A SHORT BASELINE
TRANSIENT ELECTROMAGNETIC METHOD
FOR USE ON THE SEA FLOOR

by

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Abstract

A towed electromagnetic system capable of mapping the electrical conductivity of the sea floor over a large area has many possible applications, including mapping Quaternary geology, and understanding the physical properties of midocean ridge hydrothermal systems in association with massive sulphide deposits. However, the electrical conductivity of seawater is usually much greater than that of the sea floor, rendering the majority of electromagnetic systems presently employed on land unsuitable for marine use.

A theoretical study of the transient step-on responses of some common controlled source, electromagnetic systems to adjoining conductive half-spaces shows that for two systems, the horizontal, in-line, electric dipole-dipole (ERER) and horizontal, coaxial, magnetic dipole-dipole (HRHR), the position in time of the initial transient indicates the conductivity of the sea floor, while at distinctly later time, a second characteristic of the transient is a measure of the larger seawater conductivity. The diagnostic separation between the two parts of the transient response does not occur for many other systems, including several commonly used for exploration on land.

The concept of apparent conductivity is defined for the transient system in terms of the time of the first arrival of the transient signal. This apparent conductivity is used to produce characteristic curves of the HRHR system response for different layered earth models.

A prototype HRHR system operating on a 100 m scale has been designed and constructed. A successful test of the system in shallow water was conducted in the coastal waters of Vancouver Island. The survey yielded 37 conductivity
measurements along three lines. The instruments were towed by a ship along the sea floor. The tow cable carried both current to the transmitter coil and the received signal back to the ship for processing. Both the shape and amplitude of the received signal are indicative of the conductivity of the bottom sediments. Inversion of the data suggest that a varying thickness of 1.2 S·m⁻¹ mud overlies rock or sediment with a conductivity of about 0.1 S·m⁻¹.

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Chapter 1
Introduction

Section 1.0 Prologue

It is the purpose of applied geophysics to reveal the geological structure hidden beneath the surface of the earth. This structure contains clues to the origin and history of the earth. It may point to the location of economic oil or mineral deposits, and it may determine the level and flow of groundwater. A knowledge of local faulting and lithological boundaries is important for the siting of geotechnical projects such as dams or waste disposal sites. To explore this concealed dimension geophysicists have developed a wide range tools which are sensitive to variations in the physical properties of geological structures. Seismic methods detect the changes and discontinuities in elastic properties by virtue of the reflection of sound waves. Gravity methods detect changes in the local gravitational field caused by variations in the density of different rock units. Electrical methods measure the disturbances in artificially applied electric or magnetic fields introduced by variations in electrical conductivity. Each technique provides information and insights not obtainable with the others. The information collected is interpreted by the geophysicist in terms of various models of the geological structure.

In the past, developmental work on electromagnetic (EM) geophysical methods has been focussed mainly on systems which can be used on land. A great number of systems have been developed and commercially deployed, operating on the ground and in the air, and on scales from a meter to hundreds of kilometers. In stark contrast, there is not a single, commercially available EM technique, operating on any scale, presently available for use on the sea floor. This thesis partially fills this void by presenting a new sea-floor EM system, operating on a scale of tens to hundreds
of meters, which measures the response of the sea floor to applied transients in the magnetic field. The new system is of modest size, relatively inexpensive, and easily deployed. The transmitter and receiver are towed along the sea floor behind a ship. Both the transmitted current and received signal are carried by the tow cable. Data can be recovered and interpreted quickly, which makes the system ideal for continuous mapping of the conductivity structure of the sea floor, in the same way that airborne EM techniques construct continuous maps of the conductivity structure of the earth in real time while in flight. This thesis is a complete description of the development process of the new transient system. I begin with the theory of the transient method when applied in a sea floor environment. The transient response is fundamentally different than that measured on land. Of several possible systems considered, two showed promise, and were selected for more detailed study. The design and construction of a working prototype followed. This prototype was tested near Vancouver Island and the recovered data were interpreted in terms of a simple layered-earth model. The experiment and its subsequent interpretation point to possible improvements in design and methodology, which may lead to a commercially useful system.

Section 1.1 EM Mapping at Sea

Even though petroleum is produced from huge deposits on the continental shelf, the sea floor represents a largely unexplored and unexploited resource base. Until recently, little interest was shown in the ocean floor environment, which, with the possible exception of manganese nodules, was assumed to be devoid of economic mineral deposits. However, the recent discovery of polymetallic sulfide mineralization on the crest of the East Pacific Rise (Hekinian et al, 1980), the Galapagos Ridge (Malahoff, 1982) and the Juan de Fuca plate (Normark et al, 1983) has spurred interest in the possibility of deep sea mining. The above deposits were located visually with submersibles and have been mapped acoustically with instrument packages such as SEABEAM (Ballard and Francheteau, 1982), which are
capable of determining bathymetry with high precision. While these methods have been able to examine surficial geology, they are incapable of adequately assessing the depth extent of the deposits and the structure of the regional geology in which they are found. Sea floor conductivity mapping using the electromagnetic method is one of the few geophysical tools suitable for this purpose.

Electromagnetic mapping on land is routinely carried out from the air. A simple apparent resistivity map is produced in real time as the line is flown. Such maps have been used to detect boundaries between different types of rock and directly identifying local, three-dimensional targets, such as base metal mineral deposits, which are much more conductive than the surrounding rock. The aim of the present work was to produce an EM mapping tool, operating on a scale on the order of a hundred meters, for use on the sea floor. Such a marine system is certainly not limited to locating mineral deposits. It also has applications in the detection of sub-sea permafrost, a resistive zone relative to its environment, and in acting as a supplementary technique to seismic surveys in offshore oil exploration. A profile of electrical conductivity as a function of depth can provide information on physical properties such as porosity, bulk density, water content, and compressional wave velocity. (Nobes et al., 1986).

1.2 The Electrical Conductivity of Marine Sediments

The physical properties which determine the electrical conductivity of shallow marine sediments are discussed by Edwards et al., (1988). The electrical conductivity of an unconsolidated, unfrozen sediment composed of non-conductive minerals depends primarily on the conductivity of the pore fluid and the porosity of the sediment. The rock conducts electricity because of the movement of ions (electrolytic conduction) in the fluid filled pores. The empirical relation between formation conductivity $\sigma_f$ and porosity $\phi$ (fractional pore volume) is given by Archie's formula.
Expressed in terms of conductivities,

\[ \sigma_f = \frac{\phi^m S^n \sigma_w}{A}, \]

where \( A \) is a constant in the range \( 0.5 < A < 2.5 \), \( S \) is the fraction of pores filled with water, \( n \) is the saturation exponent, \( \phi \) is the porosity, \( \sigma_w \) is the conductivity of the pore water. The exponent \( m \) is a constant which depends mainly on the geometry of the pore spaces and is called the cementation.

Shallow marine sediments are completely saturated with water. Consequently, the saturation factor \( S \) is unity and the equation simplifies to

\[ \sigma_f = \frac{\phi^m \sigma_w}{A}. \]

The porosity of marine sediments can vary from as much as 0.8 at the sea floor to as little as 0.1 at large depths. Since in the limit as \( \phi \) approaches 1, \( \sigma_f \) goes to \( \sigma_w \), and given that \( \phi \) varies greatly for the case of marine sediments, it is reasonable to assume that \( A \) is unity (Jackson et al., 1978). Thus only three parameters determine the conductivity of the sea sediment, \( \phi \), \( m \) and \( \sigma_w \).

The cementation \( m \) depends on the shape of the particles in the rock. In a study of artificial samples, Jackson et al. (1978) found that \( m \) increased from 1.2 for spheres to 1.9 for platy shell fragments, while for sands \( m \) fell between 1.4 and 1.6. They also discovered that particle size has very little effect on the cementation. In-situ measurements of sediment cementation factors for the shallow-sea, near-shore environment range between 1.5 and 3.

Conduction in water is almost entirely electrolytic. Consequently \( \sigma_w \) is a function of the number of ions, their charges, and their mobility. Generally, the mobility is independent of the type of ion since it depends principally on the collision rate which is similar for most molecules. The charge, however, can vary depending on the valence of the ion — for example \((\text{Mg})^{++}\) has the same conductive effect as \((\text{Na})^+\), but at half the molar concentration. The number of ions dissolved in the water is influenced by the type of rock and in the majority of marine sediments the
two dominant species are \((\text{Na})^+\) and \((\text{Cl})^-\).

The mobility and concentration of ions are both dependent on the temperature of the fluid. A rise in temperature will increase the average molecular velocity and hence the mobility of ions is greater. A semi-empirical formula for the conductivity of sea water, as a function of temperature and salinity, has been derived by Accerboni and Mosetti (1967), and is given by

\[
\sigma = \left[ A + B \frac{T^{1+k}}{1 + T^k} \right] \frac{S}{1 + S^h} e^{-\epsilon S e^{-\zeta(S-S_0)(T-T_0)}},
\]

(1.3)

where \(A = 0.21923, B = 0.012842, k = 0.0320, \lambda = 0.00290, h = 0.1243, \epsilon = 0.000978, T_0 = 20\,^\circ\text{C}, \zeta = 0.000165,\) and \(S_0 = 35\%\), where \(\sigma\) is measured in \(\text{S}\cdot\text{m}^{-1}\), \(T\) is measured in degrees \(\text{C}\), and \(S\) is the salinity in parts per thousand. As an example, for \(T = 10^\circ\,\text{C}\) and \(S = 30\%\), typical values for sea water, \(\sigma = 3.32\,\text{S}\cdot\text{m}^{-1}\).

If the temperature of a sediment is lowered to the point where some of the water begins to freeze, then the ionic mobility is greatly reduced. In contrast, the ion concentration in the remaining fraction of unfrozen liquid increases. There is a reduction of the effective porosity and an increase in the salinity of the fluid. The conductivity of the rock decreases rapidly with decreasing temperature as very resistive ice blocks the continuity of the conductive pathways in the material. The measurement of electrical conductivity could be an important tool in the detection of marine permafrost. Work in this area has already been conducted by Edwards et al., (1988).

Nafe and Drake (1957) describe one of the early studies which have attempted to relate porosity to seismic velocity empirically. They examined the variation with depth of porosity, density, and seismic velocity in both shallow and deep water marine sediment. Their compilation of the variation of porosity with depth for average deep (greater than 3 km) and shallow water (less than 200 m) is shown in Figure 1.1 (a). The curve for shallow water is replotted in Figure 1.1 (b) with logarithmic and linear scales for porosity and depth respectively. The straight line suggests an exponential variation of porosity with depth for shallow water marine

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Figure 1.1 (a) The porosity of shallow and deep water sediments as a function of sediment depth redrawn from Nafe and Drake (1957), and (b), the logarithm of the porosity of the shallow water sediments plotted against depth (reproduced from Edwards et al., 1988).
Chapter 1  Introduction

sediments. We have

$$\phi(z) = \phi(0) \exp(-sz),$$  \hfill (1.4)

where decay constant $s$ is determined from the slope of Figure 1.1 (b) as $6.15 \times 10^{-4} \text{m}^{-1}$, and $\phi(z)$ is the porosity at depth $z$. The porosity of the deep water sediment does not behave in a similar manner. However, two piecewise continuous exponential functions appear to fit the data very well. Consequently, some reduction in the number of parameters in a layered one-dimensional model representing the sediment may be achieved by allowing an exponential variation in porosity within the layer, rather than the more elementary constant porosity.

The corresponding conductivity variation may be obtained by substituting the porosity-depth relation (4) into Archie's rule. There results

$$\sigma_f(z) = \phi(0)^m \sigma_w \exp(-msz),$$  \hfill (1.5)

where $m$ is the cementation and $s$ is the porosity decay constant. Thus an exponential decrease in porosity causes a corresponding exponential decrease in conductivity.

Section 1.3  Transient Systems at Sea

For many years theoretical work on the response of EM systems on the sea floor has concentrated on frequency domain measurements. The elementary theory of the frequency-response of a crustal layer beneath a more conductive sea has been described by Bannister (1968), Coggon and Morrison (1970), Edwards, Law and Delaurier (1981), Chave and Cox (1982), and Kaufman and Keller (1983). More recently, the rationale for a broad-band time-domain electromagnetic system has been set out by Edwards and Chave (1986), who computed the response of a crustal halfspace beneath a more conductive halfspace representing seawater to a transient electric dipole-dipole system. The induced current systems produced by a transient system are well described using the concept of the "smoke ring" (Nahinian, 1979). A smoke ring grows and becomes less dense with time, yet maintains its basic shape.
In a similar manner, induced current smoke rings of the electromagnetic field excited by an event in the transmitter diffuse outwards from it through both the seawater and the relatively less conductive sea floor. The rate of electrical diffusion through a medium is inversely proportional to the conductivity of the medium, so that in the most common instance of a resistive sea bed, the electromagnetic field diffusing through the sea floor reaches the receiver first. At later times the field diffusing through the seawater arrives, and ultimately the measured field approaches the static limit.

Edwards and Chave (1986) showed that the transient coaxial electric dipole-dipole (ERER) system in contact with the sea floor is useful for determining sea floor conductivity. The normalized step-on response of this system is plotted in Figure 1.2 for a range of values of the conductivity ratio between the seawater and the sea floor. The initial step to one-half of the static limit is due to propagation in the sea floor; the second step at constant later time is due to propagation in the sea. Clearly, the position in time of the initial rise in the sea floor response is a direct measure of the sea floor conductivity. The sea floor is generally much less conductive than the sea water; the separation in time between the two parts of the transient response, and hence the resolution of sea floor conductivity, is substantial.

The theoretical curves computed by Edwards and Chave (1986) are displayed in the time domain. A similar separation of sea floor and seawater effects is also observed in the frequency domain. Electing to display results in the time domain reduces the number of curves a priori by a factor of two, as in the latter domain both amplitude and phase graphs are required. From the point of view of physics the two domains are equivalent, and perfect data collected in one domain can be transformed into the other domain using Fourier's theorem. However, in practice simple measurements in the two domains to the same degree of accuracy do not necessarily provide the same information. For example, the diagnostic late-time response of a finite conductor is observed directly in time, but the same information cannot be gleaned from frequency domain data unless accurate gradients of the
amplitude response are known at low frequency. A good analogy is found in the magnetotelluric method. Theoretically, it is possible to transform apparent resistivity data into phase data, so that it appears measuring both is redundant. Yet most protagonists of this method are quick to point out that phase data can sometimes yield additional information because the experimental error on a phase measure-
ment is often much smaller than the corresponding error on a phase datum derived from an apparent resistivity data set with its associated experimental error. The relative merits of broad band frequency and broad band transient systems depend on factors which are theoretical, statistical and practical in nature. In reality, the choice of a frequency domain system over a transient system or vice-versa could well be academic, as both systems require the transmission of a continuous, broad-band signal and its reception with a broad-band detector. The response of the earth is a deconvolution of the received signal with the transmitted signal combined with the system response; ultimately it is the choice of frequency or time domain formulations of this deconvolution process which distinguishes the time from the frequency domain character.

The question now arises, "What other electromagnetic systems are sensitive to the conductivity of the sea floor?" The theory of using EM systems to determine the electrical resistivity of continental material is reasonably well understood and documented (Hohmann and Ward, 1986; Spies and Frischknecht, 1986). Some of the more common systems that have been used with success on land, particularly in a layered earth environment, are illustrated in Figure 1.3. The coplanar magnetic dipole (HZHZ) system and the coaxial magnetic dipole (HRHR) system can be grouped together as far as the physics is concerned. They both generate only horizontal current flow in a one-dimensional earth, and are therefore relatively insensitive to thin horizontal resistive zones. The ERER system, which generates vertical, in addition to horizontal current flow, is preferred when such zones have to be mapped. The resistive layer is detected by the deflection in the electric field (and current) caused by electric charge build-up. A fifth system is also used in which the transmitter is a horizontal electric dipole and the receiver is a vertical magnetic dipole (EPHIHZ). Although both modes of current flow are generated by the electric dipole transmitter, the receiver senses only the magnetic field of the horizontal current flow so that interpretation of data obtained with this system follows the same path as for HZH or HRHR.
Cheesman et al. (1987) have calculated the responses of all of these systems when operated on the sea floor. Their behaviour in the sea is fundamentally different than on land, because the systems are now buried inside a conductive medium. Whereas HZHZ still generates only horizontal current flow, both ERER and HRHR generate and receive both modes. Furthermore, the electromagnetic fields due to induction in the crustal material are measured at or near the surface of a relatively good conductor. Consequently, a system such as HZHZ, in which a component of a field which vanishes at the surface of a good conductor is measured, is unlikely to produce data sensitive to a resistive sea floor. The analytic solution of some of the systems which do not exhibit the two-step response are included in Appendix C.

The most useful characteristic for inferring the suitability of a system for use on the sea floor from its behavior when used on land is the initial behavior of its step-on
response. The presence of a jump at time zero from zero field to a finite value is present in systems appropriate for sea floor use. For example: the ERER response jumps instantaneously (actually on a speed of light time scale) to one-half the late-time value when used at the surface of a uniform earth. In the sea, the situation is inverted; the less conductive half-space is below. Replacing the air by the weakly conductive sea floor delays and broadens the initial rise. The HRHR system has a similar characteristic, the instantaneous response is twice the late-time free-space static limit for either a resistive or a conductive sea floor. From a practical point of view, the field amplitude and delay times associated with transient systems on the hundred meter scale are similar to those currently employed by commercially available land based systems. Consequently, the construction of such a system poses no severe technical difficulties.

1.4 Outline of the Thesis

The thesis begins with an examination of the two transient systems which can be used on the sea floor. Chapter 2 deals with the HRHR system. The HRHR response for a double half-space model is derived. This is a natural starting point, because the half-space is the simplest possible model of the sea floor. From the double half-space response I introduce an apparent conductivity formula based on the arrival time of the transient. The apparent conductivity is a convenient tool for analyzing the response of the HRHR system to several other simple geological models: the resistive layer, the resistive and conductive basement, and the vertical conductive dyke.

Theoretical estimates of errors associated with the use of a practical system are obtained. Errors due to imperfect positioning of components of the system are estimated by calculating the response with the receiver raised off of the sea floor. Errors due to misalignment of the transmitter are estimated by determining the radial magnetic field produced by a vertical transient magnetic dipole transmitter.
A similar treatment of the ERER system is presented in Chapter 3. The response is in many ways similar to that of the HRHR system. The late-time amplitude of the ERER system can provide additional, independent information on the local conductivity structure. The early and late-time measurements are complementary in that each provides information in situations where use of the other measurement is difficult or impossible.

Chapter 4 discusses the design and construction of a working HRHR system prototype.

Chapter 5 describes a test of the prototype system, which was conducted in January, 1988 in the Channel Islands of Vancouver Island. Surveys run over three lines produced excellent data. Inversion of this data indicates that the bottom consists of a shallow sediment layer of varying thickness overlying a more resistive basement.

Conclusions and suggestions for the design of an improved system are included in Chapter 6.
Chapter 2

HRHR System Responses

2.0 Introduction

In order to interpret data collected with any geophysical prospecting system, it is first necessary to develop a broad understanding of the system’s response to various earth models. It is important to know the strengths, weaknesses and inherent limitations of the system so that it may be designed and deployed in an intelligent manner, and for maximum benefit.

The two transient systems which are sensitive to the conductivity of the sea floor are the coaxial horizontal magnetic dipole-dipole (HRHR) system, and the coaxial electric dipole-dipole (ERER) system. In this chapter I derive the theoretical responses of the HRHR system to a number of geologically relevant earth models. I develop a simple method for characterizing the response which is useful both for understanding the system’s behaviour, and interpreting data collected in surveys. In the following chapter I present a similar investigation into the response of the ERER system, and compare and contrast the responses of the two systems.

\[ \sigma_0 \quad \Theta \quad \Theta \quad \sigma_1 \quad \rho \]

Figure 2.1 The HRHR system.
The HRHR system, shown in Figure 2.1, consists of a transmitter (TX) and a receiver (RX), separated by a horizontal distance $\rho$. It is situated at the boundary between the sea water and a horizontally layered earth. Assuming zero initial conditions, and subject to the approximation that the magnetic effects of displacement currents may be neglected, the Laplace transform $H_\rho(s)$ of the magnetic field at the receiver for a transmitter of magnetic moment $m(s)$ is

$$H_\rho(s) = \frac{m(s)}{2\pi} [F_{TM}(s) + F_{PM}(s)],$$

where $s$ is the Laplace frequency and $m(s)$ is the Laplace transform of the source-current dipole moment. The functions $F_{TM}$ and $F_{PM}$ are Hankel transforms defined as

$$F_{TM}(s) = -s \int_0^{\infty} \frac{\mu_0}{Y^- + Y^+} \left[ \frac{J_1(\lambda \rho)}{\rho} \right] d\lambda,$$

and

$$F_{PM}(s) = -\int_0^{\infty} \frac{\mu_0}{Q^- + Q^+} \left[ \frac{\lambda J_0(\lambda \rho) - J_1(\lambda \rho)}{\rho} \right] d\lambda,$$

where $\mu_0$ is the magnetic permeability of free space, and $J_0$ and $J_1$ are Bessel functions. The functions in expressions (2.2) and (2.3) are the independent toroidal (TM) and poloidal (PM) parts of the solution, characterised by the absence of a vertical component of the magnetic and electric field respectively (Chave, 1984). The coefficients $Q^+, Q^-, Y^+$ and $Y^-$ are the inductances and impedances determined for the material above and below the HRHR system by the following recursion relationships:

$$Q_i = \frac{\mu_0}{u_i} \left[ \frac{u_i Q_{i+1} + \mu_0 \tanh(u_i d_i)}{\mu_0 + u_i Q_{i+1} \tanh(u_i d_i)} \right],$$

and

$$Y_i = \frac{u_i}{\sigma_i} \left[ \frac{\sigma_i Y_{i+1} + u_i \tanh(u_i d_i)}{\sigma_i Y_{i+1} \tanh(u_i d_i)} \right],$$

where $\sigma_i$ and $d_i$ are the electrical conductivity and thickness of the $i$-th layer above the sea-sea floor interface (for $Q^+$ and $Y^+$) or below the interface (for $Q^-$ or $Y^-$), and $u_i = \sqrt{\lambda^2 + \mu_0 \sigma_i s}$ [for simplicity, subsequent references to sea water properties...
use the subscript "0"]. When considering propagation in the air, we may no longer neglect the effect of displacement currents and use $u_a = \sqrt{\lambda^2 + s^2 \varepsilon_0 \mu_0}$.

