Canadian Journal of Physics
Revue canadienne de physique

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Journal:	<i>Canadian Journal of Physics</i>
Manuscript ID	cjp-2016-0558.R1
Manuscript Type:	Article
Date Submitted by the Author:	01-Sep-2016
Complete List of Authors:	Li, Guqiang; Lingnan Normal University
Keyword:	Tunneling radiation, Born-Infeld AdS black hole, the first law of thermodynamics, underlying unitary theory, WKB approximation

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Hawking radiation via tunneling from Born-Infeld AdS black hole

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Abstract: The tunneling radiation of particles from Born-Infeld AdS black holes is studied by using Parikh-Wilczek (PW) method and the emission rate of a particle is calculated. It is shown that the emission rate is related to the change of the Bekenstein–Hawking entropy (BHE) of the black hole and the emission spectrum deviates from the purely thermal spectrum but consists with an underlying unitary theory.

Keywords: Tunneling radiation; Born-Infeld AdS black hole; the first law of thermodynamics; underlying unitary theory; WKB approximation

PACS Nos.: 04.70.Dy, 97.60.Lf.

Forty years ago, it was proved by Hawking that the black hole can radiate thermally [1]. In the early period of time, all work was done to prove that the energy spectrum is precisely thermal spectrum [2-6]. In 2005, Hawking changed his opinion and argued that information can indeed get out of the black hole [7], which may partly be based on the PW work that treats the Hawking radiation as tunneling process [8-10]. In recent ten years, PW method was used to calculate the emission rate of particles from various black holes [11-23] and to obtain the modified spectrum. In this paper, we would like to extend PW method to Born–Infeld AdS black holes to calculate the corrected emission spectrum of particles from their event horizon and to explore the influence on the thermodynamic properties due to Born–Infeld electrodynamics, which are black hole solutions for Einstein gravity in AdS space coupled to Born–Infeld electrodynamics. In 1934, Born and Infeld [24] introduced a nonlinear electrodynamics called Born–Infeld electrodynamics to present a classical theory of charged particles with finite self-energy. The black hole solutions and their thermodynamics have been widely studied [25-36]. Because the metric contains hypergeometric function, it becomes complicated to make the analysis and the thermodynamics was once regarded as a nontrivial task to carry out completely [33-36]. To the best of our knowledge, the tunneling radiation from a Born–Infeld AdS black hole has not been covered in the literature yet.

The metric of a 4-dimensional static and spherically symmetric black hole has been proposed as [30]

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2 + r^2(d\theta^2 + \sin^2\theta d\varphi^2), \quad (1)$$

$$f(r) = 1 - \frac{2M}{r} + \frac{r^2}{l^2} + \frac{2b^2r^2}{3} \left(1 - \sqrt{1 + \frac{Q^2}{b^2r^4}} \right) + \frac{4Q^2}{3r^2} F\left(\frac{1}{4}, \frac{1}{2}; \frac{5}{4}; -\frac{Q^2}{b^2r^4}\right) \quad (2)$$

where M and Q are the ADM (Arnowitt-Deser-Misner) mass and the electric charge of the black hole, $b = (2\pi\alpha)^{-1}$ is the Born-Infeld parameter (where α is the string tension), $\Lambda = -3/l^2$ is the cosmological constant and $F(r)$ is a hypergeometric function. When $b \rightarrow \infty$, the metric (1) reduces to the Reissner-Nordström -AdS black hole.

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Solving the equation $f(r) = 0$, we get the largest positive root and denote it by r_+ , from which we can determine the event horizon radius. In terms of r_+ , the ADM mass, the Hawking temperature and the BHE of the black hole can be written as

$$M = \frac{r_+}{2} + \frac{r_+^3}{2l^2} + \frac{b^2 r_+^3}{3} \left(1 - \sqrt{1 + \frac{Q^2}{b^2 r_+^4}} \right) + \frac{2Q^2}{3r_+} F\left(\frac{1}{4}, \frac{1}{2}, \frac{5}{4}; -\frac{Q^2}{b^2 r_+^4}\right), \quad (3)$$

$$T = \frac{f'(r)}{4\pi} = \frac{1}{4\pi} \left[\frac{1}{r_+} + \frac{3r_+}{l^2} + 2b^2 r_+ \left(1 - \sqrt{1 + \frac{Q^2}{b^2 r_+^4}} \right) \right], \quad (4)$$

$$S = \frac{A}{4} = \pi r_+^2 \quad (5)$$

where $A = 4\pi r_+^2$ is the black hole horizon area.

To apply the Parikh-Wilczek method, we make the transformation as

$$dT = dt + \frac{1}{f(r)} \sqrt{\frac{r_+}{r}} dr. \quad (6)$$

Then the line element (1) becomes

$$ds^2 = -f(r)dT^2 + 2\sqrt{\frac{r_+}{r}}dTdr + \frac{r-r_+}{r} \frac{1}{f(r)} dr^2 + r^2 d\Omega^2. \quad (7)$$

Obviously, the line element (7) is well-behaved at the horizon position, which is very necessary to describe tunneling. The radial outgoing null geodesic is given by

$$\dot{r} = \frac{dr}{dT} = f(r) \left(1 + \sqrt{\frac{r_+}{r}} \right)^{-1}. \quad (8)$$

The total energy of a stationary space-time should be conserved during the emission. When particle's self-gravitation is taken into account and a particle of energy ω is emitted, the black hole mass will become $M - \omega$ and all above and following relevant equations should be used with $M \rightarrow M - \omega$. Since the metric is of spherical symmetry, so regarding the outgoing particle as an s-wave, i.e. a shell of energy is reasonable.

