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Bulk Viscosity in $F(T, T_G)$ Gravity

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Abstract

The present paper is devoted to explore the effect of bulk viscosity in the context of $F(T, T_G)$ gravity. We consider a time dependent viscosity model with a particular expression of Hubble parameter. We evaluate viscous effective equation of state parameter for three well-known $F(T, T_G)$ models. The behavior of the accelerated expanding universe is explored through the viscous equation of state parameter graphically. This parameter indicates the phantom dominated era as well as crosses the phantom divide line for all the three models. We conclude that the universe shows a transition from quintessence to phantom region in the presence of bulk viscosity.

Keywords: $F(T, T_G)$ gravity; Dark energy; Bulk viscosity.

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1 Introduction

The most remarkable achievement of cosmology is the accelerated expansion of the universe confirmed through various cosmological observations. It is observed that rapid expansion of the universe is the result of an unusual type of energy referred as dark energy (DE). This strange type of energy is equally

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dispersed in the universe whose nature is uncertain [1]. Many models of DE have been proposed to explain its nature such as quintessence, k-essence, perfect fluid, pilgrim DE models and so forth [2]. It is found that early inflation and recent expansion of the universe can be investigated through imperfect fluid without using cosmological constant [3]. Viscosity is basically a measurement of the fluid's resistance to flow and is classified into two types namely bulk and shear viscosity. Usually, bulk viscosity is related with isotropic universe while shear viscosity corresponds to anisotropic universe.

Bulk viscosity plays an important role in cosmic expansion as well as in pressure. This is considered as the result of its deviation from local thermodynamic equilibrium [4]. When the expansion (or contraction) of a system is so fast that it could not bring back the system to its equilibrium position for a small time interval, then there arises an effective pressure to balance its thermal equilibrium. Bulk viscosity is the amount of this effective pressure which disappears when the system tries to maintain its thermal equilibrium.

The idea of viscous DE models has been presented in different ways to understand evolution of the universe. Ren and Meng [5] have investigated the universe evolution through cosmological model with bulk viscosity. They described the model through an effective equation of state (EoS) parameter and gave a comparison of their solutions with type Ia Supernovae data. Tawfik and Harko [6] discussed phase transition of the viscous early universe. The notion of bulk viscosity has also been extensively studied in modified theories. Sharif and Rani [7] discussed the bulk viscosity in $F(T)$ theory, where T is the torsion scalar. Singh and Kumar [8] explored different aspects of viscosity in $f(R, \mathcal{T})$ gravity, where R stands for the Ricci scalar and $\mathcal{T}$ denotes trace of the energy-momentum tensor. The same authors [9] explored viscous and non-viscous holographic DE models in this theory.

The $F(T, T_G)$ gravity is a modified theory obtained by involving a Gauss-Bonnet term T_G in $F(T)$ theory. Kofinas and Saridakis [10] suggested a modified theory by taking a general function of T_G in the action namely $F(T_G)$ theory. It is further modified by including a function $F(T)$ in the action. It is found that $F(R)$ does not correspond to $F(T)$ theory at the level of equations. Therefore, many modifications of $F(T)$ theory have been proposed. The $F(T, T_G)$ gravity is a completely different theory as it has no correspondence with $F(T)$ or $F(R, G)$ theory. Since, this theory entirely depends upon T and T_G , so there exists no curvature formalism.

The cosmological applications and dynamical analysis of this theory prove to be very interesting [11]. Chattopadhyay et al. [12] worked on this the-

ory by taking pilgrim DE and reconstructed $F(T, T_G)$ models in flat FRW metric. Jawad et al. [13] discussed reconstruction scenario in this theory by considering a newly proposed quantum chromodynamics ghost DE in the flat FRW universe. Jawad and Debnath [14] discussed the correspondence of this theory with a new agegraphic version of pilgrim DE. We have considered different scale factors and explored the reconstruction scheme in this theory [15]. We have also constructed $F(T, T_G)$ models to study different phases of the universe and discussed their graphical behavior [16]. Many dark energy models have been discussed in $F(G)$ gravity as well [17, 18].