A current $I$ is switched on at time $t = 0$ and held constant in a one-turn coil of area $A$. The transform of the magnetic moment of the coil, used to calculate the step response, is

$$m^S(s) = IA/s.$$  \hfill (2.6)

There are a large number of different methods for the numerical inversion of the Laplace Transform. The Gaver-Stehfest technique is neither the most accurate nor the most generally applicable. One has to be certain a priori that the unknown temporal function $F(t)$ has no discontinuities, sharp peaks or rapid oscillations (these conditions are certainly fulfilled in the present diffusive processes — see for instance Fig. 1.1). However, the algorithm does offer the user the advantage of speed, simplicity, and the need to compute the transform function $F(s)$ only for real values of the Laplace variable $s$. A detailed description of the algorithm may be found in Stehfest (1970) and Knight and Raiche (1982). Expressions (2.2) and (2.3) are evaluated at ten defined real values of $s$ for every required time $t$. Increasing the number of evaluations produces only marginal improvements in accuracy, at the expense of increased computational effort.

It is convenient to convert the step response to a dimensionless form. Dimensionless time $x$ is defined as the ratio of the true time to the upper half-space time constant $\tau_0$. The time constant, defined as

$$\tau_0 = \mu_0 \sigma_0 \rho^2,$$  \hfill (2.7)

increases with the square of the transmitter-receiver separation and with the ocean conductivity, as expected for a purely diffusive process. The parameter $\alpha$ is defined as $\sigma_0/\sigma_1$, the ratio between the sea water conductivity $\sigma_0$, and the lower half-space conductivity $\sigma_1$. It is also convenient to express the magnetic field in terms of the
late-time step response $H^S_p(\infty)$, or, equivalently, the magnetic field of a static dipole given by

$$H^S_p(\infty) = \frac{I \Delta A}{2\pi \rho^3}.$$  \hfill (2.8)

![Graph showing HRHR step response for the double half-space model.](Image)

Figure 2.2 The HRHR step response for the double half-space model.

## 2.1 The Double Half-Space

The double half-space is a suitable approximation to the actual conductivity structure if the HRHR system is located in water whose depth is large in comparison to $\rho$,..
and where the ocean bottom is of uniform conductivity to a comparable depth. The relevant inductances and impedances are $Q^- = \mu_0/u_1$, $Q^+ = \mu_0/u_0$, $Y^- = u_1/\sigma_1$, and $Y^+ = u_0/\sigma_0$, where the subscript 0 refers to the sea water and the subscript 1 refers to the sea floor.

![Graph: Step Response](image)

**Figure 2.3** The step response of the HZH system.

The dimensionless step response is plotted in Figure 2.2. The responses are distinctly different for different values of $\alpha$. The character of the response is very similar to the responses of the ERER system, with an early arrival due to diffusion through the sea floor and a later arrival due to diffusion through the sea water.

The step response of the HZH system, evaluated in Appendix C, is shown in Figure 2.3 for comparison. Unlike the HRHR system, the HZH system is relatively
atively insensitive to the conductivity of the sea floor. What follows is a possible explanation for the disparity between the HRHR and HZH systems.

For both systems, at times just after the transmitter is switched on until a time of the order of $\tau_1/10$, the induced currents within the sea and seafloor circulate in the opposite direction to the transmitter current, and no magnetic field is observed at the receiver. For the HRHR system, there is a build-up of charge at the sea-seafloor interface. The rate at which this charge build-up occurs may be related to the conductivity of the sea floor, and occurs mainly from times of the order of $\tau_1/10$ to $\tau_1$. The net result is that the induced currents begin to circulate in same direction as the transmitter current, and the horizontal magnetic field at the receiver approaches twice the value due to the transmitter current alone. Since the majority of the induced current is present within the more conductive sea water, this state exists until times of the order of $\tau_0$, when the induced currents in the seawater have died away.

The cause of the relative insensitivity of the HZH system should now be clear. The induced currents flow parallel to the boundary between the sea and seawater. Since there is no charge build-up at the interface, the magnetic field produced by the induced currents continues to oppose magnetic field produced by the transmitter until times of the order of $\tau_0$.

In the interpretation of DC potential field measurements, the concept of the apparent resistivity is frequently used. If the ground were a uniform conductor, a single measurement of the potential difference and current flow would determine its resistivity. The formula for the apparent resistivity is invariably a simple one having the form $\rho = f(\Delta V/I, a_i)$, where $\rho$ is the resistivity of the ground, $\Delta V$ is the difference in potential between the two potential probes, $I$ is the current delivered by the source, and $a_i$ are characteristic linear dimensions of the array. If the ground is inhomogenous, the same formula is used to define an apparent resistivity — the resistivity which the ground would have if it were homogenous. It is possible to specify a similar definition of apparent conductivity for the sea-
Figure 2.4 The delay time $t_d$ for the double half-space model, normalized with respect to the characteristic diffusion time for seawater $\tau_0$, is plotted as a function of conductivity contrast $\alpha$. The percent error in the apparent conductivity formula due to deviations in the actual value of $t_d$ from the line $t_d = \tau_1/10$ is plotted as a dashed line.

floor transient system. Here the important measured parameter is the diffusion time through the sea floor, denoted as the position in time of the first maximum in the rate of change in the step response. Equivalently, this is the time to the first maximum in the impulse response. This time is hereafter referred to as the delay time $t_d$. If the sea floor were an homogenous half-space, measurement of the delay time would determine the conductivity of the sea floor. The delay time for a half-space of conductivity $\sigma_1$ beneath a perfectly conductive sea is derived in Appendix A, and is given by $t_d = \tau_1/10 = \mu_0 \sigma_1 \rho^2/10$, where $\tau_1$ is the characteristic diffusion time in the lower half-space. An appropriate formula for the apparent conductivity of the sea floor is thus
\[ \sigma_{app} = \frac{10t_d}{\mu_0 \rho^2}. \]  

(2.9)

The actual values of \( t_d \) for a sea of finite conductivity, normalized by the characteristic sea water diffusion time \( \tau_0 \), are plotted as the solid line in Figure 2.4 as a function of \( \alpha \). For \( \alpha = 1 \), the delay time is equal to .1 \( \tau_0 \). The delay time decreases as the conductivity contrast increases. The nearly collinear dotted line shows theoretical values for the delay time, based on the simple formula \( t_d = .1\tau_0 / \alpha = .1\tau_1 \). For small values of \( \alpha \) the actual value of \( t_d \) differs from \( \tau_1 / 10 \) by up to 11.3\%, but is less than about 3\% in error for values of \( \alpha \) greater than 5. The delay time is easily determined in practice; the receiver probe is a coil and produces a signal voltage proportional to the time derivative of the incident magnetic field. If the transmitter current is in the form of a square wave, the voltage produced by the receiver will be in the form of an impulse response.

2.2 The Sea of Finite Thickness

The double half-space is not a valid model of the system when the distance from the surface of the water to the system is small in comparison to the transmitter-receiver separation. The situation for a shallow sea of depth \( d \) is illustrated in Figure 2.5. This situation is important because propagation of electromagnetic energy through the air proceeds at the speed of light. For shallow depths, the fastest route for the signal may be vertically up through the sea water to the surface, through the air, then vertically down to the receiver. In such cases the first arrival of the signal might indicate the depth of the water instead of the conductivity of the sea floor.

The relevant inductances and impedances, calculated from expressions (2.4) and (2.5), are given by
\[ Q^+ = \frac{\mu_0}{u_0} \left[ \frac{u_0 + u_a \tanh(u_0d)}{u_a + u_0 \tanh(u_0d)} \right] \quad (2.10) \]
\[ Q^- = \frac{\mu_0}{u_1} \quad (2.11) \]
\[ Y^+ = \frac{u_0}{\sigma_0} \left[ \frac{u_0 \sigma_0 + s \varepsilon_0 u_0 \tanh(u_0d)}{s \varepsilon_0 u_0 + u_a \sigma_0 \tanh(u_0d)} \right] \quad (2.12) \]
\[ Y^- = \frac{u_1}{\sigma_1} \quad (2.13) \]

where \( u_a = \sqrt{\lambda^2 + s^2 \varepsilon \mu_0} \).

\[ \sigma = 0 \quad \text{air} \]
\[ \sigma_0 \quad \Theta \quad d \quad \Theta \quad \sigma_1 \]

Figure 2.5 The HRHR system is located in a sea of depth \( d \) and conductivity \( \sigma_0 \). The bottom is a halfspace of conductivity \( \sigma_1 \).

Figure 2.6 shows the HRHR system response as a function of \( \alpha \) for different thickness of sea water, corresponding to values of \( \rho/d \) of 2, 5, 10, 20, 50, and 100. The effect of propagation through the air becomes more pronounced as \( \rho/d \) increases.

In cases where the sea water is very shallow, such as in Figures 2.6 e) and f), the nature of the response is strongly dependent on the value of \( \alpha \). For large values of \( \alpha \), the sea floor is the fastest route for diffusion. For \( \alpha = 1000 \) the signal rises above and then drops towards the static limit, but the shape is clearly different from the corresponding curve for the double half-space response. The static limit is reached more quickly than in Figure 2.2. The signals for \( \alpha = 300 \) and \( \alpha = 100 \) arrive later and are of smaller amplitude. The trend as \( \alpha \) decreases is to later arrivals and
Figure 2.6 The effect of sea depth: The step response of the HRHR system is plotted for a range of values of the conductivity contrast $\alpha$ for six values of $\rho/d$. An increase in the ratio $\rho/d$ represents either a lessening of the depth of the sea or an increase in system spread.
smaller amplitudes. For values of $\alpha$ less than about 30, the trend appears to reverse; the initial arrival seems to occur earlier, and the amplitudes of the signals increase. The fastest route is now through the sea and air.

This is, in fact, a rather puzzling result. Why, for instance, does the signal for $\alpha = 1$ in Figure 2.6 e) appear to arrive sooner than the signal for $\alpha = 30$? After all, the model from the sea floor up is identical for both cases; the only difference is the conductivity of the lower half-space. An increase in overall conductivity appears to produce an increase in the speed of the signal. Examination of the corresponding impulse responses provides the solution.

![Graph showing impulse response](image)

Figure 2.7 The impulse response of the HRHR system in a shallow sea where $\rho : d = 50 : 1$, evaluated at several values of $\alpha$. This is the time derivative of the step response illustrated in Figure 2.6 e). The dotted curves represent negative values of the impulse response for $\alpha = 1000$, 300 and 100.
Figure 2.7 shows the impulse response of the HRHR system for $\rho/d = 50$. The plotted curves are the time derivatives of those found in Figure 2.6 e). The arrivals of both parts of the signal are evident in the impulse response.

The initial peak is from propagation through the sea floor, and its position in time is a function of $\alpha$. The amplitudes of these peaks vary; they are determined by both the amplitude of the step response and the arrival time. (Because the step response is plotted against a logarithmic time scale, a curve with a given slope on the graph has a larger time derivative if it occurs at an earlier time.)

The second, later peak, due to diffusion through the sea water and propagation through the air, occurs at the same instant, dimensionless time $2.5 \times 10^{-4}$, for all the models except $\alpha = 1000$. However, the amplitude of the peak is smaller for larger values of $\alpha$, so that the air-sea arrival is obscured on the step response graph for larger values of $\alpha$.

Why does the amplitude of the air-sea response vary as a function of the sea floor conductivity? In the double half-space model, the amplitude of the initial response depended on the value of $\alpha$. For large $\alpha$ the initial step approaches twice the late-time response, because, relative to the sea floor, the sea water appears to be a perfect conductor. The present model resembles an inverted form of the double half-space model. The air and shallow sea in concert behave like a single half-space which is more resistive than the crustal half-space below. The effective conductivity contrast increases as $\alpha$ decreases. For small values of $\alpha$ (or for very shallow seas), the effective contrast becomes large and the amplitude of the initial step response approaches twice the static limit.

The delay time, as stated above, is the time to the first maximum in the impulse response. The delay times for the present model, normalized by the characteristic diffusion time in sea water $\tau_0$, are plotted in Figure 2.8 as a function of $\rho/d$, for a range of values of $\alpha$. Moving from left to right on the graph corresponds to moving from deep to shallow water for a fixed $\rho$. Consider first the case for $\alpha = 1$. In deep water, $t_d \approx \tau_0/10$, the normal value for the double half-space model. Once
the water becomes shallower than about $0.5\rho$, the value of $t_d$ begins to decrease; the air-sea diffusion route becomes faster. For depths less than about $0.2\rho$, the arrival time becomes proportional to $d^2$. This is understandable because the dominating factor in the diffusion rate is now the distance to the surface of the water, and the characteristic diffusion time to the surface is $\tau = \mu_0 \sigma_0 d^2$. For values of $\alpha$ greater than about 3, a slightly more complex behaviour is observed. Here the impulse response possesses one or two maxima, corresponding to the arrivals of diffusion through the sea floor and the sea layer. In those cases where the diffusion fronts travel at nearly the same velocity, a single maximum is formed. Abrupt jumps in the curves for $\alpha = 3, 10$ and 30 correspond to the depths at which the peak in the
slope due to diffusion through the water becomes larger than that due to diffusion though the sea floor.

The curve for $\alpha = 1$ in Figure 2.8 represents a practical limit in the use of the delay time alone to determine the sea floor conductivity. For instance, at $\rho/d = 8.5$ the $\alpha = 1$ curve intersects the curve for $\alpha = 10$. At this depth, for conductivity contrasts less than 10, the air-sea transient arrives before the sea floor transient. This does not mean that the system would be insensitive to the conductivity of the sea floor, but that special care should be taken in the interpretation of the transient — the sea floor signal may be combined with, or follow the air-sea signal. A simple numerical relationship describes when the delay time is a useful indication of sea floor conductivity. The limiting line on Figure 2.8 is approximately described by the equation $t/\tau_0 = (d/\rho)^2$, or $t_l = \mu_0\sigma_0 d_0^2$, where $t_l$ is the limiting time. The largest value of apparent conductivity, $\sigma_l$, which may be determined from $t_d$, is given by

$$\sigma_l = \frac{10\sigma_0 d_2}{\rho^2}$$

(2.14)

In practice, the limitations imposed by shallow water are relatively slight — the contrast is generally much higher than 3 except in water-saturated muds, and the water is only occasionally so shallow that $\rho/d$ exceeds 10. In any case, the conductivity of the sea floor can still be determined, but more effort is required to extract the information.

One additional simplification may be made. The transient response of the HRHR system has been shown to consist of two parts. When the sea floor is less conductive than seawater, and the sea depth is large enough to ignore propagation through the air, the form of the response at early time is dictated principally by the electrical conductivity of the sea floor. The responses of different crustal models may be simply compared by plotting only early time responses. It is possible to replace the sea water by a perfectly conducting layer. The result is a reduction in the number of curves necessary to illustrate the salient physics. The errors introduced by substituting a perfectly conducting sea for a partially conducting one
are precisely those determined for the apparent conductivity formula above and are typically on the order of a few percent, except for values of \( \alpha \) less than 5, and even then they do not exceed 12%.

2.3 The Layer over a Half-Space

A more sophisticated model of the sea floor than the double half-space is illustrated in Figure 2.9. A sediment layer of thickness \( d_1 \) and conductivity \( \sigma_1 \) overlies a half-space of conductivity \( \sigma_2 \). This is a reasonable model for any area where sedimentation has occurred. The corresponding inductances and impedances, from expressions (2.4) and (2.5), for a perfectly conducting sea, are

\[
Q^+ = 0, \\
Q^- = \frac{\mu_0}{u_1} \left[ \frac{u_1 + u_2 \tanh(u_1 d_1)}{u_2 + u_1 \tanh(u_1 d_1)} \right], \\
Y^+ = 0, \\
Y^- = \frac{u_1}{\sigma_1} \left[ \frac{u_2 \sigma_1 + u_1 \sigma_2 \tanh(u_1 d_1)}{u_1 \sigma_2 + u_2 \sigma_1 \tanh(u_1 d_1)} \right].
\]

![Figure 2.9](image)

Figure 2.9 The layer over a half-space model. The sea is modelled as a perfectly conductive half-space.

The apparent conductivity, determined from the delay time, is plotted in Figure 2.10 as a function of \( \rho/d_1 \) for a range of values of \( \sigma_1/\sigma_2 \), and for a relatively
resistive lower half-space. The apparent conductivity has been normalized with respect to $\sigma_1$, the conductivity of the first layer. Intuitively, the apparent conductivity for this model lies between the conductivities of each of the layers. If $d_1$ is large in comparison to the system separation, then the lower half-space will have little effect on the result. If $d_1$ is very small, the apparent conductivity will approach $\sigma_2$.

The ability of the HRHR system to detect the presence of a less conductive lower half-space may be estimated using Figure 2.10. Let the error in the layer conductivity be $n\%$. If $\sigma_2$ is known, the maximum depth at which the lower half-space may be seen is found by determining the depth where the profile for that
conductivity comes within \( n \% \) of the profile for the simple half-space model, \( \sigma_1 / \sigma_2 = 1 \). For \( \sigma_1 / \sigma_2 = 10 \), and an uncertainty in \( \sigma_1 \) of 10\%, (typical values taken from the data analysis in Chapter 5), the maximum depth for detection is about one-half the system spread. Since all the profiles converge to the simple half-space case \( \sigma_1 / \sigma_2 = 1 \) for values of \( \rho / d_1 \) less than 2, it is not possible to detect a more resistive lower half-space using apparent conductivities when the lower half-space is deeper than one-half the system spread. This limit is valid if only the delay time \( t_d \) and its derived apparent conductivity are considered, since it may be possible to determine conductivities at greater depths by studying the late-time portion of the transient. Nevertheless, it is reasonable to propose a maximum 'depth of investigation' for the HRHR system, as being approximately one-half the system spread.

The response when the lower half-space is more conductive than the overlying layer has been plotted in Figure 2.11. The curves are qualitatively different than those for the more resistive lower half-space. Only for large values of \( \rho / d_1 \) does the apparent conductivity indicate the presence of a more conductive lower half-space. Why does the more conductive lower half-space influence the arrival time? Shouldn't the signal travelling through the resistive layer always arrive first? The reason is that the strength of the signal diffusing through the thin layer falls off quite rapidly with distance due to leakage into the upper and lower half-spaces. There is probably still a peak in the impulse response due to diffusion through this layer at large values of \( \rho / d_1 \), which would indicate an apparent conductivity of \( \sigma_1 \), but it is too small to be detected, and the later, stronger signal arriving through the lower half-space determines the delay time and apparent conductivity. In the field one might have a similar effect; the first arrival might be lost in the noise, so that only the lower half-space would be observed.

In any case, because it is the fastest diffusion path which determines the apparent conductivity, the HRHR system is inherently less sensitive to a layered earth model where the conductivity increases with depth than to one in which the conductivity decreases with depth.
Figure 2.11 The position in time of the maximum impulse response is plotted as a function of the thickness of the top layer $d_1$ for various ratios of the conductivities of layer and lower half-space, for a more conductive lower half-space.

2.4 The Thin Resistive Layer

A variation on the resistive basement is the thin resistive zone, illustrated in Figure 2.12. The model is similar to the half-space model shown in Figure 2.1, but includes a thin layer of thickness $d_2$ with a conductivity $\sigma_2 \ll \sigma_1$ at a depth $d_1$ beneath the sea floor. This layer could represent, for example, a thin layer of permafrost in an otherwise conductive sea floor. The relevant inductances and impedances are determined from the recursion relationships (2.4) and (2.5), but are too lengthy to include here.
To the PM mode of propagation, a very thin, resistive layer is virtually transparent since the horizontal current flow associated with this mode is not perturbed by the zone. The contribution from this mode to the magnetic field is the same as if the crust were a uniform half-space of conductivity $\sigma_1$. In contrast, the TM mode is affected because the vertical currents associated with it are deflected by the buildup of charge on the surfaces of the resistive zone.

The response of the HRHR system to the resistive layer model with $\sigma_2 = \sigma_1 / 10$ is plotted in Figure 2.13 as a function of $\rho / d_1$ for normalized layer thicknesses $d_2 / \rho$ of .01, .02, .03, .05, .10 and $\infty$. The curves show that even a very thin resistive layer can have a substantial effect on the delay time. Indeed, for the ratio of layer conductivities studied, a resistive layer whose thickness is more than 10% of the system separation is virtually indistinguishable from a lower halfspace of the same conductivity. This suggests that while the system is inherently sensitive to the depth of resistive layers, its ability to resolve the layer thickness is limited.

2.5 Three-Dimensional Models

The modelling of a large range of three-dimensional bodies contained in the sea floor is simplified greatly by the assumption that the sea layer behaves like a perfect conductor at early times. The simplifying theorem follows directly from the
Figure 2.13 The apparent conductivity response of the model shown in Figure 12 is plotted as a function of layer depth $d_1/\rho$ for various values of the normalized layer thickness $d_2/\rho$ and where the layer conductivity $\sigma_2 = \sigma_1/10$.

discussion of images in section 2.1. When a magnetic dipole transmitter is located on the boundary between a perfect conductor and a lower half-space containing an arbitrary conductivity distribution, the field measured anywhere in the half-space is identical to that which would be produced by a transmitter of twice the moment if the conductivity distribution of the lower half-space is mirror imaged in the perfect conductor. The theorem is particularly useful for calculating the response of a semi-body, for example a hemisphere. When the image of the hemisphere is added to the hemisphere itself, the whole, of course, becomes a sphere and the response of a sphere in a whole space is readily found.

The theorem is illustrated in Figure 2.14, where a transmitter of moment $m$
interacting with a hemisphere touching the interface are replaced by a transmitter of moment $2m$ interacting with a sphere in a whole space. The proof is trivial and consists of showing that the boundary conditions at the interface associated with the two models are identical. At the boundary with the perfect conductor the tangential electric field and the vertical magnetic field must vanish. In the whole space model both of these conditions are met by symmetry. The doubling of transmitter strength is demanded by the method of images.

We discuss only one application of the semibody theorem. The response characteristics of the HRHR system to a finite vertical conductive dyke, shown in Figure 2.15, is computed. Application of the theorem indicates that the equivalent problem is the computation of the transient response of an infinite sheet in a whole space to a magnetic dipole source whose axis is perpendicular to the sheet. The receiver is coaxial with the transmitter. The transmitter is at the origin and the receiver is a distance $\rho$ away. A vertical dyke of conductance $S$ is located at $\rho_d$. 

Figure 2.14 A semibody problem and its equivalent whole-space representation.
The normalized response, derived in Appendix A.3, is described by two expressions, depending on whether $\rho_d$ is less than or greater than $\rho$. If $\rho_d$ is less than $\rho$,

$$
\frac{H_\rho(s)}{H_\rho(\infty)} = \frac{\rho^3}{2} \int_0^\infty \frac{\lambda^3}{u_1} \left[ \frac{1}{1+\theta(s)} \right] \exp(-u_1 \rho) \, d\lambda,
$$

(2.19)

whereas if $\rho_d$ is greater than $\rho$,

$$
\frac{H_\rho(s)}{H_\rho(\infty)} = \frac{\rho^3}{2} \int_0^\infty \frac{\lambda^3}{u_1} \left[ \frac{1}{1+\theta(s)} \right] \exp[-u_1(2\rho_d - \rho)] \, d\lambda,
$$

(2.20)

where

$$
\theta(s) = \frac{s\mu_0 S}{2u_1},
$$

and where $H_\rho(\infty)$ is defined in expression (2.8).

