THE $\beta$-DELAYED 3-BODY BREAKUP OF $^{17}$Ne
AND ITS RELATION TO ASTROPHYSICS

by

Chun-leung Jimmy Chow

A thesis submitted in conformity with the requirements
for the degree of Doctor of Philosophy
Graduate Department of Physics
University of Toronto

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Abstract

This is a study on the feasibility of using the $\beta$-delayed 3-body decay of $^{17}$Ne to constrain the astrophysically important $^{12}$C($\alpha$, $\gamma$)$^{16}$O reaction rate at stellar energies. The relative rate of this reaction to that of $3\alpha \rightarrow ^{12}$C+$\gamma$ is believed to strongly affect the evolution of massive stars subsequent to the phase of helium burning. The usefulness of the cascade decay of $^{17}$Ne in this study hinges on the existence of $\alpha$-unbound states of $^{16}$O in the decay chain. Detection of all the daughter particles, $p+\alpha^{12}$C in triple coincidence, is necessary to optimize the kinematic information inherent in each event to discriminate against background. The first triple-coincidence measurement was accomplished in 1996, at the TRIUMF-TISOL facility in Vancouver, BC, Canada, in which the isobaric analogue state in $^{17}$F at 11.193 MeV ($J^{\pi}=\frac{1}{2}^-$) was observed to decay into three particles via three channels, viz., the 9.59 MeV ($J^{\pi}=1^-$) state in $^{16}$O, and the 2.37 ($J^{\pi}=\frac{1}{2}^+$) and 3.50/3.55 MeV ($J^{\pi}=\frac{3}{2}^-/\frac{5}{2}^+$) states in $^{13}$N. In two subsequent measurements, various techniques were incorporated to improve the efficiency in background discrimination. The connection between experiment and theory is made through extensive Monte Carlo simulations and analytic calculations within the framework of R-matrix theory. Based on a single-channel, 2-level R-matrix calculation, an optimum detection geometry has been defined to constrain the strength of the $E1$ component of the $^{12}$C($\alpha$, $\gamma$)$^{16}$O reaction. According to results from another single-channel, single-level R-matrix calculation, the 10.03 MeV state in $^{17}$F has been identified as the best candidate to constrain the strength of the $E2$ component. However, more studies are required before the feasibility of using the decay of $^{17}$Ne to constrain the strength of the $E2$ component can be established.

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Chapter 1

Introduction

1.1 Motivation

It is commonly believed that most of the elements heavier than helium have been synthesized through nuclear processes taking place in the interior of massive stars [1–3]. The studies of such nuclear processes fall in the discipline of nuclear astrophysics and, in particular, stellar nucleosynthesis. One of the major challenges of the theory of stellar nucleosynthesis has been the development of self-consistent stellar evolution models which incorporate detailed nuclear reaction networks based on accurate calculations or measurements of nuclear reaction rates. After four decades of work since the paper of B2FH (Burbidge, Burbidge, Fowler and Hoyle) [1], many of the key reaction rates have been determined. However, the rate of the radiative capture reaction $^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$ at stellar energies remained highly uncertain until very recently (see Sec. 1.4). The importance of this reaction rate in validating network calculations of nucleosynthesis in massive stars was first emphasized in Ref. [4], in which it was stated that excellent agreement between calculated elemental abundances of essentially all the intermediate mass isotopes ($16 \leq A \leq 32$) and observed solar values can be achieved if the $^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$ reaction rate is constrained to $1.7 \pm 0.5$ times a previously published value [5], which is equivalent to an astrophysical $S$ factor$^\dagger$ of $170 \pm 50$ keV·b at a

$^\dagger$It is customary in nuclear astrophysics to express the cross section as $\sigma(E) = \frac{1}{E} \exp(-2\eta)S(E)$, where $\eta = \frac{Z_1 Z_2 e^2}{h v}$ is the Sommerfeld parameter and the function $S(E)$ is defined as the astrophysical $S$ factor. It is commonly believed that, since $S(E)$ varies much less rapidly with energy than $\sigma(E)$,
center-of-mass energy of $E_{\text{cm}} = 300$ keV, the *most effective energy*\(^1\) of helium burning. This stringent *nucleosynthetic prediction* has motivated many experimentalists like those in our group to seek better laboratory methods to constrain the $^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$ reaction rate. Recently, two groups \(^7,^8\) have claimed that their results have satisfied the conditions set forth in Ref. \(^4\) (see Sec. 1.4). However, more recently, one of the authors of Ref. \(^4\) has updated the required precision to $\leq 20\%$ to reflect recent progress in network calculations, and has re-emphasized that the $^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$ reaction "is such a critically important rate for stellar evolution and nucleosynthesis that it warrants several determinations" \(^9\)\(^*\).

It is always beneficial to have independent determinations of the same quantity to confirm previous results, and to improve precision. In fact, the work relevant to this thesis, which concerns the feasibility of constraining the $^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$ reaction rate by a study of the $\beta$-delayed proton decay of $^{17}\text{Ne}$, started at a time when the stringent limits set forth in Ref. \(^4\) were not satisfied by any measurements. The reaction is thought to consist of at least a dipole ($E1$) and a quadruple ($E2$) component (see Sec. 1.3). In a study of the $\beta$-delayed $\alpha$ decay of $^{16}\text{N}$ performed a few years ago, our group\(^5\) succeeded in constraining the $E1$ component of the reaction to a precision of $27\%$ \(^10,^{11}\). However, no information on the $E2$ component was available in that study (see Sec. 1.4) and the total reaction rate remained highly uncertain. It has been suggested that a study of the $\beta$-delayed proton decay of $^{17}\text{Ne}$ will provide information to determine the strength of the $E2$ component, as well as to re-determine that of $E1$, so that the constraints on the total reaction rate can be improved \(^12,^{13}\).

---

\(*\)This is also known as the Gamov energy (see, e.g., Ref. \(^6\), p.159] for a derivation).

\(^*\)In fact, Woosley has stated a requirement of "$2\sigma$ or better"; i.e., the actual error required is less than 10\% at the 1\(\sigma\) level. \(^9\).

\(^5\)In fact, the study was performed before I joined the group in 1995.
§1.2 Helium Burning

1.2 Helium Burning

The birth of a star starts with the gravitational collapse of interstellar gases composed predominately of hydrogen, with a fair amount of $^4\text{He}$, and traces of $^2\text{H}$, $^3\text{He}$, and $^7\text{Li}$—all of which are believed to have originated from the *Big-Bang*. With the smallest charge among the nuclei ($Z = 1$), hydrogen nuclei are the first that can *tunnel* through the Coulomb barrier and initiate the so-called *pp-chains* of nuclear reactions to produce helium. For non-first-generation stars in which $^{12}\text{C}$ nuclei are present, the so-called CNO cycle is also possible for the production of helium. A thorough discussion of hydrogen burning can be found in Ref. [6, Chap. 6]. An up-to-date account of the progress in studies of nucleosynthesis in stars has been compiled in Ref. [14]. The following discussion will concentrate on the phase of helium burning—the nucleosynthetic process relevant to the $^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$ reaction, whose reaction rate measurement is the motivation behind this thesis.

Towards the conclusion of hydrogen burning in massive stars ($\geq 0.5 \, M_\odot$), much of the hydrogen has been converted to helium to form a core, the ash of hydrogen burning, in the interior. While converting gravitational energy into internal energy, the core contracts and becomes higher in density and temperature. The increase in temperature heats up the thin shell of hydrogen around the core, and hydrogen burning continues within this shell to further augment the helium core. The elevated temperature in the stellar interior also causes the outer regions to expand to many times the original radius. Consequently, the surface temperature drops and the radiation shifts to longer wavelengths. The shift in color to red and the increase in size lead the star to the so-called *red-giant* stage (see Ref. [6, Chap. 7]).

The phase of helium burning commences as a result of continual gravitational contraction of the helium core to the point where the temperature and density are suffi-

---

$^1M_\odot \approx 2 \times 10^{30}$ kg denotes one solar mass.
ciently high (T=10^8 K; ρ≈10^5 g/cm^3) for nuclear reactions among helium nuclei to take place. This temperature is equivalent to an α+^{12}C center-of-mass energy of 300 keV, the most effective energy of helium burning.

Two processes are known to dominate the phase of helium burning [6]. The first is the so-called triple-α process 3α→^{12}C+γγ, which is effectively a two-step process. In the first step, α+α→^8Be(gs), two α particles combine to form ^8Be in its ground state. The ground state of ^8Be is unstable against decay into two α particles with a lifetime of 0.968 × 10^{-16} s. This relatively short lifetime is sufficiently long for a ^8Be nucleus to capture a third α particle in the α+^8Be→^{12}C*(7.65)† process with a small but finite probability. It is interesting to note that, in order to reproduce the observed ^12C abundance, Hoyle et al. predicted the existence of the 7.65 MeV (J^π = 0^+) state in ^12C [15], which was later confirmed experimentally [16]. The triple-α process can be described symbolically as 3α→^{12}C*(7.65)→^{12}C(gs)+γγ (or e^+e^-); i.e., three α particles combine to form the 7.65 MeV state in ^12C, which decays† by cascade electromagnetic transition to the ground state.

The other dominating process in helium burning is ^12C(α, γ)^16O, the radiative capture of an α particle by ^12C. (The determination of the rate of this process in a stellar environment is the motivation behind the work in this thesis [13].) Once sufficient ^12C nuclei are produced in the triple-α process, it becomes more probable that a ^12C nucleus will capture another α particle to form ^16O through ^12C(α, γ)^16O. This process competes with the triple-α process for consumption of helium, and the relative rate between these processes is critical to the subsequent evolution of the star. If the triple-α process dominates, most helium will be converted into ^12C before any ^16O can be produced. Conversely, if the ^12C(α, γ)^16O process dominates, ^12C formed in

†Nuclear states are usually labelled by their level energies in this thesis.

†In fact, the dominant decay mode of ^12C(7.65) is α decay into ^8Be(gs), which in turn breaks up into two α particles. In only one in 2500 cases will ^12C(7.65) decay by an electromagnetic transition [6].
§1.2 Helium Burning

the triple-\(\alpha\) process will mostly be converted into \(^{16}\text{O}\), and little \(^{12}\text{C}\) will remain once all the helium nuclei are exhausted. (Except in very massive stars, and at very high temperature, the subsequent \(^{16}\text{O}(\alpha,\gamma)^{20}\text{Ne}\) process is too slow to be of importance in helium burning [17, p.196].) Therefore, the relative rate of the processes will determine the relative abundance of carbon to oxygen at the conclusion of helium burning. This relative abundance will, in turn, affect the subsequent evolution of a star in which heavier elements are synthesized, the composition of remnants if the star turns into a supernova and ultimately explodes, and even the probability of black-hole formation.

Since the gravitational contraction acts to counterbalance the thermal pressure produced by the various nuclear reaction processes, whether a star will go through certain burning stages depends strongly on its mass. For stars with mass greater than about 10 \(M_\odot\), other advanced burning stages from carbon to silicon burning can also take place until nuclei up to \(^{56}\text{Fe}\) are synthesized. Fusion reactions end at this point since the binding energy per nucleon has reached a maximum so that further reactions become endothermic and, thus, will not proceed spontaneously. As the remnant of all burning stages, an iron core is formed while shell burning keeps adding mass to it. Since no further fusion energy is available to balance the gravitational contraction of the iron core, it will eventually collapse and the star will explode as a supernova of Type II. A discussion on the theories of supernovae of Type II can be found in Ref. [6, p.439].

In order to utilize network calculations of nucleosynthesis in massive stars, the reaction rates involved in all phases of nuclear energy production need to be determined, either experimentally or theoretically [4,18,19]. It should be reiterated that the \(^{12}\text{C}(\alpha,\gamma)^{16}\text{O}\) reaction rate plays a crucial role in validating the stellar evolution models which form the basis of these calculations—the \(S\) factor of this reaction at 300 keV, \(S_{\text{tot}}(300)\), is required to be constrained to within 20% if the calculated isotopic abun-
dances are to be consistent with the observed solar values [9]. The triple-\(\alpha\) process has been claimed to be known to a precision of about 15%–20% for some time [17, p.196]. However, despite the enormous amount of experimental [20–28] and theoretical [29–33] work devoted to the determination of the \(^{12}\text{C}(\alpha, \gamma)^{16}\text{O}\) reaction rate in the last few decades, it is not until very recently that a single experiment [8] has claimed that the reaction rate has been determined to a precision of about 24% (see Sec. 1.4). Note that this result is still insufficient to fulfil the precision of \(\leq 20\%\) required to validate network calculations of nucleosynthesis in massive stars [9]. The work relevant to this thesis is, in fact, another desperate attempt to constrain this reaction rate.

### 1.3 Level Structure of \(^{16}\text{O}\)

![Partial energy level diagram of \(^{16}\text{O}\).](image)

**Figure 1.1:** Partial energy level diagram of \(^{16}\text{O}\). The \(^{12}\text{C}(\alpha, \gamma)^{16}\text{O}\) reaction rate in a stellar environment is thought to be dominated by contributions from the high energy tails of the two subthreshold states at 6.197 and 7.117 MeV. Note the absence of a level in the region relevant to the stellar environment.

A partial energy level diagram of \(^{16}\text{O}\) is shown in Fig. 1.1. The \(\alpha+^{12}\text{C}\) formation threshold is 7.162 MeV above the ground state of \(^{16}\text{O}\). The energy region relevant to helium burning (see Sec. 1.2) is also shown. It is apparent that no \(^{16}\text{O}\) state exists in or immediately above that well-defined region. In fact, the \(^{12}\text{C}(\alpha, \gamma)^{16}\text{O}\) reaction...
rate at stellar energies is thought to be dominated by the high-energy tails of two subthreshold states at 45 and 245 keV below the $\alpha+^{12}\text{C}$ formation threshold, with excitation energies of 7.117 and 6.917 MeV and $J^\pi$ values of 1$^-$ and 2$^+$, respectively. However, measurement of the $^{12}\text{C}(\alpha,\gamma)^{16}\text{O}$ cross section in the laboratory, which has been performed at energies down to $\sim1$ MeV [20–28], is dominated by the broad 1$^-$ state at 9.585 MeV, and extrapolations of the data down to stellar energies have been very unreliable due to the presence of the two subthreshold states whose properties are not well-determined in this energy region.

Thus, the $^{12}\text{C}(\alpha,\gamma)^{16}\text{O}$ reaction is thought to consist of an $E1$ (dipole) component due to the 7.117 MeV (1$^-$) state, and an $E2$ (quadruple) component due to the 6.917 MeV (2$^+$) state in $^{16}\text{O}$. Non-resonant direct capture and cascade transitions through the subthreshold states are also believed to contribute to the reaction, but to a much lesser extent compared with the resonant part [31, 34].

### 1.4 Review of Previous Work

It is impractical to directly measure the $^{12}\text{C}(\alpha,\gamma)^{16}\text{O}$ reaction at $E_{\text{cm}} = 300$ keV due to its extremely small cross section of about $10^{-17}$ b. Several groups [20–28] have attempted to perform elastic and radiative capture measurements, at energies as low as $\sim1$ MeV and extrapolate the results to astrophysical energies to constrain the value of $S_{\text{tot}}(300)$. In Refs. [22, 24–28], both absolute cross sections and $\gamma$-ray angular distributions were measured for ranges of energies and angles. In each work, the $E1$ component was separated from the total cross section based on the $\gamma$-ray angular distribution data. $R$-matrix$^\dagger$ fits were then performed on the derived $E1$ radiative-capture or elastic-scattering data to deduce the reduced $\alpha$ width of the 7.117 MeV state in $^{16}\text{O}$, and thus constrain the value of $S_{E1}(300)$. However, the results fall into the wide range

$^\dagger$See Chapter 3 and Ref. [35].
of $S_{E_1}(300) = 80$ to $320$ keV·b with large margins of error. A study of the $\beta$-delayed $\alpha$ decay of $^{16}\text{N}$, which provides information only about the $E1$ component [the $\beta$ decay does not appreciably populate the 6.92 MeV $(2^+)$ state since it is first-forbidden], has been reported by our group [10]. The $\beta$-delayed $\alpha$ spectrum was fitted simultaneously with the radiative-capture cross section data of Refs. [22, 24, 26, 27] and the elastic-scattering data of Ref. [25] (see below) with an R-matrix formalism based largely on Refs. [30, 36]. The simultaneous R-matrix fit$^*$ resulted in $S_{E_1}(300) = 79 \pm 21$ keV·b, which is still the best constraint$^+$ on $S_{E_1}(300)$ so far. Two other studies of the decay of $^{16}\text{N}$ exist in the literature [37, 38]. In Ref. [37], only singles data were taken so that $\beta$ background was dominant at low energies. The result of Ref. [38] suffers from low statistical accuracy.

The determination of $S_{E_2}(300)$ has not been as well-established as that of $S_{E_1}(300)$. It was suggested in Ref. [24] that elastic scattering measurements at a wide range of energies would help to constrain the value of $S_{E_2}(300)$. The best published work of $^{12}\text{C}(\alpha, \alpha)^{12}\text{C}$ elastic scattering on the determination of the $^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$ stellar reaction rate has been reported in Ref. [25]. However, the data appear to show strong correlation among the partial waves (as demonstrated in Ref. [34]) and the analysis yields $S_{\text{tot}}(300) = 0.31 \pm 0.11$ MeV·b with $S_{E_2}(300) = 89 \pm 30$ keV·b. In Ref. [34], a global R-matrix fit based on all available angular distributions and other primary data$^\S$ (including, of course, those of Ref. [25]) was unable to reproduce the result of Ref. [25] but could only set an upper limit of $S_{E_2}(300) < 140$ keV·b, while the value of $S_{E_1}(300)$ was shown to be well-constrained by the $\beta$-delayed $\alpha$-decay data of Ref. [10].

A group at Ruhr-Universität Bochum, Germany, has recently attempted to measure

$^*A$ simultaneous K-matrix fit was also performed in Ref. [10] from which a similar value for $S_{E_1}(300)$ was obtained, but the R-matrix fit yields smaller uncertainty. Moreover, the consensus in the literature appears to favor the R-matrix formalism.

$^+A$ result of $S_{E_1}(300) = 101 \pm 17$ keV·b has been given in Ref. [7]. However, their treatment of errors is unclear making interpretation of the results difficult (see next page).

$^\S$The use of only primary data avoids correlations among derived (secondary) data [34].


the γ-ray spectra of the $^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$ reaction at $90^\circ$ with respect to a $^{12}\text{C}$ beam using a $4''\times4''$ BGO crystal\(^\dagger\) in close geometry ($d_\gamma = 4.0$ cm) and a $2''\times2''$ BGO crystal in far geometry ($d_\gamma = 24.5$ cm), and a windowless $^4\text{He}$ gas target [39]. Due to the angular distribution of the γ ray with respect to the beam, the small crystal observed essentially the $E1$ component, while the large crystal the angle-integrated sum of the $E1$ and $E2$ components. Their analysis of data obtained with the small crystal yielded $S_{E1}(300) = 95 \pm 44$ keV·b in consistence with Ref. [10], while another analysis involving both data sets yielded the ratio $\sigma_{E2}/\sigma_{E1}$ for energies down to 0.935 MeV. In principle, if both $\sigma_{E1}$ and the ratio $\sigma_{E2}/\sigma_{E1}$ are known with high accuracy, $\sigma_{E2}$ can be determined. However, the errors for the ratio $\sigma_{E2}/\sigma_{E1}$ obtained in Ref. [39] were too large and no attempt was made to extract a value of $\sigma_{E2}$. Nevertheless, the data do suggest that the strength of the $E2$ component is comparable to that of $E1$.

Recently, a group at Caltech [7] has published their results of $S_{E1}(300) = 101 \pm 17$ and $S_{E2}(300) = 42^{+16}_{-23}$ keV·b from data obtained in α-transfer reactions. However, while a systematic error of 15% was stated without indicating how this estimation was obtained, this error did not seem to have been included in the above results. Furthermore, it is also unclear how the pseudo data points, which were artificially inserted in the $\ell = 2$ part of the analysis assuming an uncertainty of 10%, would have affected the final results. The Caltech group has also stated the result of $S_{\text{tot}}(300) = 159$ keV·b, which contains an estimated contribution from cascade transitions of $S_C(300) = 16$ keV·b taken from Ref. [34]. This is the first publication in which a determination of the $^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$ reaction rate is claimed to agree “...very well with the finding...” stated in Ref. [4]. However, their treatment of errors is unclear making interpretation of the results difficult.

\(^\dagger\)Bismuth Germanate ($\text{Bi}_4\text{Ge}_3\text{O}_{12}$), also known as BGO, is a high-Z, high-density scintillation material with high γ-ray absorption efficiency.
More recently, in an effort to better constrain the value of $S_{E2}(300)$, a group* at the University of Notre Dame [8] has performed high-precision angular distribution measurements of $^{12}\text{C}(\alpha, \alpha)^{12}\text{C}$ similar to those of Ref. [25], but with many improved features. In particular, an array of 32 detectors was used so that each angular distribution measurement was obtained in a single run, while a set of 7 detectors was rotated 5 times for each measurement in Ref. [25]. Moreover, angular distributions were measured for 402 energies between 2.6 and 8.2 MeV as compared to only 51 energies between 1.0 and 6.6 MeV in Ref. [25]. In an R-matrix analysis performed with an interaction radius of $a_c = 5.5$ fm and including partial waves up to $\ell = 5$, the Notre Dame group has derived a result of $S_{E2}(300) = 37^{+8}_{-18}$ keV-b. Despite a large error, the result of $S_{E1}(300)$ obtained from the phase-shift data was consistent with that given in Ref. [10]. Their estimation of systematic errors also appears to be reasonable. This might be the best value of $S_{E2}(300)$ ever determined. By combining their result of $S_{E2}(300)$, the value $S_{E1}(300) = 79 \pm 21$ keV-b in Ref. [10], and the estimated contribution from cascade transitions of $S_C(300) = 16 \pm 16$ keV-b in Ref. [34], the Notre Dame group obtained $S_{\text{tot}}(300) = 132^{+28}_{-32}$ keV-b. This value is consistent with $S_{\text{tot}}(300) = 170 \pm 50$ keV-b as suggested in Ref. [4]. However, the uncertainty of 24% does not fulfill the recently updated precision of less than 20% required to validate network calculations of nucleosynthesis in massive stars [9], and more work is required to resolve this long-standing problem.

1.5 $\beta$-delayed Particle Decay of $^{17}\text{Ne}$

A study of the $\beta$-delayed $\alpha$ decay of $^{16}\text{N}$ succeeded in constraining the $E1$ component of the $^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$ reaction to a good precision [10]. The usefulness of that study hinges on the existence of $\alpha$-unbound states in $^{16}\text{O}$ populated by the $\beta$ decay, including the

*This is, in fact, a collaboration involving TRIUMF, the University of Toronto, and the University of Notre Dame.
§1.5 β-delayed Particle Decay of $^{17}$Ne

high-energy tail of the subthreshold state at 7.117 MeV. This subthreshold state, as well as the one at 6.917 MeV, is also known to be populated in the β-delayed proton decay of $^{17}$Ne. Therefore, a study has been proposed [13] to determine the value of $S_{E2}(300)$, as well as that of $S_{E1}(300)$, using information provided by the decay of $^{17}$Ne.

1.5.1 Decay scheme of $^{17}$Ne

A partial decay scheme of $^{17}$Ne is shown in Fig. 1.2. The ground state of $^{17}$Ne decays by positron emission to states in $^{17}$F with a Q value of 14.53 MeV, and a half life of 109.2 ± 0.6 ms [41]. Since $^{17}$F is bound by only 0.6005 MeV against proton decay to $^{16}$O, most states in $^{17}$F populated by the β decay of $^{17}$Ne will decay by proton emission. The $^{12}$C($\alpha, \gamma$)$^{16}$O reaction threshold is 7.762 MeV above the $^{17}$F ground state. States in $^{17}$F above this threshold are energetically capable of decaying into $\alpha$-unbound states in $^{16}$O, including the high-energy tails of the sub-threshold states at 6.917 and 7.117 MeV. In fact, the prime interest of this study is in the proton decay of excited states in $^{17}$F into α-unbound states in $^{16}$O, which further decay into $^{12}$C(gs) giving rise to 3-body final states of p+$\alpha$+$^{12}$C. As is apparent from Fig. 1.2, $^{17}$F states with excitation energies above 5.819 MeV may also α decay into states in $^{13}$N. For cases in which the $^{13}$N state is above the $^{12}$C($\alpha, \gamma$)$^{16}$O reaction threshold, the $^{13}$N nucleus may further proton decay to $^{12}$C(gs) forming a 3-body final state which may not be distinguishable from that via the $^{16}$O channel. Thus, decays of $^{17}$F states via the $^{13}$N channel leading to 3-body final states will contribute to background in this study.

A thorough knowledge of the decay scheme of $^{17}$Ne is crucial to the success of this study, to understand potential background, and to estimate expected count rates. Only two studies of this decay scheme exist in the literature [42,47]. Many of the properties\footnote{Note that, since Q values are customarily defined in terms of the mass difference between neutral atoms, the actual amount of energy available for β$^+$ decay is the Q value minus 1.022 MeV, the equivalent of two units of electron mass [40, p.217].}
Figure 1.2: PARTIAL DECAY SCHEME OF $^{17}$Ne [41–46]. $^{17}$Ne(gs) $\beta$ decays into $^{17}$F. States in $^{17}$F above the $\alpha+^{12}$C formation threshold populated by the $\beta$ decay may further decay via the $^{16}$O or $^{13}$N channel into $^{12}$C(gs) giving rise to 3-body final states of $p+\alpha+^{12}$C. The $^{16}$N(gs) level relevant to a previous determination of $S_{\beta1}(300)$ [10] is also shown.
such as spin-parity and reduced width of states in $^{17}$F and $^{16}$O, and $\beta$-decay BRs$^1$ of $^{17}$Ne(gs) into states in $^{17}$F, etc., were either unknown or not well-determined. As a first step in the study, the BRs of states in $^{17}$F decaying into states in $^{16}$O were determined initially by p-$\gamma$ coincidence measurements. These measurements were complemented by a subsequent experimental run utilizing $\beta$-p and $\beta$-$\alpha$ coincidence measurements (see Sec. 2.3) to yield information about the weak branches. Measurements of angular distributions between protons and de-excitation $\gamma$-rays were also carried out from which the $J^\pi$ values of the 8.44 and 9.45 MeV states in $^{17}$F have been determined to be $\frac{1}{2}^-$. A complete account of the experimental work relevant to the above measurements can be found in the thesis of Morton [46], along with the extracted results of BRs and $J^\pi$ assignments, and comparisons with previous measurements [42,47].

1.5.2 Experimental requirement

In the $\beta$ decay of $^{17}$Ne(gs), all the strong branches (> 6%) to states in $^{17}$F are below the $^{12}$C($\alpha, \gamma$)$^{16}$O threshold; those above the threshold amount to < 15% of all decays. Therefore, states in $^{17}$F populated by the $\beta$ decay will decay predominately to bound states in $^{16}$O or to $^{13}$N(gs) resulting in final states of $\beta$+p+$^{16}$O or $\beta$+$\alpha$+$^{13}$N. Consequently, $\beta$-induced 2-body background is expected to be significant.

For our purpose, the $^{17}$F nucleus can be assumed to be initially at rest in the laboratory frame.$^4$. In principle, if two of the three particles in a 3-body breakup are detected with adequate energy and angular resolution, and the identities of the particles are known, the energy of the third particle can be deduced from kinematic calculations. However, particle identification is not always possible in practice (see Sec. 2.1.2) and, because of the expected intense $\beta$ flux, detection of all 3 daughter particles (p+$\alpha$+$^{12}$C) is required to optimize the kinematic information in each

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$^1$BR denotes branching ratio.
$^4$The $\beta$-decay Q value for all decays of $^{17}$Ne(gs) to $^{17}$F leading to 3-body final states is < 6 MeV. A simple calculation has shown that the recoil of a $^{17}$F nucleus due to a 6 MeV $\beta$ particle is < 1.4 keV.
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event to discriminate against background. Quadruple-coincidence measurement of \( \beta+p+\alpha+^{12}\text{C} \) has also been suggested as a means to suppress background. However, a study (see Sec. 2.3) has shown that triple coincidence might be an adequate requirement on condition that sufficient energy and angular resolution, in conjunction with particle identification, can be achieved. In principle, particle identification can be accomplished with time-of-flight measurement given a flight path of sufficient length.

1.5.3 The IAS in ¹⁷F and \( S_{E1}(300) \)

The isobaric analogue state (IAS) in ¹⁷F, with \( E_x = 11.1929\pm0.0023 \text{ MeV} \) and \( J^\pi = \frac{1}{2}^- \), is the lowest state in ¹⁷F with an isospin of \( T = \frac{3}{2} \) [41]. Since all states in ¹⁶O below 10.59 MeV have \( T = 0 \), and similarly all those in ¹³N have \( T = \frac{1}{2} \), particle decays of the IAS can only take place via isospin impurity [47, 48]. As a result of the isospin-forbidden transitions, the natural width of the IAS is only 0.18 ± 0.03 keV. It will be seen below that the narrow width of the IAS is advantageous in the identification of 3-body breakup events originating from this state.

The β-decay BR of ¹⁷Ne(gs) to the IAS in ¹⁷F [3.431 MeV above the \( ^{12}\text{C}(\alpha,\gamma)^{16}\text{O} \) reaction threshold] is 0.74 ± 0.14%. Considering the significant BR and the phase space available based on the large Q value, the IAS is the most likely state in ¹⁷F for which 3-body breakup could be observed. The parent state of a 3-body breakup can be identified by measuring the energies of all 3 daughter particles to obtain the total Q value, which is equal to the excitation energy of the parent state above the \( ^{12}\text{C}(\alpha,\gamma)^{16}\text{O} \) reaction threshold. Since the IAS has a very narrow width, its contribution to 3-body breakup should easily be identified in a triple-sum energy spectrum as a narrow peak at 3.431 MeV. In fact, it will be shown in Chapter 2 that the IAS is the only state in ¹⁷F for which 3-body breakup has been observed experimentally. The measurements of such events form the basis of the experimental work described in this thesis. It will also
be shown in Chapter 5 that information provided by a study of the 3-body breakup of the IAS could be used to constrain the value of $S_{E_1}(300)$.

### 1.5.4 Candidate states to constrain $S_{E_2}(300)$

As mentioned before, the usefulness of this study relies on the observation of 3-body breakup of states in $^{17}$F by virtue of the extension of the high-energy tails of the subthreshold states above the $^{12}$C($\alpha, \gamma)^{16}$O threshold. Based on the $\beta$-delayed proton BRs obtained from a preliminary analysis of the $p-\gamma$ coincidence data [46], and assuming the shape of nuclear states to be of the Breit-Wigner type, the absolute BRs of the decay of $^{17}$Ne leading to 3-body breakup of states in $^{17}$F were estimated with penetrability calculations [49]. In these calculations, a value of $\theta_{0.92}^2/\theta_{7.12}^2 = 3$ (the ratio between the dimensionless reduced $\alpha$ widths of the 6.92 and 7.12 MeV states in $^{16}$O) was assumed\(^\dagger\).

Results of these calculations showed that the 3-body breakup via the IAS$\rightarrow^{16}$O(7.12) transition was 50 times stronger than that via IAS$\rightarrow^{16}$O(6.92). However, breakup via the $^{17}$F(9.45)$\rightarrow^{16}$O(6.92) transition was shown to be 3 times stronger than that via $^{17}$F(9.45)$\rightarrow^{16}$O(7.12). Based on these results, the 9.45 MeV was identified as the best candidate state for observation of 3-body breakup to constrain the value of $S_{E_2}(300)$. A summary of the results of Ref. [49] is given in Table 1.1.

It is apparent from Table 1.1 that the 10.03 MeV state is another potential candidate to constrain the value of $S_{E_2}(300)$ since the 3-body breakup strength of the $^{17}$F(10.03)$\rightarrow^{16}$O(6.92) transition is comparable to that of $^{17}$F(9.45)$\rightarrow^{16}$O(6.92). It has also been found experimentally that the 10.03 MeV state does not populate the 7.12 MeV state at any appreciable level. In this respect, the 10.03 MeV state appears to be a better candidate than the 9.45 MeV to constrain $S_{E_2}(300)$ since the observation of the $^{17}$F(10.03)$\rightarrow^{16}$O(6.92) transition might be free of interference from any $E1$ component. A similar calculation using the more rigorous R-matrix approach has been

\(^\dagger\)A value of 2.4 was quoted in Ref. [25].
\section*{1.6 Layout of the thesis}

Three experimental runs for triple-coincidence measurements constitute the bulk of the material in this thesis. The runs are labeled by the month and year in which they were performed, i.e., Aug96, Jul97, and Nov98, respectively. Details of the experimental procedures and data analysis will be described in Chapter 2. The connection between experiments and theory is made through extensive Monte Carlo simulations and analytic calculations based on the \textit{R}-matrix formalism. The derivations of \textit{R}-matrix formulas used in this work and the algorithms for the calculations are presented in Chapter 3. Results of the Monte Carlo simulations have been superseded by the analytic calculations. However, the algorithms for the simulations, which might be used as a basis for extension into more complex simulations, are listed as an appendix. The IAS in $^{17}$F has been observed to break up into 3 particles via 3 channels. The dete-

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|}
\hline
1\textsuperscript{7}F$\rightarrow$1\textsuperscript{6}O transition & $BR_{\beta p}$ & $F_{AT}$ & $BR_{3\nu}$ \\
\hline
IAS$\rightarrow$7.12 & 9.3$\times 10^{-4}$ & 7.6$\times 10^{-5}$ & 7.1$\times 10^{-8}$ \\
IAS$\rightarrow$6.92 & 3.3$\times 10^{-5}$ & 4.2$\times 10^{-5}$ & 1.4$\times 10^{-9}$ \\
10.03$\rightarrow$7.12 & -- & -- & -- \\
10.03$\rightarrow$6.92 & 2.2$\times 10^{-4}$ & 1.5$\times 10^{-6}$ & 3.0$\times 10^{-10}$ \\
9.45$\rightarrow$7.12 & 4.4$\times 10^{-4}$ & 3.4$\times 10^{-7}$ & 1.5$\times 10^{-10}$ \\
9.45$\rightarrow$6.92 & 5.7$\times 10^{-3}$ & 7.3$\times 10^{-8}$ & 4.1$\times 10^{-10}$ \\
\hline
\end{tabular}
\caption{Results of calculations of $BR_{3\nu}$ by Iliadis to identify the best candidate states to constrain the value of $S_{E2}(300)$.}
\end{table}

$^{a}$\textit{β}-delayed p-proton BRs from preliminary results of p-γ coincidence data [44].

$^{b}$Fraction of decay above threshold obtained from the penetrability calculations.

$^{c}$Absolute 3-body decay BR ($BR_{\beta p} \times F_{AT}$)

performed using the latest results of \textit{β}-delayed proton-decay BRs [46] and the most recent value of $\theta_{6.92}^2/\theta_{7.12}^2$ [8]. In fact, as will be shown in Sec. 3.3, after the discussion of \textit{R}-matrix theory, the result of the recent calculation does indicate that the 10.03 MeV state is a better candidate than the 9.45 MeV state to constrain the value of $S_{E2}(300)$. 

