A Robust Hierarchical Control Structure for Virtual Power Plants

by

Fahimeh Kazempour

A thesis submitted in conformity with the requirements for the degree of Doctor of Philosophy
Graduate Department of Electrical and Computer Engineering
University of Toronto

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Abstract

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Fahimeh Kazempour
Doctor of Philosophy
Graduate Department of Electrical and Computer Engineering
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This dissertation discusses the principles of operation and control requirements of Virtual Power Plants (VPPs). Based on the identified characteristics and performance requirements of VPPs, this dissertation then proposes a robust hierarchical control structure to operate a microgrid in the VPP and islanded modes and to provide a smooth transition between these two modes.

Various sources of uncertainties and disturbances affecting dynamics of a microgrid with multiple distributed energy resource (DER) units in both the VPP and islanded modes of operation are identified and classified as local and interconnection uncertainties and exogenous disturbances. An integral quadratic constraint framework is introduced to mathematically represent these perturbations. Based on the obtained uncertain system model, a necessary and sufficient condition is proposed to find robust, decentralized, output-feedback, reference-tracking, local controllers (LCs) for DER units which (i) guarantee the robust stability of the overall microgrid, and (ii) ensure disturbance rejection. Rank constrained linear matrix inequalities (LMIs) are further derived to systematically design these LCs. This decentralized structure of LCs constitutes the primary level of the proposed hierarchical control structure.

At the secondary level, a robust $H_{\infty}$, multivariable, LMI-based, PI control strategy augmented with static, multivariable, LMI-based anti-windup compensator is proposed and designed to operate a microgrid in the VPP mode. The proposed control strategy (i) guarantees the stability of the overall, closed-loop, unconstrained microgrid while achieving a pre-determined disturbance attenuation level, and (ii) guarantees the stability of
the constrained microgrid while demonstrating graceful performance degradation during controller windup.

Furthermore, to operate a microgrid in the islanded mode, a decentralized, droop-augmented, robust voltage control structure is introduced that is composed of: (1) a P-δ and a Q-V droop-based power sharing controller, (2) the proposed robust LCs, and (3) an open-loop frequency controller.

This dissertation adopts frequency-domain and digital time-domain simulation tools, i.e., MATLAB and PSCAD/EMTDC platforms, respectively, to evaluate the attributes and performance of the proposed hierarchical control structure.
To my parents, Shahnaz and Mostafa

To my love, Robin
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<tr>
<td>CB</td>
<td>Circuit Breaker</td>
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<tr>
<td>DER</td>
<td>Distributed Energy Resource</td>
</tr>
<tr>
<td>DG</td>
<td>Distributed Generation</td>
</tr>
<tr>
<td>DS</td>
<td>Distributed Storage</td>
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<td>ESCR</td>
<td>Effective Short Circuit Ratio</td>
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<td>GM</td>
<td>Gain Margin</td>
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<td>GPS</td>
<td>Global Positioning System</td>
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<td>Integral Quadratic Constraint</td>
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<td>ISM</td>
<td>Islanded Mode</td>
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<td>L-C</td>
<td>Local Controller</td>
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<td>L-G</td>
<td>Line-to-Ground</td>
</tr>
<tr>
<td>L-L</td>
<td>Line-to-Line</td>
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<td>L-L-L-G</td>
<td>Line-to-Line-to-Line-to-Ground</td>
</tr>
<tr>
<td>LMI</td>
<td>Linear Matrix Inequality</td>
</tr>
<tr>
<td>LPF</td>
<td>Low-Pass Filter</td>
</tr>
<tr>
<td>MIMO</td>
<td>Multi-Input Multi-Output</td>
</tr>
<tr>
<td>Mm</td>
<td>Module margin</td>
</tr>
<tr>
<td>MMS</td>
<td>Microgrid Main Switch</td>
</tr>
<tr>
<td>ODE</td>
<td>Ordinary Differential Equation</td>
</tr>
<tr>
<td>OPF</td>
<td>Optimal Power Flow</td>
</tr>
<tr>
<td>PC</td>
<td>Point of Connection</td>
</tr>
<tr>
<td>PCC</td>
<td>Point of Common Coupling</td>
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<td>PI (controller)</td>
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<td>Phase-Locked Loop</td>
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<td>PM</td>
<td>Phase Margin</td>
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<tr>
<td>PMS</td>
<td>Power Management System</td>
</tr>
<tr>
<td>pu</td>
<td>per unit</td>
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<tr>
<td>RLC</td>
<td>Resistive (R)-Inductive (L)-Capacitive (C)</td>
</tr>
<tr>
<td>SC-MVA</td>
<td>Short Circuit Capacity in MVA</td>
</tr>
<tr>
<td>SCR</td>
<td>Short Circuit Capacity</td>
</tr>
<tr>
<td>SISO</td>
<td>Single-Input Single-Output</td>
</tr>
<tr>
<td>SPWM</td>
<td>Sinusoidal Pulse-Width Modulation</td>
</tr>
<tr>
<td>VCO</td>
<td>Voltage Control Oscillator</td>
</tr>
<tr>
<td>VPP</td>
<td>Virtual Power Plant</td>
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<tr>
<td>Abbreviation</td>
<td>Description</td>
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<td>VPP-PQ</td>
<td>Real (P) and Reactive (Q) Power-Controlled VPP</td>
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<td>VPP-PV</td>
<td>Real (P) Power and Voltage (V)-Controlled VPP</td>
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<td>VSC</td>
<td>Voltage-Sourced Converter</td>
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<td>Symbol</td>
<td>Definition</td>
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<tr>
<td>$C_{fn}$</td>
<td>capacitance of the RLC filter of the $n^{th}$DER</td>
</tr>
<tr>
<td>$dq$</td>
<td>Global ref. frame of the system</td>
</tr>
<tr>
<td>$d_nq_n$</td>
<td>ref. frame of the $n^{th}$DER</td>
</tr>
<tr>
<td>DER$_n$</td>
<td>the $n^{th}$DER</td>
</tr>
<tr>
<td>$i_{Ddq,n}$</td>
<td>negative-sequence $dq$ components of the terminal current of the $n^{th}$VSC</td>
</tr>
<tr>
<td>$i_{Tdq,n}$</td>
<td>negative-sequence $dq$ components of the output current of the $n^{th}$DER</td>
</tr>
<tr>
<td>$i_{Dabc,n}$</td>
<td>three-phase terminal current of the $n^{th}$VSC</td>
</tr>
<tr>
<td>$i_{Tabc,n}$</td>
<td>three-phase output current of the $n^{th}$DER</td>
</tr>
<tr>
<td>$i^{+}_{Ddq,n}$</td>
<td>positive-sequence $dq$ components of the terminal current of the $n^{th}$VSC</td>
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<tr>
<td>$i^{+}_{Tdq,n}$</td>
<td>positive-sequence $dq$ components of the output current of the $n^{th}$DER</td>
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<td>$i^{-}_{refdq,n}$</td>
<td>reference for $i^{+}_{Ddq,n}$</td>
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<tr>
<td>$i^{-}_{refdq,n}$</td>
<td>reference for $i^{-}_{Ddq,n}$</td>
</tr>
<tr>
<td>$L_{fn}$</td>
<td>inductance of the RLC filter of the $n^{th}$DER</td>
</tr>
<tr>
<td>$n$</td>
<td>the $n^{th}$entry</td>
</tr>
<tr>
<td>$N$</td>
<td>the total number of entries</td>
</tr>
<tr>
<td>$P_n$</td>
<td>output real power of the $n^{th}$DER</td>
</tr>
<tr>
<td>$P_{f,n}$</td>
<td>low-pass filtered measure of $P_n$</td>
</tr>
<tr>
<td>$P_{ref,n}$</td>
<td>reference for the output real power of the $n^{th}$DER</td>
</tr>
<tr>
<td>$P_{pcc}$</td>
<td>output real power of the microgrid</td>
</tr>
<tr>
<td>$P_{pcc,ref}$</td>
<td>reference for the output real power of the microgrid</td>
</tr>
<tr>
<td>$PC_n$</td>
<td>point of connection of the $n^{th}$DER</td>
</tr>
<tr>
<td>$Q_n$</td>
<td>output reactive power of the $n^{th}$DER</td>
</tr>
<tr>
<td>$Q_{f,n}$</td>
<td>low-pass filtered measure of $Q_n$</td>
</tr>
<tr>
<td>$Q_{ref,n}$</td>
<td>reference for the output reactive power of the $n^{th}$DER</td>
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<tr>
<td>$Q_{pcc}$</td>
<td>output reactive power of the microgrid</td>
</tr>
<tr>
<td>$Q_{pcc,ref}$</td>
<td>reference for the output reactive power of the microgrid</td>
</tr>
<tr>
<td>$R_{fn}$</td>
<td>resistance of the RLC filter of the $n^{th}$DER</td>
</tr>
<tr>
<td>$u^g_{DR,n}$</td>
<td>control input signals of the $n^{th}$controlled subsystem</td>
</tr>
<tr>
<td>$u^g_{pll,n}$</td>
<td>vector of reference signals for the $n^{th}$PLL</td>
</tr>
<tr>
<td>$V_{dc,n}$</td>
<td>dc-link voltage of the $n^{th}$DER</td>
</tr>
<tr>
<td>$V_{abc,n}$</td>
<td>three-phase terminal voltage of the $n^{th}$VSC</td>
</tr>
<tr>
<td>$V_{Dabc,n}$</td>
<td>three-phase output voltage of the $n^{th}$DER</td>
</tr>
<tr>
<td>$V^{+}_{dq,n}$</td>
<td>positive-sequence $dq$ components of the terminal voltage of the $n^{th}$VSC</td>
</tr>
<tr>
<td>$V^{+}_{Ddq,n}$</td>
<td>positive-sequence $dq$ components of the output voltage of the $n^{th}$DER</td>
</tr>
</tbody>
</table>
instantaneous voltage phasor measured at the PCC

\( V_{\text{pcc}} \)

negative-sequence \( dq \) components of the terminal voltage of the \( n^\text{th} \) VSC

\( V_{\text{tdq},n} \)

negative-sequence \( dq \) components of the output voltage of the \( n^\text{th} \) DER

\( |V_{\text{pcc}}| \)

magnitude of PCC voltage

\( |V_{\text{pcc,ref}}| \)

reference for the magnitude of PCC voltage

\( |V_{\text{ref},n}| \)

reference for the magnitude of the \( n^\text{th} \) DER output voltage

\( V^*_{n} \)

nominal magnitude of the \( n^\text{th} \) DER output voltage

\( \mathbf{x}^q_{\text{DR},n} \)

state vector of the \( n^\text{th} \) DER transferred into the global ref. frame

\( \mathbf{x}_{\text{cc},n} \)

state vector of the \( n^\text{th} \) built-in current controller

\( \mathbf{x}_N \)

state vector of the power network of a grid-connected microgrid

\( \mathbf{x}_{N,\text{is}} \)

state vector of the power network of an islanded microgrid

\( \mathbf{x} \)

state vector of the grid-connected microgrid at the current cont. level

\( \mathbf{x}_{p,n} \)

state vector of the \( n^\text{th} \) subsystem in VPP mode

\( \mathbf{x}_{e,n} \)

state vector of the \( n^\text{th} \) tracking error compensator

\( \mathbf{x}_{pc,n} \)

state vector of the \( n^\text{th} \) primary controller

\( \mathbf{x}_{\text{prm}} \)

state vector of the grid-connected microgrid at the primary cont. level

\( \mathbf{x}_{pcc} \)

state vector of the secondary controller in the VPP mode

\( \mathbf{x}_{\text{drp}} \)

state vector of the power sharing controllers

\( \mathbf{x}_{\text{is}} \)

state vector of the islanded microgrid closed at the current cont. level

\( \mathbf{x}^t_{p,n} \)

state vector of the \( n^\text{th} \) subsystem in the islanded mode

\( y_{p,n} \)

measured output of the \( n^\text{th} \) controlled subsystem

\( y_{\text{ref},n} \)

desired ref. command for \( n^\text{th} \) controlled subsystem

\( y \)

measured output of the overall closed-loop microgrid

\( \mathbf{z}_{p,n} \)

controlled output of the \( n^\text{th} \) controlled subsystem

\( \eta_{n} \)

uncertainty output affecting the \( n^\text{th} \) controlled subsystem

\( \delta^*_{n} \)

nominal phase angle of the \( n^\text{th} \) DER output voltage

\( \delta_{\text{ref},n} \)

reference for the phase angle of the \( n^\text{th} \) DER output voltage

\( \delta_{\text{pcc}} \)

phase angle of PCC voltage

\( \delta_{\text{pcc,ref}} \)

reference for the phase angle of PCC voltage

\( \gamma \)

the desired level of disturbance attenuation

\( \omega_{p,n} \)

exogenous disturbance affecting the \( n^\text{th} \) controlled subsystem

\( \omega_{gm} \)

gain margin crossover frequency

\( \omega_{pm} \)

phase margin crossover frequency

\( \omega_{e,n} \)

cut-off frequency of the power filter

\( \omega_{rn} \)

\( dq \)-frame rotational speed

\( \rho_{n} \)

\( dq \)-frame reference angle
\begin{itemize}
\item $\sigma_{\text{min}}$: minimum singular value
\item $\sigma_{\text{max}}$: maximum singular value
\item $\xi_n$: uncertainty input affecting the $n^{th}$ controlled subsystem
\item $\zeta_n$: uncertainty output affecting the $n^{th}$ controlled subsystem
\end{itemize}
## Symbols

- $H_\infty$: H-infinity
- $\|x\|$: Euclidean norm of vector $x$
- $\|G(s)\|_\infty$: The $H_\infty$ norm of the continuous LTI system $G(s)$
- $\mathbb{R}^n$: set of real-valued vectors with $n$ rows
- $\mathbb{R}^{n\times m}$: set of real-valued matrices with $n$ rows and $m$ columns
- $\mathbb{R}^+$: set of non-negative real numbers
- $\mathbf{L}_2[0, \infty)$: Lebesgue space of vector-valued square integrable functions, over $[0, \infty)$
- $\ast$: a term that is induced by symmetry
- $A^T$: Transpose of matrix $A$
- $A^{-1}$: Inverse of matrix $A$
- $A \geq 0$ (or $A \leq 0$): symmetric positive (or negative) semi-definite matrix $A$
- $A > 0$ (or $A < 0$): symmetric positive (or negative) definite matrix $A$
- $\det(A)$: Determinant of a matrix $A \in \mathbb{R}^{n\times n}$
- $I_n$: Identity matrix of size $n \times n$
- $\text{trace}(A)$: Trace of a matrix $A$
- $\text{rank}(A)$: rank of a matrix $A$
- $\Delta x, \Delta u$: linearized state vector and linearized control input
Chapter 1

Introduction

1.1 Background

Microgrid is a solution for coordinated integration of Distributed Energy Resources (DERs) into power networks [1]. A microgrid is defined as a cluster of loads and DERs served by a distribution system, and can operate in the grid-connected mode, islanded mode, and transition between the two [2]. The node that a microgrid is connected to the main grid is referred to as the Point of Common Coupling (PCC). In the grid-connected mode, voltage and frequency within the microgrid is often regulated by the utility grid. The balance of real and reactive powers generated or consumed by the microgrid is also often accommodated by the main grid. During brownouts or blackouts of the main grid, the microgrid has the ability to isolate itself and operate in the islanded mode. In this mode of operation, DER units are coordinated to regulate microgrid voltage and frequency while balancing the real and reactive power generation and consumption within the microgrid.

With the evolution of microgrids, the tendency to have a microgrid to actively participate in the operation of its main grid increases [3, 4] which brings about the concept of Virtual Power Plant (VPP). A VPP is defined as an operational mode of a grid-connected microgrid where DER units are coordinately controlled to ensure pre-specified services at the PCC. From this perspective, the microgrid resembles one single controllable entity with internal dynamics hidden from the upstream grid, similar to a traditional power plant. Accordingly, the microgrid can be added to the portfolios of: (1) Transmission System Operator (TSO) to assist with frequency and voltage control of the power network as well as with congestion managament and black start services [1], (2) Distribution System Operator (DSO) to assist with load balancing on the distribution system through an incentive-based approach [5], and (3) market players such as Balance Responsible Parties.
To offer the above management and market services, a microgrid should provide the following control functionalities, in the VPP mode: (1) exchanging pre-specified real and reactive powers at the PCC with the utility grid, if the utility grid is strong, (2) regulating the voltage at the PCC while exchanging a pre-specified real power with the grid, should the utility grid be weak, or (3) providing voltage and frequency control at the PCC. Throughout this thesis, the first two paradigms will be referred to as VPP-PQ and VPP-PV modes of operation, respectively. A microgrid should also be able to operate in the islanded mode by maintaining its voltage and frequency in permissible limits defined by standards, such as IEEE 1547 [7], while dynamically balancing its power generation and consumption. The key question yet to be answered is that are the existing microgrid control structures capable of operating microgrids in the VPP mode?

1.2 Statement of Problem

A control structure is required to guarantee the above-mentioned functionalities (1) through (3) by the microgrid, as a VPP. However, to the best of our knowledge, the current technical literature lacks a thorough understanding of the principles of operation and control requirements of microgrids as VPPs. This thesis seeks to identify the performance requirements of a multi-DER microgrid operating in the VPP-PQ, VPP-PV, islanded and transition modes. Subsequently, this thesis develops a robust hierarchical control structure to meet these requirements, at the quasi steady-state and transients operating conditions, despite disturbances and uncertainties.

The hierarchical structure is advantageous for controlling microgrids, due to its: (i) ability to coordinate multiple, interacting, constrained DER units, (ii) flexibility to operate the microgrid in its various modes, (iii) interoperability with communication network technologies, and (iv) time-scale separation characteristic. This characteristic enables the hierarchical structure to handle both short-term transient control functionalities and long-term quasi steady-state optimization and management algorithms. Therefore, the hierarchical control structure can pave the way for the deployment of microgrids in providing market and management services [3,8]. A schematic diagram of the hierarchical control structure which will be developed in this thesis is provided in the next section.

In addition, the control strategy must be robust since a microgrid is subjected to frequent operating point changes, parametric and topological uncertainties, unmodeled and nonlinear dynamics, and exogenous disturbances. These perturbations are imposed by the uncertain nature of both the microgrid and the host utility grid. Failure to
consider these perturbations may result in inability to provide the specified services or even instability of the system.

1.3 Microgrids: Modes of Operation and the Hierarchical Control Structure

This section provides the operational principles of the microgrid in each mode VPP-PQ, VPP-PV, and islanded modes. Also, the schematic diagram of the hierarchical control structure which will be developed in this thesis for each mode of operation is discussed.

1.3.1 VPP-PQ Mode

In the VPP-PQ mode, the microgrid is required to exchange a pre-determined amount of real and reactive power with the utility grid. Figure 1.1 shows a schematic diagram of the proposed hierarchical control structure in this mode. As depicted, the secondary control receives real and reactive power reference commands $P_{\text{pcc,ref}}$ and $Q_{\text{pcc,ref}}$ from an upper-level coordinator to whose portfolio the microgrid is added. The secondary control then distributes the required power commands to available DER units on a real-time basis. This distribution is performed based on participation factors allocated to DER units. Participation factors could be constant gains in proportion to the generation capacity of the DER units or time-dependent variables computed dynamically to ensure the maximum revenue for the microgrid. The output of the secondary controller is therefore real and reactive power reference commands for the local power controllers (LPCs) of the electronically-interfaced DER units. It is worth mentioning that in the case of having a DER unit with a rotating machine, the secondary control determines real power and voltage magnitude reference commands for the corresponding local controllers. However, this thesis only considers electronically-interfaced DERs.

The primary control is a decentralized structure composed of LPCs. The LPC should be able to track the received real and reactive power setpoints with zero steady-state tracking error and to maintain the stability of the overall microgrid during transients and despite uncertainties and disturbances. The output of the LPC is then the reference command for the output current of the DER unit. Irrespective of the mode of operation of the microgrid, all the DER units in this thesis are electronically interfaced to the grid as Current-Controlled Voltage-Sourced Converters (CC-VSCs), for overcurrent protection purposes. The power calculation box, depicted as “PQ Cal.” in Figure 1.1 calculates the instantaneous real and reactive power, using the measured instantaneous voltage and
Chapter 1. Introduction

1.3.2 VPP-PV Mode

In the VPP-PV mode, the microgrid is connected to a relatively weak utility grid (as compared with the VPP-PQ mode). Although weak, the utility grid still maintains the frequency of the microgrid. The microgrid, however, is required to support the magnitude of the voltage at the PCC to a specified value. In addition, a pre-determined amount of real power should be exchanged with the utility grid through the PCC.

Figure 1.2 shows a schematic diagram of the proposed hierarchical control structure for

Figure 1.1: Schematic diagram of a multi-DER microgrid operating in the VPP-PQ mode

current components.
Figure 1.2: Schematic diagram of a multi-DER microgrid operating in the VPP-PV mode.

The VPP-PV mode. Comparing Figs. 1.1 and 1.2, the only difference between the devised control structure in the VPP-PQ and VPP-PV modes of operation lies on the secondary control. In the VPP-PV mode, the two reference commands sent to the secondary control are the real power reference command \( P_{pcc,ref} \) and the PCC's voltage magnitude reference command \( V_{pcc,ref} \). Similar to the VPP-PQ mode, the primary control is composed of the local power controllers determining the current reference command for the built-in current controllers of the DER units.
1.3.3 Islanded Mode

In the islanded mode of operation, the devised control structure should regulate microgrid voltage magnitude and frequency while providing the power balance within the microgrid. To that end, and as depicted in Figure 1.3, a decentralized droop control structure shares the real and reactive power among the DER units based on a pre-determined criteria such as the generation capacity of DER units. The output of the droop-based power-sharing controller is voltage magnitude and phase angle setpoints for the local voltage controllers (LVCs) to follow. LVCs are designed to track the received voltage magnitude and phase angle setpoints with zero steady-state tracking error and to maintain the stability of the microgrid during transients and despite uncertainties and disturbances. The output of the LVC is the reference command for the output current of the DER unit. As it can be seen, DER units are interfaced as CC-VSCs, similar to the VPP mode.

Although microgrid voltage, frequency, and the power balance is rapidly and robustly ensured by the decentralized structure of the local droop-augmented voltage controllers, a secondary controller can be added in this structure to: (1) optimize the operation of the microgrid at the steady-state condition, (2) ensure safe reconnection of the microgrid to the utility grid once switching from islanded to the grid-connected mode is commanded, and (3) shed loads whenever required. As depicted in Figure 1.3, the output of the secondary control is the steady-state voltage magnitude and phase angle for the droop controllers to follow. It should be noted that designing the secondary control is out of the scope of this thesis.

1.4 Literature Review

The existing literature on the VPP concept can be categorized based on the targeted time-frame; short-term transients control and long-term steady-state optimization and market participation. A quick review of the literature clearly shows that the majority of works in the field of VPPs fall in the second category. The outcome of this literature is usually real and reactive power reference commands for either the overall VPP or the individual DER units and controllable loads. These reference commands are calculated by solving optimization algorithms to: compensate the imbalance error [9], optimize the value of the produced power by the VPP [10, 11], obtain the maximum load reduction for thermostatically controllable loads over a desirable control period [12], maximize the profit of VPPs composed of wind farms and incentivized electric vehicles [13] or composed of photovoltaic systems and controllable loads [14], optimize VPP operation based on
the proposed bid strategy in a joint market of energy and spinning reserve service \[15\], participate in systems frequency stabilization by providing secondary control reserve \[16\], and control the voltage of the distributed grid system through simultaneously regulating the transmission of real and reactive power and the energy management system \[17\]. These proposed optimization algorithms can be implemented at the tertiary control or be layered over the secondary control, depicted in Section \[1.3\] to readjust the power reference command of the DER units at slower time frames when the VPP has reached

Figure 1.3: Schematic diagram of a multi-DER microgrid operating in the islanded mode
its steady-state operating condition. The major underlying assumption in all these works is that the microgrid is already an stable, actively controlled physical system. Thus, these research works are centered around proposing power dispatch optimization and market participation of the VPPs. However, the major technical barrier hampering the delivery of the management and market services promised by the VPPs is the absence of an appropriate control structure to ensure stable and reliable operation of the system during transients [4].

Currently, and to the best knowledge of the author, the work of [18] is the first attempt in designing a control scheme based on VPP characteristics. In [18], universal active and reactive power flow controllers are proposed to operate DER units in the VPP mode. The proposed controller is applicable to electronically-interfaced DER units which are connected at high-, medium-, or low-voltage level power systems over short or medium length lines. Controllers are designed based on the proposed decoupled proportional-integral-derivative (PID) strategy.

In the literature, it is believed that the primary control of the VPP is similar to that of the conventional grid-connected microgrids in the sense that the output power of the DER units can be altered according to the state of the network by means of droop-based or non-droop-based LCs constituting the primary control [19]. However, it should be noted that a VPP may be composed of geographically dispersed DER units connected to the PCC through short, medium, or long transmission lines with various X/R ratios. Also, DER units in a VPP should be coordinately controlled to provide pre-determined services in a fast time-frame and despite frequent operating point changes, exogenous disturbances, parametric and topological uncertainties, and unmodelled dynamics. Therefore, to take these characteristics of the VPP into consideration, the primary control strategy of a VPP should be non-droop-based and robust to uncertainties and exogenous disturbances.

A comprehensive review of microgrids and their control techniques are provided in [20–23]. The primary control strategy reviewed in these survey papers can be categorized as droop-based and non-droop-based. Among the few existing non-droop-based primary control methods, a robust servomechanism control method is presented in [24]. This method is robust to microgrid parametric uncertainties such as load changes. This control method is suitable for the operation of a single DER unit in the islanded mode, though. In a most recent attempt, a robust, decentralized, servomechanism control system is designed for the primary control of an islanded microgrid with multiple DERs [25]. This control system guarantees a robust performance despite parametric uncertainties and the exogenous disturbance. The controller parameters are obtained by applying an optimal controller design method, proposed in [26], to minimize the expected value of a predefined
performance index. This method is suitable to track a constant reference input in spite of a constant disturbance and structural uncertainty in the plant’s nominal model caused by variation in load parameters. Other uncertainty sources, such as topological changes and unmodelled dynamics, which are frequently affecting a VPP system, are not considered in the paper. In addition, the controller synthesis procedure adopted in this paper is complex which may prevent it to be employed by the industry.

In a more comprehensive attempt, a robust hierarchical control structure is developed to operate the microgrid in both grid-connected and islanded modes [27]. The robust voltage controller employed in this work is a nonlinear sliding mode controller. As it is widely discussed in literature, systems with sliding mode controllers are exposed to chattering in the output of the system when its switching frequency is restricted [28]. This unwanted effect could deteriorate the system performance and lead to instability of the system [29]. Furthermore, the proposed control structure is still prone to the topological changes. Also, the overall stability of the microgrid which is faced to switching between controllers and multiple sources of uncertainties and disturbances is not analytically studied in this work. All these shortcomings of the existing research works call for a sophisticated robust control strategy developed based on VPP requirements.

On the other hand, the secondary controllers developed so far for microgrid applications either work to compensate for voltage and frequency deviations when the microgrid is islanded [25, 30, 31], or to manage the power flow between the microgrid and the host utility grid when the the microgrid is grid-connected [22, 32, 33]. When it comes to VPP application, the secondary control should actively coordinate DER units to ensure predetermined services, whether the microgrid is connected to a weak or strong host utility grid or it is islanded. This coordination should be performed in a fast time-frame and despite microgrid internal transients. In addition, the secondary control should be interoperable to employing quasi steady-state optimization algorithms. However, the current literature does not report such a comprehensive and flexible secondary control structure.

1.5 Thesis Objectives

As reviewed in the previous section, the existing research works at the primary control suffer from the following major shortcomings: (i) inability to address all types of uncertainties and disturbances that stem from the dynamics of both the microgrid and the host utility grid, (ii) lacking an analytically proof of the robust stability of the overall microgrid as a single controllable entity, (iii) heuristic or complex control design procedure, (iv) poor transient and disturbance rejection performance, (v) failure to address
topological changes frequently happening in a microgrid operating in the VPP or islanded mode, and (vi) dependency to a fixed, presumed X/R ratio.

At the secondary level, the existing literature are focused on developing a secondary controller for microgrids operating either in the conventional grid-connected or islanded mode. However, they: (i) lack a secondary controller specifically tailored based on VPP performance requirements in both the transient and quasi steady-state operating conditions, and (ii) fail to develop a comprehensive and flexible structure enabling the microgrid to operate in the futuristic modes of VPP-PQ, VPP-PV, or islanded and to smoothly ride through these modes.

To solve the problem stated in Section 1.2 and motivated by the limitations and shortcomings of the aforementioned research works, this thesis focuses on developing the hierarchical control structure introduced in Section 1.3 through:

1. developing robust decentralized local power and voltage controllers, at the primary level. The developed primary control strategy should be:
   - robust in order to guarantee the stable operation of a multi-DER microgrid, despite parametric and topological uncertainties, unmodelled dynamics, operating point changes, and exogenous disturbances,
   - decentralized in order to obviate the need for geographic proximity, high-bandwidth and highly reliable communication networks, and the computation burden of centralized control schemes,
   - Multi-Input Multi-Output (MIMO) in order to accommodate systems with multi-input and multi-output channels, such as DERs controlled in the $dq$ frame of reference,
   - output tracking to enable tracking of reference commands with zero steady-state error through output feedback.

2. developing a secondary controller for the operation of a microgrid in the VPP-PQ and VPP-PV modes. The developed secondary control strategy should be:
   - able to operate a microgrid, as one single entity, to meet the internal performance requirements of a microgrid as well as the pre-specified requirements at the PCC of the microgrid, despite frequent uncertainties and disturbances,
   - interoperable with optimization algorithms to ensure the optimal participation of a microgrid to the energy market and system management functionalities.
1.6 Methodology

Thesis objectives have been achieved through the following methodology:

1. A linearized state-space model of the nonlinear microgrid study system is developed in the $dq$ frame of reference. Validity of the developed linearized model is investigated through a set of simulation studies comparing the responses of the linearized model developed in MATLAB environment versus the detailed nonlinear model developed in PSCAD/EMTDC software.

2. The linearized state-space model coded in MATLAB/Simulink environment is used to: $(i)$ synthesize the linear robust decentralized output-tracking controllers, $(ii)$ perform a set of eigenvalue analyses, and $(iii)$ measure the robustness margins of the proposed control structure.

3. The detailed nonlinear microgrid study system controlled with the proposed hierarchical control structure is developed in PSCAD/EMTDC platform. This model is used to investigate and verify the robust performance of the proposed control structure under the effects which cannot be captured by the linearized model, such as harmonics, saturations, and nonlinearities associated with the actual physical system.

1.7 Thesis Outline

The remainder of this thesis is structured as follows:

Chapter 2 proposes a robust, decentralized, MIMO, dynamic, output-tracking control strategy. The proposed strategy is employed to design robust primary power and voltage controllers for each DER unit in a multi-DER study microgrid. To that end, a linearized state-space representation of the microgrid is developed. Various sources of uncertainties and disturbances affecting microgrid dynamics in both the VPP and islanded modes are identified and modeled in the IQC framework. Then a set of rank constrained LMIs is derived to find the above-mentioned robust primary controllers which guarantee the robust stability of the overall microgrid and ensure disturbance attenuation to pre-determined levels. A series of frequency-domain robustness analyses is performed in MATLAB environment to evaluate the efficacy of the proposed control strategy.
Chapter 3 investigates the transient and steady-state behavior of the robust primary power and voltage controllers developed in Chapter 2. Several time-domain scenarios are studied in PSCAD/EMTDC to show the efficacy of the proposed control structure to: (i) track the specified reference commands in the steady-state operating regime despite local and interconnection uncertainties and exogenous disturbances, and (ii) to maintain the stability of the multi-DER study microgrid of Figure 2.1 during transients.

Chapter 4 proposes secondary control schemes to operate a multi-DER grid-connected microgrid in the VPP-PQ and VPP-PV mode. An LMI-based, PI, multivariable, robust $H_\infty$ secondary control scheme is developed which guarantees the stability of the closed-loop system while meeting a specified $H_\infty$ level of disturbance attenuation. A limiter block is also embedded to limit the reference power commands based on the generation capacity of DER units. Then, an LMI-based anti-windup compensator is added to guarantee a graceful performance degradation during the input saturation. Finally, a comprehensive set of frequency-domain and time-domain studies are performed in MATLAB and PSCAD/EMTDC platforms, respectively, to verify the performance of the developed hierarchical control structure in operating a microgrid connected to a strong or weak utility grid.

Chapter 5 presents a decentralized droop-augmented robust voltage control structure to operate multi-DER microgrids in the islanded mode. The control structure, embedded in each DER unit is composed of: (1) a static droop-based power sharing controller, composed of real power-phase angle (P-$\delta$) droop and the reactive power-voltage (Q-v) droop controllers, (2) a robust local voltage controller, and (3) an open-loop frequency control, featuring independent internal oscillator synchronized by a common time-reference signal received from a global positioning system (GPS). Finally, a comprehensive set of frequency-domain and time-domain studies are performed in MATLAB and PSCAD/EMTDC platforms, respectively, to verify the performance of the developed control structure in operating an islanded microgrid.

Chapter 6 presents the overall robust hierarchical control structure to operate microgrids in the VPP-PQ, VPP-PV, islanded, and transition modes either pre-planned or accidental. The coordination of controllers that are engaged in each mode of operation as well as the switching logic to select between controllers based on the pre- and post-transition mode are provided in this chapter. Finally, various
case studies are simulated in PSCAD/EMTDC to investigate the performance of the proposed control structure in transition mode.

Chapter 7 summarizes the contributions of the thesis, presents its conclusions, and recommends future research directions.
Chapter 2

Robust Decentralized
Output-Tracking Primary Control

2.1 Introduction

This chapter presents the development of a robust decentralized output-tracking control strategy specifically customized to meet the performance requirements of a multi-DER microgrid. The proposed robust control strategy is then employed to design one local power controller and one local voltage controller for each DER unit in a study microgrid. The local power controller provides the tracking of the real and reactive power setpoints. The local voltage controller ensures tracking of the voltage magnitude and angle setpoints. This decentralized structure of local power and voltage controllers for all DER units inside a microgrid constitutes the primary level of the hierarchical control structure, as shown in Figs. 1.1, 1.2, and 1.3 of Chapter 1.

The first step in designing robust controllers for a dynamical system is identifying and modeling disturbances and uncertainties affecting that system. To that end, a multi-DER study microgrid is first introduced in this chapter and its linearized state-space model is obtained. This linearized model accounts for the excursions of the voltage amplitude and frequency of the host utility grid and also captures the dynamics of the microgrid in both the VPP and the islanded modes of operation. Using the obtained model, sources of perturbations affecting the dynamics of the multi-DER microgrid are identified as exogenous disturbances, parametric and topological uncertainties, and unmodelled high-frequency dynamics.

Knowing the type of perturbations affecting a multi-DER microgrid, the framework of integral quadratic constraint (IQC) is employed to model these perturbations. The
IQC framework is chosen in this thesis since it: (1) captures a rich class of uncertainties, disturbances, and unmodelled dynamics, (2) extracts the structural information about perturbations, therefore, the obtained robust controllers are less conservative, and (3) treats the interactions among subsystems as uncertainty, hence, the obtained controllers are robust to topological changes, [34].

In the next step, the IQC-based robust decentralized static state-feedback control strategy proposed by [35] is expanded upon in this chapter and a novel IQC-based robust decentralized dynamic output-tracking control strategy is proposed. It should be noted that although the proposed control strategy is developed based on the specifications and performance requirements of multi-DER microgrids, it is indeed a generic robust control methodology applicable to any linearized dynamical system that is subjected to energy-bounded disturbances, parametric and topological uncertainties, and unmodeled dynamics.

The design of the proposed control strategy requires solution to a set of coupled constrained Ricatti equations, which is difficult to find analytically. Therefore, as another contribution of this chapter, the proposed set of coupled Ricatti equations is re-formulated in terms of rank constrained Linear Matrix Inequalities (LMIs). Various toolboxes exist to efficiently find a numerical solution to the developed LMIs, such as YALMIP toolbox of MATLAB. In addition, the proposed LMI-based control strategy benefits from a systematic design procedure (as opposed to the conventional trial and error procedures).

The prominent features of the proposed control strategy are: (1) reference tracking with zero steady-state error in spite of uncertainties and disturbances; (2) maintaining the stability of the overall multi-DER microgrid during transients; (3) attenuating the $H_{\infty}$ norm of the transfer function from the disturbance input to the controlled output to a pre-determined level, the so-called disturbance rejection, and (4) providing a systematic and numerically efficient LMI-based design procedure. In addition, since the proposed control strategy offers disturbance rejection and since utility grid voltage and frequency are presented as disturbances in the obtained model, a smooth transition between the VPP mode and the islanded mode of operation is foreseen.

Finally, the developed LMIs are solved, in this chapter, to synthesize local power and voltage controllers for each DER unit of the study microgrid. Then, the method of balanced truncation using Hankel singular values is employed to reduce the order of the obtained local controllers while maintaining the achieved level of disturbance attenuation. To quantify the robustness of the developed controllers, a series of robustness margins are measured in MATLAB environment, including module margin, singular value analysis of the sensitivity function, maximum uncertainty bounds, and input time-delay tolerance.
2.2 Structure of the Study Microgrid

A schematic diagram of the microgrid study system is shown in Figure 2.1. The microgrid consists of 13.8-kV, three-phase, symmetrical, radial distribution feeders, connected to the utility grid through a 69-kV line. The microgrid includes four feeders, four DER units, and a set of loads. The microgrid configuration and parameters are adopted from the benchmark system of the IEEE standard 399-1997 [36].

All four DER units are assumed to be dispatchable electronically-interfaced units of a voltage rating of 0.6 kV. DER$_1$ to DER$_3$ have power ratings of 3 MVA, whereas DER$_4$ has a power rating of 0.5 MVA. Electrical parameters of the DER units are provided in Table B.1, Appendix B.

The host utility grid is represented by a 69-kV, 100-MVA short circuit capacity source with the X/R ratio of 22.2. The voltage is stepped down from 69 kV to 13.8 kV through a 15 MVA transformer $T_1$, with delta configuration at the high-voltage side and grounded star at the low-voltage side. Moreover, a three-phase 2-MVar fixed shunt capacitor bank, in delta configuration, is connected to the point of common coupling (PCC) of the microgrid to the utility grid. Loads, L1 to L7, are three-phase, balanced loads, each composed of series R and L branches. In Appendix A electrical parameters of the microgrid of Figure 2.1 including transformer parameters and configuration, the length of the distribution lines and also the type of the overhead lines, and the rated loads of feeders are given in Tables A.1, A.2 and A.3 respectively.

2.3 Structure of the DER units

Figure 2.2 shows the schematic diagram of a DER unit, the $n^{th}$DER unit, illustrating its hardware components and its input and output signals.

2.3.1 Hardware Components

As illustrated in Figure 2.2 each electronically-interfaced DER contains a DC/AC, two level, sinusoidal pulse-width modulated (SPWM) voltage-sourced converter (VSC). The DC-side voltage is assumed to be impressed at $V_{dc} = 1.2$ kV. Circuit elements $C_{fn}$ and $L_{fn}$ represent the per-phase capacitance and inductance of the low-pass $RLC$ filter that interfaces the VSC. The element $R_{fn}$ represents the combined effect of the on-state VSC switch resistances and the resistance of the $RLC$ filter.

The block $abc \rightarrow dq$ converts the input signal from $abc$ to both positive-sequence and negative-sequence $dq$ reference frame, based on the received angle $\rho_n$. The SPWM block
hosts a generalized SPWM switching process to produce six gating pulses for the VSC switches, based on the required terminal voltage of the VSC. The term generalized refers to the SPWM signal generator developed in [37], in which both positive and negative sequence components are used in generating gating pulses to balance output current of the DER units.

For overcurrent and overload protection, each DER is interfaced to the grid as a current-controlled VSC. The built-in current controller is a dual-sequence current controller which is composed of a positive-sequence and a negative-sequence PI controller, depicted in Figure 2.3 and is borrowed from [37]. The current control strategy is proportional-integral-based (PI-based) and designed in $dq$ frame of reference based on the guidelines provided in Section 8.4.1 of [38] to have a closed-loop time constant of 1.25 ms. This built-in controller: (1) regulates the positive-sequence of the output current of the DER to follow the received setpoints $i_{refd,n}^+$ and $i_{refq,n}^+$, and (2) balances the output...
current of the DER by following control commands $i_{refd,n}^- = 0$ and $i_{refq,n}^- = 0$. The output signals of the current controller are the $d$ and $q$ components of the reference command for the terminal voltage $V_{t,n}$, which is fed to the generalized SPWM switching block. Also, depicted in Figure 2.3, low-pass filters (LPFs) are used in the built-in current controller to attenuate high-frequency components of the output current and voltage of the DER. These high-frequency components are generated by the switching process of the VSC. The time constant of the filters are set to 1 ms [39].

To protect the converter, current reference commands $i_{ref,d,n}^+$ and $i_{ref,q,n}^+$ are limited by a vector magnitude limiter, as proposed in [40]. Schematic diagram of the limiter is provided in Figure 2.4.

### 2.3.2 Input and Output Signals of the DER unit

As illustrated in Figure 2.2, the two input signals $i_{ref,d,n}^+$ and $i_{ref,q,n}^+$ are the reference commands for the positive-sequence $d$ and $q$ components of the terminal current of the DER, i.e., $i_{DR,n}$ in Figure 2.2. As discussed in Figure 1.1 of Chapter 1, these current reference commands are determined by the primary controllers to either ensure a specific

![Diagram of the n-th electronically-interfaced DER unit](image_url)

**Figure 2.2:** Schematic diagram of the $n$th electronically-interfaced DER unit.
amount of real and reactive power output by the DER unit or to regulate the voltage at its point of connection \( PC_n \). Input signals \( i_{refd,n}^- \) and \( i_{refq,n}^- \) are set to zero to ensure a balanced current for the DER unit.

The input signal \( \rho_n \) is the reference angle based on which the conversion between \( abc \) and \( dq \) reference frames is performed. In the VPP mode, \( \rho_n \) is the angle of the output voltage of the DER unit measured at its \( PC_n \), i.e., \( V_{DR,n} \) in Figure 2.2, and is estimated by a phase-locked loop (PLL). In the islanded mode, however, \( \rho_n \) is generated by an independent internal oscillator synchronized by a common time-reference signal received from a global positioning system (GPS). In this way, the frequency of the microgrid is set to a desired value \[41\].

The output signals of the DER unit are the positive-sequence \( d \) and \( q \) components of the output current, i.e., \( i_{T,n}^+ \), and output voltage of the DER. These signals are also used to calculate the instantaneous real and reactive output power generated or absorbed by the DER unit as well as the instantaneous magnitude and phase angle of the output voltage of \( PC_n \) as \( P_n = 1.5(V_{DRd,n}^+\dot{i}_{Td,n}^+ + V_{DRq,n}^+\dot{i}_{Tq,n}^+) \), \( Q_n = 1.5(-V_{DRd,n}^+\dot{i}_{Tq,n}^- + V_{DRq,n}^+\dot{i}_{Td,n}^-) \), \( |V_{DR,n}| = \sqrt{(V_{DRd,n}^+)^2 + (V_{DRq,n}^+)^2} \), and \( \delta_{V,n} = tan^{-1}(V_{DRq,n}^+/V_{DRd,n}^+) \) \[38\].

Figure 2.3: Schematic diagram of the dual-sequence built-in current controller \[37\]: (a) positive-sequence controller, (b) negative-sequence controller.

Figure 2.4: Block diagram of the vector magnitude limiter.
2.4 State-Space Representation of the Study Microgrid

This section presents a linearized state-space representation of the microgrid of Figure 2.1 operating in the VPP or islanded mode. The linearized model is expressed in a global rotating \(dq\) frame that is synchronized to the PCC’s voltage. This dynamic model is used for the synthesis of appropriate primary controllers, based on the proposed control methodology. The model captures the positive-sequence dynamics only.

A single-line diagram of the microgrid is depicted in Figure 2.5, in which distribution lines and loads are represented by series \(RL\) branches, in each phase, and lumped together where applicable. The fixed-capacitor bank at the PCC is also modeled by an equivalent three-phase shunt capacitor, \(C_p\).

As Figure 2.5 indicates, the dynamics of the microgrid are governed by those of five interacting subsystems, that is power network and the four DER units. On the other hand, as Figs. 2.2 and 2.3 indicate, dynamics of each DER are governed by those of the corresponding power circuit, PLL (in the VPP mode only), and built-in positive-sequence current controller. In the following, a qualitative description on the dynamics of each part will be provided. An extensive detailed mathematical procedure is presented in Appendix B.

