Canadian Geotechnical Journal
Revue canadienne de géotechnique

Work input analysis for soils with double porosity and application to the hydro-mechanical modeling of unsaturated expansive clays

Journal:	<i>Canadian Geotechnical Journal</i>
Manuscript ID	cgj-2015-0574.R1
Manuscript Type:	Article
Date Submitted by the Author:	29-Mar-2016
Complete List of Authors:	Li, Jian; State Key Laboratory of Geomechanics and Geotechnical Engineering, Institute of Rock and Soil mechanics, Chinese Academy of Sciences Yin, Zhen-Yu; Research Institute in Civil and Mechanical Engineering, GeM UMR CNRS 6183, Ecole centrale de Nantes Cui, Yu-Jun; ENPC, Laboratoire NAVIER Hicher, Pierre-Yves; Research Institute in Civil and Mechanical Engineering, GeM UMR CNRS 6183, Ecole Centrale de Nantes
Keyword:	Double porosity, unsaturated expansive clay, work-conjugate variables, constitutive relation, capillary hysteresis

SCHOLARONE™
Manuscripts

Work input analysis for soils with double porosity and application to the hydro-mechanical modeling of unsaturated expansive clays

Jian LI^{1,2}, Zhen-Yu YIN^{2,3,*}, Yujun CUI^{3,4}, Pierre-Yves HICHER²

Affiliation:

¹ School of Civil Engineering, Beijing Jiaotong University, Beijing 100044, P. R. China

² LUNAM University, Ecole Centrale de Nantes, UMR CNRS GeM, BP 92101, 44321 Nantes Cedex 3, France

³ Key Laboratory of Geotechnical and Underground Engineering of Ministry of Education, Tongji University, Shanghai 200092, P. R. China

⁴ Ecole des Ponts ParisTech, Laboratoire Navier/CERMES, 6 et 8 avenue Blaise Pascal, 77455 Marne La Vallée, France

* Corresponding author: Dr. Zhen-Yu YIN, Tel. +33 240371588, Fax +33 240372535, Email: zhenyu.yin@gmail.com

Title: Work input analysis for soils with double porosity and application to the hydro-mechanical modeling of unsaturated expansive clays

Authors: Jian LI; Zhen-Yu YIN; Yujun CUI; Pierre-Yves HICHER

Abstract:

A mechanical approach for unsaturated expansive soils considering double porosity has been developed based on the porous media theory. In this approach, the adsorbed and the capillary water, as well as the micro and macro pores, are two distinct phases. An inter-aggregate stress considered as the work-conjugate of the macrostructural strain increment has been defined. Both physico-chemical and capillary effects of the pore water have been introduced at the macroscopic level. Other work-conjugate variables relevant for the constitutive modeling of double-porosity unsaturated media have also been identified, consisting of the modified suction as conjugate of the increment of the macrostructural degree of saturation and the microstructural effective stress as conjugate of the microstructural volumetric strain increment. A hydro-mechanical model for unsaturated expansive clays taking into account the interaction between the micro and the macro structures in expansive clays can thus be built. Based on the bounding surface concept, an anisotropic loading-collapse yield surface has been introduced to reproduce the three-dimensional mechanical behavior. In order to analyze the model capabilities, two series of laboratory tests consisting of multiple wetting and drying cycle tests on Boom clay and triaxial tests on ZY expansive clay were simulated. The comparisons between numerical and experimental results show that the model can reproduce with reasonable accuracy the mechanical behavior and the water retention characteristic of unsaturated expansive clays.

Keywords:

double porosity, unsaturated expansive clay, work-conjugate variables, constitutive relation, capillary hysteresis, anisotropy

Introduction

Geomaterials such as fissured rocks (Barenblatt et al. 1960), natural and compacted clays (Sridharan et al. 1971; Delage et al. 1982), and bentonite pellet mixtures (Hoffmann et al. 2007) have a pore size distribution with at least two dominant porosities, i.e., a double porosity. The two scales of porosity correspond to the micropores (e.g., matrix pores, intra-aggregate pores or intra-pellet pores) and macropores (fissures, inter-aggregate pores or inter-pellet pores). For expansive clays, the microstructure evolution under different loading conditions and wetting/drying paths has been studied extensively by mercury intrusion porosimetry (e.g., Delage and Lefebvre 1984; Hoffmann et al. 2007; Romero et al. 2011). These results show that the macrostructural deformation is strongly affected by the mechanical loading for aggregated clays, whereas the corresponding change in micropores is smaller and mainly reversible. On the other hand, the micropores of an expansive clay can vary significantly during wetting or drying, accompanied by a reversible volumetric change of the aggregates. However, the macrostructural deformation is always irreversible upon wetting and drying cycles and depends on the initial dry density, the external applied stresses, the range of suction, etc. Thus, the different deformation characteristics of both the micro and macro structures lead to a complex hydro-mechanical behavior of unsaturated expansive clays, which in turn leads to real challenges for constitutive modeling which can comprehensive depict these behavior.

To describe this complex hydro-mechanical behavior, the effects of the microstructural deformations need to be taken into account. The selection of appropriate stress variables for double-porosity media is the primary task. The currently proposed stress variables entail some

deficiencies. For example, Khalili et al. (2005) suggested an effective stress expression for unsaturated porous media with a double porosity by considering the contribution of each fluid pressure within the pores and fissures. This effective stress was used to depict the total deformation of the porous media rather than the microstructural deformation. Borja and Koliji (2009) suggested a similar definition based on thermodynamic principles. On the other hand, based on the effective stress principle, Gens and Alonso (1992) and Mašín (2013) suggested a microstructural effective stress for modeling the deformation of the particle element, i.e., aggregate. Besides, based on the microstructural observations, Della Vecchia and Romero (2013) suggested an expression describing the aggregate size evolution with water content and void ratio. However, the selection of the stress variables in the latter two studies was not based on thermodynamic principles.

Elasto-plastic models with isotropic loading-collapse yield surfaces for expansive clays were suggested by Gens and Alonso (1992), Sheng et al. (2004), Chen (2007) and Sun and Sun (2012). Laboratory tests under compression with different stress ratios on Jossigny silty clay and Zaoyang (ZY) expansive clay by Cui and Delage (1996), Zhan and Ng (2006) and Chen (2007), respectively, showed that the yield surface should be anisotropic under a given suction value. The soil anisotropy can be inherent (caused by particle orientation during soil formation) or induced (caused by loading paths applied to the soil or other factors). It has been shown that incorporating the anisotropy in constitutive modeling can generally improve the predictive ability for saturated and unsaturated soils (e.g., Banerjee and Yousif 1986; Wheeler et al. 2003; Stropeit et al. 2008; D'Onza et al. 2011; Al-Sharrad and Gallipoli 2014). Therefore, it may also help to introduce the anisotropy into the constitutive modeling of

expansive clays.

In this study, we have extended the work input approach to unsaturated expansive clays based on the porous media theory for materials with double porosity. Work-conjugate stress variables and strain increment variables relevant for the constitutive modeling of double-porosity unsaturated media are then identified. A novel hydro-mechanical constitutive model for unsaturated expansive soils is then formulated. An anisotropic yield surface based on the bounding surface concept is introduced to simulate the three-dimensional mechanical behavior of this type of materials. Laboratory tests on unsaturated compacted expansive clays consisting of multiple wetting and drying cycle tests on Boom clay and triaxial tests on ZY expansive clay are then simulated to analyze the model capability of reproducing the hydro-mechanical behavior of unsaturated expansive clays.

Work input considering double porosity

Baker and Frydman (2009) suggested a double porosity conceptual model for unsaturated expansive clays. In their model, the expansive clay is composed of four phases, i.e., aggregates (ag), capillary water (c), adsorbed water (ad) and air (a). The total surface of the aggregates is covered by adsorbed water, the capillary water being concentrated in the inter-aggregate pores, and air occupying the central portion of the inter-aggregate pores. At the microscopic level, the aggregate phase is made of two compositions, i.e., a solid matrix (i.e., clay platelets) and adsorbed water. This double porosity conceptual model has been adopted in this study to calculate the work input.

The power input to a representative elementary volume (REV) with a volume V and a

boundary area A is the sum of the power input at the boundary and the power coming from gravitational forces:

$$(1) \quad \int_V W dV = - \int_A \left[n_M \sum_{\alpha} S_r^{\alpha} u_{\alpha} v_i^{\alpha} \delta_{ij} + (1 - n_M) s_{ij} v_i^{ag} \right] n_j dA \\ + \int_V \left[n_M \sum_{\alpha} S_r^{\alpha} \rho^{\alpha} v_i^{\alpha} + (1 - n_M) \rho^{ag} v_i^{ag} \right] g_i dV, \quad \alpha = c, ad, a$$

where the negative sign on the area integral arises from the convention of positive compression; u^{α} is the α phase pressure; s_{ij} is the average stress in the aggregate phase with the similar definition as average stress for soil grains was adopted by Houlsby (1997) and Zhao et al. (2015); n_M is the macroporosity defined as the ratio of the macropores over the total volume of REV; S_{α} is the macrostructural degree of saturation of the α phase defined as the ratio of the volume of the α phase over the volume of the macropores; ρ^{ag} and ρ^{α} are the densities of the aggregate and the α phases, respectively; n_i is the outward normal to the surface A ; g_i is the gravitational acceleration vector; v_i and f_i^{α} are the average velocities of the aggregate and the α phases, respectively; the seepage velocity of the α phase is defined by $w_i^{\alpha} = n_M S_{\alpha} (f_i^{\alpha} - v_i)$.

Based on the porous media theory and considering the above decomposition, the total stress in the soil can be obtained by (Houlsby 1997; Borja and Koliji 2009; Zhao et al. 2010):

$$(2) \quad \sigma_{ij} = \sum_{\alpha} \left(n_M S_r^{\alpha} u_{\alpha} \delta_{ij} \right) + (1 - n_M) s_{ij}$$

And the overall soil density is given by:

$$(3) \quad \rho = \sum_{\alpha} \left(n_M S_r^{\alpha} \rho^{\alpha} \right) + (1 - n_M) \rho^{ag}$$

Substituting Eqs. (2), (3) into Eq. (1) and applying the Gauss divergence theorem lead to:

$$(4) \quad W = - \sum_{\alpha} \left(u'_{\alpha,j} w_j^{\alpha} \right) - \sum_{\alpha} \left(u_{\alpha} w_{j,j}^{\alpha} \right) - \sigma_{ij} v_{i,j}$$

where $u'_{\alpha,j}$ is the excess pore pressure gradient in the α phase. Here, the integral signs on both sides of the equation are removed, and it is assumed that the total stress equilibrium is satisfied, i.e. $-\sigma_{ij,j} + \rho g_i = 0$.