\section*{1.6 Layout of the thesis}

Three experimental runs for triple-coincidence measurements constitute the bulk of the material in this thesis. The runs are labeled by the month and year in which they were performed, i.e., Aug96, Jul97, and Nov98, respectively. Details of the experimental procedures and data analysis will be described in Chapter 2. The connection between experiments and theory is made through extensive Monte Carlo simulations and analytic calculations based on the \textit{R}-matrix formalism. The derivations of \textit{R}-matrix formulas used in this work and the algorithms for the calculations are presented in Chapter 3. Results of the Monte Carlo simulations have been superseded by the analytic calculations. However, the algorithms for the simulations, which might be used as a basis for extension into more complex simulations, are listed as an appendix. The IAS in $^{17}$F has been observed to break up into 3 particles via 3 channels. The dete-
mination of the relative BR between these channels is described in Chapter 4. The objective of the ultimate experiment is to constrain the value of $S_{\text{tot}}(300)$. Further studies are required before the feasibility of using the decay of $^{17}\text{Ne}$ to constrain the value of $S_{E2}(300)$ can be established. However, it will be shown in Chapter 5 that it is possible to constrain the value of $S_{E1}(300)$ based on the results and experimental knowledge gained from the triple-coincidence measurements. A proposed detection system for the determination of $S_{E1}(300)$ using the decay of $^{17}\text{Ne}$ will be presented. Much useful information relevant to the present study—3-body breakup kinematics, MC method, and angular correlation calculations, etc.—is compiled in the appendices.
Chapter 2

Experimental Work

Three experimental runs, performed in the period from 1996 to 1998, at the TRIUMF-TISOL facility [50] in Vancouver, BC, Canada, contributed to the work to be described below. The focus of the three experimental runs was on triple-coincidence measurements in the decay of $^{17}$Ne. The first was carried out in August of 1996 (Aug96) in which large Si detectors were used in close geometry. A simple Monte Carlo simulation was carried out prior to the experiment to define some of the design criteria for the experiment. In this run, the IAS in $^{17}$F was observed, for the first time, to break up into $p+\alpha+^{12}$C via the 9.59 MeV state in $^{16}$O, and the 2.37 and 3.50/3.55 MeV states in $^{13}$N. In a subsequent run, carried out in July of 1997 (Jul97), two of the detectors used in the previous run were replaced with double-sided silicon strip detectors (SSDs) to improve angular resolution. The improved quality of the data allowed for more stringent kinematic constraints to be applied to background discrimination. An additional run was performed in November of 1998 (Nov98) in which $\beta$ detectors were added to the Jul97 setup to facilitate quadruple-coincidence ($\beta+p+\alpha+^{12}$C) measurements in an attempt to further suppress the background. The distance between the detectors and the collector foil was also increased to enable time-of-flight (ToF) measurements in the hope of lifting an ambiguity in particle identification. Each of the 3 runs will be described in detail below, along with data analyses and results.
2.1 First Triple-Coincidence Measurement (Aug96)

In early 1996, it was realized that some beam time might be spared from a run originally scheduled for angular correlation measurements in the determination of the spin and parity of some of the excited states in $^{17}$F (see Ref. [46]). Hence, preparation for the first triple-coincidence measurement in the decay of $^{17}$Ne was underway with no confirmed beam time, and very little resources. As it turned out, eight 12-hour shifts were available in August of 1996. The run was successful in observing, for the first time, the 3-body breakup of the IAS in $^{17}$F via the 9.59 MeV state in $^{16}$O, and the 2.37 and 3.50/3.55 MeV states in $^{13}$N. The energies of the two levels near 3.5 MeV in $^{13}$N are so close that the transitions to them cannot be resolved in this experiment.

2.1.1 Monte Carlo Simulation

Prior to this experiment, a Monte Carlo (MC) program was developed to simulate the 3-body breakup of $^{17}$F* populated by the $\beta$ decay of $^{17}$Ne [51]. [Another MC simulation employing the more rigorous $R$-matrix theory (see App. B) was developed after this run.] Many of the properties pertaining to the decay of $^{17}$Ne (see Fig. 1.2), such as spins and parities, reduced widths of states in $^{17}$F and $^{16}$O, and $\beta$-decay branching ratios of $^{17}$Ne(gs) into states in $^{17}$F, etc., were either unknown or not well-determined. Consequently, an MC simulation would not seem promising in view of the lack of definite input parameters. However, it was believed that reasonably realistic estimated values could be assigned to the unknown parameters, and useful information could still be obtained from the simulation. As for parameters which had previously been measured, most of the branching ratios were drawn from Ref. [42] and some from previous measurements performed by our group. The reduced widths of the 7.117 and 9.585 MeV states in $^{16}$O were taken from Ref. [10].

In the MC program, the 3-body breakup was treated as a two-step (proton followed
by $\alpha$ decay) cascade transition (see App. A). The ground state of $^{17}\text{Ne}$ decays via $\beta^+$ emission to states in $^{17}\text{F}$ with a maximum $Q$ value of 14.53 MeV [41]. For states in $^{17}\text{F}$ above the $\alpha+^{12}\text{C}$ formation threshold, the $\beta$-decay $Q$ value is less than 6 MeV. The $^{17}\text{F}$ nucleus is assumed to be initially at rest in the laboratory frame (see footnote on p. 13). The energy distributions of states in $^{17}\text{F}^+$ were described simply as Lorentzians based on two assumptions: i) the $\beta$ phase space is constant over the width of a $^{17}\text{F}$ state; and ii) the total width is independent of energy. Note that these assumptions are justified only when the width of a state is narrow, e.g., the IAS, or when the excitation energy is large so that partial widths do not vary appreciably over the width of the resonances. States in $^{16}\text{O}^+$ were described by the Breit-Wigner single-level formula [6]

$$N(E) = \frac{\Gamma_p \Gamma_\alpha}{(E_r - E)^2 + (\Gamma_\alpha/2)^2}, \tag{2.1}$$

where $E_r$ is the level energy; $\Gamma_p$ and $\Gamma_\alpha$ are the (energy-dependent) formation and $\alpha$-decay widths given by

$$\Gamma_i(E_i) = \frac{2\hbar}{a_i} \sqrt{\frac{2E_i}{\mu_i}} P_i \theta_i^2, \quad i = p \text{ or } \alpha, \tag{2.2}$$

in which $a_i$ is the channel radius, $\mu_i$ is the reduced mass, $P_i$ is the penetration factor, and $\theta_i^2$ is the dimensionless reduced particle width; and only the lowest order of orbital angular momentum was assumed to contribute significantly. Since only the relative distribution was required in the calculation, the reduced proton width was set to unity.

It should be noted that this MC program does not account for the possible $\alpha$ decay of states in $^{17}\text{F}$ into $^{13}\text{N}$, which is expected to contribute to background in this experiment (see Sec. 1.5.1). Since this program has been superseded by the one described in App. B, no more detail will be given here other than the fact that each event is specified by the energies and directions of the 3 particles—$p$, $\alpha$, and $^{12}\text{C}$. Since it takes both a

\footnote{Five states were included, viz., the 8.44, 8.83, 9.45, 10.03, and 11.19 MeV states.}

\footnote{Six states were included, viz., the ground state, the 6.04, 6.13, 6.92, 7.12, and 9.59 MeV states.}
polar and an azimuthal angle to specify a direction in spherical coordinates, each event contains 9 parameters.

Analysis of these MC data provided valuable information about the kinematics of the 3-body breakup which was useful in the design of the experimental setup (see next section). Since the \(^{16}\text{O}\) nucleus decays in flight due to the recoil from the proton (see App. A), the angle between the \(\alpha\) particle and the \(^{12}\text{C}\) ion is not 180° as observed in the laboratory frame (see Fig. A.1). Based on the MC data, angular distributions of pairs of particles in the 3-body breakup were generated. As can be seen in Fig. 2.1, while the distributions of the p-\(\alpha\) and p-\(^{12}\text{C}\) angles are relatively independent of angle, the \(\alpha-^{12}\text{C}\) distribution contains a maximum at 160°. This angle turned out to be an important criterion in defining the geometry of the experimental setup, as will be seen in the next section.

The expected energy ranges of the particles to be observed are also of great interest. In Fig. 2.2, which shows the laboratory energy spectra of the 3 product particles based on the MC data, it is apparent that all particles of interest in this experiment will have energies of less than 3 MeV, with protons and \(\alpha\) particles encompassing very similar energy ranges. Since many of the input parameters were merely educated estimates, and only \(^{16}\text{O}\) channels known to have significant branching ratios were included in the
simulation, the details of these spectra were expected to be substantially different from those obtained in actual experiments. Nevertheless, the energy limits given by these data have provided useful information in the design of the detection system, as will be seen below. Note that the energy limits could have been deduced directly from calculations based on the energies of the states involved in the 3-body breakup of $^{17}$F. However, the inclusion of branching ratios in the MC simulation provided additional information on the energy distributions.

### 2.1.2 Experimental Setup

In principle, if two of the three particles are detected with adequate energy and angular resolutions, and the identities of the particles are known, the energy of the third particle can be deduced from kinematic calculations (see App. A). Charged particle identification can usually be accomplished with pairs of Si detectors in the so-called E-ΔE configuration [52]. The ΔE detector needs to be thin enough for the particle to punch through and be stopped in the E detector. However, in the 3-body breakup of $^{17}$F, the daughter particles are expected to have energies of less than about 3 MeV and, therefore, the E-ΔE scheme will not be applicable since most of the particles will
be stopped by the $\Delta E$ detector even with the thinnest\textsuperscript{1} Si detector available ($\sim 10 \mu m$). Particle identities can also be obtained with time-of-flight (ToF) measurements. Unfortunately, close geometry would be needed to optimize triple-coincidence detection (see below) and, thus, the length of the flight path would not be sufficient for ToF measurements. The incapability of particle identification in the experiment imposed the requirement that all 3 particles should be detected in order to maximize the kinematic information available for each event.

Since it has been determined from the MC data that the most probable value of the $\alpha-^{12}\text{C}$ angle is 160°, an experimental setup optimized for triple-coincidence efficiency must include a pair of detector elements at this angle. In the most thorough study of the decay of $^{17}\text{Ne}$ prior to this experiment [42], only double-coincidence measurements were utilized. Therefore, information on 3-body decay branching ratios was not available, but they were expected to be small. In order to ensure that triple coincidences could be observed with reasonable statistical accuracy within a limited amount of beam time, it was decided that, at the expense of angular resolution and ToF information, close geometry would be needed to maximize the solid angle subtended by the detectors.

The MC data also showed that the heavy $^{12}\text{C}$ recoils would have energies of less than 1 MeV. Accordingly, the dead-layer\textsuperscript{2} of the detectors should be as thin as possible to minimize energy loss and straggling. Furthermore, in anticipation of a strong $\beta$-induced background (see Sec. 1.5.2), each detector should also be backed with another reasonably thick (veto) detector to facilitate anticoincidence measurements in the rejection of such events, most probably $p+^{16}\text{O}+\beta$. As is apparent in Fig. 1.2, most excited states in $^{17}\text{F}$ decay by proton emission to bound states in $^{16}\text{O}$ and the energies of some of the protons from these decays are relatively high. The veto detectors would

\textsuperscript{1}A 10 $\mu m$ Si detector is able to stop protons of up to 800 keV and $\alpha$ particles of up to 3 MeV.

\textsuperscript{2}In a charged-particle detector, layers of inactive material usually exist on both sides of the semiconductor crystal which are either by-products of the manufacturing process, or electrical contacts necessary for charge collection. The energy lost in the inactive region cannot be recovered.
Table 2.1: List of Setup Design Criteria for the first triple-coincidence measurement.

1. A pair of detector elements should be placed at 160° about the collector foil to optimize detection of the $\alpha-^{12}$C pair.

2. The detectors should be installed in close geometry to optimize triple-coincidence efficiency.

3. The dead layer of the detectors should be as thin as possible to minimize energy loss and straggling, particularly for detection of $^{12}$C ions.

4. Each detector should be backed with another reasonably thick detector to facilitate anticoincidence measurements in the suppression of false triple coincidences induced by $\beta$ particles and/or high-energy protons.

also help in the suppression of such events.

The design criteria for the experimental setup, which were used in the final design, are summarized in Table 2.1. As depicted in Fig. 2.3, the setup consisted of two Ortec Ultra® 450 mm$^2 \times 300$ μm ion-implanted Si detectors and two Canberra 900 mm$^2 \times 300$ μm PIPS detectors. Both types of detectors are known to have very thin dead layers compared to the surface-barrier type. The 900 mm$^2$ detectors were backed with 700 μm detectors of the same type and size while the 450 mm$^2$ detectors were backed with 100 μm surface-barrier detectors of the same size. The thickness of 100 μm for the 450 mm veto detectors was too thin for them to be able to detect high-energy $\beta$ particles efficiently. In any case, with the limited amount of time and resources for preparation, these detectors were the best available.

The $^{17}$Ne radioactive ion beam was produced at the TRIUMF-TISOL facility. An MgO target, heated to about 1200 °C, was bombarded with the 500 MeV proton beam from the TRIUMF cyclotron at a nominal current of 1 μA. Both stable and radioactive particles were generated by spallation and fragmentation processes and diffused from the target chamber into the ionization chamber in which an ECR (electron cyclotron

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1The Ultra detectors were used in the p-γ coincidence experiment [46] prior to this run while the PIPS (Passivated Implanted Planar Silicon) detectors were courtesy of the University of Edinburgh, England.

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resonance) ion source [53] was used to ionize the particles. The ions were extracted through a slit into a mass analyzer (magnetic dipole with $\rho = 1.25$ m) which was tuned to select ion species of mass $A = 17$. Other electromagnetic components downstream (various quadruples and an electrostatic dipole) helped to deliver the ion beam to the experimental chamber inside which the detection system was located. The nominal yield of $^{17}\text{Ne}$ ions was a few times $10^4$/s.

As shown in Fig. 2.3, the detectors were installed at 4 cm from a carbon foil, which was located at the center of the setup to collect the radioactive ions. The entire detector array was tilted at 45° to the beam axis to allow for access of the ion beam to the carbon foil. An 8 mm diameter collimator was also installed upstream of the detection system to constrain the profile of the beam while retaining most of its intensity. The entire detection system was housed inside a $10''\times10''\times10''$ experimental chamber with pressure maintained at the low $10^{-5}$ torr scale with a cryogenic pump.

The nominal energy of the $^{17}\text{Ne}$ ion beam is 12 keV, and a calculation using...
TRIM [54] has shown that the range of the ion beam in carbon at this energy is about 20 nm. Accordingly, the thickness of the collector (carbon) foil was chosen to be 10 μg/cm² (≈42 nm) so that the ions would be stopped midway into the foil. Consequently, energy straggling due to the foil would be similar for particles emerging from either side of the foil. An 8-position adjustable target ladder was installed perpendicular to the beam axis at the center of the detection system for mounting of carbon foils. Two α sources, ¹⁴⁸Gd and ²⁴¹Am, providing 3.2 and 5.5 MeV α lines, respectively, were also mounted on the ladder for setting up electronics and for on-line energy calibration.

Standard NIM (Nuclear Instrument Module) and CAMAC (Computer Automated Measurement Control) electronic modules were used for signal processing and data acquisition (DAQ). In anticipation of high count rates for back-to-back p-¹⁶O and α-¹³N events (see Sec. 1.5.2), triggers were set only when logic signals were received from two adjacent detectors (double-coincidence trigger). The trigger rate was typically a few hundreds per second depending on the operation condition of Tisol. A VAX workstation running VMS was used to communicate with the CAMAC crate. The front-end software VDACS, which controlled a PDP11 (Starburst) processor in the CAMAC crate, was written in Twotran—a Fortran-based language designed specifically for use with CAMAC [55]. The data were written onto 8 mm cartridge tapes on an event-by-event basis. In order to keep the data files at a manageable size, a new run would start approximately every two hours. Each event consisted of energy and timing signals from the detectors, and a word, generated with a coincidence logic unit, which identified the detector pair(s) from which the trigger was obtained. The data analysis software NOVA [56], which is capable of generating 1- and 2-dimensional spectra with user-defined conditions, was used for on-line monitoring. A precision pulser signal was also periodically fed into the data stream to monitor the stability of the electronic system.
2.1.3 Data analysis and results

The goal of the analysis was to sort out triple-coincidence events originating from 3-body breakups of excited states in $^{17}\text{F}$ and to identify the transitions. A total of $73 \times 10^6$ raw events was collected. Offline analysis began with generating pulser spectra with NOVA to check for possible electronic gain shifts. The gains were found to be stable to within 1 ADC channel ($\sim 3$ keV) while the pulser spectra had an average FWHM equivalent to about 25 keV for all front detectors.

The parent state of a 3-body breakup can be identified by measuring the energies of all 3 daughter particles to obtain the total Q value, which is equal to the excitation energy of the parent state above the $^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$ reaction threshold (see App. A). Consequently, energy calibrations for all 3 types of particles, p+$\alpha$+$^{12}\text{C}$, were needed. The detectors were calibrated for energy with known particle groups in the decay of $^{17}\text{Ne}$ from singles and double-coincidence spectra generated with NOVA. Triple-coincidence events were identified with appropriate veto (anticoincidence) conditions, also with NOVA, and were written event-by-event to disk for further analysis with PHYSICA [57] and Fortran programs specifically developed for this analysis. Details of the analysis will be described in the following subsections.

2.1.3.1 Anticoincidence efficiency

The function of the back detectors (see Fig. 2.3) is to veto $\beta$ particles and protons that punch through the front detectors. The condition for anticoincidence is that a signal in the front detector is considered invalid if another signal is also recorded in the back detector. Care was taken to ensure that the veto signal threshold was set above the electronic pedestal to avoid false anticoincidences. A comparison of particle singles spectra obtained with and without the anticoincidence condition is shown in Fig. 2.4. It can be seen that the $\beta$ background in the low-energy region was modestly suppressed; a group of punch-through protons (due primarily to the $^{17}\text{F}(8.08)\rightarrow^{16}\text{O}(gs)+\text{p}$ transition)
in the region around 4 to 4.5 MeV was removed; a small number of counts was also removed throughout the entire energy region. The total anticoincidence efficiency was about 16%. It should be remembered that anticoincidence is useful for this experiment only in the energy region below 3 MeV. It is apparent in Fig. 2.4 that a major portion of the $\beta$ background in the region below 1 MeV was left untouched. Hence, $\beta$-induced false triple coincidences must be removed by other means such as particle identification, or kinematic constraining procedures (see Sec. 2.2.2.4).

2.1.3.2 Energy calibration

A list of particle groups used for energy calibrations, along with their corresponding transitions, is shown in Table 2.2. Energies of these particle groups were taken from Ref. [41]. A particle singles spectrum obtained with the detector F1 (see Fig. 2.3) is shown in Fig. 2.5. The four strongest proton lines (indicated by the arrows in Fig. 2.5), at energies of 1.680, 3.800, 4.598, and 5.114 MeV, respectively, were used for proton energy calibration. The centroids in ADC channel number were determined by fitting Gaussians to the proton peaks, and assuming a linear background. In fact, the proton group at 1.680 MeV overlaps with an $\alpha$ group at 1.725 MeV. In an attempt to disentangle the two groups, this peak was fitted with two Gaussians simultaneously. The result
Table 2.2: List of Particle Groups and their corresponding transitions used in energy calibration. The errors in energy are obtained from the uncertainties in level energies [41]. Note that the same transitions were used for both $\alpha$ and $^{13}$N calibrations.

<table>
<thead>
<tr>
<th>Particle</th>
<th>Energy (MeV± keV)</th>
<th>Transition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Proton</td>
<td>1.680±11</td>
<td>$^{17}$F(8.44)→$^{16}$O(6.05)+p</td>
</tr>
<tr>
<td></td>
<td>3.800±20</td>
<td>$^{17}$F(4.64)→$^{16}$O(gs)+p</td>
</tr>
<tr>
<td></td>
<td>4.598±11</td>
<td>$^{17}$F(5.49)→$^{16}$O(gs)+p</td>
</tr>
<tr>
<td></td>
<td>5.114±9</td>
<td>$^{17}$F(6.04)→$^{16}$O(gs)+p</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>1.725±10</td>
<td>$^{17}$F(8.08)→$^{13}$N(gs)+$\alpha$</td>
</tr>
<tr>
<td></td>
<td>2.001±10</td>
<td>$^{17}$F(8.44)→$^{13}$N(gs)+$\alpha$</td>
</tr>
<tr>
<td></td>
<td>3.220±60</td>
<td>$^{17}$F(10.0)→$^{13}$N(gs)+$\alpha$</td>
</tr>
<tr>
<td>$^{13}$N(for $^{12}$C)</td>
<td>0.531±10</td>
<td>$^{17}$F(8.08)→$^{13}$N(gs)+$\alpha$</td>
</tr>
<tr>
<td></td>
<td>0.616±10</td>
<td>$^{17}$F(8.44)→$^{13}$N(gs)+$\alpha$</td>
</tr>
<tr>
<td></td>
<td>0.991±60</td>
<td>$^{17}$F(10.0)→$^{13}$N(gs)+$\alpha$</td>
</tr>
</tbody>
</table>

showed that, although the two groups differ by 45 keV in actual energy, the observed centroids were within 2 ADC channels (≈6 keV) of each other due to the relatively large pulse height defect [52] for the $\alpha$ group. Since it would be difficult to associate the centroids with the particle groups, the average of the two centroids was taken for calibration of both proton and $\alpha$ energies. (However, see Sec. 2.2.2.2.) The calibration parameters for each detector were obtained by performing linear regression on the centroids, obtained with the Gaussian fits, and the known energies. The average linear correlation coefficient $R$ of the linear regressions for all front detectors was determined to be $0.99995 \pm 0.00002$. The actual error is energy dependent and was estimated to be better than 1% based primarily on the uncertainties in the level energies listed in Table 2.2.

As shown in Fig. 2.3, although the detectors F1 and F3 (also F2 and F4) are at 160°, the back-to-back angle is also covered by virtue of the extent of the detectors and the finite size of the beam spot. The calibrations for $\alpha$ and $^{12}$C energies actually took advantage of these effects by using particle groups coming from 2-body decay of $^{17}$F into the ground state of $^{13}$N. It should be kept in mind that triggers were generated.
only by events in which at least two adjacent detectors were activated. However, the rate of 2-body (back-to-back) events was so high that a usable double-coincidence spectrum could still be acquired through a chance coincidence with a $\beta$ or random particle. The particle spectrum of F1 obtained in coincidence with F3 is shown in Fig. 2.6. For $\alpha$ energy calibration, the same procedure as for protons was applied using the 3 $\alpha$ groups (indicated in Fig. 2.6), at energies of 1.725, 2.001, and 3.220 MeV, respectively. As noted earlier, the same (average) centroid for the proton group at 1.680 MeV was used for the $\alpha$ group at 1.725 MeV. The average correlation coefficient $R$ for the linear regressions for the front detectors was $0.99989 \pm 0.00002$. The error in $\alpha$ energy calibration was estimated to be about 2%.

As indicated in Table 2.2, energy calibration for carbon ions was actually obtained with $^{13}$N groups, as no major $^{12}$C groups were available in the $^{17}$Ne decay. Since the mass and charge of the two ions differ by only one unit, their ionization properties were expected to be very similar. In fact, a TRIM calculation has shown that, for energies of a few hundreds of keV, the ionization losses $dE/dx$ in Si for the two types of ion are within a few percents of each other. The $^{13}$N groups, with energies of 0.53, 0.62, and
0.99 MeV, respectively, were obtained by setting gates on the \( \alpha \) peaks in Fig. 2.6, and the same procedure as before was applied to the projected \( ^{13}\text{N} \) spectra to obtain the calibration parameters. The average correlation coefficient \( R \) for the linear regressions was \( 0.9996 \pm 0.0004 \). The energies obtained with these parameters were then corrected for \( ^{12}\text{C} \) energies based on the TrIM calculation. Due to the indirect method used, the error in carbon energy calibration was estimated to be about 4%.

2.1.3.3 Extraction of triple coincidences

Triple-coincidence events were sorted from the raw data by requiring at least 3 signals from the 4 front detectors, and with proper veto conditions. The setup offers 4 combinations of 3 detectors (see Fig. 2.3). However, the geometry of the setup dictates that only the combinations F1-F2-F3 and F1-F2-F4 are kinematically favorable in detection of triple coincidences; the other two combinations F1-F3-F4 and F2-F3-F4 have most of the active area of the detectors located within the same hemisphere.

Although the particles could not be identified directly in this experiment (see Sec. 2.1.2), their identities could be deduced to a high degree of accuracy from kinematic considerations in association with the geometry of the setup. For the combination F1-F2-F3, the 160° between F1 and F3 was designed, based on the MC data, to optimize detection of the \( \alpha^{-12}\text{C} \) pair. Therefore, for a triple coincidence detected with F1-F2-F3, the particle in F2 is most likely a proton, while the \( \alpha^{-12}\text{C} \) pair is picked up by F1 and F3. In their cm frame, the \( \alpha \) particle has 3 times the energy of the carbon ion, while in the lab frame, the recoil energy of the \( ^{16}\text{O} \) ion is only 1/16 of that of the proton (see App. A). As a consequence, the lab energies of the \( \alpha \) particle and carbon ion are not expected to be very different from their cm energies\(^1\). Therefore, it is very likely that the higher-energy member of the \( \alpha^{-12}\text{C} \) pair is an \( \alpha \) particle. A similar ar-

\(^1\)This assumption, which breaks down when the Q value of the second breakup is comparable to the \( ^{16}\text{O} \) recoil energy \( (< 200 \text{ keV}) \), is valid since \( E_{\alpha} > 1 \text{ MeV} \) for most of the events in this run.
§2.1 First Triple-Coincidence Measurement (Aug96)

![Graphs showing triple-sum energy spectra from data of Aug96](image)

**Figure 2.7:** TRIPLE-SUM ENERGY SPECTRA from data of Aug96: (a) F1-F2-F3—other than the peak at 3.4 MeV contributed by the 3-body breakup of the IAS in $^{17}$F, the rest of the spectrum is mostly false triples due to $\text{p}+^{16}\text{O}+\beta$; (b) F1-F3-F4—no obvious peak is present at 3.4 MeV—this combination is not favorable for triple-coincidence detection (see text).

argument can also be applied to the F1-F2-F4 combination. This particle-identification scheme has been used in subsequent analyses.

The triple-sum energy spectra obtained with the combinations F1-F2-F3 and F1-F3-F4 are shown in Fig. 2.7. An examination of the energy relations of the same data (see below) has shown that a majority of the events was due to $\text{p}+^{16}\text{O}+\beta$ and $\alpha+^{13}\text{N}+\beta$ false triples. The only obvious true triples were found to originate from the breakup of the IAS in $^{17}$F, with a Q value of 3.431 MeV, which is apparent in Fig. 2.7(a). The same peak is not present in Fig. 2.7(b), for the geometric reason mentioned earlier.

Density plots of energy relations obtained from the same data as for Fig. 2.7 are shown in Fig. 2.8, in which it is apparent that most of the raw triples are associated with a particle of relatively-low energy, most probably a $\beta$ particle simulating a proton, as is evident in Fig. 2.8(a). The Dalitz curves (see App. A.5), calculated for the 3-body breakup of the IAS in $^{17}$F, are also shown. These curves represent the regions within which 3-body breakup of the IAS in $^{17}$F is kinematically allowed. It is obvious in Fig. 2.8(b) that the $^{12}\text{C}-\alpha$ Dalitz curve is much less effective in discriminating against
§2.1 First Triple-Coincidence Measurement (Aug96)

Figure 2.8: Density plots of energy relations from triple-coincidence data: obtained with the F1-F2-F3 combination: (a) proton versus α energies; (b) 12C versus α energies. The closed curves represent the regions within which 3-body breakup of the IAS in 17F is kinematically allowed, while the straight lines in (b) represent the p-16O and α-13N energy ratio curves for 2-body decays.

background events compared to the p-α Dalitz curve. It is also obvious in Fig. 2.8(b) that most of the raw triples are, in fact, due to 2-body decays of 17F into either 13N+α or p+16O, as shown by the two bands of particles indicated by the dashed lines. As it turns out, only the 3 particle groups (the one at 1.4 MeV is not obvious) within the p-α Dalitz curve are associated with true triple coincidences, as will be shown below.

The final extraction of valid triple coincidences was performed by applying the conditions listed in Table 2.3. The conditions include setting limits on the IAS triple-sum energy, the Dalitz condition, constraints set by the geometry of the detection system, energy thresholds, and timing conditions. It is apparent that the constraint on the p-α angle in item 3 is rather loose (±46 deg) due to the large size of the detectors. This situation could be improved by employing segmented detectors, as will be seen in Sec. 2.2. The timing conditions in item 5 were based on the time-difference spectra shown in Fig. 2.9. A simple calculation has shown that the maximum difference in ToF for all particles of interest in this experiment, for a flight path of 4 cm, is about 20 ns. Therefore, the events outside the limits shown in Fig. 2.9, due mainly to coincidences with a β particle, could be excluded. The width of the peaks within the limits is in-
Table 2.3: List of Conditions applied to the Aug96 data in extraction of triples due to the 3-body breakup of the IAS in $^{17}$F.

1. The triple-sum energy is equal to the IAS 3-body breakup Q value: $3.431 \pm 0.150$ MeV.
2. Dalitz condition: $E_p$ and $E_\alpha$ are consistent with the Dalitz relation assuming proton and $\alpha$ energy resolutions of 30 and 40 keV, respectively (see App. A.5).
3. p-\(\alpha\) angle limits: the calculated p-\(\alpha\) angle based on the observed energies is within the angular range covered by F1 and F2, viz., $90^\circ \pm 2 \times 23^\circ$.
4. Energy thresholds (keV): $E_p > 150; E_\alpha > 300; E_c > 300$.
5. Time-difference limits (ns): $-45 < T_p - T_\alpha < 45; -58 < T_\alpha - T_c < 42$.

dicative that the timing resolution was not optimized in this experiment. Density plots of energy relations obtained after implementation of the conditions listed in Table 2.3 are shown in Fig. 2.10, in which it is apparent from a comparison with Fig. 2.8 that most of the background events have been suppressed.

Figure 2.9: Time-Difference Spectra from Aug96 data gated on the IAS triple-sum energy: (a) between proton and $\alpha$ particle; (b) between $\alpha$ particle and carbon ion.
2.1.3.4 Results and discussion

The IAS in $^{17}$F is energetically capable of decaying into 3 particles via both the $^{16}$O or $^{13}$N channels as can be seen in Fig. 1.2. The decay channels can be identified if the sequence of the breakups is known—proton followed by $\alpha$ for the $^{16}$O channel and vice versa for the $^{13}$N channel. In order to identify the transitions, Q-value spectra of the second breakup were generated from the final reduced data set for both possible decay sequences as shown in Fig. 2.11. The Q values were calculated using Eqs. (A.7) and (A.8) in App. A. In fact, if the decay sequence is correct, the Q value of the second breakup should be equal to the excitation energy of the intermediate state relative to the $^{12}$C($\alpha, \gamma$)$^{16}$O reaction threshold. Three such values, at 2.42, 0.42, and 1.56 MeV, corresponding to the 9.59 MeV state in $^{16}$O, and the 2.37 and 3.50/3.55 MeV states in $^{13}$N, are indicated in Fig. 2.11. The peaks in alignment with these excitation energies establish the identity of the transitions from which they originate, viz., the IAS→$^{16}$O(9.59)+p, IAS→$^{13}$N(2.37)+$\alpha$, and IAS→$^{13}$N(3.50/3.55)+$\alpha$ transitions. This is the first time such 3-body decay modes have been observed [58]. It is also shown in both spectra in Fig. 2.11 that contributions from the two $^{13}$N channels overlap with the
broad peak from the $^{16}$O channel. Since the second breakup Q value is unobservable, transitions for events in the overlapping region cannot be uniquely identified.

The projected proton and $\alpha$ spectra obtained from the final reduced data set are shown in Fig. 2.12. It should be pointed out that, while the 9.59 MeV state in $^{16}$O has a rather broad natural width ($420 \pm 20$ keV), those for the two intermediate states in $^{13}$N are relatively narrow (30~60 keV). It can be seen in Fig. 2.12(b) that the two proton peaks associated with the $^{13}$N channel are kinematically broadened as a result of the 3-body nature of the transitions. The fact that no kinematic broadening seem to have occurred for the two corresponding $\alpha$ groups suggests that, indeed, the transitions do go through 2 steps via an intermediate state, and the 3-body breakup does not occur simultaneously (see also discussion in App. A.6).

An attempt was made to extract the branching ratio of the IAS→$^{16}$O(9.59)+p relative to the IAS→$^{13}$N(2.37)+$\alpha$ transition from Fig. 2.12(a), by assuming Gaussian distributions for the $\alpha$ peaks, and taking into account the geometric efficiency of the two transitions due to the detection system. Since the result has been superseded by that from the Jul97 data (see Sec. 2.2), it will not be shown here.
§2.1 First Triple-Coincidence Measurement (Aug96)

Figure 2.12: Particle spectra obtained with conditions listed in Table 2.3: (a) $\alpha$ spectrum—three $\alpha$ groups associated with 3-body breakup of the IAS in $^{17}$F have been identified. The groups are labeled by the intermediate state of the transition. (b) Proton spectrum—the individual groups have been obtained by setting rough gates on the $\alpha$ groups in (a).

It should be remembered that the transition through the $^{16}$O channel is the one that relates to the $^{12}$C($\alpha, \gamma$)$^{16}$O reaction, which is the motivation behind this study. While the broad $\alpha$ peak at 1.8 MeV in Fig. 2.12(a), corresponding to the $^{16}$O channel via the 9.59 MeV state, is analogous to the major $\alpha$ peak obtained in the $^{16}$N experiment (see Fig. 10 of Ref. [10]), events due to the two transitions via the $^{13}$N channel are likely to contribute to background in this experiment. Fortunately, the angular distributions of the 3 transitions are rather different, as shown by the MC data in Fig. B.2, and it is hoped that a detection system with sufficient angular resolution would be able to discriminate against such transitions. It is also suggestive in Fig. 2.12(a), especially in the high- and low-energy regions, that some background events have survived the conditions listed in Table 2.3. It will be seen in Sec. 2.2 that better angular resolution will help to improve the efficiency of background suppression.
2.2 Measurement with SSDs (Jul97)

In the analysis of the Aug96 data, it was realized that angular resolution was compromised by the use of large-size detectors in close geometry required for optimization of triple-coincidence efficiency. Consequently, background discrimination was unsatisfactory due to loose kinematic constraints as a result of poor angular resolution (see Table 2.3). In order to remedy this situation, another run was conceived in which the two Ortec detectors would be replaced by double-sided silicon strip detectors (SSDs) to improve angular resolution. It was also found in the Aug96 analysis that $\beta$-induced false triples were the major source of background. Hence, it was thought that quadruple-coincidence measurement with the additional detection of the $\beta$ particle should also help in the suppression of background. However, the existing experimental chamber would only allow for the installation of a plastic scintillator which covered a small solid angle and, thus, only a small amount of quadruple coincidences would be recorded.

A period of thirty-one 12-hour shifts of beam time was scheduled in July of 1997 (Jul97) to study the feasibility of using SSDs in the measurement of triple coincidences, and the possibility of imposing quadruple coincidences to further improve background suppression. A total of $2.5 \times 10^8$ raw events was collected in 22.5 Gbyte of data.

The use of double-sided SSDs causes the number of detector elements to increase considerably compared to single-element detectors. Furthermore, substantial gain shifting occurred during the run so that the data were required to be divided into subsets and be calibrated separately. In anticipation of an even larger number of detector elements in the ultimate experiment, a considerable amount of time has been spent on the development of procedures to automate the calibration process. NOVA [56] was again used to sort and output potential triple-coincidence events to disk, for further analysis with PHYSICA [57] and Fortran programs specifically developed for analysis of these data. In particular, a procedure has been developed to apply kinematic constraints on
an event-by-event basis for background discrimination. In this aspect, the Jul97 data show definite improvement compared with the Aug96 data. However, it was also found that, although good angular resolution was obtained with the SSDs, that provided by the PIPS was still far from satisfactory, which limited the effectiveness of kinematic constraints in background suppression, as will be seen below.

### 2.2.1 Experimental setup

![Experimental Setup](image.png)

**Figure 2.13:** EXPERIMENTAL SETUP OF JUL97: The setup consists of two 5 cm x 5 cm x 300 μm (16 x 16 strips) SSDs and two 900 mm² x 300 μm circular PIPS detectors backed with 700 μm veto detectors. Signals from adjacent strips in the SSDs are tied together providing an 8 x 8 configuration with 64 pixels for each SSD. A 5½” x 5½” x ½” plastic scintillator is also located downstream along the beam axis for detection of β particles.

