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A Differentiable Optimization Model for the
Management of Single-Species, Even-Aged Stands

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Abstract

This paper presents a model for optimizing the management of single-species, even-aged stands. The model comprises a number of state variables, given by functions initially differentiable with respect to time, which cease to be due to the discontinuities that cause instant silvicultural treatments. Nevertheless, the proposed model maintains its differentiability with respect to the decision variables (timing, type, and intensity of each thinning, and rotation age). This allows to formulate the problem of optimal management of this type of stands as a linearly constrained smooth optimization problem, which can be efficiently solved by any derivative-based optimization method. The effectiveness of this formulation is showed by using a Sequential Quadratic Programming (SQP) algorithm to design the optimal management of *Pinus pinaster* Ait. in Asturias (NW Spain) from an economic perspective. These results are compared with those obtained with two methods that do not require derivatives. The SQP, as representative of gradient-type methods, proved robust and much more efficient than the derivative-free methods.

Keywords: forest simulation, silvicultural prescription, dynamic growth model, derivative-free optimization, sequential quadratic programming.

1 Introduction

Stand growth and yield prediction and its response to silvicultural treatments have always been a matter of interest for forest researchers, as the key to proper management is the adequate knowledge of the growth process of forest species. In this sense, empirical models are preferred to process-based models or hybrid models for single-species, even-aged stands (Burkhart and Tomé, 2012, p. 2). The former are usually categorized according to their modelling entity (Weiskittel et al., 2011, p. 53-83) as whole-stand, size-class and individual-tree models. Whole-stand models represent an adequate alternative for the management of single-species, even-aged stands, as they are able to provide accurate estimates, while data inputs can be easily obtained (Weiskittel et al., 2011, p. 53). The state-space approach (García, 1994) is still a common alternative for the development of empirical whole-stand growth models, e.g. in Canada (García, 2013), USA (García et al., 2011), Switzerland (Álvarez-González et al., 2010), or NW Spain (Gómez-García et al., 2015; Arias-Rodil et al., 2015a). Their implementation in computer programs allows simulation of different silvicultural prescriptions and evaluation of their consequences in the forest system (Burkhart and Tomé, 2012, p. 428-429).

Management of any resource requires a planning stage which implies a decision making process. In forestry, at least four planning levels may be considered: tree, stand, forest, and landscape. The stand level is frequently used to develop plans of action for those landowners who have small holdings (Bettinger et al., 2009, p. 125), in many cases planted with trees of the same species and age. The management of these stands from an economic perspective requires to decide about the optimal timber rotation and the optimal number, type, intensity, and timing of thinning.

As the number of options in the stand-level management problem is unlimited, an automatic search procedure (optimization algorithm) must be used to generate and evaluate alternatives with respect to the value of the specified objective function. A number of algorithms have been used for optimizing stand-level management, which differ mainly in whether they require discretization of the decision variables (thus reducing the number of solutions to evaluate) or

not. In the former, the search is reduced to the network of nodes defined in a discrete space of solutions where the algorithm reaches the global maximum. The problem is to strike a balance between information loss due to the discretization and the computation time required, which increases exponentially with the number of possible solutions. Dynamic programming (e.g. Arimizu, 1958; Amidon and Akin, 1968; Díaz-Balteiro and Rodríguez, 2006) and depth first search (Arias-Rodil et al., 2015b) are examples of methods of this type that have been used in stand-level optimization. Concerning approaches that do not require discretization of the decision variables, direct search methods have been considered (e.g. Cao, 2010; Pukkala and Kellomäki, 2012; Tahvonen et al., 2013). Their disadvantage is that they do not assure reaching the global optimum (Valsta, 1993), although they usually provide reasonably good results in reduced computing times, not requiring the use of derivatives. Within this group, there are methods that use a single solution vector and others that use multiple solution vectors, so-called population-based methods (Pukkala, 2009).

In this study we formulate a model to optimize the management of single-species, even-aged stands for which a dynamic growth model is available. Our growth model comprises a number of state variables, given by functions initially smooth (class C^∞), which cease to be when thinnings are simulated. These silvicultural treatments induce, in the moments they are applied, sudden changes in the values of the state variables, causing discontinuities at those moments. Nevertheless, and as shown in the following sections, the model remains differentiable with respect to the decision variables (timing, type, and intensity of each thinning, and rotation age). This allows us to formulate the problem of the optimal management of this type of stands as a differentiable optimization problem with linear constraints. This new formulation enables the use of derivative-based optimization methods, consequently opening up a huge range of possibilities to solve the problem. As an example, in this work we used a Sequential Quadratic Programming (SQP) algorithm to design the optimal management of *Pinus pinaster* Ait. in Asturias (NW Spain) from an economic perspective. We compared our approach with two direct-search methods: Hooke and Jeeves (1961) method (HJ), the most commonly used in forestry

(e.g. Roise, 1986; Miina, 1996; Pukkala et al., 2014; Pasalodos-Tato et al., 2016), and Differential Evolution (DE, Storn and Price, 1997), which turned out to be the most robust in a previous study of Arias-Rodil et al. (2015b).

2 Model formulation

2.1 Dynamic systems

A forest growth model predicts future values of certain outputs (e.g. timber volume) given inputs (e.g. type, intensity, and timing of thinning), both of which are functions of time. The state-space approach (Zadeh and Polak, 1969) was first used in forestry by García (1994), and has become a common approach for modelling forest dynamic systems (e.g. Diéguez-Aranda et al., 2006; Castedo-Dorado et al., 2007). The idea of this approach is to characterize the state of the system at any point in time in such a way that given the present state the future does not depend on the past, i.e. it is assumed that the state information is enough to determine the behaviour of the system from that time on. This state is defined by a vector of variables, and it is then assumed that two stands with the same values for these variables will behave equally regardless of how they reached that state.

