

From the definition of $\rho$, Eqns. (2.50) and (2.51), it is clear that the general form of the $2 \times 2$ density matrix can be expressed in the basis of the unit matrix $I$ and the Pauli spin matrices $\sigma_x, \sigma_y, \sigma_z$. Therefore, the density matrix can be written as

$$\rho = \frac{1}{2} (1 + \vec{P} \cdot \vec{\sigma}), \quad \quad (2.53)$$

7) In application to experimental results, it is understood that the ensemble average of the expectation value is taken.
where the normalization factor $\frac{1}{2}$ is chosen such that $\rho$ has unit trace with respect to normalized $\chi$. Furthermore, it is straightforward to show with Eqn. (2.52) that $\mathcal{P}$ is in fact the polarization of the state $\chi$, i.e.

$$\vec{\mathcal{P}} = <\vec{\sigma}>.$$ (2.54)

In the case of N-N scattering, the density matrix becomes the product of the projectile and the target-nucleon density matrices. For an unpolarized target, we can write

$$\rho_{NN} = \frac{1}{4} (1 + \vec{\mathcal{P}} \cdot \vec{\sigma}).$$ (2.55)

If we now denote the initial and final states by the superscripts $\alpha$ and $\beta$, respectively, and recalling

$$\chi^\beta = M \chi^\alpha,$$ (2.56)

the final polarization can be written as

$$\vec{\mathcal{P}}^\beta \equiv \frac{<\chi^\alpha|\vec{\sigma}^\beta|\chi^\beta>}{<\chi^\alpha|\chi^\beta>} = \frac{\text{Tr}\{\rho^\alpha_{NN} M^\dagger \vec{\sigma}^\beta M\}}{\text{Tr}\{\rho^\alpha_{NN} M^\dagger M\}}.$$ (2.57)

Here, the trace is taken over the projectile and target spin projections.

The N-N cross section for polarized beam is given by

$$I_{\text{pol}} \equiv <\chi^\beta|\chi^\alpha>$$

$$= \text{Tr}\{\rho^\alpha_{NN} M^\dagger M\}$$

$$= \frac{1}{4} \text{Tr}\{M^\dagger M\} + \frac{1}{4} \text{Tr}\{\sigma^\alpha M M^\dagger\} \cdot \vec{\mathcal{P}}^\alpha,$$ (2.58)

while the unpolarized cross section is simply

$$I_o = <\chi^\beta|\chi^\beta> \quad (\vec{\mathcal{P}}^\alpha = 0)$$

$$= \frac{1}{4} \text{Tr}\{M^\dagger M\}.$$ (2.59)
Therefore, Eqn. (2.57) can be rearranged as

\[ I^{\text{pol}} = \frac{1}{4} Tr\{(1 + \tilde{\sigma} \cdot \sigma) M^\dagger \sigma M\} = \frac{1}{4} Tr\{M^\dagger \sigma M\} + \frac{1}{4} Tr\{\sigma M^\dagger \sigma M\} \cdot \tilde{\sigma} \] \tag{2.60}

Defining the spin-transfer parameters \( D_{j'i} \) (\( j', i = 0, \hat{K}, \hat{n} \) and \( \hat{P} \)), with the primed index emphasizing the \( \beta \) channel, such that

\[ D_{j'i} = \frac{Tr\{\sigma_i^\beta M^\dagger \sigma_j^\beta M\}}{Tr\{M^\dagger M\}} \tag{2.61} \]

and \( \sigma_0 \equiv I \), the unit matrix, Eqn. (2.60) can be written in the form

\[
I_0 \left[ 1 + (D_{0'K} \quad D_{0'n} \quad D_{0'P} ) \begin{pmatrix} \rho_K^\beta \\ \rho_n^\beta \\ \rho_P^\beta \end{pmatrix} \right] \begin{pmatrix} \rho_K^\beta \\ \rho_n^\beta \\ \rho_P^\beta \end{pmatrix} = I_0 \left[ \frac{(D_{K'0})}{D_{n'0}} + \frac{(D_{K'n} \quad D_{K'P} )}{(D_{n'K} \quad D_{n'n} \quad D_{n'P} )} \begin{pmatrix} \rho_K^\beta \\ \rho_n^\beta \\ \rho_P^\beta \end{pmatrix} \right]. \tag{2.62}
\]

For the case of N-N elastic scattering, some of the \( D_{ij} \)'s are identically zero by the requirements of parity and time-reversal invariance on the scattering amplitude \( M \).\cite{Wo58}

If the scattering plane is chosen normal to the \( n \)-direction, then the only non-vanishing elements are \( D_{K'K}, D_{n'n}, D_{P'P}, D_{K'P}, D_{P'K}, D_{n'0}, \) and \( D_{0'n} \). Furthermore, in the c.m system, time-reversal invariance requires that

\[ D_{n'0} = D_{0'n} \tag{2.63} \]

and

\[ D_{K'P} = D_{P'K}. \tag{2.64} \]

Conventionally, experimental quantities are expressed in the laboratory particle frame of reference where the \( z \)-axis is in the direction of the particle's momentum. This implies that two different triads are used to describe the polarizations immediately before and after the scattering (figure 2.1):

\[
\hat{l} = \frac{\vec{P}_i}{|\vec{P}_i|} , \quad \hat{n} = \frac{\vec{P}_i \times \vec{P}_f}{|\vec{P}_i \times \vec{P}_f|} , \quad \hat{s} = \hat{n} \times \hat{l}; \tag{2.65}
\]
and
\[
\hat{\nu} = \frac{\hat{P}_f}{|\hat{P}_f|}, \quad \hat{n}' = \hat{n}, \quad \hat{s}' = \hat{n}' \times \hat{\nu}.
\] (2.66)

The Wolfenstein parameters are defined in this system of coordinates by
\[
\begin{align*}
R & \equiv D_{s's}\n
D & \equiv D_{n'n}

A' & \equiv D_{\nu't}

A & \equiv D_{s'1}

R' & \equiv D_{\nu's}
\end{align*}
\] (2.67)

while
\[
P \equiv D_{n'0}
\] (2.68)

is the polarization parameter for unpolarized beam, and
\[
A_y \equiv D_{0'n}
\] (2.69)

is the asymmetry in cross section for n-polarized beam.

Thus, in terms of the Wolfenstein parameters, Eqn. (2.62) can be written as
\[
\begin{pmatrix}
P_{s'} \\
P_{n'}
\end{pmatrix}
= \frac{1}{1 + A_y P_n}
\begin{pmatrix}
0 & D_{s's} & 0 & D_{s'1} \\
D_{n's} & 0 & D_{n'1} & 0 \\
D_{\nu's} & 0 & D_{\nu'1} & 0
\end{pmatrix}
\begin{pmatrix}
P_s \\
P_n
\end{pmatrix}.
\] (2.70)

The physical meanings of these spin observables are represented pictorially in figure 2.2.

For completeness, we quote here the transformation between the Wolfenstein parameters and the corresponding quantities defined in the c.m. system. (The c.m. coordinates are

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Figure 2.1  Diagram showing the laboratory coordinate system for the projectile (unprimed) and scattered (primed) proton. Both $\hat{n}$ and $\hat{n'}$ are normals to the scattering plane which is defined by the initial and final momentum, $\vec{P}_i$ and $\vec{P}_f$, respectively. The laboratory scattering angle, $\theta_{lab}$, is measured in the scattering plane.
Figure 2.2  Diagram showing the physical meanings of the N-N spin-transfer observables. The spin directions before and after the scattering are represented in the coordinate system of figure 2.1, shown here again in the boxed diagram on top. The spin direction associated with the incident proton indicates the state of polarization of which the incident beam is prepared; and that associated with the outgoing proton indicates the component of the final polarization that is measured.
defined in Eqn. (2.5).) These are, [By78]

\[ P = D_{n'0} \]

\[ A_y = D_{0'n} \]

\[ D = D_{n'n} \]

\[ R = -D_{P'P} \sin \alpha \sin \left( \frac{\theta}{2} \right) - D_{P'K} \sin \left( \alpha + \frac{\theta}{2} \right) + D_{K'K} \cos \alpha \cos \left( \frac{\theta}{2} \right) \tag{2.71} \]

\[ A = -D_{P'P} \sin \alpha \cos \left( \frac{\theta}{2} \right) - D_{P'K} \cos \left( \alpha + \frac{\theta}{2} \right) - D_{K'K} \cos \alpha \sin \left( \frac{\theta}{2} \right) \]

\[ R' = D_{P'P} \cos \alpha \sin \left( \frac{\theta}{2} \right) + D_{P'K} \cos \left( \alpha + \frac{\theta}{2} \right) + D_{K'K} \sin \alpha \sin \left( \frac{\theta}{2} \right) \]

\[ A' = D_{P'P} \cos \alpha \cos \left( \frac{\theta}{2} \right) - D_{P'K} \sin \left( \alpha + \frac{\theta}{2} \right) - D_{K'K} \sin \alpha \sin \left( \frac{\theta}{2} \right) \]

where

\[ \alpha = \frac{\theta}{2} - \theta_{\text{lab}} \tag{2.72} \]

and \( \theta, \theta_{\text{lab}} \) are the c.m. and lab scattering angles, respectively.

With these relations, we may obtain from Eqns. (2.63) and (2.64), respectively, the symmetry relations under time-reversal invariance:

\[ P = A_y \tag{2.73} \]

and

\[ D_{s'I} + D_{P's} = (D_{P'I} - D_{s'I}) \tan \theta_{\text{lab}} \tag{2.74} \]

Writing the scattering amplitude in the specific form

\[ M = M_0 + M_K \sigma_K + M_n \sigma_n + M_P \sigma_P \tag{2.75} \]

and explicitly taking the traces in Eqn. (2.61) over the projectile spin, it can easily be
shown that only the following are non-vanishing:

\[ I_0 = \frac{1}{2} Tr'(M_0^\dagger M_0 + M_K^\dagger M_K + M_n^\dagger M_n + M_P^\dagger M_P) , \]

\[ D_{K'K} = \frac{1}{2I_0} Tr'(M_0^\dagger M_0 + M_K^\dagger M_K - M_n^\dagger M_n - M_P^\dagger M_P) , \]

\[ D_{n'n} = \frac{1}{2I_0} Tr'(M_0^\dagger M_0 - M_K^\dagger M_K + M_n^\dagger M_n - M_P^\dagger M_P) , \]

\[ D_{P'P} = \frac{1}{2I_0} Tr'(M_0^\dagger M_0 - M_K^\dagger M_K - M_n^\dagger M_n + M_P^\dagger M_P) . \]

That is, only the diagonal \( D_{\alpha'\alpha} (\alpha = K, n, P) \) survive in this parametrization of the scattering amplitude and they are linear in

\[ D_\alpha = \frac{1}{2I_0} Tr'(M_\alpha^\dagger M_\alpha) . \]  

Here, the primed trace is over the target spin projections only. Eqns. (2.76) can be rearranged to yield the following expressions for the \( D_\alpha \)'s:

\[ D_0 = \frac{1}{4}(1 + D_{K'K} + D_{n'n} + D_{P'P}) , \]

\[ D_K = \frac{1}{4}(1 + D_{K'K} - D_{n'n} - D_{P'P}) , \]

\[ D_n = \frac{1}{4}(1 - D_{K'K} + D_{n'n} - D_{P'P}) , \]

\[ D_P = \frac{1}{4}(1 - D_{K'K} - D_{n'n} + D_{P'P}) . \]

Thus, it is clearly seen that each \( D_\alpha \) depends separately on a single component \( M_\alpha \) of the scattering amplitude, and that they are given by a combination of the spin-transfer parameters. As Bleszynski-Bleszynski-Whitten [Bl82] has pointed out, these particular combinations of the spin-transfer parameters may be useful for isolating the theoretical amplitudes in comparing calculations against experimental data.

Using Eqn. (2.71), Eqn. (2.78) may be expressed in terms of the experimentally observed Wolfenstein parameters. Thus,\(^8\)

\[ D_0 = \frac{1}{4} \left[ 1 + D_{n'n} + (D_{s's'} + D_{P'l}) \cos \left( \alpha + \frac{\theta}{2} \right) - (D_{s'l} - D_{s's'}) \sin \left( \alpha + \frac{\theta}{2} \right) \right] , \]  

---

\(^8\) The Secant factors are missing in the expressions for \( D_K \) and \( D_P \) given in [Bl82].
\[ D_K = \frac{1}{4} \left[ 1 - D_{n'\pi} + (D_{s'\pi} - D_{p\pi}) \sec \left( \alpha - \frac{\theta}{2} \right) \right], \quad (2.80) \]
\[ D_n = \frac{1}{4} \left[ 1 + D_{n'\pi} - (D_{s'\pi} + D_{p\pi}) \cos \left( \alpha + \frac{\theta}{2} \right) + (D_{s'\pi} - D_{p\pi}) \sin \left( \alpha + \frac{\theta}{2} \right) \right], \quad (2.81) \]
\[ D_P = \frac{1}{4} \left[ 1 - D_{n'\pi} - (D_{s'\pi} - D_{p\pi}) \sec \left( \alpha - \frac{\theta}{2} \right) \right]. \quad (2.82) \]

2.7 Concluding remarks

In the last section we have described the free N-N spin observables. In the spirit of the impulse-approximation, one would expect that the quasielastic scattering spin observables will be grossly described by the free N-N values possibly after taking the Fermi motion of the target nucleons into account. Since the spin observables are ratios of cross sections (see last section), it is expected that distortion effects will cancel to a certain extent. As a first order correction to the single-scattering impulse-approximation, one may incorporate some medium modifications by using a g-matrix for the scattering amplitude and then use the formalism described above. In Chapter 6, we will calculate the quasielastic spin observables using a free Fermi-gas model where the target nucleon momentum is taken into account. We will see that these spin observables are indeed reasonably described by the two-body N-N process.

In this chapter, we have concerned ourselves from the point of view that the quasielastic interaction being dominated by the two-body N-N process. At the time of this experiment, a schematic model for quasielastic scattering based on the N-N interaction was put forward which included the medium modification to the free N-N interaction due to relativistic effects – the so-called \( M^* \) effect where the effective nucleon mass was reduced in the nuclear medium due to the strong relativistic scalar potential.\cite{Hor88} This approach successfully predicted the strong quenching of the polarization parameter \( P \) relative to the free value, an effect which was borne out by our experimental data. This approach was not so successful,
however, in reproducing the other spin observables; in particular, for $D_{0^+}$, it destroyed the
good agreement we obtained with our free Fermi-gas model calculation where the Fermi
motion alone is taken into account. A more sophisticated calculation using the relativistic
RPA formalism is currently being investigated.[Sh88] Nevertheless, at this stage, it is also
interesting to compare the quasielastic spin observables with the free N-N values since the
latter provide the benchmark for medium modification. Indeed, this aspect of quasielas­
tic scattering has recently been used to investigate the hypothesized enhancement of the
longitudinal spin-isospin nuclear response with respect to the transverse component. This
effect, if observed, can be taken as evidence for pion condensation in the nuclear medium.
However, as we will show in Chapter 6, our results do not indicate such phenomenon.
Chapter 3

EXPERIMENT: GENERAL

In this chapter, we describe the experimental setup and provide general information about the experiment. More details concerning data acquisition and data reduction are given in the next two chapters.

3.1 An overview

The experiment described herein was conducted at the TRIUMF cyclotron facility using polarized proton beams accelerated to 290 and 420 MeV. The polarized proton beam was provided by a Lamb-shift polarized ion-source (POLISIS). The polarization of the source could be reversed by reversing the POLISIS solenoid fields, which was done for each cycle in periodic 3-minute intervals for each spin mode, with a short 1-minute interval for unpolarized mode. The beam intensity was controlled by slits located in the central region of the cyclotron. Beam extraction was facilitated by a carbon stripper foil; its radial position in the cyclotron determined the extracted beam energy, and its depth of insertion (Z) could also control the extracted beam intensity. Since the Z position was found to have some noticeable effect on the polarization of the extracted beam, a search for the best Z was made in the beginning and its position was then held fixed throughout the experiment.

The general layout of the TRIUMF cyclotron facility is shown in figure 3.1 and details of the proton experimental area are shown in figure 3.2. For clarity in the following discussion, schematic diagrams showing the experimental setup are shown in figures 3.3 and 3.4. To begin, the extracted beam from the cyclotron encounters a combination magnet 4VCM.
Figure 3.1 Diagram showing the general layout of the experimental area at TRIUMF.

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Figure 3.2  Diagram showing details of beam-line 4 and the Proton Hall experimental area.
in the cyclotron vault and, after a set of quadrupoles (4VQ1, 4VQ2, 4VQ3), it could pass through bending magnet 4VB1 into beam-line 4A or pass through both bending magnets 4VB1 and 4BV B2 into beam-line 4B (figures 3.2, 3.3). The beam emerging from the cyclotron was polarized in the vertical direction; however, by using the two solenoids (SOL1, SOL2) in the vault section in conjunction with the two bending magnets, sideways as well as longitudinal polarizations were made available on beam-line 4B.[Ab88]

The 4B in-beam polarimeter (IBP) was used to continuously monitor the vertical and sideways polarizations simultaneously.[Ab87] The beam current at 4B was obtained from the number of proton-proton elastic scattering events from the polyethylene target in the 4B IBP. A secondary-emission monitor (SEM) located near the end of the beam line was also used to monitor the beam current and this provided a measure of any hydrogen depletion in the polarimeter target. To measure the longitudinal polarization at 4B, a similarly designed IBP was installed in beam-line 4A (BL4A) and used to measure the transverse polarization before the beam was steered into 4B by the last bending magnet 4BV B2 (figure 3.3).

Downstream of the 4B in-beam polarimeter, a monochromatic beam was focussed on the target in the scattering chamber. Protons scattered at the target went through the scattering chamber exit port and on into the MRS spectrometer. The unscattered beam was refocused by a quadrupole doublet onto the SEM and eventually stopped in the shielded beam dump.

Three different targets were used in this experiment. The first, a 93 mg/cm² thick natural graphite target, was used to measure the $^{12}$C observables. The second was a water film of approximately 20 mg/cm² from a waterfall target used to measure the $^{16}$O observables. The third, a 233 mg/cm² thick polyethylene (CH$_2$) target, was also used to measure the known $p$-$p$ observables. These measurements, done under identical conditions to those of the corresponding quasielastic measurement, gave an estimate of the instrumental error.
Figure 3.3 A schematic diagram showing a top view of the experimental setup. The bend angles of 4VB1 and 4VB2 are 40° and 25°, respectively. BD: beam dump; Q: quadrupole; QD: quadrupole doublet.
Figure 3.1  A schematic diagram showing the arrangement of detectors: The front-end multi-wire drift-chamber (FEC) consists 2 pairs of staggered wire-planes, one pair for a horizontal coordinate and one pair for a vertical coordinate. The vertical-drift chambers, VDC1 and VDC2, each consists of an X-plane for a coordinate in the dispersive direction of the MRS dipole, and a U-plane at 30° to X for a supplementary coordinate. Both VDCs are at 45° to the central ray, which is in turn at 60.3° to the horizontal. The FPP detectors are assembled into a cage which is pivoted to meet the central ray at 90° during normal operation. The carbon analyzer is sandwiched by 4 multi-wire drift-chambers (D1, D2, D3, D4) and 2 segmented plastic scintillators (S1, S2). Up to 4 carbon slabs may be inserted into slots in the FPP cage.
in our setup.

