TRAJECTORY AND BREAKUP OF CRYOGENIC JETS IN CROSSFLOW

by

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Abstract

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This study investigated the breakup processes of subcritical cryogenic jets injected into subsonic crossflows of heated air. The crossflow speed, temperature, and jet velocity were varied to demonstrate the effect of thermal differences on a jet in crossflow. High speed back-lit photography and Mie scattering were used to examine the primary breakup regimes, trajectory, and breakup points. The breakup regimes show little change from jets in crossflow near thermodynamic equilibrium. Penetration of the jet increased with an increase in crossflow temperature. The breakup points in the streamwise direction followed trends previously observed for conventional jets. While the height of column fracture did not increase with momentum flux ratio as much as would be expected, its dependence matched that of the trajectory correlation. It is hypothesized that the observed differences are due to the development of a sheath of evaporated fluid around the main liquid core of the jet.
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Chapter 1

Introduction

Next generation aircraft engines will be required to be more efficient and climate friendly. To accomplish this goal there are several promising possibilities. One of these possibilities is the use of cryogenic fuel, which would allow the use of what would normally be gaseous fuels in aircraft, because of the increased energy to volume ratio. The two most likely cryogenic fuels are liquid natural gas and liquid hydrogen. Liquid natural gas offers a higher heating value than conventional jet fuel, as well as being much cheaper and more abundant in North America. Liquid hydrogen offers combustion without carbon emissions, in addition to potential use as a renewable resource. However cryogenic fuel cannot be used in current engines without redesign of their combustors [1, 2].

In the more immediate future, aeronautical gas turbine engines are continuously pushing to more intense thermal conditions. For example, compressor pressure ratios have been steadily increasing in order to improve the engine thermal efficiency, which is accompanied by an increase in the combustor air inflow temperature [3]. Simultaneously, jet fuel is undergoing increased thermal stressing prior to injection in order to cool various aircraft components [3]. This results in a heated liquid jet (potentially close to its boiling temperature) being injected into a much hotter air environment.

In both of these situations a fluid is injected near or at its boiling point into a much hotter environment. One way of modeling this injection is with a jet in crossflow (JICF). The JICF is a conical fluid dynamics system that exhibits rich physics and is used for many designs, consisting of a fluid injected perpendicularly into another fluid. There have been many studies investigating many aspects of JICF [4, 5]. Despite the number of studies done, there have been no studies investigating liquid JICFs injected near their boiling point into a much hotter crossflow.

This study will address a JICF injected at its boiling point into a heated crossflow through experiments on a cryogenic liquid injected into a heated crossflow. The low tem-
temperature of the cryogenic fluid allows for a large temperature differential at reasonable air

temperatures, allowing analysis of systems in varying degrees of thermal non-equilibrium.

In this paper current literature will first be reviewed for related work on jets in
quiescent, co-flow and crossflow. The objectives for the study will then be defined and
the experimental setup described. The results of this experiment are then presented and
analyzed.
Chapter 2

Literature Review

Despite the number of studies on JICFs [6, 7, 8, 9, 4, 10, 11, 5, 12, 13, 14, 15] several open questions remain regarding their controlling parameters. The turbulence in both the jet and crossflow, surface tension, velocities, densities, and several other properties all can impact the macro-scale features of a JICF [9, 12, 5]. Studies show many conflicting results on how different parameters effect the interaction of the jet with the crossflow. Despite this, there are some pervasive trends that are consistent among all studies surveyed.

2.1 Liquid Jet Breakup

In order to better understand cryogenic JICFs, it is useful to start with the most basic case of a circular non-cryogenic jet injected into quiescent air. This interaction, which has been studied since 1833, [16] consists of liquid being forced at a velocity \( U_L \) through a tube of diameter \( d_j \) and length \( L_j \) into a volume of still air.

The breakup of a liquid jet in quiescent air is classified into regimes based on the length of the liquid core (breakup length-\( L_{BU} \)) and underlying breakup mechanism [17]. This initial fracturing of the liquid column is refereed to as primary breakup. Both the Weber number and Reynolds number

\[
We_G = \frac{\rho G U_L^2 d_j}{\sigma} \quad Re_L = \frac{\rho L U_L d_j}{\mu_L}
\]  

(2.1)

are used to characterize these jets. Here \( We \) is the Weber number, \( Re \) is the Reynolds number, \( \rho \) is the density, \( \sigma \) is the surface tension, and \( \mu \) is the dynamic viscosity. The subscript \( L \), and \( G \) refer to the liquid and gas respectively. The Weber number compares the energy in the surface tension to the inertial energy of a fluid, whereas the Reynolds
Figure 2.1: Cylindrical jet behavior. Top: Stability curve, Bottom: example of visualizations (from left to right): Rayleigh regime (region B) $Re_L = 790, We_G = 0.06$; first wind-induced regime (region C) $Re_L = 5,500, We_G = 2.7$; second wind-induced regime (region D) $Re_L = 16,500, We_G = 24$; atomization regime (region E) $Re_L = 28,000, We_G = 70$ [16] (images from Leroux [17])
number represents the non-linear inertial terms relative to linear diffusion terms in the
momentum transport equations, and is a metric of whether the flow is turbulent. These
parameters depend on the diameter of the injection orifice, type of fluid injected and the
injection velocity.

The different breakup regimes can be visualized with a jet stability curve showing the
breakup length versus the jet velocity (with a constant jet diameter and fluid), see Figure
2.1. These regimes are, dripping (region A), Rayleigh (region B), first wind-induced
(region C), second wind-induced (region D), and atomization (region E). Two major
factors influence the jet breakup, the initial disturbances in the jet and the interaction
of the injected fluid with the surrounding medium. The aerodynamic forces become less
relevant at higher liquid Reynolds numbers and for liquids with a density more than
about 500 times that of the surrounding medium [18].

The dripping regime is characterized by the formation droplets directly from the
orifice. This occurs at low liquid velocities, with the maximum liquid velocity for dripping
being a function of the nozzle diameter and the liquid surface tension [16].

Once the liquid exceeds a critical velocity, a continuous liquid stream is formed and
breakup occurs beyond the orifice. Breakup in this regime is caused by Rayleigh insta-
bilities, which describe the breakup of a liquid cylinder with no external shear [19]. This
models how a disturbance with a wavelength larger than the jet diameter introduced
to the cylinder will cause wave-like features that form waists and bulges. The waists
are further pinched by surface tension, which is inversely proportional to the radius of
curvature. The disturbance grows until the amplitude of the wave is on the magnitude
of the jet diameter, at which point the jet breakups into droplets. The resulting droplets
have diameters that are on the same order of magnitude as the jet.

As jet velocity increases, the perturbations that cause Rayleigh instabilities are as-
sisted by the shear between the liquid and the surrounding gas. This results in the first
wind-induced regime [20]. Here the droplets are slightly smaller than in the Rayleigh
breakup regime, but still on the same order of magnitude, as the jet diameter. There
also may be satellite droplets (smaller droplets between the larger ones) formed, and the
drops will be less evenly distributed in location and size than in the Rayleigh regime.

In the second wind-induced regime, the effect of aerodynamic forces on breakup is
more pronounced, with stripping of small droplets off the liquid column immediately at
the jet exit. Larger fragments are pulled off and broken up through secondary breakup
further downstream [16].

The production of drops much smaller than the jet diameter and the complete dis-
ruption of the jet upon injection characterize the final breakup regime, the atomization
Figure 2.2: Quantitative separation of breakup regimes \(^a\) Ranz [22], \(^b\) Sterling and Sleicher [23] \(^c\) Miesse [24] \(^d\) Reitz [25] \(^e\) Dan et al. [26] taken from [16]

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region. Here, the condition of the jet flow prior to injection has a large impact on the breakup length. For example, highly turbulent or cavitating flows more easily entrain air, decreasing their breakup length. The droplets are formed by ligaments being stripped from the main jet and then broken apart [21].

In order to quantitatively assert boundaries on the breakup regimes, several authors have used the Reynolds number, Weber number, Ohnesorge number, \((Oh)\) and an additional computational parameter \(T\).

\[
W_e = \frac{\rho U^2 L d_j}{\sigma} \quad Oh = \frac{\mu L}{\sqrt{\rho L d_j \sigma}} \quad T = \frac{\rho L}{\rho G} \left(\frac{R e}{W e_L}\right)^2
\]

(2.2)

The Ohnesorge number describes the ratio of surface tension and viscosity. The region boundaries can be seen in Figure 2.2.

Another factor that affects breakup is the nozzle type, through its influence on the internal flow. This was investigated by Hiroyasul [27] using transparent nozzles, which enabled shadowgraph images before injection. The different types of flow within the nozzles can be seen in Figure 2.3.

Cavitation and its effect on turbulence is a major feature that affects the jet breakup. In (Figures 2.3a and b) there is no cavitation, which is the case for a sharp or sufficiently long nozzles. Note that 2.3b breaks up as a turbulent flow, while in 2.3a a laminar jet is formed in Figure 2.3a. Closed separation/cavitation bubbles occur in 2.3c and 2.3d, which affect the flow downstream through increased turbulence generation. In Figure 3e the flow does not reattach. This case is called super-cavitation and results in laminar jet at exit since there is no interaction with the walls to induce turbulence.

In cases where there is a long tube of a constant diameter upstream of the injection
point, the tube length also affects the behavior of the jet. Grant et al. [28] found that turbulent breakup was observed when the injection tube was long compared to injection diameter at much lower Reynolds numbers.

2.2 Jets in Crossflow

JICFs have been studied extensively in the past due to their many uses, such as fuel injection, cooling, and even blood flow [12, 29, 10, 7, 8, 14, 15]. There are many important features that characterize a JICF flow field. These include the breakup point and breakup mechanism of both the initial jet (primary breakup) and the resultant droplets (secondary breakup). In addition, the overall trajectory and penetration are important for design of devices using JICFs.

2.2.1 Forces and General Jet Structure

As with the jet in a quiescent atmosphere, there are two sets of forces which act on the jet, namely the surface tension and the aerodynamic forces (although it is important to note that internal turbulence of the jet may also affect the jet behavior). However, the addition of the crossflow changes how aerodynamic forces act on the jet. Isolating the aerodynamic forces gives a better indication on the influence of these forces on a JICF.
While this is not entirely possible, an approximation can be found using a gaseous JICF where there is no surface tension and very limited fluid viscosity.

The main fluid dynamic features of a gaseous JICF can be seen in Figure 4. The most prominent feature of this configuration is the counter-rotating vortex pair. These large vortices, which dominate the main body of the jet, are formed by the pressure differential from the windward to leeward side of the jet. The jet acts as an obstacle in the flow, resulting in a high-pressure zone on the windward side and low pressure on the leeward side. The shear layer vortices on the jet are caused by the jet’s velocity perpendicular to the crossflow, which generates Kelvin-Helmholtz instabilities on the jet itself. The horseshoe vortices are produced by the boundary layer of the crossflow rolling up as it passes the jet [4].