In this paper, we evaluate the EoS parameter by introducing bulk viscosity in the background of $F(T, T_G)$ gravity. To this end, we consider a viscous model composed of pressureless matter. We calculate the effective EoS parameter for this model and analyze EoS parameter graphically for different suitable values of viscosity coefficients. The paper is organized as follows. In the next section, we explain the basic formalism of this theory and construct the effective EoS parameter. Section 3 deals with three different $F(T, T_G)$ models to evaluate their viscous EoS parameters and explore their behavior graphically. We summarize our results in the last section.

2 Basic Formalism

In this section, we provide a basic formalism of $F(T, T_G)$ gravity and its field equations. Also, we discuss the bulk viscosity in this framework.

2.1 A Brief Review of $F(T, T_G)$ Gravity

In $F(T, T_G)$ gravity, the action can be defined as [10, 12]

$$S = \frac{1}{2} \int d^4x h F(T, T_G) + \int d^4x h \mathcal{L}_m, \quad h = \det(h_\mu^a) = \sqrt{|g|},$$

where the torsion scalar is expressed as

$$T_G = (K^{a_1}_{ek} K^{ea_2}_l K^{a_3}_{fm} K^{fa_4}_n - 2K^{a_1a_2}_k K^{a_3}_{el} K^e_{fm} K^{fa_4}_n + 2K^{a_1a_2}_k K^{a_3}_{el} K^{ea_4}_f K^f_{mn} + 2K^{a_1a_2}_k K^{a_3}_{el} K^{ea_4}_{m,n}) \delta^{klmn}_{a_1a_2a_3a_4},$$

with $\kappa^2 \equiv 8\pi G = 1$ and $\mathcal{L}_m$ represents the matter Lagrangian. The determinant of the Kronecker deltas is described as generalized δ . The antisymmetric

contorsion tensor K_{klm} and the torsion tensor T_{lm}^k are defined as

$$\begin{aligned} K_{klm} &= \frac{1}{2}(T_{mkl} - T_{lmk} - T_{klm}), \\ T_{lm}^k &= \Gamma_{ml}^k - \Gamma_{lm}^k = h_\mu^k(\partial_l h_m^\mu - \partial_m h_l^\mu). \end{aligned}$$

Variation of the above action in terms of the vierbein leads to the following field equations

$$\begin{aligned} 2(H^{[km]l} + H^{[lk]m} - H^{[ml]k})_{,m} + 2(H^{[km]l} + H^{[lk]m} - H^{[ml]k})C_{dm}^d + (2H^{[km]d} \\ + H^{dmk})C_{md}^l + 4H^{[dl]m}C_{(dm)}^k + T_{md}^k H^{mdl} - h^{kl} + (F - TF_T \\ - T_G F_{T_G})\eta^{kl} = 0, \end{aligned} \quad (1)$$

where

$$\begin{aligned} H^{klm} &= F_T(\eta^{km}K_{dl}^l - K^{lmk}) + F_{T_G}[\epsilon^{mprt}(2\epsilon_{def}^k K_p^{le} K_{qr}^d + \epsilon_{qdef} K_p^{ke} K_r^{ld} \\ &+ \epsilon_{ef}^{kl} K_{dp}^e K_{qr}^d)K_t^{qf} + \epsilon^{mprt}\epsilon_{ed}^{kl} K_p^{fd}(K_{fr,t}^e - \frac{1}{2}K_{fq}^e C_{tr}^q) + \epsilon^{mprt}\epsilon_{df}^{ke} \\ &\times K_p^{df}(K_{er,t}^l - \frac{1}{2}K_{eq}^l C_{tr}^q)] + \epsilon^{mprt}\epsilon_{edf}^k [(F_{T_G} K_p^{le} K_r^{df})_{,t} + F_{T_G} C_{pt}^q \\ &\times K^{le}_{[q} K_{r]}^{df}], \end{aligned}$$

and

$$h^{kl} = F_T \epsilon_{dme}^k \epsilon^{labe} K_{fa}^d K_b^{fm}, \quad F_T = \frac{\partial F}{\partial T}, \quad F_{T_G} = \frac{\partial F}{\partial T_G}.$$