As silvicultural treatments generally occur at specific times and cause instant changes in the state, we can assume that the inputs (system control variables) are given by a vector $\tilde{\mathbf{u}}$, and we denote by $\mathbf{x}(t)$ the finite-dimensional vector of the system state, and by $\mathbf{y}(t)$ the finite-dimensional vector of the system outputs. The behaviour of the system is then characterised by two functions:

1. A transition function $\mathbf{F}$

$$\mathbf{x}(t) = \mathbf{F}[\mathbf{x}(t_0), \tilde{\mathbf{u}}, t - t_0], \quad \forall t \geq 0, \tag{1}$$

2. An output function $\mathbf{g}$

$$\mathbf{y}(t) = \mathbf{g}[\mathbf{x}(t)], \quad \forall t \geq 0. \tag{2}$$

Function $\mathbf{F}$ gives the current state ($\mathbf{x}(t)$) as a function of the initial state ($\mathbf{x}(t_0)$), the system control variables ($\tilde{\mathbf{u}}$), and the elapsed time between the initial time (t_0) and the current time (t). The transition function can be obtained, in many cases, by solving the system of differential equations which describe the *rate of change* of the state variables. Function $\mathbf{g}$ gives the current value of any other variable of interest ($\mathbf{y}(t)$) as a function of the current state ($\mathbf{x}(t)$).

Both the transition and output functions are frequently used in forest growth and yield modelling. In section 2.2 we present a general dynamic model for the growth of pure, even-aged stands. We will show how, in this case, the transition and output functions are differentiable with respect to the decision variables. Moreover, from a given initial condition, we will give an explicit expression of the system outputs in terms of the decision variables, which is very useful to calculate its derivatives when seeking the optimal management of a stand.

2.2 Dynamic growth model for single-species, even-aged stands

According to García (1994), for single-species, even-aged stands in which a wide range of silvicultural treatments (various thinning regimes and initial spacings) may be applied (except heavy thinning and pruning) and for which we are interested in volumes of various products and parameters of size distributions, the state of a stand can be sufficiently described using the following variables: dominant height ($H(t)$, defined as the mean height of the dominant trees), number of stems per unit of land area ($N(t)$), and stand basal area ($G(t)$, defined as the total cross-sectional area of all stems in a stand measured at breast height, and expressed as per unit of land area); following the notation introduced in section 2.1, $\mathbf{x}(t) = (H(t), N(t), G(t))$. In this model, the state variables are given by known functions, $h, n, g : \mathbb{R}^+ \times \mathbb{R}^+ \times \mathbb{R}^+ \rightarrow \mathbb{R}$, so that, if for any $\bar{t} \in [a, b]$ we know the values of $H(\bar{t}) = \bar{H}$, $N(\bar{t}) = \bar{N}$, and $G(\bar{t}) = \bar{G}$ and no *treatment*

133 is applied in the stand during that time interval, it is verified that, for all $t \in [a, b]$,

$$H(t) = h(\bar{t}, \bar{H}, t), \quad (3)$$

$$N(t) = n(\bar{t}, \bar{N}, t), \quad (4)$$

$$G(t) = g(\bar{t}, \bar{G}, t). \quad (5)$$

134 Functions h , n , and g are often smooth (usually $h, n, g \in C^\infty(\mathbb{R}^+ \times \mathbb{R}^+ \times \mathbb{R}^+)$) and, obviously,
135 functions H , N , and G have the same smooth properties in $[a, b]$.

Moreover, as we have already indicated, from these functions we can estimate any other variable of interest (output) of the problem. For example, to estimate the merchantable stem volume to any top diameter limit $d > 0$, we have other function $v \in C^\infty(\mathbb{R}^+ \times \mathbb{R}^+ \times \mathbb{R}^+ \times \mathbb{R}^+)$ such that, if $V^d(t)$ denotes the merchantable stand volume of diameter greater than d at time t , then

$$V^d(t) = v(H(t), N(t), G(t), d), \quad t \geq 0.$$

136 Let consider now an initial stand of age t_0 , with state variables H_0 , N_0 , and G_0 , in which a
137 first thinning is done in instant $t_1 > t_0$, of intensity (proportion of stems to remove) $I_1 \in (0, 1)$
138 and type of thinning defined as a removal relation (variable that relates the number of stems
139 and the stand basal area removed and before thinning) $R_1 \in (0, 1]$ (Álvarez González et al.,
140 1999). This causes that functions N and G (not H , since it is assumed not affected by thinning
141 as no dominant trees are removed) suffer a jump discontinuity at that instant, verifying that

$$H(t_1^+) = H(t_1^-) = H(t_1), \quad (6)$$

$$N(t_1^+) = (1 - I_1)N(t_1^-), \quad (7)$$

$$G(t_1^+) = (1 - I_1 R_1)G(t_1^-), \quad (8)$$

142 where $H(t_1^+)$, $N(t_1^+)$, and $G(t_1^+)$ are the values of the functions after thinning and $H(t_1^-)$, $N(t_1^-)$,
143 and $G(t_1^-)$ the values before thinning.

Therefore, if no further thinning is carried out in the stand until time $t_2 > t_1$, combining (4)-(5) with (7)-(8) we have that, for all $t \in [t_1, t_2]$,

$$N(t) = n(t_1, N(t_1^+), t) = n(t_1, (1 - I_1)N(t_1^-), t) = n(t_1, (1 - I_1)n(t_0, N_0, t_1), t), \quad (9)$$

$$G(t) = g(t_1, G(t_1^+), t) = g(t_1, (1 - I_1 R_1)G(t_1^-), t) = g(t_1, (1 - I_1 R_1)g(t_0, G_0, t_1), t). \quad (10)$$

