Minimum age for clear-cutting native species with energetic potential in the Brazilian semi-arid

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</table>
Minimum age for clear-cutting native species with energetic potential in the Brazilian semi-arid region

Otacilio Antunes Santana

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Abstract

What is the minimum age for clear-cutting native species with energetic potential in the Brazilian semi-arid region? This was the central question of this study, which aimed at estimating the minimum cutting cycle for native woody species from an area covered by a sustainable forest management plan. Individuals (n = 240) of twelve species native to the semi-arid region were measured and wood discs were taken from them to collect data relative to: i) volume, by an industrial 3D scanner; ii) density, using a high-frequency densitometer; iii) age, by a growth ring measuring device; and iv) the lower heating value, by an oxygen bomb calorimeter and an elemental analyzer. Data were fitted to mathematical models by regression analysis to determine the relationship and significance between the variables analyzed. The estimated minimum age for the harvesting of native woody species was 47 years, determined by the age at which there was stabilization of volume growth curves over time, and an increase in density and heating power (wood retraction).

Keywords: growth rings; firewood; rotation age; bioenergy.
1. Introduction

The mean annual increment (MAI) and the current annual increment (CAI) are parameters that allow estimation of the optimal age of timber harvest under sustainable forest management, but their calculation is laborious (costly, expensive and requires a large data set) and time consuming in woody species of the Brazilian semi-arid region. Although sustainable forest management plans are in place for this region, such difficulty results in ages for clear cut cycles without a scientific basis to justify these possible cycles. Unlike rapidly growing species of *Eucalyptus* and *Pinus* that invest their biomass mainly in shoots, native woody species of the Brazilian semi-arid region have slow initial growth, investing most of their biomass initially in roots (high root to shoot ratio); this as a support structure, to reduce water loss and against herbivorous attack (from which organs find refuge in the soil) (Lima and Rodal 2010, Costa et al. 2014, Santana and Encinas 2016). In addition, the individual architecture of woody plants in the Brazilian semi-arid region is based on broad ramifications of the trunk and branches, which makes it difficult to estimate the independent variables, namely height and diameter, used in volumetric calculations (Barros et al. 2010).

In addition to shoot volume, other variables are essential for estimating the energy potential of wood. The bulk density, chemical composition (e.g. cellulose and lignin), percentage of vessels and parenchyma, moisture content and size of fibers are some parameters that influence the heating value of wood. However, these parameters are correlated and represented by the density of the wood, which has a significant direct and proportional relationship with the heating value of wood (Gebreegziabher et al. 2013). The age of the individual plant is another important factor, because, with advancing age and the stabilization of growth in height, woody individuals increase the carbon accumulation on their trunks and branches, thus increasing the value of the mass/volume ratio of the timber. Adult wood is characterized by high density, long tracheids, thick cell walls, high percentage of
latewood, low percentage of spiral grain, low percentage of nodes, high percentage of cellulose, low percentage of compression wood, high transverse shrinkage, smaller microfibril angle and greater mechanical strength (Rowell 2012).

The optimal age for harvesting a planted forest stand for wood production is when the current annual increment curve intersects the mean annual increment curve. At that point, the rate of growth in wood volume begins to reduce over time, and the economic profitability in relation to forest investment also begins to fall (Prodan, 1968). In semi-arid regions, in addition to a reduced rate of wood volume growth as trees age, wood density often increases over time as an ecophysiological strategy of native tree species to cope with moisture limitations (Liu et al. 2013, Klein et al. 2014, Santana and Encinas 2016). This wood retraction is evident in the aerial part of the plant and can enhance the root:shoot ratio even if the leaf area index continues to develop over time (Santana and Encinas 2016). Before the wood retraction occurs, a reduction of the rate of growth in wood volume and an increasing of the rate of growth in wood density are observed (Paula 1993, Santana and Encinas 2016). Thus, in semi-arid regions, the time at which wood volume growth levels off and when the density and LHV curves change their inclination angle in relation to age could represent a parameter defines the best time for harvesting wood destined for bioenergy uses (Liu et al. 2013, Klein et al. 2014). There is not an analytical or exact method for calculating this point of levelling off or for detecting a critical change in the inclination angle of wood density and LHV curves, but these trends are supported by observations in the literature (Althoff et al. 2016, Santana and Encinas 2016).