The addition of surface tension and viscosity, which would accompany the injection of a liquid, changes how the jet reacts to the crossflow. Breakup of a liquid jet is characterized by the fracture of an initial column into droplets. Prior to this breakup the pressure differential across the jet results in a round jet flattening into an oval shape as it penetrates the crossflow, with the leeward side and edges experiencing reduced pressure [29].

As the droplets are pulled from the jet, their path is then affected by their initial momentum. For this reason, larger heavier droplets tend to penetrate further into the flow while the lighter droplets tend to remain closer to the injection wall [15]. This manifests itself as increased spreading of the jet fluid relative to a gaseous jet. As the
velocity of the jet increases, the spray tends to spread out because of a larger variance between the momentum of the large and small droplets [15, 9].

2.2.2 Primary Breakup

Understanding the primary breakup modes and mechanisms is critical for using JICFs. Hence, there have been several studies looking at JICF breakup for systems near thermal equilibrium [14, 12, 29]. To describe the different breakup regimes both the Weber number and Reynolds number are used:

\[ We = \frac{\rho_\infty U_\infty^2 d_j}{\sigma} \]

\[ Re_j = \frac{\rho_j U_j d_j}{\mu_j} \]

where subscript \( \infty \) refers to the crossflow. Figure 2.5 shows the breakup regimes described.

2.2.3 Breakup of Laminar Jets

The breakup regimes of JICFs in which the liquid jet is initially laminar were first identified by Wu et al. [14]. They described four breakup regimes, which included three separate breakup types and an intermediate stage, namely column, bag, multi-mode, and shear breakup. Boundaries between these regimes were described by the Weber number
and momentum flux ratio. Sallam et al. [12] also found that the breakup regimes were primarily characterized by $We$, particularly for low $Oh$ liquids for which viscosity is not significant.

The column breakup regime (Figure 2.5a) for a JICF is analogous to the first wind-induced regime for jets in quiescent environments. Rayleigh instabilities induce oscillations in the jet column that increase in amplitude and eventually cause droplets to be pinched off. As the Weber number increases, droplets become smaller and are formed closer together.

When the Weber number increases to a range of 4-30, the breakup regime switches to bag breakup (Figure 2.5b). Due to the pressure difference across the jet it flattens out and bags begin to form. These bags expand until they burst, scattering smaller droplets than those seen during column breakup.

Increasing the Weber number further (beyond approximately 30) causes the jet to enter a regime in which both bag and shear breakup occur (Figure 5c). In the shear breakup mechanism, ligaments are pulled from the main body of the jet as a result of the shear forces between the jet and crossflow. These ligaments are largely formed in low pressure areas such as the leeward side and edges of the jet. The ligaments then breakup into smaller droplets.

At Weber numbers exceeding approximately 220, only shear breakup is observed (Figure 5d). At very high Weber numbers, the ligaments are very small, forming single droplets very shortly after being detached from the jet [10]. This results in the disintegration of the jet into a cloud of droplets.

### 2.2.4 Breakup of Turbulent Jets

The same breakup regimes occur for turbulent jets as laminar jets, although the conditions defining the boundary between regimes is different. This draws a close parallel with the jets in stagnant air, in which increasing turbulence changes the breakup of the jet.

Lee et al. [29] took measurements for several low viscosity liquid turbulent JICF ($Re = 7100-48000$). They concluded that there is no bag breakup or multimode breakup when the jet is turbulent. This was attributed to the internal turbulence overcoming surface tension and forming ligaments, even at low Weber numbers.

However, Wang et al. [30] looked at lower Reynolds numbers ($Re = 900-9800$) and found that bag breakup did occur. Their proposed transition from bag breakup to multimode breakup was at $We_G = 25$, lower than the range suggested for laminar flow by Sallam et al. [12]. They did express that the boundary was not well defined. The lower
Weber number at the transition to multimode breakup may be due to the turbulence within the fluid making it easier for ligaments to form.

Aalburg et al. [31] also looked at turbulent liquid JICF and found that an increase in Weber number moved the breakup location closer to the jet orifice in the shear breakup regime. In contrast, Wang et al. [30] found no dependence of breakup position on the Weber number in the bag breakup regime.

### 2.2.5 Trajectory

When using a JICF model in design perhaps the most important parameter is the trajectory. How the jet moves after interacting with the air is imperative for predicting the mixing of the injected fluid and crossflow.

The interaction of the turbulence in both fluids, deformation, evaporation, and surface tension makes simple analytical solutions for the mean trajectory of a JICF intractable. However, a physically grounded functional form for trajectory correlations can by analytically determined by assuming that many of these parameters are negligible. There are two main models used for practical purposes, though only the most common is reviewed here [32].

The most commonly used correlation models the jet as a stack of infinitesimally small cylinders, the shape of which never changes or rotates. The aerodynamic forces affect the speed and acceleration of each element of the liquid JICF. In this case, the force of drag acting on each cylindrical element is given by

\[
F_D = \frac{1}{2} \rho_\infty u_\infty^2 A_F C_d
\]

where \( \rho_g \) is the density of the crossflow, \( u_g \) is the speed of the crossflow, \( A_F \) is the area of the cylinder perpendicular to the flow, and \( C_d \) is a constant drag coefficient. This force acts on the mass of the liquid in the cylinder. Assuming that the cylinder initially only has a velocity in its axial direction \((y)\) the force balance gives

\[
\frac{1}{2} \rho_\infty u_\infty^2 A_F C_d = \rho_j A_x h \frac{d^2 x_j}{dt^2}
\]

where \( \rho_j \) is the density of the jet, \( A_x \) is the cylindrical cross section, \( h \) is the infinitesimal thickness of the cylinder, and \( x_j \) is the coordinate of the jet center in the crossflow direction. Rearranging gives

\[
\frac{d^2 x_j}{dt^2} = \frac{2 C_d \rho_\infty u_\infty^2}{\pi \rho_j h} \frac{1}{d_j}
\]
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Integrating and non-dimensionalising by the jet diameter gives

$$\frac{x_j}{d_j} = \frac{C_d \rho_\infty u_\infty^2 t^2}{\pi \rho_j u_j^2 d_j^2} \quad (2.7)$$

Assuming that the jet velocity in the y-direction does not change \((y_j = u_j t)\) and rearranging gives

$$\frac{y_j}{d_j} = \sqrt{\frac{\pi}{C_d q_x d_j}} \quad (2.8)$$

where \(q\) is the jet to crossflow momentum flux ratio

$$q = \frac{\rho_j u_j^2}{\rho_\infty u_\infty^2} \quad (2.9)$$

Hence, the jet trajectory is controlled by the drag coefficient and the momentum flux ratio. This basic equation allows for empirical correlation, such that results in different conditions and jet diameters can be compared. It is of note that this approximation is less appropriate after primary breakup, when the jet becomes a spray. In addition, the deformation of the jet from a circular shape to an ellipsoid changes the drag profile and the jet is expected to decelerate in the y-direction as it penetrates the crossflow. To allow for a more accurate representation of empirical results, Equation 7 generally is recast as

$$\frac{x}{d} = C q^\alpha \left(\frac{x}{d}\right)^\beta \quad (2.10)$$

where \(\alpha, \beta\) and \(C\) are empirically defined parameters.

There have been several attempts to fit this correlation to empirically measured results \([14, 12]\). However, there are several parameters not accounted for in the theoretical derivation of this functional form that may have an effect on the trajectory, such as pressure, temperature, and turbulence of both the jet and crossflow. Hence, several groups have attempted to include additional terms in Equation 2.10 \([9, 33, 6, 10]\). Regardless of the other parameters used, the momentum flux ratio has remained the most important parameter in all trajectory correlations.

The effect of crossflow temperature on the trajectory has also been looked at with mixed results. Some studies found that temperature had very little effect on trajectory \([9, 6]\). Eslamian et al. \([9]\) injected low temperature water into a heated crossflow \((298 - 573K)\). They speculated there may be a change in the trajectory with crossflow temperature, but the evaporation of smaller droplets on the periphery masked this change. Since Wu et al. \([15]\) showed the smaller droplets are concentrated closer to
the injection wall, this would imply increased crossflow temperature decreases overall penetration.

Lakhamraju et al. [10] found a decrease in penetration with increased crossflow temperature (294-500 K) when doing experiments with Jet-A and water. They also found that droplets were pulled off the jet closer to the injector as the crossflow temperature increased. It was speculated that the increased crossflow velocity required to get a similar momentum flux ratio and Weber number may be the cause of this.

An increase in the temperature at which the liquid is injected into the crossflow has also been investigated as an additional parameter affecting trajectory correlations. Lakhamraju et al. [10] found an increased liquid temperature (water 294-363 K, Jet-A 294-339 K) decreased penetration, but their results were based on limited data. Wiest et al. [34] did a study looking specifically at injecting heated Jet-A (300-520 K) into a crossflow. They also found that an increase in jet temperature decreased jet penetration, increased the spray cross-section, and produced smaller droplets.

When under pressure, liquids have a higher boiling point than at atmospheric pressure. When the liquid is at a temperature higher than its boiling point for a lower pressure and it is moved to that lower pressure, flashing occurs. This is very rapid evaporation that reduces the temperature of the evaporating liquid to its boiling point at the current pressure. When Wiest et al. [34] increased the jet temperature enough to promote flashing, it was found that the jet actually moved upstream of the injection point immediately upon injection, as shown in Figure 2.6. This test was done exclusively in the shear breakup regime for very turbulent jets.

### 2.2.6 Breakup Length

The breakup length of a JICF refers to the point at which the jet fractures into many smaller droplets. A review of the most common correlation will be presented here.

Ranger & Nicholas [35] found that the breakup time of droplets is

\[
\tau_b \propto d^2 \sqrt{\frac{\rho_j}{\rho_\infty}} \frac{1}{u_\infty}
\]

where \( d \) is the droplet diameter. This correlation was found using high speed photography in a shock tunnel. Later, Hsaing and Faeth [36] found that this correlation was not affected by the breakup regime.

