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Complete List of Authors:	Huang, Jung-Jeng; Ming Chi University of Technology, Center for General Education, Division of Natural Science Wang, Meng-Jong; Ming Chi University of Technology, Center for General Education, Division of Natural Science
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Estimate of energy constituents of the Universe in Λ CDM

Jung-Jeng Huang^a and Meng-Jong Wang^b

Center for General Education, Division of Natural Science
Ming Chi University of Technology, Taishan, New Taipei City 24301, Taiwan

Abstract. We propose that the Hubble law can be viewed as the de Broglie relation on cosmic scale. We show how the entropy of the Universe can be estimated in the Λ CDM model and its extended version, and how the quest for the maximal entropy leads to the energy constituents of the current Universe.

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1. Introduction

Modern cosmological observations have shown that around 5% of the total energy density in the Universe is the baryonic matter, about 26% is the dark matter needed for structure formation, and dominant 69% is the dark energy responsible for the recent phase of accelerated expansion.

The simplest model for dark energy is a cosmological constant. This tiny positive constant is reintroduced through several cosmological observations from supernovae, which indicate the acceleration of current expansion rate of the Universe [1, 2]. The addition of a cosmological constant to cold dark matter leads to the currently popular Λ CDM model. Although the Λ CDM model was introduced in the late 1990s as a concordance cosmology [3], it still provides a

^a e-mail: huangjj@mail.mcut.edu.tw

^b e-mail: mengw102@yahoo.com

good fit to a wide range of observational data. For example, recent WMAP and Planck data constrain the equation of state $w = -1$ for Λ , and estimate Ω_Λ to be $\sim 70\%$ of the current critical density [4, 5]. Combining latest Planck data with other astrophysical data, including Type Ia supernovae, the equation of state of dark energy is also constrained to $w = -1.006 \pm 0.045$, consistent with the expected value of a cosmological constant [6].

The paper is organized as follows. In Sect. 2, the Hubble law is viewed as the de Broglie relation on cosmic scale. Then we show how the entropy of the Universe can be estimated in the Λ CDM model, and how the quest for the maximal entropy leads to the energy constituents of the current Universe. In Sect. 3, similar approach is applied in the extended Λ CDM model to estimate the energy constituents of the Universe. The conclusion and discussion are presented in Sect. 4. Appendix A contains technical derivation of the energy constituents of the current Universe.

2. Hubble law and de Broglie relation

Assuming the homogeneous and isotropic Universe, the Hubble parameter $H = \dot{a}/a$ according to the Hubble law would be constant for any chosen scale a . In fact this is true only for the scale large enough to ignore the density fluctuation ($\delta\rho/\rho \approx 10^{-5}$).

Let $a = R/R_0$ where R denotes the radius of the Universe in this section, and R_0 its present value. In this scale the Hubble law $H = \dot{R}/R$ is well applicable. By definition, the constant expansion rate H (with the dimension of s^{-1}) is the recession velocity scaled by the radius. Likewise, the definite frequency ν (with the dimension of s^{-1}) of a plane wave is the phase velocity scaled by the wavelength.

Let us recall a plane wave with the wavelength λ and angular frequency ω :

$$\omega = v_p q = \frac{2\pi\nu_p}{\lambda}, \quad (1)$$

where q is the wave vector and v_p the phase velocity. The frequency is then

$$\nu = \frac{\omega}{2\pi} = \frac{v_p}{\lambda}. \quad (2)$$

Hence the ν is the phase velocity of the plane wave scaled by the wavelength.

Rewriting $v_p = \tilde{a}\dot{R}$ and $\lambda = \tilde{b}R$ would yield a relationship between ν and

H . This in effect is to recognize the λ or ν of the plane wave as the wave of the Universe. Furthermore, since the observable Universe is bounded by the size $d = 2R$, λ is bounded as well. This yields the stationary waves with $\lambda_n = 2R/n$, where $n = 1, 2, 3, \dots$. The lowest frequency configuration is given by $n = 1$. In terms of wave vector, the stationary condition reads $q_n = 2\pi/\lambda_n = n\pi/R$. If one ever starts with a wave packet composed of waves with various wavelengths, the boundary condition would result in discrete values and pick up only the plane waves with $\lambda_n = 2R/n$.

