AN EXPERIMENTAL INVESTIGATION OF
THE HEAT AND MASS TRANSFER OF GRAUPEL

by

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Abstract

The heat and mass transfer of graupel growing in a simulated cloud environment has been investigated experimentally. Rigidly suspended graupel were grown in a wind tunnel airstream under liquid water contents from 0.5 to 3.0 g m$^{-3}$, velocities from 1.1 to 3.0 m s$^{-1}$, ambient temperatures from -4.4 to -20.9°C, cloud droplet median volume radii from 12 to 21 µm and ambient pressures from 100 to 60 kPa. Measurements of the mass, volume, growth height, geometric shape and surface temperature with time were used to calculate the bulk collision efficiency, Nusselt and Sherwood numbers and accretion density. The bulk collision efficiency and Nusselt number were parametrized in terms of the Stokes parameter and Reynolds number respectively. The density and cone angle were parametrized in terms of the relative graupel-air stream velocity, the cloud droplet median volume radius and the surface temperature. The surface temperature measurements were made remotely with an infrared radiometer to within ± 0.2°C, and represent the first measurements of the surface temperature of growing graupel.

The bulk collision efficiency was found to be approximately 30% lower than that for ideal spheres as calculated by Langmuir and Blodgett (1946). The Nusselt number was found to be approximately 50% higher than that for smooth cones. The enhanced heat convection and mass deposition or sublimation is attributed to the roughness of the ice surface. Densities ranged between 0.16 and 0.70 g cm$^{-3}$ and were between 10 and 30% lower than those predicted by Heymsfield and Pflaum (1983) for densities lower than 0.4 g cm$^{-3}$.

The parametrizations of density, cone angle, Nusselt number and bulk collision efficiency represent complete solutions to the heat and mass transfer equations for graupel. Hence the graupel stage of the cold rain process can be fully and accurately incorporated into cloud dynamical models. Such a complete numerical description has not been previously reported.
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Chapter 1

Introduction and Overview

1.1 Purpose

The cold rain process generates precipitation through the formation and growth of ice particles within a cloud, which usually melt during their descent to the ground. An important component of the cold rain process is the formation of graupel. Graupel grow by the accretion of supercooled water droplets onto an ice embryo, such as a frozen cloud droplet or onto an ice crystal that has grown by deposition. They are white and opaque, with diameters up to 5 mm and densities typically between 0.2 and 0.7 g cm$^3$. The heat and mass transfer of an individual graupel has never been fully investigated.

The purpose of this thesis is to experimentally measure the heat and mass transfer of a graupel to a sufficient resolution to allow an understanding of how individual cloud parameters affect the graupels’ geometric characteristics and its growth. This will allow parametrizations of the dimensionless parameters that control the heat and mass transfer. Knowledge of the details of graupel formation is fundamental to an understanding of the kinetics and dynamics of precipitation so that parametrizations of graupel aspects are essential to cloud models. Graupel are also believed to be instrumental for secondary ice crystal production in clouds and consequently form an integral part of the mechanism of thunderstorm electrification. Virtually all precipitation that forms in mid and higher latitudes is dominated by the cold rain process and may therefore involve graupel formation.

1.2 Theories of the Formation Mechanisms of Graupel

The mechanisms involved in the cold rain process are shown in Figure 1.1. Central to this process is the evolution of ice crystals or frozen droplets into graupel, which in turn
Figure 1.1: Evolution of the cold rain process.
evolve into hail or rain. These mechanisms are qualitatively well understood, and each is discussed below.

In order for an ice particle to begin riming, it must grow to a minimum size so that its fall speed relative to cloud droplets enables it to collide with them. This minimum size is dependent on the crystal size and type, and the size distribution of droplets within the cloud. Pitter and Pruppacher (1974) modeled ice crystal droplet collisions and determined that a minimum size of 150 μm was required for a hexagonal plate to rime. Numerous observations (Ono, 1969; Wilkens and Auer, 1970; and Reinking, 1979) have shown that the size necessary for the onset of riming of a plane ice crystal varies from 150 to 300 μm depending on the nature of the crystal.

Observations of natural graupel are numerous and well documented (see for example Nakaya, 1954; List, 1958; Knight and Knight, 1973; Heymsfield, 1978; Locatelli and Hobbs, 1974; Takahashi and Fukuta, 1988). It is well understood that graupel formation is a result of the accretion of supercooled water droplets on an ice embryo within a cloud. Nakaya (1954) classified graupel into three categories based on their geometry and presumed formation. These included lump graupel which formed from spatial crystals, hexagonal graupel which formed from dendritic crystals, and conical graupel which he refrained from postulating an origin.

The origin of conical graupel has been a topic of considerable debate. Reynolds (1878) proposed that hailstones would form when larger ice particles overtook smaller ice particles thereby aggregating, increasing their mass and falling faster in an accelerating process. He postulated that particles striking near the graupel edge would overhang so that the hailstone grew in a conical shape, fanning out into the wind. He supported these ideas with simple riming experiments in his laboratory. Arenberg (1941) proposed that a dendritic ice crystal in a cloud would accumulate supercooled water droplets on its underside, growing in a similar manner to that discussed by Reynolds. However, Arenberg postulated a flowfield around the crystal which caused the riming to form inwards so that the ice
eventually formed a downward pointing cone. Weickman (1953) suggested that graupel embryos were composed of frozen drops. Based on observations of natural graupel and hail, List (1958, 1960) stated that graupel grew by accretion on dendritic crystals which had grown by deposition. The rime fanned out into the airflow forming a conically shaped particle with a crystal at the tip. He also showed evidence of hexagonally arranged grooves along the conical sides that reflected the original branches of the dendrites. Holyroyd (1964) proposed that the embryo of conical graupel were clusters of 2 to 6 slightly rimed needles that continued riming after aggregating together.

Knight and Knight (1973) failed to observe any ice crystal or frozen drop graupel embryos in several hundred thin sections of hailstones, for which graupel had served as the hailstone embryos. Subsequent observations of graupel from graupel showers in the Rocky mountains showed relatively few recognisable crystal embryos. Following List (1960), they postulated that graupel originate as planar or dendritic snow crystals falling with their plane axis perpendicular to the flow. However they proposed that the crystals rime at the corners so that a single crystal could have several graupel embryos growing simultaneously. Eventually the graupel break off individually or in groups so that the original crystal is not observed in graupel found at the ground. This is speculative and has not been physically observed. Reinking (1975) provided extensive observations of graupel from snowstorms in Sierra Nevada, which indicated that graupel does not develop predominately on the snow crystals with the longest lifetimes. When graupel does form on regular crystals, it does so predominantly on a select few relatively small crystals that accrete droplets at comparatively rapid rates. Reinking’s observations also indicated that graupel does not always employ crystals as embryos and hence some undetermined growth process must be operating. Heymsfield (1978) postulated that small crystals made up the majority of embryos for the summertime clouds that he studied. Harimaya (1976) expanded Nakaya’s classification scheme by summarizing the formation of graupel according to the embryo and falling behaviour of various observed and postulated particles. Hexagonal graupel were associated with slightly rimed plane crystals that fell without tumbling and which lacked
enough accretion to mask the original hexagonal shape. Lump graupel formed from heavily rimed columnar crystals, radiating assemblies or frozen drops that tumbled or rotated while falling. Conical graupel developed from heavily rimed plane crystals, columnar crystals, frozen drops or radiating assemblages that did not tumble or rotate while falling. Further, several conical graupel could grow simultaneously on a plane-type crystal and break off in a manner similar to that postulated by Knight and Knight. This classification scheme covered most observations and formation theories although made no attempt to determine any dominant formation mechanism. It also included several processes that have not been observed. Recent observations by Takahashi and Fukuta (1988) of about 1000 graupel from storms in Salt Lake City Utah and Sapporo Japan showed that 55% of the graupel did not have a clearly identifiable embryo.

Conical graupel-type particles comprised 80% of the embryos of hailstones as observed by List (1960). These and direct observations of graupel (List, 1958) led him to include graupel as a central stage in the cold rain process whereby graupel served as the embryo for hail. Carte and Kiddler (1966) observed that there were three types of hailstone embryos: clear particles, opaque particles and cones with opaque tips. The opaque and conic types comprised 75% of the embryos. Knight and Knight (1970) studied several hundred hailstones collected from 40 storms. They reported that 60% of the hailstone embryos were conical graupel, 10% were spherical and opaque and 25% were spherical and clear. The latter were believed to be frozen drops. They observed that for larger hailstones (i.e. those greater than 2.5 cm in diameter) there were virtually always distinct early growth stages ranging from 0.5 to 1.0 cm in diameter.

1.3 Conditions for the Formation of Graupel

The nature of the ice growth usually shows a distinct difference between the graupel embryo stage and the hailstone growth stage (List, 1960; Knight and Knight, 1970). Such observations have inferred that a necessary step in large hailstone formation was embryo formation in one set of cloud conditions followed by injection of the embryos into another
set of conditions more conducive to hailstone growth. This was supported by observations and simulated trajectory models by Heymsfield et al. (1980) and Heymsfield (1982). Ziegler et al. (1983) modeled hail trajectories based on observations of Oklahoma multicellular storms and found that the formation of large hail in the model was almost entirely due to injection of embryos into the major storm updraft. These graupel originated primarily in feeder cells that grew upwind of the main updraft region.

Typical cloud conditions under which graupel or hail embryos are produced have been reported by Heymsfield (1978) and Heymsfield and Hjelmfelt (1984). Average liquid water contents range from 1 to 2 g m\(^{-3}\) with peak values up to 4 to 6 g m\(^{-3}\). Updraft velocities vary from 5 to 15 m s\(^{-1}\). Temperatures within main growth regions are typically warmer than -20°C. Using doppler radars, the initial precipitation particle development (i.e. graupel) was usually observed at the -15°C level.

1.4 Other Cloud Mechanisms Involving Graupel

Other mechanisms associated with graupel growth include the production of secondary ice crystals within a cloud, and thunderstorm electrification. Although neither subject is investigated within this thesis, it is instructive to understand how graupel is involved within each process because the results of this investigation may have a direct application to cloud models investigating these phenomena.

1.4.1 Secondary Ice Crystals

Secondary ice crystals are produced in ways other than by homogeneous or heterogeneous ice nucleation. Extensive experimental work (Hallett and Mossop, 1974; Mossop and Hallett, 1974; Heymsfield and Mossop, 1984) have showed that when supercooled water droplets collide with an ice surface with a surface temperature between -3 and -8°C secondary ice crystals are produced. The splinter rate depends on the number of cloud droplets larger than 25 \(\text{\mu m}\) in diameter and the surface temperature of the rime,
with the maximum splinté rate occurring at -5°C. Similar observations have been made in field projects, theoretical studies and other experimental investigations (Mossop, 1985). It is believed that the production of ice splinters is related to the increase of pressure within a droplet as it freezes after colliding with an ice surface (Griggs and Choularton, 1983).

1.4.2 Thunderstorm Electrification

The generation and separation of electric charge and the associated growth of the electric field within a thunderstorm are produced by the interactions of graupel, supercooled droplets and ice crystals (Saunders, 1988). Charges are transferred when ice crystals hit and rebound from an ice surface which is growing by accretion of supercooled cloud droplets. Laboratory experiments (Keith and Saunders, 1990; Jayaratne et al., 1983) have shown that the magnitude of the charge transfer depends on the temperature, crystal dimension, relative velocity, liquid water content and impurity content of the water droplets. The charge transfer is believed to be caused by charge dislocations on the surfaces of the interacting ice crystal and graupel (Keith and Saunders, 1990). Observations of the microphysical, electrical and dynamical evolution of a thunderstorm by Dye et al. (1986) showed strong consistencies with the theory that collisional interactions of ice particles were responsible for the electrification observed.

An alternate theory of thunderstorm electrification also involves charge separation due to the riming of graupel. Mason (1988) has calculated that droplets making temporary grazing contact with graupel can account for sufficient charge separation to create the electric fields observed in thunderstorms.

1.5 Natural Observations of Graupel

Geometric characteristics of natural graupel have been observed and measured by several investigators. In general, measurements of mass m, diameter D and terminal velocity V are used to derive m-D, V-D and R_e-D parametrizations where R_e is the
Reynolds number. These are then used to determine drag coefficients \( C_d \) and Best number (\( \text{Be} = C_d R_e^2 \)) relationships as discussed in Heymsfield and Kajiwaka (1987). The diameter is usually taken as the diameter of the base which is representative of the area of the graupel perpendicular to the flow.

List (1958) studied several thousand graupel that fell at 2750 m on a mountain side. They ranged in diameter from 0.5 to 5 mm, with peak diameters between 1.5 and 2.5 mm. Cone angles ranged from 45 to 95° and densities from 0.5 to 0.7 g cm\(^{-3}\). In some cases the graupel embryos were observed to be dendritic crystals. The median diameter of frozen droplets on the graupel surface was measured as approximately 20 μm. The relatively higher densities imply that these graupel either grew at temperatures close to 0°C or experienced some soaking of water droplets into the ice structure at temperatures warmer than 0°C.

Zikmunda and Vali (1972) measured graupel from a mountain observatory. Densities ranged from 0.7 to 0.2 g cm\(^{-3}\) for graupel of diameters 0.5 to 2.0 mm respectively. Terminal velocities for particles larger than 1.0 mm varied from 1 to 3 m s\(^{-1}\), increasing rapidly with diameter. Cone angles ranged from 60 to 80° and the graupel were observed to fall with their apex up. Conical graupel tended to oscillate up to 20° from the vertical with frequencies up to 50 Hz. Drag coefficients were approximately twice those of smooth spheres.

Locatelli and Hobbs (1974) measured graupel at the ground from winter storms in Washington. They measured diameters from 1 to 3 mm, densities from 0.1 to 0.45 g cm\(^{-3}\) and terminal velocities from 1 to 3 m s\(^{-1}\). There was a strong correlation between increasing velocity and diameter and a weak correlation between increasing diameter and decreasing density.

Heymsfield (1978) collected graupel from a sailplane in situ in summertime convective clouds. Most were smaller than 5 mm in diameter although particles as large
as 10 mm were observed. Densities were typically less than 0.5 g cm\(^{-3}\) with a mean of 0.45 g cm\(^{-3}\). Cone angles varied from 30 to 80° with a mean of 60°. Graupel collected at the ground from storms in Japan have been measured by Maruyama (1968) and Heymsfield and Kajiwaka (1987). Maruyama found the average density to be 0.4 g cm\(^{-3}\) irrespective of size for diameters from 2 to 7 mm. The graupel were conical with a slightly rounded base and cone angles around 70°.

Variations in density measurements among different investigators reflect different cloud conditions and terminal velocities. Densities above 0.6 g cm\(^{-3}\) are more representative of graupel that have undergone soaking, melting or slow freezing while those below 0.2 g cm\(^{-3}\) are probably lightly rimed crystals that still have terminal velocities less than 1 m s\(^{-1}\). Most of the density measurements lie between 0.2 and 0.6 g cm\(^{-3}\). Terminal velocities measured in different field studies are quite consistent. There is a strong increase in velocity for increasing diameter, i.e. \(V = D^{0.6}\) (Locatelli and Hobbs, 1974). Drag coefficients tend to be larger than those measured for smooth cones in the laboratory (List and Schemenauer, 1971). The difference may be attributed to roughness of the natural ice surface and to the relative sharp edge at the base of the models.

1.6 Experimental Density Measurements of Rime

Macklin (1962) measured the density of ice formed by accretion on icing cylinders over a wide range of controlled cloud conditions. He found the density \(\rho\) could be parametrized as

\[
\rho = 0.110 \left( \frac{-r_m V_o}{T_s} \right)^{0.76}
\]

where \(r_m\) is the median volume radius of the droplet distribution in \(\mu\text{m}\), \(V_o\) the calculated droplet impact velocity in m s\(^{-1}\) and \(T_s\) the surface temperature of the ice in °C. Macklin interpreted \(r_m^3 V_o\) as a measure of the inertia which packed the accreted droplets together.
and $r_m^2 T_s$ as a measure of the ice freezing/bonding strength times the freezing rate. Similar experiments by Buser and Aufdermaur (1973) confirmed Macklin's results although they noted that densities lower than 0.17 g cm$^{-3}$ (as measured on cylinders) are probably unrealistically low because of the empty spaces between ice lobes being averaged into the volume measurement. They used spherical models to experimentally determine that a minimum value for the density of accreted ice would be approximately 0.17 g cm$^{-3}$.

Pflaum et al. (1978) grew freely floating graupel in a wind tunnel and investigated the hydrodynamic behaviour and terminal velocities of riming frozen drops and riming simulated ice crystal plates. Using the same apparatus, Pflaum and Pruppacher (1979) measured the density of graupel grown under various cloud conditions. They parametrized density as

$$\rho = 0.261 \left( \frac{-r_m V_o}{T_s} \right)^{0.38}$$

which predicts significantly higher densities for low values of $r_m V_o T_s^{-1}$ than Macklin's formula does. It is evident from their raw data (published in Heymsfield and Pflaum, 1985) that the graupel they grew were extremely small (typically less than 0.1 mg of mass accreted per experiment) raising the possibility of large systematic errors in their geometric measurements.

Saunders and Zhang (1987) included the effects of radial forces and determined for angular accelerations larger than 6 g

$$\rho = 0.17 \left( \frac{-r_m V}{T} \right)^{0.71} \left( a_o^{-0.16} \right)$$

where $V$ is the relative particle-air velocity in m s$^{-1}$, $T$ the ambient temperature in °C and $a_o$ the radial acceleration in m s$^{-2}$. For small values of $a_o$ the latter term is $= 0.5$ so that these results are similar to Macklin's. Because the impact velocity $V_o$ must be calculated,
and the surface temperature $T_s$ is usually calculated, Saunders and Zhang considered use of the airstream velocity and ambient temperature in the parametrization a simpler approach. They assumed that the ambient and surface temperatures were approximately equal and that the airstream velocity was close to the impact velocity.

1.7 Collision Efficiency of Droplets With a Rimed Target

Mossop (1976) determined the collision (collection) efficiency of a droplet spectrum with icing cylinders at velocities between $1.4$ and $3.0$ m s$^{-1}$. He found that the collision efficiencies were about $20$ to $30\%$ lower than those predicted by theoretical calculations for smooth cylinders (Ranz and Wong, 1952; Langmuir and Blodgett, 1946). He attributed this reduction to changes in the air flow pattern caused by the rough rime surface. Pflaum and Pruppacher (1976) found that their graupel grew with bulk collision efficiencies of approximately $0.3 \text{ as}$ compared with theoretical efficiencies of $0.7$. They also attributed the decrease to the rough surface texture of the rime. Keith and Saunders (1988) determined that the collision efficiency increased for a relatively low density rimed surface as compared with theoretical calculations for smooth cylinders. They attributed this to the feather-type rime growth having an increased collision efficiency for the small droplets that made up the bulk of their cloud. Although these results seemingly contradict each other, differences may be due to the different rime surfaces created by the different droplet distributions in each study.