From the form of expression (2.19), it is evident that if the dyke is positioned between the transmitter and receiver the response is not sensitive to the particular position of the dyke, but only the transmitter-receiver separation. The presence of the dyke delays the arrival of the signal at the receiver, with the amount of delay being determined by the specific conductance of the dyke, as illustrated in Figure 2.16, for a range of values of a normalized form of the dyke conductance. The plot of the fractional increment in the delay versus the dyke conductance is shown in Figure 2.17, using two different schemes for measuring the delay. When $t_{\frac{1}{2}}$, the time for the step response to rise to half of its final value, is used, the increment in time
is very nearly proportional to the conductance of the dyke. (This linear behaviour is not merely an artifact of the logarithmic plotting axes). However, when $t_d$, the time of the maximum impulse response is considered, the fractional change is much reduced for larger values of $S$. The behaviour is non-linear — a given fractional change in $S$ gives smaller fractional changes in $t_d$ when $S$ is large.

2.6 Estimates of Two Systematic Errors

Some of the important requirements of a practical system can be studied theoretically. For example, the system should be relatively immune to small errors in
Figure 2.17 The fractional incremental delay in the arrival of the signal caused by the presence of a conductive vertical dyke. Both the change in $t_d$, the time of the maximum in the impulse response, and in $t_{\frac{1}{2}}$, the time when the step response reaches half its final value, are plotted. $t_{\frac{1}{2}}$ and $t_d$ are times measured in the absence of the dyke.

positioning and orientation. In order to determine to some measure the robustness of the horizontal magnetic dipole system the effects of two such errors are investigated.

A practical system, consisting of a separate transmitter and receiver, should be resting on the ocean floor whenever measurements are made in order to minimize the noise caused by the movement of the array through the earth’s magnetic field. Nevertheless the transmitter and receiver cannot be positioned exactly at the boundary, both because of shallow layers of sediment which may cover the basement rock and which may differ little in conductivity from the seawater, and because the axes of
Figure 2.18 The normalized step-on response of the HRHR system when the receiver is raised a distance $d/\rho = .02$ off the interface (solid lines) is compared to the response when both transmitter and receiver are on the boundary (dashed lines).

the transmitter and receiver must of necessity be raised above the sea floor because of their finite size. The consequences may be investigated by raising the receiver a small distance and noting the effect on the transient response. This is accomplished through upward continuation of the field computed on the boundary. A number of cases were studied, and the example, in which the receiver is raised off the ocean floor by a distance equal to 2% of the array separation, is shown in Figure 2.18. In terms of a real distance, 2% might represent 1 m for a design separation of 50 m. A comparison between the dashed curves, which represent the fields on the boundary, and the solid curves, which represent the upward continued fields, reveals that the
effect of elevating the receiver is to increase the delay in the received step response, because the signal must travel a short extra distance through a conductive medium. The effect becomes more important for greater conductivity contrasts because the diffusion time through the sea represents a significant portion of the total diffusion time. The increment in the delay time due to the extra signal path length is not solely dependent on the conductivity of the sea but is also dependent on the sea floor conductivity. The increment actually decreases with greater conductivity contrasts, but the fractional change becomes more significant. Fortunately, the effect remains small for typical conductivity ratios up to a value of 30.

![Graph](image)

**Figure 2.19** The normalized step-on response for the null-coupled (HZHR) system computed analytically from equation (2.26) for a range of values of the conductivity ratio $\alpha$ (solid lines). The HRHR response shown in Figure 2.2 is included for comparison of signal amplitudes (dashed lines).
The effect of the misalignment of the transmitter may be determined by examining the two worst-case scenarios of misorientation. In the first case the transmitter is rotated about a vertical axis by 90°. The receiver is now at right angles to the transmitter coil axis, but the receiver axis still passes through the transmitter. It is not difficult to see by symmetry that any magnetic field at the receiver must be perpendicular to the receiver axis, so that no signal is ever detected. The only error introduced by the horizontal misorientation of a real system would be due to a cosine type variation of the HRHR component of the signal. In the second case the transmitter axis is rotated about a horizontal axis by 90°, so that the axis is now vertical, forming an HZHR system. Kaufman and Keller (1983) have derived the expression for the radial magnetic field of a vertical magnetic dipole. Their expression, in terms of the Laplace variable $s$, is

$$H_\rho(s) = \frac{m(s)}{4\pi \rho^3} (\tau_0 - \tau_1) s \left[ I_2(a\sqrt{s})K_2(b\sqrt{s}) - \frac{\alpha + 1}{\alpha - 1} I_1(a\sqrt{s})K_1(b\sqrt{s}) \right], \quad (2.21)$$

where

$$a = \frac{\sqrt{\tau_0} - \sqrt{\tau_1}}{2},$$

and

$$b = \frac{\sqrt{\tau_0} + \sqrt{\tau_1}}{2},$$

and where $I_\nu$ and $K_\nu$ are modified Bessel functions. The step response is then

$$H_\rho^S(t) = \frac{I \Delta A (\tau_0 - \tau_1)}{4\pi \rho^3} \frac{1}{2t} \exp\left(-\frac{\tau_0 + \tau_1}{8t}\right) \times \left[ I_2\left(\frac{\tau_0 - \tau_1}{8t}\right) - \frac{\alpha + 1}{\alpha - 1} I_1\left(\frac{\tau_0 - \tau_1}{8t}\right) \right]. \quad (2.22)$$

The HZHR system is null coupled. Expression (2.22) is normalized with respect to expression (2.8). The resulting expression may be directly compared with the HRHR system response to determine the significance of an error in orientation. The final dimensionless expression is then

$$\frac{H^S_\rho(x)}{H^S_\rho(\infty)} = \frac{(\alpha - 1)}{4\alpha x} \exp\left(-\frac{\alpha + 1}{8\alpha x}\right)$$
\[ x \left( I_2 \left( \frac{\alpha - 1}{8\alpha x} \right) - \frac{\alpha + 1}{\alpha - 1} I_1 \left( \frac{\alpha - 1}{8\alpha x} \right) \right). \]  \hspace{1cm} (2.23)

In Figure 2.19 the dashed and solid curves represent respectively the response of the HRHR and HZHR systems. By reciprocity, the HZHR system represents the largest cross-coupling error which may be introduced by the misalignment of either the transmitter or the receiver. The cross-coupled component never exceeds 50% of the proper signal, and for most times and conductivity ratios it is much smaller. In an actual case of misalignment the cross-coupled component of the transmitter field would be weighted by the sine of the misalignment angle, and the in-line component by the cosine of the misalignment angle. It is likely that this angle would be quite small, on the order of a few degrees, consequently, errors associated with the introduction of a cross-coupled component should be small. Moreover, these errors would affect mainly the amplitude of the received signal; since the HZHR response shows the same separation of response as a function of \( \alpha \) as the HRHR system, the delay times are largely unaffected.

2.7 Summary

The splitting of the HRHR system response into two parts, one due to diffusion through the sea floor and one due to diffusion through the sea water, should permit the unambiguous determination of both conductivities. The rate of diffusion is closely correlated with the conductivity of the medium through which diffusion occurs. This result has allowed the definition of an apparent conductivity based on delay time, and the subsequent production of characteristic curves, in a manner analogous to the way characteristic curves are produced for DC resistivity mapping on land, using apparent resistivities. By determining the delay times as a function of system spread, and deriving apparent conductivities, the investigator may learn something of the underlying conductivity structure by comparing experimentally determined profiles of apparent conductivity with profiles found on the characteristic curves.
The thickness of the overlying layer of seawater is an important constraint on the use of the first peak in the impulse response to determine sea floor conductivity, since diffusion through the seawater coupled with propagation through the air may proceed more quickly than diffusion through the sea floor. A simple approximate expression (2.14) describes the largest apparent conductivity which may be inferred directly from the delay time. In shallower waters the sea floor conductivity may still be determined by examination of the full transient.

A number of earth models were investigated using the concept of apparent conductivity. In the layer over a resistive half-space model, the apparent conductivity decreases as the system spread increases, tending towards the conductivity of the lower half-space for large values of $\rho$. In contrast, the apparent conductivity is relatively insensitive to a more conductive underlying half-space, because the resistive upper layer remains the fastest path for diffusion of the signal.

The thin resistive layer embedded in a relatively conductive half-space is detected by virtue of a decrease in the apparent conductivity with increasing system spread, but the thickness of the layer is poorly resolved. The presence of a vertical dyke between the transmitter and receiver introduces an extra delay in the signal, with a corresponding increase in apparent conductivity. If the increment in the delay is measured with respect to $t_{1/2}$, the time for the step response to rise to half its final value, then the increment is proportional to the conductance of the dyke.

Finally, two experimental errors which might occur while conducting an actual survey were evaluated. Raising the transmitter or receiver off the sea floor slightly increases the delay time because the signal must travel a short distance through the conductive sea water before entering the sea floor. Rotating the transmitter or receiver results in a reduction in the amplitude of the signal, or the introduction of cross-coupling with other components of the transmitted signal. In general the effect of these errors on the system is minor, and can be safely ignored.
Chapter 3
ERER System Responses

3.0 Introduction

The coaxial horizontal electric dipole-dipole system has been used for many years in geophysical exploration of the sea floor (Chave et al., 1989). Just a few years after the DC (direct current) resistivity method was developed by the Schlumberger brothers, it was used for the first resistivity survey over water (Schlumberger et. al., 1934). More recently oceanic DC resistivity surveys have been used for detection and delineation of sulphide mineral bodies (Frances, 1977, 1984), and for the determination of the depth to permafrost zones (Corwin, 1983). In general, though, the method is restricted to regions where the conductivity of the sea floor is comparable to, or greater than the conductivity of seawater.

A controlled-source electric dipole-dipole method working in the frequency domain has been developed by C.S. Cox and others at the Scripps Institution of Oceanography. The system has been successfully deployed in the deep ocean on several occasions (Spiess et al., 1980, and Young and Cox, 1981). The transmitter is a 500-1000 m cable terminated with 15 m steel electrodes. The receiver is typically a string of potential electrodes spanning 500-1000 m. The system transmits a number of frequencies from 1/16-16 Hz and records the amplitude and phase variations as a function of frequency.

Presently, our group at the University of Toronto is developing a transient electric dipole-dipole system (Everett et al., 1989). The system underwent its first tests in June, 1988 when measurements were taken at seven locations across the Strait of Georgia, in British Columbia.
I present here the basic theory of the \( \alpha \tau \)-earth response of a transient electric dipole-dipole (ERER) system and present graphs and results which may be of use in the design and interpretation of future ERER surveys.

\[
\begin{array}{c}
\text{TX} \\
\sigma_0 \\
\hline
\hline
\sigma_1 \\
\hline
\text{RX} \\
\rho
\end{array}
\]

Figure 3.1 The ERER system.

The ERER system shown in Figure 3.1 consists of a transmitter (TX) and a receiver (RX), separated by a horizontal distance \( \rho \). The calculation of the response of the system in a horizontally layered earth parallels that of the HRHR system found in Chapter 2. The Laplace transform \( E_\rho(s) \) of the electric field at the receiver

\[
E_\rho(s) = \frac{j(s)}{2\pi} [F_{TM}(s) + F_{PM}(s)],
\]

where \( F_{TM} \) and \( F_{PM} \) are Hankel transforms defined as

\[
F_{TM}(s) = -\int_0^\infty \frac{Y^- - Y^+}{Y^- + Y^+} \left[ \lambda J_0(\lambda \rho) - \frac{J_1(\lambda \rho)}{\rho} \right] d\lambda,
\]

and

\[
F_{PM}(s) = -s \int_0^\infty \frac{Q^- Q^+}{Q^- + Q^+} \left[ \frac{J_1(\lambda \rho)}{\rho} \right] d\lambda,
\]

The inductances \( Q^+ \) and \( Q^- \), and the inductances \( Y^+ \) and \( Y^- \) are defined as in expressions 2.4 and 2.5.

A current \( I \) is switched on at time \( t = 0 \) and held constant in a transmitter of length \( \Delta l \). The transform of the dipole moment is

\[
j_\mathcal{S}(s) = I\Delta l/s.
\]

Although the diffusion time constant \( \tau \) is common to both the ERER and HRHR system, the signal amplitude normalization is fundamentally different. The
amplitude of the late-time magnetic field is independent of the conductivity of the sea or the sea floor. The amplitude of the late-time electric field, however, is
determined by the value of \( \alpha \), the ratio of the seawater and sea floor conductivities,
and, in the double half-space model, is equal to

\[
E_p(\infty) = \frac{I \Delta I}{\pi \sigma_0 \rho^3} \left( \frac{2\alpha}{1 + \alpha} \right).
\] (3.5)

The normalization constant chosen is the late-time value for a wholespace of con-
ductivity \( \sigma_0 \), or equivalently, the double half-space model with \( \alpha = 1 \). Normalized
values of the late-time electric field therefore vary between 1 and 2 for values of
alpha between 1 and infinity.

The variation in the late-time field forms the basis for a more conventional
apparent conductivity formula, discussed below. This is in addition to apparent
conductivity based on delay time, discussed in Chapter 2, which remains valid for
the ERER system.

3.1 The Double Half-Space

The double half-space is a suitable approximation to the actual conductivity struc-
ture if the ERER system is located in water whose depth is large in comparison to
\( \rho \), and where the ocean bottom is of uniform conductivity to a depth comparable
to \( \rho \). The relevant inductances and impedances are \( Q^- = \omega_0 / u_1 \), \( Q^+ = \omega_0 / u_0 \),
\( Y^- = u_1 / \sigma_1 \), and \( Y^+ = u_0 / \sigma_0 \). The responses are plotted in Figure 1.1. An ana-
lytic approximation, first derived by Edwards and Chave (1986), and modified for
greater accuracy, is described in Appendix B.

In Chapter 2 an apparent conductivity for the HRHR system was developed
based on the delay time, i.e., the time to the first maximum in the impulse response.
The delay time, calculated in Appendix A.2 for a perfectly conductive sea (\( \alpha = \infty \)),
was determined to be exactly equal to the lower half-space time constant \( \tau_1 \), divided
by 10. This relationship between \( t_d \) and \( \tau_1 \) remained a good approximation for finite
values of $\alpha$, even $\alpha = 1$. The same type of procedure is repeated for the ERER system in Appendix B, with surprisingly different results. For values of $\alpha$ very near to unity, the relationship $t_d = \tau_1/10$ remains valid. However, for large values of $\alpha$, the value in the denominator increases to about 15.4. The actual values of $t_d$ for a sea of finite conductivity are plotted as the solid line in Figure 3.2 as a function of $\alpha$. The error is the difference between the measured delay time and that calculated using expression (2.9). If the 15.4 is used instead of 10, the error still remains substantial over much of the range of $\alpha$. The variation in the relation between $t_d$ and the apparent conductivity $\sigma_{app}$ makes characteristic curves based on a perfectly conductive sea layer less accurate. Nevertheless, the information supplied.
by the curves remains useful. As in the case of the HRHR system, exactness could be regained at the loss of generality by creating a number of different type curves for different values of $\alpha$.

3.2 The Sea of Finite Thickness

The double half-space is not a valid model of the system when the distance from the surface of the water to the system is small in comparison to the transmitter-receiver separation. The situation for a shallow sea of depth $d$ is illustrated in Figure 3.3. The relevant inductances and impedances are the same as for the HRHR system, found in expressions (2.10-13).

![Diagram](image)

Figure 3.3 The ERER system is located in a sea of depth $d$ and conductivity $\sigma_0$. The bottom is a halfspace of conductivity $\sigma_1$.

Figure 3.4 shows the ERER system response as a function of $\alpha$ for different thickness of sea water, corresponding to values of $\rho/d$ of 0.5, 1, 2, 5, 10, and 20. All curves have been normalized with respect to the late-time amplitude to be found in a whole-space, the same normalization as in Figure 1.1. The first three sets of curves are essentially the same as for the double half-space model. Diffusion through the sea floor is indicated by a step to 1.0 at a time determined by the specific value of $\alpha$. The diffusion through the sea water brings the response to the static limit, and occurs at the same time, $t/\tau_0 \approx .1$ for all contrasts. The late-time amplitude
Figure 3.4 The effect of the sea depth: The step response of the ERER system is plotted for a range of values of the conductivity contrast $\alpha$ for six values of $\rho/d$. An increase in the ratio $\rho/d$ represents either a lessening of the depth of the sea or an increase in system spread. (Note the variation in the vertical scale.)
is determined by the conductivity contrast, and is approximately proportional to \(2\alpha/(1 + \alpha)\).

![Diagram showing delay time vs. \(\rho/d_0\)](image)

Figure 3.5 The position in time of the maximum impulse response is plotted as a function of the ratio of sea depth \(d\) to system separation \(\rho\) for a range of values of the conductivity contrast \(\alpha\).

As with the HRHR system, a decrease in the thickness of the sea layer provides a faster path for signals through the sea and air. The movement of the seawater response to earlier times may be seen in the responses for \(\rho/d = 5, 10\) and 20. If the layer is thin enough, diffusion through the seawater and air occurs as quickly or more quickly than diffusion through the sea floor and the delay time may no longer be used as an indicator of the sea floor conductivity. The size of the early time step remains constant at 0.5, independent of the depth of the sea. The late-time
amplitude, however, increases for shallower sea depths. This occurs because at late times the current is concentrated in the thinner sea layer.

The delay time $t_d$ for the ERER system has been plotted in Figure 3.5 for various values of $\alpha$ as a function of $\rho/d$. As with the HRHR system delay time shown in Figure 2.8, the diagonal line running down and across the graph demarcates the range of depths and conductivity contrasts within which the delay time may be used to calculate apparent conductivity.

![Graph showing $2\pi \sigma_0 \rho^3 E_{DC} / |I|_d$ as a function of $\rho/d$ for different values of $\alpha$.]

Figure 3.6 The normalized value of the late-time electric field $E_{DC}$ is plotted for a range of values of $\alpha$ as a function of the ratio of sea depth to system separation $\rho/d$.

DC resistivity surveys on land use late-time measurements of the electric potential in order to determine an apparent resistivity (or equivalently, apparent con-
ductivity). Because current flow is confined to the earth, the formula is a simple closed form expression. In the case of a sea-floor system, the formula becomes somewhat more complex because current flows within the sea water. An appropriate formula for the apparent conductivity of the sea floor would thus take the form 
\[ \sigma_{app} = f(\Delta V/I, \rho, d, \sigma_0), \]
where \( d \) and \( \sigma_0 \) are the known depth and conductivity of the sea. The DC behaviour of the sea-floor electric dipole-dipole system is found by setting \( s = i\omega \) to zero in (3.1) and setting \( j(0) = I\Delta l; j(s) = 0, s \neq 0 \). The poloidal mode disappears, and we are left with

\[
E^{DC} = -\frac{1}{2\pi} \int_0^\infty \frac{\lambda^2 J_i(\lambda \rho)}{\sigma_1 + \sigma_0 \tanh(\lambda d)} \, d\lambda, \tag{3.6}
\]

or, letting \( \zeta = \lambda \rho \), and rearranging terms,

\[
2\pi \sigma_0 \rho^3 \left( \frac{E^{DC}}{I\Delta l} \right) = -\int_0^\infty \frac{\zeta^2 J_i(\zeta)}{\alpha^{-1} + \tanh(\zeta \rho/d)} \, d\zeta. \tag{3.7}
\]

The integral in expression (3.7) is not analytic, and it is not possible to solve for \( \sigma_1 \) or \( \alpha \) directly in terms of the other variables. However, by plotting the right-hand side of expression (3.7) as a function of \( \rho/d \) for different values of \( \alpha \), as in figure 3.6, the value of \( \alpha \) for a specific value of \( E^{DC} \) may be determined graphically, since all values on the left-hand side are known.

The ERER system thus provides two simple indicators for the conductivity of the sea floor, a transient indicator — the delay time, and a static indicator — the DC electric field. It is interesting to note that while the ability of a transient measurement to determine the conductivity decreases for larger values of \( \rho/d \), the sensitivity of the DC measurement increases. Both indicators are virtually useless under certain conditions. The limitation on the transient indicator is found from the line for \( \alpha = 1 \) in Figure 3.5, since this directly shows the arrival of the sea water signal as a function of \( \rho/d \). This limit is plotted in Figure 3.7 as a dashed line. The delay time is a good indicator of the apparent conductivity for all combinations of \( \alpha \) and \( \rho/d \) above and to the left of this line. For the DC measurement it is not possible to define a simple cut-off point, since the ability to resolve the higher conductivity
Figure 3.7 The delay time $t_d$ is a valid indication of the lower half-space conductivity for combinations of $\alpha$ and $\rho/d$ above the dashed line. The late time electric field $E^{DC}$ will resolve the lower half-space conductivity for combinations of $\alpha$ and $\rho/d$ found beneath the solid line indicating the percentage error in the measurement of $E^{DC}$.

Contrasts depends on the error in the measurement of $E^{DC}$. For instance, given a certain value of $\rho/d$, the relative difference between $E^{DC}$ if $\alpha = 10$ or $\alpha = \infty$ may be only 1%. If the systematic errors involved in the measurement were about 1%, we would say that the limiting contrast measurable by the system is $\alpha = 10$, since higher values of $E^{DC}$ give values of $\alpha$ indistinguishable from $\infty$. Using this criterion, a set of curves corresponding to errors of 1%, 2%, 5%, 10% and 25% have been plotted in Figure 3.7. The use of $E^{DC}$ to determine $\sigma_1$ is valid for all combinations of $\alpha$ and $\rho/d$ below these lines, depending on the accuracy of the system.
3.3 Responses to Other Models

It would be possible to produce for the ERER system characteristic curves similar in form to those produced for the HRHR system in Chapter 2. However, since the results are quite similar to those presented in Chapter 2, that work is not repeated here. Indeed, in the case of the dyke, the transient response is identical except for the dimensional constants.

3.4 Summary

The transient behaviour of the two systems is virtually identical, and the same conclusions drawn about the HRHR system may be applied to the ERER system.

There are, however, several important differences between the two systems. The ratio between the delay time and the conductivity of the crustal half-space varies by too large a degree to permit the use of a simple apparent conductivity formula accurate for values of \( \alpha \) from 1 to 1000. Characteristic curves created assuming a perfectly conductive sea layer are of less use, since they may be inaccurate for real contrasts.

