Canadian Journal of Physics
Revue canadienne de physique

On formation of long-living states

Journal:	<i>Canadian Journal of Physics</i>
Manuscript ID	cjp-2016-0227.R1
Manuscript Type:	Article
Date Submitted by the Author:	13-Jun-2016
Complete List of Authors:	Ivlev, Boris; Universidad Autonoma de San Luis Potosi
Keyword:	Particle, String, Interaction, Spectrum, Polaron

SCHOLARONE™
Manuscripts

On formation of long-living states

Boris I. Ivlev

*Instituto de Física, Universidad Autónoma de San Luis Potosí,
San Luis Potosí, 78000 Mexico*

The motion of a particle in the potential well is studied when the particle is attached to the infinite elastic string. This is generic with the problem of dissipative quantum mechanics investigated by Caldeira and Leggett [1]. Besides the dissipative motion there is another scenario of interaction of the string with the particle attached. Stationary particle-string states exist with string deformations accompanying the particle. This is like polaronic states in solids. Our polaronic states in the well are non-decaying and with continuous energy spectrum. These states may have a link to quantum electrodynamics.

PACS numbers: 03.65.-w, 03.65.Ge

Keywords: particle, string, interaction, spectrum, polaron

I. INTRODUCTION

Discrete energy levels of the electron in a potential well become of a finite width under the interaction with photons. This provides finite lifetimes of levels with respect to photon emission. Photons are emitted until the electron reaches the ground state level. This level has zero width. It is slightly shifted (the Lamb shift) due to the electron-photon interaction [2].

Discrete levels can be of different lifetimes. When the certain level is long-living this results in possibility of laser generation. Broadening of the energy level is characterized by the imaginary part of its energy. The level widths in various atoms are known [3].

There is a question which does not seem conventional. Are there some conditions to get long-living states with the continuous energy spectrum in a potential well? Such states would provide unusual laser generation. This phenomenon may occur solely due to interaction with a reservoir since in quantum mechanics levels in a well are discrete.

As shown in the paper, the answer to that question is positive. The motion of a particle in the well is studied when the particle is attached to the infinite elastic string. The moving particle causes elastic waves of the string which carry away the energy. The energy dissipation provides friction motion of the particle. This problem is generic with one in paper [1] studying dissipative quantum mechanics. See also [4–19].

Besides such dissipative motion there is another scenario of interaction of the string with the attached particle. That regime is not dissipative. The joint particle-string state is stationary and string deformations accompany the particle. This reminds polaronic state in a solid when the electron is “dressed” by phonons [20]. Our polaronic states in the well are continuously distributed in energy which does not have an imaginary part. This means that polaronic states of all their energies are non-decaying.

One can qualitatively explain why photons (string

sound waves) are not emitted by polaronic states. This is due to the “hard” connection of the particle to the string. Emission of waves would result in oscillations of the string including the point of particle attachment. This increases the particle kinetic energy preventing it to lose its total energy and therefore resulting in non-decaying states.

The issue is that nature allows the continuous non-decaying energy spectrum in a potential well. This conclusion is based on the exact solution of the particle-string problem obtained in this paper. There is no contradiction to quantum mechanics since the particle is coupled to the reservoir.

In this paper in the quantum case the exact solution for a particle attached to an elastic string is obtained.

II. PARTICLE ON A STRING

Below we study the elastic string placed along the z axis. Transverse displacements of the string are $R(z, t)$ as shown in Fig. 1. The particle of the mass m is attached to the string at the point $z = 0$ and moves together with the string. The particle coordinate is $x(t) = R(0, t)$. The potential energy $U(x)$ depends on the particle coordinate only.

The energy of the string with the particle has the form

$$E = \int dz \left\{ \frac{1}{2} [\rho + m\delta(z)] \left(\frac{\partial R}{\partial t} \right)^2 + \frac{\rho s^2}{2} \left(\frac{\partial R}{\partial z} \right)^2 + \delta(z)U(R) \right\}. \quad (1)$$

In Eq. (1) ρ is the mass density of the string and s is the velocity of elastic waves. The second term is the elastic energy. The absence of the string corresponds to the limit $\rho = 0$ and then the energy takes its usual form

$$E = \frac{m}{2} \left(\frac{dx}{dt} \right)^2 + U(x). \quad (2)$$

FIG. 1: Particle can move along the x -direction only. This particle, acted by the potential $U(x)$, is attached to the elastic string with displacements $R(z)$. The particle displacement is $x = R(0)$. Exact states of the system are localized. In the absence of the string two discrete levels in the well are shown.