The mass conservation equations of aggregate, capillary water, adsorbed water and air phases can be expressed as:

$$(5) \quad \begin{cases} \frac{\partial[(1-n_M)\rho^{ag}]}{\partial t} + \text{div}[(1-n_M)\rho^{ag}v_i] = c^{ag} \\ \frac{\partial(n_M S_r^c \rho^c)}{\partial t} + \text{div}(n_M S_r^c \rho^c f_i^c) = c^c \\ \frac{\partial(n_M S_r^{ad} \rho^{ad})}{\partial t} + \text{div}(n_M S_r^{ad} \rho^{ad} f_i^{ad}) = c^{ad} \\ \frac{\partial(n_M S_r^a \rho^a)}{\partial t} + \text{div}(n_M S_r^a \rho^a f_i^a) = 0 \end{cases}$$

where c^α is the mass exchange rate of the α phase with the other phases, and the value of c^α is positive when the α phase obtains additional mass from other phases. At the microscopic level, the aggregate phase is made of the solid matrix (i.e., clay platelets) and the adsorbed water. Therefore, c^{ad} can be decomposed into c_{ag}^{ad} and c_c^{ad} , which are the rates of mass exchange of the adsorbed water phase in macropores with the aggregate phase and the capillary water phase, respectively. The equation $c^c + c^{ad} + c^{ag} = 0$ needs to be satisfied.

Substituting the seepage velocity of the α phase into Eq. (5) and noting $v_{i,i} = -\dot{\epsilon}_v$ lead to:

$$(6) \quad \begin{cases} w_{i,i}^c = S_r^c \dot{\epsilon}_v - n_M \dot{S}_r^c - S_r^c (1-n_M) \frac{\dot{\rho}^{ag}}{\rho^{ag}} + S_r^c \frac{c^{ag}}{\rho^{ag}} + \frac{c^c}{\rho^c} \\ w_{i,i}^{ad} = S_r^{ad} \dot{\epsilon}_v - n_M \dot{S}_r^{ad} - S_r^{ad} (1-n_M) \frac{\dot{\rho}^{ag}}{\rho^{ag}} + S_r^{ad} \frac{c^{ag}}{\rho^{ag}} + \frac{c^{ad}}{\rho^{ad}} \\ w_{i,i}^a = S_r^a \dot{\epsilon}_v - n_M \dot{S}_r^a - S_r^a (1-n_M) \frac{\dot{\rho}^{ag}}{\rho^{ag}} + S_r^a \frac{c^{ag}}{\rho^{ag}} - n_M S^a \frac{\dot{\rho}^a}{\rho^a} \end{cases}$$

where $\dot{\epsilon}_v$ is the total volumetric strain increment. Here, it is assumed that the spatial variations of n_M and S_α are negligible (Houlsby 1997).

Based on the first equation of Eq. (5), the density change rate of the aggregate phase can be expressed as:

$$(7) \quad \dot{\rho}^{ag} = \rho^{ag} \frac{\dot{n}_M}{1-n_M} + \frac{c^{ag}}{1-n_M} + \rho^{ag} \dot{\epsilon}_v$$

Because the macroporosity is defined by $n_M = e_M / (1+e)$, its increment can be obtained by:

$$(8) \quad \dot{n}_M = \frac{\dot{e}_M}{1+e} - n_M \frac{\dot{e}}{1+e} = -\dot{\epsilon}_v^M + n_M \dot{\epsilon}_v$$

where $\dot{\epsilon}_v$ is the sum of the macro and micro structural volumetric strain rates (corresponding to inter-aggregate macropores and intra-aggregate micropores), i.e., $\dot{\epsilon}_v = \dot{\epsilon}_v^M + \dot{\epsilon}_v^m$; $\dot{\epsilon}_v^M$ is defined by $\dot{\epsilon}_v^M = -\dot{e}_M / (1+e)$ and $\dot{\epsilon}_v^m$ is defined by $\dot{\epsilon}_v^m = -\dot{e}_m / (1+e)$; the macro void ratio e_M is defined as the ratio of the volume of the macropores over solid volume; the micro void ratio e_m is defined as the ratio of the volume of the micropores over the solid volume; the total void ratio e is the sum of e_M and e_m . Alternative definitions of macro void ratio and strain rates were adopted by Mašin (2013). Using these definitions, the expression of work input is deduced in Appendix A. Substituting Eq. (8) into Eq. (7) leads to:

$$(9) \quad \dot{\rho}^{ag} = \frac{\rho^{ag} \dot{\epsilon}_v^m + c^{ag}}{1-n_M}$$

Furthermore, the mass exchange between the aggregate and the adsorbed water phases leads to a microstructural volume change. Thus, the micro volumetric strain rate can be expressed as:

$$(10) \quad \dot{\epsilon}_v^m = -\frac{\dot{V}^{ag}}{V} = \frac{(c_{ag}^{ad} / \rho^{ad}) V}{V} = c_{ag}^{ad} / \rho^{ad}$$

where V^{ag} is the volume of the aggregates in the REV.

Substituting Eqs. (6), (9) and (10) into Eq. (4) gives the work input to unsaturated expansive soils considering double porosity as:

$$\begin{aligned}
 (11) \quad W = & -u'_{c,j} w_j^c - u'_{ad,j} w_j^{ad} - u'_{a,j} w_j^a + (\sigma_{ij} - u_c S_r^c \delta_{ij} - u_{ad} S_r^{ad} \delta_{ij} - u_a S_r^a \delta_{ij}) \dot{\varepsilon}_{ij}^{M'} \\
 & - n_M (u_a - u_c) \dot{S}_r^c - n_M (u_a - u_{ad}) \dot{S}_r^{ad} - \left(u_c - u_{ad} \frac{\rho^c}{\rho^{ad}} \right) \frac{\dot{c}^c}{\rho^c} \\
 & + u_a n_M S^a \frac{\dot{\rho}^a}{\rho^a} + (p - u_{ad}) \dot{\varepsilon}_v^{m'}
 \end{aligned}$$

where $\varepsilon_{ij}^{M'}$ is the macrostructural strain tensor; p is the total mean stress. The first three terms on the left side of the equation represents the work caused by the seepage of the pore fluid relative to the aggregate phase; the fourth term represents the work caused by the deformation of the macrostructure; the fifth and sixth terms represent the works caused by the change in the degree of saturation of the capillary water and adsorbed water phases in macropores, respectively; the seventh term represents the work caused by the mass exchange of the capillary water phase with the adsorbed water phase; the eighth term represents the work caused by the compression of the air phase; the ninth term represents the work caused by the deformation of the microstructure.

The interpretation of some key terms in Eq. (11) is as follows. Firstly, the fourth term in Eq. (11) implies that the stress variable work-conjugate of the macrostructural strain increment is $\sigma_{ij}^* = (\sigma_{ij} - u^a \delta_{ij}) + S_r^c (u^a - u^c) \delta_{ij} + S_r^{ad} (u^a - u^{ad}) \delta_{ij}$, which is termed as the inter-aggregate stress. Different from Bishop's stress, the additional term $S_r^{ad} (u^a - u^{ad}) \delta_{ij}$ in the inter-aggregate stress considers the physico-chemical effect between the absorbed water and the solid matrix. Note that a similar inter-particle stress variable considering the physico-chemical effect was proposed by Lu and Likos (2006) based on particle-scale

equilibrium analyses. Secondly, the fifth and sixth terms imply that the stress variables work-conjugate of $\dot{S}_r^c$ and $\dot{S}_r^{ad}$ are $n_M(u_a - u_c)$ and $n_M(u_a - u_{ad})$, respectively. These stress variables are similar to the modified suction proposed by Houlsby (1997). However, the water content change related to the adsorbed water phase has been separated from the capillary water phase in the macropores. Thirdly, the ninth term implies that the stress variable work-conjugate of the microstructural volumetric deformation is $(p - u_{ad})$, which implies the effect of the external applied stresses and adsorbed water pressure on the size of the aggregates.

Constitutive modeling

Choice of stress and strain variables

Selecting stress and strain variables is the first step for establishing a constitutive model. On the basis of thermodynamics theory, the stress variables can be chosen from the expression of the input work. Based on the proposed input work expression for unsaturated expansive soils considering double porosity, several groups of work-conjugate variables have been chosen to develop an elasto-plastic constitutive model. In the derivation of the work input considering double porosity, the adsorbed water has been separated from the capillary water. Thus, it is necessary to obtain the adsorbed water pressure and the adsorbed water content within the soil. On the one hand, the pulsed NMR technique has been increasingly adopted to evaluate the content of adsorbed water and capillary water in geomaterials (Tian and Wei 2014). To our knowledge, however, the adsorbed water pressure cannot be measured directly, limiting the application of Eq. (11). Therefore, it has been assumed that the adsorbed water in macropores

is part of the aggregates (Baker and Frydman 2009). Then, Eq. (11) can be simplified to:

$$(12) \quad \begin{aligned} W = & -u'_{c,i} w_i^c - u'_{a,i} w_i^a - u'_{ad,i} w_i^{ad} + (\sigma'_{ij} + s S_r^M \delta_{ij}) \dot{\epsilon}_{ij}^M \\ & - n_M s \dot{S}_r^M + u_a n_M S_r^a \frac{\dot{\rho}^a}{\rho^a} + \left[p' + \frac{\rho^{ad}}{\rho^c} s + u_a \left(1 - \frac{\rho^{ad}}{\rho^c} \right) \right] \dot{\epsilon}_v^m \end{aligned}$$

where σ_{ij}^* is the modified inter-aggregate stress defined by $\sigma_{ij}^* = \sigma'_{ij} + s S_r^M \delta_{ij}$; s^* is the modified suction defined by $s^* = n_M s$; s is the matric suction defined by $s = u^a - u^c$; S_r^M is the macrostructural degree of saturation (water volume in macropore over the macropore volume) which is equal to the “effective degree of saturation” S_r^e by Alonso et al. (2010). The definition of σ_{ij}^* is similar to Bishop’s stress, and the effective stress parameter is S_r^M . Alonso et al. (2010) stated that the effective stress parameter of Bishop’s stress should be chosen as the effective degree of saturation S_r^e , rather than the total degree of saturation for unsaturated clays. Furthermore, the adsorbed water in macropores has been assumed as part of the aggregates; thus, the physico-chemical effect in the inter-aggregate stress has been omitted.