In 2013, 9.1% of the Brazilian energy matrix was based on wood used as firewood and charcoal, which has increased the demand for this resource by approximately 5% per year (Brasil 2013). In the semi-arid region of the state of Pernambuco, the extracted wood is used as an energy source of the Polo Gesseiro do Araripe (Araripina Microregion), which has 39
gypsum mines and 869 industries to cement manufacture, which fill 73% of their energy
requirements with wood, but the small manufacturers use 100% wood to meet their energy
needs. The production of a 10,000 kg of plaster requires 0.5 st native wood, which has an
annual demand of 5.5·10⁴ million kg/year (Silva 2008/2009). The sustainable production of
wood fuel would require a total area of 155,000 hectares specifically for the plaster industry,
with a cutting cycle ranging from 10 to 15 years. But the Northeastern Plants Association
(APNE 2014) has the registration of 94 sustainable forest management plans in the state of
Pernambuco, covering an area of 39,748 hectares, insufficient to meet the energy demands of
the Polo, which grows 10% (Sindugesso 2011) to 25% per year, in addition to a growing
ceramics industry (CPRH 2014).

The goal of this study was to estimate the minimum age for a clear cutting cycle of
native woody species from an area governed by a sustainable forest management plan in the
semi-arid region, according to requirements for implementation and maintenance of Annual
Production Unit, described by the Normative Instruction of the Brazil Ministry of the

2. Materials and Methods
The wood analyzed came from a forest management farm (Figure 1A) in the state of
Pernambuco (8°35’S and 37°59’W), having a semi-arid climate with annual rainfall below
1,000 mm (550 mm in 2015, APAC, 2016) and mean annual temperature greater than 27°C,
or a BSh climate according to the Köppen classification (Peel et al. 2007). This farm is
located on the ‘Sertaneja Meridional e Raso da Catarina’ geomorphological depression. The
soils have been classified into four classes: Haplic Cambisol, Salic Gleysol, Yellow Oxisol,
Haplic Planosol, and the vegetation has been classified into Closed Arboreal, Open Arboreal
and Shrub structural classes (Aguiar et al. 2013).
The vegetation analyzed came from the clearcutting of experimental plots in a sustainably managed forest of the Semi-Arid Forest Management Network (MMA 2009). The twelve predominant tree species have a cumulative importance value index of 96%, measured in 20 plots of 50 x 100 m (10 ha), with a 1,271 stems/ha density (diameter at breast height ≥ 1 cm) (Aguiar et al. 2013), evaluated according to the Permanent Plot Measurements Protocol of the Semi-Arid Forest Management Network, and Rodal et al. (1992).

The species and number of evaluated individuals of each species were: *Acacia kallunkiae* J.W. Grimes & Barneby, Fabaceae (n = 13); *Acacia piauhiensis* Benth., Fabaceae (n = 15); *Anadenanthera colubrina* (Vell.) Brenan. Var. *cebil* (Griseb.) Reis, Fabaceae (n = 27); *Aspidosperma pyrifolium* Mart., Apocynaceae (n = 35); *Poincianella pyramidalis* (Tul.) L. P. Queiroz, Fabaceae (n = 12); *Croton blanchetianus* Baill., Euphorbiaceae (n = 9); *Erythrina velutina* Willd., Papilionoideae (n = 17); *Jatropha elliptica* (Pohl.) Mull.Arg., Euphorbiaceae (n = 33); *Maytenus rigida* (Mart.) Benth., Celastraceae (n = 16), *Mimosa caesalpiniiifolia* Benth, Fabaceae (n = 8); *Myracrodruon urundeuva* All., Anacardiaceae (n = 29); and *Peltophorum dubium* (Spreng.) Taub., Fabaceae (n = 26).

