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Experimental and numerical study on flexural behavior of UHPFRC beams with low reinforcement ratios

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beams with low reinforcement ratios

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ABSTRACT

Flexural behaviors of reinforced ultra-high-performance fiber-reinforced concrete (UHPFRC) beams were experimentally and numerically investigated in terms of reinforcement ratio. To do this, four UHPFRC beams with different reinforcement ratios (0\%–1.71\%) were fabricated and tested. Since we focused on the placement technique of steel reinforcing bars, only smaller number of reinforced UHPFRC beams was deliberately considered. Test results indicated that with an increase in the reinforcement ratio, post-cracking stiffness and load carrying capacity were increased, whereas first cracking load was decreased. The cracking behavior was characterized by numerous vertical micro-cracks up to near the peak, followed by crack localization with a gradual decrease in load carrying capacity. The number of cracks and average crack spacings were marginally influenced by the reinforcement ratio. Sectional analysis incorporating a linear compressive model and tension-softening curves obtained from inverse analyses and direct tensile test were performed and verified through comparison with the experimental moment-curvature responses.

Keywords: Ultra-high-performance fiber-reinforced concrete; Reinforcement ratio; Flexure; Tension-softening curve; Sectional analysis

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1. Introduction

Ultra-high-performance fiber-reinforced concrete (UHPFRC), developed in the mid-1990’s, exhibits superior strength, i.e., compressive strength is greater than 150 MPa, and design value of tensile strength is 8 MPa (AFGC 2013), ductility, durability, fatigue performance, and energy absorption capacity (Richard and Cheyrezy 1995). These excellent properties can be obtained by optimizing granular mixture without coarse aggregate, resulting in the homogenization of microstructure, by using a very low water-to-binder ratio (W/B), by incorporating high volume contents of steel fibers (i.e., 2% by volume steel fibers are generally adopted for the UHPFRC commercially available in North America), and sometimes by applying heat treatment with steaming at 90°C and 95% relative humidity for 48 or 72 hours (Graybeal 2008; Yoo 2014).

Due to its excellent properties, many studies (Graybeal 2008; Yang et al. 2010; Yang et al. 2011; Noshiravani and Brühwiler 2013; Yoo et al. 2015c; Ferrier et al. 2015; Yoo et al. 2016a) have been carried out to evaluate the structural behavior of reinforced UHPFRC especially subjected to flexure. Graybeal (2008) performed structural test of a full-scale prestressed AASHTO Type II girder made of UHPFRC and proposed a flexural design philosophy for the prestressed UHPFRC girder. Yang et al. (2010) investigated the bending behavior of UHPFRC beams with low reinforcement ratios. They noted that the UHPFRC beams tested provided ductile response with the ductility indices ranging from 1.60 to 3.75, and the beams with placed at the end exhibited a better performance than those with placed at the mid-length. Yoo et al. (2016a) performed the flexural tests of UHPFRC beams reinforced with glass fiber-reinforced polymer (GFRP) and steel bars. In their test results, GFRP bar-reinforced UHPFRC beams satisfied the service crack width criteria and deformability limit by CAN/CSA S806 and CAN/CSA-S6 because of its strain-hardening characteristics, and they recommended to use GFRP bars with UHPFRC rather than to use hybrid reinforcements (GFRP + steel bars) with UHPFRC. Ferrier et al. (2015) also examined the flexural behavior of UHPFRC beams with carbon fiber-reinforced polymer (CFRP) and GFRP bars and reported that the use of CFRP bars was efficient in improving the flexural stiffness than the use of GFRP bars because of its higher elastic modulus.

Even though UHPFRC provides superb mechanical and structural performances mentioned above, the real application of this new type of construction material has been restricted by several reasons such as high cost and lack of design and analysis techniques. Firstly, because of its high material cost, several
researchers (Habel et al. 2007; Voort 2008; Yang et al. 2011; Noshiravani and Brühwiler 2013) have focused on the use of layered and/or prestressed members with optimized UHPFRC quantities. In particular, Habel et al. (2007) experimentally verified that the application of a UHPFRC layer in conventional RC elements clearly improves the service stiffness, reduces the deflection at a given load, decreases the crack widths and spacings, and delays the macro-crack formation, compared to the original RC element. Voort (2008) noted that a prestressed UHPFRC beam requires only half the section depth of ordinary reinforced or prestressed concrete beams, which in turn reduced its weight by 70% or more. Thus, the use of UHPFRC can result in a longer span structure with reduced member sizes as compared with ordinary concrete by significant reduction in self-weight and structural size.