It is perhaps worthwhile to note that the use of water (H₂O) as the primary target for the $^{16}$O data introduced background proton-proton elastic scattering events into the $^{16}$O quasielastic spectrum near the centre of the quasielastic peak. To partially remedy this situation, a recoil-scintillator was mounted in the scattering chamber to reject events matching the kinematics of $p$-$p$ elastic scattering, which eliminated the background $p$-$p$ events in the final $^{16}$O($p,p'$) spectrum. However it should be noted that the use of this front-end veto-scintillator did not guarantee the usefulness of the quasielastic $^{16}$O data at the region of the $p$-$p$ elastic peak, since the recoil-scintillator could not differentiate the inclusive $p$-$^{16}$O quasielastic events from the $p$-$p$ elastic events.

A set of front-end wire-chambers at the entrance to the MRS spectrometer were used together with two drift-chambers on the MRS focal plane to provide full ray-tracing capability with projection back to the target. The two focal-plane drift-chamber detectors also allowed ray-tracing into the focal-plane polarimeter (FPP), which consisted of four multi-wire drift chambers (D1, D2, D3, D4), two segmented plastic scintillators (S1, S2), and a carbon analyzer (C) in the order D1, S1, C, D2, D3, D4, S2 as shown in figure 3.4. The signal from S1 was used as a timing reference for the MRS electronics, while the coincidence of S1 and S2 was used as a local trigger for the FPP electronics (details in Sections 4.4 and 4.5).

3.2 Spin precession in dipoles and solenoids

In order to measure the complete set of Wolfenstein parameters, incident beams polarized, respectively, in the sideways, longitudinal and vertical directions were needed. For sideways or longitudinal polarization, the vertically polarized beam at extraction from the
cyclotron was precessed in the vault section of beam-line 4 by the combined action of the
two solenoids (SOL1 and SOL2) and the two dipole bending magnets (4VB1 and 4BVB2).
Schematically, these vault solenoids precess the spin about the longitudinal axis while the
vault dipoles precess the spin about the vertical, so that together they rotate the vertical
polarization into either a sideways or a longitudinal polarization. The magnetic fields of
the bending magnets were of course fixed by the beam momentum since they had to steer
the beam into beam-line 4B. However, as the solenoids were interleaved with the dipoles
(figure 3.3), it was possible to obtain sideways or longitudinal polarization by adjusting the
magnetic fields in the solenoids alone. Of course, for vertical polarization, the currents in
these solenoids were switched off and the beam drifted through them.

For the first part of this experiment (420 MeV), a solenoid, JANIS, was installed
in beam-line 4B upstream of the 4B IBP. Its function was simply to rotate the normal
polarization through 90° to give a sideways polarization without the need of using the vault
solenoids. For sideways polarizations, each (vault or JANIS) solenoid current could be
reversed to induce a 180° spin rotation about the solenoid axis. Therefore, data that were
taken with forward and reverse solenoid currents could be combined in such a way that
the contribution from a small longitudinal component in the initial polarization could be
eliminated from the final polarization. (See Section 5.6.)

The precession angles in the dipoles and the solenoids are given by the following ex­
pressions.[Ba59] For solenoids,

\[ \theta_{\text{sol}} = \frac{g \mu_N}{\hbar \gamma \beta_c} \int B dL , \]  

and, for dipoles,

\[ \theta_p = \gamma \left( \frac{g}{2} - 1 \right) \theta_b . \]  

Here, \( \gamma \) and \( \beta \) are the conventional relativistic kinematic variables, \( \frac{g}{2} \) is the magnetic moment
of the proton in $\mu_n = \frac{eb}{2mc}$ nuclear magneton units, and $\theta_b$ is the proton's bend angle through the dipole. The maximum strength $\int BdL$ of these superconducting solenoids was 2.2 T-m.

3.3 The in-beam polarimeters

In this experiment, two in-beam polarimeters (IBPs) were used which were capable of measuring both the vertical (normal) and sideways polarizations simultaneously.[Ab87] In each IBP two sets of detectors were installed, one in the vertical plane and the other in the horizontal plane. Each assembly consisted of two telescopes located on either side of the target, at 17° and at 70° (figure 3.5). Each telescope consisted of two plastic scintillators, of which the one downstream was tilted at 37° to the telescope axis. This tilt was chosen such that the solid angle covered by each telescope was to first order independent of small variations of the beam position. The 70° recoil angle was chosen to match the kinematics of proton-proton elastic scattering at 17°.

A motor driven target ladder in each polarimeter moved the polarimeter target into the beam. Four targets were mounted in each polarimeter: two of these were polyethylene (CH₂) foils of different thicknesses; a thin one (1.36 mg/cm²) for normal running and a thicker one (5.11 mg/cm²) for occasions when increased counting rates were desired (for example, for beam polarization diagnostics at low beam current), a zinc sulfide (ZnS) screen for locating the beam visually on television and a graphite (¹²C) target for measuring the $¹²C(p,2p)$ background in each polarimeter (see below).

As mentioned before, one of these polarimeters was installed in beam-line 4B a. 4BT1 where it was used to continuously monitor the vertical and sideways components of the beam polarization during normal running. The second one, installed in beam-line 4A, was used occasionally to measure the longitudinal component of the beam polarization at 4B.
Figure 3.5  A schematic diagram showing a pair of telescope counters and their associated recoil counters. The rear counter in each telescope is tilted at an angle of 37° to the telescope axis.
The principle was to measure the transverse components of the beam polarization before and after the last bending magnet 4BVB2 which precessed the spin about the vertical axis. Since the bend angle of 4BVB2 was known (25°), a complete knowledge of the vector polarizations before and after 4BVB2 could be obtained.[Ga86]

Figure 3.6 shows schematically a left-right assembly and the associated logic circuit. A legitimate event was defined by the coincidence of all three signals coming from a counter telescope and its associated recoil counter. On the other hand, the 43 nanosecond (ns) delay-coincidence of the recoil and telescope signals provided a measure of the accidental coincidences caused by multiple events occurring within cyclotron beam bursts arriving in 43 ns intervals. Both coincident events and accidental events were counted separately; the corresponding polarimeter count was taken to be the number of coincident events minus the accidentals. Accidental events were less than 0.5% at all times. In the following, it is understood that the accidentals are already subtracted from the total number of coincidences.

Ideally, the normal and sideways polarization components are given by [Hos68]

\[
\begin{align*}
\mathcal{P}_n^0 &= \frac{1}{A_y} \left( \frac{L - R}{L + R} \right) \\
\mathcal{P}_s^0 &= \frac{1}{A_y} \left( \frac{U - D}{U + D} \right)
\end{align*}
\] (3.3)

Here, \(L, R, U, D\) denote the left-, right-, up-, down-polarimeter counts, respectively.

In practice, however, there will be instrumental asymmetries such that the observed polarimeter counts \((L, R)\) are related to the unbiased counts \((L_0, R_0)\) by

\[
\begin{align*}
L &= L_0(1 + \varepsilon_n) \\
R &= R_0(1 - \varepsilon_n)
\end{align*}
\] and

\[
\begin{align*}
U &= U_0(1 + \varepsilon_s) \\
D &= D_0(1 - \varepsilon_s)
\end{align*}
\] (3.4)

The instrumental asymmetries, \(\varepsilon_n\) and \(\varepsilon_s\), may be expressed in the case of an unpolarized beam \((L_0 = R_0; U_0 = D_0)\)

\[
\varepsilon_n = \frac{L - R}{L + R} \quad \text{and} \quad \varepsilon_s = \frac{U - D}{U + D} \quad (\text{unpolarized})
\] (3.5)

Thus, \(\varepsilon_n\) and \(\varepsilon_s\) were measured intermittently in the experiment during unpolarized POLISIS cycles.

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Figure 3.6  Diagram showing the logic circuit defining the real and the accidental events. Shown here are the left-right events; the up-down events are defined analogically.
Using Eqns. (3.3) and (3.4), the normal polarization after correction for instrumental asymmetry is given by

\[ P_n^0 = \frac{1}{A_y} \left( \frac{L_0 - R_0}{L_0 + R_0} \right) = \frac{1}{A_y} \left( \frac{\frac{L}{1 + \varepsilon_n} - \frac{R}{1 - \varepsilon_n}}{\frac{L}{1 + \varepsilon_n} + \frac{R}{1 - \varepsilon_n}} \right) = \frac{1}{A_y} \left( \frac{\frac{L}{L + R} - \varepsilon_n}{\frac{R}{L + R}} \right). \]  

(3.6)

Similarly, for the sideways polarization,

\[ P_s^0 = \frac{1}{A_y} \left( \frac{U_0 - D_0}{U_0 + D_0} \right) = \frac{1}{A_y} \left[ \frac{\left( \frac{U - D}{U + D} \right) - \varepsilon_s}{1 - \varepsilon_s \left( \frac{U - D}{U + D} \right)} \right]. \]  

(3.7)

Here, the asymmetry function \( A_y \) of proton-proton elastic scattering is obtained from the existing database. However, due to the carbon content in polyethylene (CH\(_2\)) targets, background quasielastic \(^{12}\)C(p,2p) contributions must be subtracted from the raw CH\(_2\) events. The number of events due to proton-proton scattering from the CH\(_2\) IBP-target may be written as

\[ L(pp) = L(CH_2) - L(^{12}\text{C}) = L(CH_2) \left[ 1 - \frac{L(^{12}\text{C})}{L(CH_2)} \right] \equiv L(CH_2)\alpha_L, \]  

(3.8)

with similar expressions for R, U and D events. Consequently, the beam polarizations are given by

\[ P_n^0 = \frac{1}{A_y(pp)} \left[ \frac{L(CH_2)\alpha_L - R(CH_2)\alpha_R}{L(CH_2)\alpha_L + R(CH_2)\alpha_R} \right], \]  

(3.9)

and,

\[ P_s^0 = \frac{1}{A_y(pp)} \left[ \frac{U(CH_2)\alpha_U - D(CH_2)\alpha_D}{U(CH_2)\alpha_U + D(CH_2)\alpha_D} \right]. \]  

(3.13)

In this experiment, \( \alpha_L, \alpha_R, \alpha_U \) and \( \alpha_D \) were found by using data obtained with the carbon and polyethylene targets and normalizing to the respective ratios of SEM counts (beam flux) and target thickness. In principle, the number of \(^{12}\)C(p,2p) background events could change as the number of target nuclei were being depleted from the IBP target spot during the experiment. However, for the small beam currents used, the changes in these ratios were found to be negligible.
3.4 The Medium Resolution Spectrometer

The Medium Resolution Mass Spectrometer (MRS) served a dual purpose in this experiment: (i) to momentum-analyze the scattered protons and; (ii) to precess the spin of the scattered protons about the horizontal (sideways) magnetic field. The MRS consists of a quadrupole and a dipole magnet of radius 2.2 meters and bend angle 60.3° in the vertical plane. The whole structure, including the front-end and focal-plane detectors, the focal-plane polarimeter, as well as the electronics, is stationed on a platform which could be rotated about the scattering chamber 4BT2. Connection to the scattering chamber is made with a flexible bellows onto one of the many exit ports on the cylindrical wall of the scattering chamber. Particles of the same charge-to-mass ratio but of different momenta are radially dispersed by the spectrometer. Particles of the same momentum-to-charge ratio but of different masses are distinguished by their relative time-of-flight through the MRS and their energy-loss in the first plastic scintillator (S1) in the FPP.

Although the focal-plane polarimeter was analyzing only in its transverse plane, the spin precession in the MRS dipole enabled the determination of the full vector polarization of the scattered proton. Since the bend plane of the MRS dipole is in the vertical, the normal and longitudinal components of the scattered proton polarization precessed about the sideways axis. This precession angle was determined from a knowledge of the particle's momentum and its bend angle through the MRS dipole. The momentum of each proton was given by its position on the focal plane. However, owing to the focusing property of the MRS quadrupole-dipole combination, a proton of the same momentum but entering the MRS at a different vertical angle underwent a different bend angle. Thus, in order to determine the precession angle, we had to ray-trace the proton trajectory through the spectrometer. This ray-tracing information was also used to discriminate background events.

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not originating from the primary target and, furthermore, it allowed small aberrations in
the MRS "optics" to be software-corrected during data analysis.

3.5 Ray-tracing – the front-end and focal-plane wire-chamber detectors

In order to ray-trace an event from the focal plane back to the target position, it was
sufficient for us to obtain (i) the spatial coordinates at the entrance of the spectrometer, (ii)
the exit angles in the bend plane (angle $\theta$) and in the non-bend plane (angle $\phi$), and (iii)
the spatial coordinates at the focal plane. These required that particles be detected at the
entrance and exit of the spectrometer. Front-end detection was facilitated by a multi-wire
proportional chamber (FEC) which is made up of four wire planes, two planes for each
spatial coordinate. These are the X0 and X0' planes for a vertical coordinate, and the
Y0 and Y0' planes for a horizontal coordinate. Each wire plane is made up of 16 anode
wires, 5 mm apart, interleaved by 16 cathode wires, and sandwiched between two cathode
planes. All four wire planes are spaced 6 mm apart and the first plane was about 62 cm
from the target position. Each pair of the primed and unprimed coordinate planes are
offset transversely by half the anode-wire spacing to resolve the drift direction ambiguity
and enable interpolation between anode wires using drift times (Section 5.1). The spatial
resolution for front-end detection is better than 0.5 mm.

Particle detection at the focal plane was facilitated by two vertical drift chambers
(VDC1 and VDC2), which are oriented at 45° to the central ray (optic axis), closely match­
ing the tilt of the focal plane. Each VDC consists of two wire planes: X1, U1 in VDC1; X2,
U2 in VDC2. The X wires are directed sideways to give a coordinate in the bend plane,
while the U wires, slanted at 30° to X, provide the supplementary coordinate. There are
160 anode wires, 6 mm apart, for each wire plane in VDC1; and 176 wires per plane in
VDC2. The orientation of the VDCs with respect to the central ray and the slanted U planes with respect to the X’s implied that a number of adjacent wires in each plane would be triggered by each proton passing through the chambers. The timing signals from adjacent wires were used to determine the X and Y coordinates in each VDC (Section 5.1). The spatial resolution of the VDCs in the bend plane direction had been measured previously to be about 150 μm FWHM and the angular resolution was < 0.1°.

3.6 The focal-plane polarimeter

The transverse components of the polarization of the protons at the focal plane were measured by analyzing the asymmetry for an auxiliary scattering at the focal-plane polarimeter (FPP). The design of the FPP was very different from the in-beam polarimeters in that, while the beam position was fixed within small fluctuations at the IBPs, the incident particles at the FPP came from different focal-plane positions and differed in a range of angles $\theta$ and $\phi$. In other words, the polar as well as the azimuthal angle of scattering at the FPP carbon analyzer had to be determined relative to each proton’s trajectory. In order to determine the asymmetry of the auxiliary scattering at the FPP, it was therefore necessary to reconstruct each scattering event.

Each event at the FPP carbon analyzer could be reconstructed using information provided by the VDCs and the four FPP drift chambers (figure 3.4). For each event, the angle of scattering from the carbon analyzer was determined and a test was made to decide whether the event was within the acceptance of the FPP wire chambers. For a valid event, its contribution to left-right and up-down asymmetry was then determined from the azimuthal angle of scattering with respect to the incident direction at the analyzer. (See Section 5.3.2 for details.)
The analyzing-power for inclusive (i.e. elastic and inelastic) scattering from a graphite target was used in this polarimeter. The thickness of the carbon analyzer was chosen to optimize the product of yield and analyzing-power. At the intermediate energies between 200 to 500 MeV, this analyzing-power peaks at a lab scattering angle $\sim 10^\circ$.\[AG83\], \[Ran82\] At small angles below $< 2^\circ$, multiple small-angle scattering dominates the scattering cross section and, at large scattering angles $> 20^\circ$, both the cross section and the analyzing-power drop. The FPP at TRIUMF was therefore designed to give an angular acceptance of $20^\circ$ in both $\theta$ and $\phi$ directions.

Each of the four FPP drift chambers (D1, D2, D3, D4) was made up of two orthogonal anode wire planes equally spaced between three cathode planes. There were 110 anode wires for the X-coordinate ($\theta$ direction), and 60 wires for the Y-coordinate, in each chamber. The anode wires were 8.128 mm apart; equally spaced in between them were pairs of cathode wires 0.5 mm apart placed back-to-back above and below the anode wire plane (figure 3.7). This pairwise design of the cathode wires was found to have the effect of enhancing the electric field in the surrounding area, thereby increasing the cathode wire signals by 40 to 50%.[He87]

The whole assembly of horizontal drift chambers, the carbon scatterer, and the two plastic scintillators (S1, S2), was mounted in a cage. With the exception of S2, (which is bolted on top of the cage), individual components were inserted into slots to facilitate fast servicing. In operation, the cage was oriented orthogonal to the optic axis of the spectrometer so as to optimize the $\theta$ and $\phi$ acceptance, (figure 3.4). Up to four rectangular slabs of graphite scatterer could be inserted immediately upstream of D2. As mentioned above, the optimum thickness for the analyzer was determined by the energies of the protons. In this experiment, for incident protons of 420 MeV and quasielastic scattering through a lab angle of $23.5^\circ$, the final energy of protons across the 100 MeV wide quasielastic peak
Figure 3.7  Schematic diagram showing the pairwise design of the cathode wires for the FPP drift-chambers.
was centred at about 340 MeV. In this case, a thickness of 7.5 cm made up of two slabs (4.0 cm and 3.5 cm) was appropriate. For 290 MeV incident energy protons scattering at 29.5°, a single 3.5 cm slab was used.
Chapter 4

DATA ACQUISITION

The design of the MRS data acquisition system is based on the philosophy of recording event-by-event raw data from the experiment. In this experiment, a Data General Eclipse S/200 minicomputer was used with the TRIUMF DACS data acquisition program to record the entire data stream onto magnetic storage tapes. Later, these tapes were replayed off line with a more sophisticated analysis program. Of course each event (or optionally a fraction of the events) was also analyzed on line for diagnostics.

4.1 Event types

There were two types of events in the data stream: Type-1 and Type-2. Type-2 events, which were LAM-triggered computer readouts of the various TDC and ADC modules, constituted the main stream of data. These could be either real scattering events or LED (light-emitting-diode) simulated events, the latter being dummy events generated at a rate proportional to the accidental coincidence of a pulser-generated signal (known as the PULSER) with the in-beam polarimeter L-R signals. The PULSER rate could be adjusted to increase or decrease the number of LED events and the requirement of coincidence with the IBP signals ensured that the LED event rate was proportional to the instantaneous beam current. The ratio between the number of LED events recorded by the computer and the number of PULSER-IBP coincidences was a measure of the computer deadtime.