The imaginary part of the action for a particle crossing the horizon outwards from the initial radius r_i to the final radius r_f can be expressed as

$$\text{Im} Z = \text{Im} \int_{r_i}^{r_f} L dT = \text{Im} \int_{r_i}^{r_f} p_r dr = \text{Im} \int_{r_i}^{r_f} \int_0^{p_r} dp_r dr \quad (9)$$

where p_r is canonical momentum conjugate to r . Taking the Hamilton equation into account, we have

$$\dot{r} = \left. \frac{dH}{dp_r} \right|_r \quad (10)$$

where $(dH)_r = dM$. Changing the variable from the momentum to the energy and switching the order of integration yield

$$\text{Im} Z = \text{Im} \int_{M_i}^{M_f} \int_{r_i}^{r_f} \frac{dr}{\dot{r}} dM = \text{Im} \int_{M_i}^{M_f} \int_{r_i}^{r_f} \frac{1 + \sqrt{r_+/r}}{f(r)} dr dM \quad (11)$$

where $M_i = M$; $M_f = M - \omega$. It is easy to find that the integrand is singular at the point $r = r_+$. The integral can be evaluated by deforming the contour around the pole. Doing the r integral, we obtain

$$\text{Im} Z = -2\pi \int_{M_i}^{M_f} \frac{1}{f'(r_H)} dM. \quad (12)$$

From Eq. (3), we obtain

$$\frac{dM}{dr_+} = \frac{1}{2} + \frac{3r_+^2}{2l^2} + b^2 r_+^2 \left(1 - \sqrt{1 + \frac{Q^2}{b^2 r_+^4}} \right). \quad (13)$$

In the process to obtain (4) and (13), we have used the equality

$$F\left(\frac{1}{4}, \frac{1}{2}, \frac{5}{4}; x\right) - 4xF'\left(\frac{1}{4}, \frac{1}{2}, \frac{5}{4}; x\right) = \frac{1}{\sqrt{1-x}} \quad (14)$$

where $x = -\frac{Q^2}{b^2 r_+^4}$.

Substituting Eqs. (4) and (13) into Eq. (12), we have

$$\text{Im} Z = -\pi \int_{r_i}^{r_f} r_+ dr_+ = \frac{\pi(r_i^2 - r_f^2)}{2} = -\frac{\Delta S}{2} \quad (15)$$

where $\Delta S = S(M - \omega) - S(M)$ is the increment of the BHE of the black hole during the particle's tunneling process.

Upon WKB approximation, the relationship between the tunneling probability and the imaginary part of the action is described by $\Gamma \sim \exp(-2 \text{Im} Z)$ [37], so

$$\Gamma \sim \exp(\Delta S). \quad (16)$$

The emission spectrum obviously deviates from the pure thermal spectrum but consists with an underlying unitary theory.

If we expand ΔS in ω , the tunneling rate (16) can be rewritten as

$$\Gamma \sim \exp(\Delta S) = \exp\left(-\beta\omega + \sum_{n=2}^{\infty} a_n \omega^n\right) \quad (17)$$

where

$$a_n = \frac{1}{n!} \left. \frac{d^n(\Delta S)}{d\omega^n} \right|_{\omega=0} \quad (18)$$

and $\beta = a_1 = \frac{1}{T}$ is the inverse of the Hawking temperature. The first term in Eq. (17) gives the familiar thermal Boltzmann factor $\exp(-\beta\omega)$ for the emanating radiation, the others are the corrections resulting from the response of the background geometry to the emission of a quantum, which can easily be calculated to any desired order in $\omega \rightarrow 0$ and are indicative of a “greybody” factor in the emission spectrum. The existence of these correction terms means that it is probable that we can obtain other information from the spectrum in addition to the temperature, that is, the corrected spectrum is not purely thermal.

Further, it is easy to verify from Eqs. (3)-(5) that the first law of thermodynamics is satisfied, that is

$$dM = TdS + \phi dQ \quad (19)$$

where ϕ is the electrostatic potential and Q is the charge of the black hole. Here we only consider the emission of uncharged particles, the charge of the black hole does not change and $dQ = 0$, so Eq. (19) can be reduced to

$$dM = TdS. \quad (20)$$

Eq. (20) is an incorporation of the energy conservation law $dM = dQ_h$ (where Q_h is the heat quantity and no forces including the electric field force do work) and the second law of thermodynamics $dS = dQ_h / T$. The energy conservation is suitable for any process, but the equation $dS = dQ_h / T$ is only valid for a reversible process ($dS > dQ_h / T$ for an irreversible process). That is, the emission process has been treated as a reversible one in PW tunneling framework.

Acknowledgments

This research is supported by Guangdong Natural Science Foundation (Grant Nos. 2015A030313789 and 2016A030307051).

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