Most investigators assume that the collision efficiencies of their droplet spectra relative to graupel are equal to the theoretically calculated values for smooth spheres or cylinders. This is because of the difficulty of uncoupling the liquid water content from the collision efficiency when attempting to measure the liquid water content by collisional means. At velocities larger than $15$ to $20$ m s$^{-1}$ this assumption is a good approximation (Makkonen and Stallibrass, 1987). This is probably because the ice surface is relatively smooth for the higher densities created when the impacting droplets have high impact velocities. At low velocities (1 to 5 m s$^{-1}$) however, the surface roughness of the relatively
lower density rime may have effects on the collision efficiency of the accreting droplets. The few studies in this area have apparently contradictory results.

1.8 Intentions and Organization of this Study

There are significant gaps in our understanding of the cold rain process, especially with respect to the fundamentals of graupel growth. Experiments by Pflaum and Pruppacher (1979), Dong and Hallett (1986), Griggs et.al. (1984) and Cober (1987) can only be considered preliminary, because none obtained a sufficient resolution of all the major growth parameters to allow their uncoupling. Mass transfer characteristics such as collision efficiency, Sherwood number (for convective transfer) and certain geometric properties of growing graupel are either unknown or disagreed upon. These parameters control the growth calculations in numerical models (Beheng, 1978), affect the calculations involving doppler radar measurements of precipitation and are an important inclusion in the equations for models investigating cloud dynamics (Farley, 1987).

No heat transfer characteristics such as surface temperature and Nusselt number have been previously reported. The surface temperature is required for parametrization of the rime density, drives the mass transfer by sublimation, controls secondary ice crystal production and partially controls electrification of clouds by regulating the charge separation. It has virtually always been calculated from theoretical considerations, although some attempts have been made to measure it with thermocouples.

An experimental investigation of the parameters that control the heat and mass transfer of graupel growing in a simulated cloud environment is described in this thesis. The measurements are accompanied by theoretical calculations of the heat and mass transfer following List (1963) and Schemenauer (1972). Specifically, measurements of mass growth rate, volume, density, geometric shape and surface temperature are related to variations of ambient cloud temperature, pressure, velocity, droplet size distribution and liquid water content. The experimental data are parametrized and used to calculate the bulk
collision efficiency, Nusselt number and Sherwood number. The result is a description or parametrization of each variable in the heat and mass transfer equations to a resolution and accuracy not previously achieved.

The organization of this thesis is as follows. Chapter 2 is a discussion of the theory of the heat and mass transfer. The experimental apparatus, calibrations and procedure are described in Chapter 3. Chapter 4 presents the results, preliminary analysis and parametrizations. Chapter 5 presents a detailed analysis of the dimensionless parameters and Chapter 6 states the summary and conclusions.
Chapter 2

Heat and Mass Transfer Theory

2.1 Introduction

In this chapter the theoretical heat and mass transfer equations are developed and discussed. It will be shown that all of the parameters incorporated in the equations are known or can be measured with the exception of three: the bulk collision efficiency of the droplet distribution with the graupel, and the dimensionless heat and mass transfer parameters (Nusselt and Sherwood numbers). Equations will be derived that show how the unknowns can be determined.

2.2 Heat Transfer by Convection

For a fluid of velocity \( V \) and temperature \( T_s \) flowing over a graupel with surface area \( A \), and temperature \( T_{s*} \), the convective heat transfer is given by

\[
Q_{cc} = h A_s (T_s - T_{s*})
\]

where \( h \) is the average heat convection coefficient for the entire graupel surface. Theoretically \( h \) is expressed within the definition of the average Nusselt number

\[
N_u = \frac{h L}{k_a}
\]

where \( N_u \) is averaged over the entire surface, \( L \) is a characteristic length and \( k_a \) is the thermal conductivity of the fluid. The Nusselt number is equal to the dimensionless temperature gradient at the surface and provides a measure of the convective heat transfer occurring at the surface. It is a dimensionless number which by boundary layer theory (Incropera and DeWitt, 1985) can be shown to be a function only of the Reynolds number, \( R_e \), and the Prandtl number, \( P_r \) (definitions for \( R_e \) and \( P_r \) are in Appendix A).
Equation 2.1 can be rewritten as

\[ Q_{cc} = \frac{k_a N_u}{D} A_p (T_s - T_a) \]  

where \( D \) is the diameter of the graupel base, \( k_a \) is the thermal conductivity of air and \( T_a \) is the ambient air temperature. Roughness factors are often added to account for increased mass transfer due to surface roughness (List, 1963; Schemenauer, 1972). In Equation 2.3 the roughness factor would be inherent within the Nusselt number.

### 2.3 Mass and Heat Transfer by Deposition or Sublimation

The mass transfer from a surface by convection of vapour (deposition or sublimation) is analogous to the heat transfer by convection. By definition deposition represents the change of state from gas to solid while sublimation is the change of state from solid to gas. The convective mass transfer rate is given by

\[ M_{ds} = h_m A_p (\rho_s - \rho_v) \]  

where \( h_m \) is the average mass transfer convection coefficient, \( \rho_s \) is the mass vapour density at the surface and \( \rho_v \) is the ambient mass vapour density of the fluid. Theoretically \( h_m \) is expressed within the definition of the average Sherwood number

\[ S_h = \frac{h_m L}{D_a} \]  

where \( L \) is a characteristic length and \( D_a \) is the mass diffusivity of the fluid. The Sherwood number for mass transfer is analogous to the Nusselt number for heat transfer. Physically it is equal to the dimensionless concentration gradient at the surface. It can be shown theoretically to be a function only of the Reynolds number and Schmidt number \( S_e \). \( S_e \) is defined in Appendix A.

Thermodynamic equilibrium exists at the interface of the surface and the fluid so that the vapour at the surface is assumed to be saturated at the temperature of the surface.
If the fluid is a gas such as water vapour, the ideal gas law can be applied.

\[ \rho_v = \frac{e}{R_v T_a} \]  

(2.6)

where \( \rho_v \) is the vapour density, \( e \) is the vapour pressure, \( R_v \) is the gas constant for water vapour and \( T_a \) is the ambient temperature. Equation 2.4 becomes

\[ M_{DS} = \frac{S_h D_{wa} A_z}{D R_v} \left( \frac{e_{st}(T_s)}{T_s} - \frac{e_a(T_a)}{T_a} \right) \]  

(2.7)

where \( e_{st}(T_s) \) is the saturated vapour pressure (with respect to ice) at the surface at the temperature of the surface, \( e_a(T_a) \) is the ambient vapour pressure in air, \( D \) is the diameter of the graupel base and \( D_{wa} \) is the diffusivity of water vapour in air.

For surface temperatures close to the ambient temperature, \( 1/T_s \) can be removed from each term. For a graupel in an unsaturated vapour field, the mass transfer rate by deposition or sublimation is then given by

\[ M_{DS} = \frac{S_h D_{wa} A_z}{D R_v T_a} \left( e_{st}(T_s) - U_w e_{sa}(T_a) \right) \]  

(2.8)

where \( e_{sa}(T_a) \) is the vapour pressure saturated with respect to water at the ambient temperature and \( U_w \) is the relative humidity of the air. The corresponding heat transfer rate is then

\[ Q_{DS} = L_s M_{DS} \]  

(2.9)

where \( L_s \) is the latent heat of sublimation.

2.4 Heat and Mass Transfer From Accretion

As a graupel falls relative to the cloud droplets, it will sweep out a volume through the cloud. As the supercooled droplets within the volume collide with the graupel, the ice
surface will cause the droplets to freeze to it. The mass growth rate, $M_{CP}$, due to this geometric accretion is given by

$$M_{CP} = E_b W_f A_b V$$  \hspace{1cm} (2.10)$$

where $W_f$ is the liquid water content of the cloud, $A_b$ is the area of sweepout by the graupel (equal to $\pi D_b^2/4$), $V$ is the velocity the graupel falls relative to the droplets and $E_b$ is the bulk collection efficiency.

Droplets which collide with the ice surface are assumed to stick so that the coalescence efficiency is equal to 1. The bulk collection efficiency (the bulk collision efficiency times the coalescence efficiency) can be defined as the ratio of the liquid water content that actually collides and coalesces with a falling graupel to the total water mass within the sweepout volume. With a coalescence efficiency of 1, the bulk collection efficiency and bulk collision efficiency are equal. Because some droplets follow the streamlines around the graupel, $E_b$ is less than 1. The $E_b$ for a droplet distribution will not generally equal the collision efficiency for a droplet of given diameter with a graupel.

The heat transfer associated with the freezing of the supercooled droplets can be described by two terms, a cloud particle warming term and a latent heat of freezing term. As the droplets are cooler than the graupel surface, they will warm (after accreting to the surface) at the expense of the surface. The heat transfer rate for warming is given by

$$Q_{CP} = E_b W_f A_b V C_w (T_s - T_a)$$  \hspace{1cm} (2.11)$$

where $C_w$ is the specific heat of water, $T_s$ is the surface temperature and $T_a$ is the droplet temperature which is assumed equal to the ambient air temperature. The heat transfer rate due to latent heat release is given by

$$Q_f = E_b W_f A_b V L_f = M_{CP} L_f$$  \hspace{1cm} (2.12)$$

where $L_f$ is the latent heat of fusion.
2.5 Heat and Mass Balance Equations

The total mass transfer rate is the combination of accretion and mass convection. Combining Equations 2.8 and 2.10 gives

\[ \frac{dM}{dt} = E_b \frac{W_f A_b V}{D R_v T_a} \left( \frac{e_s(T_s) - U_w e_s(T_a)}{100} \right) \]

where \( \frac{dM}{dt} \) is the total mass transfer rate. If the saturation vapour pressure over the graupel is larger than the ambient vapour pressure then sublimation will occur, representing a mass loss. This accounts for the first minus sign in Equation 2.13.

The conservation of energy requires that the total heat transfer rate must balance. As radiation transfer is negligible in comparison to the other terms, the heat gained by the latent heat of freezing will balance the heat lost by convection, sublimation and warming of accreted cloud droplets. Combining Equations 2.3, 2.9, 2.11 and 2.12 gives

\[ 0 = \frac{N_a A_s k_g}{D} (T_s - T_a) + \frac{L S_v D_{wa} A_s}{D R_v T_a} \left( \frac{e_s(T_s) - U_w e_s(T_a)}{100} \right) + E_b \frac{W_f A_b V C_w (T_s - T_a)}{100} - E_b \frac{W_f A_b V L_f}{100} \]

Equations 2.13 and 2.14 describe the complete heat and mass transfer of a graupel growing in a cloud environment.

2.6 Experimental Determination

Enough of the parameters in Equations 2.13 and 2.14 are known or can be experimentally measured to reduce the number of unknowns to two. The parameters \( R_v \), \( L_v \), \( k_g \), \( L_v \), \( D_{wa} \), and \( C_w \) are physical properties of the air or water vapour. The last four are slightly dependent on the ambient temperature (see parametrizations given in Pruppacher and Klett, 1978). The cloud conditions \( T_s \), \( P \) and \( W_f \), and the velocity \( V \) are controlled and
measured in laboratory experiments. The particle diameter $D$ and geometric properties $A_b$ and $A_s$ can be measured experimentally. Hence, Equations 2.13 and 2.14 can be simplified to

$$\frac{dM}{dt} = Z_1 E_b - Z_2 S_h \left( e_{sa}(T_s) - U_w e_{sa}(T_a) \right)$$

2.15

$$Z_3 N_a (T_s - T_a) + Z_4 S_h \left( e_{sa}(T_s) - U_w e_{sa}(T_a) \right) + Z_5 E_b (T_s - T_a) - Z_6 E_b = 0$$

2.16

respectively, where $Z_1$ to $Z_6$ are constants incorporating most of the known parameters.

Clouds are typically saturated so that the relative humidity $U_w$ equals 1. In a wind tunnel it may be less than 1 and must be measured or calculated theoretically. The graupel surface temperature has not previously been measured directly, however a new thermal imaging system now allows such measurements. Saturation vapour pressures are well known and have been parametrized in terms of temperature (Pruppacher and Klett, 1978). The total mass growth rate can be measured experimentally. Incorporating these parameters leaves three unknowns: the bulk collision efficiency $E_b$, the Nusselt number $N_a$ and the Sherwood number $S_h$.

The functional dependencies of the average Nusselt and Sherwood numbers are $N_a = f(R_e, P_c)$ and $S_h = f(R_e, S_c)$, respectively. Ranz and Marshall (1952a and b) showed that the Nusselt and Sherwood numbers for evaporating spherical droplets are given by

$$N_a = 2.00 + 0.6 P_c^{1/2} R_e^{1/2}$$

$$S_h = 2.00 + 0.6 S_c^{1/2} R_e^{1/2}$$

2.17

The 2.00 represents the limit of transfer by conduction and can be neglected (or incorporated in the $f(R_e)$ parametrization) for $N_a$ and $S_h$ greater than 20. Similarity theory between heat convection and mass convection shows that for a particular geometric situation, the functional dependence of the Nusselt and Sherwood numbers are of the same
form. That is, if $N_u = cP_e^n R_e^m$, then $S_h = cS_e^n R_e^m$ where $c$, $n$ and $m$ are constants. This implies that

$$\frac{N_u}{S_h} = \left( \frac{P_e}{S_e} \right)^n$$

2.18

For engineering or experimental applications similar to the growing of ice in a wind tunnel, the value of $n$ equals $1/3$ (Incropera and DeWitt, 1985). Within the range of atmospheric temperatures and pressures, the Prandtl and Schmidt numbers are constant and equal to 0.71 and 0.62, respectively. Therefore, the Sherwood number can be expressed as a fraction (0.95) of the Nusselt number so that Equations 2.15 and 2.16 reduce to two equations with two unknowns: the bulk collision efficiency and the Nusselt number. Neither of these factors has previously been measured for graupel growing under typical atmospheric conditions.
Chapter 3

Apparatus, Calibrations and Experimental Procedure

3.1 Introduction

Laboratory experiments on the heat and mass transfer of graupel were carried out in an icing tunnel and consisted of producing a specific cloud environment in which graupel embryos were introduced and allowed to rime. After a specified time the resulting graupel were removed from the tunnel and analyzed. The apparatus required for controlling each cloud parameter, the corresponding calibrations and the experimental procedures and errors are discussed in this chapter.

3.2 Wind Tunnel Description

The wind tunnel facility is located at the McLennan Physical Laboratories of the University of Toronto and is described in detail by List et al. (1987). A schematic diagram of the wind tunnel is shown in Figure 3.1. The tunnel is composed of square and rectangular aluminum ducts through which air is forced to flow. The ducts form a closed loop so that the air is continuously recycled. A 3.7 kW motor (1) drives a centrifugal fan (2) which takes in horizontally moving air and exhausts it downwards (3). Turning guides (4) divert the airflow upwards into a contraction (5). This contracts the airflow, funnelling it into a measuring section (6) where the graupel growth experiments are performed. The air then passes through a return duct (7) which directs it back into the fan intake, completing the closed circuit. The tunnel has a double wall construction prior to and through the measuring section. The inner wall encloses the experimental region, isolating it from the velocity and temperature gradients within the outer ring.
3.3 The Measuring Section

3.3.1 Components

The graupel experiments were carried out in the measuring section. This section consisted of two sets of walls: aluminum outer walls (6b) and plexiglass inner walls (6a). A schematic and photograph of the measuring section are shown in Figure 3.2. The outer walls consisted of four aluminum plates (31 cm wide by 45 cm high) which together form a rectangular duct. These plates comprised the mounting bases for the particle illumination system, a camera mounting system, a graupel suspension system, and a door allowing access to the inner wall.

Small cracks in the inner walls (i.e. between adjoining plates) originally allowed interactions between the inner and outer air flows. Because these interactions could have significant effects on the graupel growth they were eliminated by sealing all the holes and cracks in the inner walls with tape and Plasticine.

3.3.2 The Graupel Suspension System

As discussed in Chapter 1, graupel embryos in natural clouds have often been observed to be hexagonal or dendritic crystals. To simulate an embryo, a 1 mm diameter hexagonal plate was cut out of mylar plastic (80 μm thick) and glued to the end of a 200 μm diameter stainless steel wire. The wire was thin enough to have a minimum effect on the airflow around the graupel while being thick enough to rigidly suspend the graupel in the airstream without bending or allowing excessive vibrations. During the heat transfer experiments the wire was found to have no significant effects on the heat transfer of the graupel. At a distance of 4 cm from the hexagonal plate, the wire was run through a 10 cm long wooden dowel and soldered to a 1 mm thick copper wire mounted in the top of the dowel. The copper wire allowed the dowel assembly to be rigidly held by a brass clamp 15 cm above the hexagonal plate, and effectively grounded the graupel particle. The
Figure 3.2: a) Schematic of the measuring section and surroundings. 1: Outer access door; 2: Inner access door; 3: Observation windows; 4: Camera; 5: Lens and extension tubes; 6: Graupel suspension system; 7: Strobe lights; 8: Illumination slits; 9: Outer wall; 10: Inner wall; 11: Graupel growth locations; 12: Water injection control system; 13: Pressure control system; 14: Temperature control system; 15: Computer display of monitored tunnel conditions.
reasoning behind the latter effect is discussed in Section 3.9. During experiments, three dowel assemblies were simultaneously suspended in the measuring section with a 2.5 cm spacing between neighbouring embryo-plates. This ensured that growing graupel were separated by at least six particle diameters for most of their growth so that the airflow around one particle did not effect the growth of another.

3.3.3 The Photographic System

The camera system consisted of a 55 mm micro lens with a Nikon M extension tube and a Nikon F camera. This system is particularly well designed for close-up photography within the wind tunnel because it allows an on-film magnification ratio of between 0.5 and 1.0 while allowing the camera lens to be from 11.1 to 6.3 cm away from the photographed object. This allowed the lens to be positioned between the inner and outer walls of the measuring section so that it was as close to the graupel as it could be without interfering with the airstream through the inner measuring section. A 2X magnifying eyepiece was used for fine focusing on the edge of the graupel. The depth of field was of the order of 8 mm so that most or all of the graupel surface was in good focus within a single photograph. Kodak 400 Tmax film was used.

The particle illumination system consisted of 2 Strobolume 1540-P strobe lights which produced a high intensity flash of 10 μs duration. The flash angle was directed at ± 135° which is the optimum angle for observing cloud droplets, and which illuminated the graupel with side and back lighting.

3.4 Velocity Control and Calibration

The velocity of the airflow is determined by the speed of the fan (2 in Figure 3.1) which is in turn controlled by a speed variator (8). The fan rotation rate is variable between 360 and 1800 RPM with the corresponding airflow velocity ranging from 5 to 30 m s⁻¹ in the measuring section. This range was not appropriate as graupel terminal velocities range
from 1 to 5 m s\(^{-1}\). A wooden board (9) was perforated with an array of 5 mm holes and positioned at the beginning of the double wall construction. This forced most of the airflow through the outer ring so that the airflow in the measuring section was reduced to between 1.0 and 4.0 m s\(^{-1}\) with a corresponding velocity range in the outer ring of 7 to 30 m s\(^{-1}\).

The velocity of the airflow was originally measured to within 0.1 m s\(^{-1}\) with a Prandtl tube held in the centre of the measuring section and was calibrated against the venturi pressure drop of the flow across the contraction area. The pressure drop was measured by two nipples (10) mounted flush with the inner walls and connected to a Setra Systems Model 264 low differential pressure transducer. With this calibration, the voltage output of the transducer was monitored by the tunnel computer system which displayed the velocity corresponding to that in the centre of the measuring section. Therefore, the velocity could be monitored during experiments without the Prandtl tube. The velocity was recorded before and after each experiment using the monitoring program of the computer.