Type-1 events were readouts from the various scaler modules which contained counters for the in-beam polarimeter signals, drift chamber wire-plane signals, PULSER-IBP coincidence signals and the front-end processor pulsed diagnostic signals. The accumulated

48
counts were read at a specified frequency (once every 5 seconds for this experiment), so that the counting rates for the last counting period could also be obtained by subtracting the previous values. The scalers could be reset by issuing a CAMAC command through the host computer. The trigger rates of the wire-planes were useful for on-line diagnostics.

4.2 The hardware window on the acceptance solid-angle

As has already been mentioned, a proton of the same momentum but entering the spectrometer at a different vertical angle will undergo a different bend-angle due to the focusing property of the spectrometer. Since the spin precession angle is directly related to the bend angle through the MRS dipole (Eqn. (3.2)), and since the scattering asymmetry at the FPP carbon analyzer is, by definition, an average over the ensemble of protons within a momentum bin, it follows that a variation in bend angle (due to different entrance angles to the spectrometer) for a given momentum will introduce an uncertainty in the measured polarization.

For 340 MeV protons passing through the spectrometer, the precession angle of the central ray is (Eqn. (3.2))

\[ \theta_p = 1.79 \left( 1 + \frac{340}{938.28} \right) 60.3^\circ = 147.1^\circ , \]

and the change in precession angle \( \Delta \theta_p \) for a change in bend angle \( \Delta \theta_b \) is

\[ \Delta \theta_p = 2.43 \Delta \theta_b . \]

For 210 MeV protons,

\[ \theta_p = 1.79 \left( 1 + \frac{210}{938.28} \right) 60.3^\circ = 132.1^\circ , \]

and

\[ \Delta \theta_p = 2.19 \Delta \theta_b . \]
The exit angle, \( \theta \), at the focal plane can be estimated from the entrance angle \( \theta_I \) at the front-end using the beam optics of the MRS. For a small beam spot at the target position, as was the case for this experiment,

\[
\theta \simeq R_{22} \theta_I
\]

where \( R_{22} \simeq -2.66 \). The bend angle \( \theta_b \) is therefore

\[
\theta_b = \theta - \theta_I \simeq (R_{22} - 1) \theta_I \simeq -3.66 \theta_I
\]

As the typical front-end acceptance was about ±1.8° in opening angle about the central ray, it was therefore magnified to ±6.6° in bend angle, and thus ±16° and ±14° in precession angles for 340 MeV and 210 MeV protons, respectively.\(^9\)

This ±15° range in precession angle is large and we had therefore chosen to narrow down the front-end acceptance to ±0.7° in the central region of the front-end chamber by turning on only the central wires in the front-end trigger. As the kinematical correlation between the scattering angle and the focal-plane position cannot be discerned for the quasielastic broad continuum, and in order to avoid excessive averaging in scattering angle, the MRS front-end horizontal acceptance was also restricted by the front-end trigger to ±0.7°. Our solid angle of acceptance was therefore only 15% of that for the usual MRS running mode. However, we did not sacrifice event rate or suffer from increased background since the large number of events from the continuum easily saturated the focal-plane polarimeter front-end processor (Section 4.8).

It was, however, desirable to keep the front-end vertical acceptance centred about the scattering plane. This was done by trimming the vertical acceptance off line during data analysis until the \( \theta \) angle at the focal plane was centred about the central bend angle (figure

\(^9\) The energies are chosen here to illustrate the case of 80 MeV excitation for our incident energies; for higher-energy outgoing protons, the variation in \( \theta_b \) increases correspondingly.
4.1).

On the other hand, in order to determine the central region of the spectrometer's front-end horizontal acceptance, the MRS quadrupole was momentarily turned off, during the initial setup period of the experiment, so that only the central rays were brought to the focal plane (figure 4.2). The horizontal FEC hardware window was then set about this central value. As it turned out, this horizontal front-end acceptance unassisted by the focussing quadrupole was about ±0.7° in opening angle from the target position. Thus, choosing a smaller front-end acceptance also avoided the effects of the MRS quadrupole magnetic field on the proton spin.

4.3 The front-end and focal-plane triggers

There were four 16-channel programmable discriminators (LeCroy 4415) used for the four front-end (FEC) wire-planes (figure 4.3). The 16 wires in each wire plane were channeled into one of these modules – one channel per wire, one module for each plane. These programmable units provided flexibility for setting the range of active wires in each plane, i.e. for setting the hardware windows on the front-end acceptance.

The FEC X0 and X0' signals, and separately the Y0 and Y0' signals, were fed into a logical OR unit (LeCroy 4564), and then the ORed-X and ORed-Y signals were channeled into a logic array (LeCroy 2365) which was programmed to select the desired front-end trigger (FETRIG) condition. In order that the hardware window on the FEC be effective, a logical AND was required on the the input X and Y signals, (FETRIG –3); similarly, the front-end-veto signal (inverted) with the X and Y signals, (FETRIG –11), was used to include the front-end veto-scintillator during 16O data-taking with the waterfall target (Section 3.1),

A logical AND of the signals from the plastic scintillators S1 (signal S1) and S2 (signal
Figure 4.1   Histogram showing the distribution of the exit angle with respect to the central ray.
Figure 4.2 Histograms showing the effect of the MRS quadrupole on the front-end horizontal acceptance. The distribution of the scattering angle is shown on the top diagram when the quadrupole is turned off; and on the bottom diagram when the quadrupole is on. In both cases, the hardware window is set at the cut-off points seen in the bottom histogram.
S2) in the focal-plane polarimeter and from the X wire-plane in VDC1 (signal X1TR), were used as the "top-end" trigger for the spectrometer. The width of the S1 signal was the shortest; therefore it served as the reference timing signal for the common-stop drift-chamber TDCs and the common-start LeCroy 2228A TDC. The latter was used, for example, to measure timing with respect to the front-end chamber and the front-end recoil-scintillator. The timing from "top to bottom" (TTB) provided time-of-flight information on the particles passing through the spectrometer.

Since the whole focal plane (in the dispersion direction) was illuminated by the quasi-elastic continuum, the X1TR signal was used (in the first running period) to narrow the focal-plane active area to better match the focal-plane polarimeter acceptance. This was done by physically disconnecting some of the X wires in VDC1 at the high-momentum end of the focal plane. For the low-momentum end, a focal-plane veto-scintillator was used to reject focal-plane events at this edge. For the first running period only, (420 MeV, $^{12}$C and $^{16}$O: P, D$n$-$n$, D$s'$,$s$, D$p$,$p$) 35% of the focal-plane area was masked by these means.

This masking of the focal plane, however, was not essential since each event was tested for cone acceptance in the FPP during data analysis (Section 5.3.2). Nevertheless, the efficiency fell significantly near the far ends of the focal-plane and the masking was done only in the first running period to give the maximum number of useful data within the scheduled period of time.
4.4 The Master Trigger-Latch

A flow-diagram of the Master Trigger-Latch configuration is shown in figure 4.3. The principle of operation is as follows: For each event,

(i) The Master Trigger is activated by a coincidence between the front-end signal (FETRIG), the S1, S2 signals, the X1TR signal, and the FPPOL signal from the focal-plane polarimeter (see below).

(ii) If the data acquisition is “enabled” and if the latch is not set, the Master Trigger sets the Busy Gate which is then latched from further events until a CLEAR is issued to reset the latch.

(iii) The system generates a Fast Clear if the focal-plane veto-scintillator is triggered or if this is not the N-th event of an N-times prescaler. In this way, the system rejects events that trigger the focal-plane veto-scintillator (Section 4.3) and in any case only passes one out of N Master Triggered events regardless of other auxiliary conditions.

In this experiment, there was no prescaling (N=1) except when the focal-plane polarimeter front-end processor (J-11) was instructed to pass every event unconditionally while cross sections were being measured. In the latter case, N was set to 10 to reduce computer deadtime.
Figure 4.3  A simplified diagram showing the basic structure of the Master Trigger-Latch configuration. For a list of units, see table 4.1

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### Table 4.1

**LeCroy Modules product description.**

<table>
<thead>
<tr>
<th>Model</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>222</td>
<td>Dual Gate and Delay Generators</td>
</tr>
<tr>
<td>365</td>
<td>Majority Logic Unit w/Veto</td>
</tr>
<tr>
<td>429A</td>
<td>Quad Mixed Logic Fan-In/Fan-Out</td>
</tr>
<tr>
<td>622</td>
<td>Quad 2-Fold Logic Unit</td>
</tr>
<tr>
<td>2228A</td>
<td>Octal NIM Input TDC</td>
</tr>
<tr>
<td>2365</td>
<td>Octal Logic Matrix</td>
</tr>
<tr>
<td>4415</td>
<td>16 Channel Non-Updating Discriminator</td>
</tr>
<tr>
<td>4418</td>
<td>16 Channel Programmable Logic Delay/Fan-Out</td>
</tr>
<tr>
<td>4431</td>
<td>8 Channel 4-Bit Programmable Prescaler</td>
</tr>
<tr>
<td>4564</td>
<td>16 to 64-Fold OR Logic Unit</td>
</tr>
<tr>
<td>4616</td>
<td>16 Channel ECL-NIM-ECL Level Converter</td>
</tr>
</tbody>
</table>

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4.5 The focal-plane polarimeter trigger-latch

A flow-diagram of the focal-plane polarimeter trigger-latch configuration is shown in figure 4.4. The principle of operation is as follows:

(i) If the Master Latch is not set and data acquisition is “enabled,” a coincidence of the S1 and S2 signals activates the FPPOL trigger. This signal is sent to the Master Trigger input as mentioned above. Any further FPP triggers are inhibited for 3 microseconds.

(ii) If the Master Latch is not set in 250 nanoseconds, a fast clear is issued to all the FPP modules. However, if the Latch is set within the specified time period, meaning that the event satisfied the Master Trigger condition, the FPP fast clear is blocked.

(iii) If the Master Latch remains set for 3 microseconds, i.e. no Fast Clear is issued by the Master Trigger/Latch electronics, an External Interrupt is sent to the focal-plane polarimeter front-end processor (J-11) which then reads the FPP modules and preprocesses the data.

(iv) If the event is rejected by the J-11, it issues a Clear Latch signal to the system (figure 4.3) through an output register (CES 1320); if the event is accepted, and provided the CAMAC DCU (data control unit) has finished reading the last event (Mail-box = 0), the J-11 transfers data into its memory buffer, and then raises a LAM signal to the DCU (see below). If the DCU has not finished reading the last event (Mail-box = 1), the J-11 waits a time-out period for the Mail-box to reset; if the ready signal does not arrive within the time-out period, the J-11 issues a Clear Latch signal through its output register (CES 1320).
Figure 4.4 A simplified diagram showing the FPP Trigger-Latch configuration.
4.6 LAM event handling

Upon receiving a LAM signal from the MRS/FPP electronics, the CAMAC DCU (data control unit) responds as follows:

(i) It clears the LAM signal and then reads the fixed-length portion of the data stream from the MRS electronics (Section 5.1).

(ii) It then issues a DCU Clear Latch signal to the system (figure 4.3). This step frees the FPP electronics for the next event so that the efficiency of the FPP front-end processor (J-11) is maximized. (It should be noted that the LeCroy 4298 controller (see below) will not transfer new data into the 4299 memory unit until the latter has been read and cleared.)

(iii) Next, the DCU proceeds to read the memory buffer of the J-11. In completion, it signals the J-11 through the Mail-box register.

(v) The DCU then proceeds to read a LeCroy 4299 memory module for the drift-chamber readouts (see below), for the front-end chamber (FEC) and focal-plane drift-chambers (VDCs). Afterwards, it clears the 4299 buffer for the next event.

4.7 The MRS and FPP drift-chamber readout systems

First of all, it should be pointed out that the design of the detector readout system for the FPP drift-chambers was very different from that for the FEC and VDC chambers. The staggered design (offset double planes) of the FEC and the 45° tilt of the VDCs with respect to the optic axis provided means for determining the hit position without ambiguity on the drift direction (Section 5.1). As mentioned before, the FPP chambers had to be oriented orthogonal to the central ray to optimize the azimuthal acceptance – this precluded the option of tilted planes like those in the VDCs. On the other hand, the staggered design would mean doubling the number of wire planes which was not economical. Therefore, a very
different readout system was devised to resolve the left-right ambiguity on drift direction. Moreover, a delay-line type readout was used for these FPP drift chambers instead of the more expensive FEC and VDC readouts that have one TDC channel per wire.

The FEC and VDC drift chamber readout system consists of twenty-three 32-channel LeCroy 4290 TDCs in a special NIM crate. A total of 736 wires from the front-end chamber (FEC) and the focal-plane drift chambers (VDC1 and VDC2) were channeled into these LeCroy TDCs in sequence – one TDC channel per wire. These TDCs were operated in the common-stop mode; in this experiment, the reference signal was given by the S1 trigger scintillator. Readout was done using a LeCroy 4298 Drift Chamber TDC Controller to transfer information from the wires fired into a LeCroy 4299 memory module which acted as a read buffer for the acquisition computer.

For the focal-plane polarimeter (FPP) drift-chambers (D1, D2, D3 and D4), the anode wires for each of the eight wire planes (D1X, D1Y; D2X, D2Y; D3X, D3Y; D4X, D4Y) were connected to a delay-line, i.e. one anode delay-line per plane. The two ends (designated as L and R) of each anode delay-line (figure 4.5) were channeled into a LeCroy 2228A TDC; one TDC module was used for the X-planes and a second one for the Y-planes. These 2228A TDCs were used in the common-start mode, and they were locally triggered by the coincidence of the S1 and S2 scintillators. In each wire-plane, the difference in times ($T_{L-R}$) for each pair of L/R TDC channels spanned the wire spectrum, while the sum of times ($T_{L+R}$) gave the relative drift times (Section 5.5.1). On the other hand, the cathode sense wires in each wire plane were connected alternatively to Odd and Even bus lines which were channeled separately into two ADCs (figure 4.5). The left-right ambiguity for the drift direction in each plane was resolved by the observation that the induced signal on the nearer cathode was larger by about 15%. Timing from the ODD cathode was also used.
Figure 4.5  Schematic diagram showing the anode delay-line readout and cathode ODD and EVEN bus-lines of a FPP wire-plane. The L and R anode signals were fed into separate TDC channels while the ODD and EVEN cathode signals were channeled into different ADCs. The ODD cathode signal was also connected to a TDC for timing.
to double check the drift time in each plane.\textsuperscript{10)}

4.8 The FPP front-end processor

A major consideration in data acquisition for FPP experiments was related to the fact that for more than 90\% of the events the protons were scattered by less than $2^\circ$ at the carbon analyzer. These small angle events would create a tremendous load on the CAMAC highway and increase the computer deadtime. To circumvent this problem, a microprocessor (Starburst J-11) was used in this polarimeter to reject the small-angle polarimeter events.

The J-11 was programmed to read data from the FPP modules and to decide if an event was useful. The J-11 passed unconditionally the LED pulser generated events as well as FPP prescaled events. FPP prescaled events were defined as one out of every hundred of the events presented to the J-11, and as they were passed unconditionally, they consisted mainly of unscattered events and were reserved for off-line diagnostic purposes only. Other events were tested for conditions on scattering angle, collinearity, and checksum before they were passed on.

The scattering angle at the analyzer was roughly estimated by the J-11 in units of wires squared, using

\begin{equation}
\delta x = (D_{1x} + D_{4x} - 2 \cdot D_{2x})^2 ,
\end{equation}

in the X-Z plane, and similarly

\begin{equation}
\delta y = (D_{1y} + D_{4y} - 2 \cdot D_{2y})^2 ,
\end{equation}

in the Y-Z plane, and, more generally, by

\begin{equation}
\delta (xy) = (D_{1x} + D_{4x} - 2 \cdot D_{2x})^2 + (D_{1y} + D_{4y} - 2 \cdot D_{2y})^2 .
\end{equation}

\textsuperscript{10)} The choice of using the ODD or the EVEN timing signal is arbitrary but equivalent since the cathode signals were induced by the anode avalanche.
Here, $Dlx$ denotes the triggered X wire in FPP chamber D1, etc. Each of the above $\delta$ quantities were compared in turn to a chosen lower limit (10 wire$^2$ for this experiment). An event was rejected if none of these estimated angles was greater than the preset value.

An anode wire could be misidentified if two background signals occurred at anode wires near the two ends of an anode delay-line. Such background signals could be identified by the checksum on drift time.\[Ha87] The checksum was defined as the difference between the drift time (of a hit in a wire-plane) obtained from the anode-wire timing signals and that from the ODD cathode-wire timing signal. This test was applied to each of the eight wire planes. In this experiment, the upper limit on checksum was 50 nanoseconds.

A background signal might have a shorter drift time than the true signal and therefore could be the first signal to arrive at the two ends of an anode delay-line. Such false events could not be identified by the checksum in drift-times. However, a straight-trajectory (collinearity) test using the hit positions at D2, D3 and D4 could eliminate these bad events in these wire-chambers. The X and Y coordinates were tested separately on collinearity:

$$\Delta X = |D2X + D4X - 2 \cdot D3X| ,$$  \hspace{1cm} (4.4)

and

$$\Delta Y = |D2Y + D4Y - 2 \cdot D3Y| .$$  \hspace{1cm} (4.5)

An upper limit was set on these quantities, which ideally should be zero for a straight trajectory passing through D2, D3 and D4. In this experiment, this upper limit was 1 mm.
4.9 The J-11 parameters\(^{11}\)

In order to calculate the scattering angle, the collinearity and the checksum, the raw data from the FPP drift-chambers had to be processed. This required a set of parameters to be loaded into the J-11 memory before it could function.

In a calibration run before the experiment, the carbon analyzer in the FPP was removed and the FPP wire chambers were uniformly illuminated by *flooding* the focal plane with a continuum of protons. In analyzing the calibration data, a procedure was followed in which the wire number as a function of \(T_{L-R}\) was determined by *counting* the wires in a \(T_{L-R}\) histogram. Assuming the separation between anode wires was constant (8.128 mm), the hit position \((D = X \text{ or } Y)\) was given in units of the anode-wire spacing (8.128 mm) as a function of \(T_{L-R}\) by

\[
D = B_0 + B_1 \cdot T_{L-R} + B_2 \cdot (T_{L-R})^2 .
\]

The additional offsets for D2 and D3 with respect to D1 and D4 were then included to bring about proper alignment of the four chambers. A "drift-table" for each wire plane was also constructed for converting drift-times into drift-distances (Section 5.5).