As a consolation, the variation in late-time amplitude as a function of both sea depth and sea floor conductivity provides a means of determining conductivity structure which is complementary to the time delay method. This extra method is no more than the DC resistivity method applied on the sea floor.
Chapter 4

The Prototype HRHR System

4.0 Introduction

Several factors led to my choice of the coaxial magnetic (HRHR) system over the coaxial electric (ERER) system. An electric system must necessarily be in direct electrical contact with sea water. A magnetic system is a sealed unit, completely isolated from the sea water, and so avoids many of the problems caused by water leakage and corrosion found in electrical systems. The motion of a magnetic probe through the earth's magnetic field produces a substantial noise component, while the motion of electrodes produces streaming potentials. It is possible to eliminate the noise in the magnetic receiver by taking measurements with the probe stationary on the sea floor, but not in the electrodes because of water currents. Our group has had a great deal of experience building and working with magnetic receivers, but virtually none in the construction and use of marine potential electrodes. Choosing the magnetic system allowed us to modify an existing, operational probe instead of building potential electrodes from scratch. Finally, it must be acknowledged that a great deal of work has already been done by C.S. Cox and others at the Scripps Institution of Oceanography designing, building and testing their own frequency-based electrical dipole-dipole system. Construction of the untried magnetic system offered greater scope for innovation and scientific investigation.

The possible applications of a short-baseline transient system are numerous, and include assessing off-shore placer mineral deposits, mapping quaternary geology, determining the depth to permafrost layers and in deeper water, studying the physical properties of mid-ocean ridge hydrothermal regimes and associated massive sulphide deposits. Once it was evident that the coaxial magnetic configuration
was a suitable choice for sea-floor EM, I designed and constructed a prototype as a practical test of the method. The prototype HRHR system was designed to operate from a small boat, be of modest size, and have moderate power requirements. The transmitter and receiver coils are towed behind the boat on a single cable. A separation of 50-200 m between the transmitter and receiver was chosen. It is difficult to record transients over much larger distances because of the rapid attenuation of the signal as a function of distance. Over much shorter distances the transients travel too quickly to be easily recorded; in addition, for distances less than about 5 m, the effects of displacement currents may no longer be ignored.

4.1 Design Criteria

Two factors overshadowed all others in the design of the HRHR prototype system; sensitivity, and frequency response. The range over which signals can be detected is determined by the moment of the transmitter and the sensitivity of the receiver coil. The quality of the transients transmitted and received is a function of the frequency response of both the transmitter and receiver.

A current \( I \) flowing through an \( N \)-turn coil of area \( A \) produces on its axis at a distance \( \rho \) a magnetic field \( B \) equal to

\[
B = \frac{\mu}{4\pi} \left[ \frac{2IA}{\rho^3} \right]
\]  

(4.1)

For a fixed transmitter-receiver separation, the product \( IAN \) must be increased in order to increase the magnetic field at the receiver. There is considerable leeway in the choice of appropriate components, once the combination necessary to allow detection has been determined.

The nature of the transient measurement requires sufficiently good frequency response, both in the transmitter and receiver, that the earth response may be deconvolved from the received signal. The transmitter must create a suitably abrupt transient in the magnetic field, and the receiver must be able to retain the higher frequency components of the response.
As a first-order approximation, the magnetic dipole transmitter may be considered as a simple RL circuit, with the inductance \( L \) and resistance \( R \) in series. If a constant voltage \( V \) is applied at \( t=0 \), the current \( I(t) \) flowing in the circuit will be equal to

\[
I(t) = \frac{V}{R} \left[ 1 - \exp(-t/\tau_c) \right], \tag{4.2}
\]

where the time constant \( \tau_c \) is equal to \( L/R \). The value of \( \tau_c \) should be of similar magnitude to, or smaller than, the delay time constant, \( \tau_d = \mu_0 \sigma \rho^2 / 10 \) in order to allow the separation of the effects of the current step-on from the effects of diffusion. For \( \alpha = 10 \), and a transmitter-receiver separation of 50 m, \( \tau \approx .3 \) ms. Minimizing the value of \( L/R \) entails making the circuit more resistive, less inductive, or both. The resistance \( R \) of the system may be adjusted as required by inserting a series element into the circuit. The inductance of system is largely a function of the number and spacing of the turns on the coil, and cannot be decreased once the coil is built.

The requirements of good frequency response and large magnetic moment are difficult to satisfy simultaneously. For instance, increasing the resistance of the transmitter circuit may decrease the rise-time of the transient signal, but also decreases the signal strength. Signal strength, which should be maximized, is proportional to the number of turns \( N \), but the self inductance, which should be minimized, is proportional to \( N^2 \).

Physically, the transmitter coil had to be a shape conducive to towing, and small enough to fit inside the elevators of the physics building. The electrical and physical factors are inter-related, and the design of a working prototype required many trade-offs between them.

4.2 The Transmitter

The equipment which supplies the current to the transmitter coil consists of three basic components: a power source, waveform generator, and voltage transformer.
The power source is two automotive lead-acid batteries wired in parallel which can provide a DC current of 40 A at 12 V continuously for approximately one hour, and much longer when operated intermittently.

The ocean bottom magnetometer high current transmitter was designed and built by the University of Toronto Physics Electronics Workshop, and has previously been used by our group in the Beaufort Sea in the ICE-MOSES Experiment (Edwards et al, 1988). In simplistic terms, the transmitter applies a constant voltage double-pole double-throw FET switch capable of energising the transformer primary with up to 40 A.

A standard 1:10 power transformer located on the ship is used to boost the voltage supplied to the transmitter coil. (The transformer was originally designed for use with 60 Hz electrical power, but operated satisfactorily at much higher frequencies.) The voltage must be increased in order to drive as much current through the transmitter coil as possible.

The transmitter coil is a single-layer cylindrical coil, since the self-inductance of a cylindrical coil is much less than that of a ring. The self-induction of this coil is specified by Nagaoka’s formula (Grover, 1946, p.143). Nagaoka’s formula is

\[ L = 0.002\pi^2 a \left( \frac{2a}{b} \right) N^2 K, \]

where \( a \) is the coil radius, \( b \) its length, and \( K \) is a function of the shape ratio \( 2a/b \), and is an adjustment for end effects. Holding the diameter and number of turns fixed, the inductance is proportional to \( K/b \). For a coil 1 m in diameter, the inductance is reduced by 40% if the length is increased from 1 m to 2 m.

Choosing a coil of large diameter has the double benefit of keeping \( N \) small, while maximizing coil area \( \pi a^2 \). The coil chosen has 100 turns of wire wound evenly along a cylindrical form 1 m in diameter and 2 m long. For \( 2a/b = 0.5 \), \( K = .818 \), so that the calculated inductance of the coil, calculated from expression (4.3), is 8.1 mH. The resistance of the coil wire itself is just 2.65Ω. The majority of the circuit resistance is found in the towing wire, with 17Ω, and in a large power resistor put
in series, which could be adjusted to add between 4Ω and 49Ω. The total resistance could thus be varied from 24Ω to 69Ω, giving rise-times of .34 ms to .12 ms.

The cylindrical shape is hydrodynamically advantageous, because the long axis maintains itself parallel to the direction in which it is pulled. The open ends allow water to flow through, which reduces drag.

Figure 4.1 The HRHR transmitter and receiver coils.

The prototype transmitter coil was designed and built by Questa Design, of Scarborough, Ontario, and is shown in Figure 4.1. The coil is a hollow fiberglass cylinder with a 5 cm thick wall in which is embedded 100 turns of #14 AWG insulated wire, wound along its length. The wire is doubled; one is a spare in case the other becomes broken or damaged. The outer surface has a brilliant orange GEL-COAT® finish, which is abrasion resistant, smooth and aesthetically pleasing. The front end is somewhat tapered to prevent the leading edge from digging into the bottom and becoming stuck. Three interior radial fins provide strength and hydrodynamic streamlining. They also help stop the transmitter from spinning and twisting the tow cable. At the front of the transmitter coil is a hollow water-tight junction box, to which the tow cable is connected. A large stainless-steel spring
provides strain relief for the cable connection to the coil by increasing the radius of curvature in the wire. Inside the junction box the current-carrying wires are split off from the receiver wires. The transmitter coil wires and receiver coil wires connect to feed-throughs on the back of the junction box with underwater plugs.

The receiver wires run down a central hollow tube along the axis of the transmitter. They terminate in a large connector at the back of the transmitter coil, which anchors the receiver cable for towing. The entire transmitter coil apparatus weighs about 1000 N in air (a mass of about 100 kg), but is only slightly negatively buoyant in water.

4.3 The Receiver

![Diagram of receiver coil](image)

Figure 4.2 The receiver coil in a cutaway view, with ferrite core, separated windings and signal guard architecture.

The magnetic receiver was originally designed, constructed and tested by James Lee (Lee, 1985) for use as a three-component down-hole magnetic probe. The coil's high sensitivity and excellent response at high frequencies made it an ideal instrument to use in the HRHR prototype. Both transverse-field sensor coils have been removed, leaving the axial coil. The axial coil, shown in Figure 4.2, consists
of a central ferrite rod (1.27 cm diameter × 15.4 cm length) on which 10,000 turns of #36 AWG wire have been wound in five sections. The inductance of the sensor is 16.5 H and the resonant frequency is about 10.4 kHz, which implies that the equivalent parallel capacitance is about 14 pF. The total effective area is about 252 m². The capacitance was reduced by using five coils in series rather than a single large coil and through the technique of signal guarding. Theoretically, by splitting the coil into five sections, the net interwinding capacitance is reduced by a factor of 25, since each section will have one-fifth the capacitance of a single, large coil and the capacitances of the five sections add in series. The use of active signal guarding reduces capacitances between the winding and the shield, and between the winding and the core. Strips of foil around the the inside and outside of the coils are maintained at potentials close to those of the adjacent sections of coil by means of a resistor voltage divider network powered by the output of the preamplifier. The low potential difference between the coil and its surroundings reduces the capacitive attenuation at higher frequencies. The signal guarding in this coil effectively doubles its bandwidth. The entire sensor is surrounded by a layer of brass foil, which is grounded and serves as a shield to prevent the coil from responding to the electric field. The overlapping edges of the foil are not in electric contact to avoid forming closed circuits around the axis which would reduce the coil’s bandwidth.

The preamplifier of the axial coil, shown in Figure 4.3, is a simple single-input, non-inverting precision low-noise operational amplifier (AD OP-27G produced by Analog Devices) which boosts the output voltage 16.8 times. The output voltage also drives the signal guarding of the coil. Power to the amplifier is supplied by two CMOS programmable micropower voltage regulators (ICL7663/7664 produced by Intersil, see Figure 4.5), which are in turn supplied by two nominal 9 V batteries. These power supplies have a low insertion loss; typically about 1.3 V at an operating current of a few milliamps. The axial coil and preamplification circuitry is housed inside a fiberglass tube with an outer diameter of 41.3 mm (1.625") and an inner diameter of 31.8 mm (1.25") and is designed to easily withstand fluid pressures
Figure 4.3 The preamplifier circuitry. The non-inverted output of the coil is amplified 16.8 times and is also used to drive the signal guarding network, which has five 400 Ω resistors in series.

up to 1 km (fluid pressure: 10 MPa, bursting pressure of tube as estimated from dimensions and material properties: 120 MPa)(Lee, 1986).

The remaining electronics, particularly the power supplies, are housed in a 55 cm aluminum tube, with an inner diameter of 2.7 cm, which is attached to the fiberglass tube housing the coil.

Following preamplification, the signal is amplified 50 times using one side of a JFET-input operational amplifier (TLO82 produced by Texas Instruments, see Figure 4.4).

The current supplied to the transmitter, and the signal returned to the surface from the receiver, are carried in separate conductors in the tow cable. The wires, however, lack shielding, so that a considerable voltage is induced in the signal wires when the current switches polarity. This switching transient is larger in magnitude, and of longer duration, than the received signal, and would normally completely hide the desired signal. The usual solution to such a problem would be to purchase a new cable with shielded conductors or optical fibre — a considerable expense. Instead, an
analogue delay line was installed to delay the transmission of the received signal until after the bulk of the switching transient has passed. The Reticon RD 5106 Analog Delay Line, shown in Figure 4.4, is a 256 sample bucket brigade device. Driven by an external clock, internal sample-and-hold provides a smooth stair step output over each sample period. These devices are manufactured using N-channel silicon-gate technology to fabricate a chain of MOS transistors and storage capacitors.
into a bucket-brigade charge-transfer device. The clock is a programmable crystal oscillator (PX0-1000 produced by Statek), shown in Figure 4.5. This chip was programmed to produce a 250 kHz square wave by setting pin P4 to on and pins P1-3 and P5-6 to off. As two clock cycles are required per sample, the sampling frequency is 125 kHz, and the total delay time should be 256/125 kHz or 2.048 ms.

![Circuit diagram](image)

**Figure 4.5** The power supply and clock circuitry. The 7663 provides an output voltage $V_{\text{OUT}} = (1 + R_2/R_1)V_{\text{SET}}$, where $V_{\text{SET}} = 1.2V$. For the clock, $R_1 = 10k\Omega$, $R_2 = 3.6k\Omega$ and $V_{\text{OUT}} \approx 4.5V$. For the delay line and buffer transistor (see Fig. 6), a second 7663 has $R_1 = 10k\Omega$, $R_2 = 1.2k\Omega$ and $V_{\text{OUT}} \approx 12V$. The clock is programmed to produce a 250 kHz square wave output.

The delay line is preceded by a 32.4 kHz two-pole, low-pass anti-aliasing filter. The output of the delay line is externally buffered (see Figure 4.6) because the internal current is inadequate to drive the low AC coupled output impedance. Following the buffer, a second 32.4 kHz low-pass filter is used to smooth the sampling steps introduced by the delay line. A follower is then used to drive the signal wire, which has considerable capacitance. A transorb — two zener diodes put back-to-back — is used to limit voltage spikes coming down the line. At the surface, another transorb protects the DATA 6000 digital analyzer (see section 4.5) from voltage spikes coming up the line. A 10 kΩ resistor across the DATA 6000 input and an additional 100Ω resistor in series with the signal combine with the 700Ω resistance at the output of the receiver to divide the voltage by a factor of 1.08.
Figure 4.6 The output from the delay line is buffered, then filtered. The output is isolated with a follower and is shielded from voltage spikes coming down the line with a transorb.

Power is supplied using four 9 V batteries. Two provide a ground and ±9 V to the operational amplifiers, the coil signal guards and power supplies, while the other two provide 0-18 V for the delay line.
The entire probe is inserted in a polycarbonate tube for protection from abrasion while it is pulled along the sea floor. The separation between the transmitter and receiver is determined by the length of the cable joining them. Three cables were constructed; 50, 100 and 200 metres in length.

4.4 Receiver Coil Calibration

The receiver coil was calibrated in the Geophysics Electronics Shop at the University of Toronto. Because of the amount of signal amplification performed within the receiver coil, it was not possible to perform a calibration using the entire receiver as deployed at sea — the amount of noise in the laboratory environment made measurements difficult at very low and very high frequencies. Furthermore, the presence of the delay line makes it difficult to determine phase shifts between the input and output — the phase rotates through 360° every time another period fits into the delay interval as frequency is increased. In addition there are phase shifts caused by the coil, amplifiers and filters. For these reasons the calibration was performed in three stages, and involved separate measurements of the coil and preamps, main amplifiers, and the delay line.

The calibration coil consisted of a square coil measuring .71 m on the side with 10 turns, put in series with a 1 kΩ resistor and driven with a sine wave voltage by a Goodwill Instrument Co. GFG-8016D function generator. The receiver coil was initially placed at the center of the calibration coil in order to produce the largest possible response. The voltage signal produced after the preamp stage by the receiver coil was compared to the voltage measured across the 1 kΩ resistor on a Phillips PM3207 analogue oscilloscope. The input current to the calibration coil was determined from the voltage measured across the 1 kΩ resistor. From this and the experimental configuration, the magnetic field produced at the receiver could be calculated. Dividing this value into the output voltage gives an output for the coil measured in V/nT.
Both the amplitude ratios and phase lag were measured for frequencies from 100 Hz to 100 kHz. The amplitude data from these measurements is somewhat suspect because the proximity of the two coils means that strong gradients in the magnetic field are found within the receiver coil. In order to correct for this, a single measurement of amplitude was made with the receiver coil several meters away, where gradients are somewhat reduced. The amplitude values obtained in the first measurements were then scaled to agree with this value.

Figure 4.7 The calibration curve for the HRHR system prototype receiver coil. This frequency response includes the amplitude in V/nT (solid line) and phase (dotted line) response of the coil, preamp, amplifiers and filters in the coil, as well as the amplitude response of the delay line, but omits the phase shifts introduced by the delay line when it delays the signal.

In order to measure the voltage response of the rest of the circuitry, the effects of the delay line must be accounted for. In addition to the rapid phase rotation introduced by the $\approx 2$ ms delay time, there is some attenuation through the delay
line at high frequency. (This additional attenuation is separate from that produced by the 32.4 kHz filters placed before and after the delay line). A voltage signal was applied at the point marked SIGNAL in Figure 4.4. This was compared with the voltage at the point marked OUTPUT in Figure 4.6. Separate measurements were made for amplitude and phase. The amplitude measurements were made with the delay line in place. However, for the phase measurements the delay line was removed — essentially shorting the wires to IN and OUT on the RD5106 in Figure 4.4. The final calibration curve, shown in Figure 4.7, shows the full response in V/nT of the coil, excluding the time delay, and was calculated by multiplying together the coil and electronics calibration curves and then interpolating between measured values. Apart from the purposeful exclusion of the delay itself, the transfer function does not take into account the affects of transmission through the tow cable. This could not be obtained because the cable remains in storage on a winch at the Pacific Geoscience Center in British Colombia.

4.5 Method

The transmitter produces a 12 V square-wave voltage output, which is transformed to 120 V to drive the transmitter coil. A commutation frequency of 100 Hz was chosen. This frequency is low enough to include the majority of a transient event, as well as the delay time, but high enough so that a large number of responses could be obtained and stacked in a short period of time.

The transmitter and receiver are towed along the sea floor by a cable attached to the ship’s winch. A length of cable substantially greater than the water depth must be payed out to ensure that the transmitter remains in contact with the bottom. In order to eliminate noise produced by motion of the receiver coil through the earth’s magnetic field, measurements are made with the system stationary on the sea floor.

Received signals are digitally sampled at 100 kHz by the Data 6000 digital analyzer. A 1024 point, 10.24 ms record contains just over one full wavelength
of information. The current transmitter provides a trigger pulse to synchronize successive traces for stacking. Stacking of the signal is done by the Data 6000 in real time, and a 512 record stack takes 67 seconds to collect. After the stacked signal has been acquired, it is transferred to a floppy disk. A record of the current waveform is provided by measuring the voltage across a .38 $\Omega$ power resistor. A 30 kHz anti-aliasing filter is applied to all signals by the Data 6000 before they are recorded.

4.6 Summary

The HRHR system prototype has been designed for short-baseline surveys, with typical separations of 50-200 m between the transmitter and receiver coils. The transmitter coil, the cable joining the transmitter and receiver coils, the receiver coil sheath, the receiver coil output electronics, and various junction boxes and connectors were newly constructed for this experiment. The remainder of the system was constructed from off-the-shelf type equipment, with minor modifications, which made the entire project quite economical. The system has low power consumption, less than 500 W, and can be run intermittently for several hours from two automotive batteries. Its modest size and weight allow it to be deployed from small research vessels by a small number of personnel.
Chapter 5

The First Field Trials of the Prototype HRHR System

5.1 The Survey Area

The first field tests of the prototype HRHR system were conducted in January, 1988, in the Trincomali Channel, among the Gulf Islands at the southeast end of Vancouver Island, at about 48° 55'N, 123° 27'W (see Figure 5.1). The local sea bottom consists of bedrock overlain by varying thicknesses of mud, ranging from zero to tens of meters, and was chosen so that the response of the system to varying thicknesses of sediment could be ascertained.

5.2 Survey Procedure

An initial survey of the area was conducted using a 6 kHz acoustic sounder to determine the topography, and, to an approximate extent, the sedimentary cover of the bottom rocks. Three different lines in the survey area were chosen with reference to the local bottom conditions. Line A was located in an acoustically featureless area where there appeared to be a uniform, very thick layer of mud. Line B was chosen because it traversed areas where the mud appeared to vary in thickness. Line C contained areas where it was thought that bare rock might be exposed. The acoustic records are difficult to reproduce, and too large to display in the body of the text. Photocopies of the records for Lines A, B and C are included in a folder on the back cover.

During the course of a survey, the ship, the C.S.S. Vector, maintained a constant velocity along the survey line. Measurements, however, had to be made with the system stationary on the bottom. This was accomplished by paying out wire while
Figure 5.1 The survey area. The symbols indicate the approximate position of recording sites. Stations are numbered from left to right; Line A has stations 1-10, Line B has stations 11-15, and Line C has stations 16-20. Contours indicate the depth of the sea in meters.

the measurements were being made, and recovering wire while the ship moved to the next site. Two stacked records were obtained at each site. Slip rings were not used in the winch cable attachment, and because the cable was continuously payed out
during measurements, it was necessary to detach, untwist and re-attach it before the second measurement was made. The number of stacks was limited by the need to keep the twisting of the cable to a minimum.

5.3 Data Processing

![Graph showing signal over time]

Figure 5.2 A typical stacked record; this from station 1b on Line A.

A typical stacked record, from station 1b on Line A, is shown in Figure 5.2. The approximately 10 ms record contains two distinctive types of features: a large, multiply spiked event occurring at 0 ms and 5 ms, and a smoother event of much
smaller amplitude at 2 ms and 7 ms. The large-amplitude event is the voltage transient induced in the receiver wire by the switching transmitter current. The smaller event is the delayed signal returned by the receiver coil. The character of the switching transient is directly related to the time rate of change of the current flowing to the transmitter. In Figure 5.3, the shape of the transient in the receiver wires is compared to the time derivative of the current flowing to the transmitter. The curves have been scaled to the same amplitude for comparison. The gross features of the two curves are very much alike, but the second and subsequent peaks in the signal curve appear to occur sooner after the first peak.

![Graphs showing dI/dt and signal over time](image)

*Figure 5.3* The time derivative of the measured transmitter current is compared to the switching transient induced on the receiver wires.

