Canadian Geotechnical Journal
Revue canadienne de géotechnique

**Numerical investigation of the earth pressure coefficient
along the central line of backfilled stopes**

Journal:	<i>Canadian Geotechnical Journal</i>
Manuscript ID	cgj-2016-0165.R3
Manuscript Type:	Note
Date Submitted by the Author:	02-Sep-2016
Complete List of Authors:	Sobhi, Mohamed Amine; École Polytechnique de Montréal, Department of Civil, Geological and Mining Engineering Li, Li; École Polytechnique de Montréal, Department of Civil, Geological and Mining Engineering Aubertin, Michel; École Polytechnique de Montréal,
Keyword:	Mines, Backfill, Earth pressure coefficient, Numerical modeling, Rankine

(cgj-2016-0165R3)

**Numerical investigation of the earth pressure coefficient
along the central line of backfilled stopes**

Mohamed Amine Sobhi, Li Li*, and Michel Aubertin

Research Institute on Mines and the Environment
Department of Civil, Geological and Mining Engineering
École Polytechnique de Montréal
C.P. 6079, Succursale Centre-Ville, Montréal, PQ, H3C 3A7 Canada

*Corresponding author. Tel: 1-514-340-4711 #2408; fax: 1-514-340-4477;
e-mail: li.li@polymtl.ca

Technical note (cgj-2016-0165R3) resubmitted to *Canadian Geotechnical Journal*

Submitted: March 2015
Evaluation received: May 2015
Revised version submitted: March 2016
Second evaluation received: May 2016
Updated (Revised) version submitted: June 2016
Third evaluation received: July 2016
Re-updated version submitted: August 2016
Forth evaluation received: August 2016
Fifth version submitted: September 2016

Numerical investigation of the earth pressure coefficient along the central line of backfilled stopes

Mohamed Amine Sobhi, Li Li*, and Michel Aubertin

Abstract: The earth pressure coefficient K , defined as the horizontal to vertical normal (effective) stresses ratio (σ_h/σ_v), is a key parameter in analytical solutions for estimating the stresses in backfilled stopes. In the case of vertical stopes, the value of K has sometimes been defined using the at-rest earth pressure coefficient K_0 , while others have applied Rankine's active earth pressure coefficient K_a . To help clarify this confusing situation, which can lead to significantly different results, the origin and nature of the at-rest and Rankine's active coefficients are first briefly recalled. The stress state in backfilled stopes is then investigated using numerical simulations. The results indicate that the value of K can be close to K_a for cohesionless backfills along the vertical central line (CL) of vertical stopes, due to sequential placement and partial yielding of the backfill. For inclined stopes, simulations show that the ratio between the minor and major principal stresses (σ_3/σ_1) along the CL in the backfill, which differs from σ_h/σ_v , can also be close to K_a . A simple expression is shown to represent well the horizontal to vertical stresses ratio $\sigma_h/\sigma_v (= K)$ along the central line of such inclined stopes. A discussion follows on the effects of backfill properties and simulation approach.

Key-words: Mines, Backfill, Earth pressure coefficient, Numerical modeling, Rankine.

*Corresponding author. Tel: 1-514-340-4711 #2408; fax: 1-514-340-4477; e-mail: li.li@polymtl.ca

Résumé: Le coefficient de poussée des terres K , défini comme le rapport entre les contraintes normales (effectives) horizontale et verticale (σ_h/σ_v), est un paramètre clé utilisé dans les solutions analytiques pour estimer les contraintes dans des chantiers remblayés. Pour les chantiers verticaux, la valeur de K est occasionnellement définie selon le coefficient de poussée des terres au repos K_0 , alors que d'autres ont plutôt utilisé le coefficient de poussée des terres actif de Rankine K_a . Pour clarifier cette situation ambiguë, qui peut conduire à des résultats assez différents, l'origine et la nature des coefficients au repos et actif sont d'abord brièvement rappelés. L'état des contraintes dans des chantiers remblayés est évalué à partir de simulations numériques. Les résultats indiquent que cette valeur peut être proche du coefficient K_a pour les remblais sans cohésion le long de la ligne de centre dans les chantiers verticaux, en raison de la mise en place progressive et de l'atteinte d'un état de plasticité partiel du remblai. Pour les chantiers remblayés inclinés, les résultats montrent que le rapport entre les contraintes principales mineure et majeure (σ_3/σ_1) le long de la ligne de centre dans le remblai, qui diffère du rapport σ_h/σ_v , peut aussi être proche du coefficient de poussée des terres actif de Rankine. Une expression simple peut être utilisée pour représenter le rapport entre les contraintes horizontale et verticale $\sigma_h/\sigma_v (= K)$ le long de la ligne centrale des chantiers remblayés inclinés. Une discussion suit en lien avec l'effet des propriétés du remblai et de l'approche pour mener les simulations.

77

Mots-clés: Mines, Remblai, Coefficient de poussée des terres, Modélisation numérique, Rankine.

79

80 Introduction

81 Stope backfilling is widely used in Canada and elsewhere, in part because it helps reduce the
 82 environmental impact of mining operations by returning mining wastes underground
 83 (Bussière 2007; Benzaazoua et al. 2008). But the main purposes of mine backfill are to
 84 improve ground stability, provide a safer work space, reduce ore dilution, and help control the
 85 air flow (Hassani and Archibald 1998; Darling 2011; Hambley 2011).

86 A key issue for the design of backfilled stopes is the determination of the stresses in the
 87 backfill. A few analytical expressions have been proposed to this effect. These are mostly
 88 based on the approach adopted by Marston (1930), who made use of the Janssen (1895)
 89 arching theory to calculate the vertical load on buried conduits in backfilled trenches. The
 90 corresponding vertical stress can be expressed as follows:

$$91 \quad (1) \quad \sigma_v = \frac{\gamma B}{2K \tan \delta} \left[1 - \exp\left(-\frac{2K \tan \delta}{B} h\right) \right]$$

92 where B (m) is the width of the opening, h (m) is the depth in the fill, γ (kN/m³) is its unit
 93 weight, δ (°) is the friction angle along the interfaces between the backfill and wall material,
 94 and K (-) is the earth pressure coefficient which is defined as the ratio between the horizontal
 95 (σ_h) and vertical (σ_v) normal effective stresses (kPa). For narrow openings, eq. (1) typically
 96 leads to vertical stresses that are much smaller than the overburden stresses, due to a load
 97 transfer from the soft backfill to the stiff walls, thus producing an arching effect (e.g., Handy
 98 and Spangler 2007).

