The Role of Oral Communication In Accessing and Assessing Mathematical Understanding: Case Studies of Primary School Teachers’ Perceptions of Teaching Mathematics and Teaching Literacy

By

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A Thesis submitted in conformity with the requirements for the Degree of Doctor of Philosophy
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Abstract

The study investigated primary teachers’ perspectives on teaching mathematics and
teaching literacy. The focus was on oral communication strategies to see if perspectives
could be harmonized by building on teachers’ greater comfort with teaching literacy.

Case studies in a small suburban GTA school provided qualitative data through
classroom observation, on-going conversations about observed teaching episodes, semi-
formal interviews with the teachers and principal at the beginning and end of the study, and
one participant’s blog and research report.

Participants’ teaching experience ranged from 10 to 25+ years from kindergarten to
grade 7. During the study (2013-2014), members of the school staff were organized into
teaching partners by grade level to teach math through inquiry with an emphasis on
communicating mathematical ideas. Evidence collected from grades 1 and 3 lead to the
following findings: (1) Essential resources for teaching math effectively that teachers need
and want, are available; However, teachers are unaware of their existence; (2) Teachers who try to implement reform strategies without understanding how they work do not achieve the desired result; 3) A teacher with well developed processes for making sense of mathematics, can identify gaps in student understanding by relating student behaviour to her own processes; (4) When a teacher contrasts his/her sense-making teaching strategies in literacy and mathematics, he/she can better identify areas of dissatisfaction in his/her math instruction and possible strategies to try; (5) A teacher who is able to think about teaching goals in more general terms may come to see parallel objectives between certain teaching strategies in math and non-math subjects; (6) Teaching mathematics through inquiry requires students to have a solid grounding in early literacy as well as early mathematics; and (7) Reporting requirements that ask for student achievement in mathematics on a strand by strand basis encourage teachers to teach the subject strand by strand.

The study has implications for effective professional development, teachers learning math content and developing teaching materials, improving teacher confidence and the development of mindful reform practice. Suggestions for stakeholders to facilitate teachers’ reform practice are included at the end of the study.
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Chapter 1 - Introduction

1.1 Setting the Stage

Spoken language is the most explicit and most commonly used form of communication among human beings. We use it to describe our experiences of the physical world and communicate our thoughts about it. Almost everyone in the world can speak, because they have been practicing since infancy, and the general societal expectation is that everyone will speak. Our sophisticated use of language is also one of the markers that separate human beings from life forms of lesser intelligence (D’Andrade, 2002; Silva, McGee, & Williams, 1983).

Given that language and intelligence are linked, and mathematics can be viewed "as a particularly clear and concentrated example of the activity of human intelligence" (Skemp, 1989, p. 25), what if we looked at Mathematics as a type of language that describes the world? Admittedly, it is a highly efficient language dealing with more abstract ideas, but there are many parallels. In mathematics, we use symbols and constructions of symbols to describe ideas, just as we do in the language we commonly use for everyday communication. Since language and mathematics both describe our ideas about our world, admittedly on different levels of abstraction, it may be possible to approach self-proclaimed non-math people from a language-learning perspective to increase their openness to mathematics and perhaps, mathematical understanding.

When a young child learns to speak, he/she already has an intuitive understanding of objects and concepts before putting words to them, through his/her experiences of the world. For example, when a child has been sitting in various chairs since the earliest moments of
awareness, he/she has an intuitive understanding of ‘chair’ by the time he/she learns to use the word.

Young children gain a clear idea of ‘larger’ and ‘smaller’ from their physical experiences of the world, because almost everyone and everything that surrounds them is larger than they are (Neményi, 1997). Polányi (1958) refers to implicitly understood ideas we cannot express in words as tacit knowledge. As we mature, we learn to put words to many of our implicitly understood ideas and start to express them through the common language of our society.

If we consider this process of how language acquires meaning for the individual, we may notice that it is consistent with strategies advocated in reform mathematics practice (NCTM, 2000) to develop mathematical understanding. The language acquisition model implies that the mathematics will be more meaningful for students if they have an implicit understanding of the concepts, patterns and processes they are to learn, before formal math language is introduced.

In the language example, if we want to teach the word ‘chair’ to a person who has no experience of chairs, because he/she comes from a society where the common practice is to sit on the floor, the student can memorize the word, even associate it with the four legged object, but he/she will not associate the same meaning to the word as someone who is used to sitting on a chair. Similarly in mathematics, reform practice advises that activities and manipulatives be used to introduce ideas to learners through experience, rather than starting instruction with verbal explanations.

The use of models and pictures provides … opportunity for understanding and conversation. Having a concrete referent helps students develop understandings that are clearer and more easily shared. (NCTM Standards, 2000, p. 197)
The suggestion that models, pictures, and concrete referents be used to facilitate mathematical conversations and understanding is the basis for including rich activities and games in early math instruction. Discussions arising from these activities have students talking about their constructions and strategies. In so doing, they are learning to put words to their implicit understandings.

As students articulate their mathematical understanding in the lower grades, they begin by using every-day, familiar language. This provides a base on which to build a connection to formal mathematical language. (NCTM, 2000 p. 63)

When meaningful understanding has been solidified within the everyday vocabulary of the learner, the time has come to teach students the mathematical vocabulary for their newly understood ideas.

The use of mathematical symbols should follow, not precede, other ways of communicating mathematical ideas. In this way, teachers help young students relate their everyday language to mathematical language and symbols in a meaningful way. (NCTM, 2000, p. 131)

The use of words to meaningfully express mathematical ideas is only part of the role that oral communication plays in the mathematics classroom. The complementary, crucial role of spoken language in the mathematics classroom is that of interactive tool for facilitating and focusing mathematical reasoning. This role of spoken language in the mathematics classroom teaches learners to participate in established mathematical practices.

As a result of expressing ideas orally, for everyone to hear, the teacher gains ready access to student understanding. This allows him/her to gauge student understanding, probe more deeply, and/or immediately adapt instruction, in response to the assessed need.
1.2 Statement of the problem

If we accept that effective oral communication in the mathematics classroom teaches students to participate in established mathematical practices and thereby deepens students’ understanding of mathematical ideas, it follows that teachers’ skilful use of spoken language in the mathematics classroom is pivotal to the effective implementation of reform mathematics practice (Ball et al., 2008; Forman, 1996; Lampert & Cobb, 2003; NCTM, 2000). In describing assessment centered classrooms, where the teacher uses formative assessment to gauge student understanding, gives immediate feedback, and gives students a chance to revise their thinking, Donovan and Bransford (2005) stress "the need to provide frequent opportunities to make students’ thinking visible as a guide for both the teacher and the student in learning and instruction" (p. 13).

Ongoing, effective oral communication in the classroom is one way to provide opportunities for making student thinking visible, and giving immediate feedback. However, to manage classroom dialogue successfully:

Teachers must know how to orchestrate effective classroom discussions; use effective questioning techniques; select, adapt and design assessment tasks that elicit evidence of learning; and provide feedback that moves learners forward and activates students as the owners of their own learning. (JRME Research Committee, 2013, p. 348)

Therefore, helping teachers and teacher candidates develop the necessary skills to facilitate oral communication in their mathematics classrooms should be of concern to practitioners, educators and researchers. In this context, oral communication would be used to: (i) help students engage in mathematical practices, (ii) deepen mathematical understanding, and (iii) conduct ongoing formative assessment.
The literature suggests that several key factors must be in place to enable meaningful oral communication practices in the mathematics classroom (Munster, 2014; Wilhelm, 2014). To ensure that students are willing to share their ideas, classroom norms must allow students to feel safe from ridicule. NCTM Standards (2000) advocates that “teachers should build a sense of community in classrooms so students feel free to express their ideas honestly and openly, without fear of ridicule” (p. 268).

To facilitate mathematical reasoning through classroom dialogue, Ball and her colleagues (2008) parse the work involved, to make teachers’ work visible:

The three practices – naming and using names, making and interpreting claims, and evaluating mathematical assertions – make visible what is involved in making mathematics work in the classroom. With these three practices in focus, teachers can reconfigure classroom talk for the purpose of engaging students in mathematical work. These practices offer ways of laying out the mathematical work to be done. They provide ways to focus on who talks, what gets talked about, and how it gets talked about. (p. 41)

The use of language as described above is not a well-developed phenomenon (Ball, Lewis & Thames, 2008; Hill, 2010; Lampert & Cobb, 2003) in the average mathematics classroom. It hinges on a multitude of factors: the teacher’s own beliefs about the nature of mathematics, the teaching of mathematics, his/her content knowledge, and comfort level in evaluating student answers, or using them to drive the lesson forward. In fact, little is known about teacher experiences in this area. How connected a teacher’s everyday language is to his/her mathematics vocabulary, and how comfortable he/she is with the process of mathematical reasoning, will influence his/her ability to create an atmosphere where ideas are freely expressed. To encourage risk-taking, and facilitate the clarification and reasonable evaluation of claims by the group, is an even more challenging undertaking (Hill, 2010; Hill, Ball & Schilling, 2008; Sleep, 2012). These factors can be complex and problematic to implement, and both theoretical and practical problems can be revealed in the process.
Research related to the facilitation of meaningful mathematical dialogue in classrooms where mathematics is taught through inquiry (Van de Walle et al., 2010) has very specific recommendations for the use of oral communication in setting up the complex task (Jackson et al., 2013), and the teacher’s role in orchestrating the debriefing discussion (Smith & Stein, 2011). Teachers’ work involved in engaging students in mathematical reasoning in the elementary classroom (Ball, Lewis & Thames, 2008) has also been clearly documented. In order for teachers to effectively build these recommended strategies into their practice, they need to perceive this information as more accessible to them. One of the intents of this study is to facilitate this perception of accessibility.

This study addresses issues associated with teachers’ role in helping learners express their understanding of mathematical ideas (conveyed to them in meaningful, concrete ways, through rich mathematical tasks), in their own words before mathematical language is associated with those understandings. It will explore and describe teachers’ experiences as they facilitate oral communication in their elementary mathematics program to give expression to students' new understandings and translate them into mathematical language. It will describe the techniques teachers use to promote oral communication, why they choose to use them, and how they use them.

The study will also describe oral communication strategies teachers use in teaching non-math subjects, particularly literacy, to explore possible bridging elements between these different applications. Through observation and interviews, teachers will be asked to reflect on the purpose of their chosen communication strategies and how well they are working in the specific application. They will also be asked to reflect on any perceived similarities or
differences in the purpose and achieved goals of the strategies in teaching literacy and teaching mathematics.

By contrasting their use of teaching strategies in these different disciplines, teachers may get a clearer picture of their areas of dissatisfaction in their mathematics practice. Finally, teachers will be asked how their work is supported and resourced, what dilemmas arise, how they deal with the dilemmas, and what areas need further development.

1.3 Purpose of the Study

The purpose of this study is to discover and document the similarities and differences in participants’ perspectives on teaching mathematics and teaching literacy. Comparison of these perspectives may shed light upon areas of perceived need for strengthening the less favoured subject area. It may also reveal previously unsuspected areas of similarity on which to build, in order to advance the teacher’s practice in the less favoured subject area.

The focus of the study will be on oral communication strategies, to document teachers’ practices in teaching mathematics as well as literacy. It will look at oral communication strategies employed in teaching literacy, and analyze them in terms of purpose and goals achieved. It will also document teachers’ progress as they work to teach mathematics through inquiry; how they use oral communication strategies to elicit student ideas, deepen students’ mathematical understanding, and facilitate and focus their students’ mathematical reasoning. This study will help identify teachers’ challenges in promoting oral communication in the mathematics classroom, and identify conditions, which support professional growth in the areas of facilitating oral communication to deepen mathematical understanding and develop students’ mathematical reasoning.
1.4 **Significance of the Study**

This study provides mathematics educators, department heads, and mathematics consultants, insight into the beliefs and practices of teachers learning to use oral communication strategies to engage students in reasoning practices and deepen mathematical understanding through classroom dialogue. It explores ways teachers might advance their own mathematics practice through reflection on their teaching practices in other subject areas. It identifies areas where teachers need support and increased knowledge to help in their implementation of new dialogic practices. The study is also a source of information for mathematics teachers who are attempting to facilitate oral communication in their classrooms for engaging students in mathematical reasoning, and deepening mathematical understanding.

1.5 **Research Questions**

A case study approach is used to pursue an understanding of the complex phenomena surrounding the implementation of oral communication strategies for deepening mathematical understanding and fostering mathematical reasoning in the elementary classroom. Teachers’ perspectives on the communication strategies they use in teaching mathematics and the communication strategies they use in teaching literacy will be explored, with a view to helping teachers identify areas where they feel that their instruction in the less favoured subject is lacking. The thick, rich description characteristic of a qualitative approach is necessary for capturing the meaning perspectives of participants and their culture as they construct their world around them. The following questions have been developed to guide the study:
1. What teaching practices are used to strengthen the connections between spoken language and mathematical ideas?

2. How is students’ ability to reason facilitated through dialogue?

3. What kinds of challenges do teachers encounter as they facilitate classroom dialogue for deepening mathematical understanding?

4. What support and assistance do teachers need for facilitating oral communication?

5. What are teachers’ perceived needs for doing mathematics in order to teach math well through inquiry?

These five questions form the basis for the development of this study and provide insight into: (1) the effective facilitation of oral communication for voicing intuitive mathematical understandings, (2) the facilitation of classroom dialogue as a tool for developing students’ ability to reason, (3) the kinds of problems that arise as teachers work at implementing these practices, (4) where teachers find support for their developing practices, and (5) how self-perceived teacher efficacy is affected by teachers’ own competence in doing mathematics.

1.6 Background of the Researcher

My early math education started in Budapest, Hungary where I attended primary school until my family immigrated to Canada when I was in grade 4. I have vivid memories of grade one activities with red and blue counters, which taught me to subitize. I believe those activities are the reason I tend to count by fives, where others might count by ones or twos. In grades five and six, my mathematical skills blew my Canadian classmates out of the water; an immigrant boy from Israel and I were considered the class geniuses in math. By middle school, however, I was just keeping up, because I did not really get ‘the new math’ as
it was taught here. In high school, we had a former engineer for a math teacher, who did a
good job of teaching trigonometry and differential calculus, but proofs were not so obvious
to me.

Then my first year lecturer at University completely put me off integral calculus. I
still wanted to be a scientist, but based on my first year experience, did not feel that math was
my strong suit. So I finished off my formal math education with a second year statistics
course, and went into Chemistry. Some years later, when my daughter was in elementary
school, I was disappointed to find that she was being taught to count by ones on her fingers,
and despite being a very smart little girl, was learning to hate math.

More years passed, before I realized that I was in a particularly advantageous position
to contribute to early math education in Ontario. During a study leave looking at early
science education in Budapest, I came across a vibrant grade three class, enthralled with
reform mathematics. Instruction in this classroom was far in advance of anything I had
personally experienced, but with the perspective of the mature scientist, I was immediately
captivated by the ease with which those children played with serious mathematical ideas like
functions, relations, simple algebraic equations, geometry, probability, and combinatorics.
The experience convinced me that, if all students learned mathematics in an engaging
manner, it would also help them love science, because it would all make sense to them.

I subsequently enrolled at the Ontario Institute for Studies in Education (OISE) at the
University of Toronto to learn more formally about reform mathematics practice. As I
became more knowledgeable about the topic, I started to incorporate reform methods into my
chemistry teaching, and have done so for years. However, it is not enough to put these
methods to good use myself. I would like to help elementary teachers build their reform mathematics practice, and in doing this research, I think I have found a way.

In one of my OISE courses, I came across an article for teaching to read meaningfully (Palincsar, 1984), which advocated techniques I considered akin to mathematical. In a series of studies designed to improve text comprehension, the author identified four techniques for instruction, expected to improve text comprehension: prediction, question generation, summarization and clarification. Each technique requires students to revisit the text and self-monitor understanding:

To generate a question, the student must first identify key information in the text, frame that information in the form of a question, and self-test for understanding and recall... when clarifying, students note when they have experienced a failure in comprehension, identify the source of that breakdown, and take appropriate steps (e.g., rereading, reading ahead, or asking for assistance) to restore meaning. Predicting, in particular, provides the opportunity for students to activate relevant background knowledge or schemata. Finally, summarizing focuses attention on integrating information across sentences, paragraphs, and pages of text. (p. 77)

These strategies for understanding text seemed very similar to strategies used in understanding word problems (read and re-read), and the process of making mathematical claims, which often have to be clarified and made more precise. When we are learning to estimate, we are often asked to make predictions based on relevant background knowledge, and the estimate can later be verified by comparison to the calculated values. The calculated answer to a word problem has to be integrated back into the circumstances of the problem, to ensure that it makes sense. Noting these parallels started me thinking about helping math phobic individuals connect to mathematics using strategies they may associate with teaching literacy. I was particularly encouraged by the author’s statement that:

The instructional format designed to impart and refine the use of these strategies … is best represented as a dialogue between teachers and students in which participants take turns assuming the role of teacher. The individual who is
teaching is responsible for leading the dialogue, which is structured by the four strategies. (p. 77)

In concept, this statement is practically identical to what Ball, Lewis and Thames (2008) advocate for engaging students in mathematical reasoning: "These practices offer ways of laying out the mathematical work to be done. They provide ways to focus on who talks, what gets talked about, and how it gets talked about." (p. 41).

If such similar teaching strategies work in both subjects, it might be a way to help math phobic individuals. If oral communication strategies can be used to enhance math phobic teachers’ understanding of the mathematical process, they might come to see mathematics as a different language for describing the world. In view of many elementary teachers’ superior confidence in teaching literacy, such a perspective may have the potential to move their mathematics teaching practices in the reform direction.

As a chemist, my training was focused on quantitative methods. Through a qualitative methods course taken at OISE, I discovered qualitative methods. Before this project, I had used qualitative methods in conducting the research for a pilot project, my Masters thesis, and a research paper in educational policy and evaluation. Though this experience with qualitative methods was admittedly limited, it gave me insight into the potential of qualitative methods. It showed me that qualitative methods can capture the rich detail of artistic and interdisciplinary characteristics associated with phenomena, that is beyond the scope of quantitative methods.

During data collection for my Masters project, and to a lesser extend through OISE research courses, I developed some skill in conducting semi-structured and group interviews. Note-taking and observation skills have been practiced over a long scientific career in academia, which also ingrained the importance of active observation with an open mind.
Based on this background, I am committed to gaining insight into the experiences of others while making a conscious effort to be aware of and minimize any biases I may bring to the table. With ten years of OISE influence on my teaching, and my conscious efforts to incorporate reform strategies into my practice, I believe I have a good understanding of what constitutes effective reform practice, and how difficult it is to achieve, even with the strongest motivation and best intentions.

Although I have not taught in schools, I have taught hundreds of undergraduates in a variety of contexts, ranging from seminar groups of 20 students, to laboratory groups of 16, to lectures of more than 200. In the 'Innovative teaching methods in chemistry' course I teach, I have worked with many students to develop effective ways of communicating abstract concepts, build on informal understanding, and connect the concrete to the abstract. I believe these are the principles that form the basis of all successful reform mathematics methods. The effective use of oral communication strategies to develop mathematical reasoning through classroom dialogue can be viewed as the culmination of successful reform practice. This is because a teacher needs to be completely confident and comfortable in his/her own content knowledge to be able to judge the validity of student ideas on the spot, and use them to drive the lesson toward his/her intended goals.

1.7 Limitations of the Study

This study examined two pairs of teaching partners and their work around the use of oral communication strategies in advancing their reform practice in teaching elementary Mathematics. Facilitating teachers’ reform practice is an ongoing concern, and any insight into hurdles teachers must overcome is useful. However, the fact that teachers worked together as partners was a function of the Principal’s awareness of research surrounding the
successful implementation of reform strategies, and is not necessarily common practice. In addition, the small sample size will make generalizations beyond cases with similar characteristics difficult.

Although large generalizations will be hard to make, this study will give insight into the way these teachers and principal perceive the impact of oral communication on reform math practice, as well as their beliefs about teaching mathematics as compared to teaching literacy. While the study may illumina specific hurdles in teachers’ attitudes toward teaching mathematics, how some of these hurdles may be overcome may not be as clear. The student population is not representative of all schools, but as a rather diverse and somewhat disadvantaged population, it may bring to light specific problems that need to be addressed in many classrooms. How to successfully address them may also be the function of a further study. In addition, study participants will be able to reveal information about influences that support their efforts to promote communication in the mathematics classroom and ones that impede them. It is anticipated that this information will provide a means of helping teachers expedite their reform practice.

1.8 Format of this Thesis

The information communicated in this thesis is organized into five chapters. Chapter one gives an introduction to the research problem, specifies the questions investigated, provides some information about the researcher’s background, and situates the research in the larger context of curriculum studies and teacher development.

Chapter two reviews the literature relevant to grounding the concepts that are central to this study. I start by considering the relationship between language and mathematics, then I review the relationship between classroom dialogue and the elements of mathematics
education reform. In the discussion of this relationship, the inherent dilemmas created by the extensive use of teacher monologue in the classroom that is typical of traditional practice, become apparent. This leads to a consideration of the specific, multifaceted nature of meaningful mathematics dialogue.

The focus is on the role classroom dialogue plays in the emergence of meaning, and teaching students to participate in established mathematical practices. This focus constitutes a relatively new approach to math education. The discussion on classroom dialogue highlights the importance of the teacher's role as facilitator, and the role of teacher beliefs in the successful implementation of classroom dialogue.

Teacher competencies and confidence as mitigating factors in implementation are also considered. The literature review concludes with related studies of teachers’ experiences with dialogue in the mathematics classroom, and elements of professional development that have been shown to foster teacher change. Examination of these studies points to areas inviting further exploration.

Chapter Three gives a detailed description of the components of the research methodology used to carry out this study. It starts with the rationale for taking a qualitative approach and goes on to detail the methods used. It covers the research setting, including entry into the setting, the selection of participants, the role of the researcher, data collection, data analysis and ethical considerations. This chapter also addresses measures taken to ensure the credibility of the study (Miles & Huberman, 1994).

Chapter four presents the cases of four primary school teachers working in pairs as teaching partners. It describes relevant background information for each participant and his/her pertinent teaching strategies. It also describes supports and limitations teachers
reported concerning their work, and the perceptions of the need to know mathematics they voiced.

Chapter five presents an overview of what all the cases revealed in a cross-case analysis. I then discussion the data collected in the context of the research questions. Each question is answered with a combined perspective from the cross-case analysis. This perspective is then supported with data elicited from the individual cases. The findings that emerge from this process are linked to relevant literature. Implications of the findings generate questions that serve as a guide to avenues of further research. In addition, they form the basis for suggestions to stakeholders for facilitating teachers’ reform practice.
Chapter 2 – Review of the Relevant Literature

2.1 Overview

Mathematics education can be viewed in different ways. It can be seen in a traditional sense, as the training of the young to master certain algorithms for specific purposes, or it can be seen as teaching students to participate in the process of mathematical reasoning (Lampert & Cobb, 2003). The need for meaningful dialogue in the mathematics classroom becomes apparent when mathematics education is considered from the latter perspective; as the enculturation of students into the mathematical discourse and reasoning of mathematically literate adults.

The process of mathematical reasoning and discourse is defined from several perspectives (Arlo & Skovsmose, 2002; Ball, 2008; Lakatos, 1976; Polya, 1957), and the convergence of meaning is used to underscore the need for meaningful dialogue in the enculturation process. In this context, I consider some of the significant factors to be addressed in investigating the role of oral communication in teaching students to engage in mathematical reasoning and discourse.

To start, I review how the language of mathematics can be made more accessible to new students of mathematics or non-mathematicians, so they can view mathematics as a language that describes the world. This process of improving accessibility focuses on the role of spoken language in the evolution of mathematical understanding from the concrete to the abstract. The conversation is expanded to consider how meaningful classroom dialogue facilitates and supports this evolution of mathematical meaning.

Next, the role of teacher beliefs in the importance of language and communication in teaching mathematics is considered. The teacher’s crucial role in generating and facilitating
meaningful dialogue is discussed, from the perspective of what should be done, and what obstacles can arise. This leads to consideration of the role of teachers’ content knowledge, and knowledge of mathematics for teaching.

The final section looks at how to bring about sustainable change in teachers’ practice for facilitating meaningful dialogue in the mathematics classroom. Motivations for change, supportive environments, and sustainable forms of professional development are described in this section, with an emphasis on the need for long term, high quality professional development in bringing about sustainable change in teacher practice.

2.2 The Language of Mathematics

The language of mathematics is a highly efficient way of expressing ideas, which has been made increasingly efficient over the millennia by the most intelligent minds. As Skemp (1989) states “...mathematics may now be seen as one of the most powerful and adaptable mental tools man(kind) has made for its own use, collectively, over the centuries. (p. 26)”

As the result of the refined ideas of some of the most intelligent minds throughout human history, many mathematical expressions and algorithms in use today are far removed from the phenomena they describe. For the purposes of the new learner, a lot of the connections between the physical phenomena being described and the abstract symbols, ideas and algorithms used to depict the phenomena, have been lost. For example, 27 maple leaves can be counted individually \((1+1+1\ldots+1 = 27)\), where the student can see a direct correlation between the concrete objects and the notation used. The leaves can be packaged into groups of three, and the student can count the number of groups \((1,1,1) + (1,1,1)\ldots + (1,1,1) = 9\), then multiply \(9 \times 3 = 27\).
This notation is still close enough to the concrete elements being described for the early student of mathematics to follow, especially if the bridge of grouping and counting the groups is supplied. If the packages of three are grouped into bundles, each bundle containing three packages \([(1,1,1), (1,1,1), (1,1,1)] + [(1,1,1), (1,1,1), (1,1,1)] +[(1,1,1), (1,1,1), (1,1,1)],\) and the bundles are counted, we can connect the concrete to the more abstract notation of exponents: \(3^3 = 27.\) As this example shows, the efficiency of mathematical notation removes it from the concrete elements the notation describes.

Effective mathematics instruction must help students build these bridges if the language of mathematics is to be meaningful for them. The lack of such connecting bridges may be one of the main reasons many people fail to make sense of the math they learn, and may also be at the root of the lack of expectation is some societies that everyone will do mathematics (Mighton, 2003; NCTM, 2000; Skemp, 1989).

Since mathematics in the primary grades seems deceptively simple and people who teach it are not required to have much mathematical training, it should not be surprising that many do not possess the in-depth mathematical understanding reform practice requires (Ball, Thames & Phelps, 2008; Hill, 2010; Hill, Ball & Schilling, 2008). At the same time, their general education has provided significant experience in the language arts, so most elementary school teachers are good at language, and comfortable with their language skills. By cross referencing Palinscar’s (1984) techniques for teaching to read meaningfully, and Ball, Lewis and Thames (2008) strategies for engaging students in mathematical reasoning through classroom dialogue, it may be possible to harmonize teachers’ perspectives on teaching literacy and teaching mathematics.
As many elementary teachers are highly accomplished literacy instructors, contrasting these teaching strategies may help them better identify areas where they feel dissatisfaction about their own math instruction. Alternatively, it may reduce their anxiety about teaching mathematics if they perceive the objectives and strategies described by these authors to have similar intent and execution.

Ball, Lewis and Thames (2008) "...offer ways of laying out the mathematical work to be done (that) focus on who talks, what gets talked about, and how it gets talked about" (p. 41) through the practices they advocate to engage students in mathematical reasoning through classroom dialogue.

Both the reading and the math instructional processes described require that students make predictions, that students support their ideas with reasons, that student ideas be clarified and ultimately scrutinized by the group as a whole, and that students be made aware of their own learning processes (Donovan & Bransford, 2005). Although, in the early elementary grades, meta-cognition is beyond children’s psychological development (Piaget & Inhelder, 1969), their thinking can be made evident to the teacher through their constructions (Neményi, 2003, personal communication). This is one reason that the meaningful use of manipulatives is so important in the early elementary classroom.

2.3 Teacher Conceptions, Knowledge, and Instructional Practices

To promote the use of language skills in teaching mathematics, teachers must come to realize that meaningful mathematical work can and should be done orally for the most part in the elementary grades.

Language is as important to learning mathematics as it is to learning to read. …. Students’ developing communication skills can be used to organize and consolidate their mathematical thinking. Teachers should help students learn how
to talk about mathematics, to explain their answers, and to describe their strategies. (NCTM, 2000, p. 128)

In the early elementary grades children are just learning to write, and their most developed mode of communication is oral. Therefore it is important for them to talk about their mathematical understandings and ideas, to make the knowledge meaningful.

Because mathematics is so often conveyed in symbols, oral and written communication about mathematical ideas is not always recognized as an important part of mathematics education. Students do not necessarily talk about mathematics naturally; teachers need to help them learn how to do so. (NCTM, 2000, p. 60)

Having children talk about their mathematical ideas and understandings is a critical step in the emergence of mathematical meaning (Lampert & Cobb, 2003; Moschkovich, 1999; Walkerdine, 1988; Webb, 1991). In addition, it enhances student learning by sharing ideas openly and often. It gives teachers more efficient and more accurate access to students’ learning for the purpose of gauging student understanding. Such immediate awareness of student learning helps them modify their instruction to make it more responsive to students’ level of understanding. Awareness and immediate instructional response are both essential features of formative assessment (Donovan & Bransford, 2005; NCTE, 2013).

In order to increase the use of meaningful dialogue in their mathematics classrooms, however, it is not enough for teachers to adopt these beliefs. They must also feel comfortable enough about their own knowledge of the subject to understand, evaluate and incorporate students’ orally expressed ideas into the lesson in a manner that will help the lesson move forward. This means that their efficacy in facilitating classroom dialogue is contingent upon their content knowledge, their knowledge of how to teach mathematics, their beliefs in how students best learn mathematics (Beswick, 2007; Bray, 2011), and their knowledge of their students, as well as awareness of classroom interactions (Sherin, Jacobs & Phillip, 2011).
“Teaching mathematics well involves creating, enriching, maintaining, and adapting instruction to move toward mathematical goals, capture and sustain interest, and engage students in building mathematical understanding.” (NCTM, 2000, p. 18).

The concept of pedagogical content knowledge; specialized knowledge in the subject matter that teachers need to teach it well, was introduced by Schulman in 1986. Building on Schulman’s idea, Ball and colleagues (Ball, Thames & Phelps, 2008; Hill, 2010; Hill, Ball & Schilling, 2008) developed a framework for the content knowledge elementary math teachers need in the work of teaching. Mathematical knowledge for teaching includes the ability to judge relative difficulty of problems, so that the teacher can select the form of a problem most appropriate to his/her learners’ skill level. It includes the ability to explain the reasoning behind common algorithms and the knowledge of how to represent operations with manipulatives to clarify their meaning for students.

Using the instrument created by Ball et al. (2008) for eliciting content knowledge for teaching, Copur-Gencturk (2015) has investigated the relationship between teachers’ content knowledge for teaching and their instructional practices. The study concentrated on math specific aspects of instruction, such as choice of task and questioning strategies. Participants were enrolled in a two and a half year master of education degree program that included courses in mathematics, science and educational theory over the course of the study, so investigators had a means of gauging participant learning. The study found, that teachers who noticeably increased their content knowledge for teaching, made instructional changes to increase their students’ conceptual understanding.

The results suggest that gains in teachers’ mathematical knowledge corresponded to the quality of their inquiry-based lessons, mathematical sense-making agenda, and classroom climate. (Copur-Genctruk, 2015, p. 297)
One of the teachers who chose to teach for greater understanding made an interesting comparison to teaching literacy. In her explanation for why she chose to teach for greater understanding, the participant emphasized the importance of sense-making in teaching reading, but felt that the emphasis on sense-making in teaching mathematics was often lacking:

In reading, you are always saying, “Think about what you are reading. You are not just reading the words. If you just read the words, then it is not really reading.” You can read really fluently, and if you are not thinking about it, then you are not really reading. Okay? But – but we never tell kids to think in math. (Copur-Gencturk, 2015, p. 304)

Teachers who completed the program with limited content knowledge for teaching tended to alter the structure of their instruction by including hands-on activities or more group work. However, they did so without improving their teaching in terms of working to enhance their students’ conceptual understanding.

2.3.1 Using Classroom Dialogue to Teach Mathematical Reasoning

Effective manipulative use and explanations can constitute isolated instances of mathematics content delivery, if instruction in the fundamental glue of mathematical reasoning is omitted or ignored (Lampert & Cobb, 2003). “Mathematical reasoning is the foundation for the construction of mathematical knowledge” (Ball, Lewis & Thames, 2008, p. 41), and classroom dialogue is one of the most efficient ways of teaching reasoning.

With these three practices in focus (naming and using names, making and interpreting claims, and evaluating mathematical assertions) teachers can reconfigure classroom talk for the purpose of engaging students in mathematical work. (Ball, Lewis & Thames, 2008, p. 41)

Expressing ideas orally gives all members of the class simultaneous access to them, and the teacher, as facilitator, can ask for clarification, can facilitate reasoning, and he/she can keep the discussion on topic. These advantages of classroom dialogue make it a vital

The increased verbalization in the elementary classroom proposed here focuses on language use by students for the purpose of thinking about their own thinking. It advocates that students should articulate their mathematical ideas in their own words. The ideas they express should always be justified orally. The ideas may then need to be refined through questioning and revision, where other students in the group give feedback, as the class as a whole evaluates the validity of the mathematical ideas.

As the more advanced definitions of mathematical reasoning below show, these are the essential elements of mathematical reasoning – the basis of all fundamental mathematical work. Surely the ultimate aim of reform mathematics, is for student to develop these thought processes.

In defining classroom dialogue for learning mathematics, Arlo and Skovsmose (2002):

introduce the notion of the dialogic act. Such an act involves making an inquiry, running a risk and maintaining equality…..getting in contact, locating, identifying, advocating, thinking aloud, reformulating, challenging, and evaluating – all exemplify dialogic acts. (p. 16)

The dialogic acts delineated by Arlo and Skovsmose (2002) are consistent with elements that constitute the pattern of mathematical reasoning, more formally identified by Lakatos (1976):

(1) primitive conjecture; (2) proof (‘a rough thought experiment or argument, decomposing the primitive conjecture into sub-conjectures or lemmas’); (3) global counter-conjectures; and (4) proof re-examined: the ‘guilty lemma’ to which the global counterexample in a ‘local’ counterexample is spotted. (p. 127)

Looking at these descriptions of mathematically meaningful classroom dialogue at different levels of instruction; elementary school (Ball, 2008), high school (Arlo & Skovsmore, 2002), and advanced mathematics (Lakatos, 1976), it becomes apparent that the
authors quoted all focus on the elements that constitute mathematical reasoning. There is an evolutionary process from beginner to accomplished mathematician, but the elements of reasoning are present throughout, and as Ball’s work (2008) suggests, the seeds must be planted in the early elementary grades.

2.3.2 Classroom Dialogue as Reform Practice

Mathematical reasoning is at the heart of communicating about mathematical ideas at all levels of instruction (Ball et al., 2008; Polya, 1962; Lakatos, 1976). As Lampert and Cobb (2003) emphasize, oral communication plays an essential role in the development of mathematical reasoning, particularly in the early grades:

If school lessons are to involve learners doing mathematical work, classrooms will not be silent places where each learner is privately engaged with ideas. If students are to engage in mathematical argumentation and produce mathematical evidence, they will need to talk or write in ways that expose their reasoning to one another and to their teacher. These activities are about communication and the use of language. (p. 237)

In learning to communicate mathematical ideas, students are learning to make connections that elevate their understanding beyond the learning of isolated mechanical skills to be used for specific applications. At the highest level of understanding, they are learning to participate in established mathematical practices. Therefore, embracing the language/communications approach to mathematics education constitutes a very different idea of what it means to learn mathematics than was traditionally accepted. “Classrooms in which mathematical communication and language development are focal are not typical” (Lampert & Cobb, 2003, p. 246).

Furthermore, if learning to communicate mathematical ideas is closely tied into developing mathematical understanding, then the types of communication students and
teachers can engage in are defined by their level of mathematical understandings. It follows that students develop more sophisticated mathematical understandings as they learn to communicate their reasoning (Simon & Blume, 1996). As students can only learn from the opportunities they are afforded, the kinds of mathematical conversations teachers orchestrate in their classrooms play a determining role in developing student understanding.

In an effort to map teachers’ evolving reform practice, Munter (2014) has developed rubrics to quantify aspects of teachers’ Visions of High Quality Math Instruction (VHQMI). The rubric pertaining to classroom discourse he applies to participant interviews identifies four sub-dimensions within the patterns and structure of classroom discourse:

Responses at the lowest level (Level 1) suggest a one-way, teacher-to-student pattern of talk such as the initiation-response-evaluation (IRE) pattern described by Mehan (1979). Other levels of this code are used when the participant indicates that talk should not be limited to a teacher-to-student pattern (Level 1) but should occur between students (Level 2), whether that is in small groups only (Level 2) or in whole-class settings (Level 3) and whether the teacher’s role in such conversations is central (Level 3) or decentralized (Level 4). (p. 603)

Munter’s (2014) rubric attempts to quantify how teachers’ ability to orchestrate classroom dialogue changes as their reform practice advances. It is a transition from the central role of teacher as repository of all knowledge in the traditional classroom, to that of teacher as facilitator of student driven argumentation in the reform classroom.

The dates of many of the references in this section show that the math education community has been aware for some time that meaningful dialogue in the mathematics classroom is important for creating meaning as mathematical understandings emerge. The following paragraphs show why dialogue is important, how it helps meaning emerge, and identifies important parameters of student talk. However, if teachers are unaware of this work, how can they advance their practice purposefully?
Web (1991) found that students who gave more elaborate explanations of mathematical reasoning seemed to do better. Hiebert and Wearne (1993) proposed that the quality of teacher questions also mattered. Higher level teacher questions, asking students to explain and analyze, seem to correlate with higher achievement. In Cobb's (1995) analysis of what makes small group relationships productive in a classroom culture where mathematical reasoning and communication were valued, he found two key elements: (1) students must develop an adequate basis for mathematical communication, and (2) students must establish a relatively symmetrical relationship, where no student is considered a mathematical authority. The lack of authority ensures that explanations are supported by reasoning, rather than simply the re-telling of what is already accepted as known.