More specifically, in the expression of wave vector

$$q = \frac{2\pi}{\lambda}, \quad (3)$$

let $\lambda = 2R/\alpha_1$, so we have

$$q = \frac{\alpha_1\pi}{R} = \frac{\alpha_1\pi H}{\dot{R}}, \quad (4)$$

where α_1 is a dimensionless parameter to be determined by observations. In fact, with bounded waves we have $\alpha_1 = n$. In deriving the last expression, the Hubble law has been used. Recall that at a given epoch the Hubble parameter H is a universal constant. Accordingly, by eq. (4), q is also a universal constant.

The recession velocity at R may be evaluated by $\dot{R} = HR$. For the Universe of the present epoch ($t = t_0$) it is given by

$$\dot{R}_0 = H_{obs,0} R_{obs,0} \cong 3.2c \cong \pi c, \quad (5)$$

where $H_{obs,0}$ and $R_{obs,0}$ from current observations [7-9] read in natural units

$$H_0 = 67.3 \text{ km s}^{-1} \text{ Mpc}^{-1} = 1.44 \times 10^{-42} \text{ GeV} / \hbar, \quad (6)$$

$$R_{obs,0} = 4.4 \times 10^{26} m (28.5 Gpc \text{ or } 93 Gly) = 2.2 \times 10^{42} GeV^{-1} \hbar c . \quad (7)$$

Let the general expression for $\dot{R}$ be written as

$$\dot{R}(t) = \alpha_2(t) \pi c , \quad (8)$$

where the dimensionless parameter $\alpha_2(t)$ changes in cosmic time according to the Friedmann-Lemaître equation. For the present epoch, α_2 is determined to be $\alpha_2(t_0) \cong 1$. The momentum corresponding to q is a constant and therefore gives rise to a sinusoidal de Broglie wave with the de Broglie relation:

$$p = \hbar q = \frac{2\pi\hbar}{\lambda} = \frac{h}{\lambda} . \quad (9)$$

Using the Hubble law (8), (4) becomes

$$q = \frac{\alpha_1 \pi H}{\dot{R}} = \left(\frac{\alpha_1}{\alpha_2} \right) \frac{H}{c} . \quad (10)$$

Likewise, the de Broglie relation (9) becomes

$$p = \frac{\alpha_1 \pi \hbar H}{\dot{R}} = \left(\frac{\alpha_1}{\alpha_2} \right) \frac{\hbar H}{c} . \quad (11)$$

Through the Hubble law $\dot{R} = HR$, the p in the de Broglie relation (11) is related to the expansion rate and the recession velocity of the space, while through the expression (8), the p is related only to the expansion rate of the space. Thus the Hubble parameter H becomes a measure of the de Broglie momentum p , and the Hubble law may be viewed as the de Broglie relation on cosmic scale. To verify, it can be shown that through (4) the Hubble law $\dot{R} = HR$ reduces to the de Broglie relation. In this case the momentum p is, apart from the parameters α_1 and α_2 , a combination of $\hbar$, c and H .

The angular frequency of the de Broglie wave is given by

$$\omega = v_p q = v_p \left(\frac{\alpha_1}{\alpha_2} \right) \frac{H}{c} . \quad (12)$$

Let $v_p = \alpha_3 \dot{R}$, where α_3 is a dimensionless constant. Then (12) becomes

$$\omega = \left(\frac{\alpha_1 \alpha_3}{\alpha_2} \right) \frac{\dot{R}H}{c} \equiv \frac{\beta}{c} RH^2, \quad (13)$$

where $\beta \equiv \alpha_1 \alpha_3 / \alpha_2$. With α_1 , α_2 and α_3 being all dimensionless, β is dimensionless and will be determined later. The corresponding de Broglie energy then reads

$$E_{dB} = \hbar \omega = \frac{\beta \hbar}{c} RH^2. \quad (14)$$

To understand E_{dB} and determine β , let us consider a black hole in the expanding Universe, which has the mass $m = 4\pi r^3 \rho_m / 3$ enclosed within the sphere of radius r . Here the energy density $\rho_m \gg \rho_\Lambda$ (i.e. Λ is negligible). With the homogeneity and isotropy of the space, and the negligible curvature constant ($K \equiv 0$) assumed, the Friedmann-Lemaître equation becomes

$$H^2 = \frac{2Gm}{r^3} + \frac{\Lambda}{3} \equiv \frac{2Gm}{r^3}. \text{ Then we have}$$

$$E_{dB} = \frac{\beta \hbar}{c} r H^2 \equiv \frac{\beta \hbar}{c} \frac{2Gm}{r^2}. \quad (15)$$

