Canadian Journal of Civil Engineering
Revue canadienne de génie civil

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Journal:	<i>Canadian Journal of Civil Engineering</i>
Manuscript ID	cjce-2016-0246.R3
Manuscript Type:	Article
Date Submitted by the Author:	01-Dec-2016
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Keyword:	Long-span bridges; coupled flutter; full-order analysis; single-parameter searching; simultaneous iteration.

Full-order and single-parameter searching analysis of coupled flutter instability for long-span bridges

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Draft

Abstract: A full-order and single-parameter searching method (F–S method) for analyzing coupled flutter instability of long-span bridges is proposed based on the full-discretized model of structure. Based on the proper approximation of the circular frequency of complex modes, the characteristic equation of the full-order system is expressed as a complex generalized eigenvalue equation which contains only two variables. The equation is used for flutter analysis by solving the complex generalized eigenvalue problem with an efficient simultaneous iteration method directly. Since its computation is reliable and efficient, the application of the proposed method on the flutter problems of long-span bridges is practical. Moreover, the flutter analysis is performed for Jiangyin Yangtse-river suspension bridge with 1385m main span in the completed stage to illustrate the reliability and effectiveness of the proposed method.

Keywords: Long-span bridges; coupled flutter; full-order analysis; single-parameter searching; simultaneous iteration.

1. Introduction

Following the collapse of the Tacoma Narrows Bridge in 1940, it was recognized that wind stability (flutter and divergence) is one of the design governing criteria of long-span bridges. With the development of longer cable-supported bridges, the static wind force could cause large deformation of structures in the completed and construction stages. This large deformation may remarkably alter the tangent stiffness and especially the attack angle along the bridge axis, and therefore both the mechanical structures and the aerodynamic forces would be affected. Accordingly, the flutter responses of bridge structures could be significantly changed. Moreover, the structural forms of long-span bridges are becoming more and more complex. The long-span bridges with inclined, cross hangers, mono-duo cables or some controlling systems have been applied. The natural characteristics of these bridge

structures could be very complex and the analysis of the flutter problems would be difficult.

The prediction of flutter behavior of such a large and complex system would require a refined analytical method that could work on the full model within the computational resources. At present, some numerical methods for analyzing flutter problems have been developed. These methods can be divided into two types. One type of methods is based on the modal coordinates of structure and is called the multimode flutter analysis. This type of methods converts the large physical system that consists of the structure and the self-excited forces into a generalized system, which contains a few degrees of freedom, using the natural modes of structure. The multimode flutter analysis is based on the fact that the flutter responses of long-span bridges are mainly composed of the first low-frequency natural modes of structures. This type of methods is very attractive due to their efficiency and has been widely employed. Scanlan (1978) and Scanlan and Jones (1990) established a basic theory for the multimode flutter analysis of long-span bridges and suggested a mode-by-mode approach. Xie and Xiang (1985) employed a planar model of unsteady aerodynamic forces and proposed a state-space method for multimode flutter problems. Agar (1989) converted the flutter motion equation into the eigenvalue problem of a real unsymmetric matrix. Namini and Albrecht (1992) proposed the PK-F method for multimode flutter problems. The method is used to solve iteratively nonlinear equations and can provide the information about the variations of structural dynamic behavior with the wind speed. Other researchers converted the flutter motion equation into various characteristic equations and solved the eigenvalue problems by different approaches (Jain et al. 1996; Boonyapinyo et al. 1999; Chen et al. 2000 and 2008; Hua et al. 2007; Bartoli and Mannini 2008; Vu et al. 2011; Omenzetter 2012; Mannini et al. 2012).

The other type of method for flutter prediction is built on the full-order physical model of bridges. Compared with the multimode flutter analysis, it is unnecessary to select the natural

modes in the full-order analysis. Therefore, the full-order analysis could be more suitable to the flutter analysis for the bridges having more complex structural forms such as the suspension bridge at the construction stage and the long-span bridges with inclined, cross hangers, mono-duo cables or some controlling systems. Miyata et al. (1988, 1995) first presented the direct flutter analysis but the effect of structural damping was not discussed. Dung et al. (1998) further extended the direct flutter analysis and solved the characteristic equations by the mode tracing method, but the effect of structural damping on the flutter velocity was still not considered effectively. Ge and Xiang (2008) also proposed a 3D-flutter analysis of long-span bridges by full-mode approach. These full-order flutter analyses are two-parameter searching methods and their computation efforts are large. Therefore they are rarely employed in the actual flutter analysis of long-span bridges.