Forman (1996) views mathematical learning as "an apprenticeship into the discourse and reasoning practices of mathematically literate adults" (p. 116), and refers to the specialized speech used in mathematical discourse as mathematics register. When she compared traditional classrooms with small group, problem solving approaches, she found that students had more opportunities to learn to use the mathematics register in reform classrooms.

It is important to note that mathematics register is not learned as a separate language, but develops as an integral part of everyday speech in effective classroom discussions. The participative and evolutionary process of acquiring mathematics register, then, depends on plentiful effective classroom dialogue. Students need opportunities to connect language, then mathematical symbols, to evolving meaning, so that their participation in mathematical reasoning can also evolve. The evolving, integral process of acquiring mathematics register
is consistent with the idea that in order to successfully apply newly acquired knowledge, it needs to be integrated into one's existing schema (Skemp, 1989).

When mathematical learning is viewed as participation in established mathematical practices rather than the acquisition of skills for specific purposes, attention becomes focused on the mathematical elements of classroom dialogue. The teacher's role as facilitator and supporter of mathematically productive discussions then becomes pivotal. When mathematical learning is viewed as participation in established mathematical practices, individual learning is seen as a function of students' developing mathematical argumentation in social interactions. This approach:

see(s) talking and writing to be aspects of doing mathematics and regard(s) the classroom as a community of learners, led by the teacher, in which learners are socialized to accept new norms of interaction and learn new meanings for mathematical words and symbols as they work together on problems. (Lampert & Cobb, 2003, p. 240)

In order to relate meaningful mathematical talk to learning, we need to consider the kinds of curriculum and instruction that will support such talk, and we need to see "talk" itself as a reflection of the learning that has been mediated by the social and cultural context.

2.3.3 Teaching Mathematics Through Inquiry – Providing Opportunities for Meaningful Mathematical Dialogue

The approach known as ‘teaching mathematics through inquiry’ is open to interpretation and can be implemented in a variety of ways, depending on teacher perceptions and beliefs. The participants in this inquiry interpreted it as problem based instruction where students work in pairs or small groups, on tasks that are intended to generate mathematical ideas and discussions. The choice of task by the teacher, and the way in which he/she facilitates classroom discussion around the task, determine what opportunities to learn
students are afforded. The curriculum that prompts high-level discussions is based on mathematical tasks of high cognitive demand (Stein, Grover & Henningsen, 1996) with several entry points. Such complex tasks are often introduced in the context of the three-part lesson (van de Walle, Folk, Karp, & Bay-Williams, 2010). In the three-part lesson, the task is introduced, students work on solving it, and then the teacher orchestrates a whole class discussion where students articulate their mathematical reasoning and connect their solutions to mathematical ideas and representations (Franke, Kazemi & Battey, 2007; Hiebert at al., 1997).

Tasks of high cognitive demand afford students the most opportunities to learn mathematics as participation in established mathematical practices; reasoning, justification and generalization. Low cognitive demand tasks teach students to memorize facts and solve problems mechanically. Students learn to solve routine problems using demonstrated procedures, but the connections between the solutions and relevant mathematical ideas evade them. Hiebert et al. (2003; 2005) have found that mathematical activity in U.S. middle school classrooms tends to be procedural, and even when high level cognitive demand tasks are selected, teachers often implement them in low-level ways (Stein & Kaufman, 2010). In studying mathematics teachers’ enactment of cognitively demanding tasks, Wilhelm (2014) found that “teachers’ mathematical knowledge for teaching and conceptions of teaching and learning mathematics were contingent on one another and significantly related to teachers’ enactment of cognitively demanding tasks” (p. 637).

It appears that the cognitive demand of a high level task was more likely to be maintained by teachers with more advanced knowledge for teaching and more sophisticated conceptions of teaching /learning mathematics. However, Wilhelm also found that teachers
in her study with more teaching experience, were more likely to pose a low level task over a task of high cognitive demand. She saw this as consistent with Remillard and Bryan’s (2004) findings that more experienced teachers were less likely to implement new reform curricula in the intended ways.

Jackson et al. (2013) found that task set up by the teacher has a significant impact on task accessibility for students, and required math talk in the set up, to ensure meaningful mathematical discussions in the third, debriefing part of the lesson:

We focus on the first phase of a lesson, in particular the practice of setting up complex tasks so that all students are able to productively engage in solving the task and thereby participate in and learn from the third phase of the lesson, the concluding whole-class discussion. (p. 648)

A high quality task set-up solicits detailed input from many students so the teacher can assess how well students understand key features of the task, and determine what kinds of supports students need for meaningful engagement (Boaler, 2002). It also provides an opportunity to develop common language for describing key task features, mathematical ideas and relationships. Students’ use of common language in this context indicates that they have developed a taken as shared understanding (Cobb, Yackel & Wood, 1994), of the important features of the task. Teachers can use different talk moves (Chapin et al., 2003) to support students in developing common language. He/she can re-voice student language, ask students to restate ideas in their own words, ask students to add onto peers’ ideas, and mark certain ideas as important.

**2.3.4 The Emergence of Meaning Through Classroom Dialogue**

When visual representations are used to help establish a key idea, it is essential that students have the opportunity to develop the common language that describes the
representation. According to Moschkovich (1999): “objects do not provide extra-linguistic clues. The objects and their meanings are not separate from language, but rather acquire meaning through being talked about and these meanings are negotiated through talk” (p. 14).

The role of language in the emergence of mathematical meaning was described more generally by Walkerdine (1988) in her paradigmatic analysis. Her analysis was so groundbreaking, because she showed that when instruction is based on activities with material objects and actions, mathematical understanding does not arise directly from the physical manipulations, as commonly assumed. According to her analysis, mathematization (the process of coming to see a concrete situation in mathematical terms) involves a series of subtle shifts in which both talk and writing play a crucial role.

In one of her key examples she analyzes the teacher-student interaction involved in teaching elementary addition. She identifies the following stages in the emergence of meaning when students are taught to add by manipulating two groups of blocks: (i) The concrete stage corresponds to the action of physically pushing two groups of blocks together, while talking about “putting them together”. (ii) By re-describing the action as “two and four make...” understanding shifts to the concrete/iconic stage. (iii) If a drawing of three circles is used to organize the action, so that the two sets of blocks are initially placed in separate circles, and then moved together into the third circle, the iconic stage has been reached. (iv) If the blocks themselves are then replaced with drawings of blocks in the circles, and the student can talk about them in the same terms, the mathematizing transition has reached the iconic/abstract stage.

As Walkerdine's (1988) analysis revealed, the emergence of mathematical understanding is very dependent on the talk associated with the manipulation of physical
objects, because it is the talk that helps make sense of the actions. So the process of mathematization as described by Walkerdine, involves (i) learning how to use words that go with sets of objects, and (ii) learning how to use words that go with actions on those objects (putting together, taking away, sharing equally, etc.) In more general terms, the mathematization process involves learning a series of symbols, how to act on the symbols, and how to talk about those actions.

In the early stages of the mathematization process, instructional activities focus on helping students see and talk about material objects, actions and events in mathematical terms. The mathematical ideas to be discussed soon become abstract, so that only experiences and actions remain concrete. There are no physical objects that directly correspond to the concepts. In Walkerdine’s example, the blocks and groups of blocks can be pointed to directly, when discussed as physical objects.

However, the abstract idea of the number of blocks, or the mental action of adding numbers cannot be pointed to directly. Sfard (2000) describes the world of mathematical ideas as a virtual reality, to distinguish it from the world of physical objects. Walkerdine’s work shows that Sfard’s ‘virtual reality’ does not spring forth naturally from physical reality. Indeed, effective mathematics teachers need to provide bridging connections between students’ concrete world and even the most elementary of abstract math concepts. For an example of a current application, see Pace and Ortiz (2015).

During these bridging activities, oral and written communications play an essential role in helping make sense of the activities as mathematics, and bringing about understanding. If meaning is not well established during the early years, in later years, functional language analysis (Fang & Schleppegrell, 2008) can be used:
to offer teachers explicit ways to focus on language itself to help students comprehend and critique the advanced texts of secondary reading.\(p. \text{iii}\)

...Solving a word problem involves both linguistic knowledge and mathematical knowledge...

Mathematics texts are often challenging to read, and functional language analysis goes beyond key words to help students recognize how language works together with mathematical symbols, equations and graphics to construct mathematical meanings. (p. 83)

However, at the beginning stages of learning mathematics, students build on their informal understandings, and must use everyday language to describe their mathematical ideas about the activities they engage with. Cobb et al. (1997) have looked at the gradual emergence of mathematical meaning through the lens of reflective discourse. According to these authors, classroom discourse that helps student thinking move from the concrete through the iconic to the abstract has specific characteristics. It typically consists of repeated shifts between what teacher and students say and do, and their subsequent, explicit discussion of that talk and action.

The structure of reflective discourse as described by Cobb et al. (1997) is comparable to psychological accounts of mathematical development (Dubinsky, 1991; Freudenthal, 1983; Gray & Tall, 1994; Pirie & Kieren, 1994; Sfard, 1991). These authors describe the structure of mathematical development as actions or processes that are transformed into mental mathematical objects. This is an iterative process where previous mental manipulations become the object of discussion.

Talking about mathematics in this way might give rise to opportunities for students to learn by reflecting on and objectifying prior activity...as a consequence...students develop what might be termed a mathematizing attitude that involves organizing the results of prior mathematical reasoning by searching for patterns. (Lampert & Cobb, 2003, p. 241)

An example would be the evolving use of a table of values in the following example. The table might initially be used to record numerical data. Recording the data in a table
organizes it, so in a later activity, the table might be used to look for patterns in the tabulated data, and subsequently, to advance one's reasoning. This example shows the central role written symbols play in moving reflective discourse from concrete to abstract.

The significant role of written symbols in generating reflective dialogue is consistent with Gravemeijer's (1997) general analysis of the connecting role symbols play in moving mathematical understanding from concrete, informal mathematical activity, to abstract mathematical reasoning. It is also compatible with Dörfler's (2000) discussion of symbolic records as a support for the development of abstract mathematical reasoning with understanding.

The process of mathematization must be allowed to develop at each student's individual pace. If the teacher or curriculum pushes for mathematization too fast, students often abandon reasoning in favor of rote learning. The latter is faster and easier in the short term. However, once students’ reasoning is separated from their concrete understanding, the teacher’s attempts to engage them in reflective discourse frequently degenerates into a guessing game (of what does the teacher want to hear?) That is why teachers need to continually assess whether students are able to reflect on their mathematical activity. To aid in this assessment, students might be asked to apply the mathematical relationship being discussed to a concrete example. For instance, students might be asked to make up a word problem that corresponds to a mathematical sentence.

The idea for this type of formative assessment is derived from Pirie and Kieren's (1994) theory of the growth of mathematical understanding, and is used in McClain & Cobb's (1998) notion of 'folding back discourse'. Students’ ability to apply mathematical relationships to concrete examples helps develop understanding in two ways. It grounds
students' increasingly abstract mathematical reasoning, and it enhances their abilities to communicate that reasoning more effectively.

   This is important, because beginning students often find it very difficult to see beyond the concrete details of the exercise to elicit the mathematical relationships (McNair, 1998). To help students develop sufficient understanding to relate the concrete details of the situation to the mathematical relationships, teachers need to talk about operations in mathematical relationships in ways that connect them to applications. This type of teacher talk helps students start to understand mathematical ideas that go beyond the concrete details of the specific example (Stevens & Hall, 1998).

   As students move from informal understanding to formal mathematics where the learning is facilitated by classroom dialogue, confusion can arise when informal definitions are mixed with formal ones. The advantage of using informal language as understanding is being constructed, is that it relates students more closely to the ideas being discussed. Problems can arise, because local classroom practices may be unintelligible to people not familiar with them, and may eventually cause confusion even for the students using local terminology.

   In classrooms that attempt to support ‘real’ inquiry in math and science, ...it is sometimes necessary to develop a working definition of some phenomenon...that will be changed as understanding increases. In such classrooms, the development of one's own symbols, measures, and terminology is often required, with complex and poorly understood difficulties often arising. (O'Connor, 1998, p. 43)

   When student-negotiated terminology is used to promote students’ sense-making, teachers sometimes face the dilemma of managing their own contradictory goals. This kind of difficulty is illustrated in Schoenfeld’s (2008) analysis of Ball’s decision-making during the '0 is odd and even' lesson:
Ball needs to know what her students believe about the evenness conjecture if they are to be working on related conjectures later in the day. (Should she) let Nathan's observation pass and reorient the class to her agenda of commenting on the previous day’s meeting, (or should she) pursue the central mathematical issue raised by Nathan: whether six is even, odd, or ‘special’. (p. 74)

In order to facilitate mathematically meaningful dialogue in the classroom, teachers need to give simultaneous attention to mathematical content and social processes. Lampert’s (1990; 2001) research on pedagogical design seems to successfully bring both elements together. She incorporated the elements of mathematical reasoning as defined by Pólya (1962) and Lakatos (1976) into lesson design, and organized whole-class discussions around student solutions. She referred to their different answers as ‘conjectures’ until the class as a whole checked and validated the various solution strategies. During these discussions, the mathematical importance of interpretations, assumptions, and conditions was made visible to students, and they were pushed to revise definitions within their own terms of reference.

Lampert’s (1991) achievements in this context are made more remarkable by the fact that she was working with students who were conditioned to expect procedural instruction. To them, learning math was about practicing problems for which the teacher had shown them the solution strategy. Such student expectations would make it even more difficult to slow down instruction enough to open up discourse so that students could participate in evaluating their own thinking.

Her work shows the teacher's role in engaging students in mathematical reasoning as an on-going process of reasserting mathematical norms and values through negotiated meaning (Voigt, 1995; Yackel & Cobb, 1996). Throughout the course of instruction, she repeatedly transferred responsibility to students for ensuring that their work made mathematical sense (Blunk, 1998). She also reinforced genre (Rittenhouse, 1998) and instilled respect in students for each other's ideas (Weingard, 1998).
2.3.5 Important Elements in Facilitating Whole-Class Discussions

The idea of the three-part lesson as a frequently used strategy for the implementation of complex mathematical tasks was introduced in a previous section. The significant role of language in the initial set-up of the mathematical task was also discussed. Another important consideration in the effectiveness of the lesson both as mathematics, and as an instrument for generating meaningful mathematical talk, is the teacher’s ability to maintain the cognitive demand of the task throughout the lesson. The task will afford students the most opportunity to learn if its cognitive demand is maintained in presenting the task, during its implementation, and right through to the end of the debriefing class discussion.

The cognitive demand of the task can be lowered at any stage, and in any number of ways (Henningsen & Stein, 1997; Stein et al., 1996). For instance, the way the teacher clarifies or even alters the expectations of the original task (Wilhelm, 2014) can completely alter the cognitive demand. If the teacher’s expectations are unclear to students, or classroom norms are unproductive, it may be impossible to maintain cognitive demand. If the task is inappropriate for student capabilities, or the teacher provides insufficient scaffolding, students may flounder, and the teacher may feel it necessary to reduce the cognitive demand to allow students to make some headway with the task.

In the 3-part lesson format much of the teaching is done in the third part of the lesson, during the teacher orchestrated whole-class discussion; the debriefing of the task. In an effective lesson, this discussion must accomplish two very important mathematical objectives. It has to clarify for students how their various solution strategies are related, and connect them to the key mathematical ideas represented by the task (Smith & Stein, 2011).
In a reform setting the discussion must also be driven by student ideas, and progress toward the goals of the lesson.

To orchestrate a meaningful discussion that accomplishes these aims, the teacher must carefully observe and make notes on student activity during the second part of the lesson, when students work on the problem in small groups or partners. Based on his/her notes and observations, the teacher then selects the student solutions to be featured. He/she orders them into a sequence that will make the solutions accessible to most students and build toward the goals of the lesson (Smith & Stein, 2011). For example, the sequence might start with a simple, common strategy that most students would recognize, and proceed to more complex solutions through a logical progression that will facilitate the connection of the strategies by building on previous understandings.

During the student presentations, the teacher facilitates discussion to clarify mathematical relationships among the various solution strategies, and connect them to the key mathematical ideas underlying the task. 'Re-voicing' is a useful technique for keeping the conversation mathematical, and moving towards the teacher's mathematical goals during these debriefing discussions. In re-voicing, the teacher rephrases a student's statement to highlight aspects of the contribution that will clarify meaning, or context, or simply connect it to previous student contributions (Ball et al, 2008; O'Connor & Michaels, 1996). The teacher also elicits alternative solutions/explanations, and facilitates analysis of incorrect student contributions, using the ‘error’ to delve more deeply into the mathematics (Mousley & Sullivan, 1996).
2.4 Teacher Development

I have discussed the teacher’s role in facilitating students' induction into the practice of mathematics. In order to be able to facilitate students’ participation in this manner, teachers need to develop their own mathematical understanding in the same way (Copur-Gencturk, 2015; Wilhelm & Kim, 2015). They need to learn to participate in established mathematical practices. Learning to do mathematics in this way is a long-term endeavour that requires on-going support. That is why long term, high quality professional development for teachers is the most effective way to support the development of reform practice that has a measurable effect on student learning (Guskey, 2000; McMeeking et al., 2012). According to Loucks-Horsley and Matsumoto (1999), high quality professional development provides this kind of support, since it must include subject immersion with hands-on learning from professionals in the field. It must provide a curriculum of knowledge and materials teachers can use, must examine their practice in the classroom, and provide a system of collaboration centering on learning communities.

Indeed, research has shown that long term and consistent participation in high quality professional development activities does influence teachers' attitudes and their practice (Banilower et al., 2006; Garet et al., 2001). Teacher efficacy grows with content knowledge, practice, and experience (Ball, Lubienski & Mewborn, 2001). As teachers learn to employ meaningful activities, learn to create an atmosphere conducive to learning, and learn to gauge how much time students need to learn given concepts, they can have a positive impact on student learning (Lasley, Siedentop & Yinger, 2006).
2.4.1 Considerations in Achieving Sustainable Teacher Change

One of the driving forces behind sustainable teacher change is an internal motivation to learn more about one’s practice. Change usually happens when teachers identify a problem in their practice. According to Attard (2007), teachers should be committed to ongoing self-reflection and investigation in order to identify deficiencies in their practice. Day (1999) recommends that teachers reflect on how well their beliefs match their practices, because it is the recognition of dissonance between beliefs and practice that reveals the need for change.

Sustaining and spreading educational change is difficult (Hargreaves & Shirley, 2009). According to these authors, change needs to be driven by people working together towards a shared, following a coherent vision with the common purpose of improving student learning and achievement. They recommend that change be guided by “sustainable leadership” (p. 95) and promote professional cultures of trust, co-operation and responsibility, through networking with peers and the support of mentors.

An important aspect of sustaining teacher change is that the teacher must feel ownership over the change being implemented and the decisions that are driving it (Day, 1999). If the teacher feels that the change is imposed upon him/her, or has beliefs about teaching that are contrary to the proposed change, any changes made will be superficial and not sustainable. Only when learning is aimed to address teacher needs, will it be transformative and result in growth.

One reason that change in teaching practice is so hard to achieve, is that teachers who engage in changing their practice make themselves extremely vulnerable. The risk they take in pursuing change often results in discomfort (Attard, 2007; Reid & Zach, 2010a). It is
much easier to continue with a routine practice than to make changes that may not work out. However, collaboration and an atmosphere of trust can ease fears associated with change.

### 2.4.2 The Role of Collaboration and Dialogue

According to Hargreaves and Shirley (2009), “collaborative structures are strongly associated with increased student success and improved retention among new teachers.” (p. 92). While Attard (2007) is a strong advocate of reflection to instigate teacher change, he also suggests that discussions with colleagues can lead to change. Such teacher-to-teacher discussion can trigger new ideas and prompt teachers to consider new perspectives.

In her analysis of the central role of dialogue in teacher professional development, Penlington (2008) points out specific components of dialogue that are necessary to bring about teacher change. In order to effectively foster teacher change, she found that teacher-to-teacher dialogue must push participants to confront subliminal and hidden ideas. Internal, reflective dialogue allows one to access ideas that are already accepted or familiar. Only when we engage in dialogue with others, do we gain access to ideas that are new to us, and alternative perspectives.

However, learning based on debate and dissonance is not usually supported by our social mores. It is more usual for educators to share experiences in ways that build a culture of agreement (Males, Otten, & Herbal-Eisenmann, 2010; Penlington, 2008). To promote productive dissonance, therefore, schools need to develop a culture where disagreement is not considered a personal attack, but necessary for advancing teaching practice. It is dissonance that pushes us to examine, discuss and change our assumptions about teaching and learning. Penlington (2008) also advocates for a strong relationship of trust and confidence between teachers, that is able to support the critical questioning productive
dissonance requires. Such a relationship is built up over time, as repeated discussions with
the same people are more likely to build trust. After a short period of interdependence, these
critical discussions should foster independence, as the practitioner develops autonomy and
self-confidence (Day, 1999).

Based on their study of professional development structures, Mesler-Parise and
Spillane (2010) report that the best predictor of teacher change is teacher engagement in
collaborative, subject specific professional development opportunities. Long-term
collaborative efforts are consistent with Penlington’s (2008) contention that effective
discussions occur when participants have the opportunity to develop trust, and feel safe in
raising multiple points of view.

The above discussion has focused on important aspects of professional development
for teaching mathematics. It is telling that the determining elements for professional
development efforts that work in teaching literacy are essentially the same. The 2013
Report: Remodeling Literacy Learning Together, published by the National Centre for
Literacy Education, indicated that “educators’ most powerful professional learning
experiences came from working with their colleagues”. Some of the key findings of the
report include:

- Teachers feel ill-prepared to help their students achieve the new literacy standards
- Working with peers is the most valued support for standards implementation
- Positive changes are occurring most where teachers are actively involved in the
  renovation
- Teachers feeling most comfortable tend to be those more frequently working with others
to analyze student work, design curriculum, and create assessments
• Teachers engaged in cross-discipline conversation about literacy are making greater shifts in their instruction.

Based on these findings, the report makes three recommendations, which can be seen as applicable to improving professional development efforts in any field:

• Provide educators with more shared time for planning and professional learning about elevating (literacy) learning for all students,
• Encourage and support educators to take initiative in designing and using innovative (literacy) teaching resources that are appropriate for their students, and not rely on prepackaged programs or solutions, and
• Draw upon the insights, skills, and experience of everyone with a stake in improving (literacy) learning to help students achieve more.

Cross-disciplinary efforts between math and literacy educators are not new. “Math Is Language Too” (Whitin & Whitin, 2000), was published jointly by the National Council of Teachers of Mathematics (NCTM) and the National Council of Teachers of English (NCTE) and written “to demonstrate the common beliefs about learning that cut across the fields of mathematics and language” (p. 8). The authors advocate for both the oral and written communication of ideas to advance student understanding, and promote a “common emphasis on inquiry (in all disciplines, that) supports children to use ideas from one subject area to explain their work in another” (p. 11).

2.4.3 Model Lessons

Many teachers report that seeing an effective demonstration, in real classroom situations, of the techniques they are supposed to learn, would be inspirational. Hargreaves
and Shirley (2009) found that demonstration is an effective way to support teachers’ professional development. Grierson and Gallagher (2009) studied demonstration classrooms in an Ontario School board. In reference to a literacy lesson demonstrated by an expert teacher, they found that teachers who observed the demonstration made changes in their classroom practices. The authors report that observing a real teacher implement the reform pedagogy in a real classroom in their own school board, changed teachers perspective; it made them feel more positive about the reasonableness of implementing the changes in their own classrooms. Grierson and Gallagher (2009) report that, after observing the lesson and debriefing with the demonstration teacher, the participants in the study made changes to the physical set up in their classroom, their literacy center, how they conducted guided reading, and how they assessed student learning. The teachers also reported that the experience motivated them to engage in more self-reflection and to set goals for their own practice.

The investigators pointed to several conditions that made this kind of professional development conducive to teacher change. The learning occurred in a safe environment where risk-taking was encouraged. Participants were at least acquainted with the practices modeled by the demonstration teacher, because they had encountered them in other professional development sessions, and they were all recommended by the Ontario government. In addition, the demonstration teacher provided support after the demonstration by means of e-mail. Additional follow-up support would have been appreciated by participants as they worked to implement the proposed changes. Two participants reported that they abandoned the changes they had initiated, because they found them too difficult to manage without more follow-up support.
2.5 Summary

The process of mathematical reasoning is defined in essentially the same way from various perspectives. Pólya (1957) uses a math educator’s perspective, Lakatos (1976) defines a mathematician’s perspective, Arlo and Skovsmose (2002) talk in terms of high school classroom dialogue, and Ball, Lewis and Thames (2008) approach from a primary teaching perspective. The convergence of these perspectives can be seen as a general agreement on what to teach. The question of how to teach it is considered from Forman’s (1996) perspective. She conceptualizes mathematics education as learning to participate in mathematical discourse and reasoning practices in the manner of mathematically literate adults. The commonality of the definitions of mathematical reasoning and Forman’s perspective on how to teach it, make the need for effective oral communication in the mathematics classroom apparent.

To see what constitutes mathematically meaningful classroom dialogue and how it can be facilitated, various contributing factors are considered. The role of language in the emergence of mathematical meaning (Cobb, 1997; Sfard, 2000; Walkerdine, 1988) is described. Teacher beliefs that language and communication in teaching mathematics is important (Cobb, 1994; NCTM, 2000), and the role of teachers’ mathematics knowledge for teaching (Ball, et al., 2008a; Hill, Ball & Schilling, 2008; Hill, 2010; Copur-Gencturk, 2015) are considered. These theoretical perspectives are then situated in the context of best practices.

Teaching mathematics through inquiry requires the use of complex tasks with multiple entry points that afford students most opportunities for quality mathematical discourse (Lampert & Cobb, 2003). This type of problem based instruction is implemented
using the format of the three part lesson (van de Walle, Folk, Karp, & Bay-Williams, 2010). The role of math talk in setting up the task (Jackson et al., 2013) and maintaining the cognitive demand of the task throughout the lesson (Wilhelm, 2014) are emphasized. The teacher's targeted activities required for the successful debriefing the lesson (Smith & Stein, 2011) are discussed.

The final section considers how to bring about sustainable teacher change in teachers’ practice for facilitating meaningful dialogue in the mathematics classroom. Motivations for change, supportive environments, and forms of professional development are described. The need for long term, high quality professional development (Banilower et al., 2006; McMeeking, 2012) is rationalized, with particular attention to the role of teacher-to-teacher dialogue (Penlington, 2008), and collaborative discussion (Melser Parise & Spillane, 2010). Collaborative Inquiry based professional development strategies advocated by NCTE (2013) for teaching literacy are included to show the strong parallels in objectives, teacher perspectives, and success criteria in achieving professional competence in the two disciplines.
Chapter 3 – Methodology and Methods

This chapter provides the rationale for using a qualitative approach in carrying out this inquiry, and organizing the inquiry into case studies. It outlines my role as a researcher, gives an overview of the research setting, the design of the inquiry, methods of data collection, data analysis procedures, ethical issues, and limitations of the research.

3.1 Research Design

In studying any aspect of teaching and/or the evolution in the practice of individual teachers, qualitative methods are thought to best elicit the rich information in context needed for understanding the phenomena being studied. By studying an evolving, “functioning body” (e.g., a child, a classroom of children, an event) (Strake, 2004, p. 444), one defines a ‘case’. Each case is a bounded unit, an entity. In multi-case research, the cases need to be similar in some ways, but the individual case must command most of the researcher’s attention during work on that case (Strake, 2005).

A case may be studied qualitatively, quantitatively or by mixed methods, but a case study is intensive, concentrating on the specifics of the case:

case studies comprise more detail, richness, completeness, and variance – that is depth – for the unit of study… case studies stress ‘developmental factors’, meaning that a case typically evolves in time, often as a string of concrete and interrelated events…case studies focus on ‘relation to environment’, that is, context. (Flyvbjerg, 2011, p. 301)

Although the case study defines the unit of study rather than the methods by which the phenomena are studied, its concentration on detailed, contextual investigation produces the kind of data that result in meaningful understanding:
the case study produces the type of concrete, context-dependent knowledge that research on learning shows to be necessary to allow people to develop from rule-based beginners to virtuoso experts. (Flyvbjerg, 2011, p. 302)

Carefully chosen experiments, cases and experience can add much to the understanding of specific phenomena. Traditionally qualitative research has concentrated on understanding specific phenomena in context, rather than striving for generalizations:

Discovering universals or laws of human behavior is not a fundamental goal of interpretivist research. Interpretivism eschews the idea of ever finding absolute and universal truth. Instead, it strives for local truth, hermeneutic understanding, multiple perspectives, phronetic knowledge, or some other form of limited, context bound, and conditional knowledge. (Willis, 2008, p. 222)

There are those who argue that carefully chosen cases and experience can bring about understanding that contributes to scientific development. George and Bennett (2005) have demonstrated strong links between case studies and theory development, especially through the study of deviant cases.

According to Flyvbjerg (2011), there is an emerging trend in research where scholars trained in both qualitative and quantitative methods are allowing the problems to drive the research rather than the methodology. These researchers choose to employ the methods that are best suited to answering the research questions at hand. Having argued that the case study is the best methodology to use for organizing the current project, I will now advance the arguments for employing qualitative methods.

Effective teaching must satisfy three objectives simultaneously. On the one hand, it has to communicate the prescribed curriculum to learners in a manner that is meaningful to them. At the same time, it must comply with the goals of the curriculum writers, and finally, it must remain true and accurate to the knowledge base.
As fundamental as curriculum is, no curriculum teaches itself. The curriculum is always mediated. It is in the description and improvement of teaching that the arts have a special contribution to make. (Eisner, 1991, p. 43)

Like the language interpreter, the teacher has to first understand the concepts accurately for him/herself, then translate them into forms of communication that will be understandable and meaningful to students. According to Eisner (1991), this process constitutes an art form:

Teaching is artistic in character in many of the ways in which all art is artistic: it provides a deep sense of aesthetic experience to both perceiver and actor when it is well done (Eisner 1982). It requires the teacher to pay attention to qualitative nuance – tone of voice, the comportment of students, the pervasive quality of the teaching episode. It requires the teacher to attend to matters of composition in order to give the day or lesson coherence. It often requires flexibility in aims and the ability to exploit unforeseen opportunities in order to achieve aims that could not have been conceptualized beforehand. (p. 44)

As Eisner explains, teaching, when effectively done, is nuance dependent and must be flexible to 'exploit unforeseen opportunities'. Only qualitative research methods are responsive enough to elucidate, record, recognize and interpret such delicate and intricate processes. With qualitative research methods, we can explore classroom events and norms; how students and teachers construct their world and make sense of it. We can elicit and record teachers' beliefs and their instructional practices. In the process, we may be able to shed light on key elements for facilitating classroom dialogue that promotes reasoning. In exploring teachers’ perspectives on specific dialogue promoting strategies in teaching literacy as well as teaching mathematics, we may be able to probe more deeply into teachers’ experiences with math reform initiatives.

By contrasting the language strategies they use, why they use them and how they use them in literacy and mathematics, teachers may be better able to identify areas where they still feel dissatisfaction about their math instruction. Qualitative research methods seem best
suited to gather the nuanced detail necessary to give a deep understanding of teachers’ perspectives in these respects, and thereby help answer the research questions that shape this inquiry.

The answers to the research questions must be constructed from information gained through interviews, note-taking, document analysis, log books and classroom observation over an extended period. “Three data-gathering techniques dominate in qualitative inquiry: participant observation, interviewing, and document collection” (Glesne, 1999, p. 31). These methods were used to document teachers’ perceived successes and failures and provide the rich detail necessary to answer the research questions.

Though the study concentrates on the beliefs and practices of specific teachers in specific classrooms, the environment in which they are situated – their school, colleagues and superiors will likely exert considerable influence on their successes and motivations. The understandings gained about language use and classroom dialogue in these circumstances can provide useful information both for the participants and teacher educators.

3.2 Participants

Since qualitative research is about collecting rich detail of a particular instance rather than a statistically significant sample, purposeful sampling seemed appropriate for this inquiry (Flyvbjerg, 2011; Glesne, 1999; Patton, 1990). “Maximum variation sampling selects cases that cut across some range of variation… (and) searches for common patterns across great variation” (Glesne, 1999, p. 29).
3.2.1 Criteria for Selection

In considering the desirable characteristics of research participants, two different types of candidates seemed suitable. Teachers with significant training in reform methods would be more likely to have sufficient confidence and skill to implement classroom dialogue as a means of doing “fundamental mathematical work” (Ball, 2008). Therefore, these types of participants would be most likely to have sufficiently advanced reform practice to develop students’ mathematical reasoning through classroom discourse. They will also have some basis for comparison when they reflect upon their practices in teaching literacy and teaching mathematics.

The other type of suitable participant would have considerable literacy training. A candidate with this background would provide an ideal perspective to investigate how quickly and effectively teachers can advance their mathematical practice by means of the proposed reflect and contrast strategy. Their strong background and their extensive repertoire in communication and reasoning strategies they commonly use in teaching literacy would provide significant information for comparison with their emerging mathematics practice. As participants identify areas of dissatisfaction with their mathematics practice more clearly by means of this contrasting process, they may also discover that some of the strategies they already use could be applied more frequently in a mathematics context.

Since participants represent a wide range of experience and perspectives on reform practice, they need to show a willingness to explore and promote the use of oral communication in their mathematics classrooms. Regardless of their current level of reform practice, potential participants should want to improve upon it. They should be open to implementing oral communication strategies that develop students’ mathematical reasoning.
Desirable pre-dispositions include the attitudes that mathematics is about making sense, that it is best learned through participation in mathematical reasoning and carried out as a group activity. When students learn mathematics as participation in reasoning, the group as a whole must decide on the validity of mathematical claims. These attitudes and practices are best adopted from the beginning of school mathematics.

All participants were in learning mode, as the entire staff of Haven P.S. was pursuing a goal of ‘collaborative inquiry in mathematics’ with a strong emphasis on communicating mathematical ideas. This was a Board initiated goal, and facilitated by the school Principal in many ways. She organized the staff into teaching partners by grade level, had the partners develop criteria for achieving the stated goals at their own grade level, and encouraged them to work together on lesson development and implementation. The Principal’s own interest in, and experience with mathematics education research, aided her efforts to develop staff expertise through these organizational measures (Principal Interview).

Participants were all working on teaching math through inquiry and facilitating communication of mathematical ideas in their classrooms. They realized that, in the primary grades, the most fluid mode of communication was oral. Therefore, many of their teaching strategies were language based. In grade 3, they regularly had students discuss ideas in small groups and report on small group activity. They expected oral explanations and worked hard to create a classroom atmosphere where students would willingly voice their ideas as an accepted and expected part of classroom participation.

3.2.2 Selection of Participants

Teacher participants represented very different stages of mathematical understanding in both content knowledge and knowledge for teaching mathematics. All were experienced
teachers (10 – 25\textsuperscript{+} years of experience), but only one had training in reform methods. A second participant was self-taught in manipulative-based teaching, a third had good content knowledge but minimal exposure to reform methods, and one year of experience teaching mathematics. The fourth participant expressed the need to understand mathematics more deeply, and his reform training consisted of one year of collaboration with a colleague experienced in reform methods. The data generated by this range of experience and training provided several significant implications for how teachers’ work might be facilitated to improve their reform practice.

Participants were recruited from the same school through word of mouth. My initial contact in the school was Sydney, whom I had met in a graduate degree math education class. She brought her teaching partner to the study, and he asked around for additional participants. That brought Anne to the study, and eventually her teaching partner, Roma. Initially, Roma was hesitant due to her insecurity about teaching mathematics. Once she understood that I was also looking for expertise in teaching literacy, she willingly participated, because, she felt she had a good deal to offer in that respect. In the end, her participation led to significant insights.

3.3 Data Collection

In this study, I initially intended to document the practices, beliefs and concerns of teachers as they facilitate dialogue in their mathematics classrooms. The idea was to ascertain how they use oral communication strategies in teaching mathematics to promote mathematical reasoning and deepen mathematical understanding. Early in the study, my goal broadened to include teachers’ perspectives on using oral communication strategies to promote reasoning in teaching non-math subjects as well as in teaching mathematics. This
would allow teachers to compare how they used oral communication strategies in teaching non-math subjects to how they used them in teaching mathematics. The expansion of perspective was prompted by the spontaneous declaration of each grade 1 participant during her first interview, that she had a preference for teaching literacy over mathematics.

Therefore, classroom observation was not restricted to mathematics class, because I had an intuitive feeling that the application of oral communication strategies in non-math subjects would be informative. I now suspected that an understanding of teachers’ application of oral communication strategies in general might help to build on teachers’ strengths. Indeed, it revealed that participants used oral communication strategies to promote reasoning in teaching reading and social studies, with ease.

These observations led me to consider that facilitating reflection on participants’ use of oral communication strategies in the subject they preferred to teach, and having them compare their use of oral communication strategies in the less favoured subject, might provide a way to probe more deeply into teachers’ experiences with math reform initiatives. Since three of the four teachers I interviewed were highly trained and accomplished instructors in literacy, this technique of comparing and contrasting their use of oral communication strategies helped them better identify areas where they felt insecure about, or dissatisfied with, their mathematics instruction.

Therefore, the research had to be conducted in a way that identified practices, elicited beliefs, promoted reflection, and shed light on concerns through multiple forms of data collection. Data collection methods included a preliminary interview to establish rapport, present the project and answer prospective participants’ questions. Classroom observation over several weeks (Nov – Dec 2013 for most participants and a follow-up in April 2014 for
one participant) was used to discover and document teacher practices, classroom and lesson organization, tasks provided, and levels of student participation. These observations formed the basis of the on-going, post-observation discussions that elicited teacher perspectives and promoted reflection. A post interview at the end of the observation cycle was used to confirm the data collected, and fill in any perceived gaps.