As shown in Fig. 2.13, the experimental setup† for this run was built upon the Aug96 configuration by replacing the two Ortec detector telescopes (see Fig. 2.3) with double-sided (16 x 16) silicon strip detectors‡. The strips on the front and back side of an SSD are oriented orthogonally. Three-dimensional measurement of charged-particle trajectories can be achieved with readouts from both sides, and additional information

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†With minor modifications, this setup was also used subsequent to this run in a study of the 3-body decay of ⁹C [59,60].

given by the location of the detector, with angular resolution limited only by the strip pitch, if all strips are read out individually. The distance between the collector foil and the detectors was also increased to 6 cm (from 4 cm) to provide an improvement in angular resolution. Note that this distance is still too short for ToF measurement to be useful. In fact, this was the maximum distance allowed by the existing 10"×10"×10" experimental chamber. The angles between the detectors were also modified from the Aug96 configuration in order to fit the chamber, and to obtain the maximum possible distance. However, the 160° angle between a PIPS and an SSD, which was taken to optimize detection of the α-¹²C pair (see Sec. 2.1.1), was retained. It should also be noted that, while veto detectors were retained for the PIPS detectors, anti-coincidence measurement was considered unimportant for the SSDs. In the Aug96 data analysis, anti-coincidence measurement was shown to be ineffective in the energy range of interest in this experiment. Furthermore, the improved angular resolution given by the SSD should provide kinematic constraints stringent enough to discriminate against false triples caused by punch-through particles. The Si detector array also occupied most of the space in the chamber. In order to install a β detector to facilitate quadruple-coincidence measurements, another chamber of the same size was coupled downstream to the existing chamber to house a 5¹⁄₂"×5¹⁄₂"×1" plastic scintillator which subtended a solid angle of about 1% of 4π.

The same VAX workstation and CAMAC crate as in the previous run were used for data acquisition. However, while operation of the PIPS detectors remained the same, the signals from the SSDs necessitated some special attention. Each SSD requires 32 electronic channels to fully utilize its position capability. Instead of being mounted on metal housings, as for most single-element Si detectors, the semiconductor crystal of an SSD is fabricated on a printed circuit board (PCB) on which a 34-way connector is mounted to allow for connections to preamplifiers with ribbon cables. While the use
of standard NIM modules is still feasible, such a method would be prohibitive, both in cost and in manpower. Instead, *micro-electronics*, developed at Rutherford Appleton Laboratory in collaboration with the Nuclear Physics group at the University of Edinburgh, U.K. [61] (RAL electronics), were used. The low-cost high-density circuitry employed by the RAL electronics allows for hundreds of channels to be realized within compact modular systems at a reasonable price. The readouts from an SSD are fed into a motherboard which houses 32 charge-sensitive preamplifiers (CSPA). Ribbon cables connect the outputs from the CSPA motherboard to shaping amplifiers (SA), which are also mounted on motherboards. Each SA motherboard has capacity for 8 modules. A maximum of 16 SA motherboards can be accommodated within a 19" KM-6 sub-rack providing a total of 128 channels in a compact system. The SAs deliver both analogue and logic (ECL) signals, which can be fed into ADCs and TDCs for digitization, and into a NIM-based logic circuit for event-trigger determination. Parameters such as gain, pole-zero cancellation, discriminator threshold, and gate width, etc., can be adjusted by plug-in resistors or trimmers, either individually on each SA or collectively on the SA motherboard. A schematic diagram of the electronics setup for the operation of SSDs can be found in Fig. D.1 in App. D. Fabrication details and specifications of the RAL electronics are described in Ref. [61].

The setup was designed with 2 double-sided (16 × 16) SSDs which require 64 electronics channels to fully utilize their position capability. However, only 32 channels of RAL electronics were available at the time of the run. In order to make use of both SSDs, at the cost of degrading angular resolution, signals from adjacent strips were tied together providing effectively an 8 × 8 configuration with 64 pixels for each SSD. Events were again triggered by adjacent detectors for the reason explained in Sec. 2.1.2. A single particle intercepted by an SSD, with energy above the discriminator threshold, will generate both a front and a back signal. Therefore, 4 signals are normally generated
when 2 particles hit the same SSD. However, if the two particles hit the same front or back strip, only 3 signals will be generated\(^\dagger\). By requiring \(2 \leq n \leq 4\), where \(n\) is the number of signals derived from a single SSD, provision was also made for triggering of double-hit events (see Sec. 2.2.2.3).

As in the run of Aug96, NOVA was used for online monitoring, and triggered events were written event-by-event onto 8 mm data cartridges for subsequent off-line analysis. Since only a small number of RAL electronics channels would be activated for a single event, except for the case of an event triggered by the electronic pulser signal, the front-end software *Twotran* was set to operate the ADCs and TDCs in zero-suppression mode so that only non-zero data words would be written out. Layouts of the electronic setup are shown in App. D—these were prepared for a run carried out in November of 1998 (see Sec. 2.3); the later run differs only in the addition of more \(\beta\) detectors plus a thick Si detector, and in the detail of the master trigger circuit.

It should be mentioned that, towards the end of the run, the total leakage current of the SSDs was found to have increased considerably from 0.3 \(\mu\)A at the beginning of the run to over 2 \(\mu\)A. This could be attributed to radiation damage\(^\ddagger\) to the detectors during the course of the run. An attempt was made to cool down the vacuum chamber but the leakage current was only slightly reduced. In fact, in the experiment for a study of \(^9\text{C}\) \([59,60]\) subsequent to this run (see footnote on p. 39), it was necessary to keep the SSDs at about 4 °C, by connecting the holders directly to a coldtrap, for them to operate properly.

\(^\dagger\)Particle multiplicities of greater than 2 are not of interest in this experiment.

\(^\ddagger\)This is a consequence of the modification of the lattice structure by the irradiation giving rise to vacancies, divacancies, vacancy-impurity complexes, etc., in the semiconductor crystal. These defects change the physical parameters of the crystal which is usually reflected as an increase in leakage current. Some of these defects might self-anneal with time at room temperature \([62,63]\), and the annealing process can be accelerated by baking the crystal at a few hundred °C.
Figure 2.14: Coordinate system and convention of strip labeling used for the Jul97 experiment setup. The detectors are on the xy plane; the front strips are parallel to the z axis; and the ion beam is along the x axis.

2.2.2 Data analysis and results

2.2.2.1 Geometric effect

As a first step in data analysis, gain stability was checked by generating pulser spectra for all runs (total: 271). The maximum shift in the pulser centroid was found to be 25 ADC channels (~80 keV). In order to minimize this effect, the entire data set was sequentially divided into 8 groups with the condition that the gain shift within each group should be less than 5 ADC channels (2~3 channels (<10 keV) in most cases).

Figure 2.14 depicts the location of the detectors with respect to the carbon foil and the convention for labeling the strip. It is obvious that the effective area of the beam spot, as seen by each front strip, decreases with increasing strip number in SSDL (decreasing strip number in SSDR). It is also obvious that, due to the planar geometry of the SSDs, the solid angle subtended by each strip with respect to the carbon foil also decreases with strip away from the center of the SSD. A set of singles spectra obtained
with the front strips of SSDL for one of the 8 groups of data is shown in Fig. 2.15, along with a plot of the number of counts in each strip. It can be seen that similar features were exhibited by all the spectra. However, the observed number of counts in each spectrum shows that the effective area of the beam spot outweighs the varying solid angle subtended by each strip, as the latter effect should be approximately\(^1\) symmetric about the center of the detector, but a rapid decrease in counts is seen in strips on the side further away from the beam axis. Similar spectra were also obtained for SSDR, which indicated that all the front strips were working as expected.

Attention should be drawn to the \(\alpha\) group indicated by the arrows in Fig. 2.15. This group, at 1.725 MeV, was mentioned in Sec. 2.1.3.2 as overlapping with the 1.680 MeV proton group (next to and in a higher ADC channel than the indicated \(\alpha\) group) in the PIPS spectrum. It can be seen that the two groups have been resolved in the SSD spectra. This is due to the existence of a thick dead layer\(^2\) in the SSDs causing the \(\alpha\) particles to lose more energy than the protons. Furthermore, the apparent variation, in both shape and energy, of the \(\alpha\) peak is the combined effect of the effective thickness of i) the carbon foil, which is a function of the angle of an emerging particle; and ii) the dead layer of the SSD, which is a function of the incident angle of a particle onto the SSD. This combined effect, which depends on the relative location of an individual strip and the carbon foil, causes the \(\alpha\) group to exhibit a varying degree of energy straggling in different strips.

The above observations suggest that, in the ultimate experiment, the dead layer of the SSDs should be minimized\(^3\), and the SSDs should not be used in close geometry in order to minimize varying response of the individual strips. The angular region in which particles need to emerge from the carbon foil at small angles to its surface should

---

\(^1\)The symmetry will be exact if the beam spot is a point source.

\(^2\)The dead layer of an SSD consists of 0.2~0.3 \(\mu\)m of aluminum and 0.4~0.5 \(\mu\)m of heavily-doped p\(^+\) implantation (boron) [64, 65].

\(^3\)By custom order, both the p\(^+\) implantation and Al layers can be reduced to 0.1 \(\mu\)m [64].
Figure 2.15: PARTICLE SINGLES SPECTRA OF FRONT STRIPS obtained with SSDL for calibration group No.6. The arrows indicate the 1.725 MeV $\alpha$ group which changes in shape and observed energy in different strips due to the change in the effective thickness of the SSD dead layer and carbon foil. The smaller plot shows the total number of counts in each spectrum.
also be avoided to minimize energy straggling due to the effective thickness of the foil. It should be noted that the energy of protons is much less affected due to their small mass and charge.

The geometric effect for the back strips is different from that for the front strips due to the difference in their orientations. As can be seen in Fig. 2.14, the effective area of the carbon foil as seen by each back strip does not vary appreciably, and the effective thicknesses of the carbon foil and the dead layer are also very similar\(^1\). Since the other geometric effect, the varying solid angle subtended by each strip with respect to the carbon foil, has been shown to be small for the front strips, the same should be true for the back strips so that the variation in count rate is not expected to be large. A set of singles spectra of back strips, obtained from the same data set as for Fig. 2.15, is shown in Fig. 2.16, in which it can be seen that all the back strips exhibit very similar features. It is also clear that the shape and location of the 1.725 MeV \(\alpha\) group does not vary appreciably in different strips. The count rates are also shown to be rather even, with slight fluctuations (especially b3) due primarily to the varying \(\beta\) response of each strip. All the above observations are consistent with the expected geometric effects.

A careful comparison between Fig. 2.15 and 2.16 shows that the energy resolution obtained with front strips is better than that with back strips. Note that there was a difference in gain of about 30% between signals derived from front and back strips, and the spike at about 1200 ch. in the front strip is sharper than that at about 850 ch. in the back strip spectra. This is attributable to the fact that most particles are stopped closer to the front of an SSD so that charges need to traverse through the entire depletion region before they can be collected by the back strips. In view of the

\(^1\)Certainly, for particles hitting the portion of a back strip further away from the beam axis, the effective thickness of the carbon foil increases. However, the same effect exists in all back strip, so they are functionally very similar.
Figure 2.16: PARTICLE SINGLES SPECTRA OF BACK STRIPS obtained with SSDL for calibration group No.6. The smaller plot shows the total number of counts in each spectrum.
better energy resolution they provide, only signals from the front strips are used for the final determination of energy and timing of a particle in this experiment; signals from back strips are used only for identification of the activated pixel (see the treatment of double-hit events in Sec. 2.2.2.6).

Another way of checking if an SSD functions properly is to check the distribution of hits among the pixels. However, interpretation of the hit patterns will be difficult when a large background is present. Thus, this will be postponed until the background has been removed.

### 2.2.2.2 Energy calibration

A list of particle groups and their corresponding transitions used in energy calibration is shown in Table 2.4. This table should be compared with Table 2.2 for the run of Aug96. The differences in the choice of particle groups for calibration are due mainly to the different detection properties between the SSD and PIPS as will be seen below. Individual calibration was required for a total of 34 elements for each of the 8 groups of data.

For the purpose of gain-matching, known proton groups at energies of 0.864, 1.680, 4.598, and 5.114 MeV [41], were used for the SSD, whereas only the latter three were used for the PIPS since the 0.864 MeV (~250 ADC channel) proton group was overwhelmed by the $\beta$ tail in the PIPS spectra. In the determination of their centroids in ADC channel number, the proton peaks were again assumed to be Gaussians with linear background. Linear regressions were then performed on the centroids, and the known energies, to obtain a set of gain-matching parameters. A typical value of the linear correlation coefficient for the linear regressions is 0.9999995 for both the SSD

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*The choice of the 0.864 MeV group is prompted by the improved $\beta$ background in the SSD spectrum. The 3.800 MeV group, used in calibration of the Aug96 data, has been excluded, due to its broadness, to render the calibration more accurate.

*Eight signals from both sides of each of the two SSDs and two from the PIPS.
Table 2.4: List of particle groups and their corresponding transitions used in energy calibration of Jul97 data. The errors in energy are obtained from the uncertainties in level energies [41]. Only two $\alpha$ groups were available for calibration due to the low number of counts obtainable from individual strips.

<table>
<thead>
<tr>
<th>Particle</th>
<th>Purpose</th>
<th>Energy (MeV±keV)</th>
<th>Transition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Proton</td>
<td>Gain matching</td>
<td>0.864±11</td>
<td>$^{17}F(8.44)\rightarrow^{16}O(6.92)+p$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1.680±11</td>
<td>$^{17}F(8.44)\rightarrow^{16}O(6.05)+p$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>4.598±11</td>
<td>$^{17}F(5.49)\rightarrow^{16}O(gs)+p$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>5.114±9</td>
<td>$^{17}F(6.04)\rightarrow^{16}O(gs)+p$</td>
</tr>
<tr>
<td>Proton</td>
<td>SSD</td>
<td>0.864±11</td>
<td>$^{17}F(8.44)\rightarrow^{16}O(6.92)+p$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>3.270±2</td>
<td>$^{17}F(IAS)\rightarrow^{16}O(7.12)+p$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>4.198±2</td>
<td>$^{17}F(IAS)\rightarrow^{16}O(6.13)+p$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>4.274±4</td>
<td>$^{17}F(IAS)\rightarrow^{16}O(6.05)+p$</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>SSD</td>
<td>1.725±10</td>
<td>$^{17}F(8.08)\rightarrow^{13}N(gs)+\alpha$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2.301±3</td>
<td>$^{17}F(IAS)\rightarrow^{13}N(2.37)+\alpha\rightarrow p+^{12}C+\alpha$</td>
</tr>
<tr>
<td>$^{13}N$</td>
<td>$^{12}C$ in PIPS</td>
<td>0.531±10</td>
<td>$^{17}F(8.08)\rightarrow^{13}N(gs)+\alpha$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.616±10</td>
<td>$^{17}F(8.44)\rightarrow^{13}N(gs)+\alpha$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.991±10</td>
<td>$^{17}F(10.0)\rightarrow^{13}N(gs)+\alpha$</td>
</tr>
</tbody>
</table>

and PIPS. In view of the large number of spectra to be processed ($8 \times 34$), and in anticipation of an even larger number in the ultimate experiment, a considerable amount of time has been spent in the development of software routines, both in PHYSICA [57] and Fortran, to automate the processes of locating the peaks, fitting the Gaussians, and extracting calibration parameters.

The gain-matched energy spectra of one of the front strips and a PIPS for the entire data set are shown in Fig. 2.17. While the two spectra exhibit similar features, obvious differences are seen in the low-energy region. This can be attributed to at least two effects: i) the thick dead layer of the SSDs inhibits the detection of low-energy heavy particles, mostly $^{16}O$; and ii) as a $\beta$-particle is scattered within the active volume of an SSD, it becomes less likely to deposit enough energy in a single strip to generate a signal above the discriminator threshold. The effect of the dead layer can also be seen from the different response of the two types of detector to protons and $\alpha$ particles. As mentioned earlier, the 1.680 MeV proton group and 1.725 MeV $\alpha$ group are resolved.

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in the SSD, but not in the PIPS spectra, which is obvious in Fig. 2.17. Also indicated in Fig. 2.17 are the proton peaks associated with 2-body decays of the IAS in $^{17}$F (see Table 2.4). It is obvious that the proton energy resolution obtained with the SSD is better than that with the PIPS. As expected, proton energy resolution is not affected by the SSD dead layer to any significant extent, while that obtained with the PIPS is degraded, due mainly to the large-size of the detectors. The FWHM of the IAS proton peaks is about 25 keV for the SSDs and 40 keV for the PIPS. The width of these peaks is also indicative of the intrinsic energy resolution of the detectors, as the natural width of the IAS (0.18 keV) is negligible in comparison with the experimental resolution.

It was mentioned in the Aug96 data analysis that particle identities could only be deduced indirectly through the geometry of the setup in association with kinematic considerations (see Sec. 2.1.3.2). The situation for the Jul97 run was similar except that the thick dead layer acted to preclude the SSDs from possible carbon-ion detection.

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\[\text{Figure 2.17: Gain-matched singles spectra: (a) is from a single front strip of SSDL while (b) is from PIPL (see Fig. 2.13). Note the peak(s) indicated by the solid arrows, which are resolved in (a) but not in (b). The dashed arrows indicate peaks due to 2-body decay of the IAS in }^{17}\text{F.} \]

---

\[\text{The dead layer in PIPS detectors is typically } < 50 \text{ nm.}\]
as a consequence of the heavy ions \((E < 1 \text{ MeV})\) losing most of their energies to it. Therefore, it is reasonable to assume that, in a 3-body breakup, the particles observed in the SSDs are either protons or \(\alpha\) particles while the carbon ions are detected by one of the PIPS. Under these assumptions, both proton and \(\alpha\) energy calibrations are required for the SSDs whereas only that for carbon is needed for the PIPS.

In principle, proton energy calibration had already been accomplished in the gain-matching process, as proton groups were used. In order to take advantage of the better statistical accuracy given by the summed spectra, proton groups at 3.270, 4.198, and 4.274 MeV (see Fig. 2.17), corresponding to 2-body decays of the IAS in \(^{17}\text{F}\) (see Table 2.4), were used to recalibrate the SSDs for proton energies. The reason for choosing the 3 proton groups is obvious—accurate calibration can be obtained with these peaks whose widths are narrow by virtue of the natural width of the IAS. The proton line at 0.864 MeV was also used in the recalibration to constrain the low-energy region. Although this recalibration amounted to a correction of only about 1%, the benefit is more in cross-checking the gain-matching process for consistency.

With the segmented SSDs, only two \(\alpha\) transitions (see Table 2.4) were found to provide enough statistical accuracy to allow for calibration of each strip individually. In fact, while \(\alpha\) particles from the first transition could be obtained from the gain-matched spectra, the second peak had to be extracted from triple-coincidence data, which have even more limited numbers of counts, especially for strips making larger angles to the beam axis. It should also be noted that, with the minimum number of data points (2) for a linear calibration, there would not be any degree of freedom in obtaining the parameters. Hence, it is impossible to quantify the accuracy of the calibration for \(\alpha\) energies. Nevertheless, since the two data points encompass a major part of the energy region of interest in this experiment, the calibration should be adequate for the present purpose.
As mentioned above, only carbon energy calibration is required for the PIPS. It should be remembered from the Aug96 calibration that spectra of $^{13}$N groups, which were obtained by setting gates on the corresponding $\alpha$ groups in the (double) coincidence spectrum (see Fig. 2.6), were used in the calibration of carbon energies (see Sec. 2.1.3.2). The same method was applied here.

2.2.2.3 Extraction of triple coincidences

Triple-coincidence events obtained with the present setup can be categorized into two types: 

a) a single hit is observed in each of the two SSDs in coincidence with a hit in at least one of the PIPS, and 

b) two hits are observed in a single SSD in coincidence with a hit in the PIPS at 160°.

With the assumption that only protons and $\alpha$ particles can be observed in the SSDs, the major difference between types a and b is in the range of angles between the trajectories of the proton and $\alpha$ particle—59° to 151° for type a and 6° to 61° for type b events—limits set by the geometry of the system. Due to the difference in their data structure, the two types were sorted and analyzed separately.

The sorting conditions for events of both types are listed in Table 2.5.

Events of type a

For events of type a, condition i) guarantees that each SSD records one, and only one, particle; ii) enforces triple while allowing the possibility of quadruple coincidences; iii) vetos events with punched-through particles; iv) and v) set energy limits to include only events of interest in this experiment. For each event which fulfilled the listed conditions, the ADC and TDC channel numbers of each detector element, and the addresses corresponding to the activated strips in the case of the SSDs, were written out, along with a word which identified the detector pair(s) from which the trigger was obtained. A total of $5.3 \times 10^5$ potential type a triple coincidences was identified.

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1 The limit of 6° is set by the strip pitch.

2 Each bit in the word is associated with a certain type of trigger, e.g., SSDL+PIPL or SSDL+SSDR. Note that an event can be triggered by more than one type of trigger.
Table 2.5: List of conditions applied in sorting of events of type a and b.

**Type a**

i) one and only one signal from both sides of each SSD

ii) a signal from at least one of the front PIPS

iii) no signal from the back PIPS of the same telescope

iv) the energy\(^a\) recorded in the front PIPS is less than 1.5 MeV

v) the triple-sum energy\(^a\) is less than 4 MeV

**Type b**

i) at least 1, and at most 2, signal(s) from each side of an SSD

ii) at least 3, and at most 4, signals from an entire SSD

iii) a signal from the front PIPS at 160° with the SSD

iv) no signal from the back PIPS of the same telescope

v) the energy\(^a\) recorded in the front PIPS is less than 1.5 MeV

vi) the triple-sum energy\(^a\) is less than 4 MeV

vii) no signal from the other SSD or PIPS

\(^a\)based on energies calibrated with protons

As noted earlier, the geometry of the Jul97 setup was too close for ToF measurement to be possible. It was also found that the electronic setup was not optimized for timing resolution. Nevertheless, a study of the relation among the timing signals could still be helpful in eliminating random coincidences. However, setting limits at this stage of the analysis would not be effective while immense background was still present. Instead, a minimal timing condition was implemented. This condition requires valid TDC signals\(^1\), defined as signals within the ranges provided by the TDCs, to be associated with all 4 ADC signals from the SSDs, and at least one from the PIPS. The implementation of this condition reduced the number of events by two-third to \(1.8 \times 10^5\). A kinematic-constraint procedure, described below in Sec. 2.2.2.4, was then applied to this set of data to obtain the final result.

\(^1\)A TDC takes a start and a stop signal, and outputs a bit pattern which represents the time between the occurrences of these two signals. After receiving a start signal, if no stop signal is present within the range allowed by the TDC, the output becomes invalid (overflow). It was found that a majority of the raw events contained at least one of these overflow signals.
Events of type \( b \)

In events of type \( b \), two particles are observed in a single SSD in triple coincidence with the PIPS at 160°. It is possible for two particles to hit the same front strip but different back strips, or vice versa\(^1\) (see discussion on p. 41). Therefore, provision should also be made for acceptance of such events. As can be seen in Table 2.5, the conditions ensure that only events associating with the SSD-PIPS pairs at 160° are accepted. The first two conditions guarantee that at most two particles are observed in the SSD; iii) to v) are similar to those implemented in the sorting of type \( a \) events; and the last condition is to exclude type \( a \) events. A set of over \( 7 \times 10^6 \) type \( b \) events was identified.

A study of the energies and correlation between strips has shown that a majority of these events were, in fact, generated by 2-body breakups in which a single particle traversing the inter-strip regions in an SSD caused the charges to be shared between adjacent strips and gave rise to false double hits (see also Ref. [66]). The additional requirement of valid TDC signals associated with all valid ADC signals was very effective in removing these false events, most likely due to one of the two signals being below the discriminator threshold. Note that the ADC and discriminator thresholds need not be the same. The reduced set of \( 1.2 \times 10^5 \) events was further processed with the kinematic-constraint procedure described in the next section.

2.2.2.4 Kinematic-constraint procedure

In order to discriminate by kinematic constraints against background, a procedure was developed to process the data on an event-by-event basis\(^2\). The procedure [67] consists of minimizing a function which depends on the observed energies of the particles, the locations of the detector elements, and the uncertainties of these observables. The

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\(^1\)Events in which two particles go into the same pixel will be lost.

\(^2\)The algorithm of this procedure was first put forward by N.P.T. Bateman, and has been largely modified by myself and extended in this thesis.
magnitude of this function, which is similar to a $\chi^2$, reflects the goodness of an event. Note that this quantity should not be treated as strictly a $\chi^2$ since the distributions of some of the variables are not necessarily normal. Nevertheless, a large value of this function associated with any event can be taken as an indication that the event is kinematically unfavored.

Numerous forms of kinematic function have been tested. The major differences among these functional forms are in the treatment of linear momentum conservation and the choice of physical quantities to be constrained. The final form consists of only 6 terms grouped below into 3 components:

$$S = \frac{S_1 + S_2 + S_3}{6},$$

(2.3)

where

$$S_1 = \sum_{i=p,a,c} \frac{(E_i - e_i)^2}{\sigma_{ei}^2},$$

$$S_2 = \frac{(\Theta_{p\alpha} - \theta_{p\alpha})^2}{\sigma_{\theta p\alpha}^2} + \frac{(\Theta_{pc} - \theta_{pc})^2}{\sigma_{\theta pc}^2},$$

$$S_3 = \frac{[(\sum_{i=p,a,c} e_i) - Q_{3b}]^2}{\sigma_{Q_{3b}}^2}.$$

In $S_1$, $e_i$ are the observed energies with uncertainties $\sigma_{e_i}$ and $E_i$ are the fitted energies. In $S_2$, $\theta_{p\alpha}$ ($\theta_{pc}$) is the angle between the centers of the detector elements in which the p-$\alpha$ (p-$^{12}$C) pair have been observed, $\Theta_{p\alpha}$ ($\Theta_{pc}$) is the angle between the trajectories of the proton and $\alpha$ particle (proton and $^{12}$C) calculated from the fitted energies, and $\sigma_{\theta p\alpha}$ ($\sigma_{\theta pc}$) is the uncertainty in $\theta_{p\alpha}$ ($\theta_{pc}$). The angular uncertainties are taken as the differences between the angles subtended by the centers of a pair of detector elements and the maximum angle subtended by the pair. Note that the $\alpha$-$^{12}$C angle, $\theta_{ac}$, does not provide an additional constraint since the sum of $\theta_{p\alpha}$, $\theta_{pc}$, and $\theta_{ac}$ should add up to

---

*Values of 12, 32, and 38 keV for $\sigma_p$, $\sigma_\alpha$, and $\sigma_e$, respectively, have been used. $\sigma_p$ was obtained from proton decays of the IAS in singles spectra; $\sigma_\alpha$ was deduced from the observed width of the 2.3 MeV $\alpha$ peak from the IAS$\rightarrow^{15}$N$(2.37)+\alpha$ transition; $\sigma_e$ was deduced from the width of the IAS triple-sum energy peak by assuming that the 3 $\sigma$'s could be added in quadrature.
In $S_3$, $e_i$ are as in $S_1$, and $Q_{3b}$ is the 3-body breakup Q value. On an event-by-event basis, the CERN package MINUIT [68] is used to obtain the minimum of $S$, which is returned as the average of the 6 terms. Note that, in a strict sense, only the 3 energies are variables in the minimization, while the other 3 quantities are calculated from the fitted energies (see App. A).

In a previous form of the function, the energies and directions of the particles were allowed to vary independently, and the total momentum was the dominating term of the function which was calculated explicitly from the fitted directions and energies. Consequently, the resulting fitted values did not necessarily conserve momentum, and the function was, in essence, a measure of the degree of violation of momentum conservation. In the present function, on the other hand, the only variables in the minimization are the energies of the particles, and momentum conservation is implicit in the calculation of $\theta_{pa}$ and $\theta_{pc}$ from the fitted energies. The inclusion of the 3-body breakup Q value as a constraint is also very effective in distinguishing events from a given parent state in $^{17}$F. It should also be noted that the effect of the beam spot has not been included in the present function. In the previous function, the directions of the trajectories were varied independently and, therefore, the fitted values were very sensitive to the location of the decay vertex. However, in the present function, since the constraints are in the angles between pairs of trajectories, and not in the directions of individual ones, the location of the decay vertex is a secondary effect which can be absorbed in the uncertainties of $\theta_{pa}$ and $\theta_{pc}$. In fact, a simple calculation has shown that, with the geometry as described in Fig. 2.13, if the decay vertex is shifted from the center to the edge of a beam spot of 4 mm radius, $\theta_{pa}$ varies by about 0.2% and $\theta_{pc}$ by about 2%.
2.2.2.5 Results for events of type $a$

According to a Monte Carlo study (see App. B.4), for events of type $a$ in the geometry as described in Fig. 2.13, the $\alpha$ particle and $^{12}\text{C}$ ion are always detected by the SSD-PIPS pair at 160° with the proton picked up by the other SSD. With the additional constraints due to the dead layer of the SSDs (see Sec. 2.2.2.2), particle identification becomes unambiguous; e.g., for an event which involved PIPL, SSDR would have detected the $\alpha$-particle, SSDL the proton, and PIPL the carbon.

The function $S$ in Eq. (2.3) was calculated for each event in the final reduced data set as described in Sec. 2.2.2.3. For quadruple events which involved both PIPS detectors (amounting to about 1% of the reduced data set), $S$ was calculated for detector combinations involving each PIPS in turn, and the one with the lower value of $S$ was retained. The result shown in Fig. 2.18 is obtained from a subset of the data and it is clear that the value of $S$ for most events is less than unity near the 3-body breakup $Q$ value of the IAS (3.431 MeV). It is also clear that there is a considerable separation between these events and the band of supposedly false events at higher $S$, which extends across a much larger range of energies. The separation between these events and those on either side of the minimum is also fairly distinct. Accordingly, it is reasonable to set an upper limit of unity on $S$ as a criterion for extraction of kinematically valid events. The energy relation between protons and $\alpha$ particles for the same subset of data, with no limit on $S$, is shown in Fig. 2.19. Note the discriminator threshold of about 250 keV for the proton energy, which was necessary to keep the trigger rate at a manageable level. It is also apparent in Fig. 2.19 that there is a substantial amount of background, due predominately to $\beta$ particles, extending to about 500 keV in proton energy.

It should be remembered that only a minimal requirement of valid TDC signals (see Sec. 2.2.2.3) has been implemented; the correlations among these signals should be further exploited at this point. Relevant to this consideration are 5 timing signals.
§2.2 Measurement with SSDs (Jul97)

Figure 2.18: Result of kinematic procedure: $S$ versus triple-sum energy. The quantity $S$ reflects the goodness of an event.

Figure 2.19: Energy relation between protons and $\alpha$ particles along with the Dalitz curve (see App. A.5).

(4 from the front and back strips of each SSD and 1 from the PIPS) from which 5 difference spectra can be generated—2 between the front and back strips of each SSD and 3 between pairs of detectors. Figure 2.20 shows the time-difference (TD) spectra

Figure 2.20: Time-difference spectra obtained with (bottom row) and without (top row) an upper limit of unity set on the kinematic function $S$: (a) and (b) front and back strips of each SSD; (c) proton and $\alpha$; (d) proton and carbon; (e) $\alpha$ and carbon.

*Only signals from front strips are used in consideration of time correlations between detectors.

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generated with and without an upper limit of unity set on the kinematic function $S$, from data obtained with the SSDL-SSDR-PIPR combination. The decrease in width of all the TD spectra in the bottom row of Fig. 2.20 is indicative of the effectiveness of the kinematic constraints in extraction of valid events. Since the timing signals were not optimized, it would be difficult to determine valid limits on the TDs. Furthermore, as their spreads are already rather narrow, little can be done to further constrain the TDs associated with only the SSDs. Nevertheless, for the two TD spectra involving the PIPS (T$_f$$_p$–T$_f$$_c$ and T$_f$$_a$–T$_f$$_c$), limits as shown in the plots were set to remove events which were believed to be erroneous—the TDs in the two spectra for these latter events were much larger than the limits obtained from ToF calculations.

The result of imposing an upper limit of unity on $S$, along with the moderate timing constraints, is shown in Fig. 2.21, in which it is clear from a comparison with Fig. 2.19 that most of the background events have effectively been removed. Figure 2.22 shows the projected $\alpha$ spectrum which can be compared with Fig. 2.12(a) in p. 37 obtained for the Aug96 run. Since the geometries of the two setups were not the same, different types of detectors were used, and different methods were employed in constraining
§2.2 Measurement with SSDs (Jul97)

(a) SSDR — protons

(b) SSDL — α particles

Figure 2.23: Hit patterns obtained with the final reduced data set of type α. Note that the left (right) hand side of SSDL (SSDR) is closer to the beam axis.

the kinematics, a direct comparison between these two data sets would be difficult. Nevertheless, implementation of the kinematic procedure to the Jul97 data was more complete, and the much improved angular resolution given by the SSDs provided more stringent constraints compared to those given by the Aug96 setup. Therefore, insofar as background discrimination is concerned, the Jul97 α spectrum is superior to the one from Aug96.

As mentioned before, the proper functioning of an SSD can be substantiated by a study of the distribution of hits among the pixels. It is also useful to check if these so-called hit patterns are consistent with the expected kinematics. It should be remembered in the analysis that particle identities were deduced from the geometry of the setup, properties of the detectors, and consideration of kinematics (see App. B.4); e.g., for a triple coincidence obtained with SSDL-SSDR-PIPR, the SSDL-PIPR pair is expected to pick up the α-12C pair, and SSDL the proton. Hit patterns of the SSDs, generated from the final reduced data set with the detector combination just mentioned, are shown in Fig. 2.23. It is apparent in Fig. 2.23(b) that most of the α particles hit the SSD near its center, which supports the assertion in Sec. 2.1.1 that the most probable angle between the α-12C pair in the 3-body breakup of 17F* is 160°. However, the hit pattern for the protons is expected to be more evenly distributed (see
§2.2 Measurement with SSDs (Jul97)

the MC angular distributions in Fig. 2.1). This is also confirmed by Fig. 2.23(a), with the variation in counts attributable primarily to the varying solid angles subtended by the pixels, and their different angles relative to the beam axis.

The same three 3-body breakup transitions as from the Aug96 data have been observed. Presumably, the relative BR's among these transitions\footnote{The first determination of the branching ratio of the IAS$\rightarrow^{16}$O(9.59)+p relative to that of the IAS$\rightarrow^{13}$N(2.37)+α transition was performed with the Aug96 data, but the result has since been superseded by the more recent one obtained with the Jul97 data.} could be deduced from Fig. 2.22. However, it should be noted that this figure was generated with a software threshold of 300 keV on the proton energy. As it turns out, this threshold plays a crucial role in the determination of the relative BR's, as will be seen in Chapter 4.

2.2.2.6 Results for events of type \textit{b}

In events of type \textit{b}, there are three possibilities as to how two particles hit a single SSD: i) two distinct front and back strips are hit giving a total of 4 signals; ii) the particles hit the same front or back strips so that 2 of the signals are summed giving only 3 signals; and iii) both particles hit the same pixel giving only 2 summed signals. The last possibility would fail the sorting conditions listed in Table 2.5. Based on an MC calculation, events with 3 signals should amount to about 20% of all true events of type \textit{b}. However, there is an immense background for these events due to the inter-strip effect mentioned in Sec. 2.2.2.3, even with the requirement of valid TDC signals, which makes them difficult to analyze. Since it was believed that no information would be given by these data in addition to those given by data with 4 signals, the subset of data with 3 signals was neglected. Thus, only events with 4 signals were analyzed.

In events of type \textit{a}, the single front-back pair of signals from each SSD uniquely determines the location of the hit. However, for type \textit{b} events with 4 signals, the determination of the locations and identities of the two hits is complicated by the fact that either of the front signals can be associated with either of the back signals,
and either of the front-back pair can be associated with a proton or an $\alpha$ particle, giving a total of 4 possible strip-particle associations. This problem is demonstrated in Fig. 2.24.

The same kinematic procedure as for type $a$ was applied to this set of data. For each event, the kinematic function $S$ was calculated for all 4 possible strip-particle combinations and the one with the smallest value was retained. A plot of $S$ versus triple-sum energy is shown in Fig. 2.25, in which it can be seen that most of the events with triple-sum energy near 3.43 MeV (the 3-body breakup Q-value of the IAS) also have values of $S$ less than unity, although the separation between good events and background is not as distinctive as that for type $a$ data.

Again, in the extraction of final data, an upper limit of unity was set on $S$ along with the same TD conditions as shown in Fig. 2.20. Since the energies of all particles in these data were relatively high (> 500 keV), no software threshold was needed. The $E_p - E_\alpha$ energy relation spectra before and after implementation of the above conditions are shown in Fig. 2.26, along with the Dalitz curve (see App. A.5). It can be seen that all the valid events are located just above the lower limit of the Dalitz curve, which is
expected since both the protons and α particles have been detected by a single SSD so that the angle between their trajectories is small. At the lower limit, this angle equals zero. Events in which the α particle goes in a direction close to that of the carbon ion would be located just below the upper limit of the Dalitz curve.