The power circuit of the DER, is represented by a three-phase RLC branch, shown in Figure 2.2. A differential equation relates the vectors of the fundamental components of the VSC terminal voltage (\(V_{t,n}\)), VSC terminal current (\(i_{DR,n}\)), DER’s output voltage (\(V_{DR,n}\)), and DER unit output current (\(i_{T,n}\)), in a \(dq\) reference frame \([42]\). In the VPP mode, a PLL is employed here to estimate the angle of the output voltage of the DER and is based on the phase angle estimator described in \([39]\). Therefore, in the VPP mode, the state variable describing the dynamics of the DER combined with its corresponding PLL is represented by \(\Delta x_{DR,n}^g \in \mathbb{R}^6\) for \(n = 1, ..., 4\), in which \(\Delta x_{DR,n}^g = [\Delta i_{DRd,n}^g, \Delta i_{DRq,n}^g, \Delta v_{DRd,n}^g, \Delta v_{DRq,n}^g, \Delta \theta_n, \Delta \omega_{rn}]^T\). In this state vector, \(\Delta \omega_{rn}\) is the deviations in the angular frequency at the \(PC_n\) and \(\Delta \theta_n\) indicates its corresponding angle. In the islanded mode, on the other hand, the state variable of the \(n^{th}\)DER is obtained as \(\Delta x_{DR,n}^g = [\Delta i_{DRd,n}^g, \Delta i_{DRq,n}^g, \Delta v_{DRd,n}^g, \Delta v_{DRq,n}^g]^T \in \mathbb{R}^4\). Also, the superscript \(g\) indicates that output current and voltage components of the DER expressed in the corresponding local \(d_nq_n\) frame are transferred to the global \(dq\) frame. The \(d\) axes of the local \(d_nq_n\) and the global \(dq\) frames of reference are aligned with the voltage phasor measured at \(PC_n\) and at the PCC, respectively \([39]\).

The positive-sequence current controller is composed of two PI compensators
and four LPFs, depicted in Figure 2.3 (a). The dynamics of each PI compensator can be captured by a first-order differential equation. The dynamics of the four LPFs are also described by first-order differential equations [39]. The state variable describing the dynamics of the built-in controller and the in-loop LPFs are represented by \( \Delta x_{cc,n} \in \mathbb{R}^6 \). The subscript “cc, n” denotes the \( n^{th} \) current controller.

In the power network, distribution lines and loads are represented by series RL branches. The fixed-capacitor bank at the PCC bus is also modeled by an equivalent three-phase shunt capacitor. The dynamic of the power network is then represented by differential equations corresponding to the three-phase RL and the shunt capacitor branch, in the global \( dq \) frame [39]. It should be noted that the second difference between the dynamic model of a microgrid in the VPP and islanded modes of operation roots from the difference in the dynamic model of its power network. In the VPP mode, in addition to the microgrid internal currents and PCC voltage, the current transferred to the utility grid through feeder \( TL_1 (i_u) \) and the voltage at \( Bus_u (V_u) \), Figure 2.5, contribute to the dynamics of the power network. Therefore, the state variable of the power network are obtained as \( \Delta x_N \in \mathbb{R}^{22} \) where:

\[
\Delta x_N = [\Delta i_1^d \Delta i_1^q \Delta i_2^d \Delta i_2^q \Delta i_3^d \Delta i_3^q \Delta i_4^d \Delta i_4^q \Delta i_5^d \Delta i_5^q \Delta i_{u,d} \Delta i_{u,q} \\
\Delta i_{T1,d}^d \Delta i_{T1,q} \Delta i_{T2,d} \Delta i_{T2,q} \Delta i_{T3,d} \Delta i_{T3,q} \Delta i_{T4,d} \Delta i_{T4,q} \Delta v_{pcc,d} \Delta v_{pcc,q}]^T,
\]

and \( i_{n,dq} \) and \( i_{Tn,dq} \) are distribution lines and DER unit output currents, Figure 2.5. Also, the subscript \( N \) represents the power Network state vector.

In the islanded mode, the microgrid is separated from the host utility grid. Therefore, deviations in the utility grid current and voltage, i.e., \( \Delta i_u \) and \( \Delta V_u \), do not participate in the dynamical equations governing the power network dynamics. Thus, the state variable of the power network are obtained as \( \Delta x_{N,is} \in \mathbb{R}^{20} \) where:

\[
\Delta x_{N,is} = [\Delta i_1^d \Delta i_1^q \Delta i_2^d \Delta i_2^q \Delta i_3^d \Delta i_3^q \Delta i_4^d \Delta i_4^q \Delta i_5^d \Delta i_5^q \Delta i_{T1,d} \\
\Delta i_{T1,q} \Delta i_{T2,d} \Delta i_{T2,q} \Delta i_{T3,d} \Delta i_{T3,q} \Delta i_{T4,d} \Delta i_{T4,q} \Delta v_{pcc,d} \Delta v_{pcc,q}]^T.
\]

### 2.4.1 State-Space Representation for the VPP Mode

In the VPP mode, following the procedure provided in Appendix B and combining the dynamic models of the above-mentioned five subsystems, based on their input-output relationship, the integrated state-space representation of the microgrid at the built-in
current controller level is obtained as:

\[
\Delta \dot{x} = A \Delta x + B_{c1} \Delta u_{DR,1} + B_{c2} \Delta u_{DR,2} + B_{c3} \Delta u_{DR,3} + B_{c4} \Delta u_{DR,4} \\
+ B_{pll1} \Delta u_{pll,1} + B_{pll2} \Delta u_{pll,2} + B_{pll3} \Delta u_{pll,3} + B_{pll4} \Delta u_{pll,4} + B_d \Delta v_u + B_w \Delta \omega_s, \tag{2.1}
\]

in which

\[
\Delta x = [\Delta x_{g,DR,1}^T, \Delta x_{g,DR,2}^T, \Delta x_{g,DR,3}^T, \Delta x_{g,DR,4}^T, \Delta x_{cc,1}^T, \Delta x_{cc,2}^T, \Delta x_{cc,3}^T, \Delta x_{cc,4}^T, \Delta x_N^T]^T,
\]

\[
\Delta u_{DR,n} = [\Delta i_{ref,d,n}, \Delta i_{ref,q,n}]^T,
\]

\[
\Delta u_{pll,n} = [\Delta \theta_{ref,n}, \Delta \omega_{ref,n}]^T.
\]

for \( n = 1, ..., 4 \). Deviations in the voltage and frequency of the host utility grid are represented by \( \Delta v_u \) and \( \Delta \omega_s \). Also, \( \Delta u_{DR,n} \) are the control inputs while the \( \Delta u_{pll,n}, \Delta v_u \), and \( \Delta \omega_s \) are time-varying exogenous disturbances.

When designing a decentralized control structure, the overall system model should virtually be partitioned into subsystems for which local controllers will be synthesized. Provided in Figure 2.5 our microgrid is partitioned into five virtual subsystems: four controlled subsystems, described by the state vectors \( \Delta x_{p,n} = [\Delta x_{g,DR,n}^T, \Delta x_{cc,n}^T]^T \) for \( n = 1, ..., 4 \), and one passive power network subsystem, described by the state vectors \( \Delta x_N \). Partitioning (2.1) based on the aforementioned state vector arrangements, the state-space model of the \( n^{th} \) controlled subsystem one obtains:

\[
\Delta \dot{x}_{p,n} = A_{nn} \Delta x_{p,n} + B_{c,nn} \Delta u_{DR,n} + \\
\sum_{j=1}^{4} [A_{nj} \Delta x_{p,j} + A_{nN} \Delta x_N + B_{c,nj} \Delta u_{DR,j}] + B_d \Delta v_u + B_w \Delta \omega_s + B_{npll} \Delta u_{pll,n}, \tag{2.2}
\]

for \( n = 1, ..., 4 \). This decentralized representation of the microgrid will be used in the upcoming sections to identify sources of uncertainties and disturbances.

### 2.4.2 State-Space Representation for the Islanded Mode

Following the detailed procedure provided in Appendix B and combining the dynamic models of the above-mentioned five subsystems, based on their input-output relationship, the integrated state-space representation of the islanded microgrid at the built-in current controller level is obtained as:
\[
\Delta \dot{x}_{is} = A_{is}\Delta x_{is} + B_{c1,is}\Delta u_{DR,1} + B_{c2,is}\Delta u_{DR,2} + B_{c3,is}\Delta u_{DR,3} + B_{c4,is}\Delta u_{DR,4}, \tag{2.3}
\]

in which \(\Delta x_{is} = [\Delta x_{g,DR,1}^T, \Delta x_{g,DR,2}^T, \Delta x_{g,DR,3}^T, \Delta x_{g,DR,4}^T, \Delta x_{T,cc,1}^T, \Delta x_{T,cc,2}^T, \Delta x_{T,cc,3}^T, \Delta x_{T,cc,4}^T, \Delta x_{N,is}^T]^T\) is the state vector and \(\Delta u_{DR,n} = [\Delta i_{refd,n}, \Delta i_{refq,n}]^T\) is the control input.

Partitioning the overall interconnected system to its five subsystems provided in Figure 2.5, the state-space model of the \(n^{th}\) controlled subsystem, for \(n = 1, ..., 4\), is obtained as:

\[
\Delta \dot{x}_{p,n}^{is} = A_{n,n}^{is}\Delta x_{p,n}^{is} + B_{c,nn}^{is}\Delta u_{DR,n} + \sum_{j=1}^{4} \left[ A_{nj}^{is}\Delta x_{p,j}^{is} + B_{c,nj}^{is}\Delta u_{DR,j} + A_{nN}^{is}\Delta x_{N,is}^{is} \right]. \tag{2.4}
\]
2.5 Sources of Perturbation in a Microgrid

The previous section was devoted to representing the physical microgrid via a mathematical model describing the behaviour of the system around an operating point. This obtained model is called as the *nominal model*. In practice, there are discrepancies between the microgrid and its nominal mathematical model. The discrepancies are due to: (1) the lack of exact information on the structure of microgrid and its components, (2) the time-varying nature of the loads, generation units, topology; (3) operating point changes, and (4) the external unknown inputs affecting the dynamics of the microgrid, such as excursions in the voltage and frequency of the utility grid. These discrepancies with their unknown nature are referred to as perturbations and are classified as uncertainties and disturbances.

The microgrid of Figure 2.5 is composed of four DER subsystems (local controllable subsystems) coupled together through an uncertain time-varying subsystems of power network (interacting subsystem). Uncertainties and disturbances, generally called as perturbations, experienced by this system can be classified as:

1) Uncertainties in the local subsystem, e.g., deviations in the operating point of a DER unit. These uncertainties exist in the first two terms in the left hand side after the equality sign of \(2.2\) and \(2.4\).

2) Uncertainties in the interactions among subsystems including (i) all of the above-mentioned uncertainties existing in the neighbouring DER units and (ii) topological changes such as sudden connection or disconnection of loads, lines, and generation units and also connection status of the microgrid to the utility grid. The second two parts of \(2.2\) and \(2.4\), lumped together in the \(\sum\) operator, constitute the interconnection uncertainties.

3) Disturbances including deviations in the utility grid voltage \(\Delta v_u\), utility grid frequency \(\Delta \omega_s\), and \(\Delta u_{\text{pll},n}\).

After identifying the sources of uncertainties, a suitable mathematical model of the uncertain system should be formed so that all the existing perturbations can be accounted for in system robustness analysis and controller design.

Local uncertainties in the \(n\)th controlled subsystem can be modeled as \(E_{p,n}\xi_n(t)\) in which \(\xi_n(t) = \psi_n(t, \zeta_n(.))\) is the uncertainty input. This dynamic perturbation is driven only by a nonlinear time-varying function \(\psi_n\) on a subspace of \(\zeta_n\) formed by the state vector and control input of the \(n\)th controlled subsystem as:

\[
\zeta_n(t) = H_{p,n}x_{p,n}(t) + G_nu_{DR,n}(t),
\]  

(2.5)
in which \( \zeta_n(.) \) is known as uncertainty output.

Interconnection uncertainties, on the other hand, can be modeled as \( L_{p,n} \eta_n(t) \). The input \( \eta_n = \phi_n(t, \zeta_j(.)) \) describes the effect of the \( j \)th subsystem on the \( n \)th subsystem, for all \( j = 1, \ldots, N \) and \( j \neq n \), in which \( \phi_n \) is a nonlinear time-varying function defined on the subspace \( \zeta_j(.) \) of the \( j \)th subsystem.

Finally, all the external inputs of \( \Delta u_{pl,n}, \Delta v_u, \) and \( \Delta \omega_s \) are lumped together to form the exogenous disturbances of \( w_{p,n} = [\Delta u_{pl,n}^T, \Delta v_u^T, \Delta \omega_s^T]^T \). The effect of the exogenous disturbance on the system can be described as \( F_{p,n}w_n(t) \), in which \( F_{p,n} = [B_d, B_w, B_{pl}] \).

Figure C.1 in Appendix C illustrates a large-scale system composed of two interconnected subsystems under the effect of these three group of perturbations.

Using the above-mentioned models for the three group of perturbations, (2.2) (and similarly (2.4)) can be re-formulated to represent the \( n \)th controlled subsystem as:

\[
\dot{x}_{p,n}(t) = A_{p,n}x_{p,n}(t) + B_{p,n}u_{DR,n}(t) + E_{p,n}x_n(t) + L_{p,n}\eta_n(t) + F_{p,n}w_{p,n}(t),
\]

in which matrices \( A_{p,n} \) and \( B_{p,n} \) represent the certain known parts of matrices \( A_{nn} \) and \( B_{c,nn} \) in (2.2) (and similarly (2.4)). Guidelines on choosing the appropriate subspace describing local and interconnection uncertainties \( \xi_n(t) \) and \( \eta_n(t) \) are provided in the upcoming Section 2.7.

**Remark 1.** It is worth mentioning that the prefix \( \Delta \) is omitted in (2.6) for a less cluttered notations. However, it is to be remembered that all the state-space vectors include variables perturbed with respect to steady-state values.

### 2.6 The Proposed IQC-Based Robust Decentralized Output-Tracking Primary Control

Now that uncertainties and disturbances affecting the dynamics of the microgrid are identified and modeled, this section proposes a robust decentralized output-tracking control strategy for the primary level to: (1) track the received setpoints and attenuate the effect of disturbances to a desired level, i.e., disturbance rejection, during the steady-state operation, and (2) maintain the stability of the overall microgrid during transients despite disturbances and local and interconnection uncertainties.

The main result which will be presented in this section shows that if: (1) the local uncertainty \( \xi_n(.) \) and the interconnection uncertainty \( \eta_n(.) \) satisfy certain IQCs, i.e., falling into the category of the so-called admissible uncertainties, and (2) disturbances
are energy-bounded satisfying $w_{p,n}(\cdot) \in L_2[0 \infty)$, then a set of necessary and sufficient conditions will be obtained which guarantee the two control objectives, mentioned above.

It should be noted that although the proposed control strategy is inspired from the characteristics and performance requirements of microgrids, it is indeed a generic robust decentralized control strategy. This strategy is applicable to any dynamical system subjected to uncertainties and disturbances, so far as these perturbations satisfy the assumptions of the proposed control theory.

### 2.6.1 Structure of the Proposed Primary Control

Inspired by the dynamical model of the $n^{th}$ controlled subsystem of the study microgrid in (2.6) and without loss of generality, let us suppose that there is a dynamical system in the form of (2.7), subjected to local and interconnection uncertainties as well as disturbances, in the following form:

\begin{align*}
\dot{x}_{p,n}(t) &= A_{p,n}x_{p,n}(t) + B_{p,n}u_{DR,n}(t) + E_{p,n}\xi_n(t) + L_{p,n}\eta_n(t) + F_{p,n}w_{p,n}(t), \\
y_{p,n}(t) &= C_{p,n}x_{p,n}(t) + D_{p,n}w_{p,n}(t),
\end{align*}

(2.7)

in which $y_{p,n}$ is the measured output of the $n^{th}$ subsystem.

The proposed structure for the $n^{th}$ robust, dynamic, and output-tracking local controller is composed of two parts:

1) a decentralized tracking error compensator:

\begin{equation}
\dot{x}_{e,n}(t) = (y_{p,n}(t) - y_{ref,n}),
\end{equation}

where $y_{ref,n}$ is the desired reference command, and $x_{e,n}$ is the state vector of the $n^{th}$ error compensator.

2) a decentralized dynamic stabilizing controller:

\begin{align*}
\dot{x}_{pc,n}(t) &= A_{pc,n}x_{pc,n}(t) + B_{pc,n}x_{e,n}(t), \\
{u}_{DR,n}(t) &= C_{pc,n}x_{pc,n}(t) + D_{pc,n}x_{e,n}(t),
\end{align*}

(2.9)

where $x_{pc,n} \in \mathbb{R}^{n_{pc,n}}$ is the state vector of the $n^{th}$ primary controller and $A_{pc,n}, B_{pc,n}$,
$C_{pc,n}$ and $D_{pc,n}$ are controller matrices, of appropriate dimensions, to be found.

### 2.6.2 Theory of the Proposed Primary Control

In this section, the final and summarized version of the proposed control theorem is presented. Due to page limits, the proof of the proposed theorem, featuring the expansions and the added Ricatti equations to the control methodology presented in [35], are provided in Appendix C.

Augmenting the subsystem (2.7) with the error compensator (2.8) results in:

$$\begin{align*}
\dot{x}_n(t) &= A_n x_n(t) + B_n u_{DR,n}(t) + E_n \xi_n(t) + L_n \eta_n(t) + F_n w_n(t), \\
y_{p,n}(t) &= C_y,n x_n(t) + D_y,n w_n(t), \\
z_n(t) &= C_n x_n(t) + D_n u_n(t),
\end{align*}$$

where $z_n(t)$ is defined as the $n^{th}$ controlled output.

Let us assume that the input and output uncertainties $\xi_n(.)$ and $\zeta_n(.)$ have the following structures:

$$\begin{align*}
\zeta_n &= [\zeta_n,1, \ldots, \zeta_n,k_n] ; \\
\xi_n &= [\xi_n,1 \cdots \xi_n,k_n] ; \\
H_n &= \begin{bmatrix} H_{n,1}^T & \cdots & H_{n,k_n}^T \end{bmatrix}^T ; \\
G_n &= \begin{bmatrix} G_{n,1}^T & \cdots & G_{n,k_n}^T \end{bmatrix}^T ; \\
\zeta_{n,v} &= H_{n,v} x_{p,n}(t) + G_{n,v} u_{DR,n}(t); \\
\xi_{n,v}(t) &= \psi_{n,v}(t, \zeta_{n,v}(.)); \quad v = 1, \ldots, k_n.
\end{align*}$$

**Definition 1 (Admissible Local and Interconnection Uncertainties).** Let us also assume that the control input $u_n(.)$, interconnection input $\eta_n(.)$, uncertainty input $\xi_n(.)$, and the disturbance input $w_n(.)$ are all locally integrable over $(0 \ t^*)$. Then, local and interconnect-
tion uncertainties are admissible if they satisfy the IQCs (2.12), where \(d_{n,1} \geq 0, \ldots, d_{n,k_n} \geq 0\) and \(\hat{d}_1 \geq 0, \ldots, \hat{d}_N \geq 0\) are constants determining the bound on the energy of local and interconnection uncertainties, respectively [34]:

\[
\int_0^{t^*} ||\xi_n(t)||^2 \, dt \leq \int_0^{t^*} ||\zeta_n(t)||^2 \, dt + d_{n,v}
\]

\[
\int_0^{t^*} ||\eta_n(t)||^2 \, dt \leq \sum_{\mu=1,\mu \neq n}^N \int_0^{t^*} ||\zeta_\mu(t)||^2 \, dt + \hat{d}_n
\]

(2.12)

**Assumption 1.** For all \(n = 1, \ldots, N\), the pair \((A_n, B_n)\) is stabilizable, \(D^T_n D_n > 0\) and the pair

\[(A_n - B_n (D^T_n D_n)^{-1} D^T_n C_n, (I - D_n (D^T_n D_n)^{-1} D^T_n) C_n)\]

(2.13)

is detectable. The pair \((A_n, C_{y,n})\) is also detectable, \(D_{y,n} D_{y,n}^T > 0\) and the pair

\[(A_n - E_n D^T_{y,n} (D_{y,n} D_{y,n}^T)^{-1} C_{y,n}, E_n (I - D^T_{y,n} (D_{y,n} D_{y,n}^T)^{-1} D_{y,n}))\]

(2.14)

is stabilizable.

Now, consider a collection of coupled algebraic Riccati equations as:

\[
(A_n - B_n R_n^{-1} \tilde{D}^T_n \tilde{C}_n)^T X_n + X_n (A_n - B_n R_n^{-1} \tilde{D}^T_n \tilde{C}_n) + \\
\tilde{C}_n (I - \tilde{D}_n R_n^{-1} \tilde{D}^T_n) \tilde{C}_n - X_n (B_n R_n^{-1} B^T_n - \tilde{B}_{2,n} \tilde{B}^T_{2,n}) X_n = 0,
\]

(2.15)

\[
(A_n - \tilde{B}_{2,n} \tilde{D}^T_{2,n} \Gamma_{y,n}^{-1} C_{y,n}) Y_n + Y_n (A_n - \tilde{B}_{2,n} \tilde{D}^T_{2,n} \Gamma_{y,n}^{-1} C_{y,n})^T + \\
Y_n (\tilde{C}^T_n \tilde{C}_n - C^T_{y,n} \Gamma_{y,n}^{-1} C_{y,n}) Y_n + \tilde{B}_{2,n} (I - \tilde{D}^T_{2,n} \Gamma_{y,n}^{-1} \tilde{D}_{2,n}) \tilde{B}^T_{2,n} = 0.
\]

(2.16)

where \(\tau_{n,v} > 0, v = 1, \ldots, k_n\), and \(\theta_n > 0, n = 1, \ldots, N\), are given constants. Also,

\[
\tau_n = [\tau_{n,1}, \ldots, \tau_{n,k_n}]^T; T_n = \text{diag} [\tau_{n,1}, \ldots, \tau_{n,k_n}]; \bar{\theta}_n = \sum_{j=1, j \neq n}^N \theta_j.
\]
\[
\begin{bmatrix}
C_n \\
(\tau_{n,1} + \bar{\theta}_n)^{1/2}H_{n,1} \\
\vdots \\
(\tau_{n,k_n} + \bar{\theta}_n)^{1/2}H_{n,k_n}
\end{bmatrix};
\]
\[
\begin{bmatrix}
D_n \\
(\tau_{n,1} + \bar{\theta}_n)^{1/2}G_{n,1} \\
\vdots \\
(\tau_{n,k_n} + \bar{\theta}_n)^{1/2}G_{n,k_n}
\end{bmatrix};
\]
\[
\begin{bmatrix}
\bar{C}_n = \tau_{n,1} + \bar{\theta}_n)^{1/2}H_{n,1} \\
\vdots \\
(\tau_{n,k_n} + \bar{\theta}_n)^{1/2}H_{n,k_n}
\end{bmatrix};
\]
\[
\begin{bmatrix}
\bar{D}_n = (\tau_{n,1} + \bar{\theta}_n)^{1/2}G_{n,1} \\
\vdots \\
(\tau_{n,k_n} + \bar{\theta}_n)^{1/2}G_{n,k_n}
\end{bmatrix};
\]
\[
\bar{R}_n = \bar{D}'_n \bar{D}_n; 
\]
\[
\bar{B}_{2,n} = \begin{bmatrix} E_n T_n^{-1/2} & \theta_n^{-1/2} L_n & \gamma^{-1/2} F_n \end{bmatrix};
\]
\[
\bar{D}_{2,n} = [D_{y,n} T_n^{-1/2}, 0, 0]; 
\]
\[
\Gamma_{y,n} = \bar{D}_{2,n} \bar{D}'_{2,n}
\]
\[
(2.17)
\]

**Theorem 1.** The overall dynamical system composed of \(N\) coupled subsystems (2.7) is absolutely stabilizable with disturbance attenuation level \(\gamma\) via decentralized dynamic controllers (2.8) and (2.9), for \(n = 1, \ldots, N\), if and only if there exist constants \(\tau_n \in \mathbb{R}^{k_n}\) and \(\bar{\theta} \in \mathbb{R}^N\) such that (2.15) has a solution \(X_n = X_n^T \geq 0\) and (2.16) has a solution \(Y_n = Y_n^T \geq 0\) satisfying: (i) spectral radius condition \(\rho(X_n Y_n) < \gamma^2\) and (ii) Hurwitz stability criterion for the following four matrices: (1) \(A_n - B_n R_n^{-1} (B_n^T X_n + D_n^T \bar{C}_n)\), (2) \(A_n - B_n R_n^{-1} (B_n^T X_n + \bar{D}_n^T \bar{C}_n) + \bar{B}_{2,n} \bar{B}_{2,2,n} X_n\), (3) \(A_n - (Y_n C_{y,n} + \bar{B}_{2,n} \bar{D}_{2,n}) \Gamma_{y,n}^{-1} C_{y,n} + Y_n C_n^T C_n\), and (4) \(A_n - (Y_n C_{y,n} + \bar{B}_{2,n} \bar{D}_{2,n}) \Gamma_{y,n}^{-1} C_{y,n}\).

Further, the matrices of the controllers are given as:

\[
A_{pc,n} = A_n + B_n C_{c,n} - B_{c,n} C_{y,n} + (\bar{B}_{2,n} - B_{c,n} \bar{D}_{2,n}) \bar{B}_{2,2,n} X_n,
\]
\[
B_{pc,n} = (I - Y_n X_n)^{-1} (Y_n C_{y,n}^T + \bar{B}_{2,n} \bar{D}_{2,n}) \Gamma_{y,n}^{-1},
\]
\[
C_{pc,n} = -R_n^{-1} (B_n^T X_n + \bar{D}_n^T \bar{C}_n), 
\]
\[
D_{pc,n} = 0.
\]
\[
(2.18)
\]

**Proof.** The proof is provided in Appendix [B].

**Remark 2.** A detailed definition of the absolute stability of a dynamical system is provided in [35]. The closed-loop system (2.7), (2.8), and (2.9) is also said to achieve disturbance attenuation level \(\gamma\) if for all \(n = 1, \ldots, N\), \(x_n(0) = 0\) and \(x_{pc,n}(0) = 0\) and the following \(H_\infty\) norm bound condition is satisfied:
where $z(.) = [z_1, ..., z_N]$ and $w(.) = [w_1, ..., w_N]$ and $\Xi$ and $\Psi$ are the sets of admissible local and interconnection uncertainties [35].

The assumption of $w_n(.) \in L_2[0, \infty)$ is valid here since the disturbance input $w_n(.)$ is only composed of the deviations of the external inputs from their nominal values.

### 2.6.3 Design Procedure of the Proposed Primary Control

Analytically solving coupled Ricatti equations (2.15) and (2.16) subject to constraints (i) and (ii) in Theorem 1 is often so hard, if not impossible [43]. Therefore, this section re-formulates the equations into rank constrained LMIs to enable a numerically efficient design procedure.

**Theorem 2.** The overall dynamical system composed of $N$ coupled controlled subsystems (2.7) is absolutely stabilizable with disturbance attenuation level $\gamma$ via decentralized dynamic controllers of the form (2.8) and (2.9) if there exist matrices $\tilde{X}_n = \tilde{X}_n^T > 0$, $\tilde{Y}_n = \tilde{Y}_n^T > 0$, $F_{x,n}$, $F_{y,n}$ and constants $\tilde{\tau}_{n,v} > 0$, $\tau_{n,v} > 0$, $\tilde{\theta}_n > 0$, and $\theta_n > 0$, for all $v = 1, ..., k_n$, and $n = 1, ..., N$, satisfying the following rank-constrained LMIs:

$$
\begin{bmatrix}
N_n & F_{x,n}^T D_{n}^T + \tilde{X}_n C_n^T & Q_{n,1} & \cdots & Q_{n,k_n} \\
* & -I & 0 & \cdots & 0 \\
* & \cdots & -\Theta_{n,1}^{-1} & \cdots & 0 \\
* & \cdots & \cdots & \cdots & \cdots \\
* & \cdots & \cdots & \cdots & -\Theta_{n,k_n}^{-1}
\end{bmatrix} < 0 \quad (2.20)
$$

$$
\begin{bmatrix}
M_n & F_{y,n}^T D_{y,n} + \tilde{Y}_n E_n & \tilde{Y}_n L_n & \tilde{Y}_n F_n \\
* & -T_n & 0 & 0 \\
* & \cdots & -\theta_n & 0 \\
* & \cdots & \cdots & -\gamma^2
\end{bmatrix} < 0 \quad (2.21)
$$

$$
\begin{bmatrix}
\tilde{X}_n & I \\
\tilde{\gamma}^2 & \tilde{Y}_n
\end{bmatrix} > 0 \quad \begin{bmatrix}
\tilde{X}_n & I \\
-\tilde{\gamma}^2 & \tilde{Y}_n
\end{bmatrix} > 0, \quad (2.22)
$$
\[ \begin{bmatrix} \tilde{\tau}_{n,v} & 1 \\ 1 & \tau_{n,v} \end{bmatrix} \geq 0, \quad \begin{bmatrix} \tilde{\gamma} & 1 \\ 1 & \gamma \end{bmatrix} \geq 0, \quad \begin{bmatrix} \tilde{\theta}_n & 1 \\ 1 & \theta_n \end{bmatrix} \geq 0, \tag{2.23} \]

\[ \text{rank} \left[ \begin{bmatrix} \tilde{\tau}_{n,v} & 1 \\ 1 & \tau_{n,v} \end{bmatrix} \right] \leq 1, \text{rank} \left[ \begin{bmatrix} \tilde{\gamma} & 1 \\ 1 & \gamma \end{bmatrix} \right] \leq 1, \text{rank} \left[ \begin{bmatrix} \tilde{\theta}_n & 1 \\ 1 & \theta_n \end{bmatrix} \right] \leq 1. \tag{2.24} \]

where \( \tilde{\tau}_{n,v} = \tau_{n,v}^{-1}, \tilde{\theta}_n = \theta_n^{-1}, \tilde{\gamma} = \gamma^{-1}, \tilde{X}_n = X_{n}^{-1}, \tilde{Y}_n = Y_{n}^{-1}, \tilde{\tau}_{n,v} = \tau_{n,v}^{-1}, \tilde{\theta}_n = \theta_n^{-1}, \tilde{\gamma} = \gamma^{-1} \), and

\[
N_n = \tilde{X}_n A_n^T + A_n \tilde{X}_n + F_{x,n}^T B_n^T + B_n F_{x,n} + \tilde{\theta}_n L_n F_n^T + \tilde{\gamma}^2 F_n^T F_n^T - \tilde{\tau}_{n,1} E_{n,1} E_{n,1}^T - ... \\
M_n = \tilde{Y}_n A_n + A_n^T \tilde{Y}_n + E_{y,n}^T C_{y,n}^T + C_{y,n} E_{y,n} + C_n^T C_n + (\tau_{n,1} + \tilde{\theta}_n) H_n^T - ... \\
+ (\tau_{n,k_n} + \tilde{\theta}_n) H_{n,k_n}^T, H_{n,k_n} \\
- \tilde{\tau}_{n,k_n} E_{n,1} E_{n,k_n}^T \\
Q_{n,v} = [F_{x,n}^T G_{n,v}^T + \tilde{X}_n H_{n,v}^T, ..., F_{x,n}^T G_{n,v}^T + \tilde{X}_n H_{n,v}^T, (N \text{ enteris})] \\
\Theta_n^{-1} = \text{diag} \left[ \tilde{\tau}_{n,v} I, \tilde{\theta}_1 I, ..., \tilde{\theta}_{n-1} I, \tilde{\theta}_{n+1} I, ..., \tilde{\theta}_N I \right], \quad v = 1, ..., k_n
\]

**Proof.** Detailed proof is provided in Appendix C.

Therefore, to find primary control matrices (2.18), rank constrained LMIs (2.20) through (2.24) can be solved.

### 2.7 Primary Control Design for the Microgrid

In this section, based on the proposed robust decentralized control strategy, two types of local controllers will be designed for each DER unit of the study microgrid of Figure 2.1: power controller and voltage controller. These local controllers constitute the primary level of the proposed hierarchical control structure. Electrical parameters of the microgrid, including transformer parameters, impedance of the distribution lines, and the rated loads are given in Appendix A. Also, the steady-state operating-point, around which the linearization of the dynamical model of the microgrid is performed, are presented in Table E.1 in Appendix E.
2.7.1 The IQC-Based Robust Power Controller Design

The $n$th primary power controller regulates the real and reactive output power of the $n$th DER, i.e., $P_n$ and $Q_n$ in Figure 2.2 at their specified reference commands. A block diagram of the robust power controller, corresponding to (2.8) and (2.9), is depicted in Figure 2.6. As depicted, the inputs to this controller are $PQ_n = [P_n, Q_n]$ (instantaneous real and reactive power generated by the DER) and $PQ_{ref,n} = [P_{ref,n}, Q_{ref,n}]$ (power setpoints). The outputs of this controller are the reference signals $i_{ref,dn}^+$ and $i_{ref,qn}^+$ for the positive-sequence built-in current controller to follow.

![Figure 2.6: Block diagram of the $n$th IQC-based robust power controller](image)

To design primary power controllers, the uncertainty output $ζ_n(·)$ and the uncertainty input $ξ_n(·)$ for $n = 1, \ldots, 4$, described in Section 2.5, are chosen such that $ζ_n = ξ_n$. This choice satisfies the integral quadratic constraints in (2.12). By considering the matrix $E_n = I$, the next step is to select the subspace of the state vectors and control inputs $ζ_n(·)$ on which local uncertainties affect the dynamics of the corresponding subsystem.

It should be noted that the elements of the matrices $A$ and $B_{ci}$ in (2.1) and (2.3) are nonlinear functions of: (1) $R$, $L$, and $C$ elements of the loads, filters, transformers, and lines, (2) time constants of LPFs, and (3) gains of the PLL and built-in current controllers. The nonlinearity comes from the inverses and multiplications between the parameters in each element of these two matrices. Among all the parameters participating in the elements of matrices $A$ and $B_{ci}$, the load parameters undergo the maximum variation. Let us assume a variation in $R$ and $L$ parameters of all the loads, from 50% to 500% of their nominal values. This variation results in 5.41% and 2.23% variations in the norm of matrices $A_i$ and $B_{c,i}$ in (2.2) from their nominal values, respectively, for $i = 1, \ldots, 4$. This variation means that uncertainties affecting the state and the controlled input vectors of each DER are lumped together and have formed the norm-bounded additive uncertainties in the form of $A_i → A_i + ΔA_i$ and $B_{c,i} → B_{c,i} + ΔB_{c,i}$, in which $∥ΔA_i∥ = 0.0541∥A_i∥$ and $∥ΔB_{c,i}∥ = 0.0223∥B_{c,i}∥$. Therefore, as one possible choice, the matrices $H_{p,i}$ and $G_i$, for
\begin{equation}
\int B_{pc,n} V_{dqref,n} + \int C_{pc,n} A_{pc,n} + \int x_{cn} x_{en} D_{pc,n} i_{refdq,n}
\end{equation}

Figure 2.7: Block diagram of the IQC-based robust voltage controller

$i = 1, \ldots, 4$, in (2.5) which form the uncertain subspace are selected as $H_{p,i} = 0.0541A_i$ and $G_i = 0.0223B_{c,i}$.

The controlled output $z_i$ is defined as $z_i(t) = w_e I x_{e,i}(t) + w_u I u_i(t)$ in which $w_e$ and $w_u$ represent the weights on the integral of the tracking error and the control effort and are selected as 1 and 0.01, respectively. The disturbance attenuation level is chosen as $\gamma = 1$. After solving matrix inequalities (2.20) through (2.24) in YALMIP toolbox [44], local controllers are obtained for the DER units as in (2.9) with $x_{c,i} \in \mathbb{R}^{10}$.

### 2.7.2 The IQC-Based Robust Voltage Controller Design

Should the microgrid operate in the islanded mode, the voltage at the PCC needs to be regulated. To that end, a secondary control determines the output voltage magnitude and angle setpoints for the $n$th DER unit, as $|V_{ref,n}| \angle \delta_n$. These setpoints can be transformed to the $dq$ frame of reference to generate $dq$-based reference values, as $V_{dref,n} = |V_{ref,n}| \cos \delta_n$ and $V_{qref,n} = |V_{ref,n}| \sin \delta_n$. The devised local voltage controller then regulates $d$ and $q$ components of the output voltage, measured at $PC_n$, to the received reference commands.

A block diagram of the local robust voltage controller, corresponding to (2.8) and (2.9), is depicted in Figure 2.7. As shown, the output of this control box is the reference signals $i_{dref,n}^+$ and $i_{qref,n}^+$ for the positive-sequence built-in current controllers to follow.

Design parameters such as uncertainty output $\zeta_i(\cdot)$, the uncertainty input $\xi_i(\cdot)$, and the controlled output $z_i$ are chosen as described in Section 2.7.1. The four local controllers are obtained for the DER units as in (2.9) with $x_{c,i} \in \mathbb{R}^{10}$, after solving matrix inequalities (2.20) through (2.24) in YALMIP toolbox [44].
2.7.3 Reduced-Order Primary Controller

As explained in Section 2.7, the obtained local controllers of each DER are in the order of \( x_{pc,i} \in \mathbb{R}^{10} \). Considering the computational costs and difficulty of real-time simulation and implementation of high-order controllers, reducing the order of controllers is of interest. The goal here is to produce a low-dimensional controller that has response characteristics, especially the robust performance, as close to that of the original system as possible.

Among all the existing methods of order reduction, \([45–48]\), the method of balanced truncation is chosen in this thesis. The main motivation is that not only does this method maintain asymptotic stability of the reduced-order system but it also provides an error bound on the difference between the full and reduced order systems \([47]\). Therefore, the disturbance attenuation level defined in (2.19) will remain bounded.

Full details and guidelines on how to select the appropriate order of reduction based on: (1) Hankel singular values of the controller, (2) time-domain approximation error, and (3) frequency-domain approximation error is provided in Section D.1, Appendix D. Based on these three criteria, an order reduction of \( r = 5 \) is selected to achieve the best trade-off between the simplicity of the reduced controller and the approximation errors.

Using the balanced order-reduction algorithm provided in Chapter 6 of \([47]\), the decentralized controllers of (2.8) and (2.9), calculated in Section 2.7, are reduced to the order of \( r = 5 \) and provided in Appendix D.

The responses of the full and reduced order controlled subsystem 1 to a 20% step change in the real power reference is compared in MATLAB environment and depicted in Figure 2.8 (a) which confirms the proximity of the dynamic behaviour of the full and reduced systems. Figure 2.8 (b), on the other hand, compares the singular value plots of the transfer function of the closed-loop subsystem 1 for both full and reduced order controllers. As it can be seen in Figure 2.8 (b), the maximum value of the singular value plot is remained at 0 dB which means that the disturbance attenuation level is preserved to the desired value \( \gamma = 1 \).

Performing the same set of analyses results in the similar order of reduction for the IQC-based robust voltage controller, as provided in Appendix D.

2.8 Robust Performance Analysis

This section seeks to quantitatively measure the robustness of the closed-loop microgrid of (2.1) and (2.3) controlled by the obtained reduced-order primary controllers of (2.8) and (2.9), through four robustness measures performed in MATLAB environment.
Chapter 2. Robust Decentralized Primary Control

2.8.1 Module Margin

The multi-DER microgrid of Figure 2.1 has a Multi-Input Multi-Output (MIMO) nature. When analyzing a MIMO system, a suitable robustness margin which ratifies the limitations of the classical phase and gain margins is introduced as module margin [48]. The module margin is defined as the minimal distance between the generalized Nyquist locus and the critical point $-1 + 0j$ and can be computed as the $H_\infty$ norm of the sensitivity transfer function:

$$ ||S(s)||_\infty = ||(1 + G_p G_c)^{-1}||_\infty \leq \frac{1}{M_m} $$

(2.25)

in which $G_p$ is the transfer function equivalence of the system of (2.1) from the input of $U(s)^T = [U_{DR1}^T, U_{DR2}^T, U_{DR3}^T, U_{DR4}^T]$ to the output of $Y(s)^T = [P_1, Q_1, P_2, Q_2, P_3, Q_3, P_4, Q_4]$. Also, $G_c$ represents the transfer function of the set of robust power primary controllers of (2.8) and (2.9), for $i = 1, ..., 4$, from the input of $U_c(s) = Y_{ref}(s) - Y(s)$ to the output of $Y_c(s) = U(s)$, in which $Y_{ref}(s)^T = [P_{ref,1}, Q_{ref,1}, P_{ref,2}, Q_{ref,2}, P_{ref,3}, Q_{ref,3}, P_{ref,4}, Q_{ref,4}]$.

The desired module margin should be in the range of $M_m \in (0, 1)$. The closer the module margin to the unity, the better the obtained robustness margin. As shown in Chapter 2 of [48], a proper module margin also implies proper gain and phase margins based on the relationships $PM \geq M_m$ and $GM \geq 1/(1 - M_m)$. The obtained module margin for the closed-loop system of (2.1), (2.8), and (2.9) is 0.99 for the designed primary power controllers, which results in $PM \geq 57.2^\circ$ and $GM = \infty$. For the case of the designed primary voltage controllers $M_m = 0.975$. This clearly demonstrates the robust stability of the closed-loop system in both cases.
2.8.2 Sensitivity Function Singular Values

Figure 2.9 shows the singular value plot of the sensitivity transfer function, defined at the left hand side of (2.25), for the case of power controller. Depicted in this figure, $S(s) \rightarrow 0$ at low frequencies for command tracking and disturbance rejection and $S(s) \rightarrow 1$ at high frequencies for sensor noise rejection and robustness to high-frequency unmodeled dynamics. This confirms the robust performance achieved using the developed control strategy. Similar results are obtained for the case of voltage controller.

2.8.3 Maximum Uncertainty Bounds

As explained in Section 2.7, multiple sources of uncertainties in each subsystem are lumped into a single complex additive perturbation $\Delta A_i$ and $\Delta B_{c,i}$, when designing the local controllers. Now, the goal in this section is to determine the maximum uncertainty bound on each specific parameter of the power network and DER units which is contributing in the uncertainty. Assume that matrices of the local controllers are fixed to the ones obtained previously. Also, parameters of the plant all are fixed except the load parameter $R_l$, filter parameter $L_{s1}$, and built-in current controller integral gain $K_{ii,1}$, each changing one at a time. A MATLAB-based incremental algorithm is then used to find the maximum uncertainty bound. This maximum uncertainty bound can be defined as the maximum allowable variation in the system parameters from their nominal values before which all the eigenvalues of the closed-loop system at the primary level remain on the left hand side plane, i.e. the stability of the controlled system is ensured. Us-
ing this incremental algorithm, the maximum uncertainty bounds are calculated as: (i) \(0.78 \leq \frac{R_{l1}}{R_{nom}} \leq 1.238\), (ii) \(0.1 \leq \frac{L_{s1}}{L_{nom}} \leq 6.88\), and (iii) \(0.21 \leq \frac{K_{ii,1}}{K_{nom,1}} \leq 41.6\). Variations in the power consumed by load \(l_1\), associated with the above-mentioned change in the resistive part of load \(l_1\), is \(-0.25 \leq \frac{S_{nom,1} - S_1}{S_{nom,1}} \leq 0.18\), in which \(S_1\) is the consumed apparent power.

### 2.8.4 Input Time-Delay Tolerance

In practical systems, there are often time delays ignored in the process of mathematical modeling of the system [41]. These delays may originate from the latency in the actuators (delay caused by VSC switches), sensors (acquisition time), control (calculation time), and field networks (time required to transfer signals from the controller to the plant). Assuming the existence of a time delay from the local controller output to the DER system control input, the input time-delay tolerance is another essential robustness margin which needs to be measured. The following definition of is offered in [25].

**Definition 2.** Given a plant \(\dot{x} = Ax + Bu\), \(y = Cx\) controlled by a controller \(\dot{x}_c = A_c x_c + B_c y\), \(u = C_c x_c\), assume that the closed-loop system is asymptotically stable, and let \(u(t)\) be replaced by \(u(t - \beta)\), corresponding to a time delay of \(\beta\) s. Then if there exists \(\beta > 0\) such that the closed-loop system remains stable \(\forall \beta \in [0, \beta]\), the closed-loop system has an input time-delay tolerance of \(\beta > 0\).

The obtained local controllers provide a tolerance of 7.2 ms, for the case of power controller, and 6.8 ms, for the case of voltage controller. Considering that the switching frequency of the VSC is 3.06 kHz (period of 0.32 ms), the obtained primary controllers present acceptable robustness to the time delay, as well.

### 2.9 Conclusion

In this chapter, a robust decentralized output-tracking MIMO control strategy is proposed, designed and applied to design robust primary power and voltage controllers. The developed controllers are implemented in the primary level of a two-level hierarchical control structure used to operate a microgrid in its various modes.

To that end, a generic microgrid study system is first identified and its state-space linearized representation is derived. Then, various sources of uncertainties and disturbances affecting the microgrid in both the VPP and islanded modes are modeled using the IQC framework. Then, a set of necessary and sufficient conditions are derived for the existence
of a new class of robust decentralized output-tracking controllers, which provides absolute stability of uncertain systems with guaranteed disturbance rejection. These conditions are derived as a set of coupled constrained algebraic Riccati equations. To provide a systematic approach toward synthesizing controllers, these equations are transformed to rank constrained LMIs which can be efficiently solved in one of the existing numerical toolboxes.