In order to differentiate the left-right ambiguity using the difference in the induced signals on the neighbouring cathode wires, the cathode wires were positioned with respect to the anode wires to within ±7\(\mu\)m. In practice, software corrections were applied to the induced signals for each anode cell and the corrected ODD and EVEN signals for each drift cell were parametrized in terms of the raw signals by

\[
\begin{align*}
\text{EVEN}(a, i) &= \text{EVEN}_{\text{raw}}(a,i) - A_{\text{even}}(a) , \\
\text{ODD}(a, i) &= [\text{ODD}_{\text{raw}}(a,i) - A_{\text{odd}}(a)] \cdot \text{GAIN}(a, i) ,
\end{align*}
\]

where \(A_{\text{even}}(a)\) and \(A_{\text{odd}}(a)\) denote the "offsets" for EVEN and ODD cathode wires, respectively, of wire plane \(a\); while \(\text{GAIN}(a, i)\) denotes the Gain of the ODD cathode wire in

\(^{11}\) The discussion here will be brief as more details are given in Section 5.5.
The separation of the ODD and EVEN signals after the software correction is shown in figure 4.6. The 'V' shape opening up about the 45° line indicates the difference in the induced signals on the ODD and EVEN wires, with the two bands corresponding to the two different drift directions. The convergence at small signals corresponds to hits near an anode wire where the separation is not well defined, but this ambiguity on drift direction (for track positions that are close to an anode wire) makes very little difference to the calculated coordinate.

The “drift-table,” the “offset” parameters, and the coefficients in Eqns. (4.6) and (4.7) were input into the J-11 during startup as were a set of constants for TDC dispersion matching. The TDC dispersion is the intrinsic difference between the timing of a pair of TDC channels connected to the two ends of an anode readout delay-line, which were determined before the experiment by feeding a common-stop signal to each 2228A TDC module. For each event accepted by the J-11, the delay-sum $T_{L+R}$ and the delay-difference $T_{L-R}$ were computed with dispersion matching before they were passed into the data stream.
Figure 4.6  A scatter-plot showing the distribution of the correlated ODD and EVEN induced signals in a FPP detector plane. The opening up of the two distinctive bands indicates the difference in induced signals when the proton is closer to one of the ODD or EVEN cathodes of a drift-cell. The convergence at small signals corresponds to protons passing close to an anode wire.

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Chapter 5

DATA ANALYSIS

Data replay and analysis was done with a computer code LISA running in a Digital VAX/VMS 8650 computer. The code LISA was executed as a “subprocess”; the main process and the subprocess shared two “common global sections” so that information could be passed between them during an interactive session. An “event loop” was initiated at startup and each event went through the following sequence. First of all, a procedure was called to decode the data stream. Here, the fixed-length portion of the current event was stored directly into a data array, while the variable-length front-end and focal-plane wire-chamber raw data were immediately decoded for the respective wire-chamber coordinates. Next, the scattering event was reconstructed using information derived from the wire-chamber coordinates. Then, each event was subjected to a sequence of Condition testing functions that were used to identify and select or reject the event; the respective number of identified events was counted. If the event was selected, then a procedure was called to perform calculations essential to the final reduction of the data. The appropriate user-specified histograms (or scatter-plots) were then incremented before the event-loop was reiterated.

5.1 The decoding routine

As each event was decoded from the data stream, it was first identified for its event-type. For a type-1 event which consisted of readings of the scalers, the counting rates were obtained by subtracting off the previous scaler readings from the new readings. These “rates” and “scalers” were stored into separate data arrays.
For a LAM triggered type-2 event, the raw data could be separated into a fixed-length portion and a variable-length portion. The variable-length portion was so called because its length may vary event by event according to the number of triggered wires in the wire-chamber detectors. On the other hand, the fixed-length portion consisted of time-of-flight TDC readouts, scintillator ADC readouts, and focal-plane polarimeter data transferred from the J-11. In total, this fixed-length segment occupied the first 71 words of an event in the raw data stream. A list of these data words is given in table 5.1. For each event, these data words were identified by the decoding routine and copied into an array (each element was called a Coordinate in LISA) starting with the DCR (digital coincidence register) word but skipping all the termination codes. The DCR word contained a ‘1’ in the 7th bit for POLISIS “spin-up” events, in the 8th bit for “spin-down,” in the 9th bit for “spin-off” (i.e. unpolarized), and in the 12th bit for LED simulated events (PULSF:RS). In practice, LED simulated events were detected early in the decoding procedure so that the rest of the calculations could be bypassed once a bit was set in the Hitpattern word for later reference (see below).

The front-end and focal-plane wire-chamber readouts were in word 72 to the end of each event record. These were immediately decoded to extract the wire-plane coordinates. (In practice, each coordinate was declared Equivalent to a LISA Coordinate so that they could be used to form histograms.)

Not every event produced good hits in the wire-chambers. Events that failed to produce a trigger in one of the wire-planes were termed “missings.” On the other hand, clusters of signals that were caused by more than one particle track going through a wire-plane were termed “multiples.” For the “missing” and “multiple” events identified in the process of extracting the corresponding coordinates from the wire-chamber detectors, a bit was set in the appropriate bit of the Hitpattern word (table 5.2). The drift-chamber decoding...
Table 5.1

List of the first 71 raw data words of an event in the data-stream.

1: (most-significant-byte) Termination code
   (least-significant-byte) Event length
2: (most-significant-byte) Event type
   (least-significant-byte) + ...
3: ... 24-bit DCU sequence number
4: DCR

*note: Number 5 – 16 from MRS electronics.*

<table>
<thead>
<tr>
<th></th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>Termination code</td>
</tr>
<tr>
<td>6</td>
<td>TDC Top-to-Bottom (TTB)</td>
</tr>
<tr>
<td>7</td>
<td>TDC RF</td>
</tr>
<tr>
<td>8</td>
<td>TDC MRS dipole entrance slit veto scintillator</td>
</tr>
<tr>
<td>9</td>
<td>TDC MRS dipole vessel liner scintillator</td>
</tr>
<tr>
<td>10</td>
<td>TDC S1</td>
</tr>
<tr>
<td>11</td>
<td>TDC front-end veto scintillator</td>
</tr>
<tr>
<td>12</td>
<td>Termination code</td>
</tr>
<tr>
<td>13</td>
<td>ADC MRS dipole entrance slit veto scintillator</td>
</tr>
<tr>
<td>14</td>
<td>ADC MRS dipole vessel liner scintillator</td>
</tr>
<tr>
<td>15</td>
<td>60 Hz scaler</td>
</tr>
<tr>
<td>16</td>
<td>60 Hz scaler</td>
</tr>
</tbody>
</table>

... cont'ed
Note: Number 17 - 71 from FPP electronics.

17: Termination code
18: J-11 prescaler N-value
19: D1X $T_{L-R}$  
20: D1X $T_{L+R}$  
21: D2X $T_{L-R}$  
22: D2X $T_{L+R}$  
23: D3X $T_{L-R}$  
24: D3X $T_{L+R}$  
25: D4X $T_{L-R}$  
26: D4X $T_{L+R}$  
27: D1Y $T_{L-R}$  
28: D1Y $T_{L+R}$  
29: D2Y $T_{L-R}$  
30: D2Y $T_{L+R}$  
31: D3Y $T_{L-R}$  
32: D3Y $T_{L+R}$  
33: D4Y $T_{L-R}$  
34: D4Y $T_{L+R}$  
35: TDC D1X ODD  
36: TDC D1Y ODD  
37: TDC D2X ODD  
38: TDC D2Y ODD  
39: TDC D3X ODD  
40: TDC D3Y ODD  
41: TDC D4X ODD  
42: TDC D4Y ODD  
43: ADC D1X ODD  
44: ADC D1Y ODD  
45: ADC D2X ODD  
46: ADC D2Y ODD  
47: ADC D3X ODD  
48: ADC D3Y ODD  
49: ADC D4X ODD  
50: ADC D4Y ODD  
51: ADC S1 (high-momentum segment)  
52: ADC S1 (low-momentum segment)  
53: ADC D1X EVEN  
54: ADC D1Y EVEN  
55: ADC D2X EVEN  
56: ADC D2Y EVEN  
57: ADC D3X EVEN  
58: ADC D3Y EVEN  
59: ADC D4X EVEN  
60: ADC D4Y EVEN  
61: ADC S2 (low-momentum, small-Y1 segment)  
62: ADC S2 (high-momentum, small-Y1 segment)  
63: ADC S2 (high-momentum, large-Y1 segment)  
64: ADC S2 (low-momentum, large-Y1 segment)  
65: TDC S1 (high-momentum segment)  
66: TDC S1 (low-momentum segment)  
67: TDC S2 (low-momentum, small-Y1 segment)  
68: TDC S2 (high-momentum, small-Y1 segment)  
69: TDC S2 (high-momentum, large-Y1 segment)  
70: TDC S2 (low-momentum, large-Y1 segment)  
71: TDC Master-Trigger
routine also allowed software-simulated hardware windows to be put on each wire-plane coordinate; if an event was found to be outside the range of the specified window, the corresponding "out-of-range" bit was also set in the Hitpattern word. This Hitpattern for the wire-chambers provided information on the detection efficiency; the latter was needed for the absolute normalization of cross sections.

The front-end chamber (FEC) coordinates were obtained in the following manner. Since the LeCroy 4290 TDCs for the front-end and focal-plane wire-chambers were used in the common-stop mode, the drift time $t$ to an anode wire was related to the TDC value $T$ by

$$T = t_0 - t$$

(5.1)

where $t_0$ is the delay-time before the common-stop signal arrived. For a good hit on a pair of staggered planes (figure 5.1), the mean drift distance was evaluated by

$$\bar{d} = \frac{1}{2}(d + d') = \frac{1}{2} \left( \frac{t}{\Delta} + \frac{\Delta - t'}{\Delta} \right) l = \frac{1}{2} \left( -\frac{T}{\Delta} + \frac{\Delta + T'}{\Delta} \right) l$$

(5.2)

where $\Delta$ denotes the drift time from cathode to anode (37 ns for the FEC.)

The FEC wires were numbered with direct correspondence to consecutive TDC addresses (relative channel numbers) as shown in figure 5.2. This numbering scheme provided a convenient means to determine the drift direction: the sign of drift was given by $2(N' - N) + 1$. In the decoding routine, the $X$ or $Y$ coordinate was measured from wire 0 of the unprimed plane and the averaged drift distance, $\bar{d}$, was added or subtracted from the unprimed hit anode-wire position according to the drift direction.

For the focal-plane vertical drift chambers (VDCs), a particle trajectory would typically trigger several anode wires as shown in figure 5.3. Notice that since the drift direction was perpendicular to the VDC wire-planes, there were no ambiguities to the drift direction. The drift distance to an anode wire was determined from the common-stop TDC values
Table 5.2

Description of bits in the Hitpattern word. Note: order between the FEC and VDC planes may vary depending on the “routing” scheme for each experiment.

<table>
<thead>
<tr>
<th></th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>Missing hit in FEC coordinate X</td>
</tr>
<tr>
<td>1</td>
<td>Missing hit in FEC coordinate Y</td>
</tr>
<tr>
<td>2</td>
<td>Missing hit in VDC coordinate U1</td>
</tr>
<tr>
<td>3</td>
<td>Missing hit in VDC coordinate X2</td>
</tr>
<tr>
<td>4</td>
<td>Missing hit in VDC coordinate U2</td>
</tr>
<tr>
<td>5</td>
<td>Missing hit in VDC coordinate X1</td>
</tr>
<tr>
<td>6</td>
<td>(not used)</td>
</tr>
<tr>
<td>7</td>
<td>(not used)</td>
</tr>
<tr>
<td>8</td>
<td>Multiple cluster in FEC coordinate X</td>
</tr>
<tr>
<td>9</td>
<td>Multiple cluster in FEC coordinate Y</td>
</tr>
<tr>
<td>10</td>
<td>Multiple cluster in VDC coordinate U1</td>
</tr>
<tr>
<td>11</td>
<td>Multiple cluster in VDC coordinate X2</td>
</tr>
<tr>
<td>12</td>
<td>Multiple cluster in VDC coordinate U2</td>
</tr>
<tr>
<td>13</td>
<td>Multiple cluster in VDC coordinate X1</td>
</tr>
<tr>
<td>14</td>
<td>(not used)</td>
</tr>
<tr>
<td>15</td>
<td>LED simulated event (PULSER)</td>
</tr>
<tr>
<td>16</td>
<td>X0 position outside window</td>
</tr>
<tr>
<td>17</td>
<td>X0' position outside window</td>
</tr>
<tr>
<td>18</td>
<td>Y0 position outside window</td>
</tr>
<tr>
<td>19</td>
<td>Y0' position outside window</td>
</tr>
<tr>
<td>20</td>
<td>U1 position outside window</td>
</tr>
<tr>
<td>21</td>
<td>X2 position outside window</td>
</tr>
<tr>
<td>22</td>
<td>U2 position outside window</td>
</tr>
<tr>
<td>23</td>
<td>X1 position outside window</td>
</tr>
<tr>
<td>24</td>
<td>FEC-time outside window</td>
</tr>
<tr>
<td>25</td>
<td>VDC-time outside window</td>
</tr>
</tbody>
</table>
Figure 5.1  A schematic diagram showing a drift-cell in the FEC $Y_0$ and $Y_0'$ planes. The drift-times are towards the respective anode wires as indicated in the figure. Note the convention with which $d$ and $d'$ are measured.
Figure 5.2 A schematic diagram showing the convention for numbering the FEC wires in a pair of staggered planes. Note that the sequence $0, 1, 2, \ldots, 14, 15, 15', 14', \ldots, 2', 1', 0'$ corresponds to 32 consecutive TDC channels.
as in Eqn. (5.1). From the trigonometry depicted in figure 5.3 and using three adjacent triggered wires having the largest TDC values (shortest drift-times), the position where the trajectory intersects the wire-plane can be determined:

\[
X = l' \cdot N - \frac{l'}{2} \left( \frac{T_{N+1} - T_{N-1}}{T_{\text{min}} - T_{\text{max}}} \right)
\]

Here, the \(T\)’s are given by Eqn. (5.1), \(N\) is the relative TDC channel number with respect to the first wire in the wire-plane, and \(l' = 8\) mm for these VDCs.

As noted before, the \(U\) wires in these VDCs were slanted at 30° to \(X\). This choice (rather than orthogonal wires) enabled each particle track passing through a \(U\) plane to produce a cluster of triggered wires in the same manner as shown in figure 5.3. The \(U\) coordinates were evaluated analogous to Eqn. (5.3).
Figure 5.3 A schematic diagram showing the pattern of triggered wires resulting from the passage of a proton through a VDC detector plane. Note that the 45° tilt of each VDC with respect to the central ray of the spectrometer is responsible for this hit pattern. The shortest drift-distance corresponds to the largest TDC time since the LeCroy 4290 TDCs were used in the common-stop mode.
5.2 The primary scattering reconstructed

As we have mentioned before, an important characteristic of this experimental setup was the ray-tracing capability (Section 3.5). First of all, we discuss the imaging between the front-end and the focal-plane coordinates.

5.2.1 The focal-plane coordinate

The focal-plane coordinate, $X_F$, in the dispersion direction was determined event by event using information provided by the VDCs. Figure 5.4 is an analytical diagram showing the focal-plane position as a function of the parameters $\delta$ and $F_f$. We have

$$\frac{S_{v12}}{\Delta X_{12}} = \tan \theta_V = \frac{z}{X_1 + X_{1\text{off}} - X_F}, \quad (5.4)$$

$$\tan \delta = \frac{F_f - z}{X_F}, \quad (5.5)$$

and

$$\Delta X_{12} = X_1 + X_{1\text{off}} - X_2. \quad (5.6)$$

Eliminating $z$ from these equations, the focal plane coordinate can be expressed in the following form:

$$X_F = \frac{S_{v12} \cdot (X_1 + X_{1\text{off}}) - (F_f \cdot \Delta X_{12})}{S_{v12} - \Delta X_{12} \cdot \tan \delta}. \quad (5.7)$$

Here, $F_f = -109.15$ mm, $\tan \delta = -0.06$, $S_{v12} = 272.00$ mm, $X_{1\text{off}} = 373.15$ mm before the X1 wire-plane was “extended,” or $X_{1\text{off}} = 277.55$ mm after the X1 wire-plane was “extended” during developments after the first running period.
Figure 5.4 An analytical diagram showing the focal plane with respect to the VDCs. The parameters $\delta$ and $F_f$ determine the position of the focal plane. The VDCs are each 1050 mm $\times$ 300 mm (X by Y), the distance $F_f$ was $-109.15$ mm and $\tan \delta = -0.06$ so that in reality the tilt of the focal plane was very close to that of the VDC's and it was entirely below VDC1 by about 100 mm.
5.2.2 The $\theta$ and $\phi$ angles at the focal plane

With reference to figure 5.4, the exit angle, $\theta_V$, in the bend plane with respect to the VDCs was given by

$$\tan \theta_V = \frac{S_{v12}}{\Delta X_{12}}.$$  \hspace{1cm} (5.8)

Since the central ray was at an angle of $45^\circ$ with respect to the VDCs, it was useful to define the quantity

$$\Theta = \theta_V - 45^\circ$$  \hspace{1cm} (5.9)

so that $\Theta$ was centred at zero for a front-end vertical acceptance symmetrical about the central ray (figure 4.1).

Finally, the exit angle $\theta$ in the bend plane, measured with respect to the horizontal is

$$\theta = \Theta + \theta_0,$$  \hspace{1cm} (5.10)

where $\theta_0$ denotes the bend angle of the central ray (figures 4.3 and 5.5) which was known for this spectrometer:

$$\theta_0 = 60.3^\circ \pm 0.3^\circ.$$  \hspace{1cm} (5.11)

The Y coordinates at the VDCs were evaluated using the X and U coordinates. As the X and U planes in each VDC were separated by a small distance ($\delta V = 26.25$ mm), precautions were made to correct for parallax. With reference to figure 5.6, the X coordinate projected onto the U plane was given by

$$X_{1u} = X1 - \frac{\Delta X_{12} \cdot \delta V}{S_{v12}},$$  \hspace{1cm} (5.12a)

and

$$X_{2u} = X2 - \frac{\Delta X_{12} \cdot \delta V}{S_{v12}}.$$  \hspace{1cm} (5.12b)
Figure 5.5  A diagram showing the correlation between the vertical angle of scattering, $\theta_I$, at the target position and the bend angle through the spectrometer. Note that due to the focusing property of the spectrometer, other things being equal, a proton of the same momentum but scattered to the "up" azimuthal direction undergoes a smaller bend angle compared with that of the central ray, and inversely for a proton scattered "down."

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Figure 5.6  A diagram showing the separation between the X and U detector planes in the VDCs. The quantity $\epsilon_x$ is the correction for parallax given by the second term in Eqn. (5.12).
Since the U wires were slanted at 30° to the X wires, the Y coordinate at each VDC was given by

\[ Y_1 = \sqrt{3} \cdot X_{1u} - 2 \cdot U_1 , \quad (5.13a) \]

and

\[ Y_2 = \sqrt{3} \cdot X_{2u} - 2 \cdot U_2 . \quad (5.13b) \]

Here, we followed the convention of using a left-handed coordinate system for these VDC coordinates such that larger X corresponds to lower momentum, with the z-axis pointing downstream (figure 5.7).