According to the definition of the matric suction (Derjaguin et al. 1987; Tuller et al. 1999), the suction defined by $s = u^a - u^c$ in Eq. (12) is only the capillary part of the matric suction. Its adsorptive part has been ignored, which limits the range of application of Eq. (12). Therefore, this equation is appropriate when the value of the pore water potential is greater than that corresponding to the cavitation of the capillary water. At equilibrium state, the potential of capillary water in the macropores is equal to the potential of adsorbed water in the micropores. In this pore water potential range, the macropores are unsaturated and filled by capillary water and air, and the capillary potential is always equal to $-(u^a - u^c)$. The

cavitation tension of fined-grained soils is usually higher than 100 kPa. For example, the cavitation tension of Barcelona silt is equal to 1-2 MPa (Gens et al. 1995). When the system is subjected to drying by exposing it to an external water potential equaled to or lower than that corresponding to the cavitation of the capillary water, cavitation of the capillary water in the macropores occurs. The above mechanism was broadly discussed by Romero et al. (1999) and Baker and Frydman (2009).

Under the triaxial condition, the stress variables and work-conjugate strain increments are chosen as:

$$(13) \quad [p^*, q, s^*, \hat{p}]^T, [d\varepsilon_v^M, d\varepsilon_s^M, dS_r^M, d\varepsilon_v^m]^T$$

where p^* is the mean inter-aggregate stress defined by $p^* = p' + sS_r^M$; p' is the net mean stress; q is the deviator stress; $\hat{p}$ is the microstructural effective stress defined by $\hat{p} = p' + \chi s$ with $\chi = \rho^{ad} / \rho^c \approx 1.3$; $d\varepsilon_s^M$ is the shear strain increment. The stress variable work-conjugate to the microstructural volumetric deformation has been modified. Because the proposed model is an incremental generalized stress-strain relationship, the constant part of the microstructural effective stress omitted. The simplified definition of $\hat{p}$ is similar to that proposed by Gens and Alonso (1992) except for the constant coefficient in the second term. Based on an extensive experimental database of different bentonites available in published literature, Mašin and Khalili (2015) pointed out that both of loading and change in pore water pressure affect the mechanical behaviour of aggregates. The simplified definition of $\hat{p}$ is consistent with the above conclusion.

Application of the bounding surface concept

In order to account for the deformation and water retention behavior under triaxial compression, a hydro-mechanical model for unsaturated expansive clay is being suggested. Three groups of bounding surfaces have been built in order to represent the mechanical and water retention behavior. The first is the loading-collapse (LC) bounding surface describing the irreversible contraction deformation caused by the increase of the inter-aggregate stress and the decrease of the suction. The second is the microstructural stress increase and decrease (MI and MD) bounding surfaces suggested to describe the macrostructural deformation caused by the microstructural volumetric strain as first proposed by Gens and Alonso (1992). The third is the suction increase and decrease (SI and SD) bounding surfaces controlled by the variation of the macrostructural degree of saturation.

LC bounding surface

Laboratory tests under triaxial compression with different stress ratios were carried out by Cui and Delage (1996), Zhan and Ng (2006) and Chen (2007) on various types of unsaturated soils, i.e., Jossigny silty clay and Zaoyang (ZY) expansive clay respectively, at given suction values. As shown in Figure 1, the initial LC yield surfaces in the $(p':q)$ plane at different suctions can be plotted as a group of rotating ellipses under triaxial condition. Thus, using an expression modified from the one proposed by Dafalias (1982) and Wheeler et al. (2003), the formulation of the LC yield curve is expressed as (see Figure 2):

$$(14) \quad f_{LC} = [\bar{q} - \alpha(\bar{p}^* + p_s^*)]^2 - (M^2 - \alpha^2)(\bar{p}_m^* - \bar{p}^*)(\bar{p}^* + p_s^*) = 0$$

where the stress point $(\bar{p}^*, \bar{q})$ lies on the LC bounding surface and is defined according to a radial mapping rule so that it has the same stress ratio as the current stress point, defined by

$\eta^* = \bar{q}/(\bar{p}^* + p_s^*) = q/(p^* + p_s^*)$; p_s^* represents the tensile strength with $p_s^* = \mu/M$; μ is the intercept of the critical state line in the $(p^* : q)$ plane; M is the slope of the critical state line in the $(p^* : q)$ plane; $\bar{p}_m^*$ and α are the hardening parameters defining the size and the inclination of the LC bounding surface in the $(p^* : q)$ plane.

The maximum inter-aggregate stresses on the LC bounding surface at a given suction, $\bar{p}_m^*(s)$, can be determined by the expression of the normal consolidation line at different suction values (Wheeler and Sivakumar, 1995):

$$(15) \quad \bar{p}_m^*(s) = \bar{p}_m^*(0)^{\frac{\lambda(0)-\kappa}{\lambda(s)-\kappa}} \exp \left[\frac{N(s) - N(0)}{\lambda(s) - \kappa} \right]$$

where $\lambda(s)$ and $N(s)$ are the slope and intercept of the normal consolidation line in the $(p^* : q)$ plane.

MI and MD bounding surface

The expression of the MI and MD bounding surfaces are (see Figure 3) (Gens and Alonso 1992):

$$(16) \quad \begin{cases} f_{MI} = \hat{p} - \hat{p}_I = 0 & \hat{p} > 0 \\ f_{MD} = \hat{p}_D - \hat{p} = 0 & \hat{p} < 0 \end{cases}$$

where $\hat{p}_I$ and $\hat{p}_D$ are the hardening parameters controlling the position of the MI and MD bounding surfaces in the $(p' : s)$ plane, respectively. According to the definition of the microstructural effective stress, the MI and MD bounding surfaces are inclined in the $(p' : s)$ plane. The slopes of the yield lines are related to the density ratio between adsorbed and capillary water, different from the one proposed by Gens and Alonso (1992).

SI and SD bounding surfaces

The water retention behavior in unsaturated soils could not be satisfactorily modeled within classical elasto-plasticity due to the non-existence of a pure elastic domain in the zone delimited by the main wetting and drying curves of SWRC. Several authors have modeled the water retention behavior using bounding surface plasticity (Li 2005; Liu and Muraleetharan 2006; Raveendraraj 2009). In the suggested model, the bounding surface concept has been applied to SWRC. The SI and SD bounding surfaces are expressed as (see Figure 4):

$$(17) \quad \begin{cases} f_{SI} = s^* - s_I^* = 0 & s^* > 0 \\ f_{SD} = s_D^* - s^* = 0 & s^* < 0 \end{cases}$$

where s_I^* and s_D^* are the hardening parameters controlling the position of the SI and SD bounding surfaces, respectively, in the $(p'; s^*)$ plane.

Stress-strain relationship

It is assumed that the total volumetric deformation of an expansive clay is composed of the macrostructural and aggregate size changes, the shear strain of the aggregates being ignored. In addition, it is assumed that the micropores are saturated with adsorbed water; thus the value of the total degree of saturation can be obtained from the macrostructural degree of saturation.

The generalized total strains can then be defined by:

$$(18) \quad \begin{cases} \varepsilon_v = \varepsilon_v^M + \varepsilon_v^m \\ \varepsilon_s = \varepsilon_s^M \\ S_r = (e_M / e) S_r^M + (e_m / e) \end{cases}$$

where ε_s is the total shear strain; S_r is the total degree of saturation. According to the method proposed by Alonso and Gens (1992), the plastic macrostructural volumetric strain is decomposed into two parts: The first one represents the contracted or collapse deformation caused by the loading or wetting; the second one represents the macrostructural deformation

caused by the coupling effect of aggregates swelling or shrinkage. Then, the strain increments can be expressed as:

$$(19) \quad \begin{cases} d\varepsilon_v^M = d\varepsilon_v^{Me} + d\varepsilon_{v(LC)}^{Mp} + d\varepsilon_{v(MI/MD)}^{Mp} \\ d\varepsilon_s^M = d\varepsilon_s^{Me} + d\varepsilon_{s(LC)}^{Mp} + d\varepsilon_{s(MI/MD)}^{Mp} \\ -dS_r^M = -dS_r^{Me} - dS_{r(SI/SD)}^{Mp} - dS_{r(LC)}^{Mp} - dS_{r(MI/MD)}^{Mp} \end{cases}$$

where $d\varepsilon_v^{Me}$, $d\varepsilon_{v(LC)}^{Mp}$ and $d\varepsilon_{v(MI/MD)}^{Mp}$ are the elastic macrostructural volumetric strain increment, plastic macrostructural volumetric strain increments due to the stress point approaching the LC and the MI/ MD bounding surfaces, respectively; $d\varepsilon_s^{Me}$, $d\varepsilon_{s(LC)}^{Mp}$ and $d\varepsilon_{s(MI/MD)}^{Mp}$ are the elastic macrostructural shear strain increment, plastic macrostructural shear strain increments due to the stress point approaching the LC and the MI/ MD bounding surfaces, respectively; dS_r^{Me} , $dS_{r(SI/SD)}^{Mp}$, $dS_{r(LC)}^{Mp}$ and $dS_{r(MI/MD)}^{Mp}$ are the elastic increment of the macrostructural degree of saturation and the plastic increments of the macrostructural degree of saturation due to the stress point approaching SI/SD, LC and MI/MB bounding surfaces, respectively.

Elastic behavior

The elastic strain increments are given by:

$$(20) \quad \begin{cases} d\varepsilon_v^{Me} = \frac{\kappa}{\nu} \frac{dp^*}{p^*} \\ d\varepsilon_s^{Me} = \frac{2\kappa(1+\nu)}{9\nu p^*(1-2\nu)} dq \\ -dS_r^{Me} = \frac{\kappa_w}{s^* + p_{ref}^w} ds^* \\ d\varepsilon_v^m = \frac{\kappa_m}{\nu} \frac{d\hat{p}}{\hat{p} + p_{ref}^m} \end{cases}$$

where κ is the slope of the elastic swelling line in the $(\nu : \ln p^*)$ plane; ν is the total

specific volume; ν is the Poisson's ratio; κ_w is an additional elastic constant; κ_m is the microscopic elastic constant; p_{ref}^w and p_{ref}^m are reference stresses.

