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Complete List of Authors:	RAO, V.U.M. ; Andhra University, Applied Mathematics Sireesha, K.V.S.; GITAM University, Engineering Mathematics
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Axially Symmetric Holographic Dark Energy model with generalized Chaplygin gas in Brans-Dicke Theory of Gravitation

V.U.M.Rao¹ & K. V. S. Sireesha²

¹Department of Applied Mathematics, Andhra University, Visakhapatnam, India

²Department of Engineering Mathematics, GITAM University, Visakhapatnam, India

umrao57@hotmail.com

Abstract: In this paper, we have investigated spatially homogeneous anisotropic axially symmetric holographic dark energy cosmological model with generalized Chaplygin gas is obtained in a scalar tensor theory of gravitation proposed by Brans and Dicke [1]. To obtain a determinate solution of the field equations we have used a power law between the metric potentials. It has been found that the anisotropic distribution of dark energy leads to the present accelerated expansion of Universe. All the models obtained and presented here are expanding, non-rotating and accelerating. Also some important features of the models including look-back time, distance modulus and luminosity distance versus red shift with their significances are discussed.

Keywords: Axially Symmetric metric, Holographic Dark energy, Chaplygin gas & Brans– Dicke theory.

1. Introduction:

The holographic principle emerged in the context of black-holes, where it was noted that a local quantum field theory cannot fully describe the black holes (Enqvist et al. [2]). Some long standing debates regarding the time evolution of a system, where a black hole forms and then evaporates, played the key role in the development of the holographic principle. Cosmological versions of holographic principle have been discussed in various literatures (Tavakol and

Ellis [3]). Easter and Lowe [4] proposed that the holographic principle be replaced by the generalized second law of thermodynamics when applied to time-dependent backgrounds and found that the proposition agreed with the cosmological holographic principle. Numerous cosmological observations have established the accelerated expansion of the Universe (Wang et al. [5], Gong [6]). Since it has been proven that the expansion of the Universe is accelerated, the physicists and astronomers started considering the dark energy. Cosmological observations indicated that at about 2/3 of the total energy of the Universe is attributed by dark energy and 1/3 is due to dark matter (Zhang [7]). The nature of the dark sector of the universe (i.e., dark energy and dark matter) remains a mystery. An economical and attractive idea to unify the dark sector of the universe is to consider it as a single component that acts as both dark energy and dark matter. One way to achieve the unification of dark energy and dark matter is by using the so-called Chaplygin gas. The pure Chaplygin gas or generalized Chaplygin gas is a perfect fluid which behaves like a pressure less fluid at an early stage and a cosmological constant at a later stage. In recent times, considerable interest has been stimulated in explaining the observed dark energy by the holographic dark energy model (Enqvist et al. [2], Zhang [7]).

Another way to study dark energy arises from holographic principle that states that the number of degrees of freedom related directly to entropy scales with the enclosing area of the system. In that case the total energy of the system with size L should not exceed the mass of the same black hole size. It means that, $L^3 \rho_A \leq LM_p^2$, where ρ_A is the quantum zero-point energy density which comes from UV cut off Λ , also M_p denotes Planck mass. The largest L is required to saturate this inequality. Then its holographic dark energy density is given by the following expression, $\rho_A = \frac{3C^2 M_p^2}{L^2}$ where C is free dimensionless parameter which commonly considered as a constant, while there is possibility to consider non-constant C (Radicella and Pavon [8], Saadat [9]). Based on cosmological state of holographic principle, the holographic model of dark

energy has been proposed and studied widely in the literature (Li [10], Guberina et al.[11], Setare [12,13,14], Setare and Vagenas [15]). In that case holographic model of dark energy based on Chaplygin gas are also interesting subject of study (Setare [16,17] & Sadeghi et al. [18]).

Holographic dark energy is the nature of DE can also be studied according to some basic quantum gravitational principle. According to this principle Susskind [19], the degrees of freedom in a bounded system should be finite and does not scale by it volume but with its boundary era. Here ρ_Λ is the vacuum energy density. Using this idea in cosmology we take ρ_Λ as DE density. The holographic principle is considered as another alternative to the solution of DE problem. This principle was first considered by 't Hooft [20] in the context of black hole physics. In the context of dark energy problem though the holographic principle proposes a relation between the holographic dark energy density ρ_Λ and the Hubble parameter H as $\rho_\Lambda = H^2$, it does not contribute to the present accelerated expansion of the universe. Granda and Olivers [21] have proposed a holographic density of the form $\rho_\Lambda \approx \alpha H^2 + \beta \dot{H}$, where H is the Hubble parameter and α, β are constants which must satisfy the conditions imposed by the current observational data. They showed that this new model of dark energy represents the accelerated expansion of the universe and is consistent with the current observational data. Granda and Olivers [22] have also studied the correspondence between the quintessence, tachyon, k -essence and dilation dark energy models with this holographic dark energy model in the flat FRW universe. Recently, Kiran et al. [23,24] have studied minimally interacting dark energy models in scalar tensor theories. Adhav et al. [25] have discussed interacting dark matter and holographic dark energy in Bianchi type-V Universe.