It should be noted that, although functions $N(t)$ and $G(t)$ are not continuous in $[0, t_2]$ (they have a jump discontinuity in t_1), functions¹

$$H(t) = h(t_0, H_0, t), \quad t \in \mathbb{R}^+, \quad (11)$$

$$N_0(t) = n(t_0, N_0, t), \quad t \in D_0, \quad (12)$$

$$N_1(t_1, I_1, t) = n(t_1, (1 - I_1)n(t_0, N_0, t_1), t), \quad (t_1, I_1, R_1, t) \in D_1, \quad (13)$$

$$G_0(t) = g(t_0, G_0, t), \quad t \in D_0, \quad (14)$$

$$G_1(t_1, I_1, R_1, t) = g(t_1, (1 - I_1 R_1)g(t_0, G_0, t_1), t), \quad (t_1, I_1, R_1, t) \in D_1, \quad (15)$$

have the same smooth properties as h , n , and g in their respective domains: $D_0 = \{t \in \mathbb{R}, t \geq 0\}$, $D_1 = \{(t_1, I_1, R_1, t) \in \mathbb{R}^4, t \geq t_1 \geq 0, I_1 \in (0, 1), R_1 \in (0, 1]\}$.

It is essential to note that this type of functions are those required in managing a stand: $H(t)$ provides the dominant height at any instant $t > 0$, $N_0(t)$ and $G_0(t)$ give the number of stems and the stand basal area until the first thinning, and $N_1(t_1, I_1, t)$ and $G_1(t_1, I_1, R_1, t)$ provide the number of stems and the stand basal area from the first thinning ($t \geq t_1$), but considering that the first thinning was done in instant t_1 , with an intensity I_1 and a type of thinning given by R_1 .

This type of functions constitute the transition function of the model $(\mathbf{F})$, for which we can give an explicit expression. For example, if only one thinning is going to be applied, then

¹For simplicity, we will use the same notation for the function $N_0(t)$ (respectively $G_0(t)$) and the initial state variable N_0 (respectively G_0).

158 $\tilde{\mathbf{u}} = (t_1, I_1, R_1)$ and

$$\mathbf{x}(t) = \mathbf{F}[\mathbf{x}(t_0), \tilde{\mathbf{u}}, t - t_0] = \begin{cases} (H(t), N_0(t), G_0(t)) & \text{if } t < t_1 \\ (H(t), N_1(t_1, I_1, t), G_1(t_1, I_1, R_1, t)) & \text{if } t \geq t_1 \end{cases} \quad (16)$$

159 In a similar way, if we have the output function $\mathbf{g}$, we can also give an explicit expression of
 160 the system outputs in terms of the initial state ($\mathbf{x}(t_0) = (H_0, N_0, G_0)$), the control variables ($\tilde{\mathbf{u}}$),
 161 and the initial (t_0) and current time (t). For example, we have the following expressions for the
 162 merchantable stand volume to a top diameter limit d in different situations:

163 1. At any time before the first thinning,

$$V_0^d(t) = v(H(t), N_0(t), G_0(t), d), \quad t \in D_0. \quad (17)$$

164 2. At any time after the first thinning, but considering that it was done in t_1 with an inten-
 165 sity I_1 and a type of thinning given by R_1 ,

$$V_1^d(t_1, I_1, R_1, t) = v(H(t), N_1(t_1, I_1, t), G_1(t_1, I_1, R_1, t), d), \quad (t_1, I_1, R_1, t) \in D_1. \quad (18)$$

166 3. Removed in the first thinning,

$$V_1^{r,d}(t_1, I_1, R_1) = V_0^d(t_1) - V_1^d(t_1, I_1, R_1, t_1), \quad (t_1, I_1, R_1) \in \mathbb{R}^+ \times [0, 1] \times [0, 1]. \quad (19)$$

167 **Remark 1.** *Generically, the volume removed in the i^{th} thinning will be:*

$$V_i^{r,d}(t_1, I_1, R_1, \dots, t_i, I_i, R_i) = V_{i-1}^d(t_1, I_1, R_1, \dots, t_{i-1}, I_{i-1}, R_{i-1}, t_i) - V_i^d(t_1, I_1, R_1, \dots, t_i, I_i, R_i, t_i), \quad (20)$$

168 being the V_i^d functions given, in analogy to (18), as

$$V_i^d(t_1, I_1, R_1, \dots, t_i, I_i, R_i, t) = v(H(t), N_i(t_1, I_1, \dots, t_i, I_i, t), G_i(t_1, I_1, R_1, \dots, t_i, I_i, R_i, t), d), \quad (21)$$

169 where, in turn, N_i and G_i are given, as in (13) and (15), by the recurring formula

$$N_i(t_1, I_1, \dots, t_i, I_i, t) = n(t_i, (1 - I_i)N_{i-1}(t_1, I_1, \dots, t_{i-1}, I_{i-1}, t_i), t), \quad (22)$$

$$G_i(t_1, I_1, R_1, \dots, t_i, I_i, R_i, t) = g(t_i, (1 - I_i R_i)G_{i-1}(t_1, I_1, R_1, \dots, t_{i-1}, I_{i-1}, R_{i-1}, t_i), t). \quad (23)$$

170 As defined, it is evident that all functions N_i and G_i have the same smoothness as n and g
 171 respectively. Moreover, by assuming that $v \in C^\infty(\mathbb{R}^+ \times \mathbb{R}^+ \times \mathbb{R}^+ \times \mathbb{R}^+)$, that smoothness with
 172 the smoothness of h , determine the smoothness of all functions V_i^d and $V_i^{r,d}$.

173 2.3 Optimal stand management

174 The optimal management of a stand implies selecting the silvicultural prescription that max-
 175 imizes the specified objective function (e.g. mean annual volume increment, soil expectation
 176 value). For that purpose, we search the number of thinnings that must be applied ($n_t \in \mathbb{N} \cup \{0\}$),
 177 the timing ($t_i \geq 0, i = 1, \dots, n_t$) of each thinning, its intensity ($I_i \in (0, 1), i = 1, \dots, n_t$) and type
 178 (defined by its removal relation $R_i \in (0, 1], i = 1, \dots, n_t$), and finally the timing of clearcutting
 179 ($t_{n_t+1} \geq 0$). Technologic, logistic or economic restrictions limit the possible number of thinnings
 180 and, therefore, considering that n_t must be a *small* number (generally lower than 5), we can
 181 eliminate it from the decision variable and maximize the objective function for each value of n_t
 182 accepted (instead of one problem we will have to solve several, but in all of them n_t will be a
 183 given value).