A Thermal Systems Incorporated hot wire anemometer was used to measure the velocity and turbulent profiles across the measuring system. Figure 3.3 shows the north-south and east-west velocity profiles at 1.1 and 2.8 m s\(^{-1}\). Across the central 12 by 12 cm of the measuring section the velocity never varied by more than 5\% of its value in the centre. Similarly, the turbulence (root mean square of the velocity) was measured to be less than 0.5\% within the same area. Within 1 cm of the walls the velocity rapidly dropped off, as would be expected within the boundary layer that would form along the wall.

3.5 Temperature and Pressure Control

The temperature of the tunnel is conditioned by a set of cooling elements (11 in Figure 3.1) and heating elements (12). The cooling elements are controlled by a compressor which can maintain the tunnel temperature between -17 and -30°C within 0.5°C. The heating elements are regulated by a variable current controlled resistive element which, when the compressor is set on its minimum cooling rate, allows the tunnel temperature to
Figure 3.3: North to south and east to west velocity profiles across the measuring section, for velocities of 2.8 m s\(^{-1}\) and 1.1 m s\(^{-1}\). \(\Delta\) indicates the locations graupel were suspended during experiments.
be maintained between -3 and -17°C within 0.5°C. Conduction between the walls of the wind tunnel and the laboratory air is minimized as the tunnel is insulated with polystyrene rigid board and soft fibreglass strips (13). Only parts of the measuring section lack a thorough insulation covering. The high velocities in the outer ring ensured that the inner measuring section was cooled from the outside inwards. This isolated the inner measuring section from any outside warming effects and ensured that the temperature across the measuring section was constant. The tunnel air temperature was measured at six positions with thermocouples and platinum thermistors (15). The ambient temperature in the measuring section was measured by a thermocouple and was recorded before and after each experiment. During the experiments riming on the thermocouple mount caused the thermocouple measurement to be up to 1.5°C warmer than the ambient temperature.

The pressure in the wind tunnel was reduced and maintained by an Edwards Model 40 vacuum pump (16). The ambient pressure could be maintained between 30 and 100 kPa within 1 kPa. Below 60 kPa however, the vacuum pump (as it pumped to overcome the small leaks in the wind tunnel) began to have a significant influence on the flow through the measuring section. Consequently pressures below 60 kPa were not used in experiments. Presently, this is an important limitation since graupel in the atmosphere can form at altitudes with pressures as low as 30 kPa. The pressure was measured with an absolute pressure transducer, with its output displayed through the computer monitoring system.

3.6 Water Injection System

3.6.1 Spraying Nozzle

The water injection system (14) consisted of a Spraying Systems Co #1/8 JJ miniature air atomizing nozzle with a J1050 fluid cap. Pressurized air and water flows are fed into opposite ends of a small chamber, and as the water is ejected through a small hole in the top of the chamber, the air flow atomizes the water into droplets. The resulting droplet size distribution is highly dependent on the pressurization of the nozzle air flow.
The water line was heated with Nichrome wire to prevent freezing of the water and to avoid a buildup of ice around the nozzle outlet. The entire nozzle was enclosed in a wooden aerofoil to insulate it from the tunnel environment, and grounded to prevent a buildup of electric charge. The nozzle was positioned 2.0 m upstream of the centre of the measuring section where the experiments were performed.

3.6.2 Droplet Size Distribution

The droplet size distribution produced was measured by the Magnesium Oxide Technique (May, 1950; Cober, 1987). Glass strips 2.3 mm wide were coated with a layer of magnesium oxide and were exposed to the tunnel air flow (at 2 m s⁻¹) for 1 to 2 seconds. Droplets colliding with the MgO layer created craters with diameters directly proportional to the diameter of the droplet. A projecting microscope was used to photograph the craters’ images. A particle counter was used to analyze photographs of the crater size distributions in terms of uniform diameter bins of approximately 4 μm in width. The bin width depended on the magnification of the photograph of the craters. Using the theoretical collision efficiencies of Ranz and Wong (1952), the measured droplet size distribution was adjusted to take into account the different collision efficiencies of the droplets. A histogram showing the standard droplet spectrum used in the experiments is shown in Figure 3.4. Droplet size distributions can be characterized by their mean radius, mean volume radius or median volume radius as shown in Table 3.1 for the droplet distributions used in different experimental conditions.

Droplets smaller than 7 μm in diameter cannot be measured by the MgO technique because the collision efficiency of droplets smaller than 7 μm with a 2.3 mm glass slide is zero. Measurements with a Forward Scattering Spectrometer Probe (FSSP) showed that similar nozzles with a higher air atomization rate than the rates used in these experiments had less than 3% of the liquid water content contained in droplets < 8 μm. In these experiments over 90% of the mass was contained in the droplets larger than the median diameter of 17 μm. Therefore, the inability to measure droplets smaller than 7 μm was not
Figure 3.4: Histogram of the droplet size distribution and the cumulative mass distribution, for the standard cloud conditions. The distribution was measured using the magnesium oxide technique. The median volume diameter is 30 μm.
Table 3.1: Mean droplet radius, \( r \), mean volume radius, \( r_v \), and median volume radius, \( r_m \), for each nozzle atomization (air flow rate) used in the experiments, as based on magnesium oxide measurements. The number of droplets for each spectrum are also listed. The nozzle air flow rate is through a #605 Matheson flowmeter.
3.6.3 Liquid Water Content

The liquid water content was calculated from the water injection rate. Water injected by the nozzle is swept up by the tunnel airflow and subsequently travels through the measuring section. By geometric considerations the liquid water content in the measuring section can then be given by

\[ W_f = \frac{F}{A \cdot V} \quad \text{(3.1)} \]

where \( W_f \) is the liquid water content in \( \text{g m}^{-3} \), \( A \) is the cross sectional area of the measuring section in \( \text{m}^2 \), \( F \) is the nozzle water flow rate in \( \text{g s}^{-1} \) and \( V \) is the airflow velocity in \( \text{m s}^{-1} \). Equation 3.1 is valid only if the following inherent assumptions are properly accounted for:

1) uniformity of the liquid water content across the measuring section;
2) evaporation of the droplet spectrum;
3) rime on the walls of the wind tunnel; and
4) large droplets having larger fall velocities than the upward speed of the airflow below the contraction, so that they fall below the nozzle.

To test the first assumption, a coarse screen composed of 1 mm wire with a 2.5 by 2.5 cm spacing was positioned in the measuring section and allowed to rime under typical experimental conditions \( (T_r = -16°C, \ V = 2.0 \text{ m s}^{-1}, \ W_f = 1.9 \text{ g m}^{-3}, \ P = 100 \text{ kPa}) \). As shown in Figure 3.5 the ice accumulation and texture were quite consistent across the entire measuring section. To quantify this, 1 mm icing disks were positioned independently at different positions in the measuring section and allowed to rime. The mass of ice accreted was measured to vary from 3 to 10% depending on the icing conditions, and under the majority of experimental conditions varied less than 5% with respect to the average. The east side of the measuring section always had more icing than the west side. This variation was caused by the spray from the nozzle and its interaction with the airflow of the tunnel.
Figure 3.5: Ice accumulation on a 2.0 mm thick, 2.5 by 2.5 cm screen suspended in the measuring section to examine the spatial distribution of the liquid water content. The icing conditions were: $T_s = -16.0\,^\circ\text{C}$, $P = 101$ kPa, $V = 2.0$ m s$^{-1}$, $W_t = 1.9$ g m$^{-3}$, $r_m = 13 \,\mu$m and $t = 25$ minutes.
The cases with the largest variation were associated with the largest droplet sizes and highest relative velocities (the conditions under which the droplets had the most inertia).

Measurements of the riming which occurred on the walls throughout the measuring section indicated that 10 to 15% of the liquid water content rimed onto the walls. Perturbations on the walls caused by tape, holes, screws holding the plexiglass, and rime outcrops would tend to cause the boundary layer along the wall to be turbulent. Riming on the walls implies that the boundary layer is turbulent because droplet trajectories in the airflow are able to reach the wall. Blasius solution (Currie, 1974) to uniform flow over a flat plate indicates that the boundary layer thickness in the measuring section would average about 1 cm. This is equivalent to 11% of the cross sectional area of the measuring section and is consistent with 10 to 15% of the liquid water riming on the walls. There is more riming on the walls near the base of the measuring section than near the top, implying that the water within it is depleted gradually. Icing tests at different heights in the measuring section showed that the mass of water within the central 10 by 10 cm area is constant, implying there is no \( W_f \) gradient towards the walls. Therefore, no correction to Equation 3.1 is necessary to account for icing on the walls.

The largest droplets observed with MgO coated slides were approximately 80 \( \mu \text{m} \) in diameter. These droplets have a terminal velocity of 19 cm s\(^{-1}\). With a measuring section velocity of 2 m s\(^{-1}\), the tunnel airflow velocity below the contraction at the level of the water injection system is 24 cm s\(^{-1}\). Hence, at velocities larger than 2 m s\(^{-1}\) the largest droplets do not fall out of the airflow. This was visually confirmed through a port hole in the tunnel at the level of the nozzle. For experiments at 1.1 and 1.5 m s\(^{-1}\) however, large droplets were observed to fall below the nozzle. Using the measured droplet mass distribution, the liquid water content was adjusted to account for the removal of the largest droplets. This adjustment accounted for 8% of the mass at 1.5 m s\(^{-1}\) and 16% at 1.1 m s\(^{-1}\).

Evaporation of the droplet spectrum accounted for approximately 5% of the liquid water content. The liquid water content was adjusted accordingly, as is discussed in detail.
in Section 3.7.

3.7 Relative Humidity

The relative humidity is an important component in the heat transfer equations for a growing graupel. Unfortunately, it is not directly controllable inside the wind tunnel. Ideally the air within the tunnel should be saturated with respect to water at the ambient temperature. Attempts were made to measure the relative humidity using thermocouples as wet and dry bulb thermometers and the Psychometric equations given by Iribarne and Godson (1981). However, for psychrometry to give correct results the velocity over the bulbs should be 4 to 10 m s\(^{-1}\), which is significantly higher than in these experiments. Further, drifting of the thermocouple preamplifier zeros allowed the wet and dry bulb thermocouples to only be consistent within 0.2°C. This would result in a 9% error in the relative humidity calculation at -15°C. During experiments, this method indicated a relative humidity of between 95 and 125% with respect to ice, depending on the icing conditions. Such results were insufficient and since no other measuring technique was available, the relative humidity was estimated using a theoretical model which simulated the thermodynamics of the wind tunnel.

As the airflow in the tunnel passes through the cooling elements, the air will saturate with respect to ice at the temperature of the cooling elements. The cooling elements were typically 1 to 1.5°C cooler than the nozzle ambient temperature. Therefore at a nozzle ambient temperature of -15°C, ice saturation at the cooling elements would correspond to a relative humidity of 77% with respect to water at the nozzle location. For a droplet of temperature \(T_d\) and radius \(r_d\) in an environment of temperature \(T_e\) and relative humidity \(U_w\), the heat transfer of the droplet is given by (Pruppacher and Klett, 1978)
\[ \frac{4 \pi c_p \rho_w r_d^3}{3} \frac{dT_d}{dt} = \frac{4 \pi r_d D_w L_v}{R_v} \left( \frac{U_w e_{sa}}{T_a} - \frac{e_{sd}}{T_d} \right) + 4 \pi r_d k_a (T_a - T_d) \] 3.2

where \( e_{sa} \) is the saturation vapour pressure at the ambient temperature and \( e_{sd} \) is the saturation vapour pressure over the surface of the droplet at the droplet temperature. Equation 3.2 states that the total heat transfer of the droplet is equal to the heat transferred due to evaporation or condensation plus the heat transferred due to conduction. Convection is not considered because the Reynolds numbers of the droplets are small enough that they can be assumed to be suspended by the flow. As the droplets injected by the nozzle were initially at 10 °C, they would cool until they reached an equilibrium temperature, \( T_{de} \), which was cooler than the ambient temperature \( T_a \). After reaching equilibrium the heat lost by evaporation would balance the heat gained by conduction. Equation 3.2 can be iterated to solve for the droplet relaxation times. Figure 3.6 shows the relaxation times for droplets initially at 10°C to cool to within 0.01°C of -15°C under typical tunnel conditions. When the velocity in the measuring section is 2 m s\(^{-1}\), the droplets take about 4 s to travel from the nozzle to the measuring section so that all the droplets reach the equilibrium temperature. Once the droplets are in equilibrium, Equation 3.2 can be solved to show the well known parabolic decay of droplets by evaporation in a subsaturated vapour field (Pruppacher and Klett, 1978). The evaporation of droplets increases the relative humidity and slows the rate of evaporation.

Similar calculations for an entire droplet spectrum were used to calculate the relative humidity in the wind tunnel. Within a theoretical model, droplets in a simulated distribution were injected into the tunnel airflow, and the heat and mass transfer effects due to the droplet temperature relaxations were calculated. The droplets were subsequently moved upwards to the contraction while being mixed with the ambient airstream. Finally the distribution was moved through the contraction and through the measuring section. The velocity of the airflow governed the length of the simulation. At each step, the heat and mass transfers were calculated and the liquid and vapour water contents were adjusted.
Figure 3.6: Droplet radius versus the relaxation time required for a droplet to cool from 10°C to within 0.01°C of -15°C, for a relative humidity of 0.75. These conditions would be representative for a droplet injected into the tunnel.
accordingly. The final liquid and vapour water contents in the measuring section were output along with the relative humidity and the net changes to the droplet spectrum. The relative humidity was calculated to range from 88 to 96% with respect to water depending on the ambient conditions. About 5% of the injected liquid water content was calculated to evaporate, and Equation 3.1 was adjusted accordingly.

3.8 Surface Temperature

An Agema Infrared Systems Thermovision 800 thermal imaging system (made available by the Ontario Laser and Lightwave Research Centre) was used to remotely measure the surface temperature of graupel growing in the wind tunnel. The system consisted of a Mercury Cadmium Telluride detector that covered the 8 to 12 μm spectral range and a 12° Germanium lens. With a 32 mm extension tube, the lens had a working distance of 17 cm. The spot size of the detector was measured to be 2.0 ± 0.5 mm in diameter, which was sufficient to resolve the graupel surface. The spot size was determined by using blackbody cavities surrounded by a low emissivity surface with a small hole through the surface into the cavity. This assembly was placed in a relatively cooler environment. Holes smaller than the spot size were not well resolved by the imaging system.

A sealed Germanium window with an anti reflection coating was mounted in the outer tunnel wall for surface temperature measurements at low pressures. The window had a transmission of more than 95% of the 8 to 12 μm radiation. The entire measuring section was painted flat black to shield it from any outside radiation and to minimize internal reflection. In this way the measuring section acted like a blackbody cavity that surrounded the graupel. Agema states a system sensitivity of 0.07°C at 30°C object temperature. In the 8 to 12 μm range of the blackbody spectrum this corresponds to a sensitivity of 0.13°C at -15°C.
The temperature displayed by the Agema was calculated from a number of parameters including the calibration relating detector voltage to detected radiation flux, the assumed emissivity of the object, and the assumed emissivity of the surroundings. The temperature of the surroundings was measured with thermocouples to within ± 0.25°C. The emissivity of the measuring section is 1.0, given the assumption that it is a blackbody cavity. In these experiments however, it was set at 0.96, although this caused no significant error. The emissivity of graupel was assumed to be equal to the emissivity of ice which is 0.96. Although the calibration range is from -20 to 1500°C, at the lower temperatures the infrared flux is considerably smaller than at warmer temperatures, making the calibrations more prone to systematic errors. At -15°C the temperature field originally displayed by the Agema showed a systematic variation across the field of view from -16 to -12°C. A graupel image typically cut across 2 to 4°C of background temperature variation making direct measurements impossible. The variation was caused by the extension tubes and lens mounting components of the Agema. Their relatively warmer temperatures introduced a systematic background radiation into the detector. The background variation was reduced to less than 2°C by cooling the camera body and lens system with dry ice to approximately -15°C. An image of the background temperature field is shown in Figure 4.34. The Agema software allowed temperature images to be subtracted, so that by subtracting images taken before and during an experiment, the net temperature change of the graupel was measurable to within 0.2 °C. This error is in effect the precision error of the system. The absorption coefficient of ice from 8 to 12 μm is 10³ cm⁻¹ (Hobbs, 1974) so that 98% of incident radiation is absorbed within 40 μm. Therefore the radiation measured by the detector is representative of the graupel surface temperature.

3.9 Electrification of the Droplet Cloud

An electrical double layer exists at the surface of an air-water interface (Loeb, 1958). When the water surface is disrupted as in the atomization of a water stream, the resulting droplets are electrically charged (Iribane, 1972). Therefore, when using atomizing
nozzles to create cloud droplets, it is important to ensure that the droplets do not have sufficient charge to affect their collision efficiency with graupel.

To estimate the charging, the charge associated with a mass of atomized water was measured using Keithly 614 and 414S electrometers. The current from the nozzle to ground was measured during atomization as was the current through an icing cylinder held in the measuring section. As expected (Iribarne, 1972), the nozzle current was positive while the current through the cylinder was negative. The current was proportional to the water flow rate and the droplet atomization rate. The maximum current corresponded to a charge of 1 to $2 \times 10^6$ esu charge per $20 \mu m$ diameter droplet. Schlamp et al. (1976) theoretically showed that the change in collision efficiency of cloud droplets colliding with droplets larger than $140 \mu m$ in diameter was negligible for electric charges up to $10^4$ esu per droplet. Since the experiments here involve 1000 $\mu m$ targets and higher relative velocities than considered by Schlamp, the effects of the estimated charging are expected to be negligible. By applying a potential across the nozzle the charge to mass ratio could be increased from a factor of +3 times to -2 times the charging at zero potential difference. The mass growth rate and graupel geometry were unaffected by the droplet charging produced by such potentials, implying that the nature of the droplet charging had no measurable effect on the ice accretion. This is an important observation because it is widely believed that the difference between the density measurements of Macklin (1962) and Pflaum and Pruppacher (1979) resulted from Mackins droplets being charged as a result of being created from atomization nozzles.

3.10 Experimental Basis

Cloud parameters which affect the growth of graupel include temperature, pressure, droplet size distribution, liquid water content, and relative humidity. Graupel parameters that should be measured experimentally include density, height, mass, volume, diameter, shape, cone angle, rotation frequency and embryo diameter. Knowledge or measurements of these parameters, the terminal velocity of the graupel and the graupel surface
temperature allow a complete solution to the heat and mass transfer equations and hence, a determination of the bulk collision efficiency and the Nusselt number. Experiments investigating the heat transfer through surface temperature measurements and the mass transfer were performed independently, since it was not feasible to perform both types of measurements simultaneously.

Individual mass transfer experiments consisted of growing three graupel simultaneously for a specific period of time under a specific set of cloud conditions. A series of experiments consisted of between 7 and 17 individual experiments of different durations. Different series reflect variations in a single cloud parameter from a standard set of conditions. The standard cloud conditions consisted of an ambient temperature of -16°C, an ambient pressure of 100 kPa, an airflow velocity of 2.0 m s\(^{-1}\), a liquid water content of 2.0 g m\(^{-3}\), an embryo diameter of 1.0 mm and a droplet distribution characterized by a mean radius of 10.4 μm, a mean volume radius of 12 μm and a median volume radius of 15 μm. The cloud conditions for each series are listed in Table 3.2. In total there were 25 series comprising 265 mass transfer experiments.