At the black hole event horizon (Schwarzschild radius) $r_s = 2Gm/c^2$,

$$E_{dB} \equiv \frac{\beta \hbar}{c} \frac{2Gm}{r_s^2} = \frac{\beta \hbar}{c} \frac{2Gm}{(2Gm/c^2)^2} = \frac{\beta \hbar c^3}{2Gm}. \quad (16)$$

Here E_{dB} may be identified with the energy of the Hawking radiation emitted at r_s provided that $\beta = 1/4\pi$:

$$E_{dB} \equiv \frac{\beta \hbar c^3}{2Gm} \Big|_{\beta=1/4\pi} = \frac{\hbar c^3}{8\pi Gm} \equiv E_H. \quad (17)$$

The temperature defined by

$$T_{dB} \equiv E_{dB} / k_B \quad (18)$$

is identified with the Hawking temperature at r_s . Here, k_B is the Boltzmann constant.

The entropy of the black hole is given by

$$\begin{aligned}
 S &\equiv \int_0^m dS = \int_0^m \frac{dE_{m'}}{T_{dB}} = \int_0^m \frac{c^2 dm'}{E_{dB} / k_B} \cong \int_0^m \frac{G dm'^2}{\beta \hbar c / k_B} \\
 &= \frac{G m^2}{\beta \hbar c} k_B = \frac{(2Gm/c^2)^2 c^3}{4\beta G \hbar} k_B = \frac{r_s^2 c^3}{4\beta G \hbar} k_B, \quad (19)
 \end{aligned}$$

where $E_{m'}$ is the energy of the matter content enclosed within the sphere of radius $r' \equiv 2Gm'/c^2$. Note also that (19) may be identified with the Bekenstein-Hawking entropy equation provided that $\beta = 1/4\pi$:

$$S \cong \frac{\pi r_s^2 c^3}{G \hbar} k_B \equiv S_{BH}. \quad (20)$$

In what follows, we further illustrate how the general entropy S for a system with non-negligible Λ and variable R can be estimated in the Λ CDM model. We also show how the quest for the maximal entropy $S_{\max}$ may lead to the energy constituents of the Universe: $(\hat{\Omega}_m, \hat{\Omega}_\Lambda) = (1/3, 2/3)$.

The Friedmann-Lemaître equation with the mass $M = 4\pi R^3 \rho_m / 3$ enclosed within the radius R of the observable Universe reads

$$H^2 = \frac{2GM}{R^3} + \frac{\Lambda}{3} \equiv \frac{2GM}{R^3} (1 + b_\Lambda R^3), \quad (21)$$

$$b_\Lambda R^3 = \frac{\Lambda/3}{2GM/R^3} = \frac{\rho_\Lambda}{\rho_m}, \quad (22)$$

$$1 + b_\Lambda R^3 = \frac{\rho_m + \rho_\Lambda}{\rho_m} = \frac{\rho_c}{\rho_m} = \frac{1}{\Omega_m}. \quad (23)$$

With $dM = \frac{3MdR}{R}$ and $E_{dB} = \frac{\beta \hbar}{c} RH^2 = \frac{\beta \hbar}{c} \left(\frac{2GM}{R^2} \right) \frac{1}{\Omega_m}$, we find that

$$\begin{aligned}
 dS &= \frac{dE_m}{E_{dB} / k_B} = \frac{c^2 dM}{E_{dB} / k_B} \text{ can be integrated to yield the entropy} \\
 S &= 3\Omega_m \frac{R^2 c^3}{4\beta G \hbar} k_B. \quad (24)
 \end{aligned}$$

In terms of E_{dB} , (24) may be written as

$$S = \frac{3R^2 c^3}{2E_{dB}} k_B. \quad (25)$$

It can be shown (see Appendix A) that the quest for the maximal entropy

$$S_{\max} = \frac{R^2 c^3}{4\beta G\hbar} k_B = \left(\frac{1}{4\pi\beta} \right) \frac{\pi R^2 c^3}{G\hbar} k_B \quad (26)$$

leads to

$$\Omega_m = 1/3 \equiv \hat{\Omega}_m. \quad (27)$$

Here $\hat{\Omega}_m$ denotes the Ω_m leading to $S_{\max}$. With $\beta = 1/4\pi$, (26) has the expression of the Bekenstein-Hawking entropy equation. By $\hat{\Omega}_m + \hat{\Omega}_\Lambda = 1$ at the present epoch, we have $\hat{\Omega}_\Lambda = 1 - \hat{\Omega}_m = 2/3$ and arrive at the energy constituents:

$$(\hat{\Omega}_m, \hat{\Omega}_\Lambda) = (1/3, 2/3), \quad (28)$$

which is the configuration corresponding to the maximal entropy of the Universe.

