Canadian Journal of Physics
Revue canadienne de physique

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Journal:	<i>Canadian Journal of Physics</i>
Manuscript ID	cjp-2016-0518
Manuscript Type:	Article
Date Submitted by the Author:	25-Jul-2016
Complete List of Authors:	Abbas, G.; The IslamiaUniversity of Bahawalpur Shah, Hassan ; COMSATS Institute of Information Technology - Abbottabad Ahmad , Zahid ; COMSATS Institute of Information Technology - Abbottabad
Keyword:	Gravitational Collapse, Matching Conditions, Radiating Source, Conformal flate, Thermodynamics

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Some Analytic Models of Plane Symmetric Radiating Collapse

G. Abbas¹ *; Hassan Shah² † and Zahid Ahmad² ‡

¹ Department of Mathematics, The Islamia University of Bahawalpur, Bahawalpur, Pakistan

² Department of Mathematics, COMSATS Institute of Information Technology, Abbottabad, KPK, Pakistan.

Abstract

This paper deals with the analytical solutions of the field equations in the presence of radiating plane symmetric source. For this purpose we have solved the field equations as well junction conditions by the imposing the conformal flatness conditions. The effective adiabatic index (that determine the stability of the system) has been calculated for the present radiating source. It has been found that effective adiabatic index remains invariant through out the matter distribution. In order to study the thermal behavior of the source, we have discussed the thermal profile of the source and found that in the absence of no dissipation from the system the temperature of the system remains constant.

Keywords : Gravitational Collapse, Matching Conditions, Radiating Source

*abbasg91@yahoo.com

†hassanshah13@yahoo.com

‡zahidahmad@ciit.net.pk

1 Introduction

In the context of modern astronomy the gravitational collapse is the destruction of a massive star (which have much larger mass than the solar mass) under the effects of its own gravitational pull. In general relativity and relativistic astrophysics it is an active research area to determine the final outcome of gravitational collapse. According to the theorems of Hawking and Penrose, it has been suggested that the final fate of the gravitational collapse of a massive star is a spacetime singularity [1]. The study of gravitational collapse started from the pioneer work of Chandrasekhar [2]. Later on, Oppenheimer and Snyder [3] investigated that spherically symmetric homogeneous dust collapse leads to the formation of black hole (BH). Initially, it was argued that homogeneity and spherical symmetry of the collapsing model are responsible for the formation of BH. However, it was found [4] that inhomogeneous dust collapse would end as shell crossing and shell focusing singularities.

The non-spherical geometries of a star have been investigated by different researchers using cylindrical and plane symmetries. The main motivation to study the non-spherical geometry is the existence of cylindrical gravitational waves. The pioneering work on cylindrical symmetry was done by Bronnikov and Kovalchuk [5] and found some exact solutions. The dust spheroid collapse was numerically studied by Shapiro and Teukolsky [6] considering cylindrical spacetime. They concluded that the end product of collapse was either a BH or a naked singularity depending on the spheroid compactness. Barrabes et al. [7] investigated the exact solution for collapsing convex shell. Herrera and Santos [8] studied the gravitational collapse and gravitational waves. Also, the static cylinders and their conformal flatness conditions have been addressed by Herrera et al. [9]. The shear free radiating spherical collapse and some more general models of collapse have been discussed by Herrera and his collaborators [10]-[13] using the approximate solutions with conformal flatness conditions. Abbas et al. [14] examined the plane symmetric source with the condition of conformal flatness. They found approximate solutions to the field equations.

Sharif and Abbas [15] studied expanding and collapsing scalar field thin shell. Di Prisco et al. [16] examined Shearfree cylindrical gravitational collapse. Maharaj and Govender [17] investigated the radiating collapse with vanishing Weyl stresses, by taking spherically symmetric spacetime. They found exact solutions of Einstein field equations. Sharif and Ahmad [18] stud-

ied plane symmetric gravitational collapse using junction conditions. They concluded that in some cases the plane collapse in some other cases the plane expand. In this paper, we extended the work done by Abbas et al. [14] for the exact model of collapse. The paper is arranged as follows: In section 2, the field equations are derived. We present conformally flat solutions in section 3. In section 4, we suggest an exact model and calculated the temperature profile. Finally the summary of the work done has been presented in last section.