In addition to these basic experiments, 16 additional experiments were performed where two or more cloud parameters were varied from their standard values. The purpose of these investigations was to examine the nature of relatively high density graupel (i.e. > 0.5 g cm\(^{-3}\)). The cloud parameters observed to individually affect the density were varied to increase the graupel ice density, and coupled to magnify the densification.

Heat transfer experiments involved measuring the surface temperature of individual graupel growing under specific sets of cloud conditions analogous to those in the mass transfer series. The surface temperature was monitored continuously so that there was no requirement for experiments of different durations. In total 89 heat transfer experiments were performed.
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Table 3.2: Experimental cloud conditions used for each series of experiments. Variations in a cloud parameter from the standard set of conditions (series 1, 18 and 27) are shown in bold type.
In total approximately 450 experiments were performed. The average duration of an experiment was about 30 minutes so that between 10 and 20 could be performed in one day. All experiments were completed over a three month period.

3.11 Experimental Technique

For a given velocity, it took 1.5 hours to adjust the temperature of the wind tunnel to an equilibrium state after which the temperature could be maintained to within 1 degree for long periods of time. Over the course of a single experiment it was usually possible to maintain the temperature within 0.3°C. Changes to the velocity or temperature settings would require up to one hour before equilibrium was reestablished. The velocity could be set to within ± 0.05 m s⁻¹ after which no further fine tuning was required for the remainder of the day.

Prior to an experiment, three graupel embryos (hexagonal plates suspended on a wire-dowel assembly) were positioned in the measuring section and the camera was focused on the central particle. The embryos were left for one to two minutes to attain thermal equilibrium. Approximately 30 seconds before beginning an experiment the nozzle air atomization flowmeter was opened to its maximum flowrate. About 15 seconds later, the nozzle water injection flowmeter was opened and set to the flowrate required for a specific liquid water content. At time zero, the air flowrate was reduced to the correct atomization required for a specific droplet size distribution. The initial 10 to 15 seconds of highly atomized water injection were required to ensure that a thin ice coating formed on the embryo. Without this initial coating, drops would sometimes begin to grow without freezing as the embryos swept out liquid water.

During each experiment photographs were taken every 1 to 2 minutes. The temperature, pressure (if other than atmospheric) and water injection systems were adjusted if required to ensure that large variations did not occur. The pressure was maintained within ± 1 kPa. The water injection could be controlled to within ± 0.05 g m⁻³ although
uncertainties in the evaporation calculations and the water spatial variations could make the liquid water content error as large as ± 0.15 g m⁻³. The computer tunnel monitoring program continually monitored the growth time, ambient temperature, ambient pressure, dynamic pressure, velocity, ice and dry bulb temperatures and the corresponding relative humidity, nozzle ambient temperature, temperature of the injected droplets and the photograph number (for the film). These were recorded before, after and at one minute intervals during the experiment to give a complete record of the experimental conditions.

At the end of an experiment a final photograph was taken, the water injection was shut off and the final conditions were recorded. While still within the airflow of the measuring section each graupel and stem was transferred to a wooden thermos which had been stored at -15°C. The thermos was used to transfer the graupel to a walk-in cold room maintained at -10°C. No melting or significant evaporation of the graupel occurred during this process. In the cold room the volume was measured using Archimedes principle (List, 1960; Knight and Heymsfield, 1983) by forcibly immersing the graupel in a container of mercury of density 13.65 g cm⁻³ and measuring the displaced mass using an analytical balance. Each graupel was then photographed with a scale to allow future geometric measurements. The final graupel mass was measured on an analytical balance at room temperature, during which the graupel melted.

During experiments where the surface temperature was measured only one graupel was grown at a time. The geometric properties were not measured because the thermal imaging equipment was only available for three weeks of experiments. Thermal images of the graupel were recorded prior to, at one minute intervals during, and after each experiment. Each experiment was of a duration equivalent to the longest time of the mass transfer experiment performed under the same cloud conditions. A continual record of the cloud conditions was recorded through the computer tunnel monitoring program.
3.12 Experimental Measurement Errors

The analytical balances used for measuring mass and displaced mass were accurate to within ± 1x10⁻⁵ g and ± 1x10⁻⁴ g, respectively. However, both balances worked under adverse conditions. The mass measuring balance had to be enclosed in a Faraday cage to measure better than ± 1x10⁻³ g. The cage isolated it from the electric fields of the other instruments in the wind tunnel room, and allowed the mass to be measured within an error of ± 5x10⁻⁵ g which was usually less than 1%. The displaced mass measuring balance worked in a cold room at -10°C where the temperature changed continuously over ± 4°C. This prevented the balance from being reliable to better than ±1x10⁻³ g. Meniscus effects of the mercury caused displaced mass measurements to be reproducible to ± 3x10⁻³ g. This implied an error in the volume determination of ± 0.30 mm³, which was usually < 2%.

For any mass or volume measurement, 5 to 9 graupel from 2 to 3 experiments were analyzed and the results averaged. Standard deviations varied from 3 to 10% of the mean values, so that the errors in the mean varied from 2 to 4% for all experiments. These errors are significantly higher than the measurement errors and reflect variations in the liquid water and velocity conditions across the measuring section and between experiments.

Since the density showed no variation with growth time over a specific set of cloud conditions, it was averaged over 25 to 50 individual measurements. The standard deviation varied from ± 0.01 to ± 0.02 g cm⁻³ with the corresponding error on the mean varying from ± 0.002 to ± 0.004 g cm⁻³.

The photographically determined heights and diameters were measured to within ± 0.1 mm. The standard deviations were ± 0.2 mm with an error on the mean of ± 0.1 mm. Cone angles could be measured to within ± 2° with standard deviations of ± 6°. Errors on other variables are discussed in the analysis in Chapters 4 and 5.
Chapter 4

Experimental Heat and Mass Transfer Results and Discussion

4.1 Photographic Measurements

The time evolution of graupel growth was recorded with photographs taken every one to two minutes during each experiment. Figure 4.1 shows a sequence of photographs taken at two minute intervals for a graupel growing under the standard set of cloud conditions (defined as $T_e = -16^\circ C$, $P = 100$ kPa, $W_e = 2.0$ g m$^{-3}$, $V = 2.0$ m s$^{-1}$, median volume radius = 15 $\mu$m and embryo diameter = 1.0 mm). The sweepout of a cloud volume is clearly shown by the absence of droplets downstream (i.e. above) of the graupel. No icing was ever observed downstream on the suspension wire within 2 mm of the embryo. This sufficiently ensured that icing on the stem near the embryo did not affect the graupel growth. Sequences of photographs allowed an estimation of the amount of deposition or sublimation occurring on the sides of the graupel. These estimates confirmed the relative humidity calculations for the air in the wind tunnel, and are discussed in Chapter 5. The diameters of the droplets in focus were not resolvable from the photographs.

Photographs taken in the cold room were used to measure the final diameter, height, geometric shape and cone angle of each graupel. The growth height was used in parametrizations involving the graupel geometry because it was well defined photographically and directly represented the amount of accretion. Other researchers typically measure diameter as the representative geometric parameter because of its relation to the Reynolds number. Here, growth height measurements showed significantly less scatter than did measurements of the diameter or cone angle, because the rough outer edges of the graupel were prone to ice fingers and growth perturbations, making the latter quantities difficult to define. For a single graupel, an average of several diameters around the circumference would be required to obtain a representative diameter.
Figure 4.1: Graupel grown at $T_a = -16.0^\circ$C, $W_r = 2.0$ g m$^{-3}$, $V = 2.0$ m s$^{-1}$, $r_m = 14.8$ $\mu$m, $P = 100$ kPa for growth times of 2, 4, 6, 8, 10 and 12 minutes.
Cone angles for graupel grown under similar conditions (i.e. between 20 and 40 graupel) were averaged to obtain a representative cone angle with a typical standard deviation of ± 6°. These average cone angles were used when comparing geometries between graupel grown under different conditions. The diameter was determined through measurements of the growth height and the cone angle, and used to calculate the Reynolds number.

4.2 Height Time Relationship

Each series of experiments consisted of several individual experiments with the same cloud conditions but different durations. The growth height, \( h \), was defined as the length of ice accretion from the initial embryo to the base of the graupel. Figure 4.2 shows a plot of growth height versus time for graupel grown under the standard cloud conditions. Each data point represents an average of measurements from between 3 and 8 graupel. The growth height in mm was parametrized as

\[
h = a_h t^{b_h}
\]

where \( t \) is in minutes; and \( a_h \) and \( b_h \) are coefficients of the fit. For the curve in Figure 4.2, \( a_h \) equals 0.56±0.03 and \( b_h \) equals 0.89. Height does not increase linearly with time, indicating a reduction of the bulk collision efficiency of the droplet distribution with increasing graupel diameter (or height). For comparison, assuming a cone angle of 55° and using the theoretical collision efficiencies of Ranz and Wong (1952), the simple geometric sweepout of a similar droplet distribution with a disk shaped base would yield a height dependence of approximately \( t^{0.93} \). If the base was assumed to be spherical, the theoretical height dependence would be \( \propto t^{0.81} \).
Figure 4.2: Variation of growth height with time for graupel grown under the standard conditions: $T_s = -16.0^\circ$C, $W_f = 2.0$ g m$^{-3}$, $V = 2.0$ m s$^{-1}$, $r_m = 14.8$ $\mu$m, $P = 100$ kPa. The data points are from Series 1, 18 and 27.
4.3 **Density Determination**

Figure 4.3 shows the mass-volume variation for the graupel grown in each experiment with the standard cloud conditions. The slope is 0.26±0.01 g cm$^{-3}$ and is equivalent to the graupel density. The constant slope implies that density is independent of time and size within the resolution of these experiments. Small volumes tend to be slightly overestimated because meniscus effects of the mercury around the wire suspension stem (in the volume measurement) become more significant at smaller volumes.

At a constant air velocity, a density dependence on growth time might be expected because the collision efficiency depends on diameter. As a graupel diameter increases, the collision efficiencies of the smaller cloud droplets are reduced more than the efficiencies of the larger droplets. Therefore, the median volume radius of the effective droplet distribution (i.e. the droplets which actually collide with the graupel) increases with increasing graupel diameter. Using the theoretical collision efficiencies of Ranz and Wong (1952), and the standard droplet distribution, a graupel growing from 2.0 to 5.5 mm in diameter would cause the effective median volume radius to change from 15 to 16.5 μm. Since the accretion density depends on the median volume radius as discussed in Chapter 1 and shown in Section 4.12, this would increase the accretion density from 0.26 to 0.28 g cm$^{-3}$. The experimental density measurements are precise to ±0.01 or ±0.02 g cm$^{-3}$, so any densification resulting from a change in the graupel size is not resolvable in these experiments.

4.4 **Mass and Volume Relationships with Time**

The time dependence of mass and volume for the graupel grown under the standard cloud conditions are shown in Figures 4.4 and 4.5 respectively. The errorbars represent the ±2 to 4% standard deviation on the mean values, with each data point representing the mean of between 3 and 8 separate measurements. The mass m in mg, and volume $V_L$ in mm$^3$, were parametrized as
Figure 4.3: Variation of mass with volume for graupel grown under the standard conditions: $T_{s} = -16.0^\circ$C, $W_{r} = 2.0$ g m$^{-3}$, $V = 2.0$ m s$^{-1}$, $r_{m} = 14.8$ µm, $P = 100$ kPa.
Figure 4.4: Variation of mass with time for graupel grown under the standard conditions:

\[ T_s = -16.0^\circ C, \ W_r = 2.0 \text{ g m}^{-3}, \ V = 2.0 \text{ m s}^{-1}, \ r_m = 14.8 \mu m, \ P = 100 \text{ kPa}. \]
Figure 4.5: Variation of volume with time for graupel grown under the standard conditions: $T_s = -16.0^\circ$C, $W_f = 2.0$ g m$^{-3}$, $V = 2.0$ m s$^{-1}$, $r_m = 14.8$ µm, $P = 100$ kPa.
\[ m = a_m t^{b_m} \quad 4.2 \]
\[ V_L = a_v t^{b_v} \quad 4.3 \]

where \( a_m, b_m, a_v \), and \( b_v \) are coefficients of the fits and \( t \) is in minutes. Since the density is invariant with time, the coefficients \( b_m \) and \( b_v \) should be equal, and the ratio \( a_m/a_v \) should equal the density in mg mm\(^{-3}\) which is equivalent to g cm\(^{-3}\). The fits shown in Figures 4.4 and 4.5 have \( b_m = b_v = 1.94 \), \( a_m = 0.111 \pm 0.005 \) and \( a_v = 0.43 \pm 0.02 \). The ratio \( a_m/a_v \) equals 0.26 \( \pm 0.01 \) which is equal to the slope of the \( m-V_L \) graph. The errors on the \( a_m \) and \( a_v \) coefficients are estimated at \( \pm 5\% \), based on the \( \pm 2 \) to \( 4\% \) errors on the data points and on the quality of the fits.

These parametrizations imply that the mass and volume growth rates are proportional to \( t^{0.94} \). Theoretically the mass growth rate is proportional to the bulk collision (collection) efficiency \( E_b \) times the sweepout area of the graupel \( A_s \). Because the collision efficiency decreases with increasing diameter, the increasing sweepout area must be the dominant cause of the accelerating mass growth rate.

### 4.5 Curve Fit Coefficients

For each set of cloud conditions (i.e. each experimental series) the growth height \( h \) in mm, the mass \( m \) in mg and volume \( V_L \) in mm\(^3\), were parametrized in terms of time \( t \) in minutes, in the form

\[ x = a_x t^{b_x} \quad 4.4 \]

where \( x \) can represent \( h, m \) or \( V_L \), and \( a_x \) and \( b_x \) are coefficients of the fits. The fit coefficients for each series and the chi-squared errors are given in Table 4.1. The coefficients listed are not the best possible fits as determined by a least squares fitting routine. Instead, the fits were adjusted so that the \( b_x \) coefficients were consistent across several series while the fits were still close to the best fits. For example, the best fits for
<table>
<thead>
<tr>
<th>Series</th>
<th>Condition</th>
<th>No. Data Pts.</th>
<th>$a_n$ mm min$^{-1}$</th>
<th>$\chi^2$/df (h)</th>
<th>$a_m$ mg min$^{-1}$</th>
<th>$\chi^2$/df (m)</th>
<th>$a_v$ mm$^3$ min$^{-1}$</th>
<th>$\chi^2$/df ($V_L$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$W_f = 2.0$ g m$^{-3}$</td>
<td>6</td>
<td>0.577</td>
<td>0.8</td>
<td>0.118</td>
<td>0.9</td>
<td>0.46</td>
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<tr>
<td>2</td>
<td>$W_f = 1.0$ g m$^{-3}$</td>
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<td>0.321</td>
<td>3.5</td>
<td>0.033</td>
<td>4.5</td>
<td>0.13</td>
<td>1.0</td>
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<tr>
<td>3</td>
<td>$W_f = 3.0$ g m$^{-3}$</td>
<td>5</td>
<td>0.823</td>
<td>1.4</td>
<td>0.268</td>
<td>2.2</td>
<td>1.04</td>
<td>1.1</td>
</tr>
<tr>
<td>4</td>
<td>$W_f = 0.5$ g m$^{-3}$</td>
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<td>0.172</td>
<td>0.6</td>
<td>0.010</td>
<td>1.9</td>
<td>0.040</td>
<td>4.5</td>
</tr>
<tr>
<td>7</td>
<td>$V = 1.5$ m s$^{-1}$</td>
<td>6</td>
<td>0.503</td>
<td>0.7</td>
<td>0.064</td>
<td>0.5</td>
<td>0.31</td>
<td>1.5</td>
</tr>
<tr>
<td>8</td>
<td>$V = 2.5$ m s$^{-1}$</td>
<td>7</td>
<td>0.621</td>
<td>0.4</td>
<td>0.178</td>
<td>1.1</td>
<td>0.60</td>
<td>0.9</td>
</tr>
<tr>
<td>9</td>
<td>$V = 3.0$ m s$^{-1}$</td>
<td>6</td>
<td>0.652</td>
<td>1.3</td>
<td>0.234</td>
<td>0.4</td>
<td>0.66</td>
<td>0.4</td>
</tr>
<tr>
<td>10</td>
<td>$V = 1.1$ m s$^{-1}$</td>
<td>9</td>
<td>0.410</td>
<td>1.7</td>
<td>0.024</td>
<td>1.5</td>
<td>0.16</td>
<td>5.0</td>
</tr>
<tr>
<td>11</td>
<td>$r_m = 21.0$ μm</td>
<td>7</td>
<td>0.528</td>
<td>0.4</td>
<td>0.140</td>
<td>0.4</td>
<td>0.45</td>
<td>0.9</td>
</tr>
<tr>
<td>12</td>
<td>$r_m = 12.3$ μm</td>
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<td>0.588</td>
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<td>1.8</td>
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<tr>
<td>13</td>
<td>$r_m = 17.4$ μm</td>
<td>7</td>
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<td>1.2</td>
<td>0.123</td>
<td>0.8</td>
<td>0.42</td>
<td>0.5</td>
</tr>
<tr>
<td>14</td>
<td>$T_s = -10.8$ °C</td>
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<td>3.0</td>
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<tr>
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<td>$T_s = -4.4$ °C</td>
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<td>0.300</td>
<td>1.7</td>
<td>0.024</td>
<td>1.5</td>
<td>0.040</td>
<td>5.9</td>
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<tr>
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<td>0.782</td>
<td>0.6</td>
<td>0.139</td>
<td>1.3</td>
<td>0.69</td>
<td>1.5</td>
</tr>
<tr>
<td>18</td>
<td>$T_s = -16.0$ °C</td>
<td>10</td>
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<td>0.102</td>
<td>1.0</td>
<td>0.40</td>
<td>0.9</td>
</tr>
<tr>
<td>24</td>
<td>$P = 90$ kPa</td>
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<td>0.583</td>
<td>0.7</td>
<td>0.120</td>
<td>1.4</td>
<td>0.47</td>
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</tr>
<tr>
<td>25</td>
<td>$P = 80$ kPa</td>
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<td>0.584</td>
<td>0.4</td>
<td>0.115</td>
<td>1.6</td>
<td>0.43</td>
<td>1.6</td>
</tr>
<tr>
<td>26</td>
<td>$P = 70$ kPa</td>
<td>7</td>
<td>0.546</td>
<td>0.6</td>
<td>0.105</td>
<td>1.0</td>
<td>0.40</td>
<td>2.3</td>
</tr>
<tr>
<td>27</td>
<td>$P = 100$ kPa</td>
<td>7</td>
<td>0.554</td>
<td>1.1</td>
<td>0.108</td>
<td>0.7</td>
<td>0.43</td>
<td>2.1</td>
</tr>
<tr>
<td>28</td>
<td>$P = 60$ kPa</td>
<td>6</td>
<td>0.510</td>
<td>0.7</td>
<td>0.098</td>
<td>0.9</td>
<td>0.51</td>
<td>0.5</td>
</tr>
<tr>
<td>30</td>
<td>$r_m = 19.9$ μm</td>
<td>7</td>
<td>0.554</td>
<td>0.4</td>
<td>0.131</td>
<td>1.0</td>
<td>0.43</td>
<td>1.2</td>
</tr>
<tr>
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<td>$T_s = -5.8$ °C</td>
<td>5</td>
<td>0.286</td>
<td>0.5</td>
<td>0.034</td>
<td>0.8</td>
<td>0.066</td>
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</tr>
</tbody>
</table>

Table 4.1: Best fit coefficients $a_n$, $a_m$ and $a_v$ assuming that $b_n = 0.89$ and $b_m = b_v = 1.53 + 0.1 V + 0.014 r_m$ (Equation 4.10). Chi squared over the number of degrees of freedom ($\chi^2$/df) is a measure of the quality of the fit. S indicates the series with standard conditions.
the m-t curves at different liquid water contents had $b_m$ coefficients of 1.92, 2.03, 1.92 and 1.94 for liquid water contents of 0.5, 1.0, 2.0 and 3.0 g m$^3$ respectively. Since there was no obvious trend in $b_m$ with $W_r$, $b_m$ was fixed at 1.94, and the fits were recalculated to determine the $a_m$ coefficients, which were then parametrized in terms of $W_r$. The errors on the fits in Table 4.1 are incorporated into the errors in the $a_x$ coefficient, which are estimated at $\pm 5\%$. With these errors, the curve fits are acceptable representations of the experimental data to within the experimental errors, as will be evident from the figures of the next five sections.