99 Equation (1) has been applied to vertical mine backfilled stopes using $\delta = \phi$, where ϕ (°) is
 100 the internal friction angle of the (cohesionless) backfill (Aubertin et al. 2003). This
 101 assumption implies that the strength along the fill-wall interfaces is controlled by the backfill
 102 strength, hence considering that yielding occurs in the fill material rather than directly along
 103 the walls (due to their very rough surface).

This type of formulation has also been extended for inclined backfilled stopes (e.g., Caceres 2005; Ting et al. 2011, 2014; Jahanbakhshzadeh 2016).

The effects of cohesion and of a three-dimensional geometry (Li et al. 2005), pore water pressures (Li and Aubertin 2009*a*, 2009*b*), and non-uniform stress distribution across the stope width (Li and Aubertin 2008, 2010) have also been introduced in modified expressions based on eq. (1). Other 3D formulations, also related to eq. (1), have been proposed by Van Horn (1964), Winch (1999), and Pirapakaran and Sivakugan (2007*a*).

All these solutions (and others) use the lateral earth pressure coefficient, K , as a state parameter to evaluate the stresses in backfilled openings. The value of K can significantly affect the calculated stresses within the backfill (Li et al. 2003). There is however an uncertainty regarding how K should be defined for backfilled stopes. This issue is addressed in the following, considering first the basic concept behind K and then analyzing the stresses along the central line of stopes using results from numerical simulations. Although the calculations presented here neglect the effect of pore water pressure, the same concept can easily be extended to cases where the backfill contains water that generates positive pressures (e.g., Li and Aubertin 2009*a*, 2009*b*).

Earth pressure coefficient K

The well-known earth pressure coefficient concept was originally developed in soil mechanics to analyse the stress state in natural deposits and the pressure on retaining walls (e.g., Coulomb 1776; Rankine 1857; Jaky 1944; Lambe and Whitman 1979; Terzaghi et al. 1996).

The corresponding earth pressure coefficient K is usually defined as the ratio between the horizontal (σ_h) and vertical (σ_v) normal stresses, which are also the minor (σ_3) and major (σ_1) principal stresses in a normally consolidated soil deposit with a horizontal surface (i.e. $\sigma_h/\sigma_v = \sigma_3/\sigma_1$).

When a soil in a semi-infinite space is under static equilibrium, the vertical (principal) stress, σ_v , can be expressed as follows:

$$(2) \quad \sigma_v = \gamma h$$

The horizontal stress is proportional to the vertical stress, so one can write:

$$(3) \quad \sigma_h = K \sigma_v$$

Equations (2) and (3) constitute the traditional overburden solution for estimating the stresses in a soil deposit in geotechnical engineering. Parameter K is usually called the (lateral) earth pressure coefficient, or a close variant of this term (e.g., Bowles 1992; CGS 2006; McCarthy 2007; Holtz et al. 2011).

For a normally consolidated soil at-rest (i.e. where vertical loads do not induce horizontal strain), $K = K_0$. Although the value of K_0 can theoretically be defined from Poisson's ratio μ (e.g., Aysen 2005; Blight 2010), geotechnical engineers generally use the following semi-empirical expression proposed by Jaky (1944), which relates the at-rest earth pressure coefficient to the (effective) internal friction angle (ϕ) of the soil:

$$(4) \quad K_0 = 1 - \sin \phi$$

A number of researchers have shown that Jaky's expression (eq. (4)) represents well the measured horizontal to vertical stresses ratio for normally consolidated soils (e.g., Al-Hussaini and Townsend 1975; Ladd et al. 1977; Mesri and Hayat 1993).

When a soil (or fill) submitted to vertical loading is retained by a smooth vertical wall of limited stiffness, there will be some horizontal movement that may result in a reduction in the horizontal stress σ_h (compared with the K_0 condition). The Mohr circle defining the new stress state can then meet the Coulomb yield surface (e.g., McCarthy 2007; Sobhi et al. 2014). In this case, the soil is said to have reached an active state following yielding. The corresponding horizontal to vertical stresses ratio can then be defined with Rankine's active earth pressure

coefficient, K_a , which can be expressed as follows for a horizontal surface (e.g., CGS 2006; Das 2010):

$$(5) \quad K_a = \frac{\sigma_h}{\sigma_v} = \frac{\sigma_3}{\sigma_1} = \frac{1 - \sin \phi}{1 + \sin \phi} = \tan^2 \left(45^\circ - \frac{\phi}{2} \right)$$

Equations (4) and (5) have both been used to calculate the stresses in backfilled openings, including mine stopes. It is however unclear which one (if any) of these two expressions should be applied in such cases.

Assessing the value of K for mine backfilled stopes

As stated above, analytical solutions developed for estimating the stresses in backfill placed in mine stopes are usually based on Marston's approach (eq. (1)). For vertical backfilled stopes, some numerical calculations and indirect measurements have indicated that Rankine's active earth pressure coefficient (eq. (5)) is close to the horizontal to vertical stresses ratio along the central line (e.g., Li et al. 2003, 2005; Li and Aubertin 2009a, 2009b; Ting et al. 2012). Using the value of coefficient K_a also agrees with Marston's (1930) results for the loads on conduits in backfilled trenches (see also McCarthy 1988; Handy and Spangler 2007).

An illustration of how the value of K may influence the stress state in a backfilled opening is shown in Fig. 1, using experimental results taken from Pirapakaran and Sivakugan (2007b) and the 3D arching solution of Li et al. (2005), which is a direct extension of eq. (1). Both the at-rest K_0 (eq. (1)) and Rankine's active K_a (eq. (5)) coefficients have been used to calculate the vertical stresses shown in the figure. As can be seen, the experimental results indicate that there is a pronounced arching effect, with the measured vertical stresses being much smaller than the overburden pressures (especially at depth). It can also be observed that a better agreement between the experimental measurements and analytical solution is obtained in this case with Rankine's active earth pressure coefficient. This agrees with earlier analyses of

experimental and analytical results presented by Li et al. (2005), which showed that a good agreement can be obtained for $K = K_a$.