During the first interview, I asked participants about their beliefs regarding the role of dialogue in the mathematics classroom, their views on mathematics education, and resources that help them facilitate effective mathematical dialogue in their classrooms, as well as hurdles they face in promoting it. Participants’ references to teaching literacy were spontaneous contributions, not elicited by direct questioning in the first interview. These spontaneous declarations indicated to me the importance in participants’ perceptions of the oral communication strategies they used in teaching reading/writing and social studies. As the project evolved, I attempted to build on these perceived strengths, by promoting reflection on, and comparison of, the way sense-making teaching strategies were used in math and non-math subjects. Data on this aspect of teachers’ work was collected through observation and post-observation discussions.

Throughout the study, the tasks used to generate classroom discussion were documented. An effort was made to connect/relate practices of various teachers, to check for connectivity and community in the practices of the school.

Although interviews can be conducted outside the classroom, it is helpful to discuss an observed class immediately after the observation period if possible, while items of note are still fresh in everyone’s mind. On-going informal discussions were conducted with teacher participants, between the opening and wind-up formal interview.
Each teacher was asked to comment on episodes of classroom dialogue that occurred during classroom observation, discussing his/her use of tasks to promote dialogue, and any comments or concerns regarding a particular episode. These brief one-on-one discussions were intended to shed additional light on events from the participant`s perspective, and promote reflection.

As the initial data was collected, it was used to refine the research questions, help connect to theories, and in general, engage in progressive problem solving (Hammersley & Atkinson, 2007). This was an iterative process, which led to the emergence of order and understanding (Morse, 1994).

3.3.1 Preliminary Interview - Invitation to Participate

The purpose of the initial interview was to explain the project to participants, and elicit and address any questions or concerns they had. In this interview, they were also assured of anonymity and confidentiality and given a detailed explanation of expectations as outlined in the letter requesting informed consent. After receiving these reassurances and explanations, each participant was asked to sign a consent-to-participate form. The length of the initial interview ranged from 45 minutes to an hour. The interview was recorded on audiotape and transcribed.

A basic principle of conducting qualitative interviews is to provide a structure within which respondents can express their own understandings in their own words (Patton, 2002). The first interview helped to capture how and why the teacher was using classroom dialogue, and what tasks or prompts in the participant's opinion, were best suited for facilitating it. It also set the stage for future discussion. Participants were asked about experiences that led
them to their current beliefs about mathematics education in general, and their role as a mathematics teacher in particular.

The first interview provided an important opportunity to gain an initial insight into the participant's views of his/her world. Although one may later infer from participants’ actions, their direct statement of beliefs is definitely more accessible, and provides a framework for those inferences:

We cannot observe how people have organized the world and the meanings they attach to what goes on in the world. We have to ask people questions about those things. The purpose of interviewing, then, is to allow us to enter into the other person's perspective. The assumption is that that perspective is meaningful, knowable, and able to be made explicit. (Patton, 2002, p. 341)

These initial interviews sought to identify to what extent each teacher was committed to promoting classroom dialogue and the purpose for which (s)he claimed to use it. They also served to address the issues of each teacher’s knowledge, beliefs, and attitudes about mathematics and mathematics learning, and how these related to his/her experience with facilitation of classroom dialogue. As an interviewer, I kept the interview questions open ended and listened to the answers actively and attentively. This was the surest way to perceive and record participants' information and point of view accurately. According to Eisner (1998), this type of attitude promotes a smoother flowing interview as well. “It is surprising how much people are willing to say to those whom they believe are really willing to listen” (p. 183).

3.3.2 Motivation in Teaching Decisions

Shulman (1986) suggests that, in order to understand the choices teachers make in classrooms, the grounds for their decisions and judgments about students, and the cognitive processes through which they select and sequence their actions while teaching, it is necessary
to study their thought processes before, during, and after teaching. Schonfeld (2008) emphasizes the complex character of in-the-moment decision making teachers engage in, explaining that competing priorities the observer may not be aware of can confound the observer who is attempting to understand a teacher's decisions during observation. Therefore, to get a closer, more direct perspective on each teacher's feelings about, and attitude towards, facilitating classroom dialogue for teaching mathematical reasoning, a direct access to participants' decision-making criteria is required.

It was not feasible to conduct an on-going interview with participants, but it was often possible to have brief dialogues after several observed dialogic sessions. At these on-the-spot talks, I asked participants to comment on their approaches to facilitating classroom dialogue, and reflect on how well each episode worked. Notes on these brief talks were recorded as addenda to the field-notes. I recorded the date of the dialogue, the type of activity used to prompt dialogue, and comments on how it worked.

In the field-notes I kept throughout the project, I recorded observations of classroom activities, described settings, dates and times of interviews, and recorded my own thoughts, impressions, and ideas. Note-taking is one of the important implements in the qualitative researcher’s toolbox, because notes provide the reminder, the quotations, the details that make for credible description and convincing interpretation (Eisner, 1998). Though the circumstances of an interview or observation may be relevant, this information is not readily visible in the transcript of the interview. "What researchers record when they take notes depends initially upon their ability to perceive what is meaningful and significant this too is the act of imagination at work" (Eisner, 1998, p. 188). With an extended study such as this, it
was necessary to keep a record of thoughts and ideas as they occurred, so they would not get lost in the abyss of time between observation cycles.

3.3.3 Observation

Each participant was observed three times a week over a four-to-six week observation cycle, using the classroom observation protocol shown in Appendix F. Each observation period lasted an entire morning or afternoon, encompassing the math class and reading/writing, social studies, or science. Most classroom observation was recorded by means of field notes. One grade 3 math class was also audio recorded.

The classroom setting provided contextual information for how each participant implemented dialogue promoting activities, and how flexible (s)he was in allowing student ideas to drive the lesson. Classroom observation was also an excellent way to gauge student response, level of engagement, and levels of student understanding. These are important elements in describing dialogic activities, and the extent to which the dialogue is mathematically significant. Copious field-notes were taken and then transcribed.

Classroom observation allowed me to describe the setting as well as the activity, and allowed for the placement of people in context. As Patton (1990) suggests, classroom observation allows the researcher to better understand the context within which the program operates, as well as notice things that may routinely escape participants. Classroom observation allows the researcher to see not only what people do, but how they do it. Schon`s (1983) concept of reflection-in-action describes an essential aspect of teacher`s practice:

The lesson plan must be put aside then, or else it must become a rough ground plan for action, a skeleton around which the teacher develops variations according to her on-the-spot understanding of the problems of particular students.
Curriculum becomes an inventory of themes of understanding and skills to be addressed rather than a set of materials to be learned. Different students present different phenomena for understanding and action. Each student makes up a universe of one, whose potential problems, and pace of work must be appreciated as the teacher reflects-in-action on the design of her work. (p. 333)

The idea that a good teacher responds to the emerging elements of a situation holds particularly well in reform mathematics practice, especially when classroom dialogue is an important vehicle for constructing understanding. Observation should help connect and contextualize teachers’ descriptions of activities and practices unfamiliar to the researcher.

When time allowed, observation episodes were followed by a post-observation discussion of varying length, shortly after the observation, while the events were fresh in both the participant’s and the observer’s mind. When immediate discussion was not possible, a follow-up was sought as soon as feasible. Due to teachers’ very busy days, with numerous people and commitments pulling them in different directions, flexibility in timing was key to respecting teachers' schedules, while satisfying the requirements of the research.

3.3.5 Final Interview

A final interview of approximately 30 - 40 minutes with each participant completed his/her observation cycle. Each interview was audio-taped and transcribed. The main purpose of the final interview was to confirm the data that had been gathered and organized from the previous interviews and field-notes. One of the participants had blogged about implementing Bansho as part of a classroom investigation during the year prior to this study. Since much of her learning about oral communication strategies in teaching mathematics had occurred during that experience, her blogs and TLLP research report were also used as sources, and data garnered from them were also confirmed in the final interview. Before conducting the final interview with each participant, the data collected through his/her
participation was analyzed, so that any obvious gaps in details, inconsistencies or misunderstandings could be clarified.

3.4 Data Analysis

In a multiple case study, there are two levels of analysis. On one level the data gathered from each individual case is analyzed, maintaining the integrity and individuality of the case. On another level, a cross-case analysis is carried out, which is concerned with the contributions that all the cases make to the study. Strake (2005) refers to the overview of all the cases that comprise a study, as the quintain.

3.4.1 Single Case Analysis

Each case study is divided into sub-sections that organize the data around relevant themes that emerged from the information collected about the case. The questions asked of participants were aimed at eliciting information on: general background of the participant and setting, the participant's view of mathematics and classroom dialogue, his or her general use of classroom dialogue, specific examples of how the participant employs classroom dialogue, frustrations and or stumbling blocks the participant has faced, and sources of support for the participant.

Transcription of interview tapes and observation notes was an ongoing process throughout the project and done soon after each interview or observation was completed. During transcription of the tapes, as I repeatedly listened to, and reflected on an individual's answers to interview questions, there were times when new elements/nuances dawned on me. In these instances, as new questions inevitably arose, they were asked to elicit further information and sometimes to clarify collected information. These secondary realizations
and details made previously unnoticed gaps visible. The subsequent new understandings and discoveries that were fed back into the loop of inquiry, helped to refine the findings.

Transcripts were coded with the aid of NVivo 10 to elicit patterns in the following types of information: (1) to characterize what participants were doing to promote classroom dialogue; (2) to determine the extent to which they allowed the dialogue to evolve, to see how often this happened, and identify descriptions of specific examples of dialogic episodes; (3) to identify participants’ attitudes toward teaching mathematics and their attitudes toward teaching literacy, and their reasons for their preferences (4) to identify teaching strategies that were used for similar purposes and/or served similar functions in mathematics and literacy; (5) to identify desired and available supports and (6) to identify participants’ perceived need for more mathematical understanding (content and/or pedagogy).

As a method for systematically examining the status of the research and determining where it should be going, I wrote regular field memos to myself in the field journal. These were short and to the point notes that recorded Progress, Problems and Plans. Analyzing the status of the work under these headings helped to review it succinctly and make realistic plans. Reflecting on both the research process and data collected helped develop new questions, new hunches, and sometimes, new ways of approaching the research.

Data analysis is not an isolated, end-of-process activity, but is best pursued as an ongoing activity. On-going data analysis leads to fresh realizations and discoveries that can be fed back into the inquiry promptly and often lead to clarification. In support of this idea, Hammersley and Atkinson (2007) specifically state that "In ethnography the analysis of the data is not a distinct stage of the research" (p. 174), and Merriam (2009) concurs:

The final product (of the research) is shaped by the data that are collected and the analysis that accompanies the entire process. Without ongoing analysis, the data
can be unfocused, repetitious, and overwhelming in the sheer volume of material that needs to be processed. Data that have been analyzed while being collected are both parsimonious and illuminating. (p. 171)

This type of early data analysis, done while the data is being collected, helps focus and shape the inquiry as it proceeds. Consistent reflection on the data helps to organize it, and eventually meaning begins to emerge. Glesne (1999) gives several helpful suggestions for techniques to organize data so the story the data tell starts to unfold:

Writing memos to yourself, developing analytic files, applying rudimentary coding schemes, and writing monthly reports will help you learn from and manage the information you are receiving. (p. 130)

I found keeping a reflective field log to be a useful supplement to the field memos that analyzed overall progress. As reflection on data can occur subconsciously even while one is sleeping or working on something else, these thoughts are illusive and best recorded immediately. Once they are saved in black and white, reflective thoughts and ideas can be easily recalled for future use.

An additional way to keep track of useful information and thought is the use of analytic files. Organized into generic categories initially, (e.g. interview questions, people, places) they soon evolve into relevant, specific files on the social process being investigated. NVivo 10 promotes this type of organization by the way nodes function. A subjectivity node to help monitor one's subjectivity, a node for quotations from the literature, as well as a node for thoughts on introductory and concluding chapters were created and found useful.

3.4.2 Cross-case Analysis

In the data analysis of studies based on multiple cases, tension arises between attention given to individual cases and attention given to the collection of cases that comprise the study as a whole. Strake, (2005), who dubs the overview of all the cases that comprise a
study, a quintain, refers to this tension as the “case-quintain dilemma”. He encapsulates the essence of cross-case analysis as follows:

The main activity of cross-case analysis is reading the case reports and applying their Findings of situated experience to the research questions of the Quintain. These research questions guide the multi-case study of the program or phenomenon. We expect to create and modify general understandings on the basis of the Case’s experience. The analysis is not simply a matter of listing the Case Findings pertinent to each research question, because, to some extent, the Findings need to keep their contextual meaning during the authoring of the multi-case report. (Strake, 2005, p. 47)

The three approaches Strake (2005) suggests for conducting analysis across cases differ in the depth of analysis given to individual cases. The first approach does most to maintain the situation of the individual cases. For a second approach, he suggests that data can be meaningfully reduced by merging findings if a large number of cases threaten to generate an overwhelming amount of data. The third approach favours quantitative analysis, when there is less interest in the situation of individual cases. All three approaches generate tentative assertions based on rating the prominence, ordinariness, utility and importance of case findings, merged findings, or factor clusters, respectively.

In generating tentative assertions using approach 1, Strake (2005) emphasizes that:

(the researcher) has taken the Themes one by one to see what the Case Findings have provided, but she has continued to remember the situationality of the Case through its Findings. (p. 58)

In generating theme-based assertions, the cross-case analyst looks at how strongly the findings of each case support the given theme, whether the case is ordinary or contrary or exceptional, and how important the case findings are for supporting the research questions framing the study. Once analysis is completed, the assertions are organized to present the strongest message based on the available evidence. “Putting the Assertions together is a time
of interpretation and composition” (Strake, 2005, p.76). It may happen, however, that researchers are strongly persuaded, even when strong empirical evidence may be lacking.

It is rare to find strong evidence for an Assertion. Most Assertions are based on compelling persuasion; at least, compelling to the researcher. Compelling persuasion is called “evidence” here. There usually will be data of relevance, some of it perhaps compelling, in the Cases that did not contribute to the statement of Findings. Somehow the case researchers (or the Quintain manager) became persuaded this Assertion is warranted. (Strake, 2005, p. 75)

So, even though the aim of the cross-case analysis is to present the overall findings of the study in a way that is supported by the individual cases, the research analyst may seek to persuade in his/her interpretation and composition of the presentation.

3.4.3 Coding Schemes

A coding scheme helps organize the data by making patterns visible. It can develop a more specific focus for the study, or clarify what questions to ask so they become more relevant. In this project, the coding scheme helped divide information into four different categories: teacher attitudes, teaching strategies, curriculum integration, and teacher supports. This initial, rough configuration of the data allowed the main framework of the study to emerge. Each of these data fields was refined through the variety of sub-codes that emerged as I worked through participants’ information. While some new sub-codes were introduced with each additional participant, much of the information fit into the sub-codes developed in the analysis of the first participant’s data. When all the coding was done, and nodes were organized to start eliciting the story, it became clear that some of the collected information was only tangential to the purpose of the study.

In the initial stages of data collection, I collected and recorded all data from all participants, to establish the norm for teaching strategies and practices employed by
participants. As more and more data accumulated, some participants’ data started to be repetitive, while others continued to feed new information to the purpose of the study.

Coding progressively sorts and defines those pieces of data that are applicable to the research purpose (Merriam, 2009). *NVivo*’s built-in coding mechanism allowed me to sort observation notes, interview transcripts, notes from relevant literature and log entries into like-minded data clumps called nodes in *NVivo*, creating an organizational framework. Each of these parent nodes was further coded into numerous sub-codes, called child nodes. Eventually the relevant nodes were organized into meaningful sequences that formed the sub-headings in the presentation of the various cases described in this thesis.

The structure of *NVivo* allows for the hierarchical organization of codes as they are created. In addition, it allows one to enter a definition of the code name under the ‘properties’ parameter. Therefore, aside from a planning chart, a separate code book for the development and working-out of a code scheme was not necessary.

When most of the data had been collected, I began the process of analytic coding. This process gave shape to the data by eliciting themes and patterns and showed how participants’ efforts, though linked in purpose, resulted in vastly different outcomes.

Coding means identifying what appears to be important as one reads the data. Each identified ‘important idea’ was given a code name, leaving as much content around each data bit as possible. In reading the data, ideas supporting the same main idea were coded to child nodes of the relevant parent node. Further child nodes were established to distinguish different perspectives on the same idea. For example, the node of ‘student preparedness’ would have a separate child node for positive and another one for negative evidence.
The NVivo trainer cautioned against ‘viral coding’, by which she meant the use of the same child nodes under multiple parent nodes. For example: since I was comparing attitudes towards teaching literacy with attitudes towards teaching math, I initially intended to have a literacy as well as a numeracy child node for several teaching strategies. However, the trainer advised me to revise the coding framework to avoid such an eventuality, as the repetitive child nodes may well be major themes and would serve better as parent nodes.

Once all the coding was completed, the next phase of data analysis was to arrange the major themes into a logical order. The major nodes and sub-nodes that seemed to belong together were grouped, as I reflected on what had been learned, made new connections and gained new insights.

My computer was of significant assistance in the compilation and manipulation of data, record keeping, and communication. Microsoft Word was used to digitize the information from interviews, field notes and field journal, references, support documents and to write this thesis. NVivo 10 was used code the data, sort it, and look for complex structural relationships.

3.4.4 Rich Description

The aim of research is to contribute to greater understanding in the field of study. In order to help one's audience make sense of the work, and realize that it is important to them, the researcher must transform, contextualize and connect the new information to accepted understandings in the field so that the data becomes a meaningful part of the knowledge base. Wolcott (1994) as quoted in Merriam (2009) depicts description, analysis and interpretation as three processes that are often used to transform research data in this way.
Description draws heavily on field notes and interview transcripts, staying close to the originally recorded data, to recreate the feel of the episode being discussed. The primary role of rich description is to situate the audience in the moment. It thus evokes emotion in the audience by allowing them to feel that they are part of the situation. The narratives used in descriptive analysis specifically focus on details that support the key findings of the study.

3.5 Ethical Issues

Obtaining written consent from each participating teacher prior to beginning the research followed ethical guidelines in place at the University of Toronto. For interviews outside school time, no separate administrative approval is required, but the permission of the school principal was obtained prior to classroom observation in the participating school. This research focused on teacher practice and classroom norms not the evaluation of individual student work, so an expedited ethical review was undertaken to that end. Approval of the School Board Ethics Review Committee was obtained as well, before classroom observation commenced.

Personal ethical issues to consider include the time participants will devote to the study, and the risk they take in allowing their work to be investigated. These sensitive areas require a carefully considerate approach, as assurances that the investigation is not evaluative, does not prevent participants from wondering what impression they are creating.

The researcher must create an atmosphere where participants feel free to express their thoughts and ideas without fear of embarrassment, just as this kind of atmosphere is indispensable in the reform mathematics classroom. Even so, participants may sometimes regret revelations they make during classroom observation or during an interview, feeling vulnerable (Eisner, 1998). Although the researcher gives assurances of privacy,
confidentiality and anonymity when participants are recruited, for full ethical conduct, the researcher must also ensure that participants' points of view is given due consideration, that participants are conversant with the researcher's interpretations of their actions, that they have the opportunity to comment on the researcher's interpretation(s); and that those comments are given due consideration (Glesne, 1999). I sought this type of feedback from participants on an on-going basis through discussion. I repeatedly asked for participants' opinions, and asked them to validate, refine, or refute my perceptions and interpretations throughout the study. When I finished writing the cases, I asked participants to give me feedback for incorporation.

3.6 Limitations of the Study

Since the findings are based on case studies of individual teachers, it may not be possible to generalize them to every teacher in every school. Further, the limited sample size may constitute only a glimpse into the classrooms of teachers pursuing reform practice in a committed way. Nevertheless, these findings are significant as they describe the kinds of challenges encountered by teachers working with conviction and commitment at raising the level of their reform practice. Therefore the concerns that arise will be of interest to other teachers and teacher educators. The case study approach yields valuable information about the details of the phenomenon of classroom dialogue as a means of developing students’ ability to reason and deepening mathematical understanding. It also reveals valuable information about these specific teachers’ attitudes and practices in teaching literacy, for possible openings into advancing their reform mathematics practice.

The case study is a particularly useful methodology for exploring an area of a field of practice not well researched or conceptualized. Case study, which has as its purpose the description and interpretations of a unit of interest, can result in
abstractions and conceptualizations of the phenomenon that will guide subsequent studies. (Merriam & Simpson, 1995, p. 112)

3.7 Summary

In conclusion, this was an investigation into the effective use of oral communication strategies in teaching literacy and teaching mathematics. I explored how oral communication strategies are used to develop students’ ability to reason, and to access and assess mathematical understanding. I also sought to identify the aims and goals achieved, when oral communication strategies are used to teach reasoning in teaching literacy, and encouraged participants to reflect on these applications in their literacy instruction.

The study was carried out using qualitative methods in strict compliance with University of Toronto Ethical Guidelines in Research, and School Board Ethics Committee approval. In identifying oral communication strategies common to teaching literacy and teaching mathematics, and promoting participant reflection on the goals and results achieved with these strategies in teaching their preferred subject, it is anticipated that self-proclaimed non-math people will find this aspect of reform math practice more accessible. Therefore the results of this study will be of interest to teachers, teacher educators, educational researchers, and school and district administrators alike.
Chapter 4 – Case Studies

4.1 Introduction

This chapter explores four experienced elementary teachers in a school that promoted 'collaborative inquiry in mathematics' with a strong emphasis on communication as primary goals during the course of this study. In this chapter, I will provide a description of the school, and the work around mathematics and literacy instruction in which teachers were engaged, from the following perspectives: Teachers' self identified attitudes with respect to teaching literacy and teaching mathematics, and the respective strategies they pursue will be described. Background information about each teacher as it relates to his/her teaching will be provided.

Classroom practice, student response, challenges faced by teachers, and perceived differences as well as similarities in teaching strategies used to teach literacy and those used to teach mathematics in the early years, will be described. The principal’s reflections on communications in mathematics at her school will be included at appropriate places. Each case study will be concluded with a summary of important ideas raised by the participant.

4.2 Haven Public School

The school is a small kindergarten to grade 5 elementary school located in a very changeable, low-income neighbourhood in the suburbs of a large Canadian city. The classes range in size from 18 - 22 students, with a great deal of flux as a result of families moving in and out of the neighbourhood. The approximately 300 students at the school are largely new immigrants, with many of them English Language Learners (ELL). Their newness to the Canadian culture as well as the English language influences their learning.
The school focus had been on literacy for some years, but during the course of this study (the 2013–2014 academic year), it was on 'collaborative inquiry in mathematics', based on a Board initiative.

We are working on our collaborative inquiry this year. Our whole focus for the whole year will be on that, but we are also looking at embedding literacy through math this year. We are working on problem solving, but the most important part is through communication, so we are looking at the various math processes to do that, and we are also looking at criteria building and how well kids can understand the criteria and apply it. Because we think that if kids are more thoughtful, about the way they do math, then they'll be able to process their issues as they go through. We want them to be able to clearly understand why they are creating criteria, what the criteria are, and how they might help improve their practice... every team has developed their own criteria based on the students they are working with. (Principal interview)

The teams she refers to are determined by grade level. The teachers at one grade level work together to develop criteria for the math they will teach and how they will teach it, to address the 'communications’ and ‘collaborative inquiry in math' initiatives as they relate to their students' needs. For the most part, there are two classes at each grade level, and the four teacher participants in this study constitute two sets of teaching partners.

One set of teaching partners (Sydney and Tom) teach grade 3 and worked together on a Teacher Learning Leadership Project (TLLP) the year previous to this study (2012–2013). Their focus was on building oral communication skills, and applying the learned skills to a problem solving approach in mathematics. As part of the project, they developed interactive lessons for a variety of mathematical units, to use with the Smart-board.

The other set of teaching partner participants (Anne and Roma) teach grade 1. They collaborated, during the course of this study, to develop interactive mathematical tasks for their students in the areas of number sense and numeration. They created work-stations where students worked in partners or small groups to solve the assigned tasks. Where
students were expected to record their results, the recording sheet had sample answers on it, to ensure that students would record their results in prescribed ways.

The four participants are all experienced teachers with a Canadian or English background. They represent a range of comfort zones in teaching mathematics. Anne and Roma have expressed a distinct preference for teaching language and literacy. Both have had significant training in teaching literacy, and both have worked as literacy resource teachers. Anne has pursued professional development opportunities in the area of reform mathematics on her own initiative for more than twenty years, and Roma has very little experience teaching math, but has always enjoyed mathematics and has always been good at it.

Sydney has had extensive training from the Ministry of Education in reform mathematics methods. She had also worked as a mathematics coach for several years, interspersed with her classroom teaching. Tom’s only professional development experience in mathematics was his year co-developing lessons with Sydney for their TLLP project. Yet he was the only participant who expressed a specific preference for teaching the more concrete subjects such as math and science.

4.3 The Case of Sydney

4.3.1 Background Information

Sydney has worked for her school Board for more than 15 years. Ten of those years she has spent as a classroom teacher, four and a half as a mathematics resource consultant/instructional coach, six months as an acting VP, six months as a student work study teacher, and one year as a pre-service teacher educator at the university level. Rather than graduating from the classroom to teaching teachers, Sydney has interspersed her direct student contact
with teacher education to keep her experiences current. She has gone: “Back and forth, out for a bit in for a bath, keeps it interesting in the classroom, and authentic when I'm out doing leadership stuff” (Sydney, personal communication).

When asked about her own experiences learning mathematics, Sydney reflects on the very negative experience of a traditional style of math learning where the student is merely coping and trying to get away from the subject as quickly as possible:

I memorized what I needed in elementary without really understanding the actual math. I knew what I needed to know for tests. It started to become apparent in the intermediate grades as the math became more complicated - I was still keeping up because I am a conscientious person - certainly by high school it was a complete disaster - it was incredibly laborious for me because I didn't know what was going on. I actually dropped math in grade 10, because when you turned 16 you could sign the drop form yourself. When my mom found out she made me take one of these basic math courses the following year, but that was it for me until teacher's college. (Sydney interview)

Her attitude towards mathematics only started to change for the better when one of her math teachers at teacher’s college gave her a broader perspective on mathematics:

I had a math science tech person - her methodology excited me because she broadened for me what it was to be learning math. To me it had always been just about the numbers and memorizing formulas - in terms of my mathematical understanding I still didn't have a clue but in terms of being more open to math that definitely made an impact. I was not afraid of the math so much anymore. (Sydney interview)

This rocky start in her own introduction to learning mathematics was exacerbated by a daunting experience teaching grade 7 math, in her first year of teaching. “I was still coming from a place where I did not like math, at all actually, it was a subject I had to teach amongst the rest, so I was just trying to stay one page ahead of the students -- I shudder to think (Sydney interview).”

After that, Sydney attended professional development workshops in mathematics. She also enrolled in a Masters program at OISE, where one of her math education courses
was taught by Dr. Rina Cohen, and aimed at math phobic individuals. Dr. Cohen was a very gentle person with an extremely approachable manner, who made it her mission to bring everyone into the mathematical fold. It was her patience, the supportive atmosphere she created, and her ever gentle approach that proved so effective in breaking through the math phobia many of her students experienced before coming to her class.

Her most dire reprimand, ‘you are acting like a mathematician’, was often mistaken for a compliment by students who were experiencing their first successes with mathematics and thought she was calling them mathematicians. As students came to know her better, they eventually realized that this was her way of scolding those who acted like the mathematical solution to a problem was either simple or obvious, when most of the class was still struggling with it.

What was interesting in her class was that we were doing mathematics; I was actually learning math and understanding concepts I had not seen before. She had us use base 10 blocks to demonstrate multiplication, and it just blew my mind, all of a sudden I saw what 28 times 52 actually meant. It was a double-edged sword. It was very exciting but it also made me realize how completely clueless I was mathematically speaking. (Sydney interview)

As she continued to pursue her training in mathematics education, Sydney progressed rapidly in her confidence as well as her career:

I was in resource at the time, because after that I became a Ministry math trainer for the province and I took a lot of those Ministry math courses because that was the big push at the time. As resource consultants we had to do that. One thing led to another with that and I became considerably more confident, at least in elementary level math. Ironically, my partner is a mathematician and she laughs her head off that I do math teaching workshops because my math is a bit of a joke compared to a professional mathematician. (Sydney interview)

Her lesson plans and exemplars are posted on the Ministry website, and she has blogged extensively about teaching mathematics. Back in the classroom again, she continued her pursuit of mathematics pedagogy by participating in a Board supported research project;
Teacher Learning Leadership Program a (TLLP). A key component of the TLLP was to learn Bansho. Bansho is:

an instructional strategy that captures the development of students’ individual and collective thinking. Bansho allows students to: (i) solve problems in ways that make sense to them, (ii) build understanding of tools, strategies and concepts by listening to, discussing and reflecting on their peers’ solutions, and (iii) build understanding of concepts through explicit connection-making facilitated by the teacher’s board writing. (Bansho - Capacity Building Series, Secretariat Special edition #17)

She and her teaching partner, Tom, wrote a successful proposal for a TLLP that bought them each an interactive whiteboard (a Smart-board), and gave them release time to co-develop interactive lessons for use with the Smart-board. "So my partner and I had all this release time - one day off every 10 - 14 days, and using the curriculum and resources like Marion Small (to) develop each math unit on the Smart-board" (Sydney Interview).

To help disseminate their learning, the project required participants to blog about the Bansho method of elucidating students' mathematical ideas, and present their experiences to mathematics teachers on a broad scale, through an Ontario Association of Mathematics Educators (OAME) workshop. They also held a workshop at their school to share their findings with the other teachers on the staff.

In her own reflections about the Bansho project, Sydney identifies three types of learning she experienced: mathematics content, mathematics pedagogy, and professional development:

It has been really mind opening to be able to spend time digging into both the math itself and the pedagogy of how best to teach it to our Grade 3s this year. I've also enjoyed being able to share our professional learning with others, as we learn from them ... it was the small group learning and mentoring that I most enjoyed this year...I've also had the opportunity to network with colleagues from other school boards, many of whom have technology skills that far exceed my own. Learning from them has kept me on my toes this year. (Sydney TLLP report)
Although her early experiences with learning mathematics were far from ideal, later exposure to more talented mathematics educators opened doors for Sydney that allowed her to flourish in the area of mathematics education. She continues to expand her horizons, constantly embracing opportunities for further learning.

4.3.2 Teaching Strategies for Communication

In considering how participants communicate information to their students, Sydney identified several strategies. Though varying somewhat in modes of implementation, their purposes and outcomes followed distinct patterns. In this section, these strategies will be described in terms of purpose, implementation and outcome.

4.3.2.1 Teaching Students to Carry on a Conversation

The Bansho technique puts strong emphasis on eliciting and communicating student ideas, so Sydney wanted to ensure that her students would communicate effectively. This would require classroom norms that supported student risk taking in expressing their ideas and opinions, as well as students who had the skills to carry on conversations.

To provide the necessary safe atmosphere where students would risk voicing their ideas, she took extra pains to develop a respectful, collaborative environment through team building activities the class practiced for far longer than is commonly done:

I spend a lot of time in September doing tribes activities... Most teachers do that for a few days, I do it for a month, and I revisit it after long breaks; after Christmas, even after Thanksgiving. (Sydney interview)

As well as establishing a safe, community atmosphere, she taught the children specific conversational moves:

over the summer I read a monograph on 'grand conversations', which was specific more to literacy. The idea was that there are these talk moves you can teach
students; how to work with a partner, how to work with a group effectively, so we developed some class norms for group discussion (that faded chart is from September). (Sydney interview)

Starting with partners or small groups, the class would practice talking for 3 to 5 minutes about non-threatening, non-academic subjects, like favourite foods, family, etc. To teach students to link to others' comments, Sydney gave them sentence starters. They would practice one or two examples at a time, so students could assimilate them easily into their repertoire and would not feel overwhelmed:

By mid-October, we had learned all those 5 talk moves and they were able to talk in a group for 12 - 15 minutes at a time about a topic, taking turns, and one group spontaneously started using a talking stick they'd hand around. It was quite funny. And now (May), a group can talk for 20 - 30 minutes, unsupervised, about a topic. (Sydney interview)

Sydney found that small group conversation flowed quite easily around the rich texts and the engaging questions she asked students to talk about. However, students only seemed to apply the conversational moves effectively when they found the topic of conversation engaging. So her original purpose in teaching students these conversational moves - to apply them to mathematical discussions - did not follow automatically:

The idea with that, was to transfer to the math classroom, because we wanted them to talk about the math when we were doing the debrief to a 3 part lesson and have some intellectual debate. It didn't quite work out that way. So, they are really good at talking in a group about a text we've read, but they are still not that well developed in math. (Sydney interview)

4.3.2.2 Teaching Students to Communicate Mathematical Ideas

To teach students how to talk about their mathematical ideas in solving a problem, Sydney uses exemplars as a basis for comparison. Usually she uses sample questions from past grade 3 tests published by the provincial ‘Education Quality and Accountability Office’ (EQAO). In a whole class discussion, students look at how other students have solved a
problem, and think out loud about what makes a good answer, versus not such a good answer:

We will take an EQAO exemplar, put it on the smart-board, and have the students compare their responses and compare them to the code 10, 20, 30 40 (level 1, 2, 3, 4, basically). (Sydney interview)

She teaches students to verbalize the differences they see between their own answers and the exemplars through a gradual release of responsibility. She would model thinking aloud first, then have students discuss with a partner, then articulate independently.

So have a look at your answer. Which of these four answers is your answer most like? Why? How could we tweak your answer to be more like this? We'll do it together as a group, looking at two or three answers. (Sydney interview)

She uses a document camera to take pictures of students' answers, and project them along side each other and the exemplars:

We'll put it up there, compare it, and at the end, I always guarantee that they'll get a level 3 or 4 by the time we are done. That makes them feel good. To do that, you have to foster a mathematical learning environment where people are respectful of each other, because it is not easy to have your work ripped apart in front of your peers. A lot of work goes into that at the beginning of the year. (Sydney interview)

To model how other students communicate their thinking, she had her students watch a video of how students, in another class whose teacher uses a Smart-board and Bansho-like methods, discuss a mathematics problem. Together they observed, and in a teacher directed whole class session, discussed and analyzed the communication moves the students in the video used to make their thinking visible:

I had my students watch video of those students with a clipboard and jot down what those students were doing, ‘What are those kids doing that helps you understand what they are saying?’ so co-constructed criteria. I am trying to be very constructivist in how I elicit answers from students. I know what I want them to do, so rather than telling them, we watch the video and have them observe and notice. They like to see other kids, especially when I tell them these guys are from another city, from another school. (Sydney interview)
As a result of that analysis, they co-constructed a list of criteria students can check against to see whether their own solutions to a problem are communicated effectively. After developing and practicing an extensive list of 'look fors’ over several weeks, Sydney realized that they could synthesize all of the specific points on their list into two over-arching criteria: "Have I answered the question?", and "Does the solution show how I got the answer?"

As Langer and Applebee (1987) explain, in this process approach to learning, also referred to as constructivist, the teacher’s role as a repository of information is transitioned to a “strengthened role in eliciting and supporting students’ own thinking” (p. 77).

4.3.2.3 Using Technology in the Classroom

The Smart-board, document camera and i-pad play import roles in facilitating and keeping track of student ideas in Sydney’s classroom. As mentioned above, when discussing exemplars with the class, Sydney projects student answers along with the exemplar for easy comparison. The Smart-board also provides continuity between lessons, as anything saved, can be displayed again.

It is nice that you can record student thinking on the Smart-board. So as they are sharing their responses I can type them up there and save it in perpetuity, as opposed to having a gazillion chart papers. You can recall last week’s lesson and throw it up there. When we were doing our unit on geometry we started a vocabulary list in our first lesson, and I just kept copy and pasting the list into subsequent lessons and we added to it. It saved a lot of trees, at least. (Sydney interview)

In addition, copying and pasting allows for the comparison of geometric figures, for easily checking congruence, sorting, and other applications yet to be discovered, as the user gains experience and facility with the technology. Still in the early stages of learning to use the smart board effectively, Sydney considers that:
it opens up a lot more possibilities for demonstration as a teacher. In an ideal world the students would be using it as a learning tool, but I am not there yet. I am a new user, so I am still on a learning curve. (Sydney interview)

Asked whether the Smart-board has influenced manipulative use in her class, she is definite in her response that it has not. It becomes a parallel manipulative, providing an additional tool to facilitate and observe student thinking. She has many manipulatives available to students, and she encourages their use. She surmises that, in classrooms where manipulatives are not as widely used, the Smart-board would provide at least some interaction with the mathematics.

When Sydney and Tom started their TLLP, they were the only teachers in the school with Smart-boards, because the Ontario Ministry of Education funded purchase of the Smart-boards through the project. However, as their colleagues realized the potential of the Smart-board and got excited about it, the school made a decision to spend their computer (technology) budget on interactive white boards. Therefore, by the end of the school year, almost everyone teaching grades 3 - 5 had them, and some of the K – 2 classrooms were starting to get them.

Sydney finds that her students have very different needs because of their different levels of readiness in mathematics. She identifies the i-pad as one of the highly desirable pieces of technology to help lagging students. Sydney brought her own i-pad to the classroom to provide a fun way for students to engage in skill and drill exercises, while she conferences with others individually. In an ideal world, she would have 3 or 4 i-pads for student use.