184 Thus, once set the value of n_t , the vector of decision variables of the problem is
 185 $\mathbf{u} = (t_1, I_1, R_1, \dots, t_{n_t}, I_{n_t}, R_{n_t}, t_{n_t+1}) \in \mathbb{R}^{3n_t+1}$. The logistic and economic restrictions men-
 186 tioned also determine minimum time intervals until the first thinning ($l_1 > 0$), between thinning

187 i and thinning $i + 1$ ($l_i > 0$), and between the last thinning and the clearcutting ($l_{n_t+1} > 0$). In
188 addition, there are lower ($\underline{I}_i \in (0, 1)$, $\underline{R}_i \in (0, 1]$) and upper ($\bar{I}_i \in [\underline{I}_i, 1)$, $\bar{R}_i \in [\underline{R}_i, 1]$) bounds
189 for the intensities and removal relations of each thinning, and upper bound for the clearcutting
190 age ($T > 0$). With all of this, the set of allowable actions is given by

$$U_{ad} = \left\{ \mathbf{u} \in \mathbb{R}^{3n_t+1} \text{ such that } \begin{array}{ll} t_i - t_{i-1} \geq l_i & i = 1, \dots, n_t + 1, \\ I_i \in [\underline{I}_i, \bar{I}_i] & i = 1, \dots, n_t, \\ R_i \in [\underline{R}_i, \bar{R}_i] & i = 1, \dots, n_t, \\ t_{n_t+1} \leq T. \end{array} \right\} \quad (24)$$

191 which results closed, bounded, and convex.

192 The optimal stand management is to solve, for all possible values of n_t , the problem

$$\max_{\mathbf{u} \in U_{ad}} J(\mathbf{u}), \quad (25)$$

193 where $J(\mathbf{u})$ is the objective function of the stand, defined from functions $\mathbf{F}$ and $\mathbf{g}$. In the
194 next section we detail, as an example, a typical function J for optimizing the management of
195 single-species, even-aged stands from an economic perspective.

196 2.4 Optimal stand management from an economic perspective

197 To optimize the management of a forest stand from an economic perspective, the criterion
198 generally used is the soil expectation value (SEV, money per unit area) of all future net revenues
199 from wood production, which enables comparison of management schedules involving different
200 rotations lengths as it assumes that the revenues are reinvested (Bettinger et al., 2009, p. 41):

$$J(\mathbf{u}) = \frac{R(\mathbf{u}) - C(\mathbf{u})}{1 - \frac{1}{(1+r)^{t_{N_t}+1}}}, \quad (26)$$

where $R(\mathbf{u})$ and $C(\mathbf{u})$ are, respectively, present values of revenues and management costs, and $r \in [0, \infty)$ is a pre-specified known annual discount rate. For single-species, even-aged stands, functions $R(\mathbf{u})$ and $C(\mathbf{u})$ are given by elementary operations between H , given by (3), and functions V_i^d , N_i , and G_i , given by (21)-(23). In the next two sections we detail typical expressions for functions R and C .

2.4.1 Revenues

For $i = 1, \dots, n_t$, functions $V_i^{r,d}(t_1, I_1, R_1, \dots, t_i, I_i, R_i)$, given by (20), provide an estimate of the removed stand volume to a specified top diameter limit in the i^{th} thinning. In a similar way,

$$V_{n_t+1}^{r,d}(\mathbf{u}) = V_{n_t}^d(\mathbf{u}) \quad (27)$$

provides an estimate of the same top diameter limit volume at the rotation age ($t = t_{n_t+1}$). The revenues will be obtained by multiplying the volume by size assortments times its corresponding expected price.

In general, the following $n_a \in \mathbb{N}$ size assortments are considered:

- Type 1: stand volume with top diameter limit less than $d_1 > 0$, with a price p_1 .
- Type j (with $j = 2, \dots, n_a - 1$): stand volume with diameter between d_j and d_{j-1} , with a price p_j .
- Type n_a : stand volume with top diameter limit larger than d_{n_a-1} , with a price p_{n_a} .

If we define $d_0 = 0$ and $p_0 = 0$, the revenues obtained at each harvest i , for $i = 1, \dots, n_{t+1}$, will be

$$\sum_{j=1}^{n_a} (p_j - p_{j-1}) V_i^{r,d_{j-1}}(t_1, I_1, \dots, t_i, I_i). \quad (28)$$

219 Finally, the present value of those revenues for an $r \in [0, 1]$ annual discount rate will be

$$R(\mathbf{u}) = \sum_{i=1}^{n_t+1} \frac{1}{(1+r)^{t_i}} \left(\sum_{j=1}^{n_a} (p_j - p_{j-1}) V_i^{r, d_{j-1}}(t_1, I_1, \dots, t_i, I_i) \right). \quad (29)$$

220 2.4.2 Costs

221 Stand management implies several cost types:

222 1. **Plantation and maintenance costs before the first thinning:** they are a function
223 of plantation density and the number of trees at the time ($0 < t_0^1 < \dots < t_0^{c_0} < t_0 + l_1$) of
224 each action (i.e. management treatment). They do not depend on the decision variable of
225 the problem and, for a given stand, are denoted by a constant

$$C_{plan+main} = f_p(N_0(0)) + \frac{f_{rm}(N_0(0))}{(1+r)^{t_{rm}}} + \sum_{k=1}^{c_0} \frac{f_0^k(N_0(t_0^k))}{(1+r)^{t_0^k}}. \quad (30)$$

226 where f_p is the plantation cost, f_{rm} is the replanting and management cost of the t_{rm}
227 year, c_0 is the number of planned actions in time interval $(0, t_0 + l_1)$, and functions $f_0^k(N)$
228 provide the cost of the k^{th} action which is done before the first thinning.