The antisymmetric symbol ϵ_{mprt} has $\epsilon_{1234} = 1$, $\epsilon^{1234} = -1$, otherwise 0. The line element of FRW metric is

$$ds^2 = dt^2 - a(t)^2 \sum (dx^i)^2,$$

where $a(t) = 1/1+z$ is the scale factor in terms of redshift parameter z . For this metric, the diagonal vierbein is $h_\mu^a = \text{diag}(1, a(t), a(t), a(t))$ while its dual is written as $h_a^\mu = \text{diag}(1, a^{-1}(t), a^{-1}(t), a^{-1}(t))$. We can also have

$$T = -6H^2, \quad T_G = 24H^2(\dot{H} + H^2).$$

The corresponding field equations become

$$F - T_G F_{T_G} + 24H^3 \dot{F}_{T_G} - 12H^2 F_T = 2\rho, \quad (2)$$

$$F - 4(3H^2 + \dot{H})F_T - T_G F_{T_G} - 4H \dot{F}_T + 8H^2 \ddot{F}_{T_G} + \frac{2}{3H} T_G \dot{F}_{T_G} = -2p, \quad (3)$$

where F_T and F_{T_G} express the derivatives of F with respect to T and T_G , respectively.

2.2 Bulk Viscosity

Bulk viscosity can also be described as the modification in the pressure p term by introducing a dissipative term Π such that the effective pressure p_{eff} is given by [19]

$$p_{eff} = p + \Pi,$$

where Π represents the dissipative process and is equal to $-3H\zeta(t)$. In thermodynamical systems, the effective pressure was suggested by Eckart [20] for relativistic dissipative processes. Also, $\zeta(t)$ is the bulk viscosity function. We consider $\zeta(t) > 0$ to avoid violation of the second law of thermodynamics. We consider that the fluid has bulk viscosity so that the energy-momentum tensor with viscous term can be defined as [19]

$$T_l^k = (\rho + p_{eff})u^k u_l - p_{eff}\delta_l^k.$$

Here, $u^k = (1, 0, 0, 0)$ is the four velocity and ρ is the energy density. As we know that in the background of FRW metric, there is no contribution of shear viscosity in the energy-momentum tensor but bulk viscosity appears like an effective pressure.

By assuming $p_m = 0$, the field equations (2) and (3) can be written as

$$\begin{aligned} 3H^2 &= \rho_m + \rho_T, \\ -(2\dot{H} + 3H^2) &= p_T, \end{aligned}$$

where ρ_T and p_T are the torsion contributions defined as

$$\rho_T = \frac{1}{2}(6H^2 - F + T_G F_{T_G} + 12H^2 F_T - 24H^3 \dot{F}_{T_G}), \quad (4)$$

$$\begin{aligned} p_T &= \frac{1}{2}(-2(2\dot{H} + 3H^2) + F - 4H\dot{F}_T - 4(\dot{H} + 3H^2)F_T - T_G F_{T_G} \\ &\quad + 8H^2 \ddot{F}_{T_G} + \frac{2}{3H} T_G \dot{F}_{T_G}). \end{aligned} \quad (5)$$

The EoS parameter of a perfect fluid is equal to the ratio of its pressure and density, i.e. $\omega = \frac{p}{\rho}$ while the viscous EoS parameter turns to be

$$\omega_{eff} = \frac{p_T + 3H\zeta(t)}{\rho_m + \rho_T}. \quad (6)$$

Substituting Eqs.(4) and (5) in (6), we obtain

$$\omega_{eff} = -1 + (2\rho_m - 4\dot{H} - 4\dot{H}F_T - 4H\dot{F}_T + (\frac{2}{3H}T_G - 24H^3)\dot{F}_{T_G} + 8H^2$$

$$\times \ddot{F}_{T_G} + 3H\zeta(t))(2\rho_m + 6H^2 - F + 12H^2F_T + T_GF_{T_G} - 24H^3\dot{F}_{T_G}).^{-1} \quad (7)$$