Wu et al. [14] applied the droplet correlation to a liquid column breakup by assuming that the jet breakup time is proportional to the breakup time of an equivalent droplet, namely
Figure 2.6: Single frame Jet-A JICF with a 0.020” orifice at different injected temperatures, axis are in mm: a) $T_L = 172^\circ$C, $q = 130$ b) $T_L = 219^\circ$C, $q = 125$ c) $T_L = 233^\circ$C, $q = 125$ d) $T = 244^\circ$C, $q = 151$ [34]
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\begin{equation}
t_b = C d_j \sqrt{\frac{\rho_j}{\rho_\infty}} \frac{1}{u_\infty}
\end{equation}

This can be recast to include the momentum flux ratio, giving

\begin{equation}
t_b = C d_j \sqrt{q} \frac{1}{u_j}
\end{equation}

Assuming a constant jet velocity until breakup \((y_b = t_b u_j)\), the distance into the crossflow at which breakup occurs is then

\begin{equation}
\frac{y_b}{d_j} = C_y \sqrt{\bar{q}}
\end{equation}

To find the downstream position at which breakup occurs \((x_b)\), Equation 2.14 can be combined with Equation 2.8 to yield

\begin{equation}
\frac{x_b}{d_j} = C_y^2 \frac{C_b}{\pi} = C_x
\end{equation}

In the case of \(x_b\), the breakup location is the same regardless of the conditions.

The breakup time is often normalized by

\begin{equation}
t^* = \sqrt{\frac{\rho_j}{\rho_\infty}} \frac{1}{u_\infty}
\end{equation}

which leaves

\begin{equation}
\frac{t_b}{t^*} = C_{td}
\end{equation}

The results of these equations show that the crossflow breakup \((x_b/d_j)\) and breakup time \((t_b/t^*)\) should be constant, whereas \(y_b/d_j\) should be a function of the momentum flux ratio. These correlations have been confirmed by several experimental studies [14, 10, 12, 7], with typical results shown in Figure 2.7a. However, where the functional form remained the same, there was a significant offset between the values found in laminar and turbulent jets, as shown in Figure 2.8.

2.2.7 Vaporization

Vaporization is of special interest in the cryogenic JICF case because the jet is injected at its boiling point into a much hotter environment. Bellofiore et al. [6] did experiments on room temperature water jet injected into an elevated pressure (1-2 MPa) and temperature (300-600 K) environment. They found that, for 100 jet diameters downstream, there was
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(a) Y breakup correlation

(b) X breakup correlation

Figure 2.7: Breakup length of a jet in a crossflow

Figure 2.8: Comparison between laminar and turbulent breakup [14]
no significant vaporization of droplets. However, this is likely due to the temperature of the injected liquid being significantly below the boiling point.

Yeun and Chen [37] looked at the difference in drag of an evaporating droplet compared to non-evaporating. They found that droplet drag could still be found using the "standard curve" [38] which is a function of the Reynolds number. However, they did find that the parameters used in the Reynolds number should be found using the temperature and mass fraction

\[
T_r = T_d + (T_\infty - T_d)/3
\]

(2.18)

\[
X_r = x_d + (x_\infty - x_d)/3
\]

(2.19)

where \( x \) is mass fraction and the subscript \( r \) refers the ratio to be used.

2.3 Cryogenic Jets

Although there has been no work done on cryogenic jets in crossflow, there have been many studies on cryogenic jets in ambient and co-flow [39, 40, 41, 42]. Such studies have been mostly focused on rocket propulsion; because of this most studies looked at the transcritical - supercritical regime. There are no studies which specifically look at the breakup of a cryogenic jet in stagnant air at ambient pressure. In addition, there is no work in the public domain investigating how different nozzles or turbulence affect cryogenic jets. That said, there is value in observing the differences between cryogenic liquids and liquids near thermodynamic equilibrium.

Cryogenic liquids have very low boiling points. Since they are injected into a gas hotter than their boiling point, evaporation can become a significant factor. In addition, the cryogenic liquid is generally stored in a pressurized Dewar. If pressure fed from an equilibrium boiling temperature, the liquid would flash vaporize over the pressure reduction when injected. Because of this physical limitation, experiments must be carefully designed to ensure that cryogenic liquids are kept in the liquid phase until injected, which generally involves supercooling the liquid in the Dewar.

Despite the increased evaporation of cryogenic jets Chehroudi et al. [43] found that cryogenic jets at subcritical pressure breakup in a similar manner. In contrast, Qureshi [40] observed a large difference while studying gas entrainment in a cryogenic jet. However, their study makes no mention of flashing, and given they are using pressure fed liquid nitrogen, it is likely that flashing had a significant impact on the jet. Tani et al.
Potential core of a cryogenic jet in a co-flow compared to a jet in thermo-equilibrium in co-flow adapted from [42]

Breakup regimes of cryogenic droplets versus droplets in thermodynamic equilibrium [44]

[41] used back-lit photography to study supercooled liquid nitrogen. The nitrogen was injected into a pressurized environment about 10 K below its boiling point. The jet produced does not dissipate quickly but shows similar patterns to what would be expected for a jet at thermodynamic equilibrium.

Another characteristic that has been studied for cryogenic jets in co-flow is the potential core length. This is the liquid region that is unaffected by the surrounding fluid, and generally can be observed as a continuous dark region in backlit photography. Gautam et al. [42] compared the potential core of cryogenic jets to jets in thermodynamic equilibrium. Their results can be seen in Figure 2.9a. They postulated that the fast evaporating cryogenic liquid created a cocoon of gas around itself. This hindered the co-flow from atomizing the jet relative to non-cryogenic jets at high co-flow velocities. However, with little or no co-flow, the evaporated liquid interacted with the surrounding flow and enhanced mixing, reducing the potential core length.

Droplet evaporation has been studied for cryogenic airblast atomizers [45, 46]. Although these studies do not give a good indication of the initial breakup region, they may show the evolution of the droplets due to evaporation. It was found that the distribution of droplets changed over the first 85 jet diameters through the swift evaporation of smaller droplets. The crossflow velocity in these cases is significantly lower than in the
case of Bellofiore et al. [6] for non-cryogenic jets in crossflow however, the results show
that the smaller droplets disappear quickly while the larger droplets remain.

Cryogenic droplet breakup have been compared to conventional droplet breakup. Mayer et al. [44] found similar breakup regimes to non-cryogenic jets, although the transition Weber numbers may be shifted as shown in Fig. 9b.

2.4 Optical Measurement Techniques

Optical measurements have been used in this work, and brief overview of the relevant
theory is presented here. These measurements have the advantage of not effecting the
flow by the addition of an impedance in the jet or air stream. They do however require
optical access to the test section. As light moves through the fluid, there are several
ways in which it is affected. The light may scatter/reflect off the droplets (or gas-phase
molecules, though this is not considered here). It also may be refracted or absorbed by
the droplets, or it may be refracted through a change in temperature or density. Here
the refraction, absorption, and scattering will be used to measure different properties in
the JICF.

2.4.1 Mie Scattering

Mie scattering uses the scattering of light off particles or droplets with a characteristic
dimension much longer than the laser wavelength to determine their position. For the
planar Mie scattering from liquid droplets of interest here, a laser beam is formed into
a sheet and passed through the jet, causing the laser to be scattered off the liquid. A
camera placed perpendicularly to the laser collects the scattered laser light to display the
position of the liquid in the JICF.

As light moves through a medium it may be absorbed, scattered, or transmitted,
which is characterized in the radiative transfer equation [47].

\[(\omega \cdot \nabla)L(x, \omega) = -\sigma_t(x)L(x, \omega) + \sigma_s(x) \int_{4\pi} p(x, \omega', \omega)L(x, \omega')d\omega' + L_e(x, \omega) \quad (2.20)\]

\[\sigma_t = \sigma_a + \sigma_s \quad (2.21)\]

here \(L(x, \omega)\) is the luminosity at position \(x\) and direction \(\omega\), \(\sigma_t\), \(\sigma_s\), \(\sigma_a\) are the extinction, scattering, and absorption coefficients, \(p(x, \omega', \omega)\) is the probability of \(L(x, \omega')\) scatting in the direction \(\omega\), and \(L_e\) is the radiance emitted. The scattering plane is defined by the
vectors \( \omega \) and \( \omega' \).

The term on the left of equation (2.20) indicates the change in radiance in a specific direction. The first term on the right indicates the reduction in radiation in the direction of interest due to scattering and absorption. The second term on the right is the radiance from any direction that is scattered in the direction of interest, and the final term is the radiance emitted in that same direction. This final term is small for transparent medium at the wavelengths of interest here, and hence will be ignored.

In the case of a columnated laser sheet, the incident light will be from a single direction. If the liquid is diffuse enough that there will not be multiple scatters the laser will be the only source in Equation 2.20. Furthermore, if the detector is positioned perpendicular to the laser sheet, the only light that will be captured is the scattered light. Hence, the luminosity of the scattered light is determined by:

\[
L(x, \omega_c) = \sigma_s(x) \int \frac{4\pi}{2} p(x, \omega', \omega_c) L(x, \omega') d\omega'
\]  

(2.22)

where \( \omega_c \) is the direction perpendicular to the input light. However, since there is only luminosity coming from a single direction, the only parameters needed are the angle of the detector relative to the input light (\( \theta \)) and the angle between the detector and the scattering plane (\( \phi \)), leaving

\[
W_c = \sigma_s(x) \int \frac{4\pi}{2} p(\theta, \phi) L(x, \omega) d\omega'
\]  

(2.23)

where \( W_c \) is the power received by the camera due to a single scattering. Thus the problem reduces to understanding the probability of light scattered in each direction. The solution for this was found independently by Lorenz [48] and Mie [49] using Maxwell’s equations. The full derivation is beyond the scope of this work, however it can be found in Hulst et al. [50]. The result of this derivation shows that

\[
p(\theta, \phi) = \frac{|S_1(\theta)|^2 \cos^2 \phi + |S_2(\theta)|^2 \sin^2 \phi}{2|k|^2 C_s}
\]  

(2.24)

where \( k \) is the wavenumber of the light (\( 2\pi/\lambda \)), \( \phi \) is the angular displacement of an element of the aperture out of the scattering plane and \( C_s \) is the scattering cross section of the droplet which is related to \( \sigma_s \). The intensity of the incoming light is described by \( S \) (one for parallel and one for perpendicularly polarized light)

\[
S_1(\theta) = \sum_{n=1}^{\infty} \frac{2n + 1}{n \ast (n + 1)} (a_n \pi_n \cos \theta + b_n \tau_n \cos \theta)
\]  

(2.25)
\[ S_2(\theta) = \sum_{n=1}^{\infty} \frac{2n+1}{n*(n+1)} (b_n \pi_n \cos \theta + a_n \tau_n \cos \theta) \] (2.26)

coefficients \( \pi_n \) and \( \tau_n \) are based on the Legendre polynomials \( (P) \)

\[
\pi_n(\cos \theta) = \frac{P_n^1(\cos \theta)}{\sin \theta} = \frac{dP_n(\cos \theta)}{d(\cos \theta)}
\] (2.27)

\[
\tau_n(\cos \theta) = \frac{P_n^1(\cos \theta)}{d\theta} = \cos \theta \pi_n(\cos \theta) - \sin^2 \theta \frac{d\pi_n(\cos \theta)}{d(\cos \theta)}
\] (2.28)