The obtained rank constrained LMIs are then solved in YALMIP software \[44\], developed in MATLAB environment, to synthesize local power and voltage controllers for each DER. Also, the method of balanced truncation based on Hankel singular values is used to reduce the order of local controllers while preserving the achieved level of disturbance attenuation.

To evaluate the efficacy of the proposed robust primary controllers, various frequency-domain robustness analyses are performed in MATLAB platform. These analyses, reported in Section 2.8, confirm that the developed robust, decentralized, reduced-order, primary power (voltage) controllers achieve a desirable module margin of 0.99 (0.975), disturbance rejection at low-frequency and measurement noise rejection at high-frequency, and tolerance to input time-delay of up to 7.2 ms (6.8 ms). The performance evaluation of the primary power and voltage controllers, based on digital time-domain simulation studies performed in PSCAD/EMTDC platform is presented in the next chapter.
Chapter 3

Performance Evaluation of the Primary Control

3.1 Introduction

This chapter investigates the transient and steady-state behavior of the robust primary power and voltage controllers developed in Chapter 2. To that end, several time-domain scenarios are studied in PSCAD/EMTDC to show the efficacy of the proposed control structure to: (i) track the specified reference commands in the steady-state operating regime despite local and interconnection uncertainties and exogenous disturbances, and (ii) to maintain the stability of the multi-DER study microgrid of Figure 2.1 during transients. The robust performance of the developed primary controllers is evaluated in response to setpoint variations, load disturbances, microgrid operating point and topological changes, host utility grid voltage sag, and three-phase and single-phase line-to-ground faults at the PCC.

In the simulation results, measured quantities are presented in per-unit (pu), wherever applicable. The base values for this per-unitization are provided in Table 3.1. The subscript \( LN, \text{rms} \) denotes the line-to-neutral rms value of the signal.

<table>
<thead>
<tr>
<th>Measured Quantity</th>
<th>Point of Measurement</th>
</tr>
</thead>
<tbody>
<tr>
<td>( S_{\text{base}} ) (MVA(_{3\phi}))</td>
<td>PCC ( PC_1 ) to ( PC_3 ) ( PC_4 )</td>
</tr>
<tr>
<td>( V_{\text{base}} ) (kV(_{LN,\text{rms}}))</td>
<td>7.96</td>
</tr>
<tr>
<td>( I_{\text{base}} ) (kA(_{L,\text{rms}}))</td>
<td>0.12</td>
</tr>
</tbody>
</table>
3.2 Dynamic Performance of the Primary Power Controllers

In this series of studies, the microgrid of Figure 2.1 is assumed to be in the VPP mode. Setpoints for DER$_1$ to DER$_3$ are specified to be the generation of 2 MVA at a leading power factor of 0.98 and $DER_4$ to be the generation of 0.2 MVA at a power factor of 1. At the steady-state condition, microgrid receives 1.015 MW from the host utility grid and provides the host utility grid with 1.031 MVAr. The voltage at the PCC is maintained by the utility grid at 1 pu (13.8 kV LL-rms). The load parameters are provided in Table E.1, Appendix E.

To study the robustness of the developed power controllers, all case scenarios are performed for two utility grid conditions: strong and weak, as defined in Table 3.2. Shown in Table 3.2 as the length of the line connecting the microgrid to the utility grid increases, i.e. line TL$_1$ in Figure 2.1, the Effective Short Circuit Ratio (ESCR) measured at the PCC of the microgrid decreases which indicates a weaker utility grid. ESCR is defined as, [49]:

$$ESCR = \frac{SC_{MVA} - Q_c}{S_{\mu G}}$$

(3.1)

in which $SC_{MVA}$ is the short-circuit capacity of the utility gird measured at the PCC, $Q_c$ is the 2 MVAr rated reactive power generated by the shunt capacitor connected to the PCC, and $S_{\mu G}$ is the 9.5 MVA generation capacity of the microgrid. Utility grids with $ESCR > 5$, $2.5 < ESCR < 5$, and $ESCR < 2.5$ are considered as strong, medium strong, and weak systems, respectively.

It is worth mentioning that the primary power controllers are designed in Chapter 2 using the dynamical model of the microgrid connected to the strong utility grid. Connecting the microgrid to such a weak utility grid is itself a considerable robustness evaluation. Another important remark here is that power setpoints for the DER units are intentionally remained unchanged for the two strong and weak utility grid scenarios to achieve an accurate comparison. It is also note worthy that this chapter is devoted to the performance evaluation of the primary controllers with no secondary controller in the loop. As a result, in the weak utility grid test scenarios, microgrid voltage may temporary deviate beyond acceptable limits. This problem will be ratified in Chapter 4 by adding a secondary controller to regulate microgrid voltage at a desired value when the microgrid is connected to the weak utility grid, i.e., operating in the VPP-PV mode.
### Table 3.2: Strong to Weak Utility Grid

<table>
<thead>
<tr>
<th>Measured Quantity</th>
<th>Grid Condition</th>
<th>Strong</th>
<th>Weak</th>
</tr>
</thead>
<tbody>
<tr>
<td>TL$_1$ length (km)</td>
<td>0.8</td>
<td>20</td>
<td></td>
</tr>
<tr>
<td>ESCR</td>
<td>14.09</td>
<td>2.16</td>
<td></td>
</tr>
</tbody>
</table>

#### 3.2.1 Case 1: Reference Tracking

To demonstrate the capability of the developed power primary controllers in command tracking, a 20\% step change increase in all the four DER units setpoints is applied at $t=0.2$ s. For the sake of brevity, only the response of DER$_1$ is presented in this section.

Figure 3.1 (a) and (b) provides real and reactive output powers of DER$_1$, when the microgrid is connected to the strong utility grid. As depicted, the generated real and reactive powers settle down to their new setpoints in 7.5 ms (with 1\% criterion) with zero overshoot and zero steady-state tracking error. Based on 1\% criterion, the settling time is defined as the time required for the response curve to reach and stay within 1\% of the final value. Figure 3.1 (c) demonstrates the reactive power generated by the capacitor of the RLC filter of DER$_1$ to be 72 kVAr. The $d$ and $q$ components of the output voltage of the DER$_1$ and its instantaneous three-phase value are depicted in Figure 3.1 (d) and (f). The increase in the DER$_1$ instantaneous output current and its unfiltered terminal current are also provided in Figure 3.1 (g) and (h).

As depicted in Figure 3.1 (i) voltage at the PCC is maintained constant, at 1 p.u, by the utility grid. However, the increase in the generated real and reactive power by the four DER units has caused a decrease in the current imported from the utility grid, Figure 3.1 (j). Prior to the increase in the power setpoints, the power generated by DER$_1$ was smaller than the demands of the two loads $L_1$ and $L_2$ (Figure 2.1). Therefore, a current of 0.18 pu was passing through feeder F1 to feed these two loads. However, the 20\% increase in the generated power by DER$_1$ reduces this imported current to almost zero pu, i.e., DER$_1$ is meeting the power demands of $L_1$ and $L_2$, as depicted in Figure 3.1 (k). This observation matches the load parameters provided in Table A.3. Shown in Figure 3.1 (l), the current consumed by the load $L_1$ remain unchanged as the step change in power generation has not caused any deviation in the voltage of the buses.

Simulation results of the case in which the microgrid is connected to the weak utility grid is provided in Figure 3.2. As confirmed in plots (a) and (b), despite connection to a weak utility grid, the developed primary power controller is able to track the specified
Figure 3.1: The strong system response to a 20% step increase in real/reactive power setpoints: (a) and (b) real and reactive power generated by DER\(_1\) and their references, (c) the reactive power consumed by the RLC filter capacitor \(C_{f1}\), (d) and (e) \(d\) and \(q\)-components of the DER\(_1\) output voltage and current, (f) to (h) three-phase instantaneous output voltage, output current, and terminal current of DER\(_1\), (i) PCC voltage, (j) and (k) current going through feeders TL\(_1\) and \(F_1\), and (l) load \(L_1\) current.

setpoint. Shown in Figure 3.2 (c), (e), (f), and (i), unlike the strong utility grid case, this increase in the power generation causes an increase in the voltage at the PCC and even at the utility grid bus, \(Bus_u\), by 15%. This increase in the voltage, consequently, causes an increase in the load current as shown in plot (l). Despite power generation increase, the increase in the load currents slightly increases the current transferred through TL\(_1\) and \(F_1\) feeders, as shown in Figure 3.2 (j) and (k).

3.2.2 Case 2: Change in Microgrid Configuration

In this section, robust stability and performance of the developed control strategy to topological uncertainties is studied. At \(t = 0.2\) s., the breaker on Feeder F3 in Figure 2.1 at the PCC side, opens. This event isolates DER\(_2\) and DER\(_3\) from the microgrid feeding two loads \(L_5\) and \(L_6\). This disconnection of two DER units and two major loads changes the operating point significantly. However, to utilize the power generated by the two DER units, the transfer switch closes after three cycles at \(t = 0.25\) s, connecting \(Bus_6\) to \(Bus_5\). This causes a major change in the configuration and the dynamic model of the microgrid.

Figure 3.3 (a) to (h) demonstrates the transients in real and reactive output power
Figure 3.2: The weak system response to a 20% step increase in real/reactive power set points: (a) and (b) real and reactive power generated by DER$_1$ and their references, (c) and (d) $d$ and $q$-components of the DER$_1$ output voltage and current, (e) and (i) three-phase instantaneous voltage measured at the utility grid bus and the PCC, (f) to (h) three-phase instantaneous output voltage, output current, and terminal current of DER$_1$, (j) and (k) current going through feeders TL$_1$ and F$_1$, and (l) load L$_1$ current.

of the four DER units. When DER$_2$ and DER$_3$ isolate from the rest of the microgrid, they are left to feed 1.48 MW+j 0.38 MVAr of loads L$_5$ and L$_6$ while these two DERs are commanded to generate 3.92 MW+j0.4 MVAr. This power mismatch increases the voltage at the buses to 1.23 pu, as depicted in Figs. 3.4 (f) and (g), 3.5 (b) and (c), and 3.6 (e). Therefor, DER$_2$ and DER$_3$ have to increase their terminal voltage to be able to follow the specified power reference commands. This pushes VSCs of DER$_2$ and DER$_3$ into saturation condition due to overmodulation and therefore current reference commands and subsequently power reference commands are not followed, as depicted in Figure 3.3 (b), (c), (f), and (g) and Figure 3.4 (b) and (c). As soon as the transfer switch closes, the devised controllers recover the unsaturated operation condition and follow power setpoints within 52 ms (1% criterion). For DER$_1$ and DER$_4$, this topological change has no significant effect, as illustrated in Figure 3.3 (a), (d), (e), and (h).

The $d$ and $q$ components of the output current and voltage of the four DER units are also provided in Figure 3.4 (a) to (h). Also, the instantaneous three-phase output voltage, output and terminal current of the DER units are shown in Figure 3.5 (a) to (l). Also, Figure 3.6 demonstrates the voltage at the main buses (plots (a) to (f)), currents going through the main feeders (plots (h) to (l)), and the loads currents (plots (m) to (r)).
Figure 3.3: The **strong** System response to the topological change in the microgrid: (a) to (h) real and reactive power generated by DER\textsubscript{1} to DER\textsubscript{4}.

Figure 3.4: The **strong** System response to the topological change in the microgrid: (a) to (d) \(d\) and \(q\)-components of the four DER units output current, and (e) to (h) \(d\) and \(q\)-components of the four DER units output voltage.

The performance of the developed robust primary controllers to this topological change when the microgrid is connected to the weak utility grid is also studied. System transients are depicted in Figs. 3.7, 3.8, 3.9 and 3.10. Being connected to such a weak host utility grid and not having a secondary controller to regulate microgrid volt-
Figure 3.5: The strong System response to the topological change in the microgrid: (a) to (d) three-phase instantaneous output voltage of the four DER units, and (e) to (l) instantaneous output current and terminal current of the four DER units.

Figure 3.6: The strong System response to the topological change in the microgrid: (a) to (f) voltage at Bus1 to Bus4, (g) current going through line C5, (h) to (l) current going through feeders TL1 to F4, (m) to (r) load currents IL1 to IL7.

age, disconnection of the two DERs causes a 35% voltage drop at PC1 and PC4 and a 23% voltage increase at PC2 and PC3, as illustrated in Figure 3.8 (e) to (h). To provide the desired power setpoints despite this voltage sag, DER1 and DER4 increase their
output current to 1.2 pu and enter the saturation region, as shown in Figure 3.8 (a) to (d). Despite all these severe effects, within 60 and 135 ms after transfer switch closes, DER1/DER4 and DER2/DER3 track their power setpoints, respectively, as depicted in Figure 3.7 (a) to (h). Transients in the three-phase instantaneous voltage and current of the microgrid are also provided in Figs. 3.9 and 3.10.

This case study demonstrates the robust performance of the proposed LPCs to ride the microgrid through operating point changes, configuration and subsequently dynamic model changes, and operation in the nonlinear region caused by saturation to protect overmodulation and overcurrent.

3.2.3 Case 3: Grid Voltage Disturbance

As the third group of perturbations identified in the study microgrid in Chapter 2, robustness of the proposed controllers against exogenous disturbances also needs to be evaluated. In this case study, a 25% voltage sag happens at the utility grid bus at \( t = 0.2 \) s and lasts for 3 cycles.

Figure 3.11 (a) and (b) show the active and reactive power generated by DER1. The \( d \) and \( q \) components of the output voltage and current of DER1 are provided in plots (c) and (d). Three-phase instantaneous voltage, measured at \( PC_1 \) and the utility grid bus, are provided in plots (e) and (f). Also, three-phase instantaneous output and terminal current of DER1 are shown in plots (g) and (h). As it can be seen, the developed robust
Figure 3.8: The **weak** System response to the topological change in the microgrid: (a) to (d) $d$ and $q$-components of the four DER units output current, and (e) to (h) $d$ and $q$-components of the four DER units output voltage.

Figure 3.9: The **weak** System response to the topological change in the microgrid: (a) to (d) three-phase instantaneous output voltage of the four DER units, and (e) to (l) instantaneous output current and terminal current of the four DER units.

collectors ride the microgrid through this considerable disturbance and control the DER units to provide the specified setpoints.

System response to this voltage disturbance when the microgrid is connected to the weak host grid is shown in Figs. 3.12. As depicted in 3.12 (a) to (f), voltage sag at the
Chapter 3. Performance Evaluation of the Primary Control

3.2.4 Case 4: Three-Phase L-L-L-G Fault at the PCC

This cases study demonstrates the performance of the developed robust power controller when a three-phase temporary fault happens. At $t=0.2$ s, a three-phase line-to-ground (L-L-L-G) fault on the PCC of Figure 2.1 occurs and lasts for 5 cycles. The fault resistance is selected as 0.2 ohms (0.01 pu). The operating point prior to fault is as described in Section 3.2. To investigate the robustness of the developed primary controllers, in this scenario, no breaker is allowed to operate to clear the fault.

As depicted in Figure 3.13 (a) and (b), during the fault power setpoints cannot be provided. The reason is that during the fault, voltage at the PC drop to 0.14 pu (plot (c) and (i)). Therefore, the primary power controller asks for more output current from DER$_1$. However, the over-current protection scheme, embedded in the DER$_1$, limits the output current to 1.2 pu (plots (d), (k), and (l)). Therefore, this tracking error cannot be compensated. As soon as fault clears, real and reactive powers settle down to their original values.

utility grid does not affect voltage at the buses of the microgrid. The reason is the large impedance of the utility line (seen from the PCC) connecting the weak utility grid to the microgrid. As a result, the active and reactive power generated by DERs undergo insignificant transients, Figure 3.12 (a) and (b).
Figure 3.11: The strong system response to a 25% voltage sag at the utility grid bus: (a) and (b) real and reactive power generated by DER$_1$ and their references, (c) the reactive power consumed by the RLC filter capacitor $C_{f1}$, (c) and (d) $d$ and $q$-components of the DER$_1$ output voltage and current, (e) utility grid voltage, (f) to (h) three-phase instantaneous output voltage, output current, and terminal current of DER$_1$, (i) PCC voltage, (j) and (k) current going through feeders $TL_1$ and $F_1$, and (l) load $L_1$ current.

Figure 3.12: The weak system response to a 25% voltage sag at the utility grid bus: (a) and (b) real and reactive power generated by DER$_1$ and their references, (c) the reactive power consumed by the RLC filter capacitor $C_{f1}$, (d) and (e) $d$ and $q$-components of the DER$_1$ output voltage and current, (f) to (h) three-phase instantaneous output voltage, output current, and terminal current of DER$_1$, (i) PCC voltage, (j) and (k) current going through feeders $TL_1$ and $F_1$, and (l) load $L_1$ current.
setpoints within 42 ms. The instantaneous three-phase output voltage, output current, and terminal current of DER$_1$ are also provided in Figure 3.13 (i), (k), and (l).

Figure 3.13 (f) to (h) illustrates that although the voltage at the utility grid drops only by 12%, the fault causes a 86% voltage drop at the PCC and bus 1. Plot (j) depicts the fault current with a peak amplitude of 53.7 p.u. As shown in plot (m), the fault current is mainly provided by the utility grid and goes through line TL1. Decrease in the voltages causes a decrease in the load currents, as confirmed by plot (o). Decrease in the load currents increases the current transferred through feeder F1, plot (n).

Figure 3.14 demonstrates system response to the LLLG fault at the PCC when the microgrid is connected to the weak utility grid. The weakness of the utility grid causes a voltage drop of 97% at all the buses even the utility grid bus, Figure 3.14 (f) to (i). Depicted in plots (m) and (j), the weak utility grid cannot feed the fault current. Therefore, this current is limited to 3.5 pu only. Also, plot (e) depicts that the PLL error in the phase angle estimation, due to distorted voltage, leads to transients in the $q$ components of the output voltage and current of the DER$_1$ which itself causes longer settling time to track reactive power setpoint, depicted in Figure 3.14 (a) to (e).
Figure 3.14: The weak System response to an L-L-L-G fault at the PCC: (a) and (b) real and reactive power generated by DER1, (c) and (e) $d$ and $q$ components of the output voltage and output current of DER1, (f) to (i) three-phase instantaneous voltage measured at Busu, PCC, Bus1, and PC1, (j) fault current, (k) and (l) instantaneous output current and terminal current of DER1, (m) and (n) current going through feeders $Tl_1$ and $F_1$, and (o) load $L_1$ current.

### 3.2.5 Case 5: Single-Phase L-G Fault at the PCC

This cases study demonstrates the performance of the developed robust power controller under unbalanced grid voltage condition, when a single-phase temporary fault happens. At $t=0.2$ s, a single-phase line-to-ground (L-G) fault on the PCC of Figure 2.1 happens and lasts for 5 cycles. The fault resistance is selected as 0.2 ohms (0.01 pu). The operating point prior to fault is as described in Section 3.2. To be bale to investigate the robustness of the developed primary controllers, in this scenario, no breaker is allowed to operate to clear the fault.

Figure 3.15 shows the performance of the proposed primary controllers when facing single-phase to ground faults. As stated earlier, the built-in current controllers are required to provide balanced output currents of the DER units. The built-in current controller performance in attenuating the negative sequence and following the positive sequence reference current is illustrated in plots (d), (k), and (l). Limited amount of power capacity though has prevented the control structure to fully over-ride this severe unbalanced voltage disturbance. Saturated output current of the DER is shown in plot (d).
Figure 3.15: The strong System response to an L-G fault at the PCC: (a) and (b) real and reactive power generated by DER1, (c) to (e) d and q components of the output voltage and output current of DER1, (f) to (i) three-phase instantaneous voltage measured at Busu, PCC, Bus1, and PC1, (j) fault current, (k) and (l) instantaneous output current and terminal current of DER1, (m) and (n) current going through feeders TL1 and F1, and (o) load L1 current.

Depicted in Figure 3.15 (a), (b), and (c), the presence of the negative-sequence component leads to double frequency oscillations in the d and q components of the output voltage of the DER units. Figure 3.15 (a) and (b) provide real and reactive powers generated by DER1. During the fault, real power setpoint cannot be met since: (1) output voltage of the DER is unbalanced, and (2) the over-current protection has limited DER’s output current. However, 47 ms after fault clears, power setpoints are met.

Figure 3.15 (f), (g), (h), and (i) illustrate that although the voltage at the utility grid remain unchanged, the fault causes a relatively large unbalanced voltage, at the PCC and Bus1, due to an unsymmetrical fault. Plot (j) depicts the fault current with a peak amplitude of 76.25 p.u. As shown in plot (m), the fault current is mainly provided by the utility grid and goes though feeder TL1. Load L1 current is also provided in plot (o).

Figure 3.16 demonstrates microgrid transients, connected to the weak utility grid, during line-to-ground fault at the PCC. Being connected to such a weak utility grid and having limited power rating, the devised control structure is only able to maintain the stability of the DER but fails to provide the specified setpoint during the fault. Only 67 ms after fault clears, control commands are tracked with zero steady state error.
Figure 3.16: The weak System response to an L-G fault at the PCC: (a) and (b) real and reactive power generated by DER1, (c) to (e) d and q components of the output voltage and output current of DER1, (f) to (i) three-phase instantaneous voltage measured at Busu, PCC, Bus1, and PC1, (j) fault current, (k) and (l) instantaneous output current and terminal current of DER1, (m) and (n) current going through feeders Tl1 and F1, and (o) load L1 current.

3.3 Dynamic Performance of the Primary Voltage Controller

The main goal of this section is to evaluate the performance of the proposed robust voltage controller. Assume that the breaker of the feeder F1 in Figure 2.1 at the PCC side is open. Therefore, DER1 is operating in the islanded mode and is required to maintain Bus1 voltage within acceptable range while feeding the two loads L1 and L2 connected to Bus1. Frequency of the microgrid is controlled in an open-loop manner [41]. Running a power flow analysis for the islanded DER1 and its corresponding loads, the voltage and angle setpoints for the PC1 are obtained as $1.01 \angle -0.0229^\circ$ (pu).

It is worth mentioning that comprehensive studies when the whole microgris is isolated from the host utility grid at its PCC and the proposed primary voltage controller is augmented with a suitable droop controller will be studied in Chapter 5. This section seeks to evaluate the ability of the designed robust voltage controller in following its reference commands under various perturbations.
3.3.1 Case 1: Reference Tracking

To demonstrate the capability of the developed robust voltage controller in setpoint tracking, a 10% step change in the voltage magnitude of DER1 is applied at $t=0.2$ s. Figure 3.17 (a) and (d) show the instantaneous three-phase output voltage at the $PC_1$ and its magnitude waveform, following the command in less than 13.7 ms (1% criterion) with 3.6% overshoot and zero steady-state tracking error. Plots (b) and (c) show the instantaneous three-phase output and terminal current of DER1. The increased real and reactive power generated by DER1 are also depicted in plot (e). As shown in plot (f), using the open-loop frequency control method proposed in [41], the frequency measured at the $Bus_1$ is regulated at the specified 60 Hz. Voltage at $Bus_1$ and the two loads currents are also provided in plots (g) to (i).

Figure 3.17: System response to a 10% step change in the voltage setpoint: (a) and (d) instantaneous three-phase output voltage at the $PC_1$ and its magnitude waveform, (b) and (c) instantaneous three-phase output current of the DER1, (e) real and reactive power generated by DER1, (f) the network frequency measured at the $Bus_1$, (g) the instantaneous three-phase output voltage at $Bus_1$, (h) and (i) instantaneous three-phase currents of loads $L_1$ and $L_2$.

3.3.2 Case 2: Load change

To investigate the robustness of the developed primary voltage controller to the local uncertainties in an islanded microgrid, system response to the sudden disconnected and re-connection of load $L_1$ is simulated in this section. Prior to $t=0.2$ s the $L_1$ branch is open. At $t=0.2$ s the load $L_1$ disconnects from the system corresponding to a decrease in the power demand from 0.6 to 0.4 pu. Then, at $t=0.3$ s, $L_1$ connects back to the system.

Figure 3.18 (a) and (d) demonstrate the effectiveness of the developed voltage controllers in regulating the voltage at its desired value within 30 ms, despite transients...
caused by the sudden load connection and disconnection. The output and terminal current of DER\(_1\), the two loads currents, and the generated active and reactive power by DER\(_1\) are provide in plots (b), (c), (e), (f), (h), and (i), respectively.

### 3.3.3 Case 3: Three-Phase L-L-L-G Fault

This cases study demonstrates the performance of the developed robust voltage controller when a three-phase temporary fault happens. At \(t=0.2\) s, a three-phase line-to-ground (L-L-L-G) fault on \(Bus_1\) of Figure 2.1 happens and lasts for 5 cycles. The fault resistance is selected as 0.2 ohms (0.01 pu). To investigate the robust performance of the developed primary controllers no breaker is allowed to operate to clear the fault.

During the fault, voltage at \(Bus_1\) and subsequently at \(PC_1\) drops to 0.03 pu. As soon as fault clears out, the developed voltage controller bring the voltage back to 1 pu within 50 ms, as depicted in Figure 3.19 (a), (b) and (l). The output and terminal current of DER\(_1\), the two loads currents, and the generated real and reactive power by DER\(_1\) are provided in Figure 3.19. Figure 3.19 (l) plots fault current. As it can be seen, this current is limited to 1.2 pu. This fault current is provided by DER\(_1\). The \(d\) and \(q\) components of the converter output current along with their references are also depicted in plots (e) and (f).
Figure 3.19: System response to an L-L-L-G fault at Bus1: (a) and (b) instantaneous magnitude and three-phase waveform of output voltage at PC1, (c) and (d) real and reactive power generated by DER1, (e) to (h) d and q components and instantaneous three-phase output current and terminal current of DER1, (i) the instantaneous three-phase output voltage at the Bus1, (j) and (k) loads $L_1$ and $L_2$ currents, and (l) fault current.

3.3.4 Case 4: Single-Phase L-G Fault

This cases study demonstrates the performance of the developed robust voltage controller under unbalanced grid voltage condition, when a single-phase temporary fault happens. At $t=0.2$ s, a single-phase line-to-ground (L-G) fault on Bus1 of Figure 2.1 happens and lasts for 5 cycles. The fault resistance is selected as 0.2 ohms (0.01 pu). To be able to investigate the robustness of the developed primary controllers, in this scenario, no breaker is allowed to operate to clear the fault.

Figure 3.20 shows the performance of the proposed primary controllers when facing single-phase to ground faults. As stated earlier, the built-in current controllers are required to provide balanced output currents for the DER units. Figure 3.20 (a) and (b) provide the instantaneous three-phase output voltage, measured at the PC1, and its magnitude waveform. During the fault, voltage setpoint cannot be met since: (1) output voltage of the DER is unbalanced, and (2) the over-current protection has limited DER’s output current. However, 47 ms after fault clears, voltage setpoint is provided.

The presence of the negative-sequence component, i.e., the double frequency oscillations, can be seen in the output voltage and power of DER1 as well as load currents. The $d$ and $q$ components and the three-phase instantaneous output and terminal current of DER1 are also provided in plots (e) to (h).

Figure 3.20 (l) plots fault current. As it can be seen, this current is limited to 0.95 pu.
This fault current is provided by DER$_1$. The built-in current controller performance in attenuating the negative sequence and following the positive sequence reference current is illustrated in plots (g) and (h). Limited amount of power capacity has prevented the control structure to fully override this severe unbalanced voltage disturbance.

Figure 3.20: System response to an L-G fault at Bus$_1$: (a) and (b) instantaneous magnitude and three-phase waveform of output voltage at PC$_1$, (c) and (d) real and reactive power generated by DER$_1$, (e) to (h) $d$ and $q$ components and instantaneous three-phase output current and terminal current of DER$_1$, (i) the instantaneous three-phase output voltage at the Bus$_1$, (j) and (k) loads $L_1$ and $L_2$ currents, and (l) fault current.

### 3.4 Conclusion

This chapter demonstrates the robust performance of the proposed primary power and voltage controllers in response to setpoint variations, load disturbances, microgrid operating point and topological changes, host utility grid voltage sag, three-phase symmetrical and single-phase unsymmetrical line-to-ground faults at the PCC, and microgrid operation in the nonlinear region (input saturation due to overload condition). To worsen test scenarios even further, the performance of the primary power controllers is evaluated in the case that the microgrid is connected to a weak utility grid with ESCR=2.16 and is subjected to the above-mentioned severe disturbances.

This set of time-domain simulation studies, performed in PSCAD/EMTDC platform, confirms that the developed robust, decentralized, primary controllers are able to: (i) maintain the stability of the overall microgrid during transients, (ii) track the specified setpoints within a few cycles, with zero tracking error, and with near to zero overshoot,
and (iii) ride the microgrid through severe perturbations such as operating point variations, topological and parametric changes, and disturbances.

The study results in Section 3.2 shows that the developed power controller track the specified real and reactive power setpoints within 7.5 ms (with 1% criterion) with zero overshoot and zero steady-state tracking error. Also, these results confirm that even in the case of severe disturbances such as faults or major operating point changes which push the DERs into nonlinear regions, the devised power controllers maintain system stability and then settle down to the desired setpoint within almost 42 ms after disturbance source is cleared. The designed primary voltage controllers also demonstrate a settling time of 13.7 ms (1% criterion) with 3.6% overshoot and zero steady-state tracking error, as provided in Section 3.3. Similarly, the devised voltage controllers maintain the stability of the microgrid which is faced with severe disturbances and pushed to nonlinear regions and then retrieve the specified operating condition within 50 ms.
Chapter 4

VPP Mode: Robust Secondary Control

4.1 Introduction

This chapter develops a PI-based multivariable robust $H_\infty$ secondary control, augmented with a static anti-windup compensator, to operate a microgrid in the VPP mode. This secondary control coordinates the robust decentralized local controllers, developed in Chapter 2, to collectively provide pre-specified requirements at the PCC of the microgrid.

Conventionally, designing and tuning PI controllers rely on the designer experience and various rounds of trial and error [50]. These heuristically designed PI controllers fail to provide an acceptable performance for a wide range of operating conditions of a microgrid. Plus, a multi-DER microgrid is subject to various uncertainties and exogenous disturbances and it is generally a MIMO system which cannot be effectively decoupled to a number of SISO control loops. This chapter resolves the above-mentioned issues through an LMI-based systematic approach towards the design of a PI multivariable robust $H_\infty$ secondary control for microgrids operating in the VPP mode. The developed controller guarantees the stability of the closed-loop system while meeting a specified $H_\infty$ level of disturbance attenuation. Furthermore, the structure of the secondary control is equipped with a limiter block. This block limits the reference power commands sent to the primary controllers to the generation capacity of their corresponding DERs.

In the operating conditions where input saturation happens, a DER constantly receives the command to generate the maximum power capacity disregarding the microgrid performance at the PCC. This causes a large tracking error at the input of the PI-based controller which is continuously integrated. As the result, a large signal is generated at the
output of the PI-based controller. This windup phenomenon can significantly deteriorate the performance and even cause instability [51]. Therefore, as the next step, a multivariable anti-windup (AW) compensator is augmented with the devised MIMO $H_{\infty}$ PI-based controller to minimize adverse effects of the input saturation. A systematic LMI-based approach is presented in this chapter to design gain matrices of the AW compensator that guarantees a graceful performance degradation during the input saturation.

Using the proposed methodologies, two MIMO $H_{\infty}$ PI controllers and two AW compensators are designed for the microgrid study system to operate the microgrid in the VPP mode facing two utility grid conditions, i.e. strong with $ESCR > 5$ and weak with $ESCR < 2.5$. A comprehensive set of (1) modal analyses, i.e., eigenvalue, participation factor, and sensitivity analyses, and (2) frequency response analysis based on singular value and bode plots, is performed in MATLAB platform. These frequency-domain studies measure the robustness margins of the developed hierarchical control structure.

Finally, the following time-domain case studies are simulated in PSCAD/EMTDC platform to investigate the performance of the developed hierarchical robust control structure: (1) reference tracking, (2) accidental disconnection and re-connection of loads, (3) change in the microgrid configuration, (4) sudden outage and re-connection of DERs, (5) nonlinear load energization, (6) three-phase L-L-L-G fault, (7) single-phase L-G fault, and (8) voltage sag at the utility grid. Two grid conditions (strong and weak) are considered for the above case studies.

### 4.2 Dynamic Model of the Closed-Loop Microgrid at the Primary Level

The linearized state-space dynamical model of the grid-connected microgrid of Figure 2.1 is provided in (2.1), Chapter 2. Also, the dynamic model of the decentralized tracking error compensator along with the robust primary power controller of the $n^{th}$ DER unit, for $n = 1, ..., 4$, are provided in (2.8) and (2.9). For the ease of reference, these equations are summarized here as:
\[
\dot{x}(t) = Ax + B_{c1}u_{DR,1} + B_{c2}u_{DR,2} + B_{c3}u_{DR,3} + B_{c4}u_{DR,4} + B_{d}v_{d} + B_{w}\omega_{s},
\]

\[
y_{p,n}(t) = C_{p,n}x,
\]

(4.1)

\[
\begin{cases}
\dot{x}_{e,n}(t) = (y_{p,n}(t) - y_{ref,n}), \\
\dot{x}_{pc,n}(t) = A_{pc,n}x_{pc,n}(t) + B_{pc,n}x_{e,n}(t), \\
u_{DR,n} = C_{pc,n}x_{pc,n}(t).
\end{cases}
\]

(4.2)

Augmenting dynamical equations of the plant (4.1) with the four primary controllers (4.2) results in the dynamic model of the closed-loop system at the primary level as:

\[
\begin{cases}
\dot{x}_{prm}(t) = A_{prm}x_{prm} + B_{prm}u_{prm} + F_{prm}\omega_{prm}, \\
y(t) = C_{prm}x_{prm},
\end{cases}
\]

(4.3)

where \(x_{prm} = [x_{c}, x_{e1}, x_{e2}, x_{e3}, x_{e4}, x_{pc,1}, x_{pc,2}, x_{pc,3}, x_{pc,4}]^T\) is the state vector, \(u_{prm} = [y_{ref,1}^T, y_{ref,2}^T, y_{ref,3}^T, y_{ref,4}^T]^T\) is the control input in which \(y_{ref,n}(t) = [P_{ref,n}(t), Q_{ref,n}(t)]^T\), and \(\omega_{prm} = [u_{d}^T, v_{u}^T, \omega_{s}]^T\) is the exogenous disturbance. Also, \(y = [P_{pcc}, Q_{pcc}]^T\) is the measured output, in the case of connection to a strong utility grid, and \(y = [P_{pcc}, |V_{pcc}|]^T\) is the measured output, in the case of connection to a weak utility grid, in which the operator \(|\cdot|\) measures the magnitude of the voltage phasor at the PCC. Also:

\[
A_{prm} = \begin{bmatrix}
A & 0 & 0 & [B_{c1}C_{cp,1} & B_{c2}C_{cp,2} & B_{c3}C_{cp,3} & B_{c4}C_{cp,4}]
\end{bmatrix}
\]

\[
B_{prm} = \begin{bmatrix}
0 \\
-I \\
0
\end{bmatrix}
\]

\[
F_{prm} = \begin{bmatrix}
B_{d} & B_{w}
\end{bmatrix}
\]
4.3 Secondary Control Objectives

Depending on the strength of the host utility grid to which the microgrid is connected, the VPP mode of operation can itself be divided into two modes which is referred to as VPP-PQ and VPP-PV modes throughout this thesis. In the VPP-PQ mode, the main utility grid is strong with ESCR \( > 5 \). So, PCC’s voltage and frequency is maintained by the utility grid. In this mode, the secondary control coordinates the DERs to provide the pre-determined instantaneous real and reactive power exchanged with the utility grid.

In the VPP-PV mode, the microgrid is connected to a weak utility grid with ESCR \( < 2.5 \). In this mode, the frequency is maintained by the utility grid. DERs are coordinated by the secondary controller to exchange a specified amount of real power with the utility grid while regulating PCC’s voltage at 1 pu.

The developed robust secondary controller and the previously designed primary controllers should:

1. provide the desired pre-determined requirements at the PCC of the microgrid, at the steady-state operating conditions,

2. maintain the stability of the overall microgrid system during transients,

3. achieve accurate power sharing among DERs to provide dynamic power balance within the microgrid,

4. maintain the stability of the constrained microgrid system with a graceful performance degradation should the input saturation happens,

5. recover the unconstrained microgrid response asymptotically when possible.

It should be mentioned that the constrained system refers to the system subjected to input saturation. Also, as defined in [51], “graceful performance degradation” means achieving a pre-determined level of attenuation in the effect of input saturation on the output of PI controller.

4.4 Secondary Control Structure

Figure 4.1 depicts our proposed secondary control structure and the augmented external AW compensator to operate the microgrid in the two VPP modes. Based on the host utility grid condition, the tertiary control (often known as the distribution management
Chapter 4. VPP Mode: Robust Secondary Control

system) determines the switch SWT-VPP to select between the two modes, i.e., VPP-PQ or VPP-PV.

The PI-based PQ and PV controller blocks are MIMO $H_\infty$ PI controllers designed based on our proposed method to follow real and reactive power and real power and voltage magnitude setpoints, respectively. The $[E_{c\text{pq}}]$ and $[E_{c\text{pv}}]$ blocks in Figure 4.1 depict the two AW compensators designed for the VPP-PQ and VPP-PV modes, respectively. Figure 4.2 illustrates the schematic diagram of the PI-based controller augmented with compensation signal received from AW scheme.

The “Participation Algorithm” box in this structure shares the required real and reactive power among the DERs. This box could be from a simple sharing gain to time-dependent variables computed dynamically by an independent organization based on bid prices, availability, congestion problems, costs and other related issues [52]. In this thesis, this box is composed of constant gains which are sharing the required power between the DERs based on their generation capacity, and are equal to 6/19 for DER$_1$ to DER$_3$ and 1/19 for the DER$_4$.

A block diagram of the secondary limiter is also provided in Figure 4.3. As depicted, the power setpoint is limited to 9.5 MVA which is the summation of the maximum generation capacity of the four DERs.

4.5 Secondary Control Design

This section designs the PI-based secondary controllers and anti-windup (AW) compensators to operate the microgrid in the VPP mode. To account for the control input saturation in controller design procedure, a unified framework is proposed by [53]. This framework is based on the following two-step design paradigm: “... Design the linear controller ignoring control input nonlinearities and then add anti-windup compensation to minimize the adverse effects of any control input nonlinearities on closed loop performance [53].”

This two-step design paradigm provides a computationally efficient technique for equipping the existing unconstrained PI controllers to eliminate controller windup problems [51]. Therefore, as the first step, LMI techniques are presented to design multi-variable $H_\infty$ PI-based secondary controllers ignoring control input saturation. Then, an LMI-based approach will be adapted to design multivariable static AW compensators.
Chapter 4. VPP Mode: Robust Secondary Control

4.5.1 MIMO $H_{\infty}$ PI Control Design via LMI Approach

As stated earlier, at this stage of the design, control input saturation and AW compensator are ignored. A multivariable PI-Based controller, depicted in Figure 4.1, is of the following form:

$$u_{prm}(t) = K_{sh}K_pe(t) + K_{sh}K_i \int_0^t e(\theta)d\theta,$$

in which $e(.)$ is the tracking error, i.e., $e(t) = r(t) - y(t)$. Also, $K_{sh}$ is the sharing gain matrix associated with the participation algorithm box, Figure 4.1, as:

$$K_{sh} = \begin{bmatrix} 6/19 & 0 & 6/19 & 0 & 6/19 & 0 & 1/19 & 0 \\ 0 & 6/19 & 0 & 6/19 & 0 & 6/19 & 0 & 1/19 \end{bmatrix}^T$$

For the multi-DER microgrid of Figure 2.1, the PI controller gains $K_p$ and $K_i$ are matrices, each belonging to $\mathbb{R}^{2 \times 2}$. Also, the reference command is $r = [P_{pcc, ref}; Q_{pcc, ref}]^T$.

Figure 4.1: Schematic diagram of the multivariable PI-based secondary control with anti-windup compensator in the VPP mode.
in the VPP-PQ mode and \( r = [P_{\text{ref}}, |V|_{\text{ref}}] \) in the VPP-PV mode.

Proposed by [54], the basic idea of MIMO \( H_\infty \) PI control design is to re-formulate the PI control design problem to that of static output-feedback (SOF) control problem. Let’s define a new state vector as \( \bar{x}(t) = [x_{\text{prm}}(t)^T, \int_0^t e(\theta)d\theta]^T \) and a new output vector as \( \bar{y}(t) = [e(t)^T, \int_0^t e(\theta)d\theta]^T \). The augmented system with (4.3) and (4.4) can then be written as:

\[
\dot{\bar{x}}(t) = \bar{A}\bar{x}(t) + \bar{B}_{\text{prm}}u_{\text{prm}}(t) + \bar{B}_w\omega_{\text{prm}}(t) + \bar{B}_r r(t),
\]

\[
\bar{y}(t) = \bar{C}\bar{x}(t) + \bar{D}r(t).
\]  

where \( \bar{A} = \begin{bmatrix} A_{\text{prm}} & 0 \\ -C_{\text{prm}} & 0 \end{bmatrix} \), \( \bar{B} = \begin{bmatrix} B_{\text{prm}} \\ 0 \end{bmatrix} \), \( \bar{B}_w = \begin{bmatrix} F_{\text{prm}} \\ 0 \end{bmatrix} \), \( \bar{B}_r = \begin{bmatrix} 0 \\ I \end{bmatrix} \), \( \bar{C} = \begin{bmatrix} -C_{\text{prm}} & 0 \\ 0 & I \end{bmatrix} \), \( \bar{D} = \begin{bmatrix} I \\ 0 \end{bmatrix} \). Therefore, the control law is obtained as:

\[
u_{\text{prm}}(t) = K\bar{y}(t),
\]  

where \( K = [K_{sh}K_p, K_{sh}K_i] \in \mathbb{R}^{8 \times 4} \).

**Assumption 2.** In (4.5), the pair \( (\bar{A}, \bar{B}) \) is stabilizable and the pair \( (\bar{A}, \bar{C}) \) is detectable. Also, disturbance \( \omega_{\text{prm}}(t) \) belongs to \( L_2[0, \infty) \).

**Definition 3** (Robust \( H_\infty \) control Problem with disturbance attenuation \( \gamma \)). Given a positive constant \( \gamma \), design an SOF controller (4.6) such that the system in (4.5), with
$r(.) = 0$, is robustly asymptotically stable with $H_\infty$ disturbance attenuation $\|T_{zw}\|_\infty < \gamma$, in which

$$\|T_{zw}\|_\infty = \sup_{w_{prm}(\cdot) \in L_2(0,\infty), \|w_{prm}(\cdot)\|_2 > 0} \|\tilde{z}(\cdot)\|_2$$

(4.7)

Also, the controlled output vector $\tilde{z}$ is chosen as:

$$\tilde{z}(t) = C_z\bar{x}(t) + D_z u_{prm}(t)$$

(4.8)

where $C_z = w_x[0 \ I]$ and $D_z = w_u \ I$ represent the weighting factors.

Substituting (4.6) into (4.5) and (4.8), the overall closed-loop system controlled at the secondary level is obtained as:

$$\dot{\tilde{x}}(t) = (\bar{A} + \bar{B} K \bar{C}) \bar{x}(t) + (\bar{B} \bar{K} \bar{D} + \bar{B}_w) \omega_{prm}(t) + (\bar{B} \bar{K} + \bar{B}_r) r(t),$$

$$\tilde{z}(t) = (C_z + D_z K \bar{C}) \bar{x}(t) + D_z K \bar{D} r(t).$$

(4.9)

**Lemma 1** ([54]). Controller (4.6) is a robust $H\infty$ controller with disturbance attenuation $\gamma$ for the system (4.5) if and only if there exists a positive definite matrix $P = P^T > 0$ such that the following matrix inequality holds:

$$\begin{bmatrix}
(\bar{A} + \bar{B} K \bar{C})^T P + P (\bar{A} + \bar{B} K \bar{C}) & P \bar{B}_w (C_z + D_z K \bar{C})^T \\
* & -\gamma I & 0 \\
* & * & -\gamma I
\end{bmatrix} < 0,$$

(4.10)

It should be noted that the matrix inequality (4.10) is in the quadratic form of the unknowns $P$ and $K$. This bilinear matrix inequality (BMI) problem is nonconvex and NP-hard to solve [55]. However, for our microgrid study system in which $C_{prm}$ is of full row rank, the BMI problem (4.10) can be converted into an LMI problem which is computationally efficient to solve:

**Theorem 3.** The closed-loop dynamical system (4.9) controlled with a multivariable PI-based control (4.4) is asymptotically stable with guaranteed $H\infty$ performance $\gamma$, if there exist matrices $Q = Q^T > 0$ and $Y$, such that the following LMI holds:

$$\Psi = \begin{bmatrix}
\bar{A} Q + B Y^T + Q \bar{A}^T + Y \bar{B}^T & \bar{B}_w & Q \bar{C}_z^T + Y D_z^T \\
* & -\gamma I & 0 \\
* & * & -\gamma I
\end{bmatrix} < 0.$$

(4.11)

In this case, a possible value for the feedback matrix gain obtains as $K = Y^T Q^{-1} \bar{C}^T (\bar{C} \bar{C}^T)^{-1}$. 
**Proof.** Pre and post multiply matrix inequality \(4.10\) with the positive definite matrix \(\Pi = \text{diag}\{P^{-1}, I, I\}\). Then, defining \(Q = P^{-1}\) and \(Y = Q\bar{C}^T \bar{K}^T\) results in the LMI \(4.11\).