229 2. **Maintenance costs between cuttings:** as for the maintenance before the first thinning,
230 these costs are a function of the number of trees at the time when each action (scrub
231 clearance, low pruning...) is applied. We admit that these actions are done always between
232 the same cuttings and, for example, if between cutting i and cutting $i + 1$ two actions are
233 planned, we denote by δ_i^1, δ_i^2 the elapsed time between thinning i and each of the two
234 actions ($0 < \delta_i^1 < \delta_i^2 < (l_{i+1})$). In response to this, maintenance costs between cuttings
235 will be

$$C_{main}(\mathbf{u}) = \sum_{i=1}^{n_t} \sum_{k=1}^{c_i} \frac{f_i^k(N_i(t_1, I_1, \dots, t_i, I_i, t_i + \delta_i^k))}{(1+r)^{t_i + \delta_i^k}}, \quad (31)$$

236 where c_i is the number of planned actions in time interval (t_i, t_{i+1}) . Functions $f_i^k(N)$
237 provide the cost of the k^{th} action which is done in period (t_i, t_{i+1}) . It is evident that, if n

is sufficiently smooth, the smoothness of $C_{main}(\mathbf{u})$ is given by the smoothness of functions $f_i^k(N)$. Nevertheless, it should be noted that, if actions are done at fixed times which may vary between thinnings, function $C_{main}(\mathbf{u})$ may present discontinuities.

3. **High pruning costs:** they are given as a function ($q(N)$) of the expected number of trees at rotation age. Therefore, if high pruning is done only once at time t_{hp} , the corresponding costs will be of the form

$$C_{hp}(\mathbf{u}) = \frac{q(N_{n_t}(\mathbf{u}))}{(1+r)^{t_{hp}}}. \quad (32)$$

4. **Harvesting costs:** all thinnings and the clearcutting involve costs associated with the cut and extraction of the timber which are a function of the number of trees before cutting, the extracted volume, and also of the average size of the trees removed. The last variable may be represented by its quadratic mean diameter (D_g^r), which is obtained directly from the removed number of trees N^r and stand basal area G^r through the expression

$$D_g^r = \sqrt{\frac{40000G^r}{N^r\pi}}. \quad (33)$$

Therefore, harvesting costs are of the form

$$C_{harv}(\mathbf{u}) = \frac{s_{n_t+1}(N_{n_t}(\mathbf{u}), G_{n_t}(\mathbf{u}), V_{n_t}^0(\mathbf{u}))}{(1+r)^{t_{n_t+1}}} + \sum_{i=1}^{n_t} \frac{1}{(1+r)^{t_i}} s_i(N_{i-1}(t_1, I_1, \dots, t_{i-1}, I_{i-1}, t_i), N_i(t_1, I_1, \dots, t_i, I_i, t_i), G_{i-1}(t_1, I_1, R_1, \dots, t_{i-1}, I_{i-1}, R_{i-1}, t_i), G_i(t_1, I_1, R_1, \dots, t_i, I_i, R_i, t_i), V_i^{r,0}(t_1, I_1, \dots, t_i, I_i)), \quad (34)$$

where functions $s_i(N)$ are those of harvesting cost i .

Finally, the function of total management costs of the stand will be

$$C(\mathbf{u}) = C_{plan+main} + C_{main}(\mathbf{u}) + C_{hp}(\mathbf{u}) + C_{harv}(\mathbf{u}). \quad (35)$$

2.4.3 Analysis and numerical resolution

We assume that h , n , g , and v are sufficiently smooth (i.e. class C^∞). As stated, functions H , N_i , G_i , V_i^d , and $V_i^{r,d}$ are class C^∞ and, consequently, function R , given by (29), is also C^∞ . The smoothness of C , given by (35), is given by the smoothness of functions f_i^k , q , and s_i . For example, if these functions are class C^∞ , then C , and therefore J , given by (26), are class C^∞ . In this situation, as U_{ad} (equation (24)) is closed and bounded, the continuity of J guarantees the existence of a solution for problem (25). Moreover, because U_{ad} is convex, the uniqueness of solution would be also guaranteed if, for example, J is strictly concave.

On the other hand, the differentiability of functions f_i^k , q , and s_i is enough to guarantee the differentiability of the objective function J . In this case, problem (25) can be solved by any derivative-based optimization method (see, for example Nocedal and Wright, 2006, for a detailed presentation of these methods). The gradient of the objective function (∇J) can be easily obtained by simply using the chain rule, but the calculations can be very tedious if the involved functions are not simple enough. In those cases, a finite difference method (see, for example Nocedal and Wright, 2006, chapter 8) can be useful to approximate $\nabla J(\mathbf{u})$, whenever necessary. As an example, in section 3 we numerically solve problem (25) for a particular case.

3 Case study: Optimization of the management of *Pinus pinaster* Ait. in Asturias (NW Spain) from an economic perspective

A dynamic growth model has recently been developed for pure, even-aged stands of maritime pine (*Pinus pinaster* Ait.) in Asturias (NW Spain) (see Arias-Rodil et al., 2015a, for details), where it is one of the most important timber species, occupying 22 500 ha (almost 5% of the total forest area, MAGRAMA, 2012). The joint use of the model (properly implemented in a computer program to simulate thinning treatments) and an optimization algorithm allowed to select the best silvicultural prescriptions at stand level for managing the species in the region from an economic perspective (Arias-Rodil et al., 2015b). These authors evaluated six optimiza-

tion methods: depth-first search (DFS, see Russell and Norvig, 2009, p. 85) and five direct search methods (Hooke and Jeeves method –1961, HJ– and four population-based methods, including differential evolution –DE, Storn and Price, 1997). According to their results, after several optimization runs for various example stands (differing in site quality and plantation density), HJ yielded the highest value of the objective function in most cases and was the fastest method, although DE behaved similar and was more stable. Therefore, in this section we used the HJ and DE methods as reference to compare with our approach in stand-level optimization of economic benefit.