2 Equations of Motion

Assume that a given $4D$ plane symmetric spacetime is divided by a $3D$ hypersurface Σ into two regions interior and exterior denoted by M^- and M^+ respectively. We consider a particular plane symmetric spacetime inside the hypersurface which is given by

$$ds_-^2 = -A^2(t, z)dt^2 + B^2(t, z)[dx^2 + dy^2 + dz^2], \quad (1)$$

We number the coordinates $x^0 = t, x^1 = x, x^2 = y$ and $x^3 = z$. The radiating matter is given by following form of energy momentum tensor

$$T_{\alpha\beta} = (\mu + P)w_\alpha w_\beta + Pg_{\alpha\beta} + w_\alpha q_\beta + q_\alpha w_\beta, \quad (2)$$

where μ , P and w^α are the energy density, pressure and the four velocity of fluid, respectively, and q^α is heat flux which satisfies the relation $q_\alpha w^\alpha = 0$. We define four velocity and heat flux as follows

$$w^\alpha = \frac{1}{A}\delta_0^\alpha, \quad (3)$$

$$q^\alpha = q\delta_3^\alpha, \quad (4)$$

where $q = q(z, t)$.

The expansion scalar $\Theta = w_{;\alpha}^\alpha$ of fluid is given from (1) and (3) as follows

$$\Theta = \frac{3B_{,t}}{AB}, \quad (5)$$

The non vanishing components of the Weyl tensor $C_{\alpha\beta\gamma\delta}$ for metric (1)

are

$$C_{1212} = \frac{B^2}{3} \left[2 \frac{B_{,z}}{B} \left(\frac{A_{,z}}{A} - \frac{B_{,z}}{B} \right) - \left(\frac{A_{,zz}}{A} - \frac{B_{,zz}}{B} \right) \right], \quad (6)$$

$$C_{1212} = 2 \left(\frac{B}{A} \right)^2 C_{0101} = - \left(\frac{B}{A} \right)^2 C_{0303} = -2C_{1313}. \quad (7)$$

For the interior spacetime (1) and matter source (2), we have following set of field equations

$$G_{00}^- = \left(\frac{A}{B} \right)^2 \left[-2 \frac{B_{,zz}}{B} + \left(\frac{B_{,z}}{B} \right)^2 \right] + 3 \left(\frac{B_{,t}}{B} \right)^2 = \kappa \mu A^2, \quad (8)$$

$$G_{11}^- = - \left(\frac{B}{A} \right)^2 \left[\frac{2B_{,tt}}{B} - \frac{2A_{,t}B_{,t}}{AB} + \left(\frac{B_{,t}}{B} \right)^2 \right] + \frac{A_{,zz}}{A} + \frac{B_{,zz}}{B} - \left(\frac{B_{,z}}{B} \right)^2 = \kappa P B^2, \quad (9)$$

$$G_{22}^- = G_{11}^-,$$

$$G_{33}^- = - \left(\frac{B}{A} \right)^2 \left[\frac{2B_{,tt}}{B} - \frac{2A_{,t}B_{,t}}{AB} + \left(\frac{B_{,t}}{B} \right)^2 \right] + \frac{2A_{,z}B_{,z}}{AB} + \left(\frac{B_{,z}}{B} \right)^2 = \kappa P B^2, \quad (10)$$

$$G_{03}^- = -2 \left(\frac{B_{,t}}{AB} \right)_{,z} A = -\kappa q A B^2, \quad (11)$$

The Taub's mass function for plane symmetric spacetime is given by [13]

$$M(t, z) = \frac{(g_{11})^{\frac{3}{2}}}{2} R_{12}^{12} = \frac{B}{2} \left[\frac{B_{,t}^2}{A^2} - \frac{B_{,z}^2}{B^2} \right], \quad (12)$$