Figure 5.4 shows an enlargement of the record at the 7 ms position of Figure 5.2. The record has three principal components. The required transient, with a steep onset and gentle decay, is superimposed on a gently increasing background, which is due to the pickup from the transmitter wires. In addition, there is a feature which I call a "wavelet" which occurs at the beginning of all the transients. It consists of
a relatively high frequency disturbance, about 20 mV in amplitude, which lasts for about 0.4 ms. Its form remains relatively constant in all the records even though the desired transient changes markedly. It is likely related to the switching transient. The voltage induced on the signal wires by the switching transient surpassed two volts in amplitude, measured at the surface, and the same voltage could be expected to be present at the output of the receiver electronics. Despite voltage protection at the outputs, a small part of this large signal may have been picked up in the preamplifier or delay line circuitry, delayed two milliseconds, re-amplified and sent back at the start of the signal transient.

In order to analyze the desired transient, it is necessary to remove both the
effect of the switched current and the “wavelet”. To isolate the effect of the switched current, a measurement of the received voltage was obtained while the inputs of the receiver were shorted together. Figure 5.5 shows the voltage measured with the receiver coil removed from the system. This represents the “background” on which the desired signal was superimposed during the surveys. The peak amplitude of the transients in this background record are matched against the amplitude of the corresponding transients in the survey records, and, after appropriate scaling, subtraction of the background is performed. Adjustments are also made for small DC offsets which are 2 mV in amplitude or less.

Figure 5.5 A measurement taken with the receiver disconnected shows the voltage induced in the receiver wires by changes in the current flow through the transmitter wires.
The next processing step is the removal of the "wavelet". Fortunately, the amplitude and duration of the wavelet are both small in comparison with the transients. The wavelet is of a consistent amplitude and shape, so that it is possible to synthesize an "average wavelet" from many records, and then subtract it from the records. This average wavelet is shown in Figure 5.6.

![Wavelet Graph](image)

**Figure 5.6** The synthesized average wavelet constructed from many records.

The final stage in processing the data is to remove the time offset introduced by the delay line. This stage is very important because the position in time of the onset of the transient is one of the most important indications of the sea floor conductivity.

Here I encountered some difficulty in determining the actual offset. The current and signal were not recorded simultaneously, for two reasons. The first was that the ground for the voltage reading across the power resistor to determine the current was at a different level than earth ground, which was the ground for the receiver signal. Attempts to measure the signal and the voltage across the resistor simultaneously
inevitably crashed the Data 6000. The second reason for determining the current and signal separately was that the maximum sampling rate of the Data 6000 is 100 kHz. Sampling two channels simultaneously reduces time resolution by a factor of two, to 50 kHz. The start of the offset period is, however, marked by the beginning of the switching transient induced in the signal wire.

As stated in Chapter 4, the Reticon RD5106 delay line, with a 250 kHz clock signal, should produce a delay of 2.048 ms. Tests with the Data 6000 showed that the delay was 2.06 ms (the maximum resolution of the Data 6000 is .01 ms). Whether the calculated delay is slightly in error or the Data 6000 runs slightly too fast is not important in determining the delay. Since measurements in the field were made with the Data 6000, the expected offset will be at least 2.06 ms. Additional delays, such as that due to the finite speed of signals travelling through wires, should add less than an additional .01 ms.

Since the wavelet appears to be an artifact of the switching transient, it should provide a time marker to indicate the delayed start of the switch in current — an unexpected (though not entirely welcome) bonus. Unfortunately, the start of the wavelet is lost in the noise in the signal and is difficult to locate. It comes somewhere between 2.06 to 2.10 ms after the switching transient. A second method for determining the offset is to locate the start of a large amplitude transient. The delay time and transient amplitude are inversely related. The fastest diffusion produces the fastest changes in the magnetic field, and therefore the largest signals. The largest transient should appear at the receiver almost immediately after the current is switched. The station with the largest (and earliest) transient is station 17a on line C. Its transient appears to start 2.07 ms or 2.08 ms after the switching transient.

Of all the above methods, the measured delay of the circuit itself is most accurately determined, and it is difficult to imagine how more than an extra .01 ms could be added to the offset by other causes, such as propagation time through the tow cable. In light of the above, an offset of 2.07 ms between the initial switch in
current polarity, and the recorded transient signal is used. The uncertainty in the offset is about .01 ms, equal to the resolution of the system. Although there were two transients recorded on each record, only the second is used. All of the switching transient which precedes it is exposed, which makes it simpler to determine the onset position. Since the second switching transient begins at 5 ms, a total of 7.07 ms is subtracted from the time axis in order to position the start of the desired transient at time zero.

![Graph showing signal over time](image)

Figure 5.7 A section of the record from station 1b after the “wavelet” and the switching transient have been removed.

A fully processed transient, with background switching transient, glitch and time offset removed, is shown in Figure 5.7. This is from station 1b on Line A, and should be compared with the unprocessed signal in Figure 5.4. A total of 39 records were obtained at 20 sites on the three lines. Two of these, from stations 5a
and 5b, were of such an anomalous character in relation to the rest of the data that they were removed from further consideration. They produced an apparent signal five times larger than any of the other stations, and of the opposite polarity to the others. In addition, there was but one successful measurement made at site 2.

5.4 Modelling the Data

The theory we presented in Chapter 2 has been implemented as a computer algorithm so the the HRHR response of a layered earth model may be calculated for this particular system. The forward model response consists of three independent parts: the input current waveform, the model's impulse response, and the receiver transfer function. A particular forward model response is determined by the convolution in time domain of these three parts, or equivalently, their product in frequency domain. In theory, it is possible to recover the actual impulse response of the sea floor by deconvolving the input current and receiver transfer function from the data. This procedure, however, is less stable than calculating the full forward model. The deconvolution of the signal, which has some high-frequency noise, by the input current waveform and receiver transfer function, which are attenuated at higher frequencies, produces an impulse response with significant high-frequency noise. The full forward model calculation however, produces smoothly varying curves that are more easily compared with the data.

The impulse response for each model was calculated at 25 times and then interpolated using a cubic spline. The impulse response was more heavily sampled at early time, where it changes most rapidly. The receiver transfer function remained constant, and the input current nearly constant throughout the experiment. Some computational effort was saved in the forward modelling procedure by pre-computing the product of the input current and receiver transfer functions for later convolution with the impulse response.
Figure 5.8  Theoretical responses of the prototype HRHR system to the double half-space model are plotted for several values of the conductivity contrast $\alpha$.

The theoretical responses of the HRHR system to the double half-space model for a range of values of the conductivity contrast $\alpha$ are plotted in Figure 5.8. For larger values of $\alpha$ the response begins to reflect the character of the time derivative of the transmitter current, shown in Figure 5.3. This occurs because the sea-floor portion of the impulse response approximates a theoretical impulse function for large values of $\alpha$. The form of the magnetic field at the receiver approximates the form of the current waveform in the transmitter. The correspondence is not exact because there is also a seawater transient, the receiver does not perform a perfect differentiation of the signal, and the sea floor transient is not a perfect impulse.

The chosen value of the seawater conductivity, 3.05 S·m$^{-1}$, was determined from expression (1.3) using a salinity of 28.9 parts per thousand, and a water tem-
perature of $8^\circ$ C. These are typical values for this area in the month of January (Lawrie Law, personal comm.). The water in the channel is well mixed, and the water temperature may be considered constant from the surface to the bottom.

The data are inverted to give layered earth models using generalized linear inversion techniques. A model proposed to fit the data is characterized by one or more parameters which may include conductivities or layer thicknesses. The technique determines the changes in the forward model produced by small variations in the model parameters and then seeks to minimize the misfit between the forward model and data by adjusting the parameters. The process precedes through several iterations until a 'best fit' is achieved. Once a 'best fit' model is determined, an estimation of the resolution of individual parameters, as well as the interrelations between parameters, may be obtained using eigenparameter statistical analysis. A detailed explanation of the statistical procedure is contained in Appendix D.

The double half-space model is the simplest possible model which can be used to represent the conductivity structure of the sea-floor; a uniform sea overlies a uniform crustal half-space. The one significant model parameter is the contrast $\alpha$ between the conductivities of the two half-spaces. In order to fit the observed data, however, three more parameters had to be introduced, a scale $s$, a voltage offset $V_{\text{off}}$, and a time offset $t_{\text{off}}$.

The scale factor is required for a number of reasons. There is an uncertainty in the absolute scale of the transfer function of the receiver, since it was determined at short range where strong gradients in the magnetic field were present. Small misalignments between the transmitter and receiver, mentioned in Chapter 2, affect the received amplitude, since only a component of the desired signal is recorded. Finally, small changes in the transmitter current amplitude occur due to battery drain. Considered together, these factors may require that amplitudes of the model curves be scaled by a value $s$ which varies downward from a maximum near 1 by 10-20%.

The parameter $V_{\text{off}}$ is a small additional correction of the offset produced by
the switching transient, since the method used in Section 5.3 to remove the voltage offset is somewhat imprecise. $V_{\text{off}}$ is not likely to exceed 1-2 mV.

Finally, the time offset $t_{\text{off}}$ allows for small errors in determining the position of the start of the transient in the data. Since the delay introduced by the delay line and the position in time of the switch in the current polarity can be determined rather precisely, this error should be quite small, perhaps one digitization interval or .01 ms.

<table>
<thead>
<tr>
<th>Stn</th>
<th>$\alpha_1$</th>
<th>scale</th>
<th>$t_{\text{off}}$ (ms)</th>
<th>$e$(mV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1a</td>
<td>2.40±0.32</td>
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<td>0.080±0.036</td>
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<td>3a</td>
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<td>1.05±0.26</td>
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<tr>
<td>3b</td>
<td>2.40±0.33</td>
<td>1.08±0.25</td>
<td>0.082±0.036</td>
<td>1.43</td>
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<tr>
<td>4a</td>
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</tr>
<tr>
<td>4b</td>
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<td>0.089±0.032</td>
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<td>6a</td>
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<td>7b</td>
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<td>8a</td>
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<td>9a</td>
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<td>9b</td>
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<td>10b</td>
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Table 5.1 The best-fit double half-space model parameters for Line A data. The conductivity contrast $\alpha_1$, the scaling factor, the time offset and rms error are tabulated. Errors are those calculated using the method shown in expression (D.17) and represent coarse upper bounds on the uncertainty of parameters.

The results of the inversion modelling for Lines A, B and C, using the double half-space model have been tabulated in Tables 5.1, 5.2 and 5.3 and are displayed graphically in Appendix E, Figures E.1-3, E.4-5, and E.6-7 respectively. Coarse up-
Table 5.2 The best-fit double half-space model parameters for Line B data.

<table>
<thead>
<tr>
<th>Stn</th>
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<th>$e$(mV)</th>
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</thead>
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<td>11a</td>
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<td>12a</td>
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<td>12b</td>
<td>4.23±0.51</td>
<td>1.30±0.40</td>
<td>0.102±0.012</td>
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</tr>
<tr>
<td>13a</td>
<td>5.78±0.76</td>
<td>1.69±0.51</td>
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<td>13b</td>
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<td>14a</td>
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Table 5.3 The best-fit double half-space model parameters for Line C data.

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<th>Stn</th>
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<th>$t_{off}$ (ms)</th>
<th>$e$(mV)</th>
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<td>1.17±0.16</td>
<td>0.004±0.003</td>
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<tr>
<td>16b</td>
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<td>1.12±0.45</td>
<td>0.000±0.003</td>
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<tr>
<td>17a</td>
<td>13.4±2.9</td>
<td>1.13±0.62</td>
<td>-0.024±0.003</td>
<td>3.41</td>
</tr>
<tr>
<td>17b</td>
<td>15.3±8.9</td>
<td>0.80±0.55</td>
<td>0.022±0.005</td>
<td>4.10</td>
</tr>
<tr>
<td>18a</td>
<td>5.53±0.65</td>
<td>1.50±0.53</td>
<td>0.000±0.007</td>
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</tr>
<tr>
<td>18b</td>
<td>5.47±0.86</td>
<td>1.40±0.57</td>
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<tr>
<td>19a</td>
<td>5.34±0.73</td>
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<tr>
<td>19b</td>
<td>4.85±0.54</td>
<td>1.45±0.49</td>
<td>0.024±0.009</td>
<td>3.08</td>
</tr>
<tr>
<td>20a</td>
<td>5.22±0.66</td>
<td>1.52±0.51</td>
<td>0.000±0.008</td>
<td>2.70</td>
</tr>
<tr>
<td>20b</td>
<td>5.29±0.73</td>
<td>1.48±0.54</td>
<td>-0.003±0.008</td>
<td>2.61</td>
</tr>
</tbody>
</table>

The lower half-space conductivity profile varies both in magnitude and character between the different lines. The conductivity contrast remains relatively constant at about 2.5 along the entire length of line A. For Line B the contrast increases...
from a minimum of 2.5 at station 11a to a maximum of 5.8 at station 13c, then decreases to 2.8 at station 15b. The largest contrasts were measured on line C: about 12.5 at stations 16a and 16b, and 13.4 and 15.3 at stations 17a and 17b. The remainder of the line covered an area with an apparent contrast of about 5. The rms error ranges from as small as 0.86 mV for station 1a up to 3 mV for stations 11b and 12b. The size of the error appears to increase with the the conductivity contrast. The higher contrasts produce a poorer fit, especially at the peak of the transient, which may be due to uncertainties in the transfer function of the receiver at high frequencies. That the ‘a’ and ‘b’ recordings at the same site sometimes give markedly different results is not too surprising. When the connecting cable was untwisted between measurements, the winch was stopped. The system may have been dragged a short distance by the ship during this interval.

Some of the inversion results for the scaling factor are curious. For small values of $\alpha$, such as those found in Line A, the scaling factor varies by about 10% around a value of 1.05, which seems reasonable in light of the problems in determining the receiver transfer function, receiver orientation and current supply outlined above. Similar values are found for sites 16 and 17, where there is a large contrast. At the other sites, however, where there is a contrast of 3-6, the required scaling factor ranges as high as 1.69.

The time offset is also enigmatic. For Line A, the curves must be shifted by .08 to .10 ms from the expected delay for the best fit. Even larger shifts, up to .13 ms for station 15a, are required for Line B. Observation indicates that the correction for the analogue delay line is incorrect. In line C, however, the amount of shift is low, and even slightly negative, suggesting the analogue delay line correction for the data is correct. The delay should remain constant for all sites.

An explanation of the apparent discrepancies in scale and time is that the wrong model was used to fit the data. With the exception of Line A, those stations with the largest scale factor and largest time offset share a common mismatch between the data and the best-fit model curves (see Figures E4-7). Although the response
at early times seems to be well fitted, at late times the measured transient decays away more quickly than the model transient. The simplest explanation for this faster decay at later times is a decrease in the conductivity of the earth with depth, which would result in faster diffusion at late times. A plausible model would be one in which the conductivity of the earth decreases smoothly with depth due to the compression of sediments and reduction in fluid content (cf. expression 1.5). Alternatively, the decrease in conductivity may be abrupt, as might be expected where a thin sediment layer overlies a more resistive basement. This latter model was chosen for modelling because it is consistent with acoustic records taken on the cruise, which indicated a sharp boundary beneath the sea floor in some areas. Initial values of the conductivity of the upper sediment layer and lower halfspace for this model were chosen by examination of the double-halfspace conductivities determined earlier. In Line A the double half-space model provided an excellent fit to the data (apart from the anomalous values of $t_{off}$), and there is no indication on the acoustic records of any underlying structure. The area appears to be covered by a deep layer of mud or sediment. The conductivity contrast $\alpha_1$ of the upper sediment layer throughout the area is not likely to differ substantially from that determined for Line A, about 2.5. This corresponds to a conductivity of 1.22 S·m$^{-1}$, a reasonable value for unconsolidated mud or gravel. The higher conductivity contrasts found at the other stations are then a function of the depth to the lower resistive halfspace (cf. section 2.3). The contrast of the lower halfspace must exceed the values indicated at sites 16 and 17. Modelling determined that the best value was about 30. This corresponds to a conductivity of about 0.1 S·m$^{-1}$.

The data from Line B and Line C was re-interpreted using the conductive layer over a resistive half-space model. This time the conductivity contrast of the upper layer was fixed at 2.5, and the lower layer at 30, and the thickness $d_1$ of the top layer was allowed to vary, along with scale $s$, voltage offset $V_{off}$ and time offset $t_{off}$ as above.

The results are tabulated in Tables 5.4 and 5.5, and are displayed graphically.
Table 5.4 The best-fit layer over a half-space model parameters for Line B data. The conductivity contrast $\alpha_1$ and the depth of the top layer, the conductivity contrast $\alpha_2$ of the lower halfspace, the scaling factor, the time offset and rms error are tabulated.

<table>
<thead>
<tr>
<th>Stn</th>
<th>$\alpha_1$</th>
<th>$d_1 (m)$</th>
<th>$\alpha_2$</th>
<th>scale</th>
<th>$t_{\text{off}}$ (ms)</th>
<th>$e$(mV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>11a</td>
<td>2.5</td>
<td>—</td>
<td>30</td>
<td>1.03±0.28</td>
<td>0.068±0.035</td>
<td>1.24</td>
</tr>
<tr>
<td>11b</td>
<td>2.5</td>
<td>24.5±3.5</td>
<td>30</td>
<td>0.94±0.13</td>
<td>0.061±0.014</td>
<td>0.86</td>
</tr>
<tr>
<td>12a</td>
<td>2.5</td>
<td>22±3.1</td>
<td>30</td>
<td>0.98±0.11</td>
<td>0.071±0.014</td>
<td>0.92</td>
</tr>
<tr>
<td>12b</td>
<td>2.5</td>
<td>17±2.5</td>
<td>30</td>
<td>1.10±0.10</td>
<td>0.056±0.015</td>
<td>1.66</td>
</tr>
<tr>
<td>13a</td>
<td>2.5</td>
<td>11±2.2</td>
<td>30</td>
<td>1.40±0.14</td>
<td>0.006±0.008</td>
<td>1.24</td>
</tr>
<tr>
<td>13b</td>
<td>2.5</td>
<td>13.5±2.4</td>
<td>30</td>
<td>1.20±0.11</td>
<td>0.017±0.013</td>
<td>2.26</td>
</tr>
<tr>
<td>14a</td>
<td>2.5</td>
<td>18±2.8</td>
<td>30</td>
<td>0.99±0.10</td>
<td>0.053±0.016</td>
<td>1.04</td>
</tr>
<tr>
<td>14b</td>
<td>2.5</td>
<td>18±2.8</td>
<td>30</td>
<td>0.92±0.09</td>
<td>0.060±0.016</td>
<td>1.55</td>
</tr>
<tr>
<td>15a</td>
<td>2.5</td>
<td>23±3.2</td>
<td>30</td>
<td>1.04±0.13</td>
<td>0.072±0.014</td>
<td>0.92</td>
</tr>
<tr>
<td>15b</td>
<td>2.5</td>
<td>28±3.8</td>
<td>30</td>
<td>1.06±0.17</td>
<td>0.063±0.011</td>
<td>0.89</td>
</tr>
</tbody>
</table>

Table 5.5 The best-fit layer over a half-space model parameters for Line C data.

<table>
<thead>
<tr>
<th>Stn</th>
<th>$\alpha_1$</th>
<th>$d_1 (m)$</th>
<th>$\alpha_2$</th>
<th>scale</th>
<th>$t_{\text{off}}$ (ms)</th>
<th>$e$(mV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>16a</td>
<td>2.5</td>
<td>3.5±0.7</td>
<td>20</td>
<td>1.05±0.12</td>
<td>0.027±0.006</td>
<td>2.20</td>
</tr>
<tr>
<td>16b</td>
<td>2.5</td>
<td>3.5±0.8</td>
<td>20</td>
<td>1.09±0.13</td>
<td>0.020±0.006</td>
<td>2.33</td>
</tr>
<tr>
<td>17a</td>
<td>2.5</td>
<td>3.0±0.8</td>
<td>20</td>
<td>1.07±0.16</td>
<td>0.032±0.003</td>
<td>3.19</td>
</tr>
<tr>
<td>17b</td>
<td>2.5</td>
<td>3.0±0.8</td>
<td>20</td>
<td>0.84±0.13</td>
<td>-0.001±0.004</td>
<td>4.00</td>
</tr>
<tr>
<td>18a</td>
<td>2.5</td>
<td>11.0±2.3</td>
<td>30</td>
<td>1.20±0.13</td>
<td>0.016±0.010</td>
<td>1.37</td>
</tr>
<tr>
<td>18b</td>
<td>2.5</td>
<td>11.2±2.4</td>
<td>30</td>
<td>1.12±0.12</td>
<td>0.019±0.011</td>
<td>1.27</td>
</tr>
<tr>
<td>19a</td>
<td>2.5</td>
<td>11.5±2.8</td>
<td>30</td>
<td>1.25±0.15</td>
<td>0.001±0.010</td>
<td>1.41</td>
</tr>
<tr>
<td>19b</td>
<td>2.5</td>
<td>13.7±3.0</td>
<td>30</td>
<td>1.21±0.14</td>
<td>0.017±0.013</td>
<td>1.86</td>
</tr>
<tr>
<td>20a</td>
<td>2.5</td>
<td>12.2±2.8</td>
<td>30</td>
<td>1.22±0.14</td>
<td>0.008±0.011</td>
<td>1.61</td>
</tr>
<tr>
<td>20b</td>
<td>2.5</td>
<td>12.0±2.9</td>
<td>30</td>
<td>1.18±0.14</td>
<td>0.006±0.011</td>
<td>1.60</td>
</tr>
</tbody>
</table>

in Figures E.8-E.11. At most stations there is a substantial improvement in the rms error, which is expressed graphically by a much closer fit in the late-time responses. As the number of parameters used is the same as for the double half-space model, the improvement in fit is significant. The interpreted depth to the lower halfspace varies from about 3 m for sites 16 and 17 up to nearly 30 m for station 15b. A better fit was achieved at sites 16 and 17 using a lower half-space contrast of 20. This result
is reasonable in light of the closer proximity of the lower half-space to the sea water. Still, at stations 16a, 17a and 17b there was only a marginal improvement in fit, and there was an actual decrease in the goodness of fit at site 16b, so statistically this model is not to be favored over the uniform half-space model at these sites.

In addition to the improvement in fit there is an improvement in both the scale factor and time offset. At all stations where these values were anomalously large they have decreased substantially. Although they still remain unacceptably large at some stations, perhaps a more sophisticated, multi-layer model of the sea floor could reduce them further. In the meantime, the size of these parameters provides an indication of the misfit between the model and the true conductivity structure.

It is difficult to correlate the acoustic data from Line B with the electrical modelling. The position of the stations is not known with any precision. The ship's location at the start of each measurement is marked, but this is not the exact position of the towed system. Moreover, although an interface, possible due to the basement, can be followed along parts of Line B, it often rises or falls sharply in the vicinity of a measurement site, so estimation of its depth is difficult or impossible. Nevertheless, there is good agreement on Line C between the interpreted depths in Table 5.5 and the relative thicknesses of the top layer shown on the acoustic records. At sites 18, 19 and 20 there is a clear surface layer whose depth remains fairly constant. Between station 17 and 18 the layer is pinched out, and there is no indication of it at stations 16 and 17, indicating that the lower, more resistive basement is at or very near the surface.