### 4.5.2 Multivariable Anti-Windup Compensator Design via LMI Approach

The AW compensator scheme, shown in Figure 4.1, is basically a static linear gain matrix \(E_c\) that relates the difference between the input and output of the secondary limiter \(\nu\) to a modification signal \(\zeta\). The modification signal \(\zeta\) is then injected to the previously designed PI controller. This anti-windup scheme can be described by:

\[
\zeta = E_c \nu, \quad (4.12)
\]

in which \(\nu = \text{sat}(u) - u\). Figure 4.2 shows the multivariable PI-based secondary controller equipped with the AW compensator. As shown, when saturation happens, the PI control law is obtained as:

\[
\dot{x}_{sc} = e(t) + \zeta, \quad (4.13)
\]
\[
u_{prm}(t) = K_{sh}K_pe(t) + K_{sh}K_ix_{sc}. \quad (4.14)
\]

Also, during the input saturation, dynamical equations of the microgrid \(4.3\) change to

\[
\dot{x}_{prm}(t) = A_{prm}x_{prm} + B_{prm}\text{sat}(u_{prm}) + F_{prm}\omega_{prm}, \quad (4.15)
\]
\[
 y(t) = C_{prm}x_{prm}. \quad (4.16)
\]

Augmenting dynamical equations of the microgrid study system \(4.15\), the multivariable PI-based secondary controller \(4.13\), and the multivariable anti-windup compensator \(4.12\), the overall closed-loop system, with \(r(.) = 0\) and \(\omega_{prm}(.) = 0\), is obtained as:

\[
\dot{x}_{aw}(t) = A_{aw}x_{aw}(t) + B_{aw}\nu(t),
\]
\[
u_{prm}(t) = C_{aw}x_{aw}(t), \quad (4.17)
\]

where \(A_{aw} = \begin{bmatrix} A_{prm} - B_{prm}K_{sh}K_p C_{prm} & B_{prm}K_{sh}K_i \\ -C_{prm} & 0 \end{bmatrix}, B_{aw} = \begin{bmatrix} B_{prm} \\ E_c \end{bmatrix},
\]
\(C_{aw} = \begin{bmatrix} -K_{sh}K_p C_{prm} \\ K_{sh}K_i \end{bmatrix}^T\), and \(x_{aw} = \begin{bmatrix} x_{prm} \\ x_{sc} \end{bmatrix}\).

Equation \(4.17\) shows that when saturation happens, the signal \(\nu\) is different than
zero. In this situation, the effect of input saturation on the performance of the controlled system can be studied from the perspective of robustness analysis of dynamical systems subjected to exogenous disturbances.

The multivariable anti-windup compensator design objective is then to find a suitable compensator gain matrix $E_c$, with appropriate dimension, such that:

1. the constrained closed-loop system is stable,
2. a desired graceful performance degradation level of $\beta$ is achieved.

**Remark 3.** Let’s assume that the disturbance input $\nu$ is of finite energy which satisfies $\nu \in L_2[0, \infty)$. Also, it is clear that the appropriate selection of PI-based controller gain $K$ which satisfies LMI (4.11) results in the hurwitz stability of the the closed-loop matrix $A_{aw}$ of the constrained system in (4.17). Therefore, by means of the well-know global stability of the bounded input systems [56], the stability of the constrained system (4.17) is guaranteed.

**Theorem 4.** There exists a static anti-windup compensator with gain $E_c$ that guarantees the $H_\infty$ performance $\|T_{u_{prm}\nu}\|_\infty < \beta$ of the constrained closed-loop system (4.17), if there exit matrices $G$ and $E_c$ such that the following LMI holds:

$$
\Psi = \begin{bmatrix}
A_{aw}G + GA_{aw}^T & M_1B_{prm} + M_2E_c & GC_{aw}^T \\
* & -\beta I & 0 \\
* & * & -\beta I
\end{bmatrix} < 0. 
$$

(4.18)

where $M_1 = \begin{bmatrix} I \\ 0 \end{bmatrix}$ and $M_2 = \begin{bmatrix} 0 \\ I \end{bmatrix}$. Also, $T_{u_{prm}\nu}$ is the transfer function relating the disturbance input $\nu$ to the control signal $u_{prm}$.

**Proof.** Pre and post multiplying LMI (4.18) with the positive definite matrix $\Phi = \text{diag}\{G^{-1}, I, I\}$ and defining $X = G^{-1}$, the LMI (14) in [57] obtains. The rest of the proof is provided in [57].

### 4.5.3 Obtained Controller and Anti-Windup Compensator

The initial operating point under which PI-based secondary controllers and the anti-windup compensators are designed in the VPP-PQ and VPP-PV modes are as follows:

1. In the VPP-PQ mode, the length of the line $TL_1$ is 0.8 km which results in a ESCR=14.09.

2. In the VPP-PV mode, the length of the line $TL_1$ is 20 km which results in a ESCR=2.16.
3. In the VPP-PQ mode, the VPP is required to transfer 1.02 MVA to the utility grid at a leading power factor of 0.98. The steady-state operating point is as of given in Table E.1 Appendix E.

4. In the VPP-PV mode, the VPP is required to transfer 1.0 MW to the utility grid and to regulate PCC’s voltage at 1 pu. The steady-state operating point is as of given in Table E.1 Appendix E.

Setting the design parameters as $\gamma = 1$, $\beta = 1$, $w_x = 1$, and $w_u = 0.1$, and solving the LMIs (4.11) and (4.18) using the YALMIP toolbox in MATLAB software, the following controller matrix and anti-windup compensator gains $K_p$, $K_i$, and $E_c$ are obtained for the microgrid study system of Figure 2.1 in the VPP-PQ mode:

\[
K_p = \begin{bmatrix}
0.210 & 0.0002 \\
0.0004 & 0.054
\end{bmatrix}, \quad
K_i = \begin{bmatrix}
200.03 & -0.0052 \\
-0.0052 & 180.36
\end{bmatrix}, \quad
E_c = \begin{bmatrix}
120 & 0.150 \\
0.002 & 87
\end{bmatrix},
\]

and for the VPP-PV mode as:

\[
K_p = \begin{bmatrix}
0.180 & 0.002 \\
0.0004 & 0.122
\end{bmatrix}, \quad
K_i = \begin{bmatrix}
105.03 & -0.0004 \\
0.0004 & 1000.36
\end{bmatrix}, \quad
E_c = \begin{bmatrix}
430 & 0.000 \\
0.780 & 870
\end{bmatrix}.
\]

A transfer function representation of the MIMO PI-based controller (4.4) for the VPP-PQ mode, depicted in Figure 4.1 is given by:

\[
\begin{bmatrix}
U_p(s) \\
U_q(s)
\end{bmatrix} = \begin{bmatrix}
0.210 + 200.03/s & 0.0002 - 0.0052/s \\
0.0004 - 0.0052/s & 0.054 + 180.36/s
\end{bmatrix} \begin{bmatrix}
E_p(s) \\
E_q(s)
\end{bmatrix},
\]

and for the VPP-PV mode as:

\[
\begin{bmatrix}
U_p(s) \\
U_v(s)
\end{bmatrix} = \begin{bmatrix}
0.180 + 105.03/s & 0.002 - 0.0004/s \\
0.0004 + 0.0004/s & 0.122 + 1000.36/s
\end{bmatrix} \begin{bmatrix}
E_p(s) \\
E_v(s)
\end{bmatrix}.
\]

### 4.6 Modal Analysis of the VPP Dynamics

To investigate the dynamic behavior of the microgrid of Figure 2.1 controlled by our proposed hierarchical control structure, modal analysis of the overall closed-loop dynamic system is performed in MATLAB software tool.
4.6.1 Eigenvalues and Participation Factors

Figure 4.4 shows the eigenvalues of the open-loop microgrid of Figure 2.1 without built-in current, primary, and secondary controllers in the loop. Figure 4.4 indicates that the open-loop system is nominally stable. Dominant eigenvalues (eigenvalues closest to the imaginary axes) are located between -45 to -10 1/s. These dominant modes, the states contributing in each mode, and their relative contribution (the so called normalized participation factors [58]) are provided in Table 4.1.

Augmenting the open-loop system with the four built-in current controllers, the four primary controllers, and the secondary controller result in the overall closed-loop system ($A_{\text{closed}} \in \mathbb{R}^{100 \times 100}$). Figure 4.5 shows the eigenvalues of the overall closed-loop system in the VPP-PQ mode. The wide band of dynamic modes existing in the microgrid system can be seen in this figure.

Depicted in Figure 4.5 (e), dominant eigenvalues are located between $-40$ to $-20$ 1/s. These lowest frequency modes are mainly affected by the state variables of the PI-based secondary controller. Comparing the dominant eigenvalues of the overall closed-loop system and the open-loop system in Table 4.1, it can be seen that the designed control structure has succeeded to increase the robustness margins. The reason is that in the open-loop system, the dominant eigenvalues are closer to the imaginary axis and are dominantly affected by the output current of the DERs. As the operating point of microgrids change, these currents vary to respond to these changes. Therefore, the
Table 4.1: Dominant eigenvalues of the grid-connected open-loop microgrid

<table>
<thead>
<tr>
<th>Mode</th>
<th>Real (1/s)</th>
<th>Imagin. (rad/s)</th>
<th>Participating States</th>
<th>Normalized Part. Factors</th>
</tr>
</thead>
<tbody>
<tr>
<td>41,42</td>
<td>-44.2360</td>
<td>± 376.9900</td>
<td>$i_{DR3,d}$</td>
<td>0.1751</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$i_{DR3,q}$</td>
<td>0.1751</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$i_{DR4,d}$</td>
<td>0.1126</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$i_{DR4,q}$</td>
<td>0.1126</td>
</tr>
<tr>
<td>43,44</td>
<td>-13.1630</td>
<td>± 376.9900</td>
<td>$i_{DR1,d}$</td>
<td>0.1879</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$i_{DR1,q}$</td>
<td>0.1879</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$i_{DR4,d}$</td>
<td>0.2796</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$i_{DR4,q}$</td>
<td>0.2796</td>
</tr>
<tr>
<td>45,46</td>
<td>-10.8420</td>
<td>± 376.9900</td>
<td>$i_{DR1,d}$</td>
<td>0.1259</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$i_{DR1,q}$</td>
<td>0.1259</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$i_{DR2,d}$</td>
<td>0.3379</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$i_{DR2,q}$</td>
<td>0.3379</td>
</tr>
</tbody>
</table>

dominant eigenvalues of the open-loop system also move. However, in the closed-loop system, the dominant eigenvalues are affected by the states of the secondary controller. Practically, once these controllers are designed and installed, their parameters would rarely change. Therefore, the robustness margins (such as module margin defined in Chapter 2) which is determined by the location of the dominant eigenvalues would not be affected by the uncertainties and disturbances in the dynamical system.

Figure 4.5: Overall closed-loop system eigenvalues
Table 4.2: Dominant eigenvalues of the closed-loop study microgrid

<table>
<thead>
<tr>
<th>Mode</th>
<th>Real (1/s)</th>
<th>Imagin. (rad/s)</th>
<th>Participating States</th>
<th>Normalized Part. Factors</th>
</tr>
</thead>
<tbody>
<tr>
<td>93,94</td>
<td>-37.8958</td>
<td>± 0.0233</td>
<td>$x_{sc,1}$</td>
<td>0.4982</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$x_{sc,2}$</td>
<td>0.4981</td>
</tr>
<tr>
<td>95,96</td>
<td>-24.0007</td>
<td>± 0.0009</td>
<td>$x_{sc,1}$</td>
<td>0.2031</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$x_{sc,2}$</td>
<td>0.3474</td>
</tr>
<tr>
<td>97,98</td>
<td>-24.0003</td>
<td>± 0.0072</td>
<td>$x_{sc,1}$</td>
<td>0.4156</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$x_{sc,2}$</td>
<td>0.2163</td>
</tr>
<tr>
<td>99,100</td>
<td>-23.9993</td>
<td>± 0.0057</td>
<td>$x_{sc,1}$</td>
<td>0.2908</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$x_{sc,2}$</td>
<td>0.2908</td>
</tr>
</tbody>
</table>

The participation factor of the dominant modes of the overall closed-loop system, the states contributing in these modes, and their normalized participation factors are provided in Table 4.2. In Table 4.2, $x_{sc} = [x_{sc,1}, x_{sc,2}]^T \in \mathbb{R}^2$, corresponds to the state variables of the multivariable PI-based secondary controller. The results of the participation factor analysis show that:

- The most dominant eigenvalues of the closed-loop system, i.e., window (e) in Figure 4.5, are mainly affected by the states of the secondary controller.
- The second group of eigenvalues, i.e., window (d) in Figure 4.5 are dominantly influenced by the state variables of the DER’s output currents and load currents. This relatively low frequency modes are caused by the large time constant of the corresponding inductors and capacitors.
- Eigenvalues in windows (c) and (b), Figure 4.5 are influenced by a combination of the state variables of the current controllers, PCC voltage, utility grid current, and the state variables of the four robust primary controllers.
- Finally, the fastest dynamics of the overall closed-loop system corresponds to the state variables of the four robust primary controllers which is desirable for a controller designed to be robust to uncertainties and disturbances. These eigenvalues can be seen in window (a) in Figure 4.5.

The above observations suggest that the developed robust hierarchical control structure is a viable solution for operating the microgrid in the VPP mode.
4.6.2 Sensitivity Analysis

This section studies the sensitivity of the dominant eigenvalues of the overall closed-loop system to the changes in microgrid parameters and controller gains.

4.6.3 Sensitivity to the Load Parameters

To confirm the robustness of the developed hierarchical control structure to load uncertainties, the trajectory of the dominant eigenvalues of the overall closed-loop and the open-loop systems are investigated and compared as the load resistance and inductance $R_{l1}$ and $L_{l1}$ of the load branch $L_1$ in Figure 2.1 are changed by a coefficient of $0.01 < \alpha < 1.2$. Figure 4.6 (a) confirms the insensitivity of the dominant eigenvalues of the closed-loop system to load uncertainties. Also, Figure 4.6 (b) shows that the open-loop dominant eigenvalues move significantly toward the imaginary axis leading to reduced stability margins of the open-loop system. Both parts (a) and (b) of Figure 4.6 highlights the achieved robustness using the proposed hierarchical control structure and confirm the discussion provided in Section 4.6.1 regarding the dominant eigenvalues of the open and closed loop systems.

4.6.4 Sensitivity to the Secondary Controller Parameters

As the participation factor in Table 4.2 suggests, the dominant eigenvalues of the overall closed-loop system are sensitive to changes in the gains of the secondary controller. Let’s
assume a change by a coefficient of $0.5 < \alpha < 2$ in the gain $K$ of the control law (4.6).

Figure 4.7 illustrates negligible movements of the dominant pairs of eigenvalues toward the imaginary axis, which confirms the robustness of the developed control structure to variations in the secondary controller parameters.

### 4.6.5 Frequency Response Analysis

Let’s define the output loop transfer function of the controlled microgrid system of Figure 2.1 as $L_o(s) = G(s) K_{prm}(s) K_{sec}(s)$, in which $G(s)$ is the transfer function representation of the microgrid system controlled at the built-in current controller level in (4.1), and $K_{prm}$ is the transfer function representation of the primary controllers (4.2), and $K_{sec}$ is the transfer function representation of the secondary controller (4.4). Provided in Chapter 8 of [59], to achieve acceptable reference following and disturbance rejection, the minimum singular value of $L_o$ should have a large gain at low frequencies, i.e. $\sigma_{\min}(\omega) \gg 1$. Also, to be insensitive to measurement noise, the maximum singular value of $L_o$ should have small gain at high frequencies, i.e. $\sigma_{\max}(\omega) \ll 1$.

Figure 4.8 shows the singular value plot of $L_o$. This figure clearly shows a well-designed robust hierarchical control structure. The overall achieved bandwidth is 68 rad/s. The overall bandwidth of a closed-loop MIMO system is defined as the frequency where the smallest singular value of the output loop transfer function, i.e., $\sigma(L(j\omega))$, crosses 0 dB, for the first time, from above [48].

Also, bode plots of the loop transfer functions from $\Delta P_{pcc,ref}$ to $\Delta P_{pcc}$ and from
\[ \Delta Q_{pcc,ref} \] to \[ \Delta Q_{pcc} \] are provided in Figure 4.9 (a) and (b), respectively. The stability margins, i.e., gain and phase margins, and their crossover frequencies are presented in Table 4.3. The gain and phase margins also highlight the achieved robustness margins.

Figure 4.8: Singular value plot of the overall loop transfer function

Figure 4.9: Bode plot of the loop transfer function of: (a) \[ \frac{\Delta P_{pcc}}{\Delta P_{pcc,ref}} \] and (b) \[ \frac{\Delta Q_{pcc}}{\Delta Q_{pcc,ref}} \]

4.7 Performance Evaluation

This series of studies seeks to evaluate the time-domain performance of the developed hierarchical control structure in operating the microgrid of Figure 2.1 in the VPP mode, connected to (i) strong utility grid, and (ii) weak utility grid.
Table 4.3: Phase and gain margins and crossover frequencies obtained from Figure 4.9

<table>
<thead>
<tr>
<th>Parameter</th>
<th>$\Delta P_{pcc}/\Delta P_{ref_{pcc}}$</th>
<th>$\Delta Q_{pcc}/\Delta Q_{ref_{pcc}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>GM (dB)</td>
<td>79</td>
<td>26.98</td>
</tr>
<tr>
<td>$\omega_{gm}$ ($rad/s$)</td>
<td>3.2e6</td>
<td>1.2e3</td>
</tr>
<tr>
<td>PM ($^\circ$)</td>
<td>52.1</td>
<td>81.7</td>
</tr>
<tr>
<td>$\omega_{pm}$ ($rad/s$)</td>
<td>354.3</td>
<td>67.6</td>
</tr>
</tbody>
</table>

Remark 4. Measured quantities are presented in per-unit (pu), wherever applicable. The base values for this per-unitization are provided in Table 3.1, Chapter 3.

4.7.1 VPP Mode: Strong Utility Grid (ESCR=14.09)

In this series of studies, microgrid is operating in the VPP-PQ mode. The utility grid condition, requirements at the PCC, and the steady-state operating point of the microgrid are described in items (1) and (2) in Section 4.5.3.

4.7.1.1 Case 1: Reference Tracking

To demonstrate the capability of the developed hierarchical control structure in the VPP-PQ mode to follow the control command, a 20% step change increase in the real and reactive power setpoints at the PCC is applied at $t=0.2$ s.

Figure 4.10 (a) and (b) depicts the real and reactive output powers of the microgrid, going through the PCC, and their references. As depicted, the real and reactive powers settle down to their new set-point in 29.2 ms and 19.8 ms (with 1% criterion) with 3.75% and 0.10% overshoot and zero steady-state tracking error, respectively. The real and reactive power generated by the four DERs in response to this step increase and their reference commands are also provided in plots (e), (f), (i), and (j) of Figure 4.10.

As shown in Figure 4.10 (c), (g), and (k), despite this change in the microgrid power generation, voltage at the PCC and subsequently at the Pc of DERs is maintained constant at 1 pu by the host utility grid. The three-phase instantaneous output current of the microgrid as well as the generated output current of DER$_1$ and DER$_4$ are also illustrated in plots (d), (h), and (l). These plots clearly show the increase in the output current of the DERs to provide the excess of the required power at the PCC. Output current and voltage waveform of DER$_2$ and DER$_3$ are similar to DER$_1$ waveform, due to their identical configuration and rating. However, for the sake of brevity, they are not provided in this section.
Chapter 4. VPP Mode: Robust Secondary Control

4.7.1.2 Case 2: Load change

To investigate the robustness of the developed hierarchical control structure to the local uncertainties in a microgrid, system response to the sudden disconnection and reconnection of Load_1 is simulated in this section. Load_1 is seen as the parallel combination of L_1 and L_2 in Figure 2.1. At \( t = 0 \) s, Load_1 disconnects from the grid, causing a decrease of 0.53 pu in the power demand, and then connects back after 3 cycles at \( t = 0.25 \) s. For the sake of brevity, simulation results are reported in Section F.1, Appendix F. These simulation results indicate that despite the considerable load variations in the microgrid, the pre-specified real and reactive powers at the PCC are delivered within 35 ms and 70 ms, respectively. This depicts the robustness of the developed hierarchical control structure in response to severe load disturbances.

4.7.1.3 Case 3: Change in the Microgrid Configuration

In this section, robust stability and performance of the developed hierarchical control structure to topological uncertainties is studied. At \( t = 0.2 \) s., the breaker on Feeder F3 in Figure 2.1, at the PCC side, opens. This event isolates DER_2 and DER_3 from the microgrid feeding two loads L_5 and L_6. This disconnection of two DERs and two major
loads significantly changes the operating point of the microgrid. However, to utilize the power generated by the two DERs, the transfer switch closes after five cycles at $t = 0.283$ s, connecting Bus$_6$ to Bus$_5$. This causes a major change in the dynamic model of the microgrid.

When the breaker on Feeder F3 opens, DER$_1$ and DER$_4$, with a maximum generation capacity of 3.5 MVA, are left to feed a total load of 3.96 MW + j1.01 MVAr and to provide the utility grid with 1.00 MW + j0.20 MVAr. Clearly, these two DERs are unable to provide the required power at the PCC. Therefore, microgrid imports 0.73 MVA from the utility grid, as depicted in Figure 4.11 (a) and (f). This significant tracking error causes an input saturation at the secondary control level. Therefore, all the four DERs are commanded to generate their maximum capacity, as shown in Figure 4.11 (b) to (e).

Figure 4.12 (a), (d), (e), (h), (i), and (l) shows the $d$ and $q$ components of the output current and output voltage of DER$_1$ and DER$_4$, respectively. These plots show that: (1) the voltage at the PC$_1$ and PC$_4$ are properly regulated at 1 pu by the utility grid, and (2) the secondary limiter has limited the power setpoints of DER$_1$ and DER$_4$ to their generation capacity therefore reference currents and subsequently the output currents of these two DERs are limited to 1 pu for overcurrent protection. Figure 4.13 (a), (d), (e), (h), (i), and (l) illustrate the three-phase output voltage, output current, and terminal current of these two DERs.

On the other hand, the opening of Feeder F3 isolates DER$_2$ and DER$_3$ to feed the two loads $L_5$ and $L_6$. This isolation form an island with a total load demand of 1.48 MW + j0.38 MVAr. However, these two DERs are controlled to follow a received real and reactive power reference command not to regulate voltage or frequency at their point of connection. Not being set to regulate the voltage during the isolation period from $t=0.2$ s to $t=0.28$ s, the voltage at these two PCs increases to 1.35 pu as shown in Figure 4.12 (j) and (k) and Figure 4.13 (b) and (c). Therefore, VSCs of DER$_2$ and DER$_3$ have to increase their terminal voltage to be able to follow their power reference commands. This pushes VSCs of DER$_2$ and DER$_3$ into saturation condition due to overmodulation and therefore current reference command and subsequently power reference commands are not followed, as depicted in Figure 4.11 (c), (d), (h), and (i) and Figure 4.12 (b), (c), (f), and (g). However, as depicted in Figure 4.11 within 132 ms after the transfer switch closes and re-connects the isolated part to the rest of the microgrid, the power setpoints at the PCC are followed with zero tracking error. Figure 4.14 demonstrates the voltage at the main buses (plots (a) to (f)), currents going through the main feeders and the transfer feeder $C_5$ (plots (g) to (l)), and the loads currents (plots (m) to (r)).

This case study clearly demonstrates the robustness of the proposed hierarchical con-
Figure 4.11: System response to the topological change in the microgrid: (a) to (j) real and reactive power transferred to the utility grid and generated by DER₁ to DER₄.

Figure 4.12: System response to the topological change in the microgrid: (a) to (l) d and q components of the output currents and output voltage of the four DER units.

trol structure to major changes in the operating point (disconnection of loads and generation units) as well as topology of the dynamical system (re-connection of loads and DERs to a different bus). Also, this case study highlights the robustness of the designed
control structure in maintaining the stability of the microgrid during its operation in the nonlinear region (input saturation). Fast recovery of the devised control structure augmented with the proposed anti-windup compensator can also be concluded.

4.7.1.4 Case 4: Nonlinear Load Energization

To investigate the robust performance of the developed hierarchical control structure when the microgrid is subjected to a nonlinear load, the case of induction machine energization is studied in this section. The motor is rated at 500 hp, 2.3 kV, and connected to Bus 5 of Figure 2.1 through a 13.8 kV/2.3 kV, Yg/Δ transformer. At t=0.2 s, motor energizes from the stall under full load condition. For the sake of brevity, simulation results are provided in Section F.1, Appendix F. These simulation results indicate that the developed hierarchical control structure maintains the stability of the time-varying nonlinear system during the start-up period of the motor and keeps the tracking error within ± 7% range. Within 300 ms after motor startup, the control structure provides the utility grid with the pre-specified power with zero tracking error. The successful implementation of the generalized SPWM along with the dual internal current controllers to balance the output current of the DERs is also illustrated in this case study.
Figure 4.14: System response to the topological change in the microgrid: (a) utility grid voltage, (b) to (f) voltage at Bus$_1$ to Bus$_4$, (g) to (l) current going through feeders TL$_1$, F$_1$ to F$_4$, and C$_5$, (m) to (r) load currents $I_{L1}$ to $I_{L7}$.

4.7.1.5 Case 5: Three-Phase L-L-L-G Fault at the PCC Bus

This case study demonstrates the performance of the developed robust hierarchical control structure when a three-phase temporary fault happens. At $t=0.2$ s, a three-phase line-to-ground (L-L-L-G) fault happens at the PCC of the microgrid and lasts for 5 cycles. The fault resistance is selected as 0.2 ohms (0.01 pu). Prior to fault, the microgrid is operating under full load condition, provided in Table E.1, Appendix E. To investigate the robustness of the developed hierarchical control structure, in this scenario, no breaker is allowed to operate to clear the fault.

Figure 4.15 (a) and (b) depicts the real and reactive power provided by the microgrid at the PCC. As shown, during the fault, the microgrid is unable to provide the required power at the PCC. It comes from the fact that during the fault, voltage at the PCC and subsequently at the four PCs drops to about 0.14 pu (plots (g) to (k)). This tracking error pushes all the four DERs into saturation providing 1.2 pu, as illustrated in Figure 4.15 (n) to (q). Figure 4.15 (m) and (r) shows that the fault current of 50 pu is mainly provided by the utility grid through the feeder TL1.

Depicted in Figure 4.15 (a) and (b), after 42 ms that fault clears, the developed
control structure successfully provides the pre-determined real and reactive power at the PCC. This clearly illustrates the capability of our developed robust hierarchical control structure to ride through such a major disturbance, as an L-L-L-G fault.

4.7.1.6 Case 6: Single-Phase L-G Fault at the PCC Bus

This cases study demonstrates the performance of the developed robust power controller under unbalanced voltage condition, when a single-phase temporary fault happens. At t=0.2 s, a single-phase line-to-ground (L-G) fault on the PCC of Figure 2.1 happens and lasts for 5 cycles. The fault resistance is selected as 0.2 ohms (0.01 pu). The operating point prior to fault is as described in Section 4.7.1.5. To investigate robustness of the developed hierarchical control structure, in this scenario, no breaker is allowed to operate to clear the fault.

Figure 4.16 shows the performance of the proposed hierarchical control structure when facing unbalanced voltage disturbances. As stated earlier, DERs are equipped with generalized SPWM and dual built-in current controllers to provide balanced output current for the DERs. Provided in Figure 4.16 (n) to (q), this devised control structure is
able to balance the output current of the DERs despite considerable unbalanced output voltage (plots (h) to (k)). Limited amount of power capacity though has prevented the control structure to fully over-ride this severe unbalanced voltage disturbance. Saturated output current of the DERs to 1.2 pu is also illustrated in Figure 4.16 (n) to (q).

The presence of the negative-sequence component in the unbalanced voltage at the PCC and PCs of the DERs results in double frequency oscillations in the output power, generated by the DERs and transferred to the main utility grid, as illustrated in Figure 4.16 (a) to (f). Within 47 ms after fault clears, power setpoints at the PCC are met. Figure 4.16 (m) and (r) depict that the fault current with an amplitude of 49.75 p.u. is provided by the utility grid.

4.7.2 VPP Mode: Weak Utility Grid (ESCR=2.16)

In this series of studies, microgrid is operating in the VPP-PV mode. The utility grid condition, requirements at the PCC, and the steady-state operating point of the microgrid are described in items (1) and (3) in Section 4.5.3.

Figure 4.17 depicts the performance of the microgrid connected to this considerably
weak utility grid, with $ESCR=2.16$, but controlled by the control structure developed for the microgrid operation in the VPP-PQ mode. At $t=0.1\ s$, DERs are enabled to export power to the power network. As it can be seen, the hierarchical control structure developed for the VPP-PQ mode fails to maintain an acceptable performance for the microgrid connected to the weak utility grid. It necessitates switching to the VPP-PV mode hierarchical control structure. It is worth mentioning that during this case study, no breaker is allowed to operate, solely for the purpose of simulation.

4.7.2.1 Case 1: Reference Tracking

To demonstrate the capability of the developed hierarchical control structure in the VPP-PV mode to follow the control command, a 10% step change increase in the PCC voltage setpoint is applied, at $t=0.35\ s$. The real power setpoint remains unchanged to 1 MW. Transients in the real and reactive power transferred to the utility grid as well as the instantaneous voltage magnitude measured at the PCC is provided in Figure 4.18 (a), (b), and (g), respectively. As demonstrated, the controlled microgrid system follows the voltage setpoint within 47 ms (1% criterion) with 1.8 % overshoot and zero steady-state tracking error.

It is worth mentioning that since the loads in the microgrid study system are modeled as RL branches, increase in the voltage causes increase in their power demand. Therefore, while the real power transferred to the utility grid is regulated to its setpoint, the real
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Figure 4.18: System response to 10% step increase in the PCC voltage setpoint: (a) to (f) output real and reactive power provided by the microgrid at the PCC and generated by DER to DER, (g) and (h) three-phase and magnitude of the instantaneous voltage at the PCC, (i) to (l) instantaneous three-phase output voltage of the DERs, (m) to (q) instantaneous three-phase output current of the microgrid and the four DERs, and (r) load \( L \) current.

The reactive power transferred to the utility grid, at the PCC, and the reactive power generated by the four DERs are also provided in Figure 4.18 (b), (d), and (f). As shown, the reactive power is increased to respond to the increase in loads power demand, Figure 4.18 (c) and (e).

Figure 4.18 (h) to (l) shows the instantaneous three-phase voltage measured at the PCC and PC to PC. Also the instantaneous three-phase output current of the microgrid and the four DERs are shown in plots (m) to (q). As depicted, the output current of the DERs are increased in response to this step change increase in the voltage setpoint.

4.7.2.2 Case 2: Grid Voltage Disturbance

In this section, robustness of the developed control structure against a 10% voltage sag at the utility grid is studied. At \( t=0.35 \) s, voltage of the utility grid source drops to 0.9 pu permanently. This voltage sag is kept permanent to test the performance of the VPP-PV control structure in maintaining the PCC’s voltage at 1 pu.

Figure 4.19 (g) and (h) demonstrate that while the voltage at the grid source is dropped to 0.9 pu, the devised control structure regulates the voltage at the PCC at 1 pu within 55 ms (with 1% criterion). Figure 4.19 (a) to (f) show the transients in the output
real and reactive power transferred to the utility grid through PCC and generated by the four DERs. As depicted, while the real power at the PCC follows the pre-determined reference command, the reactive power is increased in response to the dropped voltage. Figure 4.19 (i) to (l) and (n) to (q) depict the instantaneous three-phase output voltage and current of the four DERs.

4.7.2.3 Case 3: Load change

To investigate the robustness of the developed hierarchical control structure to the local uncertainties in a microgrid, system response to the sudden connection of Load1 is simulated in this section. At t=0.35 s, Load1 disconnects from the grid, causing a decrease of 0.53 pu in the power demand, and then connects back after 3 cycles at t=0.4 s. Simulation studies are reported in Section F.2, Appendix F which indicates that despite this severe load change, the real power transferred to the utility grid and the voltage at the PCC are regulated to their specified values within 83 ms and 40 ms, respectively.
Figure 4.20: System response to the topological change in the microgrid: (a) to (j) real and reactive power transferred to the utility grid and generated by DER$_1$ to DER$_4$.

4.7.2.4 Case 4: Change in the Microgrid Configuration

The change in the microgrid configuration scenario studied in this section is the same as the one described in Section 4.7.1.3. Figure 4.20 depicts the real and reactive power provided by the microgrid and the four DERs. The $d$ and $q$ components of the output current and voltage of the four DERs are provided in Figure 4.21. Also, the instantaneous three-phase output voltage, output current, and terminal current of the four DERs are shown in Figure 4.22. Also, the voltage at the main buses (plots (a) to (f)), currents going through the main feeders and the transfer feeder $C_5$ (plots (g) to (l)), and the loads currents (plots (m) to (r)) are provided in Figure 4.23. Within 86 ms and 93 ms after transfer switch closes, PCC’s voltage and the output real power of the microgrid follow their reference commands with zero tracking error, respectively. Also, since the transients in the waveforms studied in this section are similar to their counterparts that are studied in Section 4.7.1.3, no further explanation is provided in this section.

4.7.2.5 Case 5: Nonlinear Load Energization

The scenario studied in this section is similar to the one described in Section 4.7.1.4. Simulation results are reported in Section F.2, Appendix F. As provided, the specified real power and voltage setpoints are followed within 310 and 250 ms, respectively, despite
the severe transients caused by the nonlinearity of the motor energization process. The successful implementation of the generalized SPWM along with the dual internal current controllers to balance the output current of the DERs is also illustrated in this case study.
Figure 4.23: System response to the topological change in the microgrid: (a) and (b) magnitude and three-phase instantaneous voltage at PCC, (c) to (f) voltage at Bus1 to Bus4, (g) to (l) current going through feeders TL1, F1 to F4, and C5, (m) to (r) load currents IL1 to IL7.

4.7.2.6 Case 6: Three-Phase L-L-L-G Fault at the PCC Bus

This cases study is similar to the scenario described in Section 4.7.1.5. Figure 4.24 (a) and (h) depicts the real power provided by the microgrid and the instantaneous voltage magnitude at the PCC. As shown, during the fault, the microgrid is unable to provide the required real power at the PCC or to regulate the voltage at the PCC, as the PCC’s voltage drops to 0.01 pu. Within 150 ms after fault clears, the real power and voltage at the PCC follow their setpoints.

Depicted in Figure 4.24 (c) to (f), such a severe fault pushes the output of the secondary controller to saturation. However, the devised anti-windup compensator enables the control structure to recover from this saturation only 25 ms after fault clears. Also, depicted in plots (p) to (s), the overcurrent limiter, embedded inside the DER unit, has successfully limited the output current of the DERs to 1.2 pu. Therefore, fault current is mainly provided by the utility grid, as shown in plots (o) and (t).

The $d$ and $q$ components of the output current and output voltage of the four DERs
are shown in Figure 4.25

4.7.2.7 Case 7: Single-Phase L-G Fault at the PCC Bus

This cases study is similar to the scenario described in Section 4.7.1.6. Figure 4.26 shows the performance of the proposed hierarchical control structure when facing unbalanced voltage disturbances. As stated earlier, DERs are equipped with generalized SPWM and dual internal current controllers to provide balanced output current for the DERs. Provided in Figure 4.26 (p) to (s), this devised control structure is able to balance the output current of the DERs despite considerable unbalanced output voltage (plots (j) to (m)). Limited amount of power capacity, though, has prevented the control structure to fully over-ride this severe unbalanced voltage disturbance. Saturated output currents of the DERs to 1.2 pu are also illustrated in Figure 4.26 (p) to (s).

However, the presence of the negative-sequence component in the unbalanced voltage at the PCC and PCs of the DERs results in double frequency oscillations in the output power, generated by the DERs and transferred to the main utility grid, as illustrated in Figure 4.26 (a) to (f). Within 107 ms after fault clears, the real power and voltage setpoints at the PCC are met. Figure 4.26 (o) and (t) depict that the fault current with
Figure 4.25: System response to an L-L-G fault at the PCC: (a) to (h) $d$ and $q$ components of the output currents of the four DERs and their references, (i) to (l) $d$ and $q$ components of the output voltage of the four DERs.

Figure 4.26: System response to an L-G fault at the PCC: (a) to (f) output real and reactive power at the PCC and generated by the four DERs, (h) and (i) the instantaneous magnitude and three-phase voltage at the PCC, (j) to (n) the instantaneous three-phase voltage at the PC1 to PC4 and Bus, (o) to (s) the instantaneous three-phase output current of the microgrid and the four DERs, (t) the fault current.

an amplitude of 15.75 p.u. is mainly provided by the utility grid. Also, the $d$ and $q$ components of the output current and output voltage of the four DERs are shown in Figure 4.27.
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Figure 4.27: System response to an L-G fault at the PCC: (a) to (h) d and q components of the output currents of the four DERs and their references, (i) to (l) d and q components of the output voltage of the four DERs.

4.8 Conclusion

A robust $H_\infty$ multivariable PI-based secondary control augmented with static anti-windup compensator is developed in this chapter, to operate microgrids in the VPP-PQ and VPP-PV modes. An LMI-based technique is proposed to design this secondary controller which guarantees the stability of the overall closed-loop microgrid with an $H_\infty$ norm bound on the transfer function from disturbance input to controlled output. To limit the DERs power reference commands to their generation capacity, a secondary limiter block is added at the output of the PI-based controller. Furthermore, an LMI-based technique is introduced to synthesize a multivariable static anti-windup (AW) compensator. The proposed AW compensator: (1) guarantees the stability of the constrained system, i.e., the system faced to input saturation, and (2) demonstrates graceful performance degradation, i.e., achieving a specified $H_\infty$ norm bound on the transfer function from saturation error signal to the control effort.

After solving the proposed LMIs, two $H_\infty$ multivariable PI controllers and two multivariable AW compensators are synthesized to operate the study microgrid in VPP-PQ and VPP-PV modes. A comprehensive set of modal and frequency response analyses are performed in MATLAB environment to investigate the dynamic behavior of the overall closed-loop study microgrid. Eigenvalue analysis reveals that the most dominant modes of the overall closed-loop system are mainly affected by the states of the secondary controller. Therefore, the robustness margins would not be affected by the local and
interconnection uncertainties and disturbances. Sensitivity analysis once more confirms that the controlled system remains stable for a wide range of load and controller gain parametric variations. In addition, singular value and bode plots illustrate the viability of the designed control structure. For example, the loop transfer functions from $\Delta P_{pcc,ref}$ to $\Delta P_{pcc}$ shows a gain margin of 79 dB and a phase margin of 52.1 degree.

Exhaustive digital time-domain studies are also performed in PSCAD/EMTDC platform which confirm that the developed robust hierarchical control structure maintain the stability of the overall microgrid during transients and ride the microgrid through severe perturbations caused by operating point changes, parametric and topological changes, and severe disturbances such as line-to-ground faults and nonlinear load energization.

The study results in Section 4.7.1 shows that the developed control structure in the VPP-PQ mode provides a settling time of 29.2 ms and 19.8 ms (with 1% criterion) with 3.75 % and 0.10 % overshoot and zero steady-state tracking error, for tracking the real and reactive power setpoints, respectively. Also, these results confirm that in the case of severe disturbances such as faults which significantly push the secondary controller into nonlinear regions (saturation), the devised AW compensator still maintain the stability of the system and recover from the windup condition within 17.2 ms. The hierarchical control structure in the VPP-PV mode shows similar simulation results which confirm the viability of the designed control structure.
Chapter 5

Islanded Mode: Droop-Augmented Robust Voltage Control

5.1 Introduction

This chapter presents a decentralized droop-augmented robust voltage control structure to operate multi-DER microgrids in the islanded mode. Not being connected to a utility grid, voltage and frequency of the islanded microgrid should be controlled within permissible limits that are defined by standards such as IEEE 1547 [7]. Furthermore, dynamic power balance in the microgrid should be provided by sharing the real and reactive power demand among the DERs. To achieve these control and regulatory requirements, each DER unit is equipped with: a static droop-based power sharing controller, a robust voltage controller, and an open-loop frequency controller.

The droop-based power sharing controller is composed of a real power versus phase angle (P-δ) droop controller and a reactive power versus voltage magnitude (Q-V) droop controller. The phase angle and magnitude of the output voltage of the DER units are controlled to share the real and reactive power demand among the DER units, in proportion to their power ratings. Conventionally, the real power sharing is realized by drooping the frequency versus the output real power. The inherent trade-off between the real power sharing and frequency accuracy results in deviations in the steady-state frequency from its nominal value, as the operating point changes [60, 61]. Devising frequency control schemes for maintaining the frequency of an islanded microgrid within permissible limits while sharing the required real power among multiple DER units in the microgrid is still an ongoing subject of research [62, 66]. In recent years, however, the availability of GPS-disciplined oscillators [67, 68] has made accurate open-loop control of...
frequency a possibility [41]. Therefore, this thesis employs a P-δ droop controller and an open-loop frequency control scheme to: (1) maintain microgrid frequency at the desired value without the need for frequency restoration schemes and (2) ensure the real power balance within the microgrid.

The outputs of the droop-based power sharing controller are the magnitude and the phase angle set points for the output voltage of the DER units. These setpoints are then sent to the robust voltage controllers which were designed in Chapter 3, based on the proposed IQC-based decentralized output-tracking robust control methodology. The devised robust voltage controllers track voltage magnitude and phase angle set points with zero steady-state error and maintain the stability of the islanded microgrid during transients despite disturbances, parametric and topological uncertainties, and high-frequency unmodeled dynamics. The output of the voltage controller is the d and q components of the reference current for the built-in current controllers embedded in DER units. Similar to the grid-connected mode discussed in Chapter 4, the DER units are interfaced to the grid as current-controlled VSCs, for overcurrent protection.

The open-loop frequency controller features an independent internal oscillator synchronized by a common time-reference signal received from a GPS [41]. By employing this controller, microgrid frequency is fixed and cannot deviate during transients.

Finally, the efficacy of the developed droop-augmented robust voltage control structure in operating the multi-DER microgrid system of Figure 2.1 in the islanded mode is shown through a comprehensive set of frequency-domain and time-domain simulation studies, performed in MATLAB and PSCAD/EMTDC platforms, respectively. This set is composed of: (1) eigenvalue and participation factor analysis, (2) frequency response analysis, (3) sudden load change, (4) change in the configuration of the microgrid, (5) sudden disconnection and re-connection of a DER unit, (6) nonlinear load energization, (7) three-phase L-L-L-G fault at the PCC, and (8) single-phase L-G fault at the PCC.

5.2 Control Structure

Figure 5.1 depicts the control structure, employed for each DER unit, to operate the microgrid in the islanded mode. As depicted, this structure is composed of three parts.

1) The power sharing controller determines the magnitude and phase angle setpoints for the output voltage of the DERs, according to the real power-phase angle (P-δ) and the reactive power-voltage magnitude (Q-V) droop characteristics. As shown in Figure 5.1, the inputs to the n\textsuperscript{th} power sharing controller are the instantaneous real and reactive output power of the n\textsuperscript{th} DER, calculated from the output voltage and current components.
measured at the PC of the \( n \)th DER unit. These instantaneous power components are then passed through low-pass filters (LPFs) to attenuate high-frequency distortion components in the measured instantaneous components \[42\]. The transfer function of these LPFs where \( \omega_{c,n} \) is the cut-off frequency of the filter is

\[
P_{f,n} = \frac{\omega_{c,n} P_n}{s + \omega_{c,n}}, \quad Q_{f,n} = \frac{\omega_{c,n} Q_n}{s + \omega_{c,n}},
\]

(5.1)

The filtered powers are then fed to the following droop characteristics to share the required real and reactive power among DERs:

\[
\delta_{\text{ref},n} = \delta_n^* - D_{P,n} P_{f,n}, \quad \hat{V}_{\text{ref},n} = V_n^* - D_{Q,n} Q_{f,n},
\]

(5.2)

in which \( \delta_{\text{ref},n} \) and \( \hat{V}_{\text{ref},n} \) are the set points for the phase angle and magnitude of the \( n \)th DER’s output voltage, measured at PC\(_n\). The static droop gains are shown by \( D_{P,n} \) and \( D_{Q,n} \). Also, \( \delta_n^* \) and \( V_n^* \) are the nominal phase angle and magnitude of the voltage at PC\(_n\), respectively, determined by performing a power flow analysis at the steady-state operating condition of the microgrid.