From the Maxwell’s equations, light is treated like a wave. In the far field, which is where measurement can be made, \( a_n \) and \( b_n \) are the coefficients of the outgoing spherical waves.

\[
a_n = \frac{n_{med} \psi_n'(y) \psi_n(x) - n_p \psi_n(y) \psi_n'(x)}{n_{med} \psi_n(y) \zeta_n(x) - n_p \psi_n(y) \zeta_n'(x)}
\] (2.29)

\[
b_n = \frac{n_p \psi_n'(y) \psi_n(x) - n_{med} \psi_n(y) \psi_n'(x)}{n_p \psi_n(y) \zeta_n(x) - n_{med} \psi_n(y) \zeta_n'(x)}
\] (2.30)

\[
\psi_n(z) = z j_n(z)
\] (2.31)

\[
\zeta_n(z) = z(j_n(z) - iy_n(z))
\] (2.32)

\[
x = \frac{2\pi r n_{med}}{\lambda} \quad \text{and} \quad \frac{2\pi r n_p}{\lambda}
\] (2.33)

The end result of this is that the light scattered by droplets is very difficult to interpret because the scattered intensity depends on the scattering angle, incident angle and size of the droplet. For example, the dependence of \( |S(\theta)|^2 \) of perpendicularly polarized light can be seen in Figure 2.10 as calculated by MiePlot [51]. Further complications arise for non-spherical scatterers, which would be the case for practical droplets and ligaments. Hence, the Mie scattering images collected here are used to detect the presence of the liquid, not to quantify the amount of liquid.
2.4.2 Backlit Photography

Backlit photography is a technique used when it is difficult to light an object in a conventional manner. A light is placed behind the object to be photographed. This causes the photograph to be exposed in areas where there is nothing blocking the light. The object shows up as the dark part of the image. In a semi-transparent medium (such as water or liquid nitrogen), some light may pass through the medium causing a dimming in that area. This technique has been used by several studies to find structure in fluid interactions such as JICF [34, 13] and Rayleigh-Taylor mixing [52, 53] as it allows for a very crisp image of interacting fluids.
Chapter 3

Objectives

This study aims to fill the gap in current experimental data for cryogenic JICFs. This data may also be useful for the prediction of any jet that is injected near its boiling point into a much hotter environment. To study this system, a liquid nitrogen jet was injected into a heated crossflow. The parameters varied in this study were the jet velocity, crossflow velocity and crossflow temperature. Based on these tests, the major objectives are:

1. The breakup regimes for conventional JICF are well known. However it is unknown if a cryogenic JICF will breakup in the same regimes at the same conditions. This study aims to categorize this breakup into different regimes and find the parameters that separate these regimes.

2. Several correlations exist for the trajectory of JICF. However there are no trajectory correlations for jets that are very near their boiling points. The trajectory of the liquid nitrogen jet will be studied and a correlation found for both penetration and centerline trajectory.

3. The breakup points have been predicted and measured for conventional JICF. The breakup point for a cryogenic jet will be found and compared to correlations for conventional JICFs to determine if there are any differences.

There is a lack of experimental data for cryogenic JICFs against which computational fluid dynamics simulations (CFD) can be validated. This study was therefore setup such that the resultant data could be used as a validation data set for CFD.
Chapter 4

Experimental Apparatus

The apparatus for this experiment consists of a wind tunnel, injector plate, settling chamber, and fluid feed systems. The primary design challenge is ensuring the liquid nitrogen remains a liquid with no bubbles in an environment that is several hundred degrees higher than its boiling point.

4.1 Tunnel

The main tunnel is a rectangular stainless steel tube with inner dimensions of 32.6 mm by 38.9 mm (Figure 4.1). There is a 450 mm long initial section with no ports that allows the flow to become uniform before injection of the jet. The test section is optically accessible through three borosilicate windows. Two walls perpendicular to the injector’s wall are entirely transparent, allowing an unobstructed view directly through the test section for measurements such as backlit photography and Mie scattering. The window on the opposite side of the tunnel from the injector fits into a beveled slot in a metal frame. This frame is then fitted snugly into the slot in the upper wall. The injector plate (described below) is mounted through the fourth wall. All windows and the injector plate use Permatex red RTV as a gasket to ensure a seal with the tunnel. Seals made from this material are suitable up to 616 K.

4.2 Air Preparation

Compressed air from a central air compressor was used to achieve a flow rate of up to 4000 SLPM. Upon delivery to the lab, the air was metered through a Bronkhorst F-206BI mass flow controller, which was used to achieve the desired wind speeds. Next, the air
Chapter 4. Experimental Apparatus

25

Figure 4.1: Test channel

passed through a Sylvania Sure Heat Max to raise its temperature. The heater is rated at 36.0 kW and has a maximum temperature of 973 K. At the maximum air flow rate in the current system, the maximum achievable temperature is 630 K. After heating, the air entered a settling chamber, which removed any non-uniformities in the air. A nozzle connected the settling chamber into the main wind tunnel.

4.3 Liquid Nitrogen Feed System

The liquid nitrogen must be handled with care before injection to prevent any vaporization in the lines. The Dewar that contains the liquid nitrogen is vacuum insulated, which results in little heat transfer. Even with this very slow heating, the liquid stored in the Dewar will boil and raise its pressure. As pressure increases, the boiling temperature of liquid nitrogen also increases. Thus, with the increase in Dewar pressure, the liquid nitrogen within heats so that it is hotter than would be possible under atmospheric conditions. If the heated nitrogen were to be pushed out into a lower pressure, it would flash vaporize. In the case of a system, such as in this experiment, partial vaporization would occur each time a component introduced a pressure drop, such as in the mass flow meter. To ensure a fully liquid stream upon injection, the liquid in the Dewar therefore must be supercooled.

To supercool the liquid, the pressure within the Dewar was released and the nitrogen
subjected to atmospheric pressure. The fluid then flash boiled until it was at the boiling temperature of nitrogen at atmospheric pressure. The Dewar was then sealed and repressurized from a canister of gaseous nitrogen. The resultant stream of nitrogen will not flash unless it is subject to pressures that are lower than atmospheric or heated on route to injection.

The introduced pressure from the canister forced the liquid nitrogen out of the Dewar and through the system. From the Dewar, the liquid nitrogen going to the test section flowed through vacuum insulated tubing into a basin of liquid nitrogen containing various flow elements. First, the flow was split into a jet and cooling flow, the former of which was measured using a mass flow meter (Sponsler MF20). The two lines were then reconnected coaxially so that the jet flow was inside the cooling flow, using a modified Swagelok T in which one of the sides has a pass through that allowed a 1/16 pipe to be inserted and sealed.

The coaxial line leaving the container was another vacuum insulated hose, with the inner tube being PTFE. PTFE can be used with standard fittings, is flexible, and can withstand cryogenic temperatures [54]. At the jet injector plate (see below), the inner tube is connected to the inner pipe used for injection by a 1/16 Swagelok connector, with a custom made ferrule allow for the 1/16 tube to connect to the smaller injector pipe. See Figure 4.2 for graphical description of the nitrogen flow.

After starting the experiment, several flow-through times with liquid nitrogen were needed to cool the system before a consistent liquid flow was observed. While the system was cooling, boiling of the liquid nitrogen occurred. Flow rates through both the jet stream and cooling stream therefore were limited by the choke points of the injection orifice and entrance to the injection plate respectively. Until liquid nitrogen reaches these points, the flow rate was restricted by the increased volume of the gaseous nitrogen, which caused the cooling process to be slow. The system required at least a half hour to cool down to a point where uninterrupted liquid flow was observed.

4.4 Injector Plate

Liquid nitrogen was injected into the crossflow through a 0.5 mm inner tube. Several other studies have used this jet diameter [14, 15, 10, 12, 33, 55], which allows the current study to be compared without scaling. The dimensions of the plate are shown in Figure 4.3b. Given the size of this orifice, the effects of the crossflow boundary layer could be quite significant. Therefore, the injector was raised on a sharp-edged plate above boundary layer of the tunnel. This allows injection of the jet to be just downstream of
Figure 4.2: Liquid nitrogen flow path, dotted lines indicated gaseous flow, solid lines indicate liquid flow

the new boundary layer formed at the upstream edge of the plate.

Raising the injector plate adds additional cooling challenges. To ensure there is no evaporation as the jet flow moves through the heated crossflow to the injector plate, the cooling liquid nitrogen surrounds the jet stream until 0.5 mm before injection. Cooling flow can be seen in blue and the jet flow in red in Figure 4.3b.

The injection plate is situated vertically in the wind tunnel. The cooling liquid nitrogen moves upward through the hollow plate, exiting through an insulated hose. Due to this orientation, liquid nitrogen can build up a head in the plate before the hose, with the evaporated nitrogen drawn upward. This ensures the jet stream is surrounded by liquid nitrogen until injection.

The two halves of the injector plate were machined with a CNC machine then brazed together in a vacuum furnace with copper 102. The smaller tubes attached to the plate were laser welded. A series of standoffs were used to elevate the injector plate. These standoffs required screw holes on the top of the plate. To create a smooth surface, flat head screws were used with recessed holes. The recesses were deeper than required to bring the top of the screws below the level of the plate. Adhesive was then used to fill these holes and give the top of the plate a smooth finish.
4.5 Tunnel flow characterization

An additional consideration is the effect of tubes beneath the injector plate on the flow field. PIV therefore was done to characterize the flow field around the injector plate, and thus find the minimum distance from the injector tubes to the plate’s leading edge required to have an acceptable flow field perturbation while keeping the boundary layer on the plate small. PIV is a way to characterize the flow field of a fluid by taking two pictures of small particles illuminated by a laser sheet. The displacement of the particles between the two photos can be analyzed to find their velocity. If the Stokes number is small enough, the velocity of the particles can be assumed to be the velocity of the fluid in that location [56].

The flow was seeded with ~ 1 μm titanium dioxide particles, which were illuminated by two Nd:YAG lasers (Spectra Physics INDI). These two lasers were made coincident through the use of a polarizing beam cube, and power equalized through observation of the photographs and adjustment of q-switch timing. The camera used was an Andor Zyla 5.5 with a resolution of 2560 x 2160. Measurements were taken at 10 Hz, which was the maximum pulse rate of the lasers. The images were processed with LaVision Davis using their adaptive windowing scheme. The final interrogation boxes were 64 x 64 px with a 75% overlap, corresponding to a spatial resolution of 0.20 mm and 0.20 mm vector spacings.

Initially, the jet was injected ~3mm from the edge of a blunt leading edge plate.
The resultant flow field from this setup showed a significant deformation in the flow field above the plate.