3 Viscous Fluid and $F(T, T_G)$ Models

Many authors have considered various forms of bulk viscous coefficient to discuss the cosmological models. Here we analyze the properties of DE by considering bulk viscosity coefficient with pressureless fluid [21]-[23] as

$$\zeta(t) = \zeta_0 + \zeta_1 \frac{\dot{a}}{a} + \zeta_2 \frac{\ddot{a}}{a} = \zeta_0 + \zeta_1 H + \zeta_2 (\dot{H} + H^2), \quad (8)$$

where ζ_0, ζ_1 and ζ_2 are positive coefficients. The motivation for choosing this bulk viscosity is that the viscosity phenomenon is associated with velocity and acceleration. We know that exact form of the viscosity is not known, so we discuss cosmological issues through a parameterized bulk viscosity which is a linear combination of three terms. The second term $\zeta_1 H$ describes bulk viscosity proportional to the Hubble parameter H , i.e., expansion ratio of the universe while the third term $\zeta_2 \frac{\ddot{a}}{a}$ relates bulk viscosity to accelerated expansion of the universe. The model with first two terms has been discussed in [5]. Thus, a linear combination of these two with acceleration term $\ddot{a}$ may provide better results.

To investigate the behavior of ω_{eff} , we consider a bouncing scale factor for which the Hubble parameter is defined as [24]

$$H(t) = \frac{h_0}{(t_r - t)^\beta}, \quad (9)$$

where t_r and $h_0 > 0$ are constants. Bouncing scenario presents an inflationary universe which shows a transition from contracting ($H(t) < 0$) to an expanding epoch ($H(t) > 0$). Also, $H(t)$ diverges at $t = t_r$. When $t < t_r$, the scale factor increases with positive Hubble parameter and the universe expands. For $t > t_r$, the scale factor decreases with negative Hubble parameter and it represents a contraction. Also, when we consider $\beta = 0$ in Eq.(9), $H(t)$ coincides with the de Sitter space which contains no singularity. For $\beta = 1$, the scale factor becomes $a(t) = a_1(t_r - t)^{-h_0}$ which corresponds to future singularity known as big-rip singularity. Substituting $\beta = 1$ in Eq.(9), we obtain torsion tensor as $T = \frac{-6h_0^2}{(t_r - t)^2}$ with $t_r - t = (1 + z)^{\frac{1}{h_0}}$. Using these

Figure 1: Plot of viscous EoS parameter (ω_{eff}) versus z for model I.

values in the continuity equation $\dot{\rho}_m + 3H\rho_m = 0$ for dust case, we obtain

$$\rho_m = \rho_{m_0}(t_r - t)^3 h_0 = \rho_{m_0}(1+z)^3, \quad (10)$$

where ρ_{m_0} is an arbitrary constant. Next, we consider three $F(T, T_G)$ DE models to investigate the behavior of viscous EoS parameter ω_{eff} .

3.1 Model I

First we consider the model given by [11]

$$F(T, T_G) = -T + \gamma_1 \sqrt{T^2 + \gamma_2 T_G}, \quad (11)$$

where γ_1 and γ_2 are non-zero coupling constants. For $\gamma_1 = 0$ or $\gamma_2 = 0$, this model corresponds to teleparallel theory. Substituting Eqs.(10) and (11) in (7), we have

$$\begin{aligned} \omega_{eff} = & -1 + (2\rho_{m_0}(1+z)^{3+\frac{2}{h_0}}(36h^4 + 24\gamma_2 h_0^3(1+h_0))^{\frac{1}{2}} + 168h_0^3\gamma_1 + 8\gamma_1 \\ & \times \gamma_2 h_0^2(h_0 - 1) + 3h_0(z+1)^{\frac{3}{h_0}}(36h^4 + 24\gamma_2 h_0^3(1+h_0))^{\frac{1}{2}})(\zeta_0(1+z)^{\frac{2}{h_0}} \\ & + \zeta_1 h_0(z+1)^{\frac{1}{h_0}} + \zeta_2 h_0(1+h_0))(2\rho_{m_0}(1+z)^{3+\frac{2}{h_0}}(36h^4 + 24\gamma_2 h_0^3(1 \\ & + h_0))^{\frac{1}{2}} - 108\gamma_1 h_0^4 + 12\gamma_1\gamma_2 h_0^3 - 12\gamma_1\gamma_2 h_0^4)^{-1}. \end{aligned}$$