In general, Sydney finds that technological innovations like the Smart-board, document camera and i-pad make learning much more engaging for students, and teaching far more efficient for teachers.
When asked to compare teaching strategies she uses to teach literacy with teaching strategies she uses to teach mathematics, she says that student needs dictate that she teach a very differentiated program in both subject areas, so her approach is very similar:

I actually do not think it is all that different ... because of the diverse needs the student have here. I've got stage 1 ESL (to) students reading well beyond grade level. So I teach a very differentiated literacy program. At the beginning of the year I taught all students some learning skills, but if you walk into my class now, everybody is doing their own thing ... my teaching has become very student focused based on individual needs. I think the same is true of teaching mathematics. There are some fundamental skills you want to teach, like how to communicate their thinking or reasonableness, or some of the math processes at the beginning of the curriculum document, but in terms of the actual mathematical skills, by grade 3 their needs are so divergent I am not sure you can teach a whole class math lesson where everybody does the same thing. (Sydney interview)

To develop differentiated instruction, Sydney uses resources like Marian Small’s “Good Questions” to find open-ended questions that are accessible at many levels:

that is good, you know, providing a rich task with multiple entry points, but you still have in some cases, students who cannot access that yet. So it is a great way of teaching the vast majority of the students in your class, but I still have students who are struggling - I have to catch them up. (Sydney interview)

Therefore, questions around supports (available as well as desirable ones) form key points of discussion. What kinds of supports are available to help teachers remediate those students who are behind, and what kinds of supports would they like to have, that are not currently available to them?

4.3.3 Support for Teachers' Work

This section describes supports for teachers, identifies some areas where processes and requirements currently in place hinder teachers' work, and suggests desirable assistance
that would increase teachers' effectiveness. To learn about Sydney’s perspective on teacher supports, I reviewed her blog.

4.3.3.1 Co-workers and Professional Development

Having worked as a math trainer for years, Sydney has a significant amount of experience in both receiving and delivering professional development. She has mentored colleagues as well, and taught pre-service. Collaborating with Tom on the TLLP research project provided opportunities for more personal development. She had to immerse herself in the mathematics to understand it at a very practical, applied level. Only with such deep understanding could she develop pedagogies to present the material to her students in the form of interactive lessons.

She would have liked to visit classrooms of other practitioners who were actively engaged with eliciting student thinking, but only found such meaningful contacts on-line. The learning experiences she appreciated most were the opportunities to share her learning with colleagues at the Ontario Association of Mathematics Educators’ (OAME) annual conference, and discuss effective practices with people who are on the job daily:

When (else) do I get to meet with colleagues down the hall and across the province, on a regular basis, so that effective practices can be meaningfully shared amongst the people who use them daily with real students in real classrooms? (Sydney Blog, 5/17/2013)

If we consider that the most effective learning occurs as a result of interaction with one's peers, as Vygotsky (1978) proposes, then it is easy to understand why Sydney found these experiences most gratifying.
4.3.3.2 Administrative Impact on Teachers’ Work

This section describes existing administrative definitions, requirements, processes and forms that exercise significant impact on teachers' work. As Sydney has gained experience teaching mathematics, she has come to see that the strands of math cannot be isolated from each other if a unified knowledge structure is to develop in students' understanding. Yet, the curriculum defines the strands on a stand-alone basis, and reporting on student progress requires that teachers comment specifically and separately, on student competence in each of the strands.

The trouble of course is that as the thinkers in the intellectual organizations and the practitioners in the classrooms come to recognize how to do things well, they/we are stifled by archaic institutional structures. I am not in favour of reporting on discrete strands in math, because over the years, I have come more and more to believe that the skills developed in one strand of math are interconnected with other strands. A good, rich problem may require the use of tools and skills from a variety of strands. (Sydney Blog, 5/23/2013)

The very structure of the curricular definitions in terms of strands is conducive to predispose teachers to teach mathematics unit by unit, strand by strand, especially those practitioners who are not very knowledgeable about math pedagogy, and neither comfortable with, nor experienced at teaching mathematics. The current definitions put a great deal of emphasis on individual expectations that are perhaps more easily recognized and evaluated by the novice:

The math processes and “big ideas” in math are critically important to a cohesive presentation of the math program by teachers; we need to stop focusing so much on individual expectations, and rather consider how we can best weave these together to facilitate our students in developing a sense of the grander scheme of things. Already some schools and school boards are using the math processes rather than the overall or specific expectations to report on progress in math. (Sydney Blog, 5/17/2013)
The integration of ideas across mathematical strands and processes is key to a cohesive presentation of the mathematics program, as the Ontario Ministry’s Mathematics Curriculum implies:

The development of mathematical knowledge is a gradual process. A continuous, cohesive program throughout the grades is necessary to help students develop an understanding of the “big ideas” of mathematics – that is, the interrelated concepts that form a framework for learning mathematics in a coherent way. (Ontario Mathematics 1 – 8, p. 4)

It follows that serious attention must be devoted to changing the administrative requirements used to track and report on student achievement. A good evaluation process detects and reflects the ideals of the curriculum it evaluates (Kirkpatrick, 1994). It must also allow for detailed reporting on individual strengths and weaknesses, rather than promote generic plug-ins ostensibly applicable to everyone, but not serving anyone faithfully. Regarding Ontario Report Cards, Sydney laments:

How am I to realistically track and report a discrete mark for five individual strands of math in a program that effectively integrates across strands and processes? How do I transform my detailed, individualized observations of each student’s strengths and needs into a generic chunk of text that can be plugged into the limited box provided on a report card designed for the masses? (Sydney Blog, 5/17/ 2013)

Existing administrative requirements for reporting student progress seriously lag behind what we now understand to be a pedagogically sound presentation of the curriculum. As Sydney mentions, some schools and school boards are evolving toward a more integrated system of evaluation. However, as all evolutionary processes, progress in this area is slow, and the Board she works for in a large urban center, has some catching up to do.
4.3.3.3 Student Readiness

Many of the problems Sydney encounters in teaching her grade 3 students are related to lack of adequate student preparation. The large gaps in basic skills many students struggle with at this particular school, have forced her to adopt a differentiated mode of instruction:

For me as a teacher, a frustration has always been that students arrive without those basic skills, for whatever reason... I find that by the time they get to grade 3, I have a number of students who are still struggling with basic skills. So how do you remediate that, while still providing a rich program for students who are ready to go deeper with it? (Sydney interview)

In Sydney’s ideal world, every child entering a class would have the necessary basic skills to build on. At the very least, children lacking basic skills could be dealt with one-on-one to help them catch up, because she feels that teaching is so much more effective when it addresses individual needs. The remedial work could be done either by the teacher, or another mathematically competent adult in the classroom:

whether it is a parent volunteer, or a co-teacher, or a special ed. teacher, who can co-facilitate the class with you, because I do like the idea of students working on what they need and want to work on. (Sydney interview)

4.3.3.4 Resource Materials

Reform practice relies on student interaction with the mathematics; on using a variety of manipulatives to help visualize the problem, generate ideas they can verbalize, and thus solve the problem collectively (NCTM, 2000; Ontario Mathematics Curriculum 1 – 8, 2005). Most elementary schools have a storage area where class sets of manipulatives are kept, with the idea that several classes can share the same set of manipulatives. Teachers are expected to borrow different sets as and when their class needs them. This arrangement does not sit well with teachers who integrate manipulative use into their lessons on a regular basis. Sydney draws a parallel with literacy:
research shows you are supposed to have a classroom library of at least 1000 books for independent reading. Who's running down the hall all the time to get the manipulatives? I think they need to live in the classroom. I've been very lucky. In some of the schools, I have had class sets of manipulatives, but in other cases I've bought my own. This entire wooden cart is actually mine. 80% of the content in the blue bins is mine. Over the years, I've just bit the bullet every year and bought a few more things. I cannot be running down the hall (every minute) to the storage locker to get the set of base 10 blocks. (Sydney interview)

4.3.4 Teachers Knowing and Doing Mathematics:

Sydney thinks that doing and teaching mathematics are different, and not only do you need to understand the math, but you need additional skills to teach math well. Research in mathematics education identifies three kinds of knowledge necessary for teaching mathematics well: content knowledge, specialized content knowledge and pedagogical content knowledge (Hill, 2010; Hill et al. 2001; Hill & Ball, 2004). Teachers need to know the mathematics everyone needs to know, classified as common content knowledge. For example, knowing, how to add or multiply. They also need specialized content knowledge, which is mathematical knowledge only teachers need to know. An example might be the ability to assess whether a student invented strategy for multiplying two digit numbers would always work. Thirdly, teachers need to have pedagogical content knowledge, which refers to mathematical knowledge in relation to students. An example of pedagogical content knowledge is knowledge of common solution strategies elementary students might use.

In reference to her grade three students, Sydney is dismayed to find that many have such severe gaps in their mathematical understanding that they cannot grapple with the grade three material. She wonders what went on in grades one and two to leave students’ understanding so deficient. Could it be that they were memorizing instead of understanding? That can easily happen, as it is often difficult to distinguish a student who understands from a willing, well-trained student in the early grades (Skemp, 1989). However, if it was the
teaching that left something to be desired, Sydney is quick to add: 'not because they had bad teachers, but because they were coming from a background where they lack the mathematical knowledge to teach math well’ (Sydney Interview).

Aside from the intricate knowledge and skill set required to teach math well that few elementary teachers have the opportunity to develop, Sydney points to our societal attitude towards mathematics as an underlying factor for the general lack of expertise in math education at the elementary level:

- as a society, we have not honoured enough the importance of teachers understanding the math. Imagine if we said you do not need to be able to read and write in order to teach reading and writing. That would never happen. I think we do a real disservice to kids if we blindly go into teaching math without having any sense of the math itself. (Sydney interview)

Although the parallel she draws with literacy instruction seems extreme, her point may not be far off the mark when it comes to math instruction, as her own experiences with her first grade seven class attest.

As someone whose start in mathematics was disheartening and a start more typical of people who come to hate math, Sydney has come a long way in her attitude towards mathematics. She thinks that her rocky start has increased her appreciation of the importance of teaching mathematics well, as well as her ability to do so. She feels that her own early difficulties in understanding the subject help her find ways to engage students with mathematics; to facilitate their developing a better understanding of it.

Sydney has shown a pattern of enthusiasm for mathematics triggered by personal experiences. Every time an episode of professional development has inspired her, her students have benefited. She has gone back to her classroom and taught an amazing lesson:

- The presenters were strong in math and excited about it and showing us these resources....Suddenly I realized what a square number was. I got really excited
and I came back and I taught the best math class to my grade 7 all year because I was excited about it. That has been typical of my journey. Every time I learn something new in math my kids benefit from it. (Sydney Interview)

Her learning process has been an evolution catalyzed by increasingly mathematically oriented individuals in her life.

Her first experiences learning mathematics were off-putting, but as she came across better teachers, she started to change her perspective and began to understand what she had only memorized earlier. She realizes that her progress owes much to the superior pedagogy of good math teachers, and acknowledges the positive influence of all those who have helped her. Sydney pinpoints the beginning of her evolution as being triggered by the efforts and attitude of her math/science instructor at teacher's college. That instructor expanded Sydney’s perspective on what constitutes mathematics by helping her see that math is much more than just numbers and procedures to be memorized in isolation. Her professors at her graduate school provided the safe classroom atmosphere needed for student participation, as well as introducing her to manipulative use that allowed her to see how some of her memorized algorithms worked. Finally, she appreciates the efforts of the many professional development workshop leaders who have inspired her, taught her useful reform strategies, and have generally helped develop her reform practice. As she continues to learn, she is also grateful to the colleagues with whom she has collaborated, and those who have allowed her to refine her own practice by helping them learn.

Over the years, as her own familiarity with, and understanding of, mathematics continues to grow Sydney has come to believe that teaching mathematics to the general population is important:

- to increase the intellectual capacity of a society. I think math helps you to examine the world in ways that text-based literacy alone doesn't. I am laughing to
hear myself say that, because I am such an arts advocate in general, but I think you need both. But I do think there is a mathematical literacy that has been sadly lacking and undervalued in North America and we are losing out because of that.

She credits her close personal association with a mathematician for coming to realize:

how much those of us who lack a mathematical literacy are missing in the world. We are not able to think about some things in certain ways because our brains just aren't developed that way. It is part of a complete healthy nutritious breakfast.

You need both the math and the language. (Sydney interview)

4.3.5 Summary

Sydney is an experienced teacher who claims to be a strong arts advocate. Under the influence of increasingly effective teachers, her interest in, and understanding of, mathematics has evolved from math phobia to teacher trainer in reform methods for the Ministry of Education. She says she moves back and forth between the classroom and leadership roles to keep the examples real and the teaching fresh. My work with Sydney largely elicited her experiences with a classroom research project to learn Bansho. The following themes emerged from this work:

- Sydney has a very constructivist approach to teaching. She believes in developing co-constructed criteria with students, and a gradual release of responsibility. For example, she taught her students how to communicate their mathematical thinking by having them watch and take notes on a video of students communicating their ideas in math. Then she facilitated a whole class discussion to elicit the different strategies they observed.

- When she prepared for implementing Bansho, Sydney wanted to ensure that students would have the conversational skills to debrief the math lessons. Therefore she taught her students conversational moves so conversation would flow easily. However, she
found that the skills did not transfer to math as she had hoped, and concluded that they have to be taught in context.

- In Sydney’s opinion, technological aids such as the Smart-board and i-pad, are useful for increasing student engagement, and make teaching more efficient.

- Sydney would like to provide every student the opportunity to work on what he/she needs and wants to work on. In aid of this goal, she would like another competent adult in the class to co-facilitate.

- Challenges she finds in the work often relate to serious gaps in student preparation. She teaches a diverse group representing a wide range of needs, so she teaches a very differentiated program in both math and literacy.

- Sydney strongly feels that teachers doing math has been undervalued in our society, and we do a disservice to children when their teachers do not have the content knowledge and the pedagogical content knowledge to teach math well.

- Sydney is frustrated with an existing administrative reporting structure that requires the discrete evaluation of topics in math that should be integrated. She thinks it counterproductive to the delivery of an integrated curriculum and an accurate evaluation of student work. She thinks teaching an integrated curriculum leads to more cohesive understanding, and would like to evaluate and report on the whole picture, believing that to be a more accurate representation of student achievement.
4.4 The Case of Tom

4.4.1 Background Information

Tom was trained in Great Britain, but has been working in this Board for ten years, eight at the current school. He has taught grades 4 and 5 in the past, but this is his fourth consecutive year teaching grade 3. He chose to do it again, because, in the previous school year, he and a colleague (Sydney) developed a series of interactive mathematics lessons for the Smart-board, and he wanted to make more use of them. He says he prefers to teach math and science to literacy, because, in his opinion, the goals to achieve in a particular lesson are more easily distinguishable and more readily defined in these concrete subjects:

I like things that are fairly concrete. The thing I like about math is that you can have a specific goal for that day and you are working towards that. The difficulty with teaching reading or writing is that everybody has their own individual goal. So sometimes you have kids who are really struggling to read, and then you have one tiny very specific goal, but it is such a small part of the picture of how they are doing with reading, and I find that a little overwhelming because it is not as discreet. In Math you are just, I am going to teach this unit, and this unit, this unit, this unit. (Tom Interview)

4.4.2 The Challenges of Teaching Math through inquiry

The above reflection seems to indicate that Tom has a more highly developed sense of learning increments in literacy than in mathematics. He perceives the necessity of minor, individual student goals in reading, but seems comfortable with working toward whole class goals in mathematics. He teaches math unit by unit, where each unit corresponds to one mathematical strand. So he does not integrate the strands in his teaching.

On the other hand, he does reflect that many of his current students would benefit from a mathematics program that was subdivided into much smaller, more easily conquerable stages of achievement:
I just have a feeling that some of these kids would benefit from a math program that is broken down into much, much, muuuuch smaller increments, where every small increment builds on another, so every day they can experience some success, so that they can feel competent and start to see that ‘ok I can do math’... I am not terrible at this, I can do this, I can understand this. The problem-based approach is well and good, but if every single day the kid is sitting there ‘oh great, here we go again, another day where I do not understand what I am supposed to be doing’, then it can be somewhat damaging to their motivation. (Tom Interview)

Tom expresses ambivalence about the appropriateness of teaching mathematics as inquiry to his students, a strategy he refers to as the ‘problem based approach’. His reasoning is based on his perception that this approach is overwhelming for many of them, and makes them feel that they cannot do mathematics.

Asked how much professional development he has had in mathematics, Tom replies: "If it wasn't for this project (learning to use Bansho funded by the Ministry through the Teacher Learning and Leadership Program), not nearly enough." So having time to look at the curriculum in detail, learn more mathematics, and construct lessons to address specific expectations as part of the TLLP project definitely helped him develop personal familiarity with and understanding for the curriculum. Working with Sydney, an experienced math trainer, also helped him build his pedagogical repertoire in elementary mathematics, but he definitely feels the need for additional training and development.

When he talks about developing the interactive lessons for the whiteboard, he makes reference to the copious amounts of time spent searching the internet to develop the thorough understanding of geometric shapes that would allow him to teach a lesson driven by student ideas when he uses the Bansho format.
4.4.2.1 Teaching Students to Communicate Mathematical Ideas

The Bansho format is an inquiry-based instructional technique used in Japanese schools, where a record of the entire lesson is made visible on the board. Bansho means ‘board writing’. The lesson is based on a rich problem with multiple entry points that is presented to students. Student solutions to the problem generate the data that is explored and used to distil the important concepts the teacher wants to get across. In the Bansho process, students work on a rich problem, the teacher posts their solutions, and elicits the key mathematical ideas from the solutions through class discussion. The main points are then summarized, and students are given additional problems to practice. The Bansho is a structured way of implementing the iterative process necessary for the emergence of mathematical meaning through reflective discourse (Cobb et al, 1997; Lampert & Cobb, 2003). It is structured to ensure that repeated shifts occur between what teachers and students say and do, and their subsequent, explicit discussion of that talk and action.

The students do the work, then the teacher does the teaching... So the idea with Bansho is that everything from that day is visible: the minds-on activity, the lesson problem, the student work, the summary, and then the independent practice is all visible as one thing. (Tom Interview)

The key technological support for the Bansho structured lesson is the Smart-board. It is pivotal to keeping all of the ideas visible in several ways. Student answers are projected onto the Smart-board, and, as ideas are voiced during discussion, additional comments are added by the teacher. The Smart-board also allows for carrying forward ideas from previous lessons, which can be used to provide prompts about previous work.

In today's lesson, we had those vocabulary blocks we have been pasting from lesson to lesson so that those words are up there to jog their memory: right angles, parallel sides. Then we had those words across the bottom, words they can use to contrast and compare. Then I was trying to model it form them. How might you
use these words? Here are some sentences...You are not going to see them just pick it up. You have to build it up. (Tom Interview)

Tom provides sentence starters and models how to employ vocabulary in context to scaffold his students in expressing their mathematical ideas.

The three-part lesson structure implicit in Tom’s application of the Bansho technique also scaffolds students in their inquiry. In this lesson structure, the minds-on activities directly relate to the lesson problem. They read through the lesson problem as a class, and talk about it, so Tom can ensure that students understand what they are supposed to do before they do it.

Examples of how Tom implements the 3-part lesson are taken from his lesson on describing polygons. He conducted the entire lesson with the intent of eliciting student input. The students first reviewed the idea of a right angle, which had been discussed in the previous lesson. The summary of that lesson was projected onto the whiteboard, and Tom asked the students to gather on the carpet to discuss, with an elbow partner, what they had learned about right angles in the previous lesson. After a few minutes of small group discussion, he elicited student answers in a whole-class discussion to reinforce the concept of the right angle, and how to determine if there is one in the figure. This is illustrated in the transcript excerpt (lines 1 – 11) below. (Appendix H gives the transcript of the full lesson.)

1  Mr. Tom:  How can you tell if an angle is a right angle?
2  Jay:  like an ‘L’ ‘makes an L shape
3  Mr. Tom:  where would you find a right angle?
4  Al:  on the Smart-board (projected)
5  Mr. Tom:  where in a shape?
6  Nora:  on the edge
7  Mr. Tom:  add to that, Shane?
8  Shane:  around a corner [Mr. Tom aside: Sit and listen.] When the vertical (side) is straight (up and down), and the horizontal (side) is straight ( across)
9  Mr. T:  not always vertical and horizontal, but often. Where they come together is a vertex and the inside is an angle; a right angle.
Then the class worked on describing a rectangle (lines 12 – 19), a parallelogram (lines 20 – 36), and a right triangle (lines – 37 – 69) without naming the figures. For each figure, students were asked to talk about the shape with an elbow partner (lines 12 – 14, 37 – 38), then Tom led them in a whole-class discussion to elicit the points he wanted students to learn. Throughout the process, Tom modeled and corrected vocabulary use (lines 10, 11, 19, 29, 30, 31) and kept reiterating the evolving list of characteristics they were using to describe the shapes. One important feature of the discussion was Tom’s encouragement of risk taking. He deliberately drew student attention to the great value of incorrect ideas, because they make everyone think:

So what I like is, that Shane isn’t afraid to put his hand up and have a guess. It doesn't matter if he is right or wrong, because then it starts the conversation. What was good about that, was that as soon as he said ‘I don't think there are any right angles, it started everyone else’s brains moving, and thinking, hang on a second, I am not sure I agree with him. I think it does. Whereas if only one person says yeah, there is a right angle, then everyone else stops talking. So thank you, Shane, for being brave.  (lines 41-47)

Whole class sharing of ideas varied in length from 4 to 11½ minutes. It was interspersed with small group/partner discussions and activities. Student discussion was animated, but never too loud, and Tom was very good at regaining the class’s attention quickly.

The review of the right angle and the description of the rectangle, parallelogram and right triangle were all warm up to the lesson problem of the day (lines 15-66):

70  Mr. Tom:  O.K.  Here is today’s lesson problem.  [a lot of response; some enthusiasm] I
don’t need a lot of comments when I put the lesson problem up, as soon as I put it
up, you should just start to read it.  O.K.?  [garbled student voices, reading the
problem?  ‘I’m stuck on a word’.]  Everyone’s listening, then, not talking.  In 5, 4,
3, 2, 1.  [student voices silent]  Thank you.  Here is the lesson problem:
75  Choose a shape and describe it in as many ways as you can without using the
shape’s name. Record your work.
...You may recognize it as very similar to what we just did here. The one
difference being that now
Student: (shouts out) you have to record your work
Mr. Tom: which means you have to put it down on paper. Here are the shapes you are going
to choose from. [student chatter]

Tom took about 12 minutes in a whole-class discussion to relate the problem to the
warm up exercises, and elicit vocabulary students would need (lines 86 – 108). Many of the
key words were also projected onto the Smart-board in the vocabulary box that is
accumulated and expanded during the lesson, and carried forward from lesson to lesson.

During the problem solving portion of the lesson, Tom walked around the room,
monitoring students’ work. He asked probing questions and made suggestions to keep
students on track and focused. He also chose the pieces of student work he would ask
students to present and explain during the debriefing part of the lesson. The shapes presented
were: K (parallelogram), J (right triangle), A (square), in that order. During the debriefing,
he took pains to summarize the attributes of each shape that students had talked about (lines
134 – 139, 140 – 144, 163 – 176, 197 – 204, 232 – 243) several times, in an effort to
emphasize the different features that need to be considered. The transcript in Appendix H
shows the quality of the dialogue during debriefing: The most striking features about this
dialogue is Tom’s repetition of key points, where students are required to give short answers
to his prompts as he works to reinforce the main ideas he want students to grasp.

The debriefing of the student solutions took almost 12 minutes. During the
discussion, Tom did have to remind students several times that they should be listening (lines
121 - 127, 151, 180 – 188, 194 – 196, 205 – 207) or talking about the problem, because he
could call on anyone, and whoever was called on had better have something relevant to say.
In Tom’s opinion, the most challenging part of the lesson is the debriefing. He says that students find it difficult to listen to each other, and even share their ideas in front of everyone. No matter how well established the classroom norms are for this process, there are some students in Tom’s class who are incapable of staying focused long enough to make it work to his satisfaction:

I am into the second year of this, I think I am getting better at it, but you can see it on a daily basis, right? The problem is, that that part of the lesson always ends up being too long. That is one of the reasons it is not working, because they find it so difficult to attend during that part of the lesson, for whatever reason. You see a few of them engaging and asking questions, but you really have to pull out the bag of tricks to get them to really engage. So that is where the 'turn to an elbow partner' comes in, and that kind of thing. (Tom Interview)

One of Tom’s frequently used engagement strategies has students sharing their mathematical ideas by discussing the question with an ‘elbow partner’. This can present both a solution and a bigger challenge. In the privacy of a one-on-one conversation, students may be more willing to share their thoughts. However, once they are talking in partners or engaged in small group discussions, how can the teacher ensure that they keep talking about the mathematical problem?

It is hard. It is not that they are talking about the math all the time. You just have to try to hold them accountable in some way. Sometimes I will say 'I want you to think about this question I ask, by yourself. Then turn to an elbow partner and share with them. Then I may ask anyone in the class, so make sure you will have something to say. (Tom Interview)

Tom says that teaching the children to figure out how they came up with a solution and explain their method can be even more challenging. Often their automatic answer is that they know the answer because they did it in their head. The gap between that answer and a detailed, written, or oral explanations of an actively demonstrated solution, takes many hours of hard work on the teacher’s part.
The student friendly version of the criterion for written explanations of mathematical solutions states that someone else looking at the work should be able to understand what the student has done. Tom thinks:

this is kind of vague, so then you have this criterion, but then you have to exemplify it numerous times and then they know what it means. Because some of them think they've shown their work, but then nobody else would be able to tell them, just by looking at their work, what they have done. They know that they need to show what they did with the numbers. If it is an addition question, show how you added the numbers. Did you use a number line? Did you use an algorithm? How did you do it? and do not just do one thing in your head, and then make something up on the paper. What you have on the paper has to be what you did in your head. So at least I feel we have got them to that point. (Tom Interview)

It is not enough to exemplify different methods of doing an operation like addition for students to learn to document their reasoning. To facilitate mindful student use of the various strategies, the teacher has to bring to students’ attention the different specific strategies that have been used in class to accomplish the given operation. One of the problems Tom has encountered in requiring students to explain their work is that students, who do not have the metacognitive maturity to figure out how they solved the problem, will actually make things up to satisfy the requirement. Alternatively, they may work the problem in a less efficient way, so they can document their work.

Sometimes it can actually get in the way of them learning, because they are thinking so much about how am I going to put this down on paper, that they actually start using methods that are not the most efficient. Because they know how to do it in their head, and they are doing it well, but they are having such a hard time communicating it on paper, that they'll show you something completely different. (Tom Interview)

Tom thought that trying to learn mathematics by a problem solving approach with an emphasis on communicating their thinking was proving to be overwhelming for several of his students. In his opinion, students felt overwhelmed due to their serious knowledge gaps
in both literacy and mathematics. Perhaps because, by grade 3, the knowledge gaps in literacy have become more pronounced than those in mathematics, Tom felt that he could get more of a handle on teaching his students mathematics on a broad scale, than trying to make headway in their reading with students who were very weak readers.

Tom expressed a preference for teaching mathematics, because of his clarity on curriculum expectations in mathematics, and his feelings of satisfaction as he worked toward clear teaching goals in his mathematics lessons. This clarity was directly related to the copious amounts of time and dedication he had lavished on studying the curriculum and developing interactive mathematics lessons as part of the TLLP project he and Sydney participated in:

So when we did this project, we started with the curriculum and every lesson was based on a curriculum expectation. So yeah, when you have that specific goal in mind, and you are working towards that for the day, you know exactly what it is you want them to learn by the end of that lesson, you feel like you are accomplishing something. Whereas you get kids who come in as very weak readers, and you might sit and read with them a dozen times in reading conference, and just feel like they are not making any progress, and that can be both overwhelming and just a bit draining. (Tom Interview)

When reflecting on the problem-based approach to teaching mathematics, Tom used three criteria for evaluating student success: autonomy, relationships and competence.

So there is some level of autonomy, because they get to choose - because the questions are open ended, at least some of the time they get to choose what strategy they use to solve it, but like I said, that can be overwhelming for them. Relationships, that is not specific to this way of teaching math, your relationship with the kids arches everything, and then the competence thing is the biggest thing here, I think. A lot of kids struggle with math, they do not like math, they do not see themselves as being good at math, already by the time they are in grade 3. If they do not see themselves as competent, they are not motivated to try and to do well. And because this method of teaching math is pretty overwhelming for them, I think it is really damaging to that part of their motivation. (Tom Interview)
Overall, Tom seemed quite conflicted about his preference for teaching mathematics to his students. He said he preferred to teach math, because the goals in his math lessons were so well defined, and working towards those goals in a lesson gave him feelings of accomplishment. At the same time, he strongly believed that the problem-based approach to mathematics was overwhelming for many of his students, because of their serious gaps in basic skills. Thus, having chosen to teach mathematics by a problem based approach to students with serious knowledge gaps, the feelings of accomplishment he should have been experiencing were often frustrated.

### 4.4.2.2 Technology in the Classroom

The Smart-board and document camera were the technological tools Tom used to facilitate his mathematics lessons. In his second year of using his TLLP developed lessons on the Smart-board at the time, he had acquired some facility with the technology, easily cloning geometric figures, manipulating them to prove congruence, pasting and manipulating text, etc. To drive the lesson forward, he prompted student ideas through questioning, and allowed students to work with the board. This seemed to be very motivating for them, dragging and dropping figures to sort according to attributes, marking parallel sides and right angles onto figures.

The Smart-board set-up was very useful for showing students’ work to everyone, and helped jog students’ memory when they stood to describe what they had done. Nevertheless, Tom felt that it took too long to do this kind of whole group discussion and keep everyone’s attention on the work. From what I saw, he was right, in that he had a number of students whose attention strayed. However, that happened throughout the day regardless of the topic, not just during the debriefing of the math problem. The same children often distracted each
other instead of doing the assigned activity, and hadn’t even started yet when others had practically finished.

Tom also instructed his students in the computer lab once a week. During the period of classroom observation, they were working on written communications. They blogged on an on-going basis.

The school administration had recently made a decision to keep the students as current as possible with social media. In one of their sessions, Tom opened a twitter account for the class. He then facilitated a class tweet to one of the grade 5 classes about their activities that day. The way he elicited ideas from students about what the content should be, and then helped them pare the content down to the allowed number of words, was a wonderful exercise in questioning and reasoning. On other occasions in the computer lab, they worked on their blogs. Choosing the topic of their blog, and connecting related ideas to elaborate on their chosen topics was very revealing about the state of students’ communication skills and attention span. Some students were very productive, while others just played with the technology, and barely wrote a sentence during the entire session. All of the students, however, were engaged with the computer for the entire 40 minutes they spent in the computer lab.

In another example of Tom’s efforts to use technology in the classroom, he managed to procure a number of tablets for one of the research stations in the Social Studies Unit on pioneer days. They were mainly used to access Kriehoff paintings about pioneer life; information that could have been provided via books or prints. However, the opportunity to use the tablets made students eager to work at that particular research station, and they learned to find information as well as interpret it.
Overall, Tom has found that the use of electronic equipment for manipulating the data to be presented serves to increase student engagement. They may not always be on topic or very productive, but the use of the electronic equipment increases the probability that students interact with the teaching material to some extent.

4.4.3 Supporting Student Learning

4.4.3.1 Student Readiness

Tom seemed very sensitive to the problems his student grappled with, both academically and personally. He saw students' academic problems as often exacerbated by, if not rooted in their home situations, and found the emotional needs of the children very demanding.

The most obvious roadblock to teaching mathematics through collaborative inquiry with a focus on communication was that his students often lacked communication skills:

on average, the students here, in our classrooms, do not have particularly well developed oral language skills. So one of the difficulties we are having with teaching math this way, is that our students' oral language skills are not that great, but in order to engage in a meaningful discussion to communicate in the math classroom, it kind of presupposes that you are able to at least hold a simple conversation with somebody with a little bit of back and forth. (Tom Interview)

He found it particularly hard to hold students’ attention during the debriefing section at the end of the lesson. Many of the students were not really interested in listening to each others’ ideas, and seemed completely disengaged at that point. It wasn’t just that the session may have been too long, but had to do with students’ lack of oral language comprehension:

part of the inattention, I think, is that they are habituated to not understanding much of what is being said when people are talking; especially adults. So they just automatically turn off. I've seen examples of that throughout my teaching career, and I do not know if it is true, it may be just a gut feeling, but I honestly believe that that is the case. It is not even that they are doing it consciously any
more. It is just their default position, because that has been their life experience from a very young age. (Tom Interview)

With such large gaps in communication skills, mathematical understanding did not even enter the picture. Children’s lack of language and literacy made it difficult for them to understand the question, let alone the mathematics. Then, to figure out how to communicate their answer, was an additional burden for the students. Overall, many of Tom’s students found this kind of lesson cognitively overwhelming, even when mathematics was taken out of the equation.

They cannot read and understand the question because their reading skills are not great. They cannot decide what to do to answer the question. Even if they figure it out, their mathematical knowledge is insufficient to do the math that is required. Then they’ve got to think about how to communicate this, so the lesson goal you had in mind (I wanted them to add two 2-digit numbers together) sometimes just gets lost in the mix. (Tom Interview)

Students’ gaps in mathematical understanding played a significant role in the kinds of problems they found accessible. The lack of consolidated knowledge precluded the use of problems that incorporated more than one mathematical strand:

So you have to be very careful of the problems you choose, and make sure they have the pre-requisite skills consolidated, otherwise the lesson can very easily fall apart. Especially if the aim of your lesson is to really teach them about measurement, but then, in doing that question they had to add or subtract numbers, and they didn't know how to do that. Then you are in big trouble, because they are not learning about measurement at all, but have to take a few steps back. O.K. Let us figure out how to add these numbers together. (Tom Interview)

There were major issues of lack of preparedness for the students Tom dealt with. By grade 3, students were grappling with large gaps in basic skills in both literacy and mathematics. Many students had problems with oral language. These may have been problems specific to this particular student population, but they did underscore the fact that
communication skills need to be in place to learn mathematics through inquiry, particularly when there is the added expectation that students communicate their mathematical thinking.

In an overarching effort to remediate his students’ knowledge gaps, Tom has introduced several experiential learning modules into his grade three curriculum. He believes that the children will learn more if they get exposure to real-life applications of the concepts he is teaching them. To this end, he had the class plant and tend a vegetable garden, which they occasionally harvest. With the help of a colleague, he then facilitates the cooking of a meal using the produce, which they all share. One of the reasons he made the extra effort to put in the garden is that he thinks it important for these city children to see the entire process of food production from seed to plate.

They used a significant amount of mathematics in planning the garden; how big to make the garden if they wanted so many different types of vegetables? how many patches to create, how many plants to get per patch if each plant required a certain amount of space, and so on. He also fosters an attitude of responsibility and perseverance with the on-going work tending the garden entails. Finally, harvesting the fruits of their labours and making a meal of it, gives these city children a sense of the continuity and life-cycle of plants very few of them would normally be exposed to in their circumstances.

To provide additional real-life experiences for his students, Tom also takes the class on as many field trips as possible on a very limited budget.

I'd love to do more trips, but you have to be wary of what you are asking of the families. $16 for some families is a lot of money. It is a chunk out of their grocery money or whatever. (Tom Interview)
4.4.3.2 Facilitating Student Reasoning

Although Mathematics is the discipline whose practice is most deeply rooted in logic and reasoning, not many elementary teachers teach mathematics as participation in mathematical reasoning. Doing so requires a very deep and connected understanding of the mathematics they teach (Ball, Lewis & Thames, 2008; Lampert & Cobb, 2003). However, logic and reasoning are not restricted to mathematics. Non-mathematical subjects also make use of reasoning practices in order to organize, to make connections, to draw conclusions. Since participants in this study were at various stages of building their mathematics teaching practice, it seems reasonable to explore their facilitation of student reasoning in subjects other than mathematics as well.

The lesson explored to illustrate Tom’s facilitation of student communication and reasoning strategies is a Social Studies Unit he delivered as an inquiry, rooted in student experience. As the Social Studies Unit addressed pioneer days, the class visited Pioneer Village as one of the important field trips of the year (This was the $16 trip referred to in the quote above). They went at the beginning of the unit, so the children would have personal exposure to ways of doing things 200 years ago, and they could refer back to their experiences throughout the unit. In preparation for the visit, Tom had the children think about what they wanted to know about pioneer days, and ask questions that would elicit the information they were interested in:

We introduced them to the idea of what 200 years ago meant. We created a timeline; so we are going to be learning about people who lived here 200 years ago. So what questions do you have about how people lived back then? That was the only prompt we really gave them, and they actually came up with some pretty good questions. When we came back from the trip, we started to categorize their questions about school and food and such. (Tom Interview)
Asking good questions is key to productive communications, because questions are the catalysts that ensure that thinking occurs, and that conversation flows (Robbins, 1997). The ability to ask a good question requires the questioner to identify the information he/she needs to find out, and phrase the question in a way that will elicit the required information. (Neményi, personal communication) Tom guided his students by organizing their questions into categories, setting up research stations that addressed those categories, and giving them headings in their research journal to prompt further questioning. Most importantly, he facilitated his students' success by focusing on their interests:

I think it helped that we really focused on what it was like for children back then, we really guided them in that direction. I think that has made it easier for them to connect to what life might have been like and to ask good questions. (Tom Interview)

Collecting complete and meaningful answers to their questions was challenging for the grade three students, even though Tom had his teacher candidate model how to find and construct detailed answers to their questions at the start of the unit.

Some of the girls had been asking whether girls wore dresses back then, so Tom’s teacher candidate showed them how to find the information, and how to expand upon it. The simple answer is yes, but providing details, makes the answer more interesting and informative:

In fact, they had two dresses; one they wore to church on Sunday, and the other they wore the rest of the week. They wore crinolines under their dresses and bonnets on their heads. Then, to think more broadly about the topic, you might talk about what the boys wore. (Tom Social Studies Interview)

In essence, she modeled how to link ideas, so that one initial question allowed students to learn a lot about a topic. Both Tom and Sydney were keen on teaching their
students to link ideas and carry on conversations, to prepare them for debriefing mathematical learning.