Nonetheless, many authors working on the behavior of fills in bins and in trenches or behind retaining walls have been using $K = K_0$ in analytical solutions to define the stresses (e.g., Take and Valsangkar 2001; Blight 2006). This assumption was also adopted for backfilled stopes (e.g., Winch 1999; Pirapakaran and Sivakugan 2007a, 2007b; Fahey et al. 2009; Singh et al. 2010, 2011; Ting et al. 2011). In such cases, the use of $K = K_0$ may appear to be justified because the walls are very stiff, so the horizontal displacements (and strains) in the backfill should be very small (or nil), hence leading to an at-rest state. This assumption could however be questioned because the high compressibility of the fill, its sequential placement, and the presence of the rock walls can lead to some stress rotation and differential lateral strains under vertical loading, as indicated for instance by numerical simulations performed by Falaknaz (2014; see also Falaknaz et al. 2015b). In the following, additional analyses are performed using numerical simulations to assess the earth pressure coefficient along the central line (CL) of backfilled stopes with parallel vertical and inclined walls.

Numerical simulations

Numerical simulations have been performed to assess the stress ratio in backfilled stopes, considering the effect of stope geometry and backfill properties. The focus is placed here on the earth pressure coefficient $K (= \sigma_h/\sigma_v)$ along the central line of the stopes, where the effects of the shear stresses developing along the walls are minimal. The finite element software SIGMA/W (GEO-SLOPE 2010) was used to run the simulations; additional details are provided in Sobhi (2014).

Figure 2a shows a backfilled stope model, which includes a void space of 0.5 m above the fill surface. The backfill is considered homogeneous and isotropic and it obeys the classical

elasto-plastic model with the Coulomb yield criterion; the rock mass is isotropic, homogenous and linear elastic. A typical stress regime for the Canadian Shield is applied, where the natural vertical in-situ stress is calculated based on the overburden weight and the horizontal natural stress is twice the vertical stress. An example of a numerical model built in SIGMA/W is shown in Fig. 2b.

Each numerical simulation was conducted in three steps: i) model construction to obtain a rock mass space submitted to the external and self-weight stress regime; ii) excavation of the stope and release of elastic strains in the rock mass; iii) backfilling of the stope in four layers (similarly to Li and Aubertin 2009c). There is no interface element along the rock walls in the models; this may influence the magnitude of the stresses obtained with SIGMA/W (El Mkadmi et al. 2014), but it does not significantly affect the stress ratio being assessed here (Sobhi 2014).

The reference case configuration consists of a 6 m wide and 45 m high vertical stope surrounded by the elastic rock mass characterized by $E_r = 30$ GPa (Young's modulus), $\mu_r = 0.3$ (Poisson's ratio) and $\gamma_r = 27$ kN/m³ (unit weight). The stope is filled with a cohesionless backfill characterized by $E = 300$ MPa, $\mu = 0.2$, $\gamma = 18$ kN/m³, $\phi = 30^\circ$ (friction angle) and $\psi = 0^\circ$ (dilation angle, with a non-associated flow rule).

In addition to the reference case, various other simulations have been conducted with one parameter varied at a time in order to evaluate its influence on the earth pressure coefficient K . Details on the reference case and other simulated cases are summarized in Table 1.

Numerical results

Reference case

Figure 3 shows the vertical and horizontal stresses distribution (Fig. 3a) along the CL of the vertical stope for the reference case, together with the iso-contours of the horizontal σ_h (Fig.

3b) and vertical σ_v (Fig. 3c) stresses obtained with SIGMA/W. From Figure 3a, it can be seen that there is a nonlinear stress increase with depth. Across the width of the slope, the horizontal (Fig. 3b) and vertical (Fig. 3c) stresses tend to decrease from the slope center to the walls. These correspond well to typical stress distribution in backfilled openings due to arching effect (Li et al. 2003; Li and Aubertin 2008, 2009c; Falaknaz 2014; Falaknaz et al. 2015a).

The earth pressure coefficient K is calculated as the ratio between the horizontal and vertical stresses along the central line obtained with the code SIGMA/W. The numerical results giving the variation of coefficient K with depth along the CL of the vertical backfilled slope for the reference case configuration is shown in Fig. 4. The Rankine's active coefficient $K_a = 0.33$ (Eq. (5) with $\phi = 30^\circ$) and the at-rest coefficient based on Jaky's equation $K_0 = 0.5$ (Eq. (4) with $\phi = 30^\circ$) are also shown on Fig. 4. It is seen that the value of K obtained from the numerical simulation is close to K_a along most of the CL. However, the value of K tends toward Jaky's (eq. (4)) at-rest coefficient K_0 in the upper part of the slope.

These numerical results correspond well to the tendency reported by Thompson et al. (2012), based on measurements performed at the Cayeli mine, located in the province of Rize, Turkey; these indicated that the effective stresses in the backfill in the lower central portion of a slope were close to an active state while the upper portion was near to the at-rest state.

Effect of slope geometry

Figure 5 shows the variation of $K (= \sigma_h/\sigma_v)$ along the CL of vertical slopes width B ranging from 4 m to 10 m (Case 1, Table 1). One sees that the earth pressure coefficient remains very close to Rankine's active earth pressure coefficient K_a (eq. (5)) for all simulations. As was the case for Case 0 (Fig. 4), the simulated value of K appears somewhat overestimated by Jaky's equation (eq. (4)), except near the top.

The variation of the earth pressure coefficient $K = \sigma_h/\sigma_v$ along the CL of backfilled stopes with walls having various inclination angles α (between 90° to 50°) is shown in Fig. 6. It is seen that the value of coefficient K is very sensitive to angle α . There is a significant increase in the value of K with a decrease of the wall inclination angle. This tendency agrees with the numerical simulations results of Li and Aubertin (2009c), who showed that the horizontal stresses are almost insensitive to the stope inclination while the vertical stresses tend to decrease when α decreases. The observed trend also agrees with those reported by Li et al. (2012, 2013) and Li and Aubertin (2015) for trenches with inclined walls.

This increase in the value of K with a decrease of angle α was thus expected, but the magnitude of this effect was not necessarily anticipated.