A. Classical limit

In the classical limit the function $R(z, t)$ is a macroscopic variable. It obeys the equation obtained by the variation of the functional (1)

$$[\rho + m\delta(z)] \frac{\partial^2 R}{\partial t^2} - \rho s^2 \frac{\partial^2 R}{\partial z^2} + U'(R)\delta(z) = 0. \quad (3)$$

The integration of the singular part in Eq. (3) results in

$$m \frac{d^2 x}{dt^2} + U'(x) - \rho s^2 \left(\left. \frac{\partial R}{\partial z} \right|_{z=+0} - \left. \frac{\partial R}{\partial z} \right|_{z=-0} \right) = 0. \quad (4)$$

At $z \neq 0$ the wave equation holds

$$\frac{\partial^2 R}{\partial t^2} - s^2 \frac{\partial^2 R}{\partial z^2} = 0 \quad (5)$$

with the solution

$$R(z, t) = x \left(t - \frac{|z|}{s} \right). \quad (6)$$

This solution, in the form of two waves running from the point $z = 0$, corresponds to the condition $R(0, t) = x(t)$. The dynamic equation for x gets the form

$$m \frac{d^2 x}{dt^2} + U'(x) + \eta \frac{dx}{dt} = 0, \quad (7)$$

where $\eta = 2\rho s$. Eq. (7) corresponds to the particle with friction provided by the radiation mechanism. The energy of the moving particle is carried away by sound waves from the point $z = 0$.

The system of particle and string, described by the energy (1), is generic with the model of Caldeira and Leggett where the particle is coupled to the infinite set of oscillators [1]. Results for of Euclidean action, describing tunneling, is the same for the both approaches as shown in Appendix A. One can also show that the description of particle dynamics, following from (1), is the same as in [1]. In particular, in the thermal limit the same Fokker-Planck equation can be derived in the model (1).

FIG. 2: (a) Dissipative motion of the particle in the well. The electron-photon interaction provides a finite width of the upper level resulting in the photon emission. (b) Polaronic states. Energy spectrum of the particle, attached to the string, is continuous and non-decaying (zero imaginary part of energy).

B. Discrete coordinates

It is convenient to consider the string of the finite length L . In the final results the limit $L \rightarrow \infty$ is taken. According to this, one can use the Fourier series

$$R(z, t) = \frac{1}{L} \sum_n R_n(t) \exp \left(\frac{2\pi i n}{L} z \right). \quad (8)$$

In the representation (8) the energy (1) takes the form

$$E = \frac{\rho}{2L} \sum_n \left(\dot{R}_n^2 + \omega_n^2 R_n^2 \right) + \frac{m}{2} \dot{x}^2 + U(x), \quad (9)$$

where $\omega_n = 2\pi|n|s/L$. In Eq. (9) the R -part is not separated from the x -part due to the connection condition following from (8)

$$x(t) = \frac{1}{L} \sum_n R_n(t). \quad (10)$$

It is convenient to separate the total R_n by two parts

$$R_n = \frac{r_n + i s_n}{\sqrt{2}}, \quad (11)$$

where $r_{-n} = r_n$ and $s_{-n} = -s_n$. Then the energy (9) reads

$$E = \frac{\rho}{2L} \sum_{n=1}^{\infty} (\dot{s}_n^2 + \omega_n^2 s_n^2) + \frac{m}{2} \dot{x}^2 + U(x) + \frac{\rho}{2L} \left\{ \sum_{n=1}^{\infty} (\dot{r}_n^2 + \omega_n^2 r_n^2) + \frac{1}{2} \dot{r}_0^2 \right\}. \quad (12)$$

As follows from (10),

$$r_0 = xL\sqrt{2} - 2 \sum_{n=1}^{\infty} r_n. \quad (13)$$

We see that the x -mode is connected to r -mode only. The s -mode is independent and corresponds to usual string waves. For this reason it can be omitted.

C. Dissipative quantum mechanics

In multi-dimensional quantum mechanics for variables x and r_n ($n = 1, 2, \dots$) one can subsequently integrate out the variables r_n by simple Gaussian integration [1]. As a result, the effective action in terms of x will get the known Caldeira-Leggett form [1]. That action allows to describe dissipative quantum systems, that is interacting with reservoir. The classical dissipative limit corresponds to Eq. (7). This problem is well investigated in literature [4–15] and we do not repeat here known results. Instead one can focus on another aspect of interaction with a reservoir.

