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Reconsideration of the Overlap Region in Smooth Shallow Open Channel Flows

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Abstract

In this paper, data sets for mean velocity distributions in smooth shallow open channel flow are reconsidered to evaluate the characteristics of the overlap region and estimate the friction velocity (u_*). Both new and existing velocity measurements are used in the analysis. The velocity profiles are obtained using laser Doppler velocimetry and particle image velocimetry at typical Reynolds numbers (20,000 - 60,000) achieved in laboratory flumes. Validation of the estimated u_* values using different forms of power law is established by comparing these values with the ones available in literature. Also, the Reynolds shear stress distribution based on two-dimensional measurements validate the estimated u_* . The availability of new data sets allows one to verify the usefulness of the log-law and evaluate the log-law constants. Different fitting methods; least squares, derivative and scattered methods are used to evaluate the value of von Kármán coefficient. It is found that the value of κ obtained from the least squares method varies between 0.35 and 0.51 and depends on the Reynolds number. This refutes the conventional constant value assumption for the von Kármán coefficient ($\kappa = 0.41$). By considering the Prandtl's mixing-length theory, the present values of the von Kármán coefficient are used to evaluate the mixing length distributions. The mixing length distributions in smooth open channel flow are found to depend on Re_θ .

Keywords: eddy viscosity; friction velocity, power law; log-law; mixing length; smooth open channel flow; velocity measurement; von Kármán coefficient

1 Introduction

The properties of near-bed flows have been investigated because of their importance in many engineering applications. The limited availability of large-scale experimental facilities, the exorbitant costs involved in operating such facilities and the difficulties associated with acquiring reliable measurement data have resulted in the vast majority of laboratory experiments being conducted at relatively lower Reynolds numbers. Furthermore, it is critical to better understand the near-wall flow over the bed at relatively low Reynolds numbers because of its importance in numerical analysis of turbulent flows. Although a significant body of literature has been developed using direct numerical simulation (DNS) to solve the Navier–Stokes equations, its use is still mostly restricted to flows at low Reynolds numbers. Therefore, to develop predictive methods for calculating near-wall turbulence, one must continue to rely on appropriate turbulence models. Model developers must carefully formulate and validate their turbulence models using experimental studies and databases. This provides additional motivation for experimentalists to continue to design high-quality experiments in an effort to refine the near-bed data in turbulent boundary layer flows. Also, as a conceptual study, flow over a smooth bed is still of interest to many researchers in both open channel and turbulent boundary layer flow (Roussinova *et al.* 2008; Afzal *et al.* 2009; Gaudio and Dey 2013; Segalini *et al.* 2013).

In the near-bed region, the flow strongly depends on the wall shear stress (τ_w), density (ρ), kinematic viscosity (ν) and vertical distance from the wall (y). As such, the appropriate velocity and length scales are friction velocity $u_* (= \sqrt{\tau_w/\rho})$ and the viscosity dependent characteristic length ν/u_* , respectively. The wall bounded flow is typically divided into different regimes as shown in Figure 1. In the viscous sublayer, u_* can be easily determined if sufficient velocity measurements are available in this region. In the outer region, the effect of viscosity is inconsequential and the scales depend on the free stream velocity (U_e) and a global length scale such as the boundary layer thickness (δ). The theoretical description of the turbulent boundary layer provides for the existence of an overlap region at sufficiently high

Reynolds number, This overlap region, where the inner and outer regions meet, can be described by a proper mathematical functional form (logarithmic or power) from which the friction velocity can be estimated. The specific form of the scaling law in the overlap region depends on the assumptions made in the course of the matching process. Barenblatt (1993) discussed the logarithmic law and the power law derivations using similarity and asymptotic considerations of the kinematic quantities. The log-law assumes that the velocity gradient is independent of fluid viscosity at arbitrarily large Reynolds number. In contrast, the power law assumes that the velocity gradient remains dependent on dynamic viscosity, in other words, at arbitrarily large Reynolds number the effect of the viscosity remains not only in the viscous sublayer, but also in the entire boundary layer (Barenblatt and Prostokishin 1993). One of the controversial questions is whether the Reynolds number independent logarithmic law or the Reynolds number dependent power law is the more appropriate velocity scaling law in the overlap region. Experimental and numerical studies are available to support both descriptions.

Another difficulty in using the log-law method to determine u_* is identifying where the logarithmic region starts and ends, especially at low Reynolds numbers (Marusic *et al.* 2013). In contrast, recognition of the start and end points of the power law region is not that critical. One may consider that the power law is more useful in the determination of the friction velocity since the number of unknown parameters is less than that in the log-law.

In many flow situations, the viscous sublayer is thin and the commonly used velocity measuring instruments are not capable of acquiring accurate velocity information. Different forms of power law have been used to describe the velocity distribution and determine the bed shear stress. In the boundary layer with zero pressure gradient, George and Castillo (1997) considered the inner layer and outer layer and developed a power law velocity profile. Barenblatt *et al.* (1993, 1997) proposed the power law scaling for the dimensionless mean streamwise velocity distribution, considering viscous effects, given by

$$U^+ = C (y^+)^{\alpha}. \quad (1)$$

where $U^+ = U/u_*$ and $y^+ = yu_*/\nu$. Several researchers have expressed the parameters C and α in Eq. (1) as functions of Reynolds number, while Balachandar *et al.* (2002) has proposed constant values (Table 1).

Another method, similar to the Clauser chart method, was proposed by Buschmann and Meinert (1999). This method determines the friction velocity using the data in the overlap region and a form of the power law given by Eq. (1). Since only data in the overlap region are used, velocity measurements in the viscous sublayer are not required. To estimate the friction velocity, they proposed to curve fit the dimensional linear form of Eq. (1) and calculate the slope of the line (α_1) and intercept value (β_1) in the equation

$$\ln U = \alpha_1 \ln y + \beta_1. \quad (2)$$

Using the values obtained for α_1 and β_1 , u_* is calculated as

$$u_* = \exp\left(\frac{\beta_1 + \alpha_1 \ln \nu - \ln C_1}{\alpha_1 + 1}\right). \quad (3)$$

where $C_1 = \frac{5\alpha_1 + \sqrt{3}}{2\alpha_1}$.