For the stope geometry and backfill properties considered here, the earth pressure coefficient K for vertical ($\alpha = 90^\circ$) and sub-vertical ($\alpha = 80^\circ$) stopes along the CL is well represented by the Rankine's active earth pressure. Jaky's at-rest earth pressure coefficient (eq. (4)) is less representative of the simulations results, except may be for $\alpha = 60^\circ$ (but this agreement may be fortuitous)

The results shown in Fig. 6 furthermore indicate that the expressions presented above cannot directly be used to represent the earth pressure coefficient $K (= \sigma_h/\sigma_v, \text{ along the CL})$ when the wall inclination angle varies significantly from 90° ; the difference with the value of K_a becomes particularly important for $\alpha \leq 70^\circ$ (for the conditions analyzed here). This aspect will be further discussed below.

Additional analysis of the numerical simulations results indicates that for stopes with inclined walls, the horizontal and vertical stresses along the CL are not principal stresses, contrarily to vertical stopes (where $\sigma_h/\sigma_v = \sigma_3/\sigma_1$).

The ratio of the principal (effective) stresses $K_{ps} = \sigma_3/\sigma_1$ along the CL was evaluated for various stope inclination angles α . As it can be seen on Fig. 7, the value of K_{ps} is close to

Rankine's active coefficient K_a , except very near the top of the backfill. This suggests that the backfill may be fairly close to an active state along the CL of inclined stopes (when the latter state is defined from the principal stresses).

Effect of backfill properties

Various characteristics of the backfill have been varied in the following simulations to assess their influence on the stress state and on the value of K . It is first noted that the numerical calculations have shown that there is no influence of the backfill Young's modulus on the lateral earth pressure coefficient for E between 3 MPa to 3 GPa (results not shown here). There is however a significant effect of Poisson's ratio μ on the stress state (Li and Aubertin 2009c; Falaknaz et al. 2015b; Jahanbakhshzadeh 2016); this aspect will be discussed briefly below.

As the internal friction angle ϕ of the backfill (for a constant μ value) influences the stress state in stopes (Li and Aubertin 2009c), it was also expected to affect the value of coefficient K . Figure 8 shows that the value of K given by the simulations is usually fairly close to Rankine's active earth pressure coefficient (eq. (5)) for values of ϕ going from 10° to 40° , particularly in the deeper part of the stope.

An expression for coefficient K in inclined stopes

In the case of narrow inclined stopes, the simulations showed that the horizontal and vertical stresses along the CL are not principal stresses (because of the induced shear stresses). The results shown above have also shown that the ratio of the principal stresses $K_{ps} (= \sigma_3/\sigma_1)$ is close to K_a .

The numerical results further indicated that the earth pressure coefficient K ($= \sigma_h/\sigma_v$) is very sensitive to the slope walls inclination angle α . This aspect is not taken into account in the equations presented above.

An analysis of the simulations results has shown that the following expression represents well the value of the earth pressure coefficient as a function of the slope inclination α and the backfill internal friction angle ϕ :

$$K = K_\alpha = \frac{1 + K_a}{2} + \frac{1 - K_a}{2} \cos 2\alpha \quad (6)$$

where α ($50^\circ \leq \alpha \leq 90^\circ$) is the slope wall inclination angle and K_a is Rankine's active earth pressure coefficient. Equation (6) defining K_α ($= \sigma_h/\sigma_v$) is based on an expression presented by Ting et al. (2011), but which does not consider Rankine's active state (and which also includes an additional term that is a function of the friction angle along the walls; this term is not considered here).

Figure 9 shows a comparison between the value of K_α calculated with eq. (6) and the earth pressure coefficient K ($= \sigma_h/\sigma_v$) obtained from the numerical simulations for different slope inclination angles, for a backfill internal friction angle of 30° . It is seen that a good agreement is obtained for α varying from 50° to 90° .

Additional numerical simulations were performed using an internal friction angle $\phi = 20^\circ$ and 40° for the backfill (see Sobhi 2014). Again, a good agreement is obtained between the earth pressure coefficient K_α calculated with eq. (6) and those obtained from the numerical simulations along the central line.

These results thus suggest that eq. (6) can be used to determine the earth pressure coefficient K along the CL of inclined slopes backfilled with cohesionless material.

Discussion

The numerical modeling results presented above indicate that the active earth pressure coefficient K_a could be applied to assess the stresses along the CL of vertical backfilled stopes with solutions based on Marston's equation (eq. (1)), except very near the top surface where the backfill is closer to an at-rest state. This suggests that K should also be expressed as a function of the depth h , as shown by the recent work of Jahanbakhshzadeh (2016).

The results specifically show that there is an increase in the value of K near the top of the backfill. This could be related to the effect of backfill placement (added in layers), which tends to produce vertical and horizontal stresses that can be initially much higher than the overburden. Near the surface, the excess stresses in the backfill can be released in the vertical direction, but these are partly locked in the horizontal direction, hence producing a higher horizontal to vertical stress ratio (i.e. larger value of K).

As mentioned previously, the trends provided by the numerical results have also been confirmed (in part) by those provided by field measurements at the Cayeli mine, located in the province of Rize, Turkey (Thompson et al. 2012). Similar experimental observations, which point in the same direction, have also been reported for the Goldcorp Inc.'s Red Lake Mine (Hughes et al. 2010) and for laboratory tests on physical models (Fig. 1; see also Pirapakaran and Sivakugan 2007b; Ting et al. 2012).

More laboratory work on this aspect can be needed. But, these numerical and experimental results may appear to be going against intuition for backfilled stopes, as it could be expected that the very stiff rock walls would produce an at-rest state ($K = K_0$) in the openings (as is postulated by some).

This apparent conundrum may be resolved, at least partially, by recalling that Rankine's active state is related to yielding of the material (e.g., McCarthy 2007; Sobhi et al. 2014). A movement of the confining walls is not a necessary condition to reach an active state in a backfilled stope. Such active (yielding) state can also be reached as a result of other

mechanisms, such as a variation of the deviatoric stresses in the very soft (deformable) backfill following sequential placement, which may bring the stress state close to or in contact with the yield surface (especially at depth). This is illustrated in Fig. 10, which shows that a (plastic) yielding state is reached in a large part of the backfill after complete filling of the stope (based on a simulation result with the elasto-plastic model). When the stresses in the backfill reach the (plastic) yield criterion, the stress state will be controlled by the adopted plasticity theory and the stress ratio may approach an active state in the stope.

