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Undrained stiffness anisotropy from Hollow Cylinder experiments on four Eocene-to-Jurassic UK stiff clays

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1 **Abstract**

2 The paper describes the anisotropic undrained stiffness behaviour of four medium-plasticity
3 heavily overconsolidated UK stiff marine clays as revealed through Hollow Cylinder testing.
4 The experiments contributed to two broader studies on stiff-to-hard London, Gault,
5 Kimmeridge, and Oxford clay strata. They involved static and dynamic testing of multiple
6 high quality natural specimens sampled at similar depths from inland sites. This paper
7 explores the directional dependency of the clays' highly non-linear undrained stiffness
8 characteristics. New data-analysis approaches are outlined that allow the stiffnesses
9 associated with one dimensional vertical, horizontal or pure horizontal shear modes to be
10 isolated in complex undrained stress paths. In the presented experiments, loading progressed
11 from in-situ stresses to reach ultimate failure at a range of final major principal stress
12 orientation angles α (defined in the vertical plane) while keeping fixed values of the
13 intermediate principal stress ratio, b . The tests reveal strong undrained stiffness anisotropy
14 that can impact significantly on the prediction and understanding of ground deformation
15 patterns in numerous geotechnical engineering applications.

16 **Keywords:**

17 Clays, Laboratory tests, Stiffness, Anisotropy, Stress path

18 Introduction

19 Much of the UK's civil engineering infrastructure and several of its major cities are built on
20 stiff overconsolidated, geologically aged, clays. The clays' highly non-linear stiffness
21 characteristics have an important bearing on geotechnical engineering applications, ranging
22 from tunnels to foundations and deep excavations; see for example Jardine et al. (2005). It is
23 known that the stiff clays have anisotropic micro and macro-structures (see for example
24 Hight et al. 2007 and Hosseini-Kamal et al. 2014) and that stiffness anisotropy can influence
25 ground movement patterns markedly. Finite element simulations by Simpson et al. (1996) and
26 Addenbrook et al. (1997) indicated how accounting for the London clay's marked degree of
27 stiffness anisotropy is critical to making realistic predictions for the settlement troughs
28 developed above tunnels bored in this stratum. While sometimes addressed in research, the
29 influence of anisotropy on non-linear stiffness behaviour is rarely addressed in practical
30 design. Laboratory and field techniques exist to measure anisotropy within the (very small
31 strain) linear elastic range, but a lack of reliable information on stiffness anisotropy over the
32 full non-linear range that affects most engineering analyses is one factor that hinders practical
33 advances in ground movement assessment. Potts and Zdravkovic (2001), for example, noted
34 that a lack of reliable data regarding how the shear modulus G_{vh} degrades with shear strain
35 limits the analysis of ground deformations around tunnels. Due to their typically low
36 permeabilities, the clays' piezometric responses to large scale perturbations often involve
37 only minor pore pressure equalisation during construction; this paper therefore explores the
38 clays' non-linear stiffness anisotropy under undrained conditions. Laboratory experiments
39 with HCAs (Hollow Cylinder Apparatus) are reported on high quality samples of Jurassic to
40 Eocene marine clays that were deposited in broadly similar marine environments. The
41 sampling locations and $\approx 10\text{m}$ depths were chosen to reduce potential local effects of tectonic
42 disturbance, post-depositional weathering and glacial or tree action.

43 The paper combines analysis of HCA experiments conducted by Brosse (2012) on Gault clay
44 sampled near Cambridge, Kimmeridge clay from Steventon (Oxfordshire), and Oxford clay
45 from Elstow (Bedfordshire) along with earlier tests conducted by Nishimura (2006) that
46 contributed to the London clay Heathrow Terminal 5 campaign (Hight et al. 2007). The
47 studies on Gault, Kimmeridge and Oxford clay are detailed in Wilkinson (2011), Brosse
48 (2012) and Hosseini-Kamal (2012). Hosseini-Kamal et al. (2014) present geotechnical
49 profiles for the sites where samples of the latter three strata were taken by careful block and
50 Geobor-S rotary coring techniques.

51 Tables 1 and 2 summarise some key aspects of the properties of the four suites of clay
52 specimens' on which the HCA experiments were conducted, listing their geotechnical
53 descriptions, estimated in-situ effective stresses, meso and micro-structures, index properties,
54 peak triaxial compressive undrained shear strengths (from carefully K_0 reconsolidated,
55 100mm diameters specimens) and residual angles of shearing resistance, as determined in
56 ring shear experiments. Perhaps surprisingly, Hosseini-Kamal et al. (2014) report no clear
57 correlation between the clays' large strain behaviours and their variable ages or burial depths.
58 After removing as far as possible the potential effects of other depositional and post-
59 depositional factors, they concluded that meso-structure was the most important determinant
60 of the natural clays' large scale yielding behaviour. In this context, the meso-scale falls
61 between the micro- and macro-scales. While the micro-scale is relevant to particle level
62 phenomena and the macro-scale concerns features whose dimensions are comparable to those
63 of built structures (such as faults, extensive joints, etc.), meso-structure features are visible by
64 eye and yet sufficiently small for their effects to be perceived and treated as homogenised
65 characteristics of representative soil elements. Fissures, bedding, laminations and bioturbated
66 features are common examples of meso-structure. The Oxford clay samples had clear bedding
67 features, but no fissures, and developed markedly higher triaxial compression shear strengths

68 and post-peak brittleness. The peak triaxial shear strengths of natural samples of the other
69 clays were strongly affected by their greater fissure intensity. All four soils are markedly
70 brittle in shear. Gapsarre et al. (2007a), Hight et al. (2007) and Hosseini-Kamal et al. (2014)
71 show that any tendency to dilate after undergoing 'gross shear yielding' (which we term Y_3 to
72 distinguish this feature from earlier small strain yielding phenomena) is truncated by the
73 formation of new discontinuities, the re-mobilisation of any existing discontinuities or
74 fissures, or a combination of the two processes. The test elements are unable to progress
75 towards global critical states and the specimens develop slickensided residual shear
76 mechanisms if the shearing is continued to sufficiently large displacements.

77 The London clay's drained elastic anisotropy was explored by Gasparre et al. (2007b) in
78 terms of effective stress compliance matrices that were established by:

- 79 a) Anh-Minh's (2007) uniaxial drained HCA tests employing high resolution local
80 strain measurements over a wide range of strain and
- 81 b) Gasparre's (2005) very small-strain axial and radial drained triaxial probing
82 experiments combined with multi-axis shear wave velocity measurements,
83 following the techniques of Pennington et al. (1997) and Kuwano and Jardine
84 (1998), (2007).

85 Hosseini-Kamal (2012) applied the same approach, which implicitly assumes strain rate
86 independency, to measure the elastic anisotropic properties of the Gault, Kimmeridge and
87 Oxford clays.

88 This paper explores the directional dependency of all four clays' non-linear undrained pre-
89 failure behaviour. An analysis is made of the HCA experiments that allows the undrained
90 stiffnesses associated with vertical and horizontal compression as well as pure shear
91 deformation modes to be identified from tests in which loading progressed from in-situ
92 stresses to failure at a range of final major principal stress axis orientations (defined as α the

orientation of the σ_1 axis to the vertical), while also maintaining fixed intermediate principal stress conditions.