With fixed $b_x$ coefficients, the $a_x$ values will be dependent upon the cloud conditions. The effects of liquid water content, velocity, temperature, droplet size distribution and pressure on the nature of the ice growth and on the coefficients of the parametrization are discussed in the next five sections.

4.6 Effects of the Liquid Water Content

To investigate the effects of the liquid water content on the growth of graupel, $W_r$ was systematically varied while all other conditions were held constant at their standard values. Series of between 7 and 13 experiments were performed for each liquid water content of 0.5, 1.0, 2.0 and 3.0 g m$^3$. Figure 4.6 shows the mass-volume relationships for each series. The density, as represented by the slope of the line, is independent of $W_r$, which implies that the surface temperature increase due to an increase in $W_r$ is not significant enough to increase the graupel density. Figures 4.7, 4.8 and 4.9 show respectively, the height, mass and volume variations with growth time. The curves shown are for the fits given in Table 4.1. The fit coefficients were parametrized as
Figure 4.6: Variation of mass with volume for liquid water contents of 0.5, 1.0, 2.0 and 3.0 g m$^{-3}$ at $T_a = -16.0^\circ$C, $V = 2.0$ m s$^{-1}$, $r_m = 14.8$ µm and $P = 100$ kPa.
Figure 4.7: Variation of growth height with time for different liquid water contents at $T_s = -16.0^\circ C$, $V = 2.0$ m s$^{-1}$, $r_m = 14.8$ $\mu$m and $P = 100$ kPa.
Figure 4.8: Variation of mass with time for different liquid water contents at

\[ T_s = -16.0^\circ\text{C}, \; V = 2.0 \text{ m s}^{-1}, \; r_m = 14.8 \mu\text{m} \; \text{and} \; P = 100 \text{ kPa}. \]
Figure 4.9: Variation of volume with time for different liquid water contents at

\[ T_s = -16.0^\circ C, \ V = 2.0 \ m \ s^{-1}, \ r_m = 14.8 \ \mu m \ and \ P = 100 \ kPa. \]
\[ b_h = 0.89 \]

\[ a_h = 0.081 \frac{W_f^{0.91}}{\rho} \]

\[ b_m = b_v = 1.94 \]

\[ a_m = 0.033 W_f^{1.92} = a_v \rho \]

where \( W_f \) is in g m\(^{-3} \) and \( \rho \) is the graupel density in g cm\(^{-3} \). The errors in the parametrized fits will be discussed in Section 4.11. The cone angle and geometric shape were observed to be independent of the liquid water content, with cone angles varying between 50 and 60° with an average of 54°. The growth height coefficient \( a_h \) varies almost linearly with \( W_f \), as it would for simple geometric sweepout. The small difference is probably due to the decrease in the bulk collision efficiency as the graupel changes size and shape. Theoretically, the mass growth rate is proportional to the sweepout area of the graupel and to the liquid water content of the air. The conical geometry causes the sweepout area to increase as the mass increases, which further enhances the rate of accretion. Hence, the mass growth rate accelerates with increasing \( W_f \) because of the coupling of an increased droplet sweepout per unit time and the consequent accelerating sweepout area.

### 4.7 Effects of the Airflow Velocity

The relative velocity between undisturbed droplets in the airflow and the suspended graupel is assumed to equal the velocity in the measuring section because it was always at least 10 times the terminal velocity of the largest droplets. The effects of velocity \( V \) on the ice growth were examined for velocities of 1.1, 1.5, 2.0, 2.5 and 3.0 m s\(^{-1} \) while all other cloud conditions were held constant at their standard values. The mass volume variation for each of the five experimental series shows that an increased velocity causes an increased densification (Figure 4.10). As discussed by Macklin (1962), the inertia of the
Figure 4.10: Variation of mass with volume for different velocities at $T_s = -16.0^\circ$C, $W_t = 2.0 \text{ g m}^{-3}$, $r_m = 14.8 \text{ m} \mu \text{m}$ and $P = 100 \text{ kPa}$. The data points for the standard velocity ($V = 2.0 \text{ m s}^{-1}$) are not shown.
droplets is proportional to $V$, so that with an increased inertia the packing and spreading of droplets in the ice deposit is increased. A density-velocity parametrization is discussed in Section 4.12. The geometry of the graupel was also observed to vary with $V$ as shown in the photographs in Figure 4.11. For a velocity of 1.1 m s$^{-1}$, the cone angle averaged $35^\circ \pm 3^\circ$ and the base was quite flat. For increasing velocities the base became slightly more rounded and the cone angle increased so that at 3.0 m s$^{-1}$ it averaged $66^\circ \pm 6^\circ$. At higher velocities, an increased packing of the higher inertia droplets near the ice edge would cause more of an overhang and therefore a larger cone angle.

Figures 4.12, 4.13 and 4.14 show respectively, the height, mass and volume variations with growth time for each series. The coefficients of the curve fits were parametrized as

$$b_h = 0.89$$

$$a_h = 0.041 \frac{W_f^{0.91} V}{\rho}$$

$$b_m = b_v = 1.74 + 0.1 V$$

$$a_m = 0.0105 W_f^{1.92} V^{1.62}$$

where $V$ is in m s$^{-1}$. The $W_f$ term comes from a variation of up to 10% in $W_f$ between different velocity series. The variation is caused by differences between series in the water injection, differences in evaporation of the spectra, and by fallout of the large droplets. The height coefficient $a_h$ varies linearly with velocity as would be the case for simple geometric sweepout. The mass growth rate is strongly dependent on velocity so that both $b_m$ and $a_m$ are proportional to $V$. A higher velocity leads to an increased liquid water sweepout per unit time, the consequential accelerating sweepout area, and an increased cone angle, as compared to the accretion rate at a lower velocity. Each trend reinforces the others in a multiplicative fashion.
Figure 4.11: Graupel grown at $T_s = -16.0^\circ$C, $W_f = 2.0 \text{ g m}^{-3}$, $r_m = 14.8 \mu\text{m}$ and $P = 100 \text{ kPa}$ for a) $V = 3.0 \text{ m s}^{-1}$ for 7 minutes; b) $V = 2.5 \text{ m s}^{-1}$ for 7 minutes; c) $V = 2.0 \text{ m s}^{-1}$ for 11 minutes; d) $V = 1.5 \text{ m s}^{-1}$ for 13 minutes; e) $V = 1.1 \text{ m s}^{-1}$ for 13 minutes. The initial embryo is a disk 1 mm in diameter.
Figure 4.12: Variation of height with time for different velocities at $T_s = -16.0^\circ C$,

$W_f = 2.0 \text{ g m}^{-3}$, $r_m = 14.8 \text{ \mu m}$ and $P = 100 \text{ kPa}.$
Figure 4.13: Variation of mass with time for different velocities at $T_s = -16.0^\circ$C,

$W_f = 2.0 \text{ g m}^{-3}$, $r_m = 14.8 \text{ m} \mu \text{m}$ and $P = 100 \text{ kPa}$.
$V_L$ versus time for different $V$

Figure 4.14: Variation of volume with time for different velocities at $T_a = -16.0^\circ$C,

$W_f = 2.0\, \text{g m}^{-3}$, $r_m = 14.8\, \mu\text{m}$ and $P = 100\, \text{kPa}$. 

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4.8 Effects of the Droplet Size Distribution

A droplet spectrum can be characterized by the median volume radius, \( r_m \), which is the radius for which half of the droplet spectrum's mass is incorporated in larger droplets. To investigate the effects of a droplet spectrum on graupel growth, five series of experiments were performed with different droplet distributions (under the standard cloud conditions). These series had median volume radii of 21, 19.9, 17.4, 14.8 and 12.3 ± 1 μm (see Section 3.6.3). The mass-volume relationship for each series (Figure 4.15) indicates an increase in the accretion density for increasing \( r_m \). The relatively larger droplets will have a larger inertia causing an increased packing and spreading of the freezing droplets on the graupel surface. A density parametrization with respect to the \( r_m \) is discussed in Section 4.12. Figure 4.16 shows photographs of graupel grown under different droplet distributions. The cone angle increases from 45°±5° when the \( r_m \) was 12.3 μm, to 68°±7° for an \( r_m \) of 21 μm. As with the velocity results, the cone angle increases with the inertia of the droplets.

Graphs of height, mass and volume versus growth time are shown in Figures 4.17, 4.18 and 4.19, respectively. The fit coefficients were parametrized as

\[
b_h = 0.89
\]

\[
a_h = 0.025 \frac{W_f^{0.91} r_m^{0.43}}{\rho}
\]

\[
b_m = b_v = 1.73 + 0.014 r_m
\]

\[
a_m = 0.0053 W_f^{1.92} r_m^{0.64}
\]

where \( r_m \) is in μm. Theoretically, the growth height for simple geometric sweepout is given by
Figure 4.15: Variation of mass with volume for different median volume radii at

\[ T_s = -16.0^\circ C, \ W_f = 2.0 \text{ g m}^{-3}, \ V = 2.0 \text{ m s}^{-1} \text{ and } P = 100 \text{ kPa}. \] The data points for the standard median volume radius \( r_m = 14.8 \mu m \) are not shown.
Figure 4.16: Graupel grown at $T_s = -16.0^\circ C$, $V = 2.0 \text{ m s}^{-1}$, $W_f = 2.0 \text{ g m}^{-3}$ and $P = 100 \text{ kPa}$ for a) $r_m = 21.0 \mu\text{m}$ for 8 minutes; b) $r_m = 17.4 \mu\text{m}$ for 8 minutes; c) $r_m = 12.3 \mu\text{m}$ for 10 minutes. The initial embryo is a disk 1 mm in diameter.
Figure 4.17: Variation of height with time for different median volume radii at

\[ T_s = -16.0^\circ C, \ W_f = 2.0 \, \text{g m}^3, \ V = 2.0 \, \text{m s}^{-1} \text{ and } P = 100 \, \text{kPa}. \]
Figure 4.18: Variation of mass with time for different median volume radii at

\[ T_s = -16.0^\circ C, \ W_f = 2.0 \text{ g m}^{-3}, \ V = 2.0 \text{ m s}^{-1} \text{ and } P = 100 \text{ kPa}. \]
Figure 4.19: Variation of volume with time for different median volume radii at
\[ T_s = -16.0^\circ C, \ W_r = 2.0 \ g \ m^3, \ V = 2.0 \ m \ s^{-1} \] and \( P = 100 \ kPa \).
\[ \frac{dh}{dt} = \frac{E_b W_f V}{\rho} \]

where \(E_b\) is the bulk collision efficiency (defined in Chapter 2). In Equation 4.8 there is no explicit dependence on \(r_m\), although \(E_b\) implicitly depends on \(r_m\). In the parametrizations, the \(r_m^{0.43}\) and \(r_m^{0.64}\) terms incorporate the change in \(E_b\) with a changing droplet diameter. Over the droplet spectra investigated, the theoretical bulk collision efficiency varied by 20 to 40%. Therefore the mass growth rate was less dependent on the droplet distribution than on other parameters such as the liquid water content or velocity. Doubling the liquid water content or velocity doubles the mass growth rate, whereas doubling the median volume radius from 10 to 20 \(\mu\)m only increases the mass growth rate by 35%.

4.9 Effects of the Ambient and Surface Temperatures

Series of between 5 and 13 experiments were performed under the standard cloud conditions with air temperatures of -21, -16, -10.8, -7.0, -5.8 and -4.4°C, with respective surface temperatures of -20, -15, -9.8, -6.2, -5.2 and -3.8°C (see Section 4.15). The results will be discussed with respect to the surface temperature because it physically represents the freezing temperature of the accretion, although in these experiments, the surface and ambient temperatures differed by at most 1°C. Figure 4.20 shows the mass-volume relationship for each series. The accretion density is a function of the surface temperature with the density increasing with increasing temperature. At warmer temperatures the droplets spread more while freezing (Mason, 1971) causing fewer and smaller air spaces between the frozen droplets which make up the ice structure. The density should approach that of pure ice (0.915 g cm\(^{-3}\)) as the temperature approaches 0°C. A density-temperature parametrization is discussed in Section 4.12. The opacity of a graupel is caused by the air capillaries and bubbles within the ice structure (Mason, 1971; Hobbs, 1974). The number of bubbles is proportional to the supercooling so that the larger the supercooling the more bubbles in the droplet after freezing. Graupel grown at warmer temperatures have fewer capillaries and appear more glossy and transparent because the capillary and bubble
Figure 4.20: Variation of mass with volume for different ambient temperatures at

\[ W_f = 2.0 \, \text{g m}^{-3}, \, V = 2.0 \, \text{m s}^{-1}, \, r_m = 14.8 \, \mu\text{m} \text{ and } P = 100 \, \text{kPa}. \]

The data points for the standard temperature \( T_* = -16^\circ\text{C} \) are not shown.
structure is less effective at scattering light. This can be observed in photographs (Figure 4.21) of graupel grown at different temperatures. The graupel grown at -3.8°C and -5.2°C are glossy and transparent while those grown at temperatures colder than -6.2°C are white and opaque. It is difficult to visually distinguish the opacity between graupel formed at different temperatures below -10°C.

The geometric shape and cone angle are also related to the temperature. At -3.8°C and -5.2°C, the graupel bases were rounded and shaped more spherically, whereas at temperatures colder than -6.2°C they were relatively flat. The cone angle increases from 31°±3° at -3.8°C to 55°±6° at -15°C and decreases again to 46°±4° at -20°C. At warmer temperatures, the increased spreading of the freezing droplets causes the droplets to spread around the ice edge, producing a smaller overhang and a decreased cone angle. At colder temperatures there may be a balance between the nature and speed of the freezing and the inertial effects of the droplet. For example, at -20°C the droplets freeze faster and pack less than at -15°C, so that the graupel has a smaller density and smaller cone angle. Since at both temperatures the droplets freeze approximately as spheres (Dong and Hallett, 1986), the reduced droplet packing at -20°C (caused by the faster freezing) will lead to the reduced cone angle.

The height, mass and volume variations with growth time are shown in Figures 4.22, 4.23 and 4.24 respectively. The fit coefficients were parametrized as

\[ b_h = 0.89 \]

\[ a_h = 0.081 \frac{W_f^{0.91}}{\rho} \]

\[ b_m = b_v = 1.94 \]
Figure 4.21: Graupel grown at $W_i = 2.0 \text{ g m}^{-3}, V = 2.0 \text{ m s}^{-1}, r_m = 14.8 \mu\text{m}, P = 100 \text{ kPa}$
for a) $T_a = -20.9^\circ\text{C}$ for 5 minutes; b) $T_a = -10.8^\circ\text{C}$ for 11 minutes; c) $T_a = -7.0^\circ\text{C}$
for 15 minutes; d) $T_a = -5.8^\circ\text{C}$ for 17 minutes; e) $T_a = -4.4^\circ\text{C}$ for 17 minutes. The
initial embryo is a disk 1 mm in diameter.
Figure 4.22: Variation of height with time for different ambient temperatures at

\[ W_r = 2.0 \text{ g m}^{-3}, \quad V = 2.0 \text{ m s}^{-1}, \quad r_m = 14.8 \text{ \&m}, \quad P = 100 \text{ kPa}. \]
Figure 4.23: Variation of mass with time for different ambient temperatures at

\[ W_f = 2.0 \text{ g m}^{-3}, \ V = 2.0 \text{ m s}^{-1}, \ r_m = 14.8 \text{ m} \mu\text{m} \text{ and } P = 100 \text{ kPa}. \]
Figure 4.24: Variation of volume with time for different ambient temperatures at

\[ W_r = 2.0 \text{ g m}^{-3}, \ V = 2.0 \text{ m s}^{-1}, \ \tau_m = 14.8 \mu\text{m and } P = 100 \text{ kPa.} \]
\[ a_m = W_j^{1.92} \left( -0.0073 - 0.0036 T_s - 0.000072 T_s^2 \right) \]

where \( T_s \) is the surface temperature in °C. The height parametrization is independent of the surface temperature as would be expected from simple geometric sweepout. An implicit temperature dependence comes through the density dependence. The mass growth rate increases with decreasing temperature. As the temperature decreases, the density decreases so that for a given mass, the volume will be greater for a low density graupel particle than for a high density one. This implies a larger sweepout area and faster mass growth rate. The cone angle variation with temperature also has an effect on the mass growth rate, with larger cone angles causing a faster increase in the sweepout area.

The temperature parametrizations reflect the different behaviour of the freezing droplets over the temperature range studied. At warmer temperatures the spreading of the droplets and relatively slow freezing rate are responsible for high densities, narrow cone angles and a small mass growth rate. As the temperature cools, the rate of freezing increases until it is fast enough that the droplets freeze almost immediately upon contacting the ice surface, with the inertial effects of the droplets becoming negligible. Therefore, there should be a limiting temperature after which the density, geometry and cone angle do not change significantly with a further decrease in temperature. Experiments with spherical models by Buser and Aufdermaur (1973) have shown that this would produce an ice density of about 0.17 g cm\(^{-3}\). As a comparison, for the experimental series at -20°C the accretion density was 0.20 g cm\(^{-3}\). Differentiating the mass parametrization equation with respect to temperature yields a maximum mass growth rate at approximately -25°C with a corresponding density of 0.18 g cm\(^{-3}\). The experimental mass growth rate and density seem to asymptotically approach a constant value as the temperature decreases below -20°C, and this is consistent with the limiting density of Buser and Aufdermaur and with the observations of Macklin (1962).
4.10 Effects of the Ambient Pressure

The effects of ambient pressure on the graupel growth were examined for pressures of 100 (atmospheric), 90, 80, 70 and 60 kPa. It was not possible to investigate pressures lower than 60 kPa because perturbations to the airflow by the vacuum pump became unacceptably large. The degree of atomization of the droplets (for fixed air and water flowrates) depended on the ambient pressure surrounding the nozzle. As it decreased, the atomization of the droplets increased so that the mean droplet radius decreased. At each pressure, using MgO measurements a nozzle air flowrate was found which created a droplet distribution similar to the standard distribution at atmospheric pressure. For pressures of 100, 90, 80, 70 and 60 kPa, nozzle air flowrates of 75, 73, 71, 68 and 65 mm gave droplet distributions with median volume radii of 15, 14, 16.5, 15 and 17.5 ± 2 μm, respectively.