3. Extended Λ CDM model

In this section we estimate further the energy constituents of the current Universe in the extended Λ CDM model. Let $a \equiv \frac{R}{R_0}$ such that $\dot{a} \equiv \frac{\dot{R}}{R_0}$, $\ddot{a} \equiv \frac{\ddot{R}}{R_0}$ and $H = \frac{\dot{R}}{R} = \frac{\dot{a}}{a}$. Then the Friedmann-Lemaitre equation becomes

$$H^2 = \frac{2GM}{a^3} + \frac{\Lambda}{3a^\delta} + \frac{\gamma}{a^4} = \frac{2GM}{a^3} (1 + b_\Lambda a^{3-\delta} + b_\gamma a^{-1}), \quad (29)$$

where δ is the parameter to measure possible relics of slow-roll fields, γ is the parameter to measure the hot dark matter and/or dark radiation, and

$$M = \frac{4\pi a^3}{3} \rho_m, \quad (30)$$

$$b_\Lambda a^{3-\delta} = \frac{\Lambda / 3a^\delta}{2GM / a^3} = \frac{\rho_\Lambda}{\rho_m}, \quad (31)$$

$$b_\gamma a^{-1} = \frac{\gamma / a^4}{2GM / a^3} = \frac{\rho_\gamma}{\rho_m}, \quad (32)$$

$$1 + b_\Lambda a^{3-\delta} + b_\gamma a^{-1} = \frac{\rho_m + \rho_\Lambda + \rho_\gamma}{\rho_m} = \frac{1}{\Omega_m}. \quad (33)$$

In (29), the density parameters for radiation and curvature $\Omega_r \cong \Omega_K \cong 0$ are assumed. The de Broglie energy defined with (14) then reads

$$E_{dB} = \frac{\hbar}{4\pi c} aH^2 = \frac{\hbar}{4\pi c} \frac{2GM}{a^2} (1 + b_\Lambda a^{3-\delta} + b_\gamma a^{-1}) = \frac{\hbar}{4\pi c} \frac{2GM}{a^2} \frac{1}{\Omega_m}. \quad (34)$$

The entropy of the Universe then becomes (cf. (24) and (25))

$$S = \int \frac{dMc^2}{E_{dB} / k_B} = 3\Omega_m \frac{\pi a^2 c^3}{G\hbar} k_B = \frac{3Mc^2 k_B}{2E_{dB}}. \quad (35)$$

Let $\hat{\Omega}_\Lambda$, $\hat{\Omega}_m$ and $\hat{\Omega}_\gamma$ denote the energy constituents for $S_{\max}$. Solving

$\frac{dS}{da} = 0$ (or $\frac{dE_{dB}}{da} = 0$) for $b_\Lambda a^{3-\delta}$ yields the condition:

$$b_\Lambda a^{3-\delta} = \frac{2 + 3b_\gamma a^{-1}}{1 - \delta}. \quad (36)$$

Here, (36) may be recast as:

$$\hat{\Omega}_m = \frac{1 - \delta}{3 - \delta + (4 - \delta)b_\gamma a^{-1}}, \quad (37)$$

or

$$\hat{\Omega}_\Lambda = \frac{2\hat{\Omega}_m + 3\hat{\Omega}_\gamma}{1 - \delta} = \frac{2\hat{\Omega}_m + \hat{\Omega}_\gamma}{1 - \delta}, \quad (38)$$

or

$$\delta = \frac{(1 - 3\hat{\Omega}_M - \hat{\Omega}_\gamma)}{(1 - \hat{\Omega}_M)}, \quad (39)$$

where $\hat{\Omega}_M \equiv \hat{\Omega}_m + \hat{\Omega}_\gamma$. Using (37) and (38), it is easy to see that the energy

constituents in the Λ CDM model (28) are recovered by setting $\delta = \gamma = 0$.