Finally, the angle \( \phi \) in the Y direction is given approximately by

\[ \tan \phi = \frac{Y_2 - Y_1}{\sqrt{S_{12}^2 + \Delta X_{12}^2}} \quad (5.14) \]

where we have approximated the effective separation of the Y planes by the trajectory length across the X planes (figure 5.6).

### 5.2.3 The target coordinates

For a given momentum, the vertical target angle, \( \theta_I \), and coordinate, \( X_I \), may be related to the angle \( \theta \) by

\[ \theta = R_{21} X_I + R_{22} \theta_I , \quad (constant \ momentum) \quad (5.15) \]

where \( R_{21} \) and \( R_{22} \) are the transport coefficients in the language of beam optics (figure 5.5). However, for a finite momentum acceptance to the spectrometer, the effective length of the spectrometer varies about the central momentum. In other words, the exit angle \( \theta \) will also depend on the momentum of the particle. In practice, a fictitious angle, \( \theta_{pc} \), was introduced
Figure 5.7 A diagram showing the geometry of the U-wires with respect to the X-wires. The triangle at the bottom shows the sign convention for the X, Y and U coordinates.
as a function of the focal-plane position (momentum) and an empirical constant, \( \Theta_{xf} \), such that

\[
\theta_{pc} = \theta + \Theta_{xf} \cdot (X_F - X_F^0)
\]  

(5.16)

where \( X_F^0 \) was an offset chosen such that \( \theta_{pc} = \theta \) for the central ray (or momentum). With \( \theta_{pc} \) and the transport coefficients taken for the central momentum, Eqn. (5.15) was used to project \( \theta_{pc} \) back to the target:

\[
\theta_{pc} = R_{21}X_I + R_{22}\theta_I .
\]  

(5.17)

The projected target position, \( X_I \), was then considered independent of the focal-plane position.

For each event, the angle \( \theta_I \) was given in terms of \( X_{FEC} \) (evaluated from \( X_0 \) and \( X_0' \) with Eqn. (5.2)) and \( X_I \) by

\[
X_{FEC} = X_I + l \theta_I
\]  

(5.18)

where \( l \), the distance of the FEC from the target position, was 62 cm. Thus, eliminating \( \theta_I \) from Eqns. (5.17) and (5.18), the vertical target coordinate was given by

\[
X_I = (X_{FEC} - \frac{l}{R_{22}} \theta_{pc}) \frac{R_{22}}{R_{22} - LR_{21}} ,
\]  

(5.19a)

or,

\[
X_I = (X_{FEC} + X_{I0} \cdot \theta_{pc}) \cdot X_{I} .
\]  

(5.19b)

For the horizontal target coordinate, \( Y_I \), since the angle \( \phi \) (parallel to the dipole field) was uncorrelated with the momentum of the particle, \( Y_I \) was obtained by replacing \( X_{FEC} \), \( \theta_{pc} \), \( R_{21} \) and \( R_{22} \) in Eqn. (5.19a) with \( Y_{FEC} \), \( \phi \), \( R_{43} \) and \( R_{44} \), respectively. Thus,

\[
Y_I = (Y_{FEC} - \frac{l}{R_{44}} \phi) \frac{R_{44}}{R_{44} - LR_{43}} ,
\]  

(5.20a)

or,
\[ Y_f = (Y_{fec} + Y_{f\phi} \cdot \phi) \cdot Y_{if} \]  

(5.20b)

The constants \( X_{f\theta}, X_{if}, Y_{f\theta} \) and \( Y_{if} \) could be obtained from the known transport coefficients; however in practice, the parameter \( X_{f\theta} \) was tuned by plotting \( X_f \) against \( \theta_{pc} \) and varying \( X_{f\theta} \) until a horizontal band was obtained. Similarly, \( Y_{if} \) was adjusted by plotting \( Y_f \) against \( \phi \).

### 5.2.4 The bend angle through the spectrometer

For each event, the bend angle, \( \theta_b \), was given by

\[ \theta_b = \theta - \theta_f \]  

(5.21)

where \( \theta_f \) was obtained by eliminating \( X_f \) from Eqns. (5.17) and (5.18):

\[ \theta_f = \left( \frac{1}{R_{22} - LR_{21}} \right) \theta_{pc} - \left( \frac{R_{21}}{R_{22} - LR_{21}} \right) X_{fec} \]  

(5.22)

Hence,

\[ \theta_b = \theta - \left( \frac{1}{R_{22} - LR_{21}} \right) \theta_{pc} + \left( \frac{R_{21}}{R_{22} - LR_{21}} \right) X_{fec} \]  

(5.23)

and, substituting for \( \theta_{pc} \) with Eqn. (5.16),

\[ \theta_b = \left( \frac{R_{22} - LR_{21} - 1}{R_{22} - LR_{21}} \right) \theta + \left( \frac{R_{21}}{R_{22} - LR_{21}} \right) X_{fec} - \left[ \frac{\Theta_{xf} \cdot (X_F - X_F^p)}{R_{22} - LR_{21}} \right] \]  

(5.24)

The empirical value for \( \Theta_{xf} \) was \(-0.0074^\circ\text{mm}^{-1}\) as determined by varying \( \Theta_{xf} \) until \( \theta_{pc} \) was independent of \( X_F \): i.e., a scatter-plot of \( \theta_{pc} \) against \( X_F \) displayed a horizontal band along the central value of 60.3°. The last term in Eqn. (5.24) was zero at the central region of the focal plane, and in any case it was less than \( \pm 1^\circ \) for the whole region of our data. Thus, although the correction term \( \Theta_{xf}(X_F - X_F^p) \) on \( \theta \) was ad hoc in nature, the change in \( \theta_b \) due to this term was small.
5.2.5 The precession angle through the spectrometer

The precession angle, $\theta_p$, of the spin of each proton passing through the spectrometer was a function of its focal-plane position (momentum) and its bend angle, $\theta_b$, through the spectrometer:

$$\theta_p = \left( 1 + \frac{T}{mc^2} \right) \left( \frac{\gamma}{2} - 1 \right) \theta_b ,$$

(5.25)

where $T = E - mc^2$ is the kinetic energy of the proton to be determined from the focal-plane position, and $\frac{\gamma}{2} = 2.793$ is the proton magnetic moment.

In the final reduction of data to extract the Wolfenstein parameters, the functional values of $\sin \theta_p$ and $\cos \theta_p$, respectively, in each momentum bin were averaged. During data analysis, these functions were evaluated event by event and the functional values were accumulated in $X_F$ (momentum) bins. The respective ensemble averages were then obtained by normalizing to the number of events in each bin.

5.3 The FPP scattering reconstructed

In order to reconstruct event-by-event the scattering vertex at the FPP carbon analyzer, we need to ray-trace the incoming and outgoing trajectories at the analyzer. The incoming trajectory was reconstructed by using the focal-plane VDCs and the first FPP wire-chamber D1, while the outgoing trajectory was obtained by using the three FPP wire-chambers, D2, D3 and D4, downstream of the analyzer.

5.3.1 The VDC-FPP geometry

Since the VDCs were at an angle of 45° to the central ray while the FPP wire-chambers were orthogonal to the the latter, it was necessary to bring the VDC coordinates to a
The FPP reference frame was chosen such that the FPP wire-chamber X and Y axes were part of the triad; Z was zero at the D1X wire-plane and positive downstream of D1X. Figure 5.8 shows the orientation of the VDCs in this reference frame. There were three fixed reference points in the system: (i) the origin of the FPP frame, O, (ii) the high-momentum edge of VDC2, O', and (iii) the point, P, where D1X intersects the line passing through O' and orthogonal to the VDCs. Thus, the distance \( \overline{OP} \) was a constant in the system:

\[
\overline{OP} = D1X + \frac{X}{\cos \alpha} = constant. 
\] (5.26)

The angle each event made with the FPP Z-axis was related to the angle it made with the VDCs by

\[
\theta_{in} = \theta_V - (90^\circ - \alpha), \quad (5.27)
\]

or,

\[
\tan \theta_{in} = \frac{\tan \theta_V \tan \alpha - 1}{\tan \theta_V + \tan \alpha}. \quad (5.28)
\]

With reference to figure 5.8,

\[
\tan \theta_V = \frac{Y}{\mathcal{H}} = \frac{S_{v12}}{\Delta X_{12}}, \quad (5.29)
\]

and

\[
Y = F - X \cdot \tan \alpha, \quad (5.30)
\]

\[
\mathcal{H} = X1 + X_{1\text{off}} + X, \quad (5.31)
\]

so that

\[
X = \frac{F \cdot \Delta X_{12} - S_{v12} \cdot (X1 + X_{1\text{off}})}{S_{v12} + \Delta X_{12} \cdot \tan \alpha}. \quad (5.32)
\]

12) In this discussion, we will ignore the technical details concerning the fact that a unit of length for the VDCs (50 \( \mu \)m) was different to that of the FPP wire-chambers (0.1 mm). In all occasions when numerical values are quoted, the physical units are always given.
where
\[ \Delta X_{12} = X_1 + X_{1off} - X_2 \]  \hspace{1cm} (5.33)

The separation between the X and U planes in the VDCs ($\delta_V = 26.25$ mm) was different from the separation between the X and Y planes in the FPP ($\delta_D = 9.65$ mm). Therefore, for ease of relating the FPP and VDC quantities, it was useful to project the VDC U coordinates and the FPP Y coordinates onto the respective X planes.\(^{13}\) For the FPP wire-chambers, these are given by

\[
D1Y_x = D1Y - \epsilon , \hspace{1cm} (5.34a)
\]
\[
D2Y_x = D2Y - \epsilon , \hspace{1cm} (5.34b)
\]
\[
D3Y_x = D3Y - \epsilon , \hspace{1cm} (5.34c)
\]
\[
D4Y_x = D4Y - \epsilon , \hspace{1cm} (5.34d)
\]

where
\[ \epsilon = \delta_D \cdot \frac{D4Y - D2Y}{(D4Z - D2Z)} , \hspace{1cm} (5.35) \]

and \((D4Z - D2Z)\), the separation between D2 and D4, was 400 mm (table 5.3). For the VDCs, the analog of Eqn. (5.12) yields

\[
Y_{1x} = Y1 - \frac{(Y2 - Y1) \cdot \delta_V}{S_{v12}} , \hspace{1cm} (5.36a)
\]

and

\[
Y_{2x} = Y2 - \frac{(Y2 - Y1) \cdot \delta_V}{S_{v12}} . \hspace{1cm} (5.36b)
\]

Furthermore, to bring each VDC Y coordinate into the FPP coordinate system, an offset was determined for each VDC such that

\[
\bar{Y1} = Y_{1x} + Y_{1off} , \hspace{1cm} (5.37a)
\]
\[
\bar{Y2} = Y_{2x} + Y_{2off} . \hspace{1cm} (5.37b)
\]

\(^{13}\) See footnote 13, pg 88.
Figure 5.8  An analytical diagram showing the geometry of the VDCs with respect to the FPP. The point O is the origin of the FPP coordinate system. The relative orientation is determined by $F$ and $\alpha$. 

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Here and henceforth, the overline is used to denote VDC quantities expressed in the FPP reference frame.

Finally, the coordinates for the origins of VDC1 and VDC2 in the FPP reference frame were obtained by

\[ \overline{X_{10}} = \overline{OP} + X_{1off} \cdot \cos \alpha - F \cdot \sin \alpha , \]  
(5.38)

\[ \overline{Y_{10}} = Y_{1off} , \]  
(5.39)

\[ \overline{Z_{10}} = -(X_{1off} \cdot \sin \alpha + F \cdot \cos \alpha ) , \]  
(5.40)

and,

\[ \overline{X_{20}} = \overline{X_{10}} + S_{v12} \cdot \sin \alpha - X_{1off} \cdot \cos \alpha , \]  
(5.41)

\[ \overline{Y_{20}} = Y_{2off} , \]  
(5.42)

\[ \overline{Z_{20}} = \overline{Z_{10}} + X_{1off} \cdot \sin \alpha + S_{v12} \cdot \cos \alpha . \]  
(5.43)

The constants, \( \alpha \), \( F \), \( X_{1off} \), \( Y_{1off} \) and \( Y_{2off} \), were determined in a calibration procedure to be described in Section 5.5.3.

5.3.2 The Vertex coordinates and the angles of scattering

With the origins of VDC1 and VDC2 determined in the FPP reference frame, the VDC coordinates were expressed in the FPP frame as

\[ \overline{X_1} = \overline{X_{10}} + X_1 \cdot \cos \alpha , \]  
(5.44)

\[ \overline{Y_1} = \overline{Y_{10}} + Y_{1z} , \]  
(5.45)

\[ \overline{Z_1} = \overline{Z_{10}} - X_1 \cdot \sin \alpha , \]  
(5.46)

and,

\[ 14) \text{ See footnote 13, pg 88.} \]
Table 5.3

Physical dimensions of FPP components.

<table>
<thead>
<tr>
<th>Drift chambers</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>D1–D4</td>
<td>anode wire spacing</td>
<td>0.8128 cm</td>
</tr>
<tr>
<td></td>
<td>active area</td>
<td>89.4 cm (X) × 48.8 cm (Y)</td>
</tr>
<tr>
<td></td>
<td># anode wires</td>
<td>X: 110; Y: 60</td>
</tr>
<tr>
<td></td>
<td>sep. of X,Y planes</td>
<td>0.97 cm</td>
</tr>
<tr>
<td></td>
<td>distance D1–D2</td>
<td>40 cm</td>
</tr>
<tr>
<td></td>
<td>distance D2–D3</td>
<td>20 cm</td>
</tr>
<tr>
<td></td>
<td>distance D3–D4</td>
<td>20 cm</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Trigger scintillators</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>S1</td>
<td>active volume</td>
<td>78 cm × 24 cm × 0.64 cm</td>
</tr>
<tr>
<td></td>
<td>(x × y × z)</td>
<td>(52 cm + 52 cm) × 62 cm × 0.64 cm</td>
</tr>
<tr>
<td></td>
<td>distance D1–S1</td>
<td>22 cm</td>
</tr>
<tr>
<td></td>
<td>distance D4–S2</td>
<td>15 cm</td>
</tr>
</tbody>
</table>

| Carbon analyzer | area (X × Y) | 98 cm × 45 cm |
|                | thickness    | 4.5 cm, 3 × 3 cm |

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\[ X_2 = X_2^0 + X_2 \cdot \cos \alpha , \]  
(5.47)

\[ Y_2 = Y_2^0 + Y_2 \cdot \cos \alpha , \]  
(5.48)

\[ Z_2 = Z_2^0 - X_2 \cdot \sin \alpha . \]  
(5.49)

During data analysis, a straight-trajectory test was made on the information provided by VDC1, VDC2 and D1:

\[ \delta_x^l = X_2 - X_1 - (D1X - X_1) \times \frac{Z_1 - Z_2}{Z_1}, \]  
(5.50a)

\[ \delta_y^l = Y_2 - Y_1 - (D1Y - Y_1) \times \frac{Z_1 - Z_2}{Z_1}. \]  
(5.50b)

A window was put on \( \delta_x^l \) and \( \delta_y^l \), respectively, so that only well-defined incident tracks to the FPP would be accepted. Likewise, a similar test was put on the outgoing track after the carbon analyzer using information from D2, D3 and D4:

\[ \delta_x^n = D2X + D4X - 2 \cdot D3X , \]  
(5.51a)

\[ \delta_y^n = D2Y + D4Y - 2 \cdot D3Y . \]  
(5.51b)

For well-defined incoming and outgoing trajectories at the carbon analyzer, the incoming vector, \( \vec{v}_{in} \), and outgoing vector, \( \vec{v}_{out} \), were obtained by

\[ \vec{v}_{in} = (D1X - X_1, D1Y - Y_1, D1Z - Z_1) , \]  
(5.52)

and

\[ \vec{v}_{out} = (D4X - X_2, D4Y - Y_2, D4Z - Z_2) . \]  
(5.53)

Once the incoming and outgoing vectors were determined, the scattering angle, \( \theta_s \), at the carbon analyzer could be obtained:

\[ \cos \theta_s = \frac{\vec{v}_{in} \cdot \vec{v}_{out}}{|\vec{v}_{in}| \cdot |\vec{v}_{out}|} . \]  
(5.54)
The incoming and outgoing vectors, however, did not necessarily intersect each other due to finite resolution of the wire-chamber detectors. In practice, the vertex was defined as the mid-point on the distance-of-closest-approach. A unit vector orthogonal to both \( \vec{v}_{in} \) and \( \vec{v}_{out} \) was given by

\[
\hat{\omega} = \frac{\vec{v}_{in} \times \vec{v}_{out}}{|\vec{v}_{in} \times \vec{v}_{out}|}.
\] (5.55)

The distance of closest-approach, \( \varrho \), was simply the projection onto \( \hat{\omega} \) from any vector, \( \vec{S} \), joining two points on \( \vec{v}_{in} \) and \( \vec{v}_{out} \), say, the coordinates at D1 and D2, respectively. Thus, writing

\[
\vec{S} = (D2X - D1X, D2Y - D1Y, D2Z - D1Z),
\] (5.56)

\( \varrho \) was given by

\[
\varrho = \vec{S} \cdot \hat{\omega}.
\] (5.57)

Furthermore, denoting the end-points of \( \varrho \) by \( \varrho_i \) and \( \varrho_f \), respectively, we could write

\[
\varrho_i = \overrightarrow{D1} + \eta_i \hat{v}_{in},
\] (5.58)

\[
\varrho_f = \overrightarrow{D2} + \eta_f \hat{v}_{out},
\] (5.59)

where \( \eta_i \) and \( \eta_f \) were determined from the identity

\[
\vec{S} = \eta_i \hat{v}_{in} + \eta_f \hat{v}_{out} + \varrho \hat{\omega}.
\] (5.60)

Here, \( \hat{v}_{in} \) and \( \hat{v}_{out} \) are used to denote the normalized \( \vec{v}_{in} \) and \( \vec{v}_{out} \), respectively; also, \( \overrightarrow{D1} \equiv (D1X, D1Y, D1Z) \), and \( \overrightarrow{D2} \equiv (D2X, D2Y, D2Z) \). Thus, taking dot-products of Eqn. (5.60) with \( \hat{v}_{in} \) and \( \hat{v}_{out} \), respectively,

\[
\vec{S} \cdot \hat{v}_{in} = \eta_i + \eta_f \cos \theta_c,
\] (5.61)

\[
\vec{S} \cdot \hat{v}_{out} = \eta_f + \eta_i \cos \theta_c,
\] (5.62)
we obtained

\[ \eta_i = \frac{1}{\sin^2 \theta_c} ( \vec{S} \cdot \vec{v}_{in} - \vec{S} \cdot \vec{v}_{out} \cos \theta_c ) \quad (5.63) \]

and

\[ \eta_f = \frac{1}{\sin^2 \theta_c} ( \vec{S} \cdot \vec{v}_{out} - \vec{S} \cdot \vec{v}_{in} \cos \theta_c ) \quad (5.64) \]

With Eqns. (5.52) to (5.64), the vertex vector, \( \vec{V} \), was determined by

\[ \vec{V} = \frac{1}{2} ( \vec{\theta}_i + \vec{\theta}_f ) \quad (5.65) \]

The asymmetry of scattering at the FPP was measured by the azimuthal distribution of scattered events at the carbon analyzer. In practice, the incident angle was small enough\(^{15}\) such that, for computational purposes, the Z-axis of the FPP reference frame was chosen as the \( l'' \)-axis\(^{16}\) of the \( (s'', n'', l'') \) coordinate system defined in figure 5.9. The correspondence between the FPP frame of reference and the conventional \( (s'', n'', l'') \) coordinate system is

\[
\begin{align*}
& \{ s'' \leftrightarrow Y \} \\
& \{ n'' \leftrightarrow X \} \\
& \{ l'' \leftrightarrow Z \}
\end{align*}
\]

(5.66)

For each event, the azimuthal angle of scattering, \( \phi_c \), was measured by the angle the scattering plane made with the \( s'' \)-axis (figure 5.10). Since \( \hat{\omega} \) is a normal to the scattering plane, it follows that

\[ \cos \phi_c = \frac{\hat{\omega} \cdot \hat{Y}}{ |(\hat{\omega} \cdot \hat{X}) \hat{X} + (\hat{\omega} \cdot \hat{Y}) \hat{Y}| } \quad (5.67) \]

and

\[ \sin \phi_c = -\frac{\hat{\omega} \cdot \hat{X}}{ |(\hat{\omega} \cdot \hat{X}) \hat{Y} + (\hat{\omega} \cdot \hat{Y}) \hat{Y}| } \quad (5.68) \]

\(^{15}\) With the chosen front-end solid-angle, the angle \( \Theta \) was less than \( \pm 3^\circ \) at half-maximum as shown in figure 4.1, pg 52.