The scant information of design and analysis techniques is crucial for the limited structural applications of UHPFRC. This is mainly caused by the consideration of tensile properties carrying a portion of the external loads. The tensile properties of UHPFRC, distinct from those of conventional concretes with and without fibers due to the superior fiber bridging at the crack surfaces, are characterized by a linear elastic behavior up to cracking, followed by a postcracking strain-hardening behavior with dispersed micro-cracks and softening behavior with crack localization (Habel et al. 2006). The portions of postcracking strain-hardening and softening are the difference with the conventional concretes and of interest in the design of structural elements made by UHPFRC. Two international recommendations (AFGC 2013; JSCE 2004) have been most widely adopted to design and predict the structural UHPFRC elements. Even though these two recommendations have the same concept to obtain tensile stress block (determining tensile stress-crack opening displacement (COD), $\sigma$-$w$, curve $\rightarrow$ transferring COD to strain $\rightarrow$ obtaining tensile stress-strain, $\sigma$-$\varepsilon$, curve), the predicted results are different because of disparate characteristic length, which requires to transfer COD to strain. In addition, UHPFRC is normally characterized by strain-hardening in tension with multiple micro-cracks when fibers are well aligned in the direction of tensile load (Yoo et al. 2016b). However, significant decrease of strain capacity, leading to a strain-softening behavior, was also observed in dog-bone UHPFRC specimens when concrete was casted in the vertical direction of tensile load (Bastien-Masse et al. 2016), due to poor fiber orientation. Unfortunately, no requirement exists in terms of the placement method for both material specimens used to suggest the tension-softening curve (TSC) and structural elements.

Accordingly, research effort discussed herein focused on investigating the structural behaviors of
UHPFRC beams reinforced with deformed steel reinforcing bars (rebars) and predicting their behaviors using the sectional analysis. Our specific objectives were: (1) to evaluate the effect of reinforcement ratio on the flexural behaviors including load carrying capacity, ductility, and cracking response and (2) to predict the moment-curvature behaviors using the sectional analysis, including the TSCs obtained from inverse analysis and direct tensile tests, based on two international recommendations (AFGC 2013; JSCE 2004) and placement methods.

2. Experimental program

2.1 Materials, mixture proportions, and mixing sequence

Canadian Type 10 Portland cement (ASTM Type I) and silica fume were used as the cementitious materials. The W/B was used as 0.2. Silica sand with a grain size smaller than 0.5 mm was adopted as a fine aggregate. Silica flour with 98% SiO$_2$ at 2 µm diameter was also added in the mixture as filler, and 1.6% of polycarboxylate superplasticizer (SP) was incorporated to provide adequate fluidity and viscosity for uniform fiber dispersion. In order to improve its homogeneity, coarse aggregate was excluded from the mixture. To improve the tensile performance, 2% by volume of straight steel fibers (fiber diameter/fiber length \((d_f/L_f) = 0.2/13 \text{ mm/mm}) were included, similar to previous studies (Yang et al. 2010; Yoo et al. 2015a). The detailed mixture proportions are summarized in Table 1.

Since UHPFRC has very low W/B and high fineness admixtures without coarse aggregate, the mixing sequence is different from that adopted for other concretes used previously as follows. First, cement, silica fume, silica sand, and silica flour were premixed for 10 min. Thereafter, water premixed with SP was poured into the dry state and mixed for another 10 min. When the mixture exhibited adequate fluidity and viscosity, the steel fibers were dispersed and mixed for another 5 min. In order to fabricate all the material and structural specimens by using the same UHPFRC mixture, a 1,200 L capacity twin-shaft batch mixer was adopted in this study.

Material and structural specimens were both covered with plastic sheets immediately after casting and cured at atmosphere temperature for the first 48 h. Thereafter, the specimens were demolded and steam-cured at high temperature \((90 \pm 2{^\circ}\text{C}) for 3 days. All of the specimens were then stored in the laboratory until testing.
2.2 Mechanical tests

Three cylindrical specimens (φ 100 × 200 mm) were fabricated and tested as per ASTM C 39 (ASTM 2014). The uniaxial load was monotonically applied using a universal testing machine (UTM) with a maximum capacity of 3000 kN, and the compressive strain was measured by using a compressometer equipped with three linear variable differential transformers (LVDTs). Three-point bending test was performed as per JCI-S-002-200 (JCI 2003) using three prisms (100 × 100 × 400 mm). All prismatic beams were fabricated by placing concrete at the corner and allowed it to flow. A 30-mm notch (0.3 × height of beam) was adopted at the mid-length using a diamond saw. A load was monotonically applied using a UTM with a maximum load capacity of 250 kN, and two LVDTs were installed on both sides of the beam to measure the average mid-span deflection. A clip gage with a capacity of 10 mm was located at the notch to measure crack mouth opening displacement (CMOD). The detailed test setups for compression and flexure tests were described previously (Yoo 2014).

Direct tensile test was conducted using a notched dog-bone specimen. A 10-mm notch was applied on both sides at the center of the specimen. Two clip gages with a capacity of 10 mm were located at the notches to measure CMOD as described in a previous study (Kang and Kim 2011). To avoid secondary flexural stress and to assure centric loading condition, the test setup was designed with so-called pin-fixed ends (Kanakubo 2006). The alignment of the test specimen was carefully checked using a plumb before testing. A UTM with a maximum load capacity of 250 kN was also used to apply a load under displacement control.