Limitations

The results presented above illustrate how numerical simulations can be used to help with the validation and calibration of analytical solutions. Some reservations must nonetheless be expressed when reviewing the calculations presented here. One of these relates to the value of the Poisson's ratio μ , which is taken as a constant here ($\mu = 0.2$). It is thus considered as being independent from the internal friction angle ϕ of the backfill in the simulations. However, the work of Jaky (1944) on the at-rest coefficient, and of various others since then (e.g., Holtz et al. 2011), suggests that both parameters (μ and ϕ) should be related for consistency (with the elasto-plastic model), so that a unique value of K_0 would be obtained from either eq. (4) or the well-known expression $K_0 = \mu/(1-\mu)$ when the backfill responds according to the commonly used elasto-plastic (Mohr-Coulomb) model. Work is ongoing to evaluate if testing results obtained on granular materials supports such inter-related values for μ and ϕ . Applying the corresponding relationship between μ and ϕ in the simulations can influence the stress state (Falaknaz 2014; Falaknaz et al. 2015b; Jahanbakhshzadeh 2016), but the analysis presented above on the value of K remains relevant.

Other limitations of this work pertain to some of the characteristics of the finite element code used for these calculations, which have been addressed by El Mkadmi et al. (2014) and

Sobhi (2014). For instance, the analyses conducted by Sobhi (2014) indicate that the magnitude of the stresses in the stope may depend on the number of layers used to place the backfill in the modelled opening and on the conditions imposed along the rock walls (i.e. presence or absence of interface or transition elements). However, these factors do not influence the value of the stress ratio σ_h/σ_v obtained from the numerical simulations. The results presented here are thus representative of the conditions being evaluated to assess the value of K .

Also, it should be recalled that the focus of this study was placed on the earth pressure coefficient along the CL of backfilled stopes, where the principal stresses would typically act along the vertical and horizontal axes (in plane strain). It is known however that the stresses and earth pressure coefficients are not uniform across the width of the openings (Li and Aubertin 2008, 2010). Additional work is underway to assess the value of K across the width and near the walls of backfilled stopes.

Interface elements were not included in the models used here. Recent simulations performed by Liu et al. (2016a, 2016b) tend to indicate that the introduction of interface elements is usually not required when the surface of the walls are very rough (irregular), or when the friction angle along the walls is close to that of the backfill; the latter condition has been observed from recent laboratory tests results (e.g., Fall and Nasir 2010; Koupouli et al. 2016). Nonetheless, this aspect still needs some more work.

Finally, it should be recalled that eq. (6) was formulated for inclined stopes, while eq. (1) was developed for vertical stopes. Their combined use may thus be questioned, as these are based on different assumptions and conditions. Nonetheless, as is often the case with other types of empirical formulations used in geotechnical engineering (such as Jaky's equation for K_0), it is deemed reasonable to introduce the value of K given by eq. (6) into eq. (1) to obtain a preliminary estimate of the stresses along the CL of inclined stopes.

398

399 **Conclusion**

400 This study presents the results of a numerical investigation conducted with a finite element
401 code to evaluate the effect of slope geometry and backfill properties on the lateral earth
402 pressure coefficient $K (= \sigma_h/\sigma_v)$ within backfilled slopes. The simulations results indicate that
403 the value of coefficient K along the CL of vertical backfilled slopes can be close to Rankine's
404 active coefficient K_a , except near the fill surface. This suggests that the backfill along the CL
405 may be approaching an active state in vertical slopes. As the rock walls do not move
406 significantly, the simulations results also suggest that the backfill state cannot be solely
407 associated with the displacement of these walls. A yielding state, leading to Rankine's active
408 condition, appears to be induced in some parts of the slope due to the natural response of the
409 very soft backfill, which can generate increased (and non-uniform) deviatoric stresses that
410 tends to mobilize the yield strength following filling and settlements. This tendency applies to
411 vertical and inclined slopes. In the latter case however, it has been observed that it is the ratio
412 of the principal stresses $K_{ps} = \sigma_3/\sigma_1$ (in 2D) that seems to be approaching Rankine's active
413 coefficient K_a along the inclined CL of the slope. In addition, an empirical equation is
414 proposed for estimating the earth pressure coefficient $K = K_\alpha = \sigma_h/\sigma_v$ along the CL of inclined
415 slopes backfilled with a cohesionless backfill.

416

417 **Acknowledgment**

418 The authors acknowledge the financial support of the Natural Sciences and Engineering
419 Research Council of Canada (NSERC 402318), the Institut de recherche Robert-Sauvé en
420 santé et en sécurité du travail (IRSST 2013-0029), Fonds de recherche du Québec — Nature
421 et technologies (FRQNT 2014-MI-183747, 2015-MI-191676), and the industrial partners of
422 the Research Institute on Mines and the Environment (RIME UQAT-Polytechnique;

http://rime-irme.ca). Particular thanks are also expressed to the Editor, Associate Editor and anonymous reviewers for their constructive comments.

References

- Al-Hussaini, M., and Townsend, F. 1975. Stress deformation of sand under K_0 conditions. *In* Proceedings 5th Panamerican Conference on Soil Mechanics and Foundation Engineering. Vol. 1, pp. 129-136.
- Aubertin, M., Li L., Arnoldi, S., Belem, T., Bussière, B., Benzaazoua, M., and Simon, R. 2003. Interaction between backfill and rock mass in narrow stopes. *In* Proceedings, Soil and rock America 2003. Edited by P. J. Culligan, H. H. Einstein, and A. J. Whittle. Verlag Glückauf Essen (VGE), Essen, Germany. Vol. 1, pp. 1157–1164.
- Aysen, A. 2005. Soil mechanics: Basic concepts and engineering applications, Taylor & Francis, New York.
- Benzaazoua, M., Bussière, B., Demers, I., Aubertin, M., Fried, É., and Blier, A. 2008. Integrated mine tailings management by combining environmental desulphurization and cemented paste backfill: Application to mine Doyon, Quebec, Canada. *Minerals Eng.*, **21**: 330–340.
- Blight, G. 2006. Assessing loads on silos and other bulk storage structures: Research applied to practice. Taylor & Francis, Rotterdam, Netherlands.
- Blight, G. 2010. Geotechnical engineering for mine waste storage facilities. CRC Press, Netherlands.
- Bowles, J.E. 1992. Engineering properties of soils and their measurement, 4th Edition. McGraw-Hill.
- Bussière, B. 2007. Colloquium 2004: Hydro-geotechnical properties of hard rock tailings from metal mines and emerging geo-environmental disposal approaches. *Canadian Geotechnical Journal*, **44**(9): 1019–1052.
- Caceres, C. 2005. Effect of delayed backfill on open stope mining methods. M.S. thesis, University of British Columbia, Vancouver, B.C.
- CGS 2006. Canadian foundation engineering manual, 4th Edition. Canadian Geotechnical Society.