\(^{16}\) By definition, this should be along the particle’s momentum; however, for small \( \Theta \), the correction to the azimuthal angle, \( \phi_c \), is small.
Figure 5.9  A diagram showing the coordinate systems for the incoming proton beam, 
\((s,n,l)\); the outgoing proton after the primary scattering, \((s',n',l')\); at the focal plane,
\((s'',n'',l'')\); and after the subsequent auxiliary scattering in the FPP, \((s''',n''',l''')\).
Here, \( \hat{X} \) and \( \hat{Y} \) are the unit vectors along the X and Y axes of the FPP reference frame.

It should be stressed, however, that the finite azimuthal acceptance of the FPP made necessary that events falling outside the 180° symmetry acceptance of the FPP be discarded. In practice, a “cone test” was used to require that the full 360° azimuthal angle of scattering was within the FPP acceptance – a sufficient but not necessary condition to guarantee \((\phi_c, \, 180° + \phi_c)\) acceptance. This was done by requiring the cone of revolution of the outgoing vector \((\vec{v}_{\text{out}})\) about the incident vector \((\vec{v}_{\text{in}})\) be contained within the active region of the FPP. In practice, the coverage of D4 to this cone was tested.

### 5.4 Event-by-event calculations

Before we go on, it is perhaps worthwhile to recapitulate some of the items we have discussed so far. We have already described the data-decoding routine and the event-reconstruction analysis. For the FPP in particular, we have also described a few useful criteria to discern bad events, such as: the checksum on drift-times; the three-point collinearity of track positions for the incident and for the outgoing trajectories; the azimuthal acceptance (“cone test”); etc. We have also obtained other important Coordinates (in LISA’s language) such as: the angles \( \theta_f \), \( \theta \), and \( \phi \); the focal-plane position \( X_F \); the front-end coordinates \( X_{FEC} \) and \( Y_{FEC} \); and the target coordinates \( X_I \) and \( Y_I \). For the FPP, we have: the FPP vertex coordinates; the distance of closest approach between the incoming and outgoing trajectory at the carbon analyzer; the polar angle of scattering \( \theta_c \); and the azimuthal angle of scattering \( \phi_c \).

In the next step of our analysis, these Coordinates were subjected to windows so that a selection could be made on those events having reasonable values for the respective Coordinates. For example, a window was put on the Z-coordinate of the FPP scattering
Figure 5.10  Diagram showing the lab coordinate systems before and after the analyzer. As in figure 5.9, $i''$ is in the direction of the incoming momentum $k_c$, while $l'''$ is in the direction of the outgoing momentum $k'_c$. Here, $\hat{n}''' = \frac{k_c \times k'_c}{|k_c \times k'_c|}$ is normal to the scattering plane, and $s''' = \hat{n}''' \times \hat{l}'''$. Also, $\cos \theta_c = \hat{l}'' \cdot \hat{l}'''$ while $\cos \phi_c = \hat{n}''' \cdot \hat{n}'''$ and $\sin \phi_c = -\hat{s}'' \cdot \hat{n}'''$. 

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vertex position to accept only those scattering events for which the scattering took place within the region of the carbon analyzer;\(^{17}\) also, windows were put on \textit{checksum}, on the collinearity \(\delta\)-quantities, on the bend angle \(\theta_b\), on the target position \((X_I, Y_I)\), on the FEC coordinates, etc. Furthermore, the DCR word (Section 5.1) was bit masked to determine the POLISIS spin-mode for each event. Finally, good events were separated into the three POLISIS spin-categories.

Once we had reconstructed the event and determined if it was useful, the next step of the event-by-event analysis was to do calculations essential to the final reduction of the data.

First of all, the spin precession angle of each good event was calculated. This was done by evaluating the bend angle with Eqn. (5.24) and substituting into Eqn. (5.25), where the kinetic energy had to be determined from the focal-plane position of the proton. In Section 5.5.4, we shall describe a calibration procedure with which the momenta were mapped into the focal-plane positions.

Due to the proton's penetration into the carbon analyzer before the scattering took place, the \textit{effective} scattering-energy of the proton was evaluated by subtracting the energy-loss for a proton travelling through half the thickness of the carbon analyzer. (This correction was \(~ 10\) MeV.) Then, as a function of the effective energy and scattering angle \(\theta_c\) at the carbon analyzer, the analyzer-power \(A_{yc}\) was obtained with a polynomial parametrized to existing experimental data\cite{Ran82} for similar carbon thicknesses.\(^ {18}\) For each event, the left-right and up-down asymmetries were weighted with the analyzing-power, respectively:

\[
\Upsilon_C = \cos \phi_c \cdot A_{yc}, \quad (5.69)
\]
\[
\Upsilon_S = \sin \phi_c \cdot A_{yc}. \quad (5.70)
\]

\(^{17}\) Multiple scattering, for instance, would cause stray vertexes.

\(^{18}\) \textit{Inclusive} analyzing-power was used (Section 3.6).
These quantities were accumulated event by event into histograms as distributions in $X_F$ (momentum), with a separate histogram for each quantity according to POLISIS "spin-up", "spin-down" or "spin-off." Also, the respective normalization factor, $(A_{yc})^2$, was accumulated in a similar fashion event-by-event.

In the final step of the event-by-event analysis procedure, histograms were incremented according to whether the event satisfied the condition associated with the respective histogram. Histograms with no assigned conditions were always incremented. On the other hand, automatic incrementation could also be suppressed for special applications such as our use in accumulating the quantities

$$\begin{align*}
\sin \theta_p \\
\cos \theta_p \\
\cos \phi_c \cdot A_{yc} \\
\sin \phi_c \cdot A_{yc} \\
(A_{yc})^2
\end{align*}$$

(5.71)

for each POLISIS spin-mode. In this case, each quantity was calculated for each event and added to the appropriate channel of the respective histogram.

Eventually, when all the data for each beam polarization were analyzed, the Conditions and histograms constituted all the information needed to extract the observables from the experiment. This will be taken up in Section 5.6. But before we move on, let us elaborate on some calibration procedures needed in the preceding event-by-event analysis.

5.5 Calibration Procedures

There were a number of calibrations to be done before data analysis could be carried out. These involved: the determination of Gains and Offsets for decoding the FPP wire-chamber information; the making of the drift-table which gave time-to-distance conversion;
the determination of the VDC-FPP geometrical constants defined in Section 5.3.1; and the mapping of the focal plane as a function of momentum. Furthermore, the central scattering angle at the primary target had to be determined for the experiment, and the focal-plane response was needed for cross section measurements.

5.5.1 The FPP software parameters

The discussion in Section 4.9 concerning the J-11 data preprocessing applies also to data analysis. In order to convert the anode delay-line time-differences, $T_{L-R}$, into absolute positions, a mapping between these two quantities was needed. The delay-time difference was related to the physical location of the anode wires along the readout delay-line (figure 4.5). Figure 5.11 shows as an example the picket-fence structure of a particular anode wire-plane. By measuring the centroid of $T_{L-R}$ for each picket, and making use of the fact that the anode wires in each FPP wire-plane were 8.128 mm apart, (meaning that the wire-numbers could be converted into distance,) the centroid $T_{L-R}^{(N)}$ values for each picket could be used to determine the coefficients of a quadratic polynomial relating the wire-number to the time-difference:

$$N = B_0 + B_1 \cdot T_{L-R}^{(N)} + B_2 \cdot \left( T_{L-R}^{(N)} \right)^2 . \quad (5.72)$$

After the coefficients were determined (for each individual plane), this equation could then be used to convert the time-differences into distances in units of the wire spacing (8.128 mm).

As it has already been mentioned in Section 4.9, the ODD-EVEN cathode wire signals (pulse heights) had to be adjusted for Gains and Offsets in order to deduce the drift direction from the difference in the induced ODD-EVEN signals. These quantities were obtained as follows. For the ideal case, a scatter-plot of the ODD against EVEN signals for a single drift
Figure 5.11  Distribution of anode delay-time difference revealing the picket-fence spectrum of anode-wire positions in a detector plane.

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cell would be a 'V' shape symmetrical about the 45° line as shown by the dashed lines in figure 5.12. The degree of deviation from this ideal case could be measured by the amount the vertex was offset from the origin, and by the amount the symmetry axis was tilted with respect to the 45° line (figure 5.12).

Since the cathode wires of each individual drift-cell within a wire-plane may not be perfectly aligned (symmetrical) about the anode-wire, it is therefore desirable to introduce Offsets and Gains between the ODD and EVEN cathode wires of each drift-cell so that all the drift-cells in a detector plane can be treated uniformly. In practice, a common Offset was given to all the ODD wires in a detector plane, and a common Offset for the EVEN wires. Thus, when all the signals from a wire-plane were plotted on the same scatter-plot of ODD against EVEN, the Offsets were determined from the position of convergence (figure 5.13).

With reference to figure 5.13, the deviation of the symmetry line for each drift cell from the 45° line was caused by the Gain between the respective pair of ODD-EVEN wires. If a histogram is made for the distribution of induced signal difference, i.e. ODD minus EVEN, for a drift-cell, the two drift directions will appear as two centroids in the distribution. The valley between the two centroids should ideally be about the position of zero-difference (figure 5.14). The algorithm for determining the Gain factor for each ODD-EVEN pair was simply to require the corrected signals for the two drift-directions be symmetrical about zero. (In other words, equal chance for both drift-directions for an uniformly illuminated drift-cell.) In practice, the corrected values were given the form

\[
\text{EVEN}(a, i) = \text{EVEN}_{\text{raw}}(a, i) - \Lambda_{\text{even}}(a),
\]

\[
\text{ODD}(a, i) = [\text{ODD}_{\text{raw}}(a, i) - \Lambda_{\text{odd}}(a)] \cdot \text{GAIN}(a, i),
\]

as already stated in Eqn. (4.7); the ODD-minus-EVEN values were then normalized by
Figure 5.12 A diagram showing the ideal (dashed lines) correlation between the ODD and the EVEN signals induced in each drift-cell of an FPP detector plane. In practice, the signals are offset from the origin and tilted with respect to the 45° line (dotted lines). Software corrections must be applied to each drift-cell so that all the drift-cells in a detector plane respond symmetrically.
Figure 5.13 A diagram showing the induced cathode signals in a FPP detector plane. The Offsets for ODD ($\Lambda_{odd}$) and EVEN ($\Lambda_{even}$) wires are determined by the vertex position. Note that the V-shape is filled in because of the different gains for different drift-cells in a detector plane.
Figure 5.14 Distribution of the induced ODD-EVEN signal difference in a FPP detector plane after correction for Gains of the individual drift-cells. The dotted curves are gaussian fits to the distributions – the centroids are located at $\Gamma = -16$ and $\Gamma = 15$, respectively. Note that the left-right ambiguity is differentiated by the separation of the peaks, not by the symmetry. The goodness of separation is judged by the degree of overlap between the adjacent tails.
ODD-plus-EVEN to yield a distribution

\[ \Gamma = \frac{\text{ODD} - \text{EVEN}}{\text{ODD} + \text{EVEN}}. \tag{5.75} \]

The gain factor, GAIN\((a, i)\), of the respective ODD cathode for the \(i\)th drift cell\(^{19}\) in wire plane \(a\) was then determined from the shift \((\epsilon)\) of the valley in the \(\Gamma\) distribution:

\[ \text{GAIN} \approx 1 - 2\epsilon. \tag{5.76} \]

A histogram for the corrected \(\Gamma\)-distribution is shown in figure 5.14. The scatter-plot showing the two bands of differentiated left and right drift-directions was shown in figure 4.6 (pg 67).

### 5.5.2 The FPP drift-table

When the FPP wire-chambers were uniformly illuminated (say, with the quasielastic continuum) the distribution of drift-distance was uniform for each drift-cell. In this case, by measuring the distribution in drift-time, the drift-distance could be inferred by integrating the area under the distribution. Figure 5.15 is a typical drift-time distribution for one of the FPP wire planes. Note that a region of non-linearity was observed in the small drift-time region corresponding to hit positions close to an anode wire. This effect was however automatically accounted for when the drift-time distribution, \(\frac{\partial N}{\partial t}\), was integrated to obtain the drift-distance in units of the cathode-anode wire spacing (4.064 mm):

\[ s(t) = \frac{1}{s_{\text{max}}} \int_{t_{\text{min}}}^{t} \frac{\partial N}{\partial t} dt \]

\[ s_{\text{max}} = \int_{t_{\text{min}}}^{t_{\text{max}}} \frac{\partial N}{\partial t} dt \tag{5.77} \]

where \(t_{\text{min}}, t_{\text{max}}\) were the minimum and maximum values of \(T_{L+R}\), respectively. A "drift-table" showing the conversion between drift-time and drift-distance is shown in figure 5.16.

\(^{19}\) Note that each cathode wire belongs to two drift-cells so that the number of gain factors to be determined in each plane is the same as the number of anode wires in that plane.
Figure 5.15 A histogram on drift-times for a continuously illuminated detector plane. Note that the non-linearity at small drift-times will be taken into account when the area under the histogram is integrated to yield a "drift-table."
Figure 5.16  Graph showing the drift-time to drift-distance conversion for a particular FPP detector plane.
5.5.3 Establishing the VDC-FPP geometry

For the reconstruction of the FPP scattering vertex, the geometrical constants, $\alpha$, $F$, $X_{1\text{off}}$, $Y_{1\text{off}}$ and $Y_{2\text{off}}$, in Section 5.3.1 relating the VDC and FPP wire-chambers had to be determined before data analysis could be carried out. This was done by using “straight-through” data taken (at the beginning of each experimental period) with the carbon analyzer removed from the FPP.\textsuperscript{20} In a tuning procedure before analyzing the FPP scattering data, the “straight-through” data were replayed to determine the best values for the VDC-FPP geometrical constants.

First of all, the “zeros” for the FPP wire-plane coordinates had to be aligned between D1, D2, D3 and D4. This was done by using D2 and D3 as the reference and finding the offsets for D1 and D4:

\begin{align*}
D1X & \leftarrow D1X + D1X_{\text{off}} , \\
D1Y & \leftarrow D1Y + D1Y_{\text{off}} , \\
D4X & \leftarrow D4X + D4X_{\text{off}} , \\
D4Y & \leftarrow D4Y + D4Y_{\text{off}} .
\end{align*}

The “straight-through” quantities\textsuperscript{21}

\begin{align*}
\delta_{123}^x & = D1X + 2 \cdot D3X - 3 \cdot D2X , \\
\delta_{123}^y & = D1Y + 2 \cdot D3Y - 3 \cdot D2Y , \\
\delta_{234}^x & = D2X + D4X - 2 \cdot D3X , \\
\delta_{234}^y & = D2Y + D4Y - 2 \cdot D3Y ,
\end{align*}

\textsuperscript{20} The scintillator S1 was not removed during “straight-through” data taking, but its effect was negligible so that the volume enclosed by the four FPP wire-chambers essentially formed a free drift space.

\textsuperscript{21} Note that the D1-D2 separation is twice that of D2-D3, D3-D4. See table 5.3 (pg 92)
were constructed so that with \( D1X_{off}, D1Y_{off}, D4X_{off} \) and \( D4Y_{off} \) properly chosen, \( \delta_{123}, \delta_{234}^x, \delta_{234}^y \) and \( \delta_{234}^z \) respectively, were centred about zero. These corrections were applied to the respective quantities before they were used in the data analysis.

After this self-consistency between the FPP coordinates had been established, the VDC-FPP geometry could be found. In this procedure, the angle \( \theta_{in} \) (Eqns. (5.27) to (5.33)) was cross checked with the value given by D2 and D4:

\[
\tan \theta_{24} = \frac{D4X - D2X}{(D4Z - D2Z)}.
\]

Here, the value for \( X_{1off} \) was twiddled until the difference

\[
\delta_\theta = \tan \theta_{in} - \tan \theta_{24}
\]

assumed a minimum width and was centred about zero.

Next, the constant \( F \) was twiddled until a minimum width was obtained for the distribution in \( \overline{OP} \) (Eqns. (5.26) and (5.32)). The search for \( X_{1off} \) and \( F \) was then iterated until reasonable convergence was obtained. For the angle \( \alpha \), the FPP was mechanically oriented at 45° to the VDCs; here, a value of 45.2° was found to give reasonably good results in our calibration.