We plot ω_{eff} versus the redshift parameter z in Figure 1. We fix the range of redshift parameter from 0 to 5 and choose the remaining parameters as $\zeta_0 = 0.005$, $h_0 = 2$, $\gamma_1 = -0.5$, $\gamma_2 = 2$ and $\rho_{m_0} = 1$. The lower curve ($\zeta_1 = 0.1 = \zeta_2$) shows that $\omega_{eff} \rightarrow 0$ for large values of z . When $z > 2.2$,

Figure 2: Plot of constant ω_{eff} versus z for model I.

it corresponds to matter dominated era and represents quintessence era in the range $1 < z \leq 2.2$. For $z = 1$, $\omega_{eff} = -1$ while ω_{eff} crosses the phantom divide line when z is less than 1. Similarly, in the upper curve ($\zeta_1 = 0.005 = \zeta_2$), the EoS parameter starts from matter dominated era, enters in the quintessence era, crosses the phantom divide line at $z = 0.8$ and enters into the phantom region. To evaluate the constant viscous EoS parameter, we substitute $\zeta_1 = 0 = \zeta_2$ in Eq.(8). The graphical behavior of constant ω_{eff} is shown in Figure 2. Its behavior is the same as for the viscous EoS parameter ω_{eff} in the range $0 \leq z \leq 5$. We see that the phantom divide line is crossed in both cases for $\zeta_0 = 0.82$ as well as $\zeta_0 = 0.05$.

3.2 Model II

The modification of the model I leads to the following model [25]

$$F(T, T_G) = -T + \gamma_1 \sqrt{T^2 + \gamma_2 T_G} + \eta_1 T^2 + \eta_2 T \sqrt{|T_G|}, \quad (12)$$

where $\gamma_1, \gamma_2, \eta_1$ and η_2 are non-zero coupling constants. Substituting Eqs.(10) and (12) in (7), we have

$$\begin{aligned} \omega_{eff} = & -1 + (2\rho_{m_0}(1+z)^{3+\frac{4}{h_0}} \sqrt{6}(1+h_0)^{\frac{1}{2}} \chi - 8h_0 \sqrt{6}(1+h_0)^{\frac{1}{2}}(z+1)^{\frac{2}{h_0}} \chi \\ & - 48\eta_2(1+h_0)^{\frac{1}{2}} h_0^{\frac{5}{2}} \chi - 480\sqrt{6}\gamma_1 h_0^3(z+1)^{\frac{2}{h_0}}(1+h_0)^{\frac{1}{2}} - 8\sqrt{6}\eta_2 h_0^{\frac{5}{2}}(1 \\ & + h_0)\chi + 144\eta_1 \sqrt{6} h_0^3(1+h_0)^{\frac{1}{2}} \chi - 16(z+1)^{\frac{6}{h_0}} h_0^2(1+h_0)^{\frac{3}{2}} \sqrt{6}\gamma_1 - 24 \\ & \times \eta_2 h_0^{\frac{5}{2}}(z+1)^{\frac{4}{h_0}}(1+h_0)\chi + 72\gamma_2 h_0^{\frac{7}{2}} \chi + 8h_0^2 \gamma_1 \sqrt{6}(z+1)^{\frac{2}{h_0}}(1+h_0)^{\frac{1}{2}} \end{aligned}$$

Figure 3: Plot of ω_{eff} versus z for model II. Figure 4: Plot of constant ω_{eff} versus z for model II.