The above discussion indicates that many power law functions are presently in use with different coefficients. These coefficients are generally Reynolds number dependent, which is a point of major disagreement between power law and log-law theories (Buschmann and Meinert 1999 and Vinuesa *et al.* 2014).

Apart from the power law method, the total stress method has been successfully used to determine the bed shear stress for both smooth and rough wall boundary layers. The total stress method is based on the assumption that a region of constant shear stress, equal to the wall shear stress, exists in the inner part of the boundary layer. The total stress is calculated at the plateau of the Reynolds shear stress profile in the overlap region of the boundary layer by summing the viscous and turbulent stress contributions. Since the viscous shear stress is negligible in the outer region, the Reynolds shear stress ($-\overline{u'v'}$) is the only dominant component for the total shear stress, described by the equation

$$-\overline{u'v'} = u_*^2 \left(1 - \frac{y}{h}\right). \quad (4)$$

Since direct shear measurement is not available, the present study is concerned with applying the various power law methods to estimate the bed shear stress and then revisiting the log-law equation by evaluating the von Kármán coefficient (κ) and log-law constant (B) in uniform shallow open channel flow (OCF). For this purpose, new experiments were conducted in a smooth open channel flow using laser Doppler velocimetry (LDV) and particle image velocimetry (PIV), covering a depth Reynolds number over the range $20,000 < Re_h < 60,000$ normally encountered in laboratory flumes, where $Re_h = \frac{hU_e}{\nu}$ is based on the water depth, h . The validity of the power law method was established not only with other open channel flow data available in the literature (Nezu and Rodi 1986; Kirkgöz 1989) but also with u_* obtained from the Reynolds shear stress distribution method available from the two component velocity measurements. Different procedures for calculating κ are explored using the velocity distributions from the new experiments. Finally, by evaluating the eddy viscosity and implementing the Prandtl's mixing length theory, the dependency of the von Kármán coefficient on Reynolds number is discussed.

2 Experimental setup

The present experiments were conducted in a rectangular cross-section open channel flume at the Hydraulic Engineering Laboratory in the Centre for Engineering Innovation (CEI) at the University of Windsor. A schematic of the experimental flume is shown in Figure 2. The flume is 0.80 m deep, 1.22 m wide (b) and 16 m long. An upstream settling tank, flow straightener and turbulence stilling arrangements are used to minimize the entrance effects and lower the inlet turbulence levels. A false bottom was designed and installed inside of the flume to allow for control of the slope of the bed. A downstream weir controls the water depth in the flume. The sidewalls in the mid-portion of the flume are made of transparent acrylic plates to facilitate optical access. The measurement section is located 4.5 m downstream of the flow straightener. The flow is tripped with a wire attached to the bed

spanning the width of the flume at 2.5 m upstream of the measuring section, which allows for faster transition to a fully developed turbulent regime.

Velocity measurements were carried out along the centreline of the flume using two-dimensional laser Doppler velocimetry (LDV) and particle image velocimetry (PIV) techniques. The LDV system uses a 300mW Ar-Ion laser. The transmitting probe was fit with a 800-mm focal length lens and mounted on an adjustable tilting table. The measuring volume, based on the Gaussian intensity distribution cut off point, was $0.39 \times 0.39 \times 16.4 \text{ mm}^3$. This facilitated measurements to be made fairly close to the bed. The LDV probe was mounted on a two-dimensional computer controlled traverse system driven by stepper motors with a position accuracy of $\pm 0.01 \text{ mm}$ for each direction. The LDV system was operated in the backward scatter mode.

The PIV system consists of dual pulse Nd:YAG lasers of 532 nm wavelength and 50 mJ/pulse. The laser sheet was formed through a 500 mm spherical lens and expanded through a -15 mm cylindrical lens. The light sheet was oriented vertically in (x - y) plane at the centreline of the flume. The laser sheet was illuminated from the top of the flume while the optical axis of the camera was positioned perpendicular to it. The images were recorded using a PowerViewPlus 4 MP CCD camera with a resolution of 2048×2048 pixels operating in dual capture mode. A 70 – 300 mm Nikkor lens was mounted on the camera and adjusted to give the desired field-of-view. Two thousand image pairs were acquired at a frequency of 1.0 Hz. The images were analyzed using the commercial software Insight 4G, with a 44×44 pixel interrogation area and 50% overlap using a Fast Fourier Transform (FFT) correlator. The correlation peak was located within subpixel accuracy using a Gaussian curve-fitting method. This process yielded a final interrogation area with size of 22×22 pixels. For the average field-of-view of $90 \text{ mm} \times 90 \text{ mm}$ and camera resolution of 2048×2048 pixels the spatial resolution of the present PIV measurements is 0.50 mm/pixel. After the correlation analysis was complete, velocity outliers were rejected using the Cellular Neural Network method with a variable threshold as proposed by Shinnee *et al.* (2004). This technique uses

information about the local velocity gradient in the flow to make a suitable choice for the local threshold. The rejected vectors (less than 5%) were primarily located at the edges of the field-of-view. The rejected vectors were replaced using a Gaussian-weighted interpolation scheme.

The flow was seeded with spherical silver coated glass particles (mean diameter of 10 μm) to facilitate the velocity measurements. All measurements were made in the central plane of the channel, where it is generally accepted that the effect of any secondary flow is negligible. It should be noted that secondary flows are not expected to play any major role since the channel aspect ratio (b/h) is greater than five (Nezu and Nakagawa 1993). A summary of important test parameters is given in Table 2. Experiments T1-T5 were conducted with the LDV, while T6 and T7 were conducted using PIV. Here, Froude number (F_r) is defined based on the average velocity.