In order to determine \( Y_{1off} \) and \( Y_{2off} \), the following tangents were constructed with a "straight-through" trajectory:

\[
\tan \phi_1 = \frac{D1Y_z - \overline{Y1}}{Z1},
\]

\[
\tan \phi_2 = \frac{D1Y_z - \overline{Y2}}{Z2}.
\]

\[
\tan \phi_{24} = \frac{D4Y_z - D2Y_z}{(D4Z - D2Z)}.
\]

The best values for \( Y_{1off} \) and \( Y_{2off} \) were then obtained by minimizing, respectively,

\[
\delta_1 = \tan \phi_{24} - \tan \phi_1
\]

and
\[ \delta_2 = \tan \phi_{24} - \tan \phi_2 . \]  

(5.88)

5.5.4 Mapping momentum to the focal plane

In order to determine the proton momentum \( P \) as a function of the focal-plane position \( X_F \), the focal-plane dispersion

\[ D(B) = \frac{\delta X_F}{\delta P} \]  

(5.89)
as a function of the MRS dipole field strength \( B \) was obtained near the energy range of interest with the known energy separations between the \(^{12}\text{C}\) ground and the first two excited states. Furthermore, the focal-plane position of the \( p-p \) elastic peak, of momentum \( P_{pp} \), was measured as a function of \( B \). In the data analysis, the \( X_F \) channel for the \( p-p \) elastic peak, \( X_{pp}^{pp}(B) \), was used as a reference such that the momentum of each proton was calculated from its \( X_F \) position relative to \( X_{pp}^{pp}(B) \) using the dispersion \( D(B) \) at that MRS dipole field; that is

\[ X_F = X_{pp}^{pp} + D \ln \left( \frac{P}{P_{pp}} \right) \]  

(5.90)
or

\[ P = P_{pp} \cdot \exp \left( \frac{X_F - X_{pp}^{pp}}{D} \right) . \]  

(5.91)

5.5.5 The determination of MRS scattering angle

The central scattering angle, \( \theta_{lab} \), at the primary target was determined by using the cross-over angle of \( p-p \) elastic scattering and \( p-^{12}\text{C} \) elastic scattering.

First of all, we had to determine the \( Y \) position of the central ray at the front-end chamber (FEC). This was done during the setup period of the experiment by momentarily
turning off the MRS quadrupole so that the central rays were isolated by the narrow Y acceptance (figure 4.2). This central Y coordinate, \( Y^0_{FEC} \), was then used to set the front-end window about the central scattering angle.

By using the CH\(_2\) target, the \( p-p \) elastic peak and the \( p^{12}\)C elastic peak could be brought into the focal plane by setting the MRS dipole field. The \( Y_{FEC} \) channel at the cross-over point, indicated in figure 5.17, corresponds to the angle of scattering, \( \theta_{ex} \), where the momenta of the respective elastic \( p-p \) and \( p^{12}\)C scattered protons have the same ratio as that of the respective MRS field settings. Hence, knowing the distance, \( l \), between the FEC and the target position (62 cm), the central scattering angle could be determined from\(^{22)\}

\[
\theta_{lab} = \theta_{ex} + \tan^{-1} \left( \frac{Y_{FEC}^e - Y^0_{FEC}}{l} \right).
\]

5.5.6 The focal-plane response

For the measurement of the quasielastic cross sections, it was necessary for us to determine the focal-plane “response” of the MRS since we were utilizing its full focal-plane acceptance. This was done by mapping the \( p-p \) elastic cross section across the focal plane and fitting the relative yield to a fourth-order polynomial as a function of the focal-plane position. The variation in the relative yield was found to be within 5%.

\(^{22)\} The sign convention of the FEC was chosen such that larger Y corresponded to smaller scattering angles.
Figure 5.17  Shown here is a superposition of two scatter-plots of scattering angle against focal-plane positions, one for $p-p$ elastic scattering and one for $p-^{12}C$ scattering. The cross-over point of the respective elastic peaks, marked by 'cx' in the figure, was used to determine the central scattering angle of the experiment.
5.6 Final data reduction

The asymmetry of the FPP scattering is defined with respect to the normal $\hat{n}''$ of the reaction plane (figure 5.10). The angular distribution $I(\theta_c, \phi_c)$ for the scattering of incoming protons of polarization $\vec{P}''$ is given in terms of the unpolarized distribution $I_0(\theta_c)$ by

$$I(\theta_c, \phi_c) = I_0(\theta_c)\{1 + A_{y_c}(\theta_c)\vec{P}'' \cdot \hat{n}''\}$$

$$= I_0(\theta_c)\{1 + A_{y_c}(\theta_c)[P_n'' \cos \phi_c - P_s'' \sin \phi_c]\}. \quad (5.93)$$

For each momentum bin $\Delta P_c$, the $\phi_c$ distribution may be summed in a $\Delta \theta_c$ bin to give

$$\int_{\Delta P_c} \int_{\Delta \theta_c} d\phi_c \sigma(\theta_c, \phi_c) \cos \phi_c = \int_{\Delta P_c} \int_{\Delta \theta_c} d\phi_c \sigma_0(\theta_c) = \int_{\Delta P_c} \int_{\Delta \theta_c} 2\pi \cdot \sigma_0(\theta_c). \quad (5.94)$$

Similarly, since the analyzing-power may be taken to be constant within a $\Delta \theta_c$ bin, Eqns. (5.93) and (5.94) yield:

$$\int_{\Delta P_c} \int_{\Delta \theta_c} d\phi_c \sigma(\theta_c, \phi_c) \cos \phi_c = P_{n''} \int_{\Delta P_c} \int_{\Delta \theta_c} \sigma_0(\theta_c) A_{y_c}(\theta_c) \cdot \pi$$

$$= \frac{1}{2} P_{n''} \int_{\Delta P_c} \int_{\Delta \theta_c} d\phi_c \sigma(\theta_c, \phi_c) A_{y_c}(\theta_c) \quad (5.95)$$

and

$$\int_{\Delta P_c} \int_{\Delta \theta_c} d\phi_c \sigma(\theta_c, \phi_c) \sin \phi_c = -P_{s''} \int_{\Delta P_c} \int_{\Delta \theta_c} \sigma_0(\theta_c) A_{y_c}(\theta_c) \cdot \pi$$

$$= -\frac{1}{2} P_{s''} \int_{\Delta P_c} \int_{\Delta \theta_c} d\phi_c \sigma(\theta_c, \phi_c) A_{y_c}(\theta_c). \quad (5.96)$$

In practice, the data were sorted into momentum bins where the contribution of each event to the scattering asymmetry was weighted by the analyzing-power $A_{y_c}(\theta_c)$. Thus, Eqns. (5.95) and (5.96) were transformed into the following expressions to give the two independent transverse polarizations:

$$P_{n''} = 2 \cdot \frac{\sum[A_{y_c}(\theta_c) \cdot \cos \phi_c]}{\sum[A_{y_c}(\theta_c)]^2}, \quad (5.97)$$

and
where the sum is over all events in each momentum bin from the special histograms described in Section 5.4. Note that in the transformation from the \((s'', n'', l'')\) reference frame to the FPP reference frame, \(\sin \phi \) is replaced by \(\cos \phi \), and vice versa, see Eqns. (5.66) to (5.68).

All three components of the polarization after the primary scattering were obtained by projecting (or precessing) the transverse components at the FPP back to the target position. Since the spin precession was about the sideways axis as the bend angle, \(\theta_b\), was in the vertical plane, the polarization in the \((s'', n'', l'')\) frame was related to that in the \((s', n', l')\) frame by

\[
P_{s''} = P_{s'} \quad ,
\]
\[
P_{n''} = +P_{n'} \cos \theta_p + P_{l'} \sin \theta_p \quad ,
\]
\[
P_{l''} = -P_{n'} \sin \theta_p + P_{l'} \cos \theta_p \quad .
\]

Depending on the POLISIS spin mode, the incident beam polarization was given by

<table>
<thead>
<tr>
<th>POLISIS UP</th>
<th>POLISIS DOWN</th>
</tr>
</thead>
<tbody>
<tr>
<td>s-pol</td>
<td>((P_s^0, 0, 0))</td>
</tr>
<tr>
<td>n-pol</td>
<td>((0, P_n^0, 0))</td>
</tr>
<tr>
<td>l-pol</td>
<td>((0, 0, P_l^0))</td>
</tr>
</tbody>
</table>

Thus, using Eqn. (2.70) (pg 22) for each POLISIS spin-mode in each beam polarization and making use of the fact that the polarization for POLISIS "spin-up" and "spin-down" were equal in magnitude (to about 1%), the Wolfenstein parameters were obtained by the following expressions.

(i) For n-polarized beam,

\[
D_{n'0} = \frac{1}{\langle \cos \theta_p \rangle} \left[ \frac{P_{n''}^1 + P_{l''}^1}{2} + P_n^0 D_{0'n} \left( \frac{P_{n''}^1 - P_{l''}^1}{2} \right) \right] ,
\]
\[ D_{n'n} = \frac{1}{P_0^0 (\cos \theta_p)} \left[ \frac{P_{n''}^1 - P_{n''}^1}{2} + P_0^0 D_{0'n} \left( \frac{P_{n''}^1 + P_{n''}^1}{2} \right) \right] \] ; \quad (5.104)

(ii) for s-polarized beam,

\[ D_{s's} = \frac{1}{P_0^s (\sin \theta_p)} \left( \frac{P_{s''}^1 - P_{s''}^1}{2} \right) \] , \quad (5.105)

\[ D_{l's} = \frac{1}{P_0^l (\sin \theta_p)} \left( \frac{P_{l''}^1 - P_{l''}^1}{2} \right) \] , \quad (5.106)

\[ D_{n'0} = \frac{1}{(\cos \theta_p)} \left( \frac{P_{n''}^1 + P_{n''}^1}{2} \right) \] ; \quad (5.107)

and, (iii) for l-polarized beam,

\[ D_{l'l} = \frac{1}{P_0^l (\sin \theta_p)} \left( \frac{P_{l''}^1 - P_{l''}^1}{2} \right) \] , \quad (5.108)

\[ D_{s'l} = \frac{1}{P_0^s (\sin \theta_p)} \left( \frac{P_{s''}^1 - P_{s''}^1}{2} \right) \] \quad (5.109),

\[ D_{n'0} = \frac{1}{(\cos \theta_p)} \left( \frac{P_{n''}^1 + P_{n''}^1}{2} \right) \] . \quad (5.110)

Here, \( P_{n''}^1 \) and \( P_{n''}^1 \) denote the FPP \( P_{n''} \) polarization during POLISIS “spin-up” and “spin-down,” respectively. Similarly, \( P_{s''}^1 \) and \( P_{s''}^1 \) denote the FPP \( P_{s''} \) polarization during POLISIS “spin-up” and “spin-down,” respectively. Eqns. (5.103) through (5.110) were evaluated using Eqns. (5.71), (5.97) and (5.98).

For sideways beam polarization, the two sets of data obtained with “solenoid forward” and “solenoid reverse,” respectively, (Section 3.2) were combined to cancel the contribution from a spurious longitudinal component (< 2%). Assuming a small longitudinal component, the beam polarization was given by:

\[
\begin{align*}
\text{POLISIS UP} & \quad \text{POLISIS DOWN} \\
\text{solenoid “forward”} & \quad (P_0^0, 0, P_0^0) & \quad (-P_0^0, 0, -P_0^0) \\
\text{solenoid “reverse”} & \quad (-P_0^0, 0, P_0^0) & \quad (P_0^0, 0, -P_0^0)
\end{align*}
\] \quad (5.111)
With Eqns. (2.70) and (5.111), it can be shown that

\[ D_{\alpha a} = \frac{1}{2} [D_{\alpha a}(\text{forward}) - D_{\alpha a}(\text{reverse})] \]  
\[ D_{\nu a} = \frac{1}{2} [D_{\nu a}(\text{forward}) - D_{\nu a}(\text{reverse})] \]  
\[ D_{n\nu a} = \frac{1}{2} [D_{n\nu a}(\text{forward}) + D_{n\nu a}(\text{reverse})] \]  

For the quasielastic scattering cross sections \( I(\omega) \), the experimental values were given by

\[ I(\omega) = \frac{C \cdot A}{C_0 \cdot \mathcal{E} \cdot \mathcal{L} \cdot N_A \cdot \rho t \cdot \Omega} \]  

where

- \( C \) = number of scattered protons in each momentum bin,
- \( A \) = Atomic mass of target (g mole\(^{-1}\)),
- \( C_0 \) = number of incident protons,
- \( \mathcal{E} \) = overall wire-chamber efficiency,
- \( \mathcal{L} \) = live-time of computer acquisition,
- \( N_A \) = Avogadro number,
- \( \rho t \) = target thickness presented to beam (g cm\(^{-2}\)),
- \( \Omega \) = solid angle of scattering.

Using Eqn. (2.60) for the polarized cross section, namely

\[ I^{\text{pol}} = I_0(1 + A_y P_n^{01}) \]  

the unpolarized cross section, \( I_0 \), was obtained by

\[ I_0(\omega) = \frac{1}{2} \left[ \frac{I^{\text{pol}T}(\omega)}{1 + A_y(\omega)P_n^{01T}} + \frac{I^{\text{pol}I}(\omega)}{1 + A_y(\omega)P_n^{01I}} \right] \]  

or,
\[ I_0(\omega) = \frac{I^1(\omega) + I^\dagger(\omega)}{2} \quad (P^0_n = -P^\dagger_n), \quad (5.117b) \]

where \( I^1(\omega) \) denotes the polarized cross section for spin-up protons, etc. Also, the asymmetry of scattering \( A_y \) was obtained by

\[ A_y(\omega) = \frac{I^1(\omega) - I^\dagger(\omega)}{-I^1(\omega)P^0_n + I^\dagger(\omega)P^\dagger_n}, \quad (5.118a) \]

or,

\[ A_y(\omega) = \frac{I^1(\omega) - I^\dagger(\omega)}{P^0_n [I^1(\omega) + I^\dagger(\omega)]} \quad (-P^0_n = P^\dagger_n \equiv P_n). \quad (5.118b) \]

Finally, the double-differential cross section, \( \frac{d^3\sigma}{d\Omega dT p'} \), was obtained by

\[ \frac{d^3\sigma}{d\Omega dT p'} = \frac{I(\omega)}{\Delta \omega} \quad (5.119) \]

where \( \Delta \omega \) denotes the width of the respective energy bin where the number of counts, \( C \), was taken.

### 5.7 The Control Experiment

In order to demonstrate the degree of accuracy of our analysis procedure, and to reveal if there was any significant amount of systematic error inherent in our experimental setup, we measured in a control experiment the same spin observables for \( p-p \) elastic scattering. For these measurements, we used a polyethylene (CH\(_2\)) target of 233 mg cm\(^{-2}\) thick. The results of this control experiment are compared in table 5.4 to the values obtained from phase-shift analysis.[SAID] The excellent agreement here demonstrates that our technique of measuring these spin observables was capable of producing precise results, and that any systematic errors are small.
Table 5.4
Results from experiment and phase-shift analysis (SP88) for \( p-p \) elastic spin observables at \( q=1.9 \text{ fm}^{-1} \) and incident energies of 290 MeV (column 2 and 3), and 420 MeV (column 4 and 5).

<table>
<thead>
<tr>
<th></th>
<th>290 MeV</th>
<th></th>
<th>420 MeV</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>measured</td>
<td>p-s</td>
<td>measured</td>
<td>p-s</td>
</tr>
<tr>
<td>( P )</td>
<td>0.24 ± 0.02</td>
<td>0.25</td>
<td>0.39 ± 0.03</td>
<td>0.40</td>
</tr>
<tr>
<td>( D_{n'n} )</td>
<td>0.46 ± 0.03</td>
<td>0.46</td>
<td>0.67 ± 0.03</td>
<td>0.62</td>
</tr>
<tr>
<td>( D_{s's} )</td>
<td>0.28 ± 0.03</td>
<td>0.31</td>
<td>0.42 ± 0.02</td>
<td>0.42</td>
</tr>
<tr>
<td>( D_{ll} )</td>
<td>0.10 ± 0.03</td>
<td>0.12</td>
<td>0.31 ± 0.05</td>
<td>0.31</td>
</tr>
<tr>
<td>( D_{s'l} )</td>
<td>0.12 ± 0.02</td>
<td>0.14</td>
<td>0.25 ± 0.03</td>
<td>0.20</td>
</tr>
<tr>
<td>( D_{l's} )</td>
<td>−0.25 ± 0.03</td>
<td>−0.23</td>
<td>−0.29 ± 0.03</td>
<td>−0.27</td>
</tr>
</tbody>
</table>

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Chapter 6

RESULTS AND DISCUSSION

Results for the double differential cross section, \( \frac{d^2 \sigma}{d\Omega dE} \); the Asymmetry of scattering, \( A_y \); the Polarization, \( P \); and the Wolfenstein parameters, \( D \equiv D_{n'n}, R \equiv D_{\nu'}, A \equiv D_{\nu'1}, R' \equiv D_{\nu'2}, \) and \( A' \equiv D_{\nu'3} \) for inclusive quasielastic scattering of protons from \(^{12}\text{C}\) at a central momentum transfer of \( q = 1.9\ \text{fm}^{-1} \) and incident energies of 290 and 420 MeV, are shown in figures 6.1 to 6.4. Results for \( P, D_{n'n}, D_{\nu'}, \) and \( D_{\nu_1} \) for \(^{16}\text{O}\) at 420 MeV are shown in figure 6.5. We have also indicated in these figures the corresponding isospin-averaged free N-N values evaluated at the energy-loss \( \omega = \frac{s^2}{2m} \).

The experimental results for \(^{16}\text{O}\) can be compared with those of \(^{12}\text{C}\) measured at the same momentum transfer and incident energy. This is shown in figure 6.6 where the solid triangles represent the \(^{16}\text{O}\) data, which can be compared with the \(^{12}\text{C}\) data represented by the solid circles. It is seen immediately that the results for \(^{12}\text{C}\) and \(^{16}\text{O}\) are almost identical.

In relation to the relativistic impulse-approximation model for quasielastic scattering of Horowitz et al.,\[Hor86b\],[Hor88\] it is interesting to note that the polarization parameter \( P \) is quenched from the free N-N value as predicted by the RIA model; the isospin-averaged free value for \( P \) is 0.32 while the RIA prediction is 0.25. It should also be noted that this quenching is observed to be practically the same for both \(^{12}\text{C}\) and \(^{16}\text{O}\), which is also predicted by the model since “strong absorption” pushes the reaction to the lower-density nuclear surface.

As is to be expected for inclusive quasielastic scattering, the spin observables exhibit almost no structure other than a smooth variation as a function of the energy-loss across the broad continuum. It is also noticed that despite the fact that each of these spin observables

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Figure 6.1  Inclusive p-$^{12}$C quasielastic scattering observables at $q=1.9$ fm$^{-1}$ and incident lab. energies of 290 MeV (on the left) and 420 MeV (on the right). The unpolarized double-differential cross section (top) and the Asymmetry of scattering (bottom) are shown as a function of the outgoing proton's energy. The dotted lines indicate the corresponding free N-N values evaluated at $\omega = q^2/2m$. 

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Figure 6.2  Inclusive $^{12}$C($p,p'$) quasielastic scattering spin observables at $q=1.9$ fm$^{-1}$ and incident lab. energies of 290 MeV (on the left) and 420 MeV (on the right). The Polarization parameter $P$ (top) and the Wolfenstein parameter $D \equiv D_{n'n}$ (bottom) are shown as a function of the outgoing proton's energy. The dotted lines indicate the corresponding free N-N values evaluated at $\omega = q^2/2m$. 