Even so, many of the students thought they were done after making a single point, so they had to be reminded several times, ‘Do not think you are finished. Look for additional answers.’ Tom made considerable effort to ensure that students looked for several pieces of evidence in answering their research questions. He introduced them to methods used by historians for discovering and gathering information about the past, and told them that they were now historians; they had to use the evidence at the research stations to find the answers to their questions about how pioneers had lived.

In conducting this Social Studies Unit on pioneer days as an inquiry, Tom exposed his grade 3 students to the processes of evidence identification, gathering and interpretation. He also reminded them consistently to be persistent in their efforts. Inquiry and persistence are two determining attitudes that significantly support mathematical work based on the problem solving model:

one of the ultimate goals of education is to teach kids to be persistent and keep going even when things are difficult. So when you use open questions, when there is not just one correct answer, and kids do not just sit there trying to guess the one correct answer in the teacher's head, at least hopefully means that they'll keep working and trying, because they know that there are lots of possible answers for them to work towards. (Tom Interview)

The expectation that teaching material be presented through inquiry in the social sciences supports learning in science and mathematics, because inquiry requires students to use such mathematical processes as identifying and sorting information. It requires reasoning to make predictions, analysis of the collected information, and the drawing of conclusions.
Tom worked hard to present many of his lessons as inquiries, in the belief that exposing his students to the processes involved in learning through inquiry would benefit them in many subjects for years to come. However, he found it challenging to conduct inquiries in many topics, due to the dearth of age appropriate sources for grade 3 students to use in their inquiries. In reference to the Social Studies Unit on Pioneer Days, he said:

One of the specific expectations in this unit was looking for different sources to find information about the past. ... So one of the teacher prompts was ’what can you learn about pioneer life from the journals of Suzanna Moody?’ or Laura Secord, - ... There is no way a 7 or 8 year old child is going to read one of these journals and understand any of it. Not least, because it is written in a vernacular that we do not use anymore. There are lots of examples of that sort of thing, where the suggestion is not age appropriate. (Tom Interview)

According to Tom, there are insufficient age appropriate resources available to support teachers’ efforts to present even the social studies curriculum as inquiry. It seems that the drive to facilitate student reasoning has a strong presence in the general direction and requirements of the curriculum, but additional resources need to be developed to make implementation more feasible.

4.4.4 Challenges to Teacher Efficacy and Professional Development

Opportunities for professional development in the educational field are usually driven from the top down, with people in key administrative positions providing the direction and funding for this type of training. One’s home institution can play an important supporting role, depending on the interests and expertise of the principal and colleagues.

The Ministry has really got involved in the schools and driving these kinds of professional development initiatives (TLLP) in the last six years or so. For the first six years, it was focused almost exclusively on literacy, and here it was almost always based on reading. Now we’ve moved into math; I am guessing almost every school in the Board is working on some similar initiative. (Tom interview)
In Tom’s case, the most immediate assistance was available from Sydney, a colleague who had experience as a teacher trainer in math education. Their collaborative work on the TLLP to learn Bansho gave him exposure to, and experience with, a good deal of early math pedagogy. More broad based support was provided by his school principal, who holds a Masters degree in mathematics education, and has a special interest in early numeracy. Her expertise and experience revolve around remediating children’s gaps in basic numeracy skills at an early age.

When I did my Masters at OISE, I researched some of the work of Robbie Case on central conceptual structures, and I found that a really interesting concept. At the time, I was teaching grade 5, and what I wanted to know was how to fix the problem in grade 5, because you cannot go back to the beginning. So what we are going to do here, is target the K - 2 group, using hands-on learning, and games, and we are going to be using the program 'number world' developed by a PhD student who studied with Robbie Case. (Principal Interview)

The Board driven goal of developing teacher expertise in teaching mathematics took the form of a central theme adopted by Haven Public School. All teachers worked towards the collective theme of ‘collaborative inquiry in mathematics’, with a focus on teaching students to communicate their thinking. To enhance the probability of effective communication, their intent was to embed literacy instruction into the mathematics curriculum delivery. Since the entire staff was focused on this initiative, they approached it as a team:

We are focusing on math, and we decided to focus on communication. I am guessing in most classes it is fairly similar: so, if we teach them how to explicitly communicate their thinking in math, then they’ll get better at it, is the idea. One of the reasons for teaching math through problem solving is that as well as learning about the concepts and procedures in math, they learn to communicate their thinking. So everyone in the school has developed some kind of criteria against which you can judge a student's response; how well they've communicated their thinking. (Tom Interview)
In Tom’s case, the bulk of the team work took place during the previous academic year, when he and Sydney developed the interactive math lessons for the Smart-board. He continued to use the lessons during the period of this study, and shared them with his current teaching partner. Unfortunately, the current teaming was less a collaboration than an expert-novice relationship (Cobb, 1995; Smith, 2007), since the new teaching partner was very new to the profession, had no mathematics pedagogy to speak of, and had not participated in the learning process of developing the materials.

During the early stages of their collaboration, Tom and Sydney looked for opportunities to observe other teachers using the problem based approach to teach mathematics. They suspected it would be difficult to draw the learning out of the children during debriefing, and they wanted to see it well done.

That part of the lesson when you bring them all back together on the carpet and they are supposed to be sharing and talking and engaging, is really challenging. We knew it was going to be the most difficult part of teaching math this way, so we wanted to go and see how it was working in other schools, and actually found it very difficult to find any schools that were using this method of teaching math very effectively, to be honest. (Tom Interview)

Collegial help may have been sparse, and the student population may have large gaps in their basic skills, but their Principal was committed to breaking free of these constraints. She recognized students’ deep seated problems, realized that they were dealing with deprivations that started long before students reached grade 3, and she has worked hard to remedy their situation.

Even when they start kindergarten, not having consolidated the basic number sense or counting that they should have by that point, and then they just fall further and further behind. We recognize that this is a real problem in this school. Our principal recognizes this, it is what she did her masters on, it is something she has thought a lot about; how do you remediate the basic skills for these kids who do not have them. We are trying different approaches. Last year, we took all the grade 3s and split them into ability groups, we did it once a week, and 6 staff
members, including the principal, would work with small groups (3-12 at most) and they would work on different things. Same topic but the work would be based on learning. (Tom Interview)

In summary, once the School Board turned its attention to developing teachers’ skills for teaching mathematics in the early grades, it set forth and funded initiatives to make that happen. Tom participated in a Board sponsored TLLP to develop interactive math lessons for the Smart-board in collaboration with a colleague. He benefited doubly from his participation in the TLLP. It gave him an opportunity to delve into the mathematics, and the colleague he worked with had extensive experience and expertise in mathematics pedagogy. Tom’s professional development opportunities were further supported by a Principal who was herself interested in early mathematics education.

Recognizing that they were educating a population of students with serious knowledge gaps in both literacy and mathematics, the Principal worked hard to facilitate the bridging of those gaps. Tom happily participated in the remedial teaching efforts organized by the Principal, but was often frustrated in his personal efforts to teach his students mathematics based on a problem solving approach, due to these same knowledge gaps.

When you read the Ministry guidelines on effective mathematics instruction, they say that grade 3 is where you start teaching about problem solving. And by the end of grade 3 they are writing EQAO, so you are almost expecting them to become expert problem solvers in the space of a year. But when the kids are coming into grade 3 - at the end of grade two, some kids struggling to count by 2s, then you are a little bit hooped. (Tom Interview)

In Tom’s assessment, the students he teaches at this school do not possess an adequate knowledge base in either math or literacy to handle learning math through inquiry. In addition, he does not feel that he has been taught, or shown how to teach math through inquiry in an effective manner, especially considering the problems he deals with in his classroom, on a daily basis:
There are these videos on the Ministry web-site, these highly scripted lessons where there are 6 students sitting in front of the teacher, supposedly showing you how to do it. Well, that is not showing you how to do it in a classroom like mine, where I've got students constantly being disruptive and not paying attention and not engaging in what we are doing, and have poor math skills and poor literacy skills. (Tom Interview)

Tom reflects that it would certainly help to have both the time and the opportunity to pursue the professional development he needs to learn to do this properly. He would jump at the chance to observe practitioners who have acquired expertise in teaching math through inquiry. He is particularly interested in learning how to facilitate debriefing through classroom discussion, and doing so in real classroom situations:

The opportunity to see real teachers, in real classrooms, do these things effectively; that is what is really going to convince people that this is the right way to teach math, if indeed it is. Because that is, what we are not seeing. (Tom Interview)

Tom’s contention that demonstration lessons in real classrooms would constitute an effective form of professional development is supported in the literature. Grierson and Gallagher (2009) found that many teachers consider demonstration classrooms the best form of professional development. Although this type of demonstration is not supported over the long term, it does tend to inspire, and can therefore have tangible effects on teacher practice.

Though Sydney and Tom found very few opportunities to observe other practitioners, as their TLLP project progressed, teachers came to observe their classes. Their efforts to promote better communication skills in their grade 3 students through the communication moves they taught them garnered significant interest:

We had people watching our literacy program, because we were incorporating this framework of the daily five... we had people watching a few math lessons as well. I think there has to be a whole ton more of that. They have to find us release time where someone will cover your class while you go watch someone who has been practicing this for awhile and is doing it effectively. I really, really think that we need a lot more of that. (Tom Interview)
Preparing lessons that work without a textbook is also very time consuming, as Tom discovered during the TLLP experience. When contrasting the inquiry method with teaching out of a textbook, he reflects:

It is definitely a very complex way of teaching math. You have really got to know the math. And that is the thing about teaching out of the textbook, you can pretty much get by, especially in the younger grades, by just reading the explanations in the textbook. It is not the best way of teaching, but you can pretty much get by just being one step ahead of the kids. (Tom Interview)

Through this TLLP experience, Tom learned that, before teaching mathematics as inquiry, he had to understand the mathematics in a connected way:

to teach math this way,… one of the major learning for us last year was that we really had to make sure that we understood what we were doing. And not only what we were doing that day, but how does this math link onto what we’ve already done, and how does it link onto what we are going to do next? (Tom Interview)

To create teaching materials that communicated the concepts effectively required a lot of planning and forethought. That was the only way he would be able to facilitate discussion driven by student ideas. He felt he needed to anticipate what students might come up with to be able to guide the discussion towards his intended goals for the lesson:

You cannot just do this on the fly. You have to choose the lesson problem very specifically, you have to think about what the students might do, is this question sufficient to draw out the learning I want to draw out that day? You have to guess what the students might come up with, so that when you are debriefing, it is not just random. It cannot be random, because then it is just a disaster. (Tom Interview)

The choice of a challenging yet accessible problem, the thorough planning of the lesson, the anticipation of student responses, are all consistent with best practices advocated by NCTM (Smith & Stein, 2011) for orchestrating meaningful dialogue in the mathematics classroom.
Tom was ambivalent about the value of teaching math through inquiry, and considered that educational research is frequently on a different page from educational practice:

What you are seeing is a major problem in education. It is just the disconnect between theory and practice. When you are reading about 'reform math' (or whatever you want to call it) everything you are reading about makes sense in terms of the research that is being done. But often when it is applied, it almost never works out as it is supposed to; as people think it should. (Tom Interview)

In Tom’s experience, his grade three students were overwhelmed by the cognitive load of learning math through inquiry. He identified the gaps in his students’ knowledge in literacy as well as math as the root of their inability to cope. He also realized that the learning objectives in math need to be defined in much smaller increments to allow students to feel successful on an on-going basis. These realizations indicate that Tom is a good teacher, but one who needs a lot more professional development, both in content knowledge, and mathematical knowledge for teaching, to teach math well.

Based on the vast amount of time the preparation for inquiry based math lessons took, Tom also expressed the concern that it is unrealistic to expect a teacher without all the release time he and Sydney had for developing the lessons, to teach like this on a regular basis:

The only way we were able to learn to teach math this way, and then to develop the materials we needed, was the release days we took. We probably spent a dozen days developing the lessons we are using this year that other teachers simply do not have. So it is completely unrealistic to expect other teachers to be teaching this way on a regular basis. It takes a long of time, especially if you do it properly. If you take the idea of lesson study... (Tom Interview)

The loss of a number of PD days over the last few years also means that much of the professional development available to elementary teachers takes place at staff meetings:

So you are talking about an hour and 15 minutes after school, when people are exhausted - but we have some things there, to do with math and to do with communication. So...in some ways the support is there, it is just finding the time to access it. (Tom Interview)
At the elementary level, two of the PD days are allotted to writing report cards, which is welcome according to Tom, because it can be an overwhelming task. In doing the report cards, one of the problems with reporting on achievement in mathematics is:

that you have to report on each math strand separately on the report card. So I have to give them a mark for geometry, a mark for data management, a mark for number sense and numeration, a mark for measurement, a mark for patterning and algebra. In an ideal world you might do that (overlap the strands) to make it more meaningful. (Tom Interview)

In other words, the requirement of reporting on each of the strands separately is interfering with presenting the material in an integrated way. Lack of strand integration in the presentation of the material decreases the likelihood of children developing a unified knowledge structure in mathematics.

In Tom’s opinion, the challenges to teacher efficacy and teacher professional development are largely attributable to limited opportunities and severe time constraints. The lack of opportunity to actually see what competent teaching of math through inquiry looks like is a huge challenge. The highly scripted exemplars posted by the Ministry do not address important and problematic situations that arise in real classroom, and there is no time or opportunity to observe the classrooms of effective practitioners.

The second challenge concerns the relentlessness of the ongoing and administrative demands on a teacher’s time. He feels that this lack of time limits the pursuit of professional development opportunities, and precludes the development of expertise and resources that would enable teachers to teach mathematics through collaborative inquiry requiring the communication of mathematical ideas.
4.4.5 Summary

Tom is a dedicated, moderately experienced teacher who is sensitive to his students’ needs. He makes a lot of effort to present the curriculum in a connected, experiential way, so what students are learning will be meaningful to them. My work with Tom explored his teaching practices in mathematics as well as social studies. I encouraged him to reflect on teaching strategies he uses in non-mathematics subjects, for the purpose of comparing them to teaching strategies he uses in mathematics. By contrasting the strategies like this, he might more easily identify areas of need or dissatisfaction in his mathematics practice, and possibly use his expertise in social studies instruction to help advance his mathematics teaching. The themes that emerged from Tom’s teaching practices were:

- Tom thinks that it is important for children to experience real life applications of what they are learning about. So he takes students on relevant field trips, keeps a vegetable garden, and teaches social studies through inquiry.
- Tom feels frustrated by the lack of age appropriate resources for teaching subjects like social studies through inquiry.
- He would like to see what effective teaching of math through inquiry looks like in a real classroom, contending with real problems.
- Tom reports that the release time he and Sydney had in the TLLP project allowed him to delve into the math and the curriculum in a way that the average teacher does not have the time to do.
- He believes that time constraints pose very significant challenges
  - for pursuing professional development opportunities, as well as
Tom reflects that many of his current students are incapable of learning math through inquiry due to the serious knowledge gaps in their early math and early literacy.

He is ambivalent about teaching math through inquiry to them, because he feels that this teaching strategy is overwhelming for them, and makes them feel that they cannot do mathematics.

He intuitively felt that many of his students would benefit from a math program where the learning objectives were much more finely divided, allowing them to experience small successes on a daily basis.

Tom used technology in the classroom to increase student engagement.

He identified two challenges to teaching the math as integrated strands:

- the requirement that student achievement in each strand be evaluated and reported on separately.
- the severe gaps in many of his students’ basic mathematical and literacy skills that prevented them from handling problems that address multiple strands at one time.

4.5 The Case of Anne

4.5.1 Background Information

Anne has been teaching kindergarten to grade 2 for more than 25 years. She earned a Masters’ degree at OISE in the early 1990s in the Department of Curriculum, Teaching and
Learning, and was interested in Dr. James Cummins’ work on second language learning. Subsequently, she worked in ESL, and was a resource teacher for 5 years: “I was in program support, supporting teachers with their literacy planning and their literacy understanding, so I am more a literacy person than a math one (Anne Interview).”

Though she self-identifies as ‘more of a literacy person’, because she has more formal training in teaching literacy, she has worked diligently to improve her mathematics practice on her own. Early in her career, even before she had a grade 1 class, she took a session on ‘Mathematics Their Way’ by Mary Baratta-Lorton. It was a one week course during the summer. In retrospect, Anne feels that the teaching philosophy and techniques she learned there have stood her in good stead over the years:

I find her influence has carried on. She was very much into using manipulatives, using objects. Now it is a given that children are going to use manipulatives. But at that time the idea of using objects, of actually sorting with real things, of finding ways of showing your thinking was revolutionary.....in terms of understanding around composing and decomposing numbers, she had many ways of showing it….With that course I saw, that especially with young students, you had to show the same concept in many different ways for them to really understand it. (Anne Interview)

In terms of her own mathematical understanding, Anne makes two points. She claims she had no difficulties with mathematics until grade 13, so she is confident and competent in early mathematics. However, she has no patience for endless practice questions. “Once I got it, I didn’t see the sense of practicing.” (Anne Interview)

Over the years, she has accumulated many mathematics teaching resources which she incorporates into her teaching, and widely encourages her students to use. She constantly strives to improve her mathematics teaching practice, but finds time constraints in the lesson itself make it difficult to get children to express their observations as much as she would like. During the debriefing of an activity, she will often encapsulate the main points instead of
getting the children to tell her. She commented several times during classroom observation 'I should have had them tell me, but we were out of time'.

### 4.5.2 Communication in Mathematics

Teaching grade 1 entails providing a lot of relevant experiences for children to make sense of the information they need to assimilate. The teacher must facilitate students’ acquisition of the attitudes, habituation of the processes, and practice of the skills they will need to pursue further learning. Anne thinks that the age group she deals with has to ‘work through it’ to understand many of the concepts she has to teach them. Consequently, many of her teaching strategies are connected to physical activities, and are play related.

### 4.5.2.1 Teaching Students to Communicate Mathematical Ideas

Anne agrees that mathematics is about problem solving, but, at the grade 1 level, she sees it as a balance between problem solving and creating appropriate problems to solve. She also feels that, in grade 1, the teacher has to do a lot of modeling of the skills students will need to solve problems. For example, in cases of students whose number sense is weak, this might involve habituating processes that involve one-to-one correspondence, to give students the physical experience they need to make sense of the process:

> Which is sometimes a little more explicit teaching, for more experiences. In grade 1, it might be using 10-frames, and maybe sitting in a whole class with a 10-frame and 10 counters and just every time I ring the bell, put the counter on. And just experiencing counting up to 10, and what it looks like counting back to zero. And now show 6, show 7. Maybe later they are doing some problem solving and using the 10-frame as a tool. They might bring that experience to how to use it. (Anne Interview)

An important aspect of scaffolding manipulative use involves giving the children time to explore the tool on their own before asking them to do mathematical work with it.
When I was taking the 'mathematics their way' course, with any sort of materials you were going to use, she would have them do a week of free exploration. Children need to explore first, to get some of the play out; even base 10 blocks. Other years in my grade 1, I've had base 10 blocks out as a bin to play with. And they build with base 10 blocks. So later, when I want them to use it in a more formal math format, some of that play is out of the way. And I will say, 'now I want you to show the number 27 with it'. But if you do not let the children explore the materials before you expect them to use it in a math context, they are going to spend half an hour figuring out how the material works. So you have to give them that time to explore the material. (Anne Interview)

She then makes extensive use of manipulatives to familiarize children with the many forms a given quantity can take. Working with her teaching partner, Roma, they made up manipulative based activities for students to practice counting by 2s, and 10s, composing and decomposing numbers by representing a specified quantity in different ways, deciding on relative quantities (more, less and equal), engaging in a number based board game, even bowling (where they have to count the number of pins knocked down/still standing). The manipulatives and recording sheets for each activity were stored in plastic tubs, so they refer to the activity centers as ‘math tubs’. Two to four students would be assigned to each tub during small group activity. There were ten to twelve unique activity tubs for each topic; half of them used in Anne’s class, the other half in Roma’s.

When students had mastered the tubs their class started with, the classes switched tubs. The two teachers were in constant communication regarding student progress with the math tub activities, learning from each other how to make them work better.

In facilitating students’ experience with the many physical manipulations and different representations of various quantities, Anne was planting the seeds that prepared her students for an intuitive understanding of addition and subtraction.

In January and February, I will be getting into addition and subtraction equations. So in November-December, we do a fair bit of composing/decomposing numbers with materials without the equation, and talking about what does the number 10
look like, 'it can be 4 + 6', '3 and 7', and talking about number combinations. So
that later, when I work through computations and actually record it with symbols,
hopefully all that experience will come into play in understanding. (Anne
Interview)

Furthermore, she ensures that her students develop a unified understanding of
addition and subtraction through the kinds of representations she has them work with. For
eexample, using only red and yellow counters, she asks students to represent the quantity 8 in
different ways. Looking at the various bicolour displays, and describing them in different
ways, she is implicitly connecting the operations of addition and subtraction:

One thing I like to do a lot once I get to equations, is the idea that, if I can see 3
and 7 in different color cubes, is getting to what are the four equations? Seeing
that connection of addition and subtraction all along. That it is not 'let us learn
addition' and 'let us learn subtraction, but it is seeing that they are two sides of the
coin. I think the more we play with numbers, and look at it as just pulling them
apart and putting them together, that is all addition and subtraction.(Anne
Interview)

Anne also makes a consistent effort to show her students that using numbers can
improve efficiency, by incorporating number applications into classroom norms. Part of this
practice is based on the prescribed curriculum; they discuss the date, the temperature, the
number of students present, every morning. However, a significant component of this
practice is based on Anne’s own conviction that students need to see numbers incorporated
into everyday living, so they can come to appreciate the important role numbers play in
organizing activities and improving efficiency.

I think children in primary can see how math can help them solve a problem if we
are trying to figure out how much pizza to order for a pizza party for the class. If
we are trying to figure out how many students are here today. It is important for
us to try to build in as many real life contexts for them as possible, so they see
that it connects to real life. (Anne Interview)

In her commitment to showing students how numbers can increase efficiency, she goes to the
extent of digitizing students’ materials and assigning student numbers:
I number all my students, and I number all their materials, such as their agendas. So when I try to find out whose agenda is missing, we sequence the agendas, so I can quickly say, 'oh, #11 is missing'. And that is seeing an application where numbers are faster than looking at the names. (Anne Interview)

Anne incorporates numbers into daily activities, models and encourages manipulative use, all in an effort to set her students up for successful problem solving. Even before students have learned operations in a formal way, Anne and Roma introduce problems that can be solved based on the composing/decomposing numbers activities they have practiced through math tubs. One of the math problems they implemented towards the end of November, dealt with number sense. After considerable work on composing and decomposing 10 through paired and small group activity in the form of the math tubs, Anne and Roma presented the following problem to their students: “Draw a picture of all the members of your household if there are only 10 feet living at your house.” Anne tried it with her class in the morning, she and Roma discussed how it had gone, then Roma presented it to her class in the afternoon.

When she presented the problem, Anne reminded students of several things:

Remember we were talking about math thinking after math tubs yesterday. We looked at the pictures I took during math tubs, and talked about what you were doing on the picture. After we do this problem, you can talk about how you did it. She modeled how to approach the problem through a personal example. She also showed students how to illustrate the answer with a quick sketch:
If I think about me, I use a stick figure. At my house there is me and my dog.” She put tally marks under each foot for easy counting. Then she asked: Do I have 10? (No.)

When you solve the problem you can make it up. You can think about people. How many feet does one person have?

(2)

How many feet do fish and snakes have? (0) You can show them on your picture, but they count as zero.

Could I have three dogs at my house? I can count their feet by two’s 2, 4, 6, 8, 10, 12. That is more than 10. Could we all live in my house? (No.)

You already know the answer: 10 feet. The question doesn’t ask how many feet? When you show your math thinking you can use words, pictures and numbers. But remember, these are math pictures. Do not take a lot of time drawing a beautiful picture, because you will not have time for more math thinking.

After the students had been working awhile, Anne saw that many of them were having trouble answering the question, because they were literally depicting the people and animals living at their house. So she announced:

These are pretend families. When you draw on the sheet of paper you have, you can probably fit four different ways of showing 10 feet. What counting tools can we use to help us?

- student 1: the hundreds chart
- student 2: a rekenrek (an arithmetic rack with two rows of 10 beads)
- student 3: easy count
- student 4: cubes.

You are welcome to use anything in the room.

As she circulated among the working children, Anne made suggestions for improving the documentation of solutions. She told one boy to add some numbers to his drawing to show what he meant. She urged a couple of girls to start recording their solutions rather than repeatedly pushing the beads on the rekenrek by two’s as they recited family members.

After some minutes, she asked students to stop and stand behind their chairs.
Sometimes when working on math, it helps to see other people’s work. Put your hands behind you and walk around to see what others are doing. After you have visited some other people, go back to your desk and keep working.

During debriefing, she projected the images of several solutions and conducted detailed discussions around them. One of the girls had the number 2 under each person. When asked to talk about her solution, she recited the members of her family: Mom and Dad is 4, plus me is 6, plus my aunt is 8, plus my grandma is 10.

An incorrect solution with only 4 people and no animals on it, was used to show how to check if the solution is correct. During the process of checking, Anne also facilitated self-correction by the student. She recognized half formed ideas, and verbalized encouragements for fleshing them out. Throughout the discussion she commended organization and clear work, while recognizing the importance of evolving ideas:

It looks like Adam crossed some things off, which is o.k., because you can change your mind, or think of new things. On the second side, you got a little more organized, and it is easier to see what you were doing when you are organized.

After several other students described their work and Anne facilitated self-correction where necessary, she addressed the author of a very organized piece of work:

You were very organized on your paper. It is easy to see what you were doing, because it is so organized. You divided it into four sections with these lines, so it is easy to see the different solutions.

The student did not want to talk about his work, so she analyzed it for the class:

I think here he is thinking about his family and counting by two’s to count feet. Here he has two animals with four feet (4+4) and a person (+2), maybe himself, so that also makes 10. Here he was thinking about his friends and 2+2+2+2+2 also makes 10.

In discussion with me after this incident, Anne expressed disquiet about sharing the last set of solutions with the class despite the reluctance of the author to talk about his work:
Maybe I should have respected Henry’s wishes and not shown his solutions, but they were the best in the class and I wanted to share them with the others.

In reference to facilitating the student expression of mathematical thinking, Anne identified factors peculiar to grade 1 students that are limiting in some areas of communication:

At this stage, some of the students in grade 1 cannot use writing as an easy way to express their thinking. Even with oral, I find the challenge is having the right question to ask the student, and that is something we are always working on. If you have a discussion, they find it hard to listen to each other. So that takes class team building, because they do not necessarily show respect to each other when they are speaking, and really listen to the ideas of the person.... and you can only listen for so long. So you have to build in strategies so that not every person is speaking at the same time. Maybe you are hearing a member of a group, or maybe you are hearing a few students at one time. (Anne Interview)

Developing classroom norms to habituate students to have patience with their classmates and listen to each other’s ideas is one thing. But how does one keep the conversation on mathematics, or even facilitate student engagement in meaningful mathematical dialogue at this age?

I think maybe, looking for problem solving opportunities as they come up during the day. Certainly planning ahead is always good. But if something comes up, saying, ‘oh, I wonder if we can figure that out,’ or taking the time to ask someone, 'how did you figure that out?’ So if one of the students notices something and it has a math context, I can ask 'why do you think that?’ 'how did you know that?’, But the biggest enemy is time. Taking the time to find those opportunities and get into it. Even when we are finishing a lesson and coming back to talk together, you never build in as much time as you need. So you just start talking about it, and it is lunch, or recess, or time to move on. (Anne Interview)

Anne is an experienced teacher with an integrated understanding of primary mathematics and extensive knowledge for teaching mathematics to young learners by means of manipulative based activities. As such, she knows that when students learn mathematics, they do not distinguish between strands; it is the curriculum writers who do that:
The math strands are arbitrary words we put on math ideas. Students do not think, 'I'd better stop thinking about numeration, now I have to think about geometry'. And numeration really flows over everything. When you are looking at the number of sides and angles on geometric shapes, you are still bringing in numbers, you are still comparing numbers, it still comes into so many other areas. I think part of it is, if we are teaching students to think as mathematicians, it is approaching any situation, as 'what is the math here?' so it is not 'what is the geometry here?' it is 'what math can we pull out of this situation?' (Anne Interview)

4.5.2.3 Technology in the Classroom

Anne did not have a Smart-board in her classroom yet, but she used a computer and data projector combination daily, for presenting material and modeling activities she wanted students to participate in. She also used a document camera to project student work when the class was debriefing an activity. She frequently used her personal i-pad to photograph student activity during small group work. During the debriefing of the lesson, she would often project one of her i-pad records and ask the student pictured to explain what he/she was doing. She scaffolded student reporting on an activity by reminding him/her what the activity was about.

The i-pad record also served to remind her of student success at activity centers that did not require students to complete a recording sheet. She found this visual reminder useful when it came to evaluating student progress:

One of the challenges is documentation of their learning. So often we are having them do something on paper so that later, when we are trying to figure out who was able to do what, we have something physical to look at. When we plan a set of 5 math tubs (activity centers), not all of them have a recording sheet. In this round of 5 right now, 2 do not have recording sheets, so I try to get around to take pictures of those ones, and not so much of the other ones. But it is very easy to get to writing report cards and realize that there are some students you just do not have a good handle on what they know right now. So then you have to pull the student individually and try to elicit more information. (Anne Interview)
In addition, Anne projected Internet-based information to facilitate class discussion. When discussing the seasons, she projected the weather report and facilitated a discussion of the meaning of temperature, emphasizing the variation in temperature:

<table>
<thead>
<tr>
<th>Yesterday</th>
<th>Today</th>
<th>Thursday</th>
<th>Friday</th>
<th>Saturday</th>
<th>Sunday</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2°C</td>
<td>0°C</td>
<td>6°C</td>
<td>9°C</td>
<td>10°C</td>
<td>13°C</td>
</tr>
</tbody>
</table>

-2°C is colder than 0. This afternoon, it will be +2. Tomorrow, it will be warmer... let us look at Sunday. It will be 13 degrees. Is it getting warmer or colder?

By questioning students, she established that the increasing numbers meant that the weather was getting warmer. Talking about the numbers lead to the definition and use of a thermometer, which allowed her to demonstrate an interactive program she found on the Internet that depicts typical human activities at specific temperatures. Through questioning, she had the children predict the temperature on a hot day, then she set the on-screen thermometer to the temperature they suggested (30°C). In response, the program changed the scene from a snowman building activity to scantily clad people frolicking on a sunny beach. “In a flurry of enthusiasm, students asked for different temperature settings (Fieldnotes)”.

Anne used a variety of computer applications she had found on the Internet to engage her students. Many were related to the applications of numbers, some straight out counting, to reinforce number shapes and number sequence. For example, on student request, she frequently projected a count-down app. that started from 60 (because grade 1s like large numbers) during the settling down phase while students entered the classroom after lunch. Many of the students would join in the count-down, counting out loud, along with the application as it ran.
She also employed computer games to engage children in literacy activities. For example, to drill spelling in a fun way, she used a computer game that was similar to hangman, except that score was kept with a mouse trying to snatch cheese from under the nose of a sleeping cat. She projected ten words students had learned, to provide some scaffolding. The unknown word was one of the projected words. Then she started the game, having students take turns to guess possible letters in the unknown word. When student guesses were repetitive or not productive, she would discuss strategy with students. As the number of wrong letter guesses increased, the cat went through stages of waking up. One eye blinked, the tail moved, the eye opened, the other eye blinked, etc. In the examples I observed, the children always got the word before the cat was fully awake.

4.5.2.4 Teaching Literature versus Teaching Mathematics

A difference in comfort level between teaching literacy and teaching mathematics started to emerge when we got onto the topic of differentiated instruction. This arose when Anne explained that students entering a class, even in grade 1, have a range of competence levels in the expected skills. A significant amount of informal mathematical understanding is expected to have evolved by the time students enter grade 1. As grade 1 represents their third year of schooling for most students, even a limited amount of formal understanding might be present in some. They have certainly had a variety of experiences, and sufficient time has elapsed to magnify differences in students’ rate of development. In Anne’s experience, there are several significant differences in competency, which she specifies as:

Some come in with strong number sense on a conceptual level. What a set looks like, which is a bigger set, which has fewer, some have strong rote counting skills, some students easily form numerals, read numerals and write numerals, some may recognize a few numerals, maybe are not yet counting with 1-to-1 correspondence. Generally, you expect students entering grade 1 to have 1-to-1
correspondence, but some of them do not. Certainly, in September, we are still
doing some beginning counting strategies just to see who is doing what, and who
knows what. (Anne Interview)

Given a new class of students, Anne has the students engage in activities that will
allow her to see students’ different stages of development, so she can gear subsequent work
to their needs:

September is a lot of observing and just seeing what they can do. We generally
do a lot of sorting and patterning in September, so as you are walking around, you
might observe some of the (counting) strategies they have. Roma and I have
talked quite a bit about differentiation this year. We both feel that we are quite
effective at differentiation at literacy, but it is harder to differentiate in math.
(Anne Interview)

On probing, she reflected that, in mathematics, she is ‘teaching the curriculum more
than the individual student’, because it was easier for her to identify the next steps the student
needed to conquer in order to progress in his or her reading or writing. In teaching math, she
was not so clear on the incremental structure of student understanding.

I guess it is easier for me to see literacy on a continuum. I understand that math is
similarly on a continuum, but in literacy I have a better sense of here is reading
skills on a continuum, and I look at where each student is, and where they are
heading. I have less of a sense of that when I look at my students as math
learners. Of where they are and where they are heading. (Anne Interview)

During classroom observation, I saw small group activities of varying difficulty as
well as activities that had different access points, to accommodate students of different
competence levels. So it was not that class activities did not incorporate differentiation, it
was more that the differentiation was not as sophisticated and individually geared as it may
have been in literacy.

Some easy differentiation; sometimes when I've had a recording sheet, I've asked
a student to do half of it. Or sometimes when we are representing a number, we
will plan some numbers for some students and smaller numbers for other students.
Certainly as we plan, we try to think of our students as we know them. And who's
going to struggle with this activity, and who might need an extension. But it is easier for me to differentiate in literacy than it is in math. (Anne Interview)

Students seemed to be assigned to the work stations in a strategic manner, with less advanced children working at the easier activities in many cases. However, students working in pairs often had significantly different levels of competence, though the activities themselves provided a variety of access points that could accommodate students of varying abilities. Anne was still testing for the optimum pairings of her current students, and was slowly working towards pairings of similar abilities:

Each time you try a different combination, because we are looking at both personality and learning style as well as the skills they've shown us so far. Ultimately I think you want students of a similar ability, or at least close in ability, because if you do go too far apart, I think one will take over. (Anne Interview)

One aspect of teaching literacy and teaching math that converged for Anne, was that part of student response, that is based on the student’s personality. The student’s desire to feel safe versus his/her willingness to take a risk will determine his/her actions. Whether it is the challenge of having an estimate that exactly corresponds to the calculated value, or getting the spelling right, in Anne’s experience, the student’s willingness to take a chance is a huge determinant in how he/she approaches the task:

The challenge with estimation is that as much as we teach them that it is a smart guess and it is not going to be the same as your actual measurement, they want it to be the same. And I have had years of students erasing or crossing out the number, and changing their estimate so it is the same as their measurement ... When children are starting to write...a lot of it is personality. Some of them are quite happy taking a risk in writing the first letter they can hear in a word, and some children only want to have correct spelling ... Or if you are reading a book and you do not know a word, well, make a smart guess. Look at the picture, look at the first letter, what makes sense? So I think that idea of smart guesses comes in everywhere. (Anne Interview)
The ‘smart guesses’ she refers to in teaching literacy rely on reasoning. Teaching children a strategy of looking at the illustration, deciphering the first letter, and deciding what makes sense when guessing what the unknown word might be, is teaching them to reason. In essence, strategies like this reinforce mathematical thinking, if we consider that mathematics is all about reasoning.

4.5.3 Supports and Challenges

4.5.3.1 Co-Workers and Professional Development

The collaboration with Roma on their math manipulative based student activities, their on-going dialogue about the activities, the problem solving, and the various teaching strategies to elicit student mathematical ideas, have been a rewarding experience for Anne. She feels that she and Roma have a very similar approach to teaching, and finds it easy to work with her:

I think one thing that is helping this year, is that my teaching partner and I are doing basically the same lessons, and we are not usually right at the same time. Whichever one of us does the lesson first, says 'this is what happened', and we can learn from each other that way, because if one of us does it in the morning, and the other is in the afternoon, or even a day apart. So that really helps: talking through what worked with another class. (Anne Interview)

The grade 1 teachers were also part of a learning community facilitated by the Superintendent. Teachers from three smaller schools, where the opportunities of team sharing within the school were limited, worked together. At the grade 1 level, Anne and Roma worked with grade 1 teachers from the two other schools, in an abbreviated lesson-study format. They had one day of release time to watch a Marion Small web cast, with time built in to stop and have local discussions. On the same day, the group planned a lesson
jointly, which was subsequently taught by each group member. Everyone was to observe, and be observed by, at least one other group member during lesson delivery:

> It is always interesting to see how someone else approaches something. Teaching is very insular, and you always assume that what you are doing is what everyone else is doing. (Anne Interview)

The limited opportunity for classroom observation did provide a glimpse into fellow practitioners’ approach to implementing the joint lesson, even though the primary objective of the classroom observation portion of the project was to see student response: “When we observed, the focus was on observing the students it wasn't observing each other as teachers (Anne Interview).”