3.1 Simulation of stand dynamics

The dynamic stand growth model for *Pinus pinaster* in Asturias represents an example of the model explained in section 2.2 in which model functions are (see Arias-Rodil et al., 2015a, for details):

$$h(\bar{t}, \bar{H}, t) = \frac{41.40}{1 - \left(1 - \frac{41.40}{\bar{H}}\right) \left(\frac{\bar{t}}{t}\right)^{1.325}}, \quad (36)$$

(37)

$$n(\bar{t}, \bar{N}, t) = \bar{N} e^{-4.296 \cdot 10^{-3}(t-\bar{t})}, \quad (38)$$

(39)

$$g(\bar{t}, \bar{G}, t) = \frac{\frac{1}{2} \left(\bar{G} + \sqrt{\bar{G}^2 + 880468 \bar{G} \bar{t}^{-2.255}} \right)}{1 + \frac{220117}{\frac{1}{2} \left(\bar{G} + \sqrt{\bar{G}^2 + 880468 \bar{G} \bar{t}^{-2.255}} \right)} t^{-2.255}}, \quad (40)$$

(41)

$$v(H, N, G, d) = 0.6677 G^{0.9789} H^{0.8440} e^{-0.3427 \left(\sqrt{\frac{40000G}{N\pi}} \right)^{-2.949}} d^{3.313}. \quad (42)$$

In this model, each stand is characterized by its site index (S , defined as the dominant height of the stand at a reference age of 20 years) or its dominant height H_0 at any moment $t_0 > 0$,

and by the number of trees N_0 at t_0 . From these values, H_0 (if needed) and G_0 (if it is not calculated from inventory data) are obtained in the following manner:

$$H_0 = h(20, S, t_0), \tag{43}$$

$$G_0 = 0.005790S^{1.030}N_0^{0.3971}t_0^{1.057}. \tag{44}$$

3.2 Optimal stand management

3.2.1 Decision variables

In the present case study, the decision variable indicated in section 2.3 included the rotation age t_{n_t+1} and the timing t_i and intensity I_i of thinnings. The maximum number of thinning operations considered was three ($n_t \leq 3$), given that only 1–2 are usually applied in Asturias (Rodríguez et al., 2007, p. 44). The removal relation R_i , in this case defined as $R_i = \frac{G_r/G_{bt}}{N_r/N_{bt}}$ (Álvarez González et al., 1999), relates the number of stems and the stand basal area removed (N_r and G_r) and before thinning (N_{bt} and G_{bt}). It was dropped from the decision variable to simplify the analysis of results. Therefore, it was fixed to a value of 0.9 for the first thinning, which represents a combination of uniform and low thinning, and to 0.75 for subsequent thinnings, which describe an intermediate thinning (a combination of low and crown thinning), respectively (Kerr and Haufe, 2011, pp. 20-23).

In this case, the vector of decision variables of the problem results $\mathbf{u} = (t_1, I_1, \dots, t_{n_t}, I_{n_t}, t_{n_t+1}) \in \mathbb{R}^{2n_t+1}$, for $n_t = 0, 1, 2, 3$.

3.2.2 Constraints

As stated in section 2.3, based on the study of Arias-Rodil et al. (2015b) several restrictions were considered in the optimization process:

- Minimum thinning intensity of 15% ($I_i = 0.15$), because of logistic and economic considerations.

- Maximum thinning intensity of 45% ($\bar{I}_i = 0.45$), to prevent the stand from becoming too sensitive to wind throw or snow breakage (Castedo-Dorado, 2009).
- Minimum interval between consecutive cuttings of $l_i = 5$ years, to prevent too-frequent cutting, which may harm the residual stand due to damage arising from the use of harvesters or forwarders (Han and Kellogg, 2000). This constraint is also related to both wind and, in the interval between consecutive thinning operations, snow protection and residual stand damage (Arias-Rodil et al., 2015b).
- Maximum rotation age of $T = 100$ years, because of model limitations (Arias-Rodil et al., 2015b).

3.2.3 Objective function: soil expectation value

Selection of the best management prescription from an economic perspective consists of maximizing function J given by (26). Next, we detail the revenues and costs associated with this case study. The interest rate was set to 4%.

Revenues The timber assortments considered in the present study and their roadside prices are a simplification of those used by Arias-Rodil et al. (2015b) (here we did not consider the requirement of log length –see Table 1), which were defined after consulting various forest associations and timber companies of Asturias and nearby regions (three timber assortments, $n_a = 3$, with the following distribution: $d_1 = 15$, $p_1 = 25$, $d_2 = 40$, $p_2 = 40$, $p_3 = 60$).

[Table 1 about here.]

Costs The following cost functions (€ ha^{-1}) used by Arias-Rodil et al. (2015b) were considered (all but functions $c(N)$, $\lambda_1(N)$, $\lambda_2(N)$, $\mu_1(N)$, and $\mu_2(N)$, which were made *smooth* to avoid discontinuities):

1. Plantation and maintenance before the first thinning: $c_0 = 2$,

$$f_p(N) = 604.1 + 43.93\sqrt{\frac{N}{100}} + 1.134N \text{ (plantation, } t_p = 0), \tag{45}$$

$$f_{rm}(N) = 150 + 0.217N \text{ (replanting and scrub clearance, } t_{rm} = 1), \tag{46}$$

$$f_0^1(N) = 0.217N \text{ (scrub clearance, } t_0^1 = 2), \tag{47}$$

$$f_0^2(N) = 949.9 + 0.5268N \text{ (scrub clearance and low pruning, } t_0^2 = 12). \tag{48}$$