2.3 Test setup and instrumentation

A total of four UHPFRC beams with various reinforcement ratios were fabricated and tested. For all test beams, concrete was carefully placed at one end of the beam and allowed it to flow. This could provide uniform fiber orientation and dispersion and improve structural performance of UHPFRC beams, according to the test results of Yang et al. (2010). Even if using several specimens for each variable is good for obtaining reliable test data, it is limited due to the high material costs. Thus, we very carefully fabricated all the beams by applying identical placement method in order to improve the reliability of the test data obtained instead of increasing the number of test specimens. The details of geometry and reinforcement are shown in Fig. 1. All test beams were 2900 mm long with a rectangular cross-section of...
200 × 270 mm. These were reinforced with one or two layers of reinforcement. The effective depths of the one layer and the two layers were 240 and 222.5 mm, respectively. Yoo et al. (2015b) reported that the deformed steel rebar with a yield strength of 522.7 MPa embedded in UHPFRC yields at the embedment length of two times the rebar diameter \( L_e = 2d_s \) before pullout. Thus, the requirement for development length was satisfied without the end hook of longitudinal rebars. The main variable was the longitudinal reinforcement ratio. The specimens were divided into two series; a beam series reinforced without steel rebar (UH-N) and a beam series reinforced with steel rebars (UH-\( \rho \)). The letters \( \rho \) indicated the reinforcement ratio. For example, UH-0.53% indicates the UHPFRC beams with a reinforcement ratio of 0.53%. Deformed steel rebar with a nominal diameter of 12.7 mm was used as longitudinal reinforcement for all test beams. The properties of used steel rebar is summarized in Table 2.

Four-point static loading was applied using a UTM with a maximum load capacity of 2000 kN, as shown in Fig. 2. The load was applied monotonically in small increments while the loads, deflections, and strains were recorded. At each loading stage (20 kN interval), the number of cracks and average crack spacing were also recorded. The crack width was measured using a crack width comparator. The mid-span deflection and support settlements were measured using three LVDTs, and electrical resistance strain gages were attached to the center of all longitudinal reinforcements to determine the yielding point. In addition, four LVDTs were affixed to the side surface on the beam in pure moment region to measure the strains at different height. These four LVDTs were located at distances of 30 mm (T1), 70 mm (T2), 135 mm (C1), or 240 mm (B1) from the top fiber of the beam, as shown in Fig. 2(c).

3. Test results and discussion

3.1 Mechanical behaviors (compression and flexure)

Fig. 3(a) shows compressive stress-strain curves of all cylindrical specimens. In Table 3, the average compressive strength, strain capacity (strain at the peak), and elastic modulus were found to be 196.7 MPa, 0.0044, and 47.8 GPa, respectively. UHPFRC exhibited a very linear stress-strain curve up to failure. Test specimens were failed in a brittle manner with a rapid load decrease just after the peak load. However, insignificant fragmentation was observed due to the confinement effect by the fibers.

The flexural load-CMOD curves of all test beams are shown in Fig. 3(b). All beams showed a higher load carrying capacity after the first cracking and a very ductile post-peak softening behavior.
average flexural strength was found to be 35.6 MPa and the corresponding deflection and CMOD were found to be 0.63 and 1.05 mm, respectively.

Since the above cylindrical and prismatic specimens were all fabricated by using the UHPFRC mixture identical to that used for structural beams, the mechanical properties given in Table 3 can be considered as representative values for numerical analysis.

3.2 Structural behavior of reinforced UHPFRC beams

3.2.1 Cracking behavior

The crack patterns at the loading step just before the peak load are shown in Fig. 4(a), while the crack patterns at failure are shown in Fig. 4(b). For all test beams, very fine vertical flexural cracks were predominantly appeared perpendicular to the direction of the maximum principal stress induced by pure moment within the constant moment region at initial stage. As the applied load increased, initial flexure crack propagation outside the pure moment region was observed. In contrast to previous normal- and high-strength concrete beams, very fine vertical micro-cracks were only appeared for UHPFRC beams until near the peak load as shown in Fig. 4(a). The maximum crack widths obtained just before the peak loads were approximately 0.06 mm for all test beams. This indicated that up to the peak load, UHPFRC resisted a part of tensile stress induced by the external load, because it includes a high volume content of steel fibers, leading to strain-hardening behavior. The load carrying capacities of all test beams were gradually reduced with the widening of a specific single crack (named “crack localization”). For UH-N, the load carrying capacity was rapidly decreased with the widening of one of the vertical flexure cracks. For UH-0.53% and UH-1.06%, the flexural-shear crack originated as the vertical crack outside the loading points appeared at near the peak load. Subsequently, the load carrying capacity was gradually decreased with the widening of the flexural-shear crack. In the case of UH-1.71%, the shear and flexural crack widths were simultaneously increased at near the peak load, but it was finally failed by the rupture of outer longitudinal steel rebar since the shear resistance increased due to the improved dowel action from double layer reinforcing. The number of cracks in all test beams increased with increasing the applied load, whereas the average crack spacing decreased with increasing the applied load, as shown in Fig. 5. The depth of crack propagation at the ultimate state was obviously reduced with an increase in the reinforcement ratio (in Fig. 4(b)), whereas both the number of cracks and average crack spacing were
insignificantly affected by the reinforcement ratio (in Fig. 5). From the observations, it can be noted that the number of cracks and average crack spacing of UHPFRC beams were more influenced by the fiber ratios rather than the passive reinforcing bars.