Typical estimates of uncertainty in the mean velocity at different vertical locations for each set are reported in Table 3. The uncertainty analysis was based on the methodology outlined by Moffat (1988). The highest uncertainty estimates in the mean velocity occur near the wall ($y^+ < 15$).

3 Friction velocity results

In this study, the different power law methods defined in Table 1, proposed by Barenblatt (1993), Zagarola and Smits (1998), Kailasnath (1993) and Balachandar *et al.* (2002), have been adopted to determine u_* . To validate our implementation, these methods were first used to estimate the friction velocity from previous smooth open channel flow experiments of Nezu and Rodi (1986), Kirkgoz (1989) and Steffler *et al.* (1983, 1985). Our calculated values for friction velocity obtained from the different power laws are reported in Table 4. All power laws used in Table 4 produced estimates of the friction velocity with less than 4% difference from the previously reported (estimated) values. The power law methods are easy to implement numerically, since just one unknown parameter (u_*) has to be

determined. On the other hand, as will be discussed in Section 4, the composite log-law based on either Coles' or Krogstad's wake function requires the determination of three parameters.

The power law methods used above and that due to Buschmann and Meinert (1999) were also applied to the present measurements. Table 5 presents the estimates for the friction velocities for each experimental set described in Table 2. It can be observed that u_* determined by applying the different power law methods to the measured data are in good agreement with each other. Although this does not ensure the accuracy of the results, it does confirm that the present measurements are consistent with those of earlier studies. In addition, the Reynolds stress distributions for some of the data sets from the two dimensional measurements were used to validate the friction velocity with the Reynolds stress distribution method. The bed shear stress has been evaluated from the slope of the best fit line of the Reynolds stress profile above the bed. Table 6 shows that the friction velocity results from the Reynolds stress distribution method are in good agreement with the values obtained from the power law methods.

4 Revisiting the log-law method

von Kármán (1930) described the mean velocity distribution in the outer region of the boundary layer of pipe flow in a logarithmic form (Pope 2000). Other researchers, including Coles (1956), subsequently modified the log-law equation by adding another term known as a wake function, yielding the modified logarithmic equation for the velocity profile as

$$U^+ = \frac{1}{\kappa} \ln y^+ + B + \frac{2\Pi}{\kappa} w(\eta). \quad (5)$$

The coefficient κ is referred to as the von Kármán constant, and is considered to have a universal value of 0.41. The value of the constant B depends on flow type, geometry and wall conditions. Discussion about the universality of κ and B is presented in section 4.1. In Eq. (5), $w(\eta)$ is the wake function and $\eta = y/\delta$, where δ is the boundary layer thickness defined as the distance from the wall where the velocity is equal to 99% of the free stream velocity. The parameter Π in Eq. (5) determines the strength of the wake. For the zero pressure gradient

turbulent boundary layer, the value of Π has been regarded as a constant. However, Afzal *et al.* (2009) have shown that Π is dependent on the nature of the near-wall perturbations imposed on the boundary layer. In the case of open channel flow, the value of Π and the wake function can also be influenced by the presence of the free surface, especially in shallow flow (Balachandar *et al.* 2002). Different functional forms have been proposed for the wake function in the case of turbulent boundary layer flow over a smooth wall, with the most commonly used forms being those proposed by Coles (1956),

$$w(\eta) = \sin^2\left(\frac{\pi}{2}\eta\right) \quad (6)$$

and by Krogstad *et al.* (1992),

$$w(\eta) = \frac{1}{2\Pi}[(1 + 6\Pi) - (1 + 4\Pi)\eta]\eta^2. \quad (7)$$

Nezu and Rodi (1986) and Kirkgöz (1989) have checked the validity of the Coles wake function as applied to OCF and obtained values of $\Pi = 0.20$ and 0.10 , respectively. By applying Coles' equation, Cardoso *et al.* (1991) noted that Π varies between -0.02 and 0.27 in smooth uniform OCF. The wake function (Eq. (7)) proposed by Krogstad *et al.* (1992) has been validated for flow in open channels by Tachie *et al.* (2000) and Afzal *et al.* (2009). The value of Π for smooth OCF has been reported to be between 0.10 and 0.16 based on different channel aspect ratios (Roussinova *et al.* 2008). Furthermore, bed roughness has also been shown to influence the value of the wake parameter Π (Krogstad *et al.* 1992).

4.1 von Kármán constant or von Kármán coefficient?

The value of κ can be very important in industrial applications. Most turbulence models used in commercial computational fluid dynamics software rely on the assumption that the near-wall flow can be described by Eq. (5). However, Spalart (2006) has suggested that a 2% decrease in κ produces a 1% decrease in the overall drag estimate. In this case, aircraft manufacturers would prefer to perform simulations using a smaller value of κ , since the fuel consumption would appear to be more economical than a competitor's estimate with a higher value of κ (George 2007).

For many years, turbulence researchers were content to use $\kappa = 0.41$, and thereby implicitly accepted that κ was a universal constant. However, different approaches to determine the von Kármán constant from velocity measurements have been proposed and, in the earliest field experiments by Sheppard (1947) and Pasquill (1950), κ was found to be 0.46 and 0.37, respectively. Later, in the well-known Kansas experiments, a smaller value of $\kappa = 0.35$ was obtained by Businger *et al.* (1971) for atmospheric boundary layers.

There have also been attempts to mathematically compute κ for flow over a smooth wall with homogeneous turbulence. For instance, Long *et al.* (1993) analyzed the turbulent flow in smooth pipes and obtained $\kappa = 0.408$. Lo *et al.* (2005) estimated κ for wall-bounded turbulence based on the similarity with homogeneous constant shear turbulence, and obtained $\kappa = 0.42$. Gaudio and Dey (2013) reported that κ is not universal in flows with bed and suspended load transport. To date, the research community has not been able to reach a consensus on a single value for κ , with the values ranging between 0.36 and 0.44 for different canonical flow cases. Table 7 shows values of κ from several sources. It has only very recently become more common to refer to κ as the von Kármán coefficient (rather than a constant), thereby allowing for a pressure gradient dependence (Nickels 2004; Nagib and Chauhan 2008).