- 449 Coulomb, C.A. 1776. Essai sur une application des règles de maximis & minimis à quelques
450 problèmes de statique, relatifs à l'architecture. Mémoires de mathématique & de physique présentés
451 à l'Académie Royale des Sciences par divers savans & lûs dans ses assemblées, **7**: 343-382.
- 452 Darling, P. (editor) 2011. SME Mining Engineering Handbook, Third Edition. SME, Littleton.
- 453 Das, B.M. 2010. Principles of geotechnical engineering, 7th Edition. Cengage Learning.
- 454 El Mkadmi, N., Aubertin, M., and Li, L. 2014. Effect of drainage and sequential filling on the
455 behavior of backfill in mine stopes. Canadian Geotechnical Journal, **51**(1): 1-15.
- 456 Fahey, M., Helinski, M., and Fourie, A. 2009. Some aspects of the mechanics of arching in backfilled
457 stopes. Canadian Geotechnical Journal, **46**(11): 1322-1336.
- 458 Falaknaz, N. 2014. Analysis of geomechanical behavior of two adjacent backfilled stopes based on
459 two and three dimensional numerical simulations. Ph.D thesis, École Polytechnique de Montréal,
460 Canada.
- 461 Falaknaz, N., Aubertin, M., and Li, L. 2015a. Numerical analyses of the stress distribution in two
462 neighbouring backfilled stopes. International Journal of Geomechanics, **15**(6): 04015005.
- 463 Falaknaz, N., Aubertin, M., and Li, L. 2015b. A numerical investigation of the geomechanical
464 response of adjacent backfilled stopes. Canadian Geotechnical Journal, **52**: 1507–1525.
- 465 Fall, M., and Nasir, O. 2010. Mechanical behaviour of the interface between cemented tailings backfill
466 and retaining structures under shear loads. Geotechnical and Geological Engineering, **28**(6): 779–
467 790.
- 468 GEO-SLOPE 2010. Stress-Deformation Modeling with SIGMA/W 2007. An Engineering
469 Methodology, 4th Edition, GEO-SLOPE International Ltd, Canada.
- 470 Handy, R.L., and Spangler, M. 2007. Geotechnical engineering: soil and foundation principles and
471 practice. McGraw Hill Professional.
- 472 Hambley, D.F. 2011. Backfill mining. In SME mining engineering handbook. Edited by P. Darling.
473 SME, Littleton. Vol 1, pp. 1375–1384.
- 474 Hassani, F., and Archibald, J. 1998. Mine backfill (CD-ROM). Canadian Institute of Mining,
475 Metallurgy and Petroleum, Montreal.

- 476 Holtz, R.D., Kovacs, W.D., and Sheahan, T.C. 2011. An Introduction to Geotechnical Engineering.
477 Pearson Education, Limited.
- 478 Hughes, P., Pakalnis, R., Hitch, M., and Corey, G. 2010. Composite paste barricade performance at
479 Goldcorp Inc. Red Lake Mine, Ontario, Canada. *International Journal of Mining, Reclamation and*
480 *Environment*, **24**(2): 138-150.
- 481 Jahanbakhshzadeh, A. 2016. Analyse du comportement géomécanique des remblais miniers dans des
482 excavations souterraines inclinées. Ph.D. thesis, École Polytechnique de Montréal, Montréal, QC.
- 483 Jaky, J. 1944. The coefficient of earth pressure at rest. *Journal of the Society of Hungarian Architects*
484 *and Engineers*, **78**(22): 355-358.
- 485 Janssen, H.A. 1895. Versuche über getreidedruck in gilozellen. *Verein Deutscher Ingenieure*, **39**:
486 1045-1049.
- 487 Koupouli, N.J.F., Belem, T., Rivard, P., and Effenguet, H. 2016. Direct shear tests on cemented paste
488 backfill–rock wall and cemented paste backfill–backfill interfaces. *Journal of Rock Mechanics and*
489 *Geotechnical Engineering* (available online).
- 490 Ladd, C.C., Foote, R., Ishihara, K., Schlosser, F., and Poulos, H.G. 1977. Stress deformation and
491 strength characteristics, state of the art report. *In Proceedings of the Ninth International Conference*
492 *on Soil Mechanics and Foundation Engineering*. Vol. 2, pp. 421-494.
- 493 Lambe, T.W., and Whitman, R.V. 1979. *Soil mechanics*. John Wiley & Sons.
- 494 Li, L., and Aubertin, M. 2008. An improved analytical solution to estimate the stress state in sub-
495 vertical backfilled stopes. *Canadian Geotechnical Journal*, **45**(10): 1487-1496.
- 496 Li, L., and Aubertin, M. 2009a. Influence of water pressure on the stress state in stopes with
497 cohesionless backfill. *Geotechnical and Geological Engineering*, **27**(1): 1-11.
- 498 Li, L., and Aubertin, M. 2009b. A three-dimensional analysis of the total and effective stresses in
499 submerged backfilled stopes. *Geotechnical and Geological Engineering*, **27**(4): 559-569.
- 500 Li, L., and Aubertin, M. 2009c. Numerical investigation of the stress state in inclined backfilled
501 stopes. *International Journal of Geomechanics*, **9**(2): 52-62.
- 502 Li, L., and Aubertin, M. 2010. An analytical solution for the nonlinear distribution of effective and
503 total stresses in vertical backfilled stopes. *Geomechanics and Geoengineering*, **5**(4): 237-245.