The final proton and α spectra are shown in Fig. 2.27. The IAS→\(^{13}\)N(2.37)+α transition, one of the 3 prominent transitions observed in type α events, is not seen in these events—its observation is not kinematically favored by the geometry. The peak at 1.4 MeV in the α spectrum in Fig. 2.27 (1 MeV in the proton spectrum) is identified as corresponding to the IAS→\(^{13}\)N(3.50/3.55)+α transition. The other sharp peaks, about 1.3 MeV in both spectra, arise from events in which protons and α particles were falsely identified by the kinematic procedure. This could be attributed to the large solid angle subtended by the PIPS detectors so that the constraints were not sufficiently stringent, and the proton and α energies not sufficiently different when observed with this geometry, for the procedure to work effectively. Due to pulse-height defects [52, p.238], misidentified protons (α particles) would have their energies shifted upward (downward). Note that the contribution from the IAS→\(^{16}\)O(9.59)+p transition

![Figure 2.26: ENERGY RELATION between proton and α particle (a) before and (b) after applying an upper limit of unity on S and TD limits as shown in Fig. 2.20. The Dalitz curve (dashed) is also shown (see App. A.5).](image)
appears as a broad peak underneath the sharp peaks in both spectra.

The above results suggest that events of type \( b \) are not useful for the ultimate experiment for the following reasons: i) contribution from the IAS\( \rightarrow ^{16}\text{O}(9.59)+\text{p} \) channel is not prominent; ii) the IAS\( \rightarrow ^{13}\text{N}(3.50/3.55)+\alpha \) channel will likely interfere with the measurement; iii) background suppression will not be as effective as for events of type \( a \); and iv) ambiguity in particle identification will render data analysis more difficult. It will be shown in Sec. 2.3 that, by moving the detectors further away from the collector foil, events of type \( b \) can be kinematically suppressed.

2.2.2.7 Hints from \( \beta \)-gated quadruples

The presence of the scintillator paddle installed downstream of the Si detector array (see Fig. 2.13) allowed a small amount of quadruple coincidences \( (\beta+\text{p}+\alpha+^{12}\text{C}) \) to be collected. With detection of the additional \( \beta \) particle, which was known to have caused a substantial amount of background, it was hoped that these events would provide hints on important features of good events with less interference from background.

Figure 2.28 shows the raw triple-sum energy spectrum of quadruple coincidences in comparison with that obtained from triple coincidences of type \( a \). While contribution
from the breakup of the IAS at 3.43 MeV is obvious in both spectra, the background has been strongly suppressed in the quadruple-coincidence spectrum. From this result, it was thought that detection of the $\beta$ particle would be essential in suppression of background, and another experimental run was devised in which $\beta$ detectors covering a much larger solid angular ($\sim$25% of $4\pi$) would be installed to study this effect. As it turned out, the requirement of quadruple coincidence was not as essential as was first thought, as will be seen in Sec. 2.3.2.1.
2.3 Quadruple-coincidence and ToF Measurements

Prompted by the unsatisfactory discrimination against background in the Jul97 data, due partly to the intense $\beta$ flux which gave rise to false triple coincidences, and partly to loose kinematic constraints as a consequence of poor angular resolution due to the use of large-size PIPS detectors, another setup was devised to address these issues. It was felt that the requirement of quadruple coincidence would improve the suppression of accidental coincidence, and Time-of-Flight (ToF) measurements would render unambiguous identification of particles. For this run, a total of 35 12-hour shifts of beam time was scheduled in November and December of 1998. The detection system was similar to that used in the run of Jul97 except for the replacement of the single scintillator paddle with 4 slightly larger ones to be placed in much closer geometry, and the addition of a thick Si detector for generation of singles particle spectra. An increase in the distance of the Si detectors from the collector foil was necessary for ToF measurement, and more space was needed to accommodate the plastic scintillator detectors. In fact, a new vacuum chamber was required to house the modified detection system. Much effort was expended in the fabrication of this new chamber. Unfortunately, many problems were encountered during the course of the run making it much less productive than had been expected. Nevertheless, the results still provided useful information for the ultimate experiment.

2.3.1 Experimental setup

It was mentioned in the Jul97 setup that the existing $10'' \times 10'' \times 10''$ vacuum chamber was too small for ToF measurement to be possible. Furthermore, no space was available for installation of scintillator paddles for $\beta$ detection in close geometry. In order to facilitate both ToF and quadruple-coincidence measurements, a new $24'' \times 24'' \times 20''$
§2.3 Quadruple-coincidence and ToF Measurements (Nov98)

An experimental chamber was built. With the idea of recycling some of the covers of the existing 10"×10"×10" chamber, the new chamber was designed to have more than 20 openings, which turned out to be a major problem. While setup of the detection system was made easy by the many openings providing access to the inside of the chamber, the large amount of O-ring surfaces required to seal these openings increased the chance of potential vacuum leaks tremendously. Much effort was spent in testing the chamber for leaks—during the process, some of the joints were found to require re-welding, and all of the O-ring surfaces needed to be polished. A major overhaul of the system at beam line B of TiSOL was also needed to accommodate the new chamber—the supporting structure, coupling peripherals to the existing beam pipe, the target ladder, electronic feed-throughs, and detector holders all needed to be manufactured or modified. With the much increased volume, a cryogenic pump with a larger capacity was also required. Ironically, much more effort was spent in preparation for this run compared to the previous two, but the least amount of data was acquired, due to numerous instrumental problems which will be detailed below.

The detection system is depicted in Fig. 2.29. The same Si detectors as for the Jul97 run were used with the distance from detectors to the collector foil increased to 10 cm (from 6 cm in Jul97) to enable ToF measurement. In addition, a 1500 μm Si surface-barrier (SSB) detector was placed at a distance of 20 cm from the collector foil, and at an angle of 20° to the beam axis. This thick detector served two purposes: i) to accumulate low-energy particle spectra in which protons and α particles could be separated by ToF measurement; and ii) to obtain a high-quality spectrum of the weak proton groups at energies greater than 6 MeV. Some details of the 17F decay scheme were still uncertain, particularly due to overlap between low-energy proton and

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†This chamber was designed by an engineer at TRIUMF and manufactured at The Ohio State University, thanks to Prof. R.N. Boyd.

†Detectors with thickness of ≤300 μm, which could stop protons of up to about 6 MeV, were used in previous runs.
Figure 2.29: EXPERIMENTAL SETUP for ToF and quadruple-coincidence measurements (Nov98). The same Si detectors as for the Jul97 run were used. In addition, four 4"×8"×4/4 scintillator paddles were installed just outside the Si-detector array for β detection. Light guides (not shown) connected the paddles along the 8" side to photomultiplier tubes located just outside the chamber. A 1500 μm × 100 mm² SSB detector was placed at a distance of 20 cm from the collector foil, and at an angle of 20° to the beam axis, for generation of singles particle spectra.
α groups which were obscured by the tail of the intense β flux through the various
detectors used in previous runs. Since the data obtained with the SSB detector are
more relevant to Morton's thesis, they were analyzed by him [46].

In order to enable quadruple-coincidence measurements, four 4"×8"×1" scintillator
paddles were installed just above and below the Si detector array covering a total solid
angle of about 25% of 4π. Light guides (not shown in Fig. 2.29) connected the paddles
along the 8" side to RCA 8578 photomultiplier tubes (PMT) located just outside of the
chamber. The logic signals derived from these scintillators, presumably triggered by
β particles, were also useful in providing the start or stop signal for ToF measurement†.

It has been noted in Sec. 2.2.1 that, towards the end of the Jul97 run, leakage
current in the SSDs was observed to have increased considerably due presumably to
radiation damage; in the 9C experiment (see footnote on page 39), it was necessary to
keep the SSDs at about 4 °C for them to operate properly. Although they might have
self-annealed [69] after being idle for over a year since the 9C run, operating the SSDs
at lower temperature was still desirable to lower the leakage current. For this purpose,
a LN₂-cooled trap was installed above the chamber to cool the SSDs by connecting the
trap to the detector holders with braided copper wires. The filling cycle of the trap was
about 2 hours, and the temperature, monitored with a digital thermometer thermally
coupled to the PCB of the SSDs, was maintained at about —10 °C throughout the run.

An event-trigger scheme similar to that of the Jul97 run was used, with the addition
of a signal from the SSB detector. Signals from the scintillator paddles were not
used in determination of the event trigger. The DAQ was also upgraded to a PC
which operated on LINUX with a 300 Mhz Pentium III processor. The same CAMAC
crate and electronic modules were used, but a new DAQ software, MIDAS (Maximum

†The relativistic β particles provide essentially a zero time reference point for ToF measurements
of other particles. Furthermore, the risetime of pulses obtained with plastic scintillators is typically a
few hundreds of picoseconds whereas that with Si detectors is a few nanoseconds.
Integrated Data Acquisition System) [70] was employed. This is a versatile software, developed for small to medium scale experiments, which has all the functionalities of VDACS (used in the previous two runs) but with many additional features, such as beam line and high voltage controls, and controlling and monitoring of an experiment through a web browser. The additional features were not useful for the present run. MIDAS also comes with its own subprogram (analyzer) for data analysis. However, since NOVA is a program with which the experimenters were more familiar, it was again used for online monitoring.

As mentioned before, the data obtained in this run were not as satisfactory as had been expected, both in quantity and quality, due to problems which were reflected in analysis of the data, as will be described in the next section. Since the method of analysis is very similar to that applied to the Jul97 data, processes such as energy calibration, data reduction, and details of implementing the kinematic-constraint procedure, will not be repeated.

2.3.2 Data analysis and results
2.3.2.1 Quadruple- versus Triple-coincidences

A major difference of the Nov98 setup compared with that of Jul97 was the avoidance of the 180° angle covered by the PIPS-SSD pairs at 160° as a result of the increase in distance of the detectors from the collector foil (see Fig. 2.29). Consequently, 2-body+/β/random false triples were greatly suppressed. (It should also be noted that, as another consequence of the present geometry, events of type b (see Sec. 2.2.2.3) were not allowed kinematically.) The improvement in background suppression resulted more from this effect than from the requirement of an additional β coincidence. This is demonstrated in the triple-sum energy spectra in Fig. 2.30, obtained from raw triple and quadruple coincidences, in which it can be seen that the requirement of quadruple coincidence only lowered the count rate without providing obvious improvement.
in suppression of background. This result is in contradiction with the speculation that quadruple coincidences would be an essential requirement for the experiment (see Sec. 2.2.2.7); in a geometry in which 2-body back-to-back events do not contribute, triple coincidence seems to be a sufficient requirement. Therefore, information provided by the $\beta$ paddles will be ignored in most of the analyses described below, except in a comparison of the final proton and $\alpha$ spectra in Sec. 2.3.2.4, and the discussion of ToF measurement in Sec. 2.3.2.5. Note that the number of quadruples is about 25% of that of the triples, reflecting the solid angle covered by the scintillator paddles.

**2.3.2.2 Problem with logic signals from a PIPS detector**

As mentioned before, triple-coincidence events were actually obtained with triggers from adjacent detectors (double-coincidence trigger), i.e., SSDL+SSDR, SSDL+PIPL, or SSDR+PIPR\(^\dagger\). The two types of trigger involving the PIPS detectors would seem to be redundant since the geometry dictated that any triple coincidences would involve both SSDs (see Fig. 2.29). The inclusion of these triggers served to accept triple-coincidence events in which one of the signals from the SSDs fell below the discriminator threshold. In fact, the requirement of valid TDC signals associated with all ADC

\footnote{Combinations of 3 detectors which involve both PIPS have the active areas of the detectors lying within one hemisphere. Therefore, events which involved both PIPS were not kinematically favored.}
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Signals in triple-coincidence events, which was implemented in the Jul97 data analysis, is equivalent to requiring triggers from SSDL+SSDR+PIPL or SSDL+SSDR+PIPR. This condition was shown to be very effective in eliminating false triple coincidences (see Sec. 2.2.2.3).

However, the above condition could not be applied to the present data due to an abnormal logic signal associated with one of the PIPS detectors—in the set of raw triple coincidences, the number of valid TDC signals associated with PIPL is only 45% of that with PIPR. A check on the singles and triple-sum energy spectra, and discriminator threshold settings, showed no obvious difference, other than the number of counts, between the two detector combinations. One possible explanation could be a faulty timing filter amplifier (TFA) which delivered intermittent signals to the discriminator (some of the TFAs were found to be faulty during set up of the electronics), or it could have been caused by a bad cable connection. Since this problem was realized only after the setup was dismantled, its cause could not be traced.

A way to circumvent the above problem in the analysis is to neglect logic signals from both of the PIPS detectors. By requiring that the SSDL+SSDR trigger be present in all triple-coincidence events, the problem of inconsistent count rates between the two combinations of detectors due to triggers involving the PIPS detectors is virtually eliminated. Without the TDC signals, coincidences caused by random signals in the PIPS detectors could not be accounted for. However, with a yield of \( \sim 10^5 \text{ }^{17}\text{Ne/s} \), the rate of random coincidences is not expected to be significant (see App. A.8) so that the above method is justified, and will be used in subsequent analysis.

2.3.2.3 Problem with vacuum and/or MgO target

The run was carried out with the new 24"x24"x20" chamber with over 20 covers. The pressure inside the chamber, although at the low \( 10^{-5} \text{ torr} \) scale initially, was found to be at \( 4 \times 10^{-4} \text{ torr} \) at the end of the run. The amount of O-ring surfaces present as a
result of the improper design of the chamber was definitely the cause of this problem. The $^{17}$Ne beam could have partially scattered off the residual gas inside the chamber before it reached the collector foil and, therefore, decreased in intensity. This effect was at least partly accountable for the decrease in trigger rate, which was at a few hundred Hz initially, but dropped gradually during the run to a few tens. The decrease in trigger rate could also be attributable to the gradual degradation of the MgO target used in producing the $^{17}$Ne beam at TISOL. Consequently, the amount of data acquired was much less than had been anticipated.

2.3.2.4 Extraction of the final result

By neglecting signals from the $\beta$ paddles (see Sec. 2.3.2.1), the only difference between the data of Jul97 and the present data is in the geometry of the Si detector array. With straightforward modification of parameters relevant to the geometry of the setup, the kinematic-constraint procedure of Sec. 2.2.2.4 was applied to the present data. The result is shown in Fig. 2.31 in which it is obvious that, as was the case for the Jul97 data, the value of the kinematic function $S$ is less than unity for most of the events with triple-sum energy near the IAS 3-body breakup Q value of 3.43 MeV, and the separation between these events and background is also fairly distinct, particularly for those on the high-energy side of the minimum.

The p-$\alpha$ energy relation of raw triple coincidences is shown in Fig. 2.32(a), in which it can be seen that events outside the Dalitz curve are distributed more randomly than those obtained in the run of Jul97 (see Fig. 2.19). In fact, it has already been shown in the triple-sum energy spectra in Fig. 2.30 that the only prominent peak is the one at 3.4 MeV which sits on a relatively weak background. Evidently, except for the strong $\alpha$ group associated with the $^{17}$F(8.08)$\rightarrow^{13}$N(gs)+$\alpha$ transition (appears as a horizontal and a vertical band at about 1.7 MeV in Fig. 2.32(a)), other 2-body transitions do not seem to contribute significantly to false triples. With the improved angular resolution
compared with that obtained in the Jul97 data, the kinematic constraints should be capable of suppressing most, if not all, of these background events.

After setting an upper limit of unity on $S$, an energy threshold of 500 keV for all particles, and appropriate time-difference gates (for SSD only, see Sec. 2.3.2.2), a $p$-$\alpha$ energy spectrum, as shown in Fig. 2.32(b), was obtained. Note that the $\alpha$ group at 2.3 MeV, corresponding to the $\text{IAS} \rightarrow ^{13}\text{N}(2.37)+\alpha$ transition as seen in the previous two runs, is barely visible in Fig. 2.32(a) and has been suppressed totally in Fig. 2.32(b)—the geometry of the present setup does not favor detection of this transition.

Figure 2.32: Energy relation between protons and $\alpha$ particles: (a) raw triple coincidences (the dashed line is a Dalitz curve (see App. A.5)); (b) after setting a limit of unity on $S$, a proton energy threshold of 500 keV, and appropriate time-difference gates on TDC signals from the SSDs.
The projected triple-coincidence proton and α spectra are shown in Fig. 2.33 along with those obtained from quadruple coincidences. With the avoidance of the 180° angle, which resulted in a very effective suppression of false triples due to 2-body+3/5/3-body fractures, and the decrease in solid angle subtended by each detector element, which provided more stringent kinematic constraints, the spectra in Fig. 2.33 are believed to be the cleanest yet obtained. If the nature of the background were the same for triple and quadruple coincidences, the ratio of the latter to the former would be directly proportional to the solid angle covered by the scintillator paddles. As indicated in Fig. 2.33, this ratio is about 20% which, considering the low number of counts, is reasonably consistent with the solid angle coverage of the scintillator paddles of about 25% of 4π. It could also be explained as an improvement in background suppression with the requirement of quadruple coincidence. Again, with the low statistical accuracy, this may not be considered as conclusive.

Particle groups belonging to the IAS→16O(9.59)+p and IAS→13N(3.50/3.55)+α transitions are apparent in Fig. 2.33. However, without the presence of the IAS→13N(2.37)+α transition, a comparison of the present result with those obtained from the two previous runs will not be straightforward.

Fig. 2.33: Final triple-coincidence spectra of (a) protons and (b) α particles from data of Nov98.
2.3.2.5 ToF measurement

One of the objectives of this run was to test if protons and $\alpha$ particles could be separated by ToF measurements. It should be remembered that the success of ToF measurements in this experiment requires good timing signals provided by $\beta$ particles detected in the scintillator paddles. A ToF$^*$ versus (proton) energy spectrum, obtained with one of the strips in SSDL and one of the scintillator paddles, is shown in Fig. 2.34, in which a few $\alpha$ groups are seen to be barely separable from the much stronger proton groups with shorter ToF. The timing resolution obtained with the SSDs was about 3 ns. It is obvious that this timing resolution is inadequate for a clean separation of protons and $\alpha$ particles at a flight distance of 10 cm. This result shows that the flight path needs to be increased, and the timing signals optimized, for ToF measurement to be useful in this experiment.

It was shown in Sec. 2.3.2.1 that only about 1500 quadruple coincidences were acquired. It should be pointed out that ToF values need to be calculated for each pair of strip+paddle, since the electronic delays, either intrinsic to the electronic modules or

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*The logic signals from the RAL shaping amplifiers were, in fact, generated with leading edge discriminators (LED) [52, p.318]. The timing of these signals is pulse-height dependent. This effect can be corrected in software (*walk* adjustment). The ToF values shown in Fig. 2.34 have not been *walk* corrected.
due to the cables which connect them, are not identical for each electronic channel, and severe degradation of timing resolution would result if signals from all combinations of strip+paddle were analysed simultaneously without correction for the variations in timing. Consequently, the statistical accuracy is much too low (<20 counts for each pair of strip+paddle) for the ToF technique\(^1\) to be applied to the set of quadruple coincidences.

It turned out that ToF measurement was much more useful in this run in generating high-quality proton and \(\alpha\) spectra with the 1500 \(\mu\)m SSB detector located at 20 cm. The small solid angle subtended, i.e., small variation in flight distance, and the much longer flight distance provided by this detector, enabled clean proton and \(\alpha\) spectra to be extracted (see the thesis of Morton [46]).

### 2.3.2.6 Summary

With the decrease in solid angle subtended by the detector elements and the relatively low trigger rates, far fewer triple coincidences were acquired than had been anticipated. The poor statistical accuracy and the instrumental problems encountered during the course of the run make it difficult to draw firm conclusions from the results. Nevertheless, at least the following can be stated:

1. The avoidance of the 180° angle covered by the PIPS-SSD pairs at 160° is essential in the suppression of 2-body+/\(\beta\)/random false triples.

2. The requirement of quadruple coincidence does not seem to improve the suppression of background significantly compared with that of triple coincidence.

3. The flight distance needs to be increased, and timing signals optimized, in order to make the ToF technique useful in triple-coincidence measurements.

\(^1\)This technique requires a sufficient number of events so that particle groups can be formed distinctively and gates set to separate them.
Chapter 3

R-matrix Calculations

The $\beta$-delayed 3-body decay of $^{17}\text{Ne}$ is so complex that an analytic description of the process is very difficult. Monte Carlo methods are especially useful in this situation. In fact, this method (see Sec. 2.1.1) was used to tackle the problem of the 3-body breakup of $^{17}\text{F}^*$ using a Breit-Wigner approach which included all the strong branches but ignored the $^{13}\text{N}$ channel [51]. The simulation was subsequently extended to an R-matrix approach focusing only on the breakup of the IAS in $^{17}\text{F}$—the only state populated by the $\beta$ decay of $^{17}\text{Ne}(gs)$ from which 3-body breakup has been observed experimentally—through both the $^{16}\text{O}$ and $^{13}\text{N}$ channels. However, it has since been realized that, by focusing only on the IAS, the problem is sufficiently simple that analytic calculations are possible. This approach has indeed been used to determine the relative BRs of the three observed 3-body breakup channels from the IAS, as will be described in Chapter 4. By modifying the single-channel multi-level R-matrix formula (see Sec. 3.1.2.2) to account for the differences in angular distributions between the emitted radiations for decays originating from different feeding channels, calculations have also been performed to study the interference between the 9.585 and 7.117 MeV states in $^{16}\text{O}$, and the results have been used to design an optimum detection geometry to constrain the value of $S_{E1}(300)$, as will be described in Chapter 5.

It should be noted that, if the $\beta$-decay branch$^1$ or other 3-body breakup channels are

---

$^1$Because of their light mass, $\beta$ particles are more susceptible to multiple scattering [52] making their trajectories unpredictable. Monte Carlo methods are best suited for simulation of such processes.
included, the problem becomes more complicated and a Monte Carlo approach might be more appropriate. As mentioned above, a Monte Carlo study has been carried out to simulate the 3-body breakup of the IAS. However, the results from this study have been superseded by those from the analytic calculations, which are the subject of this chapter. Nevertheless, the Monte Carlo simulation method is described in App. B in the hope that it might be useful as a basis for extensions into more sophisticated descriptions of the $\beta$-delayed 3-body breakup processes.

### 3.1 The R-matrix Formalism

Only the essential features of the R-matrix theory which are relevant to the present work are described here. A comprehensive treatment of the theory can be found in [35]. A less formidable treatment is presented in [71]. The following discussion is based largely on the above two references.

We consider nuclear reactions of the type

$$a + A \rightarrow B^* \rightarrow C + c, \quad (3.1)$$

in which a projectile nucleus $a$ is scattered by a target nucleus $A$ forming a compound nucleus $B^*$, which subsequently breaks up into two nuclei $C$ and $c$. In an actual scattering experiment, there can be only a limited number of entrance channels ($a+ A$), determined by the properties of the beam and target, but all energetically-allowed exit channels ($C + c$) contribute to the total cross section. Elastic scattering occurs when the entrance and exit channels are identical.

The usefulness of the theory lies in the easy way in which it allows the extraction of nuclear parameters, such as level energies and widths, from experimental data. In this respect, the theory is a practical one. It starts with two basic assumptions: i) the total system is separable into various pairs of nuclei ($C + c$), defined as reaction channels or alternatives; ii) each channel is associated with an interaction (channel)
radius $a_c$. By labeling the intrinsic spins of nuclei $C$ and $c$ as $I_C$ and $I_c$, and their relative orbital angular momentum as $\ell$, each channel is specified by a set of quantum numbers $(\alpha, s, \ell, J, M)$, where $s = I_C + I_c$, $J = s + \ell$ with component $M$, and $\alpha$ contains all the other quantum numbers pertaining to the channel. The significance of the channel radius $a_c$ is in the assumption that the configuration space is divisible into an *internal* ($r < a_c$) and an *external* ($r \geq a_c$) region; nuclear (strong) interactions among the nucleons take place only within the internal region, while the well-known Coulomb interaction exists in the external region.

### 3.1.1 Derivation of the $R$ function

By limiting the applicability of the theory only to low energies ($E_{cm} < \text{a few tens of MeV}$), non-relativistic quantum mechanics is valid. We illustrate the essence of the theory by considering the simple case of elastic (i.e., single channel) scattering of spinless particles. The extension to the multi-channel case involves more of the mathematics than the physics of the theory.

#### 3.1.1.1 The internal wave function

The modified radial wave function$^\dagger$, $u(r) = r R(r)$, in the internal region satisfies the Schrödinger equation

$$\frac{\hbar^2}{2\mu} u''(r) + [E - V(r)] u(r) = 0,$$

where $\mu = \frac{M_C M_c}{M_C + M_c}$ is the reduced mass; $V(r)$ is a central potential which might also include a centrifugal component; and a prime denotes differentiation with respect to $r$.

We now impose a boundary condition on the surface at $r = a_c$:

$$\left. \frac{r X'_\lambda(r)}{X_\lambda(r)} \right|_{a_c} = b_c, \quad \text{or} \quad X'_\lambda(a_c) = \frac{b_c X_\lambda(a_c)}{a_c},$$

---

$^\dagger$The total wave function can be treated as of the form $\Psi = \sum_c \psi_c u_c$, where $u_c$ pertains to the radial part, and $\psi_c$ is the remaining part of the wave function which contains all the other quantum numbers. It is the radial part that contains all the physical content of the theory [71].

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where \( X_\lambda \) is an eigenfunction with eigenenergy \( E_\lambda \), and \( b_c \) is a boundary constant. This condition allows the acquisition of a complete set of eigenstates from which \( u(r) \) is expandable; i.e.,

\[
u(r) = \sum_\lambda A_\lambda X_\lambda(r).
\]

(3.4)

The set of \( X_\lambda(r) \) also satisfies the Schrödinger equation

\[
\frac{\hbar^2}{2\mu} X_\lambda''(r) + [E_\lambda - V(r)]X_\lambda(r) = 0.
\]

(3.5)

By multiplying Eq. (3.2) by \( X_\lambda \) and Eq. (3.5) by \( u \), taking the difference, and integrating over the internal region, we obtain

\[
\frac{\hbar^2}{2\mu} \int_0^{a_c} [X_\lambda u'' - u X_\lambda''] \, dr = (E_\lambda - E) \int_0^{a_c} u X_\lambda \, dr.
\]

(3.6)

Integrating the integral on the left by parts, we have

\[
\frac{\hbar^2}{2\mu} [X_\lambda u' - u X_\lambda']_{a_c} = (E_\lambda - E) \int_0^{a_c} u X_\lambda \, dr.
\]

(3.7)

It is apparent in Eq. (3.7)\(^1\) that the integral on the right is just the projection of \( u \) onto the eigenfunction \( X_\lambda \); i.e.,

\[
\int_0^{a_c} X_\lambda u \, dr = A_\lambda
\]

(3.8)

such that Eq. (3.4) is valid. By substituting Eqs. (3.3) and (3.8) into Eq. (3.7), we get

\[
A_\lambda = \frac{\hbar^2}{2\mu a_c (E_\lambda - E)} \left[ a_c u'(a_c) - b_c u(a_c) \right].
\]

(3.9)

By putting Eq. (3.9) back into (3.4), we obtain an expression for the wave function \( u(r) \) in terms of its value and derivative at the surface \( r = a_c \):

\[
u(r) = G(r, a_c) \left[ a_c u'(a_c) - b_c u(a_c) \right],
\]

(3.10)

\(^1\)To obtain this result, use was made of the condition that the functions \( u(r) \) and \( X_\lambda(r) \) should remain finite at the origin so that their contributions at the origin must vanish (see, e.g., Ref. [72, p.274]).
where \( G(r, a_c) \) is defined as
\[
G(r, a_c) \equiv \frac{\hbar^2}{2\mu a_c} \sum_{\lambda} \frac{X_{\lambda}(r)X_{\lambda}(a_c)}{E_{\lambda} - E}. \tag{3.11}
\]
In the special case that \( G(r, a_c) \) is evaluated at the surface \( r = a_c \), it is called an R function:
\[
R = G(a_c, a_c) = \frac{\hbar^2}{2\mu a_c} \sum_{\lambda} \frac{X_{\lambda}^2(a_c)}{E_{\lambda} - E} \equiv \sum_{\lambda} \frac{\gamma_{\lambda}^2}{E_{\lambda} - E}, \tag{3.12}
\]
where
\[
\gamma_{\lambda}^2 = \frac{\hbar^2}{2\mu a_c} X_{\lambda}^2(a_c) \tag{3.13}
\]
is defined as the reduced width belonging to the eigenstate \( \lambda \). In the more general multi-channel case, the R function extends to an R matrix such that
\[
R_{cc'} = \sum_{\lambda} \frac{\gamma_{\lambda} \gamma_{c'}^{\lambda}}{E_{\lambda} - E}, \tag{3.14}
\]
where \( R_{cc'} \) is an element of the matrix which connects the channels \( c \) and \( c' \).

Using Eq. (3.10), we can obtain an expression for the logarithmic derivative of \( u \) at \( r = a_c \) times the channel radius:
\[
\frac{a_c u'}{u} \bigg|_{a_c} = \frac{1 + b_c R}{R}. \tag{3.15}
\]
This expression will be useful below in matching the wave functions in the two regions. Furthermore, using Eq. (3.10) again,
\[
R = \sum_{\lambda} \frac{\gamma_{\lambda}^2}{E_{\lambda} - E} = \left[ \frac{a_c u'}{u} - b_c \right]^{-1}_{a_c}; \tag{3.16}
\]
i.e., the R function depends on the logarithmic derivative of \( u \) at \( r = a_c \), and the two constants \( a_c \) and \( b_c \). In practice, the level properties \( \gamma_{\lambda} \) and \( E_{\lambda} \) are the parameters of interest in a nuclear reaction. Equation (3.16) shows explicitly the dependence of the extracted parameters on the constants \( a_c \) and \( b_c \), which are arbitrary at this point. The essential point is that the values of the extracted nuclear parameters depend strongly
on the specific values of the channel radius and boundary constant, or sets of such quantities in the multi-channel case. In practice, $a_c$ must be determined from a best fit to experimental data (see, e.g., Ref. [10]). A useful starting value of $a_c$ in a fitting procedure is [40]

$$a_c = r_0 \left( A_c^{1/3} + A_C^{1/3} \right),$$

(3.17)

where $r_0 \approx 1.2$ fm, and $A_c$ and $A_C$ are the mass numbers of the nuclei $c$ and $C$. The boundary constant, however, is not a physical quantity, so quantities such as reaction cross sections and phase shifts should not depend on it. In fact, it has been shown that, for a set of parameters obtained with a particular value of $b_c$, an equivalent set of parameters corresponding to a new boundary constant $b'_c$ can be calculated by a linear transformation [73]. In practice, the choice of $b_c$ is not as arbitrary as it seems, as will be shown after the introduction of the shift function $\Delta_{c\ell}$ in Sec. 3.1.1.3.

3.1.1.2 The external wave function

In the external region, we assume the potential $V$ to arise only from the Coulomb interaction, with a centrifugal contribution due to the relative orbital angular momentum $\ell$; i.e.,

$$V(r) = \frac{Z_cZCe^2}{r} + \frac{\ell(\ell + 1)\hbar^2}{2\mu r^2}. \quad (3.18)$$

It is convenient to write the radial part of the Schrödinger equation in this region as [35, p.269]

$$u''_c(\rho_c) - \left[ \frac{\ell(\ell + 1)}{\rho_c^2} + \frac{2\eta_c}{\rho_c} + 1 \right] u_c(\rho_c) = 0; \quad \rho_c = k_c r, \quad (3.19)$$

where a prime denotes differentiation with respect to $\rho_c$ with wave number $k_c$, the upper (lower) sign is for positive- (negative-) energy channels, and $\eta_c$ is the Sommerfeld parameter given by

$$\eta_c = \frac{Z_cZCe^2}{\hbar v}; \quad v = \frac{\hbar k}{\mu}. \quad (3.20)$$

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For open (positive-energy) channels, the general solution of Eq. (3.19) can be expressed as a linear combination of incoming wave $I_c^+$ and outgoing wave $O_c^+$ [35, p.273]:

$$u_c = I_c^+ - U_{cc} O_c^+,$$

where $U_{cc}$ is a collision function\(^1\), and the positive superscript signifies open channels. The functions $I_c^+$ and $O_c^+$ are customarily expressed in terms of the regular and irregular Coulomb functions $F_c$ and $G_c$ [35, p.269]:

$$I_c^+ = (G_c - iF_c) e^{i\omega_c}; \quad O_c^+ = I_c^{+*},$$

where

$$\omega_c = \sum_{n=1}^{l} \arctan \eta_c / n$$

is the Coulomb phase shift. For closed (negative-energy) channels, the Whittaker function $W$, i.e., an exponentially-decaying Coulomb function, should be used in place of the normal Coulomb functions [35, p.349].

3.1.1.3 Solving the boundary-value problem

It is useful to define the logarithmic derivative of the outgoing wave function in the following way (the prime denotes differentiation with respect to $\rho_c$):

$$L_c = \frac{\rho_c O'_c}{O_c} \bigg|_{a_c} = S_c + iP_c.$$  

The real and imaginary parts of $L_c$ can be written in terms of the Coulomb or Whittaker functions and their derivatives with respect to $r$ [35, p.271]:

$$S_c^+ = \frac{\rho_c (F_c F'_c + G_c G'_c)}{F_c^2 + G_c^2} \bigg|_{a_c};$$  

$$S_c^- = \frac{\rho_c W'_c}{W_c} \bigg|_{a_c};$$

\(^1\)For the multi-channel case, the function extends to a collision matrix of which the element $U_{cc}$ relates the scattering amplitude from entrance channel $c'$ to exit channel $c$. 

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\[ P_c^+ = \frac{\rho_c}{F_c^2 + G_c^2} \] ; and \hspace{1cm} \text{(3.27)}

\[ P_c^- = 0. \] \hspace{1cm} \text{(3.28)}

We are now in a position to express the collision function in Eq. (3.21) in terms of the \( R \) function. This is accomplished by matching the logarithmic derivatives of the internal and external wave functions at the channel surface \( r = r_c \). Using Eq. (3.15),

\[ \left[ \frac{a_c du/dr}{u} \right]_{r_c} = \left[ \frac{\rho_c u'}{u} \right]_{r_c} = \frac{1 + b_c R}{R} = \frac{\rho_c (I_{c}^{+} - U_{cc}^+ \mathcal{O}_c^{+})}{(I_{c}^{+} - U_{cc}^+ \mathcal{O}_c^{+})}, \] \hspace{1cm} \text{(3.29)}

where a prime is understood to be differentiation with respective to \( \rho_c \). Therefore, using Eq. (3.24) to solve for \( U_{cc} \) in Eq. (3.29),

\[ U_{cc} = \Omega_{cc} \cdot \frac{1 - R(L_c^* - b_c)}{1 - R(L_c - b_c)}; \quad \Omega_{cc} = \frac{I_{c}^{+} \mathcal{O}_c^{+}}{I_{c}^{+} \mathcal{O}_c^{+}} \] \hspace{1cm} \text{(3.30)}

in which \( \Omega_{cc} \) is a unit-modulus complex number expressible as [35, p.271]

\[ \Omega_{cc} = e^{2i(\omega_c - \phi_c)}, \] \hspace{1cm} \text{(3.31)}

where \( \omega_c \) is the Coulomb, and \( \phi_c \) the hard sphere, scattering phase shift. Noting that both \( R \) and \( b_c \) are real, the fraction in Eq. (3.30) is also a unit-modulus complex number expressible as

\[ \frac{1 - R(L_c^* - b_c)}{1 - R(L_c - b_c)} = e^{2i\alpha_c}, \] \hspace{1cm} \text{(3.32)}

where, using Eq. (3.24),

\[ \alpha_c = \arctan \left[ \frac{\text{Im}[1 - R(L_c - b_c)]}{\text{Re}[1 - R(L_c - b_c)]} \right] = \arctan \left[ \frac{P_c R}{1 - (S_c - b_c) R} \right]. \] \hspace{1cm} \text{(3.33)}

Therefore, rewriting Eq. (3.30) in terms of Eqs. (3.31) and (3.32), we obtain

\[ U_{cc} = e^{2i\alpha_c} e^{2i(\omega_c - \phi_c)} = e^{2i\delta_c}, \] \hspace{1cm} \text{(3.34)}

where \( \delta_c \) is the total phase shift given by

\[ \delta_c = \arctan \left[ \frac{P_c R}{1 - (S_c - b_c) R} \right] + \omega_c - \phi_c. \] \hspace{1cm} \text{(3.35)}
By assuming that only a single level $\lambda$ dominates the phase shift so that

$$R \approx \frac{\gamma^2}{E_\lambda - E},$$  
(3.36)

and ignoring the Coulomb phase shift $\omega_c$, we obtain

$$\delta_c = \arctan \left[ \frac{P_c \gamma^2}{(E_\lambda - E) - (S_c - b_c) \gamma^2} \right] - \phi_c. $$  
(3.37)

We now define

$$\Gamma_{c\lambda} = 2P_c \gamma^2, $$  
(3.38)

$$\Delta_{c\lambda} = -(S_c - b_c) \gamma^2 $$  
(3.39)

and rewrite Eq. (3.37) as

$$\delta_c = \arctan \left[ \frac{\Gamma_{c\lambda}/2}{E_\lambda - E + \Delta_{c\lambda}} \right] - \phi_c. $$  
(3.40)

The above equation is the one-level approximation to the phase shift, which is useful in fitting elastic scattering phase-shift data such as those in Refs. [8,25]. The reason for defining $\Gamma_{c\lambda}$ and $\Delta_{c\lambda}$ as in Eqs. (3.38) and (3.39) will be apparent below.