Recently Chaplygin gas (CG) is considered in the literature as one of the prospective candidate for DE which however was first introduced in 1904 in

aerodynamics. Although it contains a positive energy density it is referred as an exotic fluid due to its negative nature of pressure. CG may be described by a complex scalar field originating from generalized Born–Infeld action. The equation of state for CG is given by $p = -A/\rho$, where ‘ A ’ is a positive constant. It is known from cosmological observations that CG does not permit a viable cosmology. Consequently, a generalized Chaplygin gas (GCG) is proposed in the literature, (Billic et al. [26], Bento et al. [27]) the equation of state for the GCG is given by, $p = -A/\rho^\alpha$ where $0 \leq \alpha \leq 1$. At high energy GCG behaves almost like a pressure less dust whereas at low-energy regime it behaves like a DE, its pressure being negative and almost constant. Thus GCG smoothly interpolates between a non relativistic matter dominated phases in the early Universe with a DE dominated phase in the late Universe. This interesting property of GCG has motivated cosmologists to consider it as a candidate for unified dark matter and DE models on the other hand modification of the underlying theory of gravitation, however, can be thought of from a fundamentally different perspective.

In the last few decades there has been much interest in alternative theories of gravitation, especially the scalar tensor theories proposed by Brans and Dicke [1], Nordvedt [28], Barber [29] & Saez and Ballester [30] etc. Brans and Dicke [1] scalar-tensor theory of gravitation introduces an additional scalar field ϕ beside the metric tensor g_{ij} and a dimensionless value coupling constant ω . This theory tends to general relativity for large value of the coupling constant ($\omega > 500$). In this theory the scalar field has the dimension of inverse of the gravitational constant and its role is confined to its effects on gravitational field equations.

Brans-Dicke field equations for the combined scalar and tensor field are given by

$$G_{ij} = -8\pi\phi^{-1}T_{ij} - \omega\phi^{-2}\left(\phi_{,i}\phi_{,j} - \frac{1}{2}g_{ij}\phi_{,k}\phi^{,k}\right) - \phi^{-1}(\phi_{i;j} - g_{ij}\phi_{;k}^{,k}) \quad (1.1)$$

$$\text{and } \phi_{,k}{}^k = 8\pi(3 + 2\omega)^{-1}T \quad (1.2)$$

where $G_{ij} = R_{ij} - \frac{1}{2}Rg_{ij}$ is an Einstein tensor, R is the scalar curvature, ω and n are constants, T_{ij} is the stress energy tensor of the matter and comma and semicolon denote partial and covariant differentiation respectively.

Also, we have energy – conservation equation

$$T^{ij}{}_{,j} = 0 \quad (1.3)$$

Several aspects of Brans-Dicke cosmology have been extensively investigated by many authors. Rao et al. [31] have obtained exact Bianchi type-V perfect fluid Cosmological models in Brans-Dicke theory of gravitation. Rao et al. [32] have obtained axially symmetric string cosmological models in Brans – Dicke theory of gravitation. Rao and Vijaya Santhi [33] have discussed Bianchi type-II, VIII and IX magnetized cosmological models in Brans – Dicke theory of gravitation. Rao and Sireesha [34,35,36] have studied a higher-dimensional string cosmological model in a scalar-tensor theory of gravitation, Bianchi type-II, VIII and IX string cosmological models with bulk viscosity in Brans-Dicke theory of gravitation and axially symmetric string cosmological model with bulk viscosity in self creation theory of gravitation respectively. Recently, Rao et al. [37] have obtained LRS Bianchi type-I dark energy cosmological model in Brans-Dicke theory of gravitation. Rao and Sireesha [38] have investigated Bianchi type-II, VIII & IX cosmological models with strange quark matter attached to string cloud in Brans-Dicke and General theory of gravitation. Rao and sireesha [39] have discussed Two fluid cosmological models in Bianchi type-II, VIII & IX space times in Brans-Dicke [1] theory of gravitation. Recently, Rao et al. [40] have studied five dimensional FRW string cosmological model with bulk viscosity in Brans-Dicke [1] theory of gravitation.

This paper is outlined as follows. In Sect. 2, we have obtained the Brans - Dicke field equations for axially symmetric metric in the presence of holographic dark energy cosmological model with generalized Chaplygin gas. In Sect. 3, we have obtained the solution of the field equations along with the correspondence between the holographic and generalised Chaplygin gas model of dark energy. We also discuss some of the features of this model including effective EoS and the evolution of energy density between DE and DM. In Sect.4, we discuss some important properties of the model. Some conclusions are presented in the last section.