3 Conformally Flat Solution

In this section, we apply the conformal flatness to metric (1) i.e., all the components of the Weyl tensor must vanish from (6) and (7) we see that if $C_{1212} = 0$ this condition is fulfilled, hence we have

$$\left(\frac{BA_{,zz}}{AB} - \frac{AB_{,zz}}{AB} \right) - \frac{2B_{,z}}{B} \left(\frac{BA_{,z}}{BA} - \frac{AB_{,z}}{AB} \right) = 0, \quad (13)$$

The above equation gives

$$A = B (C_1(t)z + 1). \quad (14)$$

For the isotropy of pressure, we have from $G_{11} = G_{33}$, the following relation

$$\frac{B_{,zz}}{B_{,z}} - 2 \frac{B_{,z}}{B} = 0, \quad (15)$$

The solution of above differential equation is

$$B = \frac{1}{C_2(t)z + C_3(t)}, \quad (16)$$

where C_2 and C_3 are functions of t .

Here, we matched the interior spacetime with the exterior spacetime that is described by plane symmetric vaidya spacetime given by

$$ds_+^2 = \frac{2m(\nu)}{z} d\nu^2 - 2d\nu dz + z^2(dy^2 + dz^2), \quad (17)$$

from Eqs.(1) and (17) on hypersurface M along with field equations (8)-(11) and the mass function (12), we get

$$B_\Sigma = z_\Sigma, \quad (18)$$

$$P_\Sigma = (qB)_\Sigma, \quad (19)$$

$$M(\nu) = \frac{B}{2} \left[\frac{B_{,t}^2}{A^2} - \frac{B_{,z}^2}{B^2} \right]_\Sigma. \quad (20)$$

Using Eqs. (10), (11), (14), (16) and (19), we obtain

$$\begin{aligned} & \left[\ddot{C}_2 z + \ddot{C}_3 - \frac{3}{2} \frac{(\dot{C}_2 z + \dot{C}_3)^2}{C_2 z + C_3} - (C_1 \dot{C}_3 - \dot{C}_2) \right. \\ & \quad \left. - \frac{2\dot{C}_1 z (\dot{C}_2 z + \dot{C}_3)}{C_1 z + 1} + \frac{(C_1 z + 1)^2}{2(C_2 z + C_3)} \right. \\ & \quad \left. (C_1 C_2^2 z + 3C_2^2 - 2C_1 C_2 C_3) \right] = 0. \end{aligned} \quad (21)$$

4 Exact Model

A simple approximate solution has been proposed by Abass et al.[14] which satisfy the junction condition (21) for the arbitrary functions $C_1(t)$, $C_2(t)$ and $C_3(t)$. They considered the following forms

$$C_1 = \epsilon c_1(t), \quad C_2(t) = 0, \quad C_3(t) = \frac{a}{t^2}, \quad (22)$$

where $0 < \epsilon < 1$, $0 \leq t \leq -\infty$ and $a > 0$ a constant proportional to the total mass in the plane.

The junction condition (21) is exactly solvable for the particular choices of the arbitrary functions. Here we present an exact solution for (21) we assume

$$C_2(t) = 0. \quad (23)$$

Using Eq. (23) in Eq. (21), we get

$$C_3 \ddot{C}_3 - \frac{3}{2} \dot{C}_3^2 - \left[\frac{2\dot{C}_1 z}{C_1 z + 1} + C_1 \right] C_3 \dot{C}_3 = 0. \quad (24)$$

Now using the transformation $C_3(t) = u^{-2}$ in Eq. (24), we obtain

$$\ddot{u} - \left[\frac{2\dot{C}_1 z}{C_1 z + 1} + C_1 \right] \dot{u} = 0. \quad (25)$$

If we assume $C_1 = C$ where C is constant. Then (24) reduces to

$$\ddot{u} - C\dot{u} = 0. \quad (26)$$

The differential equation (26) has the solution in the original function C_3 as

$$C_3(t) = (\beta_1 + \beta_2 e^{Ct})^{-2}. \quad (27)$$

where β_1 and β_2 are integration constants.