### 3.2.2 Load–deflection (or –strain) behaviors

Since UHPFRC exhibits strain-hardening behavior, it is hard to detect the first cracking point of UHPFRC beams including steel rebars from either of load-deflection curve and visual inspection. Therefore, in this study, six strain gages with a length of 60 mm were glued to the bottom surface in pure moment region to detect the first cracking point. In accordance with a previous study (Lim et al. 1999), the first cracking point was determined when the strain measured from the strain gages suddenly changed in response to the first crack occurrence.

As summarized in Table 4, the first cracking load decreased with the increase in the number of outer steel rebars. The least first cracking load was found to be 59.4 kN for UH-1.06% (four outer steel rebars), which was approximately 3.7–9.2% lower than those obtained from other beams including lower number of outer steel rebars. Based on a previous study performed by Yoo et al. (2014), UHPFRC exhibited very high early-age autogenous shrinkage, and higher steel rebar ratio resulted in higher residual tensile stress generated in UHPFRC by restraint of shrinkage. Therefore, the decreased first cracking load in the beams with higher number of outer steel rebar was caused by the increased residual tensile stress in UHPFRC prior to the external load due to the restraint of shrinkage.

The applied load versus mid-span deflection curves are shown in Fig. 6. In addition, the loads and mid-span deflections at the first crack, yielding of steel rebar, and peak load are summarized in Table 4. All test beams showed similar linear load-deflection behaviors before the first cracking. After that, non-linear load-deflection behaviors were obtained, and higher reinforcement ratio provided higher post-cracking stiffness until the peak load. The loads at yielding of steel rebar and at peak load were increased with reinforcement ratio. The highest load carrying capacity was found to be 250 kN for UH-1.71%, approximately 10–81% higher than those of the beams with lower reinforcement ratios. The load at yielding of steel rebar was also increased with increasing the reinforcement ratio. This is mainly resulted from a higher post-cracking stiffness, which requires a higher load to provide an identical strain in steel rebar, at a higher reinforcement ratio. Higher deflection at the peak load was obtained by including steel rebars.
rebars, whereas it was insignificantly influenced by the reinforcement ratio. The deflection at the peak load was found to be 9.6 mm for UH-N, approximately 28–30% lower than those obtained from beams with steel rebars. These results indicated that the flexural performance of UHPFRC beams was improved by including steel rebars in the tensile zone. In addition, the post-peak behavior was substantially affected by the reinforcement ratio. The slope in the descending branch of load-deflection curve was mitigated by increasing the reinforcement ratio, and particularly the specimen UH-1.71% exhibited no obvious decrease in the load carrying capacity after the peak load until steel rebar rupture, exhibiting almost perfectly plastic behavior.

The load-strain responses obtained from the LVDTs on the side surface of the beams are shown in Fig. 7. The load-strain data of UH-N was not obtained due to technical problem during the test, and similar shape of load-strain curve was obtained regardless of the reinforcement ratio. In Fig. 7, negative strain indicated the compressive strain, while positive strain indicated the tensile strain. The compressive strains were developed at positions of T1 and T2 until the peak load, while the tensile strain was developed at a position of B1 until the peak load. The strain at position of C1 showed almost zero strain until approximately 100 kN, then changed to tensile strain. This strain change is on account of the propagation of tensile cracks from the bottom to the top and the upward shift of neutral axis in the cross-section. The abrupt strain change (with a slight decrease of load) after initiation of cracking was not observed at position of B1 in contrast to the reinforced ultra-high-strength concrete beams without fiber (Yoo and Yoon 2015c). This is because the high volume contents of steel fibers at the crack planes can resist the applied tensile load immediately after cracking.

3.2.3 Ductility

Several forms of ductility such as deflection, curvature or rotational ductility are available. In this study, the deflection ductility was used to characterize the ductility of test beams. This ductility index can be obtained by dividing the mid-span deflection at the peak load by the mid-span deflection at the yielding of tensile longitudinal reinforcement. Thus, the ductility index is simply calculated by Eq. (1).

\[ \mu_p = \frac{\Delta_p}{\Delta_y} \]  

(1)
where $\Delta_p$ is the mid-span deflection at the peak load and $\Delta_y$ is the mid-span deflection at the yielding of steel rebar.

Furthermore, Shin et al. (1989) suggested Eq. (2) for evaluating deflection ductility when the RC beams continued to sustain the load well beyond the peak flexural load.

\[ \mu_p = \frac{\Delta_u}{\Delta_y} \]

where $\Delta_u$ is the mid-span deflection at the ultimate (80% of peak load in the descending branch).

The ductility indices obtained from Eqs. (1) and (2) are summarized in Table 4. The ductility $\mu_p$ of UHPFRC beams with steel rebars decreased with the increase in reinforcement ratio. This result was in accordance with the findings of many researchers (Shin et al. 1989; Ashour 2000; Bernardo and Lopes 2004). However, in contrast to $\mu_p$, the second ductility $\mu_u$ was improved with the increase in the reinforcement ratio. In particular, UH-1.71% showed much higher $\mu_u$ of 3.98 than $\mu_u$ of 2.12 for UH-0.53% and $\mu_u$ of 2.14 for UH-1.06%, respectively. This because the load carrying capacity of UH-1.71% well sustained after the peak load, compared to those of UH-0.53% and -1.06% (see Fig. 6). For the beams with low reinforcement ratios, the peak load can be improved by the strain-hardening behavior of UHPFRC, exhibiting a higher tensile load carrying capacity after first cracking. Therefore, a decrease in the load carrying capacity was observed in the UH-0.53% and -1.06% after the peak point. However, in the case of UH-1.71%, the increase of peak load by its strain-hardening effect was mitigated due to the higher reinforcement ratio, which means that the steel rebars dominated the post-cracking flexural behavior, and thus, no significant decrease in the post-peak load carrying capacity was obtained. Consequently, the specimen UH-1.71% exhibited almost perfect plastic post-peak behavior and much higher ultimate ductility $\mu_u$ than others.