Figure 4.25 shows the mass-volume relationship for the four lower pressure series. The slope (or density) is constant and equal to the density at atmospheric pressure. This implies that the nature of the ice accretions is the same at each pressure and confirms that the droplet distributions at the different pressures are fairly consistent. There was no significant difference observed in the graupel geometry or cone angle for different pressures. The cone angles varied between 50 and 60°, which were the same as at atmospheric pressure.

Figures 4.26 and 4.27 show respectively the height and mass variations with growth time for each series. The atmospheric pressure results are shown for comparison. There is a 20% decrease in the mass growth rate between 90 and 60 kPa, although the atmospheric pressure results are not consistent with this trend. The 70 to 90 kPa results are all within 5 to 10% of the atmospheric pressure results so it is difficult to distinguish between a specific trend and the experimental errors. It is unclear why a reduction in pressure would decrease the mass growth rate. At lower pressures, the reductions in density of the air should tend to increase the collision efficiency of the droplets because the air would have a reduced ability to transfer momentum to the droplets and deflect them around the graupel.
Figure 4.25: Variation of mass with volume for ambient air pressures of 100, 90, 80, 70 and 60 kPa at $T_a = -16^\circ C$, $W_r = 2.0$ g m$^{-3}$, $V = 2.0$ m s$^{-1}$ and $r_m = 14.8$ μm.
Figure 4.26: Variation of height with time for different ambient air pressures at

$T_s = -16^\circ C$, $W_f = 2.0$ g m$^{-3}$, $V = 2.0$ m s$^{-1}$ and $r_m = 14.8$ $\mu$m. The data points for the standard pressure ($P = 100$ kPa) are not shown.
Figure 4.27: Variation of mass with time for different ambient air pressures at $T_a = -16^\circ$C, $W_r = 2.0 \text{ g m}^{-3}$, $V = 2.0 \text{ m s}^{-1}$ and $r_m = 14.8 \mu\text{m}$.
The ambient pressure has, at best, a second order effect on the mass growth rate in comparison to the other cloud parameters, for the range of pressures studied. Therefore, no parametrizations of the graupel characteristics were determined with respect to pressure. Assuming that the parametrizations at 100 kPa hold for low pressures, they will be in error by between 5 and 10% for the 70 to 90 kPa data and by up to 20% for the 60 kPa data.

4.11 Combination of the Parametrizations

For each cloud variable, parametrizations have been determined for height, mass and volume with respect to time and the cloud variable. Combining the parametrizations together gives the following equations:

$$b_h = 0.89$$

$$a_h = 0.0127 \frac{V W_f^{0.91} r_m^{0.43}}{\rho}$$

$$b_m = b_v = 1.53 + 0.1 V + 0.014 r_m$$

$$a_m = 0.0595 W_f^{1.92} V^{1.62} r_m^{0.64} \left(-0.0073 - 0.0036 T_s - 0.000072 T_s^2\right)$$

$$a_v = \frac{a_m}{\rho} \quad (4.10)$$

where $W_f$ is the liquid water content in g m$^{-3}$, $V$ is the velocity in m s$^{-1}$, $r_m$ is the median volume radius in μm, $T_s$ is the surface temperature in °C and $\rho$ is the ice density in g cm$^{-3}$.

Using the values of the cloud parameters in Table 3.2, the parametrized $a_h$, $a_m$, and $a_v$ coefficients were calculated and compared to the individual fit coefficients (given in Table 4.1). Figure 4.28 shows comparisons of the coefficients for height and mass respectively. The errorbars for the parametrized values represent a 10 to 15% error based
Figure 4.28: Comparisons between the mass-time and height-time fit coefficients, $a_m$ and $a_h$, for the best fits (Table 4.1) and the parametrized fits (Equation 4.10). The line represents a slope of 1.
on the cumulative effect of the following errors: $\Delta \rho = \pm 0.01 \, \text{g cm}^{-3}$; $\Delta T_s = \pm 0.25 \, ^\circ\text{C}$; $\Delta W_f = \pm 0.15 \, \text{g m}^{-3}$; $\Delta V = \pm 0.05 \, \text{m s}^{-1}$; and $\Delta r_m = \pm 1 \, \mu\text{m}$. The error bars shown for the individual best fit coefficients (Table 4.1) are $\pm 5\%$. This was estimated from the fact that each fit was based on data points with $a \pm 2$ to 4\% error, and was a good representation of the data to within the experimental error.

These parametrizations reflect the experimental data for variations in a single cloud parameter from a standard value. However, the exponential nature of the fits tend to amplify the growth rates when two or more cloud parameters are increased simultaneously from their standard values. This overestimates the mass, volume and height as compared to experimentally measured values. Therefore, these parametrizations are not general enough for use in a computational graupel growth model. General equations for use in such a model will be derived in Chapter 5 when the bulk collision efficiency and Nusselt numbers (dimensionless parameters that represent the mass and heat transfers) are derived from the experimental results.

### 4.12 Density Parametrization

As stated in Sections 4.7, 4.8 and 4.9 the density of the ice accretion was found to be dependent on the velocity, the droplet size distribution and the surface temperature. These relationships have been observed by other workers (Macklin, 1962; Pflaum and Pruppacher, 1979) and were discussed in Chapter 1. Following Macklin (1962), it is common to parametrize the density in terms of a variable $R_i$, where

$$R_i = \frac{-r_mV_i}{T_s}$$

where $r_m$ is the median volume radius in $\mu\text{m}$, $V_i$ is the droplet impact velocity in $\text{m s}^{-1}$ and $T_s$ is the surface temperature in $^\circ\text{C}$. Macklin interpreted $r_mV_i$ as a measure of the force that packed the droplets together, and $T_s$ as the temperature that governed the freezing rate. To investigate a wider range of the $R_i$ parameter, additional experiments were performed where
two or more cloud variables known to affect the accretion density were simultaneously varied to magnify the densification. Figure 4.29 shows the variation of $R_\i$ with density, for each experimental series and for the additional experiments. The impact velocities were calculated from the parametrizations of Rasmussen and Heymsfield (1985). The data was parametrized by

$$\rho = 0.078 + 0.184 R_\i - 0.015 R_\i^2 \quad 0.3 \leq R_\i \leq 5.5 \quad 4.12$$

where $V$ is in m s$^{-1}$, $r_m$ in μm, $T_\i$ in °C, $\rho$ is in g cm$^{-3}$, and $R_\i$ is in μm m s$^{-1}$ C$^{-1}$. The density parametrization of Heymsfield and Pflaum (1985), as based on the experimental data of Pflaum and Pruppacher (1979), is also shown in Figure 4.29. Their parametrization agrees fairly well with Equation 4.12, although it predicts significantly higher densities for $R_\i$ less than 1.5. As discussed in Chapter 1 there is reason to believe that Pflaum and Pruppacher overestimated the density of their graupel for small values of $R_\i$.

The impact velocity changes over a surface, and surface averaged calculations of $V_\i$ are not consistently defined by different workers. Therefore, it is simpler to redefine $R_\i$ with respect to the ambient velocity $V$ (as opposed to the impact velocity) so that

$$R = \frac{-r_m V}{T_\i} \quad 4.13$$

Figure 4.30 shows the variation of $R$ with the density for each experimental series and for the special experiments. The data is well described by

$$\rho = 0.051 + 0.115 R - 0.0055 R^2 \quad 1.0 \leq R \leq 8.0 \quad 4.14$$

where the units are the same as in Equation 4.12. In these experiments the air velocity is about 1.6 times the surface averaged impact velocity, as calculated from Rasmussen and Heymsfield (1985).
Figure 4.29: Density variation with $R_i$, where $R_i = -r_m \cdot \frac{V_i}{T_i}$, and $V_i$ is the surface averaged impact velocity.
Figure 4.30: Density variation with $R$, where $R = - r_m \frac{V}{T_r}$, and $V$ is the relative velocity between the droplets (airflow) and the graupel.
4.13 Graupel Geometry

In order to maintain consistency between the volume and height parametrizations, the geometry of the graupel was generalized with a simple geometric formula. After measuring approximately 850 cold room photographs of graupel, the geometry (with the exception of the -3.8 and -5.2°C experiments) was found to be adequately described by a conical shape as shown in Appendix C. Using the formulas from Appendix C, the volume and surface area can be calculated from a knowledge of the cone angle, the embryo diameter and the growth height of the accretion. Figure 4.31 shows a plot of the volume calculated through photographic measurements versus the volume measured through Archimedes principle. The best fit slope is 0.98, with the two volumes agreeing to within ± 15% for every experiment. The errors of the photographically calculated volumes primarily result from the ± 6° error on the cone angle measurements.

The graupel from the -3.8 and -5.2°C experiments require the graupel base to be approximated by a spherical sector as opposed to being mostly flat. Appendix C describes the geometry for the graupel grown in the -3.8 and -5.2°C experiments.

4.14 Cone Angle Calculation

With a geometric description, the volume and height parametrizations can be used to calculate the geometric parameters required when solving the heat and mass transfer equations, as shown in Chapter 5. At a specific time, the height and volume can be determined from the parametrizations. Then, the cone angle which is consistent with the parametrized volume and height can be calculated. The diameter, sweepout area, and the surface areas of the graupel sides and base can then be calculated and used in the heat and mass transfer equations.
Figure 4.31: Comparison between the graupel volume calculated through photographic measurements and a geometric approximation and the volume measured with Archimedes principle. The line represents a slope of 1.
Figure 4.32 shows how the cone angle varies with growth height, as determined from the parametrizations for $W_t$ and $V$. For comparison the experimentally measured cone angles are also shown. The calculations show a small decrease in the cone angle with increasing growth height, which is also observed in the experimental measurements and is thought to be caused by a change in the streamlines around the graupel as the diameter increases. The calculated cone angles are representative of the experimentally measured ones, agreeing to within ± 5°.

The cone angle calculation breaks down for growth heights smaller than 1.5 to 2.0 mm because of the exponential nature of the fits. This implies that the mass and volume parametrizations should not be extrapolated to values where the corresponding height is less than 1.5 mm. The growth height parametrization can probably be accurately extrapolated to an initial height of zero mm because of the weak power of the fit (i.e. $h \propto t^{0.89} = t$). By setting the cone angle for small growth heights equal to the maximum cone angle in the calculations (which occurs at a growth height of about 2.5 mm as shown in Figure 4.32), the problem with the cone angle calculation can be eliminated.

To avoid using the volume and height parametrizations to calculate the cone angle in a more general graupel growth model, the average cone angle for each series was parametrized as

$$c_a = 0.0183 \left( 1.35 + 2.37 (V r_m) - 0.02 (V r_m)^2 \right) \left( 25.2 - 4.84 T_s - 0.19 T_s^2 \right)$$

where $c_a$ is in degrees, $V$ in m s$^{-1}$, $r_m$ in µm and $T_s$ in °C. Figure 4.33 shows a comparison between the parametrized cone angle and the average cone angle. Equation 4.15 is not valid for temperatures warmer than -5°C where it significantly overestimates the cone angle. Because the decrease in cone angle with increasing diameter was small, it was not incorporated in Equation 4.15.
Figure 4.32: Comparison between the cone angles measured experimentally (points) and those calculated through the geometric model (curves) for the $W_f$ and $V$ series.
Figure 4.33: Comparison between the cone angle measured experimentally and that calculated from the parametrization (Equation 4.15). The line represents a slope of 1.
4.15 Surface Temperature Measurements

During heat transfer experiments, images of the surface temperature distribution of a growing graupel were recorded at one minute intervals with an Agema 800 Thermovision Thermal Imaging System. A pre-experiment background image was subtracted to give the temperature elevation over the ambient temperature across the graupel surface. Figure 4.34 shows photographs of the temperature elevation for a graupel grown under the standard cloud conditions. The background temperature image is also shown in Figure 4.34. A maximum elevation of 1.0±0.2°C occurs near the base where the accretion is occurring. Rough measurements of the base temperature indicated that it was within 0.1°C of the maximum temperature elevation measured on the sides. The temperature gradients near the edges and the apex are caused by the 2.0±0.5 mm resolution (spot size) of the detector. When an edge is within the detector's field of view, radiation from both the surface and the background are averaged together. For graupel larger than 3 to 4 mm in height, there were consistent observations that the temperature elevation decreased towards the apex. This would result from the fact that the latent heat release due to riming is the dominant heating mechanism and occurs mostly at the base. Old accretional growth would be separated from the main region of heating by newer ice growth and consequently could only warm by conduction through the new ice (assuming the heat transfer by deposition is small in comparison).

The surface temperature increased with time until the graupel size was larger than the spot size, after which there was no further temperature increase indicating that the maximum surface temperature elevation was independent of time. The images indicated that at least 75% of the graupel surface area was within 0.2°C of the maximum elevation. Therefore, the maximum temperature elevation was taken as representative of the entire surface, the cooler apex being neglected.
Figure 4.34: Background image (a) and thermal image (b) of a graupel as measured by the Agema thermal imaging system. The scale on the right is offset by -10.0°C. The images can be subtracted and the final image magnified (c). The maximum surface temperature elevation is 1.1°C over the ambient temperature. Box 1 is a 1 mm by 1 mm scale. Box 2 is approximately the size of the spot size of the detector. The temperature profiles at the bottom and side of c are taken along the two crosshair lines that intersect near the centre of the graupel image.
The surface temperature experiments were performed with the same combinations of cloud conditions as the mass transfer experiments. The liquid water content, velocity, median volume radius, temperature and pressure were independently and systematically varied from the standard set of cloud conditions. Figures 4.35 and 4.36 show changes in the maximum surface temperature elevation, $\Delta T_s$, for variations in $W_f$, $V$, $r_m$ and $T_s$. The maximum temperature elevation was parametrized as

$$\Delta T_s = 0.025 \ r_m \ W_f \ (V - 0.65) \ (0.24 - 0.09 \ T_a - 0.0026 \ T_a^2)$$

where $\Delta T_s$ is in °C. The temperature elevation increases with increasing mass growth rate because of the associated increase in the latent heat release of the freezing droplets. Decreasing temperature, increasing velocity, increasing liquid water content or increasing median volume radius all increase the mass growth rate and consequently the temperature elevation. The temperature elevation was independent of pressure which is consistent with the mass growth rate being independent of pressure. The experimental surface temperature elevation is plotted against the parametrized elevation in Figure 4.37, which shows that Equation 4.16 is correct to within the experimental error of ±0.2°C.

**4.16 High Density Experiments**

Sixteen additional experiments were performed as a preliminary investigation of higher density graupel (i.e. $\rho > 0.5 \text{ g cm}^{-3}$). The cloud conditions were chosen to enhance the densification and included all combinations of the following parameters: $T_s = -6, -8, -11$ and -15°C; $r_m = 20$ and 15 µm; $V = 3.0$ and 2.5 m s$^{-1}$. Measurements of the cone angles and densities from these experiments allowed the parametrizations discussed in Sections 4.12 and 4.13 to accurately extend to larger values of $c_s$ and $\rho$. Figure 4.38 shows graupel grown under four such sets of conditions. An increase in the curvature of the base is evident with increasing density. This is similar to that observed in the -4.4°C and -5.8°C series where it was attributed to the slower freezing rate of droplets at warmer temperatures. However, the increased curvature of the base is evident here at -10°C,
Figure 4.35: Surface temperature elevations for variations in \( W_0 \) and \( V \) from the standard conditions.
Figure 4.36: Surface temperature elevations for variations in $r_m$ and $T_a$ from the standard conditions.
Figure 4.37: Comparison between the surface temperature elevation measured experimentally and that calculated from the parametrization (Equation 4.16). The line represents a slope of 1.
Figure 4.38: Graupel grown under conditions which enhance the densification.

a) 8 minutes at $T_s = -11^\circ$C, $V = 3.0$ m s$^{-1}$, $r_m = 15$ $\mu$m, $\rho = 0.45$ g cm$^{-3}$.
b) 11 minutes at $T_s = -8^\circ$C, $V = 3.0$ m s$^{-1}$, $r_m = 15$ $\mu$m, $\rho = 0.53$ g cm$^{-3}$.
c) 11 minutes at $T_s = -8^\circ$C, $V = 3.0$ m s$^{-1}$, $r_m = 20$ $\mu$m, $\rho = 0.63$ g cm$^{-3}$.
d) 11 minutes at $T_s = -6^\circ$C, $V = 3.0$ m s$^{-1}$, $r_m = 15$ $\mu$m, $\rho = 0.65$ g cm$^{-3}$.
which indicates that the inertia of the impacting droplets also contributes to the sphericity of the graupel base.

A change in the graupel geometry with increasing density may indicate a change in the centre of growth as discussed by List (1960). Such a change may be inherent in the transition of graupel to small hail, which may also be accompanied by a change in the aerodynamic behaviour.
Chapter 5

Analysis and Discussion of the Heat and Mass Transfer

5.1 Introduction

In this chapter the Nusselt number and bulk collision efficiency will be calculated from the experimental data and parametrized in terms of the Reynolds number and Stokes number respectively. The parametrizations will be discussed and compared to other experimental measurements and theoretical estimates. Finally, a basic graupel growth model will be set up and applied to the range of experimental conditions to test its accuracy.

5.2 Application of the Experimental Data to Theory

The mass and heat transfer equations for a graupel, as developed in Chapter 2, are given by:

\[
\frac{dM}{dt} = Z_1 E_b - Z_2 N_u \Delta e
\]

\[
0 = Z_3 N_u \Delta T + Z_4 N_u \Delta e + Z_5 E_b \Delta T - Z_6 E_b
\]

where \(\frac{dM}{dt}\) is the mass growth rate, \(E_b\) is the bulk collision efficiency, \(N_u\) is the Nusselt number, and \(\Delta e, \Delta T\), and \(Z_1\) to \(Z_6\) are given by

\[
\Delta T = T_s - T_a
\]

\[
\Delta e = e_{ss}(T_s) - U_a e_{ss}(T_a)
\]

\[
Z_1 = A_b W_f V
\]
\[ Z_z = \frac{0.95 D \frac{D w a}{D T_a}}{A_z} \]

\[ Z_3 = \frac{A_z k_a}{D} \]

\[ Z_4 = \frac{0.95 L_s D \frac{D w a}{D T_a}}{A_z} \]

\[ Z_5 = A_b W_f V C_w \]

\[ Z_6 = A_b W_f V L_f \]

These variables are functions of air and vapour properties, the cloud conditions, and the graupel geometry. By defining

\[ C = \frac{Z_3 \Delta T - Z_6}{Z_3 \Delta T + Z_4 \Delta e} \]

Equations 5.1 and 5.2 can be reduced to

\[ E_b = \frac{dM}{dt} \frac{1}{Z_1 + Z_2 C \Delta e} \]

\[ N_a = -E_b C \]

respectively.