Plastic behavior related to LC bounding surface

The first part of the plastic volumetric strain increment caused by the stress point approaching the LC bounding surface is written as:

$$(21) \quad d\varepsilon_{v(LC)}^{Mp} = R_{LC} \Lambda_{LC} \frac{\partial f_{LC}}{\partial p^*}$$

where Λ_{LC} is the non-negative loading index of the LC bounding surface; R_{LC} is a scaling function controlling the hardening rate of the LC loading surface. The corresponding plastic shear strain increment is then defined as:

$$(22) \quad d\varepsilon_{s(LC)}^{Mp} = R_{LC} \Lambda_{LC} \frac{\partial f_{LC}}{\partial q}$$

The plastic change of the degree of saturation caused by the stress point approaching the LC bounding surface is given by:

$$(23) \quad dS_{r(LC)}^{Mp} = k_{ws} R_{LC} \Lambda_{LC} \frac{\partial f_{LC}}{\partial p^*}$$

where k_{ws} is the coupling parameter controlling the magnitude of the change of the degree of saturation due to the macrostructural deformation.

According to Yao et al. (2012) for overconsolidated soil, the scaling function R_{LC} can be defined as:

$$(24) \quad R_{LC} = R^{\gamma_{LC}(M_f - \eta^*)} \frac{M^{\gamma_{LC}} - \eta^{*\gamma_{LC}}}{M_f^{\gamma_{LC}} - \eta^{*\gamma_{LC}}}$$

where R represents the size ratio between the current LC stress surface and the LC bounding surface with $R = p_m^*(0)/\bar{p}_m^*(0)$; γ_{LC} is a soil parameter; M_f is the potential failure stress ratio which is equal to the ratio between the current mean unsaturated effective stress p^* and the corresponding deviator stress on the Hvorslev envelope q_f , as show in Figure 5, expressed as:

$$(25) \quad M_f = \frac{q_f}{p^*} = 6 \left[\sqrt{\frac{k}{R} \left(1 + \frac{k}{R} \right)} - \frac{k}{R} \right]$$

where $k = M^2/[12(3-M)]$.

A hardening law has been defined to describe the evolution of the bounding surface. The size change of the LC bounding surface is assumed to be related to the plastic volumetric strains, $d\varepsilon_{v(LC)}^{Mp}$ and $d\varepsilon_{v(MI/MD)}^{Mp}$:

$$(26) \quad d\bar{p}_m^*(0) = \bar{p}_m^*(0) \frac{v}{\lambda(0) - \kappa} (d\varepsilon_{v(LC)}^{Mp} + d\varepsilon_{v(MI/MD)}^{Mp})$$

A second hardening law is then introduced, describing the change of inclination of the LC bounding surface due to plastic strains, representing the change of anisotropy with plastic strains. Wheeler et al. (2003) suggested a rotational hardening law, assuming that the plastic volumetric strain attempts to drag the value of α towards an instantaneous target value χ_v that is dependent on the current value of the stress ratio. Simultaneously, the plastic shear strain is assumed to drag α towards a different target value χ_d . This rotational hardening law has been adopted in this model, defined by:

$$(27) \quad d\alpha = \omega \left[(\chi_v - \alpha) (d\varepsilon_{v(LC)}^{Mp} + d\varepsilon_{v(MI/MD)}^{Mp}) \right] + \omega_d (\chi_d - \alpha) |d\varepsilon_s^{Mp}|$$

where ω is a soil parameter which controls the relative effectiveness of the plastic shear

strain and plastic volumetric strain in determining the overall current target value for α ; ω_d is another soil parameter which controls the rate at which α approaches its current target value.

According to Wheeler et al. (2003): $\chi_v(\eta^*)=3/4\eta^*$, and according to Wheeler (1997), Kobayashi et al. (2003) and Yin et al. (2015): $\chi_d(\eta^*)=0$. Then, considering the full saturation as a special case, the parameters ω and ω_d can be determined by the methods proposed by Wheeler et al. (2003), Leoni et al. (2008) and Yin et al. (2011), as follows:

$$(28) \quad \begin{cases} \omega_d = \frac{3(4M^2 - 4\eta_{K0}^2 - 3\eta_{K0})}{8(\eta_{K0}^2 - M^2 + 3\eta_{K0})} \\ \omega = \frac{1+e_0}{\lambda(s)-\kappa} \ln \frac{10-2\alpha_{K0}\beta}{M^2-2\alpha_{K0}\beta} \end{cases}$$

where $\eta_{K0} = (1+2K_0)/[3(1-K_0)]$ is the stress ratio at K_0 -consolidated state with the assumption that $p_s^* = 0$.

The initial inclination of the LC bounding surface is expressed as (Wheeler 2003; Yin et al. 2011):

$$(29) \quad \alpha_{K0} = \frac{\eta_{K0}^2 + 3\eta_{K0} - M^2}{3}$$

Plastic behavior related to MI and MD bounding surfaces

The plastic volumetric strain increment caused by the stress point approaching the MI or MD bounding surface is given by:

$$(30) \quad \begin{cases} d\varepsilon_{v(MI)}^{Mp} = R_{MI/MD} \Lambda_{MI} \frac{\partial g_{MI}}{\partial \hat{p}} d_I & \text{for } d\hat{p} < 0 \\ d\varepsilon_{v(MD)}^{Mp} = R_{MI/MD} \Lambda_{MD} \frac{\partial g_{MD}}{\partial \hat{p}} d_D & \text{for } d\hat{p} > 0 \end{cases}$$

where Λ_{MI} and Λ_{MD} are the non-negative loading indexes of the MI and MD bounding surfaces, respectively; $R_{MI/MD}$ is the scaling function for the MI/MD bounding surface; d_I and d_D are soil parameters which control the value of $d\varepsilon_{v(MI/MD)}^{Mp}$; g_{MI} and g_{MD} are plastic potential functions. Alonso et al. (1999) proposed that the soils parameters d_I and d_D are related to overconsolidation ratio. When the value of d_D is negative, the aggregates swell into the macropores and do not contribute significantly to the global deformation. The test results by Monroy et al. (2010) for compacted London clays showed that the decrease of macroporosity is small during wetting except for wetting from suction 40 kPa to 0 kPa. Furthermore, the test results by Alonso et al. (1995) for loosely compacted clays showed the decrease of global volume during wetting from suction 700 kPa to 10 kPa. To simplify the expression of soils parameters d_I and d_D , the above behaviors during wetting are attributed to the collapse of macropores.

A non-associated flow rule has been adopted for the MI/MD bounding surface and the derivation of g_{MI} and g_{MD} gives:

$$(31) \quad \frac{\partial g_{MI/MD}}{\partial p'} = 0, \quad \frac{\partial g_{MI}}{\partial s} = +1, \quad \frac{\partial g_{MD}}{\partial s} = -1$$

Furthermore, the corresponding plastic shear strain increment was assumed to be zero when the stress point approaches the MI or MD bounding surface.

The plastic change of the degree of saturation caused by the stress point approaching the MI or MD bounding surface is given by:

$$(32) \quad dS_{r(MI/MD)}^{Mp} = k_{ws} d\varepsilon_{v(MI/MD)}^{Mp}$$

The scaling function $R_{MI/MD}$ is defined by:

$$(33) \quad R_{MI/MD} = \begin{cases} (\hat{p}/\hat{p}_I)^{\gamma_{MID}} & \text{for } d\hat{p} > 0 \\ (\hat{p}_D/\hat{p})^{\gamma_{MID}} & \text{for } d\hat{p} < 0 \end{cases}$$

where γ_{MID} is a soil parameter.

For the MI and MD bounding surfaces, the hardening parameters are $\hat{p}_I$ and $\hat{p}_D$, respectively. The change of position of the MI and MD bounding surfaces is given by:

$$(34) \quad \begin{cases} \frac{d\hat{p}_I}{\hat{p}_I + p_{ref}^m} = \frac{\nu}{\kappa_m} \left(\Lambda_{MI} \frac{\partial g_{MI}}{\partial \hat{p}} + k_{MID} \Lambda_{MD} \frac{\partial g_{MD}}{\partial \hat{p}} \right) & \text{for } d\hat{p} < 0 \\ \frac{d\hat{p}_D}{\hat{p}_D + p_{ref}^m} = \frac{\nu}{\kappa_m} \left(\Lambda_{MD} \frac{\partial g_{MD}}{\partial \hat{p}} + k_{MID} \Lambda_{MI} \frac{\partial g_{MI}}{\partial \hat{p}} \right) & \text{for } d\hat{p} > 0 \end{cases}$$

where the parameter k_{MID} controls the shifting of the MI (or MD) bounding surface due to the current stress point approaching the MD (or MI) bounding surface. In general, the range of values for the parameter k_{MID} is 0 to 1. When $k_{MID} = 0$, the hardening law becomes isotropic; when $k_{MID} = 1$, the hardening law becomes purely kinematic.

Plastic behavior related to SI and SD bounding surfaces

The plastic change of the degree of saturation due to the stress point approaching the SI or SD bounding surface is given by:

$$(35) \quad dS_{r(SI/SD)}^{Mp} = R_{SI/SD} \Lambda_{SI/SD} \frac{\partial f_{SI/SD}}{\partial s^*}$$

where $\Lambda_{SI/SD}$ is the loading index of the SI and SD bounding surfaces; $R_{SI/SD}$ is the scaling function for the MI/MD bounding surface defined by:

$$(36) \quad R_{SI/SD} = \begin{cases} (s^*/s_I)^{\gamma_{SID}} & \text{for } ds^* > 0 \\ (s_I/s^*)^{\gamma_{SID}} & \text{for } ds^* < 0 \end{cases}$$

where γ_{SID} is a soil parameter. For the SI and SD bounding surfaces, the hardening parameters are s_I^* and s_D^* , respectively.

The change of position of the SI and SD bounding surfaces is given by:

$$(37) \quad \frac{ds_I^*}{s_I^* + p_{ref}^w} = \frac{ds_D^*}{s_D^* + p_{ref}^w} = \frac{1}{\lambda_w - \kappa_w} \left(-dS_{r(SI/SD)}^{Mp} + dS_{r(LC)}^{Mp} + dS_{r(MI/MD)}^{Mp} \right)$$

where λ_w is the plastic constant for the liquid phase. To simplify the part of the model corresponding to the water retention behavior, the main drying and wetting lines are assumed to be linear in a semi-logarithmic plot. It is, however, possible to replace them by nonlinear forms (e.g., Brooks and Corey, 1964; Van Genuchten, 1980).