4. Analytical approach for predicting moment-curvature response of UHPFRC beams

4.1 Material modeling and analytical procedure

As shown in Fig. 3(a), UHPFRC showed a very linear compressive stress-strain curve up to failure.
Therefore, the compressive behavior of UHPFRC was modeled with a linear stress-strain relation up to failure in this study in contrast to conventional concretes modeled with a parabolic stress-strain relation.

For tensile modeling, the inverse analysis proposed by Uchida et al. (1995) was adopted to obtain an optimized TSC. The cohesive stress-crack opening relationship was determined using crack growth analysis based on the fictitious crack model (Hillerborg et al. 1976) and the polylinear approximation method (Kitsutaka 1997). The finite element modeling for the half of the beam was incorporated into the inverse analysis, and the center of the specimen was modeled with dense meshing to precisely estimate the crack propagation (Yoo and Yoon 2015c).

Polylinear TSC obtained from inverse analysis is shown in Fig. 8. To verify this curve, the analytical prediction was compared to experiments as shown in Fig. 3(b). The analysis fairly well predicted the experimental flexural behaviors. The discrepancy between the calculated peak load 34.3 kN and the measured peak load 33.9 kN was only 1.2%. From this, it was verified that the polylinear TSC was appropriate to predict the flexural behavior of UHPFRC beam.

To investigate the effect of placement method on the accuracy of prediction for flexural response of reinforced UHPFRC beams, two different TSCs obtained from inverse analyses in this study and a previous study (Yoo et al. 2013) were adopted. In this study, the beams were fabricated by placing concrete at one corner and leading it to flow, whereas in the previous study, the beams were fabricated by placing concrete parallel to the tensile direction to intentionally control the fiber arrangement. The TSC obtained from a direct tensile test was also considered from a previous study (Yoo et al. 2016b) to estimate its applicability. The highest tensile strength was obtained for the bilinear TSC, followed by the polylinear TSC from inverse analysis and the TSC from direct tensile test, as shown in Fig. 8. However, the difference between the tensile strengths in TSCs obtained from inverse analysis and direct tensile test in the present study was only 4.8%, thus it was noted that the TSC from tensile test can be also adopted for the numerical analysis.

In order to convert the obtained tensile stress-CMOD curve into the stress-strain curve for sectional analysis, two different design recommendations (AFGC 2013; JSCE 2004) were applied. The equations of these two design recommendations are summarized in Appendix.

The tensile stress-strain curves of UHPFRC obtained from AFGC recommendation are shown in Fig. 9(a). The specimen with concrete placed parallel to the tensile direction (bilinear TSC) exhibited higher
tensile strength and stresses corresponding to crack widths of 0.3 mm and 1% of height of the prism specimen due to a better fiber orientation at the cracked surfaces than its counterpart. On the other hand, the tensile stress-strain curve obtained from inverse analysis showed similar behavior with that obtained from direct tensile test. In addition, the ultimate tensile strain was identical in all cases (by 0.018), because the used fiber length and beam height were identical to each other. Fig. 9(b) shows the tensile stress-strain curves calculated based on JSCE recommendation. The tensile stress-strain curve obtained from inverse analysis in present study exhibited very similar behavior to that obtained from direct tensile test. From this observation, it was concluded that the TSCs obtained from both of inverse analysis and uniaxial tensile test were adoptable for sectional analysis. However, the bilinear TSC exhibited the much higher tensile strength and stress at certain strain than others. In comparison with the stress-strain curve obtained from AFGC recommendations (Fig. 9(a)), the tensile strength slightly decreased (maximum difference was 4.7%), but the ultimate strain substantially increased (maximum difference was 152.4%). This might be due to the characteristic length suggested by each recommendation was different. For the steel rebar, bilinear stress-strain curve was assumed. The yield strength and elastic modulus were used by 522.7 MPa and 200.0 GPa (Table 2), respectively.

Considering the shear component for the numerical analysis is complicated. In addition, all the tested beams were more dominated by the flexural failure modes (i.e., localization of flexural crack and rupture of longitudinal steel rebar) rather than the shear failure. Thus, a simple sectional analysis, which has been adopted by several researchers (Kooiman 2004; Thomas and Ramaswamy 2006; Yang et al. 2011) for analyzing the flexural response of fiber-reinforced concrete (FRC) beams, was used in this study. The compressive and tensile stresses at each layer can be effectively considered with the assumption of linear strain distribution (plane section remains plane), and the neutral axis depth can be calculated from force equilibrium in the cross-section. The algorithm of multi-layer method is found in a previous study (Yoo et al. 2015c)

### 4.2 Verification of sectional analysis

In order to compare the experimental and numerical results, the applied load was transformed to the moment from the linear elastic theory, Eq. (3), and the values of curvature at each loading points were simply calculated based on the neutral axis depth and the strain measured at the location of T1.
\[ M = \frac{PL}{2} \] (3)

where \( P \) is the applied load and \( L \) is the length between the support and the loading point (= 1050 mm).