For multi-DER microgrids, the droop gains are calculated based on the permitted range of variations in the magnitudes and phase angles of output voltages of the DERs and should satisfy the following conditions:

\[
P_n D_{P,n} = P_m D_{P,m} = \Delta \delta_{\text{max}}, \quad Q_n D_{Q,n} = Q_m D_{Q,m} = \Delta V_{\text{max}},
\]

(5.3)

for \( n = 1, ..., N \), \( m = 1, ..., N \), and \( n \neq m \), where \( N \) is the number of DERs inside the microgrid. \( P_n \) and \( Q_n \) are the rated real and reactive power capacity of the \( n \)th DER. \( \Delta \delta_{\text{max}} \) and \( \Delta V_{\text{max}} \) are the maximum allowable phase angle and magnitude deviations in the output voltage, respectively. Assuming that the phase angle at the PCC is \( \delta_{\text{pcc}} = 0 \), maximum allowable phase angle deviations in the DER’s output voltage is chosen as \( \Delta \delta_{\text{max}} = \pi/30 - (-\pi)/30 = 2\pi/30 \) to allow \( \sin(\delta) \approx \delta \) and \( \cos(\delta) \approx 1 \). Therefore, \( P - \delta \) and \( Q - V \) droop characteristics can be applied to determine the voltage reference of the DERs. Also, (5.3) ensures that the load demand is shared among the DERs in proportion to their power ratings.
As depicted in Figure 5.1, the output of this droop-based power sharing control determines the set points for the $d$ and $q$ components of the output voltage of the $n^{th}$ DER, based on

$$V_{dref,n} = \hat{V}_{ref,n} \cos(\delta_{ref,n}),$$
$$V_{qref,n} = \hat{V}_{ref,n} \sin(\delta_{ref,n}).$$

Signals $V_{dref,n}$ and $V_{qref,n}$ and the phase angle setpoint $\delta_{ref,n}$ constitute the outputs of the power sharing controller.

2) The voltage controller is the robust IQC-based dynamic output-tracking controller designed in Section 2.7.2, Chapter 3 to follow reference commands despite uncertainties and disturbances. To provide the specified $V_{dref,n}$ and $V_{qref,n}$, the voltage controller determines reference commands for the positive-sequence $d$ and $q$ components of the output current of the DER units, i.e., $i^{+}_{refd,n}$ and $i^{+}_{refq,n}$. These two reference commands will then be sent to the dual-sequence built-in current controller, as explained in Section 2.3.1, Chapter 2.

3) The frequency controller includes an oscillator which generates a sawtooth waveform as $\theta(t) = \int_{0}^{t} \omega_0^* d\tau$, where $\omega_0^* = 2\pi f_0^*$ is the angular frequency fixed at 377 rad/s to operate the system at the frequency of 60 Hz. The output of the frequency controller is $\rho_n = \theta(t)$ which is the reference frequency for the $n^{th}$ DER. As explained in Section 2.3.1, Chapter 2, $\rho_n$ will be used for $abc \rightarrow dq$ and $dq \rightarrow abc$ transformation. All the DER units are synchronized by a global synchronization signal communicated to the oscillators inside the frequency controllers through a GPS, \[41\].

5.3 Control Parameters

Detailed procedure on designing the robust IQC-based primary voltage controllers is provided in Section 2.7.2, Chapter 2. Matrices of the four primary voltage controllers are reported in Section D.3, Appendix D.

For designing the Q-V droop gain, i.e. $D_{Q,n}$, the allowable deviation in the output voltage is assumed to be within $\pm 5\%$. Therefore, based on (5.3), $D_{Q,n} = 0.02$ kV/MVAr, for DER$_1$ to DER$_3$, and $D_{Q,4} = 0.12$ kV/MVAr for DER$_4$. Regarding the $P - \delta$ droop gain, i.e. $D_{P,n}$, the allowable deviation in the phase angle of the output voltage is assumed to be within $\pm \pi/30$ rad. Therefore, based on (5.3), $D_{P,n} = 0.07$ rad/MW, for DER$_1$ to DER$_3$, and $D_{P,4} = 0.42$ rad/MW for DER$_4$. 
Figure 5.1: Droop-augmented robust voltage control structure for the $n^\text{th}$ DER in the islanded mode.

Power flow analysis determines the nominal voltage magnitudes and phase angles in (5.2) for PC$_1$ to PC$_4$ as $0.985 \angle 0.025$, $0.990 \angle 0.008$, $1.005 \angle 0.032$, and $0.990 \angle 0.012$, in pu and rad. The cut-off frequency for the filters of (5.1) is chosen as $\omega_{c,n} = 33.3$ rad/sec, for DER$_1$ to DER$_4$, to avoid altering controller dynamic properties while ensuring adequate filtering [69].

5.4 Eigenvalue Analysis

The eigenvalues of the islanded open-loop microgrid of Figure 2.1 without built-in current, primary voltage, and power sharing controllers in the loop, are shown in Figure 5.2 which depicts the nominal stability of the open-loop system. Dominant eigenvalues are located between $-14$ to $-10$ (1/s). These dominant modes, the states contributing in
each mode, and their normalized participation factors are provided in Table 5.1. Similar to the grid-connected case, reported in Table 4.1 of Chapter 4, the dominant eigenvalues of the islanded open-loop microgrid are affected by the output currents of the DER units.

![Figure 5.2: Eigenvalues of the islanded open-loop microgrid](image)

Table 5.1: Dominant eigenvalues of the islanded open-loop microgrid

<table>
<thead>
<tr>
<th>Mode</th>
<th>Real (1/s)</th>
<th>Imagin. (rad/s)</th>
<th>Participating States</th>
<th>Normalized Part. Factors</th>
</tr>
</thead>
<tbody>
<tr>
<td>41,42</td>
<td>-13.1777 ± 376.9991</td>
<td>$i_{DR1,d}^g$, $i_{DR1,q}^g$, $i_{DR4,d}^g$, $i_{DR4,q}^g$</td>
<td>0.1810, 0.1810, 0.2867, 0.2867</td>
<td></td>
</tr>
<tr>
<td>43,44</td>
<td>-10.8561 ± 376.9911</td>
<td>$i_{DR1,d}^g$, $i_{DR1,q}^g$, $i_{DR2,d}^g$, $i_{DR2,q}^g$</td>
<td>0.1317, 0.1317, 0.3316, 0.3316</td>
<td></td>
</tr>
</tbody>
</table>

Augmenting the integrated state-space representation of the islanded microgrid at the built-in current controller level in (2.3), the four primary voltage controllers described by (2.8) and (2.9), and the linearized equivalent of the four droop-based power sharing controllers described by (5.1), (5.2), and (5.4) based on their input-output relationship results in the overall closed-loop system with $x_{closed} \in \mathbb{R}^{96}$. Eigenvalues of the overall closed-loop islanded microgrid are shown in Figure 5.3.
Depicted in Figure 5.3 (f), dominant eigenvalues are located between $-7$ to $-2$ ($1/s$). Participation factors show that these lowest frequency modes are mainly affected by the state variables of the four droop-based power sharing controllers. This reveals that, as oppose to the open-loop system, the dominant modes are not sensitive to the output currents of DERs. Therefore, stability margins of the closed-loop system will not be affected by the disturbances or operating point changes which cause variations in the output current of the DER units. These dominant eigenvalues, the states contributing in these modes, and their normalized participation factors are provided in Table 5.2. In Table 5.2, $x_{drp} \in \mathbb{R}^8$ corresponds to the state variables of the four droop-based power sharing controllers, described by (5.1), (5.2), and the linearized form of (5.4).

![Figure 5.3: Overall closed-loop system eigenvalues](image)

Participation factors associated with the modes of the closed-loop islanded microgrid, although not reported in this chapter, show that:

- The low-frequency modes of the closed-loop islanded microgrid, i.e., windows (f), (e), and (d) in Figure 5.3, are mainly affected by the state variables of the droop-based power sharing controller. This is due to the low-frequency power filters of (5.1) that are designed to extract the fundamental components of the measured instantaneous real and reactive output power of the DER units.

- The medium-frequency modes of the closed-loop islanded microgrid, i.e., windows (b) and (c) in Figure 5.3, are dominantly influenced by the state variables of the ro-
Table 5.2: Dominant eigenvalues of the overall closed-loop study system

<table>
<thead>
<tr>
<th>Mode</th>
<th>Real (1/s)</th>
<th>Imagin. (rad/s)</th>
<th>Participating States</th>
<th>Normalized Part. Factors</th>
</tr>
</thead>
<tbody>
<tr>
<td>91,92</td>
<td>-6.2552</td>
<td>± -13.9705</td>
<td>$x_{drp,2}$</td>
<td>0.1134</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$x_{drp,3}$</td>
<td>0.3756</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$x_{drp,4}$</td>
<td>0.4817</td>
</tr>
<tr>
<td>93,94</td>
<td>-5.5408</td>
<td>± 19.7620</td>
<td>$x_{drp,1}$</td>
<td>0.1142</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$x_{drp,2}$</td>
<td>0.1051</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$x_{drp,3}$</td>
<td>0.1154</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$x_{drp,4}$</td>
<td>0.1306</td>
</tr>
<tr>
<td>95,96</td>
<td>-2.7348</td>
<td>± 5.5871</td>
<td>$x_{drp,1}$</td>
<td>0.1789</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$x_{drp,2}$</td>
<td>0.1850</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$x_{drp,3}$</td>
<td>0.2468</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$x_{drp,4}$</td>
<td>0.2531</td>
</tr>
</tbody>
</table>

bust primary voltage controllers, power network, and the built-in current controller as well as the output current of DER units.

- The high-frequency modes of window (a) in Figure 5.3 are dominantly affected by a combination of PCC’s voltage and load currents. The state variables of the four robust primary voltage controllers also affect these high-frequency modes.

All of the above observations suggest that the developed control structure is a viable solution for operating microgrids in the islanded mode.

5.5 Frequency Response Analysis

Figure 5.4 (a) shows the singular value plot of the output loop transfer function representation of the islanded microgrid from the input of $V_1^*$ to the output of $V_{DRd,1}$. The output loop transfer function of the controlled islanded microgrid of Figure 2.1 is defined as $L_o(s) = G(s)K_{prm}(s)K_{drp}(s)$, in which $G(s)$ is the transfer function representation of the islanded microgrid controlled at the built-in current controller (2.3) of Chapter 2, $K_{prm}$ is the transfer function representation of the primary voltage controllers described by (2.8) and (2.9) of Chapter 2, and $K_{drp}$ is the transfer function representation of the droop-based power sharing controllers described by the linearized equivalent of (5.1), (5.2), and (5.4).

Provided in Figure 5.4 (a), the designed control structure has achieved an acceptable reference following and disturbance rejection since $\sigma_{min}(\omega) >> 1$ at low frequencies.
Figure 5.4: The overall islanded microgrid: (a) singular value plot of the overall loop transfer function, and (b) bode plot of the loop transfer function

Also, having small gain at high frequencies, i.e. $\sigma_{\text{max}}(\omega) << 1$, guarantees the controlled islanded microgrid is insensitive to the measurement noise.

Also, bode plots of the loop transfer function $L_o$ from $V_1^*$ to $V_{DRd,1}$ is provided in Figure 5.4 (b). The phase margin and its crossover frequencies are obtained as PM=160 $^\circ$, $\omega_{pm} = 295$ rad/s, respectively. Also, the gain margin is infinite. The obtained gain and phase margins highlights the achieved robustness margins.

5.6 Performance Evaluation

A series of time-domain case studies are performed in this section to evaluate the performance of the the proposed control structure in operating the multi-DER study microgrid of Figure 2.1 in the islanded mode (ISM). In these studies, the microgrid is working under full load condition provided in Table E.1, Appendix E. Measured quantities are presented in per-unit (pu), wherever applicable. The base values for this per-unitization are provided in Table 3.1, Chapter 3.

5.6.1 Case 1: Load change

In this case study, system response to the sudden energization of Load$_1$ is studied. Load$_1$ is seen as the parallel combination of $L_1$ and $L_2$ in Figure 2.1. Prior to t=0.2 s, Load$_1$ branch is open. At t=0.2 s, the load connects to the system corresponding to an increase in the power demand from zero to 0.51 p.u ($S_{\text{base}} = 3$ MVA).
The real and reactive powers consumed by Load_1 are illustrated in Figure 5.5 (a) and (b). Transients in the instantaneous voltage measured at the four PCs and PCC are shown in Figure 5.5 (c), (g), and (k). As depicted, within 32 ms, the devised control structure regulates voltage at the PC_1 to PC_4 to 0.98, 0.99, 1.01, and 0.98 pu, respectively. Subsequently, PCC’s voltage is indirectly regulated at 0.99 pu.

The d and q components of the output current and output voltage of the four DER units as well as the instantaneous three-phase values of the output voltage, output current, and terminal current of them are shown in Figure 5.6. As depicted, the output current of the four DER units are increased to respond to the increased load demand.

The instantaneous three-phase voltage measured at the main buses of the microgrid along with the currents going through the main feeder units are depicted in Figure 5.7 (a) to (i). Figure 5.7 (f) depicts that the connection of Load_1 decreases the current which is going out to the microgrid through line F_1. It means that the current generated by DER_1 is mainly consumed by Load_1. The increase in the Load_1 current is shown in Figure 5.7 (j). Transients in the currents consumed by the other loads are also provided in plots (k) to (o).

This case study depicts: (i) the robust performance of the proposed control structure in maintaining the stability of the islanded microgrid during transients caused by this severe parametric perturbation, and (ii) the fast response of the devised control structure in retrieving the integrity of the microgrid.
Figure 5.6: Load change in the ISM: (a) to (t) \( d \) and \( q \) components and three-phase instantaneous output current, terminal current, and output voltage of DER\(_1\) to DER\(_4\).

Figure 5.7: Load change in the ISM: (a) to (f) voltage at PCC and Bus\(_1\) to Bus\(_4\), (f) to (i) current going through Bus\(_1\) to Bus\(_4\), (j) to (o) load currents \( I_{L1} \) to \( I_{L7} \).

### 5.6.2 Case 2: Change in Microgrid Configuration

In this section, topological changes in the microgrid is studied. At \( t = 0.2 \) s., the breaker on Feeder F3 in Figure 2.1 at the PCC side, opens. This event isolates DER\(_2\) and DER\(_3\) from the microgrid feeding loads \( L_5 \) and \( L_6 \). However, to utilize the power generated by
the two DERs, the transfer switch closes after three cycles at \( t = 0.25 \) s, connecting Bus\(_5\) to Bus\(_6\).

The instantaneous voltage magnitude measured at the PCC and the four PCs are shown in Figure 5.8 (a) to (d). As depicted, by the disconnection of DER\(_2\) and DER\(_3\), voltage at the PCC, PC\(_1\), and PC\(_4\) instantly drops to the vicinity of 0.6 pu and brought back up to the vicinity of 0.8 pu by the devised control structure within 50 ms. Also, voltage at the PC\(_2\), and PC\(_3\) increases to 1.2 pu. Within 155 ms after transfer switch closes, the devised control structure regulates voltage at the PC\(_1\) to PC\(_4\) to 1.03, 1.00, 0.98, and 1.01 pu, respectively. Subsequently, PCC’s voltage is indirectly regulated at 1.02 pu. Transients in the output real and reactive power generated by the four DER units are also depicted in Figure 5.8 (e) to (l).

Figure 5.9 (a) to (t) shows the \( d \) and \( q \) components of the output current and voltage of the four DER units as well as their instantaneous magnitude waveform. Magnitude waveform are plotted here rather than their three-phase counterparts solely for the purpose of the quality of the figures depicting the waveform over a longer period of time.

The instantaneous voltage magnitude measured at the main buses of the microgrid along with the currents going through the main feeder units are depicted in Figure 5.10 (a) to (i). As depicted in Figure 5.10 (h), the opening of breaker on Feeder F3 causes the current going through this feeder to drop to zero. Transients in the currents consumed by loads are also provided in Figure 5.10 (j) to (o).

This case study clearly demonstrates the effectiveness of the proposed control structure to major changes in the operating point (disconnection of loads and generation units) as well as topology of the dynamical system (re-connection of loads and DER units to a different bus).

5.6.3 Case 3: Sudden Disconnection and Re-connection of DER units

This section evaluates the performance of the developed control structure in operating the islanded microgrid facing accidental disconnection and then re-connection of DER units. At \( t=0.2\) s, DER\(_1\) disconnects from the microgrid, after filter capacitor, and then connects back at \( t=0.25\) s.

Transients in the instantaneous voltage magnitude and the output real and reactive power measured at the PCC and the four PCs are provided in Figure 5.11 (a) to (l). Prior to the outage, DER\(_1\) provides the microgrid with 1.65 MW and absorbs 0.3 MVAr from the microgrid. As shown in plots (a) to (d), at the instant of disconnection of DER\(_1\),
Figure 5.8: Topological changes in the ISM: (a) to (d) the instantaneous voltage magnitude measured at the PCC and the four PCs, (e) to (l) output real and reactive power generated by the four DER units.

Figure 5.9: Topological changes in the ISM: (a) to (t) d and q components and three-phase instantaneous output current, and terminal current of DER1 to DER4.

Voltage at PC2, PC3, PC4, and consequently at the PCC drops to the vicinity of 0.75 pu and brought back to the vicinity of 0.98 pu by the devised control structure within 40 ms. Voltage at the PC1, on the other hand, increases to 1.3 pu. Operating in an open circuit condition, the devised control structure in DER1 fails to control the voltage
Figure 5.10: Topological changes in the ISM: the magnitude waveform of the instantaneous (a) to (f) voltage at PCC and Bus 1 to Bus 4, (f) to (i) current going through Bus 1 to Bus 4, (j) to (o) load currents $I_{L1}$ to $I_{L7}$.

within the permissible range.

After 152 ms that DER$_1$ re-connects to the microgrid, the developed control structure regulate the voltage at PC$_1$ to PC$_4$ at 0.99, 1.02, 1.03, and 1.00 pu, respectively. Therefore, voltage at the PCC is indirectly regulated at 1.01 pu.

Figure 5.12 (a) to (t) shows the $d$ and $q$ components of the output current and voltage of the four DER units as well as their instantaneous magnitude waveform. Instantaneous magnitude waveform are plotted here rather than their three-phase counterparts solely for the purpose of the quality of the figures depicting the waveform over a longer period of time. Also, to provide an insight in the dynamic behaviour of the microgrid, instantaneous voltage magnitude measured at the main buses of the microgrid along with the currents going through the main feeder units are depicted in Figure 5.13 (a) to (i). Transients in the currents consumed by loads are also provided in Figure 5.13 (j) to (o).

5.6.4 Case 4: Nonlinear Load Energization

To investigate the performance of the developed control structure when the islanded microgrid is subjected to a nonlinear load, the case of induction machine energization is studied in this section. The motor is rated at 500 hp, 2.3 kV, and connected to Bus 5 of Figure 2.1 through a 13.8 kV/2.3 kV, $Yg/\Delta$ transformer. The induction motor
parameters are provided in [41]. At t=0.2 s, motor energizes from the stall under full load condition.

Figure 5.12 (a) to (c) shows the real power, reactive power, and the inrush current
Figure 5.13: Sudden outage and re-connection of DER1 in the ISM: the magnitude waveform of the instantaneous (a) to (f) voltage at PCC and Bus1 to Bus4, (f) to (i) current going through Bus1 to Bus4, (j) to (o) load currents $I_{L1}$ to $I_{L7}$.

consumed by the induction motor. The per-unit values of the electric torque, mechanical torque, and mechanical speed of the motor are demonstrated in plots (d) to (f), respectively. The per-unitization is done based on the motor ratings.

Figure 5.15 (a) to (d) show the transients in the instantaneous voltage magnitude. As shown, despite the severe disturbance due to the motor energization, the devised control structure maintains the voltage deviation within a negligible range until the motor reaches its steady-state. At $t=1.52$ s, when the steady-state operating conditions is achieved, voltage at PC1 to PC4 are regulated at 0.98, 1.02, 0.99, and 0.985 pu, respectively. Therefore, voltage at the PCC is indirectly regulated at 0.984 pu. Real and reactive powers generated by the four DER units are also shown in plots (e) to (l).

Figure 5.16 (a) to (p) shows the $d$ and $q$ components of the output current and output voltage of the four DER units as well as the instantaneous magnitude waveform of the output voltage, output current, and terminal current of them. Instantaneous magnitude waveform are plotted here rather than their three-phase counterparts solely for the purpose of the quality of the figures depicting the waveform over a longer period of time.
5.6.5 Case 5: Three-Phase L-L-L-G Fault at the PCC

This case study demonstrates the performance of the developed control structure in operating an islanded microgrid when a three-phase line-to-ground fault happens. At t=0.2 s, a three-phase line-to-ground (L-L-L-G) fault on the PCC of Figure 2.1 happens and lasts for 5 cycles. The fault resistance is selected as 0.2 ohms (0.01 pu). The operating point prior to fault is as described in Section 5.6.

As shown in Figure 5.17 (a) to (d), by the occurrence of the L-L-L-G fault, voltage at the PCC and the four PCs drops to the vicinity of 0.01 pu. Within 162 ms after fault clears out, the developed control structure regulate the voltage at PC1 to PC4 at 0.975,
Figure 5.16: Induction motor energization: (a) to (t) \(d\) and \(q\) components and three-phase instantaneous output current and output voltage of DER\(_1\) to DER\(_4\).

0.985, 0.992, and 0.985 pu, respectively. Therefore, voltage at the PCC is indirectly regulated at 0.982 pu. Transients in the output real and reactive power generated by the four DER units are also shown in Figure 5.17 (e) to (l).

As depicted in Figure 5.18, to compensate for the significant voltage tracking error, the primary voltage controllers require DER units to generate large values of output current. However, the built-in limiter embedded in the DER units limits the output current to 1.2 pu. Also, as shown in Figure 5.19 (f), the output currents of the DER units feed the fault current of 2.5 pu. Transients in the instantaneous three-phase voltage measured at the main buses of the microgrid, currents going through the main feeder units, and load currents are depicted in Figure 5.19.

5.6.6 Case 6: Single-Phase L-G Fault at the PCC

To evaluate the performance of the developed control structure when a severe unbalanced voltage disturbance, as a single-phase line-to-ground fault, happens at the PCC of the islanded microgrid. At \(t=0.2\) s, a three-phase line-to-ground (L-G) fault on the PCC of Figure 2.1 happens and lasts for 5 cycles. The fault resistance is selected as 0.2 ohms (0.01 pu). The operating point prior to fault is as described in Section 5.6.

Figs. 5.20, 5.21, and 5.22 show that the developed control structure enables the microgrid to ride through such severe unbalanced L-G fault. Depicted in Figure 5.20 (a)
Figure 5.17: L-L-L-G fault at the PCC in the ISM: (a) to (d) the instantaneous voltage magnitude measured at the PCC and the four PCs, (e) to (l) output real and reactive power generated by the four DER units.

Figure 5.18: L-L-L-G fault at the PCC in the ISM: (a) to (t) \(d\) and \(q\) components and three-phase instantaneous output current, terminal current, and output voltage of DER\(_1\) to DER\(_4\).
Figure 5.19: L-L-L-G fault at the PCC in the ISM: the instantaneous three-phase waveform of (a) to (e) voltage at PCC and Bus1 to Bus4, (f) fault current, (g) to (j) current going through Bus1 to Bus4, (k) to (o) load currents.

to (e), by the occurrence of the line-to-ground fault, voltage at phase-a of the PCC and subsequently at the buses 1 to 4 drops to the vicinity of 0.005 pu. Also as shown in Figure 5.20 (f), since fault current is only provided by the DER units, it remains limited to 5 pu. Transients in the current going through the main feeder units as well as load currents are provided in Figure 5.20 (g) to (o).

The unbalanced L-G fault causes double frequency oscillations in the microgrid voltage and subsequently in the output real and reactive power generated by DER units. This is shown in Figure 5.21. Within 85 ms after fault clears, voltages of PC1 to PC4 are regulated at 0.975, 0.985, 0.968, and 0.96 pu, respectively. Therefore, voltage at the PCC is indirectly regulated at 0.972 pu.

5.7 Conclusions

A decentralized droop-augmented robust voltage control structure is introduced in this chapter to operate multi-DER microgrids in the islanded mode. The proposed control structure is a decentralized structure of:

(1) Droop-based power sharing controllers which feature real power-phase angle (P-δ) and a reactive power-voltage (Q-V) droop controllers to share power demand among the
operational DER units in proportion to their power ratings. The output of the droop-based power sharing controller is voltage magnitude and phase angle set points for the PC's voltage of the DER units.
Figure 5.22: L-G fault at the PCC in the ISM: (a) to (t) $d$ and $q$ components and three-phase instantaneous output voltage, output current, and terminal current of DER$_1$ to DER$_4$.

(2) Robust voltage controllers which are the IQC-based robust primary voltage controller developed in Chapter 3. As stated in Chapter 3, the developed robust voltage controllers guarantee fast response with zero tracking error of the voltage magnitude and phase angle set points during the steady-state operation and maintain the stability of the islanded microgrid during transients despite disturbances and uncertainties. The output of the voltage controller is the $d$ and $q$ components of the reference current for the built-in current controllers embedded in the DER units. Similar to the grid-connected mode discussed in Chapter 4, the DER units are interfaced to the grid as current-controlled VSCs, for overcurrent protection.

(3) Open-loop frequency controllers are devised to maintain systems frequency at 60 Hz. The frequency controller employs an independent internal oscillator synchronized by a common time-reference signal received from a GPS.

The results obtained from the eigenvalue and participation factor analyses, performed in MATLAB environment, reveals that the dominant eigenvalues of the closed-loop system, located between -7 to -2 (1/s), are mainly affected by the state variables of the four droop-based power sharing controllers. This indeed means the insensitivity of the stability margins to the disturbances and operating points changes. Furthermore, the
frequency response analysis illustrates a phase margin of 160 degree and an infinite gain margin. The singular value plot also verifies that the designed control structure is a viable solution in rejecting low frequency disturbances, tracking the specified reference commands, and insensitivity to high-frequency measurement noises and unmodelled dynamics.

Finally, a set of time-domain simulation studies are performed in PSCAD/EMTDC platform to investigate the performance of the controlled microgrid when subjected to severe operating point changes and disturbances such as sudden load variations, major topological changes, accidental disconnection and re-connection of DER units, nonlinear load energization, and three-phase L-L-L-G fault and single-phase L-G fault at the PCC. Simulation results verifies that the proposed robust control structure maintains voltage and frequency stability of the islanded microgrid during transients and achieves dynamic power balance within the microgrid, despite perturbations. The fast response of the devised control structure in retrieving the operational integrity of the microgrid is also illustrated through simulation results. As an example, the scenario studied in Section 5.6.1 depicts that within 32 ms after that a 0.51 pu load connects to the microgrid, voltage at the PCC of the microgrid is regulated at 0.99 pu.
Chapter 6

Transition Mode: The Overall Robust Hierarchical Control Structure

6.1 Introduction

This chapter demonstrates the overall robust hierarchical control structure, that has been developed in this thesis. The control structure enables a multi-DER microgrid to operate in the VPP-PQ mode, VPP-PV mode, and islanded mode of operation. It also ensures a smooth transition among the modes, whether the transfer command is pre-planned or accidental. The coordination of controllers that are engaged in each mode of operation and the switching logic to select between controllers based on the pre- and post-transition mode are also provided in this chapter.

Recalling from the past four chapters, in the steady-state regime, the developed robust hierarchical control structure ensures that: (1) in the VPP-PQ mode, a pre-specified amount of real and reactive power is exchanged with the utility grid, (2) in the VPP-PV mode, a pre-specified amount of real power is exchanged with the utility grid while microgrid voltage magnitude at the PCC is regulated to a desired value, and (3) in the islanded mode, the microgrid voltage and frequency are controlled within permissible limits while the power balance in the microgrid is maintained. These requirements are met despite disturbances, parametric and topological uncertainties, operating point changes, and unmodelled high-frequency dynamics.

During a transition between microgrid modes of operation, the developed robust hierarchical control structure ensures that the stability of the microgrid is maintained
and the transition occurs smoothly. The smoothness is achieved since primary controllers are designed to reject the external disturbance of deviations in the voltage and frequency of the utility grid.

In this chapter, several time-domain scenarios are studies in PSCAD/EMTDC to show the efficacy of the proposed control structure: (i) to meet pre-specified requirements during the steady-state operation of the multi-DER microgrid of Figure 2.1 in the VPP-PQ, VPP-PV, or islanded mode, and (ii) to smoothly ride the study microgrid through mode transitions. The performance of the developed overall control structure is investigated during the following test scenarios:

• **Scenario 1**: a pre-planned islanding command is issued at a specified time instant causing a transition from the VPP-PQ or the VPP-PV mode to the islanded mode of operation.

• **Scenario 2**: a permanent three-phase L-L-L-G fault with a resistance of 0.01 pu occurs on the utility grid bus $Bus_u$ in Figure 2.1 at $t=0.2$ s. The CBs on line $TL_1$ in Figure 2.1 operate after 5 cycles, at $t=0.283$ s., to clear the fault. Islanding is detected within 3.5 cycles at $t=0.341$ s. During a time period of 141 ms, i.e., from $t=0.2$ to $t=0.341$ s, the microgrid is still commanded to operate in the VPP-PQ (VPP-PV) mode while practically it is not hosted by a strong (weak) utility grid.

• **Scenario 3**: a temporary three-phase L-L-L-G fault with a resistance of 0.01 pu occurs on the utility grid bus $Bus_u$ in Figure 2.1 at $t=0.2$ s. After 5 cycles, at $t=0.283$ s., CBs on line $TL_1$ in Figure 2.1 operate to clear the fault. After 3.5 cycles, $t=0.341$ s, islanding is detected and mode transfer switch in the control structure is commanded. Thirty cycles after the fault occurrees, at $t=0.783$ s, the microgrid reconnects to the utility grid while a 21 degree phase shift exists between the voltages of microgrid and the utility grid. No synchronization of phase sequence, frequency, or voltage of the microgrid to the host utility grid is employed. Thus, the microgrid is exposed to a severe disturbance.

• **Scenario 4**: the islanded microgrid reconnects to a strong (weak) utility grid and operates in the VPP-PQ (VPP-PV) mode. At the time of re-connection, there is a 25.5 degree phase shift between the voltages of microgrid and the utility grid. No synchronization of phase sequence, frequency, or voltage of the microgrid to the host utility grid is employed to expose the microgrid to a severe disturbance.
6.2 The Overall Control Structure

Figure 6.1 depicts the overall hierarchical control structure employed in this thesis to operate a multi-DER microgrid. The Mode Transfer (MT) command, shown in Figure 6.1, is the switching logic that selects between controllers based on the pre- and post-transition mode of operation of the microgrid. The MT command takes the value of: (i) “00” to operate the microgrid in the VPP-PQ mode, (ii) “01” to operate the microgrid in the VPP-PV mode, and (iii) “10” to operate the microgrid in the islanded mode.

It should be noted that providing schemes or operators that determine the MT command is out of the scope of this thesis. These schemes and operators can include fast and accurate islanding detection schemes [70], auto-reclosure circuit breakers (CBs) [71], and the distribution system operator. Also, it is worth mentioning that schemes to synchronize phase sequence, frequency, and voltage of the microgrid to that of the host utility grid at the time of reclosure, e.g., the method proposed in [72], is not employed in this work. Although large switching transients in microgrid voltage and current may occur at the time of re-connection of unsynchronized microgrid to the utility grid, the devised robust control structure is designed to attenuate the effect of these disturbances to a pre-determined level.

When the microgrid is connected to a strong or medium-strong utility grid, i.e., operating in the VPP-PQ mode, the microgrid is asked to exchange a specific amount of real and reactive power with the utility grid, shown as $P_{\text{pcc,ref}}$ and $Q_{\text{pcc,ref}}$ in Figure 6.1. Should the utility grid be weak, microgrid is required to regulate voltage magnitude at the PCC to a desired value of $|V_{\text{pcc,ref}}|$ while exchanging a specific amount of real power with the utility grid, i.e., operating in the VPP-PV mode. To meet the specified requirements at the PCC, in either one of the VPP modes, the secondary control, developed in Chapter 4, determines the output real and reactive power setpoints ($P_{\text{ref,n}}$ and $Q_{\text{ref,n}}$) for the local power controllers of DER units. Therefore, in selecting between the VPP-PQ and VPP-PV modes of operation, the switching in the control structure happens at the secondary control level, as shown in Figure 6.1.

In the islanded mode of operation, droop-based power sharing controllers, implemented in each DER unit, determine output voltage magnitude and phase angle setpoints for the robust local voltage controllers. These setpoints are determined to ensure that microgrid voltage remains in permissible limits and to provide dynamic power balance within the microgrid. Also, an open-loop frequency control structure is employed to regulate microgrid frequency. Therefore, when transition between VPP and islanded modes happens, two switches is commanded in the control structure: the first one selects
between local power and local voltage controllers at the primary level, the second one selects the input signal $\rho_n$ which is the reference angle based on which the conversion between $abc$ and $dq$ reference frames is performed. In the VPP mode, $\rho_n$ is the angle of the output voltage of the DER unit, i.e., $V_{DR,n}$ in Figure 6.1 and is estimated by a PLL. In the islanded mode, however, $\rho_n$ is generated by the devised frequency controller.

Both local power and voltage controllers are developed in Chapter 3 based on the proposed IQC-based robust control methodology to ensure the overall stability of the microgrid and to provide fast and accurate tracking of reference commands with zero steady-state error, despite disturbances and uncertainties. These local controllers constitute the primary level of the developed hierarchical control structure. The outputs of the primary control are setpoints for the $d$ and $q$ components of the positive-sequence of the DER’s output current. These current setpoints are then sent to the dual-sequence current controllers built in each DER unit. Therefore, irrespective of the mode of operation of the microgrid, DER units are utilized as CC-VSCs for over-current protection.

### 6.3 Performance Evaluation

This section illustrates the effectiveness of the developed overall control structure in providing: (1) the desired performance during the steady-state operation of microgrids either in the VPP or islanded mode, and (2) a fast and smooth transition between these two modes. Throughout this section, monitored quantities are presented in per-unit (pu), wherever applicable. The base values for this per-unitization are provided in Table 3.1, Chapter 3.

#### 6.3.1 Case 1: Pre-Planned Islanding from the VPP-PQ Mode

Transient behavior of the study microgrid in Figure 2.1 due to a pre-planned islanding scenario is studied in this section. Prior to islanding, the microgrid operates in the VPP-PQ mode and transfers 1.02 MVA to the utility grid at a leading power factor of 0.98 under full load condition, as provided in Table E.1 Appendix E. At $t=0.2$ s, a pre-planned islanding command is issued through opening of circuit breakers at both ends of the 69-kV utility line, causing a switch at $t=0.2$ s in the control structure from the VPP-PQ to the islanded mode.

System transients due to this islanding process are shown in Figs. 6.2 and 6.3. Figure 6.2(a) to (i) show the transients in the real and reactive power transferred to the utility grid through the PCC and the output power of the four DERs. As illustrated in Figure
6.2 (j), the devised open-loop frequency control structure regulates the frequency at 60 Hz within 53 ms (with 1% criterion). Also depicted in Figure 6.2 (k) to (o), at the instant that switching between modes is commanded, microgrid voltage drops to 0.32
Figure 6.2: Pre-planned islanding from the VPP-PQ mode: (a) and (f) real and reactive powers transferred to the utility grid through the PCC and their reference commands, (b) to (e) and (g) to (i) output real and reactive powers generated by DER1 to DER4, (j) frequency measured at the PCC, (k) to (o) magnitude waveform of the instantaneous voltage measured at the PCC and PC1 to PC4.

pu. However, within 46 ms (with 1% criterion), voltages at the PCC and PC1 to PC4 are regulated at 0.991, 0.988, 0.995, 1.00, and 0.988 pu, respectively. Transients in the instantaneous three-phase voltage measured at the PCC and PC1 to PC4 as well as the output currents of the four DERs and the current going through the feeder TL1 to the host utility grid are also provided in Figure 6.3 (a) to (j).

The study results indicate the ability of the devised control structure in meeting the requirements of a microgrid in VPP-PQ and islanded modes, and in providing a smooth and fast transition between these two modes.

6.3.2 Case 2: Pre-Planned Islanding from the VPP-PV Mode

This case studies the performance of the developed control structure when subjected to a pre-planned islanding scenario causing a transition from the VPP-PV to the islanded
mode. Prior to \( t=0.2 \) s, the microgrid operates in the VPP-PV mode and transfers 1.00 MW of real power to the utility grid and regulating PCC’s voltage at 1.00 pu under full load condition, as provided in Table E.1, Appendix E.

Figure 6.4 (a) to (i) show the transients in the real and reactive power transferred to the utility grid through the PCC and generated by the four DERs. Also shown in Figure 6.4 (j), the devised open-loop frequency control structure regulates the frequency at 60 Hz within 53 ms (with 1% criterion). Figure 6.4 (k) demonstrates that the employed control structure regulates the PCC voltage at 1 pu prior to islanding and then within 46 ms (with 1% criterion) after the islanding event and with a 13.7% overshoot, voltage at the PCC is regulated at 0.991 pu. Transients in the instantaneous three-phase voltage measured at the PCC and PC\(_1\) to PC\(_4\) as well as the output currents of the four DERs and the current going through the feeder TL\(_1\) to the host utility grid are also provided.
Figure 6.4: Pre-planned islanding from the VPP-PV mode: (a) and (f) real and reactive powers transferred to the utility grid through the PCC and the real power reference command, (b) to (e) and (g) to (i) output real and reactive powers generated by DER1 to DER4, (j) frequency measured at the PCC, (k) magnitude waveform of the instantaneous voltage measured at the PCC and its reference command, and (l) to (o) magnitude waveform of the instantaneous voltage measured at PC1 to PC4.

It is worth mentioning that the low frequency oscillations at a frequency of about 25 Hz which can be seen in the reactive power waveform in Figure 6.4 (g) to (i) is caused by interactions between the DER units and the 2-MVAR fixed shunt capacitor bank connected to the PCC in sharing the required reactive power. These oscillations consequently affect microgrid voltage as depicted in Figure 6.4 (k) to (o) causing a longer settling time and larger overshoot. Figure 6.6 shows simulation results of the same scenario except with a difference that the capacitor bank is disconnected from the PCC. As it can be seen in Figure 6.6 (g) to (i) and (k) to (o), low frequency oscillations in the reactive power waveform is removed and voltage at the PCC is regulated at 1.00 pu within 20 ms with zero overshoot.
Figure 6.5: Pre-planned islanding from the VPP-PV mode: (a) to (e) three-phase instantaneous voltage measured at the PCC and PC1 to PC4, and (f) to (j) three-phase instantaneous current going through feeder TL1 and output current of DER1 to DER4.

The study results demonstrate the ability of the devised control structure in meeting the requirements of a microgrid in both the VPP-PV and islanded modes as well as in providing a smooth and fast transition between these two modes.

6.3.3 Case 3: Unintentional Islanding from the VPP-PQ Mode

In this case study, a permanent three-phase L-L-L-G fault with a resistance of 0.2 ohm (0.01 pu) occurs on the utility grid bus, i.e., Bus_u of Figure 2.1 at t=0.2 s. The CBs at both ends of the 69-kV utility line operate at t=0.283 to clear the fault and the current of each phase of line TL1, is cleared at the first subsequent zero crossing. Islanding is detected after 3.5 cycles at t=0.341 s, which results in a mode switch in the devised control structure from VPP-PQ to a permanent islanded mode. Pre-fault operating condition is similar to Case 1.

Figure 6.7 (a) to (j) show the transients in the real and reactive power transferred to the utility grid through the PCC and generated by the four DERs. Also, the instantaneous three-phase and magnitude waveform of the voltage measured at the PCC and PC1
Figure 6.6: Pre-planned islanding from the VPP-PV mode without the shunt capacitor: (a) and (f) real and reactive powers transferred to the utility grid through the PCC and the real power reference command, (b) to (e) and (g) to (i) output real and reactive powers generated by DER$_1$ to DER$_4$, (j) frequency measured at the PCC, (k) magnitude waveform of the instantaneous voltage measured at the PCC and its reference command, and (l) to (o) magnitude waveform of the instantaneous voltage measured at PC$_1$ to PC$_4$.

to PC$_4$ are shown in Figure 6.8 (a) to (e) and Figure 6.9 (a) to (d). These plots depict that at the fault instant the microgrid voltage drops to 0.15 pu. At $t=0.283$ that CBs operate and disconnect the utility line, voltage increases to about 1.3 pu and remains until the islanding is detected. The over-voltage during this period of 3.5 cycles occurs since the control structure is still operating in the VPP-PQ mode while the microgrid is practically disconnected from the host utility grid. Within 48 ms after islanding is detected, voltage at the PCC and the four PCs are regulated at 1.03, 1.01, 1.02, 1.03, and 1.00 pu, respectively. Transients in the instantaneous three-phase and magnitude waveform of the output currents of the four DERs and the current going through the feeder $TL_1$ to the host utility grid are also provided in Figure 6.8 and Figure 6.9 (f).
Figure 6.7: Permanent islanding from the VPP-PQ mode: (a) and (f) real and reactive powers transferred to the utility grid through the PCC, (b) to (e) and (g) to (j) output real and reactive powers generated by DER$_{1}$ to DER$_{4}$. 

to (j). Figure 6.9 (e) illustrates that during the 141 ms from when fault occurs to the time islanding is detected, microgrid frequency increases to about 64.28 Hz. However, after 41.5 ms that islanding is detected and switching in control structure is commanded, microgrid frequency is regulated at 60 Hz.

This case study clearly depicts the robust performance of the proposed control structure when the microgrid faces severe disturbances such as faults and the subsequent islanding process.
6.3.4 Case 4: Unintentional Permanent Islanding from the VPP-PV Mode

In this case study, a permanent three-phase L-L-L-G fault with a resistance of 0.2 ohm (0.01 pu) occurs on the weak utility grid bus, i.e., Bus_u in Figure 2.1, at t=0.2 s. The CBs at both ends of line TL_1 in Figure 2.1 operate at t=0.283 to clear the fault and the current of each phase, going through the utility line TL_1, is cleared at the first subsequent zero crossing. Islanding is detected after 3.5 cycles at t=0.341 s, which results in a mode switch in the devised control structure from VPP-PV to a permanent islanded mode. The pre-fault operating condition is similar to Case 2.
Figure 6.9: Permanent islanding from the VPP-PQ mode: (a) to (d) magnitude waveform of the instantaneous voltage measured at the PCC and PC1 to PC4, (e) frequency measured at the PCC, and (f) to (j) magnitude waveform of the instantaneous current going through feeder TL1 and output current of DER1 to DER4.

Figure 6.10 (a) to (j) show the transients in the real and reactive power transferred to the utility grid through the PCC and generated by the four DERs. Also, the instantaneous three-phase and magnitude waveform of the voltage measured at the PCC and PC1 to PC4 are shown in Figure 6.11 (a) to (e) and Figure 6.12 (a) to (d). These plots depict that at the instant that fault happens, microgrid voltage drops to 0.39 pu. At t=0.283 that CBs operate and disconnect the utility line, voltage increases to about 1.34 pu and remains there until the islanding is detected. The over-voltage during this period of 3.5 cycles occurs since the control structure is still operating in the VPP-PV mode while the microgrid is practically disconnected from the host utility grid. Within 42 ms
after islanding is detected, voltage at the PCC and the four PCs are regulated at 1.03, 1.01, 1.02, 1.03, and 1.00 pu, respectively. Transients in the instantaneous three-phase and magnitude waveform of the output currents of the four DERs and the current going through the feeder $T L_1$ to the host utility grid are also provided in Figure 6.11 and Figure 6.12 (f) to (j). Figure 6.12 (e) illustrates that during the 141 ms from when fault occurs to the time islanding is detected, microgrid frequency deviates between 57.4 and 65.5 Hz. However, after 51.1 ms that islanding is detected and switching in control structure is commanded, microgrid frequency is regulated at 60 Hz.

The robust performance of the proposed control structure in riding the microgrid through an unintentional islanding event is clearly demonstrated in this case study.

6.3.5 Case 5: Three-Phase Fault, Fault Clearing, Islanding Detection, and Temporary Islanding

This case study is similar to Case 3 except that the fault is temporary and lasts for 5 cycles. At $t=0.2$ s fault happens. At $t=0.283$ CBs operate at both ends of the 69-kV utility line to clear the fault and the current of each phase, going through the utility line $T L_1$, is cleared at the first subsequent zero crossing. After 3.5 cycles, at $t=0.341$ s, islanding is detected and the mode switch in the control structure from VPP-PQ to islanded mode is commanded. Then, 30 cycles after fault clearance, i.e., at $t=0.783$ s, the auto-reclosure scheme embedded in the CBs reconnect the microgrid to the utility grid. This switches the control structure from the islanded to the VPP-PQ mode of operation.