Apparatus and Procedures

Principles

Hight et al. (1983) describe how varying the axial force, torque and inner and outer cell pressures applied to HCA specimens provides control over the vertical, radial, circumferential and torsional shear stresses σ_z , σ_r , σ_θ and $\tau_{z\theta}$. HCA systems can maintain $\tau_{zr} = \tau_{r\theta} = 0$ and provide four degrees of freedom that offer independent control of the p , q , α and b stress parameters defined as:

$$[1] \quad p = \frac{(\sigma_1 + \sigma_2 + \sigma_3)}{3}$$

$$[2] \quad q = \sigma_1 - \sigma_3$$

$$[3] \quad \alpha = \frac{1}{2} \tan^{-1} \left(\frac{2\tau_{z\theta}}{\sigma_z - \sigma_\theta} \right)$$

$$[4] \quad b = \frac{\sigma_2 - \sigma_3}{\sigma_1 - \sigma_3}$$

α is the angle between the major principal stress direction and the vertical. Effects of α on material behaviour are therefore direct indices of anisotropy. The intermediate principal stress ratio b can be varied between 0 and 1. An additional variable, $\alpha_{d\sigma}$, is also defined, to indicate the angle between the axis of the major principal stress increment and the vertical (see Figure 3) which, as will be discussed, was used as one of the control variables in the test series.

$$[5] \quad \alpha_{d\sigma} = \frac{1}{2} \tan^{-1} \left(\frac{2d\tau_{z\theta}}{d\sigma_z - d\sigma_\theta} \right)$$

112 *Equipment*

113 The two computer-controlled systems illustrated in Figures 1 and 2 and described by
114 Nishimura et al. (2007), Anh-Minh et al. (2011) and Brosse (2012) were employed in the
115 HCA experiments. Both sets can perform experiments under mixed stress and strain control.
116 The nominal outer diameters, inner diameters and heights of their test specimens were
117 respectively 70 mm, 38 mm and 170~190 mm for the Imperial College Resonant Column
118 Hollow Cylinder Apparatus (ICRCHCA) and 100 mm, 60 mm and 200 mm for the Imperial
119 College Hollow Cylinder Apparatus Mark II (ICHCA II). The ICRCHCA is equipped with a
120 Hardin oscillator that can perform torsional resonant column tests to obtain dynamic G_{z0}
121 shear moduli. Sample deformation is measured globally, apart from the semi-local (platen-to-
122 platen) torsional rotations, which are sensed with a proximity transducer system. The ICHCA
123 II employed a full suite of high-resolution local transducers. Axial displacements and
124 torsional rotations were measured with enhanced electro-level inclinometers, while changes
125 of the inner and outer radii were measured with a Linear Variable Differential Transformer
126 (LVDT) and a set of three proximity transducers, respectively. From these boundary
127 displacements, the average strains induced in the clay specimens were calculated according to
128 the equations given by Hight et al. (1983).

129 *Specimen preparation*

130 HCA specimens were prepared as described by Nishimura et al. (2007) and Brosse (2012).
131 First cylinders were trimmed to the specified outer dimensions with a wire saw in a rotary
132 soil lathe. The inner cylinder void was then formed by jacketing the sample in an aluminium
133 mould and using a metal working lathe to advance six drill bits of gradually increasing
134 diameters to achieve the desired inner diameter with minimal disturbance. The specimens
135 were carefully installed with (internal and external) latex membranes over systems of surface
136 filter paper drains and set on the sharp edges of shear vanes anchored into the HCAs' sintered

bronze vertical platens. The latter ensured that sufficient traction could be applied during torsional shearing to failure.

Test programme

The general scheme of the stress paths applied in the undrained HCA tests is illustrated in Figure 3. Both b and α were varied in the London clay tests by Nishimura (2006) and Anh Minh (2007) who investigated a range of depths at one site. Brosse (2012) covered three different sites and clays, but focussed on a narrower depth range and maintained $b = 0.5$ in most of her tests. Tables 3 to 6 specify each test's code, sampling depth, reconsolidation effective stresses and shearing effective stress path. The tabulated Oxford, Kimmeridge, Gault and London clay tests, which all involved samples taken from $\approx 10\text{m}$ depth and imposed $b = 0.5$ during shearing, represent a large proportion of those undertaken by Brosse (2012) and a limited sub-set of the London clay programme. All of the tabulated London clay tests were conducted in the ICRCHCA, while both sets of HCA equipment were deployed in Brosse's experiments. Brosse et al. (2016) demonstrate that the four clays are anisotropic in their undrained shear strength behaviour and Tables 3 to 6 specify the peak undrained shear strengths developed in each test.

Test procedures

All tests started by applying a cell pressure greater than the estimated in-situ mean effective stress to induce a positive pore pressure once full contact was made with the platens. The system was saturated by back pressure to acceptable undrained pore pressure response B ratios above 0.9 in most of the tests. While all the specimens had been sampled from below the water table, where they had experienced high degrees of saturation, their relatively high effective bulk moduli limited their theoretical B values to 0.95-0.97, even when perfectly saturated. Careful handling, line flushing, thorough saturation of the porous discs and an elevated back pressure helped to maximise the set-up stage B values. Maintaining the high

162 back pressure over the subsequent re-consolidation and creep stages should have led to still
163 higher final, pre-shearing, B values.

164 All specimens were reconsolidated to anisotropic effective stress states reflecting the
165 estimated in-situ values. The suction measurements on multiple triaxial and HCA samples
166 presented in Table 7 allowed K_0 values (all >1) to be estimated for each site and depth, after
167 making an allowance for deviator stress release (using the dq/dp' slopes given in Table 7) and
168 carefully establishing the in-situ vertical effective stress. The resulting range of K_0 values are
169 all close to the passive limit, as were other estimates derived from Mayne and Kulhawy's
170 (1982) equations and in-situ measurements made with a Marchetti dilatometer in
171 Kimmeridge clay and a self-boring pressuremeter in Oxford clay. Isotropic reconsolidation
172 was conducted first to reach the desired p' values and then $(\sigma_z - \sigma_\theta)/2$ was reduced, keeping
173 constant p' , to reach the final effective stress points. The axial straining experienced during
174 anisotropic reconsolidation was restricted to a maximum of 0.5% to avoid any premature
175 destructuration of the high quality samples. The first clay tested was Oxford clay and it was
176 found that the estimated K_0 values (3.2) could not be reached without over-straining the
177 specimens during re-consolidation. Trial tests showed that it was necessary to adopt a lower
178 K (1.8) and therefore a higher vertical stress (see Tables 1 and 3) than had been originally
179 estimated. When moving on to test the Gault and Kimmeridge clays, it was decided to
180 maintain the same $K_0 = 1.8$ for compatibility, noting that this value fell towards the lower end
181 of the estimated K_0 range.

182 Stiffness measurements can be severely compromised by any residual creep straining that
183 may apply prior to shearing; see Jardine (1985) or Gasparre et al. (2014). Sufficient pause
184 times were therefore imposed after the end of all consolidation stages for creep rates to
185 stabilise before commencing undrained shearing. Several days were required for the
186 volumetric and deviatoric strain rates to fall below the adopted 0.002 % per hour criterion

recommended by Jardine (1985). This rate is in order of 1/100 of the subsequent strain rate (0.15% per hour, as described later). This condition is similar to the limit Jardine (1985) recommended and Clayton and Heymann's (2001) found necessary to ensure that consistent Young's moduli are measured in triaxial compression and extension over the initial (pre- Y_1 yielding) elastic range. Allowing for further creep stabilisation may also affect the later non-linear stiffness behaviour by allowing the kinematic (Y_1 and Y_2) 'small-strain' yield surfaces identified by Jardine (1992) to adjust. However, waiting for the creep rates to reduce to much slower rates would have required the tests to be extended to prohibitively long durations (possibly many months), an option that could not be explored practically. The shearing stages involved a first step of changing b from its initial value of unity (which applies with $K_0 > 1$) to a value of 0.5 under undrained conditions, while keeping p , q and α constant. After allowing 12 hours for minor additional creep, further undrained shearing was initiated with constant p , b and $\alpha_{d\sigma}$ values. It follows that α varied during undrained shearing except in the $\alpha_{d\sigma} = 90^\circ$ tests. The $\alpha_{d\sigma}$ paths were set so that failure (taken as reaching peak q in this case) would develop at target α values of 0° , 23° , 45° , 67° and 90° respectively, which are denoted as α_f . Tables 3 to 6 present the chosen values of $\alpha_{d\sigma}$ with the corresponding actual value of α_f . The shearing stages were conducted under a mixed stress and strain control arrangement, set to give an average deviatoric strain rate of 0.15% per hour.