A remaining problem in interpreting the data is the large offset times observed for sites on Line A. There is an excellent fit to the observed waveform at all times using the double half-space model; it is impossible to improve on it using the layer over a half-space model. A solution comes from a different direction. At several stations on other lines, notably stations 17a and 17b, there is a discrepancy at very late times between the models and data. For these cases, the high contrast should produce at very late times a voltage decaying to zero. Instead, the data displays
a noticeable positive trend. This is likely due to an imperfect subtraction of the background switching transient. If such an error occurred in the processing of Line A it would be hidden, because there is still a substantial voltage at 2.5 ms. If it were present, though, it could produce the apparent time offset — lowering the tail of the transient forces the inversion routine to match the curve with a slightly higher contrast. Because the onset of this model transient occurs earlier, the whole curve must be shifted later to match the data, and an anomalous time shift is produced. Unfortunately, it is not possible to improve the data processing because the background appears to vary somewhat between stations and lines, and there was but one measurement of the switching transient with the receiver removed.

5.5 Summary

The first test of the HRHR system prototype has proved the utility of transient EM in seafloor conductivity mapping. Data from 19 sites and 37 stations has been collected, processed and interpreted in relation to both the double half-space and layer over a half-space models.

The use of unshielded conductors to carry both the transmitted current and received signal results in a great deal of interference on the signal wires, but this has been largely overcome through the use of an analogue delay line which holds the received transient until after the main effects of the switching transient have passed. Much of the remaining effect of the switching transient can be removed by measuring the voltage induced when the receiver is disconnected, and subtracting this from the records. A second manifestation of the switching transient is the noise wavelet which precedes the records. Fortunately its small amplitude and duration made it relatively easy to remove.

The theoretical response of the system to a particular model was synthesized by combining the measured input current and receiver transfer function with the impulse response of the model. This forward model was then used for inversion
modelling of the collected data. The transients vary widely in both amplitude and yet all can be closely fitted using the simple model of a conductive sediment layer of varying depth overlying a more resistive basement. The overlying sediment has a conductivity of about 1.2 S·m⁻¹ while the basement is much less conductive, at about 0.1 S·m⁻¹. In the region of Line A, the thickness of the sediment is such that the depth to the basement cannot be resolved. The thickness of the top layer would likely be greater than 25 m, since the system appears to be able to resolve the basement at nearly that depth at stations 11b and 12a. This is consistent with the acoustic survey, which showed no evidence of structure in this area. In Line B the depth to the basement varies from about 10 to 25 m. In Line C the basement is close to the surface at sites 16 and 17, but is buried by a layer of sediment 10-15 m thick at sites 18, 19 and 20, a result in agreement with the acoustic log.
Chapter 6

Conclusions

Section 6.1 General Observations

A study of some of the possible configurations for a transient, sea floor electromagnetic system reveals that two, the HRHR and ERER systems, are well-suited to the determination of sea floor conductivity if the sea floor is less conductive than seawater. The location in time of the initial rise in the transient response, called here the delay time, $t_d$, provides a direct means of calculating an apparent conductivity of the sea floor.

The separation in time of the transient response into sea floor and seawater parts enables the relevant, early time response of a number of models to be computed simply, using the approximation that the upper, sea layer is perfectly conductive. Characteristic curves based on apparent conductivities were constructed for a number of layered earth models. Their form is in many ways similar to curves based on apparent resistivity used for land-based DC resistivity measurements.

The transient responses of three layered-earth models illustrate the strengths and weaknesses of the transient method. The models consist of a uniform half-space with one additional feature: a resistive basement, a resistive zone, or a conductive basement. The systems are quite sensitive to a decrease in conductivity with depth, but generally insensitive to an increase, because the chief control on the rate of diffusion is the path of greatest electrical resistivity. If conductivity decreases at depth, faster paths become accessible as the system spread increases. If conductivity increases, the path near surface remains the most direct route, and little change in the response is observed. The response of a thin, resistive zone in a crustal half-space differs substantially from the response of the half-space alone. This result
demonstrates a fundamental difference in the induced current modes for the sea floor and land environments. The HRHR system generates only the PM mode on land; the induced current flow is horizontal in a 1-D earth. In the sea both the TM and PM modes are produced because current can cross the boundary between the sea and sea floor, where the transmitter is located.

A vertical, infinite, conductive dyke, outcropping at the sea floor, produces an additional delay in the transient response, and the increase in the delay is closely related to the conductance of the dyke. This shows that the transient system response is sensitive to the integrated conductivity between the transmitter and receiver.

In some situations where the sea layer is very shallow the fastest route for diffusion is through the seawater and air, and the delay time is no longer a valid indicator of the conductivity of the sea floor. A simple approximate expression (2.14) describes the largest apparent conductivity resolvable from the delay time given the system separation and sea depth. Even in shallow seas the sea floor signal is present, but the waveform as a whole must be examined, and the depth of the sea carefully monitored.

Errors in alignment of a transient system produce changes in signal amplitude, but not in delay, and so are not serious. On the other hand, any factor which raises the transmitter or receiver from the ocean bottom will increase the delay time and the apparent conductivity — it is important that the system remain in contact with the sea floor during operation.

The PRER system differs from the HRHR system mainly in the form of its late-time response. The amplitude of the late-time response is dependent on the conductivity structure, and in some cases may provide the operator with useful additional information. This property is most helpful in very shallow water, where the sea floor transient may be obscured by the air wave.

I chose the HRHR system for further study for several reasons. The magnetic system is sealed, and avoids problems associated with porous electrodes. Although
the HRHR probe must (at present) be stationary while measurements are taken, it
avoids problems such as stream potentials which can affect electrical systems. Our
group has more experience with magnetic coils than marine electrodes. I was able
to adapt a previous tested coil instead of designing and building marine electrodes
from scratch. Finally, I did not wish to duplicate work done already on a frequency-
based ERER system at the Scripps Institute of Oceanography; the HRHR system
allowed more scope for innovation and discovery.

The design of a practical HRHR system involves trade-offs among many con-
fllicting design requirements. The system requires a transmitter with a large mag-
netic moment, and a receiver with a high sensitivity. They must both have good
enough frequency responses that it is possible to deconvolve the impulse response
of the earth from the received signal.

The large diameter transmitter used in the prototype maximizes the signal
strength while minimizing the number of turns and resultant self-induction. The
cylindrical coil design has a better frequency response than a close-wound coil, and
makes the transmitter naturally hydrodynamic, maintaining its axis parallel to the
direction in which it is pulled.

The prototype receiver coil was a modified borehole probe. This high-frequency
coil produced a voltage signal proportional to the change in the measured magnetic
field. The chief problem with the system as first designed was the coincidence of a
very large transient, produced by the change in the current, with the desired tran-
sient recorded by the receiver. An expensive solution would have been to purchase
a new cable with fully shielded conductors, at a cost of 10$ per meter or more. The
simple, inexpensive alternative was to add an analogue delay line which delayed the
signal to a point where when the switching transient was much smaller, and more
predictable in nature.

The prototype HRHR system was constructed of a combination of newly fab-
ricated and off-the-shelf type equipment, which made the entire project quite eco-
nomical. The system draws less than 500 W of power, and can be run intermittently
off two automotive batteries for several hours. Its modest size and weight allow it to be deployed from small research vessels by a small number of personnel.

The first test of the system was very successful, with three lines of data recorded at 37 sites. The signals are of useable quality, and show a wide range of responses. Forward modelling indicates that the results are consistent with a model in which a layer of sediment 2.5 times more resistive than sea water overlies a basement about 30 times as resistive as sea water. The depth of the layer varied from 3 to 25 m. With only a few exceptions, the data was fit to an rms error of 1 or 2 mV.

Section 6.2 Recommendations

The design of the transmitter, while adequate to test basic aspects of a transient EM system, is not adequate for more practical survey work. Several improvements, both in equipment and in procedure, are recommended.

The transmitter is presently located on board the ship, along with the power supply. The frequency response of the transmitter coil is presently controlled by varying the resistance of a large power resistor on board the ship. The disadvantage is that most of the power used is lost in the resistor and tow cable. A more reasonable system would produce the transmitter current waveform electronically from a pressure vessel housed within the transmitter coil assembly. The reduction in power consumption would enable the power supply to be included inside the transmitter coil itself. The electrical load (the transmitter coil) could be more easily matched to the current supply, and bounces caused by impedance mismatches would not be a problem. The form of the input current would be much smoother. The current should then be sampled in the transmitter while the system is in operation. A smoother waveform, combined with a more accurate representation of the current, would make it simpler to invert the data later.

With the switching transient removed, it would no longer be necessary to delay the received signal. This eliminates problems encountered here of determining where
to put \( t = 0 \), as well as with unknown voltage offsets. The noise glitch found at
the start of the signals should also disappear. Transmitting the signals digitally, as
an FM signal, or using shielded cables or optical fibers, are all possible means of
reducing pickup noise.

The Data 6000 digital analyzer, while a fine instrument in its prime, could be
much improved on. A great deal of the data is lost because the machine spends
so much time doing analogue to digital conversions and stacking. Theoretically, it
should take just 5 seconds to obtain one full period (two switches) stacked 500 times
with a 100 Hz commutation frequency. The DATA 6000 requires over 90 seconds.
With the present setup, it is also not possible to sample both the current and signal
simultaneously because of ground loops. Even if that were not a problem, the scope
is limited to a 100 kHz sampling rate (adding all channels), and recording two
channels simultaneously means getting, at best, two 50 kHz records. The archaic
file control system and byzantine operation procedure make errors in operation
inevitable, and contributed to the loss of important data.

More information could be obtained using multiple receivers. Receivers could
be placed 10, 20, 50, 100 and 200 meters apart. Use could then be made of char-
acteristic curves such as those developed in Chapter 2. This would require more
sophisticated data transmission and recovery techniques to handle the increase in
information.

Improvements in the rate of data acquisition, and the reduction of noise due
to better equipment design, may soon allow measurements to be made with the
system in continuous motion. Real-time processing could be employed to calculate
apparent conductivities while the survey is run. At that time it should be possible
to produce, in a single pass, continuous maps of the conductivity structure of the
ocean floor in much the same way that maps of electrical or magnetic properties
are produced using airborne EM systems today.
References


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Appendix A

The HRHR System — Analytic Solutions

A.1 The Step Response for the Double Half-space Model

In order to test the validity of the numerical solution for the HRHR response, I have derived this approximate, analytic solution for the step response of the HRHR system for the double half-space model described in section 2.1. The analytic solution is valid for large values of the conductivity contrast $\alpha$, and remains a good approximation for values of $\alpha$ as small as 10.

The Laplace transform of the magnetic field measured at the receiver by the HRHR system is given by

$$H_\rho(s) = \frac{n(s)}{2\pi} [F_{TM}(s) + F_{PM}(s)],$$  \hspace{1cm} (A.1)

where $n(s)$ is the Laplace transform of the source current dipole moment and $F_{TM}$ and $F_{PM}$ are defined as in expressions (2.2) and (2.3). The relevant inductances and impedances, derived from expressions (2.4) and (2.5), are $Q^- = \mu_0/u_1$, $Q^+ = \mu_0/u_0$, $Y^- = u_1/\sigma_1$, and $Y^+ = u_0/\sigma_0$, where $u^2 = \lambda^2 + k^2$, and $k^2 = i\omega\mu_0\sigma$. The subscript 0 refers to the sea water and the subscript 1 refers to the sea floor. We assume throughout that $\mu_i = \mu_0$, the permeability of free space.

The modal expressions (2.2) and (2.3) become

$$F_{TM}(s) = -\frac{\mu_0\sigma_0\sigma_1 s}{\rho} \int_0^\infty \frac{J_1(\lambda\rho)}{u_0\sigma_1 + u_1\sigma_0} d\lambda,$$  \hspace{1cm} (A.2)

and

$$F_{PM}(s) = -\frac{\partial}{\partial \rho} \int_0^\infty \frac{u_0u_1}{u_0 + u_1} J_1(\lambda\rho) d\lambda.$$  \hspace{1cm} (A.3)

Expression (A.2) cannot be evaluated analytically because of the presence of multiple branch cuts. However, it may be approximated for certain cases. The
conductivity $\sigma_0$ of the upper half-space, i.e. the sea, is usually significantly greater than the conductivity $\sigma_1$ of the lower half-space, the crust, so that we may expand (A.2) in powers of $u_0\sigma_1/u_1\sigma_0$ and drop second order and higher terms. A complementary approximation may be made when the seafloor is relatively conductive ($\sigma_1 \gg \sigma_0$). It is still not possible to evaluate terms with $u$ in the numerator because of the branch cuts, so a second assumption is made: in the range of values of $\lambda$ where the significant contribution to the total integral is made, $\lambda^2 \ll |s\mu_0\sigma_0|$, so that $u_0$ may be approximated by $k_0$. This approximation is valid in the mid-field ($|s\mu_0\sigma_0| \gg 1/\rho^2 \gg |s\mu_0\sigma_1|$) and in the far field ($|s\mu_0\sigma_1| \gg 1/\rho^2$). In the near field ($1/\rho^2 \gg |s\mu_0\sigma_0|$), where the greatest discrepancy is introduced, the error inherent in the approximation may be shown to be at most of order $\sigma_1/\sigma_0$. Expression (A.2) under this approximation becomes

$$F_{TM}(s) = \frac{k_0^2}{\rho} \int_0^\infty \frac{1}{u_1} \left[ 1 - \frac{k_0\sigma_1}{u_1\sigma_0} \right] J_1(\lambda\rho) d\lambda. \quad (A.4)$$

The first and second terms in (A.4) may be evaluated using the standard integrals

$$\int_0^\infty \frac{J_1(\lambda\rho)}{u} d\lambda = \frac{1}{k\rho} \left[ 1 - \exp(-k\rho) \right], \quad (A.5)$$

and

$$\int_0^\infty \frac{J_1(\lambda\rho)}{u^2} d\lambda = \frac{1}{k^2\rho} \left[ 1 - k\rho K_1(k\rho) \right], \quad (A.6)$$

where the latter may be derived by the differentiation of Mehler’s Integral (Watson, 1966). The expression for the TM mode then becomes

$$\rho^2 F_{TM}(s) = \frac{\sigma_1}{\sigma_0} k_0\rho - k_1\rho \exp(-k_1\rho) - \sqrt{\frac{\sigma_1}{\sigma_0}} k_1^2 \rho^2 K_1(k_1\rho). \quad (A.7)$$

Expression (A.3), for the PM mode, may be evaluated by multiplying the numerator and the denominator by $u_0 - u_1$. This yields the exact expression

$$F_{PM}(s) = \frac{1}{k_0^2 - k_1^2} \frac{\partial}{\partial \rho} \left[ \int_0^\infty (u_0 - u_1)\lambda^2 J_1(\lambda\rho) d\lambda \right]$$

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\[ + \int_{0}^{\infty} (k_1^2 u_0 - k_0^2 u_1) J_1(\lambda \rho) \, d\lambda, \]  
\[ (A.8) \]

which is evaluated using the standard integrals

\[ \int_{0}^{\infty} u J_1(\lambda \rho) \, d\lambda = \frac{1}{\rho^2} \left[ k \rho + \exp(-k \rho) \right], \]  
\[ (A.9) \]

and

\[ \int_{0}^{\infty} u \lambda^2 J_1(\lambda \rho) \, d\lambda = -\frac{1}{\rho^4} \left[ k^2 \rho^2 + 3k \rho + 3 \right] \exp(-k \rho), \]  
\[ (A.10) \]

to give the following expression:

\[ F_{PM}(s) = \frac{1}{(k_0^2 - k_1^2) \rho^5} \left[ k_0 k_1 (k_0 - k_1) \rho^3 ight. \] 
\[ - \left[ k_0 (k_1^2 - k_0^2) \rho^3 + (2k_1^2 - 5k_0^2) \rho^2 \right. \] 
\[ - 12k_0 \rho - 12 \] \exp(-k_0 \rho) \] 
\[ + \left[ k_1 (k_0^2 - k_1^2) \rho^3 + (2k_0^2 - 5k_1^2) \rho^2 \right. \] 
\[ - 12k_1 \rho - 12 \] \exp(-k_1 \rho) \] \]  
\[ (A.11) \]

Both \( F_{PM}(s) \) and \( F_{TM}(s) \) contain terms without exponential behavior, which tend to infinity for large values of \( s \), or equivalently in the time domain, as \( t \) approaches zero. In fact, the field must be zero for times smaller than \( \rho/c \), where \( c \) is the speed of light. The \( 1/\rho^2 \) terms in the PM mode physically must cancel those in the TM mode, but because of the approximation introduced in the evaluation of the TM mode the cancellation is not exact. There are two satisfactory ways to resolve the problem. The first would be to replace the \( \sigma_0/(\sigma_0 - \sigma_1) \) and \( \sigma_1/(\sigma_0 - \sigma_1) \) terms in expression (A.11), the PM mode, with 1 and \( \sigma_1/\sigma_0 \) respectively. The second is to do the reverse to expression (A.7), the TM mode — replacing \( \sigma_1/\sigma_0 \) terms with \( \sigma_1/(\sigma_0 - \sigma_1) \) and terms of order unity with \( \sigma_0/(\sigma_0 - \sigma_1) \). The second method is preferable for two reasons. First, the exact nature of the PM solution is unaltered. (The TM solution already has errors of order \( \sigma_1/\sigma_0 \).) In addition, the property that the field at late times approaches the whole space field is preserved; the first method would introduce an error in the late-time amplitude of order \( \sigma_1/\sigma_0 \). The TM mode found in expression (A.7) is thus multiplied by \( \sigma_0/(\sigma_0 - \sigma_1) \) and added to
the PM mode expression (A.11), the physically unreasonable singular terms cancel and the final expression for the magnetic field becomes

\[
H_\rho(s) = \frac{m(s)}{2\pi \rho^3 (\tau_0 - \tau_1)s} \left[ -\sqrt{\tau_0 s} (\tau_1 s)^3 K_1(\sqrt{\tau_1 s}) \\
- [\sqrt{\tau_0 s} (\tau_1 - \tau_0)s + (2\tau_1 - 5\tau_0)s \\
- 12\sqrt{\tau_0 s} - 12] \exp(-\sqrt{\tau_0 s}) \\
+ [\sqrt{\tau_1 s} (2\tau_0 - \tau_1)s + (2\tau_0 - 5\tau_1)s \\
- 12\sqrt{\tau_1 s} - 12] \exp(-\sqrt{\tau_1 s}) \right], \tag{A.12}
\]

where the electromagnetic diffusion time constant for the i-th medium is

\[
\tau_i = \mu_0 \sigma_i \rho^2. \tag{A.13}
\]

The time constant increases with the square of the transmitter-receiver separation and with the given conductivity, as expected for a purely diffusive process.

Expression (A.12) may be inverted to the time domain if the Laplace transform \(m(s)\) of the source current dipole moment is specified. A current \(I\) is switched on at time \(t = 0\) in a one turn coil of area \(\Delta A\) and held constant. The Laplace transform is

\[
m(s) = I \Delta A/s. \tag{A.14}
\]

The relevant Laplace transforms of terms in (A.12) are:

\[
\begin{align*}
\frac{1}{s^2} \exp(-\sqrt{\tau s}) & \Leftrightarrow \left(t + \frac{\tau}{2}\right) \text{erfc}\left(\frac{\tau}{\sqrt{4t}}\right) - \sqrt{\frac{\tau}{\pi}} \exp\left(-\frac{\tau}{4t}\right) \\
\frac{1}{s^{3/2}} \exp(-\sqrt{\tau s}) & \Leftrightarrow \sqrt{\frac{4t}{\pi}} \exp\left(-\frac{\tau}{4t}\right) - \sqrt{\tau} \text{erfc}\left(\sqrt{\frac{\tau}{4t}}\right) \\
\frac{1}{s} \exp(-\sqrt{\tau s}) & \Leftrightarrow \text{erfc}\left(\sqrt{\frac{\tau}{4t}}\right) \\
\frac{1}{s^{1/2}} \exp(-\sqrt{\tau s}) & \Leftrightarrow \frac{1}{\sqrt{\pi t}} \exp\left(-\frac{\tau}{4t}\right) \\
\exp(-\sqrt{\tau s}) & \Leftrightarrow \frac{1}{t} \sqrt{\frac{\tau}{4\pi t}} \exp\left(-\frac{\tau}{4t}\right)
\end{align*}
\]
\[ K_1(\sqrt{\tau s}) \leftrightarrow \frac{1}{\sqrt{\tau t}} \exp \left( -\frac{\tau}{8t} \right) W_{\frac{1}{2},\frac{1}{2}} \left( \frac{\tau}{4t} \right), \]

where \( \text{erfc}(x) = 1 - \text{erf}(x) \) is the complementary error function and \( W_{\frac{1}{2},\frac{1}{2}}(x) \) is Whittaker's function, given by

\[
W_{\frac{1}{2},\frac{1}{2}}(x) = \frac{x}{\sqrt{\pi}} \exp \left(-\frac{x}{2}\right) \int_0^\infty \exp(-x\zeta) \sqrt{\frac{\zeta + 1}{\zeta}} d\zeta.
\]

The resulting step response is

\[
H_p^S(t) = \frac{I \Delta A}{2\pi \rho^3 (\tau_0 - \tau_1)} \left[ -\tau_1 \sqrt{\frac{\tau_0}{t}} \exp \left(-\frac{\tau_1}{8t} \right) W_{\frac{1}{2},\frac{1}{2}} \left( \frac{\tau_1}{4t} \right) \\
+ (\tau_0 - \tau_1 + 12t) \sqrt{\frac{\tau_0}{\pi t}} \exp \left(-\frac{\tau_0}{4t} \right) \\
- (\tau_0 + 2\tau_1 - 12t) \text{erfc} \left( \sqrt{\frac{\tau_0}{4t}} \right) \\
+ (2\tau_0 - \tau_1 - 12t) \sqrt{\frac{\tau_1}{\pi t}} \exp \left(-\frac{\tau_1}{4t} \right) \\
+ (2\tau_0 + \tau_1 - 12t) \text{erfc} \left( \sqrt{\frac{\tau_1}{4t}} \right) \right]. \tag{A.15}
\]

It is convenient to convert the step response to a dimensionless form. Dimensionless time \( x \) is defined as the ratio of the true time to the upper half-space time constant \( \tau_0 \). The parameter \( \alpha \) is the conductivity ratio \( \sigma_0 / \sigma_1 \). A unit of magnetic field is the late-time step response \( H_p^S(\infty) \), or, equivalently, the magnetic field of a static dipole given by

\[ H_p^S(\infty) = \frac{I \Delta A}{2\pi \rho^3}. \tag{A.16} \]

The dimensionless step response is then

\[
\frac{H_p^S(x)}{H_p^S(\infty)} = \frac{1}{\alpha - 1} \left[ -\frac{1}{\sqrt{x}} \exp \left(-\frac{1}{8\alpha x} \right) W_{\frac{1}{2},\frac{1}{2}} \left( \frac{1}{4\alpha x} \right) \\
+ (\alpha - 1 + 12\alpha x) \sqrt{\frac{1}{\pi x}} \exp \left(-\frac{1}{4x} \right) \\
- (\alpha + 2 - 12\alpha x) \text{erfc} \left( \sqrt{\frac{1}{4x}} \right) \right]
\]

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\[(2\alpha - 1 - 12\alpha x)\frac{1}{\pi ax} \exp\left(-\frac{1}{4\alpha x}\right)
\]
\[+(2\alpha + 1 - 12\alpha x)\text{erfc}\left(\frac{\sqrt{ax}}{4\alpha}\right)\]  \hspace{1cm} (A.17)

Figure A.1 The step-on response of the HRHR system. Both the analytic and numerical results are plotted.