To calculate \( N_a \) and \( E_b \) using these equations, the following sequence of calculations was performed. The cloud conditions of a mass transfer experiment series defined the ambient cloud conditions. The relative humidity and the corresponding ambient vapour pressure were obtained from the simulations of the thermodynamics of the wind tunnel. The heat transfer experiments (for the same cloud conditions) gave the surface temperature
elevation and the corresponding saturation vapour pressure over the graupel surface, so that \( \Delta T \) and \( \Delta e \) could be determined. The best fits for the volume-time and height-time relationships (Table 4.1) were used to calculate the cone angle, surface area, graupel diameter and sweepout area, allowing \( Z_1 \) to \( Z_6 \) to be determined. Using the best fit of the mass-time relationship, the mass growth rate, \( \text{d}M/\text{d}t \), was calculated over a time step \( \text{d}t \). The time step \( \text{d}t \) was chosen as 0.01 minutes during which the geometry of the graupel does not change significantly. Finally \( N_0 \) and \( E_0 \) were calculated from Equations 5.5 and 5.6 respectively.

5.3 Bulk Collision Efficiency Analysis

Figure 5.1 exhibits the variation of the bulk collision efficiency with graupel diameter for the \( W_r \), \( V \), \( r_m \) and \( T_a \) series. The bulk collision efficiency consistently decreases with increasing diameter. As the experiments with the shortest durations had graupel with diameters of about 2 mm, the bulk collision efficiency results were not extrapolated to diameters less than 2 mm. \( E_0 \) is independent of the liquid water content, the ambient temperature and the pressure (results for \( P \) are not shown), to within the experimental errors. The variability of the results is caused by variations in cloud parameters such as \( W_r \) and \( V \) between series and is indicative of the experimental errors. The sensitivity of the calculations to the experimental errors will be discussed in Section 5.5.

The bulk collision efficiency increases with increasing velocity and median volume radius (Figure 5.1), and decreases with increasing graupel diameter. These trends are consistent with the theoretical calculations of Langmuir and Blodgett (1946) and Ranz and Wong (1952). Langmuir and Blodgett expressed the collision efficiency in terms of the dimensionless Stokes parameter \( K \) where

\[
K = \frac{2 \rho_d V r_d^2}{9 \eta r_i}
\]  

5.7
Figure 5.1: Variation of bulk collision efficiency with graupel diameter for the $W_p$, $V$, $r_m$, and $T_p$ series.
where \( \eta \) is the dynamic viscosity of the air, \( \rho_d \) the droplet density, \( r_d \) the droplet radius, \( r_t \) the target radius and \( V \) the initial relative velocity between the droplet and the target. Physically, the Stokes parameter represents the ratio of the distance a droplet would travel through still air given an initial velocity of \( V \) and assuming Stokes law, versus the radius of the target. Since the effects of velocity, droplet size and target size are all incorporated within the Stokes parameter, Langmuir and Blodgett (1946) were able to determine parametrizations of the collision efficiency in terms of only the Stokes parameter. In the results presented here, the median volume radius is taken as representative of the droplet radius while the graupel radius represents the target radius.

The variation of the bulk collision efficiency with the Stokes parameter is displayed in Figure 5.2 for the \( W_n \), \( V \), \( r_m \) and \( T_s \) series of experiments. The results are consistent within the variability of the experimental data over the range of \( K \) between 1.5 and 10, with the exception of the 1.1 m s\(^{-1}\) data. The inconsistency of the 1.1 m s\(^{-1}\) results is believed to result from an overestimation of the relative humidity. The bulk collision efficiency was parametrized as

\[
E_b = 0.55 \log(2.51 K) \quad 1.5 \leq K \leq 10
\]

which is shown in Figure 5.3 compared with the theoretical results of Langmuir and Blodgett (1946) for an ideal sphere, and with the results of Ranz and Wong (1952) for an ideal disk. Physically, the graupel shape is both like a disk because of the flat base and like a sphere because of the rounded edges on the base and the conical shape. Therefore, the collision efficiency of the graupel would be expected to be between that of a sphere and a disk. However, although the experimental and theoretical trends are similar, the experimental collision efficiencies are about 25% lower than the theoretical efficiency for the sphere and about 40% lower than that for the disk.

A reduced collision efficiency, compared to theoretical estimates, is an important factor which is not incorporated in numerical models that simulate graupel growth.
Figure 5.2: Variation of bulk collision efficiency with Stokes parameter for the $W_p$, $V$, $r_m$ and $T_a$ series. The Stokes parameter is defined as $K = 2 \rho \frac{r_m^2}{\eta} \frac{V}{r_g}$. 

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Figure 5.3: Parametrization of the bulk collision efficiency with Stokes parameter, and comparison to other theoretical and experimental results.
Therefore, such calculations would overestimate the mass growth rate. Electrification of the
droplet spectrum could reduce the collision efficiency, as Pflaum and Pruppacher (1979)
proposed to explain why the results of Macklin (1962) did not agree with their own. As
discussed in Chapter 3, experiments were performed in the present study that showed that
droplet electrification had no significant effect on the mass growth of graupel in the wind
tunnel. Experimentally, an overestimation of the liquid water content or the droplet median
volume radius could account for the reduced collision efficiency. However, \( W_f \) or \( r_m \) would
have to be overestimated by at least 40%, which is significantly larger than the estimated
experimental error of ±5 to ±10%.

The bulk collision efficiency results are similar to those of Mossop (1976) and
Pflaum and Pruppacher (1979). Mossop (1976) determined experimentally that the collision
efficiency of a droplet spectrum with a 2 mm cylinder at velocities between 1.4 and 3
m s\(^{-1}\) (\( R_e = 300 \)) could best be described by the theoretical curves of Davies and Pectz
(1956) for cylinders with \( R_e = 10 \). Mossop's results (Figure 5.3) also predict a 20 to 30%
decrease compared to theory in the collision efficiency, which he attributed to the changes
in the air flow pattern caused by the roughness of the rime. The enhanced surface
roughness of the graupel ice surface could decrease the collision efficiency, although it is
difficult to envision how the boundary layer around a rough graupel (as compared to that
around a smooth graupel) could influence the collision efficiency of the colliding droplets
significantly. Mossop did not elaborate on the significance of his results or attempt to
provide any other explanation. Pflaum and Pruppacher (1979) offered a similar explanation
although they did not quantify their results.

Keith and Saunders (1988) showed experimentally that the collision efficiency of
small droplets (< 20 \( \mu \)m) with a low density rimed icing cylinder was larger than that
predicted by theory. They attributed this to an enhanced collection of droplets by rime
feathers growing from the ice surface. Their droplets averaged around 10 \( \mu \)m in diameter
with corresponding collision efficiencies of about 0.1, and significantly smaller Stokes
parameters (i.e. $K = 1$) than for the cases presented here. The rime feathers change the nature of the droplet-surface interactions so that their results are difficult to compare to, and may not contradict those presented here.

Makkonen and Stallibrass (1987) experimentally verified the theoretical collision efficiencies of Langmuir and Blodgett (1946) for Reynolds numbers larger than $10^5$ ($V > 20 \text{ m s}^{-1}$), and thereby showed that the application of potential flow theory was applicable for larger Reynolds numbers. At these velocities the ice surface is seemingly not able to influence the airstream enough to change the collision efficiency from that corresponding to that of an ideal smooth surface in an ideal flow.

There is no obvious explanation for why the observed collision efficiency is smaller than that predicted by theoretical calculations.

5.4 Nusselt Number Analysis

The Nusselt number variation with Reynolds number is displayed in Figure 5.4 for the $V, r_m, T_a$ and $P$ series. Figure 5.4 shows a consistent increase in $N_u$ with increasing $R_e$. For a Reynolds number of 500, the Nusselt number varies between 22 and 32, the variability being within the experimental errors as discussed in Section 5.5. There is no distinct trend in $N_u$ with respect to any cloud variable except for velocity which is incorporated within the Reynolds number. Theoretically, $N_u$ is only a function of $R_e$ and the Prandtl number, $P_r$ (Incropera and DeWitt, 1985) and is usually parametrized as

$$N_u = n R_e^m P_r^{1/3}$$

where $n$ and $m$ are constants of the fit and $N_u$ is the average Nusselt number for the entire surface. Under atmospheric conditions $P_r$ is constant and equal to 0.71 so that $N_u$ is only a function of $R_e$. 

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Figure 5.4: Variation of Nusselt number with Reynolds number for the $V$, $\tau_m$, $T_s$ and $P$ series.
For the range of Re used in the experiments, \( N_u \) was parametrized as

\[
N_u = 2.0 R_e^{0.41} \quad 300 \leq R_e \leq 1500
\]

which is shown in Figure 5.5. Schemenauer (1972) simulated the convective mass transfer with conical models suspended in an electrolytic liquid flow. He found (Figure 5.5) that the Nusselt number was given by

\[
N_u = 2.0 + 0.78 P_r^{1/3} R_e^{1/2}
\]

for a smooth conical model with a 70° cone angle and a spherical base. The Nusselt numbers presented here are about 1.5 times larger than Schemenauer’s which can be accounted for by the surface roughness of the ice. Schuepp and List (1969) experimentally showed that surface roughness can account for increases in the transfer rates of up to a factor of 2 over that of smooth surfaces.

5.5 Sensitivity of the Calculations to the Experimental Errors

To understand the observed variation in the bulk collision efficiency and Nusselt number between different experimental series, their sensitivity to small changes in individual cloud parameters was investigated. \( E_b \) and \( N_u \) were calculated for similar sets of conditions which differed by only the experimental error of one cloud variable. Figures 5.6 and 5.7 show the change in \( E_b \) and \( N_u \) for changes of \( \Delta W_r = \pm 0.15 \text{ g m}^{-3} \), \( \Delta V = \pm 0.05 \text{ m s}^{-1} \), \( \Delta (\Delta T) = \pm 0.1^\circ \text{C} \) and \( \pm 0.2^\circ \text{C} \) and \( \Delta U_w = \pm 0.02 \) and \( \pm 0.04 \). These errors cause variations in \( E_b \) of up to \( \pm 10\% \) and in \( N_u \) of up to \( \pm 30\% \). This is the same magnitude as the observed variations in the measurements shown in Sections 5.3 and 5.4. Therefore, the variations in \( E_b \) and \( N_u \) can be accounted for within the experimental errors. Since the parametrizations are based on averages of several measurements, the estimated errors on the parametrizations of \( E_b \) and \( N_u \) are \( \pm 5\% \) and \( \pm 15\% \) respectively.
Figure 5.5: Parametrization of the Nusselt number with Reynolds number, and comparison with the measurements for a smooth cone.
Figure 5.6: Sensitivity of the $E_s$ versus $K$ relationship for variations in $W_t$, $V$, $\Delta T$ and $U_w$.

The variations are equal to the estimated experimental error. The cloud conditions are $T_s = -16^\circ\text{C}$, $W_t = 2.0$ g m$^{-3}$, $V = 2.0$ m s$^{-1}$, $r_m = 15$ $\mu$m and $P = 100$ kPa.
Figure 5.7: Sensitivity of the $N_u$ versus $R_e$ relationship for variations in $W_p$, $V$, $\Delta T$ and $U_w$.

The variations are equal to the estimated experimental error. The cloud conditions are $T_s = -16^\circ C$, $W_f = 2.0 \text{ g m}^{-3}$, $V = 2.0 \text{ m s}^{-1}$, $r_m = 15 \text{ \mu m}$ and $P = 100 \text{ kPa}$.
5.6 Sensitivity of the Calculations to the Relative Humidity

The relative humidity, $U_w$, can have a significant influence on the derived Nusselt number. Calculations of $U_w$ (as discussed in Section 3.7) indicated it ranged from 0.88 to 0.96 depending on the tunnel conditions. The analysis of previous icing experiments in the wind tunnel for both graupel and hail have always assumed that $U_w$ was approximately 1.0. However, a 0.1 change in the assumed value of $U_w$ can change $N_u$ by factors of 1.2 to 10. For example, at the standard conditions, the Nusselt numbers calculated assuming a relative humidity of 0.9 and 1.0 differ by 45%.

Photographic measurements of the amount of deposition or sublimation on the graupel sides during an experiment indirectly confirmed the relative humidity calculations. Since accretion was never observed on the graupel sides, any changes to the graupel diameter downstream of the base could only be caused by deposition or sublimation. The separation point of the airflow probably occurred at the intersection of the sides and the graupel base, so that no streamline containing droplets could make contact with the graupel sides. By superimposing photographs taken after four minutes of growth and at the end of an experiment, and comparing the graupel diameters for the first four minutes of growth, the amount of deposition or sublimation from the edges could be estimated. Two graupel from each series were measured this way and no measurable deposition or sublimation was found. Since the photographs could be resolved within 1 part in 50, less than 2% of the sides of the graupel were gained or lost by sublimation or deposition. Schemenauer (1972) found that the convective transfer on the leading face could be twice as high as that on the trailing face, although his models were held with the apex into the flow. Assuming that the sublimation per unit area on the base was larger than that on the sides, the mass change through deposition or sublimation was estimated at less than 3% for any experiment.

$E_o$ and $N_u$ were calculated by balancing the heat and mass transfer equations. This allowed the ratio of the mass transfer by deposition or sublimation to the mass transfer by accretion ($M_{DS}/M_{CP}$) to be calculated. This ratio varied between -2.8% and +2.4% for all
the experimental series except the 1.1 m s\(^{-1}\) series. Problems with the 1.1 m s\(^{-1}\) series have been discussed before and it was discounted. If the relative humidity (as determined from the thermodynamics simulations of the wind tunnel) was in error by more than \(\pm 0.03\), then the ratio \(\frac{M_D}{M_{CP}}\) (as determined from the heat and mass transfer equations and the assumed relative humidity) would reflect values larger than \(\pm 3\%\). This would contradict the photographic estimates of the maximum amount of deposition or sublimation discussed above. The relative humidity calculations are therefore believed to be accurate to within \(\pm 0.03\).

5.7 Basic Graupel Growth Model

The parametrized equations for cone angle, density, bulk collision efficiency and Nusselt number were incorporated in a simple graupel growth model. Using the heat and mass transfer equations and the geometries of graupel given in Appendix C, the mass, volume, growth height and surface temperature variations with time were calculated for graupel growing under cloud conditions similar to those used in each experimental series. Figures 5.8, 5.9 and 5.10 show comparisons between the model and experimental results for the \(W_n\), \(V\) and \(T_s\) series. The model results were always within \(\pm 10\%\) of the experimental results.

By incorporating the Best number-Reynolds number parametrization of Heymsfield and Kajikawa (1987), as based on measurements from natural conical graupel, the drag coefficient and terminal velocity can be estimated from the graupel mass and diameter. In a cloud dynamical model the time evolution of the graupel terminal velocity (and related parameters \(\rho\), \(c_s\) and \(\Delta T\)) could hence be calculated. By applying the parametrizations to the heat and mass transfer equations within the model, the growth cycle of a graupel (and its effects on the cloud) would be accurately represented as it traced a trajectory through the temperature, liquid water and velocity fields of the cloud. Although such a model is beyond the scope of this work, this brief discussion shows potential applications of the parametrizations.
Figure 5.8: Comparisons of the experimental and model results, for the model incorporating the parametrizations of $E_o$, $N_o$, $\rho$ and $c_s$. The cloud conditions are $T_s = -16^\circ C$, $V = 2.0 \text{ m s}^{-1}$, $r_m = 15 \mu m$, $P = 100 \text{ kPa}$ and $W_z$ = variable.
Figure 5.9: Comparisons of the experimental and model results, for the model incorporating the parametrizations of $E_a$, $N_a$, $p$ and $c_a$. The cloud conditions are $T_s = -16^\circC$, $W_i = 2 \text{ g m}^{-3}$, $r_m = 15 \mu m$, $P = 100 \text{ kPa}$ and $V = \text{variable}$. 

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Figure 5.10: Comparisons of the experimental and model results, for the model incorporating the parametrizations of $E_o$, $N_o$, $\rho$ and $c_o$. The cloud conditions are $W_t = 2.0$ g m$^{-3}$, $V = 2.0$ m s$^{-1}$, $r_m = 15$ μm, $P = 100$ kPa and $T_s = \text{variable}$. 

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Chapter 6

Summary and Conclusions

6.1 Summary and Conclusions

This thesis describes an experimental investigation of the heat and mass transfer of graupel growing in a simulated cloud environment within a wind tunnel. The standard growth conditions consisted of a liquid water content $W_f$ of 2.0 g m$^{-3}$, velocity $V$ of 2.0 m s$^{-1}$, ambient temperature $T_a$ of -16.0°C, cloud droplet median volume radius $r_m$ of 15 μm and ambient pressure $P$ of 100 kPa. Different experiments reflected different growth conditions of graupel in nature through variations in individual cloud parameters over the following ranges: $W_f$ from 0.5 to 3.0 g m$^{-3}$; $V$ from 1.1 to 3.0 m s$^{-1}$; $T_a$ from -4.4 to -20.9°C; $r_m$ from 12 to 21 μm; and $P$ from 100 to 60 kPa. Changes in mass, volume, growth height, geometric shape, cone angle and surface temperature with time were measured and used for calculations of the density, bulk collision efficiency $E_b$, Nusselt number $N_u$ and Sherwood number $S_h$. These quantities were parametrized in terms of the growth conditions.

The major conclusion can be stated as follows:

The dimensionless heat and mass transfer parameters ($N_u$, $S_h$ and $E_b$) describing the growth of graupel within typical cloud conditions have been parametrized in terms of physical parameters. The parametrizations represent complete solutions of the heat and mass transfer equations, meaning the basic physics of graupel growth can be easily numerically simulated. This represents an important contribution towards describing the cold rain process and the embryo formation of hail. Such a complete description has not been previously reported.
The experimental results can be summarized as follows:

1. Surface temperatures of graupel, measured remotely with an infrared radiometer, ranged from 0.3 to 2.0 ± 0.2°C warmer than the ambient temperature, depending on the growth conditions (Table B2). These represent the first such measurements of the surface temperature of graupel sized particles under typical cloud conditions.

2. The Nusselt number (and Sherwood number = 0.95 $N_u$) was calculated through measurements of the mass transfer and the graupel surface temperature. $N_u$ varied between 18 and 40 and was dependent on the graupel Reynolds number, $R_e$. A parametrization of $N_u$ in terms of $R_e$ (Equation 5.10) was determined for $300 \leq R_e \leq 1500$. Comparisons with the Nusselt numbers for smooth cones indicate that the heat and mass transfer by convection and deposition are enhanced by approximately 50% because of the surface roughness of the ice.

3. The bulk collision efficiency (collection efficiency), calculated through measurements of the mass growth rate and geometric characteristics, varied between 0.8 and 0.35. It was dependent on the median volume radius $r_m$ of the cloud droplets, the relative velocity $V$ between the droplets and the graupel, and the graupel radius $r_g$. $E_b$ was parametrized in terms of the Stokes parameter $K (\propto r_m^2 V / r_g)$ over the range $1.5 \leq K \leq 10$ (Equation 5.8). For a given $K$, values of $E_b$ are 25 to 30% lower than the theoretical collision efficiencies for an ideal sphere determined by Langmuir and Blodgett (1946). The parametrization of $E_b$ allows the mass transfer by accretion for graupel to be easily incorporated into cloud dynamical models.