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Figure 6.3 As in figure 6.2 but for $R \equiv D_{s's}$ (top) and $A' \equiv D_{\pi 1}$ (bottom).
Figure 6.4 As in figure 6.2 but for $A = D_{s'}$ (top) and $-R' = -D_{s*}$ (bottom).
Figure 6.5 Inclusive $^{16}\text{O}(p,p')$ quasielastic scattering spin observables at $q=1.9$ fm$^{-1}$ and incident lab. energy of 420 MeV. Clockwise from top-left is the Polarization parameter $P$, the Wolfenstein parameters $D \equiv D_{n'nc}$, $R \equiv D_{s'c}$, and $-R' \equiv -D_{y'c}$ plotted as a function of the outgoing proton’s energy. The dotted lines indicate the corresponding free N-N values evaluated at $\omega = q^2/2m$. 

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Figure 6.6 Inclusive quasielastic proton spin observables at a central momentum transfer of 1.97 fm$^{-1}$ and incident lab energy of 420 MeV are shown as a function of the energy of the outgoing proton. The solid circles are the results for $^{12}$C$(\vec{p},\vec{p}')$, which can be compared to the $^{16}$O$(\vec{p},\vec{p}')$ results shown by the solid triangles.

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varies individually by about 20% across the continuum, the symmetry relations under time-reversal-invariance, namely

$$ P = A_y , $$

(6.1)

and

$$ D_{s'i} + D_{t's} = (D_{t'i} - D_{s's}) \tan \theta_{lab} , $$

(6.2)

are seen to be respected across the full width of the quasielastic continuum as shown in figure 6.7. This is perhaps a good indication that the N-N process still dominates the reaction even away from the top of the quasielastic peak, in accord with the suggestion that the broadening of the quasielastic peak is largely a result of the Fermi motion of the target nucleons.

As our emphasis here is on comparing the quasielastic spin observables with the N-N observables, it will be extremely interesting for us to see how far the Fermi motion of the target nucleons alone can account for the observed slopes in these spin observables across the quasielastic continuum. To this end, we have adopted a naive free Fermi-gas model where the target nucleons are assumed to be uniformly distributed in momentum up to a cut-off maximum.

The effective lab energy of scattering, \( T_{\text{eff}} \), was evaluated for each target nucleon momentum, \( \vec{P}_2 \), as

$$ T_{\text{eff}} = \frac{E_1 E_2 - \vec{P}_1 \cdot \vec{P}_2 - m^2}{m} , $$

(6.3)

where, in the lab reference frame,

$$ \vec{P}_1 \cdot \vec{P}_2 = P_1 P_2 \sin \theta_1 \sin \theta_2 \cos \phi_2 + \cos \theta_1 \cos \theta_2 , $$

(6.4)

$$ \theta_1 = \sin^{-1} \left( \frac{k_1 \sin \theta_{lab}}{q} \right) , $$

(6.5)

$$ \theta_2 = \cos^{-1} \left( \frac{\omega^2 - q^2 + 2\omega E_2}{2P_2 q} \right) , $$

(6.6)
Figure 6.7  The quantities $P - A_y$ (top) and $D_{sI} + D_{sI} - (D_{sI} - D_{sI})\tan\theta_{lab}$ (bottom) are shown here for inclusive $^{12}\text{C}(p,p')$ quasielastic scattering at $q=1.9$ fm$^{-1}$ and incident lab. energies of 290 MeV (on the left) and 420 MeV (on the right), plotted against the outgoing proton's energy. For elastic scattering, time-reversal-invariance requires both $P = A_y$ and $D_{sI} + D_{sI} = (D_{sI} - D_{sI})\tan\theta_{lab}$.
\[ E_1 = \sqrt{P_1^2 + m^2} , \quad (6.7) \]
\[ E_2 = \sqrt{P_2^2 + m^2} , \quad (6.8) \]
\[ \omega = E_1 - E_1' = \sqrt{P_1^2 + m^2} - \sqrt{k_1^2 + m^2} ; \quad (6.9) \]

\( \theta_1 \) and \( \theta_2 \) are the angles of \( \vec{P}_1 \) and \( \vec{P}_2 \) with respect to the momentum transfer \( \vec{q} = \vec{P}_1 - \vec{k}_1 \) (figure 6.8); \( \vec{k}_1 \) is the momentum of the scattered proton, and \( \theta_{\text{lab}} \) is the lab scattering angle. Here, we have followed the convention of choosing the z-axis along the incident proton momentum, \( \vec{p}_1 \).

The effective c.m. scattering angle, \( \theta_{\text{cm}}^{\text{eff}} \), was calculated by [Ha73]

\[ \theta_{\text{cm}}^{\text{eff}} = 2 \sin^{-1} \left( \frac{-\omega^2 + q^2}{2mT_{\text{eff}}} \right)^{\frac{1}{2}} . \quad (6.10) \]

Furthermore, energy and momentum conservation requires that only target nucleons with momentum above

\[ P_{\text{min}} = \left| \frac{q}{2} - \frac{\omega}{2} \sqrt{1 - \frac{4m^2}{\omega^2 - q^2}} \right| \quad (6.11) \]
can contribute.

Given a value for the momentum of the scattered proton (i.e. equivalently, given the energy-loss in the quasielastic spectrum) for each value of \( P_2 \) and \( \phi_2 \) (hence the effective scattering angle and energy), the spin observables are given by the formalism described in Section 2.6 where Eqn. (2.57) may be now written as

\[ D_{j'i} = \frac{\int_{P_{\text{min}}}^{P_{\text{max}}} P_2^2 dP_2 \int d\phi_2 \left\{ \sigma_j^{\rho} M(T_{\text{eff}}, \theta_{\text{cm}}^{\text{eff}}) \sigma_{j'}^{\rho} M(T_{\text{eff}}, \theta_{\text{cm}}^{\text{eff}}) \right\}}{\int_{P_{\text{min}}}^{P_{\text{max}}} P_2^2 dP_2 \int d\phi_2 \left\{ M(T_{\text{eff}}, \theta_{\text{cm}}^{\text{eff}}) M(T_{\text{eff}}, \theta_{\text{cm}}^{\text{eff}}) \right\}} . \quad (6.12) \]

In practice, the following form of the last equation was used:

\[ D_{j'i} = \frac{\int_{P_{\text{min}}}^{P_{\text{max}}} P_2^2 dP_2 \int d\phi_2 D_{j'i}(T_{\text{eff}}, \theta_{\text{cm}}^{\text{eff}}) I_0(T_{\text{eff}}, \theta_{\text{cm}}^{\text{eff}})}{\int_{P_{\text{min}}}^{P_{\text{max}}} P_2^2 dP_2 \int d\phi_2 I_0(T_{\text{eff}}, \theta_{\text{cm}}^{\text{eff}})} . \quad (6.13) \]

For the upper limit of \( P_2 \), we found that a value of 0.86 fm\(^{-1}\) roughly reproduced the shape of the double-differential cross section; other choices of the upper limit change the

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Figure 6.8 Diagram showing the relation between $\theta_1$, $\theta_{\text{lab}}$, $\vec{P}_1$, $\vec{k}_1$ and $\vec{q}$. 

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overall normalization of the spin observables but have no visible effects on the slopes. For instance, the results obtained by taking $P_{\min}$ alone in the calculations and those by integrating the Fermi momentum up to $1.36$ fm$^{-1}$ (nuclear matter saturation value) differed by not more than 10% in the respective calculated values, and there were no visible differences in the slopes of the corresponding results.

The calculated values from our simple free Fermi-gas model are compared with the measured quasielastic observables in figure 6.9; effects of using different $P_{\max}$'s are shown in figure 6.10. It is noticed that the general trend is reproduced except for the case of $D_{\ell}s$ and $D_{\ell'1}$ at 420 MeV. Due to the crudeness of our free Fermi-gas model, we cannot expect the results to be quantitative. Nevertheless, judging from the overall qualitative agreement as seen in the figure, it is apparent that these spin observables are indeed grossly described by single particle scattering. And, indeed, Fermi motion does seem to account for the broadening of the quasielastic peak.

Recall that the observables $D_0$, $D_K$, $D_n$ and $D_P$ are defined by Eqn. (2.79), namely

$$D_0 = \frac{1}{4} \left[ 1 + D_{n'n'} + (D_{s's'} + D_{\ell'\ell}) \cos \left( \alpha + \frac{\theta}{2} \right) - (D_{s's'} - D_{\ell'\ell}) \sin \left( \alpha + \frac{\theta}{2} \right) \right], \quad \text{(6.14)}$$

$$D_K = \frac{1}{4} \left[ 1 - D_{n'n'} + (D_{s's'} - D_{\ell'\ell}) \sec \left( \alpha - \frac{\theta}{2} \right) \right], \quad \text{(6.15)}$$

$$D_n = \frac{1}{4} \left[ 1 + D_{n'n'} - (D_{s's'} + D_{\ell'\ell}) \cos \left( \alpha + \frac{\theta}{2} \right) + (D_{s's'} - D_{\ell'\ell}) \sin \left( \alpha + \frac{\theta}{2} \right) \right], \quad \text{(6.16)}$$

$$D_P = \frac{1}{4} \left[ 1 - D_{n'n'} - (D_{s's'} - D_{\ell'\ell}) \sec \left( \alpha - \frac{\theta}{2} \right) \right]. \quad \text{(6.17)}$$

These observables are plotted for quasielastic scattering in figure 6.11 as a function of the energy of the outgoing proton. The same observables evaluated from the free Fermi-gas model described above are also shown by the solid curves. Since each $D_\alpha$ depends separately on a scattering amplitude $M_\alpha$ as given by Eqn. (2.77), and judging from the qualitative fit observed in figure 6.11, the quasielastic scattering amplitudes seem to be reasonably described by the free N-N amplitudes, with the exception of the amplitude $M_n$, for which
Figure 6.9  Calculated values (solid lines) from a free Fermi-gas model as described in the text are compared with experimental $^{12}$C($\vec{p},\vec{p}'$) data of 290 MeV (on the left) and 420 MeV (on the right).

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Figure 6.10  Shown here for comparison are the calculated free Fermi-gas values corresponding to the cases where the radial integration on $P_2$ is (i) up to 1.36 fm$^{-1}$ (dashed line); (ii) up to 0.86 fm$^{-1}$ as shown in figure 6.9 (solid line); and (iii) taken only at $P_2 = P_{\text{min}}$ (dotted line).
Figure 6.11  Inclusive $^{12}\text{C}(\vec{p},\vec{p}')$ quasielastic scattering spin observables at $q = 1.9$ fm$^{-1}$ and lab incident energy of 290 MeV (on the left) and 420 MeV (on the right). From top to bottom are the observables $D_0$, $D_K$, $D_n$ and $D_P$. Qualitative fits (solid curves) are obtained with a free Fermi-gas model as described in the text.

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we notice the slope of $D_n$ is opposite to that of the data.

The collective behaviour of the spin-isospin longitudinal and transverse interactions for symmetric, infinite nuclear matter was investigated recently under the RPA framework.[Alb88], [Alb84], [Alb82], [Alb80] The longitudinal channel of the spin-isospin interaction changes sign and becomes slightly attractive at about $q = 1$ fm$^{-1}$. It was pointed out that this would lead to a significant difference between the spin-isospin longitudinal and transverse response functions in nuclear matter. However, comparison of calculation to experimental $(p,p')$ data [Ca84], [Ca85], [Re86] did not reveal such collectivity effects. This may be due partially to the collective behaviour being less pronounced at the low density nuclear surface where the quasielastic reaction is largely localized, and to the isoscalar contribution in $(p,p')$ reactions which “dilutes” the enhancement in the ratio of the longitudinal to transverse response functions. Indeed, for a detailed comparison with the experimental quasielastic data, the coupling of the longitudinal and transverse parts of the interaction in the surface region must be taken into account.[Alb88], [Alb84]

The longitudinal and transverse response functions can be obtained from inclusive quasielastic proton spin observables through the longitudinal and transverse spin-flip probabilities, $S_L \equiv D_K$ and $S_T \equiv D_P$, given by Eqns. (6.15) and (6.17). For quasielastic scattering, these spin-flip functions have been approximated as [Ca84], [Be82]

$$I_0 S_L = I_0^{N-N} S_L^{N-N} R_L(q,\omega)N_{\text{eff}}, \quad (6.18)$$

$$I_0 S_T = I_0^{N-N} S_T^{N-N} R_T(q,\omega)N_{\text{eff}}, \quad (6.19)$$

$$I_0 = I_0^{N-N} R(q,\omega)N_{\text{eff}}, \quad (6.20)$$

where the superscribed quantities denote the respective free nucleon-nucleon values; $N_{\text{eff}}$ is the effective number of participating target nucleons and $R_L(q,\omega)$, $R_T(q,\omega)$ and $R(q,\omega)$ are the longitudinal, transverse and total nuclear response functions, respectively.[Alb88]
From our data, we have extracted the ratio

\[
\frac{R_L}{R_T} = \left( \frac{S_L}{S_T} \right) / \left( \frac{S_L^{N-N}}{S_T^{N-N}} \right)
\]  

(6.21)

of the response functions using, as an approximation, our Fermi-gas model calculations for \( S_L^{N-N} / S_T^{N-N} \). Apparently there is no enhancement of the longitudinal response over the transverse response for these inclusive quasielastic data (figure 6.12); in fact, the data indicate a relative quenching rather than enhancement. This effect is also observed in similar results obtained for \(^{208}\text{Pb}(\vec{p},\vec{p}')\) at 500 MeV and at lower \( q = 1.75 \text{ fm}^{-1}.\) [Re86] We can therefore conclude that measurements on \((\vec{p},\vec{p}')\) observables so far do not support the enhancement due to collective nuclear spin-isospin effects predicted by Alberico et al. [Alb88]
Figure 6.12  The longitudinal-to-transverse ratios of nuclear response functions, $R_L/R_T$, are shown for inclusive $^{12}$C($p,p'$) quasielastic scattering at $q = 1.9$ fm$^{-1}$ and lab incident energies of 290 MeV (on the left) and 420 MeV (on the right), plotted as a function of the outgoing proton's kinetic energy. The dotted line in this figure marks the level of unit ratio.
Chapter 7

SUMMARY

The measurement of spin observables for the continuum is a relatively new area of experimental nuclear physics. Here, we have demonstrated that very precise measurements of these observables can be obtained with the use of a focal-plane polarimeter and a large-momentum-bite spectrometer with full ray-tracing capability. In a total of 15 days over three running periods separated by 12 and 3 months, we obtained 3 sets of spin observables for inclusive quasielastic proton scattering at two incident energies. These include two complete sets of data for $^{12}\text{C}$ at 290 and 420 MeV incident kinetic energies, and a partial set of spin observables for $^{16}\text{O}$ at 420 MeV.

In operating the TRIUMF MRS spectrometer, we were careful to restrict the bend angle of particles to reduce the spread in spin-precession angle through the spectrometer. In measuring spin-transfer observables, it is also important to limit the range in vertical scattering angle to better define the scattering in the horizontal plane. To this end, we made use of the MRS electronics to constrain the front-end acceptance by invoking a hardware window on the front-end wire-chamber detector.

Since it was only shortly after the focal-plane polarimeter was put into commission that this experiment was conducted, we did a control experiment using the $p-p$ elastic scattering observables for comparison with the known values; excellent agreement was obtained.

For interpretation of the experimental results, we have concerned ourselves mainly from the point of view that the quasielastic scattering process is dominated by the knock-on interaction. At the present, a comprehensive treatment of the continuum spin observables is not available. Currently, the nuclear response of finite nuclei is being investigated, and

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a Dirac RPA calculation for these proton quasielastic spin observables will be available in the near future [Sh88].

We notice in our data that the quasielastic spin observables vary smoothly across the broad continuum. This feature of quasielastic scattering should not be overlooked since it indicates that these spin observables are relatively insensitive to nuclear structure and hence they can better isolate the reaction dynamics from nuclear structure effects. Indeed, that quasielastic scattering is relatively insensitive to nuclear structure can be seen from the striking resemblance of the $^{12}$C and $^{16}$O spin observables. Furthermore, we observe that the symmetry relations under time-reversal invariance, strictly valid only for elastic scattering, are respected by the quasielastic spin observables across the full width of the $\sim 100$ MeV continuum. Once again, this is in accord with the quasielastic N-A interaction dominated by the two-body N-N interaction. Comparison of the quasielastic spin observables to the N-N values evaluated at $\omega = \frac{3}{2}m$ reveals that these quasielastic spin observables are close to the free N-N values, with the noticeable exception of the polarization parameter, $P$, which is significantly quenched from the free value. This quenching is correctly predicted by the relativistic impulse approximation to quasielastic scattering where the strong relativistic scalar potential reduces the "effective mass" of the target nucleon. It fact, it is the interest in relativistic effects in nucleon-nucleus scattering that is partially responsible for the current interest in continuum spin observables.

Following the line of argument for the two-body nature of quasielastic scattering, and in the spirit of the impulse-approximation to nucleon-nucleus scattering, we have employed a simple free Fermi-gas model in an attempt to investigate the origin of the observed slopes in these quasielastic spin observables. Our result shows that the Fermi motion of the target nucleons alone accounts for the observed slopes to a large extent. This is in agreement with the speculation that the broadening of the quasielastic peak is a kinematical effect due to
the momentum distribution of target nucleons – a model advocated in earlier studies of quasi-two-body scaling (y-scaling) of nucleon-nucleus scattering. Indeed, the resemblance of our $^{12}$C and $^{16}$O data is indirect evidence of two-body kinematics and that the direct reaction is largely localized to the nuclear surface due to strong absorption.

Using our Fermi-gas model calculation for the quasielastic spin observables, we have also extracted the ratio of longitudinal-to-transverse spin-isospin nuclear response. Despite the fact that a small enhancement of the longitudinal response is expected by current theoretical calculations, we do not observe any such enhancement. In fact, the ratio extracted from our measurements at both incident energies, 290 and 420 MeV, taken at a momentum transfer of 1.9 fm$^{-1}$ where an enhancement is to be expected, indicates a relative quenching rather than enhancement. Therefore, we conclude that our results do not support any enhancement of the longitudinal spin-isospin nuclear response in these finite nuclei.
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Appendix A

Experimental values for inclusive quasielastic $^{12}\text{C}(\vec{p},\vec{p}')$ spin observables. Errors given are statistical only.

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Appendix B

Experimental values for inclusive quasielastic $^{16}$O$(p,p')$ spin observables. $T_{lab} = 420$ MeV, $\theta_{lab} = 23.8^\circ \pm 0.2^\circ$. Errors given are statistical only.

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Appendix C

Experimental values for inclusive quasielastic $p - ^{12}\text{C}$ double-differential cross section.

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<th>$T'_p$ (MeV)</th>
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\theta_{\text{lab}} &=& 29.3^\circ \pm 0.3^\circ & \theta_{\text{lab}} = 23.8^\circ \pm 0.2^\circ \\
\end{array}
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\[
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190.19 & 0.69 & 279.71 & 0.37 \\
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180.01 & 0.55 & & \\
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END
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FIN