From (8), (9) and (11), it follows

$$\kappa\mu = \frac{12(C\beta_2 e^{Ct})^2}{(\beta_1 + \beta_2 e^{Ct})^6 (Cz + 1)^2}, \quad (28)$$

$$\kappa p = \frac{-4C\beta_2 e^{Ct}}{(\beta_1 + \beta_2 e^{Ct})^6 (Cz + 1)} [3(C\beta_2 - 1) + (\beta_1 + \beta_2 e^{Ct})C], \quad (29)$$

$$\kappa q B = \frac{-4C^2\beta_2}{(\beta_1 + \beta_2 e^{Ct})^5 (Cz + 1)^2} e^{Ct}. \quad (30)$$

The luminosity is given by

$$B_{\Sigma} = (\beta_1 + \beta_2 e^{Ct})^2. \quad (31)$$

The four acceleration of this model is non vanishing and is given by

$$a_{\alpha} = u^{\beta} \nabla_{\beta} u_{\alpha} = \frac{A_{,z}}{A} \delta_a^3 = \frac{C}{Cz + 1}. \quad (32)$$

The effective adiabatic index is

$$\Gamma_{eff} = \frac{d \ln P}{d \ln \mu} = \left(\frac{\dot{P}}{P} \right) \left(\frac{\mu}{\dot{\mu}} \right), \quad (33)$$

It helps to explain the dynamical instability of the gravitating source at a particular instant of time. For our model, we get

$$\Gamma_{z=0} - \Gamma_{z=z_{\Sigma}} = 0. \quad (34)$$

This shows that the source collapse is homogeneous at $z = 0$ and $z = z_{\Sigma}$.

4.1 Calculation of Temperature

Here we calculate the temperature profile $T(z, t)$ for our model, using the following heat transport equation

$$\tau h^{\alpha\beta} w^{\gamma} q_{\beta;\gamma} + q^{\alpha} = -K h^{\alpha\beta} (T_{,\beta} + a_{\beta} T), \quad (35)$$

where the quantities τ , K and $h^{\alpha\beta} = g^{\alpha\beta} + w^{\alpha} w^{\beta}$ are relaxation time, thermal conductivity and projection tensor, respectively. Considering Eqs.(1)-(4) and (35), we get

$$\tau B(\dot{B}q + B\dot{q}) + qAB^2 = -K(T_{,z}A + TA_{,z}), \quad (36)$$

Using (11), (14) and (16) into (36) and considering $C_2 = 0$, we have

$$\tau(C_1 C_3 \dot{C}_3)_{,t} + C_1 \dot{C}_3 = -\frac{\kappa K}{2} [T(C_1 z + 1)]_{,z}. \quad (37)$$

see [14] for detail.

The integration of (37) gives

$$T(t, z) = F(t) + \frac{-C_1}{4\pi K(C_1 z + 1)} [\tau(\dot{C}_3^2 + \ddot{C}_3 C_3)z + \dot{C}_3 z]. \quad (38)$$

Now if there is no dissipation we have ($C_1 = 0$) from (38) it follows that $T = T_c(t)$ meaning that the temperature is homogeneous with in the fluid distribution, we will have

$$T(t, z) = T_c(t) + \frac{-C_1}{4\pi K(C_1 z + 1)} [\tau(\dot{C}_3^2 + \ddot{C}_3 C_3)z + \dot{C}_3 z]. \quad (39)$$

where C_3 is given by (27) and T_c is the central pressure.

5 Conclusion

In this paper, we have investigated the some analytic models of radiating plane symmetric source by imposing the conformal flatness condition. The application of conformal flatness condition to the underlying spacetime leads to the exact solutions of junction condition, with three temporal functions. We assumed particular values of the temporal functions which satisfied the junction conditions as well as Einstein field equations. We have calculated the adiabatic index at $z = 0$ and $z = z_\Sigma$ from this we concluded that the collapse is homogeneous.

We have also determine the temperature function for the given system by using the Maxwell-Cattaneo heat transport equation. In equation (39) the last term showed the influence of dissipation on the temperature distribution and the second term showed the relaxational effects. We have found that in the absence of no dissipation (heat flux) from the system the temperature of the system becomes same through out the matter distribution.

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