2. Metric and Energy Momentum Tensor:

We consider axially symmetric metric in the form

$$ds^2 = dt^2 - A^2(d\chi^2 + f^2(\chi)d\phi^2) - B^2 dz^2 \quad (2.1)$$

where A, B are functions of 't' and f is a function of the coordinate χ only.

The energy momentum tensors for matter and the holographic dark energy are defined as

$$T_{ij} = \rho_m u_i u_j \quad (2.2)$$

$$\text{and } \bar{T}_{ij} = (\rho_\Lambda + p_\Lambda) u_i u_j - g_{ij} \rho_\Lambda \quad (2.3)$$

where ρ_m & ρ_Λ are energy densities of matter and holographic dark energy and p_Λ is the pressure of holographic dark energy.

In a co moving coordinate system, we get

$$T_1^1 = T_2^2 = T_3^3 = 0, T_4^4 = \rho_m \text{ and } \bar{T}_1^1 = \bar{T}_2^2 = \bar{T}_3^3 = -p_\Lambda, \bar{T}_4^4 = \rho_\Lambda \quad (2.4)$$

where the quantities ρ_m, ρ_Λ and p_Λ are functions of 't' only.

3. Solutions of Field equations:

The field equations (1.1) & (1.2) for the metric (2.1), with the help of equations (2.2) to (2.4), can be written as

$$\frac{\ddot{A}}{A} + \frac{\dot{A}\dot{B}}{AB} + \frac{\ddot{B}}{B} + \frac{1}{2} \frac{\omega \dot{\phi}^2}{\phi^2} + \frac{\ddot{\phi}}{\phi} + \frac{\dot{\phi}}{\phi} \left(2 \frac{\dot{A}}{A} + \frac{\dot{B}}{B} \right) = -8\pi\phi^{-1} p_{\Lambda} \quad (3.1)$$

$$\frac{2\ddot{A}}{A} + \left(\frac{\dot{A}}{A} \right)^2 - \frac{1}{A^2} \left(\frac{f''}{f} \right) + \frac{1}{2} \frac{\omega \dot{\phi}^2}{\phi^2} + \frac{\ddot{\phi}}{\phi} + \frac{\dot{\phi}}{\phi} \left(\frac{2\dot{A}}{A} \right) = -8\pi\phi^{-1} p_{\Lambda} \quad (3.2)$$

$$\left(\frac{\dot{A}}{A} \right)^2 + \frac{2\dot{A}\dot{B}}{AB} - \frac{1}{A^2} \left(\frac{f''}{f} \right) - \frac{1}{2} \frac{\omega \dot{\phi}^2}{\phi^2} + \frac{\dot{\phi}}{\phi} \left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} \right) = 8\pi\phi^{-1} (\rho_m + \rho_{\Lambda}) \quad (3.3)$$

$$\ddot{\phi} + \dot{\phi} \left(2 \frac{\dot{A}}{A} + \frac{\dot{B}}{B} \right) = 8\pi(3 + 2\omega)^{-1} (\rho_m + \rho_{\Lambda} - 3p_{\Lambda}) \quad (3.4)$$

$$\dot{\rho}_m + \dot{\rho}_{\Lambda} + (\rho_m + \rho_{\Lambda} + p_{\Lambda}) \left(2 \frac{\dot{A}}{A} + \frac{\dot{B}}{B} \right) = 0 \quad (3.5)$$

Here the overhead dot denotes differentiation with respect to 't' and the overhead dash denotes differentiation with respect to χ .

From (3.2) & (3.3), we can observe that it is possible to separate the terms of $f(\chi)$ to one side and the terms of $A(t), B(t), \rho_m(t), \rho_{\Lambda}(t)$ & $p_{\Lambda}(t)$ to another side.

Hence we can take each part is equal to a constant. So,

$$\frac{f''}{f} = k^2, \quad k^2 \text{ is a constant.} \quad (3.6)$$

If $k=0$, then $f(\chi) = c_1\chi + c_2$, $\chi > 0$

where c_1 and c_2 are integrating constants.

Without loss of generality, by taking $c_1 = 1$ and $c_2 = 0$, we get $f(\chi) = \chi$.