Model parameters

The parameters used in the suggested model can be classified into the following categories: normal consolidation line, critical state line, elastic constants, plastic constants, plastic potential functions, and hardening laws. The tests required for determining these parameters include isotropic consolidation compression tests at different suctions, triaxial compression tests at different effective confining stresses for saturated soils, suction-controlled oedometer tests under constant volume and suction-controlled cyclic tests at low and high net mean stresses. The parameters can be determined as follows:

- (1) The slope $\lambda(s)$ and the intercept $N(s)$ of the normal compression lines are obtained from isotropic compression tests at different suctions. The expression of the parameter $\lambda(s)$ proposed by Alonso et al. (1990) is used, as follow:

$$(38) \quad \lambda(s) = \lambda(0)[(1 - r_\lambda) \exp(-\beta_\lambda s) + r_\lambda]$$

The expression of the parameter $N(s)$ is assumed to be similar to the expression of $\lambda(s)$ and it is expressed as:

$$(39) \quad N(s) = N(0)[(1 - r_N)\exp(-\beta_N s) + r_N]$$

where r_λ , β_λ , r_N and β_N are soil parameters determined by curve fitting from the normal compression lines at different suctions.

- (2) The slope M and the intercept μ of the critical state line in the $(q : p^*)$ plane are obtained from shear tests. For simplicity, M and μ are assumed to be constant, independent of the suction. This implies that the critical state line is unique for different suctions, from zero to several hundred kPa in the $(p^* : q)$ plane, as shown by Sivakumar (1993), Wheeler and Sivakumar (2000) on compacted Speswhite kaolin and by Barrera (2002) on compacted Barcelona clayey silt. Then, the stress results at or nearby the critical state of at least three triaxial compression tests at different net confining stresses and saturated state are plotted in the $(q : p^*)$ plane to obtain the slope M and the intercept μ of the critical state line.
- (3) The elastic constant κ can be determined from the unloading and reloading parts of the normal consolidation line in the $(\nu : \ln p^*)$ plane obtained by an isotropic compression test on saturated soil specimen. The water elastic constant κ_w is the slope of the scanning line in the $(S_r^M : \ln(s^* + p_{ref}^w))$ plane. The microstructural elastic constant κ_m and the parameter p_{ref}^m can be determined by mercury intrusion porosimetry tests during wetting and drying paths. Furthermore, κ_m can also be determined from wetting and drying tests for highly over-consolidated soils that have negligible macropores (Cui et al. 2002).

- (4) The water plastic constant λ_w is the slope of the main wetting and drying lines in the $(S_r^M : \ln(s^* + p_{ref}^w))$ plane under constant volume condition. It should be determined by a suction-controlled oedometer test under constant volume condition. Besides, the coupling parameter k_{ws} controls the effect of the plastic macro-volumetric strain on the degree of saturation. Therefore, it can be calibrated by an isotropic consolidation compression test at a given suction. When the suction-controlled oedometer test under constant volume is not possible, λ_w and k_{ws} can be calibrated simultaneously by at least two suction-controlled tests at different constant confining stresses.
- (5) When the stress point is far away from the LC bounding surface, the part of plastic macrostructural volumetric strain increments due to the stress point approaching the LC bounding surfaces, i.e., $d\varepsilon_{v(LC)}^p$ can be ignored. Therefore, the plastic potential parameters, d_I and d_D , can be calibrated by a suction-controlled cyclic test at low net mean stress.
- (6) There are six hardening parameters, i.e., $\bar{p}_m^*(0)$, $\hat{p}_I$, $\hat{p}_D$, s_I^* , s_D^* , k_{MID} and three scaling parameters, i.e., γ_{LC} , γ_{SID} , γ_{MID} in this model. The initial value of $\bar{p}_m^*(0)$ can be determined by an isotropic compression test at $s=0$; the initial values of $\hat{p}_I$, $\hat{p}_D$, s_I^* and s_D^* can be determined by suction-controlled tests. Besides, the hardening parameter k_{MID} and the scaling parameters γ_{LC} , γ_{SID} and γ_{MID} can be determined simultaneously by these testes, which values also affect the initial values of these hardening parameters.

Experimental validation

Brief introduction of the tests used in the simulation

Two series of tests on expansive clays were selected for simulations. The first series consists of multiple wetting-drying cycles on Boom clay by Alonso et al. (1995). The second series consists of triaxial tests on ZY expansive clay by Zhan and Ng (2006), Chen (2007) and Chen and Ng (2013).

Multiple wetting-drying cycle tests on Boom clay

Boom clay has a liquid limit of 55.9%, a plasticity index of 29.2%. Kaolinite, illite and montmorillonite contents are 20%, 30% and 10%, respectively. Oedometer tests were carried out on samples with an initial dry density of 14 kN/m^3 . The soil specimens were first loaded at constant water content, and then an initial suction of 500 or 700 kPa was applied. After equilibrium at this initial suction, multiple wetting-drying cycles were applied with suction varying from 500 or 700 kPa to 10 kPa. Tests with wetting-drying cycles under constant net mean stresses of 20, 100 and 400 kPa were simulated.

Triaxial tests on ZY expansive clay

The ZY expansive clay has a liquid limit of 50%, a plastic index of 31%, a specific gravity (G_s) of 2.67, and a clay content of 39%. Based on the USCS classification, the soil is a medium-high plastic clay. The value of the one-dimensional free swelling deformation is approximately 16% for the compacted specimen. The soil can be classified as a medium expansive soil in agreement with the criterion proposed by Sridharan and Prakash (2000). The selected tests with their experimental procedures are summarized below:

- (1) Three wetting and drying cycle tests under different constant net mean stresses (i.e., 40,

70 and 100 kPa) were selected for the simulations. The first specimen was consolidated up to a net mean stress of 40 kPa, and then subjected to a wetting and drying cycle. The suction was changed from 540 kPa to 10 kPa and then to 400 kPa. The other two specimens were firstly wetted to a suction of 200 kPa from the initial state, followed by a consolidation stage under different net mean stresses (70 and 100 kPa), and then subjected to a wetting and drying cycle during which the suction decreased from 200 kPa to 10 kPa and then increased to 350 kPa.

- (2) Fourteen fully drained shearing tests under different constant net confining stresses and suctions were selected for the simulations. Firstly, the specimens were wetted to different suctions (200, 100, 50 and 25 kPa) under a net mean stress of 20 kPa. Then, they were consolidated under net mean stresses of 50, 200 and 350 kPa at different suctions. Unlike other tests, two soil specimens wetted to zero suction were consolidated under 50 and 350 kPa. Finally, all soil specimens were sheared under drained condition at constant net confining stress and constant suction.
- (3) Six suction-controlled drained triaxial tests at different constant net mean stresses were selected for the simulations. These soil specimens were wetted at different suctions (200, 100 and 0 kPa) under a net mean stress of 20 kPa. Then, three of them were sheared to a deviator stress of 11 kPa; the other three were sheared to a deviator stress of 20 kPa under a constant net mean stress of 20 kPa. Afterwards, these specimens were compressed to a net mean stress of 350 kPa under different constant net stress ratios ($\eta = 0.55, 1.0$). Finally, they were sheared under drained condition at a constant net mean stress of 350 kPa.

Test simulations

Multiple wetting-drying cycle tests on Boom clay

According to section 2.4, all the parameters used in the subsequent simulations have been determined based on isotropic consolidation compression tests at different suction levels and on suction-controlled cyclic tests. In order to determine the model parameters of microstructure and SWRC, the results on Boom clay obtained by Romero et al. (1999) were used. The values of model parameter are summarized in Table 1. The values of the initial state parameters are given in Table 2.

Figure 6 shows the variations of the specific volume during wetting and drying cycles. A comparison between measured and calculated results shows that the suggested model can quantitatively describe the behavior of Boom clay under multiple wetting and drying cycles. An accumulated plastic contraction grows with the increase of the net confining stress. Furthermore, an equilibrium state of deformation is almost reached after the applied wetting-drying cycles. The predicted deformations at 20 and 400 kPa net confining stresses during the wetting and drying cycles are close to the test results; however, the predicted deformation at 100 kPa net confining stress is smaller than the experimental one when a unique group of parameters for different net confining stresses is used. Note that the coupling parameters d_I and d_D were assumed constant and independent of the degree of saturation and of the overconsolidation ratio in the model. Nonetheless, the nonlinear behavior upon wetting and drying cycles could be reproduced, thanks to the bounding surface concept applied to LC and MI/MD yield surfaces.

Triaxial tests on ZY expansive clay

According to section 2.4, all the parameters used in the subsequent simulations have been determined based on isotropic consolidation compression tests at different suction levels, on triaxial compression tests at different effective confining stresses for saturated soils and on suction-controlled cyclic tests. The values of model parameter are summarized in Table 3. The values of the initial state parameters are given in Table 4. Since the mercury intrusion porosimetry test results are not available, the initial micro porosity and the elastic constants for the microstructure used in the simulation cannot be directly obtained. Gens et al. (2011) suggested that the initial ratio of micro voids to macro voids is linked to the initial dry density for a given compacted clay. To determine the initial ratio for ZY expansive clay, the mercury intrusion porosimetry test results on a similar type of clay by Romero et al. (1999) and Nowamooz and Masrouri (2010) have been adopted. By means of simple interpolation, the initial ratio of micro porosity to total porosity for ZY expansive clay with initial dry densities of 1.56 Mg/m^3 has been estimated equal to 0.4. Besides, according to the results of one-dimensional free swelling, a value of $\kappa_m = 0.008$ has been chosen.

(1) Wetting and drying cycle under constant net mean stress

Figure 7 shows the variations of the specific volume and the degree of saturation for wetting and drying cycle tests. The comparison between measured and calculated results shows that the model can describe reasonably well the behavior of ZY expansive clay upon the wetting and drying cycle. Both the swelling deformation along a wetting path and the accumulated plastic swelling deformation after a wetting-drying cycle increase with the

decrease of the net confining stress. Furthermore, the hydraulic hysteresis is well reproduced by the model. The degree of saturation on the drying path is larger than the one on the wetting path at a given suction. In addition to the hydraulic hysteresis, the SWRC of a given soil also depends on the net confining stress and on the specific volume. The degree of saturation increases with the net confining stress. Besides, the bounding surface approach can reproduce the smooth transition between the scanning line and the bounding surfaces of SWRC.