D. Particle energy

Below we consider the potential in the form of harmonic oscillator $U(x) = m\Omega^2 x^2/2$. The total energy (12) goes over into two quadratic forms with respect to coordinates and their time derivatives. After diagonalization of the forms the energy (12) is

$$E = \frac{\rho}{2L} \sum_q (\dot{\eta}_q^2 + \omega_q^2 \eta_q^2), \quad (14)$$

where η_q are the certain variables as in Appendix. The frequency ω_q depends on the new wave vector q according to the relation (see Appendix)

$$\Omega^2 - \omega_q^2 = \frac{\eta\omega_q}{m} \tan\left(\frac{L\omega_q}{2s}\right). \quad (15)$$

The quantization of the new wave vector follows from (15)

$$q_n = \frac{2\pi n}{L} - \frac{2}{L} \arctan\left(\frac{msq_n}{\eta} - \frac{m\Omega^2}{\eta sq_n}\right). \quad (16)$$

Formally calculating $\partial n/\partial q_n$ one can establish the connection

$$\sum_n F(\omega_q) = \int_0^\infty d\omega \nu(\omega) F(\omega), \quad (17)$$

where the frequency distribution is

$$\nu(\omega) = \nu_0 + \frac{\eta}{\pi} \frac{m\omega^2 + m\Omega^2}{(m\omega^2 - m\Omega^2)^2 + \eta^2\omega^2}. \quad (18)$$

Here $\nu_0 = L/(2\pi s)$ is referred to the string without the particle.

The energy (14) consists of contributions of independent oscillators. Suppose all frequencies to be separated by groups $\{\omega\}_1, \{\omega\}_2, \dots$. Oscillators with frequencies of the group l are in the l th excited state. The total energy can be separated by two parts $E = E_0 + E_p$ where

$$E_p = \int_0^\infty \hbar\omega \left(\frac{1}{2} + \sum_{l=1}^\infty N_{l\omega} \right) (\nu - \nu_0)(\omega) d\omega \quad (19)$$

is the particle contribution. Each $N_{l\omega} \leq l$ accounts for the frequency distribution in the group $\{\omega\}_l$. When, for example, all oscillators are in the first excited state then only $N_{1\omega}$ is not zero and $N_{1\omega} = 1$ for all frequencies. Each separation on frequency groups corresponds to the certain energy (19). The energy spectrum is continuous.

The string energy, without the particle, E_0 is given by analogous equation but with the frequency distribution ν_0 related to the free string. The energy $E_0 \sim L$ is strongly divergent at large frequencies.

In the limit $\eta \ll m\Omega$ of small particle dissipation the particle energy $E_p^{(0)}$ of the ground state ($N_{l\omega} = 0$), according to (19), is

$$E_p^{(0)} = \frac{\hbar\Omega}{2} + \frac{\hbar\eta}{2\pi m} \ln \frac{\omega_{max}}{\Omega}. \quad (20)$$

Here the second term is due to the connection to the string. It diverges logarithmically and should be cut off by the large frequency ω_{max} . This frequency can be related to the discreteness $a \sim 1/q_{max}$ of the string $\omega_{max} \sim sq_{max}$.

Correction to the quantum mechanical ground state energy in Eq. (20) reminds the Lamb shift in hydrogen atom [2, 22]. In that case the shift is also proportional to the similar logarithm with $mc^2/\hbar$ instead of ω_{max} .

The energy spectrum, described by Eq. (19), is continuous and non-decaying (zero imaginary part of an energy). This type of states provides an example of long living ones, that is with the infinite time. In real physical systems one can investigate how other types of interaction influence the conclusion on the infinite lifetime. In reality long living states with continuous energy spectrum can be a basis for unusual laser emission with continuous frequency. In that case population inversion, required for laser generation, can be created automatically. Anyway, those phenomena need further studies.