- Li, L., and Aubertin, M. 2015. Numerical analysis of the stress distribution in backfilled trenches with inclined walls. *Indian Geotechnical Journal*, **45**(3): 278-290.
- Li, L., Aubertin, M., Simon, R., Bussière, B., and Belem, T. 2003. Modeling arching effects in narrow backfilled stopes with FLAC. *In* *FLAC and Numerical Modeling in Geomechanics – 2003. Edited by R. Brummer, P. Andrieux, C. Detournay, and R. Hart. A.A. Balkema, Rotterdam, The Netherlands. pp. 211–219.*
- Li, L., Aubertin, M., and Belem, T. 2005. Formulation of a three dimensional analytical solution to evaluate stresses in backfilled vertical narrow openings. *Canadian Geotechnical Journal*, **42**(6): 1705-1717.
- Li, L., Dubé, J.S., and Zangeneh-Madar, Z. 2012. Estimation of total and effective stresses in trenches with inclined walls. *International Journal of Geotechnical Engineering*, **6**(4), 525-538.
- Li, L., Dubé, J.S., and Aubertin, M. 2013. An extension of Marston's solution for the stresses in backfilled trenches with inclined walls. *Geotechnical and Geological Engineering*, **31**(4): 1027-1039.
- Liu, G.S., Li, L., Yang, X.C., Guo, L.J. 2016a. A numerical analysis of the stress distribution in backfilled stopes considering interfaces between backfill and rock walls. *ASCE—International Journal of Geomechanics* 06016014-1–06016014-9 (available online).
- Liu, G.S., Li, L., Yang, X.C., Guo, L.J. 2016b. A numerical analysis of the stress distribution in backfilled stopes considering non-planar interfaces between backfill and rock walls. *International Journal of Geotechnical Engineering*, **10**(3): 271-282.
- Marston, A. 1930. The theory of external loads on closed conduits in the light of the latest experiments. Bulletin No. 96, Iowa, Engineering Experiment Station, Ames, Iowa.
- McCarthy, D.F. 1988. *Essentials of soil mechanics and foundations: Basic geotechnics*. Prentice Hall, Englewood Cliffs, N.J.
- McCarthy, D.F. 2007. *Essentials of soil mechanics and foundations: Basic geotechnics*, 7th Edition. Prentice Hall Books.
- Mesri, G., and Hayat, T. 1993. The coefficient of earth pressure at rest. *Canadian Geotechnical Journal*, **30**(4): 647-666.

- 532 Michalowski, R.L. 2005. Coefficient of earth pressure at rest. *Journal of Geotechnical and*
533 *Geoenvironmental Engineering*, **131**(11): 1429-1433.
- 534 Pirapakaran, K., and Sivakugan, N. 2007*a*. Arching within hydraulic fill stopes. *Geotechnical and*
535 *Geological Engineering*, **25**(1): 25-35.
- 536 Pirapakaran, K., and Sivakugan, N. 2007*b*. A laboratory model to study arching within a hydraulic fill
537 stope. *Geotechnical Testing Journal, ASTM*, **30**(6): 496-503.
- 538 Rankine, W.J.M. 1857. On the stability of loose earth. *Philosophical Transactions of the Royal Society*
539 *of London, London*, **147**: 9-27.
- 540 Singh, S., Sivakugan, N., and Shukla, S.K. 2010. Can soil arching be insensitive to ϕ ? *International*
541 *Journal of Geomechanics*, **10**(3): 124-128.
- 542 Singh, S., Shukla, S.K., and Sivakugan, N. 2011. Arching in inclined and vertical mine stopes.
543 *Geotechnical and Geological Engineering*, **29**(5): 685-693.
- 544 Sobhi, M.A. 2014. Analyse numérique visant l'évaluation du coefficient de pression des terres et des
545 contraintes dans des chantiers remblayés au-dessus d'un pilier-dalle. M.Sc.A. thesis, École
546 Polytechnique de Montréal.
- 547 Sobhi, M.A., Li, L., and Aubertin, M. 2014. Numerical investigation of the lateral earth pressure
548 coefficient along the VCL of vertical backfilled stopes. *In GeoRegina 2014. Canadian Geotechnical*
549 *Society*.
- 550 Take, W., and Valsangkar, A. 2001. Earth pressures on unyielding retaining walls of narrow backfill
551 width. *Canadian Geotechnical Journal*, **38**(6): 1220-1230.
- 552 Terzaghi, K., Peck, R.B., and Mesri, G. 1996. *Soil mechanics in engineering practice*. John Wiley &
553 *Sons, New York*.
- 554 Thompson, B., Bawden, W., and Grabinsky, M. 2012. In situ measurements of cemented paste backfill
555 at the Cayeli Mine. *Canadian Geotechnical Journal*, **49**(7): 755-772.
- 556 Ting, C.H., Shukla, S.K., and Sivakugan, N. 2011. Arching in soils applied to inclined mine stopes.
557 *International Journal of Geomechanics*, **11**(1): 29-35.
- 558 Ting, C.H., Sivakugan, N., and Shukla, S.K. 2012. Laboratory simulation of the stresses within
559 inclined stopes. *Geotechnical Testing Journal, ASTM*, **35**(2): 280-294.

- 560 Ting, C.H., Sivakugan, N., Read, W., and Shukla, S.K. 2014. Analytical Expression for Vertical Stress
561 within an Inclined Mine Stope with Non-parallel Walls. *Geotechnical and Geological Engineering*,
562 **32**(2): 577-586.
- 563 Van Horn, D. 1964. A study of loads on underground structures. *In* Proceedings of the Symposium on
564 Soil-Structure Interaction, 8–11 June 1964. University of Arizona, Tucson, Ariz., pp. 256–282.
- 565 Winch, C. 1999. Geotechnical characteristics and stability of paste backfill at BHP Cannington mine.
566 B. E. Hons Thesis, James Cook University, Townsville, Australia.
- 567

Draft

568

569 **List of tables**570 Table 1. Characteristics of the different simulations (with $\gamma = 18 \text{ kN/m}^3$, $E = 300 \text{ MPa}$, $\mu = 0.2$ 571 and $\psi = 0^\circ$ for the backfill).