2. Maintenance between cuttings: $c_1 = \dots = c_{n_t} = 0$.

3. High pruning: $q(N) = 1.505N$.

4. Harvesting:

(a) Thinning

$$s_i(N_{i-1}, N_i, G_{i-1}, G_i, V_i^{r,0}) = 223.2 + c(D_{g,i}^r)V_i^{r,0} + \tag{49}$$

$$1.538(N_{i-1} - N_i) \left(1.182 + \lambda_1(D_{g,i}^r) + \lambda_2(N_{i-1}) + \lambda_3(N_{i-1} - N_i) \right), \tag{50}$$

where:

$$D_{g,i}^r \equiv D_{g,i}^r(N_{i-1}, N_i, G_{i-1}, G_i) = \sqrt{\frac{40000(G_{i-1} - G_i)}{(N_{i-1} - N_i)\pi}}, \tag{51}$$

$$c(D) = 5.555 - 0.08072D^{0.73}, \tag{52}$$

$$\lambda_1(D) = 0.03D^{0.7}, \tag{53}$$

$$\lambda_2(N) = 2.4 \cdot 10^{-4} N^{0.87}, \tag{54}$$

$$\lambda_3(N) = -0.02219 + 57.79 \log(N)^{-4.160}. \tag{55}$$

(b) Clearcutting

$$s_{n_t+1}(N_{n_t}, G_{n_t}, V_{n_t}^0) = 223.2 + c(D_g^{n_t+1})V_{n_t}^0 + \quad (56)$$

$$2.548N_{n_t} (1.106 + \mu_1(D_g^{n_t+1}) + \mu_2(N_{n_t}) + \mu_3(N_{n_t})), \quad (57)$$

where:

$$D_g^{n_t+1} \equiv D_g^{n_t+1}(N_{n_t}, G_{n_t}) = \sqrt{\frac{40000G_{n_t}}{\pi N_{n_t}}}, \quad (58)$$

$$\mu_1(D) = 0.07D^{0.54}, \quad (59)$$

$$\mu_2(N) = 1.9 \cdot 10^{-4} N^{0.87}, \quad (60)$$

$$\mu_3(N) = -0.02124 + 55.51 \log(N)^{-4.161}. \quad (61)$$

3.2.4 Numerical resolution

Considering that all model functions are smooth, we propose the use of a gradient-based method to solve problem (25). To be exact, we used a Sequential Quadratic Programming (SQP) algorithm, which is detailed in Nocedal and Wright (2006, chapter 18). The gradient, when necessary, was approximated by the forward-finite-difference method (see, e.g. chapter 8 in Nocedal and Wright, 2006). For comparison, we also solved the problem with the direct-search methods HJ and DE, treating the constraints with a penalty function.

The model proposed in this case study differs from that of Arias-Rodil et al. (2015b) in two aspects:

- The way in which thinning treatments were simulated. In the previous study, the first thinning was considered to be a combination of uniform and low thinning (applying the latter over the remaining trees after the former), while the subsequent thinning operations were assumed to be entirely from below. Uniform thinning was simulated by removing the same proportion of trees from each diameter class, while the low thinning operations were simulated following the approach proposed by Alder (1979). In this study, the simulation

of stand basal area removed was done using a fixed removal relation of 0.9 for the first thinning (which describes a combination of uniform and low thinning) and of 0.75 for subsequent thinnings (which is indicative of intermediate thinning).

- Functions $c(N)$, $\lambda_1(N)$, $\lambda_2(N)$, $\mu_1(N)$, and $\mu_2(N)$ used in the computation of the cutting costs. The previous formulation of these functions involved artificial discontinuities that were avoided by fitting a smooth function.

Therefore, our results are not directly comparable with those of Arias-Rodil et al. (2015b). Nevertheless, the interest of this study is to show how the new formulation of the model is more appropriate in optimization.

3.3 Numerical results

We tested the three approaches (SQP, HJ, and DE) using 16 example stands which differ in site index (S , the dominant height expected at a reference age of 20 years) and number of stems per hectare at plantation. The optimal results are shown in table 2.

[Table 2 about here.]

Concerning the optimal value of the objective function, SQP and HJ provided the same results, while DE always yielded lower values (0.44% worse on average, with a coefficient of variation of 38.1%). This finding disagrees with other studies like Pukkala (2009), where DE provided the best results for maximization of the net present value of 719 stands in North Karelia (Finland). Additionally, SQP and HJ were robust and always provided the same optimal value when repeating the optimization process from different starting values, while DE showed variability (the coefficient of variation of the optimal results for each example stand after 20 optimization runs ranged from 0.05% to 0.41%). When comparing optimization techniques, it becomes of special interest their evaluation in terms of efficiency, as they could be implemented in decision support systems, where this characteristic becomes valuable. For this, using the number of function evaluations allows a fair comparison avoiding the effect of computer capabilities or

programmer skills (Talbi, 2009, p. 62). In terms of number of function evaluations, SQP was 3.1 and 20.6 times more efficient on average across the best solutions for all example stands (coefficients of variation of 14.5% and 22.9%) than HJ and DE, respectively. It corresponds to a higher difference in CPU time used because HJ and DE algorithms include a penalty function with several ‘if’ control structures that slow down the optimization process (we do not provide times of this comparison because of its dependence on machine and programming skills).

Arias-Rodil et al. (2015b) reported that HJ method got sometimes trapped to a local optimum, which was also found by Pukkala (2009) for some stands when optimizing the net present value of the mentioned forest of 719 stands. However, when Pukkala (2009) optimized the net present value of an example stand, the HJ method proved stable. In this study, HJ method always led to the same management prescription when repeating the optimization process. This is probably because the model proposed in this study differs from that of Arias-Rodil et al. (2015b) in functions $c(N)$, $\lambda_1(N)$, $\lambda_2(N)$, $\mu_1(N)$, and $\mu_2(N)$ involved in the computation of the harvesting costs. These functions are now smooth, which avoids the artificial discontinuities that might make the algorithms stopping in local maxima.