Small-strain non-destructive resonant column tests were conducted during ICRCHCA tests to monitor the variation of the elastic $G_{z\theta}$ during reconsolidation and shearing. Brosse (2012) reports on the Resonant Column apparatus, test procedures, calibrations and test interpretation.

Stress and strain measurements

The accurate measurement, from the very early stages of shearing up to failure, of the four dimensional stress and strain states developed in HCA tests poses considerable technical

difficulties. The HCA cells illustrated in Figures 1 and 2 were set in a tightly temperature regulated laboratory. Stable, 16 bit resolution auto-ranging Analogue to Digital converters were employed to track the stress and strain measurements. Both HCA systems employed inner and outer cell pressure measurements as well as purpose-designed dual axis, pressure compensated internal load cells that measured the axial force and torque applied to the samples. These bi-axial cells were carefully calibrated for both direct and cross-sensitivity under the load paths applied employed for testing. As described above, for most of the tests, strain measurements were realised using a mix of global and semi-local (platen-to-platen) in the ICRCHCA except for the Kimmeridge clay series where local instrumentation was employed in the ICHCA II.

A key factor when attempting to measure soil stiffness at small strain is the limiting stable resolution of the stress and strain sensing systems; see for example Jardine et al. (1984) or Gasparre et al. (2014). Table 8 lists the theoretical strain resolution limits based on the instruments fitted to each apparatus. Brosse (2012) noted that the HCA instruments' scatter limited the ability to define stiffness at small strains. As described later, further filtering and post-processing has been performed to enhance the clarity of the traces recorded at very small strains.

It is well known that external strain measurements can be unrepresentative over the early stages of soil shearing tests due to apparatus compliance, potentially eccentric loading and imperfect platen-to-specimen contact. Radial strains deduced from volume strains can also be affected by straining non-uniformity and membrane penetration. Ideally, triaxial and HCA strains should be measured with high resolution local strain sensors, such as those described by Burland and Symes (1982) or Tatsuoka et al. (1997). However, such systems can be difficult to apply in HCA tests on natural stiff clays that are subjected to high cell pressures over extended durations and taken ultimately to large strain failure.

Local strain instruments were deployed in the ICHCA II tests and compliance effects were alleviated with the ICRCHCA by (i) employing platen-to-platen shear strain measurements and (ii) correcting the external axial strain measurements for the system compliance, which was calibrated carefully; Brosse (2012). Membrane penetration was not significant with the fine grained clays. Checks were made of any potential remaining strain measurement errors by comparing equivalent measurements from: the two independent HCA systems; fully instrumented triaxial tests; and by contrasting static torsional shear measurements with output from fully independent resonant column and dynamic bender element measurements. The degree of agreement seen in such checks was generally encouraging. As discussed later, the resonant column measurements tended to indicate marginally higher shear moduli than other static and dynamic G_{vh} tests. Recognizing that static stiffnesses may not have been defined as accurately as desired over the very small strain linear range, parallel triaxial tests were undertaken with very high resolution local strain sensors; see Gasparre et al. (2007b) and Hosseini-Kamal (2012). Gasparre et al. (2007b) tracked the details of how the Y_1 and Y_2 kinematic yield surfaces manifested by the London clay evolved with depth and effective stress state, while Hosseini-Kamal (2012) reported on the kinematic yielding features exhibited by the three other stiff clays. However, as noted in the introduction, this paper focuses principally on the evolution of the stiffness anisotropy manifested on various loading paths that start from in-situ K_0 conditions, covering the extended non-linear range that controls ground movements and soil-structure interaction in the field; Addenbrooke et al. (1997).

Characterising the stress-strain response

Characterising stiffness from very small strains to failure also poses theoretical difficulties. Jardine (1992), (2014) emphasises that behaviour can only be considered elastic within a very small area of the stress space available to soils that is limited by a kinematic hardening (Y_1)

262 true yield surface. The latter is dragged with the current effective stress point, growing and
263 shrinking with p' and changing in shape with proximity to the outer Y_3 surface, which
264 corresponds to the yield surface recognised in classical critical state soil mechanics.
265 Behaviour within the true Y_1 yield surface has been found to be broadly cross anisotropic in
266 vertically sedimented K_0 consolidated soils; Jardine et al. (2004), Nishimura (2014a). The
267 horizontal Young's moduli can greatly exceed the vertical values in high OCR clays;
268 Gasparre et al. (2007b), Hosseini-Kamal (2012).

269 Plastic straining commences once the Y_1 surface is engaged and becomes progressively more
270 important as straining continues along any monotonic path. An intermediate kinematic Y_2
271 surface was identified for London clay by Gasparre et al. (2007b) that marks: (i) potential
272 changes in strain increment directions, (ii) the onset of marked strain-rate or time dependency
273 and (iii) a threshold condition in cyclic tests (as noted by Vucetic 1994) beyond which
274 permanent strains (or p' reductions in constant volume tests) accumulate significantly. The
275 initial proximity to the Y_2 surface affects the extent of the Y_1 region. In the same way, the
276 location of the anisotropic Y_3 surface constrains the non-linear stiffness response. Any
277 attempt to treat the non-linear stiffness behaviour within the framework of elasticity is
278 constrained by these fundamental limitations and by possible interactions between the
279 different modes of deformation imposed by multi-axial HCA experiments.

280 Nevertheless, the cross-anisotropic elastic framework provides a simple and clear way of
281 expressing potential stiffness anisotropy and the practical impact of anisotropic pre-failure
282 behaviour can be addressed analytically by incorporating a non-linear elastic approach into
283 conventional elastic-plastic models (Addenbrooke et al. 1997). Brosse (2012) retained this
284 framework and the constitutive Equation 6 given below for her test interpretation, while also
285 highlighting aspects of her experiments that deviated from this simple theory. In Equation 6,
286 E , G and μ are respectively the Young's moduli, the shear moduli and the Poisson's ratio and

can be either effective or undrained parameters depending on the set of stresses used in Equation 6 (effective or total stresses respectively). The subscripts v and h represent the vertical and horizontal directions respectively. In the present study, it is assumed that the horizontal directions x and y can be interchanged with the radial and circumferential directions due to the axis symmetry of the model. As demonstrated by Love (1927), the relationships given in Equations 7 and 8 exist between cross-anisotropic material parameters and consequently only 5 independent parameters are required to fully describe the soil behaviour in the drained case.