There is even less consensus in the literature about the value of coefficient B in Eq. (5). In a zero pressure gradient turbulent boundary layer, traditional estimates of B range between 4.9 and 5.1. Recent estimates cover a much wider range of B from 4 to 10. Surprisingly, the lack of apparent universality of B seems to be of far less concern than the precise value of κ (George 2007).

The present study is also concerned with evaluating the von Kármán coefficient (κ) and log-law constant (B) in uniform OCF. Different methods for calculating κ are discussed here in order to revisit the log-law equation and determine whether or not κ can be considered as a universal constant in shallow OCF.

4.2 Evaluation of von Kármán coefficient

If the friction velocity is known, the von Kármán coefficient can be calculated using three methods. They are described below and are denoted as derivative, scatter and fitting methods. The corresponding von Kármán coefficient calculated by these methods is denoted as κ_{der} , κ_{S} and κ_{f} , respectively.

4.2.1 Derivative method

At relatively high Re_θ , the value of κ can be found by using the logarithmic law of the wall. Neglecting the wake function and differentiating Eq. (5) gives

$$\frac{dU^+}{dy^+} = \frac{1}{\kappa} \frac{1}{y^+}. \quad (8)$$

As a result, $\frac{dU^+}{dy^+}$ is a linear function of $\frac{1}{y^+}$, with slope $\frac{1}{\kappa}$. The variation of $\frac{dU^+}{dy^+}$ versus $\frac{1}{y^+}$ is illustrated in Figure 3 for each of the experiments in this study. The slope of the best-fit line passing through the origin represents the inverse of the von Kármán coefficient. This procedure is sensitive to noise and data scatter due to the inherent uncertainty associated with experimental data. The experimental uncertainties are further amplified when $\frac{dU^+}{dy^+}$ is calculated directly by using finite difference on the experimental data. To overcome this difficulty, a common remedy is to average multiple profiles in the log-law region. Since this remedy is not applied here, small fluctuations in $\frac{dU^+}{dy^+}$ profiles are observed in Figure 3. Throughout this paper, $\frac{dU^+}{dy^+}$ is determined numerically using a second-order central-difference scheme. In the derivative method, the log-law region is considered to occur between $70 < y^+ < 700$, where $y^+ \left(\frac{dU^+}{dy^+} \right)$ should reach a constant value (Österlund *et al.*, 2000).

The analytical expression for calculating the von Kármán coefficient (κ_{der}) can be written as

$$\frac{1}{\kappa_{\text{der}}} = \frac{1}{N} \sum_{i=1}^N \left[y_i^+ \left(\frac{dU^+}{dy^+} \right)_i \right] \quad (9)$$

where N is the number of data points located in the logarithmic region. For all experimental runs (T1-T7), the correlation coefficient of the expected line and the experimental data was above 95 %, and the values of κ_{der} are shown in Figure 3.

4.2.2 Scatter method

The scatter method, which has recently been discussed by Segalini et al. (2013), is based on neglecting the wake function and re-arranging Eq. (5) in the form

$$\kappa B = \kappa U^+ - \ln y^+. \quad (10)$$

Each pair of y_i^+ , U_i^+ values that belong to the overlap region should follow Eq. (10), with corresponding coefficients denoted as κ_s and B_s . In other words, for all pairs of (y_i^+, U_i^+) in the overlap region, B can be calculated from Eq. (10) for a specific κ value. As κ varies, the calculated B will change respectively for all pairs of (y_i^+, U_i^+) in the overlap region. Figure 4 shows that different values of B are obtained in the overlap region when $\kappa \neq \kappa_s$. For this method the overlap region is supposed to be between $30 < y_i^+ < 0.2\delta^+$. As discussed in Segalini *et al.* (2013), having a large number of velocity measurements in this region increases the accuracy of the prediction. Theoretically, in this method, obtaining a proper estimate for the von Kármán coefficient requires the collapse of all pairs of (y_i^+, U_i^+) of the overlap region to one point (κ_s, B_s) . In the practical solution, the pairs of (y_i^+, U_i^+) will lie on a short line, whose centre is located at (κ_s, B_s) and the ends are located at $(\kappa_s, B_s \pm \frac{\Delta B}{2})$. Figure 4 shows the evaluated values for κ_s and B_s for the present experiments.

4.2.3 Fitting method (least-squares method)

This method is equivalent to the classical approach used for the estimation of the von Kármán coefficient and it is mostly used to determine the shear velocity under the assumption that κ and B take universal values. Since, according to Eq. (5), the velocity profile in the logarithmic region should obey the linear relationship between U^+ and $\ln y^+$, the values of κ

and B can be determined through a least-squares linear fit (the wake function is neglected).

By minimizing the error from the least-squares, the values of κ_f and B_f can be obtained:

$$\kappa_f = \frac{N \sum_{i=1}^N \ln^2 y_i^+ - (\sum_{i=1}^N \ln y_i^+)^2}{N \sum_{i=1}^N U_i^+ \ln y_i^+ - \sum_{i=1}^N U_i^+ \sum_{i=1}^N \ln y_i^+} \quad (11a)$$

$$B_f = \frac{\sum_{i=1}^N \ln^2 y_i^+ \sum_{i=1}^N U_i^+ - (\sum_{i=1}^N \ln y_i^+) (\sum_{i=1}^N \ln y_i^+ U_i^+)}{N \sum_{i=1}^N \ln^2 y_i^+ - (\sum_{i=1}^N \ln y_i^+)^2} \quad (11b)$$

The accuracy obtained from the least-squares fit is sensitive to the number of velocity measurements in the overlap region and the lower and upper bounds of the logarithmic region. In this study, $70 < y_i^+ < 700$ is chosen for the bound of the logarithmic region. The values of κ_f and B_f obtained from the least-squares method are summarized in Table 8. Here, the regression correlation coefficient is above 99% for all tests. Table 8 also includes the estimates for κ and B obtained from the derivative and scatter methods.