Table 2 also indicates the number of thinnings of the optimal silvicultural prescription that maximizes the SEV. The general trend of the optimal prescription implies to carry out thinnings of the highest intensity allowed (45%) and cuttings at the minimum time interval (5 years, except in the three example stands of lower quality $-S = 7$ m– and higher density $-N_0 = 900, 1300, 1700$ trees ha^{-1} – in which the clearcutting is delayed 2.5, 6.5, and 11.2 years, respectively). Considering these results, it is possible to analyze the behaviour of the objective function if we fix all but two decision variables to the allowable upper and lower bounds indicated and evaluate how the objective function value varies with the value of the decision variables maintained free. For example, figure 1 shows the graph of function $J(t_1, I_1, t_1 + 5, 0.45, t_1 + 10, 0.45, t_1 + 15)$, which corresponds to the SEV when we only vary the timing (t_1) and intensity (I_1) of the first thinning for the example stand of site index $S = 13$ m and number of trees per hectare at plantation $N_0 = 1300$. As can be observed, function J is smooth for the two free variables and

there exists a straight line of maximum values. With this information, we could choose to carry out a first thinning of the maximum intensity allowed (45%) at the optimal age of 31 years or advance it to almost 28 years but doing the thinning of the minimum intensity (15%), without practically losing money. In both cases (and in any intermediate alternative), the strategy for the remaining thinnings and the clearcutting will be the same, i.e. that stated for drawing the graph of figure 1. Note that this type of analyses could be done for many other pairs of decision variables, with the obvious advantages in evaluating management prescriptions differing from the optimum.

[Figure 1 about here.]

4 Conclusions

In this study we propose a linearly constrained smooth optimization model formulation to optimize the management of single-species, even-aged stands. We exemplify this formulation for a case study of maximization of the economic profitability of *Pinus pinaster* Ait. in Asturias at stand level, solving it using a derivative-based algorithm. It is noteworthy that:

1. Whenever possible, it is advisable to use smooth models, mainly for two reasons:
 - (a) They are easier to solve because:
 - Gradient-type methods can be used, which are very efficient and robust.
 - Derivative-free methods also behave better (are more robust), as can be seen for the application of the Hooke and Jeeves (1961) method in this study as compared to the study of Arias-Rodil et al. (2015b), where HJ got trapped in local maxima.
 - (b) The solutions obtained are never *singular* (small changes in the decision variable will not cause great changes in the objective function).
2. The use of derivative-based methods in this type of models is very advisable. As exemplified, they are robust and much faster than derivative-free methods. Additionally, their

efficiency with respect to the latter methods increases with the dimension of the decision variable. Therefore, the use of derivative-based methods would help to better solve much bigger convex problems and possibly also other problems where the convexity is not guaranteed (e.g. in optimization at forest-level with multiple stands in which the dimension of the decision variable and the number of objectives may increase considerably).

3. The derivative-based method used in this study (SQP) resulted very efficient to solve this type of problems.
4. The fact that SQP and HJ always provided the same optimal value when repeating the optimization process from different starting values, suggests that problem (25) has a unique solution for the particular case of study.

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533 **Tables**

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Table 1: Assortments, dimensions, and prices of *Pinus pinaster* Ait. in Asturias (simplification of Arias-Rodil et al., 2015b, without including log-length requirements)

Assortment	Top diameter limit (d , cm)	Price (€ m ⁻³)
Veneer and plywood	$d \geq 40$	60
Sawlog	$15 \leq d < 40$	40
Pulpwood	$d < 15$	25

Table 2: Maximum value (J_{opt} , € ha⁻¹) of objective function (26) obtained with the three approaches (SQP, HJ, and DE –for the latter, the best result after 20 optimization runs is shown) for 16 example stands which differ in site index (S) and number of stems per hectare at plantation (N_0). For SQP and HJ, n_t indicates the optimum number of thinnings. The number of function evaluations (n_{eval}) used by each numerical method in every problem is also pointed out.

S	N_0	n_t	SQP		HJ		DE	
			J_{opt}	n_{eval}	J_{opt}	n_{eval}	J_{opt}	n_{eval}
7	500	3	-164.50	204	-164.50	712	-164.89	5915
	900	3	-1124.05	283	-1124.05	763	-1126.60	4305
	1300	3	-2049.88	297	-2049.88	804	-2056.97	3675
	1700	3	-2962.88	321	-2962.88	746	-2969.54	3990
10	500	2	1986.08	185	1986.07	607	1982.99	4445
	900	3	1003.96	214	1003.95	707	1001.16	4865
	1300	3	41.48	225	41.47	636	41.36	6510
	1700	3	-923.35	277	-923.35	756	-925.11	4515
13	500	1	4653.60	191	4653.60	611	4652.75	3640
	900	2	3699.48	212	3699.48	605	3666.31	4165
	1300	3	2659.33	212	2659.32	634	2651.23	4200
	1700	3	1704.19	209	1704.19	709	1700.42	4690
16	500	1	7800.39	180	7800.39	688	7798.66	3885
	900	2	6842.63	179	6842.62	757	6836.32	3920
	1300	2	5781.71	174	5781.71	555	5775.59	4025
	1700	3	4798.14	197	4798.14	652	4784.64	3990

534 List of Figures

- 535 1 Graph of function $J(t_1, I_1, t_1 + 5, 0.45, t_1 + 10, 0.45, t_1 + 15)$, the SEV when vary-
536 ing the timing (t_1) and intensity (I_1) of the first thinning (example stand of site
537 index $S = 13$ m and number of trees per hectare at plantation $N_0 = 1300$). . . . 31

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Figure 1: Graph of function $J(t_1, I_1, t_1 + 5, 0.45, t_1 + 10, 0.45, t_1 + 15)$, the SEV when varying the timing (t_1) and intensity (I_1) of the first thinning (example stand of site index $S = 13$ m and number of trees per hectare at plantation $N_0 = 1300$).