572

Draft

573

574 **List of figures**

575 **Fig. 1.** Experimentally measured average vertical stresses reported by Pirapakaran and Sivakugan
 576 (2007b) for (a) a 600 mm high and 100 mm wide square stope, and (b) a 900 mm high and
 577 150 mm wide square stope; the stresses measured in the laboratory tests are compared with
 578 those obtained from the 3D analytical solution of Li et al. (2005) using Rankine's active (K_a)
 579 and Jaky's at-rest (K_0) earth pressure coefficients. Material properties (taken from
 580 Pirapakaran and Sivakugan 2007b): $\gamma = 17.53 \text{ kN/m}^3$, $\phi = 35^\circ$, $\delta = 0.83\phi$.

581 **Fig. 2.** Schematic view of (a) a typical backfilled stope model and (b) the numerical model built
 582 with SIGMA/W.

583 **Fig. 3.** (a) Distribution of the stresses along the CL of the reference case configuration. (b) Iso-
 584 contours of the horizontal stress σ_h in the stope (increments of 10 kPa). (c) Iso-contours of
 585 the vertical stress σ_v in the stope (increments of 50 kPa). The left side of (b) and (c)
 586 corresponds to the plane of symmetry for the vertical stope (see details in Table 1, Case 0).

587 **Fig. 4.** Variation of earth pressure coefficient K along the CL of the backfilled stope for the
 588 reference case simulated with Sigma (dots); the values of K obtained from eqs. (4) to (5) are
 589 also shown in the figure (see Table 1, Case 0).

590 **Fig. 5.** Variation of lateral earth pressure coefficient along the vertical CL of the stope for various
 591 widths B ; the figure shows the values obtained from the numerical simulations (dots) and
 592 from eqs. (4) to (5) (Case 1, Table 1).

593 **Fig. 6.** Variation of lateral earth pressure coefficient along the vertical CL of the stope for various
 594 inclination angles α ; the figure shows the values obtained from the numerical simulations
 595 (dots) and from eqs. (4) to (5) (Case 2 in Table 1).

596 **Fig. 7.** Variation of the ratio $K_{ps} = (\sigma_3/\sigma_1)$ along the CL of backfilled stopes with various inclination
 597 angles α ; the figure shows the values obtained from the numerical simulations (dots) and
 598 from eqs. (4) to (5) (Case 2, Table 1).

599 **Fig. 8.** Variation of lateral earth pressure coefficient along the vertical CL of the slope for various
600 internal friction angles ϕ ; the figure shows the values obtained from the numerical
601 simulations (dots) and from Rankine's active coefficient; more details are given in Table 1,
602 Case 3.

603 **Fig. 9.** Comparison between the variation of lateral earth pressure coefficient for various slope
604 inclination angles and their corresponding K ($\phi = 30^\circ$; Case 2, Table 1).

605 **Fig. 10.** Yield state of the backfill in (a) a vertical slope and (b) an inclined slope.

606

Draft

Table 1. Characteristics of the different simulations (with $\gamma = 18 \text{ kN/m}^3$, $E = 300 \text{ MPa}$, $\mu = 0.2$ and $\psi = 0^\circ$ for the backfill)

Cases	Figures	Stope geometry		Backfill properties
		$B \text{ (m)}$	$\alpha \text{ (}^\circ\text{)}$	$\phi \text{ (}^\circ\text{)}$
Case 0	3, 4	6	90	30
Case 1	5	VAR ¹	90	30
Case 2	6, 7, 9	6	VAR	30
Case 3	8	6	90	VAR

¹VAR = Variable

Fig. 1. Experimentally measured average vertical stresses reported by Pirapakaran and Sivakugan (2007b) for a 600 mm high and 100 mm wide square stope (Stope 1), and a 900 mm high and 150 mm wide square stope (Stope 2); the stresses measured in the laboratory tests are compared with those obtained from the 3D analytical solution of Li et al. (2005) using Rankine's active (K_a) and Jaky's at-rest (K_0) earth pressure coefficients. Material properties (taken from Pirapakaran and Sivakugan 2007b): $\gamma = 17.53 \text{ kN/m}^3$, $\phi = 35^\circ$, $\delta = 0.83\phi$.

Fig. 2. Schematic view of (a) a typical backfilled stope model and (b) the numerical model built with SIGMA/W.

Fig. 3. (a) Distribution of the stresses along the CL of the reference case configuration. (b) Iso-contours of the horizontal stress σ_h in the stope (increments of 10 kPa). (c) Iso-contours of the vertical stress σ_v in the stope (increments of 50 kPa). The left side of (b) and (c) corresponds to the plane of symmetry for the vertical stope (see details in Table 1, Case 0).

Fig. 4. Variation of earth pressure coefficient K along the CL of the backfilled stope for the reference case simulated with SIGMA/W (dots); the values of K obtained from eqs. (4) to (5) are also shown in the figure (see Table 1, Case 0).

Fig. 5. Variation of lateral earth pressure coefficient along the vertical CL of the slope for various widths B ; the figure shows the values obtained from the numerical simulations (dots) and from eqs. (4) to (5) (Case 1, Table 1).

Fig. 6. Variation of lateral earth pressure coefficient along the vertical CL of the slope for various inclination angles α ; the figure shows the values obtained from the numerical simulations (dots) and from eqs. (4) to (5) (Case 2 in Table 1).

Fig. 7. Variation of the ratio $K_{ps} = (\sigma_3/\sigma_1)$ along the CL of backfilled slopes with various inclination angles α ; the figure shows the values obtained from the numerical simulations (dots) and from eqs. (4) to (5) (Case 2, Table 1).

Fig. 8. Variation of lateral earth pressure coefficient along the vertical CL of the slope for various internal friction angles ϕ ; the figure shows the values obtained from the numerical simulations (dots) and from Rankine's active coefficient; more details are given in Table 1, Case 3.

Fig. 9. Comparison between the variation of lateral earth pressure coefficient for various slope inclination angles and their corresponding K ($\phi = 30^\circ$; Case 2, Table 1).

Fig. 10. Yield state of the backfill in (a) a vertical stope and (b) an inclined stope.

Draft