Once all the lessons were taught, the group had another half day of release time to meet and debrief the taught lessons. Due to the wide range of experience and expertise among the group members, the debriefing was less helpful to Anne than she would have liked. Much time was spent on delivery strategies of more interest to novice teachers. As the description of her teaching strategies and comments show, Anne has a good understanding of how to deliver mathematical ideas in an interactive way. What she would like to find out more about, is how to elicit mathematical ideas from students orally:

> I think it could have gone deeper in our discussions in the debriefing. Where we needed to go was looking at 'what are the prompts we are using with students?' 'what is the language we are using when we are debriefing the lesson with students?' 'what language can we use to pull out their thinking?' and we just got started on that in our discussion. I think we could have taken more time on that. (Anne Interview)

4.5.3.2 Administritive Impact on Teachers’ Work

Reflecting on how the changes in the written mathematics curriculum over the years of her teaching experience have affected curriculum delivery, Anne maintains that the theory
in front of the expectations has remained consistent, but the delivery and support have varied. When she started, her Board had a local curriculum which cycled back through the strands, a concept she thinks is important. In the common curriculum, the expectations were well defined for grades 3, 6 and 9, only:

So if you were in grade 1, you had to look at the grade 3 expectations, and figure out what that might look like in grade 1...now it is much more clear for us what exactly the children need to understand. (Anne Interview)

The idea of communication in math has always been there, but in the penultimate curriculum it was much more language based:

There was a lot more focus on explaining your thinking and on math vocabulary, on being able to talk about it. So those of us who had many English language learners, found it a difficult program to teach, because the children didn't have the language, or weren't able to really explain themselves. (Anne Interview)

The way curriculum is written definitely influences delivery. Different versions can create implicit supports (cycling back through the strands) as well as new hurdles (more emphasis on language when working with ESL students). As an experienced teacher with a coherent understanding of early mathematics and significant knowledge for teaching mathematics in an experiential way, Anne finds that the reporting expectations of the curriculum pose obstacles to integrating the strands when teaching mathematics. When asked whether she integrates the mathematical strands in her teaching, she responds:

I'd like to do a better job of it. Part of it is having the January report. When it comes up, we know we have to report on four out of five strands. So what do I do if I haven't really touched on this strand? So in my whole class teaching I do not tend to be that integrated. But I try in my math tubs. I usually have more than one strand represented, so that at the same time, they are certainly working on more than one strand. (Anne Interview)
She finds it hard to balance the need to come up with specific evaluations in all the required individual strands for all students in each reporting period, with the time required to delve into topics more deeply and promote rich discussions:

Knowing that a reporting period is coming up, and we have to touch on so many concepts, so that if we really want to get into really rich discussions, then all of a sudden you realize 'I haven't touched this strand at all,' then all of a sudden you feel you have to rush through something in order to report on it. (Anne Interview)

4.5.3.3 Resource Materials

Anne has accumulated a significant number of resources over the years. “I am happy to have professional books to read on my own, to get ideas from, or video resources to watch (Anne Interview). She owns a number of math books she ‘goes back to’, and there is a Provincial Guide to Effective Instruction in Mathematics in all the strands, now available online, which is ‘fine’ according to Anne:

A lot of times it is just taking the time to go through resources and look for things. I think it is important not to get sucked into a cute activity or a fun activity, and always know that you are choosing an activity because it is what your students need at that time. (Anne Interview)

When she finds ideas appropriate to her students’ needs, she incorporates them. Pictuest is a web site popular with teachers, where many people post and look for teaching ideas. However, you have to look at the posts with a critical eye, she admonishes, and always ask yourself:

Is this what my students need right now? So one of my math tubs uses popsicle sticks to make tallies to show the 5s. I actually did find that idea there, but also knowing that that fits in with where my students are now. I wanted to reinforce the tallies more kinaesthetically. So rather than just drawing them with a pencil, I knew that placing the sticks would reinforce it in a way that some students needed. (Anne Interview)
Since her practice is largely manipulative based and involves a significant amount of problem solving, she does not like the restrictions imposed by student workbooks:

I am not a fan of student workbooks, because it pushes you into using the pages in the workbook. It has been bought for you and you feel some obligation. So last spring the principal asked if we wanted the workbooks for this year, and we had some choice. So Roma and I said no, and asked if we could have the same amount of money to buy manipulatives. So we were able to add manipulatives to our grade 1 supplies. (Anne Interview)

4.5.4 Summary

Anne is an experienced teacher with a solid content knowledge of elementary mathematics, who has developed an extensive repertoire of manipulative based activities for teaching mathematics in an interactive way. Her knowledge acquisition has been self-motivated and often self-funded. My work with Anne focused on identifying her teaching strategies for communicating mathematical ideas, and eliciting her perspectives on teaching mathematics.

The following themes emerged in Anne’s practice:

- Anne sees herself as more of a language person than a math person, because her formal education has been in literacy, and she says she has a better sense of the stages of student development in reading and writing than in mathematics.

- Anne’s mathematical practice is built on the conviction that students ‘have to work through it’, in order to understand concepts. She builds student competence in early mathematics by giving her students experiences with manipulatives with the intent of creating an intuitive understanding of mathematical concepts. She says students must come to see the manipulatives as a math tool, not as a toy. Then she has them use manipulatives to convey meaning in several ways and to connect operations.
Anne makes a conscious effort to incorporate the use of numbers into the daily activities of the classroom to show students the usefulness of numbers, and increase the amount of math talk students engage in.

Anne believes in building students’ problem solving skills from the earliest stages of math instruction. So she does a significant amount of modeling, scaffolding, and guided instruction to embed basic skills students will need for problem solving, and introduces word problems based on composing and decomposing numbers, even before they have started on operations formally.

Anne feels that lack of time is the biggest enemy in engaging students in deep mathematical dialogue. There are so many topics to cover, that if they take the time for deep dialogue on one topic, they have to rush through others.

Anne makes wide ranging, effective use of technology in her classroom in three ways. She uses it to engage student interest, to scaffold student reporting on activities, and as a record of student work that will facilitate evaluation.

For professional development, Anne enjoys comparing notes with colleagues on teaching experiences and collaborating with her grade 1 teaching partner to develop activity centers for their classes. They help each other implement them through an ongoing dialogue.

Anne felt that the Superintendent promoted professional development initiative left her wanting, because much of the debriefing time was spent on rudimentary topics. This left insufficient time for exploring the more nuanced details of eliciting student thinking that would have been of more interest to her.
• Anne has been self-driven and rather self reliant in her professional development over a long career. Her resources for personal development consist of math books she has collected, the on-line Provincial Guide to Effective Instruction in Mathematics, and the Internet, which she searches critically, for innovative apps and teaching ideas her students would benefit from.

• Anne finds student work books in math too constrictive, and prefers to spend her budget on expanding her collection of manipulatives.

• Anne has found that the evolution of the written curriculum over the years has influenced the way math is taught in both positive and negative ways. One version of the written curriculum changed the organization of the taught material, while another put much more emphasis on student explanations. For example, the organization of the locally developed curriculum some years back, supported student understanding by cycling back through the strands. A subsequent version put heavy emphasis on student explanations. That version posed additional challenges in classes where many of the students are just learning English as a Second Language.

• As an experienced teacher, Anne knows that integration of the strands is vital for students to develop a unified understanding of mathematics. However, she feels compelled to separate the strands in whole class teaching, due to the necessity of evaluating each student on each strand separately.
4.6 The Case of Roma

4.6.1 Background Information

Roma was teaching grade 1 for the second consecutive year during the 2013–2014 school year when this study was conducted. Her formal education includes a BA in music, a BA in education, and an MA in early literacy. She had been working as a literacy coach, specializing in reading recovery, for twelve years prior to her return to the classroom two years ago motivated by financial constraints in the Board. She has been in the teaching profession for 21 years, and is a particularly interesting participant, because she is somewhat of an anomaly for a primary school teacher. She professes: “I love math, love it. In grade 13 math was my highest mark, but I also loved music.” (Roma Interview) Torn between math and music for a career, she chose to do a degree in music, and wound up teaching.

With so much formal training and experience in early literacy, she prefers to teach literacy. This becomes obvious when she reflects on the daily routines in her classroom in order to incorporate more mathematics:

When the kids come in, in the morning they go to literacy activities. I am thinking - could that be math activities instead? Because you tend to focus so much of your day on literacy. Because you enjoy teaching it more. (Roma Interview)

In this reflection, Roma recognizes that some strategies she routinely employs in teaching literacy, are not used to the same extent in teaching mathematics.

4.6.2 Teaching for Understanding

Roma is a firm believer in teaching for understanding, as her statements and teaching strategies show. The teaching strategies described in the following sections are often rooted in literacy instruction, because in her reflections on how to improve her math instruction
Roma often came back to them. Literacy instruction was the knowledge she felt sure of, so that is where she started to connect new knowledge to her existing schema (Skemp, 1989).

Of all the participants, Roma devoted the most amount of time to discussing episodes of classroom observation. She gave me an extra interview to specifically talk about her strategies in teaching literacy. Therefore, all references to classroom observation in her class are supported by direct quotes to clarify what she meant by her actions.

4.6.2.1 Teaching Children to Read and Write

Many of the strategies described in the following paragraphs make specific reference to literacy instruction. Considered in a more analytical way, however, they are strategies for sense-making, for clarification, for interpretation, for fostering connections. They are the strategies used by teachers who want their students to see meaning and find significance in what they are learning.

When teaching a child to read, Roma’s goal is to have the child understand what he or she is reading. She says that simply decoding is not reading. The focus is always on the story so that the child sees the purpose of reading, and is more motivated. As the child is working on the passage, Roma will use fine tuned prompts to draw the child’s attention to visual and structural information that will help create meaning. When she introduces a book to a young reader, she might flip through most of the book with the child, discussing the illustrations, using words he/she will encounter in the story, asking for predictions based on visual information, in order to help him or her anticipate and thus make reading easier. In scaffolding like this, she is lightening the cognitive load, because the child does not have to keep so many different things in mind:
When they are just learning to read, I think it is fair for them to almost know the story first. So they can anticipate better, so that they are not dealing with all the problems at the same time. (I got to read the words, I got to figure out the story, I got to break the words apart, I got to re-read it...) too many things. But if you have a general idea of what the story is about, then that is one problem set aside. And you have the big picture in your head, so it makes it a lot easier to read.... When I was introducing the book, I was also using a lot of words that would be in the book, so you are feeding language ahead of time, so they can better anticipate. (Roma Interview)

Roma is a firm advocate for students taking books home and practicing their reading outside of the classroom. She says that using reading material of the right difficulty is as important as consistent practice. She says it is very obvious which students make the effort, because their reading improves dramatically:

I talk all the time about the importance of reading (the books they are allowed to take home), and I talk to the parents at the interview as well. So most of them do (read at home), and you can really see the ones that are doing a lot of it, because their reading just skyrockets. It is just so important to have books that are actually readable just in that zone of proximal development - so they struggle just a little bit but they can still handle it. So if you can find that zone in reading then their reading improves a lot. (Roma Interview)

Strategies she employs to make reading more meaningful include discussing the tools authors use to achieve certain effects. The focus during these discussions is typically on emotion and action. For example, she draws children’s attention to the way a word is written when a person is yelling or whispering it, as well as the vocabulary used to describe the utterance. She will also point out the variety of different verbs used to describe similar actions, and may even ask students to think of additional ones:

If you do not pay attention to why did the author do it this way? Or what is going to happen next (on a very basic level)? Then a lot of kids totally miss the fact that it is meaning that you are looking for. So I think you have to have the big picture all the time. Talking about the author's choices also helps students’ writing. They know that authors make choices. Why did the author do it this way? Why did they not choose to do this? (Roma Interview)
When children are reading a passage aloud, Roma will often ask a reader to repeat a phrase or sentence with the correct intonation:

I talk a lot about fluency and phrasing and intonation, because the more they can do that, the faster they become at reading and the more fluent they become. They have to understand what the story is about in order to read it that way. (Roma Interview)

Getting students used to grouping words when reading is an early focus in building fluency, because saying words in isolation, even if they are in sequence, loses meaning:

When they are first going into fluency, we talk about grouping words together. So it might be the words 'he said' Well you know 'he', and you know 'said', you are going 'he....said', but it is 'he said'. So when they group words, they start to get the flow of language, and reading becomes much, much, easier. (Roma Interview)

While reading aloud to students, Roma asks many questions that push students to think about what she is reading, and predict what might happen next. Sometimes she even has them infer story facts that are not explicitly stated, scaffolding student thinking by giving clues. Her motivation for employing these questioning techniques is directly related to helping the child anticipate so that reading will become easier. As these questioning practices develop students’ ability to reason, it can be argued that they support mathematical thinking as well.

In teaching students to decipher a new word, or figure out how to write it, she asks them to break it into syllables. To help students decompose a word, she has them clap it:

I want them to clap it to get the syllables in the word. Then they can break it into parts more easily. And if they actually have to physically clap, they are more likely to hear it. (Roma Interview)

Decomposing words into syllables, and then into sounds helps build on letter combinations students already know. Even if working with a single syllable word, Roma will ask the student to ‘say the word in two parts’. For example, the word ‘swing’:
'sw - ing’ I want her to hear the two parts of the word even though it is one syllable. What I am getting at there, (I was pretty sure, though I was wrong, that she knew 'ing'), so I want them to hear the parts they know. So just like the word 'fall', I know 'all' is on the word wall. She said 'I fall off the swing'. 'all' is on the word wall, so I want them to use the parts they know to get to something more complicated. Which is what I am trying to figure out how to do in math. (Roma Interview)

The words on the word wall are the 100 most commonly used words in the English language. They are accumulated at a rate of five words per week, usually of varying lengths and starting with different letters, gleaned from the poem of the week. If they are not all in the poem, Roma supplements the list with words she knows students will need. She wants students to know word wall words really well, because they constitute basic, recurring letter combinations students need to build their literacy on:

I want them to write the word wall words fast, fast, fast. If it is on the word wall, write it fast, do not get hung up on it. Work on the words that are big words. Slow down for those. But do not think about that. You know that. Write it fast, get it done. (Roma Interview)

Word wall words constitute factual information and repeating patterns that have to be memorized and at one’s fingertips as one acquires competency in literacy and tackles more complex work. Much of her work in literacy involves showing children how to manipulate letter patterns to make new words out of ones they know. In the following passage, she gives an example of how she might do that, after I suggest that the role of the word wall words is not unlike the role number facts play in mathematics:

Yeah, it is. You just know them, you have got them. Once you know them, you can do so much with other words. And when they are reading words, say they are reading the word 'day', and the reason why I always say, 'you know play, say it in 2 parts; 'pl - ay', so I will ask, do you see a part you know? And I'd cover over the d. And if they know 'ay', great. If they do not, I will write 'pl' in front of it on a piece of paper, and ask 'now do you know it'? 'play' say it in 2 parts, 'pl - ay' and then move the paper, so they see day. So that is another consideration with the word wall, it is putting words up there that can be used to take other words apart. So a lot of beginning reading and writing instruction is so linked together, and so
knowing how words work, is a big deal in grade one. Knowing how they work, How can they be taken apart? Can they be manipulated? Just like numbers. It really is the same, is it not? I have never really thought of it so much. (Roma Interview)

4.6.2.2 Teaching Students to Communicate Mathematical Ideas

Motivating students to work on mathematical activities is an important aspect of engaging them. In Roma’s estimation, most motivating strategies are related to play:

A lot of it is game related. No kid wants to sit there and do a work sheet. But I like playing that game with the triangles, I enjoy it even as an adult. Like Olive, you could hear her saying, I like this part. So she wanted the challenge, but she doesn't want the challenge of a worksheet, she wants the challenge of a game. I think the whole game structure is good. (Roma Interview)

With the math tubs she and Anne have created, much of the math teaching happens in pairs or small groups as students are rotated through the various activity centers. Roma believes that learning to articulate mathematical ideas can best be facilitated when students are helping each other solve problems, preferably in a game situation. One of the math tub activities provided the circumstances that elicited this idea. Several students were playing a game called TENS. The principle of the game is similar to dominos. The essential differences are that the pieces are triangular in shape, and each piece displays three numbers under ten; one in each corner. The players take turns fitting the pieces to the ones already placed, so that the sum of the values at touching corners equals 10. The game situation created an opportunity where students naturally talked about adding the numbers, finding the right tiles, and where they should be placed:

When they were playing this game (TENS) today, they were helping each other. I think maybe that is it. When they are helping each other, they are articulating. (Roma Interview)
As Roma reflects on student behaviour at the center, she starts to realize that the factors at the heart of student participation were the engaging nature of the game, as well as the fact that it was accessible at a variety of different ability levels: “about half way through I got up and Olive came here. And Connor was on task the entire time. The entire time, he was right into it.”

This observation is significant, because it shows the wide range of competence levels the game appeals to. Olive is the strongest student in the class so she plays the game easily, adding numbers in her head. In fact, she discarded the rekenrek with disdain, saying ‘I do not need this’. Connor on the other hand, has a very hard time concentrating. He usually wanders off to do his own thing, or else he is disrupting other people’s work. The other students at the center represented a range of intermediate abilities:

Albert was really struggling. But Sam was helping. Now, Sam, though, is a much stronger math student. If it had been someone right at Albert’s level, probably better dialogue would happen. He told him what to do. ‘how about the 7’ or whatever. But they were helping each other. I mean, Daniel and Sam were helping each other, and they are right around the same level and they were using the rekenrek, and they were showing each other, so that was good. (Roma Interview)

Roma’s evolving idea that students helping each other on mathematical tasks is the key to having them articulate mathematical ideas, solidifies into a conviction when she recalls her own experiences in learning mathematics:

I was good in math in school, I could always do it, but I think the reason was because I have an identical twin sister, and we always did our math together. So we talked about it all the time. So I think talking about it is the key. Being able to articulate what you are doing, and being able to explain it to someone else. (Roma Interview)

As she builds her practice in teaching mathematics, Roma is moving toward including more oral work and discussion into her lessons. Although she starts by giving an example
about a whole group discussion in literacy, she quickly proceeds to show how she is including the strategy in her math classes:

If we have time, we often come back and share what we've been doing, just like if they are writing, they come and share what they've written...and I didn't used to do that in math, and now I do. And I try to do that a lot more. It is not always about the centers, sometimes it might be a whole group if I have a real-life problem for them, and they go off in (usually) homogeneous pairs, - although sometimes that works and sometimes it doesn't - and then they come back and share their ideas, and we try to do it ...It is the same problem, so here is how they did it, here is how they did it, here is how they did, and you did it that way? That is like Sheila and Olive, or whatever. (Roma Interview)

Roma believes that promoting a classroom atmosphere where students are willing to voice their ideas is fundamental to generating classroom dialogue. To encourage students to share their opinions, she tries to make everyone’s ideas valued. She often reduces students’ anxiety in presenting, by conducting discussions in a think-pair-share format:

where they think to themselves what something can be, share with a partner, because it is a much lower risk environment that way, then we usually turn back and whoever wants to share can share. I do that a lot in language, I do not do it as much in math, and I should, I know. (Roma Interview)

Roma has recently started to implement a strategy to teach students to link to each other's ideas. She finds it hard going, because in grade 1, students are not generally interested in one another’s ideas, so it is a skill that has to be specifically trained. She started it as a strategy in literacy, but she thinks she’ll have to implement it in all subject areas to ingrain it sufficiently, so that students do it automatically:

I am trying to teach them to read their writing to someone else, and then the other person has to come up with one thing they liked and one question. It forces them to listen. I haven't done that in math yet. They do go off in pairs to do things in math, but there has to be a way to teach them how to listen to the other person. It is really difficult because they do not care what the other person has to say, usually. This is a very egocentric age. But if you do it in all the areas, then they get really good at doing that. Just like, they do think-pair-share pretty well now... so it is just (clap) automatic turn, then usually I just clap (clap) and they turn back.
So if you get those kinds of routines really fast, then you can focus more on the actual listening. (But) it has to be something they want to tell. So even if they are listening in math, they have to care. (Roma Interview)

In reflecting on the strategies she uses to encourage students to voice their opinions during class discussion, Roma does note that she has been using the strategies more frequently during literacy instruction, and intends to use them more frequently in math instruction.

4.6.2.3 Teaching Literature and Teaching Mathematics

Roma is an accomplished literacy coach who personally enjoys mathematics, but is only just beginning to develop her practice in teaching mathematics. She feels that she has a better handle on the fine-grained nature of literacy instruction than mathematics instruction. Therefore she is better able to help her students move from one level of achievement to the next in literacy. In her reflections on her mathematics teaching, she tries to see if she might build on strategies she has found effective in teaching literacy.

I think if you can fine-tune prompts, that will help them so that they still come to it themselves. In reading, there are different levels of prompts. So it might be a very open ended prompt, or it might be something very specific, like 'did you check the first letter?' That is really specific, but they still have to do it. Or it might be, 'what do you think is going to happen?' or 'what do you think it might be?' and that is really open-ended. So if we could do the same thing to help persistence in math, by fine-tuning the prompts, and getting it so that they see that they can actually do it. (Roma Interview)

In wanting the students to come to the answer themselves, Roma shows her commitment to constructivist pedagogy. She is able to guide students to self-correct in literacy because she has developed finely turned prompts through training and experience. In building her mathematics teaching practice, she is looking for a similarly detailed understanding of student progress to allow her to develop appropriate prompts.
When she is evaluating a student's reading, she is looking for understanding; of how the student is making meaning of the passage he or she is reading. She keeps a running record of the cues used and missed, errors made, types of information used:

I keep track of missed cues and errors. As she does it, these are my notes. Her phrasing was ok, her pace was ok, needs to work on punctuation to help her phrasing, I write down exactly what she did here. So as she read, she went 'sn-ah- o-w e-d'. So she used visual information. When she made an error did she use the meaning of the story, did she use the structure of the sentence, or did use visual information - the letters? So here I am looking, she made two errors in a row, where she didn't think about meaning, she didn't think about syntax, the structure, she solely relied on visual information. Here, she used all three, and then she self-corrected using more visual information, and more meaning information. So I am really thinking about are they using meaning, are they using structure, are they using visual information, and the most important thing is, how are they using those in a combined way in order to gain understanding from the book. (Roma Interview)

To apply this kind of analysis to students' ability to make meaning in mathematics, she cites the example of Alan at one of the activity centers:

Like Alan; he can add a page of numbers, just ba ba ba ba ba. But he had a lot of trouble with the game (TENS) he was playing here because he had to think of it differently. I have already got two, how many more do I need? He really struggled there. But he can do a sheet of adding, and you think, o.k., he can add. But he cannot think of it in any other way. (Roma Interview)

When she starts to reflect on what might be going on in the student's head, and connecting it to her own experiences in learning mathematics, she arrives at some interesting conclusions:

I do not think he is visualizing. I think a lot of it is visualizing. My sister and I were talking about this the other day. When you think about numbers to 100 in your head, you see it a certain way. And you are seeing the 10s and how it works. And I think that is what kids need to be able to do. If they are just using numbers on a page like that, they are not going to get that visualization. When they are reading, I want them to visualize. It is a big thing in reading...and we teach visualization in reading, but do we in mathematics? (Roma Interview)

Roma’s concern that her students should do math with deep understanding rather than mechanically, has generated reflection that connects to her own mathematical processes. As a
result, she is able to ask questions that will improve her teaching. Though her newness to teaching mathematics has not allowed her to realize how mathematical visualizations are facilitated, she has arrived at the conclusion that students need to visualize. She is very close to connecting this identified need to the role of work with manipulative is promoting visualization in mathematics. Using the various counting aids (rekenrek, abacus, ten-frame) subliminally implants a picture of number groupings in the decimal system. Anne and Roma have also included a variety of manipulatives in the math tub activities that allow students to build a myriad of visual representations of number quantities in their heads.

The source of disconnect for students who have learned to count by rote is that they do not associate meaning with the number sequence they have memorized. The same phenomenon emerges when students claim they can do the problem, 'just tell me which formula to use'. What they mean is that they can go through the learned mechanics of plugging numbers into an equation and manipulating them to come up with a numerical answer. However, they are incapable of translating the word problem into the required equation, because the meaning is missing.

To build meaning into their mathematics learning, it is not enough for students to use manipulatives in working the assigned problems. They must also see a reason for doing the problems. Roma is convinced that math problems have to be applicable to students’ interests and they have to be 'real' for students to be motivated. This conviction seems to bear a direct relation to her statement that in teaching students to read the focus is always on the story, so students can see the purpose of learning to read.

She contends that motivation is strongest when students really want to find the answer; when the problem is ‘real’. In trying to decide whether students should be drilled in
number facts and sequences before doing problem solving, she starts her reasoning with a literacy example that is well within her comfort zone:

I do not at all believe in teaching reading by teaching the sounds of the letters and certain sight words, and do that for a year, and then we read a real book... and that is done a lot. (Roma Interview)

She emphasizes the importance of applying skills during the learning process to real situations, as she gives an example from writing:

I think when they are writing we can practice word wall words, we can practice making words from other words, we can practice all those things, but at some point in the day you have to write real stuff. (Roma Interview)

Finally she ties it all together by applying her literacy teaching strategies to teaching math:

In math too, I guess you can practice little facts if you want, but at the end of the day, there has to be something that is real, and where the math means something. (Roma Interview)

So beyond the skill drill she provides at the activity centers, Roma tries to expose her students to real situations that require mathematical solutions on an on-going, daily basis. These usually take the form of references to classroom responsibilities or simple word problems that are relevant to students:

Even the centers I have do not necessarily mean anything. Well, I guess, counting how many kids there are, at least they are seeing that it is for a purpose. Even the numbers on the calendar, just being able to identify numbers, (I do not use flash cards), or I will ask stupid questions I know the answer to; 'which day are you the helper?' So they'll say 'the 27th'. So it has to be all useful. So I think it should all be at the same time. Even if we practice little skills by pulling them out, I think it all has to go together fast. (Roma Interview)

Roma is convinced that the math they learn has to matter to students, and the best way for that to happen, is to present the material in applied situations that students care about. She quickly reverts to her literacy teaching experience to prove her point:
In reading recovery we read real books. Always. If they are having trouble with something, I might go out of the book for a minute, get little magnet letters, work on that, put the magnet letters away, and go right back into the book. So that it is practiced in context. (Roma Interview)

Finally, Roma believes that math learning has a developmental trajectory and some operations are taught prematurely:

Anne and I think that time is a developmentally inappropriate thing to be teaching in grade 1. It is too abstract. They have to be able to tell time to the hour and the half hour. I say if they know the sequence of days, if they know how the seasons change and how that is time, and how our day works, what we do next, what the sequence of a day at school is, I think that's a lot more appropriate and that's in the curriculum. But telling the time on a clock, I think that is sort of like teaching a 2 year old to read. (Roma Interview)

4.6.3 Co-Workers and Professional Development

Since Roma has been away from classroom work for more than half of her career, and has specialized in coaching literacy, her mathematics practice has benefited significantly from working with her teaching partner. She and Anne have co-developed manipulative based activity centers and word problems for their students. They have on-going conversations to support each other in the implementation of the co-developed materials, debriefing lessons taught in tandem, and sharing resource materials. They have brought different skills, but a uniform vision of what constitutes good teaching, to the partnership. Anne has years of experience in the primary classroom, and developed skills in manipulative based instruction. Roma's own love of mathematics and her habit of discussing mathematics with her sister have stood her in good stead, as she builds her mathematics practice to teach for understanding.

The Board Superintendent and Roma’s school principal have made significant efforts to support teachers’ professional development in teaching mathematics. The organization of school staff into teaching partners by grade level was quite successful with the grade 1
teachers, as described above. The three-school abbreviated lesson study-style professional development organized by the Superintendent would have been more satisfying had it been more specifically targeted:

The people who came represented a really wide-range of experience, and the person from the Board wound up addressing the lowest common denominator. ...I found they talked about basic things I already do, whereas my question is, 'o.k. We are going for inquiry in math, what does that class look like when it is done really well? That is what I want to talk about. (Roma Interview)

In Roma’s opinion, her established process for understanding mathematics through discussion with a partner of similar understanding, would have served better in this instance:

Anne and I said that we would do better sitting together, looking at resource books, going through them and talking to each other. That would be better for us. (Roma Interview)

Roma realizes that different people have different competency levels in both content knowledge and knowledge for teaching mathematics, and she tries to be understanding of the training as it occurred. However, in actively building her knowledge for teaching mathematics, she looks for directed professional development experiences. Therefore, she has been dissatisfied with the Board promoted professional development efforts in math, because she feels that they have not been adequate for her needs. She longs for more in-depth, more detailed discussions:

I understand about subitizing, but we wound up talking a whole day about it. That is an important issue if you do not know about it, but I just wanted more out of this. It was frustrating for that reason. I wasn't their fault, it just should have been organized differently. (Roma Interview)

4.6.4 Summary

Despite her confidence in doing mathematics, Roma lacks confidence in teaching it. This is due in large part to her lack of practice. Her knowledge for teaching mathematics is
only just starting to take shape. Much of my work with Roma involved identifying her specific teaching strategies in literacy. I encouraged her to reflect upon them with a view to comparing them to teaching strategies she uses in teaching mathematics. By contrasting strategies in this way, she might more easily identify areas of need or dissatisfaction in her mathematics practice, and possibly use her expertise in literacy instruction to help advance her teaching in mathematics.

The following themes emerged in Roma’s practice:

- Roma strongly believes in teaching for understanding. She teaches sense-making strategies when teaching children to read, and believes that the problems in math have to be ‘real’ and students have to care about finding the solution to be engaged.
- She is concerned about students doing math mechanically, without deep understanding.
- Roma likes to use instructional strategies that reduce risk and promote learner engagement, particularly in promoting the articulation of student ideas. She often uses the think-pair-share strategy for this purpose.
- She believes that students articulate ideas spontaneously when they are helping each other work on a task that they are interested in.
- Based on personal experience, she thinks talking about mathematics all the time with peers is essential to making sense of it.
- Roma has a keener sense of the stages of student development in literacy instruction than in math, so she is better able to help students progress in literacy.
- Roma believes that math learning has a developmental trajectory, and some math operations are taught prematurely.
• Roma recognizes that some strategies she routinely employs in literacy instruction, she does not use to the same extent in mathematics instruction, and thinks perhaps she should.

• Roma is able to reflect upon her own processes in doing mathematics, in a way that helps her identify gaps in her students’ mathematical processes.

• Roma has a collaborative relationship for teaching math, with her teaching partner. They co-develop activity centers and word problems, and support each other in implementing the co-developed materials.

• Roma has been dissatisfied with Board sponsored professional development efforts in math. She feels they were too broadly pitched and inadequate for her needs. She longs for more in-depth, more detailed discussions of the topics she needs to learn about.
Chapter Five – Cross Case Analysis, Interpretation, and Discussion

5.1 Introduction

This chapter is organized into five sections. In the next section, I use a cross-case analysis to discuss what all the cases have to say about teachers’ perspectives on eliciting student thinking. That leads up to answering the original research questions in the subsequent section. Each question is answered from the perspective of the cross-case analysis, followed by the evidence contributed by individual cases that supports the answer. Within each answer, I make reference to core documents such as the Provincial Mathematics Curriculum, as well as relevant research. The major findings of the study are formulated in the third section. Implications of these findings and suggestions for future research are considered in the fourth section. Finally, recommendations for stakeholders involved in facilitating the work of elementary mathematics teachers who view themselves as 'more of a literacy person', are provided in the last section.

5.2 Cross-Case Analysis

In an effort to elicit as much convergent information as possible from the contributing cases, it seems reasonable to look at possible patterns in what the participants have to tell us about their efforts around the central theme the school adopted during this study. The theme was stated as ‘collaborative inquiry in mathematics’, but there was a strong, embedded emphasis on facilitating student communication of ideas. To tease out the contributing components of teachers’ efforts, it seems useful to consider these two teaching objectives individually, as well as how they are connected. Therefore, I examine teachers’ efforts to teach students to communicate their understandings, their efforts to teach mathematics as inquiry, and how these efforts might be connected.
Consider teachers’ work around helping students communicate their ideas. All participants made a big effort to teach their students to communicate. However, the bulk of their efforts were not based in mathematics. Sydney and Tom were concerned that they would have trouble eliciting students’ ideas during the debriefing part of the mathematics lesson. They therefore taught their students conversational moves to ensure they would be able to carry on extended, independent, topic based conversations. Their intent was that this skill should transfer to conversations about mathematics.

Though students eventually learned to carry on topic based conversations for 20 to 30 minutes in small groups, Sydney and Tom found that the skill did not automatically transfer to conversations about mathematics. According to the literature students’ ability to use mathematical jargon meaningfully, referred to as mathematics register, needs to be developed as a part of regular conversation in the classroom (Forman, 1996). For students to develop an adequate basis for mathematical communication, they must negotiate shared understandings, a common language, and a relatively symmetrical relationship in which none of the students is seen as a mathematics authority (Cobb, 1995; Lampert & Cobb, 2003). Furthermore, they must be taught to participate in mathematical reasoning practices, which form the core of mathematical conversations (Ball, Lewis & Thames, 2008; Lampert, 2001).

As the dates of the references show, none of this research is brand new. Teachers’ intentions were highly commendable. However, the lack of awareness of available resources that would have tailored their efforts to ensure better success sheds light on a serious problem in communication between curricular requirements and curricular support. If teachers are expected to teach students to communicate mathematical ideas effectively, they need structured, long-term instruction in how to do it.
They need to learn the language (mathematical terms) and the grammar (mathematical reasoning), and become comfortable with it, before they can be reasonably expected to facilitate such communication among their students. They must also teach their students how to talk about mathematics. To develop shared meanings with their students, about what it means to give a mathematical explanation, what a reasonable justification is, what it means to understand someone else’s explanation or justification, what it means to collaborate when learning mathematics, and so on (Cobb, Yackel & Wood, 1989). When the communication of mathematical ideas is seen as an aspect of students participating in the activities of a mathematical community, “learning to communicate as a goal of instruction cannot be cleanly separated from communication as a means by which students develop mathematical understandings” (Lampert & Cobb, 2003).

The mathematics lesson excerpt from Anne’s class on the word problem about “feet living at your house”; and the one from Tom’s class on describing geometric shapes both show the teachers’ efforts to teach the proper use of notation and mathematical vocabulary. Both teachers ask students for input, but the dialogue is very traditional, in that the teacher does most of the talking in both cases. Students give short answers to specific questions, with little or no reasoning required on their part. In addition, the teacher as authority automatically shuts down student reasoning, because in the presence of authority, there is no need for argumentation. The authority’s word is accepted as known (Cobb, 1995).

Teachers in this study seemed to do a better job of facilitating student communication and reasoning in non-mathematical subjects. Tom’s social studies lesson on the lifestyle of pioneers was conducted as inquiry to the point of students asking and answering their own research questions. What made it easier for him to facilitate students’ reasoning in this type
of lesson? Was he more comfortable facilitating student communication and reasoning because he was less intent on drilling students in new vocabulary and new concepts? Was it easier for him to facilitate reasoning in social studies because he himself was more familiar with the language, and felt more knowledgeable about the topic he was teaching? If we draw the analogy that learning to participate in mathematical reasoning is like learning to speak a new language, these kinds of explanations for teachers’ difficulties would seem reasonable.

Roma was also more at ease facilitating reasoning in non-mathematical subjects. In her case, it was reading and writing. Her need for facilitating reasoning and making connections has to do with her teaching goals. Two of Roma’s top priorities for student learning focus on students making sense of what she is teaching them, and students seeing the reason for learning it. When she teaches students to read, the focus is on the story, so they can see the value of reading. She teaches reasoning strategies that enable students to better anticipate the story-line, and thus make reading easier. In the same way, she believes that the math they are doing has to matter to students and the problems have to be real for them, if they are to engage with mathematics in a meaningful way.

In her opinion, the best way to engage students in meaningful conversation about mathematical ideas, is to have them work together on problems they want to solve. Helping each other solve a real problem they care about, will naturally result in students articulating their ideas. She believes that students need to talk about math in order to make sense of it, because her own mathematical understanding is built on an on-going conversation about mathematics with her sister, all the way through school. The literature supports the idea that mathematical meaning evolves through the repeated discussion of actions taken during engagement with mathematical tasks, particularly when students articulate their mathematical
reasoning and connect their solutions to key mathematical ideas (Franke, Kazemi & Battey, 2007; Hiebert et al., 1997).

Roma’s contention that the math has to be real, even at the earliest stages of instruction, is supported by research that demonstrates that people are capable of figuring out mathematics they need, without formal mathematical training. For example, a study of housewives in California found that the women could solve mathematical problems when comparison shopping, that they could not solve when the same problems were posed as formal mathematics (Sternberg, 1999; Lave, 1988). Similarly, Brazilian street urchins could do mathematics when selling merchandise in the streets, but they could not solve similar problems when presented in a school setting (Carraher et al, 1986; Carraher, 1985).

Anne’s perspective on the idea that the math has to be real is her belief that students can see the value of math in organizing and making things more efficient from a very young age. The emphasis in her teaching is on having students work through concepts, because in her experience young students need to be exposed to a variety of representations of a math concept in order to understand it. She therefore plants the conceptual seeds before she adds vocabulary, which is completely consistent with the literature that advocates manipulative use for presenting ideas before adding mathematical language (NCTM, 2000; Early Math Strategy, 2003). She and Roma even introduce problem solving to their grade one students based on composing and decomposing numbers, even before they cover formal operations. So they are focusing on teaching math through inquiry; when they have children engage with manipulative based activities in small groups their objective is to teach mathematics through collaborative inquiry. However, they still have to work on adding the language, the back and forth discussions, that will make the activities mathematically meaningful.
Anne has specifically stated that she wants and needs professional development on how to elicit student ideas. Her stated needs in this area, and her related actions helped shed light on some teacher challenges associated with mathematically meaningful dialogue in the classroom. Anne’s tendency to run out of time before she elicits student ideas orally limits the effectiveness of her manipulative-based lessons in generating mathematical meaning. Her habit of stating the main ideas herself instead of making students articulate them does little to support the emergence of meaning, according to the literature. Research shows that meaning emerges through the repeated back and forth between engagement with mathematical tasks and discussions of these actions between students and teacher. Specifically, the emergence of meaning is connected with students’ articulation of their mathematical ideas (Jackson et. al., 2013; Moschkovich, 1999; Walkerdine, 1988).