The above methods lead to different values of κ and B for the same flow condition (e.g., test condition T2). The difference is primarily because of the uncertainty in each method and the number of points that correspond to the log-law region. Additional details for uncertainty evaluation can be found in the study by Segalini *et al.* (2013). In the current study, the results show that the least-squares method predicts the values for κ and B with higher regression correlation coefficient value compared to the derivative method. Also it should be mentioned that one of the advantages of the least-squares method lies in the simultaneous evaluation of κ and B, unlike the derivative method. In addition, in the scatter method, B is assumed to be dependent on the value of κ but not vice versa, whereas these parameters are dependent on each other. Different κ values provide different slopes for the streamwise mean velocity profiles in the overlap region compared to the universal value. Applying the least-squares method on other smooth OCF experimental data by Nezu and Rodi (1986), Kirkgoz (1989) and Steffler *et al.* (1983, 1985) shows that the von Kármán coefficient varies between 0.35 and 0.51 and the variation of B is from 3.0 to 7.6, which is consistent with the results from the current experiments. The least-squares method is used in the remainder of this paper and the proposed values for the log-law coefficients are based on this method.

4.3 Velocity profile and log-law

The streamwise mean velocity profiles in inner scaling for the present tests T1 – T7 are shown in Figure 5. As stated above, the friction velocity is evaluated from the power law equations and the values for κ and B are obtained from the least-squares method. Moreover, for tests T1, T4 and T7, one measurement was available in the viscous sublayer that shows agreement with the viscous sublayer law ($U^+ = y^+$). This provides an endorsement of the method used for evaluating the shear velocity. On the other hand, for large y^+ values ($y^+ > 700$) the experimental data does not obey the trend of the overlap region. This is due to the wake feature which is more obvious in tests T2, T6 and T7.

The variation of von Kármán coefficient observed in this study is in agreement with the findings of other studies by Coleman (1981, 1986) and Gaudio and Dey (2013). Different relations have been proposed to define the dependency of κ on Reynolds number. The most well known equations are Simpson's (1970) interpretation and Wosnik *et al.* (2000) estimation. In the current work, the classical Prandtl's mixing-length theory is applied to evaluate the von Kármán coefficient. Determination of mixing length for different velocity profiles requires the eddy viscosity evaluation and its variation along the depth of flow.

4.4 Eddy viscosity

As discussed earlier, the most common wake functions used for the outer region are those proposed by Krogstad (Eq. (7)) and Coles (Eq. (6)). By applying these wake functions and finding the wake parameter for each velocity profile, the eddy viscosity can be obtained. Based on Reynolds number, the eddy viscosity and mixing length can change and as a result the von Kármán coefficient can be affected.

In fully developed two-dimensional open channel flow, the total shear stress can be evaluated as (Cebeci and Smith 1974):

$$-\overline{u'v'} + \nu \frac{dU}{dy} = u_*^2(1 - \eta) \quad (12)$$

Here $-\overline{u'v'}$ is the Reynolds shear stress and $\eta = y/\delta$. One of the simplest models to eliminate the Reynolds shear stress in Eq. (12) is to use the concept of eddy viscosity, defined by $-\overline{u'v'} = \nu_t \frac{dU}{dy}$. Combining this with Eq. (12) yields the following expression for the eddy viscosity:

$$\nu_t = u_*^2(1 - \eta) \left(\frac{dU}{dy} \right)^{-1}. \quad (13)$$

Eq. (13) can be used to evaluate eddy viscosity if explicit information for the velocity distribution is available. Using Coles' wake function for high Reynolds numbers, the eddy viscosity is given by

$$\frac{\nu_t}{u_* \delta} = \kappa(1 - \eta) \left(\frac{1}{\eta} + \pi \Pi \sin \eta \pi \right)^{-1} \quad (14)$$

Although the effect of the wake parameter Π on eddy viscosity is often neglected, Nezu and Rodi (1986) were among the first to show a significant effect of Π on eddy viscosity, especially in the overlap and outer regions.

Another expression for the eddy viscosity can be obtained by applying the wake function proposed by Krogstad *et al.* (1992), yielding

$$\frac{\nu_t}{u_* \delta} = \kappa(1 - \eta) \left[\frac{1}{\eta} + 2(1 + 6\Pi)\eta - 3(1 + 4\Pi)\eta^2 \right]^{-1}. \quad (15)$$

Figures 6a and 6b show profiles of eddy viscosity for all experimental cases (T1-T7; Nezu and Rodi 1986;; Kirkgoz 1989; Roussinova *et al.* 2008; Afzal *et al.* 2009) together with the curves according to Eqs. (14) and (15). As observed in Figure 6a, the eddy viscosity based on Coles' wake function (Eq. (14)) starts from zero near the bed, attains a maximum at $y/\delta \approx 0.5$ and terminates at zero for $y = \delta$. This is different from Spalart's (1988) DNS prediction at $y = \delta$. Krogstad's wake function, on the other hand, shown in Figure 6b produces results similar to Spalart's (1988) for the eddy viscosity. Also, the maximum dimensionless eddy viscosity value evaluated using Eq. (15) is 0.091, which is close to the value reported by Spalart (1988). Thus, we have established the existence of a sizable logarithmic region in all experimental data and that the eddy viscosity distribution can be evaluated using a composite

velocity profile incorporating the wake function proposed by Krogstad *et al.* (1992). Given the availability of the OCF velocity profiles, the mixing length is further examined.