The elastic cross section can be written as [35, p.274]

$$\sigma_{cc} = \frac{\pi g_c}{k^2} \left| 1 - U_{cc} \right|^2; \quad k = \frac{\sqrt{2\mu E}}{\hbar}; \quad g_c = 2\ell + 1. $$  
(3.41)

By substituting Eq. (3.30) into (3.41), we obtain

$$\sigma_{cc} = \frac{\pi g_c}{k^2} \left| \frac{(L_c - L_c)R}{1 - (L_c - b_c)R} \right|^2 = \frac{\pi g_c}{k^2} \left| \frac{2iP_cR}{1 - (L_c - b_c)R} \right|^2. $$  
(3.42)

Again, by assuming that only a single level dominates the cross section so that Eq. (3.36) is valid, and using Eq. (3.24), Eq. (3.42) can be rewritten as

$$\sigma_{cc} = \frac{\pi g_c}{k^2} \left| \frac{2iP_c \gamma^2}{(E_\lambda - E) - (S_c - b_c) \gamma^2 - iP_c \gamma^2} \right|^2. $$  
(3.43)

\[1\]In phase-shift analyses, the channels are usually labeled by the partial waves; i.e., $c = \ell$. 

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Using Eqs. (3.38) and (3.39), we obtain

\[ \sigma_{cc} = \frac{\pi g_c}{k^2} \frac{\Gamma_{c\lambda}^2}{(E_\lambda - E + \Delta_{c\lambda})^2 + (\Gamma_{c\lambda}/2)^2}. \] (3.44)

For the case of inelastic scattering in which the entrance and exit channels are different, Eq. (3.44) can be extended as

\[ \sigma_{ce} = \frac{\pi g_{c'}}{k^2} \frac{\Gamma_{c\lambda} \Gamma_{e\lambda}}{(E_\lambda - E + \Delta_{c\lambda})^2 + (\Gamma_{c\lambda}/2)^2}. \] (3.45)

The above equation is the single-level approximation to the cross section, which can be compared with the famous Breit-Wigner formula for a single resonance [74, p.392]:

\[ \sigma_{ab} = \frac{\pi g_a}{k^2} \frac{\Gamma_a \Gamma_b}{(E - E_R)^2 + (\Gamma/2)^2}. \] (3.46)

It is obvious that Eqs. (3.45) and (3.46) are of the same form, except that \( \Gamma_{c\lambda} \) is energy dependent, and the resonance energy is shifted by \( \Delta_{c\lambda} \), in the \( R \)-matrix formula. In view of these connections, \( \Gamma_{c\lambda} \) is called the formal level width, and \( \Delta_{c\lambda} \) the level shift.

The real functions \( S_c \) and \( P_c \) in Eq. (3.24) are called the shift and penetration factors, respectively. It is Eq. (3.45) that has been modified and extended in the calculations relevant to this thesis.

The choice of the boundary constant \( b_c \) has yet to be resolved. The criterion for its choice is that the one-level approximation be as accurate as possible [35, p.232]. The level shift \( \Delta_{c\lambda} \) is dependent on \( b_c \), as is apparent in Eq. (3.39), while the level width \( \Gamma_{c\lambda} \) is not. The common practice is to set \( b_c \) equal to the shift factor \( S_c(E) \) at the level energy \( E_\lambda \), i.e.,

\[ \Delta_{c\lambda}(E_\lambda) = -[S(E_\lambda) - b_c] \gamma_\lambda^2 = 0. \] (3.47)

The shift function \( \Delta_{c\lambda} \) has been shown to vary approximately linearly over a wide range of energies which often includes the width of a resonance [75]. Consequently, it can be expanded in a Taylor series about the resonance energy \( E_\lambda \) and approximated by only the first non-zero term, i.e.,

\[ \Delta_{c\lambda} \approx -(E - E_\lambda) \gamma_\lambda^2 \left. \frac{dS_c}{dE} \right|_{E=E_\lambda}. \] (3.48)
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Using the above approximation, Eq. (3.45) becomes

\[
\sigma_{cc} \approx \frac{\pi g_{cc}^{\text{obs}}}{k^2} \frac{\Gamma_{\alpha\lambda}^{\text{obs}}}{(E_{\lambda} - E)^2 + (\Gamma_{\alpha\lambda}^{\text{obs}}/2)^2},
\]

where

\[
\Gamma_{\alpha\lambda}^{\text{obs}} = \Gamma_{\alpha\lambda}(E_{\lambda}) \left( 1 + \gamma_{\lambda}^2 \frac{dS_{c}}{dE} \right)^{-1}. 
\]

\(E_{\lambda}\) and \(\Gamma_{\alpha\lambda}^{\text{obs}}\) are now the experimentally observed energy and level width for a single resonance.

3.1.2 Adaptation for decay studies

In adapting the single-level cross section formula for our purpose, it is convenient to rewrite Eq. (3.43) in the following form:

\[
\sigma_{cc} = \frac{4\pi g_{cc}}{k^2} P_{c'} \left| \frac{\gamma_{c'} \gamma_{\alpha\lambda}}{E_{\lambda} - E} \right|^2 \left| 1 - (S_{c} - b_{c} + i P_{c}) R_{cc} \right| P_{c}. \tag{3.51}
\]

The above equation can be interpreted as describing a reaction in which the entrance channel is \(c'\) with penetrability (penetration factor) \(P_{c'}\), and the exit channel is \(c\) with penetrability \(P_{c}\). Note that all the quantities in the denominator of Eq. (3.51) belong to the exit channel.

The experimental material in this thesis concerns the 3-body breakup of \(^{17}\text{F}\)\(^*\), which is a two-step decay process (see App. A), rather than a nuclear reaction. The process can be thought of as starting with a decay from the parent state \(^{17}\text{F}\)\(^*\) as an entrance channel, forming a compound nucleus \(^{16}\text{O}\) or \(^{13}\text{N}\), which then decays into \(^{12}\text{C}\) as the exit channel. A subtle point to be stressed is that, contrary to a nuclear reaction, the analogue of elastic scattering (identical entrance and exit channels) does not exist in a two-step cascade decay; i.e., the entrance (feeding) channel to form the compound nucleus is always different from the exit channel. In this respect, the definition of channel should always refer to the decay of the compound nucleus; i.e., even if there is more than one feeding channel, as long as there is only a single decay
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channel for the compound nucleus, the process can be described adequately by a single-channel approximation. However, the formula needs to be modified to incorporate the difference in angular correlation between the emitted radiations for decays originating from different feeding channels, as will be seen in Sec. 3.1.2.2.

3.1.2.1 Single-channel, single-level formula

Assuming that the breakup takes place in a 2-step sequential process (see App. A), that the parent state in $^{17}$F is identified (i.e., the 3-body breakup $Q$ value is known), and that the angle ($\theta'$ in Fig. A.1) between the two emitted radiations ($p$ and $\alpha$) is determined, then the kinematics of the 3-body breakup of $^{17}$F* into a final state of $p+\alpha+^{12}$C is completely determined once the energy of the intermediate state ($^{16}$O or $^{13}$N) is known. In fact, it is the description of the energy spectrum of the intermediate state that involves all the calculations relevant to R-matrix theory. We first assume that only a single level in $^{16}$O contributes to the decay (the $^{13}$N channel can be described in a similar way); i.e., the entrance channel is $^{17}$F*→$^{16}$O*+$p$ and the exit channel is $^{16}$O*→$^{12}$C+$\alpha$. Equation (3.51) can now be modified to describe the $^{16}$O energy spectrum:

$$N_{\lambda}(E) = A^2 P_p \left| \frac{\frac{C_\lambda}{E_\lambda-E}}{1 - (S_\alpha - b_\alpha + iP_\alpha)\frac{\lambda^2}{E_\lambda-E}} \right|^2 P_\alpha, \quad (3.52)$$

where $A^2$ is a normalization constant that is a function of the number of counts in a spectrum, $P_p$ and $P_\alpha$ are the proton and $\alpha$ penetration factors, and $C_\lambda$ is a proton-feeding parameter, which is a function of the proton-decay transition matrix element.

It should be pointed out that Eq. (3.52) cannot be used directly to fit an experimental spectrum, as was done in the $^{16}$N experiment with a similar equation [10, Eq. (4)]$^1$.

$^1$The differences between Eq. (3.52) and its counterpart in [10] are in the replacement of $P_p$ with the integrated Fermi function $f_{\beta}$, and the proton-feeding parameter $C_\lambda$ with the $\beta$-feeding parameter $B_\lambda$. Note also the absorption of the normalization constant in the definition of $B_\lambda$ in Ref. [11] which makes it dependent on the particular set of data from which the parameter is extracted.

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§3.1 The R-matrix Formalism

because the present case involves 3-body kinematics so that the observed particle spectra depend on the geometry of the detection system (see Sec. 3.2). Indeed, the equation has been used in analytic calculations of theoretical spectra which incorporate the geometry of the detection system. These calculated spectra have been used to fit the experimental $\alpha$ spectrum obtained from the Jul97 data to extract the relative branching ratios between the three observed 3-body decay channels from the IAS (see Chapter 4).

3.1.2.2 Single-channel, multi-level formula

If there is only one feeding channel, the extension to the multi-level case is simply to replace the single term in the R function and the single term in the numerator in Eq. (3.52) by summations over the levels; i.e.,

$$N(E) = A^2 P_p \left| \sum_{\lambda} \frac{\gamma_{\lambda}}{E_{\lambda} - E} \right|^2 P_\alpha.$$

Note that the interference among levels comes about naturally in the cross terms in the expansion of Eq. (3.53).

In case there are two feeding channels but the compound nucleus still decays through only a single channel, Eq. (3.53) needs to be modified as follows:

$$N(E, \theta') = A^2 \left| \sum_{c=1,2} \frac{P_{\alpha c}^{1/2} \sum_{\lambda} \frac{C^c_{\lambda}(\theta') \gamma_{\lambda}}{E_{\lambda} - E}}{1 - (S_\alpha - b_\alpha + iP_\alpha) \sum_{\lambda} \frac{\gamma_{\lambda}}{E_{\lambda} - E}} \right|^2 ; \quad C^c_{\lambda} \omega_c(\theta') = C^c_{\lambda c} \mathcal{W}_{\lambda c}(\theta'),$$

where an extra index $c$ is required to label the feeding channels, and $\mathcal{W}_{\lambda c}(\theta')$ is a function (normalized to unity) which depends on the angular distribution (see App. C) between the two emitted particles for a cascade transition originating from a particular feeding channel. Note that the major difference between Eqs. (3.53) and (3.54) is in the modification of the feeding factor $C^2_{\lambda c}$ by the angular distribution function $\mathcal{W}_{\lambda c}(\theta')$. On expanding Eq. (3.54) and integrating over $\theta'$, the squared terms will resemble Eq. (3.53), while the cross term describes the interference between the two
3.2 Calculations of Laboratory Spectra

As mentioned in Sec. 3.1.2.1, Eq. (3.52) cannot be used directly to fit an experimental spectrum, as was done in the $^{16}$N experiment with a similar equation, since the present case involves 3-body kinematics so that the observed particle spectra depend on the geometry of the detection system. A typical detection system for the observation of 3-body breakups is depicted in Fig. 3.1. For simplicity, we assume that all detectors are SSDs similar to those used in the runs of Jul97 and Nov98. The three parameters which completely specify the system are: i) $\angle_{pa}$—the angle between the proton and $\alpha$ detectors, ii) $\angle_{ac}$—the angle between the $\alpha$ and carbon detectors, and iii) $\angle_{1/2}$—the half angle\(^1\) subtended by a detector. An actual detection system might contain numerous combinations of 3 detectors and, in principle, particle spectra can be obtained by

\(^1\)For a circular detector, the half-angle is proportional to the radius; for a square detector, the half-angle is proportional to half the length of an edge.

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Figure 3.1: A TYPICAL SYSTEM of 3 SSDs for measurement of triple coincidences.
Table 3.1: List of essential variables used in the R-matrix calculations of laboratory particle spectra.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Q$</td>
<td>3-body breakup Q value (= 3.431 MeV for decays from the IAS in $^{17}$F)</td>
</tr>
<tr>
<td>$\theta_p$</td>
<td>angle between the center of a pixel in the proton detector and the $z$ axis</td>
</tr>
<tr>
<td>$E$</td>
<td>excitation energy of the intermediate state ($^{16}$O or $^{13}$N)</td>
</tr>
<tr>
<td>$\theta'$</td>
<td>angle between the two emitted radiations $p$ and $\alpha$ (see Fig. A.1)</td>
</tr>
<tr>
<td>$E_p, E_\alpha, E_c$</td>
<td>laboratory particle energies</td>
</tr>
<tr>
<td>$\phi$</td>
<td>azimuthal angle of decay plane</td>
</tr>
<tr>
<td>$N$</td>
<td>contribution to energy spectra based on the R-matrix Eq. (3.52) or (3.54)</td>
</tr>
<tr>
<td>$\Omega_p$</td>
<td>solid angle subtended by a pixel in the proton detector</td>
</tr>
</tbody>
</table>

summing contributions from all the combinations with the same geometry.

The essential variables used in the calculations of laboratory particle spectra for a definite detection geometry, and through a definite decay channel, are listed in Table 3.1. For the case of the $^{16}$O channel\(^1\), we first assume the proton detector to be a point detector on the $+z$ axis, and we place the $\alpha$ and carbon detectors on the $xz$ plane with their active surfaces towards the origin, and with the polar angles of their centers determined by $\angle_{pa}$ and $\angle_{ac}$. We further assume that the decay takes place on the $xz$ plane with the decay vertex located at the origin. With the direction of the proton fixed, the condition for a 3-body breakup to be observed is that the trajectories of both the $\alpha$ particle and carbon ion are intercepted by the respective detectors. As pointed out in Sec. 3.1.2.1, the kinematics is completely determined once $E$, $\theta'$, and the 3-body breakup Q value are determined. Equations relevant to the calculation of laboratory energies and directions of the particles using the above variables are given in App. A. If the calculated directions of the $\alpha$ particle and carbon ion are intercepted by the respective detectors, the energy bins of values $E_p$, $E_\alpha$, and $E_c$ of the respective particle spectra may be incremented by an amount $N$ obtained with Eq. (3.52), (3.53) or (3.54). Full-energy spectra can then be obtained by integrating $E$ over the range

\(^1\)Spectra for decays through the $^{13}$N channel can similarly be obtained by switching the roles of the proton and $\alpha$ particle.
§3.2 Calculations of Laboratory Spectra

determined by the 3-body breakup $Q$ value, $\theta'$ over the range $[0,\pi]$, and $\phi$ over the range subtended by the $\alpha$ and carbon detectors to account for their finite size (i.e., the breakup does not necessarily take place on the $xz$ plane). Note that, since a factor of $\sin \theta'$ should always be included in an integration of a polar angle $\theta'$, $N$ should be multiplied by this factor in incrementing the spectra.

It should be remembered that, up to this point, the proton detector has been treated as a point detector. In order to account for its finite size, the proton detector is divided into $n \times n$ pixels where $n$ is the number of readouts per side of an SSD*. Each pixel in a proton detector can then be treated as a point detector as before. A convenient way to do the calculation is to find the angle $\theta_p$ between the center of the pixel and the $z$ axis, and to rotate the $\alpha$ and carbon detectors accordingly so that the trajectory of the proton is always treated as along the $z$ axis. The total spectrum can then be obtained by summing contributions from all the pixels. Since the detector is planar so that the amount of solid angle $\Omega_p$ subtended by each pixel about the decay vertex varies, contributions from each pixel should be weighted by $\Omega_p$. (A method for calculating the solid angle subtended by a rectangular element can be found in Ref. [76].) It should be mentioned that the finite size of the beam spot has been ignored in this calculation. Results from a Monte Carlo study have shown that the error in triple-coincidence efficiency due to a beam spot of 8 mm diameter for the Jul97 geometry is about 3% (see Sec. 4.1.5.2). This discrepancy is small since the solid angles covered by the detectors is large relative to the size of the beam spot; if the amount of solid angle covered by the detectors were relatively small, one would have to integrate over the beam spot to account for its finite size. However, this integration might not be useful in practice since the distribution within the beam spot might not be determinable.

The algorithm for the above calculation of laboratory particle spectra is listed in

---

* $n$ also reflects the angular resolution of the system, which affects the accuracy of the calculation.
Table 3.2: Algorithm for generation of laboratory particle spectra.

1. place the proton detector on the +z axis, and the α and carbon detectors on the xz plane with their active surfaces towards the origin, and with the polar angles of their centers determined by $\angle_{pa}$ and $\angle_{ac}$ (see Fig. 3.1)
2. divide the proton detector into $n \times n$ pixels
3. loop through all the proton detector pixels in step 2: determine $\theta_p$ and $\Omega_p$
4. rotate the α and carbon detectors by $\theta_p$ so that the pixel in step 3 is centered along the z-axis
5. loop through all possible values of $E$ determined by $Q$ [step size = 1 keV]
6. loop through all possible values of $\theta'$ in the range $[0, \pi]$ [step size = 0.02 ($\approx 1^\circ$)]
7. compute $N$ using Eq. (3.52), (3.53) or (3.54)
8. loop through the range of azimuthal angles $\phi$ subtended by the α and carbon detectors [step size = 0.02 ($\approx 1^\circ$)]
9. compute energies and directions of all particles in the lab frame (relevant equations are given in App. A)
10. determine if both α and $^{12}$C are intercepted by the respective detectors
11. if step 10 is true, increment the energy bins with values of $E_p$, $E_\alpha$, and $E_c$ of the spectra of the respective particles by an amount $N \times \Omega_p \times \sin \theta'$
12. return to step 8 until all steps of $\phi$ are exhausted.
13. return to step 6 until all steps of $\theta'$ are exhausted
14. return to step 5 until all steps of $E$ are exhausted
15. return to step 3 until all of the pixels in the proton detectors are exhausted
16. convolute the spectra with energy resolutions
17. output spectra

*See Table 3.1 for definitions of variables.

Table 3.2. A flow chart of the algorithm is also shown in Fig. 3.2, in which it can be seen that the algorithm contains four iterative loops and, depending on the step sizes and solid angles subtended by the detectors, the calculation can be rather time-consuming. The step sizes indicated in Table 3.2 have been shown to obtain satisfactory results within a reasonable amount of time for the various detection systems described in this thesis. (The step sizes might need to be adjusted for other systems with very different geometry.) This method has been used for calculations of theoretical spectra in Chapters 4 and 5 in the determination of relative BRs between the three observed 3-body breakup channels from the IAS, and in the study of the interference between
the 9.858 MeV ($J^\pi = 1^-$) and 7.117 MeV ($J^\pi = 1^-$) states $^{16}\text{O}$. It should be pointed out that, if a state in $^{17}\text{F}$ other than the IAS is considered, one would have to integrate over the natural width of the state to obtain the appropriate particle spectra.

**Figure 3.2:** FLOW CHART for generation of laboratory particle spectra. See Table 3.1 for definitions of variables.
§3.3 Calculations of Absolute 3-body Decay BR

3.3 Calculations of Absolute 3-body Decay BR

The results of a calculation of the absolute 3-body decay BR using a Breit-Wigner approach [49] in the determination of the best candidate state in $^{17}\text{F}$ to constrain the value of $S_{E2}(300)$ have been shown in Sec. 1.5.4. A similar calculation using the more rigorous R-matrix approach is presented in this section. The objective of this exercise is to calculate the relative strength ($F_{AT}^\dagger$) of the $\beta$-delayed proton decay of specific states in $^{17}\text{F}$ to the 7.117 MeV ($J^\pi = 1^-$) and 6.917 MeV ($J^\pi = 2^+$) subthreshold states in $^{16}\text{O}$ above ($\alpha$ channel) and below ($\gamma$ channel) the $^{12}\text{C}+\alpha$ formation threshold. It should be remembered that the usefulness of the present study hinges on the extension of the high-energy tails of the two subthreshold states above the threshold. The absolute $\beta$-delayed proton-decay BRs ($BR_{\beta p}$) of states in $^{17}\text{F}$ relevant to the present study have been obtained by $p-\gamma$ and $\beta$-p coincidence measurements\(^3\) [46]. The product $F_{AT} \times BR_{\beta p}$ gives the absolute $\beta$-delayed 3-body decay BR ($BR_{3b}$) of that state. Based on the relative values of the $BR_{3b}$'s, the best candidate state in $^{17}\text{F}$ for use to constrain the value of $S_{E2}(300)$ can be identified.

3.3.1 R-matrix formulas

Since each level and channel is considered separately, the single-channel, single-level equation (3.52) can be used. For decays above the threshold, the dominant width is $\Gamma_{\alpha \lambda}$ (i.e., $\Gamma_{\gamma \lambda} \approx 0$) and Eq. (3.52) can be written in the form

$$N_{\alpha \lambda}(E) = \frac{P_p \Gamma_{\alpha \lambda}}{(E_\lambda - E + \Delta_{\lambda \lambda}^+)^2 + (\Gamma_{\alpha \lambda}/2)^2},$$

(3.55)

\(^1\)Fraction of decay above threshold.
\(^\dagger\)Without ambiguity, the 7.117 and 6.917 MeV states will be referred to as the two subthreshold states in this section.
\(^3\)Strictly speaking, the determination of $BR_{\beta p}$ by a $p-\gamma$ coincidence measurement ignores the portion of decay above the threshold. However, since the contribution from the decay of a specific state in $^{17}\text{F}$ to the two subthreshold states in $^{16}\text{O}$ above the threshold is small ($< 10^{-4}$), the $BR_{\beta p}$ so obtained will not be very different from those obtained with a $\beta$-p coincidence measurement.
where the index $\alpha$ signifies the channel, $\lambda$ labels the two subthreshold levels, and the $+$ sign in $\Delta^+_{\alpha\lambda}$ indicates explicitly that the $\alpha$ channel is open so that Eq. (3.25) should be used to calculate the level shift. In obtaining Eq. (3.55), Eqs. (3.38) and (3.39) have been used, and the constant factor $\frac{1}{2}A^2C_\lambda$ has been omitted since only relative values are of interest here. Remembering that $E$ is the excitation energy of the state in $^{16}$O, a quantity $W_{\alpha\lambda}$ can be obtained by integrating over all possible values of $E$ for the contribution from the $\alpha$ channel; i.e.,

$$W_{\alpha\lambda} = \int \frac{P_p \Gamma_{\alpha\lambda}}{(E_\lambda - E + \Delta^+_{\alpha\lambda})^2 + (\Gamma_{\alpha\lambda}/2)^2} dE.$$  \hspace{1cm} (3.56)

Under the assumptions that the dominant width is $\Gamma_{\gamma\lambda}$ (i.e., $\Gamma_{\alpha\lambda} \approx 0$) and the dominant level shift is $\Delta_{\alpha\lambda}$ for the $\gamma$ channel (below the threshold), an expression similar to Eq. (3.56) can be written:

$$W_{\gamma\lambda} = \int \frac{P_p \Gamma_{\gamma\lambda}}{(E_\lambda - E + \Delta^-_{\alpha\lambda})^2 + (\Gamma_{\gamma\lambda}/2)^2} dE,$$  \hspace{1cm} (3.57)

where the $-$ sign in $\Delta^\mp_{\alpha\lambda}$ indicates explicitly that the $\alpha$ channel is closed so that Eq. (3.26) should be used in the calculation of the level shift. In analogy with Eq. (3.38), the energy-dependent formal $\gamma$ width is defined as [30, Eq. (22)]

$$\Gamma_{\gamma\lambda} = 2E^{2\ell+1}_\gamma \gamma^2_{\gamma\lambda} \equiv 2P_p\gamma^2_{\gamma\lambda},$$  \hspace{1cm} (3.58)

in which $\ell$ is the multipolarity of the $\gamma$ ray, and $\gamma^2_{\gamma\lambda}$ is the reduced $\gamma$ width of the level $\lambda$. Note that the $\gamma$ channel is affected by the closed $\alpha$ channel via a non-zero shift factor given by Eq. (3.26). The relative strength $F_{AT}$ between the $\alpha$ and $\gamma$ channels can then be obtained by taking the ratio between Eqs. (3.56) and (3.57).

### 3.3.2 Calculations and results

Values of $F_{AT}$ are calculated only for 3-body breakups going through the two subthreshold states from the IAS, 9.45 MeV and 10.03 MeV states in $^{17}$F, since these states are believed to contribute most strongly to 3-body breakups. The parameters
§3.3 Calculations of Absolute 3-body Decay BR

Table 3.3: List of essential parameters used in the calculations of $F_{AT}$.  

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_p$</td>
<td>channel radius$^a$ for proton decay</td>
<td>4.4 fm</td>
<td>$= r_0(16^{1/3} + 1^{1/3})$</td>
</tr>
<tr>
<td>$a_\alpha$</td>
<td>channel radius for $\alpha$ decay</td>
<td>6.5 fm</td>
<td>Ref. [10]</td>
</tr>
<tr>
<td>$\lambda = 1$: Breakups through 7.117 MeV ($J^\pi = 1^-$) state</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\gamma_{71}$</td>
<td>reduced $\gamma$ width amplitude</td>
<td>$8.76 \times 10^{-6}$ MeV$^{-1}$</td>
<td>Ref. [41]</td>
</tr>
<tr>
<td>$\gamma_{\alpha 1}$</td>
<td>reduced $\alpha$ width amplitude</td>
<td>0.0794 MeV$^{1/2}$</td>
<td>Ref. [10]</td>
</tr>
<tr>
<td>$\ell_{p 1}$</td>
<td>$\ell$ value of proton</td>
<td>0 or 2</td>
<td>—</td>
</tr>
<tr>
<td>$\ell_{\alpha 1}$</td>
<td>$\ell$ value of $\alpha$ particle</td>
<td>1</td>
<td>—</td>
</tr>
<tr>
<td>$\lambda = 2$: Breakups through 6.917 MeV ($J^\pi = 2^+$) state</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\gamma_{72}$</td>
<td>reduced $\gamma$ width amplitude</td>
<td>$1.75 \times 10^{-6}$ MeV$^{-2}$</td>
<td>Ref. [41]</td>
</tr>
<tr>
<td>$\gamma_{\alpha 2}$</td>
<td>reduced $\alpha$ width amplitude$^b$</td>
<td>$0.1123$ MeV$^{1/2}$</td>
<td>$\sqrt{2} \times \gamma_{\alpha 1}$ Ref. [8]</td>
</tr>
<tr>
<td>$\ell_{p 2}$</td>
<td>$\ell$ value of proton</td>
<td>1 or 3</td>
<td>—</td>
</tr>
<tr>
<td>$\ell_{\alpha 2}$</td>
<td>$\ell$ value of $\alpha$ particle</td>
<td>2</td>
<td>—</td>
</tr>
<tr>
<td>$\Gamma_{9.45}$</td>
<td>natural width$^c$ of 9.45 MeV state in $^{17}$F</td>
<td>200 keV</td>
<td>Ref. [41]</td>
</tr>
<tr>
<td>$\Gamma_{10.0}$</td>
<td>natural width of 10.03 MeV state in $^{17}$F</td>
<td>170 keV</td>
<td>Ref. [41]</td>
</tr>
</tbody>
</table>

$^a$ $r_0 = 1.25$ fm is taken.

$^b$ Recently, the ratio of the dimensionless reduced widths $\theta_{6.92}^2/\theta_{7.12}^2 \approx 2$ was obtained in Ref. [8].

$^c$ The width of the IAS is negligible.

used in the calculations are listed in Table 3.3 along with their sources. While $\ell_{\alpha 1}$ and $\ell_{\alpha 2}$ are unique, $\ell_{p 1}$ and $\ell_{p 2}$ can take on two values; since only the relative strength of 3-body decay is important in this exercise, only calculations involving the lowest order of $\ell_{p 1}$ and $\ell_{p 2}$ will be given below. Furthermore, since the 9.45 and 10.03 MeV states have non-negligible natural widths, integrations over their widths are necessary in the calculations involving these states.

The results are shown in Table 3.4 along with the absolute 3-body breakup BRs ($BR_{3b}$) deduced from the most recent results of $\beta$-delayed proton-decay BRs [46]. The present results should be compared with those of Table 1.1, which were obtained with a Breit-Wigner approach using preliminary results of $\beta$-delayed proton-decay BRs [44] and an estimated ratio of the dimensionless reduced widths of $\theta_{6.92}^2/\theta_{7.12}^2 = 3$, while a value of 2 has been used in the present calculations [8]. Both the previ-
Table 3.4: Results of calculations of absolute 3-body decay BRs of the IAS, 10.03, and 9.45 MeV states in $^{17}$F using an R-matrix approach. Only results with the lowest order of proton $\ell$ values are given.

<table>
<thead>
<tr>
<th>$^{17}$F$\to^{16}$O transition</th>
<th>$BR_{\beta p}^a$</th>
<th>$F_{AT}^b$</th>
<th>$BR_{3b}^c$</th>
</tr>
</thead>
<tbody>
<tr>
<td>IAS$\to$7.117</td>
<td>1.14x10$^{-3}$</td>
<td>7.2x10$^{-5}$</td>
<td>8.2x10$^{-8}$</td>
</tr>
<tr>
<td>IAS$\to$6.917</td>
<td>4.2x10$^{-5}$</td>
<td>2.6x10$^{-5}$</td>
<td>1.1x10$^{-9}$</td>
</tr>
<tr>
<td>10.03$\to$7.117</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>10.03$\to$6.917</td>
<td>2.7x10$^{-4}$</td>
<td>1.1x10$^{-6}$</td>
<td>2.8x10$^{-10}$</td>
</tr>
<tr>
<td>9.45$\to$7.117</td>
<td>1.8x10$^{-3}$</td>
<td>3.6x10$^{-7}$</td>
<td>6.5x10$^{-10}$</td>
</tr>
<tr>
<td>9.45$\to$6.917</td>
<td>5.3x10$^{-3}$</td>
<td>5.4x10$^{-8}$</td>
<td>2.9x10$^{-10}$</td>
</tr>
</tbody>
</table>

$^a$ Most recent results of $\beta$-delayed proton BR[46].

$^b$ Obtained by taking the ratio of Eqs. (3.56) and (3.57).

$^c$ Absolute 3-body decay BR ($BR_{\beta p} \times F_{AT}$)

ous and present results indicate that it is hopeless to use the $^{17}$F(IAS)$\to^{16}$O(6.917) transition to constrain the value of $S_{E2}(300)$ since the 3-body decay strength of the $^{17}$F(IAS)$\to^{16}$O(7.117) transition is more than 70 times (50 times based on the previous results) stronger. The previous results also showed that the strength of the $^{17}$F(9.45)$\to^{16}$O(6.917) transition was about 3 times stronger than that of $^{17}$F(9.45)$\to^{16}$O(7.117) while, on the contrary, the present results show that it is only about half as strong. Based on the present results, it does not seem that the 9.45 MeV state is a good candidate for use in constraining the value of $S_{E2}(300)$. However, the present results also show that the strength of the $^{17}$F(10.03)$\to^{16}$O(6.917) transition is comparable to that of $^{17}$F(9.45)$\to^{16}$O(6.917); in fact, they can be taken as equivalent if their uncertainties are considered. Furthermore, as mentioned in Sec. 1.5.4, it has also been found experimentally that the 10.03 MeV state does not populate the 7.117 MeV (1$^-$) state with any appreciable strength [46]. Therefore, based on the present results and the fact that the observation of the $^{17}$F(10.03)$\to^{16}$O(6.917) transition might be free of interference from any $E1$ component, the 10.03 MeV state is identified as the best candidate for use to constrain the value of $S_{E2}(300)$. 

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Chapter 4

Relative Branching Ratio and Proton-Feeding Parameters

In the run of Aug96, the IAS in $^{17}\text{F}$ was first observed to break up into 3 particles via the 9.59 MeV state in $^{16}\text{O}$, and the 2.37 and 3.50/3.55 MeV states in $^{13}\text{N}$. Since the BR of the IAS→$^{13}\text{N}(2.37)+\alpha$ relative to that of the IAS→$^{16}\text{O}(7.12)+\text{p}$ channel had been determined to be $10.5\pm0.8 : 16.7\pm0.5$ in a previous run, similar quantities for the other two channels could be extracted from the triple-coincidence data. However, the margin of error on the relative BR via the IAS→$^{16}\text{O}(9.59)+\text{p}$ channel extracted from the Aug96 data was large for at least three reasons: i) the inability to effectively suppress the false triple coincidences induced by an intense $\beta$ background; ii) the energy distribution of the nuclear states were described simply as Gaussians; and iii) the effect of energy threshold was not treated properly. Triple-coincidence data of better quality would be needed to improve this measurement.

The use of SSDs in the run of Jul97 provided triple-coincidence data of much better quality due to the improvement in angular resolution\footnote{Even better angular resolution was obtained in the run of Nov98 (Sec. 2.3), but the data suffered from poor statistical accuracy and, therefore, were not used in the determination of BRs.} (see Sec. 2.2). The process for the extraction of the final experimental $\alpha$ spectrum from the Jul97 triple-coincidence data has been described in Sec. 2.2.2.5. In addition, instead of treating the spectral shapes simply as Gaussians, an $\alpha$ spectrum from each individual channel was calculated.
§4.1 Determination of Relative BR

using the single-channel, single-level formula derived in Sec. 3.1.2.1. These spectra were then used to fit the experimental spectrum to extract the relative BRs.

In this chapter, the process of extraction of the relative BRs will be described along with error analysis. The calculations of proton-feeding parameters based on the extracted BR of the IAS→\(^{16}\)O(9.59)+p channel relative to that of IAS→\(^{16}\)O(7.12)+p will also be given. The decay channels will henceforth be labeled by the energy of the intermediate state, e.g., IAS→\(^{16}\)O(9.59)+p is referred to as the 9.59 channel or transition.