E. Fluctuation dissipation theorem

As follows from Eq. (10),

$$x = \frac{\sqrt{2}}{L} \sum_q u_{0q} \eta_q. \quad (21)$$

The mean squared value of x , after the average on fluctuations, is

$$\langle x^2 \rangle = \frac{2}{L^2} \sum_q \frac{\eta^2 \omega_q^2}{(m\omega_q^2 - m\Omega^2)^2 + \eta^2 \omega_q^2} \langle \eta_q^2 \rangle. \quad (22)$$

Besides quantum fluctuations one can consider also thermal ones. It follows from (14) that

$$\langle \eta_q^2 \rangle = \frac{L\hbar}{2\rho\omega_q} \cot \frac{\hbar\omega_q}{2T} \quad (23)$$

as for a harmonic oscillator. One should use the density of states ν_0 . The mean squared displacement (22) corresponds to the usual fluctuation dissipation theorem

$$\langle x^2 \rangle = \frac{i\hbar}{2\pi} \int_{-\infty}^{\infty} \cot \frac{\hbar\omega}{2T} \frac{d\omega}{m\omega^2 - m\Omega^2 + i\eta\omega}. \quad (24)$$

F. Distribution of string displacements

In Fig. 1 the instant string deformation is schematically shown. The mean displacements of the particle and string coordinates are zero. There is a reason to look at the mean squared displacements.

As follows from Eqs. (8), (10), and (13), the string displacement is

$$R(z) = x + \frac{\sqrt{2}}{L} \sum_{n=1}^{\infty} r_n \left[\cos \left(\frac{2\pi n z}{L} \right) - 1 \right]. \quad (25)$$

With the use of Eqs. (B2) and (21) it follows

$$R(z) = \frac{\sqrt{2}}{L} \sum_q \eta_q \sum_{n=1}^{\infty} u_{nq} \left[\cos \left(\frac{2\pi n z}{L} \right) - 1 \right]. \quad (26)$$

Accounting for Eqs. (B11) and (B10) one can obtain

$$R(z) = \frac{\sqrt{2}}{L} \sum_q \eta_q u_{0q} \left[\cos \frac{z\omega_q}{s} + \frac{m(\Omega^2 - \omega_q^2)}{\eta\omega_q} \sin \frac{|z|\omega_q}{s} \right] \quad (27)$$

The mean squared displacement of the string is

$$\begin{aligned} \langle R^2(z) \rangle &= \frac{2\hbar}{\pi\eta} \int_0^{\infty} \frac{d\omega_q}{\omega_q} \left(\frac{\rho\omega_q}{\hbar L} \langle \eta_q^2 \rangle \right) \left\{ \sin^2 \frac{z\omega_q}{s} \right. \\ &\quad \left. + u_{0q}^2 \left[\cos \frac{2z\omega_q}{s} + \frac{m(\Omega^2 - \omega_q^2)}{\eta\omega_q} \sin \frac{2|z|\omega_q}{s} \right] \right\}. \quad (28) \end{aligned}$$

The first term in (28) is particle independent and relates to the string only. This part is logarithmically divergent at large momenta. The second two terms are related to the particle. For the ground state $\langle \eta_q^2 \rangle = \hbar L / (2\rho\omega_q)$ and at large distances $\eta s / (m\Omega^2) \ll z$

$$\langle R^2(z) \rangle = \langle R^2(z) \rangle_0 + \frac{\hbar}{2\pi m\Omega^2} \frac{s}{|z|}, \quad (29)$$

where the first term is particle independent and originates from the first term in (28)

$$\langle R^2(z) \rangle_0 = \frac{2\hbar}{\pi\eta} \int_0^{q_{max}} \frac{dq}{q} \sin^2 qz \simeq \frac{2\hbar}{\pi\eta} \ln z q_{max}. \quad (30)$$

We include into (30) also the effect of the s -mode which doubles the result.

One can see from (29) that the particle, attached to the string, results in its displacements localized along the string. This reminds a polaronic state of the electron in solids [20].

G. Dissipative motion versus polaronic states

Usually the electron interaction with photons (string waves in our case) results in a finite width of upper states and therefore in the photon emission. This is the dissipative motion of the particle since emitted photons propagate to the infinity carrying away the energy. The corresponding state is nonstationary which in the classical limit is referred to the electron dissipative motion (6), (7).

In this paper a different scenario (polaronic states) is studied. The particle energy spectrum (19) is continuous as in Fig. 2(b). E_p is an exact energy of particle-string states with zero imaginary part. So these states are not decaying that is the particle cannot emit photon (propagating sound wave along the string) as in Fig. 2(a). The ground state energy in Fig. 2(b) is determined by Eq. (20).