Using the initial PIV, it was determined a knife edge of 7 mm should be added to the plate; this can be seen in Figure 4.3. Figure 4.4 shows profiles of the mean and root-mean-squared velocity at different axial positions along the plate with the knife edge for flow rates of 1,100 and 4,000 SLPM at atmospheric temperature. Disturbances caused by the plate’s leading edge result in a slightly non-uniform velocity profile at the jet exit ($x_j = 0$ mm), but this is small compared to the turbulent fluctuations.
4.6 Diagnostics

Two diagnostic techniques were used to examine the cryogenic JICF. Mie scattering was used to determine the trajectory of the centerline of the jet. Backlit photography was used to observe the breakup regimes and breakup point. It was also used as a verification for the Mie scattering based trajectory. Both diagnostics used a Photron SA5 camera with a resolution of 1024 x 1024. The camera image covered an area of 32mm x 32mm giving a pixel resolution of 31 µm x 31 µm. In each test case, the flow rates and temperatures were stabilized, then the backlit photography was taken, followed by the Mie scattering measurements.

Mie scattering was done with a Nd:YAG (Spectra Physics INDI) at 532 nm with a pulse width of 7 ns. The laser was first formed into a sheet and then directed along the center of the tunnel from the downstream end. This geometry ensured the entire jet could be captured by preventing reflection caused by impinging the laser sheet directly on the injector plate. The mirror directing the laser sheet into the tunnel was protected from the air leaving the tunnel by a glass barrier. The images were taken at 10 Hz, the maximum frequency of the laser.

The backlit photography was lit with a Cree CXA 1310 High-Density LED Array, the light from which was focused using a Tamron SP 180 mm camera lens, allowing most of the LED power to be used when capturing images. Images were taken at 7000 Hz for 1/7th of a second for a total of 1000 images. The exposure time was 256 ns, giving a crisp image. To enable these short exposure times, the aperture on the lens was fully open. This resulted in a short depth of focus centered on the centerline of the jet. As droplets spread out after breakup, they left the focal plane and could become indistinct.

4.7 Experimental Conditions

Three parameters were varied over the conditions tested. The first was the crossflow velocity which was controlled with a mass flow controller (Brinkhorst F-206BI) and limited by the capacity of the compressor supplying the air. Second was the jet velocity, which was induced by pressuring the Dewar and measured through a mass flow meter (Sponsler MF20). This flow was limited by the capacity of the mass flow meter and the minimum pressure needed to provide sufficient cooling flow. The final parameter varied was the crossflow temperature. The power of the inline heater (Sylvania Sure Heat Max) limited the maximum temperature; the minimum temperature was the temperature received from the compressor.
While these were the physical parameters controlled, the jet trajectory and breakup are controlled by the various non-dimensional parameters introduced in Section 2, namely the Weber number, Reynolds number, and jet-to-crossflow momentum flux ratio. In addition, temperature effects may be important in this cryogenic system, so a reduced temperature

\[ \theta = \frac{T_\infty}{T_j} \]  

is introduced. The ranges of parameters spanned can be seen in Table 4.1.

### Table 4.1: Experimental Conditions

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>Liquid velocity ((u_j))</td>
<td>4.9-21.7 m/s</td>
</tr>
<tr>
<td>Liquid Reynolds number ((Re_j))</td>
<td>8100-29000</td>
</tr>
<tr>
<td>Crossflow velocity ((u_\infty))</td>
<td>10-90 m/s</td>
</tr>
<tr>
<td>Crossflow Reynolds number ((Re))</td>
<td>22000-65000</td>
</tr>
<tr>
<td>Weber number ((We))</td>
<td>6-270</td>
</tr>
<tr>
<td>Liquid/gas momentum flux ratio ((q))</td>
<td>10-1000</td>
</tr>
<tr>
<td>Crossflow temperature ((T_\infty))</td>
<td>300-580 K</td>
</tr>
<tr>
<td>Temperature ratio ((\theta))</td>
<td>3.8-7.5</td>
</tr>
</tbody>
</table>
4.8 Liquid Nitrogen Properties

Many calculations in the following chapters require the properties of liquid nitrogen. For reference, the relevant properties are provided in Table 4.2.

Table 4.2: Properties of Liquid Nitrogen, all properties are at atmospheric pressure, and the boiling temperature at that pressure

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Boiling Temperature</td>
<td>77.36 K [57]</td>
</tr>
<tr>
<td>Density</td>
<td>807.4 kg/m(^3) [57]</td>
</tr>
<tr>
<td>Viscosity</td>
<td>0.166e-3 Pa.s [57]</td>
</tr>
<tr>
<td>Refractive Index (546 nm)</td>
<td>1.1990 [58]</td>
</tr>
<tr>
<td>Latent Heat of Vaporization</td>
<td>199.26 kJ/kg [57]</td>
</tr>
<tr>
<td>Surface Tension</td>
<td>8.87 mN/m [57]</td>
</tr>
</tbody>
</table>
Chapter 5

Results & Discussion

5.1 Analysis Considerations

The significant temperature differential between the jet and crossflow caused several problems that needed to be addressed during analysis. To ensure the results were not affected by experimental conditions outside the parameters defined, the tests exhibiting spurious effects were identified and removed from consideration.

The air was dried before being used for the crossflow. However, liquid nitrogen cools the injection plate to near 77 K. This low temperature causes moisture to freeze to the plate, even out of this relatively dry air. As a consequence, ice built up on various parts of the plate over the course of long duration experiments. The areas showing ice included the top and bottom of the plate, the tubes bring the liquid nitrogen to and from the plate, and for low-temperature crossflows, the windows within a couple of millimeters of the plate. The ice on the top of the injection plate was particularly problematic, as it changes where the jet begins to interact with the crossflow. The ice was removed from the injection plate before each test in an attempt to mitigate its effect, and the liquid nitrogen flow was cut every twelve tests to allow all the ice to melt. Despite these precautions, there was some ice build up on the injection surface when measurements were taken. Cases where there was a significant build-up of ice were not used for further analysis. In all other cases, the origin in the jet direction was taken to be the outer surface of the ice layer, which was determined case-by-case from the backlit photographs. See Figure 5.1 for examples of icing on the injection plate.

At higher crossflow temperatures, the metal tunnel expanded. This expansion changed where the jet was injected when compared to calibration images used to register the camera field-of-view to the geometry. The difference was particularly noticeable on days when several different temperatures were tested in succession. To account for this change, the
Chapter 5. Results & Discussion

(a) Test case with excessive ice  
(b) Test case with acceptable icing

Figure 5.1: Icing comparisons, note that the Weber numbers are not consistent between the two cases

backlit photographs were analyzed to find the starting point in the crossflow direction. This point was determined using an algorithm that found the center of the jet in the first few millimeters post-injection.

The other main challenge associated with injecting a fluid at or near its boiling point is flashing. Flashing can drastically change the dynamics of the jet, as the formation of gaseous bubbles within the jet expand and promote fragmentation. The liquid nitrogen is cooled to its boiling temperature at atmospheric pressure before the start of the experiment, however, if additional heating occurs between the tank and the injection, flashing may occur.

To identify flashing in the test cases, comparisons were made to the breakup regimes defined by Corn et al. [8]. They showed two new regimes associated with flashing in JICFs. The regime with the most flashing shows no liquid core and a ‘V’ shape immediately upon injection. All test cases in this regime encountered in this experiment were removed from the results analyzed. See Figure 5.2 for an example of flashing JICF in this study. Corn et al. [8] also identified a transition region between no flashing and flash breakup. This regime shows a substantial decrease in penetration, although breakup looks very similar to conventional JICF breakup. Kerosene jet fuel was used in their experiments, so it is unclear if this reduction in penetration is caused by the flash
evaporation of some of the components of the fuel with lower boiling points, or would be apparent in a single component JICF as well. It should be noted that this regime may be present in the results of this study.

5.2 Breakup Regimes

Breakup regimes were categorized for the presently studied cryogenic JICFs, to visualize any changes from conventional JICFs. Backlit images were used for this analysis. The high-speed images allowed not only the analysis of the breakup at single points in time, but also how the breakup evolves through time.

Three of the breakup regimes observed for jets in thermal equilibrium also were observed in this study; column breakup was not observed, but the Weber numbers achieved in this study were not low enough to expect this regime [14, 12]. At lower Weber numbers, surface waves on the liquid column led to the formation of bags that subsequently broke into droplets. As the Weber number was increased, ligaments were pulled from the surface of the column in addition to the bags, representing the multi-mode breakup regime. With a further increase in Weber number, purely shear breakup was observed. Further increases in Weber number decreased the size of the ligaments until droplets where drawn directly from the surface of the jet.
However, even after viewing the images it was unclear, in some test cases, which regime the jet should be categorized as. Ligaments from burst bubbles may be mistaken for ligaments pulled from the surface of the jet in the transition region between bag and multimode breakup. In addition, bags become quite small and difficult to identify when moving from the multimode to shear breakup regime. These bags often were created and burst within one frame, making it difficult to determine objectively if a bag had been formed. Despite these difficulties, every effort was made to be consistent with the descriptions of Sallam et al. [12]. Examples of each breakup regime and how they evolve through time can be seen in Figure 5.3.

The test cases were categorized into bag, multi-mode, or shear breakup. The dependence of breakup mode on Weber number and momentum flux ratio is shown in Figure 5.4. Due to the lower limit of the liquid nitrogen mass flow meter, lower liquid nitrogen momentum flux ratios could not be achieved at lower Weber numbers. As a result, tests were run at high momentum flux ratios where many of the test points impinged on the far wall of the tunnel. These impinging cases are identified in Figure 5.4, as the impact of impingement on breakup mode was unclear \textit{a priori}.

Transitions between regimes are well described by the Weber numbers. However, the transition Weber number does change as the momentum flux ratio increases, particularly for the transition between the bag and multi-mode regimes. This is attributed to the increased turbulence experienced by the jet as the jet velocity increases with increased momentum flux ratio at a constant Weber number.

At the lowest momentum flux ratios and thus, for a constant Weber number, the lowest turbulence conditions, the transition from bag to multimode breakup is about $We = 25$ ($Re_j \approx 9600$). This is slightly less than the transition Weber number observed by Sallam et al. [12] for laminar jets although it corresponds well to the value found by Wang et al. [30] for turbulent jets ($Re_j = 900-9800$). It is interesting to note that Lee et al. [29] did not observe bag or multimode breakup for turbulent JICFs ($Re_j = 7100-48000$).

The transition from multimode to shear breakup is at about $We = 70$ ($Re_j \approx 18000$) much lower than $We = 220$ found by Sallam et al. [12] for laminar JICF. The change is attributed to the turbulence within the jet facilitating the formation of ligaments and thus promoting shear breakup. No studies have been found that show the transition from multimode to shear breakup for thermal equilibrium turbulent JICFs, and it therefore is not possible to compare the value to previous studies.