Harvested trees were analyzed for volumetry (Figure 1B), densitometry (Figure 1C), dating (Figure 1C) and calorimetry (Figure 1D). The volume of the wood (m$^3$) was measured by an industrial 3D scanner (3D scanner - SK-DK-FX - four Lens, Foshan Shangke Machinery Co., Ltd., Guangdong Province, China), in which all parts (trunk + branches) were reduced to 1 m length (Figure 1B) in a mobile studio in locus. The density of the wood and the age (growth rings) were measured from sectioned discs, 2 cm thick, by means of high-frequency densitometry (Schinker et al. 2003), in LignoStation™ using LignoScop (Rinntech, Heidelberg, Germany) (Figure 1C) to scan surfaces using cameras attached to the microscope, and LignoScan to scan the wood surface using high-frequency (1600 dpi) with 0.001 mm precision (Shchupakivskyy et al. 2014). Standard procedures (Fritts 1976, Lisi et al. 2008)
and COFECHA software (The University of Tennessee, Knoxville, Grissino-Mayer 2001) were used for cross-dating and standardization. The irreguality of inter-annual rain, or of another climate event, could be revised by the crossdating analysis. This analysis could identify growth zone and false growth rings in an inter-annual period (Pagotto et al. 2015). The equipment performed 1,200 factorial (1,200!) crossdating analyzes (240 studied individuals with five replicates each).

The Higher Heating Value (HHV) of the wood was calculated by collecting data measured with a calorimeter (C6000 global standards 2/10 IKA®, Staufen, Germany) (Günther et al. 2012). Samples were ground to a maximum size of 25 µm. Lower Heating Value (LHV) was determined by subtracting the heat of vaporization of the water vapor from the higher heating value. The water vapor was determined by variance of the hydrogen content of the sample measured through the Element-Analyzer (Vario EL, Elementar Analysesystem GmbH, Langenselbold, Germany) (Figure 1D). These procedures followed the standards DIN EN ISO 1716-2009, DIN 51900-1 2000, DIN 51900-3 2005 (Günther et al. 2012) and ABNT/NBR 8633/84.

The relationships between variables (Y = volume and X = density; Y = volume and X = age; Y = lower heating value and X = density; Y = density and X = age; Y = lower heating value and X = volume; and Y = lower heating value and X = age) were performed by fitting the data to a wide array of mathematical growth models (Table 1) using regression analysis to calculate a coefficient of determination ($R^2$), root-mean-square error (RMSE), significance level (p) and the fit curve; and the selection of the best fit model (based on maximizing $R^2$, minimizing RMSE, and minimizing p) for each case (Zar 1999). The regression analysis was preceded by the D’Agostino Normality test (D'Agostino et al. 1990) for each variable to validate statistical premises. The models were chosen as indicated by Kleinbaum et al. (2013). The possible multicollinearity among variables was calculated by the Farrar-Glauber test.
(Farrar & Glauber 1976). Adjustments, graphics, statistical deviation and coefficient of variation were calculated using Statistica 12 (Statsoft, Dell, Tulsa, USA).

3. Results

The mean age of the study population was 46 years (CV% = 15.2%; Figure 2A), ranging from 98 years (Peltophorum dubium) to 5 years (Croton blanchetianus). The mean volume was 0.12 m$^3$ (CV% = 11.9%; Figure 2B) per individual tree, with individuals ranging in size from a minimum volume of 0.003 m$^3$ (Croton blanchetianus) to a maximum volume of 0.40 m$^3$ (Acacia kallunkiae). The mean wood density was 0.8 g · cm$^{-3}$ (CV% = 7.1%; Figure 2C), varying between 0.4 g · cm$^{-3}$ (Jatropha elliptica) and 1.05 g · cm$^{-3}$ (Acacia kallunkiae). The mean lower heating value was 4,863 kcal · kg$^{-1}$ (CV% = 6.2%; Figure 2D), ranging from 3,783 kcal · kg$^{-1}$ (Acacia piauhiensis) to 5,697 kcal · kg$^{-1}$ (Erythrina velutina).