$$[6] \quad \begin{Bmatrix} \delta \varepsilon_r \\ \delta \varepsilon_\theta \\ \delta \varepsilon_z \\ \delta \gamma_{\theta z} \\ \delta \gamma_{zr} \\ \delta \gamma_{r\theta} \end{Bmatrix} = \begin{bmatrix} \frac{1}{E_h} & -\frac{\mu_{hh}}{E_h} & -\frac{\mu_{vh}}{E_v} & 0 & 0 & 0 \\ -\frac{\mu_{hh}}{E_h} & \frac{1}{E_h} & -\frac{\mu_{vh}}{E_v} & 0 & 0 & 0 \\ -\frac{\mu_{hv}}{E_h} & -\frac{\mu_{hv}}{E_h} & \frac{1}{E_v} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{G_{hv}} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{G_{vh}} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{2(1+\mu_{hh})}{E_h} \end{bmatrix} \begin{Bmatrix} \delta \sigma_r \\ \delta \sigma_\theta \\ \delta \sigma_z \\ \delta \tau_{\theta z} \\ \delta \tau_{zr} \\ \delta \tau_{r\theta} \end{Bmatrix}$$

$$[7] \quad \frac{\mu_{vh}}{E_v} = \frac{\mu_{hv}}{E_h}$$

$$[8] \quad G_{hh} = \frac{E_h}{2(1+\mu_{hh})}$$

Up to four sets of independent stress/strain relationships can be measured in HCA tests. Resolving the material parameters from such measurements usually requires that specific stress paths are followed to obtain a given parameter. For example, stress probe tests often measure how the four strain components respond to changes applied to just one stress component at a time. However, in the undrained case, the soil deforms at constant volume

and the additional relationships given in Equations 9 and 10 apply (Gibson 1974). The number of independent parameters required to describe the constitutive relationship therefore reduces to three. The superscript U designates undrained in the following parameter sets.

$$[9] \quad \mu_{vh}^U = \frac{1}{2}$$

$$[10] \quad \mu_{hh}^U = 1 - \mu_{hv}^U = 1 - \frac{1}{2} \frac{E_h^U}{E_v^U}$$

Combining Equations 6, 9 and 10 and moving to axially symmetric coordinates gives the relationships in Equations 11 to 14. Applying these expressions to the present undrained HCA tests made it possible to resolve the variation of the secant values of the three independent parameters E_v^U , E_h^U and G_{vh} from the stresses imposed and the strains measured.

$$[11] \quad E_v^U = \frac{2\delta\sigma_z - \delta\sigma_r - \delta\sigma_\theta}{2\delta\varepsilon_z}$$

$$[12] \quad E_{hr}^U = \frac{\delta\sigma_r(\delta\sigma_r - 2\delta\sigma_z) - \delta\sigma_\theta(\delta\sigma_\theta - 2\delta\sigma_z)}{\delta\varepsilon_r(\delta\sigma_r + \delta\sigma_\theta - 2\delta\sigma_z) + \delta\varepsilon_z(\delta\sigma_\theta - \delta\sigma_z)}$$

$$[13] \quad E_{h\theta}^U = \frac{\delta\sigma_r(\delta\sigma_r - 2\delta\sigma_z) - \delta\sigma_\theta(\delta\sigma_\theta - 2\delta\sigma_z)}{\delta\varepsilon_\theta(\delta\sigma_r + \delta\sigma_\theta - 2\delta\sigma_z) + \delta\varepsilon_z(\delta\sigma_r - \delta\sigma_z)}$$

$$[14] \quad G_{vh} = \frac{\delta\tau_{z\theta}}{\delta\gamma_{z\theta}}$$

For transversely isotropic media, the horizontal Young's moduli deduced from radial strain measurements (E_{hr}^U) should be equal to those deduced from circumferential strain measurements ($E_{h\theta}^U$) as the properties are considered isotropic within the horizontal plane. In the present paper, the two values obtained independently from Equations 12 and 13 were averaged to provide the best estimates for E_h^U . The E_v^U and E_h^U parameters could be resolved in all the tests, while G_{vh} could only be resolved in those involving continuous rotations of

the principal stress axes, as in the $\alpha_{d\sigma} = 23^\circ, 45^\circ$ and 67° experiments. Whenever possible, independent measurements of G_{vh} were also made during shearing with the ICRCHCA's built in resonant column system.

Experimental outcomes

Data processing

The main focus of this paper is on the anisotropic undrained stiffness characteristics observed in the HCA experiments conducted on the four clays. Figures 4a and 4b present one typical test that involved principal stress rotation to $\alpha_f = 23^\circ$ with $b = 0.5$ on Oxford clay, showing how the key stress and strain components varied over its full shearing stage. Applying Equations 11 to 13 gave the two undrained overall (elastic-plastic) secant Young's moduli trends with strain displayed in Figure 5. The results indicate an anisotropic response, but the degree of instrument scatter experienced at very small strains led to uncertainty about the, presumably linear, initial stiffness response. The impact was greatest for the horizontal undrained Young's moduli, because their derivation depends on two strain variables while the E_v^U and G_{vh} stiffness expressions only rely on one strain component each.

Further post processing of the raw data has been carried out that involved:

- Least square linear regressions over the strain range up 0.005 % in order to determine best fitting initial E_{v0}^U , E_{h0}^U and G_{vh0} values. These moduli do not necessarily coincide with true elastic values but define the parameters up to the lowest strain limit at which stiffness could be defined meaningfully given the available strain measurement resolution and signal scatter. It is possible that the stiff clays' engaged their Y_1 limits at smaller strains.
- A Savitzky–Golay filter was applied over the next decadal strain range (from 0.005% to 0.05%) to the ε_r , ε_θ , σ_z , σ_r and σ_θ data sets. The filter performed a piece wise fitting

of polynomial functions by least square regression; order 2 functions were adopted in this study. This approach, among those attempted, led to the most consistent smoothing and clearest presentation of the data. It should be noted, however, that adopting other appropriate filtering methods should lead to the same overall conclusions regarding the non-linear stiffness of the four clays studied.

- No further processing was applied to the larger strain data.

Although laborious, the reprocessing led to the clearer non-linear stiffness trends exhibited as black continuous curves in Figure 5, where the initial linear regression best fit stiffness values for test OA2305 are also shown.

A key consideration when reporting the non-linear anisotropy is the choice of strain parameter with which each elastic-plastic stiffness trend should be correlated. In conventional triaxial tests, vertical stiffness degradation is usually correlated with increasing axial strain ε_z , or shear strain invariant $\varepsilon_s = 2(\varepsilon_1 - \varepsilon_3)/3$, which equals ε_z in undrained tests. General invariant measures of deviatoric strain are often employed to define octahedral shear stiffness variations in numerical analysis; see for example Potts and Zdravkovic (1999), (2001). It is less clear how anisotropic non-linear behaviour should be treated and Brosse (2012) assessed several options including: (i) plotting all stiffnesses against octahedral shear strain, (ii) relating a given stiffness with the “predominant” influential strain (for example E_v^U with ε_z), and (iii) other combinations. No single choice led to an ideal synthesis of the recorded data, but the clearest trends were found by correlating the secant vertical, horizontal and torsional shear stiffnesses with the ε_z , $(\varepsilon_r + \varepsilon_\theta)/2$ and $\gamma_{z\theta}$ strains respectively (Option (ii) above). Summary plots with these axes are presented in Figures 6 to 17 showing the secant stiffness trends from undrained HCA tests that brought the four clays to failure (starting from nominally K_0 initial conditions) at final σ_1 inclinations (α_f) from the vertical between 0° to 90° while maintaining $b = 0.5$. Also shown are elastic G_{vh} moduli determined from concurrent

resonant column tests, as interpreted using the Ashmawy and Drnevich (1994) three degree of freedom model, accounting for apparatus compliance.