This paper proposes a novel full-order method for analyzing coupled flutter instability of long-span bridges. The model of self-excited forces that contains 18 flutter derivatives is employed and expressed in a complex-number form. The characteristic equation of the full-order system, which is expressed as a complex generalized eigenvalue equation and contains only two variables, is used for flutter analysis by solving the complex generalized eigenvalue problem with an efficient simultaneous iteration method directly. The proposed method is a single-parameter searching method that is more stable and efficient than the two-parameter one.

2. General formulation

The self-excited forces per unit span are expressed in Scanlan's extended format below (Scanlan 1978; Scanlan 1993):

$$L_{se}(t) = \frac{1}{2} \rho U^2 (2B) \left(KH_1^* \frac{\dot{h}}{U} + KH_2^* \frac{B\dot{\alpha}}{U} + K^2 H_3^* \alpha + K^2 H_4^* \frac{h}{B} + KH_5^* \frac{\dot{p}}{U} + K^2 H_6^* \frac{p}{B} \right) \quad (1a)$$

$$D_{se}(t) = \frac{1}{2} \rho U^2 (2B) (KP_1^* \frac{\dot{p}}{U} + KP_2^* \frac{B\dot{\alpha}}{U} + K^2 P_3^* \alpha + K^2 P_4^* \frac{p}{B} + KP_5^* \frac{\dot{h}}{U} + K^2 P_6^* \frac{h}{B}) \quad (1b)$$

$$M_{se}(t) = \frac{1}{2} \rho U^2 (2B^2) (KA_1^* \frac{\dot{h}}{U} + KA_2^* \frac{B\dot{\alpha}}{U} + K^2 A_3^* \alpha + K^2 A_4^* \frac{h}{B} + KA_5^* \frac{\dot{p}}{U} + K^2 A_6^* \frac{p}{B}) \quad (1c)$$

where ρ is the air density; U is the mean wind velocity; $B(=2b)$ is the bridge deck width; $K(=\omega B/U)$ is the reduced frequency and $V^* (=U/\omega B)$ is the reduced velocity, which ω is the circular frequency of vibration; h , p , and α are the vertical, lateral, and torsional displacements, respectively; the over-dot denotes the partial differentiation with respect to time t ; and H_i^* , P_i^* , A_i^* ($i=1\sim6$) are the non-dimensional flutter derivatives, which are functions of the reduced frequency and depend on the geometrical configuration of the bridge section and the approach flow. The positive direction of the aerodynamic forces and the displacements is shown in Fig. 1.

Fig. 1. Aerodynamic forces acting on bridge deck

For the bluff bridge deck sections, the flutter derivatives can be determined experimentally. This is actually done through a system identification method in the frequency domain or in the time domain using free-vibration or forced-vibration tests in a wind tunnel (Sarkar et al. 1994, Iwanmoto and Fujino 1995). The measurement of free vibration is relatively simple and is often applied. At present a two-degree-of-freedom section model of the bridge deck is widely used to identify the flutter derivatives H_i^* and A_i^* ($i=1\sim4$). The drag and components associated with lateral motion are generally negligible, but may become important for certain bridge deck configuration. The quasi-steady theory is invoked to consider the effects in the absence of the measured results in wind tunnel (Miyata et al. 1994).

$$P_1^* = -\frac{1}{K} C_D, \quad P_2^* = \frac{1}{2K} C_D', \quad P_3^* = \frac{1}{2K^2} C_D'$$

$$P_5^* = \frac{1}{2K} C_D', \quad H_5^* = \frac{1}{K} C_L, \quad A_5^* = -\frac{1}{K} C_M$$

$$P_4^* = P_6^* = H_6^* = A_6^* = 0 \quad (2)$$

where C_L , C_D , and C_M are the static lift, drag and moment coefficients (referred to deck width B), respectively; and $C_D' = dC_D/d\alpha$.

The expressions (1) are the real-number form of self-excited forces. In complex-number notation, the corresponding expressions of aerodynamic forces are (Starossek 1998):

$$L_{se}(t) = \omega^2 \rho B^2 (C_{Lh} h + C_{Lp} p + BC_{L\alpha} \alpha) \quad (3a)$$

$$D_{se}(t) = \omega^2 \rho B^2 (C_{Dh} h + C_{Dp} p + BC_{D\alpha} \alpha) \quad (3b)$$

$$M_{se}(t) = \omega^2 \rho B^2 (BC_{Mh} h + BC_{Mp} p + B^2 C_{M\alpha} \alpha) \quad (3c)$$

where C_{rs} ($r = D, L, M$; $s = h, p, \alpha$) are the complex-number coefficients of self-excited forces.