Now the field equations (3.1) to (3.5) will reduce to

$$\frac{\ddot{A}}{A} + \frac{\dot{A}\dot{B}}{AB} + \frac{\ddot{B}}{B} + \frac{1}{2} \frac{\omega \dot{\phi}^2}{\phi^2} + \frac{\ddot{\phi}}{\phi} + \frac{\dot{\phi}}{\phi} \left(2 \frac{\dot{A}}{A} + \frac{\dot{B}}{B} \right) = -8\pi\phi^{-1} p_{\Lambda} \quad (3.7)$$

$$\frac{2\ddot{A}}{A} + \left(\frac{\dot{A}}{A} \right)^2 + \frac{1}{2} \frac{\omega \dot{\phi}^2}{\phi^2} + \frac{\ddot{\phi}}{\phi} + \frac{\dot{\phi}}{\phi} \left(\frac{2\dot{A}}{A} \right) = -8\pi\phi^{-1} p_{\Lambda} \quad (3.8)$$

$$\left(\frac{\dot{A}}{A} \right)^2 + \frac{2\dot{A}\dot{B}}{AB} - \frac{1}{2} \frac{\omega \dot{\phi}^2}{\phi^2} + \frac{\dot{\phi}}{\phi} \left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} \right) = 8\pi\phi^{-1} (\rho_m + \rho_{\Lambda}) \quad (3.9)$$

$$\ddot{\phi} + \dot{\phi} \left(2 \frac{\dot{A}}{A} + \frac{\dot{B}}{B} \right) = 8\pi(3 + 2\omega)^{-1} (\rho_m + \rho_{\Lambda} - 3p_{\Lambda}) \quad (3.10)$$

$$\dot{\rho}_m + \dot{\rho}_{\Lambda} + (\rho_m + \rho_{\Lambda} + p_{\Lambda}) \left(2 \frac{\dot{A}}{A} + \frac{\dot{B}}{B} \right) = 0 \quad (3.11)$$

Among the above five field equations (3.7) to (3.11), the first four equations are independent involving six unknowns $A, B, \rho_m, \rho_{\Lambda}, p_{\Lambda}$ and ϕ . Hence, in order to get a deterministic solution we take the following linear relationship between the metric potentials A and B , i.e., $A = B^n$ (3.12)

where n is an arbitrary constant.

From equations (3.7), (3.8) & (3.12), we get

$$(n-1) \frac{\ddot{B}}{B} + 2n(n-1) \frac{\dot{B}^2}{B^2} - \frac{\dot{B}\dot{\phi}}{B\phi} = 0, \quad n \neq 1 \quad (3.13)$$

The continuity equation can be obtained as

$$\dot{\rho}_m + \dot{\rho}_{\Lambda} + \left(\frac{2\dot{A}}{A} + \frac{\dot{B}}{B} \right) (\rho_m + \rho_{\Lambda} + p_{\Lambda}) = 0 \quad (3.14)$$

The continuity equation of the matter is

$$\dot{\rho}_m + \left(\frac{2\dot{A}}{A} + \frac{\dot{B}}{B} \right) \rho_m = 0 \quad (3.15)$$

The continuity equation of the holographic dark energy is

$$\rho'_\Lambda + \left(\frac{2\dot{A}}{A} + \frac{\dot{B}}{B} \right) (\rho_\Lambda + p_\Lambda) = 0 \quad (3.16)$$

The barotropic equation of state

$$p_\Lambda = \omega_\Lambda \rho_\Lambda \quad (3.17)$$

From equation (3.13), we get

$$B = \left[k_2 (at + b)^{\frac{r+n-1}{n-1}} \right]^{\frac{1}{2n+1}} \quad (3.18)$$

$$\phi = (at + b)^r \quad (3.19)$$

From equations (3.12) & (3.18), we get

$$A = \left[k_2 (at + b)^{\frac{r+n-1}{n-1}} \right]^{\frac{n}{2n+1}} \quad (3.20)$$

The holographic dark energy density are given by

$$\rho_\Lambda = \frac{2}{\alpha - \beta} \left(\dot{H} + \frac{3\alpha}{2} H^2 \right) \quad (3.21)$$

where H is the Hubble parameter, α and β are constants which must satisfy the restrictions imposed by the current observational data.

From equations (3.18)-(3.21), we get

the holographic dark energy density

$$8\pi\rho_\Lambda = \frac{2a^2}{3(\alpha - \beta)} \left[\left(\frac{1-n-r}{n-1} \right) (at + b)^{\frac{-[r+2(n-1)]}{(n-1)}} + \frac{\alpha}{2} (at + b)^{\frac{-2[r+n-1]}{(n-1)}} \right] \quad (3.22)$$

From equations (3.9) & (3.18)-(3.22), we get

$$8\pi\rho_m = \left\{ \left[\left(\frac{n(n+2)a^2}{(2n+1)^2} \right) (at+b)^{\frac{-[r(3-n)+2(n-1)]}{(n-1)}} + \frac{(n+1)ra^2}{(2n+1)} (at+b)^{\frac{-[r(2-n)+2(n-1)]}{(n-1)}} - \frac{\omega}{2} r^2 a^2 (at+b)^{2-r} \right] \right. \\ \left. - \frac{16\pi a^2}{3(\alpha-\beta)} \left[\left(\frac{1-n-r}{n-1} \right) (at+b)^{\frac{-[r+2(n-1)]}{(n-1)}} + \frac{\alpha}{2} (at+b)^{\frac{-2[r+n-1]}{(n-1)}} \right] \right\} \quad (3.23)$$