Young's moduli degradation

Figures 6 to 13 present E_v^U and E_h^U degradation traces for each of the twenty-five constant $\alpha_{d\sigma}$ HCA tests. The Kimmeridge series is the only set in which strains were measured locally. Of these, KA0005 presents a softer E_v^U trend than its companions, probably because of a malfunction in one of its axial strain sensors. The resulting reliance on just one electro-level device rendered the stiffness calculations liable to significant potential errors in the event of any tilting or non-axial misalignment.

Truly elastic transversely isotropic materials should develop stiffness (E_v^U , E_h^U , G_{vh}) degradation curves that are independent of the shearing path ($\alpha_{d\sigma}$) and HCA test curves that lie in a narrow band for each clay. However, the experimental Young's modulus curves vary with $\alpha_{d\sigma}$ and the horizontal stiffness trends show a wider spread than the vertical equivalents. E_h^U depends on two strain measurements (ε_z and ε_r or ε_θ , see Equations 12 and 13) and this introduces greater scatter at small strains than the single measurement required for E_v^U . The deviation from elastic theory reflects the in-elastic and plastic nature of the straining developed once the very small strains required to engage the Y_1 surface have been exceeded. While unique elastic E_{vmax} , E_{hmax} and G_{vhmax} may well apply within Y_1 for each clay, these values could not be resolved statically in most of the HCA tests described herein.

Close inspection shows no overall systematic trend of variation with $\alpha_{d\sigma}$ in the Young's moduli stiffness curves. The only exceptions are the $\alpha_f = 67^\circ$ experiments, whose E_h^U and E_v^U curves consistently fall below their companions. Brosse (2012) reviewed possible reasons for this outcome. Firstly, while random heterogeneity is always present within natural samples, it is highly unlikely to be the cause of such a consistent pattern in the four soils. In the same way, it is true that the $\alpha_f = 67^\circ$ experiments all predominantly involved torsion and only

396 minor axial and horizontal straining, creating scope for the calculated moduli to be affected
 397 significantly by even minor cross sensitivity between the torsional and other strain
 398 components. However, the same features apply to the $\alpha_f = 45^\circ$, which do not present similarly
 399 anomalous E_h^U and E_v^U stiffness degradation curves. Brosse (2012) concluded that the
 400 'unusually soft' $\alpha_f = 67^\circ$ trends most likely result from the relationship between the in-situ
 401 stress state, the stress path direction and the clays' anisotropic Y_3 yield surfaces. The
 402 reconsolidation paths took the effective stress points relatively close to passive failure. As
 403 noted earlier and shown in Tables 3 to 6, the clays' have anisotropic undrained strengths S_u
 404 that often present relatively low S_u in tests that apply torsion. Figure 18 presents the Oxford
 405 clay tests' shear strength envelope in $\tau_{z\theta} - (\sigma_z - \sigma_\theta)/2$ space. Shearing to $\alpha_f = 67^\circ$ involved
 406 applying a combination of extension and torsional straining. As illustrated in Figure 18, the
 407 combination of (i) the shape of the Y_3 yield surfaces and (ii) the starting in-situ stress state led
 408 to the $\alpha_f = 67^\circ$ stress path offering the shortest and most direct route towards the failure
 409 envelope, explaining the faster development of strain, as shown by the strain contours in
 410 Figure 18. Brosse (2012) concluded that combining these factors led to the clays' Y_1 , Y_2 and
 411 Y_3 surfaces being engaged at particularly early stages in all $\alpha_f = 67^\circ$ tests.

412 Non-linear analyses of practical boundary value problems are often modelled with parameters
 413 calibrated against stiffness degradation curves from triaxial compression tests; Jardine et al.
 414 (2005). Comparison of the HCA E_v^U results with curves from triaxial compression tests
 415 starting from similar stress levels indicates that the latter fall towards the upper range of the
 416 HCA tests; Brosse (2012). Analyses using isotropic non-linearity calibrated against triaxial
 417 compression curves may then over-estimate the E_v^U stiffness response, while under-
 418 estimating E_h^U values over the operational strain range of even routine geotechnical
 419 engineering applications that involve vertical and lateral loading as well as principal stress
 420 axis rotation, such as embankments, footings or excavations; Jardine (1994).

421 *Stiffness anisotropy*

422 Comparison the E_v^U and E_h^U degradation curves reveals a hierarchy of stiffness anisotropy
423 among the four heavily over-consolidated stiff clays. The degree of anisotropy can be
424 quantified by the evolution of the E_h^U/E_v^U ratio with strain as presented in Figure 19 for $\alpha_f =$
425 0° , 45° and 90° (23° and 67° were omitted as their curves are similar to the 0° and 90°
426 respectively). The ratio between the two Young's moduli remains relatively stable up to at
427 least 0.01% strain and, while varying at larger strain, it remains above unity up to 0.1% strain
428 (except for Kimmeridge clay). As expected for these heavily over-consolidated deposits, all
429 four clays are stiffer in the horizontal direction than in the vertical. The largest variations in
430 E_h^U/E_v^U are observed in $\alpha_f = 45^\circ$ tests. Oxford clay presents the highest ratio (2-3)
431 independently of the direction of shearing, while Kimmeridge shows the lowest (1-1.5) and
432 the Gault and London clays exhibit intermediate degrees of stiffness anisotropy. The linearly
433 fitted, initial average stiffness values for each soil (excluding the anomalous $\alpha_f = 67^\circ$ tests)
434 given in Table 9 reflect the same hierarchy of stiffness anisotropy.

435 The exact reasons for the observed hierarchy of stiffness anisotropy are not fully clear. The
436 clays' particle-scale properties and micro-fabrics are likely to be influential; see Nishimura
437 (2014a). However, meso-structure or fissuring does not appear to affect stiffness greatly,
438 provided the test specimens are under sufficient confinement to keep any discontinuities fully
439 closed; see Costa-Filho (1980) or Gasparre et al. (2007b). In the same way, stress-induced
440 anisotropy does not appear to have a major influence under the conditions investigated, as the
441 geologically aged clays' stiffnesses are relatively insensitive to pre- Y_3 effective stress
442 changes; Hosseini-Kamal (2012). While all four clays were tested after consolidation to
443 similar K_0 s and effective stress states, they manifested a wide range of degrees of stiffness
444 anisotropy.

It is noteworthy that this hierarchy appears to correlate well with the strength anisotropy observed in terms of peak ϕ' (or t/s' ratio) with Oxford clay presenting the highest $(t/s'_{max})/(t/s'_{min})$ ratio at peak, Kimmeridge the lowest and Gault and London clays in between values.

Torsional shear stiffness degradation curves

Secant torsional shear stiffness traces of $G_{z\theta}$ (i.e. G_{vh}) with $\gamma_{z\theta}$ are given in Figures 14 to 17 from the three torsional tests in the 'constant $\alpha_{d\sigma} - b = 0.5$ ' series on each clay. Also shown are the curves from HCA simple shear tests that started from the same estimated in-situ stresses. The resulting four groups of $G_{z\theta} - \gamma_{z\theta}$ curves fall in a narrow band for each soil demonstrating a convincing correlation that is not influenced systematically by the other strain components or the direction of shearing $\alpha_{d\sigma}$. Comparing the static G_{vh0} values with the E_{v0}^U values in Table 9 shows that G_{vh0}/E_{v0}^U falls close to the 1/3 ratio expected for an isotropic soil. Nishimura (2014b) found the same result in tests on six sedimentary clays, with apparently little sensitivity to applied stress states.