The relationships between the real-number and complex-number aerodynamic coefficients can be established by comparing the corresponding expressions of aerodynamic force. The following relations are found:

$$C_{Lh} = H_4^* + iH_1^*, C_{Lp} = H_6^* + iH_5^*, C_{L\alpha} = H_3^* + iH_2^* \quad (4a)$$

$$C_{Dh} = P_6^* + iP_5^*, C_{Dp} = P_4^* + iP_1^*, C_{D\alpha} = P_3^* + iP_2^* \quad (4b)$$

$$C_{Mh} = A_4^* + iA_1^*, C_{Mp} = A_6^* + iA_5^*, C_{M\alpha} = A_3^* + iA_2^* \quad (4c)$$

Comparing the real-number force expressions (1) with the equivalent complex-number expressions (3), it is obvious that the complex-number equations are more compact. The reason is that the complex-number coefficients naturally represent the phasing between displacements and displacement-induced aerodynamic forces. In the real-number notation, the velocity terms are included in order to account for phasing properly. Although the two forms of self-excited forces are equivalent in essence, the presented method will exhibit that the complex-number expressions (3) are more beneficial. Moreover, the phasing between displacements and aerodynamic forces becomes quite obvious in the complex-number form.

The governing equation of motion of a bridge structure excited by aerodynamic forces is given in a matrix form by

$$\mathbf{M}\ddot{\mathbf{X}} + \mathbf{C}\dot{\mathbf{X}} + \mathbf{K}\mathbf{X} = \mathbf{F}_{se} + \mathbf{F}_b \quad (5)$$

where $\mathbf{M}$, $\mathbf{C}$, and $\mathbf{K}$ are the structural mass, damping, stiffness matrices, respectively; $\mathbf{X}$ is the nodal displacement vector; $\mathbf{F}$ indicates the nodal force vector; each dot denotes the partial differentiation with respect to time t ; and the subscript se and b represent the self-excited and turbulence-induced buffeting force components, respectively. It is assumed that the buffeting forces have no influence on the aerodynamic stability and are excluded in the following parts.

In the FEM analysis, the distributed forces of a bridge deck are converted into equivalent nodal loads at member ends as follows:

$$\mathbf{F}_{se}^e = \omega^2 \mathbf{A}_{se}^e \mathbf{X}^e \quad (6)$$

where the superscript e represents the local coordinates of the member (see Fig. 2). $\mathbf{A}_{se}^e$ is a 12 by 12 aerodynamic matrix of the member, and has lumped and consistent forms as the mass matrix. The lumped aerodynamic matrix of a bridge deck member with length L is

$$\mathbf{A}_{se}^e = \begin{bmatrix} \mathbf{A} & \mathbf{0} \\ \mathbf{0} & \mathbf{A} \end{bmatrix} \quad (7)$$

where

$$\mathbf{A} = \frac{1}{2} \rho B^2 L \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & C_{Lh} & C_{Lp} & BC_{L\alpha} & 0 & 0 \\ 0 & C_{Dh} & C_{Dp} & BC_{D\alpha} & 0 & 0 \\ 0 & BC_{Mh} & BC_{Mp} & B^2 C_{M\alpha} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (8)$$

Fig. 2. Positive directions of 12-degree-of-freedom space frame member

Since the aerodynamic forces are non-conservative, the aerodynamic matrix of the member is generally unsymmetrical and is a function of the reduced frequency. When the aerodynamic matrices of the members are transformed into the structural global coordinates and are

assembled, then

$$\mathbf{F}_{se} = \omega^2 \mathbf{A}_{se} \mathbf{X} \quad (9)$$

where $\mathbf{A}_{se}$ is the structural aerodynamic matrix. Obviously, $\mathbf{A}_{se}$ is a complex matrix.