Data reflecting the relationship between volume and density, and between volume and age, were well fit to sigmoidal models (5 parameters) significantly ($R^2 >$ 0.86; RMSE < 0.05; $p < 0.001$; Figure 3A and 3B, Table 2). In both relationships, volume began to rise sharply (curve growth $> 45^\circ$ slope relative to the dependent variable) to the point where it became constant (curve growth $< 5^\circ$ slope in relation to the dependent variable) and then slowed relative to growth of the independent variable (density or age). The relationship between the lower heating value (LHV) and density was the most significant ($R^2 >$ 0.97; RMSE < 0.03; $p < 0.001$), directly and significantly increasing with increasing density (Figure 3C, Table 2). Multicollinearity was not found in the models tested ($p > 0.800$). Considering the relationship between density and LHV with the age of individuals, data fitted to the exponential growth model (double and with five parameters), in which, from a given year, 47 years for density and 49.5 for LHV, there is a levelling off of the curve (Figure 3D and 3F; Table 2), with a more marked increase in the values of these variables (curve growth $> 45^\circ$ slope relative to the
dependent variable) than previously (growth curve ≈ 30° slope relative to the dependent
variable). As for the relationship between LHV and volume, data fitted to the Chapman model
(four parameters) (Figure 3E; Table 2), indicating that in the higher volume growth phase, the
increase in LHV value is less pronounced (curve growth < 15° slope in relation to the
dependent variable) than when the volume growth begins to stabilize (growth curve > 45°
slope in relation to the dependent variable).

The minimum age for clear cutting cycle in an area of Sustainable Forest Management
Plan in the semi-arid region, with the destination of wood with energy purposes, suggested by
the variables analyzed, was from 46.8 (47) years (Figure 3B). This is the average tree age at
which there is a stabilization of the volume growth (curve growth < 5° slope in relation to the
dependent variable), and pronounced growth of the LHV (from 49.5 years; Figure 3F) and
density (from 47 years; Figure 3D).

4. Discussion

One limitation of this study lies in the impracticality of working with a larger sample size,
given the time required for cutting and loading for transportation (about 24 hours), making it
difficult to scan other woody individuals in the semi-arid region. However, 240 individuals
were measured with five replicates of each measurement or analysis, improving the reliability
and accuracy of data collected.

Our findings corroborate what has already been reported for the area (Aguiar et al.
2013), which before the Sustainable Forest Management Plan, had reduced logging (> 20
individuals / ha / year). The values of volume, density, and LHV varied within the 95%
confidence intervals reported in the literature for the mean values of the species analyzed:
volume (from biomass) from 0.001 m³ (Santana and Souto 2006) to 0.50 m³ (Silva and
Sampaio 2008); density from 0.3 g · cm⁻³ (Lima and Rodal 2010) to 1.2 g · cm⁻³ (Paula 1993);
and LHV from 3,800 kcal · kg\(^{-1}\) (Medeiros Neto et al. 2012) to 5,800 kcal · kg\(^{-1}\) (Quirino et al. 2004).

The rapid growth in the volume of tree species, over the years, and a subsequent stabilization of this growth is commonly observed in the literature, not only for fast growing plantation species (Encinas et al. 2011), but also for native species (Felker 1986, Lima 1986, Vico et al. 2015, Santana and Encinas 2016). This stabilization is considered as the saturation period of the plant individual, both by limited environmental resources, by competition with others in their environment, but also by individual senescence marked by the genetics of the species (Hunt 1982, Felker 1986, Soares et al. 2011). The onset of the saturation, senescence or slow growth process, as shown in Figure 3B from 47 years, is marked in forestry science by the beginning of the cutting cycle for vegetation intended for production of firewood and charcoal (Rode et al. 2014, Althoff et al. 2016). When the volume tends to stabilize, other variables, such as density (Figure 3D) and heating value (Figure 3F), which have a high correlation \((R^2 > 0.97)\), continue to grow in value. While a saturation of those values was not observed in this work, it is, however, emphasized in the literature (Hunt 1982, Felker 1986). This reduction of the rate of growth of the wood volume and an increasing of the rate of growth of the wood density and lower heating value (LHV) infer in the begin of wood retraction. Thus, from the age of 47, the harvesting is strongly indicated.