From equations (3.7), (3.8) & (3.18)-(3.20), we get

$$8\pi p_\Lambda = \left\{ \left(\frac{3n+1}{2n+1} - \frac{n^2+7n-2}{(2n+1)^2} \right) \frac{a^2}{2} (at+b)^{\frac{-[r(3-n)+2(n-1)]}{(n-1)}} + \left(\frac{[2r-(\omega+2)r^2]a^2}{2} \right) (at+b)^{2-r} - \right. \\ \left. \frac{[4n+1]ra^2}{2(2n+1)} (at+b)^{\frac{-[r(2-n)+2(n-1)]}{(n-1)}} \right\} \quad (3.24)$$

From equations (3.17), (3.22) & (3.24), we get

$$w_\Lambda = \frac{p_\Lambda}{\rho_\Lambda} = \frac{\left\{ \left(\frac{3n+1}{2n+1} - \frac{n^2+7n-2}{(2n+1)^2} \right) \frac{a^2}{2} (at+b)^{\frac{-[r(3-n)+2(n-1)]}{(n-1)}} + \left(\frac{[2r-(\omega+2)r^2]a^2}{2} \right) (at+b)^{2-r} - \right. \\ \left. \frac{[4n+1]ra^2}{2(2n+1)} (at+b)^{\frac{-[r(2-n)+2(n-1)]}{(n-1)}} \right\}}{\frac{2a^2}{3(\alpha-\beta)} \left[\left(\frac{1-n-r}{n-1} \right) (at+b)^{\frac{-[r+2(n-1)]}{(n-1)}} + \frac{\alpha}{2} (at+b)^{\frac{-2[r+n-1]}{(n-1)}} \right]} \quad (3.25)$$

The coincident parameter is

$$r = \frac{\rho_\Lambda}{\rho_m} = \frac{\frac{2a^2}{3(\alpha-\beta)} \left[\left(\frac{1-n-r}{n-1} \right) (at+b)^{\frac{-[r+2(n-1)]}{(n-1)}} + \frac{\alpha}{2} (at+b)^{\frac{-2[r+n-1]}{(n-1)}} \right]}{\left\{ \left(\frac{n(n+2)a^2}{(2n+1)^2} \right) (at+b)^{\frac{-[r(3-n)+2(n-1)]}{(n-1)}} + \frac{(n+1)ra^2}{(2n+1)} (at+b)^{\frac{-[r(2-n)+2(n-1)]}{(n-1)}} - \frac{\omega}{2} r^2 a^2 (at+b)^{2-r} \right\} \\ - \frac{16\pi a^2}{3(\alpha-\beta)} \left[\left(\frac{1-n-r}{n-1} \right) (at+b)^{\frac{-[r+2(n-1)]}{(n-1)}} + \frac{\alpha}{2} (at+b)^{\frac{-2[r+n-1]}{(n-1)}} \right]} \quad (3.26)$$

Correspondence between the holographic and generalised Chaplygin gas model of dark energy:

To establish the correspondence between the holographic dark energy with Generalised Chaplygin gas dark energy model, we compare the EoS and the dark energy density for the corresponding models of dark energy. The pressure and the density of the Generalised Chaplygin gas is given by

$$p_{ch} = -\frac{A}{\rho_{ch}^l} \quad (3.27)$$

$$\rho_{ch} = -\frac{[B + Aa^{3(1+l)}]^{\frac{1}{1+l}}}{a^3} = \left[A + \frac{B}{a^{3(1+l)}} \right]^{\frac{1}{1+l}} \quad (3.28)$$

where a is the average scale factor of the universe and A, B, l are positive constants with $0 < l \leq 1$.

Now following Setare [16] we assume that the origin of the dark energy is a scalar field ϕ , so

$$\rho_{\phi} = \frac{1}{2}\dot{\phi}^2 + V(\phi) = \left[A + \frac{B}{a^{3(1+l)}} \right]^{\frac{1}{1+l}} \quad (3.29)$$

$$p_{\phi} = \frac{1}{2}\dot{\phi}^2 - V(\phi) = \frac{-A}{\left[A + \frac{B}{a^{3(1+l)}} \right]^{\frac{l}{1+l}}} \quad (3.30)$$

$$w_{ch} = \frac{p_{ch}}{\rho_{ch}} = \frac{-A}{\rho_{ch}^{1+l}} = \frac{-A}{A + \frac{B}{a^{3(1+l)}}} \quad (3.31)$$