3. Full-order flutter analysis

Based on the preceding part, the governing equation of motion is expressed in the complex form as

$$\mathbf{M}\ddot{\mathbf{X}} + \mathbf{C}\dot{\mathbf{X}} + \mathbf{K}\mathbf{X} = \omega^2 \mathbf{A}_{se} \mathbf{X} \quad (10)$$

Let $\mathbf{X} = \mathbf{R}e^{st}$, where $\mathbf{R}$ is the complex modal response of the system including the structure and airflow; its corresponding complex frequency $s = (-\xi + i)\omega$ (where ξ and ω are the damping ratio and circular frequency of the complex mode, respectively, and $i^2 = -1$), substituting that into Eq. (10) yields

$$(s^2 \mathbf{M} + s \mathbf{C} + \mathbf{K} - \omega^2 \mathbf{A}_{se}) \mathbf{R} e^{st} = 0 \quad (11)$$

For the exponential e^{st} is never zero, the characteristic equation is given as

$$(s^2 \mathbf{M} + s \mathbf{C} + \mathbf{K} - \omega^2 \mathbf{A}_{se}) \mathbf{R} = 0 \quad (12)$$

where s is the eigenvalue of the system and $\mathbf{R}$ is the corresponding eigenvector of the system.

Eq. (12) would have a nontrivial solution for $\mathbf{R}$, provided that the matrix with the parentheses on the left-hand side is singular. In fact, this equation represents what is known as the quadratic eigenvalue problem. Its solution gives $2n$ eigenvalues and corresponding eigenvectors, where n is the total number of degree of freedom. However, several low-frequency complex modes of the system are needed to take into account in the actual flutter analysis of long-span bridges. Therefore, it is only necessary to solve some eigenvalues and corresponding eigenvectors of the characteristic equation.

Considering the fact that the damping ratios of the system (positive or negative) are small, the circular frequency of a complex mode is approximately given by $\omega = -si$ (where $i^2 = -1$),

inserting this into Eq. (12) yields

$$[s^2(\mathbf{M} + \mathbf{A}_{se}) + s\mathbf{C} + \mathbf{K}]\mathbf{R} = 0 \quad (13)$$

The vibration corresponding to the self-excited force expressions given by Eq. (3) is harmonic and constant-amplitude. The effect of the damping ratio on self-excited forces is still an open problem for further discussion. It has been noted that the damping ratio of the system is equal to zero (i.e. $\omega = -si$) at the critical flutter state, so the above approximation is proper and does not have any influence on the critical flutter state and flutter velocity.

It is appropriate to transform the quadratic eigenvalue problem to a linearized form, since it allows for the use of methods applicable to the eigenvalue solution in an undamped case. To obtain the linearized form, the additional equations are supplied in an interesting way:

$$(\mathbf{M} + \mathbf{A}_{se})s\mathbf{R} - (\mathbf{M} + \mathbf{A}_{se})s\mathbf{R} = 0 \quad (14)$$

Eqs. (13) and (14) can be combined into the following matrix equation:

$$\mathbf{A}\mathbf{Y} = s\mathbf{B}\mathbf{Y} \quad (15)$$

where

$$\mathbf{Y} = \begin{Bmatrix} s\mathbf{R} \\ \mathbf{R} \end{Bmatrix} \quad (16a)$$

$$\mathbf{A} = \begin{bmatrix} \mathbf{M} + \mathbf{A}_{se} & \mathbf{0} \\ \mathbf{0} & -\mathbf{K} \end{bmatrix} \quad (16b)$$

$$\mathbf{B} = \begin{bmatrix} \mathbf{0} & \mathbf{M} + \mathbf{A}_{se} \\ \mathbf{M} + \mathbf{A}_{se} & \mathbf{C} \end{bmatrix} \quad (16c)$$

Eq. (15) is a generalized eigenvalue equation and it can be used for the full-order flutter analysis of long-span bridges in which an efficient simultaneous iteration method is introduced to solve this eigenvalue problem in the following part.

Because the employed expressions of self-excited forces in Eq. (3) do not contain any term in U , neither does Eq. (15), and thus it needs to be solved for only two variables, s and K . Then a process of two-parameter searching is avoided for the flutter analysis. After fixing K ,

the procedure of solving the above generalized eigenvalue problem with the simultaneous iteration method is as follows:

1. Compute the structural aerodynamic matrix $\mathbf{A}_{se}$ corresponding to the reduced frequency K , the initial values of eigenvectors are set to $\mathbf{V}_1$.
2. Enter the iteration loop, determine the complex matrix $\mathbf{A}$ and $\mathbf{B}$, solve $\mathbf{A}\mathbf{V}_{k+1} = \mathbf{B}\mathbf{V}_k$ for $\mathbf{V}_{k+1}$.
3. Compute $\mathbf{G}^* = \mathbf{V}_k^T \mathbf{V}_k$ and $\mathbf{H}^* = \mathbf{V}_k^T \mathbf{V}_{k+1}$.
4. Solve $\mathbf{G}^* \mathbf{Q}^* = \mathbf{H}^*$ for $\mathbf{Q}^*$.
5. The matrix $\mathbf{Q}^*$ is analyzed by standard linear eigensolver, i.e. $\mathbf{Q}^* \mathbf{Y}_{k+1} = \mathbf{Y}_{k+1} \mathbf{\Lambda}_{k+1}^{-1}$, where the elements of diagonal matrix $\mathbf{\Lambda}_{k+1}$ are the m approximate eigenvalues of Eq. (15).
6. Compute $\mathbf{Z}_{k+1} = \mathbf{V}_{k+1} \mathbf{Y}_{k+1}$, $\mathbf{Z}_{k+1}$ is normalized to the matrix $\bar{\mathbf{Z}}_{k+1}$, where the columns of normalized matrix are the approximate eigenvectors.
7. When the relative error of the i th element of $\mathbf{\Lambda}_{k+1}$ and $\mathbf{\Lambda}_k$ is below tolerance, i.e. $|\lambda_{k+1}^i - \lambda_k^i| < tol$, where the tolerance value is nearly zero and i indicates the traced complex mode, stop the iteration loop; otherwise, $\mathbf{V}_k = \bar{\mathbf{Z}}_{k+1}$ and $k=k+1$, return to step 2.

When the reduced frequency K is known, the eigenvalues s and corresponding eigenvectors $\mathbf{R}$ of the characteristic equation can be solved from Eq. (15) by the simultaneous iteration method, and

$$s = (-\xi + i)\omega \quad (17a)$$

$$\mathbf{R} = \mathbf{a} + \mathbf{b}i \quad (17b)$$

The eigenvectors $\mathbf{R}$ are the complex modal shapes of the system. In a prescribed complex modal shape, the magnitude and phase of the k th nodal displacement are given as

$$|\mathbf{R}_k| = \sqrt{\mathbf{a}_k^2 + \mathbf{b}_k^2} \quad (18a)$$

$$\phi_k = \tan^{-1}(b_k/a_k) \quad (18b)$$

The critical flutter state can be identified from the complex modes of system. If the damping ratios of complex modes are positive the system is stable; if one damping ratio is equal to zero the system is neutrally stable; if one damping ratio is negative the system is unstable. Fortunately, the actual flutter always occurs on the low-frequency modes, therefore only few complex modes of system are required to discuss in the full-order flutter analysis. The flutter analysis is to find the critical state that one damping ratio of the system is zero through searching the reduced velocity V^* . The corresponding circular frequency is the flutter circular frequency ω_f and the critical flutter velocity can be computed by $U_{cr} = B\omega_f/K$. A simple automatic searching procedure can be employed to find the lowest critical flutter velocity.

To understand the participation of natural modes of structure in the flutter motion, the generalized modal coordinates of flutter mode are computed through

$$\mathbf{R}_f = \Phi \mathbf{q}_f \quad (19)$$

Multiply (19) with $\Phi^T \mathbf{M}$ on the left, yields

$$\mathbf{q}_f = \Phi^T \mathbf{M} \mathbf{R}_f = \mathbf{a}_f + \mathbf{b}_f i \quad (20)$$

So in the complex flutter modal shape, the magnitude and phase of the k th natural mode are given as

$$|\mathbf{q}_{fk}| = \sqrt{a_{fk}^2 + b_{fk}^2} \quad (21a)$$

$$\phi_{fk} = \tan^{-1}(b_{fk}/a_{fk}) \quad (21b)$$

In the actual flutter analysis of long-span bridges, the critical flutter state is determined through searching the reduced velocity from low to high. The initial values of $\mathbf{R}_1$ at the first reduced velocity are generally set to the first m natural modal shapes of structure. It has been noted that the complex eigenvalues and eigenvectors of the characteristic equation vary

continuously with the reduced velocity. Thus it is proper and efficient that the initial values at each reduced velocity are set to the convergent results at the last reduced velocity.