As shown in Figs. 6.13, 6.14, and 6.15, system transients during the fault, fault clearing, and islanding detection are similar to their counterparts in Section 6.3.3. When the microgrid is operating in the islanded mode, voltage at the PCC drifts with respect to that of the main utility grid. At the instant of reconnecting the islanded microgrid to the utility grid, this phase shift is 21 degree. Therefore, this reclosure resembles an out-of-phase synchronization of two voltage sources which can cause a severe disturbance. However, as depicted in Figs. 6.13, 6.14, and 6.15, the developed control structure effectively rejects this severe disturbance and provides the utility grid with the predetermined real and reactive power within 80 ms and 110 ms, respectively (with 1% criterion).

6.3.6 Case 6: Transition from Islanded to VPP-PQ Mode

This case studies the performance of the developed control structure in providing a smooth transition from the islanded to the VPP-PQ mode of operation. Prior to this
transition, the microgrid study system of Figure 2.1 is operating in the islanded mode under full load condition, provided in Table E.1 Appendix E. At t=0.2 s, the islanded microgrid reconnects to the strong utility grid (described in Chapter 4 with ESCR=14.09) and provides it with 1.02 MVA at a leading power factor of 0.98. It should be noted that at the time of re-connection of the microgrid to the utility grid, there is a 18.5 degree phase shift between the voltage of the PCC and Bus_u. However, no synchronization procedure is adopted which results in exposing the microgrid to a severe disturbance.

Depicted in Figure 6.10 (a) and (f), output real and reactive powers of the microgrid follow their setpoints within 56 ms and 85 ms (with 1% criterion), respectively. Real and
Figure 6.11: Permanent islanding from the VPP-PV mode: (a) to (e) three-phase instantaneous voltage measured at the PCC and PC1 to PC4, and (f) to (j) three-phase instantaneous current going through feeder TL1 and output current of DER1 to DER4.

reactive power generated by the four DER units are also shown in plots (b) to (e) and (g) to (i). Figure 6.16 (k) depicts that despite this disturbance, voltage at the PCC drops to 0.94 pu and then settles down at 1.00 pu within 41 ms (with 1% criterion). Shown in Figure 6.16 (j), microgrid frequency measured at the PCC increases to 60.5 Hz due to this mode transition and settles down to 60 Hz within 45 ms. Transients in the instantaneous three-phase voltage and output current, measured at the PCC and the four PCs of the DER units, are also provided in Figure 6.17 (a) to (j). This case study clearly depicts the superiority of the proposed robust control structure in riding a microgrid through severe disturbance of unsynchronized re-connection of an islanded microgrid to a strong utility grid.

It is worth mentioning that the low frequency oscillations at a frequency of about
25 Hz which can be seen in the reactive power waveform in Figure 6.16 (f) to (i) is caused by the interactions between the DER units and the 2-MVAR fixed shunt capacitor bank connected to the PCC in sharing the required reactive power. Figure 6.18 shows simulation results of the same scenario except with a difference that the capacitor bank is disconnected from the PCC. The comparison of Figure 6.16 and Figure 6.18 shows that disconnecting the capacitor bank reduces the settling time of the reactive power from 85 ms to 62 ms (with 1% criterion). However, DER units are required to increase the generation of reactive power from 0.025 pu to 0.2 pu in the VPP-PQ mode.
6.3.7 Case 7: Transition from Islanded to VPP-PV Mode

This case studies the performance of the developed control structure in the transition mode from the islanded to the VPP-PV mode of operation. Prior to this transition, the microgrid study system of Figure 2.1 is operating in the islanded mode under full load condition, provided in Table E.1, Appendix E. At t=0.2 s, the islanded microgrid reconnects to the weak utility grid (described in Chapter 4 with ESCR=2.16) providing it with 1.00 MW real power while regulating the voltage at the PCC at 1.00 pu. At the time of re-connection, there is a 25.5 degree phase shift between the voltage of the PCC and Busᵢ. However, no synchronization procedure is allowed to act to face the microgrid to a severe disturbance.

Figure 6.19 (a) to (i) show the transients in the real and reactive power transferred to the utility grid through the PCC and generated by the four DERs. As depicted, the output real power follows its reference command within 60 ms. Figure 6.19 (k) depicts
that despite this disturbance, voltage at the PCC drops to 0.74 pu and then settles down at 1.00 pu within 64 ms (with 1% criterion). Shown in Figure 6.19 (j), microgrid frequency measured at the PCC deviates between 59.1 and 62.2 Hz due to this mode transition and settles down to 60 Hz within 39 ms. Transients in the instantaneous three-phase voltage and output current, measured at the PCC and the four PCs of the DER units, are also provided in Figure 6.20 (a) to (j). This case study clearly depicts the superiority of the proposed robust control structure in riding a microgrid through severe disturbance of unsynchronized re-connection of an islanded microgrid to a weak utility grid.
Figure 6.15: Temporary islanding from the VPP-PQ mode: magnitude waveform of the instantaneous voltage measured at the PCC and PC1 to PC4, (e) frequency measured at the PCC, and (f) to (j) magnitude waveform of the instantaneous current going through feeder TL1 and output current of DER1 to DER4.

6.4 Conclusions

This chapter provides the overall hierarchical control structure that is developed level by level in the course of the past three chapters. Based on the utility grid condition, a microgrid may be required to switch between VPP-PQ, VPP-PV, or islanded modes of operation. Accordingly, a switch in the control structure is commanded to ensure desired requirements in each mode of operation is provided. After providing the switching logic in the control structure based on the pre- and post-transition modes, in this chapter, several case studies are performed in PSCAD/EMTDC to investigate the performance of
The overall control structure.

The case scenarios studied in this chapter include: (1) a pre-planned islanding scenario from the VPP-PQ or VPP-PV to the islanded mode, (2) three-phase fault at the strong (weak) utility grid side, detection, and permanent islanding from the VPP-PQ (VPP-PV) mode with no automatic re-closure, (3) three-phase fault at the utility grid side, detection, and temporary islanding, and (4) unsynchronized transition from the islanded mode to the VPP-PQ or VPP-PV mode. The obtained simulation results clearly illustrate that the developed robust hierarchical control structure enables a multi-DER microgrid to: (1) meet performance requirements during the steady-state operation in every operational mode, and (2) ride through transition between modes, despite rather severe disturbances, such as line-to-ground faults and unsynchronized connection of microgrid to the utility.
Figure 6.17: Transition from islanded to the VPP-PQ mode: (a) to (e) three-phase instantaneous voltage measured at the PCC and PC\textsubscript{1} to PC\textsubscript{4}, and (f) to (j) three-phase instantaneous current going through feeder TL\textsubscript{1} and output current of DER\textsubscript{1} to DER\textsubscript{4}.

grid. To add to the severity of the disturbances, time periods of 5 cycle and 3.5 cycle are incorporated for fault clearing time and islanding detection time, respectively. It means that during a timespan of 8.5 cycles, controllers are commanded to operate in the VPP-PQ (VPP-PV) mode while practically the microgrid is not hosted by a strong (weak) utility grid. This pushes the controllers to the nonlinear region of operation (saturation).

The islanding scenarios from the VPP-PQ and VPP-PV to the islanded mode, studied in Sections 6.3.1 and 6.3.2 show a settling time of 53 ms and 46 ms for the PCC’s frequency and voltage to be regulated at 60 Hz and 0.991 pu, respectively. On the other hand, when the unsynchronized transition from the islanded to the VPP-PQ mode occurs, simulation results in Case 6 Section 6.3.6 depicts a settling time of 56 ms and 85 ms for the microgrid’s output real and reactive power. Also, Case 7 studied in Section 6.3.7 shows that in the unsynchronized transition from the islanded to the VPP-PV mode, microgrid’s output real power follows its reference command within 60 ms while
the voltage at the PCC is regulated at 1.00 pu within 64 ms.

In addition, the effect of interactions between the DER units and the 2-MVAR fixed shunt capacitor bank connected to the PCC in sharing the required reactive power is studied in this chapter. As simulation results in Sections 6.3.2 and 6.3.6 illustrate, these interactions cause a low-frequency oscillation at a frequency of about 25 Hz on the reactive power waveform. These oscillations consequently increase the settling time in following the reactive power or voltage reference commands from 62 ms to 85 ms and from 20 ms to 46 ms, respectively. However, disconnecting the shunt capacitor necessitates DER units to allocate a higher percentage of their capacity to the generation of reactive power, as discussed in Sections 6.3.2 and 6.3.6.
Figure 6.19: Transition from the islanded to the VPP-PV mode: (a) and (f) real and reactive powers transferred to the utility grid through the PCC and the real power reference command, (b) to (e) and (g) to (i) output real and reactive powers generated by DER\textsubscript{1} to DER\textsubscript{4}, (j) frequency measured at the PCC, (k) magnitude waveform of the instantaneous voltage measured at the PCC and its reference command, and (l) to (o) magnitude waveform of the instantaneous voltage measured at PC\textsubscript{1} to PC\textsubscript{4}.
Chapter 6. Transition Mode: The Overall Control Structure

Figure 6.20: Transition from the islanded to the VPP-PV mode: (a) to (e) three-phase instantaneous voltage measured at the PCC and PC1 to PC4, and (f) to (j) three-phase instantaneous current going through feeder TL1 and output current of DER1 to DER4.
Chapter 7

Summary and Conclusions

7.1 Summary

This thesis provides a thorough understanding of the principles of operation and control requirements of virtual power plants. It also develops a robust hierarchical control structure to operate multi-DER microgrids in the VPP-PQ, VPP-PV, islanded, and transition modes. To that end, a linearized state-space representation of a study microgrid with multiple electronically-interfaced DER units and loads is first developed. Then, various sources of uncertainties and disturbances affecting dynamics of the microgrid in both the VPP and islanded modes are identified and classified as parametric and topological uncertainties, unmodelled dynamics, and exogenous disturbances. These groups of perturbations are modeled in an IQC framework. Based on the obtained uncertain system model, a necessary and sufficient condition is proposed to find robust, decentralized, output-feedback, reference-tracking, local controllers for DER units. These local controllers guarantee the robust stability of the overall microgrid and ensure disturbance attenuation to a pre-determined level. The proposed necessary and sufficient condition is derived in terms of coupled constrained algebraic Riccati equations which is then transferred into rank constrained LMIs. The proposed LMIs can be efficiently solved in one of the existing numerical toolboxes to design a local power and a local voltage controller for each DER unit. These local controllers constitute the primary level of the proposed hierarchical control structure.

At the secondary level, a robust $H_{\infty}$, multivariable, LMI-based, PI control strategy augmented with static, multivariable, LMI-based anti-windup compensator is proposed and designed for the study microgrid. The proposed secondary control coordinates local power controllers at the primary level to operate a microgrid in the VPP-PQ and VPP-PV modes. The proposed control strategy: (1) guarantees the stability of the overall, closed-
loop, unconstrained microgrid while achieving a pre-determined disturbance attenuation level, and (2) guarantees the stability of the constrained microgrid while demonstrating graceful performance degradation during controller windup.

Furthermore, to operate a microgrid in the islanded mode during brownouts and blackouts of the host utility grid, a decentralized, droop-augmented, robust voltage control structure is introduced in this thesis. The proposed control structure in this mode is constituted of: (1) $P-\delta$ and $Q-V$ droop-based power sharing controllers, (2) the proposed IQC-based robust voltage local controllers, and (3) open-loop frequency controllers featuring independent internal oscillators synchronized by a common time-reference signal received from a GPS.

Finally, the overall hierarchical control structure that is developed level by level in the course of the thesis is presented. Based on the utility grid condition, a microgrid may be required to switch between VPP-PQ, VPP-PV, or islanded modes of operation. Accordingly, a switch in the control structure is commanded to ensure desired requirements in each mode of operation is provided. Therefore, the switching logic in the control structure based on the the pre- and post-transition modes are provided. Throughout the thesis, various frequency-domain and digital time-domain studies are performed in MATLAB and PSCAD/EMTDC platforms to evaluate the attributes and performance of the designed hierarchical control structure.

### 7.2 Qualitative Conclusions

This thesis concludes that:

- The hierarchical control structure is a viable solution for the operation of microgrids in the VPP mode due to its inherent (1) ability to coordinate multiple, constrained, interacting DER units in a microgrid, (2) interoperability with communication network technologies, and (3) time-scale separation characteristics which enables it to handle both short-term transient control functionalities as well as long-term quasi steady-state management and optimization services.

- The primary control should be a decentralized structure of the local controllers implemented in DER units to obviate the need to high-bandwidth and highly reliable communication networks and to make it scalable for the future installations of DER units.

- A microgrid, operating in either the VPP or the islanded mode, is subjected to parametric and topological uncertainties, exogenous disturbances, and unmodelled
dynamics. Therefore, the primary control which is responsible for stable and reliable operation of the microgrid should be robust against these perturbations.

- Although widely ignored in the previous research works, a particular emphasis should be placed on the robustness of the primary control to topological uncertainties since: (1) configuration changes frequently occur in microgrids with multiple interacting DER units and loads transiting between modes of operation, and (2) this feature adds to the modularity of the primary control and makes it independent of a particular microgrid configuration.

- For today’s microgrids in which multiple, interacting, constrained DER units need to be coordinated in a real-time basis to provide pre-specified services, the conventional heuristic control design methods entailing multiple rounds of trial and error is no longer a recommended procedure. Instead, systematic design procedures should be provided for both the primary and secondary control design. These procedures find control parameters which satisfy certain necessary and sufficient or sufficient only conditions which analytically prove the robust stability of the overall microgrid.

### 7.3 Quantitative Conclusions

This thesis concludes that:

- The method of balanced truncation using Hankel singular values is a promising method for reducing the order of controllers, e.g., from 10 to 5, while maintaining asymptotic stability of the reduced-order system and the achieved level of disturbance attenuation. The performed step response analysis, singular value plots, and numerical frequency- and time-domain approximation errors, in MATLAB environment, verify that reducing the order of controllers from 10 to 5 achieves a time- and frequency-domain approximation error of 5.6 % and 2.4 %, respectively, and preserves the disturbance attenuation level to its pre-determined value.

- The designed local power controllers at the primary level maintain the robust stability of the microgrid during transients and ensure reference tracking with zero steady-state error. This is confirmed through a set of:

  1. Modal analyses performed in MATLAB environment. These analyses show a module margin of 0.99, a sensitivity function singular value plot with $S(s) \rightarrow 0$
at low frequencies ensuring reference tracking and disturbance rejection and $S(s) \rightarrow 1$ at high frequencies ensuring measurement noise rejection, and a maximum tolerable input time-delay of 7.2 ms.

2. Time-domain simulation performed in PSCAD/EMTDC environment. These analyses show the real and reactive power reference tracking within 7.5 ms (with 1% criterion) with zero overshoot and zero steady-state tracking error. Simulation results also confirm that even in the case of severe disturbances, e.g., faults, which push the DERs into nonlinear operational regions, due to saturation in the current limiter and the subsequent current controller windup, the devised power controllers maintain system stability and then settle down to the desired setpoint within 42 ms after disturbance source is cleared.

Similar results are concluded for the proposed local voltage controllers.

- The developed hierarchical control structure in the VPP-PQ mode maintains the stability of the closed-loop microgrid during transients and meets the pre-specified requirements at the PCC in a fast and robust fashion. This is confirmed through a set of:

  1. Modal analyses performed in MATLAB environment. Eigenvalue and participation factor analyses show that the dominant eigenvalues of the closed-loop system are located between -40 to -20 1/s and are mainly affected by the state variables of the PI-based secondary controller. Therefore, robustness margins of the overall controlled system are insensitive to parametric and operating point changes of the microgrid, as confirmed by the performed sensitivity analysis. The viability of the designed control structure is also confirmed through singular value and bode plots, e.g., a gain margin of 79 dB and a phase margin of 349 degree are obtained for the loop transfer functions from $\Delta P_{\text{pcc,ref}}$ to $\Delta P_{\text{pcc}}$.

  2. Time-domain simulation studies performed in PSCAD/EMTDC platform. These studies demonstrate the robust performance of the devised hierarchical control structure in operating the microgrid in the VPP-PQ mode when the system is subjected to disconnection and re-connection of loads and DER units, change in the microgrid configuration, nonlinear load energization, three-phase line-to-ground fault, and single-phase line-to-ground fault. Simulation results indicate that the developed control structure in the VPP-PQ mode provides a settling time of 29.2 ms and 19.8 ms (with 1% criterion) with 3.75 % and
0.10% overshoot and zero steady-state tracking error, for tracking the real and reactive power setpoints, respectively. Also, these results confirm that in the case of severe disturbances such as faults which significantly push the secondary controller into nonlinear regions (saturation), the devised anti-windup compensator maintains the stability of the system and recover from the windup condition within 17.2 ms after the disturbance source is cleared.

Similar results are concluded for the devised control structure in the VPP-PV and islanded modes.

- The devised control structure provides a smooth transition between VPP and islanded modes since deviations in the frequency and voltage of the utility grid are considered as exogenous disturbances. This feature is verified through multiple time-domain simulation studies including pre-planned islanding, three-phase fault at the utility grid side and the subsequent permanent or temporary islanding, and unsynchronized transition from islanded mode to the VPP mode. For example, a settling time of 56 ms and 85 ms for the microgrid’s output real and reactive power is observed when an unsynchronized transition from the islanded to the VPP-PQ mode occurs.

- The overcurrent and overload protection schemes implemented in the DER units successfully limit DER’s output current to 1.2 pu in the worst-case fault scenarios such as line-to-ground faults.

- The employed limiter block at the secondary level successfully limits the real and reactive power reference commands sent to the local controllers to the maximum generation capacity of their corresponding DER units.

### 7.4 Contributions

The main contributions of this dissertation are as follows:

- Introducing the IQC framework for modelling uncertainties and disturbances due to the ability of this framework in capturing a rich class of perturbations including parametric and topological uncertainties, disturbances, and high-frequency unmodelled dynamics. In addition, since the IQC framework extracts the structural information about perturbations, the designed robust controllers are foreseen to be less conservative.
• Developing robust, decentralized, MIMO, output-tracking strategy for the local power and voltage controllers implemented at the primary level. The developed control strategy (i) provides the overall stability of a multi-DER microgrid subjected to parametric and topological uncertainties, unmodelled dynamics, and exogenous disturbances, and (ii) guarantees meeting a pre-specified level of disturbance attenuation.

• Proposing a PI-based, multivariable, robust $H_\infty$ secondary control augmented with multivariable anti-windup compensator to operate a multi-DER microgrid in the VPP-PQ and VPP-PV mode. The proposed secondary control scheme guarantees the stability of the closed-loop system while meeting a specified $H_\infty$ level of disturbance attenuation, for the unconstrained system, and provides a graceful performance degradation during the input saturation while maintaining system stability.

• Proposing a numerically efficient, LMI-based, systematic design procedure for both the primary and secondary controllers.

• Providing an analytical proof for the stability of the overall closed-loop microgrid controlled by the proposed hierarchical control structure.

• Developing a decentralized droop-augmented robust voltage control structure to operate multi-DER microgrids in the islanded mode. This control structure is composed of: (1) static droop-based power sharing controllers with real power-phase angle (P-δ) droop and reactive power-voltage (Q-v) droop controllers, (2) a robust local voltage controller, and (3) an open-loop frequency control, featuring independent internal oscillator synchronized by a common time-reference signal received from a global positioning system (GPS).

7.5 Future Research Directions

In continuation of this thesis, the following subjects are suggested for future studies:

• Implementing optimization algorithms in the participation algorithm block at the secondary control. The implemented optimization algorithms can readjust the power reference commands of the DER units at the quasi steady-state operating condition to ensure their optimal operation.
• Developing a tertiary controller which coordinates multiple VPPs connected to a host utility grid to ensure optimal participation of each VPP in the market and management functionalities.

• In this work, each DER unit is equipped with a dual-sequence current controller which is composed of a positive-sequence and a negative-sequence PI controller. Currently, the reference command for the negative-sequence controller is kept constant at zero to balance the output current of the DER units during unbalanced conditions in the microgrid. An immediate extension to this work is to determine the negative-sequence current references by a controller embedded at the secondary control to actively compensate imbalances in microgrid voltage or output power. In this way, the VPP starts supporting the host utility grid with power and voltage quality issues, as well.
Appendix A

Electrical parameters of the Study System

The electrical parameters of the study system, including transformer parameters and configuration, the length of the distribution lines and also the type of the overhead lines, and the rated loads of feeders are given in Tables A.1, A.2, and A.3, respectively.

Table A.1: Parameters of Transformers T-1 to T-8 and T-DERs

<table>
<thead>
<tr>
<th>Transformer</th>
<th>Rated MVA</th>
<th>Primary kV</th>
<th>Secondary kV</th>
<th>Z(%)</th>
<th>X/R ratio</th>
<th>Impedance (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>T-1</td>
<td>15.00</td>
<td>69.00</td>
<td>13.8</td>
<td>8.00</td>
<td>17.00</td>
<td>0.31+j5.32</td>
</tr>
<tr>
<td>T-2</td>
<td>2.50</td>
<td>13.8</td>
<td>2.4</td>
<td>5.75</td>
<td>10.00</td>
<td>2.28+j22.88</td>
</tr>
<tr>
<td>T-3</td>
<td>3.75</td>
<td>13.8</td>
<td>2.4</td>
<td>5.50</td>
<td>12.00</td>
<td>2.22+j14.62</td>
</tr>
<tr>
<td>T-4</td>
<td>3.75</td>
<td>13.8</td>
<td>5.5</td>
<td>5.50</td>
<td>12.00</td>
<td>1.22+j14.62</td>
</tr>
<tr>
<td>T-5</td>
<td>1.25</td>
<td>13.8</td>
<td>0.48</td>
<td>4.50</td>
<td>6.00</td>
<td>5.9184+j35.51</td>
</tr>
<tr>
<td>T-6</td>
<td>1.25</td>
<td>13.8</td>
<td>0.48</td>
<td>4.50</td>
<td>6.00</td>
<td>5.9184+j35.51</td>
</tr>
<tr>
<td>T-7</td>
<td>1.50</td>
<td>13.8</td>
<td>2.4</td>
<td>5.50</td>
<td>6.50</td>
<td>5.57+j36.24</td>
</tr>
<tr>
<td>T-8</td>
<td>1.50</td>
<td>13.8</td>
<td>2.4</td>
<td>5.50</td>
<td>6.50</td>
<td>5.57+j36.24</td>
</tr>
<tr>
<td>T-DR4</td>
<td>0.60</td>
<td>13.8</td>
<td>0.6</td>
<td>5.00</td>
<td>10.00</td>
<td>0.83+j8.30</td>
</tr>
<tr>
<td>T-DR1</td>
<td>4.50</td>
<td>13.8</td>
<td>0.6</td>
<td>5.00</td>
<td>11.00</td>
<td>1.0+j11.00</td>
</tr>
<tr>
<td>T-DR2</td>
<td>4.00</td>
<td>13.8</td>
<td>0.6</td>
<td>5.00</td>
<td>10.00</td>
<td>0.12+j1.25</td>
</tr>
<tr>
<td>T-DR3</td>
<td>3.00</td>
<td>13.8</td>
<td>0.6</td>
<td>5.00</td>
<td>10.00</td>
<td>1.60+j16.01</td>
</tr>
</tbody>
</table>
Table A.2: Parameters of Conductors of the Study System

<table>
<thead>
<tr>
<th>Conductor</th>
<th>Length (km)</th>
<th>Line Impedance (pu)</th>
</tr>
</thead>
<tbody>
<tr>
<td>T-L1</td>
<td>0.80</td>
<td>0.9054+j1.2861</td>
</tr>
<tr>
<td></td>
<td>266.8 MCM</td>
<td></td>
</tr>
<tr>
<td>F-1</td>
<td>2.06</td>
<td>2.9180+j3.4222</td>
</tr>
<tr>
<td></td>
<td>ACSR 4/0</td>
<td></td>
</tr>
<tr>
<td>F-2</td>
<td>4.83</td>
<td>6.8417+j8.0238</td>
</tr>
<tr>
<td></td>
<td>ACSR 4/0</td>
<td></td>
</tr>
<tr>
<td>F-3</td>
<td>1.8</td>
<td>2.5497+j2.9902</td>
</tr>
<tr>
<td></td>
<td>ACSR 4/0</td>
<td></td>
</tr>
<tr>
<td>F-4</td>
<td>2.06</td>
<td>2.9180+j3.4222</td>
</tr>
<tr>
<td></td>
<td>ACSR 4/0</td>
<td></td>
</tr>
<tr>
<td>C-1</td>
<td>0.189</td>
<td>0.5318+j0.3387</td>
</tr>
<tr>
<td></td>
<td>ACSR 1/0</td>
<td></td>
</tr>
<tr>
<td>C-2</td>
<td>1.78</td>
<td>5.0081+j3.3511</td>
</tr>
<tr>
<td></td>
<td>ACSR 1/0</td>
<td></td>
</tr>
<tr>
<td>C-3</td>
<td>0.362</td>
<td>1.0185+j0.6487</td>
</tr>
<tr>
<td></td>
<td>ACSR 1/0</td>
<td></td>
</tr>
<tr>
<td>C-4</td>
<td>6.2</td>
<td>8.7822+j10.2997</td>
</tr>
<tr>
<td></td>
<td>ACSR 4/0</td>
<td></td>
</tr>
<tr>
<td>C-5</td>
<td>1.1</td>
<td>3.0949+j1.9712</td>
</tr>
<tr>
<td></td>
<td>ACSR 4/0</td>
<td></td>
</tr>
<tr>
<td>C-6</td>
<td>15.34</td>
<td>21.7291+j25.4834</td>
</tr>
<tr>
<td></td>
<td>ACSR 4/0</td>
<td></td>
</tr>
<tr>
<td>C-7</td>
<td>0.201</td>
<td>0.5655+j0.3602</td>
</tr>
<tr>
<td></td>
<td>ACSR 1/0</td>
<td></td>
</tr>
<tr>
<td>C-8</td>
<td>0.362</td>
<td>1.0185+j0.6487</td>
</tr>
<tr>
<td></td>
<td>ACSR 1/0</td>
<td></td>
</tr>
<tr>
<td>C-9</td>
<td>0.128</td>
<td>0.3601+j0.2294</td>
</tr>
<tr>
<td></td>
<td>ACSR 1/0</td>
<td></td>
</tr>
</tbody>
</table>

Table A.3: Load Parameters of the Study System

<table>
<thead>
<tr>
<th>Load Number</th>
<th>Maximum Power (MW+jMVAr)</th>
<th>Load Impedance (pu)</th>
</tr>
</thead>
<tbody>
<tr>
<td>L-1</td>
<td>1.8+j0.45</td>
<td>5.22+j1.31</td>
</tr>
<tr>
<td>L-2</td>
<td>0.7+j0.18</td>
<td>13.4+j3.45</td>
</tr>
<tr>
<td>L-3</td>
<td>1.25+j0.55</td>
<td>6.80+j3.00</td>
</tr>
<tr>
<td>L-4</td>
<td>1.12+j0.08</td>
<td>8.80+j0.25</td>
</tr>
<tr>
<td>L-5</td>
<td>0.75+j0.08</td>
<td>13.18+j1.21</td>
</tr>
<tr>
<td>L-6</td>
<td>0.73+j0.30</td>
<td>11.6+j5.00</td>
</tr>
<tr>
<td>L-7</td>
<td>0.87+j0.33</td>
<td>10.00+j3.84</td>
</tr>
</tbody>
</table>
Appendix B

Details of the Microgrid Model

This Appendix presents a detailed systematic approach to derive a small-signal linearized model of the study system shown in Figure 2.1. The modeling approach adopted in this thesis is based on [73].

B.1 Dynamic Model of the Study Microgrid

To synthesize controllers and also to analyze dynamics of the microgrid system of Figure 2.1 deriving its dynamical model is needed. A single-line diagram of the study system is depicted in Figure B.1. Distribution lines and loads are represented by three-phase \( RL \) elements and lumped together wherever applicable. The fixed-capacitor bank at the PCC bus is also modeled by an equivalent three-phase shunt capacitor \( C_p \).

To present a systematic approach toward deriving the dynamic model, the study system of Figure B.1 is virtually partitioned into five subsystems: four DER units and one power network. Detailed procedures for deriving dynamic models of subsystems, in \( dq \) frame of reference, are provided in this appendix. These dynamic models are then used to obtain a small-signal model of the microgrid through linearization around an operating point.

B.1.1 Dynamic Model of DER Units

A schematic diagram of the \( n \)th DER is provided in Figure 2.2 and is repeated here as Figure B.2 for the ease of reference. Dynamic model of the \( n \)th DER unit can be obtained by augmenting dynamic models of (i) the converter power circuit, (ii) the PLL, and (iii) the built-in current controller. The proposed model in this appendix captures the positive-sequence dynamics only. Table B.1 provides ratings and the main parameters of
all the four DER units.

The power calculation block, depicted in Figure B.2 uses the \(d\) and \(q\) components of the converter output current and voltage to compute the real and reactive output powers of the \(n^{th}\) DER unit, based on the following formula:

\[
P_n = 1.5(V_{d,n}i_{d,n} + V_{d,n}i_{d,n})
\]

\[
Q_n = 1.5(V_{q,n}i_{d,n} - V_{d,n}i_{q,n})
\]
B.1.1.1 Dynamic Model of DER unit Power Circuit

Figure B.3 illustrates the power circuit of \( n^{th} \) DER. A mathematical description of this circuit, in the \( abc \) reference frame is given by:

\[
\begin{align*}
    v_{tn,abc} &= R_{sn}i_{DRn,abc} + L_{sn}\frac{d}{dt}i_{DRn,abc} + v_{DRn,abc}, \\
    c_{fn}\frac{d}{dt}v_{DRn,abc} &= i_{DRn,abc} - i_{Tn,abc}.
\end{align*}
\]  

(B.1)

Table B.1: Ratings and parameters of the DER unit

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Parameter Name</th>
<th>DER(_2) - DER(_3) Value</th>
<th>DER(_4) Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inverter nominal power</td>
<td>( S_n )</td>
<td>3 ( MV A )</td>
<td>0.5 ( MV A )</td>
</tr>
<tr>
<td>Filter inverter-side inductance</td>
<td>( L_f )</td>
<td>0.1 mH</td>
<td>0.57 mH</td>
</tr>
<tr>
<td>Filter inverter-side resistance</td>
<td>( R_f )</td>
<td>2.4 mΩ</td>
<td>21.6 mΩ</td>
</tr>
<tr>
<td>Filter capacitor</td>
<td>( C_f )</td>
<td>500 ( \mu F )</td>
<td>87.7 ( \mu F )</td>
</tr>
<tr>
<td>RLC filter cutoff frequency</td>
<td>( f_c )</td>
<td>711.7 ( HZ )</td>
<td>711.7 ( HZ )</td>
</tr>
<tr>
<td>Grid frequency</td>
<td>( f )</td>
<td>60 ( HZ )</td>
<td>60 ( HZ )</td>
</tr>
<tr>
<td>Inverter commutation frequency</td>
<td>( f_{com} )</td>
<td>3060 ( HZ )</td>
<td>3060 ( HZ )</td>
</tr>
<tr>
<td>DC bus voltage</td>
<td>( V_{DC} )</td>
<td>1200 V</td>
<td>1200 V</td>
</tr>
<tr>
<td>Closed-loop time constant</td>
<td>( \tau_i )</td>
<td>1.25 ms</td>
<td>1.25 ms</td>
</tr>
</tbody>
</table>
where \(v_{tn,abc}, i_{DRn,abc}, v_{DRn,abc}\), and \(i_{Tn,abc}\) are vectors of the fundamental components of the converter terminal voltage, converter current, converter bus voltage, and current passing through DER’s transformer, respectively.

\[
\begin{align*}
\begin{bmatrix}
V_{ta,n} \\
V_{tb,n} \\
V_{tc,n}
\end{bmatrix}
&= \begin{bmatrix}
R_{ta,n} & L_{ta,n} & i_{DRn,a} \\
R_{tb,n} & L_{tb,n} & i_{DRn,b} \\
R_{tc,n} & L_{tc,n} & i_{DRn,c}
\end{bmatrix}
\begin{bmatrix}
V_{DRn,a} \\
V_{DRn,b} \\
V_{DRn,c}
\end{bmatrix}
+ \begin{bmatrix}
i_{Tn,a} \\
i_{Tn,b} \\
i_{Tn,c}
\end{bmatrix}, \\
\begin{bmatrix}
\frac{d}{dt}i_{DRn,dn} \\
\frac{d}{dt}i_{DRn,qn} \\
\frac{d}{dt}v_{DRn,dn} \\
\frac{d}{dt}v_{DRn,qn}
\end{bmatrix}
&= \begin{bmatrix}
A_{i,DRn} & B_{V_{DRn}} & B_{i_{tn}} & B_{v_{tn}} \\
A_{v,DRn} & B_{i_{DRn}} & B_{i_{tn}} & B_{v_{tn}}
\end{bmatrix}
\begin{bmatrix}
i_{DRn} \\
v_{DRn}
\end{bmatrix}
+ \begin{bmatrix}
\frac{1}{L_{sn}} & 0 & 0 & 0 \\
0 & \frac{1}{L_{sn}} & 0 & 0
\end{bmatrix}
\begin{bmatrix}
v_{tn} \\
i_{tn}
\end{bmatrix},
\end{align*}
\]

Equation (B.2) can be written in a matrix form as:

\[
\begin{align*}
\dot{i}_{DRn} &= A_{i,DRn}i_{DRn} + B_{V_{DRn}}v_{DRn} + B_{i_{tn}}i_{tn} + B_{v_{tn}}v_{tn}, \\
\dot{v}_{DRn} &= A_{v,DRn}v_{DRn} + B_{i_{DRn}}i_{DRn} + B_{i_{tn}}i_{tn} + B_{v_{tn}}v_{tn},
\end{align*}
\]

where \(i_{DRn,dn}, i_{DRn,qn}, v_{tn,dn}, v_{tn,qn}, v_{DRn,dn}, v_{DRn,qn}, i_{Tn,dn},\) and \(i_{Tn,qn}\) are the \(d_n\)-axis and \(q_n\)-axis components of the converter current, terminal voltage and bus voltage, respectively. The speed of rotation (angular frequency) of the local \(d_nq_n\) frame of reference is depicted as \(\omega_{rn}\).

Equation (B.2) can be written in a matrix form as:

\[
\begin{align*}
\dot{i}_{DRn} &= A_{i,DRn}i_{DRn} + B_{V_{DRn}}v_{DRn} + B_{i_{tn}}i_{tn} + B_{v_{tn}}v_{tn}, \\
\dot{v}_{DRn} &= A_{v,DRn}v_{DRn} + B_{i_{DRn}}i_{DRn} + B_{i_{tn}}i_{tn} + B_{v_{tn}}v_{tn},
\end{align*}
\]

where \(A_{i,DRn} = \begin{bmatrix} R_{ta,n}/L_{ta,n} & \omega_{rn} & -R_{ta,n}/L_{ta,n} \\
-\omega_{rn} & R_{ta,n}/L_{ta,n} & \omega_{rn} \\
0 & 0 & 0 \end{bmatrix}, \quad B_{V_{DRn}} = \begin{bmatrix} 1/L_{sn} & 0 & 0 \\
0 & -1/L_{sn} & 0 \\
0 & 0 & 1/C_{fn} \end{bmatrix}, \quad B_{i_{tn}} = \begin{bmatrix} 1/L_{sn} & 0 & 0 \\
0 & 1/C_{fn} & 0 \\
0 & 0 & -1/C_{fn} \end{bmatrix}, \quad B_{v_{tn}} = \begin{bmatrix} 0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \end{bmatrix}.
\]
B.1.1.2 Dynamic Model of the Phase-Locked Loop

In a DER unit, a PLL is employed to estimate the angle of the terminal voltage of the DER. The PLL system is based on the phase angle estimator described in [39]. As depicted in Fig B.4, the model includes a loop filter (a PI controller) and a Voltage Control Oscillator (VCO). The linearized model of the PLL system in the frequency-domain is:

\[
\omega_{rn} = k_{pll,n}(k_{pw,n} + \frac{k_{iw,n}}{s})(\theta_{ref,n} - \theta_n)
\]  

(B.4)

or, in the time domain:

\[
\frac{d}{dt}\omega_{rn} = k_{pll,n}k_{pw,n}\omega_{ref,n} - k_{pll,n}k_{pw,n}\omega_{rn} + k_{pll,n}k_{iw,n}(\theta_{ref,n} - \theta_n)
\]  

(B.5)

where

\[
\frac{d}{dt}\theta_{ref,n} = \omega_{ref,n} \\
\frac{d}{dt}\theta_n = \omega_{rn}
\]  

(B.6)

![Figure B.4: Linearized model of the PLL system](image)

B.1.1.3 Dynamical Model of the Current Controller

The positive-sequence part of the built-in current controller includes two PI compensators as depicted in Figure B.5. The superscript “f” denotes the filtered quantities. \( T_{vn} \) and \( T_{in} \) are the time constants of the LPFs. Dynamic model of the current controller is then obtained as:
Appendix B. Details of the Microgrid Model

\[
\frac{d}{dt} v_{i,dn} = i_{\text{ref}d,n} - i_{\text{dn}},
\]
\[
\frac{d}{dt} v_{i,qn} = i_{\text{ref}q,n} - i_{\text{qn}},
\]

\[(B.7)\]

Figure B.5: Schematic diagram of the positive-sequence built-in current controller

Figure B.6 illustrates the control block diagram of the CC-VSC system in Figure B.2.

B.1.2 Dynamic Model of the Power Network

The following set of differential equations represents the power network of Figure B.1 in the abc reference frame.

\[
R_1 i_1 + L_1 \frac{di_1}{dt} = v_{\text{pcc}} - v_{B1},
\]
\[
R_2 i_2 + L_2 \frac{di_2}{dt} = v_{\text{pcc}},
\]
\[
R_3 i_3 + L_3 \frac{di_3}{dt} = v_{\text{pcc}} - v_{B3},
\]

\[(B.8)\]
Appendix B. Details of the Microgrid Model

Figure B.6: Control block diagram of a current-controlled VSC system

\[
R_4 i_4 + L_4 \frac{di_4}{dt} = v_{pcc} - v_{B4},
\]

\[
R_5 i_5 + L_5 \frac{di_5}{dt} = v_{B6} - v_{B3},
\]

\[
R_u i_u + L_u \frac{di_u}{dt} = v_u - v_{pcc},
\]

\[
R_{T1} i_{T1} + L_{T1} \frac{di_{T1}}{dt} = v_{DR1} - v_{B1},
\]

\[
R_{T2} i_{T2} + L_{T2} \frac{di_{T2}}{dt} = v_{DR2} - v_{B3},
\]

\[
R_{T3} i_{T3} + L_{T3} \frac{di_{T3}}{dt} = v_{DR3} - v_{B6},
\]

\[
R_{T4} i_{T4} + L_{T4} \frac{di_{T4}}{dt} = v_{DR4} - v_{B4},
\]

\[
C_p \frac{dv_{pcc}}{dt} = i_u - i_1 - i_2 - i_3 - i_4.
\]

Transferring the set of differential equations (B.8) to the global dq frame of reference, that is synchronized to the PCC’s voltage, yields:
\[
\begin{align*}
\frac{d}{dt} i_{1,d} &= \frac{1}{L_1} (v_{pcc,d} - v_{B1,d}^g) - \frac{R_1}{L_1} i_{1,d} + \omega_s i_{1,q}, \\
\frac{d}{dt} i_{1,q} &= \frac{1}{L_1} (v_{pcc,q} - v_{B1,q}^g) - \frac{R_1}{L_1} i_{1,q} - \omega_s i_{1,d}, \\
\frac{d}{dt} i_{2d} &= \frac{1}{L_2} (v_{pcc,d}) - \frac{R_2}{L_2} i_{2d} + \omega_s i_{2q}, \\
\frac{d}{dt} i_{2q} &= \frac{1}{L_2} (v_{pcc,q}) - \frac{R_2}{L_2} i_{2q} - \omega_s i_{2d}, \\
\frac{d}{dt} i_{3,d} &= \frac{1}{L_3} (v_{pcc,d} - v_{B3,d}^g) - \frac{R_3}{L_3} i_{3,d} + \omega_s i_{3,q}, \\
\frac{d}{dt} i_{3,q} &= \frac{1}{L_3} (v_{pcc,q} - v_{B3,q}^g) - \frac{R_3}{L_3} i_{3,q} - \omega_s i_{3,d}, \\
\frac{d}{dt} i_{4,d} &= \frac{1}{L_4} (v_{pcc,d} - v_{B4,d}^g) - \frac{R_4}{L_4} i_{4,d} + \omega_s i_{4,q}, \\
\frac{d}{dt} i_{4,q} &= \frac{1}{L_4} (v_{pcc,q} - v_{B4,q}^g) - \frac{R_4}{L_4} i_{4,q} - \omega_s i_{4,d}, \\
\frac{d}{dt} i_{5,d} &= \frac{1}{L_5} (v_{B6,d}^g - v_{B3,d}^g) - \frac{R_5}{L_5} i_{5,d} + \omega_s i_{5,q}, \\
\frac{d}{dt} i_{5,q} &= \frac{1}{L_5} (v_{B6,q}^g - v_{B3,q}^g) - \frac{R_5}{L_5} i_{5,q} - \omega_s i_{5,d}, \\
\frac{d}{dt} i_{u,d} &= \frac{1}{L_u} (v_{u,d}^g - v_{pcc,d}) - \frac{R_u}{L_u} i_{u,d} + \omega_s i_{u,q}, \\
\frac{d}{dt} i_{u,q} &= \frac{1}{L_u} (v_{u,q}^g - v_{pcc,q}) - \frac{R_u}{L_u} i_{u,q} - \omega_s i_{u,d}, \\
\frac{d}{dt} i_{T1,d} &= \frac{1}{L_{T1}} (v_{DR1,d}^g - v_{B1,d}^g) - \frac{R_{T1}}{L_{T1}} i_{T1,d} + \omega_s i_{T1,q}, \\
\frac{d}{dt} i_{T1,q} &= \frac{1}{L_{T1}} (v_{DR1,q}^g - v_{B1,q}^g) - \frac{R_{T1}}{L_{T1}} i_{T1,q} - \omega_s i_{T1,d},
\end{align*}
\]
\[ \begin{align*}
\frac{d}{dt} T_{2,d} &= \frac{1}{L_{T2}} (v_{DR2,d}^g - v_{B3,d}^g) - \frac{R_{T2}}{L_{T2}} i_{T2,d} + \omega_s i_{T2,q}, \\
\frac{d}{dt} T_{2,q} &= \frac{1}{L_{T2}} (v_{DR2,q}^g - v_{B3,q}^g) - \frac{R_{T2}}{L_{T2}} i_{T2,q} - \omega_s i_{T2,d}, \\
\frac{d}{dt} T_{3,d} &= \frac{1}{L_{T3}} (v_{DR3,d}^g - v_{B6,d}^g) - \frac{R_{T3}}{L_{T3}} i_{T3,d} + \omega_s i_{T3,q}, \\
\frac{d}{dt} T_{3,q} &= \frac{1}{L_{T3}} (v_{DR3,q}^g - v_{B6,q}^g) - \frac{R_{T3}}{L_{T3}} i_{T3,q} - \omega_s i_{T3,d}, \\
\frac{d}{dt} T_{4,d} &= \frac{1}{L_{T4}} (v_{DR4,d}^g - v_{B4,d}^g) - \frac{R_{T4}}{L_{T4}} i_{T4,d} + \omega_s i_{T4,q}, \\
\frac{d}{dt} T_{4,q} &= \frac{1}{L_{T4}} (v_{DR4,q}^g - v_{B4,q}^g) - \frac{R_{T4}}{L_{T4}} i_{T4,q} - \omega_s i_{T4,d}, \\
\frac{d}{dt} v_{pcc,d} &= \frac{1}{C_p} (i_{u,d} - i_{1d} - i_{2d} - i_{3d} - i_{4d}) + \omega_s v_{pcc,q}, \\
\frac{d}{dt} v_{pcc,q} &= \frac{1}{C_p} (i_{u,q} - i_{1q} - i_{2q} - i_{3q} - i_{4q}) - \omega_s v_{pcc,d}.
\end{align*} \]

where $\omega_s$ is the synchronous speed of rotation of the global $dq$ frame of reference.