The agreement between tests' $G_{z\theta}$ curves is encouraging. The ICRCHCA tests' all relied on platen-to-platen $\gamma_{z\theta}$ measurements. The Kimmeridge clay's ICHCA II trends, which were derived from local shear strain measurements, agree closely with that from the ICRCHCA simple shear test, verifying that platen-to-platen $\gamma_{z\theta}$ measurements match the local shear strain response. Also presented in Figures 14 to 17 are the dynamic resonant column system G_{vhmax} measurements, which involve far smaller strain perturbations than the static torsional shear tests and relatively fast (≈ 150 Hz) cycling. If the material behaviour was rate independent and elastic then the RC G_{vhmax} measurements and static G_{vh0} values should converge. However, the latter are lower, typically by around 15% to 25%, than the former. Possible explanations include: the static tests underestimating true elastic values that may apply at strains too small

to resolve reliably in the HCAs, rate and frequency effects, or imperfections in RC interpretation theory. For comparison, bender elements tests carried out on triaxial samples tended to give lower values than the RC, except for Oxford clay where they were about 20% higher.

The G_{vhmax} determinations made at various stages of static shearing indicate a gradual and limited reduction as shearing progresses that apparently reverses after about 1% torsional shear strain. However, shear bifurcations usually started to appear at such strains that invalidate the assumption of sample continuity on which the resonant column test interpretation relies.

Summary and conclusions

This paper has outlined how anisotropic non-linear stiffness characteristics were investigated and evaluated from suites of 25 undrained HCA tests on high quality natural samples of four geologically old UK stiff clays. The key conclusions are:

1. Manipulation of the theory for transversely isotropic elastic media led to simple expressions for the E_v^U , E_h^U and G_{vh} moduli that could be evaluated from the main run of 21 HCA tests that progressed from K_0 initial conditions to failure at a range of σ_1 orientations while maintaining $b = 0.5$. G_{vh} moduli could also be assessed from four accompanying HCA simple shear tests.
2. The clays all showed steep falls in secant stiffness with strain. The non-linearity of their responses was characterised by correlating the observed degradation of E_v^U , E_h^U and G_{vh} with the measured ε_z , $(\varepsilon_r + \varepsilon_\theta)/2$ and $\gamma_{z\theta}$ strains respectively.
3. The initial segments of the stiffness strain curves were affected by significant scatter. The small-strain stiffness measurements were enhanced by applying linear stress-strain fitting up to 0.005% strain and then further filtering including polynomial fitting over the next decade of strain change. It may not have been possible with the

- instruments deployed to resolve the initial (within Y_1) linear stiffnesses exactly, but the processing clarified the vertical and torsional shear data sets considerably. The small-strain horizontal stiffnesses remained subject to greater spreading because they relied on two independent sets of scattering strain measurements.
4. The resulting trends indicated a hierarchy of non-linear stiffness anisotropy in the four clays, which was generally independent of the σ_l orientations followed in each test. Oxford clay presents the highest E_h^U/E_v^U ratio and Kimmeridge clay the lowest. Gault and London clays exhibit intermediate degrees of stiffness anisotropy.
 5. The secant stiffness E_h^U and E_v^U degradation curves varied with $\alpha_{d\sigma}$ and so deviated from the behaviour expected for ideal non-linear cross-anisotropic materials, reflecting the plastic nature of the straining developed once the Y_1 and Y_2 kinematic yield surfaces have been engaged. There was no apparent overall systematic trend of variation with shearing direction, except for the $\alpha_f = 67^\circ$ direction experiments, which always presented a softer, more steeply degrading curve than tests with other α_f angles. This trend was attributed to the $\alpha_f = 67^\circ$ paths engaging the anisotropic Y_1 to Y_3 yield surfaces earlier than the other modes as a result of the anisotropic shape of the yielding surfaces.
 6. The torsional shear stiffness data presented the least scatter and the clearest trends, indicating $G_{z\theta} - \gamma_{z\theta}$ trends that were practically independent of the shearing stress paths. However, the interpreted initial static (linear) values of $G_{z\theta}$ typically fell 15 to 25% below the 'elastic' values found at much smaller strains in concurrent high frequency resonant column tests. This may be due to: a possibly very limited elastic strain range, frequency dependency in the four clays tested or imperfections in RC interpretation procedures.

518 7. The strong non-linearity and anisotropy revealed by the HCA tests has important
519 implications for engineering practice in the strata studied; affecting ground movement
520 predictions and soil-structure interaction responses in applications ranging from
521 tunnelling to the settlements and group action of friction piles; Jardine et al. (2005).
522 The above experimental and interpretative approaches may be refined if higher resolution
523 static strain measurement systems can be deployed to bring still greater convergence between
524 static and dynamic measurements and improve stiffness definition (especially for E_h^U) at very
525 small strains.

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List of symbols

α	Angle between the vertical and the direction of the major principal stress σ_1
$\alpha_{d\sigma}$	Angle between the vertical and the direction of the increment in $\Delta\sigma_1$
α_f	Angle between the vertical and the direction of σ_1 axis at peak q
$\varepsilon_1, \varepsilon_2, \varepsilon_3$	Major, intermediate and minor principal strains respectively
$\varepsilon_z, \varepsilon_r, \varepsilon_\theta$	Axial, radial and circumferential strains respectively
γ	Bulk unit weight
$\gamma_{z\theta}$	Shear strain in the axial-circumferential plane
ϕ'	Effective angle of shearing resistance
$\sigma_1, \sigma_2, \sigma_3$	Major, intermediate and minor principal stresses respectively
$\sigma_z, \sigma_r, \sigma_\theta$	Axial, radial and circumferential stresses respectively
μ_{vh}	Poisson ratio for the influence of increments of vertical stress on horizontal strain
μ_{hv}	Poisson ratio for the influence of increments of horizontal stress on vertical strain
μ_{hh}	Poisson ratio for the influence of increments of horizontal stress on horizontal strain
$\tau_{z\theta}, \tau_{zr}, \tau_{r\theta}$	Shear stresses in the axial-circumferential, axial-radial and radial-circumferential planes respectively
B	Coefficient of saturation
b	Intermediate principal stress factor
e	Void ratio
E_v, E_h	Vertical and horizontal Young's moduli respectively
G_{hh}	Shear modulus in horizontal plane

G_{hv}, G_{vh}	Shear moduli in vertical plane
$G_{z\theta} (=G_{vh})$	Shear moduli in the axial-circumferential (vertical) plane
HCA	Hollow Cylinder Apparatus
ICRCHCA	Imperial College Resonant Column Hollow Cylinder Apparatus
ICHCA II	Imperial College Hollow Cylinder Apparatus Mark II
K_0	Coefficient of earth pressure at rest
OCR	Overconsolidation Ratio
p'	Mean effective stress
p'_0	In-situ mean effective stress
PL, LL	Plastic Limit, Liquid Limit
q	Deviatoric stress $(=\sigma'_1 - \sigma'_3)$
s'	$(\sigma'_1 + \sigma'_3)/2$
S_u	Undrained shear strength
SBPM	Self-Boring Pressuremeter
t	$(\sigma'_1 - \sigma'_3)/2$
TC	Triaxial Compression test
TE	Triaxial Extension test
v	Specific Volume

Figures Captions

Figure 1. Schematic diagram of the Imperial College Resonant Column Hollow Cylinder Apparatus - ICRCHCA (modified from Nishimura 2006; Brosse 2012)

Figure 2. Schematic diagram of the Imperial College Hollow Cylinder Apparatus Mark II - ICHCA II (Anh-Minh 2007)

Figure 3. Schematic of the undrained stress paths followed during shearing from in-situ stresses in the HCA ‘constant $\alpha_{d\sigma} - b=0.5$ ’ test series.