Now adding (3.29) and (3.30), we get

$$\dot{\phi}^2 = \left[A + \frac{B}{a^{3(1+l)}} \right]^{\frac{1}{1+l}} - \frac{A}{\left[A + \frac{B}{a^{3(1+l)}} \right]^{\frac{l}{1+l}}} \quad (3.32)$$

Again subtracting (3.30) from (3.29), we get

$$V(\phi) = \frac{1}{2} \left[A + \frac{B}{a^{3(1+l)}} \right]^{\frac{1}{1+l}} + \frac{A}{2 \left[A + \frac{B}{a^{3(1+l)}} \right]^{\frac{l}{1+l}}} \quad (3.33)$$

Now we assume that the holographic dark energy density is equivalent to the Generalised Chaplygin gas energy density.

Therefore using equations (3.22) and (3.29), we get

$$B = a^{3(1+l)} \left\{ \left[\left(\frac{1-n-r}{n-1} \right) \frac{2a^2}{3(\alpha-\beta)} (at+b)^{\frac{-[r+2(n-1)]}{(n-1)}} + \frac{2a^2\alpha}{6(\alpha-\beta)} (at+b)^{\frac{-2[r+n-1]}{(n-1)}} \right]^{l+1} - A \right\} \quad (3.34)$$

From equations (3.25) & (3.31), we get

$$w_\Lambda = \frac{p_\Lambda}{\rho_\Lambda} = \frac{\left\{ \left(\frac{3n+1}{2n+1} - \frac{n^2+7n-2}{(2n+1)^2} \right) \frac{a^2}{2} (at+b)^{\frac{-[r(3-n)+2(n-1)]}{(n-1)}} + \left(\frac{[2r-(\omega+2)r^2]a^2}{2} \right) (at+b)^{2-r} - \frac{[4n+1]ra^2}{2(2n+1)} (at+b)^{\frac{-[r(2-n)+2(n-1)]}{(n-1)}} \right\}}{\frac{2a^2}{3(\alpha-\beta)} \left[\left(\frac{1-n-r}{n-1} \right) (at+b)^{\frac{-[r+2(n-1)]}{(n-1)}} + \frac{\alpha}{2} (at+b)^{\frac{-2[r+n-1]}{(n-1)}} \right]} \\ = \frac{-A}{\rho_{ch}^{1+l}} = \frac{-A}{A + \frac{B}{a^{3(1+l)}}} \quad (3.35)$$

From equation (3.35), we get

$$A = \left\{ \frac{[4n+1]r a^2}{2(2n+1)} (at+b)^{\frac{-[r(2-n)+2(n-1)]}{(n-1)}} - \left(\frac{3n+1}{2n+1} - \frac{n^2+7n-2}{(2n+1)^2} \right) \frac{a^2}{2} (at+b)^{\frac{-[r(3-n)+2(n-1)]}{(n-1)}} - \left(\frac{[2r-(\omega+2)r^2]a^2}{2} \right) (at+b)^{2-r} \right\} \left[\left(\frac{1-n-r}{n-1} \right) \frac{2a^2}{3(\alpha-\beta)} (at+b)^{\frac{-[r+2(n-1)]}{(n-1)}} + \frac{2a^2\alpha}{6(\alpha-\beta)} (at+b)^{\frac{-2[r+n-1]}{(n-1)}} \right]^l \quad (3.36)$$

Using equation (3.36) in (3.34), we get

$$B = a^{3(1+l)} \left[\left(\frac{1-n-r}{n-1} \right) \frac{2a^2}{3(\alpha-\beta)} (at+b)^{\frac{-[r+2(n-1)]}{(n-1)}} + \frac{2a^2\alpha}{6(\alpha-\beta)} (at+b)^{\frac{-2[r+n-1]}{(n-1)}} \right]^l \left\{ \left(\frac{1-n-r}{n-1} \right) \frac{2a^2}{3(\alpha-\beta)} (at+b)^{\frac{-[r+2(n-1)]}{(n-1)}} + \frac{2a^2\alpha}{6(\alpha-\beta)} (at+b)^{\frac{-2[r+n-1]}{(n-1)}} + \left(\frac{3n+1}{2n+1} - \frac{n^2+7n-2}{(2n+1)^2} \right) \frac{a^2}{2} (at+b)^{\frac{-[r(3-n)+2(n-1)]}{(n-1)}} + \left(\frac{[2r-(\omega+2)r^2]a^2}{2} \right) (at+b)^{2-r} - \frac{[4n+1]r a^2}{2(2n+1)} (at+b)^{\frac{-[r(2-n)+2(n-1)]}{(n-1)}} \right\} \quad (3.37)$$