4.1 Determination of Relative BR

4.1.1 R-matrix calculation and fit expression

The algorithm for calculation of laboratory spectra for the 3-body breakup of the IAS in \(^{17}\)F has been described in Sec. 3.2. The single-channel, single-level R-matrix formula Eq. (3.52) derived in Sec. 3.1.2.1 was again modified (as it was in the calculation of \(F_{AT}\) in Sec. 3.3) and used in step 7 of the algorithm (Table 3.2) to obtain the energy spectrum of the intermediate states for the 3 observed channels; i.e.,

\[
N_\lambda(E) = \frac{P_i \Gamma_f}{(E_\lambda - E + \Delta)^2 + (\Gamma_f/2)^2},
\]

where \(P_i\) is the penetrability of the feeding channel, \(\Gamma_f\) is the width of the exit channel obtained with Eq. (3.38), the constant factor \(\frac{1}{2}A^2C_\lambda\) has been set to unity (since only relative values are of interest here), and \(\lambda\) labels the 3 intermediate states \(^{16}\)O(9.59), \(^{13}\)N(2.37), and \(^{13}\)N(3.50). Essential parameters used in the R-matrix calculations are listed in Table 4.1 along with their sources. The calculations based on the algorithm described in Sec. 3.2 resulted in three sets of proton and \(\alpha\) laboratory particle spectra, as shown in Fig. 4.1, with the area of each spectrum normalized to unity.

\[\text{The 3.55 MeV state (5/2\(^+\)) has been omitted in the calculation. Since the 3.50/3.55 transition is relatively weak, the omission of the 3.55 MeV state is not expected to affect the result significantly.}\]
Table 4.1: List of Parameters used in the R-matrix calculations of spectra for the intermediate states.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda = 1$: IAS$\rightarrow^{16}\text{O}(9.59)+p$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$E_x$</td>
<td>Excitation energy$^a$</td>
<td>9.562 MeV</td>
<td>Ref. [10]</td>
</tr>
<tr>
<td>$a_{p1}$</td>
<td>proton decay channel radius$^b$</td>
<td>4.4 fm</td>
<td>$r_o(16^{1/3} + 1^{1/3})$</td>
</tr>
<tr>
<td>$a_{\alpha 1}$</td>
<td>$\alpha$ decay channel radius$^c$</td>
<td>6.5 fm</td>
<td>Ref. [10]</td>
</tr>
<tr>
<td>$\gamma_{9.59}$</td>
<td>reduced $\alpha$ width amplitude</td>
<td>0.471 MeV$^{1/2}$</td>
<td>Ref. [10]</td>
</tr>
<tr>
<td>$\ell_{p1}$</td>
<td>proton $\ell$ value$^d$</td>
<td>0</td>
<td>—</td>
</tr>
<tr>
<td>$\ell_{\alpha 1}$</td>
<td>$\alpha$-particle $\ell$ value</td>
<td>1</td>
<td>—</td>
</tr>
<tr>
<td>$\lambda = 2$: IAS$\rightarrow^{13}\text{N}(2.37)+\alpha$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$E_x$</td>
<td>Excitation energy</td>
<td>2.369 MeV</td>
<td>Ref. [77]</td>
</tr>
<tr>
<td>$a_{\alpha 2}$</td>
<td>$\alpha$ decay channel radius$^c$</td>
<td>6.5 fm</td>
<td>$r_o(13^{1/3} + 4^{1/3})$</td>
</tr>
<tr>
<td>$a_{p2}$</td>
<td>proton decay channel radius$^b$</td>
<td>4.1 fm</td>
<td>$r_o(12^{1/3} + 1^{1/3})$</td>
</tr>
<tr>
<td>$\theta_{2.37}^2$</td>
<td>dimensionless reduced proton width$^e$</td>
<td>0.54</td>
<td>Ref. [77]</td>
</tr>
<tr>
<td>$\ell_{\alpha 2}$</td>
<td>$\alpha$-particle $\ell$ value</td>
<td>1</td>
<td>—</td>
</tr>
<tr>
<td>$\ell_{p2}$</td>
<td>proton $\ell$ value</td>
<td>0</td>
<td>—</td>
</tr>
<tr>
<td>$\lambda = 3^f$: IAS$\rightarrow^{13}\text{N}(3.50)+\alpha$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$E_x$</td>
<td>Excitation energy</td>
<td>3.499 MeV</td>
<td>Ref. [77]</td>
</tr>
<tr>
<td>$\theta_{3.50}$</td>
<td>dimensionless reduced proton width$^e$</td>
<td>0.031</td>
<td>Ref. [77]</td>
</tr>
<tr>
<td>$\ell_{\alpha 3}$</td>
<td>$\alpha$-particle $\ell$ value</td>
<td>2</td>
<td>—</td>
</tr>
<tr>
<td>$\ell_{p3}$</td>
<td>proton $\ell$ value</td>
<td>1</td>
<td>—</td>
</tr>
</tbody>
</table>

$^a$ Note that this value is different from 9.585 due to the choice of the boundary constant in obtaining the reduced widths such that $\Delta(E_x) = -[S(E_x) - b]\gamma_{9.59}^2 = 0$ [see Eq. (3.47)].

$^b$ $r_o = 1.25$ fm is taken for the proton channels.

$^c$ $r_o = 1.67$ fm is taken for the $\alpha$ channels based on $r_o(16^{1/3} + 1^{1/3}) = 6.5$ fm, the best-fit value obtained in Ref. [10].

$^d$ Only lowest order of $\ell$ is included in the calculation.

$^e$ This value is converted to the reduced width of Eq. (3.13) in the calculation via $\gamma^2 = \theta^2 \times (\frac{3\hbar^2}{\mu a^2})$ [30, Eq. (7)].

$^f$ The channel radii used are the same as for $\lambda = 2$. 

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§4.1 Determination of Relative BR

In principle, each data point $N_{\text{exp}}(i)$ in the experimental $\alpha$ spectrum can be fitted with the expression

$$N_{\text{cal}}(i) = \sum_{j=1}^{3} f_{j} N_{j}(i),$$

(4.2)

where the $N_{j}$ correspond to the ordinates of the $\alpha$ spectra in Fig. 4.1, and the $f_{j}$ are to be determined from the fit. The $\chi^{2}$ of a fit can be obtained in the usual way as

$$\chi^{2} = \sum_{i=1}^{n} \frac{[N_{\text{cal}}(i) - N_{\text{exp}}(i)]^{2}}{N_{\text{exp}}(i)}$$

(4.3)

where $n$ is the number of data points in the spectrum. The CERN package MINUIT was used in the minimization of Eq. (4.3) in order to determine the best values of $f_{j}$, which were in turn corrected for detection efficiencies (see below) to obtain the BRs.
4.1.2 Proton energy threshold and detection efficiency

It is apparent in Fig. 2.19 that the proton spectrum is superimposed upon a substantial $\beta$ background which extends to as high as 500 keV. Such background would give rise to $\beta + \alpha + ^{12}$C triple coincidences and hence falsely enhance the $\alpha$ spectrum. Therefore, a software threshold needs to be set at about the 500 keV level in order to minimize this effect. However, the proton spectrum of the 2.37 transition in Fig. 4.1 shows that the protons have energies which peak at about 400 keV and extend to as low as 200 keV, thus overlapping largely with the $\beta$ background. While a low threshold setting will result in a falsely enhanced $\alpha$ spectrum, too high a threshold will result in a very small contribution from the 2.37 channel and thus render the fit unreliable. Consequently, a compromise needs to be made in setting the threshold.

An additional effect of the proton energy threshold in the determination of the relative BRs arises from the fact that the geometric detection efficiency of triple coincidences via the 2.37 channel is a strong function of this threshold, while that of the other two channels is not, as can be seen in Fig. 4.2. Since the efficiency of the 2.37 channel plays a crucial role in the determination of the relative BRs of the other two 3-body breakup channels (see Sec. 4.1.4), great care must be taken to make sure that

![Graphs showing calculated geometric detection efficiencies of the three 3-body decay channels](image-url)
the appropriate efficiency is used in the calculation.

4.1.3 Proton energy threshold adjustment

Figure 4.3 shows the result of an attempt to fit experimental and calculated \( \alpha \) spectra of the same proton energy threshold of 300 keV. The difference in number of counts between the spectra is about 15%.

Figure 4.3 shows the result of an attempt to fit a calculated \( \alpha \) spectrum, with proton energy threshold of 300 keV, to the experimental spectrum (Fig. 2.22) with the same threshold. It is obvious that, while the region below about 1.8 MeV is quite well fitted, the fit is far from satisfactory for the region between 1.8 and 2.2 MeV. This discrepancy was first attributed to random background in that region. However, it has since been realized that much better fits can be obtained if the experimental proton energy threshold, henceforth to be referred to as \( E_0 \), is adjusted to account for uncertainties in the energy calibration in the low energy region. The threshold in the calculated spectrum will be referred to as \( E'_0 \). In order to determine the required amount of adjustment to \( E_0 \), experimental proton spectra of fixed \( E_0 \) were fitted with calculated spectra for \( E'_0 = E_0 \pm 200 \) keV in steps of 20 keV. Figure 4.4 shows the results of a typical set of fits with \( E_0 = 420 \) keV, in which it is clear that a good fit can be obtained with an adjustment of \( E_0 \) by +100 keV. In fact, this is true for any experimental proton spectrum with \( E_0 > 400 \) keV. Accordingly, it is instructive to see if the same is also true for the \( \alpha \) spectrum. Indeed, Fig. 4.5 shows the result of fitting
4.1 Proton energy threshold and detection efficiency

![Figure 4.4: Reduced $\chi^2$ of fitting proton spectra versus adjustment in proton energy threshold.](image1)

![Figure 4.5: Reduced $\chi^2$ of fitting $\alpha$ spectra versus adjustment in proton energy threshold.](image2)

Calculated spectra of varying $E'_0$ to the experimental $\alpha$ spectrum with $E_0 = 420$ keV and, evidently, the adjustment of $E_0$ by $+100$ keV is also valid for the $\alpha$ spectrum.

### 4.1.4 Results

In view of the above observation, experimental $\alpha$ spectra with $E_0$ from 250 to 600 keV have been fitted with calculated spectra with $E'_0 = E_0 + 100$ keV. The result is shown in Fig. 4.6 in which Fig. 4.6(a) suggests that $E_0 = 400$ keV should provide the best fit between experimental and calculated spectra. Note that, although $\chi^2_\nu$ is even smaller for $E_0 > 525$ keV, the fact that the contribution from the 2.37 channel has been greatly suppressed at such a value (see proton spectra in Fig. 4.1) excludes the validity of these fits in the determination of the relative BRs. It is also apparent in Fig. 4.6(b) that the calculated relative BR between the 9.59 and 2.37 transitions decreases (almost linearly) with increasing $E_0$ in the range of 450 to 600 keV. This can be understood as an artifact of the fits where the contribution from the 2.37 channel is supposed to be small (see proton spectra of Fig. 4.1) but the minimization routine forces $f_2$ to values higher than they should be in order to better fit the spectra. Figure 4.7 shows the result of fitting the calculated spectrum, obtained with $E'_0 = 500$ keV, to the experimental...
§4.1 Proton energy threshold and detection efficiency

Figure 4.6: Results of fitting experimental α spectra with proton energy threshold of $E_0$ to calculated spectra with $E_0' = E_0 + 100$ keV: (a) reduced $\chi^2$ versus $E_0$, (b) $f_1/f_2$, the relative BR between the 9.59 and 2.37 transitions (corrected for efficiency), versus $E_0$.

spectrum with $E_0 = 400$ keV. A linear background has also been assumed in the fit but its contribution is very small as is obvious in Fig. 4.7. This fit will be referred to as the best fit.

We will henceforth refer to the relative BR of the 9.59, 2.37, and 3.50 transitions as $R_1$, $R_2$, and $R_3$, respectively. It should be remembered that $R_2$ has already been determined from the ratio 10.5/16.7 obtained in a previous run. $R_1$ and $R_3$ can now be calculated from

$$R_j = \frac{f_j/\eta_j}{f_2/\eta_2} \cdot \frac{10.5}{16.7} \cdot 100 ; \quad j = 1 \text{ or } 3,$$

where $f_j$ are the fitted parameters in Eq. (4.2), $\eta_j$ are the relative efficiencies, and the factor of 100 is included so that the BR of the 7.12 transition has a value of 100. The result is $R_1 : R_2 : R_3 = 13.3(9) : 62.8(7) : 1.5(2)$, as given in Fig. 4.7. Note that these values have been obtained using geometric efficiencies for the individual channels calculated with $E_0' = 500$ keV.

As given in Fig. 4.7, the value of 3.87 for $\chi^2_\nu$, the reduced $\chi^2$ defined as $\chi^2/\nu^\dagger$, where $\nu$ is the degree of freedom of the fit given by the number of data points minus the number of parameters obtained from the fit.
Figure 4.7: Result of fitting the calculated spectra with $E_0' = 500$ keV to the experimental spectrum with $E_0 = 400$ keV. The circles represent the experimental spectrum; the solid line is the calculated total spectrum and the dashed lines are calculated contributions from the individual channels, plus a linear background.

is higher than the statistically expected value of unity, which seems to indicate that the single-level, single-channel R-matrix approximation is not adequate to describe the experimental data. Indeed, a many-level calculation will be needed to account for interferences among nearby states; e.g., the 7.12 and 9.59 transitions are known to interfere with each other (see Sec. 4.2). However, the distortion in spectral shapes due to the calculation is small compared to those due to instrumental effects (see Sec. 4.1.5). Furthermore, since only relative values are extracted from the data, the results might be less sensitive to small distortions in the spectral shapes. Judging from the moderate quality of the data, which is reflected by the relatively large systematic error as will be seen in Sec. 4.1.5, the value of $\chi^2$ is not unreasonable for the fit.

4.1.5 Error analysis

It should first be noted that the assignment of an error to the fitted relative BRs obtained in the previous section, and to any fitted parameters in general, is a very subjective matter. It is apparent in Fig. 4.6(a) that, since the derivative of $\chi^2$ at the vicinity of the local minimum at $E_0 = 400$ keV is relatively large, the statistical error in

\[ \chi^2 = 3.87 \]

\[ E_0 = 400 \text{ keV}; E_0' = 500 \text{ keV} \]

\[ \text{relative BR of } 9.59 : 2.37 : 3.50/3.55 \]

\[ 13.39 : 62.87 : 1.52 \]

\[ \text{if parameters such as reduced particle widths are to be extracted from the data, a correct description of spectral shapes will be essential.} \]
§4.1 Proton energy threshold and detection efficiency

the relative BRs due to the fit is expected to be relatively small. However, an estimate of
the magnitude of the systematic error is not as obvious due to the various sources
of error involved. Nevertheless, certain instrumental effects are known to have affected
the quality of the data considerably so that the systematic errors are not expected to
be small. The two types of error will be investigated separately.

4.1.5.1 Statistical errors

Two sources contribute to the statistical error: (1) the uncertainty in the relative BR
between the 2.37 and 7.12 transitions, i.e., $R_2/100$; and (2) the statistical uncertainty
in fitting the spectra. The calculation of the first source of error is straightforward.
Since $R_2$ is determined by the ratio $10.5\pm0.8 : 16.7\pm0.5$, its uncertainty due to this
ratio is calculated with the usual method of error propagation. The relative error is

$$\delta_{R_2} = \sqrt{\left(\frac{0.8^2}{10.5^2} + \frac{0.5^2}{16.7^2}\right)} = 0.08(2). \tag{4.5}$$

Since $R_1$ and $R_3$ have been calculated [Eq. (4.4)] based on the ratio 10.5/16.7, their
errors due to this ratio are

$$\sigma_{R_j}^{(1)} = R_j \delta_{R_2}; \quad j = 1 \text{ or } 3. \tag{4.6}$$

Thus, $\sigma_{R_1}^{(1)} = 1.1(0)$ and $\sigma_{R_3}^{(1)} = 0.1(2)$.

The error due to the fitting procedure can be obtained from the covariance matrix
(see, e.g. Ref. [78], p.121-3) calculated by MINUIT [68] at the $1\sigma$ level. Cautions have
been taken, by using different minimization algorithms provided by MINUIT and by
varying the initial values, to make sure that the absolute minimum has been found.
Based on results of the best fit, we obtain $f_1 = 15817 \pm 1331$, $f_2 = 14685 \pm 1274$, and
$f_3 = 792 \pm 351$. Therefore, the relative errors in the relative BRs due to the fit are

$$\delta_{f_j/f_2} = \sqrt{\left(\frac{\sigma_{f_j}^2}{f_j^2} + \frac{\sigma_{f_2}^2}{f_2^2}\right)}; \quad j = 1 \text{ or } 3. \tag{4.7}$$

\footnote{We denote relative errors by $\delta$ and actual errors by $\sigma$.}
Hence, $\delta_{f_1/f_2} = 0.121$ and $\delta_{f_3/f_2} = 0.452$. The large error in $f_3/f_2$ can be attributed to the small magnitude of $f_3$ relative to $f_1$ and $f_2$; a large variation in $f_3$ does not affect the value of $f_1$ and $f_2$ significantly during the minimization process. Thus, the errors due to the fit are $\sigma_{R_1}^{(2)} = 1.6(2)$ and $\sigma_{R_3}^{(2)} = 0.6(9)$.

Since the two sources of error are independent, they can be added in quadrature resulting in total statistical errors of $\sigma_{R_1}^{\text{stat}} = 1.9(6)$ and $\sigma_{R_3}^{\text{stat}} = 0.7(0)$.

### 4.1.5.2 Systematic errors

At least 3 sources contribute to the systematic error: (1) uncertainties due to the kinematic procedure in extracting the final experimental spectrum; (2) uncertainty in proton energy threshold; and (3) uncertainties in geometric detection efficiency due to uncertainties in locations of detectors and the finite size of the beam spot. The error due to (1) was estimated by fitting the calculated spectra, with $E_0' = 500$ keV, to experimental spectra with $E_0 = 400$ keV but with the upper limit on the kinematic parameter varied by 20% of $S = 1$, i.e., from 0.8 to 1.2. Based on these fits, $R_1$ and $R_3$ were then calculated and compared with the values obtained with $S \leq 1$. The results show a variation of 0.65% in $R_1$ and 1.45% in $R_3$. Accordingly, we quote $\sigma_{R_1}^{(1)} = 0.08(7)$ and $\sigma_{R_3}^{(1)} = 0.02(2)$. These results show that the relative BRs are not very sensitive to the upper limit of the kinematic parameter.

In order to estimate the errors due to the uncertainty in proton energy threshold, the experimental spectrum at $E_0 = 400$ keV was fitted with calculated spectra with $E_0' = 500 \pm 10$ keV, corresponding to a spread in $\chi^2$ of about 20%. Based on these fits, $R_1$ and $R_3$ were again calculated and compared with the values obtained with the best fit. The results show that $R_1$ varies by 16.2% and $R_3$ by 12.3%. Therefore, we quote $\sigma_{R_1}^{(2)} = 2.1(7)$ and $\sigma_{R_3}^{(2)} = 0.1(9)$. Indeed, the errors in the relative BRs are rather sensitive to the proton energy threshold.

The uncertainties in geometric detection efficiency are due to 2 factors: a) the
uncertainties in the locations of the detector elements, which are about 2°; and b) the effect of the finite size of the beam spot. In order to estimate the error due to a), detection efficiencies were calculated with the detectors moved by 2° in turns. This new set of efficiencies was then compared with that obtained with the detectors at the original positions and the largest deviations were taken as errors. This calculation results in errors in detection efficiency of 2.5%, 15.9%, and 12.3% for the 9.59, 2.37, and 3.50 channels, respectively. It is apparent that the efficiency of the 9.59 channel is much less sensitive to the locations of the detectors than is the efficiency of the other two channels. This can be attributed to the fact that the system has been designed so that the angle between the proton and α detectors is 160°, which is the most probable (lab) angle between the proton and α particle for decays from the 9.59 channel, whereas the corresponding angle is 167° for the 2.37 and 148° for the 3.50 channel (see App. B, Fig. B.2).

From an inspection of the collector foil at the end of the experiment, the beam spot was determined as roughly circular with a diameter of about 8 mm. However, an analytical calculation of the error in efficiency due to a finite beam spot is difficult for two reasons: i) the actual distribution of the ions within the beam spot is not known; and ii) the calculation involves transformation and rotation of coordinate systems which are tedious to handle analytically. Instead, a Monte Carlo simulation was performed to study this effect assuming the ions were distributed normally within the beam spot. The results from the Monte Carlo study show that, compared to results obtained with a point-like beam spot, the efficiency is altered by 2.6%, 1.7%, and 3.1%, respectively, for the 9.59, 2.37, and 3.50 channels. These errors are then added in quadrature to those obtained from the consideration of the uncertainties in the locations of the detectors. The results are 3.6%, 16.0%, and 12.7% for the 3 channels, respectively. With these uncertainties in detection efficiency, we first calculate the relative error in $\eta_2/\eta_j$, which
Table 4.2: List of errors from individual sources in the determination of relative BRs.

<table>
<thead>
<tr>
<th>Type</th>
<th>( R_1 = 13.3(9) )</th>
<th>( R_2 = 62.8(7) )</th>
<th>( R_3 = 1.5(2) )</th>
<th>Source of error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Statistical</td>
<td>( \sigma_{R_1}^{(1)} )</td>
<td>( \sigma_{R_1}^{(2)} )</td>
<td>( 5.1(6) )</td>
<td>10.5/16.7 ratio</td>
</tr>
<tr>
<td></td>
<td>1.1(0)</td>
<td>1.6(2)</td>
<td>0.1(2)</td>
<td>( \chi^2 ) fit</td>
</tr>
<tr>
<td>Total:</td>
<td>( \sigma_{R_1}^{syst} )</td>
<td></td>
<td>5.1(6)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>1.9(6)</td>
<td></td>
<td>0.7(0)</td>
<td></td>
</tr>
<tr>
<td>Systematic</td>
<td>( \sigma_{R_2}^{(1)} )</td>
<td>( \sigma_{R_2}^{(2)} )</td>
<td>( \sigma_{R_3}^{(3)} )</td>
<td>0.08(7)</td>
</tr>
<tr>
<td></td>
<td>0.08(7)</td>
<td></td>
<td>( - )</td>
<td>Kinematic procedure</td>
</tr>
<tr>
<td></td>
<td>2.1(7)</td>
<td></td>
<td>( - )</td>
<td>Proton energy threshold</td>
</tr>
<tr>
<td></td>
<td>2.2(0)</td>
<td></td>
<td>0.1(9)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( \sigma_{R_3}^{syst} )</td>
<td></td>
<td>( - )</td>
<td>Detection efficiency</td>
</tr>
<tr>
<td>Total:</td>
<td>( \sigma_{R_3}^{syst} )</td>
<td></td>
<td>( - )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>3.0(9)</td>
<td></td>
<td>0.3(6)</td>
<td></td>
</tr>
</tbody>
</table>

is the factor, relevant to efficiencies, used in Eq. (4.4) to calculate the relative BRs:

\[
\delta_{\eta_j} = \sqrt{\left( \frac{\sigma_{\eta_1}^2}{\eta_1^2} + \frac{\sigma_{\eta_2}^2}{\eta_2^2} \right)} ; \quad j = 1 \text{ or } 3 .
\]  

Hence, \( \delta_{\eta_1} / \eta_1 = 0.164 \) and \( \delta_{\eta_2} / \eta_2 = 0.204 \), so that \( \sigma_{R_1}^{(3)} = 2.2(0) \) and \( \sigma_{R_2}^{(3)} = 0.3(1) \).

Since the 3 sources of systematic error are independent of one another, they can be added in quadrature to obtained the final total systematic errors of \( \sigma_{R_1}^{syst} = 3.0(9) \) and \( \sigma_{R_3}^{syst} = 0.3(6) \).

### 4.1.5.3 Final results

The contribution from each source of error, both statistical and systematic, are summarized in Table 4.2. We quote the final results as \( R_1 = 13.4 \pm 2.0 \pm 3.1 \), \( R_2 = 62.9 \pm 5.2 \), and \( R_3 = 1.5 \pm 0.7 \pm 0.4 \), relative to a value of 100 for the IAS\( \rightarrow ^{16}\text{O}(7.12) + p \) transition. Note that, insofar as this determination of relative BRs is concerned, the systematic error does not apply to \( R_2 \).

### 4.2 Relative Proton-feeding Parameters

In a single-channel, single-level consideration of the IAS\( \rightarrow ^{16}\text{O}(9.59) + p \) transition, the \( ^{16}\text{O} \) energy spectrum can be described by Eq. (3.52) or, more specifically,

\[
N_\lambda(E) = \frac{P_p C_{\lambda \ell}^2 \Gamma_{\alpha \lambda}}{(E_\lambda - E + \Delta_{\alpha \lambda})^2 + (\Gamma_{\alpha \lambda}/2)^2},
\]  

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where \( C_{2}^2 \) is the proton-feeding parameter, and the normalization factor \( A^2 \) is set to unity since only relative values are of interest here. The subscript \( \ell \) in \( C_{\ell} \) is necessary in this case since there are two possible values of \( \ell \) (0 or 2) for the incoming channel \([J^{\pi} = \frac{1}{2}^{-}\) for the \( ^{17}\text{F}\)(IAS) and \( J^{\pi} = 1^{-}\) for the \( ^{16}\text{O}(9.59) \) state]. If the 7.12 MeV state \((J^{\pi} = 1^{-}) \) in \( ^{16}\text{O} \) is also considered, the single-channel, two-level formula [Eq. (3.54)] should be used, or more specifically (again, with \( A^2 = 1 \)),

\[
N(E, \theta') = \sum_{\ell=0,2} \frac{P_{\ell}^2 \sum_{\lambda} C_{\ell}^{(\theta')\gamma_{\lambda}} P_{\ell}^{1/2} \frac{P_{\ell}^{1/2}}{E_{\lambda} - E}}{1 - (S_{\alpha} - b_{\alpha} + iP_{\alpha}) \sum_{\lambda} \frac{\gamma_{\lambda} \gamma^{(\ell)}}{E_{\lambda} - E}}^2 \cdot C_{\ell}^{2(\theta')} W_{\ell}(\theta'), \tag{4.10}
\]

where the feeding channels are labeled by the \( \ell \) values of the proton. In reality, the proton-feeding amplitudes \( C_{\ell} \) should be quantities to be determined simultaneously by fitting a theoretical (calculated) spectrum to an experimental spectrum. However, angular correlation information given by a set of p-\( \gamma \) coincidence data [46] has shown that the IAS\( \rightarrow ^{16}\text{O}(7.12)\)+p transition has almost pure \( \ell = 2 \), while the set of triple-coincidence data of Jul97 indicates that \( \ell = 0 \) is more probable for the IAS\( \rightarrow ^{16}\text{O}(9.59)\)+p transition as shown by the results in Sec. 4.1 where a good fit has been obtained with the assumption of \( \ell = 0 \) (see Table 4.1), the lowest order orbital angular momentum. The 7.12 (9.59) MeV state will henceforth be labeled as \( \lambda = 1 \) (2), in consistence with Ref. [10]. Accordingly, we assume \( C_{10} \) and \( C_{22} \) to be vanishingly small. We further assume that the interference between the two levels is relatively small; the relative proton-feeding amplitudes, \( C_{12} \) and \( C_{20} \), can hence be deduced separately using a single-level, single-channel approximation. For the IAS\( \rightarrow ^{16}\text{O}(7.12)\)+p transition, since the 7.12 MeV state is below the \( \alpha + ^{12}\text{C} \) formation threshold, \( \gamma \) decay dominates. Therefore, the \( ^{16}\text{O} \) energy spectrum can be described by Eq. (3.57), which has been used in Sec. 3.3 in the calculation of the strength of the \( \gamma \) channel in the determination of \( F_{\gamma} \).
Table 4.3: Results of relative proton-feeding amplitudes based on relative BRs obtained in Sec. 4.1 and p-γ coincidence measurements from Ref. [46]. The first index in C refers to levels while the second one refers to proton \( \ell \) values. \( C_{10} \) and \( C_{22} \) are assumed to be vanishingly small.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>source of error</th>
</tr>
</thead>
<tbody>
<tr>
<td>( C_{10} )</td>
<td>( \approx 0 )</td>
<td>–</td>
</tr>
<tr>
<td>( C_{12} )</td>
<td>( 8.7 \pm 0.3 )</td>
<td>IAS( \rightarrow ) 16O(7.12)+p BR</td>
</tr>
<tr>
<td>( C_{20} )</td>
<td>( 5.8 \pm 1.6 )</td>
<td>( R_1 ) in Sec. 4.1</td>
</tr>
<tr>
<td>( C_{22} )</td>
<td>( \approx 0 )</td>
<td>–</td>
</tr>
</tbody>
</table>

Based on Eq. (3.57) and assuming a spectrum normalized to a total of 100 counts\(^\dag\), we have

\[
C_{12}^2 = 100 \left( \int \frac{P_2 \Gamma_1}{(E_1 - E + \Delta \alpha_1)^2 + (\Gamma_1/2)^2} dE \right)^{-1} \tag{4.11}
\]

In principle, the high energy tail of the 7.12 MeV state above the formation threshold, where \( \alpha \) decay dominates, should also be included in the calculation, but its contribution is too small, as was seen in the calculation of \( F_{AX} \), to be of significance here.

In the case of the IAS\( \rightarrow \) 16O(9.59)+p transition, since the 9.59 MeV state is well above the formation threshold, \( \alpha \) decay dominates, so that Eq. (3.56) should properly describe the process. Therefore, using the value of \( R_1 \) obtained in Sec. 4.1, we obtain

\[
C_{20}^2 = R_1 \left( \int \frac{P_0 \Gamma_{\alpha 2}}{(E_2 - E + \Delta \alpha_2)^2 + (\Gamma_{\alpha 2}/2)^2} dE \right)^{-1} \tag{4.12}
\]

In this case, the \( \gamma \) width is assumed to be negligible. The final results of the above calculations are listed in Table 4.3.

### 4.3 Summary

A redetermination of the relative BR of the IAS\( \rightarrow \) 16O(9.59)+p 3-body breakup channel (first determined with the Aug96 data), as well as a first determination of that of IAS\( \rightarrow \) 13N(3.50)+\( \alpha \), has been carried out using the Jul97 triple-coincidence data in conjunction with single-channel, single-level R-matrix calculations in the description of the

\(^\dag\)This is the normalization used in obtaining the relative BRs in Sec. 4.1.4 in p. 107.
3-body breakup mechanism. It was found that the best fit between experimental and calculated spectra was obtained by adjusting the experimental proton energy threshold by $\pm 100$ keV to account for the uncertainty in energy calibration in the low energy region. A thorough error analysis has also been performed. The BR is determined to be $13.4 \pm 2.0 \pm 3.1$ for the IAS$\rightarrow^{16}\text{O}(9.59)+\text{p}$ channel, and $1.5 \pm 0.7 \pm 0.4$ for the IAS$\rightarrow^{13}\text{N}(3.50)+\alpha$ channel, relative to a BR of 100 for the IAS$\rightarrow^{16}\text{O}(7.12)+\text{p}$ channel. Relative proton-feeding amplitudes between the IAS$\rightarrow^{16}\text{O}(9.59)+\text{p}$ and IAS$\rightarrow^{16}\text{O}(7.12)+\text{p}$ channels have also been calculated resulting in $C_{12} : C_{20} = 8.7 \pm 0.3 : 5.8 \pm 1.6$. These results will be used in Chapter 5 in a single-channel, two-level R-matrix calculation to study the interference between these two channels in the 3-body breakup of the IAS.
Chapter 5

Interference Between the 7.117 and 9.585 MeV States in $^{16}$O

As mentioned in Sec. 1.1, the reduced $\alpha$ width of the 7.117 MeV ($1^-$) state in $^{16}$O has been measured in a study of the $\beta$-delayed $\alpha$ decay of $^{16}$N with an error of 27% [10,11]. It was also pointed out that no information about the $E2$ component was obtained in that experiment since the $\beta$ decay of $^{16}$N does not populate the 6.917 MeV ($2^+$) state. However, it has since been realized that, by using the $\beta$-delayed proton decay of $^{17}$Ne, an experiment similar in principle to the case of $^{16}$N might provide information to constrain the $E2$ component, as well as the $E1$, of the $^{12}$C($\alpha, \gamma$)$^{16}$O reaction [12,13]. A partial decay scheme of the IAS in $^{17}$F, along with that of $^{16}$N(gs), relevant to measurements in constraining the $E1$ component is shown in Fig. 5.1, in which it can be seen that, instead of being populated by a $\beta$ decay in the case of $^{16}$N, the $^{16}$O states are populated by a proton decay in the case of $^{17}$Ne. The difference in the feeding channel makes the case of $^{17}$Ne more complicated, and the experiment much more challenging, due to the involvement of 3-body kinematics (see App. A).

The determination of the proton-feeding parameter of the IAS$\rightarrow$$^{16}$O(9.585)+p channel relative to that of IAS$\rightarrow$$^{16}$O(7.117)+p was described in Chapter 4, where it was also noted that the interference between these two channels could be studied by means of a single-channel, two-level $R$-matrix calculation. The results from this calculation would provide information for the design of an optimum experimental setup, and an
estimation of the required count rates, for a measurement of the reduced \( \alpha \) width of the 7.117 MeV state in \( ^{16}O \). It is hoped that a measurement of the same parameter using the decay of \( ^{17}Ne \), being complimentary to the result obtained in the case of \( ^{16}N \), will help in reducing the error of \( S_{E1}(300) \) to a level of about 20%.

### 5.1 R-matrix calculation

The algorithm for calculation of laboratory particle spectra has been described in Sec. 3.2. Since both the 7.117 and 9.585 MeV states in \( ^{16}O \) are involved, the single-channel, 2-level formula derived in Sec. 3.1.2.2 [Eq. (3.54)] should be used in step 7 of the algorithm in Table 3.2 to calculate the energy spectrum of the \( ^{16}O \) state. Moreover, the proton-feeding amplitudes \( C_{\lambda \ell} \) obtained in Sec. 4.2 and the angular correlation functions \( \mathcal{W}_{\lambda \ell}(\theta') \) obtained in App. C should be used, and with the normalization constant \( A^2 \) omitted; i.e.,

\[
N(E, \theta') = \left| \sum_{\ell=0,2} \frac{P_\ell^{1/2} \sum_\lambda \frac{C_{\lambda \ell}(\theta') \gamma_\lambda}{E_\lambda - E} P_\alpha^{1/2} P_{\alpha}^{1/2}}{1 - (S_\alpha - b_\alpha + iP_\alpha) \sum_\lambda \frac{\gamma_\lambda^2}{E_\lambda - E}} \right|^2 \; ; \; \; C_{\lambda \ell}^2(\theta') = \mathcal{W}_{\lambda \ell}(\theta') C_{\lambda \ell}^2 . \tag{5.1}
\]
§5.1 Single-channel, 2-level R-matrix Calculation

Table 5.1: List of parameters used in the single-channel, 2-level R-matrix calculations of energy distribution of the intermediate state including both the 7.117 ($\lambda=1$) and 9.585 ($\lambda=2$) states.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_{p1}$</td>
<td>proton decay channel radius$^a$</td>
<td>$4.4 \text{ fm}$</td>
<td>$r_o(A_1^{1/3}+A_2^{1/3})$</td>
</tr>
<tr>
<td>$a_{\alpha 1}$</td>
<td>$\alpha$ decay channel radius$^b$</td>
<td>$6.5 \text{ fm}$</td>
<td>Ref. [10]</td>
</tr>
<tr>
<td>$\ell_\alpha$</td>
<td>$\alpha$-particle $\ell$ value for both channels</td>
<td>$1$</td>
<td>—</td>
</tr>
<tr>
<td>$E_1$</td>
<td>Excitation energy</td>
<td>$7.117 \text{ MeV}$</td>
<td>Ref. [10]</td>
</tr>
<tr>
<td>$\gamma_1$</td>
<td>reduced $\alpha$ width amplitude</td>
<td>$0.0794 \text{ MeV}^{1/2}$</td>
<td>Ref. [10]</td>
</tr>
<tr>
<td>$\ell_{p2}$</td>
<td>proton $\ell$ value</td>
<td>$2$</td>
<td>Ref. [46]</td>
</tr>
<tr>
<td>$C_{12}$</td>
<td>proton-feeding parameter</td>
<td>$0.31$</td>
<td>Sec. 4.2</td>
</tr>
<tr>
<td>$E_2$</td>
<td>Excitation energy$^c$</td>
<td>$10.006 \text{ MeV}$</td>
<td>Ref. [10]</td>
</tr>
<tr>
<td>$\gamma_2$</td>
<td>reduced $\alpha$ width amplitude$^d$</td>
<td>$0.329 \text{ MeV}^{1/2}$</td>
<td>Ref. [10]</td>
</tr>
<tr>
<td>$\ell_{p0}$</td>
<td>proton $\ell$ value</td>
<td>$0$</td>
<td>Sec. 2.2</td>
</tr>
<tr>
<td>$C_{20}$</td>
<td>proton-feeding parameter</td>
<td>$0.20$</td>
<td>Sec. 4.2</td>
</tr>
</tbody>
</table>

$^a$ $r_o = 1.25 \text{ fm}$ is taken for the proton channels.
$^b$ Value obtained with best fit in Ref. [10].
$^c$ Note that $E_2$ is different from 9.585 due to the choice of the boundary constant in obtaining the reduced widths such that $\Delta_1(E_1) = -[S(E_1) - b/\gamma_1^2] = 0$ for the 7.117 MeV state [10] [see Eq. (3.47)].
$^d$ This value is different from that listed in Table 4.1 for the same reason as given in footnote $c$.