4. The density of accretion, $\rho$, is dependent on $r_m$, $V$ and the graupel surface temperature $T_g$. A parametrization relating $\rho$ to $R (= -r_m V / T_g)$ has been determined (Equation 4.14) and is valid over $1.0 \leq R \leq 8.0$. Although similar observations and parametrizations have been reported (Macklin, 1962; Pflaum and Pruppacher, 1979; Heymsfield and Pflaum, 1985), the results presented here avoid limitations inherent in the
previous approaches and are considered significantly more accurate over the range of R investigated. There is also evidence that as the temperature decreases below -20°C the density (for constant $r_m$ and V) approaches a constant value of approximately 0.18 g cm$^{-3}$ which is consistent with the prediction of Buser and Aufdermaur, 1973.

5. The geometric shape of a graupel growing on a fixed stem can be described as a cone with a flat base and rounded edges (Appendix C). This is accurate to within ±10% by volume provided $\rho < 0.45$ g cm$^{-3}$. For larger $\rho$ the curvature of the base increases with increasing $\rho$ and at $\rho = 0.6$ g cm$^{-3}$ the graupel was best approximated as a cone with a base equivalent to a half sphere.

6. The cone angle $c_*$ of a graupel growing on a fixed stem is dependent on the cloud droplet median volume radius, the velocity of the droplets relative to the graupel and the graupel surface temperature. Higher inertia of impacting droplets causes larger cone angles, while warmer surface temperatures cause slower droplet freezing rates and narrower cone angles. A parametrization relating $c_*$ to $r_m$, $T_s$ and V has been determined (Equation 4.15).

7. The effects of pressure on graupel growth are at most second order as compared to the effects of $T_s$, $W_o$, $r_m$ and V. Measurements of the heat and mass transfer under pressures from 100 to 60 kPa did not vary with pressure beyond the experimental error, although trends in both $E_b$ and $N_b$ were qualitatively evident with decreasing pressure.

8. The electrical charging of the water droplet spectrum, caused by the disruption of a water stream through an atomizing nozzle, did not effect the growth rate or characteristics of graupel grown in the wind tunnel within the resolution of the measurements. This contradicts claims that atomizing nozzles cannot be used for icing experiments, as given by Pflaum and Pruppacher (1979) who proposed that the results of Macklin (1962) were biased by charging of his droplet spectrum.
9. The relative humidity during icing experiments in the wind tunnel is less than 1. Theoretical calculations based on the heat and mass transfer of droplets in unsaturated and cooler ambient air determined the relative humidity $U_w$ to be between 0.88 and $0.96 \pm 0.03$ depending on the icing conditions. The relative humidity affects the evaporation of the cloud droplets and hence the liquid water content, and affects the sublimation and deposition of water vapour on the graupel surface. Previous tunnel icing studies on hail always assumed that $U_w$ equalled 1 so that analysis of the hailstones' heat and mass transfer was somewhat in error.

Measurements of the heat and mass transfer of graupel have allowed a fairly complete parametrization of the graupel stage of the cold rain process, which can provide an essential component to cloud dynamical models. The experiments have also provided a solid foundation for the heat and mass transfer of hail where the situation is complicated by surface water skins and shedding of droplets.

6.2 Future Experiments

To allow a broader understanding of graupel growth and the cold rain process, the experiments presented here could be expanded to investigate several other processes. Experiments at temperatures between -20 and -30°C would determine whether the density approaches a constant value and verify an important concept in accretional growth. An extensive series of experiments are required between -1 and -8°C to provide a more detailed understanding of the cone angle, density and geometric shape dependence on temperature. This is important because at temperatures warmer than about -8°C, these parameters change rapidly with temperature. Measuring the effects of velocities between 0 and 1.5 m s$^{-1}$ would allow an understanding of the initial ice crystal-graupel stage of the cold rain process. Experiments with a mono sized droplet distribution would give a more in depth understanding of the effects of droplet size on density, collision efficiency and geometric shape. A more accurate investigation of the growth at low pressure would allow the effects of pressure, if any, to be quantified. Major graupel related topics for which the
icing tunnel will require few modifications include investigating the production of secondary ice crystals associated with riming graupel and the related charging and charge separation from graupel growing under typical thunderstorm conditions.
Appendix A

List of Symbols

\( a_o \) \( \text{m s}^{-2} \) radial acceleration

\( a_h \) \( \text{mm min}^{-1} \) coefficient of height time curve fit

\( a_m \) \( \text{mg min}^{-1} \) coefficient of mass time curve fit

\( a_v \) \( \text{mm}^3 \text{ min}^{-1} \) coefficient of volume time curve fit

\( A \) \( \text{m}^2 \) area of measuring section of wind tunnel (0.178 by 0.178 m)

\( A_b \) \( \text{m}^2 \) sweepout area of the base of a graupel

\( A_s \) \( \text{m}^2 \) total surface area of a graupel

\( b_h \) --- exponent coefficient of height-time curve fit

\( b_m \) --- exponent coefficient of mass-time curve fit

\( b_v \) --- exponent coefficient of volume-time curve fit

\( B_e \) --- Best number = \( C_d R_e^2 \)

\( c_a \) \( \text{deg} \) cone angle of graupel

\( \bar{c}_a \) \( \text{deg} \) average cone angle for a series of experiments

\( C \) --- coefficient for \( N_u \) - \( E_b \) relationship

\( C_d \) --- drag coefficient

\( C_w \) \( \text{J kg}^{-1} \text{K}^{-1} \) specific heat of water

\( D \) \( \text{m} \) graupel diameter (defined as the diameter of the graupel base)

\( D_{wa} \) \( \text{m}^2 \text{ s}^{-1} \) water vapour diffusivity in air

\( D_s \) \( \text{m}^2 \text{ s}^{-1} \) mass diffusivity

\( e \) \( \text{Pa} \) vapour pressure

\( e_o(T_o) \) \( \text{Pa} \) vapour pressure at temperature \( T_o \)

\( e_{sa}(T_o) \) \( \text{Pa} \) saturation vapour pressure over water at temperature \( T_o \)

\( e_{sd}(T_d) \) \( \text{Pa} \) saturation vapour pressure over water at droplet temp. \( T_d \)

\( e_{si}(T_i) \) \( \text{Pa} \) saturation vapour pressure over ice at surface temperature \( T_i \)
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Unit</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>E_b</td>
<td>---</td>
<td>bulk collision efficiency (bulk collection efficiency)</td>
</tr>
<tr>
<td>F</td>
<td>g s^{-1}</td>
<td>flow rate of water through the water injection nozzle</td>
</tr>
<tr>
<td>g</td>
<td>m s^{-2}</td>
<td>acceleration due to gravity</td>
</tr>
<tr>
<td>h</td>
<td>mm</td>
<td>graupel growth height of ice accretion</td>
</tr>
<tr>
<td>h</td>
<td>W m^{-2} K^{-1}</td>
<td>average convection heat transfer coefficient</td>
</tr>
<tr>
<td>h_m</td>
<td>m s^{-1}</td>
<td>average convection mass transfer coefficient</td>
</tr>
<tr>
<td>k_a</td>
<td>W m^{-1} K^{-1}</td>
<td>thermal conductivity of air</td>
</tr>
<tr>
<td>K</td>
<td>---</td>
<td>Stokes parameter = ((2 r_d^2 \rho_w \nu)/(\eta \tau_g))</td>
</tr>
<tr>
<td>L</td>
<td>m</td>
<td>characteristic length scale</td>
</tr>
<tr>
<td>L_f</td>
<td>J kg^{-1}</td>
<td>latent heat of fusion (freezing and melting)</td>
</tr>
<tr>
<td>L_v</td>
<td>J kg^{-1}</td>
<td>latent heat of vaporization (evaporation and condensation)</td>
</tr>
<tr>
<td>L_s</td>
<td>J kg^{-1}</td>
<td>latent heat of sublimation (sublimation and deposition)</td>
</tr>
<tr>
<td>m</td>
<td>mg</td>
<td>graupel mass</td>
</tr>
<tr>
<td>dM/dt</td>
<td>kg s^{-1}</td>
<td>total mass growth rate</td>
</tr>
<tr>
<td>M_{CP}</td>
<td>kg s^{-1}</td>
<td>mass growth rate by accreted cloud droplets</td>
</tr>
<tr>
<td>M_{DS}</td>
<td>kg s^{-1}</td>
<td>mass growth rate by deposition and sublimation</td>
</tr>
<tr>
<td>N_u</td>
<td>---</td>
<td>Nusselt number (dimensionless temperature gradient at the surface)</td>
</tr>
<tr>
<td>P</td>
<td>Pa</td>
<td>ambient pressure</td>
</tr>
<tr>
<td>(P_f)</td>
<td>---</td>
<td>Prandtl number = (\nu/\alpha) = ratio of momentum and thermal diffusivities</td>
</tr>
<tr>
<td>Q_{CC}</td>
<td>J s^{-1}</td>
<td>heat transfer rate by convection</td>
</tr>
<tr>
<td>Q_{DS}</td>
<td>J s^{-1}</td>
<td>heat transfer rate by evaporation and sublimation (deposition)</td>
</tr>
<tr>
<td>Q_{CP}</td>
<td>J s^{-1}</td>
<td>heat transfer rate by warming of rimed droplets to surface temperature</td>
</tr>
<tr>
<td>Q_f</td>
<td>J s^{-1}</td>
<td>heat transfer rate by freezing of accreted cloud droplets</td>
</tr>
<tr>
<td>r</td>
<td>\mu m</td>
<td>mean radius of a droplet distribution</td>
</tr>
<tr>
<td>r_d</td>
<td>m</td>
<td>radius of a droplet</td>
</tr>
</tbody>
</table>
\( r_g \) m radius of a graupel
\( r_m \) \( \mu \text{m} \) median volume radius of droplet distribution
\( r_o \) mm initial embryo radius of a graupel
\( r_t \) m radius of a target (i.e. a disk, sphere or cylinder)
\( r_v \) \( \mu \text{m} \) mean volume radius of a droplet distribution
\( \Gamma \) \( \text{m} \mu \text{m} \text{s}^{-1} \text{C}^{-1} \) density parametrization coefficient = \(-r_m \) \( V / T_s \)
\( R_e \) --- Reynolds number = \( \text{V}L/\nu \)
\( R_i \) \( \text{m} \mu \text{m} \text{s}^{-1} \text{C}^{-1} \) density parametrization coefficient = \(-r_m \) \( V_i / T_s \)
\( R_v \) J kg\(^{-1} \) K\(^{-1} \) gas constant for water vapour
\( S_c \) --- Schmidt number = \( \nu/D_w \) = ratio of momentum and mass diffusivities
\( S_h \) --- Sherwood number (dimensionless concentration gradient at the surface)
\( t \) min time
\( T_s \) °C ambient surface temperature
\( T_d \) °C droplet temperature
\( T_{de} \) °C droplet equilibrium temperature
\( T_s \) °C graupel surface temperature
\( U_w \) --- relative humidity
\( V \) m s\(^{-1} \) airflow velocity
\( V_L \) mm\(^3\) graupel volume
\( V_i \) m s\(^{-1} \) droplet impact velocity with a graupel surface
\( W_f \) g m\(^3\) liquid water content
\( Z_1 \) kg s\(^{-1} \) coefficient for mass transfer by accretion
\( Z_2 \) kg Pa\(^{-1} \) s\(^{-1} \) coefficient for mass transfer by sublimation and deposition
\( Z_3 \) J K\(^{-1} \) s\(^{-1} \) coefficient for heat transfer by convection
\( Z_4 \) J Pa\(^{-1} \) s\(^{-1} \) coefficient for heat transfer by sublimation and deposition
\( Z_5 \) J K\(^{-1} \) s\(^{-1} \) coefficient for heat transfer by accreted droplets
$Z_0 \quad J \, s^{-1}$ coefficient for heat transfer by accreted droplets freezing

$\alpha \quad m^2 \, s^{-1}$ thermal diffusivity

$\Delta \psi \quad Pa$ vapour pressure difference $= e_w(T_s) - U_w \, e_{ts}(T_s)$

$\Delta P \quad Pa$ experimental error on ambient pressure

$\Delta r_m \quad \mu m$ experimental error on median volume radius

$\Delta T \quad ^\circ C$ temperature elevation of graupel surface over ambient temperature

$\Delta T_a \quad ^\circ C$ experimental error of ambient temperature

$\Delta T_s \quad ^\circ C$ experimental error of surface temperature

$\Delta V \quad m \, s^{-1}$ experimental error of velocity

$\Delta W_f \quad g \, m^{-3}$ experimental error of liquid water content

$\Delta(\Delta T) \quad ^\circ C$ error of graupel surface temperature elevation

$\Delta \rho \quad g \, cm^{-3}$ experimental error on ice density measurement

$\nu \quad m^2 \, s^{-1}$ momentum diffusivity

$\rho \quad g \, cm^{-3}$ density of ice accretion

$\bar{\rho} \quad g \, cm^{-3}$ average density of ice for an experimental series

$\rho_s \quad kg \, m^{-3}$ vapour density in ambient air

$\rho_v \quad kg \, m^{-3}$ vapour density at surface of graupel

$\rho_v \quad kg \, m^{-3}$ density of vapour

$\rho_w \quad kg \, m^{-3}$ density of water

$\sigma_\alpha \quad deg$ standard deviation on cone angle measurement

$\sigma_h \quad mm$ standard deviation on growth height measurement

$\sigma_m \quad mg$ standard deviation on mass measurement

$\sigma_v \quad mm^3$ standard deviation on volume measurement

$\sigma_p \quad g \, cm^{-3}$ standard deviation on average density measurement

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Appendix B

Summary of the Experimental Data

Table B1 lists the raw data from the mass transfer series of experiments. The condition listed represents the single cloud parameter that was varied from the standard cloud conditions. Table B1 shows the growth time $t$ in minutes, number of graupel analyzed $\#$, final mass $m$ in mg, final volume $V_L$ in mm$^3$, final growth height $h$ in mm, cone angle $\theta$ in degrees, and their standard deviations $\sigma_m$, $\sigma_V$, $\sigma_h$ and $\sigma_\theta$ respectively.

Table B2 lists the data from the heat transfer experiments, and the averages of the geometry related parameters. For each series the surface temperature $T_s$ in °C, graupel temperature elevation over the ambient temperature $\Delta T$ in °C, average density $\rho$ in g cm$^{-3}$ and average cone angle $\theta$ in degrees are shown. The average density and cone angle values are calculated from the data in Table B1.
<table>
<thead>
<tr>
<th>Ser</th>
<th>Condition</th>
<th>t/min</th>
<th>#</th>
<th>m mg</th>
<th>σ_m mg</th>
<th>V_L mm³</th>
<th>σ_V mm³</th>
<th>h mm</th>
<th>σ_h mm</th>
<th>c_a deg</th>
<th>σ_a deg</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>W_r = 2.0 g m³</td>
<td>3</td>
<td>6</td>
<td>1.04</td>
<td>0.08</td>
<td>5.1</td>
<td>0.5</td>
<td>1.6</td>
<td>0.1</td>
<td>58</td>
<td>5</td>
</tr>
<tr>
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<td>$\Delta T$ °C ± 0.2</td>
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<td>$\sigma_p$ g cm$^{-3}$</td>
<td>$\bar{\alpha}$ average degrees</td>
<td>$\sigma_\alpha$ deg.</td>
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<td>-5.2</td>
<td>0.6</td>
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Appendix C

Graupel Geometry

This appendix illustrates the geometric shapes used to represent the graupel grown in the icing tunnel experiments. The formula for each component of the surface area $S$ and volume $V$ are given. Figure C1 shows the geometry describing graupel with densities less than 0.45 g cm$^{-3}$ (representing 23 of the 25 experimental series). Figure C2 shows the geometry representing graupel with densities greater than 0.55 g cm$^{-3}$ (i.e. the $T_s = -3.8$ °C and $T_s = -5.0$ °C series). There were few graupel with densities larger than 0.45 g cm$^{-3}$, so no attempts were made to couple the degree of sphericity of the base to the density of accretion.
\[ S_{\text{top}} = \pi r_o^2 \]
\[ S_{\text{sph}} = 2\pi r_3 h_2 \]
\[ S_{\text{cone}} = \pi \left( r_o + r_1 \right) \sqrt{(r_1 - r_o)^2 + h_1^2} \]
\[ S_{\text{base}} = \pi b^2 \]
\[ S_{\text{tot}} = S_{\text{sph}} + S_{\text{cone}} + S_{\text{top}} + S_{\text{base}} \]
\[ V_{\text{cone}} = \frac{\pi}{6} h_1 \left( r_1^2 + r_o^2 + r_o r_1 \right) \]
\[ V_{\text{sph}} = \frac{\pi}{6} h_2 \left( 3 r_1^2 + 3 b^2 + h_2^2 \right) \]
\[ V_{\text{tot}} = V_{\text{sph}} + V_{\text{cone}} \]

\[ \theta = \text{defined} \]
\[ r_o = \text{defined} \]
\[ h_0 = \frac{r_o}{\tan(\theta)} \]
\[ h_{\text{ice}} = \text{defined} = h_1 + h_2 \]
\[ r_T = h_{\text{ice}} + h_0 \]
\[ r_1 = r_T \sin(\theta) \]
\[ h_1 = \frac{r_1}{\tan(\theta)} - h_0 \]
\[ h_2 = h_{\text{ice}} - h_0 \]
\[ d = \frac{h_{\text{ice}}}{2} \]
\[ c = d - h_2 \]
\[ r_3 = \sqrt{r_1^2 + c^2} \]
\[ b = \sqrt{r_3^2 - d^2} \]

**Figure C1**: Geometry of a graupel with a flat base and a spherical edge. This is a good approximation for graupel with densities less than 0.45 g cm\(^{-3}\), and represents most of the graupel studied.
\[ S_{\text{top}} = \pi r_0^2 \]
\[ S_{\text{cone}} = \pi (r_0 + r_1) \sqrt{(r_1 - r_0)^2 + h_1^2} \]
\[ S_{\text{sph}} = 2\pi r_1^2 \]
\[ S_{\text{tot}} = S_{\text{top}} + S_{\text{sph}} + S_{\text{cone}} \]

\[ V_{\text{cone}} = \frac{\pi}{3} (r_1^2 + r_0^2 + r_0 r_1) \]
\[ V_{\text{sph}} = \frac{4\pi}{6} h_2^3 \]
\[ V_{\text{tot}} = V_{\text{sph}} + V_{\text{cone}} \]

\[ \theta = \text{defined} \]
\[ r_0 = \text{defined} \]
\[ h_0 = \frac{r_0}{\tan(\theta)} \]
\[ h_{\text{ice}} = \text{defined} = h_1 + h_2 \]
\[ r_1 = (h_0 + h_{\text{ice}}) \left( \frac{\tan(\theta)}{\tan(\theta) + 1} \right) \]
\[ h_1 = \frac{r_1}{\tan(\theta)} - h_0 \]
\[ h_2 = r_1 \]

Figure C2 : Geometry of a graupel with a half sphere base. This is a good representation for graupel with densities greater than 0.55 g cm\(^{-3}\), such as those grown at temperatures warmer than -5°C.
References