4.5 Mixing length

The mixing length ℓ_m is defined as a turbulent length scale that the fluid parcel will travel, conserving its properties, before completely mixing with the surrounding fluid. Based on Prandtl's mixing-length theory, the eddy viscosity is evaluated from the velocity gradient dU/dy and the mixing length as

$$\nu_t = \ell_m^2 \frac{dU}{dy}. \quad (16)$$

In Eq. (16), the mixing length depends on the nature of the flow and in general, is space dependent (Nezu and Nakagawa 1993). The distribution of ℓ_m in boundary layer flows has been conventionally described by two separate regions. The first region is the outer region where δ is assumed as the length scale and the outer mixing length $(\ell_m)_o$ is assumed to be proportional to δ ($\ell_m \sim \delta$)_o. In the inner layer, the variation of the mixing length is assumed to be proportional to the distance from the wall ($\ell_m \sim y$)_i. Based on von Kármán's postulation, the proportionality coefficient is κ and the mixing length in the inner region can be determined from the location to the wall ($\ell_m = \kappa y$)_i. From the latter assumption, the log-law of the wall was proposed (Cebeci and Smith 1974). Figure 7 shows the variation of normalized mixing length with increase of y/δ , where the mixing length is calculated based on the eddy viscosity obtained from Eq. (15).

As expected, the mixing length changes for different locations, confirming its space dependency. Figure 7 shows that the mixing lengths approach similar values near the wall for all experiments, when the wall distance is normalized in outer scaling. Near the bed at $y/\delta < 0.2$, all mixing length profiles collapse on to each other, since the effect of the wake parameter in this region is insignificant. Far from the bed ($y/\delta > 0.2$), the mixing length profiles vary at different wall distances because of changes in Reynolds number and wake

parameter. Also, the effect of the von Kármán coefficient on the mixing length cannot be distinguished well in Figure 7 since the mixing length distribution is plotted in outer scaling.

Figure 8 shows the near-wall variation of normalized mixing length with y^+ for the range of data near the wall. The linear relation between ℓ_m/δ and y^+ confirms the first assumption for the log-law equation. Here, the linear relation between ℓ_m/δ and y^+ specifies a slope κ_1 (i.e. $\ell_m/\delta = \kappa_1 y^+$) that is not constant but varies according to Re_θ . Figure 8 shows that as Re_θ increases the coefficient κ_1 decreases, which will ultimately change the von Kármán coefficient. In other words, Figure 8 shows that the von Kármán coefficient is Reynolds number dependent, contradictory to the classical assumption that all the constants in the log-law equation (Eq. (5)) are independent of Reynolds number.

4.6 Variation of von Kármán coefficient

Based on the present LDV measurements and the results discussed above, the classical claim that the von Kármán coefficient has a universal constant value can be refuted, at least in open channel flows. Although many studies show that κ varies with Reynolds number (e.g., Zanon et al. 2003; Segalini et al. 2013), there is still considerable discussion about the relationship between the von Kármán coefficient and Reynolds number.

As discussed above, Figure 8 shows the variation of dimensionless mixing length with y^+ . This figure elucidates the contribution of the von Kármán coefficient in the variation of mixing length for different test cases. This contribution is identified as parameter κ_1 , and its variation with Re_θ is presented in Figure 9. The Reynolds number (Re_θ) range in this study is between 330 and 6200. Figure 9 shows that κ_1 decreases sharply as Re_θ increases. For high Re_θ , κ_1 reaches an asymptotic value of approximately 1.6×10^{-4} . It should be mentioned that based on the definition for mixing length in the inner region, the von Kármán coefficient is related to κ_1 by

$$\kappa = \kappa_1 \delta^+ \quad (17)$$

In Eq. (17), δ^+ is known as the dimensionless boundary layer thickness, which explicitly reveals that there is a separation between the largest and smallest length scales in the flow. δ^+ is directly proportional to Re_θ and its variation is shown in the inset of Figure 9. By considering Eq. (17) and Figure 9, the variation of von Kármán coefficient with Re_θ can be simplified and expressed as

$$\kappa \approx 0.2725 Re_\theta^{0.031} + 42.78 Re_\theta^{-0.969} . \quad (18)$$

It should be mentioned that Eq. (18), which is derived for near-uniform smooth OCF in a specific range of Reynolds number ($Re_\theta = 300$ to $6,200$), is another way to predict the Kármán coefficient rather than using a constant assumption.

5 Conclusions

The evaluation of the bed shear velocity and log-law coefficients in smooth open channel flow are re-examined in this study for $Re_h < 60,000$. The main findings of the present study can be summarized as follows:

- The friction velocity has been evaluated accurately from the streamwise velocity profile at different Reynolds numbers by applying various forms of power law in open channel flow. The power law methods are straightforward to implement numerically as it involves only one unknown.
- The present study discusses various methods used to determine the von Kármán coefficient. Analytical expressions have been developed by different methods for fitting the streamwise velocity profiles. Values of κ and B were estimated based on the least-squares method, since this method has a higher regression correlation and also is more accurate compared to the derivative and scatter methods.
- The mixing length in open channel flow is evaluated based on the eddy viscosity distribution assuming a composite velocity profile incorporating the wake function proposed by Krogstad *et al.* (1992). The mixing length is found to vary with wall distance, as reported earlier by Nezu and Rodi (1986). This also confirms that the size of

eddies is proportional to their distance from the wall. In the outer region, far from the bed ($y/\delta > 0.2$), the flow Reynolds number and wake strength influence the mixing length. In contrast, in the inner region, the mixing length is only a function of the distance from the wall and Re_θ . The Reynolds number effect on the inner mixing length is presented as the coefficient κ_1 and δ^+ which, when multiplied together gives the von Kármán coefficient κ . In other words, the von Kármán coefficient is not constant and is dependent on Reynolds number.