$$\begin{aligned}
 & - 3h_0\sqrt{6}(z+1)^{\frac{5}{h_0}}(1+h_0)^{\frac{1}{2}}\chi(\zeta_0(z+1)^{\frac{2}{h_0}} + \zeta_1h_0(z+1)^{\frac{1}{h_0}} + \zeta_2h_0(1 \\
 & + h_0)))(2\rho_{m_0}(1+z)^{3+\frac{4}{h_0}}\sqrt{6}(1+h_0)^{\frac{1}{2}}\chi + 18\sqrt{6}h_0^2(z+1)^{\frac{2}{h_0}}(1+h_0)^{\frac{1}{2}}\chi \\
 & - 6\sqrt{6}h_0^2(1+h_0)^{\frac{1}{2}}\chi - \gamma_1\sqrt{6}(z+1)^{\frac{2}{h_0}}(1+h_0)^{\frac{1}{2}}\chi + 72h_0^{\frac{7}{2}}\eta_2(1+h_0) \\
 & - 72\sqrt{6}h_0^4\gamma_1(z+1)^{\frac{2}{h_0}}(1+h_0)^{\frac{1}{2}} - 180\sqrt{6}\eta_1h_0^4(1+h_0)^{\frac{1}{2}}\chi + 144\eta_2h_0^{\frac{7}{2}}(1 \\
 & + h_0)^{\frac{1}{2}}\chi + 12h_0^3\gamma_1(z+1)^{\frac{2}{h_0}}\sqrt{6}(1+h_0)^{\frac{3}{2}} - 36h_0^{\frac{7}{2}}\eta_2(1+h_0)\chi + 12\sqrt{6} \\
 & \times \gamma_1(z+1)^{\frac{5}{h_0}}(1+h_0)^{\frac{1}{2}}h_0^3 + 36\eta_2h_0^{\frac{7}{2}}\chi)^{-1}, \quad (13)
 \end{aligned}$$

where $\chi = (36h_0^4 + 24\gamma_2h_0^3(1+h_0))^{\frac{1}{2}}$. Here, we take $\gamma_1 = -2$, $\gamma_2 = -1$, $\eta_1 = 2$, $\eta_2 = 4$, $\zeta_0 = 0.005$, $h_0 = 2$ and $\rho_{m_0} = 1$. The plot of ω_{eff} versus z for model II is shown in Figure 3. We can easily observe that the curve ($\zeta_1 = 0.1 = \zeta_2$) indicates the matter dominated era for large values of redshift parameter z , it corresponds to quintessence era for lower values of z and at $z = 3.1$, the

Figure 5: Plot of ω_{eff} versus z for model III.

curve crosses the phantom divide line. The viscous EoS parameter enters into the phantom era for $z < 3.1$ and it remains in the phantom era as z approaches to 0. The curve ($\zeta_1 = 0.05 = \zeta_2$) represents approximately the same behavior as the upper graph. Here, the curve crosses the phantom divide line at exactly $z = 3$. The viscous EoS parameter remains in the phantom region as we decrease the values of ζ_1 and ζ_2 . For $\zeta_1 = 0 = \zeta_2$ in Eq.(13), the constant viscous EoS parameter represents its behavior in Figure 4. Here, we consider two values of the constant ζ_0 as 0.82 and 0.05 to examine the behavior of constant ω_{eff} . In both cases, constant ω_{eff} exhibits transition from quintessence to phantom era.

3.3 Model III

Finally, we consider the model in the following form [25]

$$F(T, T_G) = -T + \gamma_1(T^2 + \gamma_2 T_G) + \gamma_3(T^2 + \gamma_4 T_G)^2. \quad (14)$$