The analytic solution has been plotted in Fig. A.1 against a numerical solution calculated using the Gavers-Stehfest algorithm. The analytic solution is valid at all times for values of \(\alpha\) greater than about 10. The analytic solution requires approximately one-tenth the computational effort of the numerical solution. (59 seconds vs 11:49 on a MicroVax II for 51 times).
A.2 The Delay Time

In section 2.1 I define a delay time \( t_d \) as the time of the maximum impulse response. Equivalently, this is the time where the derivative of the step response is greatest. There is no exact analytic solution to the general double half-space problem, but by specifying either a wholespace (\( \sigma_0 = \sigma_1 \)), or a perfectly conductive sea, the delay time may be determined analytically. These two situations bracket the range of conductivity contrasts important to the operation of the HRHR system.

First, consider the wholespace solution. Setting \( \sigma = \sigma_0 = \sigma_1 \) in expression (A.1), the modal expressions become

\[
F_{TM}(s) = -\frac{\mu_0 \sigma s}{2\rho} \int_0^\infty \frac{J_1(\lambda \rho)}{u} \, d\lambda,
\]

and

\[
F_{PM}(s) = -\frac{1}{2} \frac{\partial}{\partial \rho} \int_0^\infty u J_1(\lambda \rho) \, d\lambda.
\]

The integrals may be evaluated using expressions (A.5) and (A.9) to yield

\[
2\rho^3 F_{TM}(s) = -k \rho + k \rho \exp(-k \rho),
\]

and

\[
2\rho^3 F_{PM}(s) = k \rho + (2 + k \rho) \exp(-k \rho).
\]

The final expression for the magnetic field, in terms of the time constant \( \tau \), is

\[
H_\rho(s) = \frac{m(s)}{\sqrt{\pi s} \rho^3} (1 + \sqrt{s}) \exp(-\sqrt{s}).
\]

The Laplace transform \( m^I(s) \) of a current switched on at time \( t = 0 \) for a time \( dt \) in a one-turn coil of area \( \Delta A \) is

\[
m^I(s) = I \, dt \Delta A.
\]

Inverting to time domain,

\[
H^I_\rho(t) = \frac{I \, dt \Delta A \tau}{t^{5/2}} \exp\left(-\frac{\tau}{4t}\right).
\]
The time of the impulse response, \( t_d \), is found by setting the derivative of expression (A.24),

\[
\frac{\partial H(t)}{\partial t} = \frac{I d t \Delta A \tau}{4 t^9/2} [\tau - 10 t] \exp \left( -\frac{\tau}{4 t} \right),
\]

(A.25)

to zero. For finite values of \( t \), this occurs only when

\[
t_d = \frac{\tau}{10} = \frac{\mu_0 \sigma \rho^2}{10}.
\]

(A.26)

For very large values of \( \alpha \), the sea behaves as a perfect conductor. Setting \( \sigma_0 = \infty \) in expression (A.1) yields equations identical to (A.18) and (A.19), except for the factor of \( \frac{1}{2} \), which is missing. Although both extremal models have the same delay time, numerical analysis presented in Section 2.2 shows that for values of the conductivity contrast between 1 and \( \infty \) the delay time varies somewhat both below and above the value given in expression (A.26).

### A.3 The Infinite Dyke

The semibody theory developed in Section 2.5 states that when a magnetic dipole transmitter is located on the boundary between a perfect conductor and a lower half-space containing an arbitrary conductivity distribution, the field measured anywhere in the half-space is identical to that which would be produced by a transmitter of twice the moment if the conductivity distribution of the lower half-space is mirror imaged in the perfect conductor. To illustrate the use of the semibody theory, the response of the HRHR system to a semi-infinite vertical dyke was investigated. I present here the derivation of the responses of the HRHR system, given in expressions (2.19) and (2.20).

An infinite dyke of conductance \( S \) is located in a wholespace of conductivity \( \sigma_1 \). The transmitter and receiver are a distance \( \rho_s \) apart, and the distance between the transmitter and the dyke is \( \rho_d \). The situation is illustrated in Figure A.2.

The primary electric field produced by a one-turn coil of moment \( m \) is
Figure A.2 The infinite dyke model.

\[ E^P_\phi = \frac{i \omega \mu_0 m}{4\pi} \frac{r}{R^3} (1 + k_1 R) e^{-k_1 R}, \quad (A.27) \]

where \( \rho \) is the distance along the axis, and \( r \) is the radial distance away from the axis, and \( r^2 + \rho^2 = R^2 \). Expression (A.27) may be recast into the following equivalent form:

\[ E^P_\phi = \frac{i \omega \mu_0 m}{4\pi} \int_0^\infty \frac{\lambda^2}{u_1} e^{-u_1 \rho} J_1(\lambda r) \, d\lambda \quad (A.28) \]

The secondary field produced at the dyke is

\[ E^S_\phi(\rho_d) = \frac{i \omega \mu_0 m}{4\pi} \int_0^\infty \frac{\lambda^2}{u_1} A(\lambda) e^{-u_1 \rho_d} J_1(\lambda r) \, d\lambda, \quad (A.29) \]

where \( A(\lambda) \) is a function to be determined.

Therefore, for \( \rho < \rho_d \), region 1, the secondary electric field is

\[ E^{S1}_\phi = \frac{i \omega \mu_0 m}{4\pi} \int_0^\infty \frac{\lambda^2}{u_1} A(\lambda) e^{-u_1 (2\rho_d - \rho)} J_1(\lambda r) \, d\lambda, \quad (A.30) \]

and, for \( \rho > \rho_d \), region 2, the secondary electric field is

\[ E^{S2}_\phi = \frac{i \omega \mu_0 m}{4\pi} \int_0^\infty \frac{\lambda^2}{u_1} A(\lambda) e^{-u_1 \rho} J_1(\lambda r) \, d\lambda. \quad (A.31) \]
The total field $E_T^φ$ is the sum of the primary and secondary fields: in region 1,

$$E_T^{φ1} = \frac{iωμ_0 m}{4π} \int_0^∞ \frac{λ^2}{u_1} \left[ e^{-u_1 ρ} + A(λ)e^{-u_1(2ρ_d - ρ)} \right] J_1(λr) \, dλ,$$  \hspace{1cm} (A.32)

and, in region 2,

$$E_T^{φ2} = \frac{iωμ_0 m}{4π} \int_0^∞ \frac{λ^2}{u_1} \left[ 1 + A(λ) \right] e^{-u_1 ρ} J_1(λr) \, dλ.$$

Now, from Maxwell’s equations,

$$\text{curl } E_φ = -\frac{∂B}{∂t} = -iωμ_0 H_r,$$

therefore

$$\frac{∂E_φ}{∂ρ} = iωμ_0 H_r.$$

From expressions (A.32) and (A.35), the radial magnetic field for $ρ < ρ_d$ is

$$H_r^1 = \frac{m}{4π} \int_0^∞ λ^2 \left[ e^{-u_1 ρ} - A(λ)e^{-u_1(2ρ_d - ρ)} \right] J_1(λr) \, dλ,$$  \hspace{1cm} (A.36)

and, from expressions (A.33) and (A.35), the radial magnetic field for $ρ > ρ_d$ is

$$H_r^2 = \frac{m}{4π} \int_0^∞ λ^2 \left[ 1 + A(λ) \right] e^{-u_1 ρ} J_1(λr) \, dλ.$$

By Ampere’s Law,

$$\oint H \, dl = J_s = SE_φ = H_r^1 - H_r^2$$

(A.38)

Substituting expressions (A.32), (A.36) and (A.37) into expression (A.38) and taking the limit as $ρ$ approaches $ρ_d$,

$$1 - A(λ) - 1 - A(λ) = \frac{iωμ_0 S}{u_1} [1 + A(λ)].$$

(A.39)

Therefore

$$A(λ) = -\frac{θ(ω)}{1 + θ(ω)},$$

(A.40)

where

$$θ(ω) = \frac{iωμ_0 S}{2u_1}.$$
The electric field at the receiver is

$$E^{T2}_\phi = \frac{i\omega \mu_0 m}{4\pi} \int_0^\infty \frac{\lambda^2}{u_1} \left[ \frac{1}{1 + \theta(\omega)} \right] e^{-u_1 \rho} J_1(\lambda r) \, d\lambda \quad (A.42)$$

To determine the horizontal magnetic field at the receiver, we use

$$i\omega \mu_0 H_\rho = \frac{1}{r} \frac{\partial}{\partial r} [r E_\phi]. \quad (A.43)$$

Since

$$\frac{1}{r} \frac{\partial}{\partial r} r J_1(\lambda r) = \lambda J_0(\lambda r), \quad (A.44)$$

we get

$$H_\rho(r, \rho_s) = \frac{m}{4\pi} \int_0^\infty \frac{\lambda^3}{u_1} \left[ \frac{1}{1 + \theta(\omega)} \right] e^{-u_1 \rho_s} J_0(\lambda r) \, d\lambda \quad (A.45)$$

On the axis, $r = 0$, and $J_0(0) = 1$, so

$$H_\rho(\rho_s) = \frac{m}{4\pi} \int_0^\infty \frac{\lambda^3}{u_1} \left[ \frac{1}{1 + \theta(\omega)} \right] e^{-u_1 \rho_s} \, d\lambda, \quad (A.46)$$

which is the dimensionalized form of expression (2.19). If $\rho_d > \rho_s$ (that is, the dyke is not between the transmitter and the receiver) the procedure is very similar, and will not be repeated here.
Appendix B

The ERER System — Analytic Solutions

B.1 The Step Response for the Double Half-Space Model

This analytic solution for the ERER system in a double half-space is a modification of that presented by Edwards and Chave (1986). An additional term in the solution provides increased accuracy for small values of $\alpha$.

The Laplace transform of the electric field measured at the receiver by the ERER system is given by

$$E_{p}(s) = \frac{j(s)}{2\pi} [F_{TM}(s) + F_{PM}(s)], \quad (B.1)$$

where $j(s)$ is the Laplace transform of the source current dipole moment and $F_{TM}$ and $F_{PM}$ are defined as in expressions (3.2) and (3.3). The relevant inductances and impedances, derived from expressions (2.4) and (2.5), are $Q^{-} = \mu_{0}/u_{1}$, $Q^{+} = \mu_{0}/u_{0}$, $Y^{-} = u_{1}/\sigma_{1}$, and $Y^{+} = u_{0}/\sigma_{0}$, where the subscript 0 refers to the sea water and the subscript 1 refers to the sea floor.

The modal expressions become

$$F_{TM}(s) = -\int_{0}^{\infty} \frac{u_{0}u_{1}}{u_{0}\sigma_{1} + u_{1}\sigma_{0}} \left[ \lambda J_{0}(\lambda \rho) - \frac{J_{1}(\lambda \rho)}{\rho} \right] d\lambda, \quad (B.2)$$

and

$$F_{PM}(s) = -\frac{1}{\rho(\sigma_{0} - \sigma_{1})} \int_{0}^{\infty} (u_{0} - u_{1}) J_{1}(\lambda \rho) d\lambda. \quad (B.3)$$

Expression (B.2) cannot be evaluated analytically because of the presence of multiple branch cuts. However, if the conductivity $\sigma_{0}$ of the upper half-space, the sea, is significantly greater than the conductivity $\sigma_{1}$ of the lower half-space, the
crust, we may proceed as in Appendix A and expand (B.2) in powers of $u_0 \sigma_1/u_1 \sigma_0$ and drop second order and higher terms:

$$F_{TM}(s) = -\frac{1}{\sigma_0} \int_{0}^{\infty} \left[ u_0 - \frac{u_0^2 \sigma_1}{u_1 \sigma_0} + \frac{k_0^2 \sigma_1^2}{u_1^2 \sigma_0^2} \right] \left[ \lambda J_0(\lambda \rho) - \frac{J_1(\lambda \rho)}{\rho} \right] d\lambda. \quad (B.4)$$

The third term above is not included in the original derivation, and provides additional accuracy for smaller conductivity contrasts.

![Graph](image.png)

**Figure B.1** The step-on response of the ERER system. The numerical solution (solid lines) and the approximate analytical solution (dashed lines) are plotted for several values of conductivity contrast $\alpha$.

The Hankel transforms may now be evaluated using standard integrals, and
the final expression for the electric field, including both modes, becomes

\[
E_\rho(s) = \frac{j(s)}{2\pi\sigma_0} \left[ \frac{1}{\rho^3} \left[ \sqrt{\tau_0 s} + 1 \right] \exp(-\sqrt{\tau_0 s}) 
+ (\tau_1 s + \sqrt{\tau_1 s} + 1) \exp(-\sqrt{\tau_1 s}) \right] 
+ \sqrt{\frac{\sigma_1}{\sigma_0}} \frac{\partial}{\partial \rho} \frac{1}{\rho^2} \sqrt{\tau_1 s} K_1(\sqrt{\tau_1 s}) \right], \tag{B.5}
\]

A current \( I \) switched on at time \( t = 0 \) and held constant has the source transform

\[
j_S(s) = I\Delta l/s. \tag{B.6}
\]

A unit of the electric field is the late-time step response \( E_\rho^S(\infty) \) for a static dipole in a whitespace (\( \alpha = 1 \)) given by

\[
E_\rho^S(\infty) = \frac{2I\Delta l}{\pi\sigma_0 \rho^3} \tag{B.7}
\]

The resulting dimensionless step response is then

\[
\frac{E_\rho^S(x)}{E_\rho^S(\infty)} = \text{erfc} \left( \sqrt{\frac{1}{4x}} \right) + \sqrt{\frac{1}{\pi x}} \exp \left( -\frac{1}{4x} \right) 
+ \text{erfc} \left( \sqrt{\frac{1}{4\alpha x}} \right) + \sqrt{\frac{1}{\pi \alpha x}} \left( 1 + \frac{1}{2\alpha x} \right) \exp \left( -\frac{1}{4\alpha x} \right) 
+ \frac{1}{\alpha \sqrt{x}} \exp \left( -\frac{1}{8\alpha x} \right) \left[ \left( 1 - \frac{1}{2\alpha x} \right) W_{1/2,1/2} \left( \frac{1}{4\alpha x} \right) - \frac{1}{4\alpha x \sqrt{\pi}} K_1 \left( \frac{1}{8\alpha x} \right) \right]. \tag{B.8}
\]

(Note that the dimensionless time \( x \) is defined in terms of \( \tau_0 \), not \( \tau_1 \) as in Edwards and Chave (1986)). Due to the concentration of current in the more conductive medium, the late-time amplitude is proportional to \( 2\sigma_1/(\sigma_0 + \sigma_1) \). The response is plotted in dashed lines in Figure B.1 for a range of values of \( \alpha \). The numerical solution is plotted in solid lines for comparison. The numerical and analytic solutions are very close for values of \( \alpha \) greater than 10. For lower values of \( \alpha \) it is the analytic solution which is in error (the numerical solution for \( \alpha = 1 \) matches the exact analytical whitespace expression).
B.2 The Delay Time

As in Appendix A, we define a delay time $t_d$ as the time of maximum impulse response. Again, there is no exact analytic solution to the double half-space problem, but solutions may be obtained for $\alpha = 1$, the wholespace response, and for $\alpha \to \infty$, the perfectly conductive sea.

For the wholespace response, $\sigma_0 = \sigma_1 = \sigma$, and the terms in expression (B.1) become

\[ F_{TM}(s) = -\int_{0}^{\infty} \frac{u}{2\sigma} \left[ \lambda J_0(\lambda \rho) - \frac{J_1(\lambda \rho)}{\rho} \right] d\lambda, \]  
(B.9)

and

\[ F_{PM}(s) = -\frac{s\mu_0}{2\rho} \int_{0}^{\infty} \frac{J_1(\lambda \rho)}{u} d\lambda. \]  
(B.10)

Expression (B.9) is evaluated using (A.9) and the standard integral

\[ \int_{0}^{\infty} u\lambda J_0(\lambda \rho) d\lambda = -\frac{\exp(-k\rho)}{\rho} [1 + k\rho], \]  
(B.11)

while expression (B.10) is evaluated using (A.5). Thus

\[ 2\sigma \rho^3 F_{TM}(s) = k\rho + [2 + k\rho] \exp(-k\rho) \]  
(B.12)

and

\[ 2\sigma \rho^3 F_{PM}(s) = -k\rho + k\rho \exp(-k\rho). \]  
(B.13)

The final expression for the electric field, in terms of the time constant $\tau$, is

\[ E_{\rho}(s) = \frac{j(s)}{\pi \sigma \rho^3} (1 + \sqrt{\tau s}) \exp(-\sqrt{\tau s}). \]  
(B.14)

This is of the same form as expression (A.22) for the wholespace response of the HRHR system. The delay time will thus be the same as well,

\[ t_d = \tau/10 = \frac{\mu_0 \sigma \rho^2}{10} \]  
(B.15)

Determining the response for a perfectly conducting sea is not a well posed problem, since both $F_{TM}$ and $F_{PM}$ tend to infinity as $\sigma_0 \to 0$. Instead, the problem may be solved by considering the case where $\alpha \to \infty$. This was done by Edwards
and Chave (1986). They derived the dimensionless response for large values of \( \alpha \).

It is

\[
\frac{E^I(x)}{E^S(\infty)} = \frac{\Delta x}{2\sqrt{\pi x}} \left[ \frac{1}{2} \sqrt{\frac{1}{x}} \exp\left(-\frac{1}{4x}\right) + \sqrt{\frac{1}{\alpha x}} \left( \frac{1}{4\alpha x} \right) \exp\left(-\frac{1}{4\alpha x}\right) \right] .
\]

(B.16)

For large values of \( \alpha \), the first term of the expression in brackets in (B.16) vanishes. The derivative of the remainder,

\[
\frac{\partial}{\partial x} \frac{E^I(x)}{E^S(\infty)} = \frac{\Delta x}{4\sqrt{\pi x}} \left[ \frac{x}{16} \right] \exp\left(-\frac{1}{4x}\right) ,
\]

(B.17)

vanishes for non-zero values of \( x \) given by \((9 \pm \sqrt{41})/40\). The first value is the desired delay time, while the second is the maximum negative slope in the normalized step response where it falls back to 1.0 after the initial rise to about 1.3. The first value, approximately equal to 1/15.4, suggests a delay time defined by

\[
t_d = \frac{\tau}{15.4} = \frac{\mu_0 \sigma_1 p^2}{15.4} .
\]

(B.16)

This, it should be noted, is substantially different from the delay for the whitespace model, given in expression (B.15).
Appendix C

The Transient Response of Some other Systems

C.1 The Vertical Magnetic Dipole-Dipole (HZHZ) System

The analytical frequency response of the double half-space model to the vertical coplanar magnetic dipole-dipole system on the interface may be found in Kaufman and Keller (1983). In terms of the Laplace variable \( s \) it is

\[
H_z(s) = \frac{m(s)}{2\pi \rho^3 (\tau_0 - \tau_1)s} \left[ \left( 9 + 9\sqrt{\tau_0 s} + 4\tau_0 s + (\sqrt{\tau_0 s})^3 \right) \exp(-\sqrt{\tau_0 s}) - \left( 9 + 9\sqrt{\tau_1 s} + 4\tau_1 s + (\sqrt{\tau_1 s})^3 \right) \exp(-\sqrt{\tau_1 s}) \right]. \quad (C.1)
\]

The step response may be found by inserting the transform of the source current found in expression (A.19) into expression (C.1) and inverting using standard integrals. In terms of true time \( t \) it is

\[
H^S_z(t) = \frac{9I \Delta A}{2\pi \rho^3 (\tau_0 - \tau_1)} t \left[ \left( 1 - \frac{\tau_0}{18t} \right) \text{erfc} \left( \frac{\sqrt{\tau_0}}{4t} \right) + \sqrt{\frac{\tau_0}{\pi t}} \left( 1 + \frac{\tau_0}{9t} \right) \exp \left( -\frac{\tau_0}{4t} \right) - \left( 1 - \frac{\tau_1}{18t} \right) \text{erfc} \left( \frac{\sqrt{\tau_1}}{4t} \right) - \sqrt{\frac{\tau_1}{\pi t}} \left( 1 + \frac{\tau_1}{9t} \right) \exp \left( -\frac{\tau_1}{4t} \right) \right]. \quad (C.2)
\]

The late-time step response is given by

\[
H^S_z(\infty) = -\frac{I \Delta A}{4\pi \rho^3}. \quad (C.3)
\]

The dimensionless step response is then

\[
\frac{H^S_z(x)}{H^S_z(\infty)} = \frac{18\alpha x}{\alpha - 1} \left[ \left( 1 - \frac{1}{18\alpha x} \right) \text{erfc} \left( \sqrt{\frac{1}{4\alpha x}} \right) - 114 \right].
\]
Figure C.1 The step response of the HZHZ system.

\[
\begin{align*}
+ \sqrt{\frac{1}{\pi \alpha x}} \left(1 + \frac{1}{9 \alpha x}\right) \exp\left(-\frac{1}{4 \alpha x}\right) &- \left(1 - \frac{1}{18 x}\right) \text{erfc}\left(\sqrt{\frac{1}{4x}}\right) \\
- \sqrt{\frac{1}{\pi x}} \left(1 + \frac{1}{9 x}\right) \exp\left(-\frac{1}{4 x}\right)
\end{align*}
\]  
(C.4)

This expression is exact; there is no requirement for a large conductivity contrast. The response is plotted in Figure B.1 for a range of values of \( \alpha \).

C.2 The EPHIHZ System.