The essential parameters used in the calculation of the energy spectrum of the $^{16}\text{O}$ state are listed in Table 5.1. In particular, the reduced $\alpha$ widths obtained in the $^{16}\text{N}$ experiments are used [10], and the proton feeding parameters have been normalized such that

$$\int_0^{2\pi} (C_{12}'^2 + C_{20}'^2) \sin \theta' d\theta' = \int_0^{2\pi} [\mathcal{W}_{12}(\theta')C_{12}^2 + \mathcal{W}_{20}(\theta')C_{20}^2] \sin \theta' d\theta' = 1. \quad (5.2)$$

Note also that, since $C_{10}$ and $C_{22}$ are assumed to be negligibly small, i.e., a particular feeding channel only populates a single level ($\ell_{p}=2$ to the 7.117 MeV state and $\ell_{p}=0$ to the 9.585 MeV state), the R function in the denominator of Eq. (5.1) should contain only the term relevant to the particular feeding channel; the summation would be needed if $C_{10}$ and $C_{22}$ are non-zero.
5.2 In Search of an Optimum Geometry

As described in Sec. 3.2 and depicted in Fig. 3.1, the geometry of a typical 3-element detection system for the observation of 3-body breakups is completely determined by the three parameters $Z_{pa}$, $Z_{ac}$, and $Z_{1/2}$. The two major criteria for defining an optimum geometry are to maximize the triple-coincidence efficiency and the interference effect. In the following sections, the effects of varying the values of $Z_{pa}$, $Z_{ac}$, and $Z_{1/2}$ on the two criteria will be explored in an attempt to determine an optimum geometry for detection of the interference between the 7.117 and 9.585 MeV states in $^{16}$O.

5.2.1 Detection efficiency

In order to gain ideas about the relation between the angles $Z_{pa}$ and $Z_{ac}$, and the triple-coincidence efficiency, Eq. (5.1) was first evaluated for all possible values of $E$ and $\theta'$. This was accomplished by stepping through the range of $^{16}$O excitation energies $E$ between the $\alpha+^{12}$C formation threshold [7.762 MeV relative to $^{17}$F (gs)] and the IAS level energy (11.193 MeV), and through the angle $\theta'$ (see Fig. A.1) in the range $[0, \pi]$. The energy and angular step sizes were chosen to be 5 keV and 1°, respectively, resulting in a total of about $1.2 \times 10^5$ data points. The angles between the particle trajectories in the laboratory frame, i.e., $\angle_{p-a}$, $\angle_{p-^{12}$C}, and $\angle_{^{12}$C}$, as well as the particle energies, were also calculated for each data point. By a 2-dimensional binning of $N$ (the contribution calculated with Eq. (5.1) for each data point) as a function of both $\angle_{p-a}$ and $\angle_{^{12}$C}$, a density plot was obtained as shown in Fig. 5.2. Apparently, 3-body kinematics does give rise to a functional dependence between the two angles as can be seen in Fig. 5.2—for each value of $\angle_{p-a}$, there is a corresponding value of $\angle_{^{12}$C}$ such that the triple-coincidence detection efficiency is maximized.

\footnote{We refer to the angles between particles as $\angle_{p-a}$, $\angle_{p-^{12}$C}$, and $\angle_{^{12}$C}$, and the angles between detectors as $\angle_{pa}$, $\angle_{pc}$ and $\angle_{ac}$.}
If efficiency is of paramount importance for the experiment, the setup must include \( \angle_{pa} = 104^\circ \) and \( \angle_{ac} = 160^\circ \) (indicated in Fig. 5.2) as setup parameters to maximize detection of triple coincidences. In fact, all 3 runs for triple-coincidence measurements mentioned in this thesis did take advantage of this fact, although they were based on results from Monte Carlo studies (see Sec. 2.1.1 and App. B) instead of analytic calculations. The setup parameters used in the 3 runs are listed in Table 5.2. Although the \( p-\alpha \) angle of 90\(^\circ\) for the Aug96 and Nov98 setups was different from the most efficient angle of 104\(^\circ\), it can be seen from the projection of \( \angle_{p-\alpha} \) in Fig. 5.2 that the dependence of efficiency on this angle is relatively weak about the maximum so that the two setups were indeed very nearly optimized for efficiency. In fact, the smaller value of \( \angle_{pa} \) did help in reducing energy and angular straggling due to the collector foil (see Sec. 2.2.2.1), at the expense of a small decrease in efficiency. The half angles \( \angle_{1/2} \) subtended by the detectors are also listed in Table 5.2. Note that, for all the 3 runs, the detectors used for detection of protons or \( \alpha \) particles were different from those for carbon ions, due mainly to the differences in masses and charges of the particles. This will also be assumed for the setup to be proposed below.
Table 5.2: Setup Parameters for the 3 triple-coincidence experiments. Note that different types of detectors were used for detection of protons or α particles, and for carbon ions.

<table>
<thead>
<tr>
<th>Experimental run</th>
<th>( \angle_{pa} )</th>
<th>( \angle_{ac} )</th>
<th>( \angle_{1/2} (p &amp; \alpha) )</th>
<th>( \angle_{1/2} (^{12}\text{C}) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aug96</td>
<td>90°</td>
<td>160°</td>
<td>16.7°</td>
<td>23.0°</td>
</tr>
<tr>
<td>Jul97</td>
<td>104°</td>
<td>160°</td>
<td>22.6°</td>
<td>15.8°</td>
</tr>
<tr>
<td>Nov98</td>
<td>90°</td>
<td>160°</td>
<td>14.0°</td>
<td>9.6°</td>
</tr>
</tbody>
</table>

5.2.2 The \( p-\alpha \) and \( \alpha-^{12}\text{C} \) angles

As depicted in Fig. 5.1, it is the high-energy tail of the 7.117 MeV state in \(^{16}\text{O}\) that extends above the threshold to contribute to \( \alpha \) decays. Therefore, \( \alpha \) particles from decays via this state are expected to be predominately of energies of up to a few hundreds of keV in the cm frame of the \(^{16}\text{O}\). In most cases, since the velocity of the recoiling \(^{16}\text{O}\) is relatively small, the \( \alpha \) energy will not be greatly modified when observed in the lab frame. Consequently, another criterion for an optimum geometry to observe the interference effect is that it must allow for observation of \( \alpha \) particles of energies as low as practicably possible\(^1\). In order to investigate the dependence of observed \( \alpha \) energies on the angles \( \angle_{pa} \) and \( \angle_{ac} \), density plots were produced with \( N \) as functions of \( \alpha \) energies and \( \angle_{pa} \), and \( \angle_{ac} \), respectively, as shown in Fig. 5.3, obtained from the same set of data mentioned in Sec. 5.2.1. As can be seen in Fig. 5.3(a), \( \angle_{pa} \) should be at about 45° to optimize observation of low-energy \( \alpha \) particles. However, as shown in Fig. 5.3(b), \( \angle_{ac} = 160° \) (the most efficient angle) is still valid for observation of low-energy \( \alpha \) particles.

As shown by the dashed box in Fig. 5.2, any set of 3 detector elements with definite sizes and shapes will cover a boxed region of angular space in the plot. The spectral shape and range of particle energies observed with such a system depend strongly on the sizes, shapes, and locations of the detector elements. In order to investigate the interference effect more accurately, the algorithm described in Sec. 3.2 was used to

\(^1\)Based on data from the 3 experimental runs described in this thesis, the \( \alpha \)-energy threshold should be about 400 keV for the types of detectors used.
§5.2 In Search of an Optimum Geometry

Figure 5.3: DENSITY PLOTS to determine the relations of $\alpha$ energy to the p-$\alpha$ and $\alpha$-${}^{12}\text{C}$ angles: (a) $N$ as a function of $\alpha$ energy and the p-$\alpha$ angle; (b) $N$ as a function of $\alpha$ energy and the $\alpha$-${}^{12}\text{C}$ angle.

calculate laboratory particle spectra for a 3-element setup with definite values of $\angle_{pa}$, $\angle_{ac}$, and $\angle_{1/2}$. The elements were assumed to be of equal size and square in shape (such as the SSDs used in the runs of Jul97 and Nov98) to simplify the calculation. This method will henceforth be referred to as the proper method of calculation. As suggested by the result in Fig. 5.3(b), $\angle_{ac}$ was chosen to be 160°, with $\angle_{1/2}$ set at 9° (for reasons to be explained later), and with various values of $\angle_{pa}$, in the calculation. The $\alpha$ spectra resulting from the calculation are shown in Fig. 5.4(a). Apparently, the most efficient combination of $(\angle_{pa}, \angle_{ac}) = (104^\circ, 160^\circ)$ is not a favorable configuration since it does not allow for observation of $\alpha$ particles of less than about 1.1 MeV. It also suggests that the region of angular space with $\angle_{pa} < 60^\circ$ should be covered by the detection system in order that the low-energy region can be observed.

As shown in Fig. 5.4(a), it would be almost impossible to observe the interference peak experimentally, as was the case of $^{16}\text{N}$ [10], since it is below 200 keV and of intensity many orders of magnitude weaker than that of the main peak. The reason is that the BR of the IAS$\rightarrow^{16}\text{O}(7.117)+p$ channel is greater than that of IAS$\rightarrow^{16}\text{O}(9.585)+p$ by less than one order of magnitude (see Fig. 5.1 and Chapter 4) while, in the case of $^{16}\text{N}$, the $\beta$ decay to the 7.117 MeV state is stronger than that to the 9.585 MeV


§5.2 In Search of an Optimum Geometry

\[ \frac{Z_2}{Z} = 9^\circ \]

\[ \frac{Z_\infty}{Z} = 160^\circ \]

\[ V = V \]

\[ 2.5 \]

4.0

6.5

7.5

\[ J_D < 2 \]

\[ 104.0 \]

\[ 1.5 \]

2.0

0.5

1.5

0.0

1.0

0.5

1.0

\[ E_a \ (\text{MeV}) \]

\[ E_a (\text{MeV}) \]

\[ \text{Fig u re 5.4: R ESU L T S O F A proper C ALCULATION with fixed values of the half and } \alpha^{12}\text{C angles: (a) set of } \alpha \text{ spectra calculated for various values of the } p-\alpha \text{ angle with contributions from the 7.117 MeV state; (b) fractional differences between } \alpha \text{ spectra generated with and without contributions from the 7.117 MeV state.} \]

state by more than 3 orders of magnitude. Nevertheless, it is still possible to deduce properties of the 7.117 MeV state by investigating the change in spectral shape due it its presence. This fact will be explored in subsequent sections.

5.2.3 Interference and the p-\alpha angle

The effect of the presence of the 7.117 MeV state in the 3-body breakup of the IAS can be explored by using the proper method (see Sec. 5.2.2) to calculate \( \alpha \) spectra which do not contain contributions from the 7.117 MeV state, and comparing the results with those shown in Fig. 5.4(a). Figure 5.4(b) shows a plot of the absolute fractional difference\(^\dagger\), defined as \[ \left| \frac{N' - N}{N} \right| \] where \( N \) and \( N' \) correspond to the spectra with and without contributions from the 7.117 MeV state, respectively, versus \( \alpha \) energy. Interference is most prominent in the low-energy region and gradually decreases with increasing energy, which is the expected effect of a subthreshold state. It is also obvious that \( \angle_{pa} \) should be as small as possible in order to ensure the observation of the interference

\(^\dagger\)The absolute magnitude of difference should be understood in subsequent discussions, as only positive values can be displayed in a logarithmic scale.

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effect. Although the fractional differences of the two sets of data with $\angle_{pa} = 90^\circ$ and $104^\circ$ are among the largest at high $\alpha$ energy, they can be excluded due to their inability to cover the low-energy region. The data with $\angle_{pa} = 22.5^\circ$ can also be excluded due to low efficiency, especially for smaller values of $\angle_{1/2}$ (see Sec. 5.2.4). Therefore, the optimum value of $\angle_{pa}$ should be within the range of $30^\circ$ to $60^\circ$, as suggested by the results in Fig. 5.4.

5.2.4 The half angle

With the optimum (range of) values of $\angle_{pa}$ and $\angle_{ac}$ established in previous sections, the half angle $\angle_{1/2}$ remains to be determined before an optimum detection geometry can be put forward. In order to investigate the effect of $\angle_{1/2}$ on the observation of the interference effect, calculations were again performed with the proper method, with $\angle_{pa} = 45^\circ$ and $\angle_{ac} = 160^\circ$, as suggested by results from previous sections, and with values of $\angle_{1/2}$ ranging from $3^\circ$ to $15^\circ$. Cases both with and without contribution from the 7.117 MeV state were calculated. The resulting $\alpha$ spectra are shown in Fig. 5.5(a), and the fractional difference in $N$ between the two types of spectra are shown in Fig. 5.5(b). The results show that the range of observed energies increases with increasing value of $\angle_{1/2}$, which is expected since a larger phase space is covered by a larger half angle. It is also obvious that the efficiency increases with increasing values of $\angle_{1/2}$. However, as indicated in Fig. 5.5(b), the fractional difference in $N$ is insensitive to the value of $\angle_{1/2}$, despite the change in range of observable energies. The insensitivity of the interference effect to $\angle_{1/2}$ can be attributed to the fact that the ranges of angles subtended by detectors with $\angle_{1/2} < 15^\circ$ are relatively small so that the integration over the solid angles does not vary the interference effect significantly. Consequently, the determination of an optimum value of $\angle_{1/2}$ should depend much more strongly on the detection efficiency and the range of observed energy. As suggested by the results in Fig. 5.5(a), $\angle_{1/2}$ should be greater than $6^\circ$ for a good range of energy to
§5.3 An Optimum Geometry

Figure 5.5: RESULTS OF A proper calculation with fixed values of the p-α and α-¹²C angles: (a) set of α spectra calculated for various values of half angle with contributions from the 7.117 MeV state; (b) differences between α spectra generated with and without contributions from the 7.117 MeV state.

be observed, and with a reasonable efficiency. It should be pointed out that, as particle spectra can be obtained by integrating over detectors subtending large half angles, the detectors still need to be pixelated to provide sufficient angular resolution to facilitate the kinematic procedure (see Sec. 2.2.2.4) in the discrimination against background.

5.3 An Optimum Geometry

With the information given by results from Sec. 5.2, we are in a position to define an optimum system for detection of the interference between the 7.117 and 9.585 MeV states in ¹⁶O. As shown in Fig. 5.6, the values \( \angle_{pa} = 45^\circ \), \( \angle_{ac} = 157.5^\circ \), and \( \angle_{1/2} = 9^\circ \) have been proposed for the detector configuration, with additional detectors to form an array of 8 elements to provide more combinations of the same geometry. This detector configuration will henceforth be referred to as the optimum geometry. With the assumption that the type of element for detection of protons and α particles is different from that for carbon ions (see Sec. 5.2.1), the array of 8 elements is capable of providing 6 combinations of 3 elements of the same geometry.
The results in Sec. 5.2 have shown that the choice of an optimum value for each of the parameters $\angle_{pa}$, $\angle_{ac}$, and $\angle_{1/2}$, is not independent of the values for the other parameters. Nevertheless, by taking advantage of symmetry, so that more combinations of the same geometry can be obtained with the least number of elements, the reasons behind the choice of each of the parameters become apparent. It has been shown in Sections 5.2.1 and 5.2.2 that $\angle_{ac} = 160^\circ$ is the most efficient angle, which also provides a good range of observed energies. The choice of $\angle_{ac} = 157.5^\circ$ in place of the most efficient angle is, in fact, for reason of geometric symmetry. It can be seen in Fig. 5.6 that, with $\angle_{ac} = 157.5^\circ$, the proton and $\alpha$ detectors can be interchanged to provide another combination of the same geometry. Since $\angle_{1/2}$ has been chosen to be $9^\circ$ (see below), the most efficient angle is, in fact, also covered.

The determination of an optimum value for $\angle_{1/2}$ was based on the assumption that detectors similar to those used in the Jul97 and Nov98 runs, with dimensions of 5 cm $\times$ 5 cm, would be used in the ultimate experiment. Furthermore, it was realized that a flight path of about 15 cm would be needed to distinguish the $\beta$ particle from other particles using ToF techniques (see also Sec. 2.3). Accordingly, the half angle $\angle_{1/2}$ has been assigned a value of $9^\circ$. The space of $4.5^\circ$ between detectors, equiva-
§5.4 The Proposed Detection System

In order to provide clearance for the $^{17}$Ne ion beam (assumed to be along the $z$ axis) to access the collector foil (assumed to be centered at the origin on the $xy$ plane), the basic structure of Fig. 5.6 is rotated about the $x$ axis by $+20^\circ$ resulting in the configuration shown in Fig. 5.7. A larger array can be built by added another set of elements obtained by rotating the basic structure about the $x$ axis by $-20^\circ$. The entire system, as shown in Fig. 5.8, consists of 10 elements for detection of protons and $\alpha$ particles, and 6 elements for detection of carbon ions. Extra space is available for...
installation of additional detectors to improve efficiency. However, as will be shown in Sec. 5.5, the proposed setup is already adequate for performing the experiment within a reasonable period of time. Thus, additional detectors do not seem to be necessary.

A pair of $\alpha$ spectra calculated with and without contributions from the 7.117 MeV state, and with a set of 3 elements at the optimum geometry, is shown in Fig. 5.9. The entire system is capable of providing 12 combinations of the same geometry—6 from each of the 2 planes containing 8 elements. In principle, an ultimate experimental $\alpha$ spectrum can be obtained by summing contributions from all of the 12 combinations, and the resulting spectrum can be fitted with a theoretical spectrum, such as the one given in Fig. 5.9, to extract the parameters of interest. This system also provides...
§5.5 Estimate of Count Rate

It would be beneficial to have an estimate of the time frame within which an experiment could be accomplished with the proposed detection system. While it is difficult to anticipate how well an experimental spectrum can be fitted with a calculated one, an estimate of count rates can still be made based solely on the calculated spectrum. The presence of the 7.117 MeV state is not significant at the high-energy wing of the main peak, as can be seen in Fig. 5.9. As a working condition, we arbitrarily state combinations of other geometric configurations; combinations with $\angle_{pa} = 22.5^\circ$, 67.5$^\circ$, or 90$^\circ$, and $\angle_{ac} = 112.5^\circ$, 135$^\circ$, or 180$^\circ$ will also be available. However, as can be seen in Fig. 5.10, the (45$^\circ$, 157.5$^\circ$) combination is still the best configuration covering a large range of energy, and providing the highest efficiency in the low-energy region where interference is expected to be most prominent [see Fig. 5.4(b)]. Nevertheless, while the optimum geometry should provide spectra to best extract the parameters of interest, these other configurations might be able to provide extra constraints on the fit parameters, as well as serve to calibrate the system.

Figure 5.10: Set of $\alpha$ spectra provided by the proposed detection system with configurations other than the optimum one given in Fig. 5.6. The spectrum obtained with the optimum geometry is also shown (solid circles) as reference.
§5.5 Estimate of Count Rate

Table 5.3: LIST OF PARAMETERS used in the estimation of the counts required to observe the interference between the 7.117 and 9.585 MeV states using the detection system depicted in Fig. 5.8.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{17}\text{Ne}$ yield$^a$</td>
<td>$10^8/$s</td>
<td>projected</td>
</tr>
<tr>
<td>Triple-coincidence geometric efficiency</td>
<td>$1.2 \times 10^{-5}$</td>
<td>proper calculation</td>
</tr>
<tr>
<td>$^{17}\text{Ne} \rightarrow \text{IAS} + \beta$ branching</td>
<td>0.74%</td>
<td>Ref. [46]</td>
</tr>
<tr>
<td>$\text{IAS} \rightarrow ^{16}\text{O}(7.117) + p$ branching</td>
<td>18.6%</td>
<td>Ref. [46]</td>
</tr>
<tr>
<td>$\text{IAS} \rightarrow ^{16}\text{O}(9.585) + p$ branching</td>
<td>2.5%</td>
<td>Ref. [46] and Chapter 4</td>
</tr>
<tr>
<td>$\alpha$ spectrum bin size</td>
<td>20 keV</td>
<td></td>
</tr>
<tr>
<td>$\alpha$ energy resolution</td>
<td>40 keV</td>
<td></td>
</tr>
</tbody>
</table>

$^a$With a proton beam current of 1 $\mu$A on an MgO target, yields of a few times $10^5$ $^{17}\text{Ne}$/s have been realized at Tisol. The target at ISAC is projected to handle proton current of tens of $\mu$A so that a yield of $10^8$/s is not impossible [79]. Development of a new ECR ion source and target for the production of radioactive Ne ions is underway at ISAC.

that the two spectra need to be resolved to a level of $2\sigma^\dagger$ at 1.65 MeV, the maximum point of the main peak, for a spectrum to be considered as statistically acceptable for this experiment. In fact, under this condition, the two spectra will be separated by more than $2\sigma$ for most of the energy bins on the low-energy wing of the main peak (see below), and thus the presence of the 7.117 MeV state is guaranteed to be strong enough for its properties to be properly constrained in the fitting procedure.

The parameters used in the estimation of the required number of counts are listed in Table 5.3. In particular, the yield of $^{17}\text{Ne}$ is projected from that which has been realized at Tisol, the triple-coincidence efficiency per set of 3 detectors is obtained from a proper calculation using the optimum geometry, and the BRs are quoted from Ref. [46]. Based on this calculation, the triple-coincidence rate is $\sim 0.2$ count/s. The entire system, with 12 sets of the same geometry, will be able to observe about 2 count/s.

At a $2\sigma$ separation between the two spectra, the required number of counts is about 1.7 million which, based on the above assumptions, requires an acquisition period $^\dagger\sigma$ is the standard deviation defined as the square root of the number of counts in an energy bin.
§5.5 Estimate of Count Rate

# counts required: $\sim 1.7 \times 10^6$

Figure 5.11: A replot of the spectra in Fig. 5.9 normalized to the required number of counts such that the two spectra with and without contribution from the 7.117 MeV state are separated by $2\sigma$ at 1.65 MeV. The spectra have been convoluted with a resolution of 40 keV; the error bars represent $1\sigma$; and the number of counts in each energy bin in the spectrum with contribution from the 7.117 MeV state has been randomized with a Gaussian distribution to simulate statistical fluctuation.
§5.5 Estimate of Count Rate

Figure 5.12: Separation between the two spectra in Fig. 5.11 in units of $\sigma$.

of about 9 days. Figure 5.11 shows a replot of Fig. 5.9 normalized to the required number of counts, of which the number of counts in each energy bin in the spectrum with contribution from the 7.117 MeV state has been randomized with a Gaussian distribution to simulate statistical fluctuation. The separation of the two spectra in units of $\sigma$ is shown in Fig. 5.12, in which it can be seen that most of the energy bins below 1.65 MeV have separations considerably greater than $2\sigma$, i.e., the presence of the 7.117 MeV state should be strong enough for its properties to be constrained in a fitting procedure.

Based on the above estimation and a projected yield of $10^8$ $^{17}$Ne/s (see footnote in Table 5.3), we conclude that an experiment with the proposed setup to observe the interference between the 7.117 and 9.585 MeV states in $^{16}$O is feasible. Even if the yield is as low as a few times $10^7$ /s, sufficient counts can still be accumulated within a few weeks of beam time.
Chapter 6

Conclusions

A study on the feasibility of using the $\beta$-delayed proton decay of $^{17}\text{Ne}$ to constrain the rate of the astrophysically important $^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$ reaction has been underway at the TRIUMF-TISOL facility. The usefulness of the study hinges on the existence of $\alpha$-unbound states in $^{16}\text{O}$ populated by the proton decay of states in $^{17}\text{F}$, which subsequently decay into $^{12}\text{C}(\text{gs})$ giving rise to 3-body final states of $p+\alpha+^{12}\text{C}$. States in $^{17}\text{F}$ with excitation energies above 5.819 MeV may also $\alpha$ decay into states in $^{13}\text{N}$ and, for those above the $^{12}\text{C}+\alpha$ formation threshold, the $^{13}\text{N}$ nucleus may further proton decay to $^{12}\text{C}(\text{gs})$ forming 3-body final states which may not be distinguishable from those formed via the $^{16}\text{O}$ channel, and thus would contribute to background. Furthermore, since states in $^{17}\text{F}$ populated by the $\beta$ decay of $^{17}\text{Ne}$ decay predominately to bound states in $^{16}\text{O}$ or to $^{13}\text{N}(\text{gs})$ resulting in final states of $\beta+p+^{16}\text{O}$ or $\beta+\alpha+^{13}\text{N}$, $\beta$-induced 2-body background is expected to be significant.

A thorough knowledge of the decay scheme of $^{17}\text{Ne}$ is crucial to the success of this study, to understand potential background, and to estimate expected count rates. Many properties of the decay scheme of $^{17}\text{Ne}$ relevant to this study—such as $\beta$ decay branchings of $^{17}\text{Ne}$, $\beta$-delayed proton and $\alpha$ branchings of states in $^{17}\text{F}$ (particularly the IAS), and $J^*$ values of some of the states in $^{17}\text{F}$—have been determined with $\beta$-$p$, $\beta$-$\alpha$, or $p$-$\gamma$ coincidence measurements [46].

Due to the anticipated intense $\beta$ flux, and the possible ambiguity in particle identifi-
cation, detection of all 3 product particles (p+α+^{12}C) is required to optimize the kinematic information inherent in each event to discriminate against background. The experimental work in this thesis is based on three experimental runs on triple-coincidence measurements performed in the period from 1996 to 1998. The first run was carried out in August of 1996 (Aug96—Sec. 2.1) in which large-size PIPS detectors were used in close geometry. The run succeeded in the observation, *for the first time*, of the β-delayed 3-body breakup of the IAS in ^{17}F into the 9.585 MeV state in ^{16}O, and the 2.365 and 3.502/3.548 MeV states in ^{13}N. However, the poor angular resolution given by the setup rendered background suppression unsatisfactory. In order to improve the situation, another run was carried out in July of 1997 (Jul97—Sec. 2.2) in which two of the PIPS detectors were replaced by double-sided silicon strip detectors; a β detector which covered a small solid angle was also added to allow for quadruple-coincidence measurement. In order to take advantage of the improved angular resolution, a procedure was developed in which a kinematic function was minimized on an event-by-event basis in the extraction of triple coincidences. The same three 3-body breakup channels of the IAS were observed, and the background level was also largely improved but was still unsatisfactory due mainly to the large-size PIPS detectors used in the detection of carbon ions. Only the events of type a (a single hit on each SSD) were found to be useful while intense 2-body background was found in events of type b (double hits in a single SSD). The quadruple-coincidence data obtained with the β detector also hinted at the possibility that the additional detection of the β particle might help in background suppression. Prompted by the unsatisfactory background level in the run of Jul97, another setup was devised and the run was carried out in November and December of 1998 (Nov98—Sec. 2.3). The same particle detectors were used but with their distance from the collector foil increased from 6 to 10 cm to improve angular resolution and to facilitate ToF measurement. Large-size β detectors covering a solid
angle of $\sim25\%$ of $4\pi$ were also installed around the particle-detector array to facilitate quadruple-coincidence measurement with a reasonable detection efficiency. While results were similar compared to those of the previous two runs, the unexpectedly low number of events obtained, due to various instrumental problems occurring during the run, made it difficult to draw firm conclusions from the results. Nevertheless, it was found that: i) the avoidance of the $180^\circ$ angle covered by the PIPS-SSD pairs at $160^\circ$ is essential in the suppression of $2$-body+$\beta$/random false triples; ii) the requirement of quadruple coincidence does not seem to improve background suppression significantly; and iii) the flight distance needs to be increased (from $10$ cm), and timing signals optimized, to make the ToF technique useful in triple-coincidence measurements.

The connection between experiments and theory is made through extensive Monte Carlo simulations and analytic calculations based on the $R$-matrix formalism. MC simulations, which focused on the study of the $3$-body breakup mechanism of the three experimentally observed decay channels of the IAS (see App. B), were carried out first to generate laboratory particle spectra. It was then realized that, with the focus only on the IAS, the problem was sufficiently simple that analytic calculations of laboratory particle spectra would be possible. The results from the analytic calculations have superseded those from the MC simulations. Using a single-channel, single-level approximation in calculations of the laboratory particle spectra of the $3$ observed $3$-body breakup channels of the IAS, the branching ratios via the $9.59$, $2.37$, and $3.55$ channels were determined to be $R_1 = 13.4\pm2.0\pm3.1$, $R_2 = 62.9\pm5.2$, and $R_3 = 1.5\pm0.7\pm0.4$, respectively, relative to a value of $100$ for the $\text{IAS} \rightarrow ^{16}\text{O}(7.12)+\text{p}$ transition. Based on the relative BR between the $\text{IAS} \rightarrow ^{16}\text{O}(9.59)+\text{p}$ and $\text{IAS} \rightarrow ^{16}\text{O}(7.12)+\text{p}$ transitions, the relative proton-feeding amplitudes for the two channels were determined with a single-channel, single-level $R$-matrix calculation which was extended to include the $\gamma$ channel, and with the assumption that the proton $\ell$ values were $2$ and $0$ for the $7.117$...
and 9.585 channels, respectively. The resulting proton-feeding parameters were then used in a series of single-channel, two-level $R$-matrix calculations to study the interference between the 7.117 and 9.585 MeV states in $^{16}$O in the 3-body breakup of the IAS. The results from these calculations provided information for the design of an optimum geometry for a possible determination of the reduced $\alpha$ width of the 7.117 MeV state, and hence for a constraint on the value of $S_{E1}(300)$. An estimation of count rates has shown that this experiment could be accomplished within a few days of beam time at ISAC, which is projected to deliver a yield of $10^8$ $^{17}$Ne/s.

The 9.45 MeV state in $^{17}$F had been identified as the best candidate state for use to constrain the value of $S_{E2}(300)$ based on results from a previous calculation of absolute 3-body breakup branchings of states in $^{17}$F using a Breit-Wigner approach, $\beta$-delayed proton branchings from a preliminary analysis of a set of p-$\gamma$ coincidence data, and an estimated value of the ratio $(\theta_{6.92}^2/\theta_{7.12}^2)$ between the reduced widths of the 6.917 (2$^+$) and 7.117 (1$^-$) MeV states. However, a recent calculation employing an $R$-matrix approach, and using the most recent values of branching ratios and the ratio $\theta_{6.92}^2/\theta_{7.12}^2$, has shown that the 10.03 MeV state has a 3-body breakup strength which is comparable to that of the 9.45 MeV state. Moreover, the observation of the $^{17}$F(10.03)$\rightarrow^{16}$O(6.92)$+p$ transition might be free of interference from any $E1$ component, since it was also found experimentally that the 10.03 MeV state does not feed the 7.117 (1$^-$) MeV state to any appreciable extent. Thus, the 10.03 MeV state should replace the 9.45 MeV state as the best candidate for use to constrain the value of $S_{E2}(300)$. Nonetheless, 3-body breakups from both the 10.03 and 9.45 MeV states, as well as any other states which have a non-negligible 3-body breakup strength, such as the IAS, will be observed within the same experiment, and only an analysis of the data can prove if the above assertion is correct.

It should be pointed out that the calculated 3-body breakup BR of the $^{17}$F(10.03)$\rightarrow$...
$^{16}$O(6.92)+p transition ($\sim 3 \times 10^{-10}$) is two orders of magnitude weaker than that of IAS$\rightarrow ^{16}$O(7.12)+p. Moreover, the fact that the 10.03 MeV state is a broad state also makes data analysis more complicated. Consequently, the determination of the strength of the $E1$ component is a necessary prerequisite to gain experience on potential backgrounds and experimental methods, as well as on development of data analysis algorithms, before a determination of the $E2$ component can be proved to be feasible.
Appendix A

Issues on 3-body Sequential Decay

In this appendix, some of the aspects of 3-body sequential decays will be discussed within the framework of non-relativistic kinematics. The physics is simple but the results are very useful in the interpretation of triple-coincidence data. While the concepts are applicable to any 3-body 2-step cascade transitions, the specific reference to the decay of $^{17}$F makes the discussion more easily related to the material of this thesis.

A.1 Energies

We consider the spontaneous 3-body decay of $^{17}$F*, at rest initially, into final states of p+$\alpha$+$^{12}$C. The process, described schematically in Fig. A.1, is assumed to take place in two sequential steps. We first assume that the decay goes through an $^{16}$O resonance. In the first breakup, the proton and $^{16}$O ion travel back-to-back, with Q value $Q_1$, and conservation of energy gives\(^\dagger\)

$$Q_1 = E_p + E_r , \quad (A.1)$$

\(^\dagger\)Strictly speaking, the excitation energy of the $^{16}$O ion should also be included in the calculation. However, its effect ($< 0.05\%$) is too small to be of any significance here.
where \( r \) denotes quantities pertaining to the recoiling \( ^{16}\text{O} \) ion, while conservation of momentum gives\(^1\)

\[
\begin{align*}
p_r^2 &= p_p^2; \\
2m_r E_r &= 2m_p E_p; \\
E_r &= E_p \frac{m_p}{m_r} \approx E_p \frac{m_p}{m_\alpha + m_c}.
\end{align*}
\] 

Subsequently, the \( ^{16}\text{O} \) ion decays in flight by \( \alpha \) emission. The \( \alpha \) particle and carbon ion travel back-to-back with Q value \( Q_2 \) in their center-of-momentum (cm) frame, and their cm energies are related by

\[
Q_2 = E'_\alpha + E'_c; \quad E'_c = E'_\alpha \frac{m_\alpha}{m_c},
\]

while the sum of their lab energies is given by

\[
E_\alpha + E_c = Q_2 + E_r.
\]

By substituting Eq. (A.4) into Eq. (A.6), we obtained

\[
E_\alpha + E_c = Q_2 + E_p \frac{m_p}{m_\alpha + m_c}.
\]

\(^1\)Strictly speaking, \( m_r = m_\alpha + m_c - E_b \) where \( E_b \) is the binding energy of the \( ^{16}\text{O} \) ion. Its effect is again too small to be of any significance here.
Q_2 can be calculated from Eq. (A.7) if the energies and identities of all 3 particles are known. Since the sequence of the two breakups is indeterminable, it is equally valid to assume that the process goes through an $^{13}$N resonance in which case the $\alpha$ decay takes place first. In this case, we would have

$$E_p + E_c = Q'_2 + E_\alpha \frac{m_\alpha}{m_p + m_c}.$$ (A.8)

Since $Q_2$ and $Q'_2$ are usually not equal, they might be used to identify the transitions.

### A.2 Angles between particle trajectories

By the law of momentum conservation, the 3 momentum vectors have to form a closed triangle, as shown in Fig. A.2, from which the following can be established with the cosine law,

$$p_p^2 + p_\alpha^2 - p_c^2 = 2 p_p p_\alpha \cos \theta.$$ (A.9)

Rewriting Eq. (A.9) in terms of energies, we obtain

$$m_p E_p + m_\alpha E_\alpha - m_c E_c = 2 \sqrt{m_p E_p m_\alpha E_\alpha} \cos \theta.$$ (A.10)

A similar equation can also be obtained for the angle $\phi$,

$$m_p E_p + m_c E_c - m_\alpha E_\alpha = 2 \sqrt{m_p E_p m_c E_c} \cos \phi.$$ (A.11)

If the energies of the 3 particles are measured, Eqs. (A.10) and (A.11) can be used to calculate the angles between pairs of trajectories. Note that the angle between the trajectories of the proton and $\alpha$ particle is $\pi - \theta$ and between those of the $\alpha$ particle and $^{12}$C is $\theta + \phi$. 

![Momentum conservation in 3-body decay](image)
A.3 Kinematic parameters

The present problem contains 9 parameters\(^1\): 3 energies \((E_p, E_\alpha, E_c)\), 2 angles \((\theta, \phi)\), 3 masses \((m_p, m_\alpha, m_c)\), and \(Q_2\). Thus, assuming that the identities (masses) of the particles are known, and 3 of the 6 remaining quantities are measured, the kinematics of the entire problem can be determined by solving Eq.’s (A.4), (A.10), and (A.11). In practice, however, the identities of the particles may not be known so that the measurement of more than 3 quantities will be needed. Furthermore, over-determining the kinematics will also improve background suppression.