All participants realized that they wanted and needed more professional development to be able to facilitate classroom dialogue for the purpose of eliciting student ideas and engaging students in mathematical reasoning. Tom said he wanted to see accomplished practitioners demonstrate how to teach math through inquiry in a real classroom. He and Sydney searched to find classrooms in their school district where student communication of mathematical ideas was successfully facilitated. Tom said they were hard put to find any, while Sydney found one practitioner on-line, who worked in another urban center hundreds of miles away.

Anne and Roma were frustrated with the professional development activity they participated in, because it stopped short of addressing their specific interests/concerns related to the facilitation of classroom dialogue. They wanted to talk about the details of eliciting student thinking; what language to use, the specifics of questioning techniques that result in
the student articulation of mathematical ideas. According to Anne, they had just gotten started on that conversation when time ran out.

Three of the four participants either showed in their actions, or specifically stated that they were more comfortable and more confident in facilitating student reasoning in non-math subjects. When asked to reflect upon the reason for this, two of them ascribed it to having a much clearer sense of the stages of student progress in literacy than in mathematics. They all said they needed more professional development to understand what was required of them, and how to facilitate student reasoning and elicit student thinking in mathematics. To help participants see that they already use some working strategies in this area, I encouraged them to reflect on the strategies they employ in facilitating student reasoning and eliciting student thinking in non-math subjects. When asked to contrast their use of those strategies in mathematics, they seemed to get a clearer picture of what is missing or unsatisfactory to them in their mathematics practice.

5.3 Research Questions

The overview of the findings presented in the cross-case analysis above will now be refined to tease out the details that specifically address the research questions that guided this study:

1. What teaching practices are used to strengthen the connections between spoken language and mathematical ideas?
2. How is students’ ability to reason facilitated through dialogue?
3. What kinds of challenges do teachers encounter as they facilitate classroom dialogue for deepening mathematical understanding?
4. What support and assistance do teachers need for facilitating oral communication?
5. What are teachers’ perceived needs for doing mathematics in order to teach mathematics well through inquiry?

The first two questions, which deal with facilitating oral communication and reasoning, will be framed in the context of the communication promoting strategies advocated by the Ministry's mathematics curriculum document for grades 1 – 8. They will be answered by reference to examples of study participants’ teaching practices that show how they implement these recommended strategies. The strategies relevant to communication will be addressed under question 1, while those dealing with reasoning will be addressed under question 2. Questions 3 - 5, dealing with problems faced in implementation, desired support, and teachers' perceived need for being able to do the mathematics themselves, respectively, will be addressed by direct reference to the cases as presented in chapter 4.

### 5.3.1 What teaching practices are used to strengthen the connections between spoken language and mathematical ideas?

In order for mathematical meaning to emerge, students must engage with mathematical tasks of high cognitive demand (Hiebert et al, 2003; 2005; Stein, Grover & Henningsen, 1996), articulate their mathematical reasoning, and connect their solutions to key mathematical ideas (Franke, Kazemi & Battey, 2007; Hiebert at al., 1997). The iterative process of shifting repeatedly, between engagement with a mathematical task and the discussion of actions taken during engagement, are the basis for establishing meaning (Boaler, 2002; Cobb, Yackel & Wood, 1994; Jackson et. al., 2013; Lampert & Cobb, 2003; Moschkovich, 1999; Walkerdine, 1988). In recognition of these research findings, the
Ontario Mathematics Curriculum Document specifically states that communication is an important part of students' process for acquiring and refining mathematical understanding:

> Communication is an essential process in learning mathematics. .... Teachers need to be aware of the various opportunities that exist in the classroom for helping students to communicate. (math18curr.pdf, p. 17)

The curriculum document also makes specific recommendations for strategies teachers can use to facilitate mathematical communication among students. Most teachers use the Curriculum document to guide their work rather than accessing research findings directly. Therefore, the following paragraphs describe how study participants implement specific communication strategies advocated by the Ministry Mathematics Curriculum document. All participants made significant efforts to help students acquire proper mathematical notation, learn mathematical vocabulary, and use both correctly. Efforts to facilitate student articulation of mathematical ideas, or to involve students in mathematical reasoning practices were less clear. The following paragraphs give evidence from each case to support these statements.

In the excerpt from her lesson about ‘feet living at your house’, we saw that Anne modeled how to represent one possible solution to the word problem with sketches and tally marks to facilitate the counting for her grade 1 students. At that point in the school year, her students were still composing and decomposing numbers. The word problem required students to show different representations of the quantity ten in an applied way. Since students had not yet covered formal operations, the modeling Anne did served two purposes. First, in the circumstances, it was tantamount to modeling the proper use of symbols, and second, it modeled how this kind of question can be answered. Her actions were in complete agreement with the Ontario Math Curriculum document, which recommends that teachers
“model proper use of symbols, vocabulary, and notations in oral, visual, and written form”.

This recommendation is consistent with research on teacher supports for developing a common language (Chapin et al., 2003), and making sense of visual representations through the development of common language (Moschkovich, 1999).

Modeling proper use of vocabulary and notation were prominent components of Tom’s lesson on describing polygons. The smart board supported his lesson delivery, so he had posted the vocabulary accumulated over several lessons. In the lesson described, he had students talk about geometric shapes based on their attributes, without naming the figure.

On another occasion, he had students find, among a group of geometric figures, the ones with parallel lines, the ones with right angles, and possible combinations of these parameters. These actions are examples of his direct implementation of communication strategies advocated by the Provincial Math Curriculum, which exhorts teachers to “ensure that students begin to use new mathematical vocabulary as it is introduced (with the aid of a word wall; by providing opportunities to read, question, and discuss)”. These recommendations are based on the necessity for developing a common language that establishes a taken as shared understanding (Chapin et al., 2003; Cobb, Yackel & Wood, 1994).

During the whole class discussion that followed, students were expected to name and point to right angles and parallel sides, and mark the characteristic with the appropriate symbol on the corresponding portion of the chosen figure. Throughout the discussion, Tom asked probing and clarifying questions to ensure students were on the right track, and provided feedback to students on their use of terminology and conventions.
Tom elicited student input throughout the lesson, giving several students the opportunity to talk at all stages of the inquiry-based lesson. He started with an elbow-partner discussion of right angles to review the main ideas of the previous lesson, then conducted a brief whole-class discussion to elicit and reinforce the salient points. He applied these points to the discussion of three different geometric figures, which he asked the students to discuss in terms of the attributes they had talked about, without naming the shape. Once he introduced the problem of the day, he spent about six minutes relating it to the warm-up exercises they had done, giving several students the opportunity to answer his direct questions. According to the literature a high quality set-up of the problem needs to elicit input from a variety of students so the teacher can gauge how much students understand and what kinds of supports he/she will need to provide (Boaler, 2003). It also requires math talk that will give students access to the problem, and ensure a meaningful mathematical discussion during debriefing (Jackson et al., 2013).

Tom made considerable efforts to establish the shared vocabulary students would need, and modeled several solutions to the problem by eliciting student answers to leading questions. These practices were consistent with research recommended strategies, in establishing shared meanings for vocabulary, and ascertaining the levels of student understanding. In addition, it provided a considerable amount of scaffolding.

He had students work on the problem in small groups, while circulating among them to observe, prod and counsel. Students were expected to voice their ideas to each other during this time. He asked three groups to present their solutions, and conducted a whole-class discussion that often elicited input to direct questions from various students. He
worked hard to keep students attentive by asking direct questions during this debriefing, but rarely required students to reason.

While all teacher participants used a think-pair-share strategy during mathematical discussions, their reasons for doing so, varied. Anne, Roma and Sydney primarily used the strategy to create a less threatening atmosphere for students to express their opinions. In Tom's case, it was also a way of holding students accountable for what they talked about during these small group discussions. He often told his students that he could pick on anyone to report from their group, so everyone had to make sure he/she had something to say about the topic under discussion.

All participants made a conscious effort to give students a variety of opportunities to answer questions during the class. As far as students were expressing their own ideas about mathematical tasks or their own reasoning, this was consistent with research on the importance of students articulating their understanding in the emergence of mathematical meaning (Lampert & Cobb, 2003; Moschkovich, 1999; Walkerdine, 1988). However, when they were giving short answers to direct prompts they were not learning to participate in mathematical reasoning.

5.3.2 How is students’ ability to reason facilitated through dialogue?

One of the sustaining pillars of mathematical understanding is the ability to reason (Lakatos, 1976; Pólya, 1957). It follows that good teaching must develop students’ ability to reason. The most effective strategies for developing students’ mathematical reasoning, involve engaging students in mathematical practices (Ball, Lewis & Thames, 2008; Lampert, 1999).
The teacher thinking out loud during a problem solving exercise can model mathematical reasoning through dialogue. However, for students to actually engage in mathematical reasoning, they must be required to make claims, justify claims, and as a group, evaluate the validity of mathematical claims that have been made in the class (Ball, Lewis & Thames, 2008). Asking probing questions that clarify and extend in a situation that consists primarily of modeling does not fully engage students in mathematical reasoning practices.

All participants knew that they needed to teach their students to reason in mathematics. They strove to do so in various ways, as the following paragraphs will show, but they all expressed the need for more professional development to be quite clear on how they could elicit student ideas and engage their students in mathematical reasoning. They made consistent and considerable attempts to employ the recommended strategies as they understood them, for teaching students to reason, and eliciting student ideas. Their implementation of these strategies put a heavy emphasis on modeling solutions with the teacher thinking out loud to exemplify reasoning processes. The student articulation of ideas was a recommended strategy that frequently fell victim to adaption, due to perceived time constraints.

When Sydney compared her students’ answers to EQAO exemplars and helped them tweak their answers to a level 3 or 4, she was modeling how to answer the question well. In the process, she made visible different levels of complexity in the answer. She also modeled how to connect ideas and extend students’ thinking, mostly by thinking out loud. In tweaking student answers, she was still modeling. To fully engage student reasoning, it would have been necessary to develop student skills to the point where students could work
through tasks by themselves to the desired level of complexity without the necessity of modeling.

When Anne discussed Henry's organized representation of his various solutions to the word problem about the number 10, she was essentially modeling how the question might be answered well, and made her reasoning visible by thinking out loud. Anne realized that it would have been better to elicit from students what they thought Henry was thinking about in his different solutions, but ceded to the tyranny of the clock. She felt the only way the important points would be voiced before time was up, was if she summarized them herself. She stated repeatedly that she felt the biggest enemy of engaging students in meaningful mathematical discussion was time. For this reason, she frequently summarized the key points she wanted to convey rather than eliciting them from students.

During the whole-class discussion that debriefed the polygon lesson, Tom solicited brief answers from students. He then spent a lot of time reviewing the features used to describe a figure, and linking the current description to that of the previous figure. He worked hard to reinforce the polygon parameters through repetition. His thinking out loud strategy consistently pointed out the features various figures had in common, and repeated the types of features to look for. His monologue was consistent with the requirement to link solutions and connect them to the mathematical ideas he wanted to convey. He lost student engagement when he did not require students to make these connections themselves. He knew he was losing the students during the debriefing, he said more than once, that he felt that part of the lesson was always too long.

In all three of these examples, the teachers were thinking out loud, while analyzing the different types of answers to specific problems they modeled for students. Students did
very little of the actual mathematical work as outlined by Ball, Lewis and Thames (2008). Though students were regularly asked to justify their answers, the context of the justification was frequently procedural rather than conceptual. During class discussion, when engagement in mathematical reasoning would have required the class as a group to evaluate mathematical assertions, students were required to give short answers to direct questions. The teachers themselves did most of the analyzing and connecting, by thinking out loud, rather than engaging the students in these reasoning processes. Teachers’ expressed desire for more professional development in this area seems warranted, to help clarify for them how to make their efforts more successful.

Even if mathematical reasoning must be developed in context, students’ ability to reason may be supported, even fostered, through a variety of activities outside of the mathematics classroom. Activities that require reasoning can range from reading strategies to research processes. Participants observed in this study, were more successful at facilitating student reasoning through dialogue outside the mathematics classroom. Roma developed her students’ reasoning when she asked them to predict the story they were reading, based on different kinds of clues. The strategies she taught them to use for decoding the words in order to anticipate and make reading easier, taught students to combine information in order to arrive at a conclusion, and thereby constituted reasoning.

Tom also supported reasoning outside the math classroom when he taught a social studies unit as inquiry. He facilitated students asking their own research questions, then he had them find the answers at various research stations. The entire process required students to reason, connect ideas, and extend their thinking.
5.3.3 What kinds of challenges do teachers encounter as they facilitate classroom dialogue for deepening mathematical understanding?

All teachers in the study were committed to facilitating the communication of mathematical ideas. This commitment was supported by a Board-sponsored theme of 'collaborative inquiry in mathematics'. The entire teaching staff of the school, organized by the principal, focused their efforts on identifying criteria for facilitating and evaluating mathematical problem solving with a heavy emphasis on the communication of mathematical ideas. In pursuit of this common goal, participants encountered three types of challenges. One was rooted in insufficient knowledge of how to elicit student mathematical ideas, or engage students in mathematical reasoning practices. Another type of challenge concerned lack of understanding of why it is important to have students articulate their mathematical ideas, and the third was based on a lack of awareness of available resources.

The grade three teachers were teaching three-part, inquiry-based lessons (van de Walle, Folk, Karp, & Bay-Williams, 2010). In this model, most of the teaching occurs during the third, debriefing part of the lesson, which is conducted orally. When Sydney and Tom embarked on their TLLP study to learn to teach math this way, they both suspected that eliciting student ideas during the debriefing part of the lesson would be the most difficult aspect of implementing this model. They therefore taught their students conversational moves to encourage facile conversation in preparation for the debriefing sessions. Though the students eventually became quite good at independently carrying on topic-based conversations for extended periods about the stories they read, this facility did not transfer to mathematical discussions. Sydney concluded that the conversational moves had to be taught
in context to transfer to mathematical conversations, and intended to try to do so in the future.

This conclusion is consistent with studies on the emergence of mathematical meaning (Boaler, 2002; Jackson, 2013; Moschkovich, 1999; Walkerdine. 1988). The teacher has to facilitate the construction of a conceptual bridge from the concrete to the abstract through meaningful talk about mathematical activity. Lampert and Cobb (2003) extend the definition of matematization by proposing that it is an iterative process. In this repeated, cyclic process, student and teacher actions become mental objects for discussion that extends reasoning based on the previous actions. The talk moves Sydney and Tom taught their students strengthened their basic skills in literacy, but needed to be reinforced in a mathematical context for students to develop associated meaning. This was true, even though students were taught strategies to communicate mathematical ideas, math vocabulary was drilled, specific problem solving strategies were modeled, and students’ own answers were tweaked to improve the quality. Teachers’ desired additional professional development would show them how to supply the crucial missing step; the engagement of students in mathematical reasoning practices, so that meaning can emerge (Ball, Lewis & Thames, 2008; Lampert, 1999).

The teacher’s preparation for and orchestration of the debriefing discussion in the third part of the three-part lesson is outlined by Smith and Stein (2011). They advise that the teacher carefully select and order the student solutions to be discussed so that the discussion can be guided to connect solutions to each other and the mathematical principles they illustrate. In Tom’s polygon description lesson, he started the discussion with two student solutions that were identical to the warm-up exercises they had done at the beginning of the
lesson. These made the discussion accessible to most students, but since they were identical to the ones that had been done earlier, they did not extend the learning.

It can also be argued that the cognitive demand of the task (Stein, Grover & Henningsen, 1996) in the polygon description lesson was reduced (Stein & Kaufman, 2010) by the fact that two of the three student solutions discussed the same figures as had been covered in the warm-up. He did link the solutions and connected them to the core mathematical concepts himself. Therefore, he was following the steps of the suggested strategy for orchestrating the debriefing dialogue. He knew he was losing students during the debriefing, but was so intent on emphasizing the points he wanted to make, that he often made them himself, rather than requiring students to make these connections themselves.

Tom also found it challenging to teach students to become aware of their own reasoning processes and communicate them. As justifying one’s answer is part of learning math through inquiry, this aspect of instruction was an integral part of what Tom felt he had to do. His frustration with this aspect of instruction colored his entire perspective on teaching math through inquiry. Because he attributed students’ difficulty to their knowledge gaps in both math and literacy he felt that having to justify their answers often hindered students’ procedural fluency. They could do the computation, but had so much trouble explaining their process that they made things up to satisfy the requirement. Students’ knowledge gaps may well have contributed to their difficulties in describing their thinking, but it is also possible that in grade 3, many of Tom’s students were not cognitively mature enough (Piaget, 1969) to engage in the metacognitive analysis required to become consciously aware of how they arrived at the answer.
In the early grades, student understanding can often be evaluated based on student construction and activity. That is one reason manipulative use to actively demonstrate solutions is highly recommended at this level. The teacher can gauge student understanding from the activity, even if the student cannot yet explain his/her thought processes. Classroom discussion is so vital during this time, because by adding language to the manipulations, the teacher builds students’ ability to put words to their actions, and thus facilitates the emergence of meaning.

Tom’s perceived challenges with students’ difficulty in justifying their processes and their attempts to compensate, points to his need for more professional development in reform methods. It is really incumbent upon the school system to give better training to a concerned, conscientious teacher like Tom. Professional development he would find useful, would make him aware of the significant role of classroom dialogue as connected to manipulative use for the emergence of meaning. It would also allow him to distinguish between procedural and conceptual justification, and to put more emphasis on the latter. His perceived challenges could also be significantly reduced by showing him that students’ age dependent metacognitive maturity may play into the problems he notices. As a conscientious teacher, he uses repeated exemplification and practice to train students to be able to do what he knows they must. His teaching would become much more rewarding if he received the professional development to understand how the reform teaching strategies he has been told to use, work together to help students overcome these difficulties.

Tom suspected that the reason his students had such difficulty with learning math through problem solving, was two-fold. Either they did not know the math they needed, or they did not have the language. Since learning math through inquiry both requires and
promotes a close connection between mathematical understanding and language use, knowledge gaps in either area would have serious consequences for student learning. A lack of consolidation in the early skill sets in mathematics, such as number sense, numeration, addition and subtraction, would hinder students in handling material presented in an integrated or applied format. To solve problems successfully, students need to have a unified knowledge structure in mathematics, because to solve a problem, they must be able to call on various aspects of their mathematical knowledge seamlessly. Therefore, the knowledge gaps Tom identified in his students posed serious roadblocks to his efforts to teach math through inquiry, with an emphasis on students communicating their mathematical ideas.

By the end of his second year of teaching math through inquiry, Tom was seriously wondering if he would continue to do so on a daily basis. His ambivalence was brought about by the conviction that the problem solving approach was causing cognitive overwhelm for many of his students. Their severe knowledge gaps in early math and literacy made many of them incapable of grappling with word problems. There are so many places in the problem solving process where students with knowledge gaps can stumble: (i) reading the question, (ii) understanding the question, (iii) identifying the data given and what is not known, (iv) deciding what operations are required, (v) performing those operations, and (vi) making sense of the calculated number in terms of the required answer. Finally Tom came to the conclusion that his students would benefit from a much more finely articulated goal sequence in the math they needed to learn. Such a goal sequence would allow students to experience and build on small successes, instead of the continual cognitive overwhelm they were experiencing, that resulted in their growing conviction that they could not do mathematics.
The grade one teachers expressed a lack of confidence in teaching mathematics through inquiry for similar reasons. They were both highly skilled teachers of literacy with a well-developed and finely tuned sense of the learning/developmental continuum in literacy. They longed for a similarly finely articulated sense of student development in mathematics, so that they could target their questioning and scaffolding with equal confidence and accuracy. Anne and Roma specifically expressed their need for this type of knowledge for teaching mathematics and Tom alluded to it when he said his students would benefit from a more finely divided set of learning goals in math.

Anne had worked diligently at developing activity-based strategies for teaching mathematics concepts over a long career. All of her math lessons incorporated manipulative use. The activity centers she had developed with Roma were all geared to small group work with manipulatives, and many of the tasks were accessible at more than one level. However, there was no discussion to speak of at any of the activity centers. She introduced six activity centers to the class at a time, essentially demonstrating the activities one after another, and explaining the sample answer(s) on the recording sheet for each activity center. She then assigned students in groups of two or three to each center, and expected them to go to it.

This very limited discussion during bulk introduction of the activities drastically reduced students’ ability to make sense of the activities as mathematics. Instead, it encouraged students to work on them as rote manipulations. The sample solutions she provided on the recording sheets at some of the centers to ensure that students recorded their work as required, drastically reduced the cognitive demand of the tasks (Stein & Kaufman, 2010). This became evident during debriefing, when she asked students probing questions to
elicit their reasoning for the answers they had recorded, and they said: “because that’s what you did.”

At the heart of Anne’s manipulative based math lessons, is the conviction that students need to see multiple representations of concepts to develop understanding. She acquired this understanding through a professional development episode early in her career, and has proceeded to make sure her students ‘work through the mathematical ideas’ by incorporating a lot of manipulative use into her math classes. The very limited amount of language she adds to the manipulative based activities corroborates that her desire for additional professional development is warranted. Professional development targeted to her needs would show her the crucial role of repeated student-teacher discussion of the activities to give them mathematical meaning. It would also impress upon her the importance of students articulating their own mathematical ideas in the emergence of meaning.

Anne makes a point of integrating number applications wherever possible throughout the school day, because she is convinced that young students can understand how useful numbers are in organizing and making things efficient. Still, she struggles with eliciting student mathematical ideas orally. She knows she should have students tell her their ideas, but regularly cites time constraints as the reason for summarizing the main points she wants to bring out of an activity during debriefing, instead of requiring students to tell her their ideas. While admittedly, teachers have a lot of ground to cover, an experienced, conscientious teacher like Anne would surely reorganize her priorities if she was fully aware why it is necessary to have students articulate their own ideas and understanding. If she understood the significance of this reform practice, she would implement it the way it was intended (Remillard & Bryan, 2004).
5.3.4 What support and assistance do teachers need for facilitating oral communication?

The need for support emerged in the following four areas: opportunities for targeted professional development, collaboration with colleagues, modification of administrative requirements, and the availability of physical resources for teaching.

The desire for targeted professional development was expressed in several ways. Three of the participants voiced the need to observe competent practitioners in action. They wanted to see someone who had mastered the art of questioning for the purpose of eliciting student ideas in mathematics. In addition, Tom voiced the need to see successful inquiry based lessons demonstrated in ‘a real classroom situation’, where the teacher had to cope with the day-to-day discipline problems he had to contend with. Grierson and Gallagher (2009), in their study on demonstration classrooms, report that teachers are often inspired to make changes in their own classrooms by seeing a real teacher implement reform pedagogy in a real classroom.

Roma was equally interested in seeing what collaborative inquiry in the classroom looks like. If she could not observe an accomplished practitioner in action, she at least wanted to talk about effective practices in a professional development setting. Consequently, she was frustrated in the debriefing of the lesson study PD she participated in, because the discussion around the type of questions that elicit student ideas and how to ask them, never happened: “O.K. We are going for inquiry in math, what does that class look like when it is done really well? That is what I want to talk about”. Roma’s desire to discuss successful practices in order to effect change, is supported by the Mesler-Parise and Spillane (2010) study on professional development structures. They report that collaborative discussion is
the best predictor of teacher change. In an ideal world, she would also like to have a reform
math specialist with her in her classroom, to discuss the kinds of things she is doing, and how
her lessons might be done more effectively. This suggestion for a sounding board in the
classroom would bode well for bringing about the desired changes in teachers’ practice, even
if the alternate classroom presence was a collaborating teacher. According to Hargreaves and
Shirley (2009), “collaborative structures are strongly associated with increased student
success and improved retention among new teachers.”

Participants found that collaborating with a like-minded colleague to develop
mathematics lessons presented as inquiry was both helpful and satisfying. Tom found that
working with Sydney in the context of their TLLP project was much more beneficial and
satisfying than simply handing off the lessons to his new teaching partner. The former
process had forged a supportive partnership, as they worked through the mathematics and
came up with the teaching materials for their Bansho lessons. He did not develop the same
connection with his new teaching partner, because the mutual struggle and support was
lacking. They did not even develop an on-going discussion around implementation with the
new teaching partner, because they were at such different stages of understanding.
Penlington (2008) contends that collegial collaborations that advance teachers’ practice, must
build trust between participants that allows them to debate and confront each other’s
understanding.

Roma and Anne were able to build a mutually supportive relationship through
collaboration. In the process of developing the activities for their math centres, they bounced
ideas off each other, shared resources, and reduced their overall workload. Developing
appropriate problems for their students to solve, and helping each other fine-tune the
problems, helped generate ideas and introduced clarity. However, it was their on-going
dialogue around the successful implementation of the teaching materials they had developed
that provided the sounding board that helped each of them enhance her practice. Mesler-
Parise and Spillane (2010) concur that collaborative discussion is the greatest predictor of
teacher change.

Mathematical understanding must be developed as a unified knowledge structure in
students’ minds, for deep understanding to occur. The experienced participants said that they
knew this to be true, but felt compelled to teach math as separate strands, by the requirement
that they evaluate and report student achievement on a strand by strand basis. Therefore a
significant support in teachers’ work would be accomplished by modifying the reporting
requirements to reflect an integrated understanding of mathematics.

Finally, before students can express mathematical ideas, they must have experience
with problem solving activities that generate ideas. Activities of high cognitive demand give
the most opportunities to learn (Hiebert et al, 2003; 2005; Stein, Grover & Henningsen,
1996). In the primary grades, such activities involve extensive manipulative use. The
experienced participants in this study had all accumulated the resources they needed in their
classrooms over the years. However, at least one participant strongly expressed the opinion
that it was necessary for these resources to be available in the classroom at all times. In her
opinion, the practice of having a storage area in the school from where a teacher can borrow
a class set of manipulatives for a limited time does not promote independent manipulative
use. Sydney used the analogy that the classroom has to have its own library of books to
promote independent reading, and she felt this was also true of manipulatives for promoting
independent use as mathematical tools.
5.3.5 What are teachers’ perceived needs for doing mathematics in order to teach math well through inquiry?

The participants in this study represented a range of ease with doing and teaching mathematics, though there did not seem to be a direct correlation between confidence in doing mathematics and confidence in teaching it. Three participants specifically expressed a personal need for either more content knowledge in mathematics, or more pedagogical content knowledge in mathematics to increase teaching efficacy. The fourth participant talked more generally about the importance of teachers knowing the math they teach to do their students justice.

The grade three teachers both had to work hard to acquire the connected mathematical understanding they needed for teaching math through inquiry. After years of professional development and extensive experience as a math trainer for the Ministry of Education, Sydney has confidence in her competence in elementary mathematics. However, she qualifies her confidence by stating that her math is minimal compared to that of a mathematician. She also expressed very strong views on the importance of teachers being able to do mathematics. As an arts advocate, she draws a parallel with literacy when she suggests that no one would reasonably expect a person who cannot read or write to teach reading and writing. To improve society’s attitude towards mathematics so that it will be sufficiently valued, she suggests that it is important to teach mathematics to increase the intellectual capacity of society; to teach people to reason, to examine the world in ways that text based literacy alone, does not allow (Mighton, 2003; Skemp, 1987).

Tom was less specifically vocal about the need for the general population to have some competency in mathematics, or the necessity for elementary teachers to be able to do
mathematics. He did, however, express the need for more personal professional development in mathematics, and stated that the teacher has to have a very deep and connected understanding of the mathematics to teach math through inquiry.

In the first interview, Tom claimed to prefer to teach the more concrete subjects like math and science. However, his idea of concrete rested on a limited experience with teaching math through inquiry. His lack of experience showed when he referred to teaching math strands in sequence, not in the form of an integrated curriculum.

His conviction that teaching objectives were more concrete in math was based on the process he and Sydney used to develop the TLLP lessons. They followed the curriculum strictly, ensuring that every lesson addressed a curriculum expectation. That made the criteria clear, because they had been recently reviewed, and specifically addressed.

The ambivalence towards teaching math through inquiry that eventually emerged from Tom's comments was based on a lack of clarity about the various stages of student development in mathematics. He intuitively felt that his students would benefit from a mathematics program with more finely divided goals, so students could experience small successes and build on them (PRIME; Neményi, 1997). However, he needs the additional professional development he asks for, to refine these intuitive feelings into useful teaching strategies. His desire for a deeper, more comprehensive knowledge of mathematics is also warranted. He may then be able to invoke his own learning processes to help him in his teaching. Additional pedagogical content knowledge would also serve him well in teaching math through inquiry.

Anne was confident about her competence in early math. Over the years she had accumulated a plethora of manipulative based tasks to use with young students. She also
knew the mathematical ideas she wanted her students to learn from the activities. However, she frequently reduced the thinking component of the activity by providing sample answers on the recording sheets. Consequently, students simply copied her solutions rather than working the task. She also limited her students’ engagement in mathematical reasoning, when she failed to elicit mathematical ideas from her students. She summarized the key ideas she wanted them to get from the activity herself, citing lack of time as a justification.

Therefore, her significant content knowledge, and knowledge of the kinds of problems and activities that are meant to convey the mathematical concepts to her students, is limited in its effectiveness by the lack of meaningful classroom dialogue associated with them. The literature shows that manipulative based activities gain mathematical meaning through associated dialogue (Boaler, 2002; Jackson, 2013; Walkerdine, 1988).

I am indebted to Anne for providing this example of the importance of teachers fully understanding the significance of the reform strategies they are encouraged to use. It is not enough to tell teachers to use certain strategies. They must understand all aspects of how they work, in order to implement them in a way that will achieve the desired effect. Teachers’ mathematical content knowledge is important for maintaining the cognitive demand of tasks through implementation (Stein & Kaufman, 2010; Wilhelm, 2014), and knowledge for teaching helps the teacher develop student understanding (Hill, 2010).

Roma was hesitant with regard to her mathematics teaching practice due to her lack of experience. However, her solid understanding of mathematics, her passion for the subject, and her ability to reflect on her own learning processes in mathematics allowed her to ask important questions that helped advance her learning for teaching mathematics. In one of the classroom episodes cited, she reflected on Alan’s ability to easily add numbers on a
worksheet, but his inability to apply that skill in a game situation. She conjectured that his problem was that he did not visualize numerical quantities, because when she thought about her own processes, they involved seeing numerical quantities in a certain way. The conjecture was prompted by a discussion she had with her twin sister on the subject.

Roma maintains that her relative ease with learning math is connected to the on-going discussions she has always had about math with her sister. As this example shows, their mathematical discussions continue to inform to this day. The importance of articulating one’s ideas in the emergence of mathematical understanding is well documented (Boaler, 2002; Jackson, 2013; Moschkovich, 1999; Walkerdine. 1988).

By reflecting on her own experiences of learning and doing math, Roma is able to identify important components of the process. When she contrasts her own processes with the processes she observes in her students, she is able to identify gaps in the student’s processes that are likely responsible for the student’s lack of understanding. Her reflective approach to teaching mathematics, combined with her ability to rely on her own processes for guidance, significantly enhance her ability to teach mathematics well.

Based on the experiences reported by these participants, the following are the perceived needs for doing mathematics in order to be effective at teaching mathematics. Teachers need a deep, connected understanding of the mathematics content they teach, in order to teach it through inquiry. Pedagogical content knowledge, such as the stages of student development in mathematics is also important, to allow teachers to scaffold student learning with nuance and confidence. Finally, a strong personal understanding of mathematics content together with well developed processes for making sense of
mathematics allow teachers to detect gaps in student learning by contrasting it with their own processes.

5.4 Summary of the Research Findings

Based on the data collected in this study, the following findings emerged:

1. A description of the stages of student development in mathematics was identified by teachers as crucial pedagogical content knowledge they needed to have to teach mathematics with confidence. Two of the participants specifically attributed their lesser comfort level with teaching mathematics as opposed to teaching literacy, in large part, to the fact that they were much less clear on the stages of student development in mathematics than in literacy. Therefore the prompts for advancing children’s understanding available to them were much less accurate, and consequently, less targeted. There is a resource available that addresses this need. PRIME (Professional Resources and Instruction for Math Educators), offered by Nelson, Canada, is a series of courses on developmental learning in elementary mathematics developed by Dr. Marion Small. The course package offers maps of the stages of student development (referred to as phases), examples of teaching strategies and phase appropriate problems. There is also a pdf available on-line from Nelson, which summarizes the research, theories and best practices in Math Education as of August 2002. However, participants were unaware of these resources.

2. It is well documented in the literature that the articulation of mathematical ideas by students is crucial in the emergence of mathematical meaning ((Boaler, 2002; Jackson, 2013; Moschovich, 1999; Walkerdine. 1988). Tom and Anne both know that it is important to elicit student ideas during class discussions in math, and they believe they
are working to facilitate the student articulation of ideas. Observation of their classes reveals, however, that their students rarely get a chance to articulate more than short responses to targeted teacher prompts. Anne routinely runs out of time for eliciting student ideas, and summarizes the important points of the lesson herself to ensure that they are voiced. She expresses feelings of guilt about not doing what she is supposed to, but continues to run into the same problem. Tom feels the debriefing part of his lesson always goes on too long, and students lose interest. Observation of his classes reveals that students’ reasoning is only minimally engaged during whole-class discussions. He usually requires students to give short answers to specific questions and proceeds to analyze and connect ideas in his own words.

In trying to elicit student ideas through specific prompts or stating the main points instead of getting students to articulate them, these teachers show that they are committed to communicating the important ideas in their lessons. They apply the recommended strategies for eliciting student ideas and engaging students in reasoning practices as they understand them. However, both teachers have expressed the need for additional professional development to practice these strategies more successfully. In allowing their classrooms to be observed, they have helped to identify the kinds of help they need. They need to understand (i) the crucial importance of having students articulate their understandings so that meaning will emerge, and (ii) the importance of having students engage in reasoning, so they will remain engaged in the discussions.

3. Roma’s solid understanding of math, and her well developed processes for making sense of mathematics allowed her to identify gaps in her student’s understanding. She was able to do this despite her inexperience with reform methods and her lack of familiarity with
the stages of student development in mathematics. By reflecting on her own processes and contrasting them with her student’s observed behaviour, she was able to identify the gap in his process that hindered his understanding. Roma’s experience would suggest that having a good understanding of the mathematics he/she teaches allows a teacher to improve his/her teaching of mathematics because he/she can analyze his/her own processes in trying to understand students’ observed struggles. Through this type of reflection and contrast, there is a good chance the teacher can identify the problem, and subsequently find ways to help the student over the conceptual hurdle.

4. Three of the participants reported that they wanted help in building their practice for promoting student reasoning and eliciting student ideas in mathematics. In observing their classes, I noticed that they routinely promoted student reasoning and the articulation of student ideas in reading and social studies. Therefore, I encouraged them to reflect on the teaching strategies they used in these non-math subjects, and contrast them with strategies they used in teaching mathematics. This technique of contrasting helped them better identify areas of dissatisfaction with their math instruction, as well as suggest possible teaching strategies to try in mathematics, that they were already using in non-math subjects.

5. Contrasting the teaching strategies used in literacy with teaching strategies used in mathematics can prompt teachers to think about their goals in teaching the different subjects in more general terms. This kind of analysis may help to narrow the gap between their divergent perceptions on teaching these subjects. When Roma reflected on why she wanted students to know the word wall words without thinking about them, she was able to reframe her concept of them as letter patterns that are recurring, widely
applicable units of literacy. Once she described them in this general way, it was not
difficult for her to see a parallel between the role of word wall words in literacy and the
role of number facts in mathematics.

Teachers who are able to find this kind of bridging perspective between their practices
have a better chance of lessening their fear of teaching mathematics.

6. Teaching mathematics through inquiry requires students to have a solid grounding in
early literacy as well as early mathematics. Knowledge gaps in oral communication
skills, for example, seriously hamper students in paying attention, understanding what is
being said, as well as expressing their own ideas. All participants commented on the
knowledge gaps students arrive with. By grade 3, the gaps can be so severe that learning
math through inquiry is overwhelming for many students, and consequently teaching
math through inquiry becomes very challenging for the teacher as well as students.

7. Two participants felt that reporting on student achievement strand by strand was counter-
productive to teaching mathematics well. As experienced teachers, they felt that the
strands should be integrated in teaching the curriculum. That would be the best way to
achieve a unified knowledge structure in student understanding. However, in order to
comply with reporting requirement they felt compelled to teach strand by strand. It can,
of course, be argued that strand by strand reporting requirements ensure that teachers
cover all the strands. Still, it has also been shown, that an integrated strand approach is
more efficient and builds better student understanding. Therefore, the work of teaching
mathematics well, would be better supported by reporting requirements that reflect
student achievement when strands are integrated.
5.5 **Implications for Future Research**

Based on a Board driven initiative, the entire staff of Haven P.S. were focused on ‘collaborative inquiry in mathematics’. Organized into teaching partners defined by grade level, they identified criteria for teaching and evaluating their students’ learning in mathematics through inquiry. In the process, some of the teaching partners forged strong partnerships, where they relied on each other for support. Roma and Anne found satisfaction in their collaboration, but both looked for more. Further research might inquire into ways participants could help each other further their practice. Specifically, how could the experience and expertise of one foster those qualities in the other? Alternatively, one might inquire into the specific qualities that contribute to creating fruitful and satisfying partnerships that move teachers in the direction of improving their reform practice. What attributes do not lead to successful partnerships?

Participants named time constraints as the reason for a variety of missed teaching opportunities in eliciting student ideas orally. Anne said that there was never enough time left for the debriefing of a lesson so she frequently summarized the important points herself, rather than requiring students to tell her what they thought. In contrast, Tom said that debriefing always took too long. He had trouble keeping students’ attention on the work during debriefing. In the end, both teachers felt that students were not interested in their peers’ ideas. Even Roma, who did a creditable job of eliciting student ideas, felt that at this young age students are not interested in what their peers have to say.