The comparison of moment-curvature responses obtained from the experiments and sectional analyses using various material models is shown in Figs. 10 and 11. The moment-curvature curves obtained from the experiments were only plotted up to the peak point because the reliability of the measured strain becomes poor after that, owing to the crack localization obtained outside of the maximum moment region. In addition, the comparison of maximum moments from the experiments and sectional analyses is summarized in Table 5.

The analytical results obtained from AFGC recommendations considering TSCs from inverse analysis and direct tensile test in present study exhibited good agreement with the test data. These analyses provided better predictions for UHPFRC beams reinforced with steel rebars than that without steel rebar (UH-N). In addition, the calculated curvature at the peak moment accurately corresponded to the test data. On the other hand, the analytical results obtained from AFGC recommendations with the bilinear TSC overestimated the moment capacity because of its higher tensile strength parameter. Therefore, it was concluded that the TSC should be carefully adopted to predict flexural behavior of reinforced UHPFRC beams in consideration of the placement method (or fiber orientation). For the JSCE recommendations, the uses of TSCs obtained from inverse analysis and direct tensile test in present study showed very similar predicted moment-curvature curves each other. The ultimate moment calculated by JSCE recommendations fairly well corresponded to the test data when using the TSCs obtained from inverse analysis and direct tensile test, whereas larger curvatures at the peak were obtained for analyses than those for experiments. Therefore, it was concluded that the characteristic length suggested by AFGC recommendations was more appropriate to predict flexural response of steel bar reinforced UHPFRC beams than that suggested by JSCE recommendations.

4.3 Discussion on using a direct “stress-strain” approach

Since the reinforced UHPFRC beams exhibited multiple micro-cracks, it is worthy to investigate the
The applicability of a pre-peak tensile stress-strain curve (before crack-localization) obtained from direct tensile tests without notch. To do this, a direct tensile stress-strain curve was first obtained from a previous study (Yoo et al. 2016b) and simply modeled as a bilinear curve up to the peak strength, which is called modeled direct “stress-strain” in Fig. 12. After the peak strength, it was assumed that the tensile stress in the modeled direct “stress-strain” curve immediately drops to zero value to determine the point where crack localization occurs. In addition, two reference tensile stress-strain curves calculated by AFGC and JSCE recommendations based on a polylinear TSC were compared.

Fig. 13 shows the comparison of experimental and numerical results based on three different tensile stress-strain models as mentioned above. It is interesting to note that the use of modeled direct “stress-strain” curve exhibited slightly stiffer moment-curvature behavior than those based on AFGC and JSCE recommendations due to the higher tensile strength and post-cracking secant modulus in Fig. 12. In the case of modeled direct “stress-strain,” since the crack localization from the bottom of the beam occurred before reaching the actual maximum moment, the decrease of moment started at the lower curvature than that from the experiments. (The moment decrease zone is caused by the assumption that tensile stress drops immediately after reaching the tensile strength, thus this is not a real behavior.) Consequently, it was noted that the tensile stress-strain model obtained from a direct tensile test using dog-bond specimens without notch can also be adopted for predicting the moment-curvature behavior of reinforced UHPFRC beams before the crack localization.

5. Conclusions

This study investigated the flexural behaviors of UHPFRC beams with various reinforcement ratios. Because we focused on the placement technique of internal steel rebars, only small number (four) of steel bar-reinforced UHPFRC beams were fabricated and tested under quasi-static loads. In addition, to predict the moment-curvature response of UHPFRC beams, the sectional analysis on the basis of AFGC and JSCE recommendations was adopted. From the above discussions, the following conclusions were drawn:

1) The first cracking load slightly decreased with an increase in the number of outer steel rebar due to the increased residual tensile stress by restraint of shrinkage. However, the higher reinforcement ratio provided higher post-cracking stiffness and yielding and peak loads.
2) In contrast to RC beams made of normal concrete, steel bar-reinforced UHPFRC beams exhibited only vertical micro-cracks until near the peak load, then the load carrying capacity was gradually reduced with the widening of a specific single crack.

3) Higher reinforcement ratio effectively controlled the crack propagation to compressive zone, whereas the number of cracks and average crack spacing were insignificantly affected by the reinforcement ratio.

4) AFGC recommendations well predicted the flexural behavior including both the maximum moment and corresponding curvature, whereas the curvature at the peak predicted by JSCE recommendations was larger than that obtained from experiments due to inappropriate characteristic length. Thus, AFGC recommendations are considered to be more appropriate for predicting the flexural behavior of reinforced UHPFRC beams than JSCE recommendations.

5) The analytical results based on the TSC obtained from different placement methods overestimated the maximum moments from the experiments due to the different fiber orientation characteristics. Thus, the TSC should be carefully adopted by considering the placement method that affects fiber orientation. The tensile model obtained from a direct tensile test without notch can also be adopted for numerical analysis before the crack localization.