Using the values of A and B in equations (3.32) and (3.33), we get the potential and dynamics of the scalar field as

$$\phi = \int \left\{ \left[\left(\frac{1-n-r}{n-1} \right) \frac{2a^2}{3(\alpha-\beta)} (at+b)^{\frac{-[r+2(n-1)]}{(n-1)}} + \frac{2a^2\alpha}{6(\alpha-\beta)} (at+b)^{\frac{-2[r+n-1]}{(n-1)}} \right] + \left[\frac{[4n+1]ra^2}{2(2n+1)} (at+b)^{\frac{-[r(2-n)+2(n-1)]}{(n-1)}} - \left(\frac{3n+1}{2n+1} - \frac{n^2+7n-2}{(2n+1)^2} \right) \frac{a^2}{2} (at+b)^{\frac{-[r(3-n)+2(n-1)]}{(n-1)}} - \left(\frac{[2r-(\omega+2)r^2]a^2}{2} \right) (at+b)^{2-r} \right] \right\}^X dt \left[\left(\frac{1-n-r}{n-1} \right) \frac{2a^2}{3(\alpha-\beta)} (at+b)^{\frac{-[r+2(n-1)]}{(n-1)}} + \frac{2a^2\alpha}{6(\alpha-\beta)} (at+b)^{\frac{-2[r+n-1]}{(n-1)}} \right]^{l-1} \quad (3.38)$$

$$V(\phi) = \frac{1}{2} \left\{ \left(\frac{1-n-r}{n-1} \right) \frac{2a^2}{3(\alpha-\beta)} (at+b)^{\frac{-[r+2(n-1)]}{(n-1)}} + \frac{2a^2\alpha}{6(\alpha-\beta)} (at+b)^{\frac{-2[r+n-1]}{(n-1)}} \right\} + \frac{1}{2} \left\{ \left[\frac{[4n+1]ra^2}{2(2n+1)} (at+b)^{\frac{-[r(2-n)+2(n-1)]}{(n-1)}} - \left(\frac{3n+1}{2n+1} - \frac{n^2+7n-2}{(2n+1)^2} \right) \frac{a^2}{2} (at+b)^{\frac{-[r(3-n)+2(n-1)]}{(n-1)}} - \left(\frac{[2r-(\omega+2)r^2]a^2}{2} \right) (at+b)^{2-r} \right] \right\}^X \left[\left(\frac{1-n-r}{n-1} \right) \frac{2a^2}{3(\alpha-\beta)} (at+b)^{\frac{-[r+2(n-1)]}{(n-1)}} + \frac{2a^2\alpha}{6(\alpha-\beta)} (at+b)^{\frac{-2[r+n-1]}{(n-1)}} \right]^{l-1} \quad (3.39)$$

The metric (2.1), in this case, can be written as

$$ds^2 = dt^2 - \left[\frac{2a^2\alpha}{6(\alpha-\beta)} (at+b)^{\frac{r+n-1}{n-1}} \right]^{2n/2n+1} (d\chi^2 + f^2(\chi)d\varphi^2) - \left[\frac{2a^2\alpha}{6(\alpha-\beta)} (at+b)^{\frac{r+n-1}{n-1}} \right]^{2/2n+1} dz^2 \quad (3.40)$$

Thus the metric (3.40) together with (3.22) - (3.26) & (3.34) – (3.39) constitutes an axially symmetric holographic dark energy cosmological model with generalized Chaplygin gas in Brans-Dicke [1] theory of gravitation.

4. Some other important properties of the model:

The spatial volume for the model is

$$V = (-g)^{\frac{1}{2}} = \left[\frac{2a^2\alpha}{6(\alpha - \beta)} (at + b)^{\frac{r+n-1}{n-1}} \right] \quad (4.1)$$

The average scale factor for the model is

$$a(t) = V^{\frac{1}{3}} = \left[\frac{2a^2\alpha}{6(\alpha - \beta)} (at + b)^{\frac{r+n-1}{n-1}} \right]^{\frac{1}{3}} \quad (4.2)$$

The expression for expansion scalar θ calculated for the flow vector u^i is given by

$$\theta = u^i_{;i} = \frac{(r+n-1)a}{(n-1)(at+b)} \quad (4.3)$$

and the shear scalar σ is given by

$$\sigma^2 = \frac{1}{2} \sigma^{ij} \sigma_{ij} = \frac{7}{18} \frac{(r+n-1)^2 a^2}{(n-1)^2 (at+b)^2} \quad (4.4)$$

The deceleration parameter q is given by

$$q = (-3\theta^{-2})(\theta_{;i} u^i + \frac{1}{3}\theta^2) = \frac{2n-r-2}{r+n-1} \quad (4.5)$$