Chave and Cox (1982) derive the expression for the vertical magnetic field of a horizontal electric dipole of moment \( pe^{i\omega t} \) oriented azimuthally to the receiver. On
the interface this expression in terms of the Laplace variable \( s \) reduces to

\[
H_r(s) = \frac{2p(s)}{4\pi s (\sigma_0 - \sigma_1)} \int_0^\infty (u_0 - u_1) \lambda^2 J_1(\lambda \rho) \, d\lambda,
\]

which may be simplified using expression (15) to give

\[
H_z(s) = \frac{p(s)}{2\pi \rho^4 s (\sigma_0 - \sigma_1)} \left[ (3 + 3\sqrt{\tau_1 s + \tau_1 s}) \exp(-\sqrt{\tau_1 s}) - (3 + 3\sqrt{\tau_0 s + \tau_0 s}) \exp(-\sqrt{\tau_0 s}) \right].
\]

A current \( I \) switched on at time \( t = 0 \) in a wire of length \( \Delta l \) and held constant has the transform

\[
p(s) = I \Delta l / s.
\]
The Step Response is then

\[ H_z^S(t) = \frac{3\mu_0 I \Delta l}{2\pi \rho^2} \frac{t}{(\tau_0 - \tau_1)} \left[ \left( 1 - \frac{\tau_1}{6t} \right) \text{erfc} \left( \sqrt{\frac{\tau_1}{4t}} \right) + \sqrt{\frac{\tau_1}{\pi t}} \exp \left( -\frac{\tau_1}{4t} \right) \ight. \\
\left. \quad - \left( 1 - \frac{\tau_0}{6t} \right) \text{erfc} \left( \sqrt{\frac{\tau_0}{4t}} \right) - \sqrt{\frac{\tau_0}{\pi t}} \exp \left( -\frac{\tau_0}{4t} \right) \right]. \quad (C.8) \]

The late time response is equal to

\[ H_z^S(\infty) = \frac{\mu_0 I \Delta l}{4\pi \rho^2}, \quad (C.9) \]

so that the dimensionless step response is given by

\[ \frac{H_z^S(x)}{H_z^S(\infty)} = \frac{6\alpha x}{\alpha - 1} \left[ \left( 1 - \frac{1}{6\alpha x} \right) \text{erfc} \left( \sqrt{\frac{1}{4\alpha x}} \right) + \sqrt{\frac{1}{\pi \alpha x}} \exp \left( -\frac{1}{4\alpha x} \right) \ight. \\
\left. \quad - \left( 1 - \frac{1}{6x} \right) \text{erfc} \left( \sqrt{\frac{1}{4x}} \right) - \sqrt{\frac{1}{\pi x}} \exp \left( -\frac{1}{4x} \right) \right]. \quad (C.10) \]

As for the HZH case, this expression is exact. The response is plotted in Figure B.2 for a range of values of \( \alpha \).

C.3 The Large Loop (HZHZC) System.

It is worthwhile to consider a variation on the HZH system in which the vertical magnetic receiver is placed at the source loop center, denoted here as the HZHZE system. Such an apparatus has been deployed in shallow seas and moved systematically over the sea floor. It could also be mounted as a “fixed wing” device on a remote operated vehicle or a submersible. For this problem it is necessary to consider the finite dimensions of the source loop explicitly rather than using a point dipole approximation. For a loop of radius \( a \) carrying an electric current \( I \) and lying on the interface between half-spaces of conductivities \( \sigma_0 \) and \( \sigma_1 \), the frequency response in terms of the Laplace variable \( s \) for the vertical magnetic field is

\[ H_z(s) = \frac{I}{a(\tau_0 - \tau_1)} \left[ (\tau_1 s + 3\sqrt{\tau_1 s} + 3)\exp(-\sqrt{\tau_1 s}) \ight. \\
\left. \quad - (\tau_0 s + 3\sqrt{\tau_0 s} + 3)\exp(-\sqrt{\tau_0 s}) \right] \quad (C.11) \]
where the time constant is defined as

\[ \tau_i = \mu_0 \sigma_i a^2. \]

The step response may be found by inserting the transform of the source current \( I/s \) into expression (C.11) and inverting to the time-domain.

\[
H^S_x(t) = \frac{3I}{a} \frac{t}{(\tau_0 - \tau_1)} \left[ \left( 1 - \frac{\tau_1}{6t} \right) \text{erfc} \left( \frac{\tau_1}{\sqrt{4t}} \right) + \sqrt{\frac{\tau_1}{\pi t}} \exp \left( -\frac{\tau_1}{4t} \right) 
- \left( 1 - \frac{\tau_0}{6t} \right) \text{erfc} \left( \frac{\tau_0}{\sqrt{4t}} \right) - \sqrt{\frac{\tau_0}{\pi t}} \exp \left( -\frac{\tau_0}{4t} \right) \right].
\] (C.12)

The late-time response is given by

\[
H^S_x(\infty) = \frac{I}{2a},
\] (C.13)

yielding the dimensionless step response

\[
\frac{H^S_x(x)}{H^S_x(\infty)} = \frac{6ax}{\alpha - 1} \left[ \left( 1 - \frac{1}{6ax} \right) \text{erfc} \left( \sqrt{\frac{1}{4ax}} \right) + \sqrt{\frac{1}{\pi ax}} \exp \left( -\frac{1}{4ax} \right) 
- \left( 1 - \frac{1}{6x} \right) \text{erfc} \left( \sqrt{\frac{1}{4x}} \right) - \sqrt{\frac{1}{\pi x}} \exp \left( -\frac{1}{4x} \right) \right].
\] (C.14)

This expression is identical to that for the EPHIZ system (although the time constant is defined differently and the late-time response is distinct).
Appendix D

Eigenparameter statistical analysis

D.1 Non-uniqueness — Eigenparameter Statistical Analysis

The investigation of the resolution by a given data set of a model parameter has to address a fundamental question of uniqueness. When one parameter of a model is changed, a change will be observed in the model type curve which passes through the data. If a certain parameter change moves the type curve, on average, just outside the errors on the data, it is tempting to state that the parameter is resolved to an accuracy which depends simply on size of this change. Unfortunately, the way in which the type curve is displaced need not be unique, and varying the value of a different model parameter, or the values of a particular group of model parameters, sometimes produces a very similar displacement. In such a case, one can not argue that the first model parameter is resolved by the data even though varying it does significantly alter the form of the type curve. The problem of this type of parameter intercorrelation is avoided by a technique known as eigenparameter statistical analysis. The method provides a very clear, unambiguous set of statements for the interpreter, or the designer, of an experiment as to what parts of his model are determined by real or synthetic data respectively. Further, if experimental or estimated errors are assigned to the data, the method provides a simple scheme for assessing the errors in the model.

Let a given model have parameters $P_j$, where $j = 1, N$. The $P_j$ are, for instance, the thicknesses and conductivities of the layered model. Let the data set, either an experimental or a synthetic one, from which the model is determined be $Y_i, i = 1, M$, and let the measured or assigned errors on the data be $e_i, i = 1, M$. 

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For a small variation $dP_j$ in a parameter $P_j$, the expected changes $dY_i$ in the data set $Y_i$ are given by the first term of Taylor's series as

$$dY_i = \sum_{j=1}^{N} A_{ij} dP_j,$$

or, in matrix notation,

$$dY = A dP,$$

where each coefficient $A_{ij}$ is simply a measure of the sensitivity of datum $Y_i$ to a change in parameter $P_j$, or the partial derivative $\partial Y_i/\partial P_j$. These derivatives may be found either analytically or numerically from the forward solution given the physics of the problem.

Expressions (D.1) and (D.2) clearly display the problem of non-uniqueness. A given change in a datum can be produced by changing any one of the model parameters provided the associated $A_{ij}$ is non-vanishing. However, it is possible to choose linear combinations $dP^*$ of the parameter changes $dP$ and corresponding linear combinations $dY^*$ of the data changes $dY$ such that expressions (D.1) and (D.2) are greatly simplified. The process of finding these combinations is through Singular Value Decomposition (SVD) of the matrix $A$. Standard software exists to write $A$ as

$$A = U \times L \times V^T. \tag{D.3}$$

The matrices $U$ and $V$ have the property that

$$U^T U = V^T V = V V^T = 1, \tag{D.4}$$

and the matrix $L$ is diagonal. If $dY^*$ and $dP^*$ are defined by the equations

$$dY^* = U^T dY, \tag{D.5}$$

and

$$dP^* = V^T dP, \tag{D.6}$$
then equation (D.2) may be written as

$$dY^* = L dP^*. $$  \hspace{1cm} (D.7)

Only one set of weights \(U\) and \(V\) permits this simplification. The matrix \(L\) contains the eigenvalues of \(A\). The vectors \(dP^*\) and \(dY^*\) are termed eigenparameters and eigendata respectively. Each eigendatum is related to a corresponding eigenparameter and only that eigenparameter through the equation

$$dY_j^* = L_{jj} dP_j^*, \quad 1 < j < N. \hspace{1cm} (D.8)$$

The problem of parameter intercorrelation is clearly avoided if parameter resolution and error assessment are considered in terms of these eigensolutions.

The error in each eigenparameter is expressed very simply in terms of the above analysis provided each datum of the data set has an independent standard error estimate \(e\) of unity. Expression (D.5) is a relationship between small changes in the original data and small changes in the eigendata. The same set of weights must relate the errors in the two data types, so that

$$e_i^* = \sum_{j=1}^{M} U_{ji} e_j. \hspace{1cm} (D.9)$$

The covariance, or mean product, of the errors \(e_i^*\) and \(e_j^*\) is

$$\text{cov}(e_i^*, e_j^*) = E(e_i^*, e_j^*) = \sum_{j=1}^{M} \sum_{k=1}^{M} U_{ji} U_{lk} E(e_j, e_k), \hspace{1cm} (D.10)$$

where \(E\) is the expectation value operator. The original data errors are independent and of unit variance. Hence, their covariance is simply given by

$$E(e_j, e_k) = \delta_{jk}. \hspace{1cm} (D.11)$$

Equation (D.10) reduces first to

$$\text{cov}(e_i^*, e_j^*) = \sum_{j=1}^{M} U_{ji} U_{ij}, \hspace{1cm} (D.12)$$
and then to

\[ \text{cov}(e_i^*, e_i^*) = \delta_{ii}, \]  

(D.13)

because the rows of the matrix \( U^T \) are orthonormal vectors. Equation (D.13) shows that the standard errors in the eigendata are also independent and also have a value of unity. Now any small change in an eigendatum is related to a corresponding small change in an eigenparameter by equation (D.8). Hence, the standard error in an eigenparameter is just the reciprocal of the corresponding eigenvalue - a remarkably simple result.

Each element \( \partial Y_i / \partial P_j \) of the Jacobian matrix \( A \) is scaled in two ways before SVD is undertaken. It is divided by \( e_i \). This has the effect of rescaling the units in which datum \( Y_i \) is measured so that its standard error is unity, as required by the theory. The element is also multiplied by \( P_j \). This has the effect of redefining the parameter \( P_j \) as \( \log P_j \), because

\[ P_j \partial Y_i / \partial P_j = \partial Y_i / \partial (\log P_j). \]  

(D.14)

The whole procedure of eigenparameter analysis clearly has very limited appeal if the eigenparameters cannot be identified as representing physically understandable combinations of the original model parameters. The use of logarithmic scaling of the model parameters makes this identification much like dimensional analysis. As an example, consider the model of a layer of resistivity \( \rho_1 \) and thickness \( t_1 \) over a half-space of resistivity \( \rho_2 \). A change in \( P_1^* \), one of the three possible eigenparameters, is related to changes in the model parameters by

\[ dP_1^* = V_{11} d(\log \rho_1) + V_{21} d(\log t_1) + V_{31} d(\log \rho_2). \]  

(D.15)

The weights \( V_{11}, V_{21} \) and \( V_{31} \) are normalised by the SVD analysis so that the sum of their squares is unity. The physical interpretation of the eigenparameter may be deduced as

\[ \rho_1^{V_{11}} t_1^{V_{21}} \rho_2^{V_{31}}. \]  

(D.16)

If the conductivity-thickness product of the layer is the 'physical interpretation' of
the eigenparameter, then \( V_{11} = -V_{21} = .707 \) and \( V_{31} = 0 \). Also, the standard error in the eigenparameter is the standard error in the logarithm of the conductivity-thickness product or, equivalently, the fractional standard error in the conductivity-thickness product itself.

The relationship between the fractional standard error in a given model parameter and the standard errors in the eigenparameters is obtained by inverting equation (D.6), having noted from equation (D.4) that the inverse of matrix \( V^T \) is just matrix \( V \). A coarse upper bound on the fractional standard error in the thickness \( t_1 \) in the example is given by

\[
(V_{11}/L_{11}) + (V_{12}/L_{22}) + (V_{13}/L_{33}),
\]

where \( L_{11}, L_{22} \) and \( L_{33} \) are the eigenvalues of the Jacobian. A fractional error in a model parameter may only be computed in this manner provided it is small compared with unity because the theory described is only valid for small changes, that is to first order. If the standard error in the parameter is predicted as being much larger than unity, due for example to a non-zero weight being divided by a small eigenvalue, then a different technique has to be adopted to find the true error bound.

D.2 Model Resolution and Hypothesis Testing

I now investigate the capability of the HRHR method to resolve the conductivity contrast in the double half-space model. I follow the methods set out by Vozoff and Jupp (1975) and Glenn and Ward (1976). For my analysis I have used a typical response from the surveys described in Chapter 5. The approximate errors assigned to the data were determined from a visual inspection of the scatter in the data.

An inverse analysis, using parameter eigenvector statistics is performed. The analysis shows how well the model parameters are constrained by the data. In general, some combinations of parameters will be well determined, some poorly.
In other words, the model in total is rarely completely defined by the data set. It is particularly important in real data inversion to convey to the interpreter the nature of the undetermined parameter combinations. The model can be altered in these directions without greatly influencing the goodness of fit of the data to the model. Consequently, if the interpreter has a hypothesis that the real earth is somewhat different from the inverted model, then he can try to alter the inverted model towards the hypothetical model by changing the undetermined parameters. In many, many cases electrical data and geological data can be made consistent by this simple exercise. Sometimes, it is logical to fix certain parameters in the model, based on a priori knowledge, and then examine whether the remainder are determined.

The eigenparameter statistics are displayed in a table. The eigenparameters are ranked from top to bottom in order of decreasing eigenvalue, increasing standard error. Each coefficient in a row is the weight with which the logarithm of each model parameter is included in the combination of parameters forming the eigenparameter. The interpretation of the eigenparameter is included under 'physics'.

My example is the double half-space model used to invert the data in Section 5.4 of Chapter 5. The data set consists of 31 points selected from the time series recorded at station 1b. The response has been sampled most heavily at early time, where the response varies most quickly. The data points are assigned a constant, conservative, error of 2.5 mV (see Figure 5.7). The model has four free parameters, the conductivity contrast $\alpha$, the scaling factor $s$, the time offset $t_{\text{off}}$, and the voltage offset $V_{\text{off}}$.

Most parameters chosen for examination, such as physical properties and dimensions, are always positive quantities. The two offsets are fundamentally different in that they are expected to take on both positive and negative values centered around zero. Because the analysis works with the natural logarithms of parameters, offset variables must be treated specially; a negative or zero value cannot be handled because the logarithms are non-real. A transformation of the variable solves this
problem. The offset is first scaled, then a new variable, equal to the exponential of the scaled offset is created. The scaling factor is required to ensure that small changes in the transformed variable, used to calculate derivatives in the eigenanalysis, correspond to small differences in the original variable. Let \( x \) be the offset variable and \( y \) the new parameter, and \( a \) a scaling factor. Then

\[
y = \exp(ax),
\]

and a small change \( y \) will be given by

\[
dy = \exp(ax) \cdot a \cdot dx,
\]

where \( dx \) is the corresponding change in \( x \). If the analysis indicates a fractional uncertainty \( dy/y \) in the parameter \( y \), this corresponds to a real (dimensionalized) uncertainty in \( x \) given by

\[
dx = \frac{1}{a \cdot y} dy.
\]

<table>
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<th>P</th>
<th>( \delta \alpha/\alpha )</th>
<th>( \delta s/s )</th>
<th>( \delta t^<em>/t^</em> )</th>
<th>( \delta V^<em>/V^</em> )</th>
<th>Physics</th>
<th>Std Error</th>
<th>Value</th>
<th>Error</th>
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<td>0.857</td>
<td>0.504</td>
<td>-0.030</td>
<td>-0.099</td>
<td>Contrast</td>
<td>0.0181</td>
<td>2.41</td>
<td>0.31</td>
</tr>
<tr>
<td>2</td>
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<td>0.852</td>
<td>-0.050</td>
<td>0.204</td>
<td>Scale</td>
<td>0.0431</td>
<td>1.10</td>
<td>0.26</td>
</tr>
<tr>
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<td>-0.115</td>
<td>0.181</td>
<td>0.959</td>
<td>V offset</td>
<td>0.230</td>
<td>-1.3 mV</td>
<td>0.88 mV</td>
</tr>
<tr>
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<td>-0.170</td>
<td>t offset</td>
<td>0.861</td>
<td>0.083 ms</td>
<td>0.038 ms</td>
</tr>
</tbody>
</table>

Table D.1 The eigenvector analysis for the double half-space model for station 1B. The asterisks denote transformed offset variables.

For instance, the time offset is expected to range from about -0.1 to 0.1 ms. I first scale this by a factor of 10, then exponentiate it. The new variable will always be positive, and will normally range from about 0.36 to 2.7. If the analysis indicates a fractional uncertainty of 0.1 when the time offset is zero (and \( y = 1 \)), the actual uncertainty is equal to 0.01 ms. For the voltage offset, no scaling is necessary, so
a is set to one. In the tables that follow, the coefficients and standard errors are those for the transformed offsets.

The eigenanalysis is shown in Table D.1. The eigenparameters are just the four free parameters, there is little mixing. The conductivity contrast has a standard error of about 2%, while the scale has a standard error of about 4%. The transformed time and voltage offsets have standard errors of 23% and 86%. The errors, calculated using expression (D.20), are also displayed.
Appendix E
Modelling Results

This appendix consists of a series of diagrams which display graphically the results of the inversion modelling. For each station, the processed transient is plotted along with the transient of the theoretical model which was found to be the most similar in the least-squares sense. (For details on the processing of the data and the modelling procedure, see chapter 5 and appendix D). The plotted points are those used in the modelling routine, and form a subset of those actually recorded. Data points are plotted as squares, while model points are plotted as diamonds. The voltage scale is maintained constant for all stations in each line for the purpose of comparison. Parameters of the best-fit models are included with each station. (For uncertainties in these values, consult Tables 5.1-5). Stations 1a to 10b are from Line A, stations 11a to 15b are from Line B, and stations 16a to 20b are from Line C. Figures E1 to E7 give the best-fit curves for the half-space model. Figures E8 to E11 give the improved curves for Line B and Line C using the layer over a half-space model.
Figure E.1 The data recorded for sites 1a to 3b on line A (squares) are plotted with best-fit double half-space models (diamonds). The conductivity contrasts $\alpha$, scaling factor $s$, offset time $t$ (in ms) and rms error $e$ (in mV) of the model are included.
Figure E.2 The data recorded for sites 4a to 7b on line A (squares) are plotted with best-fit double half-space models (diamonds). The conductivity contrasts $\alpha$, scaling factor $s$, offset time $t$ (in ms) and rms error $e$ (in mV) of the model are included.
Figure E.3 The data recorded for sites 8a to 10b on line A (squares) are plotted with best-fit double half-space models (diamonds). The conductivity contrasts $\alpha$, scaling factor $s$, offset time $t$ (in ms) and rms error $e$ (in mV) of the model are included.
Figure E.4 The data recorded for sites 11a to 13b on line B (squares) are plotted with best-fit double half-space models (diamonds). The conductivity contrasts $\alpha$, scaling factor $s$, offset time $t$ (in ms) and rms error $e$ (in mV) of the model are included.
Figure E.5  The data recorded for sites 14a to 15b on line B (squares) are plotted with best-fit double half-space models (diamonds). The conductivity contrasts $\alpha$, scaling factor $s$, offset time $t$ (in ms) and rms error $e$ (in mV) of the model are included.
Figure E.6 The data recorded for sites 16a to 18b on line C (squares) are plotted with best-fit double half-space models (diamonds). The conductivity contrasts $\alpha$, scaling factor $s$, offset time $t$ (in ms) and rms error $e$ (in mV) of the model are included.
Figure E.7  The data recorded for sites 19a to 20b on line C (squares) are plotted with best-fit double half-space models (diamonds). The conductivity contrasts α, scaling factor s, offset time t (in ms) and rms error e (in mV) of the model are included.
Figure E.8 The data recorded for sites 11a to 13b on line B (squares) are plotted with best-fit layer over a half-space models (diamonds). The conductivity contrast $\alpha_1$ of the top layer is equal to 2.5, and the conductivity contrast $\alpha_2$ of the lower half-space is equal to 30. The thickness $d_1$ of the top layer, scaling factor $s$, offset time $t$ (in ms) and rms error $e$ (in mV) of the model are included.
Figure E.9 The data recorded for sites 14a to 15b on line B (squares) are plotted with best-fit layer over a half-space models (diamonds). The conductivity contrast $\alpha_1$ of the top layer is equal to 2.5, and the conductivity contrast $\alpha_2$ of the lower half-space is equal to 30. The thickness $d_1$ of the top layer, scaling factor $s$, offset time $t$ (in ms) and rms error $e$ (in mV) of the model are included.
Figure E.10  The data recorded for sites 16a to 18b on line C (squares) are plotted with best-fit layer over a half-space models (diamonds). The conductivity contrast $\alpha_1$ of the top layer is equal to 2.5, and the conductivity contrast $\alpha_2$ of the lower half-space is equal to 20 for stations 16a, 16b,17a, and 17b, and 30 for stations 18a and 18b. The thickness $d_1$ of the top layer, scaling factor $s$, offset time $t$ (in ms) and rms error $e$ (in mV) of the model are included.
19a: $d_1=11.5$ $s=1.25$ $t=.001$ $e=1.41$

19b: $d_1=13.7$ $s=1.21$ $t=.017$ $e=1.66$

20a: $d_1=12.2$ $s=1.22$ $t=.008$ $e=1.61$

20b: $d_1=12.0$ $s=1.16$ $t=.006$ $e=1.60$

Figure E.11 The data recorded for sites 19a to 20b on line C (squares) are plotted with best-fit layer over a half-space models (diamonds). The conductivity contrast $\alpha_1$ of the top layer is equal to 2.5, and the conductivity contrast $\alpha_2$ of the lower half-space is equal to 30. The thickness $d_1$ of the top layer, scaling factor $s$, offset time $t$ (in ms) and rms error $e$ (in mV) of the model are included.