(2) Drained shearing tests under different constant net confining stresses

Figures 8 to 10 show the variations of the deviator stress, the specific volume and the degree of saturation during the triaxial tests. The comparison between the measured and calculated results shows that the model can quantitatively describe the shearing behavior of unsaturated expansive soil. The deviator stresses reach an almost steady state, i.e., the critical state, in the last phase of shearing, and the shear strength increases with the suction and the net confining stress. The predicted changes of the deviator stress with the matric suction is close the experimental results under net confining stresses of 200 and 350 kPa; however, the predicted changes of the deviator stress with the matric suction is smaller than the experimental one under a net confining stress of 50 kPa. This discrepancy at the latter condition is attributed to the assumption of a unique critical state line in the $(p^* : q)$ plane. Besides, the final value of the specific volume decreases with the suction increase under a net confining stress of 50 kPa. On the other hand, the final value of the specific volume grows with the increase of suction under net confining stresses of 200 and 350 kPa. Finally, the degree of saturation of each specimen increases with the axial strain.

Furthermore, to highlight the influence of the anisotropy, test simulations using the isotropic model (setting $\alpha_{K0} = \omega = 0$) were compared with the measured results in Figure 11. It can be seen that the calculated deviator stress for each specimen is larger when considering the anisotropy and more in agreement with the experimental data, especially at low net confining stresses. Moreover, the calculated specific volume of each specimen is larger when considering the anisotropy and, as a consequence, the predicted degree of saturation is also larger. The calculated specific volume considering anisotropy is closer to the experimental data at low matric suction. However, an opposite result is obtained at high matric suction. The evolution of the anisotropy with the matrix suction will have to be better analyzed in further studies.

(3) Drained shearing tests under constant net mean stress

Figures 12 and 13 show the variations of the deviator stress, the specific volume and the degree of saturation during anisotropic compressions at constant net stress ratios of 0.55 and 1.0, followed by shearing at constant net mean stress. A comparison between the measured and calculated results shows that the model can reproduce with reasonable accuracy the soil behavior during anisotropic compression and shearing. The deviator stress at high strain levels increases with the suction and the effect of the net stress ratio is not significant. As shown in Figure 13(a), the predicted deviator stress of 0 kPa matric suction is larger than the experimental data due to the fact that the specimen broke into pieces during shearing. Besides, the value of the specific volume increases with the suction. This phenomenon is consistent with the volume change during the drained shearing tests under constant net confining stresses of 200 and 350 kPa. Furthermore, the test simulations with the isotropic model

compared to the test results in Figures 14 demonstrate the improvement of the simulation by considering the anisotropy of the yield surface.

Conclusions

In this paper, the work input expression was deduced to unsaturated expansive clays considering double porosity, based on the porous media theory. Unlike the conventional assumptions used to derivate a work input expression for general unsaturated soils, the adsorbed water phase assumptions used to derivate a work input expression for general unsaturated soils from the capillary water phase, and the micropores were assumed to separate from the macropores. An inter-aggregate stress was then derived, which is the work-conjugate of the macrostructural strain increment, to account for both the physico-chemical and the capillary effects due to the coupling between the pore water and the other phases at the macroscopic level. Based on the input work expression, four groups of work-conjugate variables were selected to establish a constitutive model, which were (p^*, ε_v^M) , (q, ε_s^M) , (s^*, S_r^M) and $(\hat{p}, \varepsilon_v^m)$. The former three were used for describing the macrostructural behavior, and the last one was used for describing the microstructural deformation. The proposed average skeleton stress p^* is a function of the net stress, the macrostructure degree of saturation representing the capillary water content, and the suction. The expression of the microstructure effective stress $\hat{p}$ implies that the changes in the external applied stresses and suction affect the aggregate sizes.

A new hydro-mechanical model was then proposed which considers the interaction between the micro and macro soil structures. The bounding surface concept was adopted for

modeling both the stress-strain relation and the water retention curve, which leads to a simplified expression for the plastic potential parameters d_I and d_D . An anisotropic expression of the loading-collapse yield surface was also introduced in order to improve the model predictive ability under triaxial condition. Wetting and drying cycle tests and triaxial compression tests on expansive clay were selected for the simulation. The comparisons between numerical and experimental results demonstrate the ability of the model to reproduce the coupled hydro-mechanical behavior of unsaturated expansive clays.

Acknowledgments

This research was financially supported by the National Natural Science Foundation of China (Grant No. 41372285, 51579179, 51239010), the China Postdoctoral Science Foundation (Grant No. 2015M582314), and the Erasmus Mundus Programme TANDEM. These supports are greatly appreciated.

References

- Alonso, E.E., Gens, A., and Josa, A. 1990. A constitutive model for partially saturated soils. *Géotechnique*, **40**(3), 405-430. doi: 10.1680/geot.1990.40.3.405.
- Alonso, E.E., Lloret, A., Gens, A., and Yang, D.Q. 1995. Experimental behaviour of highly expansive double-structure clay. *In* 1st International Conference on Unsaturated Soils, Paris, 6-8 September 1995. *Edited by* E.E. Alonso and P. Delage. Balkema, Rotterdam,

the Netherlands. Vol. 1, pp. 11-16.

Alonso, E.E., Pereira, J.-M., Vaunat, J., and Olivella, S. 2010. A microstructurally based effective stress for unsaturated soils. *Géotechnique*, **60**(12), 913-925. doi: 10.1680/geot.8.P.002.

Al-Sharrad, M. A., and Gallipoli, D. 2014. An elasto-plastic model for unsaturated soils with evolving anisotropy. *In Proceedings of the 6th International Conference on Unsaturated Soils, UNSAT 2014, Sydney, Australia. Edited by N. Khalili, A.R. Russell and A. Khoshghalb.* pp. 433-439. doi: 10.1201/b17034-60.

Baker, R., and Frydman, S. 2009. Unsaturated soil mechanics: Critical review of physical foundations. *Engineering Geology*, **106**(1-2), 26-39. doi:10.1016/j.enggeo.2009.02.010.

Banerjee, P.K., and Yousif, N.B. 1986. A plasticity model for the mechanical behaviour of anisotropically consolidated clay. *International Journal for Numerical and Analytical Methods in Geomechanics*, **10**(5), 521-541. doi: 10.1002/nag.1610100505.

Barenblatt, G.I., Zheltov, Iu.P., and Kochina, I.N. 1960. Basic concepts in the theory of seepage of homogeneous liquids in fissured rocks. *Journal of Applied Mathematics and Mechanics*, **24**(5), 1286-1303. doi: 10.1016/0021-8928(60)90107-6.

Barrera, M.B. 2002. Estudio experimental del comportamiento hidro-mecánico de suelos colapsables (in Spanish). Ph.D. thesis, Universitat Politecnica de Catalunya, Barcelona, Spain.

Borja, R.I., and Koliji, A. 2009. On the effective stress in unsaturated porous continua with

- double porosity. *Journal of the Mechanics and Physics of Solids*, **57**(8), 1182-1193. doi: 10.1016/j.jmps.2009.04.014.
- Brooks, R.H., and Corey, A.T. 1964. Hydraulic properties of porous media. Hydrology Paper No.3, Colorado State University, Fort Collins, Colorado.
- Chen R. 2007. Experimental study and constitutive modelling of stress-dependent coupled hydraulic hysteresis and mechanical behaviour of an unsaturated soil. Ph.D. thesis, Hong Kong University of Science and Technology, Hong Kong.
- Chen, R., and Ng, C.W.W. 2013. Impact of wetting-drying cycles on hydro-mechanical behavior of an unsaturated compacted clay. *Applied Clay Science*, **86**, 38-46. doi: 10.1016/j.clay.2013.09.018.
- Cui, Y.J., and Delage, P. 1996. Yielding and plastic behaviour of an unsaturated compacted silt. *Géotechnique*, **46**(2), 291-311. doi: 10.1680/geot.1996.46.2.291.
- Cui, Y.J., Yahia-Aissa, M., and Delage, P. 2002. A model for the volume change behaviour of heavily compacted swelling clays. *Engineering Geology*, **64**(2-3), 233-250. doi: 10.1016/S0013-7952(01)00113-2.
- Dafalias, Y.F., and Herrmann, L.R. 1982. Bounding surface formulation of soil plasticity. *In* Proceedings of the International Symposium on Soils under Cyclic and Transient Loading. Edited by G.N. Pande and O.C. Zienkiewicz. Wiley, New York. pp. 253-282.
- Delage, P., and Lefebvre, G. 1984. Study of the structure of a sensitive Champlain clay and of its evolution during consolidation. *Canadian Geotechnical Journal*, **21**(1), 21-35. doi:

10.1139/t84-003.

Delage, P., Tessier, D., and Marcel-Audiguier, M. 1982. Use of the Cryoscan apparatus for observation of freeze-fractured planes of a sensitive Quebec clay in scanning electron microscopy. *Canadian Geotechnical Journal*, **19**(1), 111-114. doi: 10.1139/t82-011.

Della Vecchia, G., and Romero, E. 2013. A fully coupled elastic–plastic hydromechanical model for compacted soils accounting for clay activity. *International Journal for Numerical and Analytical Methods in Geomechanics*, **37**(5), 503-535. doi: 10.1002/nag.1116.

Derjaguin, B.V., Churaev, N.V., and Muller, V.M. 1987. *Surface Forces*. Plenum Press, New York.

D’Onza, F., Gallipoli, D., and Wheeler, S.J. 2011. Effect of anisotropy on the prediction of unsaturated soil response under triaxial and oedometric conditions. *In Proceedings of the 5th International Conference on Unsaturated Soils, Barcelona, Spain. Edited by E. Alonso and A. Gens.* pp. 787-794. doi: 10.1201/b10526-122.

Gens, A., and Alonso, E.E. 1992. A framework for the behaviour of unsaturated expansive clays. *Canadian Geotechnical Journal*, **29**(6), 1013-1032. doi: 10.1139/t92-120.

Gens, A., Alonso, E.E., Suriol, J., and Lloret, A. 1995. Effect of structure on the volumetric behaviour of a compacted soil. *In 1st International Conference on Unsaturated Soils. Edited by E.E. Alonso and P. Delage.* Paris, Balkema. Vol. 1, pp. 83-88.

Gens, A., Valleján, B., Sánchez, M., Imbert, C., Villar, M.V., and Van Geet, M. 2011.