Therefore, one might usefully inquire into developing strategies for helping students realize that they have a lot to learn from their peers. At the very least, one might inquire into developing strategies for classroom norms that foster listening to each other. The inquiry
might require analysis from a psychological perspective, to establish what kinds of attention, reflection and explanations primary students’ psychological maturity will allow. That would be a useful guide for teachers to know the forms of communications they can reasonably expect.

Participants worked hard to apply the communication strategies advocated in the Ministry of Education’s curriculum document. They taught strategies for communicating mathematical ideas, facilitated and drilled the correct use of math vocabulary, modeled how to solve various types of problems, helped students tweak their answers to improve the quality, engaged students in manipulative based activities, and used Bansho strategies to facilitate problem solving. They even taught communication moves to encourage independent, topic based small group discussion. Students did learn to carry on extended topic based discussions independently in small groups about books they read, but could not do so in mathematics. These are all necessary skills for students to acquire in order to do mathematics. The use of correct vocabulary establishes shared meaning. The consolidation of communication skills allows students to communicate their ideas more effectively. The modeling of problem solving strategies by thinking out loud can expose students to new ideas. However, in order to integrate these skills, students need to practice using these strategies on their own initiative, driven by their own ideas. They need to participate in mathematical reasoning and practices, and they need to articulate their own ideas for meaning to emerge. Therefore, a useful avenue of inquiry might be to find ways of engaging teachers in mathematical practices, and supporting their learning in the long term. That is how the educational establishment might ensure that teachers acquire the ability and habits of
mind to teach students to integrate these skills into their own problem solving strategies driven by their own ideas.

Roma represented an anomaly as a primary school teacher, because she had a solid understanding of, and a passion for mathematics. Her competence in doing mathematics and her well established processes for making sense of mathematics allowed her to ask the kinds of questions about student learning that serve to improve her math pedagogy in a meaningful way. Such meaning is often missing when teachers learn and implement reform techniques whose connection to sense-making in mathematics eludes them. For example, I have repeatedly heard teachers say: “I usually do the unit on patterning first.”, as if this was a type of exercise to be completed before moving on. After the unit is ‘done’, how much effort is made to bring to students’ attention the myriad of patterns to be found in mathematics? Roma herself has not yet made the connection that extensive work with manipulatives is a way of helping students visualize in mathematics, even though she has come to realize that it is important for students to visualize.

Therefore, one might usefully inquire how teachers can be helped to make such connections, so that the advocated reform strategies they are taught to use, are used mindfully. Engaging students in mathematical reasoning can only happen, if teachers view the teaching of mathematics as an exercise in reasoning in which they must engage their students as well as themselves. So what can be done to assist a teacher like Roma to quickly identify the strategies she needs to help her students once she identifies the learning gap? Second, what can be done to motivate teachers who do not have a solid mathematical understanding to acquire some, so they can reflect on their own processes and make similar connections?
Sydney, who has years of experience as a math trainer, and Anne, who has built her own reform practice since the 1980s, both expressed a frustration with the requirements for reporting on student achievement by strands. Their understanding of mathematics and their teaching experience tells them that the natural order of things is for the strands to be integrated. That is how students achieve a unified knowledge structure in mathematical understanding. However, they feel compelled by reporting requirements to teach at least some of the curriculum as isolated strands.

A less experienced, mathematically less knowledgeable teacher could find it easier to follow the strand by strand definition of the curriculum. The objectives of each lesson seem more concrete when taught strand by strand. However, this perception shows a lack of awareness of the stages of development in mathematical understanding. Therefore, a question to guide further inquiry might be: How can the appropriate stakeholders be convinced to support a better pedagogy, when simplified definitions are more appealing to the less experienced and less knowledgeable?

5.6 Recommendations for Stakeholders in Elementary Math Education

In light of the concerns participants raised, the following suggestions can be made to assist teachers’ work in implementing a strong reform practice built on oral communication in the mathematics classroom:

1. Two of the teachers expressed a desire to have a knowledgeable person in the classroom with them. One wanted the person to serve a co-facilitator role so students’ needs could be more individually addressed, and the other wanted discussion and feedback in order to advance her reform practice more rapidly. While the coaching programs in some Boards may not be sufficiently funded or wide-spread enough to provide the desired level of
coaching in all classrooms, perhaps a supplementary program could be set up through a local University. The proposal is for a co-op program for PhD students doing research in reform mathematics, where candidates could spend the equivalent of a work term in a school, working with several teachers.

2. Participants in this study collaborated with a teaching partner. In the context of the TLLP research project, they had a significant amount of release time to delve into the mathematics and develop teaching materials. Tom commented that preparation for teaching math through inquiry was so time consuming, that he thought it unreasonable to expect teachers without comparable release time to teach this way on a daily basis. Therefore, in schools committed to improving teachers’ reform practice, it would be useful to devise a system of scheduling that allowed simultaneous preparation time for teaching partners. It seems likely that many teaching partners would take advantage of the opportunity provided by simultaneous prep time, to collaborate on developing teaching materials, given such a school-wide commitment. Additional funding may also be necessary, to relieve teachers of some of their administrative responsibilities so they could concentrate on learning more mathematics.

3. A math specialist stream in pre-service could serve to assist in-service teachers in developing the resources they need to teach mathematics through inquiry, while dramatically increasing the number of practitioners with this type of expertise. A possible structuring for the specialist stream might include the following: a three week intensive math camp in the August of the Fall candidates start the pre-service program, based on the University of Ottawa model. The self-selected candidates who sign up for the camp would thus acquire some experience with manipulative use, get a start on
developing interactive math lessons, and begin to acquire a sense of the stages of student
development in mathematical understanding. As part of the pre-service math specialist
program, they would complete a work term at a school. They would be assigned in pairs,
to work with teachers at one grade level, to develop a complete set of mathematical
inquiry lessons for that grade.

Collaboration with the pre-service teaching partner as well as the in-service teachers
would ensure a learning community whose members could look to each other for ideas,
analysis, feedback and support. The teacher candidates would bring some theory and
familiarity with manipulative use to the collaboration as well as the support of their pre-
service mentors/instructors, while the in-service teachers would have years of teaching
experience to contribute to the partnership. In working together, each participant would
benefit from the strengths of the others. The work of developing mathematical inquiry
oriented lessons would likely be a new field of endeavour for all, so their level of
expertise in this area would be comparable. That would work in favour of a profitable
and satisfying learning experience all around.

4. Teachers expressed a desire to participate in professional development that was more
targeted to their needs. The abbreviated lesson study the grade 1 teachers participated in
was a good start, but did not provide sufficient opportunity to watch each other teach the
jointly prepared lesson. More importantly, the time allotted for debriefing would have
been better managed had participants of similar experience and concerns been allowed to
debrief together. Then each group could have pursued their own interests and concerns.
Furthermore, facilitators must be knowledgeable enough to facilitate discussions, offering
leadership. They must work to engage participants in mathematical practices, to emphasize the importance of applying the strategies discussed mindfully.

5. To assist teachers in building confidence in teaching mathematics, they need easy access to a user friendly, detailed and comprehensive description of the stages of development in early mathematical understanding that includes appropriate teaching prompts. There are courses on this topic, offered by Nelson Education, but these essential resources should be freely available as an integral part of the pre-service program, and offered on-line by The Ministry of Education as in-service. They should be widely disseminated through extensive advertising, consistent and long term promotion at professional development workshops, math educators’ conferences, and teachers’ unions.

6. To assist teachers in implementing recommended reform strategies with more success, professional development opportunities that ensure teachers’ full understanding of the strategies, including awareness of related research must be made available, widely disseminated, and offer support on a continuing basis. Anne worked hard to develop appropriate manipulative based activities for her students and she always ensured that the important mathematical ideas of the lesson were voiced. The key to the improvement she wanted to make in her practice would be to make her conversant with the research on the direct correlation between student articulation of mathematical ideas and the emergence of mathematical meaning. She could then make the adjustments in her priorities to make sufficient time for students to voice their ideas.

Tom could also raise the level of his reform practice to his desired level with the aid of such professional development and familiarity with relevant research. During debriefing he is already connecting the student solutions to each other as well as the main
mathematical ideas of the lesson. The understanding on Tom’s part, that meaning emerges when students articulate their own ideas and they participate in mathematical reasoning, will help him to reduce his own role to facilitator during the debriefing of his lessons.

7. To support teachers’ work in delivering the mathematics curriculum in a way that will develop a unified knowledge structure in students’ understanding, reporting requirements need to be modified. The curricular definitions and reporting expectations can be rewritten in a way that integrates the strands, or at least cycles back through them, over and over. Such restructuring would benefit student learning as well as help beginning and inexperienced teachers learn to teach math better, faster.

5.7 Conclusion

This dissertation introduces a new approach to interviewing for researchers investigating elementary mathematics reform efforts. When interviewing participants about their mathematics teaching, the emphasis is usually on eliciting information about strategies used in teaching mathematics, as the focus is on improving mathematics practice. While conducting this research, I noticed that participants lacked confidence in teaching reasoning in mathematics, but were doing a good job in teaching reasoning in the non-mathematics subjects they taught. Based on this observation, I asked participants to reflect on teaching strategies they used to promote reasoning in teaching non-mathematics subjects. My experience with this approach suggests that we can understand teachers’ perceptions of their mathematics teaching more deeply, if we prompt them to draw comparisons between their teaching strategies in math and their teaching strategies in literacy. Since most elementary teachers possess a rich repertoire of instructional strategies in literacy, such comparisons help
them better identify and define the areas in their math instruction where they feel that it is lacking, as well as giving them ideas for strategies to try.

The process of contrasting teaching strategies in literacy and mathematics thus has the potential to bring closer, teachers’ divergent perceptions on teaching mathematics and teaching literacy. Bringing these divergent perceptions closer may improve teachers’ perspective on teaching mathematics in two ways:

The first involves teaching strategies for sense-making. When teachers teach strategies for making sense of the reading material during literacy instruction, they are developing students’ ability to reason. Strategies that promote prediction, pattern recognition, question generation, answer justification, summarization, clarification, selection and combination of information to draw conclusions, all constitute reasoning.

Reasoning is an invisible process that goes on in an individual’s mind, until physical actions or words reveal it to the outside world. Therefore, how small group activity is completed and how the student subsequently reports on it, allows the teacher to immediately access, as well as assess student thinking. Encouraging clarifying questions, predicting, and summarizing are different strategies teachers use to make student thinking visible through oral communication. Elementary teachers seem to use these strategies more naturally when teaching non-mathematical subjects like reading or social studies. However, it can be argued that these are analytical tools based in mathematical thinking. The realization that they are teaching students to make sense of the world in both math and literacy may provide a bridging element to a more friendly perspective on teaching mathematics.

The second perspective change in favor of their mathematics practice involves teachers analyzing their teaching strategies to identify teaching objectives in general terms.
This process can lead to the realization that there are basic patterns in all subjects that students need to know, and learn to manipulate, to achieve fluency in the subject. Looking at good teaching in this generic manner may help integrate mathematics into the fabric of teachers’ practice. Such perceptual integration also has the potential to improve teachers’ perspective on teaching math, by decreasing feelings of intimidation in this regard.

In the process of identifying these potential changes in perspectives, three barriers to teachers’ work in advancing their reform practice were revealed. The implementation of recommendations for overcoming these barriers will also help teachers improve their reform mathematics practice more rapidly.

Many elementary teachers feel more comfortable teaching literacy than mathematics because they have a clearer sense of stages of student development in literacy. This allows them to help their students overcome learning hurdles more effectively, using targeted prompts. To expedite their reform practice, teachers need ready access to a detailed and comprehensive description of the stages of student development in early mathematical understanding that includes appropriate teaching prompts. This essential resource is available in the form of PRIME (Professional Resources and Instruction for Math Educators) offered by Nelson, Canada. However, it needs to be an integral part of the pre-service curriculum and available free of charge, on-line, for in-service professional development. Currently, its existence is not known to many teachers, and the cost in time and financial resources to access it, severely restricts its use.

The second barrier to teachers’ work in advancing their reform practice, concerns how they employ reform strategies. Participants in the study, as well as elementary teachers I have observed in other schools, frequently use reform strategies in a mechanical way, to
train students to do certain things, rather than to engage students in reasoning practices. They need to realize that the strategies are only effective for developing mathematical understanding, when students are involved in the work of mathematical reasoning. That, in turn, requires that teachers themselves engage in mathematical reasoning, which is the glue that holds reform strategies together. Teachers, who have a solid understanding of the mathematics they teach, are in a better position to engage their own reasoning to help their students learn.

The third obstacle identified to the optimal development of student understanding in mathematics, is the presentation of the material on a strand by strand basis. Even experienced participants, who know that the best way to develop a unified structure in student understanding is to integrate strands, feel compelled to teach strand by strand due to reporting requirements. Therefore, the modification of reporting requirements to reflect student achievement in an integrated fashion, as opposed to strand by strand, would encourage teachers to deliver the mathematics curriculum more effectively.

Teachers need help to overcome these three barriers to advancing their reform practice. They need ready access to expedited knowledge of stages of student development in early mathematics, they must be encouraged to use reform strategies mindfully, and reporting requirements on student achievement must be rewritten to encourage teachers to present the curriculum as integrated strands. These modifications in existing practices would help teachers inexperienced in teaching math, or teachers working on advancing their reform practice, learn to teach math better, faster.

In addition, the realization that teaching reasoning processes are equally applicable to literacy and mathematics, may mark the beginning to harmonizing teachers’ perspectives on
teaching these subjects. Seeing the parallel roles of specific teaching strategies may also increase the frequency with which a teacher uses them in teaching mathematics. The psychological boost of such a change in perspective, coupled with an expedited access to the stages of student development in mathematics has the potential to significantly increase the rate at which teachers learn to teach math well.
References


Bansho Board Writing (2011)  
retrieved July 2015


Copur-Gencturk, Y. (2015). The Effects of Changes in Mathematical Knowledge on Teaching: A Longitudinal Study of Teachers’ Knowledge and Instruction. *Journal for Research in Mathematics Education,* (46). 280 -


Erickson, F. (1986). Qualitative methods in research on teaching. In M.C. Wittrock (Ed.), *Handbook on research on teaching* 3e pp. 119-161. MacMillan Publishing, N.Y.


Small, M. (2010). *Big ideas from Dr. Small: creating a comfort zone for teaching mathematics.* Nelson Education, Toronto ON.


Appendix A

Interview Guide – Teacher Initial Interview

Thank you for agreeing to be interviewed. I would like to record this interview. You can choose not to answer any question that makes you uncomfortable, or stop at any time - we can go back and revise any answer if you would like to clarify any point.

1. Could you tell me a little about yourself, and your experiences learning and teaching mathematics?
   a. How long have you been teaching?
   b. What do you most like teaching? What makes it enjoyable?
   c. What is your mathematics background?
   d. What were your experiences learning mathematics in school?

2. Can you tell me about professional development initiatives (if any) you have participated in recently?
   a. Who was involved? Where was it held?
   b. What was the structure and length of support?
   c. How did you benefit from it? How could it have met your needs better?

3. What do you feel are the most important things for students to learn in math class?
   a. How would you respond to the following statements?
      i. Nothing ever changes in mathematics- there are a limited number of efficient procedures that students should learn.
      ii. Mathematics is about problem-solving
      iii. You have to study math for a long time to see how useful it is

4. Why do we teach mathematics?

5. In your experience, how do students learn best in math class?
   a. How would you respond to the following statements?
      i. Time should be spent practicing computational procedures before they are expected to understand them.
      ii. Students learn best when they work together to discover mathematical ideas.
iii. When students are working on math problems, more emphasis should be put on getting the correct answer than the process followed.
iv. Students need explicit instructions on how to solve word problems
v. Students need to make connections between the various ideas in mathematics
   How do you teach the different strands of mathematics? (Do you try to integrate them?)
vi. The best math problems can be solved in many different ways.

6. Think of a mathematics class that you taught recently that went very well. What was special about that class?

7. How much can you do to motivate students to learn mathematics?

8. To what extent can you provide alternative strategies, explanations or examples when students are confused in mathematics? Please think of a specific situation and tell me about it.

9. How much can you do to get students to follow classroom rules in mathematics?
   a. What rules do you think are important in mathematics class?
   b. What strategies do you use to get students to follow your rules?

10. What changes in mathematics instruction have you experienced as a teacher?
    a. What are your successes and weaknesses?
    b. What do you think of these changes?

11. How often do you elicit student methods for math questions?
    a. How comfortable are you in effectively managing these contributions?

12. Here at Haven P.S. how do teachers collaborate to support each other?
    a. Think of an example where the staff worked together for the students. What stands out?
    b. How often do faculty have conversations about:
       i. Managing classroom behavior or developing classroom culture.
       ii. Sharing teaching materials / assignments or student work
    c. What parts of this collaboration are most meaningful to you?
    d. Does this collaboration raise any challenges for you?
    e. How would you like to collaborate with other staff in the future?
13. What resources or supports would help you to teach math more effectively?

14. What are your goals for participating in this project?

15. Do you have any more ideas about the role of classroom dialogue in teaching mathematics, or mathematics in general that you would like to share?
Appendix B

Interview Guide - Strategies for Expressing Mathematical Ideas

Thank you for agreeing to be interviewed. I would like to record this interview. You can choose not to answer any question that makes you uncomfortable, or stop at any time - we can go back and revise any answer if you would like to clarify any point.

1. Can you tell me about some of the strategies you use to help students express their mathematical ideas?

2. What classroom norms are useful in promoting dialogue in mathematics class?

3. How do you engage students in meaningful dialogue around mathematics? Keep them talking about math?

4. What would you say is the most difficult aspect of getting students to engage in meaningful mathematical dialogue?

5. Is there a problem with students not all having the same prior knowledge? Specific probes around differentiated instruction in math as opposed to literacy, depending on participant’s answers.
Appendix C

Interview Guide – Promoting Reasoning in Social Studies

Thank you for agreeing to be interviewed. I would like to record this interview. You can choose not to answer any question that makes you uncomfortable, or stop at any time- we can go back and revise any answer if you would like to clarify any point.

1. Can you tell me about the curriculum requirements in this subject?

2. Can you give me some details about how you structured the lessons to teach this topic as inquiry?

3. What resources or supports did you enlist to implement this unit as inquiry?

4. What additional supports, if any, would have made the exercise more efficient for you and more productive for the students?

5. Can you tell me about the opportunities for promoting student discussion and student reasoning in the context of this inquiry?
Appendix D

Interview Guide – Literacy Techniques

Thank you for agreeing to be interviewed. I would like to record this interview. You can choose not to answer any question that makes you uncomfortable, or stop at any time - we can go back and revise any answer if you would like to clarify any point.

1. Ask about specific events observed during literacy teaching episodes for the purpose of:
   a. Getting the teacher’s perspective on the strategy used
   b. Describe what the teacher thought happened during the episode, in comparison to the researcher’s observations
   c. Identify the purpose of the strategy and how successfully it was used

2. Facilitate discussion around extending the strategy to facilitating teachers’ reform mathematics practice
   a. Can the teacher think of the strategy discussed in abstract terms – could the identified purpose of the strategy be applied to other subjects?
   b. Can the teacher think of a strategy with similar purpose and function in mathematics?
Appendix E

Interview Guide – Principal Interview on Early Numeracy

Thank you for agreeing to be interviewed. I would like to record this interview. You can choose not to answer any question that makes you uncomfortable, or stop at any time—we can go back and revise any answer if you would like to clarify any point.

1. Can you tell me about the central theme in math education your school is pursuing this year and how you are facilitating this work?

2. Can you give me some details about your background and perspective on early math education as it pertains to this initiative and this student population?

3. What kinds of additional supports and resources might be useful in carrying out this work?
Appendix F:

Classroom Observation Protocol

Date:
Location:
Start Time:
End Time:
Teacher:
Topic:

Research context (i.e. classroom set up, number of students, organization of desks, etc.)

Observation content:
Describe evidence of strategies for promoting oral communication and reasoning
(vocabulary promoted and how it is used, lesson and classroom organization for facilitating
discussion, scaffolding student thinking and reporting, modeling reasoning and problem
solving strategies)

Participation patterns (i.e. who speaks, when, who does not, when, etc.):
Describe the structure of the mathematical work (i.e. individual, pair, group, written, oral,
manipulatives, etc.):

Describe the teacher talk around mathematics (i.e. what types of questions are being
asked, what is the response to students’ mathematical ideas and/or mathematical errors, how
is themathematical work described, etc.)

Describe the teacher’s actions (i.e. where does the teacher stand, how does the teacher
move around the room, how does the teacher respond to struggling students, etc.)

Describe the tools the teacher uses in the classroom, and how they are used (i.e. what
manipulatives are used, by which students, and how are they accessed, are calculators
available, etc.)

Observer questions, comments, statements (i.e. for follow-up or reflections, etc.):
Appendix G

Information Letter and Consent Forms (Teachers)

This letter is a formal invitation for you to be a participant in a research project that will lead to a dissertation in fulfillment of the requirement of the degree of Doctor of Philosophy at the University of Toronto. The project will be conducted within the Curriculum, Teaching and Learning Department of the Ontario Institute for Studies in Education, University of Toronto by myself, Cecilia Kutas (ckutas@chem.utoronto.ca) and supervised by Dr. D. McDougall (dmcdougall@oise.utoronto.ca). If you have questions about your rights as a participant that needs further clarification, you can contact the Office of Research Ethics at ethics.review@utoronto.ca or 416-946-3273.

The title of the research project is "Accessing Mathematical Understanding Through Spoken Language: Case Studies of Elementary School Teachers Teaching Mathematics".

Purpose of the Study
The purpose of this study is to investigate the use of spoken language to facilitate mathematical understanding and teach mathematical reasoning in elementary classrooms. The study will work to make explicit the beliefs, practices and concerns of teachers as they facilitate classroom dialogue to make visible their students’ mathematical understanding, and hone their students’ mathematical reasoning.

Selection of Case Study Participants
I have chosen to invite you to participate in this research project because of your commitment to teach mathematics by the reform method, in a school environment where prolific classroom dialogue and discussion are the accepted norms. The fact you teach at this teacher training school indicates that you have achieved a certain level of expertise, and are willing to share it with others.

Methodology
I will be gathering data through individual interviews with each teacher involved, which I will be recording, with your permission. I will come to observe your classroom for three week periods near the beginning of the school year, and near the end of the school year. I will also be observing participants and recording two Focus Group meetings over the period of the study. Data will also be gathered through the submission of copies of dialogue prompting activities, and a teacher log book. The process will include two 3 – 4 week visits separated by several months, specifically April 2013 and October 2013.

The interviews will consist of five parts. The pre-interview will be to give you information about the nature of the research and the kinds of things that we will be discussing in subsequent interviews. It is also a time for you to ask questions about the research project. I will be making field notes at the end of the interview.
The second interview will last for approximately one hour and we will talk about your ideas on mathematics education, how you see the role of classroom dialogue in teaching mathematics, what techniques you use to facilitate and promote classroom dialogue in honing mathematical understanding, your experiences with these techniques as well as any concerns you may have surrounding the use of classroom dialogue in mathematics class. It would also be interesting to learn of your goals in promoting classroom dialogue during mathematics class. These interviews will be audio-taped so that all of the data will be captured and the awkwardness of note-taking will be eliminated. I will transcribe the tapes and thus be able to reread them and recall and reflect on our interviews.

Two to three classroom observations per week during each observation cycle will occur with each participant at times that are convenient to the participant. The intent of this observation is to see the use of classroom dialogue in action. Each observation will be followed by an informal discussion of 15 - 30 minutes to discuss the observed class. The final interview will occur towards the end of the project and we will discuss new experiences with the use of classroom dialogue, review previous issues, discuss new concerns, and review old concerns. Again, these interviews will be audio-taped.

During our focus group discussions, I will be taking short field notes and audio-taping the meetings so that I can recall the nature of the activities and discussion that occurred.

Participants will also be asked to keep a record or log book of their experiences with the facilitation of classroom dialogue over the period of the study.

Ethical Issues
Confidentiality of our discussions is extremely important to me. The trust that you place in me will not be violated. The following steps will be taken to make sure of that.
Anonymity - All proper names and identifying details relating to the school, to teachers or to pupils will be altered in my notes, my interview transcripts and in the final thesis.
Confidentiality - Field notes and interview transcripts remain confidential. They are not to be shared with others except the three members of my committee. Those members are only given the data with altered proper names and they also maintain the confidentiality of the material. The information of one participant (given to me directly) is not shared with any other participants in the project.
Protection against the possibility of evaluation - This study is not meant to evaluate teachers, but to learn more about teachers' experiences of facilitating classroom dialogue for the purposes of accessing and honing mathematical understanding. It is not possible to offer protection from a third party using the study in an evaluative way, but ensuring anonymity and confidentiality is intended to protect individual participants.
Right to Withdraw - You have the right to withdraw from this study at any time. If you should decide to withdraw, I will stop collecting data from you immediately. Any data collected to that point will be excluded from the study and destroyed.
Benefits of Participation

I hope that by participating in this study you will have a chance to voice some of your experiences. I hope that I will give you an opportunity, a forum, for exploring your own understanding of facilitating classroom dialogue and sharing that with others.

Written Consent

If you are willing to participate in this research project, please sign the following:

I am willing to participate in the research project "Accessing Mathematical Understanding Through Spoken Language: Case Studies of Elementary School Teachers Teaching Mathematics".

Signed.............................................................................................................

Dates..........................................................................................

I greatly appreciate your willingness to participate. Thank you

Cecilia Kutas Chisu
Appendix H

Transcript of Lesson on Describing Polygons

Fig. 1 classroom layout

1  Mr. Tom: How can you tell if an angle is a right angle?
2  Jay:  like an ‘L’ ‘makes an L shape
3  Mr. Tom: where would you find a right angle?
4  Al:  on the Smart-board (projected)
5  Mr. Tom: where in a shape?
6  Nora: on the edge
7  Mr. Tom: add to that, Shane?
8  Shane: around a corner [Mr. Tom aside: Sit and listen.] When the vertical (side) is
9  straight (up and down), and the horizontal (side) is straight ( across)
10 Mr. T: not always vertical and horizontal, but often. Where they come together is a
11 vertex and the inside is an angle; a right angle.
12 Mr. Tom assigns students to talk in elbow partners about a rectangle without using
13 its name. After a few minutes he recalls their attention to share with the whole
14 group.
15 Cal:  It has 4 right angles
16 Lyn:  4 vertices (one is a vertex)
17 Leila:  4 corners
18 Sam: close to another shape
19 Mr. Tom: we say ‘similar’ to a square. Do you see any L shapes in there?
Here’s another shape

Nora (shouts out): it’s a parallelogram

Mr. Tom: You’re right, but talk about it without using its name. (background noise is getting loud) 5, 4, 3, 2, 1. Share and learn.

Nora: it doesn’t have the same sides.

Mr. Tom: the same how?

Nora: like the square.

Mr. Tom: do you mean the length? All sides are not the same length?

Leila: it has no right corners.

Mr. Tom: we say angles. No right angles. Corner or vertex is not the same as angle. The angle is inside.

Leila: the side goes this way

Mr. Tom: it has some diagonals

Lyn disagrees that it has no right angles, then changes her mind. ‘It is similar to the diamond shape.’

Sam: It is all tilted.

Mr. Tom: Kind of like it is leaning

Mr. Tom asked students to describe a right triangle to an elbow partner without using its name. Then they came back to a whole class discussion that lasted [10:48]

Mr Tom: So what can you say about this one without saying its name. (mumbling) pardon? Disagreement? [there’s one] Shane can you come and point to where you think the right angle is? So what I like is, that Shane isn't afraid to put his hand up and have a guess. It doesn't matter if he is right or wrong, because then it starts the conversation. What was good about that, was that as soon as he said 'I don't think there are any right angles, it started everyone else started your brains moving, and thinking, hang on a second, I am not sure I agree with him. I think it does. Whereas if only one person says yeah, there is a right angle, then everyone else stops talking. So thank you, Shane, for being brave. There's the right angle there.

Student voice: you tried your best.

Mr Tom: What else might we say about this shape? This triangle, how else might we describe it? Sam, what do you think?

Sam: It's like a square, but it's cut in half.

Mr Tom: You know how I sometimes think about it? It's like half a sandwich. (giggles) How else can we describe it?

Nora: It has an L shape.

Mr. Tom: It's like an L. So what do we know about that? When we see an L shape in the corner, that's a right angle. Jay?

Jay: similar to a whole triangle

Mr. Tom: what do you mean by that?

Jay: That if you stick two right triangles together it makes a whole one.

Student: (shout-out): No it doesn’t!]

Mr. Tom: A whole one? So it is a triangle, are you talking about putting two of them together to make a square? So yeah, if you took one that was exactly the same size as this and put it on the other side [Nora: no] you would have a perfect square
Nora: No, that’s not what he’s saying.

Mr. Tom: so you were his partner, right? So can you clarify what he’s saying? [inaudible]

Oh, so he’s saying that if we took it like this, we could turn it into two triangles?

Mr. Tom: O.K. Here is today’s lesson problem. [a lot of response; some enthusiasm] I
don’t need a lot of comments when I put the lesson problem up, as soon as I put it
up, you should just start to read it. O.K.? [garbled student voices, reading the
problem? ‘I’m stuck on a word’.] Everyone’s listening, then, not talking. In 5, 4,
3, 2, 1. [student voices silent]. Thank you. Here is the lesson problem:

Choose a shape and describe it in as many ways as you can without using the
shape’s name. Record your work.

...You may recognize it as very similar to what we just did here. The one
difference being that now

Student : (shouts out) you have to record your work

Mr. Tom: which means you have to put it down on paper. Here are the shapes you are going
to choose from. [student chatter]

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**Fig. 2** The geometric shapes referred to in the Nov.22 – Dec 3
sequence of observed lessons

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Mr. Tom: We do not need any comments about the problem. You are just thinking about
‘how am I going to do this today.’ Here are the shapes to choose from, and
describe it. Which means to talk about it in as many ways as you can. Just before
you start to do that, let’s review some of our vocabulary. That means the words
we use. What are some of the words we use specifically with geometry, talking
about shapes? What are some of the words we’ve used in Friday’s lesson and
today’s lesson? [indiscernible student discussion] 5, 4, 3, 2, 1. [silence]

What are some of our geometry words that we’ve used so far?

Student: (shout out) vertices

Mr. Tom: Right. So vertex being one, vertices meaning more than one, meaning the
corners. Leila?

Leila: angle

Mr. Tom: Angle. We’ve used the word angle. Shane?

Shane: shape.

Mr. Tom: Shape. Cheryl?

Cheryl: figure

Mr. Tom: Figure, which is another way to say shape.
Student: (shout out) right angle?
Mr. Tom: Right angle. The main type of angle we’ve talked about. There are other kinds...Lisa?
Lisa: edge?
Mr. Tom: You are on the right line, but edge is not a word we generally use to describe this type of shape. Instead of that, we usually use the word ‘side’. But we’ll hold on to the word edge, because when we start describing different kinds of shapes, it will come back to us.
So I’m giving you 25 seconds to sit next to whoever your partner was on Friday. [loudish discussion among students to sort themselves into partners] 5, 4, 3, 2, 1. (takes about 90 seconds, then Tom still has to verify some of the partners, but students do quiet down by the time he reaches 1 in the countdown. He does sometimes hold a strategic pause between 2 and 1 to give all students a chance to comply). [10:48 min.]
Mr. Tom: Some of the words you will use to talk about it, are in this box right here. If you and your partner get through describing one shape, remembering of course to do your work so that it is [Lyn, (a warning tone)], very clear for someone else to read, then you can flip your page over and choose a second shape. Do not chase me around the room when you’re done. I’ll answer your question in a minute o.k.?
[44 seconds]
Mr Tom: 5, 4, 3, 2, 1. If you can’t hold your paper up without it flapping around, put it on the floor in front of you. What I expect when people are listening is that you listen to them. If you have any questions or anything you don’t agree with, you raise your hand, and we’ll talk about it. The whole point is we're having a discussion and learning from each other. (Lyn, move in and show me you’re listening please, because right now your body language is not showing me that.) OK girls, go ahead:
Leila: We chose K, because it has four sides and we wanted a four-sided one, so we chose that
Nora: It’s a parallelogram. And we wrote that it has 4 corners and 4 sides and it doesn’t have any right angles.
Mr. Tom: Thank you, Let’s give them a round of applause (applause). So they showed us 3 ways you can describe a shape. It doesn’t matter what the shape is, you can talk about these three things. You can talk about the number of sides. In this case it has four, but it doesn’t have to be, you can talk about the number of sides on any shape. Then you can talk about the number of corners, but of course, you can use the right word and call them vertices.
Leila: Sorry.
Mr. Tom: That’s o.k., we’re learning from you. And another thing you can talk about is, does it have any right angles or not. And if it does, how many? In this case it doesn’t have any right angles. So there are three ways you can describe any shape: Kay, how many sides does it have, how many vertices does it have, and does it have any right angles or not?
directed at Kay: How many sides does this have?
Kay: four
Mr. Tom: how many vertices?
Kay: four
Mr. Tom: Bear that in mind. (turns to next student solution) Whose is this one? Come on up.
That’s pretty good listening so far. We’re going to continue to listen. Tell us about shape J.
Jack: It has 3 vertices, (audible), 3 sides, (inaudible) this is bigger.
Mr. Tom: Can you explain to the class what you mean by not the same length? What is not the same length?
Jack: (pointing to figure) that one
Mr. Tom: When you say ‘that one’, can you be more specific? Which part of the shape are you talking about?
Jack: the side
Mr. Tom: The SIDE. So the sides are not the same length. Let’s give them a round of applause. So that was shape J, right? (rustle of paper as he posts the group’s work sheet) There it is.
So they have talked about some of the same ways as this group here. They have talked about the number of vertices. In this case 3. How many sides? What do you notice? 3 vertices, How many sides?
Student voices: three
Mr. Tom: how many sides here?
Student voices: four
Mr. Tom: How many vertices?
Student voices: four
Mr. Tom: They’ve talked about right angles. It has one right angle. So they have talked about 3 of the same things as this group has, but they have also talked about something this group did not talk about. They talked about how long the sides are. They said that the sides are not the same length. That’s another way you can talk about shapes. So this side here, is not the same length as this side here.
If we look at K (inaudible, copying K?) Now talk to an elbow partner. What can you say about the length of the sides.
[students talk, not discernible, some teacher remonstrations....]
Mr. Tom: Jack, if I ask you a sentence make sure you’ve talked about it. What can you say about the length of the sides of this shape? How long the sides are? O.K. I’ll give you a few seconds to make sure you’ve talked about it. [more student discussion] What can you say about the length of the sides of this shape? O.K. listening in 5, 4, 3, 2, 1. Taylor, what did you and your partner, partners talk about? What can you say about the length of the sides of this shape? How long the sides are? [inaudible] Well, you’ve had time to talk about it. Now you’ve got to say something. Jack?
Jack: six feet?
Mr. Tom: So you can measure how long a side is, Now 6 feet? It’s not 6 feet, I can tell you that. 6 ft is about the length of this whiteboard. How wide it is.
Student voice: Wooow
Mr. Tom: But we can talk about how long something is. Kay? [inaudible] O.K. Stop for a second. Everybody’s listening. I’m going to ask you to repeat what Kay said in a second, so make sure you are listening. Everybody needs to be listening the whole time. Go ahead. [inaudible] I think I understand what you’re saying. You are saying that this side is the same length as this one here. You know what we can do? We can put one of those on it, and we can put one here, and that tells us that they are the same length. But then this one is not the same length as that, so we can put two. So the two little marks tell us that this side is the same as this one, but it is not the same as those two. So that tells us that this shape has two pairs of sides that are the same length. This pair is the same length, and this pair is the same length, but all four are not the same. Daryl can you come and tell us about yours? Come and tell us about shape A. You have a lot to learn from Daryl, so listen. He is going to tell us important things about geometry.

Daryl’s partner: (inaudible)

Mr. Tom: Oh, o.k. are you going to tell us about it too? Go ahead.

Daryl’s partner: it has 4 sides

Mr. Tom: 4 sides? yeap,

Daryl: 4 sides are the same length

Mr. Tom: yeap, what else?

Daryl: 4 right angle

Mr. Tom: (pointing to figure) these ones right here?

Daryl’s partner: 4 vertex, vertices

Mr. Tom: right. Give these guys a round of applause. [Applause, Bravo! Bravo!] So this shape here, look at how they’ve described [Leila makes an animal sound].

Mr. Tom: Tell me what you want in a normal voice.

Leila: Go back.

Mr. Tom: What would you like me to do when I go back?

Leila: (pointing to the board) put it there

Mr. Tom: You would like me to copy and paste the shape in here?

Leila: yeah

Mr. Tom: O.k. That’s a good idea. (pastes shape, relative silence in the room). O.K. have a seat. There it is. So, What is one thing you have learned from Daryl about shape A?

Student: Shape A has more vertices

Mr. Tom: more vertices? How many vertices?

Student: it has 4 vertices.

Mr. Tom: it has 4 right angles. All of them are right angles. So he’s talked about how many right angles it has. Shane, what did you learn from Daryl?

Shane: it has 4 sides

Mr. Tom: So he talked about the number of sides, what else did he talk about? Sam?

Sam: all the sides are the same.

Mr. Tom: The sides are all the same length. So in this one they are not all the same length. This one is the same as this one, and this one is the same as this one, but in a square...and look what Daryl did here. He knew, that when you want to show
that the sides are of the same length, you put these little hash marks there. So in
this case, because they are all the same, they all get one. So here are the symbols
we are using, and I expect you to use correctly: that can show you how long a
side is, that shows you that it is a right angle. So he has talked about four
different ways you can describe a shape: how many sides, how many vertices,
how many right angles, and are the sides the same length or not. OK? So that is
four ways you can describe a shape so far. Turn to an elbow partner and talk
about how many different ways you can describe a shape that we have talked
about. [11:37 minutes]