Inserting Eqs.(10) and (14) in (7), we obtain

$$\begin{aligned} \omega_{eff} = & -1 + (2\rho_{m_0}(1+z)^{3+\frac{8}{h_0}} + 48\gamma_1 h_0^3(z+1)^{\frac{4}{h_0}} + 672\gamma_3 h_0^3\psi + 128 \\ & \times \gamma_3\gamma_4 h_0^2(1+h_0)\psi - 192\gamma_3\gamma_4 h_0^3\psi + 320h_0^2\gamma_3\gamma_4 h_0^3\psi - 3h_0(1+z)^{\frac{9}{h_0}} \\ & \times (\zeta_0(1+z)^{\frac{2}{h_0}} + \zeta_1 h_0(1+z)^{\frac{1}{h_0}} + \zeta_2 h_0(1+h_0)))(2\rho_{m_0}(1+z)^{3+\frac{8}{h_0}} \\ & - \gamma_1(1+z)^{\frac{4}{h_0}}\psi - \gamma_3\psi^2 - 12h_0^2(1+z)^{\frac{6}{h_0}} + 144\gamma_1 h_0^4(1+z)^{\frac{4}{h_0}} - 288 \\ & \times \gamma_3 h_0^4\psi + 24\gamma_1\gamma_2 h_0^3(1+h_0)(1+z)^{\frac{4}{h_0}} + 48\gamma_3\gamma_4 h_0^3(1+h_0)\psi - 192 \end{aligned}$$

Figure 6: Plot of constant ω_{eff} versus z for model III.

$$\times \gamma_3 \gamma_4 h_0^3 \psi)^{-1} \quad (15)$$

where $\psi = 36h_0^4 + 24\gamma_4 h_0^3(1 + h_0)$. The graphical behavior of the time dependent EoS parameter ω_{eff} is shown in Figure 5. For this purpose, we assume $\gamma_1 = 0.001$, $\gamma_2 = 0.002$, $\gamma_3 = -0.003$, $\gamma_4 = 0.004$, $\zeta_0 = 0.005$, $h_0 = 2$ and $\rho_{m_0} = 1$. Both curves exhibit similar behavior. The viscous EoS parameter shows matter dominated era for $z > 2.4$, equals to -1 for $z = 2.4$ and then enters in the phantom region in both cases. For constant viscous EoS parameter, we substitute $\zeta_1 = \zeta_2 = 0$ in Eq.(15) and its plot is shown in Figure 6. Thus for $\zeta_0 = 0.82$, ω_{eff} crosses the phantom boundary and behaves like quintom. When we decrease the value of viscosity coefficient ζ_0 to 0.05, ω_{eff} still remains in the phantom era.

4 Concluding Remarks

In cosmology, viscous fluid models have extensively been considered to explore expanding nature of the universe. This paper explores the behavior of viscosity by considering dust matter in the background of $F(T, T_G)$ gravity. We have assumed three well-known $F(T, T_G)$ models to evaluate the viscous EoS parameter. For all the three models, we have seen that the viscous EoS parameter achieves phantom like universe for particular values of ζ_0 , ζ_1 and ζ_2 . It remains in the phantom region if we consider constant viscosity. Thus the viscous EoS parameter represents DE dominated universe for all the models depending upon model parameters. We have also compared our results when the background is filled with a non-viscous fluid.

The non-viscous case ($\zeta = 0$) for the model I represents a universe which can result in the DE dominated era depending upon the model parameters [11]. For non-viscous fluid, our results are consistent with those given in [25]. We have also checked that the effective EoS parameter for all models show consistency with the observational data [26] given as

$$\begin{aligned}\omega &= -1.023_{-0.096}^{+0.091} \quad (\text{Planck TT+lowP+ext}), \\ \omega &= -1.006_{-0.091}^{+0.085} \quad (\text{Planck TT+lowP+lensing+ext}), \\ \omega &= -1.019_{-0.080}^{+0.075} \quad (\text{Planck TT, TE, EE+lowP+lensing+ext}).\end{aligned}$$

These constraints have been calculated by imposing different observational schemes at 95% level of confidence. We can say that the DE era may be obtained in the presence of viscous fluid. It is found that the transition of EoS parameter from quintessence to phantom is due to bulk viscosity and phantom phase is an uncertain phase. Thus we can conclude that the presence of bulk viscosity forces the universe to a darker region. Also, the combination of torsion and viscosity shows the accelerated expansion of the universe which strictly depends on the viscous coefficients of the model. The viscosity model represents different phases of the accelerating universe in DE era under the presence of viscous fluid. Finally, we conclude that bulk viscosity is capable of showing phantom-like universe in $F(T, T_G)$ gravity.

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