To model the change in breakup mode from bag breakup to multimode breakup, an energy balance was done on the creation of ligaments in the jet. This method was first
Figure 5.3: Breakup regimes and their evolution through time, the formation and breakup of bags is indicated by read circles
used by Lee and Faeth [18] to examine the ligament formation on jets in a stagnant atmosphere. It was adapted by Lee et al. [29] to find where turbulent jets in crossflow begin to breakup. The energy equation contains the kinetic energy of the fluid perpendicular to the mean jet velocity, the energy in the surface tension, and the energy that the crossflow contributes through lower pressure on the jet’s surface due to an increase in velocity as the crossflow flows around the jet

$$\frac{1}{2}\rho_j u_{lig}^2 \frac{d_{lig}^2}{4} = \frac{1}{2} C_{sp} \rho_\infty u_\infty^2 \frac{d_{lig}^2}{4} = C_{si} \sigma \pi d_{lig}$$

(5.1)

where $lig$ refers to the ligament being created, $C_{sp}$ and $C_{si}$ are constants.

The size and speed of the ligaments must be found in order to use this correlation. The velocities of the fluid creating a ligament is caused by the turbulence within the jet itself. The turbulence in a fully developed pipe flow is $v'/\bar{v} = 0.03$ [59]. The ligaments themselves must be in the inertial range of the turbulence, giving $d_{lig} = \Lambda$ [18]. The integral scale for a turbulent pipe is $\Lambda = d_{pipe}/8$ [59].

Using $u_{lig} = u'_{pipe}$ Equation 5.1 was fit for the parameters $C_{sp}$ and $C_{si}$. $C_{sp}$ is a constant giving the portion of the crossflow’s kinetic energy used to help form the ligament. This will be a negative number, as energy will be transfered to the ligament on the sides and leeward side of the jet. $C_{si}$ is a constant that account for any non spherical aspects relating to surface tension. It should be on the order of 1/2.

After fitting, $C_{sp}$ and $C_{si}$ were found to be $-0.50$ and $0.21$ respectively. The resultant curve can be seen in figure 5.5. Although the curve fits reasonably well, the portion of crossflow energy transfered to the ligaments is much higher than expected, at 50%
 compared to 7% found by Lee et al. [29]. This implies this model does not well represent all the relevant sources of energy in ligament formation.

5.3 Trajectory

The result of the thermodynamic difference between the jet and crossflow were investigated to find its effect on the trajectory, as manifested by functional dependencies in trajectory correlations. Cases that impinged on the far wall were not included in the trajectory analysis due to the unknown influence of impingement. The limitation in minimum liquid nitrogen flow rate leaves few examples of bag and multimode breakup cases that do not impinge. To prevent bias that may be introduced in the different breakup regimes, only non-impinging jets in the shear breakup regime were used for calculation of trajectory correlations.

Both backlit photographs and Mie scattering data were used to find the mean trajectory for each case. All results presented below use the Mie scattering to determine the trajectory. Trajectory correlations using the backlit images are given in Appendix I, and follow the same trends. The Mie images were used to find the center plane trajectory of the JICFs. However, as shown in Section 2.4.1, the intensity of the scattered light is dependent on the incident and scattered angle of the laser, and size of the droplet. The liquid column and non-spherical droplets further complicate how to interpret Mie scattering images. It therefore was concluded that the Mie scattering intensity could not be quantitatively interpreted. Instead, the Mie scattering was taken as an indicator of the
presence of liquid. The background of the Mie scattering was first removed subtracting a moving average and then thresholding. Any signal remaining in the resultant images was considered to be true liquid, and set to a value of unity. The average of these binarized images was taken and this average was then used to find the trajectory.

The intensity of backlit photographs is more indicative of the volume of fluid integrated across the test section width. However, it is difficult to give quantitative meaning to the intensity of these images, as the reduction in background light is dependent on the amount of liquid in the path of the light as well as the distance of the liquid from the focal plane. Approximately, the intensity can be seen as inversely proportional to the thickness of liquid in or near the focal plane, as seen in the difference between the intensities of the liquid core, bags, and ligaments. Therefore, the unnormalized intensities of the backlit photos were used to find the average. In order to use the same trajectory algorithm on the backlit photos as the Mie scattering, the backlit images were inverted.

5.3.1 Algorithm

Jet trajectories were calculated representing the centerline of the jet, as well as the windward and leeward edges. An algorithm was written to provide a consistent way of finding the trajectories. This algorithm is described in detail below such that it may be compared with other methods of finding trajectories [9, 12].

The jet trajectories were calculated starting from the injection point and following an iterative predictor-corrector algorithm to add subsequent points. Note that the centreline jet trajectory for the first 10 points (ca. 1.5 mm) was taken to be in the initial jet direction (perpendicular to the cross-flow) and therefore did not implement the corrector step. Outside of this initial region, a predicted jet centerline point was calculated based on a linear fit to the previous 25-point segment of the trajectory (or less than 25 points if fewer had been defined). Given the proposed point along the centerline trajectory, an intensity profile was obtained in the direction perpendicular to the jet trajectory at that point in a band 10 pixels wide. This profile was then integrated to get an accumulated intensity, with zero being at the windward edge of the profile. The windward, corrected centre, and leeward sides of the jet then were taken to be at 10%, 50%, and 90% of the accumulated intensity. A restriction was placed on the allowable deviation of the predicted and corrected centerline points to prevent occasional spurious data. The trends for the windward and leeward trajectories were independent of the particular intensity thresholds used.

Stopping criteria were added to ensure that trajectories were not calculated in regions
Figure 5.6: Averages images with calculated trajectories. The leeward boundary is not used for fitting.

Figure 5.7: Example of an undefined lower boundary with insufficient liquid. As the liquid moves downstream, it disperses and evaporates. This results in decreased signal in both the Mie scattering and backlit images. When the signal is low, the trajectories can become quite noisy. Hence, trajectories only were calculated if the integrated intensity along the profile at a given point exceeds a threshold value.

Furthermore, edge of the frame can alter the trajectory if a significant portion of the intensity profile is beyond the field of view. To ensure this does not happen, calculations were stopped if any trajectory was within 200 pixels of the image boundary.

In some cases, small particles were pulled from the jet (stripping), see Figure 5.7. This causes low level intensity downstream of the injection point, in both Mie and backlit photographs. Despite the low intensity levels caused by these droplets, the levels were
sufficient to blur the leeward boundary of the jet. Therefore the leeward boundary of the jet was not used for fitting.

### 5.3.2 Fits

Once the points along the upper and centerline of the jets had been found they were fit. There are generally two ways to fit a curve. The first uses the difference in output values for a given input and reduces this to a minimum. In the second, the distance from a point being fit to the fitted curve is found and minimized. Although the second technique is significantly more computationally expensive, it is used here because the jet is essentially vertical at the injection point. For near vertical lines, there can be large differences in the output \( y_j \) from the fit for a given input \( x_j \) but a small overall distance between the fit and actual point, thus reporting a large error when the fit is quite good.

It has also been suggested that different breakup regimes result in different trajectory correlations [5, 12]. Because cases that interacted with the far tunnel wall were excluded from the fitting, few cases were retained for fitting in the bag and multi-mode breakup regimes. It was therefore decided only the JICF in the shear breakup regime would be used for fitting. Fits to several different functional forms were attempted to determine which best describes the JICF trajectories. Three fundamental forms were used:

\[
\frac{y_j}{d_j} = C_1 q^{\frac{1}{2}} \left( \frac{x_j}{d_j} \right)^{\frac{1}{2}} \tag{5.2}
\]

\[
\frac{y_j}{d_j} = C_2 q^{\alpha_2} \left( \frac{x_j}{d_j} \right)^{\beta_2} \tag{5.3}
\]

\[
\frac{y_j}{d_j} = C_3 q^{\alpha_3} ln \left( 1 + \beta_3 \frac{x_j}{d_j} \right) \tag{5.4}
\]

where \( C_i, \alpha_i \) and \( \beta_i \) are constants. The first equation is theoretically based off a set of stacked cylinders as shown in Section 2.2.5, where \( C_1 \) is the coefficient of drag of the liquid column. The second is based off the first, but with the empirically fit constants to account for difference between the real system and theory used to derive Equation 5.2. This is the most commonly used correlation [5]. The third equation is based on the trajectory of a droplet in a crossflow, see Ashgriz et al. [32].

These base equations consider only momentum flux ratio and position. In this work, additional equations that included the Weber number, crossflow Reynolds number, and temperature ratio also were tested, to determine any effects of these parameters on the trajectories. This was done by multiplying the base equations by each of these terms,
Table 5.1: Fitted equations to cryogenic JICF trajectory using Equation 5.2

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Correlation</th>
<th>R-Squared</th>
</tr>
</thead>
<tbody>
<tr>
<td>Centerline</td>
<td></td>
<td></td>
</tr>
<tr>
<td>-</td>
<td>$y_j/d_j = 0.53q_j^{1/2}(x_j/d_j)^{1/2}$</td>
<td>0.75</td>
</tr>
<tr>
<td>$We$</td>
<td>$y_j/d_j = 0.48q_j^{1/2}(x_j/d_j)^{1/2}We^{0.022}$</td>
<td>0.75</td>
</tr>
<tr>
<td>$Re$</td>
<td>$y_j/d_j = 13q_j^{1/2}(x_j/d_j)^{1/2}Re^{-0.47}$</td>
<td>0.82</td>
</tr>
<tr>
<td>$\theta$</td>
<td>$y_j/d_j = 0.095q_j^{1/2}(x_j/d_j)^{1/2}\theta^{0.99}$</td>
<td>0.98</td>
</tr>
<tr>
<td>$We, \theta$</td>
<td>$y_j/d_j = 0.11q_j^{1/2}(x_j/d_j)^{1/2}We^{-0.040\theta^{1.0}}$</td>
<td>0.88</td>
</tr>
<tr>
<td>$Re, \theta$</td>
<td>$y_j/d_j = 0.18q_j^{1/2}(x_j/d_j)^{1/2}Re^{-0.077\theta^{0.92}}$</td>
<td>0.88</td>
</tr>
<tr>
<td>Windward</td>
<td></td>
<td></td>
</tr>
<tr>
<td>-</td>
<td>$y_j/d_j = 0.62q_j^{1/2}(x_j/d_j)^{1/2}$</td>
<td>0.79</td>
</tr>
<tr>
<td>$We$</td>
<td>$y_j/d_j = 0.52q_j^{1/2}(x_j/d_j)^{1/2}We^{0.037}$</td>
<td>0.79</td>
</tr>
<tr>
<td>$Re$</td>
<td>$y_j/d_j = 11q_j^{1/2}(x_j/d_j)^{1/2}Re^{-0.43}$</td>
<td>0.84</td>
</tr>
<tr>
<td>$\theta$</td>
<td>$y_j/d_j = 0.12q_j^{1/2}(x_j/d_j)^{1/2}\theta^{0.96}$</td>
<td>0.89</td>
</tr>
<tr>
<td>$We, \theta$</td>
<td>$y_j/d_j = 0.13q_j^{1/2}(x_j/d_j)^{1/2}We^{-0.022\theta^{0.97}}$</td>
<td>0.89</td>
</tr>
<tr>
<td>$Re, \theta$</td>
<td>$y_j/d_j = 0.17q_j^{1/2}(x_j/d_j)^{1/2}Re^{-0.042\theta^{0.92}}$</td>
<td>0.89</td>
</tr>
</tbody>
</table>

raised to an empirically fit parameter. Combinations of these factors were also done. However, forms simultaneously including the Reynolds number and Weber number were not considered together, as the major influence on both was the crossflow velocity.