Figure 4. Strain and stress variations with time during test OA2305 ($\alpha_f = 23^\circ$ at $b = 0.5$) – (a) Strains; (b) Stresses

Figure 5. Comparison of the undrained Young’s moduli degradation curves before and after post-processing of the data for OA2305 ($\alpha_f = 23^\circ$ at $b = 0.5$).

Figure 6. Comparison of secant E_v^U degradation curves at different α_f for Oxford clay test series.

Figure 7. Comparison of secant E_h^U degradation curves at different α_f for Oxford clay test series.

Figure 8. Comparison of secant E_v^U degradation curves at different α_f for Kimmeridge clay test series.

Figure 9. Comparison of secant E_h^U degradation curves at different α_f for Kimmeridge clay test series.

Figure 10. Comparison of secant E_v^U degradation curves at different α_f for Gault clay test series.

Figure 11. Comparison of secant E_h^U degradation curves at different α_f for Gault clay test series.

Figure 12. Comparison of secant E_v^U degradation curves at different α_f for London clay test series.

Figure 13. Comparison of secant E_h^U degradation curves at different α_f for London clay test series.

Figure 14. Comparison of secant $G_{z\theta}$ degradation curves at different α_f for Oxford clay test series.

Figure 15. Comparison of secant $G_{z\theta}$ degradation curves at different α_f for Kimmeridge clay test series.

Figure 16. Comparison of secant $G_{z\theta}$ degradation curves at different α_f for Gault clay test series.

Figure 17. Comparison of secant $G_{z\theta}$ degradation curves at different α_f for London clay test series.

Figure 18: Example of strain contours and strength envelopes in $\tau_{z\theta}-(\sigma_z-\sigma_\theta)/2$ space for the Oxford clay case. The envelopes and contours are illustrated by fitted ellipses.

Figure 19: Comparison of the variation of the Young's moduli ratio E_h^U/E_v^U with axial strain for the different soils (a) at $\alpha_f = 0^\circ$ (b) at $\alpha_f = 45^\circ$ (c) at $\alpha_f = 90^\circ$.

Figure 1. Schematic diagram of the Imperial College Resonant Column Hollow Cylinder Apparatus - ICRCHCA (modified from Nishimura, 2006; Brosse, 2012)

Figure 1
165x263mm (300 x 300 DPI)

Figure 2. Schematic diagram of the Imperial College Hollow Cylinder Apparatus Mark II - ICHCA II (Anh-Minh, 2007)

Figure 2
225x400mm (300 x 300 DPI)

Schematic of the undrained stress paths followed during shearing from in-situ stresses in the HCA 'constant $\alpha_{d\sigma} - b=0.5$ ' test series.

Figure 3
130x85mm (300 x 300 DPI)

Figure 4. Strain and stress variations with time during test OA2305 ($\alpha_f = 23^\circ$ at $b = 0.5$) – (a) Strains; (b) Stresses

Figure 4
236x309mm (300 x 300 DPI)

Figure 5. Comparison of the undrained Young's moduli degradation curves before and after post-processing of the data for OA2305 ($\alpha_f = 23^\circ$ at $b = 0.5$).

Figure 5
120x80mm (300 x 300 DPI)

Figure 6. Comparison of secant E_v^U degradation curves at different α_f for Oxford clay test series.

Figure 6
120x80mm (300 x 300 DPI)

Figure 7. Comparison of secant E_h^U degradation curves at different α_r for Oxford clay test series.

Figure 7
120x80mm (300 x 300 DPI)

Figure 8. Comparison of secant E_v^U degradation curves at different α_f for Kimmeridge clay test series.
Figure 8
120x80mm (300 x 300 DPI)

Figure 9. Comparison of secant E_h^U degradation curves at different α_f for Kimmeridge clay test series.

Figure 9
120x80mm (300 x 300 DPI)

Figure 10. Comparison of secant E_v^U degradation curves at different α_f for Gault clay test series.
Figure 10
120x80mm (300 x 300 DPI)

Figure 11. Comparison of secant E_h^U degradation curves at different α_f for Gault clay test series.

Figure 11
120x80mm (300 x 300 DPI)

Figure 12. Comparison of secant E_v^U degradation curves at different α_f for London clay test series.
Figure 12
120x80mm (300 x 300 DPI)

Figure 13. Comparison of secant E_h^U degradation curves at different α_f for London clay test series.

Figure 13
 120x80mm (300 x 300 DPI)

Figure 14. Comparison of secant $G_{z\theta}$ degradation curves at different α_r for Oxford clay test series.

Figure 14

120x80mm (300 x 300 DPI)

Figure 15. Comparison of secant $G_{z\theta}$ degradation curves at different α_f for Kimmeridge clay test series.

Figure 15
120x80mm (300 x 300 DPI)

Figure 16. Comparison of secant $G_{z\theta}$ degradation curves at different α_r for Gault clay test series.
Figure 16
120x80mm (300 x 300 DPI)

Figure 17. Comparison of secant $G_{z\theta}$ degradation curves at different α_f for London clay test series.

Figure 17
120x80mm (300 x 300 DPI)

Figure 18: Example of strain contours and strength envelopes in $\tau_{z\theta}-(\sigma_z-\sigma_\theta)/2$ space for the Oxford clay case. The envelopes and contours are illustrated by fitted ellipses.

Figure 18
99x54mm (300 x 300 DPI)

Figure 19: Comparison of the variation of the Young's moduli ratio E_h^U/E_v^U with axial strain for the different soils (a) at $\alpha_f = 0^\circ$ (b) at $\alpha_f = 45^\circ$ (c) at $\alpha_f = 90^\circ$.

Figure 19

236x309mm (300 x 300 DPI)

Tables

Table 1. Soil description and estimated in-situ stresses

Soil	Age [Mya]	In-situ stresses			Micro-structure ^b	Meso-structure ^c
		K_0^a	p'_0	$\sigma'_{v0}-\sigma'_{h0}$		
			[kPa]	[kPa]		
Oxford clay	Upper Jurassic 161-156	3.2 (3-7)	250	-223	Strong horizontal orientation of particles and shells	No significant fissuring but highly bedded. Horizontal shell beds.
Kimmeridge clay	Upper Jurassic 156-151	1.8 (1.5-3)	186	-97	No strong preferred orientation due to presence of larger silt particles	Close spaced fissuring (2-5cm spacing) sub-horizontal & 40°-60° to Vertical
Gault clay	Lower Cretaceous 112-99	1.8 (1.5-3)	163	-85	No strong preferred orientation due to presence of larger micro-fossils	Close spaced fissuring (spacing of 2-5cm) sub-horizontal and sub-vertical with frequent zones of intense fissuring. Presence of nodules
London clay (Unit B2c)	Eocene 56-49	1.8 (1.5-3)	323	-165	Moderate particle orientation	Highly fissured (spacing typically 10-20cm) in sub-vertical and sub-horizontal directions

^a The estimated chosen value is given on the first line. The range obtained from a variety of methods is indicated in brackets. Note that in the case of Oxford clay, the estimated value could not be reached without over-straining the sample and so a K_0 of 1.8 was used ultimately (see Table 3).