A.4 Conversion between lab and cm angle

The conversion between lab angle \(\theta\) and cm angle \(\theta'\) in Fig. A.1 is obtained from the sine law, which gives

\[
\sin(\theta' - \theta) = \frac{v_r}{v'_\alpha} \sin \theta
\]

\[
\sin \theta' \cos \theta - \cos \theta' \sin \theta = \frac{v_r}{v'_\alpha} \sin \theta .
\]

Therefore,

\[
\tan \theta = \frac{\sin \theta'}{v_r/v'_\alpha + \cos \theta'} .
\]

A.5 Dalitz condition

The daughter particles of a 3-body breakup may have any momenta permitted by conservation laws. However, for a definite \(Q\) value, extreme values of the momenta occur only when all three particles travel on the same line of flight. In our present case of decay of \(^{17}\)F\(^*\), for instance, the \(\alpha\) particle has minimum momentum when it

\(^1\)Only two of the three angles are needed since they should add up to \(2\pi\). Also, \(Q_1 = E_p + E_\alpha + E_c - Q_2\).
pairs with the proton to travel antiparallel to the carbon ion while the maximum occurs when it pairs with the carbon ion to travel antiparallel to the proton. Given the 3-body breakup $Q$ value $Q_{3b}$ and proton energy, these extrema can be calculated by solving for $p_\alpha$ in the quadratic equation,

$$\frac{p_p^2}{2m_p} + \frac{p_\alpha^2}{2m_\alpha} + \frac{(p_p + p_\alpha)^2}{2m_c} = Q_{3b}.$$  \hspace{1cm} (A.15)

Obviously, the two solutions for $p_\alpha$ should have opposite signs.

The parent state can be identified by measuring the energies of all 3 daughter particles to obtain the total $Q$ value ($Q_{3b} = Q_1 + Q_2 = E_p + E_\alpha + E_c$), which is equal to the excitation energy of the parent state above the $^{12}$C($\alpha$, $\gamma$)$^{16}$O reaction threshold. Triple-coincidence data can be sorted according to the parent states. In the sorting of data for a certain parent state, e.g. the IAS, only events with observed $\alpha$ energy within the extrema calculated with Eq. (A.15) should be accepted. Uncertainties in the observed energies should be accounted for in setting these limits. This is equivalent to requiring that the proton and $\alpha$ energies be within the closed curve in a Dalitz plot [80, p.784], the region within which a 3-body decay is accessible. Dalitz plots of proton versus $\alpha$ energy for the IAS and 9.45 MeV state as parent states are shown in Fig. A.3(a). This constraint is termed the Dalitz condition in this thesis. As shown in Fig. A.3(b), the carbon energy along with the total $Q$ value can also be used to calculate another closed curve. However, since the carbon energy is fixed by the $Q$ value in the first calculation, this other condition will be satisfied automatically. Furthermore, it is obvious that use of the p-$\alpha$ Dalitz plot is preferred in this case, since transitions from different parent states are well-separated.

Note that the Dalitz condition should be applied only for data reduction at the early stage of an analysis since it is not a very stringent constraint. Kinematic constraints such as those described in Sec. 2.2.2.4 should be used in the final stage of the analysis to extract valid events.
A.6 Sequential versus simultaneous 3-body decay

If the transition matrix element is independent of energy, events within the closed curve in a Dalitz plot will be evenly distributed. This would be the case if the 3-body decay took place simultaneously and no penetrabilities of the emitted particles were involved. Any structure within the Dalitz curve is indicative of the dependence of the matrix element on energy. More specifically, if the transition is sequential in nature and goes through a narrow intermediate state, a horizontal or vertical band will appear in the Dalitz plots. This band also allows the identification of the decay channel. For instance, if a horizontal band were to appear in Fig. A.3(a), the transition would have to have proceeded through the $^{16}\text{O}$ channel, in which the proton is first emitted so that its energy is not modified by the recoiling $^{16}\text{O}$; if the band were vertical, it would have to have proceeded through the $^{13}\text{N}$ channel, in which the $\alpha$ particle is first emitted.

Nature seems to favor sequential rather than simultaneous 3-body nuclear decays, at least for the energy region relevant to this work. All the data of $\beta$-delayed 3-body breakup of $^{17}\text{Ne}$ described in this work favor the sequential mechanism. A study of the $\beta$-delayed 3-body breakup of $^{9}\text{C}$ [59,60,81] and one of the $\beta$-delayed 2-proton decay of

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Figure A.3: DALITZ PLOTS of (a) proton versus $\alpha$ energy; and (b) carbon versus $\alpha$ energy, for 3-body breakups of the IAS and 9.45 MeV state in $^{17}\text{F}$. 

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§A.7 Geometric detection efficiency

$^{31}\text{Ar}$ [82] also support this assertion.

A.7 Geometric detection efficiency

A calculation of geometric detection efficiency is useful in the estimation of count rates. For detection of single particles, the geometric detection efficiency $\eta$ is proportional to the solid angle subtended by the detector. For a circular detector of radius $r$ at a distance $R$ from the collector foil, assumed to be a point, the efficiency is given by

$$
\eta_o = \frac{2\pi}{4\pi} \int_0^\alpha \sin \theta \, d\theta = \frac{1 - \cos \alpha}{2}; \quad \alpha = \arctan \frac{r}{R}
$$

(A.16)

For a square detector, such as the SSD used in the runs of Jul97 and Nov98, with length of an edge $\ell$ [76],

$$
\eta_o = \frac{4R}{4\pi} \int_0^\frac{\pi}{2} dx \int_0^\frac{\pi}{2} dy \frac{1}{(x^2 + y^2 + R^2)^{3/2}} = \frac{1}{\pi} \arctan \left( \frac{(\frac{\ell}{2})^2}{R \left( 2(\frac{\ell}{2})^2 + R^2 \right)^{1/2}} \right).
$$

(A.17)

For back-to-back coincidence in the detection of 2-body decay products, the efficiency is approximately equal to the smaller of $\eta_o$ and $\eta_o^2$, since the two particles go back-to-back, the detection of one almost\footnote{This is only approximate since effects such as energy threshold, finite size of the beam spot, and uncertainties in position of the detectors, need to be taken into account.} automatically guarantees detection of the other.

For triple coincidence in the detection of 3-body decay products with, e.g., one PIPS and two SSDs as in the setups of Jul97 and Nov98, the efficiency might be taken as $\eta_o \cdot \eta_o^2$. However, if there exists a certain angle between a pair of particles favored by the kinematics, such as the $160^\circ$ angle between the $\alpha$-$^{12}\text{C}$ pair in the 3-body breakup of the IAS in $^{17}\text{F}$ and, if the circular detector is positioned at this angle with respect to one of the square detectors, the efficiency should be approximately $\eta_o \cdot \eta_o$ (assuming isotropic angular correlation between the first and second emitted particles (see also App. C)) since, by placing a pair of detectors at the strongly favored angle, the situation is similar to a back-to-back detection, and the efficiency is approximately proportional to

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the smaller of the two solid angles subtended by the detectors. In practice, the actual
detection efficiency is always different from that calculated above due to such effects
as energy threshold, angular correlations among the daughter particles, finite size of
the beam spot, and uncertainties in the position of the detector(s). In principle, these
effects can be taken into account by using numerical integrations or MC simulations
(see App. B).

A.8 Rate of random triple coincidences

Measurement of two signals in coincidence is usually accomplished by feeding the logic
pulses, derived from the analogue signals with discriminator modules, into a coincidence
module (AND gate) which outputs another logic pulse if the two input pulses overlap in
time. The coincidence signals can be used to generate event triggers. We first consider
the measurement of a 2-body decay process, with a rate of $N_\circ$, and assume the system
to consist of two identical detectors in back-to-back geometry. We further assume that
each detector has a geometric efficiency of $\eta$ and a detection efficiency of 100%. The
rate of random (accidental) coincidence is given by [83]

$$N_f = (2N_\circ)^2 \eta^2 (2\sigma_t),$$  (A.18)

where $\sigma_t$, of the order of a few tens of nanoseconds, is the resolving time$^\dagger$ of the system,
and the factor of 2 in front of $N_\circ$ accounts for the fact that there are two daughter
particles in each decay. The true coincidence rate is given by (see discussion of 2-body
decay detection efficiency in the last section)

$$N_t = N_\circ \eta.$$  (A.19)

Therefore, the ratio of false to true coincidences is

$$\frac{N_f}{N_t} = 4N_\circ \eta (2\sigma_t).$$  (A.20)

$^\dagger$ $\sigma_t$ is approximately equal to the width of the logic pulse.
§A.8 Rate of random triple coincidences

In the triple-coincidence measurements described in this thesis, event triggers were obtained from coincidences between adjacent pairs of detectors. We make the following assumptions to specify the problem:

1) the angular correlation between the first and second emitted particles is isotropic;
2) the detection system consists of SSDL+SSDR+PIPL, as in the setup of Nov98 with PIPL and SSDR at 160° to each other;
3) the SSDs and PIPS have the same geometric efficiency \( \eta \);
4) events are triggered solely by the SSDL+SSDR pair;
5) the detectors are capable of detecting all particles, including the \( \beta \), with 100% efficiency.

With the above assumptions, the random coincidence rate due to the trigger signal is

\[
N_f' = (4N_o')^2 \eta^2 (2\sigma_t),
\]

where \( N_o' \) is the 3-body decay rate, and the factor of 4 is due to the 4 species of particles (\( \beta+p+\alpha+^{12}\text{C} \)) available in each decay. With the further assumption that the placement of PIPL at 160° to SSDR results in 100% detection efficiency of the carbon ion for 3-body breakup events in which the \( \alpha \) particle is detected by SSDR, the rate of true triple coincidences is given by (see discussion of triple-coincidence detection efficiency in the last section)

\[
N_t' = \frac{1}{2} N_o' \eta^2,
\]

in which the factor of \( \frac{1}{2} \) is due to the fact that the detection of the \( p-\alpha \) pair cannot be exchanged in the pair of SSDs. Therefore, the ratio of false to true triple coincidences is

\[
\frac{N_f'}{N_t'} = 64 N_o' \sigma_t.
\]
As a sample calculation, consider the 3-body decay of the IAS in $^{17}$F, which is populated in the $\beta$ decay of $^{17}$Ne with a branching ratio of 0.74%. Assuming a yield of $10^5$ $^{17}$Ne/s and $\sigma_t = 50 \cdot 10^{-9}$ s, Eq. (A.23) gives

$$\frac{N'_t}{N_t} = 64 (10^5 \cdot 0.74) (50 \cdot 10^{-9}) = 0.24\%.$$ (A.24)

With the many assumptions involved, the above is at best an order-of-magnitude estimate. The calculation also ignores multiple triggers. Nevertheless, the result shows that, with a yield of $10^5$ $^{17}$Ne/s, accidental coincidences do not seem to contribute significantly. However, since Eq. (A.23) shows that the ratio of accidental to true coincidences is proportional to the 3-body decay rate $N'_t$ and the resolving time $\sigma_t$, a substantial increase in the yield of $^{17}$Ne might not be beneficial, while a decrease in $\sigma_t$ might be necessary. This potential problem should be kept in mind in designing the ultimate experiment and in setting up the electronics.
Monte Carlo Algorithm

In this appendix, the algorithm used in the Monte Carlo (MC) simulation of the 3-body breakup of the IAS in $^{17}$F via the three experimentally observed channels, viz., the 9.59 MeV state in $^{16}$O, and the 2.37 and 3.50\textsuperscript{1} MeV states in $^{13}$N (see Chapter 2) is described. The results from this MC simulation have been superseded by those obtained with the analytic calculation described in Sec. 3.2. Nevertheless, it is hoped that this method can serve as a basis for extensions into more sophisticated descriptions of the $\beta$-delayed 3-body breakup process. The basic methods of Monte Carlo simulations—issues on random number generators and general algorithms in the generation of random deviates, etc—can be found in Ref. [84].

The first MC simulation [51] of the $\beta$-delayed 3-body breakup of $^{17}$F\textsuperscript{*} has been briefly described in Sec. 2.1.1. That simulation was an ambitious one in the sense that all the $^{16}$O branches known to contribute to 3-body breakups were included, while the $^{13}$N branches were ignored, in the calculation. However, as mentioned above, the simulation described in this appendix focuses only on the 3 observed 3-body breakup channels from the IAS. In terms of computational methods, the major difference between this simulation and the previous one is in the description of the energy spectrum of the intermediate state [$^{16}$O or $^{13}$N, (see App. A)]; a modified R-matrix approach has been used instead of a Breit-Wigner approach. The methods for calculating the

\textsuperscript{1}The 3.55 MeV state has been ignored. See footnote on p. 101.
kinematic variables also differ, but the differences are more in style than in principle.

The same objective is shared by the MC method and the analytic method described in Sec. 3.2, viz., generation of laboratory particle spectra in the $\beta$-delayed 3-body breakup of $^{17}$Ne. In many ways, if only a single breakup channel is considered, the two methods are similar except that whenever a variable is integrated over in the analytic method, a random deviate is generated in the MC method. In essence, the analytic method includes all the possibilities of a particular channel while the MC method only samples the possibilities; the two methods should agree if the same set of parameters is used for both methods, and good statistical accuracy is acquired in the MC method. However, the analytic method can only handle one channel at a time while, in principle, the MC method can handle any number of channels simultaneously. In this respect, the MC method is more suitable for the study of multi-channel processes than the analytic method. As mentioned in Chapter 3, if the $\beta$-decay branch is included, or the finite size of the beam spot is taken into account, the MC method is definitely more appropriate.

B.1 The Algorithm

The only calculation in the MC method involving R-matrix theory is in the description of the energy spectrum of the intermediate state. The spectra for the 9.59 MeV state in $^{16}$O, and the 2.37 and 3.50 MeV states in $^{13}$N, are calculated with Eq. (3.52). Essential parameters used in the R-matrix calculation have been listed in Table 4.1. The resulting spectra, which are effectively the probability distribution functions (pdf's) of the excitation energy of the intermediate state for the respective channels, are calculated at the beginning of the simulation and stored in memory for subsequent use in the generation of individual events (see below).

The 3-body breakup of a $^{17}$F nucleus is assumed take place in two steps [71]: proton
followed by $\alpha$ decay for the $^{16}$O channel, and *vice versa* for the $^{13}$N channels. It is obvious that, except for the difference in their observed kinematic energies, the daughter particles are identical ($p+\alpha+^{12}$C).

The $^{17}$F nucleus is assumed to be initially at rest at the origin in the laboratory frame (see footnote on p. 13). To specify a 3-body breakup event, it suffices to identify the energies and directions of the three daughter particles. Since only the IAS, with a natural width of 0.18 keV, is considered here, the 3-body breakup Q value ($Q_{3b}$) can be treated as a constant (3.431 MeV). If a state other than the IAS is considered, another random deviate must be generated to account for the natural width of the state.

As mentioned above, this simulation concerns only the three channels which have been observed experimentally. The first step in the generation of an event is to determine the decay channel ($dc$), which is accomplished by assigning each channel with a number $\lambda$, as labeled in Table 4.1, and generating this number randomly with a distribution consistent with the (estimated) relative branching ratios of the 3 channels.

The next step is to determine the excitation energy ($E_x$) of the intermediate state into which the IAS decays. As mentioned earlier, the probability density functions ($pdf's$) of the intermediate states are calculated at the beginning of the simulation and stored in memory. The respective $pdf$ for the channel determined in the first step should be used.

Once the excitation energy of the intermediate state is determined, the Q value of the first decay ($Q_1$) can be calculated:

$$Q_1 = E_{IAS} - E_x - E_{ft}$$  \hspace{1cm} (B.1)

where $E_{IAS} = 11.193$ MeV is the level energy of the IAS, and $E_{ft} = 0.601$ MeV is the energy of $^{16}$O(gs) relative to that of $^{17}$F(gs). As mentioned before, the total 3-body breakup Q value ($Q_{3b}$) is 3.431 MeV, and energy conservation requires that the Q value
of the second breakup is

\[ Q_2 = Q_{3b} - Q_1. \] (B.2)

Using the law of momentum conservation, the kinetic energy of the first daughter particle (proton in the $^{16}$O channel and $\alpha$-particle in the $^{13}$N channel), as well as that of the heavy recoil ($^{16}$O or $^{13}$N), can then be determined with $Q_1$, and the center-of-mass (cm) energies of the daughter particles in the decay of the intermediate state can be determined with $Q_2$. These two daughter particles will, of course, be back-to-back in their cm frame. Furthermore, their trajectories in the cm frame will bear a certain angular correlation to that of the daughter particle from the first decay depending on the spin of the states involved, as well as the spin and orbital angular momentum of the emitted radiations (see App. C). A calculation following the formalism described in [85] has shown that this angle ($\theta'$), as depicted in Fig. A.1, is isotropic with respect to the direction of the heavy recoil for both the IAS→$^{16}$O(9.59)+p and IAS→$^{13}$N(2.37)+$\alpha$ channels, but for the IAS→$^{13}$N(3.50)+$\alpha$ channel, $\theta'$ has a distribution of $\mathcal{W}(\theta') = 1 + P_2(\cos \theta')$, where $P_2$ is the 2nd order Legendre polynomial (see App. C). In the simulation, $\theta'$ is generated in the range $[0, \pi]$ according to the appropriate angular distribution of the respective channel. Once $\theta'$ is determined, the angle of $\vec{v}_2$ with respect to the direction of the heavy recoil ($\theta$) (see Fig. A.1), can be calculated using Eq. (A.14). The direction of $\vec{v}_3$ can be obtained in the same manner. Once the directions of $\vec{v}_2$ and $\vec{v}_3$ are known, their magnitudes can be calculated from trigonometry. Consequently, the energies of the respective particles in the laboratory frame can be calculated since the identities of the particles are known in the simulation.

After the energies of the 3 daughter particles and the relative angles between them are obtained, the directions of the trajectories relative to a fixed coordinate system have to be determined. As mentioned before, the $^{17}$F nucleus is assumed to be initially at rest in the laboratory frame and, as far as the first decay is concerned, it is a 2-body
§B.1 The Algorithm

decay and, therefore, the first daughter particle is expected to be emitted isotropically; 
i.e., the plane in which the decay takes place is isotropic with respect to a fixed coordi­
nate system. Accordingly, two isotropic random directions are generated to define the 
random plane, and the direction of the first daughter particle is set to coincide with 
one of the two random directions. Finally, the vector product between the two random 
directions defines an axis-of-rotation which is then used to establish the directions of 
the other two particles according to their angles relative to the first daughter particle. 

At the conclusion of generating an event, the energies of the 3 particles as well as their 
directions, each of which is specified by a polar and an azimuthal angle, are determined.

Simulation of the finite size of the beam spot is accomplished by assuming the 
beam spot to be circular with the decay vertices normally distributed within it. The 
locations of the trajectories obtained previously (originating from the origin) can then 
be translated for an amount equivalent to the vector distance between the decay vertex 
and the origin. The resulting trajectories can then be tested to determined if all of 
them are intercepted by a predefined detection system, and if so, the event can be 
written to a file by outputting the 3 energies, 3 azimuthal angles, and 3 polar angles. 
The simulation terminates when a predefined number of events has been generated, and 
laboratory particle spectra and angular distribution spectra can be generated from the 
data.

As input to the MC program, a text file is set up with information about the 
geometry of a predefined experimental setup, number of events to be simulated, energy 
resolutions, energy thresholds, estimated relative BRs, and the radius of the beam 
spot. This text file can be modified to simulate different experimental configurations 
and other related parameters. The algorithm for generating 3-body breakup events 
from the IAS for a predefined detection system is listed in Table B.1. The source codes

\[\text{This effect may not be realized in practice since the distribution of decay vertices within the beam spot may not be determinable.}\]
in Fortran 90 can be found in Ref. [67].

### B.2 Comparisons between MC and EXPT data

The validity of the present MC method can be demonstrated by a comparison between MC and experimental data. In particular, a comparison between an $\alpha$ spectrum obtained with the algorithm described in the last section with a proton threshold of 500 keV, and an experimental $\alpha$ spectrum obtained from the type $\alpha$ data of Jul97 with a proton threshold of 400 keV, is shown in Fig. B.1. The use of different proton energy thresholds in the comparison is to account for the uncertainty in this threshold at the low-energy region (see Sec. 4.1.3). The relative BRs between the 3 breakup channels obtained in Chapter 4 were used, and no background assumed, in generating the MC data. It can be seen in Fig. B.1 that the MC result is in reasonable agreement with the triple-coincidence data of Jul97. The minor discrepancies can be attributable to the omission of background, the uncertainties in relative BRs and energy thresholds, the improper treatment of the beam spot as a Gaussian distribution, or the uncertainties in any of the parameters listed in Table 4.1, in the MC calculation.
Table B.1: Monte Carlo Algorithm for generation of 3-body breakup events. (Text in brackets corresponds to the $^{13}$N channel. Steps labeled with * require generation of random deviates.)

1*. Determine the decay channel ($dc$)— 1) $^{16}$O–9.59; 2) $^{13}$N–2.37; 3) $^{13}$N–3.50/3.55.

2*. Determine the excitation energy ($E_x$) of the intermediate state.

3. Compute the $Q$ values of the first and second decays ($Q_1$ and $Q_2$).

4. Use $Q_1$ to compute the lab kinetic energies of proton and $^{16}$O ($\alpha$ and $^{13}$N) in the first decay.

5. Use $Q_2$ to compute the kinetic energies of $\alpha$ and $^{12}$C [proton and $^{12}$C] in the CM frame of the recoiling $^{16}$O [$^{13}$N].

6*. Determine the angle between the recoiling $^{16}$O [$^{13}$N] in the lab frame and the $\alpha$ [proton] in the CM frame ($\theta'$ in Fig. A.1).

7. Compute the direction of $\alpha$ and $^{12}$C [proton and $^{12}$C] relative to the proton [$\alpha$] in the laboratory frame.

8. Compute the energies of $\alpha$ and $^{12}$C [proton and $^{12}$C] in the lab frame.

9. Convolute the energies with resolutions.

10*. Randomize isotropically the direction of the plane in which the 3-body breakup takes place and compute the directions of the 3 daughter particles with reference to a fixed coordinate system.

11*. Assume a normal distribution of decay vertices within the beam spot; translate the coordinate system accordingly.

12. Determine if triple coincidence occurs for a predefined detection system.

13. If triple coincidence occurs, determine location of detector elements (strip numbers) if SSDs are used.

14. If triple coincidence occurs, output decay channel, energies and directions of the 3 particles, and strip numbers if SSDs are used.
B.3 Angular distribution

The distributions of angles between the daughter particles of the 3 observed 3-body breakup channels, particularly the angle between the $\alpha$ particle and carbon ion ($\angle\alpha^{12}\mathrm{C}$), are of interest since, given sufficient angular resolution, the 3 channels might be differentiated, at least to a certain extent, by selecting data within only a certain range of $\angle\mathrm{p-\alpha}$ preferred by a particular channel. The distributions of $\angle\mathrm{ac}$ for the 3 channels, obtained with a calculation using only up to step 7 of the algorithm described in Table B.1 and weighted by their respective BR obtained in Chapter 4, are shown in Fig. B.2. The distribution for the 9.59 channel is of prime interest since the reduced width of the 7.12 MeV state might be obtained with a study of the interference between the 9.59 and 7.12 channels (see Chapter 5). It can be seen in Fig. B.2 that, if $\angle\mathrm{p-\alpha}$ is chosen at 160° with a range of a few degrees, most of the contributions from the 2.37 and 3.55 channels can be eliminated. However, while contribution from the 3.55 channel is negligible within the window (note the log scale used in Fig. 2.1), that from the 2.37 channel is relatively strong, and such background must be taken into account in the analysis of data.

\footnote{The $\mathrm{p-^{12}\mathrm{C}}$ and $\mathrm{p-\alpha}$ angles do not give rise to distinctive angular distributions (see Fig. 2.1).}
B.4 Particle identification

The MC data also provide information on particle identification for events obtained with the Jul97 setup. Due to the thickness of the dead layer of the SSD as explained in Sec. 2.2.2.2 on p. 50, the $^{12}$C ions are always assumed to be detected by the PIPS, while the proton and $\alpha$ particle by the SSDs. The Jul97 setup also requires that each triple-coincidence event must involve both SSDs for kinematic reason. Consequently, the identities of the proton and $\alpha$ particle might be ambiguous. However, according to the MC data, for events of type $\alpha$ obtained with the Jul97 setup (see Sec. 2.2.2.3), the $\alpha$ particle and $^{12}$C ion are always detected by the SSD-PIPS pair at 160° with the proton picked up by the other SSD. As can be seen in Fig. B.2, the lower limit of the $\alpha$-$^{12}$C angle for the 3 observed channels is about 145°. However, the coverage of the SSD-PIPS pair at 96° has an upper limit of about 135° and, therefore, the pair is not capable of detecting any $\alpha$-$^{12}$C pair. Thus, particle identification is unambiguous. Note that this is not true for the Aug96 setup due to the closer geometry employed.
Angular Correlation in Cascade Transitions

Figure C.1: Description of angular momentum coupling of 2-step cascade transitions. The transition involves 3 nuclear states with angular momenta (intrinsic spins) $a$, $b$, and $c$; and 2 emitted particles with total angular momenta $L_1$ and $L_2$, orbital angular momenta $\ell_1$ and $\ell_2$, and intrinsic spins $s_1$ and $s_2$.

Angular correlation theory in nuclear physics deals with quantum mechanical calculations of angular distributions among radiations in multi-stage nuclear reaction or decay processes. The distribution comes as a result of the coupling of the various angular momenta of the nuclear states, as well as the absorbed or emitted radiations, involved in the process. This appendix deals only with cases of particle emissions. A schematic diagram describing the coupling of angular momenta in a 2-step particle-decay transition is shown in Fig. C.1. Let us assume that the initial, intermediate, and final states have angular momenta (intrinsic spins) $a$, $b$, and $c$, respectively, and
the two emitted particles have total angular momenta $L_1 = \ell_1 + s_1$ and $L_2 = \ell_2 + s_2$. The angular distribution of the first emitted particle with respect to the second, in the center-of-momentum (cm) frame of the intermediate state, is usually not isotropic but, rather, depends on all of the abovementioned quantities. A comprehensive treatment of angular correlation theory can be found in Ref. [85] on which the following discussion is based.

### C.1 Pure transitions

For two successive pure transitions in which the emitted particles have definite angular momenta, the angular correlation function $\mathcal{W}(\theta')$ can be expressed as

$$
\mathcal{W}_{L_1 \ell_1, L_2 \ell_2}(\theta') = (-1)^{a+b-2b_1} \frac{\hat{b}^2}{(4\pi)^2} \sum_k (-1)^{-L_1-L_2} c_{k0}(L_1 \ell_1) c_{k0}(L_2 \ell_2) \times \frac{W(b b L_1 L_1; k a) W(b b L_2 L_2; k c) P_k(\cos \theta')}{},
$$

(C.1)

where $\theta'$ is the angle between the first and second emitted particles (see Fig. A.1); $L$ denotes the quantities $L$, $\ell$, and $s$; $\hat{b}^2 = 2b + 1$; $W$ is a Racah coefficient (see, e.g., Ref. [40, Sec. B4]); $P_k$ is the $k$th-order Legendre polynomial; and $c_{k0}$ is a radiation parameter given by

$$
c_{k0}(LL') = \frac{\hat{L} \hat{L}'}{4\pi} (-1)^{L-s} (L s, L' -s | k 0); \quad \hat{L} = \sqrt{2L + 1},
$$

(C.2)

in which $(L s, L' -s | k 0)$ is a Clebsch-Gordan coefficient. The selection rule for $k$ in the summation requires that it must be even and

$$
k \leq (2\ell_1, 2L_1, 2\ell_2, 2L_2, 2b).
$$

(C.3)

Note that Eq. (C.1) has been normalized such that the coefficient of $P_0$ equals unity.

---

1 $\theta'$ is used here, in consistence with Fig. A.1.
Equation (C.1) has been used in calculations of p-α distribution functions in the cm frame of the intermediate states in the 3-body breakup of the IAS via the 7.12 and 9.59 MeV states in $^{16}$O, and the 2.37 and 3.50 MeV states in $^{13}$N. While the $\ell$ values of the emitted particles for the $^{13}$N channels are unique, that of the proton for the $^{16}$O channels can be 0 or 2. The angular momenta pertaining to each of the 4 channels are listed in Table C.1, along with the calculated distributions. The Racah and Clebsch Gordan coefficients have been calculated with subprograms available in the CERN program library [86]. The calculations have shown that, for lowest order of $\ell$ values, the distributions for transitions through the 7.12, 9.59, and 2.37 channels are isotropic (since $P_0 = 1$ is independent of angle), while that through the 3.55 channel follows a distribution of $P_0 + P_2$, as shown in Fig. C.2.

### C.2 Mixed transitions

The equation for pure transitions can be extended to include transitions in which one of the emitted particles has mixed angular momentum. Specifically, assuming that the first emitted particle has angular momentum which is a superposition of $L_1$ and $L'_1$ while the second emitted particle has pure angular momentum $L_2$ (this could be the case for the 7.12 and 9.59 channels in which the proton can have 2 possible $\ell$ values),

---

**Table C.1: List of angular momenta pertaining to the 4 channels in the (pure) cascade decay of the IAS in $^{17}$F, and the resulting angular distribution functions obtained with Eq. (C.1).**

<table>
<thead>
<tr>
<th>Channel</th>
<th>$a$</th>
<th>$b$</th>
<th>$c$</th>
<th>$L_1$</th>
<th>$L_2$</th>
<th>$\ell_1$</th>
<th>$\ell_2$</th>
<th>$s_1$</th>
<th>$s_2$</th>
<th>$W(\theta')$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{16}$O(7.12)+p ($\ell_p=0$)</td>
<td>$\frac{1}{2}$</td>
<td>1</td>
<td>0</td>
<td>$\frac{1}{2}$</td>
<td>1</td>
<td>0</td>
<td>$\frac{1}{2}$</td>
<td>0</td>
<td>0</td>
<td>$P_0$</td>
</tr>
<tr>
<td>$^{16}$O(7.12)+p ($\ell_p=2$)</td>
<td>$\frac{1}{2}$</td>
<td>1</td>
<td>0</td>
<td>$\frac{3}{2}$</td>
<td>1</td>
<td>2</td>
<td>$\frac{1}{2}$</td>
<td>0</td>
<td>0</td>
<td>$P_0 + P_2$</td>
</tr>
<tr>
<td>$^{16}$O(9.59)+p ($\ell_p=0$)</td>
<td>$\frac{1}{2}$</td>
<td>1</td>
<td>0</td>
<td>$\frac{1}{2}$</td>
<td>1</td>
<td>0</td>
<td>$\frac{1}{2}$</td>
<td>0</td>
<td>0</td>
<td>$P_0$</td>
</tr>
<tr>
<td>$^{16}$O(9.59)+p ($\ell_p=2$)</td>
<td>$\frac{1}{2}$</td>
<td>1</td>
<td>0</td>
<td>$\frac{3}{2}$</td>
<td>1</td>
<td>2</td>
<td>$\frac{1}{2}$</td>
<td>0</td>
<td>0</td>
<td>$P_0 + P_2$</td>
</tr>
<tr>
<td>$^{13}$N(2.37)+\alpha</td>
<td>$\frac{1}{2}$</td>
<td>$\frac{1}{2}$</td>
<td>0</td>
<td>$\frac{1}{2}$</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>$\frac{1}{2}$</td>
<td>$P_0$</td>
</tr>
<tr>
<td>$^{13}$N(3.50)+\alpha</td>
<td>$\frac{1}{2}$</td>
<td>$\frac{3}{2}$</td>
<td>0</td>
<td>$\frac{3}{2}$</td>
<td>2</td>
<td>1</td>
<td>0</td>
<td>$\frac{1}{2}$</td>
<td>$P_0 + P_2$</td>
<td></td>
</tr>
</tbody>
</table>
Figure C.2: Angular distributions for the cascade decay of the IAS (assumed to be pure transitions). The calculations show that, for lowest order of $\ell$ values, the angular distribution for decays through the 7.12, 9.59, and 2.37 states are isotropic, while that through the 3.55 state follows a distribution of $P_0 + P_2$.

the angular correlation function between the two particles can be expressed as [85]

$$\mathcal{W}(\theta') = |<b||L_1||a>|^2 \mathcal{W}_{L_1 L_1', L_2 L_2} + |<b||L_1'||a>|^2 \mathcal{W}_{L_1 L_1', L_2 L_2} + 2\text{Re}[<b||L_1||a><b||L_1'||a>^*] \mathcal{W}_{L_1 L_1', L_2 L_2}.$$  (C.4)

The first two terms are just contributions from pure transitions, as given by Eq. (C.1), weighted by the appropriate reduced matrix elements, i.e., the parts of the matrix elements which are invariant with respect to any coordinate system, pertaining to the first transition. The third term, which represents the interference between the two components $L_1$ and $L_1'$, is given by

$$\mathcal{W}_{L_1 L_1', L_2 L_2}(\theta') = (-1)^{a+c-2b} \hat{b}^2 (4\pi)^2 \sum_k (-1)^{L_1} c_{k0}^*(L_1 L_1') (-1)^{L_2} c_{k0}(L_2 L_2) \times W(b;b L_1 L_1'; k a) W(b;b L_2 L_2'; k c) P_k(\cos \theta'),$$  (C.5)

where $k$ is still required to be even and

$$k \leq (\ell_1 + \ell'_1, L_1 + L_1', 2\ell_2, 2L_2, 2b).$$

Equation (C.4) would be useful if $C_{10}$ or $C_{22}$ in Table 4.3 were non-zero.
Appendix D

Electronics Diagrams of Nov98

The electronics setup for Aug96 was very simple and the electronics diagrams will not be listed here. The setup for Nov98 was similar to that of Jul97, except for the addition of more $\beta$ detectors and a thick Si detector, and in the detail of the master trigger circuit. Therefore, only electronics diagrams belonging to the Nov98 setup will be listed.
Double-Sided Silicon Strip Detectors
Figure D.2: PIPS DETECTOR ELECTRONICS SETUP
Figure D.3: Scintillator paddles electronics setup
MASTER TRIGGER FIFO

PULSER (0) →
SSD-L/PIP-L CONN. (1) →
SSD-R/PIP-R CONN. (2) →
SSD-L DOUBLE HIT (3) →
SSD-R DOUBLE HIT (4) →
SSD-L/R CONN. (5) →
SSD/PIPS PRESCALED SINGLES (6) →
THK PRESCALED SINGLES (7) →
THK/BETA CONN. (8) →

→ MASTER TRIGGER FIFO

Title: Electronics Diagram (IV)

Drawn By: J. C. Chow
Project: SETUP 1998
Date: OCT 15, 1998
Time: 4 of 5
Master Trigger

Start Stop

Latch

Reset from ND027 ch#2

- Begin of run
- Resume run
- End of event readout

Computer Busy from ND027 Ch#0

- End of run
- Pause run
- Processing events

Busy

Veto

Quadrupler

Busy out

821Z

Master Trigger

Computer Busy from ND027 Ch#0

- End of run
- Pause run
- Processing events

Busy

Veto

Quadrupler

Busy out

821Z

Master Trigger

Computer Busy from ND027 Ch#0

- End of run
- Pause run
- Processing events

Busy

Veto

Quadrupler

Busy out

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- End of run
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- Processing events

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Master Trigger

Computer Busy from ND027 Ch#0

- End of run
- Pause run
- Processing events

Busy

Veto

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Busy out

821Z

Master Trigger

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- End of run
- Pause run
- Processing events

Busy

Veto

Quadrupler

Busy out

821Z

Master Trigger

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- End of run
- Pause run
- Processing events

Busy

Veto

Quadrupler

Busy out

821Z

Master Trigger

Computer Busy from ND027 Ch#0

- End of run
- Pause run
- Processing events

Busy

Veto

Quadrupler

Busy out

821Z

Master Trigger

Computer Busy from ND027 Ch#0

- End of run
- Pause run
- Processing events

Busy

Veto

Quadrupler

Busy out

821Z

Master Trigger

Computer Busy from ND027 Ch#0

- End of run
- Pause run
- Processing events

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Veto

Quadrupler

Busy out

821Z

Master Trigger

Computer Busy from ND027 Ch#0

- End of run
- Pause run
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- Processing events

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Veto

Quadrupler

Busy out

821Z

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Computer Busy from ND027 Ch#0

- End of run
- Pause run
- Processing events

Busy

Veto

Quadrupler

Busy out

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