The results of the fits were compared by their coefficient of determination, or R-squared values. This value indicates how well the resultant equation fits the results. A R-squared value of one means the equations predicts the results perfectly. The results of fitting Equations 5.2, 5.3, and 5.4 and the additional terms above can be seen in Tables 5.1, 5.2, and 5.3.

The results indicate that using the theoretically-based exponents (Equation 5.2) gives a substantially poorer fit than the other two functional forms, which is not surprising since this equation only has one free parameter. Furthermore, inclusion of two additional free parameters in base Equations 5.3 and 5.4 did not substantively increase the fitting
Table 5.2: Fitted equations to cryogenic JICF trajectory using Equation 5.3

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Correlation</th>
<th>R-Squared</th>
</tr>
</thead>
<tbody>
<tr>
<td>Centerline</td>
<td></td>
<td></td>
</tr>
<tr>
<td>-</td>
<td>$y_j = 2.5q^{0.27}(x_j/d_j)^{0.33}$</td>
<td>0.85</td>
</tr>
<tr>
<td>$We$</td>
<td>$y_j = 4.0q^{0.25}(x_j/d_j)^{0.33}We^{-0.079}$</td>
<td>0.85</td>
</tr>
<tr>
<td>$Re$</td>
<td>$y_j = 230q^{0.29}(x_j/d_j)^{0.33}Re^{-0.42}$</td>
<td>0.91</td>
</tr>
<tr>
<td>$\theta$</td>
<td>$y_j = 0.38q^{0.38}(x_j/d_j)^{0.33}\theta^{0.80}$</td>
<td>0.92</td>
</tr>
<tr>
<td>$We, \theta$</td>
<td>$y_j = 0.64q^{0.36}(x_j/d_j)^{0.33}We^{-0.087}\theta^{0.80}$</td>
<td>0.93</td>
</tr>
<tr>
<td>$Re, \theta$</td>
<td>$y_j = 4.1q^{0.36}(x_j/d_j)^{0.33}Re^{-0.17}\theta^{0.59}$</td>
<td>0.93</td>
</tr>
<tr>
<td>Windward</td>
<td></td>
<td></td>
</tr>
<tr>
<td>-</td>
<td>$y_j = 3.1q^{0.25}(x_j/d_j)^{0.34}$</td>
<td>0.89</td>
</tr>
<tr>
<td>$We$</td>
<td>$y_j = 4.8q^{0.23}(x_j/d_j)^{0.34}We^{-0.074}$</td>
<td>0.89</td>
</tr>
<tr>
<td>$Re$</td>
<td>$y_j = 170q^{0.26}(x_j/d_j)^{0.34}Re^{-0.37}$</td>
<td>0.93</td>
</tr>
<tr>
<td>$\theta$</td>
<td>$y_j = 0.60q^{0.34}(x_j/d_j)^{0.33}\theta^{0.70}$</td>
<td>0.94</td>
</tr>
<tr>
<td>$We, \theta$</td>
<td>$y_j = 0.88q^{0.32}(x_j/d_j)^{0.34}We^{-0.082}\theta^{0.71}$</td>
<td>0.95</td>
</tr>
<tr>
<td>$Re, \theta$</td>
<td>$y_j = 5.8q^{0.32}(x_j/d_j)^{0.34}Re^{-0.16}\theta^{0.50}$</td>
<td>0.95</td>
</tr>
</tbody>
</table>
Table 5.3: Fitted equations to cryogenic JICF trajectory using Equation 5.4

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Correlation</th>
<th>R-Squared</th>
</tr>
</thead>
<tbody>
<tr>
<td>Centerline</td>
<td></td>
<td></td>
</tr>
<tr>
<td>-</td>
<td>$\frac{y}{d_i} = 2.1q^{0.27}ln(1 + 1.2\frac{x_j}{d_i})$</td>
<td>0.85</td>
</tr>
<tr>
<td>$We$</td>
<td>$\frac{y}{d_i} = 3.4q^{0.25}ln(1 + 1.2\frac{x_j}{d_i})We^{-0.078}$</td>
<td>0.85</td>
</tr>
<tr>
<td>$Re$</td>
<td>$\frac{y}{d_i} = 190q^{0.29}ln(1 + 1.2\frac{x_j}{d_i})Re^{-0.41}$</td>
<td>0.91</td>
</tr>
<tr>
<td>$\theta$</td>
<td>$\frac{y}{d_i} = 0.32q^{0.38}ln(1 + 0.1.2\frac{x_j}{d_i})\theta^{0.80}$</td>
<td>0.93</td>
</tr>
<tr>
<td>$We,\theta$</td>
<td>$\frac{y}{d_i} = 0.55q^{0.36}ln(1 + 1.2\frac{x_j}{d_i})We^{-0.085}\theta^{0.80}$</td>
<td>0.93</td>
</tr>
<tr>
<td>$Re,\theta$</td>
<td>$\frac{y}{d_i} = 3.4q^{0.36}ln(1 + 1.2\frac{x_j}{d_i})Re^{-0.17}\theta^{0.59}$</td>
<td>0.93</td>
</tr>
</tbody>
</table>

| Windward |             |           |
| -          | $\frac{y}{d_i} = 2.8q^{0.25}ln(1 + 0.1.1\frac{x_j}{d_i})$ | 0.89 |
| $We$       | $\frac{y}{d_i} = 4.3q^{0.23}ln(1 + 1.1\frac{x_j}{d_i})We^{-0.073}$ | 0.89 |
| $Re$       | $\frac{y}{d_i} = 140q^{0.26}ln(1 + 1.1\frac{x_j}{d_i})Re^{-0.36}$ | 0.93 |
| $\theta$  | $\frac{y}{d_i} = 0.54q^{0.34}ln(1 + 0.1.1\frac{x_j}{d_i})\theta^{0.70}$ | 0.94 |
| $We,\theta$ | $\frac{y}{d_i} = 0.88q^{0.32}ln(1 + 1.1\frac{x_j}{d_i})We^{-0.081}\theta^{0.70}$ | 0.95 |
| $Re,\theta$ | $\frac{y}{d_i} = 5.0q^{0.32}ln(1 + 1.1\frac{x_j}{d_i})Re^{-0.16}\theta^{0.50}$ | 0.95 |
accuracy beyond that achieved by including the temperature ratio in Equation 5.2. For fits using base Equations 5.3 and 5.4, the exponents determined for the dependencies with momentum flux ratio, temperature ratio, Weber number, and Reynolds number were nearly identical.

The base forms of Equations 5.3 and 5.4 performed better than Equation 5.2 due to the additional free parameters. Inclusion of the additional parameters created similar improvements to Equation 5.2. With a single additional parameter, the temperature ratio provided the best fit, followed by the Reynolds number, and then the Weber number. Furthermore, including a second parameter (e.g. \(Re\) or \(We\)) does not significantly improve the fit, with the marginal improvement being attributed to the presence of an additional free parameter as opposed to a physical reason.

Inclusion of the Weber number in the correlations does not substantively increase the fit accuracy. This is expected since aerodynamic forces dominate surface tension forces in the shear breakup regime. In addition, since the surface tension does not change in this study, changes in the Weber number were only due to changes in the crossflow velocity. The effects of such changes are captured through changes in the momentum flux ratio.

The Reynolds number has a large effect on the goodness of fit of the JICF; as the Reynolds number increases, the penetration decreases. While this could be attributed to an increase in crossflow turbulence accelerating such behavior, this behavior should be similar to conventional JICFs which have shown little dependence on the crossflow Reynolds number. In this study, the only difference between the dependence of the Weber number and Reynolds number on the test conditions is due to viscosity changing due to a change in temperature. This may indicate the Reynolds number dependence is due to the effect of temperature through the change in viscosity.

Including the temperature ratio in the correlation improves the quality of the fit by more than any other single parameter. An increase in crossflow temperature increased the penetration of jet. This is largely in contrast to studies in which the jet fluid was not near its boiling point, which found little or no effect of crossflow temperature on penetration [9, 10, 6].

In fits that do not include the temperature ratio the dependence on the momentum flux ratio is less than would be expected in a JICF in thermodynamic equilibrium [5]. However, when the temperature ratio is included in the dependence on the momentum flux ratio increases to levels which have been observed previously for conventional JICFs [5].

The dependence on Reynolds number exhibited in the fit above is similar to the dependence of the of the coefficient of drag on the Reynolds number for a sphere. To
Table 5.4: Fitted equations to cryogenic JICF trajectory using Equation 5.4

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Correlation</th>
<th>R-Squared</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Cd(Re)$</td>
<td>$\frac{y_j}{d_j} = 2.8Cd(0.10Re)q^{0.28}(\frac{x_j}{d_j})^{0.33}$</td>
<td>0.90</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Correlation</th>
<th>R-Squared</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Cd(Re)$</td>
<td>$\frac{y_j}{d_j} = 4.1Cd(0.14Re)q^{0.25}(\frac{x_j}{d_j})^{0.34}$</td>
<td>0.93</td>
</tr>
</tbody>
</table>

...test if the coefficient of drag is the cause of this dependence the “standard curve” [38]

$$Cd(Re) = \frac{24}{Re} + \frac{2.6 \frac{Re}{5.0}}{1 + \frac{Re}{5.0}^{1.52}} + \frac{0.411 \frac{Re}{26300}^{-7.94}}{1 + \frac{Re}{26300}^{-8.00}} + \frac{Re^{0.80}}{461000} \tag{5.5}$$

which gives the coefficient of drag of a sphere based on the Reynolds number was used to calculate the coefficient of drag in the correlation equation (see Section 2.2.5). The input Reynolds number was scaled by a constant and the equation was still linearly modified by an additional constant

$$\frac{y_j}{d_j} = C_{Cd}Cd(C_{Re}Re_{j\infty})q^\alpha \left(\frac{x_j}{d_j}\right)^\beta \tag{5.6}$$

$$Re_{j\infty} = \frac{\rho_ru_{\infty}d_j}{\mu_r} \tag{5.7}$$

where $C_{Cd}$ and $C_{Re}$ are the constants modifying drag and the Reynolds number respectively. The subscript $r$ indicates the quantity is calculated as in Yeun and Chen [37] using the crossflow composition, but a temperature between the crossflow and jet, see Equation 2.18. This is done to account for the change in crossflow properties due to evaporation. The resultant fits and R-squared values can be seen in Table 5.4.