Based on the numerical results, it is obvious that the use of appropriate material models obtained by performing mechanical tests and inverse analysis is better for the practical structural design and prediction. However, if it is difficult to obtain actual material models, the design parameters proposed by AFGC and JSCE recommendations can be adopted alternatively.

Acknowledgements
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Appendix
• AFGC/SETRA recommendation:

According to AFGC/SETRA recommendations, the stress-strain curve can be determined by Eqs. (A1)–(A6) based on the stress-crack opening relations. Elastic strain, strains at crack widths of 0.3 mm
and 1% of beam height, and ultimate strain can be calculated as follows:

\[ \varepsilon_e = \frac{f_t}{E_c} \]  
\[ \varepsilon_{0.3} = \frac{w_{0.3}}{l_e} + \frac{f_t}{\gamma_{bf} E_c} \]  
\[ \varepsilon_{1\%} = \frac{w_{1\%}}{l_e} + \frac{f_t}{\gamma_{bf} E_c} \]  
\[ \varepsilon_{\text{lim}} = \frac{l_f}{4l_e} \]

where \( \varepsilon_e \) is the elastic strain, \( f_t \) is the tensile strength, \( E_c \) is the elastic modulus of concrete, \( \varepsilon_{0.3} \) is the strain at crack width of 0.3 mm, \( w_{0.3} \) is the crack width of 0.3 mm, \( l_e \) is the characteristic length (2/3×h for rectangular and T-beams), \( \gamma_{bf} \) is the partial safety factor, \( \varepsilon_{1\%} \) is the strain at crack width corresponding to 0.01H, \( w_{1\%} \) is the crack width corresponding to 0.01H, \( \varepsilon_{\text{lim}} \) is the ultimate tensile strain, and \( l_f \) is the fiber length.

The stresses at the crack widths of 0.3 mm and 1% of beam height are calculated by

\[ f_{se} = \frac{f(w_{0.3})}{K\gamma_{bf}} \]  
\[ f_{1\%} = \frac{f(w_{1\%})}{K\gamma_{bf}} \]

where \( f_{se} \) is the stress at crack width of 0.3 mm, \( K \) is the fiber direction coefficient, and \( f_{1\%} \) is the stress at crack width corresponding to 0.01H. To accurately predict the flexural behavior, \( \gamma_{te} \) and \( K \) were assumed to be 1 as in a previous study (Yang et al. 2011).

- JSCE recommendation:

  The stress-strain relation suggested by JSCE recommendations can be expressed using Eqs. (A7)–(A9)
in terms of the stress-crack opening curves.

\[ \varepsilon_{cr} = \frac{f_{tk}}{\gamma_c E_c} \]  
(A7)

\[ \varepsilon_1 = \varepsilon_{cr} + \frac{w_{1k}}{L_{eq}} \]  
(A8)

\[ \varepsilon_2 = \frac{w_{2k}}{L_{eq}} \]  
(A9)

where \( \varepsilon_{cr} \) is the factored elastic strain, \( f_{tk} \) is the tensile stress at \( w_{1k} = 0.5 \) mm, \( \gamma_c \) is the partial safety factor, \( \varepsilon_1 \) is the strain at the end of initial plateau, \( w_{1k} \) is the CMOD for which a certain stress level is retained after the first crack, \( L_{eq} \) is the equivalent specific length (\( = 0.8h \times \left[ 1 - 1/(1.05 + 6h/l_{ch}) \right] \)), \( h \) is the overall depth of beam, \( l_{ch} \) is the characteristic length (\( = G_f f_{tk}^2 \)), \( G_f \) is the fracture energy, \( \varepsilon_2 \) is the strain at \( f_{tk} = 0 \), and \( w_{2k} \) is the CMOD at zero tensile stress. Similar to the AFGC recommendations, \( \gamma_c \) was assumed to be 1 to accurately predict flexural behaviors.

JSCE recommendations (JSCE 2004; Uchida et al. 2006) suggested the values of \( f_{tk} = 8.8 \) MPa, \( w_{1k} = 0.5 \) mm, and \( w_{2k} = 4.3 \) mm for ultra-high-strength fiber-reinforced concrete with 2 vol.% of steel fibers, which has similar mixture proportions and mechanical properties to UHPFRC used in this study. Therefore, we adopted the parameters of \( w_{1k} = 0.5 \) mm and \( w_{2k} = 4.3 \) mm. However, the tensile strength parameter \( f_{tk} \) in TSC is sensitive to several factors, thus the tensile stresses at \( w_{1k} = 0.5 \) mm obtained from inverse analysis and direct tensile test were used instead of 8.8 MPa. Fracture energy was obtained from the area under TSCs obtained and summarized in Table 3.