4. Determine the structural damping matrix

The effect of structural damping on the flutter velocity should be considered accurately in the flutter analysis of long-span bridges. In order to keep the band-packed storing of large sparse matrix, the structural damping matrix is employed as the Rayleigh's orthogonal form.

$$\mathbf{C} = \alpha \mathbf{M} + \beta \mathbf{K} \quad (22)$$

where α and β are the real coefficients, which can be determined by the damping ratios and circular frequencies of structural natural modes.

It is generally believed that the damping ratios and circular frequencies of natural modes of the bridge structures are known. In terms of the consistency of the Rayleigh's damping and the modal damping, the following relationship can be established (Clough and Penzien 1975).

$$\begin{bmatrix} \xi_1 \\ \xi_2 \end{bmatrix} = \frac{1}{2} \begin{bmatrix} \frac{1}{\omega_1} & \omega_1 \\ \frac{1}{\omega_2} & \omega_2 \end{bmatrix} \begin{bmatrix} \alpha \\ \beta \end{bmatrix} \quad (23)$$

where ξ_i and ω_i ($i=1,2$) are the damping ratios and circular frequencies of the structural natural modes, respectively.

When the damping ratios and circular frequencies of structural natural modes are known, the real coefficients of the Rayleigh's damping matrix could be solved from Eq. (23) by the damping ratios and circular frequencies of two structural natural modes. It is noted that the selection of the two structural natural modes has a small influence on the real coefficients. Then it could affect slightly the result of flutter analysis. Therefore, the real coefficients should be computed with the damping ratios and circular frequencies of the two dominant natural modes in the coupled flutter motion of the long-span bridge. For instance, the first

natural mode of symmetric vertical motion and the first one of symmetric torsional motion often dominate the coupled flutter of the long-span bridges and then the real coefficients should be computed with the two natural modes.

5. Example

The proposed F–S method for analyzing flutter problems is coded into a computer program, which is able to analyze the spatial flutter problems of long-span bridges and take into consideration the nonlinear deformations under dead loads and static wind loads. To illustrate the reliability and effectiveness of the proposed method, numerical flutter analysis is performed on two typical examples: a simply supported beam structure with an idealized thin-plate section and Jiangyin Yangtse-river suspension bridge in the completed stage.

5.1 A simply supported beam structure

The first example considered is that of a simply supported beam structure with an idealized thin-plate section. Since it has a theoretical expression for the aerodynamic forces and hence it serves as a check on the proposed method. The structural properties are given as follows: span $L = 300\text{m}$; width $B = 40\text{m}$; vertical bending stiffness $EI_z = 2.1 \times 10^6 \text{ Mpa} \cdot \text{m}^4$; lateral bending stiffness $EI_y = 1.8 \times 10^7 \text{ Mpa} \cdot \text{m}^4$; torsional stiffness $GI_t = 4.1 \times 10^5 \text{ Mpa} \cdot \text{m}^4$; mass $m = 20,000\text{kg/m}$; mass moment of inertia $I_m = 4.5 \times 10^6 \text{ kg} \cdot \text{m}^2/\text{m}$; and air mass density $\rho = 1.25\text{kg/m}^3$.

Table 1. Flutter results of the beam structure

The self-excited aerodynamic lift and moment acting on the idealized thin-plate can be represented by the Theodorsen's function, and the flutter derivatives H_i^* and A_i^* ($i = 1 \sim 4$) can be determined from the Theodorsen's function (Scanlan 1993). The flutter problem of the simply supported beam is analyzed using the proposed F–S method, and the flutter results are

listed in Table 1 under the condition of zero structural damping. The analytical results of the pK-F methods (Namini 1992) are also presented in Table 1. The exact results are provided by an analytical method. The flutter velocity of the beam structure is determined at $U_f = 139.8$ m/s, and the corresponding flutter frequency is 0.38Hz. So the analytical results are very close to the exact results.

5.2 Jiangyin Yangtse-river Suspension Bridge

The flutter problem of Jiangyin Yangtse-river suspension bridge with 1385m-long main span in the complete stage shown in Fig. 3 is analyzed with the program. The bridge deck section is a streamlined box with 36.9m width and 3.0m height (see Fig. 4).