Note that for the case $\gamma \neq 0$, the constituent of dark radiation

$\hat{\Omega}_\gamma \equiv \Omega_\nu + \Omega_a$ is constrained by observations, where Ω_ν represents the possible contribution of the neutrino families and Ω_a of the hypothesized existing axions. Taking $\Omega_\nu \cong 0.0044$, $\Omega_a \cong 0.0068$ (for the sum of the three

active neutrino masses $\Sigma m_\nu = 0.20^{+0.13}_{-0.14} \text{ eV}$ and axion mass $m_a = 0.57^{+0.50}_{-0.47} \text{ eV}$)

for an instance, as reported in CMB+DR11+BAO+HST+SZ Cluster datasets [10],

one finds $\hat{\Omega}_\gamma \equiv \Omega_\nu + \Omega_a \cong 0.0044 + 0.0063 = 0.011$ such that

$$\hat{\Omega}_\Lambda = \frac{2\hat{\Omega}_M + 0.011}{1 - \delta}. \quad (40)$$

In this case $\delta = 0.064$ yields

$$(\hat{\Omega}_M, \hat{\Omega}_\Lambda) = (0.315, 0.685) = (\Omega_{M,obs}, \Omega_{\Lambda,obs}). \quad (41)$$

4. Conclusion and discussion

In this paper, we first establish the relationship between the Hubble parameter and the frequency of a sinusoidal cosmic wave. Then we propose that the Hubble law can be viewed as an indication of the de Broglie relation on cosmic scale. We also calculate the entropy of the Universe in the Λ CDM model, and obtain the energy constituents of the current Universe through the quest of the maximal entropy.

From (28) we note that the energy constituents $(\hat{\Omega}_m, \hat{\Omega}_\Lambda) = (1/3, 2/3)$

estimated in the Λ CDM model are slightly different from the results based on full-mission Planck 2015 observations

$$(\hat{\Omega}_{m,obs}, \hat{\Omega}_{\Lambda,obs}) = (0.315 \pm 0.013, 0.685 \pm 0.013) \quad [7, 8]. \quad (42)$$

This slight deviation may arise from the relics of the early Universe such as slow-roll fields, hot dark matter, and dark radiation, which would further modify the Friedmann-Lemaitre equation. We showed in Sect.3 how the extended Λ CDM model with modified Friedmann-Lemaitre equation (29) can lead to the observational energy constituents (41) of the current Universe.

We have also shown that at a given epoch, the Hubble constant H is related to the wave vector q by (4). One could start with a wave packet to associate with the wavelengths bounded by the size of the observable Universe. The boundary condition $\lambda_n = 2R/n$ or $q_n = n\pi/R$ then leads to (4).

Finally we note that the de Broglie wave used for description of the Universe in this paper has its usual meaning in the standard quantum mechanics, and it is not the pilot wave in the de Broglie-Bohm theory. For the pilot wave treatment, it has been shown that quantum potential effects are negligible when the Universe is at the epoch of well after horizon exit [11].

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Appendix A. Derivation of equation (27) in Sect. 2

Using

$$\Omega_m = \frac{1}{1 + b_\Lambda R^3} \quad (\text{A1})$$

and
$$S = 3\Omega_m \frac{R^2 c^3}{4\beta G \hbar} k_B = \frac{3c^3}{4\beta G \hbar} k_B \left(\frac{R^2}{1 + b_\Lambda R^3} \right), \quad (\text{A2})$$

$S_{\max}$ can be found by examining $S' \equiv \frac{dS}{dR}$ and $S'' \equiv \frac{d}{dR} \left(\frac{dS}{dR} \right)$. Let

$C \equiv \frac{3c^3}{4\beta G \hbar} k_B$ so that $S = C \left(\frac{R^2}{1 + b_\Lambda R^3} \right)$. The quest for $S_{\max}$ proceeds by

firstly solving $S' = 0$ through

$$\begin{aligned} S' &= C \left(\frac{2R}{1 + b_\Lambda R^3} - \frac{3b_\Lambda R^4}{(1 + b_\Lambda R^3)^2} \right) \\ &= C \left(\frac{2R + 2b_\Lambda R^4 - 3b_\Lambda R^4}{(1 + b_\Lambda R^3)^2} \right) = C \left(\frac{R(2 - b_\Lambda R^3)}{(1 + b_\Lambda R^3)^2} \right) = 0 \end{aligned} \quad (\text{A3})$$

to yield

$$b_\Lambda R^3 = 2, \quad (\text{A4})$$

$$\Omega_m = \frac{1}{(1 + b_\Lambda R^3)} = \frac{1}{3} \equiv \hat{\Omega}_m, \quad (\text{A5})$$

and then verifying $S''|_{\hat{\Omega}_m} < 0$ to reach $S|_{\hat{\Omega}_m} = S_{\max}$.