The polaronic state can arise from the dissipative state. This can happen when the wave, emitted by the particle, is reflected from the end of the string and returns. But it takes the infinite time when $L \rightarrow \infty$. In this paper we do not discuss a link from the phenomenon investigated to general mathematical aspects of statistical physics. The review on non-Markovian dynamics in open quantum systems is in Ref. [21].

Another way to form the polaronic state is to act by a non-stationary pulse on the dissipative state. The pulse has to be formed in space to reflect emitted photons which, being returned, participate in formation of the polaronic state.

III. DISCUSSION

One can qualitatively explain why photons (string sound waves) are not emitted by polaronic states. This is due to the “hard” connection to the string. Emission of waves would result in oscillations of the string including the point of particle attachment. This increases the particle kinetic energy preventing it to lose its total energy and therefore resulting in non-decaying states. These general arguments do not depend on a type of the potential where the particle moves. For this reason, the polaronic states may exist not in harmonic potentials only. This question requires further studies.

Despite only the specific example (a particle on a string) is considered one can hypothesize that the concept of polaronic states is not strictly related to the system chosen. The general issue of the paper is that nature allows the continuous non-decaying energy spectrum in a potential well. This conclusion is based on the exact solution of the particle-string problem obtained. There is no contradiction to quantum mechanics since the particle is coupled to the reservoir.

In quantum electrodynamics radiative corrections are small [2] and one can say that the electron is not connected “hard” to electromagnetic coordinates as in the

string case. Therefore shifts of discrete energy levels of the electron (the Lamb shift [2]) are small. This is accompanied by a small broadening of higher levels resulting in photons emission. The Lamb shift occurs since, due to the photon influence, the electron “vibrates” within the narrow region of 10^{-11}cm [22, 23]. In that way it probes various parts of the potential and therefore slightly changes its energy. Thus in quantum electrodynamics polaronic states are impossible at first sight. However a more accurate treatment shows that the situation can be more complicated. This is a matter for a further study.

IV. CONCLUSIONS

The motion of a particle in the potential well is studied when the particle is attached to the infinite elastic string. This is generic with the problem of dissipative quantum mechanics studied by Caldeira and Leggett [1]. Besides the dissipative motion there is another scenario of interaction of the string with the particle attached. Stationary particle-string states exist with string deformations accompanying the particle. This is like polaronic states in solids. Our polaronic states in the well are non-decaying and with continuous energy spectrum. These states may have a link to quantum electrodynamics.

Acknowledgments

I appreciate stimulating discussions with N. Pavlova and A. Pavlova. This work was supported by CONACYT through grant number 237439.

Appendix A: Connection to Caldeira-Leggett results

We consider tunneling through the potential barrier $U(x)$. In the description of this process the imaginary time $t = i\tau$ is involved. The string displacement at $z \neq 0$

$$R(z, \tau) = \frac{T}{\hbar} \int_0^{\hbar/T} d\tau_1 x(\tau_1) \left\{ 1 + 2 \sum_{n=1}^{\infty} \exp\left(-\frac{2\pi T}{\hbar s} n|z|\right) \cos\left[\frac{2\pi T}{\hbar} n(\tau - \tau_1)\right] \right\} \quad (\text{A1})$$

follows from Eq. (5) where one should put $t = i\tau$. The displacement (A1) depends on temperature T . Here $x(\tau) = R(0, \tau)$. Tunneling probability $\exp(-S_E/\hbar)$ is determined by the Euclidean action

$$S_E = \int_0^{\hbar/T} L d\tau, \quad (\text{A2})$$

where the Lagrangian L differs from the expression (1) by the formal substitution $t \rightarrow \tau$. In the expression

for $\partial R/\partial z$, following from (A1), one should integrate by parts with respect to τ_1 . That expression has to be put into Eq. (4) where $t = i\tau$. As a result, the classical trajectory in imaginary time satisfies the equation

$$m \frac{d^2 x}{d\tau^2} - U'(x) + \frac{\eta T}{\hbar} \int_0^{\hbar/T} d\tau_1 \cot\left[\frac{\pi T}{\hbar}(\tau_1 - \tau)\right] \frac{dx(\tau_1)}{d\tau_1} \quad (\text{A3})$$

which is equivalent to one of Caldeira-Leggett [1]. The Lagrangian in Eq. (A2) takes the form

$$L = \frac{m}{2} \left(\frac{dx}{d\tau} \right)^2 + U(x) - \frac{\eta}{2\pi} \frac{dx}{d\tau} \int_0^{\hbar/T} d\tau_1 \frac{dx(\tau_1)}{d\tau_1} \ln \left[\sin \frac{\pi T}{\hbar} (\tau - \tau_1) \right], \quad (\text{A4})$$

which is also equivalent to one of Caldeira-Leggett [1].