^b Wilkinson (2011)

^c Hosseini-Kamal et al. (2014)

Table 2. Basic index and strength properties

Soil	Water content	Plastic Limit	Liquid Limit	Plastic index	S_u	Residual ϕ'
	[%]	[%]	[%]	[%]	[kPa]	[Deg]
Oxford clay	24-27	32-35	64-67	30-34	200-300	10
Kimmeridge clay	16-21	18-22	46-55	28-33	80-100	7
Gault clay	28-30	28-31	71-77	40-46	70-90	10
London clay (Unit B2c)	23-25	26-27	68-70	41-44	120-170	12

Table 3. Summary of HCA tests for Oxford clay series

Test reference	Sample type	Specimen depth [mBGL]	Apparatus	Type of test ^a	Initial stresses		α_f^b [Deg]	b [-]	$\alpha_{d\sigma}$ [Deg]	S_u [kPa]
					p'_0 [kPa]	$\sigma'_{v0}-\sigma'_{h0}$ [kPa]				
OA0005	block	10	ICRCHCA	$\alpha_{d\sigma}$	250	-130	0	0.5	0	175
OA2305	block	10	ICRCHCA	$\alpha_{d\sigma}$	250	-130	22	0.5	15	145
OA4505	block	10	ICRCHCA	$\alpha_{d\sigma}$	252	-125	43	0.5	30	127
OA6705	block	10	ICRCHCA	$\alpha_{d\sigma}$	250	-130	69	0.5	57	145
OA9005	block	10	ICRCHCA	$\alpha_{d\sigma}$	252	-125	90	0.5	90	260
OA0005*	block	10	ICRCHCA	$\alpha_{d\sigma}$	252	-125	0	0.5	0	159
OASS	block	10	ICRCHCA	SS	250	-130	57	-	-	112

^a $\alpha_{d\sigma}$ refers to shearing at constant direction of principal stress increments $\alpha_{d\sigma}$; SS to simple shear tests.

^b α_f is the α angle at peak q .

Table 4. Summary of HCA tests for Kimmeridge clay series

Test reference	Sample type	Specimen depth [mBGL]	Apparatus	Type of test ^a	Initial stresses		α_f^b [Deg]	b [-]	$\alpha_{d\sigma}$ [Deg]	S_u [kPa]
					p'_0 [kPa]	$\sigma'_{v0}-\sigma'_{h0}$ [kPa]				
KA0005	rotary	11.45	ICHCA II	$\alpha_{d\sigma}$	186	-97	0	0.5	0	100
KA2305	rotary	11.17	ICHCA II	$\alpha_{d\sigma}$	186	-97	21	0.5	14	97
KA4505	rotary	11.52	ICHCA II	$\alpha_{d\sigma}$	186	-97	40	0.5	30	126
KA6705	rotary	11.94	ICHCA II	$\alpha_{d\sigma}$	186	-97	63	0.5	55	161
KA9005	rotary	11.69	ICHCA II	$\alpha_{d\sigma}$	186	-97	90	0.5	90	175
KASS	rotary	8.98	ICRCHCA	SS	186	-97	53	-	-	151

^a $\alpha_{d\sigma}$ refers to shearing at constant direction of principal stress increments $\alpha_{d\sigma}$; SS to simple shear tests.

^b α_f is the α angle at peak q .

Table 5. Summary of HCA tests for Gault clay series

Test	Sample	Specimen	Apparatus	Type	Initial stresses		α_f^b	b	$\alpha_{d\sigma}$	S_u
reference	type	depth		of	p'_0	$\sigma'_{v0} - \sigma'_{h0}$				
		[mBGL]		test ^a	[kPa]	[kPa]	[Deg]	[-]	[Deg]	[kPa]
GA0005	rotary	9.50	ICRCHCA	$\alpha_{d\sigma}$	163	-85	0	0.5	0	106
GA2305	rotary	9.76	ICRCHCA	$\alpha_{d\sigma}$	163	-85	21	0.5	15	104
GA4505	rotary	11.77	ICRCHCA	$\alpha_{d\sigma}$	163	-85	39	0.5	28	94
GA6705	rotary	10.60	ICRCHCA	$\alpha_{d\sigma}$	163	-85	65	0.5	55	112
GA9005	rotary	10.30	ICRCHCA	$\alpha_{d\sigma}$	163	-85	90	0.5	90	126
GASS	rotary	10.85	ICRCHCA	SS	163	-85	53	-		99

^a $\alpha_{d\sigma}$ refers to shearing at constant direction of principal stress increments $\alpha_{d\sigma}$; SS to simple shear tests.

^b α_f is the α angle at peak q .

Table 6. Summary of HCA tests for London clay series

Test	Sample	Specimen	Apparatus	Type	Initial stresses		α_f^b	b	$\alpha_{d\sigma}$	S_u
reference	type	depth		of	p'_0	$\sigma'_{v0} - \sigma'_{h0}$				
		[mBGL]		test ^a	[kPa]	[kPa]	[Deg]	[-]	[Deg]	[kPa]
LA0005	block	10.5 ^a	ICRCHCA	$\alpha_{d\sigma}$	323	-165	0	0.5	0	159
LA2305	block	10.5	ICRCHCA	$\alpha_{d\sigma}$	323	-165	23	0.5	15	152
LA4505	block	10.5	ICRCHCA	$\alpha_{d\sigma}$	323	-165	48	0.5	30	121
LA6705	block	10.5	ICRCHCA	$\alpha_{d\sigma}$	323	-165	67	0.5	55	191
LA9005	block	10.5	ICRCHCA	$\alpha_{d\sigma}$	323	-165	90	0.5	90	242
LASS	block	10.5	ICRCHCA	SS	323	-165	53	-	-	122

^a The calculated in-situ stresses take into account an extra 5.8m layer of gravel that was removed about 70 years prior to sampling (see Nishimura et al. 2007).

^b $\alpha_{d\sigma}$ refers to shearing at constant direction of principal stress increments $\alpha_{d\sigma}$; SS to simple shear tests.

^c α_f is the α angle at peak q .

Table 7. Range for suction measurements, slope dq/dp' in triaxial compression tests, and the resulting estimated K_0 in for each soil.

Soil	Suction range	Slope dq/dp'	K_0 range
	[kPa]	[-]	[kPa]
Oxford	200 - 220	-2.15 – -2.75	3.9 - 6.5
Kimmeridge	136 - 190	-2.4 – -2.75	1.4 - 3.2
Gault	95 - 135	-2.5 – -2.75	0.6 - 2
London ^a	200-260	Infinity (Isotropic)	1.8 - 2

^a Gasparre (2005) – suction measurements on thin-wall samples.

Table 8. Estimated strain resolution based on the resolution of the instrumentation

Strain	ICRCHCA Resolution [%] $\times 10^{-3}$	ICHCA II (Global) Resolution [%] $\times 10^{-3}$	ICHCA II (Local) Resolution [%] $\times 10^{-3}$
ε_z	0.22	0.24	0.88
ε_r	0.34	0.27	0.15
ε_θ	0.25	0.32	0.25
$\gamma_{z\theta}$	0.19	3.5	0.19
ε_v	0.16	0.2	1.38

Table 9. Initial stiffnesses calculated by linear regression up to 0.005% strain – average for each soil.

Soil	p'_0 [kPa]	E_{v0}^U [Mpa]	E_{h0}^U [Mpa]	G_{vh0} [Mpa]	E_{h0}^U/E_{v0}^U [-]	$3*G_{vh0}/E_{v0}^U$ [-]
Oxford	250	287	655	69	2.3	0.72
Kimmeridge	186	229	270	56	1.2	0.73
Gault	163	173	377	62	2.2	1.07
London	323	212	335	71	1.6	1.00