Fig. 3. Jiangyin Yangtse-river suspension bridge

Fig. 4. Cross section of the deck of Jiangyin bridge

Structural properties are as follows. cables in the center span: area = 0.4825m^2 , mass = 3974.3kg/m ; cables in the side span: area = 0.5053m^2 , mass = 4081.8kg/m , $E = 2.0 \times 10^8 \text{ kN/m}^2$. Hangers: area = 0.0064m^2 , mass = 50kg/m , $E = 1.4 \times 10^8 \text{ kN/m}^2$. A single column of towers: area = $33.389 \sim 43.148\text{m}^2$, I (longitudinal) = $371.17 \sim 847.55\text{m}^4$, I (lateral) = $137.15 \sim 183.52\text{m}^4$, J (torsional) = $299.56 \sim 479.23\text{m}^4$, $\rho = 2550\text{kg/m}^3$, $E = 3.5 \times 10^7 \text{ kN/m}^2$. Deck: area = 1.1m^2 , I (vertical) = 1.844m^4 , I (lateral) = 93.32m^4 , J (torsional) = 4.82m^4 , mass = 18000 kg/m , moment of inertia = $1.426 \times 10^6 \text{ kg} \cdot \text{m}^2/\text{m}$, overall deck width = 36.9m .

A model of the bridge deck with two degrees of freedom is used to measure the flutter derivatives H_i^* and $A_i^* (i=1 \sim 4)$ in the wind tunnel. The results at 0 degree of incidence are shown in Fig. 5. The static lift, drag, and moment coefficients of the model at different angles of incidence are measured in smooth flow (Xiang et al. 1996). The static coefficients and the derivatives of the deck section at 0 degree of incidence are $C_D = 0.0697$, $dC_D/d\alpha = 0.0$, $C_L = -0.128$, $dC_L/d\alpha = 5.5577$, and $C_M = -0.0074$, $dC_M/d\alpha = 1.2662$. Since there is no

measured result of the flutter derivatives related to the lateral motion, these flutter derivatives are calculated by the expressions given by Eq. (2) based on the quasi-steady theory. The structural damping ratio for each natural mode is assumed to 0.005.

The analyzing finite-element model of the suspension bridge is set up based on the design data. In the FE model of the bridge, both the tower and the deck are modeled with 3D beam element of 12 degrees of freedom (DOFs) in conjunction with the spine girder model, whilst the truss element is used for modeling the main cable and the hanger rods, where the geometric stiffness of the main cable due to gravity is taken into consideration in the finite-element model. The first 50 natural modes are computed by the Lanczos method, and the major modes of the bridge deck are listed in Table 2. The Sturm check on the first 50 modes is applied to prevent the missing of modes, and no mode is found missing.

Table 2. Main modes of the bridge deck in complete stage

Fig. 5. Flutter derivatives H_i^* and A_i^* of bridge deck

The automatic searching of flutter analysis is performed within the range of measured reduced velocity. Two critical flutter states are found and listed in Table 3. Furthermore, no any other flutter critical state is found in the tracing procedures for the first 15 complex modes of the system. It is noted that the lowest critical flutter velocity is 67.6m/s, and the corresponding flutter frequency is 0.241Hz. The flutter motion initiates from the first symmetric torsion mode and is obviously coupled. The variations of circular frequencies and damping ratios of the first 15 complex modes with reduced velocity are shown in Figs. 6 and 7.

The amplitude and phase shifts of the vertical, lateral and torsional displacement along the bridge axis in the flutter motion are shown in Fig. 8. The analytical results show that the coupled flutter occurs on the first-order symmetric torsional complex mode for the typical long-span suspension bridge, and that the critical flutter velocity of the first-order

antisymmetric torsional complex mode is slightly higher.

The comparison on the analytical results of the proposed method and the ones of the previous multimode flutter analysis (Ding 2002) is presented in Table 4. It has been seen that there is good agreement between the two results.

Table 3. Results of critical flutter states found

Table 4. Comparison on flutter results of the suspension bridge

Fig. 6. Frequencies versus reduced velocity

Fig. 7. Damping Ratios versus reduced velocity

Fig. 8. Amplitude and phase shifts of coupled flutter motion along the bridge axis

The relative amplitudes of the first 50 participating natural modes in the symmetric and antisymmetric coupled flutter motion are shown in Figs. 9 and 10, respectively. It is noted that modes 5, 6, and 15 are major participating modes of the symmetric coupled flutter. For the antisymmetric coupled flutter, modes 2, 4, 11, and 14 are the main participating modes. The phase shifts among those modes are all quite obvious. Although the lateral-motion modes of bridge deck also participate in the flutter motions, the degree of participation is small.