Appendix B: Reduction to independent oscillators

We introduce new variables $\xi_n = r_n$ at $n = 1, 2, \dots$ and $\xi_0 = Lx/\sqrt{2}$. The form (12) now takes the form

$$\begin{aligned} \frac{2L}{\rho} E &= \sum_{n=1}^{\infty} \omega_n^2 \xi_n^2 + \frac{2m\Omega^2}{\rho L} \xi_0^2 + \sum_{nm=1}^{\infty} (2 + \delta_{nm}) \dot{\xi}_n \dot{\xi}_m \\ &- 4\dot{\xi}_0 \sum_{n=1}^{\infty} \dot{\xi}_n + 2 \left(1 + \frac{m}{\rho L} \right) \dot{\xi}_0^2. \end{aligned} \quad (\text{B1})$$

One can apply the linear transformation

$$\xi_n = \sum_p u_{np} \eta_p, \quad (\text{B2})$$

where η_p is the new variable and matrix elements u_{np} are to be determined to get the form (B1) quadratic. The energy (B1) looks as

$$\frac{2L}{\rho} E = \sum_{pq} (A_{pq} \dot{\eta}_p \dot{\eta}_q + B_{pq} \eta_p \eta_q), \quad (\text{B3})$$

where

$$\begin{aligned} A_{pq} &= \sum_{nm=1}^{\infty} (2 + \delta_{nm}) u_{np} u_{mq} - 2 \sum_{n=1}^{\infty} (u_{0q} u_{np} + u_{0p} u_{nq}) \\ &+ 2 \left(1 + \frac{m}{\rho L} \right) u_{0p} u_{0q}, \quad B_{pq} = \sum_{n=0}^{\infty} u_{np} \kappa_n u_{nq}. \end{aligned} \quad (\text{B4})$$

Here $\kappa_n = \omega_n^2$ at $n = 1, 2, \dots$ and $\kappa_0 = 2m\Omega^2/(\rho L)$.

One should choose the matrix u_{nq} in a way to get the relations

$$A_{pq} = \delta_{pq}, \quad B_{pq} = \omega_q^2 \delta_{pq}, \quad (\text{B5})$$

where ω_q is the function to be determined. Using Eqs. (B2) and (B5) one can easily obtain

$$\eta_q = \sum_{n=0}^{\infty} \frac{\kappa_n}{\omega_q^2} u_{nq} \xi_n. \quad (\text{B6})$$

In the equation $B_{pq} - \omega_q^2 A_{pq} = 0$, which follows from (B5), one can equalize the coefficients at u_{np} to zero. This results in relations

$$2u_{0q} = \sum_{m=1}^{\infty} (2 + \delta_{nm})u_{mq} - \frac{\kappa_n}{\omega_q^2} u_{nq}, \quad n = 1, 2, \dots \quad (\text{B7})$$

and

$$2u_{0q} \left(1 + \frac{m}{\rho L} - \frac{\kappa_0}{2\omega_q^2} \right) = 2 \sum_{m=1}^{\infty} u_{mq}, \quad n = 0. \quad (\text{B8})$$

Excluding $2u_{0q}$ from (B7) and (B8) we obtain

$$\left(1 + \frac{\rho L}{m} \frac{\omega_q^2}{\omega_n^2 - \Omega^2} \right) u_{nq} = \frac{2\omega_q^2}{\omega_n^2 - \omega_q^2} \sum_{m=1}^{\infty} u_{mq}. \quad (\text{B9})$$

One can make the summation on n in the both parts of (B9) and to cancel $\sum u_{mq}$. Using the formula

$$\sum_{n=1}^{\infty} \frac{\cos n\lambda}{n^2 - a^2} = \frac{1}{2a^2} - \frac{\pi}{2a} (\sin a|\lambda| + \cos a\lambda \cot \pi a) \quad (\text{B10})$$

we obtain the relation (15) for the function ω_q . The total energy gets the form (14).