The Hubble's parameter H is given by

$$H = \frac{(r+n-1)a}{3(n-1)(at+b)} \quad (4.6)$$

The mean anisotropy parameter A_m is given by

$$A_m = \frac{1}{4} \sum_{i=1}^4 \left(\frac{H_i - H}{H} \right)^2 = \frac{2(n-1)^2}{(2n+1)^2} \quad \text{where } \Delta H_i = H_i - H \quad (i=1,2,3) \quad (4.7)$$

Look-back time-red shift: The look-back time, $\Delta t = t_0 - t(z)$ is the difference between the age of the universe at present time ($z=0$) and the age of the universe when a particular light ray at red shift z , the expansion scalar of the universe $a(t_z)$ is

related to a_0 by $1 + z = \frac{a_0}{a}$, where a_0 is the present scale factor. Therefore from (4.2), we get

$$1 + z = \frac{a_0}{a} = \left(\frac{at_0 + b}{at + b} \right)^{\frac{r+n-1}{3(n-1)}} \quad (4.8)$$

Using equation (4.6) from above equation we get the luminosity distance as

$$\Delta t = \frac{r+n-1}{3(n-1)H_0} \left(1 - (1+z)^{\frac{3(1-n)}{r+n-1}} \right)$$

where H_0 is the present value of Hubble's constant.

Fig. 1: Plot of Look-back time Δt versus redshift for $H_0 = 67.74 \text{ Km} / \text{Mpc} \cdot \text{s}$ (from SDSS-III spectroscopic survey by Grieb et al. [41]).

Figure 1 describes the behaviour of Look-back time versus Hubble's redshift. It can be seen that the Look-back time has a maximum distance of ~ 13.8 Gyrs which corresponds to age of the present universe (Ade et al. [42]),

which occurs at very high redshift values. This is because of the fact that the redshift is very large near the black hole.

Luminosity distance:

Luminosity distance is defined as the distance which will preserve the validity of the inverse law for the fall of intensity and is given by

$$d_L = r_1(1+z)a_0 \quad (4.9)$$

where r_1 is the radial coordinate distance of the object at light emission and is given by

$$r_1 = \int_t^{t_0} \frac{1}{a} dt = \left(\frac{3(\alpha - \beta)}{a^2 \alpha} \right)^{1/3} \frac{3(n-1)}{[2(n-1)-r]} (at_0 + b) \left[1 - (1+z)^{r-2(n-1)/(r+n-1)} \right] \quad (4.10)$$

From equations (4.9) and (4.10), we get

The luminosity distance

$$d_L = \left(\frac{3(\alpha - \beta)}{a^2 \alpha} \right)^{1/3} \frac{3(n-1)}{[2(n-1)-r]} a_0 (1+z) (at_0 + b) \left[1 - (1+z)^{r-2(n-1)/(r+n-1)} \right] \quad (4.11)$$

From equation (4.11), we get

The distance modulus

$$D(z) = 5 \log \left\{ \left(\frac{3(\alpha - \beta)}{a^2 \alpha} \right)^{1/3} \frac{3(n-1)}{[2(n-1)-r]} a_0 (1+z) (at_0 + b) \left[1 - (1+z)^{r-2(n-1)/(r+n-1)} \right] \right\} + 25 \quad (4.12)$$

The tensor of rotation

$w_{ij} = u_{i,j} - u_{j,i}$ is identically zero and hence this universe is non-rotational.

5. Discussion and Conclusions:

In this paper we have presented spatially homogeneous anisotropic axially symmetric holographic dark energy cosmological model with generalized Chaplygin gas is obtained in a scalar tensor theory of gravitation proposed by Brans and Dicke [1].

The following are the observations and conclusions:

- The model (3.40) has singularity at $t = \frac{-b}{a}$ for $r > (1 - n)$.
- The spatial volume increases with the increase of time 't'.
- At $t = \frac{-b}{a}$, the expansion scalar θ , shear scalar σ and the Hubble parameter H decreases with the increase of time.
- For $n \neq 1$, the model (3.40) indicates that there is certain amount of anisotropy in the Universe and for $n = 1$, one can get the isotropic model from the original equations.
- The model at initial stage represents anisotropic phase of Universe but in special case it isotropizes which is present phase of Universe.
- The matter energy density, the holographic dark energy density, the pressure of holographic dark energy and are decreases with the increase of time 't'.
- The deceleration parameter appears with negative sign for large values of 't' and also for $n < 1$. This implies that the present model represents the accelerating expansion of the universe, which is consistent with the present day observations.
- We have obtained expressions for look-back time ΔT , distance modulus $D(z)$ and luminosity distance d_L versus red shift and discussed their significance.

- We have also reconstructed the potentials and the dynamics of the scalar field for this anisotropic accelerating model of the universe.
- All the models presented here are anisotropic, non-rotating, expanding and also accelerating. Hence they represent not only the early stage of evolution but also the present universe.

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