In other dry forests on Earth, the cutting cycle of wood is ranges from 63 to 97 years, much longer than the 47 years derived here. The causes for these longer rotation periods are: i) in India, to avoid urban occupation (Agarwal et al., 2016); ii) in Australia and Africa, to conserve the habitat for large mammals (Bhadouria et al. 2016); iii) in West Africa, to reduce wildfire susceptibility and microclimate changes (Scheitera and Savadogo, 2016); iv) in Cameroon and Panama, to preserve forest stands that are used in popular religious and sacred rituals (Kemeuze et al. 2016, Seijo et al. 2016); v) in Costa Rica, India, Papua New Guinea,
and Southern Africa, to produce medicinal bioproducts (Schmiedel et al. 2016); vi) in Morocco and Argentina, to perpetuate ethnobotanical and other cultural uses of natural resources (Martínez 2015, Blanco and Carrière 2016); and vii) in China, to maintain the groundwater flow near to surface by hydraulic lift from trees (Xiao and Huang 2016). In all cases, like in this work, biodiversity preservation and firewood sustainability also were objectives of forest management (Althoff et al. 2016, Santana 2016).

5. Conclusion

The estimate of the minimum age for cutting cycle of native woody species in the evaluated area in Brazil’s semi-arid region was 47 years, determined by the age at which there was stabilization of volume growth over time, and an increase in density and heating power (wood retraction).

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Figure 1. Map showing the origin of the tree individuals studied (A); and methodological steps: volumetry (B); densitometry and dating (C) and calorimetry (D).

714x851mm (72 x 72 DPI)
Figure 2. Age (A), volume (B), density (C) and lower heating value (LHV) (D) of twelve native species analyzed in the semi-arid region.

1103x1446mm (72 x 72 DPI)
Figure 3. Relationship between the analyzed variables: volume, lower heating value (LHV), density and age of the 240 woody individuals measured.

1431x1577mm (72 x 72 DPI)
Table 1. Mathematical models used for data fit.

<table>
<thead>
<tr>
<th>Model</th>
<th>Equation*</th>
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<tbody>
<tr>
<td>Polynomial (linear)</td>
<td>$Y_i = \beta_0 + \beta_1 \cdot X_i + \epsilon_i$</td>
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<tr>
<td>Polynomial (Quadratic)</td>
<td>$Y_i = \beta_0 + \beta_1 \cdot X_i + \beta_2 \cdot X_i^2 + \epsilon_i$</td>
</tr>
<tr>
<td>Polynomial (Cubic)</td>
<td>$Y_i = \beta_0 + \beta_1 \cdot X_i + \beta_2 \cdot X_i^2 + \beta_3 \cdot X_i^3 + \epsilon_i$</td>
</tr>
<tr>
<td>Sigmoidal (3 parameters)</td>
<td>$Y_i = \frac{\beta_1}{1 + e^{\frac{X_i - \beta_0}{\beta_2}}} + \epsilon_i$</td>
</tr>
<tr>
<td>Sigmoidal (4 parameters)</td>
<td>$Y_i = \beta_0 + \frac{\beta_1}{1 + e^{\frac{X_i - \beta_0}{\beta_2}}} + \epsilon_i$</td>
</tr>
<tr>
<td>Sigmoidal (5 parameters)</td>
<td>$Y_i = \beta_0 + \frac{\beta_1}{1 + e^{\frac{X_i - \beta_0}{\beta_2}}} + \epsilon_i$</td>
</tr>
<tr>
<td>Logistic (3 parameters)</td>
<td>$Y_i = \frac{\beta_1}{1 + \left(\frac{X_i}{\beta_2}\right)^\beta} + \epsilon_i$</td>
</tr>
<tr>
<td>Logistic (4 parameters)</td>
<td>$Y_i = \beta_0 + \frac{\beta_1}{1 + \left(\frac{X_i}{\beta_2}\right)^\beta} + \epsilon_i$</td>
</tr>
<tr>
<td>Weibull (4 parameters)</td>
<td>$Y_i = \beta_0 \left[1 - e^{\left(\frac{X_i - \beta_0 - \beta_1 \ln X_i}{\beta_2}\right)^\beta}\right] + \epsilon_i$</td>
</tr>
<tr>
<td>Weibull (5 parameters)</td>
<td>$Y_i = \beta_0 + \beta_1 \left[1 - e^{\left(\frac{X_i - \beta_0 - \beta_1 \ln X_i}{\beta_2}\right)^\beta}\right] + \epsilon_i$</td>
</tr>
<tr>
<td>Gompertz (3 parameters)</td>
<td>$Y_i = \beta e^{\frac{X_i - \beta_0}{\beta_2}} + \epsilon_i$</td>
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<tr>
<td>Gompertz (4 parameters)</td>
<td>$Y_i = \beta_0 + \beta e^{\frac{X_i - \beta_0}{\beta_2}} + \epsilon_i$</td>
</tr>
<tr>
<td>Hill (3 parameters)</td>
<td>$Y_i = \frac{\beta X_i^{\beta_1}}{\beta_3^{\beta_1} + X_i^{\beta_1}} + \epsilon_i$</td>
</tr>
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</table>
Hill (4 parameters)  \[ Y_i = \beta_0 + \frac{\beta_1 X_i^{\beta_2}}{\beta_3^{\beta_4} + X_i^{\beta_5}} + \varepsilon_i \]