**B.2 Small-Signal Model of the Study Microgrid**

The first step in obtaining a small-signal representation of the overall study microgrid, in the general form of (B.10), is to linearize (B.3), (B.5), (B.6), (B.7), and (B.9) around an operating point.

\[ \Delta \dot{x} = A \Delta x + B \Delta u \quad (B.10) \]

**B.2.1 Small-Signal Model of the DER Units**

As before, the small-signal dynamic model of the DER unit is composed of the small-signal models of: (i) the power circuit, (ii) the PLL, and (iii) the built-in current controller.
Appendix B. Details of the Microgrid Model

B.2.1.1 Small-Signal Model of DER Unit Power Circuit

The small-signal dynamic model of the power circuit of each DER unit, in its local $d_nq_n$ frame of reference, is obtained by linearizing (B.3), as:

$$
\Delta \dot{i}_{DRn} = A_i,DRn \Delta i_{DRn} + B_{iDRn}^V \Delta v_{DRn} + B_{iDRn}^{\Delta v_{tn}} \Delta v_{tn},
$$

$$\Delta \dot{v}_{DRn} = A_v,DRn \Delta v_{DRn} + B_{iDRn}^V \Delta i_{DRn} + B_{iDRn}^{\Delta v_{tn}} \Delta i_{Tn}.
$$

To derive the small-signal model of the overall microgrid, state vectors $\Delta i_{DRn}$ and $\Delta v_{DRn}$ should be expressed in the global $dq$ coordinate. Figure B.7 depicts the angular relationship between the global $dq$ and the local $d_nq_n$ reference frames. The phase difference between these two reference frames and the angular velocity of the global and local reference frames are expressed as $\delta_n$, $\omega_s$, and $\omega_{rn}$, respectively.

For a counter clockwise rotation around the origin by an angle $\delta_n$, the following rotation matrix could be used:

$$
\begin{bmatrix}
  f_d^g \\
  f_q^g
\end{bmatrix} =
\begin{bmatrix}
  \cos \delta_n & -\sin \delta_n \\
  \sin \delta_n & \cos \delta_n
\end{bmatrix}
\begin{bmatrix}
  f_{dn} \\
  f_{qn}
\end{bmatrix}
$$

in which $f_n$ is a vector defined in the local $d_nq_n$ frame. Linearized expression for this rotation matrix is obtained in (B.13):

$$
\begin{bmatrix}
  \Delta f_d^g \\
  \Delta f_q^g
\end{bmatrix} =
\begin{bmatrix}
  \cos \delta_n^0 & -\sin \delta_n^0 \\
  \sin \delta_n^0 & \cos \delta_n^0
\end{bmatrix}
\begin{bmatrix}
  \Delta f_{dn} \\
  \Delta f_{qn}
\end{bmatrix} +
\begin{bmatrix}
  -f_q^0 \\
  f_d^0
\end{bmatrix} \Delta \delta_n
$$

where superscript “0” indicates the steady-state value of the quantity at the given operating point.

Equation (B.13) is employed to transfer the state vectors $\Delta i_{DRn}$ and $\Delta v_{DRn}$ in (B.11) from the local $d_nq_n$ into the global $dq$ frame of reference.
Appendix B. Details of the Microgrid Model

\[ \Delta i_{DRn} = T_n^0 \Delta i_{DRn} + i_{\delta n}^0 \Delta \delta_n, \]  
\[ \Delta v_{DRn} = T_n^0 \Delta v_{DRn} + v_{\delta n}^0 \Delta \delta_n \]  

where \( T_n^0 = \begin{bmatrix} \cos \delta_n^0 & -\sin \delta_n^0 \\ \sin \delta_n^0 & \cos \delta_n^0 \end{bmatrix} \), \( i_{\delta n}^0 = \begin{bmatrix} -i_{DRn,q}^0 \\ i_{DRn,d}^0 \end{bmatrix} \), and \( v_{\delta n}^0 = \begin{bmatrix} -v_{DRn,q}^0 \\ v_{DRn,d}^0 \end{bmatrix} \)

Solving for \( \Delta i_{DRn} \) and \( \Delta v_{DRn} \) from (B.14) yields:

\[ \Delta i_{DRn} = (T_n^0)^{-1} \left[ \Delta i_{DRn}^0 - i_{\delta n}^0 \Delta \delta_n \right], \]
\[ \Delta v_{DRn} = (T_n^0)^{-1} \left[ \Delta v_{DRn}^0 - v_{\delta n}^0 \Delta \delta_n \right]. \]  

Substituting for \( \Delta i_{DRn} \) and \( \Delta v_{DRn} \) from (B.15) into (B.11) results in:

\[ \Delta i_{DRn} = T_n^0 A_{i,DRn} (T_n^0)^{-1} \left[ \Delta i_{DRn}^0 - i_{\delta n}^0 \Delta \delta_n \right] + T_n^0 B_{i,DRn}^v (T_n^0)^{-1} \left[ \Delta v_{DRn}^0 - v_{\delta n}^0 \Delta \delta_n \right] \\
+ T_n^0 B_{i,DRn}^{\omega_\delta} v_{\omega n} + i_{\delta n}^0 \Delta \dot{\delta}_n, \]
\[ \Delta v_{DRn} = T_n^0 A_{v,DRn} (T_n^0)^{-1} \left[ \Delta v_{DRn}^0 - v_{\delta n}^0 \Delta \delta_n \right] + T_n^0 B_{v,DRn}^i (T_n^0)^{-1} \left[ \Delta i_{DRn}^0 - i_{\delta n}^0 \Delta \delta_n \right] \\
+ T_n^0 B_{v,DRn}^{i,\omega} i_{\omega n} + v_{\delta n}^0 \Delta \dot{\delta}_n, \]  

Considering Figure B.7, it can be concluded that \( \frac{d}{dt} \delta_n = \omega_r - \omega_s. \)

\[ \Delta i_{DRn} = T_n^0 A_{i,DRn} (T_n^0)^{-1} \left[ \Delta i_{DRn}^0 - i_{\delta n}^0 \Delta \delta_n \right] + T_n^0 B_{i,DRn}^v (T_n^0)^{-1} \left[ \Delta v_{DRn}^0 - v_{\delta n}^0 \Delta \delta_n \right] \\
+ T_n^0 B_{i,DRn}^{\omega_\delta} v_{\omega n} + i_{\delta n}^0 \Delta \omega_r - i_{\delta n}^0 \Delta \omega_s, \]
\[ \Delta v_{DRn} = T_n^0 A_{v,DRn} (T_n^0)^{-1} \left[ \Delta v_{DRn}^0 - v_{\delta n}^0 \Delta \delta_n \right] + T_n^0 B_{v,DRn}^i (T_n^0)^{-1} \left[ \Delta i_{DRn}^0 - i_{\delta n}^0 \Delta \delta_n \right] \\
+ T_n^0 B_{v,DRn}^{i,\omega} i_{\omega n} + v_{\delta n}^0 \Delta \omega_r - v_{\delta n}^0 \Delta \omega_s, \]  

Furthermore, \( \Delta \delta_n \) in (B.17) can be deduced as a function of both the \( PC_n \) voltage in the global \( dq \) frame of reference, i.e., \( \Delta v_{DRn}^0 \), and the PLL angle \( \Delta \theta_n \). As indicated in Figure B.8, one can write:

\[ \theta_n = \tan^{-1} \left( \frac{v_{DRn,qn}}{v_{DRn,drn}} \right) \]  

The linearized from of (B.18) is:

\[ \Delta \theta_n = \frac{-v_{DRn,qn}^0}{(v_{DRn,drn})^2 + (v_{DRn,qn})^2} \Delta v_{DRn,drn} + \frac{v_{DRn,drn}^0}{(v_{DRn,drn})^2 + (v_{DRn,qn})^2} \Delta v_{DRn,qn}. \]  

or, in a matrix form:
Figure B.8: Projection of the \( n \)th PC voltage onto its local \( dq \) frame of reference

\[
\Delta \theta_n = M_n \Delta v_{DRn}, \tag{B.20}
\]

where

\[
M_n = \begin{bmatrix}
m_{nd} & m_{nq} \\
\end{bmatrix} = \begin{bmatrix}
-\frac{v^0_{DRn,qn}}{(v^0_{DRn,dn})^2 + (v^0_{DRn,qn})^2} & \frac{v^0_{DRn,dn}}{(v^0_{DRn,dn})^2 + (v^0_{DRn,qn})^2}
\end{bmatrix}. \tag{B.21}
\]

As mentioned in (B.14),

\[
\Delta v^g_{DRn} = T^0_n \Delta v_{DRn} + v^0_{\delta_n} \Delta \delta_n \tag{B.22}
\]

Solving for \( \Delta v_{DRn} \) from (B.22) and pre-multiplying by \( M_n \) results in:

\[
M_n \Delta v_{DRn} = M_n (T^0_n)^{-1} \Delta v^g_{DRn} = M_n (T^0_n)^{-1} - M_n T^0_n B^0_{\delta n} \Delta \delta_n \tag{B.23}
\]

Substituting for \( \Delta v_{DRn} \) from (B.20) into (B.23) and then rearranging yields:

\[
\Delta \delta_n = M_n (T^0_n)^{-1} \Delta v^g_{DRn} - \Delta \theta_n \tag{B.24}
\]

Equation (B.24) provides \( \Delta \delta_n \) as a function of the desired variables. Thus, substituting for \( \Delta \delta_n \) from (B.24) into (B.17) results in:

\[
\begin{align*}
\Delta \dot{i}_{DRn}^g &= A^g_{i,DRn} \Delta i_{DRn}^g + B^g_{i,DRn} \Delta i_{DRn}^g + B^0_{i,DRn} \Delta \theta_n + T^0_n B^0_{\delta n} \Delta \delta_n + i_{\delta n}^0 (\Delta \omega_{rn}^n - \Delta \omega_s), \\
\Delta \dot{v}_{DRn}^g &= A^g_{v,DRn} \Delta v_{DRn}^g + B^g_{v,DRn} \Delta i_{DRn}^g + B^0_{v,DRn} \Delta \theta_n + T^0_n B^0_{\delta n} \Delta \delta_n + v^0_{\delta n} (\Delta \omega_{rn}^n - \Delta \omega_s),
\end{align*} \tag{B.25}
\]

where
Appendix B. Details of the Microgrid Model

\[ A_{i,D_{Rn}}^g = T_n^0 A_{i,D_{Rn}} (T_n^0)^{-1}, \]
\[ B_{v_{D_{Rn}}}^{\theta} = T_n^0 B_{v_{D_{Rn}}}^{\theta} (T_n^0)^{-1} - \left( T_n^0 A_{i,D_{Rn}} (T_n^0)^{-1} i_{\delta_n}^0 + T_n^0 B_{v_{D_{Rn}}}^{\theta} (T_n^0)^{-1} v_{\delta_n}^0 \right) M_n (T_n^0)^{-1}, \]
\[ B_{v_{D_{Rn}}}^{\theta} = \left( T_n^0 A_{i,D_{Rn}} (T_n^0)^{-1} i_{\delta_n}^0 + T_n^0 B_{v_{D_{Rn}}}^{\theta} (T_n^0)^{-1} v_{\delta_n}^0 \right), \]
\[ A_{v_{D_{Rn}}}^g = T_n^0 A_{v,D_{Rn}} (T_n^0)^{-1} - \left( T_n^0 A_{i,D_{Rn}} (T_n^0)^{-1} i_{\delta_n}^0 + T_n^0 B_{v_{D_{Rn}}}^{\theta} (T_n^0)^{-1} v_{\delta_n}^0 \right) M_n (T_n^0)^{-1}, \]
\[ B_{v_{D_{Rn}}}^{\theta} = T_n^0 B_{v_{D_{Rn}}}^{\theta} (T_n^0)^{-1}, \]

and

\[ B_{v_{D_{Rn}}}^{\theta} = \left( T_n^0 A_{v,D_{Rn}} (T_n^0)^{-1} i_{\delta_n}^0 + T_n^0 B_{v_{D_{Rn}}}^{\theta} (T_n^0)^{-1} v_{\delta_n}^0 \right). \]

B.2.1.2 Small-Signal Model of the Phase-Locked Loop

The small-signal dynamic model of the PLL, depicted Figure [B.4] is obtained from linearization of (B.5) and (B.6):

\[ \Delta \dot{\omega}_{rn} = k_{pll,n} k_{pw,n} \Delta \omega_{ref,n} - k_{pll,n} k_{pw,n} \Delta \omega_{rn} + k_{pll,n} k_{iw,n} \Delta \theta_{ref,n} - k_{pll,n} k_{iw,n} \Delta \theta_n \]  

\[ \Delta \dot{\theta}_n = \Delta \omega_{rn} \]  

B.2.1.3 Small-Signal Model of the Current Controller

The small-signal dynamic model of the \( n \)th current controllers, depicted in Fig. [B.5] are:

\[ \frac{d}{dt} \Delta v_{i,dn} = \Delta i_{ref,d,n} - \Delta i_{dn}^f, \]
\[ \frac{d}{dt} \Delta v_{i,qn} = \Delta i_{ref,q,n} - \Delta i_{qn}^f, \]  

The converter terminal voltage vectors \( v_{t,dn} \) and \( v_{t,qn} \) can be expressed in terms of the control system state variables, as:

\[ \Delta v_{t,dn} = \Delta v_{dn}^f - \omega_{rn} L_{sn} \Delta i_{dn}^f + K_{iin} \Delta v_{i,dn} + K_{pin} \Delta \dot{v}_{i,dn}, \]
\[ \Delta v_{t,qn} = \Delta v_{qn}^f + \omega_{rn} L_{sn} \Delta i_{dn}^f + K_{irn} \Delta v_{i,qn} + K_{prn} \Delta \dot{v}_{i,qn}, \]  

Also, a first order low-pass filter with the time constant \( \tau_i \) can be presented in the state-space as:
\[ \Delta \dot{x}_f = \frac{1}{\tau_i} \Delta x - \frac{1}{\tau_i} \Delta x_f \]  \hspace{1cm} (B.30)

Based on (B.28), (B.30), and (B.29), a state-space representation for the small signal dynamic model of the current controller obtains as:

\[
\Delta \dot{x}_{cc,n} = A_{cc,n} \Delta x_{cc,n} + C^{xDRn}_{cc,n} \Delta i_{DRn} + C^{vDRn}_{cc,n} \Delta v_{DRn} + B^{uDRn}_{cc,n} \Delta u_{DRn} + B^{\theta n}_{cc,n} \Delta \theta_n,
\]

\[
\Delta v_{tn} = C^{xcc,n}_{Vtn} \Delta x_{cc,n} + C^{xcc,npr}_{Vtn} \Delta i_{cc,n}
\]  \hspace{1cm} (B.31)

where \( \Delta x_{cc,n} = [\Delta v_{fd,n}, \Delta v_{fq,n}, \Delta i_{fd,n}, \Delta i_{fq,n}, \Delta v_{id,n}, \Delta v_{i,qn}]^T \), \( \Delta u_{DRn} = [\Delta i_{refd,n}, \Delta i_{refq,n}]^T \), and

\[
A_{Cin} = \begin{bmatrix} -\frac{1}{T_{vn}} & 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{1}{T_{vn}} & 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{1}{T_{in}} & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 \end{bmatrix}
\]

\[
C^{xDRn}_{Cin} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{T_{in}} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{T_{in}} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} (T_{n})^{-1}
\]

\[
C^{vDRn}_{Cin} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} (T_{n})^{-1}
\]

\[
B^{\theta n}_{Cin} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}
\]

\[
C^{xcc,n}_{Vtn} = \begin{bmatrix} 1 & 0 & 0 & -\omega r_n L_{sn} & K_{iin} & 0 \\ 0 & 1 & \omega r_n L_{sn} & 0 & 0 & K_{iyn} \end{bmatrix}
\]

\[
C^{xcc,npr}_{Vtn} = \begin{bmatrix} 0 & 0 & 0 & 0 & K_{pin} & 0 \\ 0 & 0 & 0 & 0 & 0 & K_{pnn} \end{bmatrix}
\]

and

\[
B^{\theta n}_{Cin} = C^{xDRn}_{Cin} \Delta i_{\delta n} + C^{vDRn}_{Cin} \Delta v_{\delta n}
\]
B.2.2 Small-Signal Model of the Power Network: Grid-Connected Mode

A small-signal dynamic model of the power network of Figure B.1 is obtained from linearizing the set of equations in (B.9):

\[
\frac{d}{dt} \Delta i_{1,d} = \frac{1}{L_1} (\Delta v_{pcc,d} - \Delta v_{gB1,d}^g) - \frac{R_1}{L_1} \Delta i_{1,d} + \omega_s \Delta i_{1,q},
\]

\[
\frac{d}{dt} \Delta i_{1,q} = \frac{1}{L_1} (\Delta v_{pcc,q} - \Delta v_{gB1,q}^g) - \frac{R_1}{L_1} \Delta i_{1,q} - \omega_s \Delta i_{1,d},
\]

\[
\frac{d}{dt} \Delta i_{2,d} = \frac{1}{L_2} (\Delta v_{pcc,d} - \Delta i_2) - \frac{R_2}{L_2} \Delta i_{2,d} + \omega_s \Delta i_{2,q},
\]

\[
\frac{d}{dt} \Delta i_{2,q} = \frac{1}{L_2} (\Delta v_{pcc,q} - \Delta i_2) - \frac{R_2}{L_2} \Delta i_{2,q} - \omega_s \Delta i_{2,d},
\]

\[
\frac{d}{dt} \Delta i_{3,d} = \frac{1}{L_3} (\Delta v_{pcc,d} - \Delta v_{gB3,d}^g) - \frac{R_3}{L_3} \Delta i_{3,d} + \omega_s \Delta i_{3,q},
\]

\[
\frac{d}{dt} \Delta i_{3,q} = \frac{1}{L_3} (\Delta v_{pcc,q} - \Delta v_{gB3,q}^g) - \frac{R_3}{L_3} \Delta i_{3,q} - \omega_s \Delta i_{3,d},
\]

\[
\frac{d}{dt} \Delta i_{4,d} = \frac{1}{L_4} (\Delta v_{pcc,d} - \Delta v_{gB4,d}^g) - \frac{R_4}{L_4} \Delta i_{4,d} + \omega_s \Delta i_{4,q},
\]

\[
\frac{d}{dt} \Delta i_{4,q} = \frac{1}{L_4} (\Delta v_{pcc,q} - \Delta v_{gB4,q}^g) - \frac{R_4}{L_4} \Delta i_{4,q} - \omega_s \Delta i_{4,d},
\]

\[
\frac{d}{dt} \Delta i_{5,d} = \frac{1}{L_5} (\Delta v_{gB6,d}^g - \Delta v_{gB3,d}^g) - \frac{R_5}{L_5} \Delta i_{5,d} + \omega_s \Delta i_{5,q},
\]

\[
\frac{d}{dt} \Delta i_{5,q} = \frac{1}{L_5} (\Delta v_{gB6,q}^g - \Delta v_{gB3,q}^g) - \frac{R_5}{L_5} \Delta i_{5,q} - \omega_s \Delta i_{5,d},
\]

\[
\frac{d}{dt} \Delta i_{u,d} = \frac{1}{L_u} (\Delta v_{u,d}^g - \Delta v_{pcc,d}) - \frac{R_u}{L_u} \Delta i_{u,d} + \omega_s \Delta i_{u,q},
\]

\[
\frac{d}{dt} \Delta i_{u,q} = \frac{1}{L_u} (\Delta v_{u,q}^g - \Delta v_{pcc,q}) - \frac{R_u}{L_u} \Delta i_{u,q} - \omega_s \Delta i_{u,d},
\]
\[
\begin{align*}
\frac{d}{dt} \Delta i_{T1,d} &= \frac{1}{L_{T1}} (\Delta v_{DR1,d}^g - \Delta v_{B1,d}^g) - \frac{R_{T1}}{L_{T1}} \Delta i_{T1,d} + \omega_s \Delta i_{T1,q}, \\
\frac{d}{dt} \Delta i_{T1,q} &= \frac{1}{L_{T1}} (\Delta v_{DR1,q}^g - \Delta v_{B1,q}^g) - \frac{R_{T1}}{L_{T1}} \Delta i_{T1,q} - \omega_s \Delta i_{T1,d}, \\
\frac{d}{dt} \Delta i_{T2,d} &= \frac{1}{L_{T2}} (\Delta v_{DR2,d}^g - \Delta v_{B3,d}^g) - \frac{R_{T2}}{L_{T2}} \Delta i_{T2,d} + \omega_s \Delta i_{T2,q}, \\
\frac{d}{dt} \Delta i_{T2,q} &= \frac{1}{L_{T2}} (\Delta v_{DR2,q}^g - \Delta v_{B3,q}^g) - \frac{R_{T2}}{L_{T2}} \Delta i_{T2,q} - \omega_s \Delta i_{T2,d}, \\
\frac{d}{dt} \Delta i_{T3,d} &= \frac{1}{L_{T3}} (\Delta v_{DR3,d}^g - \Delta v_{B6,d}^g) - \frac{R_{T3}}{L_{T3}} \Delta i_{T3,d} + \omega_s \Delta i_{T3,q}, \\
\frac{d}{dt} \Delta i_{T3,q} &= \frac{1}{L_{T3}} (\Delta v_{DR3,q}^g - \Delta v_{B6,q}^g) - \frac{R_{T3}}{L_{T3}} \Delta i_{T3,q} - \omega_s \Delta i_{T3,d}, \\
\frac{d}{dt} \Delta i_{T4,d} &= \frac{1}{L_{T4}} (\Delta v_{DR4,d}^g - \Delta v_{B4,d}^g) - \frac{R_{T4}}{L_{T4}} \Delta i_{T4,d} + \omega_s \Delta i_{T4,q}, \\
\frac{d}{dt} \Delta i_{T4,q} &= \frac{1}{L_{T4}} (\Delta v_{DR4,q}^g - \Delta v_{B4,q}^g) - \frac{R_{T4}}{L_{T4}} \Delta i_{T4,q} - \omega_s \Delta i_{T4,d}, \\
\frac{d}{dt} \Delta v_{pcc,d} &= \frac{1}{C_p} (\Delta i_{u,d} - \Delta i_{1d} - \Delta i_{2d} - \Delta i_{3d} - \Delta i_{4d}) + \omega_s \Delta v_{pcc,q}, \\
\frac{d}{dt} \Delta v_{pcc,q} &= \frac{1}{C_p} (\Delta i_{u,q} - \Delta i_{1q} - \Delta i_{2q} - \Delta i_{3q} - \Delta i_{4q}) - \omega_s \Delta v_{pcc,d}.
\end{align*}
\]

Based on the set of equations [B.32], a linear state-space representation of the small-signal dynamic model of the power network is:

\[
\Delta \dot{x}_N = A_N \Delta x_N + B_N^{v_{DR1}} \Delta v_{DR1}^g + B_N^{v_{DR2}} \Delta v_{DR2}^g + B_N^{v_{DR3}} \Delta v_{DR3}^g + B_N^{v_{DR4}} \Delta v_{DR4}^g + B_N^{v_{B1}} \Delta v_{B1}^g + B_N^{v_{B2}} \Delta v_{B2}^g + B_N^{v_{B3}} \Delta v_{B3}^g + B_N^{v_{B4}} \Delta v_{B4}^g + B_N^{v_u} \Delta v_u,
\]

where

\[
\Delta x_N = [\Delta i_{1d} \Delta i_{1q} \Delta i_{2d} \Delta i_{2q} \Delta i_{3d} \Delta i_{3q} \Delta i_{4d} \Delta i_{4q} \Delta i_{5d} \Delta i_{5q} \Delta i_{u,d} \Delta i_{u,q} \Delta i_{T1,d} \Delta i_{T1,q} \Delta i_{T2,d} \Delta i_{T2,q} \Delta i_{T3,d} \Delta i_{T3,q} \Delta i_{T4,d} \Delta i_{T4,q} \Delta v_{pcc,d} \Delta v_{pcc,q}]^T,
\]

and
\[
\Delta v_u = \begin{bmatrix} \Delta v_{u,d} & \Delta v_{u,q} \end{bmatrix}^T, \quad \Delta v_{DRn}^g = \begin{bmatrix} \Delta v_{DRn,d}^g & \Delta v_{DRn,q}^g \end{bmatrix}^T, \quad \Delta v_{Bn}^g = \begin{bmatrix} \Delta v_{Bn,d}^g & \Delta v_{Bn,q}^g \end{bmatrix}^T,
\]

for \(n=1, \ldots, 4\).
\[ B_{N}^{v_{DR1}} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1/L_{T1} & 0 \\ 0 & 1/L_{T1} \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, \quad B_{N}^{v_{DR2}} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1/L_{T2} & 0 \\ 0 & 1/L_{T2} \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, \quad B_{N}^{v_{DR3}} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \]
\[
B_{N}^{v_{DR4}} = \begin{bmatrix}
0 & 0 \\
0 & 0 \\
0 & 0 \\
0 & 0 \\
0 & 0 \\
0 & 0 \\
0 & 0 \\
0 & 0 \\
0 & 0 \\
\frac{1}{L_4} & 0 \\
0 & \frac{1}{L_4} \\
0 & 0 \\
0 & 0
\end{bmatrix},
\]

\[
B_{N}^{v_{B1}} = \begin{bmatrix}
-\frac{1}{L_1} & 0 \\
0 & -\frac{1}{L_1} \\
0 & 0 \\
0 & 0 \\
0 & 0 \\
0 & 0 \\
0 & 0 \\
0 & 0 \\
0 & 0 \\
0 & 0 \\
0 & 0 \\
0 & 0 \\
0 & 0
\end{bmatrix},
\]

\[
B_{N}^{v_{B3}} = \begin{bmatrix}
0 & 0 \\
0 & 0 \\
-\frac{1}{L_3} & 0 \\
0 & -\frac{1}{L_3} \\
0 & 0 \\
0 & 0 \\
0 & 0 \\
0 & 0 \\
0 & 0 \\
0 & 0 \\
0 & 0 \\
0 & 0 \\
0 & 0
\end{bmatrix},
\]
In order to derive the overall dynamic equations, variables $\Delta v^g_{Bn}$ for $n = 1, \ldots, 4$ are needed to be expressed in terms of state vectors $\Delta x_N$. From Figure B.1 one can writes:

\begin{align*}
\Delta v^{g\text{B}1}_{abc} &= R_{l1}(\Delta i_{T1} + \Delta i_1) + L_{l1} \frac{d}{dt}(\Delta i_{T1} + \Delta i_1) \\
\Delta v^{g\text{B}3}_{abc} &= R_{l2}(\Delta i_{T2} + \Delta i_3 + \Delta i_5) + L_{l2} \frac{d}{dt}(\Delta i_{T2} + \Delta i_3 + \Delta i_5) \\
\Delta v^{g\text{B}4}_{abc} &= R_{l4}(\Delta i_{T4} + \Delta i_4) + L_{l4} \frac{d}{dt}(\Delta i_{T4} + \Delta i_4) \\
\Delta v^{g\text{B}6}_{abc} &= R_{l3}(\Delta i_{T3} - \Delta i_5) + L_{l3} \frac{d}{dt}(\Delta i_{T3} + \Delta i_5)
\end{align*}

Transferring these equations into the global $dq$ frame of reference yields:

\begin{align*}
B^g_{N4} &= \begin{bmatrix}
0 & 0 \\
0 & 0 \\
0 & 0 \\
0 & 0 \\
0 & 0 \\
0 & 0 \\
0 & 0 \\
-\frac{1}{L_4} & 0 \\
0 & -\frac{1}{L_4} \\
0 & 0 \\
0 & 0 \\
0 & 0 \\
0 & 0 \\
0 & 0 \\
0 & 0 \\
\end{bmatrix}, & B^g_{N6} &= \begin{bmatrix}
0 & 0 \\
0 & 0 \\
0 & 0 \\
\frac{1}{L_5} & 0 \\
0 & \frac{1}{L_5} \\
0 & 0 \\
0 & 0 \\
0 & 0 \\
0 & 0 \\
0 & 0 \\
0 & 0 \\
0 & 0 \\
0 & 0 \\
\end{bmatrix}, & B^u_{N} &= \begin{bmatrix}
\frac{1}{L_u} & 0 \\
0 & \frac{1}{L_u} \\
0 & 0 \\
0 & 0 \\
0 & 0 \\
0 & 0 \\
0 & 0 \\
0 & 0 \\
0 & 0 \\
0 & 0 \\
0 & 0 \\
0 & 0 \\
0 & 0 \\
\end{bmatrix}.
\end{align*}
\[\Delta v_{B1,d}^g = R_{l1} \Delta i_{T1,d} + R_{l1} \Delta i_{1,d} + L_{l1} \frac{d}{dt} \Delta i_{T1,d} + L_{l1} \frac{d}{dt} \Delta i_{1,d} + L_{l1} \omega_s \Delta i_{T1,q} - L_{l1} \omega_s \Delta i_{1,q}\]

\[\Delta v_{B1,q}^g = R_{l1} \Delta i_{T1,q} + R_{l1} \Delta i_{1,q} + L_{l1} \frac{d}{dt} \Delta i_{T1,q} + L_{l1} \frac{d}{dt} \Delta i_{1,q} + L_{l1} \omega_s \Delta i_{T1,d} + L_{l1} \omega_s \Delta i_{1,d}.\]  

(B.38)

\[\Delta v_{B3,d}^g = R_{l2} \Delta i_{T2,d} + R_{l2} \Delta i_{3,d} + R_{l2} \Delta i_{5,d} + L_{l2} \frac{d}{dt} \Delta i_{T2,d} + L_{l2} \frac{d}{dt} \Delta i_{3,d} + L_{l2} \frac{d}{dt} \Delta i_{5,d} - L_{l2} \omega_s \Delta i_{T2,q} - L_{l2} \omega_s \Delta i_{3,q} - L_{l2} \omega_s \Delta i_{5,q}\]

\[\Delta v_{B3,q}^g = R_{l2} \Delta i_{T2,q} + R_{l2} \Delta i_{3,q} + R_{l2} \Delta i_{5,q} + L_{l2} \frac{d}{dt} \Delta i_{T2,q} + L_{l2} \frac{d}{dt} \Delta i_{3,q} + L_{l2} \frac{d}{dt} \Delta i_{5,q} + L_{l2} \omega_s \Delta i_{T2,d} + L_{l2} \omega_s \Delta i_{3,d} + L_{l2} \omega_s \Delta i_{5,d}.\]  

(B.39)

\[\Delta v_{B4,d}^g = R_{l4} \Delta i_{T4,d} + R_{l4} \Delta i_{4,d} + L_{l4} \frac{d}{dt} \Delta i_{T4,d} + L_{l4} \frac{d}{dt} \Delta i_{4,d} + L_{l4} \omega_s \Delta i_{T4,q} - L_{l4} \omega_s \Delta i_{4,q}\]

\[\Delta v_{B4,q}^g = R_{l4} \Delta i_{T4,q} + R_{l4} \Delta i_{4,q} + L_{l4} \frac{d}{dt} \Delta i_{T4,q} + L_{l4} \frac{d}{dt} \Delta i_{4,q} + L_{l4} \omega_s \Delta i_{T4,d} + L_{l4} \omega_s \Delta i_{4,d}.\]  

(B.40)

\[\Delta v_{B6,d}^g = R_{l3} \Delta i_{T3,d} - R_{l3} \Delta i_{5,d} + L_{l3} \frac{d}{dt} \Delta i_{T3,d} - L_{l3} \frac{d}{dt} \Delta i_{5,d} - L_{l3} \omega_s \Delta i_{T3,q} + L_{l3} \omega_s \Delta i_{5,q}\]

\[\Delta v_{B6,q}^g = R_{l3} \Delta i_{T3,q} - R_{l3} \Delta i_{5,q} + L_{l3} \frac{d}{dt} \Delta i_{T3,q} - L_{l3} \frac{d}{dt} \Delta i_{5,q} + L_{l3} \omega_s \Delta i_{T3,d} - L_{l3} \omega_s \Delta i_{5,d}.\]  

(B.41)

Thus, \(\Delta v_{Bn}^g\) can be written as:

\[\Delta v_{Bn}^g = E_{v_Bn}^{xN} \Delta x_N + E_{v_Bn}^{\dot{x}N} \Delta \dot{x}_N\]  

(B.42)

where
$E_{v_{B1}^*}^{x_N} = \begin{bmatrix} R_{l1} & L_{l1}\omega_s \\ -L_{l1}\omega_s & R_{l1} \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ R_{l1} & L_{l1}\omega_s \\ -L_{l1}\omega_s & R_{l1} \end{bmatrix}^T$, $E_{v_{B3}^*}^{x_N} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ R_{l2} & L_{l2}\omega_s \\ -L_{l2}\omega_s & R_{l2} \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ R_{l2} & L_{l2}\omega_s \\ -L_{l2}\omega_s & R_{l2} \end{bmatrix}^T$, $E_{v_{B4}^*}^{x_N} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ R_{l4} & L_{l4}\omega_s \\ -L_{l4}\omega_s & R_{l4} \end{bmatrix}^T$. 
Appendix B. Details of the Microgrid Model

\[ E_{v_{B6}}^{x_N} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -R_{l3} & -L_{l3} \omega_s \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -R_{l3} & -L_{l3} \omega_s \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ L_{l3} \omega_s & -R_{l3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ R_{l3} & L_{l3} \omega_s & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -L_{l3} \omega_s & R_{l3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \]

\[ E_{v_{B1}}^{x_N} = \begin{bmatrix} L_{l1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \]

\[ E_{v_{B3}}^{x_N} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \]
Substituting for $\Delta v^g_{Bn}$ from (B.42) into (B.33) yields the state-space representation of the small-signal dynamic model of the power network as:

$$
\Delta \dot{x}_N = A_N \Delta x_N + B^v_{D R 1} \Delta v^g_{D R 1} + B^v_{D R 2} \Delta v^g_{D R 2} + B^v_{D R 3} \Delta v^g_{D R 3} + B^v_{D R 4} \Delta v^g_{D R 4} + B^v_{B 1} E^x_{v^g_{B 1}} \Delta x_N + B^v_{B 3} E^x_{v^g_{B 3}} \Delta x_N + B^v_{B 4} E^x_{v^g_{B 4}} \Delta x_N + B^v_{B 6} E^x_{v^g_{B 6}} \Delta x_N + B^v_{u} \Delta v_u,
$$

(B.43)

B.2.3 Small-Signal Model of the Power Network: Islanded Mode

When the microgrid is operating in the islanded mode, utility grid dynamics do not contribute in the microgrid dynamics. Thus, the state vector of the power network will be modified as:
\[ \Delta x_{N,is} = \begin{bmatrix} \Delta i_{1d} & \Delta i_{1q} & \Delta i_{2d} & \Delta i_{2q} & \Delta i_{3d} & \Delta i_{3q} & \Delta i_{4d} & \Delta i_{4q} & \Delta i_{5d} & \Delta i_{5q} & \Delta i_{T1,d} & \Delta i_{T1,q} & \Delta i_{T2,d} & \Delta i_{T2,q} & \Delta i_{T3,d} & \Delta i_{T3,q} & \Delta i_{T4,d} & \Delta i_{T4,q} & \Delta v_{pcc,d} & \Delta v_{pcc,q} \end{bmatrix}^T, \]

Therefore, the state-space model of the power network in the islanded mode is deduced from \((B.32)\) by removing the utility current \(i_u\) from the equations:

\[ \dot{x}_{N,is} = A_{N,is} \Delta x_{N,is} + B_{\sigma N,is}^D \Delta v_{DR1} + B_{\sigma N,is}^D \Delta v_{DR2} + B_{\sigma N,is}^D \Delta v_{DR3} + B_{\sigma N,is}^D \Delta v_{DR4} + B_{\sigma N,is}^D \Delta v_{B1,is} + B_{\sigma N,is}^D \Delta v_{B2,is} + B_{\sigma N,is}^D \Delta v_{B3,is} + B_{\sigma N,is}^D \Delta v_{B4,is}, \]

in which

\[ A_{N,is} = \begin{bmatrix} -R_{1}/L_{1} & \omega & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0&
Appendix B. Details of the Microgrid Model

\[ B_{N,i8}^{\text{v}_g^{\text{DR}_1}} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ \end{bmatrix} \]

\[ B_{N,i8}^{\text{v}_g^{\text{DR}_2}} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ \end{bmatrix} \]

\[ B_{N,i8}^{\text{v}_g^{\text{DR}_3}} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ \end{bmatrix} \]
\[ B_{N,ls}^{v_{DR4}} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ \frac{1}{L_4} & 0 \\ 0 & \frac{1}{L_4} \\ 0 & 0 \end{bmatrix}, \quad B_{N,ls}^{v_{g1}} = \begin{bmatrix} -\frac{1}{L_1} & 0 \\ 0 & -\frac{1}{L_1} \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, \quad B_{N,ls}^{v_{g3}} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ -\frac{1}{L_3} & 0 \\ 0 & -\frac{1}{L_3} \\ 0 & 0 \\ 0 & 0 \\ -\frac{1}{L_5} & 0 \\ 0 & -\frac{1}{L_5} \\ 0 & 0 \end{bmatrix}, \]
Appendix B. Details of the Microgrid Model

\[
B_{\text{N, is}}^{v_{4}} = \begin{bmatrix}
0 & 0 \\
0 & 0 \\
0 & 0 \\
0 & 0 \\
0 & 0 \\
0 & 0 \\
0 & 0 \\
0 & 0 \\
0 & 0 \\
-\frac{1}{L_4} & 0 \\
0 & -\frac{1}{L_4} \\
0 & 0 \\
0 & 0 \\
0 & 0 \\
0 & 0 \\
0 & 0 \\
0 & 0 \\
0 & 0 \\
0 & 0 \\
0 & 0 \\
0 & 0 \\
0 & 0 \\
\end{bmatrix}, \quad B_{\text{N, is}}^{v_{6}} = \begin{bmatrix}
0 & 0 \\
0 & 0 \\
\frac{1}{L_5} & 0 \\
0 & \frac{1}{L_5} \\
0 & 0 \\
0 & 0 \\
0 & 0 \\
0 & 0 \\
0 & 0 \\
-\frac{1}{L_{T3}} & 0 \\
0 & -\frac{1}{L_{T3}} \\
0 & 0 \\
0 & 0 \\
0 & 0 \\
0 & 0 \\
0 & 0 \\
0 & 0 \\
0 & 0 \\
0 & 0 \\
0 & 0 \\
\end{bmatrix}.
\]

Also, variables \( \Delta v_{\text{Bn, is}}^{g} \) for \( n = 1, ..., 4 \) can be written as:

\[
\Delta v_{\text{Bn, is}}^{g} = E_{\text{v}_{\text{Bn, is}}}^{x_{\text{N, is}}} \Delta x_{\text{N, is}} + E_{\text{v}_{\text{Bn, is}}}^{x_{\text{N, is}}} \Delta i_{\text{N, is}} \quad \text{(B.45)}
\]

where
### Appendix B. Details of the Microgrid Model

\[ E_{v_{B1,ls}}^{x_N,ls} = \begin{bmatrix} R_{l1} & L_{l1}\omega_s \\ -L_{l1}\omega_s & R_{l1} \end{bmatrix}^{T} \]

\[ E_{v_{B2,ls}}^{x_N,ls} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}^{T} \]

\[ E_{v_{B3,ls}}^{x_N,ls} = \begin{bmatrix} R_{l2} & L_{l2}\omega_s \\ -L_{l2}\omega_s & R_{l2} \end{bmatrix}^{T} \]

\[ E_{v_{B4,ls}}^{x_N,ls} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}^{T} \]
\[
E_{v_{B6, is}}^{x_{N, is}} = \begin{bmatrix}
0 & 0 \\
0 & 0 \\
0 & 0 \\
0 & 0 \\
0 & 0 \\
0 & 0 \\
0 & 0 \\
0 & 0 \\
\end{bmatrix}^T
\]

\[
E_{v_{B1, is}}^{x_{N, is}} = \begin{bmatrix}
L_{t1} & 0 \\
0 & L_{t1} \\
0 & 0 \\
0 & 0 \\
0 & 0 \\
\end{bmatrix}^T
\]

\[
E_{v_{B3, is}}^{x_{N, is}} = \begin{bmatrix}
0 & 0 \\
0 & 0 \\
0 & 0 \\
L_{t2} & 0 \\
0 & L_{t2} \\
0 & 0 \\
0 & 0 \\
\end{bmatrix}^T
\]
Substituting for $\Delta v_{B1, is}^g$ from (B.45) into (B.44) yields the state-space representation of the small-signal dynamic model of the power network in the islanded mode as:

$$\Delta \dot{x}_{N, is} = A_{N, is} \Delta x_{N, is} + B_{N, is}^{v_B} \Delta v_{B1, is} + B_{N, is}^{v_D} \Delta v_{D1} + B_{N, is}^{v_D} \Delta v_{D2} + B_{N, is}^{v_D} \Delta v_{D3} + B_{N, is}^{v_D} \Delta v_{D4}$$

$$+ B_{N, is}^{v_B} E_{v_B, is}^x \Delta x_{N, is} + B_{N, is}^{v_B} E_{v_B, is}^{\dot{x}} \Delta \dot{x}_{N, is} + B_{N, is}^{v_B} E_{v_B, is}^{\ddot{x}} \Delta \ddot{x}_{N, is} + B_{N, is}^{v_B} E_{v_B, is}^{\dddot{x}} \Delta \dddot{x}_{N, is} \Delta x_{N, is}$$

$$+ B_{N, is}^{v_B} E_{v_B, is}^x \Delta x_{N, is} + B_{N, is}^{v_B} E_{v_B, is}^{\dot{x}} \Delta \dot{x}_{N, is} + B_{N, is}^{v_B} E_{v_B, is}^{\ddot{x}} \Delta \ddot{x}_{N, is} + B_{N, is}^{v_B} E_{v_B, is}^{\dddot{x}} \Delta \dddot{x}_{N, is}.$$  

(B.46)

**B.2.4 Small-Signal Model of the Integrated System: Grid-Connected Mode**

In this section, a small-signal dynamic model of the microgrid system, operating in the grid-connected mode, is developed by integrating the small-signal dynamic models of:
Appendix B. Details of the Microgrid Model

(i) four DER units power circuit, given by (B.25), (ii) four PLLs, given by (B.26) and (B.27), (iii) four DER units current controllers, given by (B.31), and (iv) the power network, given by (B.43), based on their input-output relationships shown in Figure B.9.

The state vector of the integrated open-loop VPP, operating in the grid-connected mode, system and the input vectors are as follows:

\[
\Delta x = [\Delta x_{DR}^{g1}, \Delta x_{DR}^{g2}, \Delta x_{DR}^{g3}, \Delta x_{DR}^{g4}, \Delta x_{cc,1}, \Delta x_{cc,2}, \Delta x_{cc,3}, \Delta x_{cc,4}, \Delta x_N]^T,
\]

\[
\Delta x_{DR}^{g} = [\Delta i_{DR}^{g1}, \Delta i_{DR}^{g2}, \Delta i_{DR}^{g3}, \Delta i_{DR}^{g4}, \Delta \omega_{ref,n}]^T,
\]

\[
\Delta u_{DR} = [\Delta u_{DR1}, \Delta u_{DR2}, \Delta u_{DR3}, \Delta u_{DR4}]^T,
\]

\[
\Delta u_{DRn} = [\Delta i_{ref,n}, \Delta \theta_{ref,n}]^T,
\]

\[
\Delta u_{pll} = [\Delta u_{pll1}, \Delta u_{pll2}, \Delta u_{pll3}, \Delta u_{pll4}]^T,
\]

\[
\Delta u_{plln} = [\Delta \theta_{ref,n}, \Delta \omega_{ref,n}]^T,
\]

for \( n = 1, \cdots, 4 \).