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List of symbols

B = log-law of the wall constant (-)

B_f = log-law of the wall constant by fitting method (-)

B_s = log-law of the wall constant by scatter method (-)

C = coefficient (-)

C_1 = coefficient (-)

F_r = Froude number based on the average velocity (-)

h = water depth (m)

Re_D = Reynolds number based on pipe diameter (D) and friction velocity (-)

Re_h = Reynolds numbers based on the water depth (h) and free stream velocity (-)

Re_R = Reynolds number based on hydraulic radius (R_h) and mean velocity (-)

Re_θ = Reynolds number based on momentum thickness (θ) and free stream velocity (-)

U = mean streamwise velocity (ms^{-1})

U_m = depth-averaged streamwise velocity (ms^{-1})

U^+ = dimensionless mean streamwise velocity, normalized by friction velocity (-)

u_* = friction velocity (m s^{-1})

U_e = free stream velocity (m s^{-1})

$-\overline{u'v'}$ = Reynolds shear stress ($\text{m}^2 \text{s}^{-2}$)

y = normal distance from the wall (m)

y^+ = normalized distance from the wall, scaled by viscous length (-)

α = coefficient (-)

α_1 = coefficient (-)

β_1 = coefficient (-)

δ = boundary layer thickness (m)

δ^+ = dimensionless boundary layer thickness based on the viscous length scale (-)

η = normalized distance from the wall, scaled by boundary layer thickness (-)

θ = momentum thickness (m)

ℓ_m = mixing length (m)

κ = von Kármán coefficient (-)

κ_1 = coefficient (-)

κ_{der} = von Kármán coefficient by derivative method (-)

κ_f = von Kármán coefficient by fitting method (-)

κ_s = von Kármán coefficient by scatter method (-)

ρ = fluid density (kg m^{-3})

ν = fluid kinematic viscosity ($\text{m}^2 \text{s}^{-1}$)

ν_t = eddy viscosity ($\text{m}^2 \text{s}^{-1}$)

Π = wake strength (-)

τ_w = wall shear stress (Pa)

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Table 1 Power law ($U^+ = C(y^+)^{\alpha}$) coefficients defined by researchers

	Researcher	C	α
Power law methods	Barenblatt (1993)	$\frac{1}{\sqrt{3}} \ln^* \text{Re}_{\delta} + 2.5$	$\frac{3}{2 \ln \text{Re}_{\delta}}$
	Kailasnath (1993)	$2.8 \text{Re}_{\delta}^{-0.077} + 2.277$	$0.314 \text{Re}_{\delta}^{-0.077}$
	Zagarola <i>et al.</i> (1997)	$0.7053 \ln \text{Re}_{\delta} + 0.3055$	$\frac{1.085}{\ln \text{Re}_{\delta}} + \frac{6.535}{(\ln \text{Re}_{\delta})^2}$
	Balachandar <i>et al.</i> (2002)	7.957	0.1551

*Reynolds number based on boundary layer thickness (δ) and depth-averaged velocity (U_m)

$$(Re_{\delta} = \frac{2\delta U_m}{\nu})$$

Table 2 Experimental details

Test	$U_e \times 10^{-2}$ (ms^{-1})	$h \times 10^{-2}$ (m)	b/h	$\delta \times 10^{-2}$ (m)	Re_{θ}	F_r	Re_h
T1	32.0	6.3	19.4	5.3	1,674	0.35	20,466
T2	24.0	15.4	7.9	8.2	1,962	0.18	36,849
T3	44.8	8.4	14.5	6.8	2,987	0.44	36,418
T4	57.9	7.2	16.9	6.3	3,790	0.61	37,306
T5	57.1	10.4	11.7	8.1	4,417	0.51	60,370
T6	15.6	7.1	17.0	5.4	839	0.17	10,563
T7	51.1	7.1	17.0	5.1	2516	0.54	33,755

Table 3 Uncertainty estimates for the mean velocity (U)

Test	Uncertainty (%)		
	Near bed ($y^+ < 15$)	Overlap region	Outer region
T1	0.8	0.5	0.4
T2	0.7	0.5	0.4
T3	0.6	0.5	0.4
T4	0.7	0.5	0.4
T5	0.5	0.5	0.4
T6	0.7	0.4	0.4
T7	0.5	0.4	0.4

Table 4 Bed shear velocity, $u_* \times 10^{-2} \text{ (ms}^{-1}\text{)}$ calculated using different power law methods

Experiments	Power law methods				Average value	Reported value	Difference (%)
	Barenblatt (1993)	Kailasnath (1993)	Zagarola and Smits (1997)	Balachandar <i>et al.</i> (2002)			
Nezu and Rodi (P1)*	0.42	0.43	0.41	0.43	0.42	0.43	2.3
Nezu and Rodi (P2)*	0.95	0.97	0.95	0.96	0.96	0.95	1.1
Nezu and Rodi (P3)*	2.18	2.20	2.21	2.22	2.20	2.20	0.0
Nezu and Rodi (P4)*	2.98	3.01	3.03	3.04	3.02	3.04	0.7
Nezu and Rodi (P5)*	4.75	4.80	4.85	4.89	4.80	4.66	3.0
Nezu and Rodi (P6)*	2.67	2.73	2.70	2.74	2.71	2.71	0.0
Nezu and Rodi (P7)*	4.05	4.09	4.11	4.12	4.09	3.99	2.5
Nezu and Rodi (P8)*	2.20	2.23	2.23	2.23	2.22	2.22	0.0
Steffler <i>et al.</i> (SRP1)*	3.34	3.38	3.38	3.36	3.37	3.35	0.6
Steffler <i>et al.</i> (SRP2)*	1.66	1.69	1.66	1.68	1.67	1.64	1.8
Kirkgoz (S2)*	0.27	0.28	0.26	0.28	0.27	0.28	3.6
Kirkgoz (S3)*	1.07	1.09	1.05	1.07	1.07	1.04	2.9
Kirkgoz (S4)*	0.60	0.61	0.59	0.61	0.60	0.59	1.7
Kirkgoz (S5)*	6.50	6.60	6.40	6.59	6.52	6.58	0.9
Kirkgoz (S6)*	0.84	0.85	0.83	0.85	0.84	0.85	1.2
Kirkgoz (S10)*	2.53	2.56	2.53	2.54	2.54	2.47	2.8
Kirkgoz (S12)*	3.82	3.87	3.82	3.83	3.84	3.82	0.5

*The number in the brackets corresponds to different flow conditions with respect to Reynolds number and Froude number.