- Hydromechanical behaviour of a heterogeneous compacted soil: experimental observations and modelling. *Géotechnique*, **61**(5), 367-386. doi: 10.1680/geot.SIP11.P.015.
- Hoffmann, C., Alonso, E.E., and Romero, E. 2007. Hydro-mechanical behaviour of bentonite pellet mixtures. *Physics and Chemistry of the Earth*, **32**(8-14), 832-849. doi: 10.1016/j.pce.2006.04.037.
- Houlsby, G.T. 1997. The work input to an unsaturated granular material. *Géotechnique*, **47**(1), 193-196. doi: 10.1680/geot.1997.47.1.193.
- Khalili, N., Witt, R., Laloui, L., Vulliet, L., and Koliji, A. 2005. Effective stress in double porous media with two immiscible fluids. *Geophysical Research Letters*, **32**(15), L15309.1-L15309.5. doi: 10.1029/2005GL023766.
- Kobayashi, I., Soga, K., Iizuka, A., and Ohta, H. 2003. Numerical interpretation of a shape of yield surface obtained from stress probe tests. *Soils and Foundations*, **43**(3), 95-103. doi: 10.3208/sandf.43.3_95.
- Leoni, M., Karstunen, M., and Vermeer, P.A. 2008. Anisotropic creep model for soft soils. *Géotechnique*, **58**(3), 215-226. doi: 10.1680/geot.2008.58.3.215.
- Li, X.S. 2005. Modelling of hysteresis response for arbitrary wetting/drying paths. *Computers and Geotechnics*, **32**(2), 133-137. doi: 10.1016/j.compgeo.2004.12.002.
- Liu, C., and Muraleetharan, K. 2006. Description of soil water characteristic curves using the bounding surface plasticity theory. *In* Proceedings of the 4th International Conference on

- Unsaturated Soils, Carefree, Arizona, April 2-6, 2006. *Edited by* G.A. Miller, C.E. Zapata, S.L. Houston and D.G. Fredlund. Vol. 2, pp. 2432-2440.
- Lu, N., and Likos, W.J. 2006. Suction stress characteristic curve for unsaturated soils. *Journal of Geotechnical and Geoenvironmental Engineering*, **132**(2), 131-142. doi: 10.1061/(ASCE)1090-0241(2006)132:2(131).
- Mašin, D. 2013. Double structure hydromechanical coupling formalism and a model for unsaturated expansive clays. *Engineering Geology*, **165**(24), 73-88. doi: 10.1016/j.enggeo.2013.05.026.
- Mašin, D., and Khalili, N. 2015. Swelling phenomena and effective stress in compacted expansive clays. *Canadian Geotechnical Journal*, **53**(1), 134-147. doi: 10.1139/cgj-2014-0479.
- Monroy, R., Zdravkovic, L., and Ridley, A. 2010. Evolution of microstructure in compacted London Clay during wetting and loading. *Géotechnique*, **60**(2), 105–119. doi: 10.1680/geot.8.P.125.
- Nowamooz, H., and Masrouri, F. 2010. Relationships between soil fabric and suction cycles in compacted swelling soils. *Engineering Geology*, **114**(3-4), 444-455. doi: 10.1016/j.enggeo.2010.06.005.
- Raveendraraj, A. 2009. Coupling of mechanical behaviour and water retention behaviour in unsaturated soils. Ph.D. thesis, University of Glasgow, Scotland, UK.
- Romero, E., Gens, A., and Lloret, A. 1999. Water permeability, water retention and

- microstructure of unsaturated compacted Boom clay. *Engineering Geology*, **54**(1-2), 117-127. doi:10.1016/S0013-7952(99)00067-8.
- Romero, E., Della Vecchia, G., and Jommi, C. 2011. An insight into the water retention properties of compacted clayey soils. *Géotechnique*, **61**(4), 313-328. doi: 10.1680/geot.2011.61.4.313.
- Sheng, D., Sloan, S.W., and Gens, A. 2004. A constitutive model for unsaturated soils: thermomechanical and computational aspects. *Computational Mechanics*, **33**(6), 453-465. doi: 10.1007/s00466-003-0545-x.
- Sivakumar, V. 1993. A critical state framework for unsaturated soil. Ph.D. thesis, University of Sheffield, UK.
- Sridharan, A., and Prakash, K. 2000. Classification procedures for expansive soils. *Proceedings of the Institution of Civil Engineers, Geotechnical Engineering*, **143**(4), 235-240. doi: 10.1680/geng.2000.143.4.235.
- Sridharan, A., Altschaeffl, A.G., and Diamond, S. 1971. Pore size distribution studies. *Journal of the Soil Mechanics and Foundations Division*, **97**(5), 771-787.
- Stropeit, K., Wheeler, S.J., and Cui, Y.J. 2008. An anisotropic elasto-plastic model for unsaturated soils. *In Proceedings of 1st European Conference on Unsaturated Soils, E-UNSAT 2008, Durham, United Kingdom, 2-4 July 2008. Edited by D.G. Toll, C.E. Augarde, D. Gallipoli and S.J. Wheeler.* pp. 625-631. doi: 10.1201/9780203884430.ch84.
- Sun, W., and Sun, D. 2012. Coupled modelling of hydro-mechanical behaviour of unsaturated

- compacted expansive soils. *International Journal for Numerical and Analytical Methods in Geomechanics*, **36**(8), 1002-1022. doi: 10.1002/nag.1036.
- Tian, H.H., and Wei, C.F. 2014. A NMR-based testing and analysis of adsorbed water content (in Chinese). *Scientia Sinica Technologica*, **44**(3), 295-305. doi: 10.1360/092013-1133.
- Tuller, M., Or, D., and Dudley, L.M. 1999. Adsorption and capillary condensation in porous media: liquid retention and interfacial configurations in angular pores. *Water Resources Research*, **35**(7), 1949-1964. doi: 10.1029/1999WR900098.
- Van Genuchten, M.Th. 1980. A closed-form equation for predicting the hydraulic conductivity of unsaturated soil. *Soil Science Society of America Journal*, **44**(5), 892-898. doi: 10.2136/sssaj1980.03615995004400050002x.
- Wheeler, S.J. 1997. A rotational hardening elasto-plastic model for clays. *In Proceedings of the 14th International Conference on Soil Mechanics and Foundation Engineering*. Hamburg, A.A. Balkema, Rotterdam. Vol. 1, pp. 431-434.
- Wheeler, S.J., and Sivakumar, V. 1995. An elasto-plastic critical state framework for unsaturated soil. *Géotechnique*, **45**(1), 35-53. doi: 10.1680/geot.1995.45.1.35.
- Wheeler, S.J., and Sivakumar, V. 2000. Influence of compaction procedure on the mechanical behaviour of an unsaturated compacted clay. Part 2: Shearing and constitutive modelling. *Géotechnique*, **50**(4), 369-376. doi: 10.1680/geot.2000.50.4.369.
- Wheeler, S.J., Näätänen, A., Karstunen, M., and Lojander, M. 2003. An anisotropic elastoplastic model for soft clays. *Canadian Geotechnical Journal*. **40**(2), 403-418. doi:

10.1139/t02-119.

- Yao, Y., and Kong, Y. 2012. Extended UH Model: Three-Dimensional Unified Hardening Model for Anisotropic Clays. *Journal of Engineering Mechanics*, **138**(7), 853-866. doi: 10.1061/(ASCE)EM.1943-7889.0000397.
- Yin, Z.-Y., Karstunen, M., Chang, C.S., Koskinen, M., and Lojander, M. 2011. Modeling time-dependent behavior of soft sensitive clay. *Journal of Geotechnical and Geoenvironmental Engineering*, **137**(11), 1103-1113. doi: 10.1061/(ASCE)GT.1943-5606.0000527.
- Yin, Z.-Y., Yin, J.H., and Huang, H.W. 2015. Rate-dependent and long-term yield stress and strength of soft Wenzhou marine clay: experiments and modeling. *Marine Georesources & Geotechnology*, **33**(1), 79-91. doi:10.1080/1064119X.2013.797060.
- Zhan, T.L., and Ng, C.W. 2006. Shear strength characteristics of an unsaturated expansive clay. *Canadian Geotechnical Journal*, **43**(7), 751-763. doi: 10.1139/t06-036.
- Zhao, C.G., Liu, Y., and Gao, F.P. 2010. Work and energy equations and the principle of generalized effective stress for unsaturated soils. *International Journal for Numerical and Analytical Methods in Geomechanics*, **34**(9), 920-936. doi: 10.1002/nag.839.
- Zhao, C.G., Liu, Z.Z., Shi, P.X., Li, J., Cai, G.C., and Wei, C.F. 2015. Average Soil Skeleton Stress for Unsaturated Soils and Discussion on Effective Stress. *International Journal of Geomechanics*, D4015006. doi: 10.1061/(ASCE)GM.1943-5622.0000610.

List of figure captions

Figure 1 Yield curves based on experimental data at various suctions by (a) Cui and Delage (1996); (b) Zhan and Ng (2006) and Chen (2007)

Figure 2 Illustration of anisotropic loading collapse bounding surface

Figure 3 Illustration of MI and MD bounding surfaces

Figure 4 Illustration of SI and SD bounding surfaces

Figure 5 Illustration of the parabolic Hvorslev envelope

Figure 6 Comparisons between the measured and computed results of the multiple wetting and drying cycle tests for Boom Clay

Figure 7 Comparisons between the measured and computed results of the wetting and drying cycle test for ZY expansive clay

Figure 8 Comparisons between the measured and computed results of drained shear tests at different suctions under net confining stress of 350 kPa

Figure 9 Comparisons between the measured and computed results of drained shear tests at different suctions under net confining stress of 200 kPa

Figure 10 Comparisons between the measured and computed results of drained shear tests at different suctions under net confining stress of 50 kPa

Figure 11 Comparisons between computed results considering anisotropy and ignoring anisotropy of drained shear tests at different suctions under constant net confining stress of 350 kPa

Figure 12 Comparisons between the measured and computed results of anisotropic consolidation under constant net stress ratio of 0.55 and drained shear tests at different

suctions under constant net mean stress

Figure 13 Comparisons between the measured and computed results of anisotropic consolidation under constant net stress ratio of 1.0 and drained shear tests at different suctions under constant net mean stress

Figure 14 Comparisons between computed results considering anisotropy and ignoring anisotropy of anisotropic consolidation under constant net stress ratio of 0.55 and drained shear tests at different suctions under constant net mean stress

Draft

Figure 1-Yield curves based on experimental data at various suctions by (a) Cui and Delage (1996);

(b) Zhan and Ng (2006) and Chen (2007)