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### Table 1 Mix proportions

<table>
<thead>
<tr>
<th>$W/B$ (%)</th>
<th>Water</th>
<th>Cement</th>
<th>Silica fume</th>
<th>Silica flour</th>
<th>Silica sand</th>
<th>SP (%)</th>
<th>Flow (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>160.3</td>
<td>788.5</td>
<td>197.1</td>
<td>236.6</td>
<td>867.4</td>
<td>2.0</td>
<td>230</td>
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</table>

[Note] SP = superplasticizer

### Table 2 Properties of deformed steel reinforcing bar

<table>
<thead>
<tr>
<th>$d_s$ (mm)</th>
<th>$A_s$ (mm$^2$)</th>
<th>$E_s$ (GPa)</th>
<th>$f_y$ (MPa)</th>
<th>$\varepsilon_y$ (mm/mm)</th>
<th>$f_{ult}$ (MPa)</th>
<th>$\varepsilon_{ult}$ (mm/mm)</th>
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</thead>
<tbody>
<tr>
<td>12.7</td>
<td>126.7</td>
<td>200.0</td>
<td>522.7</td>
<td>0.0026</td>
<td>627.6</td>
<td>0.164</td>
</tr>
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</table>

[Note] $d_s$ = nominal diameter, $A_s$ = area, $E_s$ = elastic modulus, $f_y$ = yield strength, $\varepsilon_y$ = yield strain, $f_{ult}$ = ultimate strength, and $\varepsilon_{ult}$ = ultimate strain

### Table 3 Summary of mechanical and fracture properties of UHPFRC

<table>
<thead>
<tr>
<th>Name</th>
<th>$f'_c$ (MPa)</th>
<th>$\varepsilon_u$ (mm/mm)</th>
<th>$E_c$ (GPa)</th>
<th>$f_{MOR}$ (MPa)</th>
<th>$f'_t$ (MPa)</th>
<th>$f''_t$ (MPa)</th>
<th>$f''_t$ (MPa)</th>
<th>$G^1_f$ (N/mm)</th>
<th>$G^2_f$ (N/mm)</th>
<th>$G^3_f$ (N/mm)</th>
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</thead>
<tbody>
<tr>
<td>UHPFRC</td>
<td>196.7</td>
<td>4.4×10$^5$</td>
<td>47.8</td>
<td>35.6</td>
<td>10.9</td>
<td>11.4</td>
<td>13.4</td>
<td>27.8</td>
<td>32.2</td>
<td>38.2</td>
</tr>
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</table>

[Note] $f'_c$ = compressive strength, $\varepsilon_u$ = strain at the peak, $E_c$ = elastic modulus, $f_{MOR}$ = flexural strength, $f_t$ = tensile strength, and $G_f$ = fracture energy

1 obtained from direct tensile test
2 obtained from inverse analysis
3 obtained from previous research (Yoo et al. 2013)
Table 4 Structural test results of UHPFRC beams with steel rebar

<table>
<thead>
<tr>
<th>Name</th>
<th>$P_{cr}$ (kN)</th>
<th>$\Delta_{cr}$ (mm)</th>
<th>$P_{y}$ (kN)</th>
<th>$\Delta_{y}$ (mm)</th>
<th>$P_{p}$ (kN)</th>
<th>$\Delta_{p}$ (mm)</th>
<th>$\Delta_{u}/\Delta_{y}$</th>
<th>$\Delta_{u}^{*}/\Delta_{y}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>UH-N</td>
<td>65.2</td>
<td>1.28</td>
<td>-</td>
<td>-</td>
<td>138.1</td>
<td>9.60</td>
<td>12.84</td>
<td>-</td>
</tr>
<tr>
<td>UH-0.53%</td>
<td>62.8</td>
<td>1.10</td>
<td>168.4</td>
<td>9.13</td>
<td>186.5</td>
<td>13.49</td>
<td>19.35</td>
<td>1.48</td>
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<tr>
<td>UH-1.06%</td>
<td>59.4</td>
<td>0.75</td>
<td>200.3</td>
<td>10.31</td>
<td>226.2</td>
<td>13.41</td>
<td>22.10</td>
<td>1.30</td>
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<tr>
<td>UH-1.71%</td>
<td>61.7</td>
<td>0.80</td>
<td>224.8</td>
<td>11.17</td>
<td>249.5</td>
<td>13.77</td>
<td>44.5</td>
<td>1.23</td>
</tr>
</tbody>
</table>

[Note] $P_{cr}$ = cracking load, $\Delta_{cr}$ = deflection at first crack, $P_{y}$ = load at rebar yielding, $\Delta_{y}$ = deflection at rebar yielding, $P_{p}$ = peak load, $\Delta_{p}$ = deflection at peak, and $\Delta_{u}$ = deflection at ultimate

* Deflection corresponding to 80% of peak load in descending branch
<table>
<thead>
<tr>
<th>Name</th>
<th>Moment at ultimate state (kN·m)</th>
<th>Test result / Analytical result</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Test result</td>
<td>AFGC(^1)</td>
</tr>
<tr>
<td>UH-N</td>
<td>72.50</td>
<td>66.56</td>
</tr>
<tr>
<td>UH-0.53%</td>
<td>97.91</td>
<td>93.80</td>
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<tr>
<td>UH-1.06%</td>
<td>118.76</td>
<td>121.02</td>
</tr>
<tr>
<td>UH-1.71%</td>
<td>130.99</td>
<td>141.22</td>
</tr>
<tr>
<td>Average</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Standard deviation</td>
<td>0.071</td>
<td>0.032</td>
</tr>
</tbody>
</table>

\(^1\) using polylinear TSC from inverse analysis in present study
\(^2\) using bilinear TSC from Yoo et al. (2013)
\(^3\) using TSC from direct tensile test
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