As follows from Eqs. (B8) and (B9),

$$u_{nq} = \frac{2m}{\rho L} \frac{\omega_q^2 - \Omega^2}{\omega_n^2 - \omega_q^2} u_{0q}. \quad (\text{B11})$$

Substituting these forms into Eqs. (B4), it is not difficult to check that $B_{pq} = 0$ when $p \neq q$. To get the normalization condition $B_{pp} = \omega_p^2$ (B5) one should choose the proper function u_{0q} . One can check that

$$u_{0q} = \frac{\eta\omega_q}{\sqrt{(m\omega_q^2 - m\Omega^2)^2 + \eta^2\omega_q^2}}. \quad (\text{B12})$$

We omit simple calculations.

-
- [1] A. O. Caldeira, and A. J. Leggett, *Annals of Physics* **149**, 374 (1983).
 - [2] V. B. Berestetskii, E. M. Lifshitz, and L. P. Pitaevskii *Quantum Electrodynamics* (Pergamon, New York, 1980).
 - [3] M. O. Krause and J. H. Oliver, *J. Phys. Chem. Ref. Data* **8**, No. 2 (1979).
 - [4] A. I. Larkin and Yu. N. Ovchinnikov, *Zh. Eksp. Teor. Fiz. Pis'ma*, **37**, 322 (1983) [*Sov. Phys. JETP Lett.* **37**, 382 (1983)].
 - [5] A. Schmid, *Phys. Rev. Lett.* **51**, 1506 (1983).
 - [6] V. I. Melnikov, *Zh. Eksp. Teor. Fiz.* **87**, 663 (1984) [*Sov. Phys. JETP* **60**, 380 (1984)].
 - [7] L. -D. Chang and S. Chakravarty, *Phys.* **29**, 130 (1984).
 - [8] H. Grabert, U. Weiss, and P. Hänggi, *Phys. Rev. Lett.* **51**, 1506 (1983).
 - [9] A. J. Leggett, S. Chakravarty, A. T. Dorsey, M. P. A. Fisher, A. Garg, and W. Zwerger, *Rev. Mod. Phys.* **59**, 1 (1987).
 - [10] S. E. Korshunov, *Zh. Eksp. Teor. Fiz.* **92**, 1828 (1987) [*Sov. Phys. JETP* **66**, 872 (1987)].
 - [11] B. I. Ivlev, *Zh. Eksp. Teor. Fiz.* **94**, 333 (1988) [*Sov. Phys. JETP* **68**, 1486 (1988)].
 - [12] P. Hänggi, P. Talkner, and M. Borcovec, *Rev. Mod. Phys.* **62**, 251 (1990).
 - [13] Yu. Kagan and N. V. Prokof'ev, in *Quantum Tunneling in Condensed Media*, edited by A. Leggett and Yu. Kagan (North-Holland, Amsterdam, 1992).
 - [14] A. I. Larkin and Yu. N. Ovchinnikov, in *Quantum Tunneling in Condensed Media*, edited by A. Leggett and Yu. Kagan (North-Holland, Amsterdam, 1992).
 - [15] U. Weiss, in *Series in Modern Condensed Matter Physics*, vol. 2 (World Scientific, Singapore 1993).
 - [16] L. Magazzù, D. Valenti, A. Carollo, and B. Spagnolo, *Entropy* **17**, 2341 (2015).
 - [17] D. Valenti, L. Magazzù, P. Caldara, and B. Spagnolo, *Phys. Rev. B* **91**, 235412 (2015).
 - [18] L. Magazzù, D. Valenti, B. Spagnolo, and M. Grifoni, *Phys. Rev. E* **92**, 032123 (2015).
 - [19] L. Magazzù, A. Carollo, B. Spagnolo, and D. Valenti, *J. Stat. Mechanics: Theory and Experiment* 054016 (2016).
 - [20] C. Kittel, *Quantum Theory of Solids* (John Wiley and Sons, New York, 1963).
 - [21] H.-P. Breuer, E.-M. Lain, J. Pillo, and B. Vacchini, *Rev. Mod. Phys.* **88**, 021002 (2016).
 - [22] A. B. Migdal, *Qualitative Methods in Quantum Theory* (Addison-Wesley, 2000).
 - [23] T. H. Boyer, *Phys. Rev.* **182**, 1374 (1969).