8.4×5.4 cm (600×600 DPI)

Figure 2-Illustration of anisotropic loading collapse bounding surface

6.7 × 5.8 cm (600 × 600 DPI)

Figure 3-Illustration of MI and MD bounding surfaces

6.5×5.7 cm (600×600 DPI)

Figure 4-Illustration of SI and SD bounding surfaces

6.5 × 5.7 cm (600 × 600 DPI)

Figure 5- Illustration of the parabolic Hvorslev envelope

7.5×5.8 cm (600×600 DPI)

Figure 6-Comparisons between the measured and computed results of the multiple wetting and

drying cycle tests for Boom Clay

8.2 × 6.3 cm (600 × 600 DPI)

Figure 7-Comparisons between the measured and computed results of the wetting and drying cycle test for ZY expansive clay

8.1×7.0 cm (600×600 DPI)

Figure 8-Comparisons between the measured and computed results of drained shear tests at

different suctions under net confining stress of 350 kPa

8.5 × 5.1 cm (600 × 600 DPI)

Figure 9-Comparisons between the measured and computed results of drained shear tests at different suctions under net confining stress of 200 kPa

8.5×5.1 cm (600×600 DPI)

Figure 10-Comparisons between the measured and computed results of drained shear tests at

different suctions under net confining stress of 50 kPa

8.5×5.1 cm (600×600 DPI)

Figure 11-Comparisons between computed results considering anisotropy and ignoring anisotropy of drained shear tests at different suctions under constant net confining stress of 350 kPa

8.5×5.1 cm (600×600 DPI)

Figure 12-Comparisons between the measured and computed results of anisotropic consolidation under constant net stress ratio of 0.55 and drained shear tests at different suctions under constant net mean stress

8.5×5.1 cm (600×600 DPI)

Figure 13-Comparisons between the measured and computed results of anisotropic consolidation

under constant net stress ratio of 1.0 and drained shear tests at different suctions under constant net mean stress

8.5×5.1 cm (600×600 DPI)

Figure 14-Comparisons between computed results considering anisotropy and ignoring anisotropy

of anisotropic consolidation under constant net stress ratio of 0.55 and drained shear tests at

different suctions under constant net mean stress

8.5×5.1 cm (600×600 DPI)

Tables

Table 1 Values of model parameters for Boom clay

$\lambda(0)$	r_{λ}	β_{λ}	$N(0)$	r_N	β_N	M	κ
0.130	1.0	1.0	2.10	1.08	0.006	0.69	0.00
κ_m	d_D	d_I	λ_w	κ_w	k_{ws}	p_{ref} / kPa	γ
0.025	2	2.0	0.172	0.02	0.0	100	8.0

Table 2 Initial values of soil state and hardening parameters for the multiple wetting and drying cycle tests

$e_0 \Big _{\substack{p'=20 \text{ kPa} \\ s=500 \text{ kPa}}}$	$e_0 \Big _{\substack{p'=100 \text{ kPa} \\ s=700 \text{ kPa}}}$	$e_0 \Big _{\substack{p'=400 \text{ kPa} \\ s=700 \text{ kPa}}}$	$e_m \Big _{\substack{p'=20 \text{ kPa} \\ s=500 \text{ kPa}}}$	$S_r \Big _{s^*=152 \text{ kPa}}$
2.049	1.943	1.868	0.426	0.16
$s_I^* \Big _{s^*=152 \text{ kPa}}$	$s_D^* \Big _{s^*=152 \text{ kPa}}$	$\hat{p}_I / \text{kPa}$	$\hat{p}_D / \text{kPa}$	
342	152	318	0	

Table 3 Values of model parameters for ZY expansive clay

$\lambda(0)$	r_{λ}	β_{λ}	$N(0)$	r_N	β_N	M	μ / kPa	κ
0.11	1.45	0.4	2.27	1.4	0.2	0.89	71.2	0.01
ν	κ_m	d_D	d_I	λ_w	κ_w	k_{ws}	p_{ref} / kPa	γ
0.3	0.008	5.4	3.7	0.13	0.01	2.0	0.5	8.0

Table 4 Initial values of soil state and hardening parameters for the triaxial tests

p' / kPa	s / kPa	e_0	e_m	S_r	s_l^* / kPa	s_D^* / kPa	$\hat{p}_l / \text{kPa}$	$\hat{p}_D / \text{kPa}$
20	540	0.716	0.287	0.68	180	50	700	120

Draft

Appendix A. The alternative expression of work input

Alternative definitions of macro void ratio and strain rates were adopted by Mašin (2013). Using these definitions, the expression of work input is deduced as follows.

The total, macro and micro structural volumetric strain rates are defined by:

$$(A.1) \quad \dot{\epsilon}_v = -\frac{\dot{e}}{1+e}; \quad \dot{\epsilon}_v^M = -\frac{\dot{e}_M}{1+e_M}; \quad \dot{\epsilon}_v^m = -\frac{\dot{e}_m}{1+e_m}$$

where the macrostructural void ratio e_M is defined as the ratio of macropore volume over the total volume of aggregates; the microstructural void ratio e_m is defined as the ratio of micropore volume over the solid volume. Then the following additive equation for the void ratio e hold true:

$$(A.2) \quad e = e^M + e^m + e^M e^m$$

Substituting Eq. (A.2) into the first equation of Eq. (A.1) leads to:

$$(A.3) \quad \dot{\epsilon}_v = \dot{\epsilon}_v^M + \dot{\epsilon}_v^m$$

According to the defining of macrostructural void ratio e_M , the macroporosity n_M and its rate could be expressed as:

$$(A.4) \quad n_M = \frac{e_M}{1+e_M}$$

$$(A.5) \quad \dot{n}_M = -(1-n_M)\dot{\epsilon}_M$$

Substituting Eq. (A.5) into Eq. (7) leads to:

$$(A.6) \quad \dot{\rho}^{ag} = \frac{c^{ag}}{1-n_M} + \rho^{ag} \dot{\epsilon}_v^m$$

The mass exchange between the aggregate and the adsorbed water phases leads

to a microstructural volume change. Thus, the micro volumetric strain rate can be expressed as:

$$(A.7) \quad \dot{\epsilon}_v^m = -\frac{\dot{V}^{ag}}{V^{ag}} = \frac{(c_{ag}^{ad}/\rho^{ad})\dot{V}}{V^{ag}} = \frac{c^{ag}}{(1-n_M)\rho^{ad}}$$

Substituting Eqs. (6), (A.6) and (A.7) into Eq. (4) gives the work input to unsaturated expansive soils considering double porosity as:

$$(A.8) \quad \begin{aligned} W = & -u'_{c,j}w_j^c - u'_{ad,j}w_j^{ad} - u'_{a,j}w_j^a + (\sigma_{ij} - u_c S_r^c \delta_{ij} - u_{ad} S_r^{ad} \delta_{ij} - u_a S_r^a \delta_{ij}) \dot{\epsilon}_{ij}^M \\ & - n_M (u_a - u_c) \dot{S}_r^c - n_M (u_a - u_{ad}) \dot{S}_r^{ad} - \left(u_c - u_{ad} \frac{\rho^c}{\rho^{ad}} \right) \frac{\dot{c}^c}{\rho^c} + u_a n_M S^a \frac{\dot{\rho}^a}{\rho^a} \\ & + (p - u_{ad})(1 - n_M) \dot{\epsilon}_v^m + (p - u_c S_r^c - u_{ad} S_r^{ad} - u_a S_r^a) n_M \dot{\epsilon}_v^m \end{aligned}$$

The new work input expression is the same as Eq. (11) except to the terms represented the work caused by the deformation of the macrostructure and microstructure. It is caused by the alternative definitions of macro and micro structural volumetric strain. The relationships between alternative definitions and that adopted in this paper are as follows:

$$(A.9) \quad \begin{aligned} \dot{\epsilon}_v^{M'} &= -\frac{\dot{e}_M'}{1+e} = -\frac{\dot{e}_M}{1+e} (1+e_m) - \frac{\dot{e}_m}{1+e} e_M = \dot{\epsilon}_v^M + \dot{\epsilon}_v^m n_M \\ \dot{\epsilon}_v^{m'} &= -\frac{\dot{e}_m'}{1+e} = -\frac{\dot{e}_m}{1+e} \frac{1+e_m}{1+e} = \dot{\epsilon}_v^m (1-n_M) \end{aligned}$$

The notations adopted in this paper are denoted by superscript '.

Substituting Eq. (A.9) into Eq. (A.8) leads to

$$(A.10) \quad \begin{aligned} W = & -u'_{c,j}w_j^c - u'_{ad,j}w_j^{ad} - u'_{a,j}w_j^a + (\sigma_{ij} - u_c S_r^c \delta_{ij} - u_{ad} S_r^{ad} \delta_{ij} - u_a S_r^a \delta_{ij}) \dot{\epsilon}_{ij}^{M'} \\ & - n_M (u_a - u_c) \dot{S}_r^c - n_M (u_a - u_{ad}) \dot{S}_r^{ad} - \left(u_c - u_{ad} \frac{\rho^c}{\rho^{ad}} \right) \frac{\dot{c}^c}{\rho^c} \\ & + u_a n_M S^a \frac{\dot{\rho}^a}{\rho^a} + (p - u_{ad}) \dot{\epsilon}_v^{m'} \end{aligned}$$

Eq. (A.10) is the same as Eq. (11).

The work conjugated stress to $\dot{\epsilon}_{ij}^M$ is $(\sigma_{ij} - u_c S_r^c - u_{ad} S_r^{ad} - u_a S_r^a)$ which is the same as that work conjugated stress to $\dot{\epsilon}_{ij}^{M'}$. However, an additional term represented the work caused by the deformation of the microstructure appears when the alternative definition of macrostructural void ratio are adopted. The additional work conjugated stress to $\dot{\epsilon}_v^m$ is $(p - u_c S_r^c - u_{ad} S_r^{ad} - u_a S_r^a)$ which is the same as the stress work conjugated stress to $\dot{\epsilon}_v^M$. The alternative definition of macrostructural void ratio has conceptual advantages (Mašin, 2013), however it complicate the work input terms caused by the deformation of the microstructure.

Besides, the alternative definition of microstructural volumetric strain doesn't lead to an addition term in the expression of work input. The new work conjugated stress to $\dot{\epsilon}_v^m$ is $(1 - n_M)(\sigma_{ij} - u_{ad})$. Thus the alternative definition of microstructural volumetric strain could be chosen.