Fig. 9. Relative amplitudes of natural modes in the symmetric flutter motion

Fig. 10. Relative amplitudes of natural modes in the antisymmetric flutter motion

6. Concluding remarks

This paper proposes the F–S method for analyzing the coupled flutter instability of long-span bridges based on the full-discretized model of structure. Based on the proper approximation of the circular frequency of complex mode which has no influence on the critical flutter state and flutter velocity, the characteristic equation is expressed as a complex generalized eigenvalue equation which includes only two variables. Then a process of two-parameter searching is avoided for the proposed flutter analysis. After fixing the reduced

frequency, the complex generalized eigenvalue problem can be solved by the simultaneous iteration method. In order to keep the band-packed storing of large sparse matrix, the structural damping matrix is employed as the Rayleigh's orthogonal form. The proposed method overcomes some shortcomings of the previous direct flutter analysis and is a single-parameter searching method that is more stable and efficient than the two-parameter one.

The full-order flutter analysis of a simply supported beam structure with idealized thin-plate section is performed as a numerical example. The self-excited aerodynamic lift and moment acting on the idealized thin-plate can be represented by the Theodorsen's function, so the exact results could be obtained by an analytical method. The flutter results are compared with the exact results under the condition of zero structural damping, and these results have good agreements.

The flutter problem of Jiangyin Yangtse-river suspension bridge with 1385m main span in the completed stage is also analyzed. It can be seen that the results in the complete stage with the proposed method have good agreement with the ones of the previous multimode flutter analysis. The analytical results show that the coupled flutter occurs on the first-order symmetric torsional complex mode for the typical long-span suspension bridge, and that the critical flutter velocity of the first-order antisymmetric torsional complex mode is slightly higher.

In some respects, the proposed F-S flutter analysis has some advantages relative to the multimode flutter analysis. The writers suggest that the proposed F-S method for flutter analysis could be widely employed in the actual flutter analysis of long-span bridges.

Acknowledgements

This paper is financially supported by the National Science Foundation of China (Grant 51078275 and 50738002) and Shanghai Youth Qiminxin Science and Technology Program

(Grant 08QA1406800), which are gratefully acknowledged.

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Table 1. Flutter results of the beam structure

The analyzing method	Critical wind velocity(m/s)	Flutter frequency(Hz)
The pK-F method	139.15	0.3789
The proposed F-S method	139.8	0.3800
The exact result	139.9	0.3801

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Table 2. Main modes of the bridge deck in complete stage

Mode No.	Frequency (Hz)	Mode Shape	Mode No.	Frequency (Hz)	Mode Shape
1	0.0516	S-L-1	15	0.2730	S-T-1
2	0.0891	A-V-1	16	0.3107	A-V-4
3	0.1237	A-L-1	27	0.3707	S-V-4
4	0.1316	A-V-2	30	0.4132	S-T-2
5	0.1338	S-V-1	31	0.4322	A-V-5
6	0.1883	S-V-2	36	0.4990	S-V-5
7	0.2005	A-V-3	38	0.5304	A-T-2
12	0.2468	S-L-2	41	0.5690	A-V-6
13	0.2583	S-V-3	44	0.6444	S-V-6
14	0.2677	A-T-1	45	0.6640	S-T-3

Note: S–Symmetric, A–Antisymmetric, V–Vertical, L–Lateral, T–Torsional.

Table 3. Results of critical flutter states found

Critical flutter state	Reduced velocity (U/fB)	Flutter velocity (m/s)	Flutter frequency (Hz)	Original natural modes
1	7.587	67.6	0.241	Symmetric torsion-1
2	8.822	73.7	0.226	Antisymmetric torsion-1

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Table 4. Comparison on flutter results of the suspension bridge

The analyzing method	Critical wind velocity(m/s)	Flutter frequency(Hz)
Multimode flutter analysis	67.55	0.2413
Proposed F-S method	67.6	0.241

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Fig. 1. Aerodynamic forces acting on bridge deck

Fig. 2. Positive directions of 12-degree-of-freedom space frame member

Fig. 3. Jiangyin Yangtse-river suspension bridge

Fig. 4. Cross section of the deck of Jiangyin bridge

Fig. 5. Flutter derivatives H_i^* and A_i^* of bridge deck

Fig. 6. Frequencies versus reduced velocity

Fig. 7. Damping Ratios versus reduced velocity

Fig. 8. Amplitude and phase shifts of coupled flutter motion along the bridge axis

Fig. 9. Relative amplitudes of natural modes in the symmetric flutter motion

Fig. 10. Relative amplitudes of natural modes in the antisymmetric flutter motion

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