Table 5 Estimates for $u_* \times 10^{-2}$ (ms^{-1}) for the present experiments using different power law methods

Test	Power law methods					Average value
	Barenblatt	Kailasnath	Zagarola and	Balachandar	Buschmann and	
	(1993)	(1993)	Smits (1997)	<i>et al.</i> (2002)	Meinert (1999)	
T1	1.43	1.45	1.43	1.44	1.42	$1.43 \pm 0.4 \%$
T2	1.06	1.07	1.07	1.07	1.06	$1.07 \pm 0.4 \%$
T3	1.89	1.91	1.91	1.91	1.88	$1.90 \pm 0.4 \%$
T4	2.37	2.40	2.39	2.39	2.36	$2.38 \pm 0.4 \%$
T5	2.31	2.34	2.33	2.33	2.31	$2.33 \pm 0.4 \%$
T6	0.75	0.77	0.74	0.77	0.76	$0.76 \pm 0.1 \%$
T7	2.18	2.20	2.17	2.18	2.16	$2.18 \pm 0.4 \%$

Table 6 Bed shear velocity comparison using the Reynolds stress and the power law methods

Test	$u_* \times 10^{-2}$ (ms^{-1})	
	Using total stress	Average value using the
	method	power law methods
T4	2.33	2.38
T6	0.77	0.76
T7	2.23	2.18
Roussinova <i>et al.</i> 2008 (1)	2.17	2.19
Roussinova <i>et al.</i> 2008 (2)	2.14	2.11
Afzal <i>et al.</i> 2009 (B-SM2)	1.57	1.58

Table 7 Reported values of κ from different sources

Reference	Experimental / Numerical	Reported value for κ	Details
von Kármán (1930)	Smooth pipe	0.41	Laboratory measurement
Sheppard (1947)	ABL	0.46	Field measurement
Pasquill (1950)	ABL	0.37	Field measurement
Businger <i>et al</i> (1971)	ABL	0.35	Field measurement
Tennekes (1973)	TBL	0.35	Wind tunnel
Francey and Garratt (1981)	ABL	0.38	Field measurement
Coleman (1981, 1986)	OCF	0.26-0.60	Suspended sediment flow
Nezu and Rodi (1986)	OCF	0.41	¹ $Re_R = 23,000 - 440,000$
Kim <i>et al.</i> (1987)	DNS	0.40	² $\delta^+ = 180$
Oncley <i>et al.</i> (1990)	ABL	0.364	Re-analysis of field measurements
Long <i>et al.</i> (1993)	Smooth pipe	0.408	³ $Re_D = 1,000 - 100,000$
Moser <i>et al.</i> (1999)	DNS	0.45-0.33	² $\delta^+ = 590$
Österlund <i>et al.</i> (2000)	TBL	0.38	⁴ $Re_\theta = 2,500 - 27,000$, Wind tunnel
Zanoun <i>et al.</i> (2003)	Channel flow	0.37	² $\delta^+ = 4,800$
Monty (2005)	Duct flow	0.389	² $\delta^+ = 4,000$, Smooth wall
Hoyas and Jimenez (2006)	DNS	0.41	² $\delta^+ = 2,000$
Gaudio and Dey (2013)	OCF	0.35-0.42	Sediment transport
Segalini <i>et al.</i> (2013)	TBL	0.36-0.44	⁴ $Re_\theta = 2500 - 27000$

¹ Reynolds number based on hydraulic radius (R_h) and average velocity. ($Re_R = 4R_h U_m/\nu$)

² Reynolds number based on boundary layer thickness (δ) and friction velocity ($\delta^+ = \delta u_*/\nu$)

³ Reynolds number based on pipe diameter (D) and friction velocity ($Re_D = 0.5D u_*/\nu$)

⁴ Reynolds number based on momentum thickness (θ) and free stream velocity ($Re_\theta = U_e \theta/\nu$)

Table 8 Comparison between different methods for κ and B

Test	Derivative method		Scatter method			Least-squares method		
	κ_{der}	R^2 (%)	κ_s	B_s	$\Delta B/2$	κ_f	B_f	R^2 (%)
T1	0.42	98.7	0.42	5.57	0.11	0.40	5.15	99.9
T2	0.38	95.4	0.42	5.39	0.14	0.36	3.58	99.4
T3	0.39	98.6	0.42	5.61	0.15	0.40	5.04	99.8
T4	0.41	96.0	0.42	5.51	0.19	0.40	4.89	99.4
T5	0.39	98.3	0.43	5.78	0.15	0.39	4.60	99.7
T6	0.35	96.1	0.47	6.75	0.04	0.38	4.49	99.0
T7	0.44	98.2	0.47	7.00	0.03	0.36	3.58	99.4

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Figure Captions

Figure 1: Typical turbulent flow regions in OCF

Figure 2: The experimental flume setup

Figure 3: Variation of $\frac{dU^+}{dy^+}$ vs. $\frac{1}{y^+}$ using the derivative method

Figure 4: Determination of κ_s and B_s for different experiments using scatter method

Figure 5: Streamwise mean velocity distribution in inner scale for different tests

Figure 6: Dimensionless eddy viscosity distribution based on (a) Coles' wake function and (b) Krogstad's wake function

Figure 7: Dimensionless mixing length distributions (outer scaling)

Figure 8: Dimensionless mixing length distributions (inner scaling)

Figure 9: Variation of κ_1 with Re_θ

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Typical turbulent flow regions in OCF
Figure 1
274x211mm (96 x 96 DPI)

The experimental flume setup
Figure 2
321x88mm (96 x 96 DPI)

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Determining K_s and B_s for different experiments using scatter method
 Figure 4
 969x700mm (96 x 96 DPI)

Dimensionless mixing length distributions (outer scaling)
Figure 7
329x230mm (96 x 96 DPI)

Dimensionless mixing length distributions (inner scaling)

Figure 8

331x218mm (96 x 96 DPI)

Variation of κ_1 with Re_θ
Figure 9
327x230mm (96 x 96 DPI)