Chapman (3 parameters)  \[ Y_i = \beta_0 + \beta_1 (1 - e^{-\beta_2 X_i})^{\beta_3} + \varepsilon_i \]

Chapman (4 parameters)  \[ Y_i = \beta_0 + \beta_1 (1 - e^{-\beta_2 X_i})^{\beta_3} + \varepsilon_i \]

Exponential Growth (Simple 1 parameter)  \[ Y_i = e^{\beta_1 X_i} + \varepsilon_i \]

Exponential Growth (Simple 2 parameters)  \[ Y_i = \beta_1 \cdot e^{\beta_1 X_i} + \varepsilon_i \]

Exponential Growth (Simple 3 parameters)  \[ Y_i = \beta_0 + \beta_1 \cdot e^{\beta_1 X_i} + \varepsilon_i \]

Exponential Growth (Double 1 parameter)  \[ Y_i = \beta_1 \cdot e^{\beta_1 X_i} + \beta_2 \cdot e^{\beta_2 X_i} + \varepsilon_i \]

Exponential Growth (Double 2 parameter)  \[ Y_i = \beta_0 + \beta_1 \cdot e^{\beta_1 X_i} + \beta_2 \cdot e^{\beta_2 X_i} + \varepsilon_i \]

*\( Y_i \) = observed value of the dependent variable; \( X_i \) = observed value of the independent variable; \( \beta_i \) = regression equation parameters; \( \varepsilon_i \) = fit error.
Table 2. Significant fit of data to models for six analyzed relationships: volume vs. density (A); volume vs. age (B); lower heating value vs. density (C); density vs. age (D); lower heating value vs. volume (E); and lower heating value vs. age (F).

<table>
<thead>
<tr>
<th>Model</th>
<th>Function</th>
<th>R²</th>
<th>RMSE</th>
<th>p</th>
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<tr>
<td>A</td>
<td>Sigmoidal (5 parameters)</td>
<td>$Y_i = -0.12 + \frac{0.509}{1 + e^{\left(\frac{X_i - 0.423}{0.001}\right)^{0.005}}}$</td>
<td>0.86</td>
<td>0.044</td>
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<tr>
<td>B</td>
<td>Sigmoidal (5 parameters)</td>
<td>$Y_i = 0.007 + \frac{0.031}{1 + e^{\left(\frac{X_i - 23.86}{4.257}\right)^{3.399}}}$</td>
<td>0.89</td>
<td>0.037</td>
</tr>
<tr>
<td>C</td>
<td>Polynomial (linear)</td>
<td>$Y_i = 3.736 + 1.738 \cdot X_i$</td>
<td>0.97</td>
<td>0.026</td>
</tr>
<tr>
<td>D</td>
<td>Exponential Growth (Double 5 parameters)</td>
<td>$Y_i = -0.193 + 0.248 \cdot e^{0.001 \cdot X_i} + 0.210 \cdot e^{0.019 \cdot X_i}$</td>
<td>0.91</td>
<td>0.034</td>
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<tr>
<td>E</td>
<td>Chapman (4 parameters)</td>
<td>$Y_i = 0.004 + 0.001 \left(1 - e^{6.29 \cdot X_i}\right)^{6.438}$</td>
<td>0.90</td>
<td>0.048</td>
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<tr>
<td>F</td>
<td>Exponential Growth (Double 5 parameters)</td>
<td>$Y_i = -1415.94 + 5142.98 \cdot e^{0.001 \cdot X_i} + 76.118 \cdot e^{0.044 \cdot X_i}$</td>
<td>0.94</td>
<td>0.050</td>
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Source: Author.