The quality of fit improves relative to the baseline form of Equation 5.3, but is similar to that with an exponential dependence on Reynolds number and worse than with an exponential dependence on the temperature ratio. The dependence on the momentum flux ratio and position change very little when compared to the correlation without the Reynolds number included. However, the constant preceding the Reynolds number is very small. This indicates the calculated Reynolds number does not give a good fit
to the expected drag profile and may indicate non-physical results. Consequently, the improvement in fit quality may be attributed to an additional free parameter rather than the addition of the proper drag profile.

The only major difference found between cryogenic JICF and conventional JICF is the dependence of the trajectory on temperature. It is hypothesized the cause of this change is a sheath of evaporated fluid forming around the liquid core of the column. The evaporation of the jet manifesting itself as this sheath causes the increase in penetration observed with increased temperature.

To prove this hypothesis more study is needed. The most useful information would be the temperature profile around the jet. This can be found using simultaneous shadowgraph/backlit photography and filtered Rayleigh scattering. With these two measurements, the temperature can be overlayed on the physical position of the liquid, which would indicated the presence of a colder sheath around the main jet.

5.4 Breakup Point

5.4.1 Methodology

The breakup point is defined as the point where the liquid column fractures. This feature can be difficult to find visually, especially in the shear breakup regime where the liquid column shifts to a cloud of droplets. In this study, an objective methodology was developed to ensure that the breakup points were chosen in a consistent, unbiased manner.

The backlit photos were used to find the breakup point of the JICF (note that the images are not inverted for the breakup point analysis). After the injection point was found, the ice and injection plate were removed from the picture to ensure only the liquid in the crossflow remained. A threshold was then defined as a percentage of maximum intensity in the photo. The image was binarized around this threshold. The contiguous region below this threshold and that contained the injection point was considered to be the unbroken column of the JICF. The maximum position of this column in both the initial jet direction and crossflow direction then was found. The average of these two points was taken to be the instantaneous breakup point. This breakup point was found in each of the one thousand backlit photographs for each test case. The average of these individual breakup points taken to be the average breakup point of the jet discussed hereafter.

Several thresholds were tested to ensure that the trends and values were not sig-
Figure 5.8: The effect of a change in threshold on breakup position
(a) Change in breakup points between threshold of 3% and 8% in the crossflow direction ($x_b$)

(b) Change in breakup points between threshold of 3% and 8% in the jet stream direction ($y_b$)

Figure 5.9: Change in breakup point with between 3% and 8% threshold values

significantly affected by it. Figure 5.8 shows a typical image in which was evaluated for thresholds from 1% to 10% of the maximum intensity in the image. It can be seen that there is a significant change in breakup point between the different thresholds in Figure 5.8, particularly for low threshold values. Inspection of several samples found that thresholds of 3-8% best replicated breakup points observed visually. The differences calculated mean breakup points using a 3% versus 8% threshold are shown in Figures 5.9, demonstrating a change of less than 1 mm. The results presented below use a threshold of 5%. The trends reported were insensitive to the choice of threshold.

5.4.2 Results

As with the trajectory correlations, only the jets in the shear breakup regime which did not impinge on the walls were used for breakup point calculations because the effect of the impingement and breakup regime on the breakup point is unknown. However, for reference, graphs including all impinging test cases and different regimes can be seen in Appendix II.

The mean breakup points as a function of to Weber number, temperature, and momentum flux ratio can be seen in Figures 7.1, 7.2, and 7.3, with trend lines where applicable. For each mean breakup point, the distribution of points from the individual images was roughly Gaussian. Where the trend lines are included, the error bars indicate
two standard deviations. Note that all breakup points are normalized by the jet diameter, the breakup time is calculated assuming a constant jet velocity perpendicular to the crossflow and normalized by the characteristic time presented in Equation 2.16.

Correlations were found for the breakup points and breakup time versus the momentum flux ratio. No correlation was found with the Weber number (Figure 7.3). For the temperature ratio, no correlation was observed for the breakup points, but the breakup
(a) Crossflow direction breakup versus Weber number

(b) Jet direction breakup versus Weber number

(c) Breakup time versus Weber number

Figure 5.11: Breakup parameters versus Weber number

time did show some dependence.

The breakup point in the cross-flow direction is approximately constant. This is consistent with what has been found in literature, and the value is within the range found for conventional jets in crossflow [29, 10].

The breakup in the jet flow direction is a function of the momentum flux ratio as expected. However, it increases less with the momentum flux ratio than would be expected
(a) Crossflow direction breakup versus temperature

(b) Jet direction breakup versus temperature

(c) Breakup time versus temperature

Figure 5.12: Breakup parameters versus temperature

theoretically. Interestingly, the dependency of the $y$-breakup point on the momentum flux ratio is smaller in a similar manner as the trajectory. This shows that although different than expected purely from theory, the breakup remains related to penetration. The breakup time for low $q$ is similar to that expected for conventional JICFs. However, the breakup time decreases relative to the expected value as $q$ increases. This is due to the difference in functional dependency of the $y_b$ and the characteristic time on $q$. That
is, $t^* \propto q^{1/2}$, whereas it has been experimentally determined to be proportional to a smaller exponential. This causes $t_b/t^*$ to decrease with $q$, indicating that a redefinition of the characteristic time may be warranted.
Chapter 6

Conclusions

An examination of the basic primary breakup characteristics and trajectories of cryogenic JICFs was performed. This was done injecting a liquid nitrogen jet into a crossflow of air over momentum flux ratios 10-1000, temperature ratios 3.8-7.5, Weber numbers from 6-270 and crossflow Reynolds numbers of $2 \times 10^4 - 6.5 \times 10^4$. The major conclusions of the study were:

- Breakup regimes of the cryogenic JICF were the same as those expected in a conventional JICF. Transitions between the different breakup regimes were well described by the Weber number, and the transition Weber numbers were similar to those found for JICFs near thermal equilibrium. A change in the transition Weber was observed at high momentum flux ratios and on the transition from multimode to catastrophic breakup, but this was attributed to differences in the jet turbulence rather than thermal effects.

- In addition to the conventional dependence of trajectory on momentum flux ratio, a dependence on the cross flow temperature, and crossflow Reynolds number was observed. The penetration of the jet increased with an increased crossflow temperature, and decreased with an increased Reynolds number. It is hypothesized that the dependence on Reynolds number is be due the change in viscosity with temperature, thus showing the effect of temperature instead of a distinct affect.

- The breakup of the cryogenic JICF was very similar to that of a conventional JICF. The major difference was the breakup point in the initial jet direction was a function of the momentum flux ratio to the power of 0.33 instead of one half previously. This was also manifested in the normalized breakup time. It is of note that this dependence on the momentum flux ratio was the same as that observed for
the jet trajectory. Similarly, the jet-direction breakup point for conventional JICFs scaled in the same manner as their trajectory. Thus this difference is attributed to a change in penetration.

It was hypothesized that evaporation of the cryogenic JICF created a sheath around the liquid core. This sheath thickened with an increase in crossflow temperature, resulting in an increase in penetration. It is recommend that additional study be preformed to find the temperature profile around a cryogenic JICF. This will allow a more complete analysis on the effect of evaporation and change in crossflow temperature.

A more complete analysis on the liquid injection at is also needed. This study should use several liquids injected at multiple temperatures lower and higher than their boiling points. Similar work has been done for a kerosene based jet fuel, but should be done with single element fluids.

Another pertinent study is an investigation into how the crossflow and jet turbulence affect the JICF. Given the dependence of droplet and cylinder drag on the crossflow Reynolds number, it is likely this would have a similar affect on a JICF. In addition the affect of turbulence in the jet on breakup is unclear. It has been suggested several times, including in this study, that turbulence has a significant effect on breakup. An experiment spanning laminar and turbulent regimes would allow a definitive assessment of the affect of this turbulence.
Bibliography


Chapter 7

Appendix

7.1 Appendix I: Selected Backlight Trajectory Correlations

Although Mie scattering images were primarily used for trajectory backlit images were also fit for the most commonly used base Equation, Equation 5.3. The results shown in Table 7.1 indicate the same trends the trajectories discussed in Section 5.3.

Table 7.1: Fitted equations to cryogenic JICF trajectory biased on back lit image

<table>
<thead>
<tr>
<th>Windward</th>
<th>Center Line</th>
</tr>
</thead>
<tbody>
<tr>
<td>Correlation</td>
<td>R-Squared</td>
</tr>
<tr>
<td>( \frac{y}{d_j} = 3.1q^{0.24}(\frac{x}{d_j})^{0.36} )</td>
<td>0.87</td>
</tr>
<tr>
<td>( \frac{y}{d_j} = 7.4q^{0.21}(\frac{x}{d_j})^{0.33}We^{-0.14} )</td>
<td>0.88</td>
</tr>
<tr>
<td>( \frac{y}{d_j} = 1.1q^{0.33}(\frac{x}{d_j})^{0.35}We^{-0.13}\theta^{0.75} )</td>
<td>0.95</td>
</tr>
<tr>
<td>( \frac{y}{d_j} = 0.45q^{0.37}(\frac{x}{d_j})^{0.37}\theta^{0.77} )</td>
<td>0.94</td>
</tr>
<tr>
<td>( \frac{y}{d_j} = 490q^{0.29}(\frac{x}{d_j})^{0.33}Re^{-0.47} )</td>
<td>0.94</td>
</tr>
<tr>
<td>( \frac{y}{d_j} = 18q^{0.34}(\frac{x}{d_j})^{0.35}Re^{-0.27}\theta^{0.42} )</td>
<td>0.95</td>
</tr>
</tbody>
</table>

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7.2 Appendix II: Additional Breakup Point Data

Breakup parameters including all breakup regimes and test cases that impinged on the wall of the test section are shown below. The trends seen are the same as those seen in Section 5.4. However an additional trend may be seen between the Weber number and the jet direction breakup. This trend is suspect as the effect of wall impingement is unknown.
Figure 7.1: Multiple breakup regime breakup parameters versus momentum flux ratio, cyan indicates bag breakup, red indicates multimode-breakup, and black indicates shear breakup
Figure 7.2: Multiple breakup regime breakup parameters versus Weber number, cyan indicates bag breakup, red indicates mutimode-breakup, and black indicates shear breakup.
(a) Crossflow direction breakup versus temperature

(b) Jet direction breakup versus temperature

(c) Breakup time versus temperature

Figure 7.3: Multiple breakup regime breakup parameters versus temperature, cyan indicates bag breakup, red indicates mutimode-breakup, and black indicates shear breakup