Therefore, the small-signal dynamic equation of the integrated system can be expressed as:

\[
\Delta \dot{x} = A^{aux} \Delta x + F \Delta \dot{x} + B^{aux}_{c1} \Delta u_{DR1} + B^{aux}_{c2} \Delta u_{DR2} + B^{aux}_{c3} \Delta u_{DR3} + B^{aux}_{c4} \Delta u_{DR4} \quad (B.47)
\]

+ \( B^{aux}_{pll1} \Delta u_{pll1} + B^{aux}_{pll2} \Delta u_{pll2} + B^{aux}_{pll3} \Delta u_{pll3} + B^{aux}_{pll4} \Delta u_{pll4} + B^{aux}_{d} \Delta v_{u} + B^{aux}_{w} \Delta \omega_{s} \),

in which

\[
A^{aux} = \begin{bmatrix}
A_{1}^{DR1} & 0 & 0 & 0 & A_{1}^{Ci1} & 0 & 0 & 0 & A_{1}^{N} \\
0 & A_{2}^{DR2} & 0 & 0 & 0 & A_{2}^{Ci2} & 0 & 0 & A_{2}^{N} \\
0 & 0 & A_{3}^{DR3} & 0 & 0 & 0 & A_{3}^{Ci3} & 0 & A_{3}^{N} \\
0 & 0 & 0 & A_{4}^{DR4} & 0 & 0 & 0 & A_{4}^{Ci4} & A_{4}^{N} \\
A_{N} & A_{N}^{2} & A_{N}^{3} & A_{N}^{4} & 0 & 0 & 0 & 0 & A_{NN}
\end{bmatrix}.
\]
Appendix B. Details of the Microgrid Model

Figure B.9: Block diagram of the input-output relationships of the subsystem for developing state-space equations

\[
A_{DRn} = \begin{bmatrix}
A_{i,DRn} & B_{v,DRn}^g & B_{v,DRn}^b & -j\delta_{s,n} \\
B_{v,DRn}^g & A_{v,DRn} & B_{v,DRn}^b & -j\delta_{a,n} \\
0 & 0 & 0 & 1
\end{bmatrix},
\]

\[
A_{N} = \begin{bmatrix}
0 \\
B_{v,DRn}^g \\
0
\end{bmatrix},
\]

\[
A_{Cin} = T_n B_{i,DRn}^v C_{Vin},
\]

\[
A_{N} = [0, B_{v,DRn}^g, 0, 0].
\]

for \( n = 1, ..., 4 \) and \( A_{NN} = A_{N} + B_{v,DRn}^{g_1} E_{v,DRn}^{x_1} + B_{v,DRn}^{g_2} E_{v,DRn}^{x_2} + B_{v,DRn}^{g_3} E_{v,DRn}^{x_3} + B_{v,DRn}^{g_4} E_{v,DRn}^{x_4} + B_{v,DRn}^{g_5} E_{v,DRn}^{x_5}. \)

Also,

\[
B_{v,DRn}^{x_1} = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix},
\]

\[
B_{v,DRn}^{x_2} = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix},
\]
\[ B_{c,DR3}^{xN} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{C_{f3}} \cos \delta^0_3 & \frac{1}{C_{f3}} \sin \delta^0_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{C_{f3}} \sin \delta^0_3 & -\frac{1}{C_{f3}} \cos \delta^0_3 & 0 & 0 & 0 \end{bmatrix}, \]

\[ B_{c,DR4}^{xN} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{C_{f4}} \cos \delta^0_4 & \frac{1}{C_{f4}} \sin \delta^0_4 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{C_{f4}} \sin \delta^0_4 & -\frac{1}{C_{f4}} \cos \delta^0_4 & 0 & 0 \end{bmatrix}. \]

Also

\[ F = \begin{bmatrix} 0 & 0 & 0 & 0 & F^{C_11}_{i} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & F^{C_22}_{i} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & F^{C_33}_{i} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & F^{C_44}_{i} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \]

where \( F^{C_{in}}_i = T_{m}^{0} B_{vtn}^{p_{tn}} C_{vtn}^{p_{tn}} \) and \( f_{NN} = B_{N}^{v_{B1}} E_{B1}^{v_{B1}} + B_{N}^{v_{B3}} E_{B3}^{v_{B3}} + B_{N}^{v_{B4}} E_{B4}^{v_{B4}} + B_{N}^{v_{B6}} E_{B6}^{v_{B6}} \).

Besides,

\[ B_{aux}^{p_{pl1}} = \begin{bmatrix} B_{pl1} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad B_{aux}^{p_{pl2}} = \begin{bmatrix} B_{pl2} \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad B_{aux}^{p_{pl3}} = \begin{bmatrix} B_{pl3} \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad B_{aux}^{p_{pl4}} = \begin{bmatrix} B_{pl4} \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \]

where

\[ B_{pl,n} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, k_{pl,n}, k_{iw,n}, k_{pw,n} \]

for \( n = 1, \ldots, 4. \)
Appendix B. Details of the Microgrid Model

\[
B_{c1}^{aux} = \begin{bmatrix}
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
B_{uDR1}^{C11}
\end{bmatrix}, \quad B_{c1}^{aux} = \begin{bmatrix}
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
B_{uDR2}^{C12}
\end{bmatrix},
\]

\[
B_{c3}^{aux} = \begin{bmatrix}
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
B_{uDR3}^{C13}
\end{bmatrix}, \quad B_{c3}^{aux} = \begin{bmatrix}
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
B_{uDR4}^{C14}
\end{bmatrix},
\]

\[
B_{d}^{aux} = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & B_{N}^{v}\n\end{bmatrix}^T,
\]

\[
B_{w}^{aux} = \begin{bmatrix}
B_{w1} & B_{w2} & B_{w3} & B_{w4} & 0 & 0 & 0 & 0 & 0
\end{bmatrix}^T,
\]

\[
B_{wn} = \begin{bmatrix}
i_{δ_n} & i_{δ_n} & 0 & 0
\end{bmatrix}^T \text{ for } n = 1, ..., 4.
\]

Equation (B.47) can be rewritten as:

\[
(I - F) \Delta \dot{x} = A^{aux} \Delta x + F \Delta \dot{x} + B_{c1}^{aux} \Delta u_{DR1} + B_{c2}^{aux} \Delta u_{DR2} + B_{c3}^{aux} \Delta u_{DR3} + B_{c4}^{aux} \Delta u_{DR4} + B_{d}^{aux} \Delta v_u + B_{w}^{aux} \Delta \omega_s,
\]

or

\[
\Delta \dot{x} = (I - F)^{-1} (A^{aux} \Delta x + F \Delta \dot{x} + B_{c1}^{aux} \Delta u_{DR1} + B_{c2}^{aux} \Delta u_{DR2} + B_{c3}^{aux} \Delta u_{DR3} + B_{c4}^{aux} \Delta u_{DR4} + B_{d}^{aux} \Delta v_u + B_{w}^{aux} \Delta \omega_s),
\]

Finally, the linearized state-space dynamical model of the grid-connected microgrid is
Appendix B. Details of the Microgrid Model

obtained as:

\[
\Delta \dot{x} = A \Delta x + B_{c1} \Delta u_{DR1} + B_{c2} \Delta u_{DR2} + B_{c3} \Delta u_{DR3} + B_{c4} \Delta u_{DR4} \\
+ B_{pl1} \Delta u_{pl1} + B_{pl2} \Delta u_{pl2} + B_{pl3} \Delta u_{pl3} + B_{pl4} \Delta u_{pl4} + B_d \Delta v_u + B_w \Delta \omega_s.
\]

where

\[
A = (I - F)^{-1} A^{aux}, \\
B_{cn} = (I - F)^{-1} B_{cn}^{aux}, \\
B_{plin} = (I - F)^{-1} B_{plin}^{aux}, \\
B_d = (I - F)^{-1} B_d^{aux}, \\
B_w = (I - F)^{-1} B_w^{aux},
\]

B.2.5 Small-Signal Model of the Integrated System: Islanded Mode

In the islanded mode, the state vector of the integrated microgrid system obtains as:

\[
\Delta x_{is} = [\Delta x_{DR1}^g, \Delta x_{DR2}^g, \Delta x_{DR3}^g, \Delta x_{DR4}^g, \Delta x_{cc,1}, \Delta x_{cc,2}, \Delta x_{cc,3}, \Delta x_{cc,4}, \Delta x_{N,is}]^T,
\]

Therefore, the small-signal dynamic equation of the integrated system in the islanded mode can be expressed as:

\[
\Delta \dot{x}_{is} = A_{is}^{aux} \Delta x_{is} + F_{is} \Delta \dot{x}_{is} + B_{c1,is}^{aux} \Delta u_{DR1} + B_{c2,is}^{aux} \Delta u_{DR2} + B_{c3,is}^{aux} \Delta u_{DR3} + B_{c4,is}^{aux} \Delta u_{DR4} \\
+ B_{pl1,is}^{aux} \Delta u_{pl1} + B_{pl2,is}^{aux} \Delta u_{pl2} + B_{pl3,is}^{aux} \Delta u_{pl3} + B_{pl4,is}^{aux} \Delta u_{pl4}. \quad (B.48)
\]

in which
\[ \begin{align*}
A_{is}^{aux} &= \begin{bmatrix}
A_{1}^{DR1} & 0 & 0 & 0 & A_{1}^{Gi1} & 0 & 0 & 0 & A_{1,N,is}^{N,\text{is}} \\
0 & A_{2}^{DR2} & 0 & 0 & 0 & A_{2}^{Gi2} & 0 & 0 & A_{2,N,is}^{N,\text{is}} \\
0 & 0 & A_{3}^{DR3} & 0 & 0 & 0 & A_{3}^{Gi3} & 0 & A_{3,N,is}^{N,\text{is}} \\
0 & 0 & 0 & A_{4}^{DR4} & 0 & 0 & 0 & A_{4}^{Gi4} & A_{4,N,is}^{N,\text{is}} \\
A_{1,N,is} & A_{2,N,is} & A_{3,N,is} & A_{4,N,is} & 0 & 0 & 0 & 0 & A_{NN,N,is}^{N,\text{is}} 
\end{bmatrix}, \\
A_{N,is}^N &= \begin{bmatrix}
0 \\
B_{N,is}^{v_{B1}} \\
0 \\
0 
\end{bmatrix}, \quad A_{N,is}^n = [0, B_{N,is}^{v_{B2}}, 0, 0].
\end{align*} 
\]

for \( n = 1, ..., 4 \) and \( A_{NN,N,is} = A_{N,is} + B_{N,is}^{v_{B1}} E_{N,is}^{x_{B1}} + B_{N,is}^{v_{B2}} E_{N,is}^{x_{B2}} + B_{N,is}^{v_{B3}} E_{N,is}^{x_{B3}} + B_{N,is}^{v_{B4}} E_{N,is}^{x_{B4}}. \)

Also,
\[ \begin{align*}
B_{N,is}^{x_{B1}} &= \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & c_{j1} & \cos \delta^0_1 & \frac{1}{c_{j1}} & \sin \delta^0_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & c_{j1} & \sin \delta^0_1 & -\frac{1}{c_{j1}} & \cos \delta^0_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 
\end{bmatrix}, \\
B_{N,is}^{x_{B2}} &= \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & c_{j2} & \cos \delta^0_2 & \frac{1}{c_{j2}} & \sin \delta^0_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & c_{j2} & \sin \delta^0_2 & -\frac{1}{c_{j2}} & \cos \delta^0_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 
\end{bmatrix}, \\
B_{N,is}^{x_{B3}} &= \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & c_{j3} & \cos \delta^0_3 & \frac{1}{c_{j3}} & \sin \delta^0_3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & c_{j3} & \sin \delta^0_3 & -\frac{1}{c_{j3}} & \cos \delta^0_3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 
\end{bmatrix}, \\
B_{N,is}^{x_{B4}} &= \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & c_{j4} & \cos \delta^0_4 & \frac{1}{c_{j4}} & \sin \delta^0_4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & c_{j4} & \sin \delta^0_4 & -\frac{1}{c_{j4}} & \cos \delta^0_4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 
\end{bmatrix}. 
\end{align*} \]

Also,
\[ F_{is} = \begin{bmatrix}
0 & 0 & 0 & F_{1}^{Ci1} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & F_{2}^{Ci2} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & F_{3}^{Ci3} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & F_{4}^{Ci4} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\end{bmatrix} f_{NN,N,is}. \]
where \( f_{N_{i},i} = B_{N_{i},i}^{N_{i}} E_{N_{i},i}^{N_{i}} + B_{N_{i},i}^{B_{3_{i},i}} E_{B_{3_{i},i}}^{B_{3_{i},i}} + B_{N_{i},i}^{B_{4_{i},i}} E_{B_{4_{i},i}}^{B_{4_{i},i}} + B_{N_{i},i}^{B_{6_{i},i}} E_{B_{6_{i},i}}^{B_{6_{i},i}} \).

Equation (B.48) can be rewritten as:

\[
(I - F_{i}) \Delta \dot{x}_{i} = A_{is}^{aux} \Delta x_{is} + F_{is} \Delta \dot{x}_{is} + B_{c1_{is}}^{aux} \Delta u_{DR1} + B_{c2_{is}}^{aux} \Delta u_{DR2} + B_{c3_{is}}^{aux} \Delta u_{DR3} + B_{c4_{is}}^{aux} \Delta u_{DR4} + B_{c2_{is}}^{aux} \Delta u_{pl1} + B_{c3_{is}}^{aux} \Delta u_{pl2} + B_{c4_{is}}^{aux} \Delta u_{pl3} + B_{c4_{is}}^{aux} \Delta u_{pl4},
\]

or

\[
\Delta \dot{x}_{is} = (I - F_{is})^{-1} (A_{is}^{aux} \Delta x_{is} + F_{is} \Delta \dot{x}_{is} + B_{c1_{is}}^{aux} \Delta u_{DR1} + B_{c2_{is}}^{aux} \Delta u_{DR2} + B_{c3_{is}}^{aux} \Delta u_{DR3} + B_{c4_{is}}^{aux} \Delta u_{DR4} + B_{c2_{is}}^{aux} \Delta u_{pl1} + B_{c3_{is}}^{aux} \Delta u_{pl2} + B_{c4_{is}}^{aux} \Delta u_{pl3} + B_{c4_{is}}^{aux} \Delta u_{pl4}),
\]

Finally, the linearized state-space dynamical model of the islanded microgrid is obtained as:

\[
\Delta \dot{x}_{is} = A_{is} \Delta x_{is} + B_{c1_{is}} \Delta u_{DR1} + B_{c2_{is}} \Delta u_{DR2} + B_{c3_{is}} \Delta u_{DR3} + B_{c4_{is}} \Delta u_{DR4} + B_{pl1_{is}} \Delta u_{pl1} + B_{pl2_{is}} \Delta u_{pl2} + B_{pl3_{is}} \Delta u_{pl3} + B_{pl4_{is}} \Delta u_{pl4},
\]

where \( A_{is} = (I - F_{is})^{-1} A_{is}^{aux} \), \( B_{cn_{is}} = (I - F_{is})^{-1} B_{cn_{is}}^{aux} \), and \( B_{pln_{is}} = (I - F_{is})^{-1} B_{pln_{is}}^{aux} \).

### B.3 Model Verification

The accuracy of the stability analysis and controller synthesis for a dynamical system depends on the accuracy of the mathematical model used. Therefore, this section seeks to verify the accuracy of the derived dynamical model of the study microgrid. To that end, the linearized state-space dynamical model of the grid-connected microgrid (B.48) is coded in MATLAB environment and its response to a step change increase in the current reference command is simulated. Similar scenario is then simulated for a detailed nonlinear model of the study microgrid of Figure 2.1 developed in PSCAD/EMTDC environment.

Prior to disturbances, DER1 to DER3 are commanded to track the references of \( i_{refd,n} = 1 \text{ kA} \) and \( i_{refq,n} = 0.2 \text{ kA} \) for \( n = 1, ..., 3 \) and DER4 to track the references of \( i_{refd4} = 0.5 \text{ kA} \) and \( i_{refq4} = 0.0 \text{ kA} \). At \( t = 0.2 \text{ sec} \), a 10% step change increase happens to both the \( d \) and \( q \) components of the reference command of DER1. Simulation results obtained in MATLAB and PSCAD/EMTDC platforms, depicting the \( d \) and \( q \) components of the output currents of the four DERs and the bus voltage of \( DER_1 \), are compared in
Figure B.10: DERs response to step change increase in the reference current of DER\textsubscript{1}: (a) the \textit{d} components, (b) the \textit{q} components.

Figs. B.10 and B.11. As it can be concluded, the analytically developed linear model reflects the dynamics of the detailed nonlinear system with acceptable accuracy.

Figure B.11: The \textit{d} and \textit{q} components of $PC_1$ voltage
Appendix C

Integral Quadratic Constraint Uncertainty Description

C.1 Background

In order to design the desired robust primary controller, the first step would be identifying the sources of uncertainties and disturbances affecting the system under study. Then, a suitable mathematical description should be employed to model these uncertainties and disturbances. Based on this uncertain system model, the robust controller will then be developed.

Suitability of the uncertain system model is evaluated not only by its comprehensiveness to model various sources of uncertainties impacting the system, but also by the tractability of the robust control problem corresponding to this uncertain system model. Many different types of uncertainty description can be identified in the literature, such as parametric, value-bounded, polytope-bounded, and element-bounded uncertainties or uncertainties satisfying a matching condition \[\text{[34]}\]. Each of these uncertainty models addresses just limited types of uncertainties affecting a VPP system.

A comprehensive uncertainty model which allows for considering a wide class of perturbations, with the minimum required information, is the Integral Quadratic Constraint (IQC) framework \[\text{[74]}\].

The IQC uncertainty description originates in the work of Yakubovich who treated the stability problem for systems with advanced nonlinearities \[\text{[75]}\]. This uncertainty framework allows for a wider class of perturbations, either in the local subsystems or in the interactions among them. Nonlinear and time varying uncertainties, saturation, exogenous disturbances, stochastic noise, and uncertain time delays could also be modelled...
by IQCs. Extracting structural information about perturbations is another advantage of modelling uncertain systems using IQCs which results in designing less conservative controllers [34].

Using the IQC uncertainty description, interactions among subsystems can also be considered as an uncertainty. Therefore, the decentralized controllers designed based on this method do not rely on a specific configuration. This property is highly desirable for a VPP system which is subject to frequent topological changes. In addition, since DERs in a VPP are operated by different owners, treating the interconnections as uncertain perturbations is more motivated. These unique properties of the IQC framework make it promising to model various types of uncertainties and disturbances in a VPP.

A wide body of research can be found on developing robust controllers for various types of uncertain systems all described by IQCs, including [76–79]. However, these works consider a single uncertain system and are not suitable for developing decentralized LCs for interconnected subsystems of a large-scale system.

Reference [35] presents necessary and sufficient conditions for the existence of decentralized static state-feedback controllers using IQC uncertainty description. In this work, apart from the local internal uncertainties and inter-subsystem interaction uncertainties, each subsystem is also driven by an exogenous uncertainty input signal. However, in a VPP system, measuring the entire state vector or estimating them through observers imposes high costs and implementation complexities. Therefore, output-feedback robust LCs are more desirable.

In [43], robust decentralized dynamic output-feedback switching controllers are designed for a class of uncertain Markov jump parameter systems. The interconnected system is subjected to uncertain disturbances and random changes in its parameters which are modelled using a continuous time finite-state Markov chain. While local and interconnection uncertainties are modelled using IQCs, this research work fails to guarantee robust performance to the exogenous disturbances.

The methodology proposed in [43] is applied for designing decentralized power system stabilizers for interconnected power systems, with modifications to include the variations in the parameters around the operating point, in [80]. However, as indicated in [81], in practical power systems, it is difficult to implement the switching controllers as unwanted transients may arise due to switching. Therefore, [81] proposes a decentralized robust controller using minimax linear quadratic output-feedback control design approach for static synchronous compensators (STATCOMs). The IQC model used in this paper does not contain exogenous disturbances. Besides, as discussed before, LCs of a VPP should be able to track time-varying set points.
As it can be concluded, the problem of designing IQC-based dynamic output-tracking decentralized controllers still needs to be solved. Thus, this dissertation generalizes the robust decentralized static state-feedback control strategy presented in [35] to propose a robust, decentralized, dynamic, output feedback, reference tracking control strategy for a general dynamic uncertain system composed of multiple interacting subsystems.

C.2 Schematic Diagram of the Uncertain System

Provided in Sections 2.5 and 2.6.2, the $n^{th}$ subsystem of an uncertain dynamical system composed of $N$ subsystems, for $n = 1, ..., N$, and affected by local and interconnection uncertainties $\xi_n(t)$ and $\eta_n(t)$ and exogenous disturbance $w_n(t)$ can be described as:

$$
\dot{x}_n(t) = A_n x_n(t) + B_n u_n(t) + E_n \xi_n(t) + L_n \eta_n(t) + F_n w_n(t),
$$

$$
y_{p,n}(t) = C_{y,n} x_n(t) + D_{y,n} w_n(t),
$$

$$
\zeta_n(t) = H_n x_n(t) + G_n u_n(t),
$$

$$
\xi_n(t) = \psi_n(t, \zeta_n(.)),
$$

$$
\eta_n(t) = \phi_n(t, \zeta_j(.)), \text{ for } j = 1, ..., N, j \neq n
$$

Figure C.1 shows the first and the second subsystems of the overall uncertain system illustrating their input-output relationships and their local and interconnection uncertainty blocks.

C.3 Proof of Theorem 1

Proof. (i) $\Rightarrow$ (ii). Let us follow the lines of the proof of Theorem 9.3.1 in [34] and generalize it for the case of designing output-tracking local controllers. Equation (9.3.24) in [34] is re-written here as:

$$
\sup_{\bar{w}(\cdot) \in L_2[0,\infty), ||\bar{w}(\cdot)||_2 > 0} \frac{||\bar{z}(\cdot)||_2^2}{||\bar{w}(\cdot)||_2^2} < 1 - \delta_2 \epsilon
$$

(C.1)

where $\bar{w}(\cdot) = [T_1^{1/2} \xi_1(\cdot), \sqrt{\theta_1} \eta_1(\cdot), \gamma w_1(\cdot), ..., T_N^{1/2} \xi_N(\cdot), \sqrt{\theta_N} \eta_N(\cdot), \gamma w_N(\cdot)]$ is the disturbance input of the closed-loop large-scale system composed of the N controllers of (2.9) and the open-loop system of the form
Appendix C. Integral Quadratic Constraint Uncertainty Description

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Figure C.1: The large-scale closed-loop uncertain system

\[
\dot{x} = Ax(t) + \bar{B}_1 u(t) + \bar{B}_2 \bar{w}(t); \quad x(0) = 0;
\]
\[
\bar{z} = \bar{C} x(t) + \bar{D} u(t);
\]
\[
\bar{y} = \bar{C}_y x(t) + \bar{D}_y \bar{w}(t)
\]

\text{(C.2)}

where \(A = \text{diag}[A_1, ..., A_N]; \bar{B}_1 = \text{diag}[B_1, ..., B_N] \bar{B}_2 = \text{diag}[\bar{B}_{2,1}, ..., \bar{B}_{2,N}], \bar{C} = \text{diag}[\bar{C}_1, ..., \bar{C}_N], \bar{D} = \text{diag}[\bar{D}_1, ..., \bar{D}_N], \bar{C}_y = \text{diag}[\bar{C}_{y,1}, ..., \bar{C}_{y,N}], \bar{D}_y = \text{diag}[\bar{D}_{2,1}, ..., \bar{D}_{2,N}].\) The constant \(\epsilon\) in (C.1) is a constant satisfying

\[
0 < \epsilon \leq \frac{1 + \sum_{n=1}^{N} (\theta_n + \sum_{k=1}^{k_n} \tau_{n,k,n})}{\max_{n=1,...,N}(\theta_n, ||T_n||, \gamma^2)}.
\]

Therefore, (C.1) implies that for \(n = 1, ..., N,\)

\[
\sup_{\bar{w}_n(\cdot) \in L_2[0,\infty)} \frac{||z_n(\cdot)||_2^2}{||\bar{w}_n(\cdot)||_2^2} < 1 - \delta \epsilon \quad \text{(C.3)}
\]

where \(\bar{w}_n(\cdot) = [T_{n}^{1/2} \zeta_n(\cdot), 0, \gamma w_n(\cdot)]\) is the disturbance input for the closed-loop system composed of the controller (2.9) with the initial condition \(x_{pc,n}(0) = 0\) and the open-loop
Appendix C. Integral Quadratic Constraint Uncertainty Description

System

\[ \begin{align*}
\dot{x}_n &= A_n x_n(t) + B_n u_n(t) + \bar{B}_{2,n} \bar{w}_n(t); \quad x_n(0) = 0; \\
\bar{z}_n &= \bar{C}_n x_n(t) + \bar{D}_n u_n(t); \\
\bar{y}_n &= \bar{C}_{y,n} x_n(t) + \bar{D}_{2,n} \bar{w}_n(t).
\end{align*} \]

(C.4)

Condition (C.3) implies that \( u_n \) of the controller (2.9) solves the disturbance attenuation problem associated with system (C.4). Now, since according to Assumptions 1 pairs \((A_n, B_n)\) and (2.14) are stabilizable and pairs \((A_n, C_{y,n})\) and (2.13) are detectable, according to Lemmas 3.2.4 and 3.2.5 of [34], for each \( n = 1, ..., N \) the algebraic Riccati equations (2.15) and (2.16) both admit minimal non-negative definite stabilizing solutions \( X_n \) and \( Y_n \) which satisfy the condition \( \rho(X_n Y_n) < \gamma^2 \) such that four matrices in Theorem 1 are stable. It means the collection of the constants \( \tau_{n,v} \) and \( \theta_n \) determined from the S-procedure satisfy the conditions in Theorem 1.

\[(ii) \Rightarrow (i). \] Consider the system (C.2) and the matrices \( X = \text{diag}[X_1, ..., X_N] \) and \( Y = \text{diag}[Y_1, ..., Y_N] \). Since entries \( X_n \) and \( Y_n \) for \( n = 1, ..., N \) satisfy conditions in Theorem 1 by letting \( R = \text{diag}[R_1, ..., R_N] \), it concludes that matrices:

\[ A - \bar{B}_1 R^{-1}(\bar{B}_1' X + \bar{D}' \bar{C}) A - \bar{B}_1 R^{-1}(\bar{B}_1' X + \bar{D}' \bar{C}) + \bar{B}_2 \bar{B}_2' X, \]

\[ A - (Y \bar{C}_y + \bar{B}_2' \bar{D}_y) \Gamma_y^{-1} \bar{C}_y + Y \bar{C}' \bar{C}, \]

are stable, where \( \Gamma_y = \bar{D}_y \bar{D}_y' \). It also follows that non-negative definite matrices \( X \) and \( Y \) solve Riccati equations:

\[ (A - \bar{B}_1 R^{-1} \bar{D}' \bar{C})' X + X (A - \bar{B}_1 R^{-1} \bar{D}' \bar{C}) + \bar{C}' (I - \bar{D} R^{-1} \bar{D}') \bar{C} - X (\bar{B}_1 R^{-1} \bar{B}_1' - \bar{B}_2 \bar{B}_2') X = 0 \]

\[ (A - \bar{B}_2 \bar{D}_y \Gamma_y^{-1} C_y) Y + Y (A - \bar{B}_2 \bar{D}_y \Gamma_y^{-1} C_y)' + Y (\bar{C}_y \bar{C}_y' - \bar{C}_y \Gamma_y^{-1} \bar{C}_y) Y + B_2 (I - \bar{D}_y \Gamma_y^{-1} D_y) \bar{B}_2^T = 0 \]

Since the pairs \((A, \bar{B}_1)\) and \((A - \bar{E} \bar{D}_y (\bar{D}_y \bar{D}_y')^{-1} \bar{C}_y, \bar{E} (I - \bar{D}_y' (\bar{D}_y \bar{D}_y')^{-1} \bar{D}_y)\) are stabilizable and pairs \((A, \bar{C}_y)\) and \((A - \bar{B}_1 (\bar{D}' \bar{D})^{-1} \bar{D}' \bar{C}, (I - \bar{D} (\bar{D}' \bar{D})^{-1} \bar{D}') \bar{C})\) are detectable, then it follows from \( H_\infty \) control theory (e.g., see Theorem 3.2.3 of [34]) that controllers of the form (C.5) solve a standard output feedback \( H_\infty \) control problem defined by the system (C.2) and the \( H_\infty \) norm bound (C.1). Here, \( E = \text{diag}[E_1, ..., E_N] \), and \( A_c = \text{diag}[A_{c,1}, ..., A_{c,N}] \), \( B_c = \text{diag}[B_{c,1}, ..., B_{c,N}] \), \( C_c = \text{diag}[C_{c,1}, ..., C_{c,N}] \), and \( D_c = \text{diag}[D_{c,1}, ..., D_{c,N}] \).
In other words, there exist a constant $\epsilon > 0$ such that the closed-loop system of the form (C.6)

\[
\dot{x}_{cl}(t) = A_{cl}x_{cl}(t) + B_{cl}\bar{w}(t),
\]
\[
\dot{z}(t) = C_{cl}x_{cl}(t), \bar{y}(t) = C_{y,cl}x_{cl}(t) + \bar{D}_y\bar{w}(t). \tag{C.6}
\]

satisfies the condition $||\bar{z}(.)||_2^2 - ||\bar{w}(.)||_2^2 < -\epsilon||\bar{w}(.)||_2^2$ for all $\bar{w}(.) \in L_2[0,\infty)$ and $x(0) = 0, x_{pc}(0) = 0$. Here, $x_{cl} = \begin{bmatrix} x \\ x_{pc} \end{bmatrix}$ and $A_{cl} = \begin{bmatrix} A & B_1C_c \\ B_2C_{y,n} & A_c \end{bmatrix}$, $B_{cl} = \begin{bmatrix} \bar{B}_2 \\ B_2D_y \end{bmatrix}$, $C_{cl} = [\bar{C}, \bar{D}C_{cl}]$, and $C_{y,cl} = [\bar{C}_y, 0]$. From the Strict Bounded Real Lemma 3.1.2, [34], these conditions are equivalent to the existence of positive definite symmetric matrices $\bar{X} > X$ satisfying the matrix inequality $A_{cl}^T\bar{X} + \bar{X}A_{cl} + \bar{X}B_{cl}B_{cl}^T\bar{X} + C_{cl}C_{cl}^T \leq -\epsilon I$ for some $\epsilon > 0$. From this inequality and using $x_{cl}'\bar{X}x_{cl}$ as a candidate Lyapunov function, it can be verified that $\epsilon||x_{cl}(.)||_2^2 + ||\dot{z}(.)||_2^2 \leq ||\bar{w}(.)||_2^2 + x_{cl}'(0)\bar{X}x_{cl}(0)$ for all $\bar{w}(.) \in L_2[0,\infty)$. From this point again the line of proofs of Theorem 9.3.1 in [34] can be followed. \hfill \Box

### C.4 Proof of Theorem 2

**Proof.** Following the lines of the well known Strict Bounded Real Lemma, [34], one can conclude that the coupled Riccati equations of (2.15) and (2.16) have stabilizing solutions $X_n \succeq 0$ and $Y_n \succeq 0$ if and only if there exist matrices $\bar{X}_n > 0$ and $\bar{Y}_n > 0$ such that

As proposed by [43], consider the following two inequalities instead of the Riccati equations (2.15) and (2.16):

\[
(A_n - B_nR_n^{-1}\bar{D}_n^T\bar{C}_n)^T X_n + X_n(A_n - B_nR_n^{-1}\bar{D}_n^T\bar{C}_n) + \bar{C}_n^T(I - \bar{D}_nR_n^{-1}\bar{D}_n^T)\bar{C}_n - X_n(B_nR_n^{-1}B_n^T - \bar{B}_2n\bar{B}_2n^T)X_n < 0 \tag{C.7}
\]

\[
(A_n - \bar{B}_2n\bar{D}_2n^T\Gamma_{y,n}^{-1}C_{y,n})Y_n + Y_n(A_n - \bar{B}_2n\bar{D}_2n^T\Gamma_{y,n}^{-1}C_{y,n})^T + Y_n(\bar{C}_n^T\bar{C}_n - C_n^T\Gamma_{y,n}^{-1}C_{y,n})Y_n + \bar{B}_2n(I - \bar{D}_2n\Gamma_{y,n}^{-1}\bar{D}_2n)\bar{B}_2n^T < 0 \tag{C.8}
\]
Now, multiply (C.7) with $\tilde{X}_n = X_n^{-1}$ and (C.8) with $\tilde{Y}_n = Y_n^{-1}$, from left and right and then add

$$
[F_{x,n}^T + B_n R_n^{-1} + \tilde{X}_n (\bar{D}_n^T \bar{C}_n)^T R_n^{-1}] R_n [F_{x,n}^T + B_n R_n^{-1} + \tilde{X}_n (\bar{D}_n^T \bar{C}_n)^T R_n^{-1}]^T
$$

$$
[F_{y,n}^T + \tilde{Y}_n \bar{B}_{2,n} \bar{D}_{2,n}^T \Gamma_{y,n}^{-1} + C_{y,n} \Gamma_{y,n}^{-1}] \Gamma_{y,n} [F_{y,n}^T + \tilde{Y}_n \bar{B}_{2,n} \bar{D}_{2,n}^T \Gamma_{y,n}^{-1} + C_{y,n} \Gamma_{y,n}^{-1}]^T
$$

to the left-hand side of the obtained inequalities. Here, $F_{x,n}$ and $F_{y,n}$ are LMI variables of appropriate dimensions. Now, by applying Schur complement [82] to the above obtained inequalities together with (2.17), LMIs (2.20) and (2.21) are obtained. Furthermore, the spectral radius condition in Theorem 1 is equivalent to the LMIs (2.22). Besides, conditions $\tilde{\tau}_{m,v} > 0$, $\tau_{n,v} > 0$, $\tilde{\tau}_{n,v} \tau_{n,v} = 1$, $\theta_n > 0$, $\theta_n > 0$, $\tilde{\theta}_n \theta_n = 1$, $\tilde{\gamma} > 0$, $\gamma > 0$, and $\tilde{\gamma} \gamma = 1$ are equivalent to the rank constrained LMIs (2.23) and (2.24) [43].
Appendix D

Order of Reduction and the Reduced Primary Control Parameters

This appendix provides a guideline on how to select the appropriate order of reduction which yields the best trade-off between the simplicity of the reduced controller and approximation errors. Also, it presents the reduced-order controller matrices designed in Section 2.7.3 Chapter 2.

D.1 Order of Reduction

To choose the appropriate order of reduction, the Hankel singular values of the full-order robust power controller designed in Section 2.7.1 is obtained and plotted in Figure D.1. The size of each Hankel singular value is a relative measure of the contribution of each state in the input-output behavior of the dynamic system [48]. As Figure D.1 suggests, the first six states make the most contribution in the controller dynamics.

To quantify the accuracy of the reduced system, a time-domain and a frequency-

Figure D.1: Hankel singular value plot of the full-order power controller
Table D.1: Approximation errors of the reduced power controller $LC_1$

<table>
<thead>
<tr>
<th>error</th>
<th>$r=1$</th>
<th>$r=2$</th>
<th>$r=3$</th>
<th>$r=4$</th>
<th>$r=5$</th>
<th>$r=6$</th>
<th>$r=7$</th>
<th>$r=8$</th>
<th>$r=9$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\varepsilon_t$</td>
<td>$\infty$</td>
<td>$\infty$</td>
<td>$\infty$</td>
<td>0.2261</td>
<td>0.0560</td>
<td>0.0014</td>
<td>0.0010</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>$\varepsilon_f$</td>
<td>$\infty$</td>
<td>$\infty$</td>
<td>$\infty$</td>
<td>0.2488</td>
<td>0.0245</td>
<td>0.0006</td>
<td>0.0005</td>
<td>0.0003</td>
<td>0.0002</td>
</tr>
</tbody>
</table>

Domain approximation error are defined as:

$$\varepsilon_t = \sqrt{\frac{1}{n_t} \sum_{j=1}^{n_t} |P_{\text{full},i}(t) - P_{\text{red},i}(t)|^2_j}$$ (D.1)

$$\varepsilon_f = \|G_{c,i}^{\text{full}}(s) - G_{c,i}^{\text{red}}(s)\|_\infty$$ (D.2)

where $i = 1, \ldots, 4$ represent the $i^{th}$ local controller number. The real power generated by DER$_i$ in the microgrid controlled with full order and the reduced order primary controllers are depicted by $P_{\text{full},i}^{\text{DER}}$ and $P_{\text{red},i}^{\text{DER}}$, respectively. Also, $n_t$ represents the number of sampling points in the time-domain response of $P_{\text{DER},i}$. The transfer function of the $i^{th}$ local controller is shown by $G_{c,i}(s)$.

Table D.1 summarizes the time-domain and the frequency-domain approximation errors versus the order of the reduced controller $r$ for the $LC_1$ power controller. The sign $\infty$ in the first three columns means the instability of the reduced controlled system. Similar results can be achieved for the rest of LCs.

D.2 The Obtained Reduced Primary Power Controllers

The decentralized primary power controllers of the form (2.9), calculated in Section 2.7, are reduced to the order of $r = 5$ and provided in this section.

$$A_{c_1} = \begin{bmatrix} -0.9996 & 0.0000 & 0.0248 & 0.0110 & 0.0119 \\ 0.0000 & -0.9997 & -0.0081 & 0.0181 & -0.0022 \\ -0.0339 & 0.0149 & -1.4710 & -0.4169 & -1.2580 \\ -0.0176 & -0.0238 & -0.0950 & -0.9748 & -0.2477 \\ 0.0171 & 0.0008 & 1.0570 & 0.7305 & 1.3580 \end{bmatrix}, B_{c_1} = \begin{bmatrix} 15.1310 \\ -7.9044 \\ -1530.0012 \\ 193.5856 \\ 511.8845 \end{bmatrix},$$
Appendix D. Order of Reduction and the Reduced Primary Control Parameters

\[ A_2 = \begin{bmatrix} -0.8996 & -0.0001 & 0.0223 & 0.0099 & 0.0107 \\ 0.0004 & -0.8997 & -0.0073 & 0.0162 & -0.0020 \\ -0.03054 & 0.0134 & -1.3237 & -0.3752 & -1.1319 \\ -0.01589 & -0.0215 & -0.0855 & -0.8773 & -0.2228 \\ 0.0154 & 0.0097 & 0.9516 & 0.6574 & 1.2226 \end{bmatrix}, B_{c2} = \begin{bmatrix} 13.6213 \\ -7.1138 \\ -1376.7485 \\ 174.1311 \\ 460.64639 \end{bmatrix}, \]

\[ A_3 = \begin{bmatrix} -1.2994 & -0.0001 & 0.0323 & 0.0143 & 0.01554 \\ 0.0001 & -1.2996 & -0.0106 & 0.0235 & -0.0028 \\ -0.0441 & 0.0194 & -1.9120 & -0.5419 & -1.6350 \\ -0.0229 & -0.0310 & -0.1235 & -1.2672 & -0.3219 \\ 0.02234 & 0.0010 & 1.3745 & 0.9496 & 1.7660 \end{bmatrix}, B_{c3} = \begin{bmatrix} 19.6657 \\ -10.1242 \\ -1988.6534 \\ 251.5634 \\ 665.3016 \end{bmatrix}, \]

\[ A_4 = \begin{bmatrix} -0.2998 & -0.0000 & 0.0074 & 0.0033 & 0.0035 \\ 0.0000 & -0.2999 & -0.0024 & 0.0054 & -0.0006 \\ -0.0101 & 0.0044 & -0.4412 & -0.1250 & -0.3773 \\ -0.0053 & -0.0071 & -0.0285 & -0.2924 & -0.0742 \\ 0.0051 & 0.0002 & 0.3172 & 0.2191 & 0.4075 \end{bmatrix}, B_{c4} = \begin{bmatrix} 4.5404 \\ -2.3712 \\ -458.9161 \\ 58.0437 \\ 153.5487 \end{bmatrix}, \]

\[ C_{c1} = \begin{bmatrix} -0.003175 \\ -0.002914 \end{bmatrix}, D_{c1} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}. \]

\[ C_{c2} = \begin{bmatrix} -0.0028 \\ -0.0026 \end{bmatrix}, D_{c2} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}. \]

\[ C_{c3} = \begin{bmatrix} -0.0041 \\ -0.0037 \end{bmatrix}, D_{c3} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}. \]

\[ C_{c4} = \begin{bmatrix} -0.0009 \\ -0.0008 \end{bmatrix}, D_{c4} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}. \]
## D.3 The Obtained Reduced Primary Voltage Controllers

The decentralized primary voltage controllers of the form (2.9), calculated in Section 2.7, are reduced to the order of $r = 5$ and provided in this section.

$$A_{c_1} = \begin{bmatrix} -0.9995 & 0.0000 & -0.0036 & 0.0069 & -0.0024 \\ 0.0000 & -0.9995 & -0.0072 & -0.0035 & -0.0015 \\ 0.0030 & 0.0110 & 0.8211 & -0.0341 & -0.0594 \\ -0.0105 & 0.0029 & 0.0350 & 0.8237 & 0.0474 \\ -0.0002 & -0.0041 & 0.0717 & 0.0251 & 1.009 \end{bmatrix}, B_{c_1} = \begin{bmatrix} 101.7000 \\ 47.9800 \\ -1530.0012 \\ -1864.0000 \\ -406.1000 \end{bmatrix},$$

$$A_{c_2} = \begin{bmatrix} -0.8895 & 0.0012 & -0.0039 & 0.0081 & -0.0012 \\ 0.0100 & -0.9105 & -0.0152 & -0.0055 & -0.0055 \\ 0.0034 & 0.0242 & 0.7995 & -0.0341 & -0.0634 \\ -0.0147 & 0.0043 & 0.0412 & 0.7837 & 0.0547 \\ -0.0000 & -0.0033 & 0.0512 & 0.0153 & 0.9960 \end{bmatrix}, B_{c_2} = \begin{bmatrix} 98.0000 \\ 48.0100 \\ -1460.0012 \\ -1264.0000 \\ -387.1000 \end{bmatrix},$$

$$A_{c_3} = \begin{bmatrix} -1.0094 & -0.0012 & 0.0642 & 0.0645 & 0.0852 \\ 0.0032 & -1.0296 & -0.0534 & 0.0687 & -0.0100 \\ -0.0441 & 0.0194 & -1.0091 & -0.5419 & -0.6350 \\ -0.0229 & -0.0510 & -0.0235 & -1.2321 & -0.0019 \\ 0.02234 & 0.0016 & 0.0745 & 0.6490 & 1.1860 \end{bmatrix}, B_{c_3} = \begin{bmatrix} 154.0320 \\ 62.3773 \\ -1988.8412 \\ -2423.7659 \\ -527.9637 \end{bmatrix},$$

$$A_{c_4} = \begin{bmatrix} -0.2998 & -0.0000 & -0.0010 & 0.0020 & -0.0007 \\ 0.0000 & -0.2998 & -0.0021 & -0.0010 & -0.0004 \\ 0.0009 & 0.0032 & 0.2463 & -0.0102 & -0.0178 \\ -0.0031 & 0.0008 & 0.0105 & 0.2471 & 0.0142 \\ -0.0001 & -0.0012 & 0.0215 & 0.0075 & 0.3025 \end{bmatrix}, B_{c_4} = \begin{bmatrix} 30.4977 \\ 14.3947 \\ 458.9733 \\ -559.3306 \\ -121.8377 \end{bmatrix},$$

$$C_{c_1} = \begin{bmatrix} -0.003175 & 0.002431 & -0.2588 & 0.01614 & -0.1141 \\ -0.002914 & -0.002284 & -0.07474 & -0.2236 & -0.05083 \end{bmatrix}, D_{c_1} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}. $$

$$C_{c_2} = \begin{bmatrix} -0.0018 & -0.0017 & 0.0358 & -0.0130 & 0.0137 \\ 0.0016 & -0.0018 & 0.0130 & 0.0353 & -0.0027 \end{bmatrix}, D_{c_2} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}. $$
$C_{c3} = \begin{bmatrix}
-0.0025 & -0.0024 & 0.04980 & -0.0180 & 0.0191 \\
0.0022 & -0.0025 & 0.0181 & 0.0490 & -0.0036
\end{bmatrix}, \quad D_{c3} = \begin{bmatrix}
0 \\
0
\end{bmatrix}$.

$C_{c4} = \begin{bmatrix}
-0.0005 & -0.0005 & 0.0113 & -0.0041 & 0.0043 \\
-0.0000 & -0.0005 & 0.0041 & 0.0112 & -0.0008
\end{bmatrix}, \quad D_{c4} = \begin{bmatrix}
0 \\
0
\end{bmatrix}$. 
Appendix E

Steady-State Operating Points

The steady-state power generation and consumption for the selected operating point of the microgrid study system of Figure 2.1 are provided in this appendix. The operating point in the VPP-PQ, VPP-PV, and islanded modes are provided in the columns $2^{nd}-3^{rd}$, $4^{th}-5^{th}$, and $6^{th}-7^{th}$ of the Table E.1 respectively.

Table E.1: Steady-state power generation and consumption for the selected operating point

<table>
<thead>
<tr>
<th>Component</th>
<th>PQM Gen.</th>
<th>PQM Cons.</th>
<th>PVM Gen.</th>
<th>PVM Cons.</th>
<th>ISM Gen.</th>
<th>ISM Cons.</th>
</tr>
</thead>
<tbody>
<tr>
<td>DER$_1$</td>
<td>2.24 MW</td>
<td>-</td>
<td>1.94 MW</td>
<td>-</td>
<td>1.87 MW</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>0.07 MVAr</td>
<td>-</td>
<td>0.25 MVAr</td>
<td>-</td>
<td>-0.18 MVAr</td>
<td>-</td>
</tr>
<tr>
<td>DER$_2$</td>
<td>2.24 MW</td>
<td>-</td>
<td>1.94 MW</td>
<td>-</td>
<td>1.81 MW</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>0.07 MVAr</td>
<td>-</td>
<td>0.25 MVAr</td>
<td>-</td>
<td>-0.14 MVAr</td>
<td>-</td>
</tr>
<tr>
<td>DER$_3$</td>
<td>2.24 MW</td>
<td>-</td>
<td>1.94 MW</td>
<td>-</td>
<td>1.55 MW</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>0.07 MVAr</td>
<td>-</td>
<td>0.25 MVAr</td>
<td>-</td>
<td>0.18 MVAr</td>
<td>-</td>
</tr>
<tr>
<td>DER$_4$</td>
<td>0.37 MW</td>
<td>-</td>
<td>0.32 MW</td>
<td>-</td>
<td>0.23 MW</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>0.01 MVAr</td>
<td>-</td>
<td>-0.05 MVAr</td>
<td>-</td>
<td>-0.13 MVAr</td>
<td>-</td>
</tr>
<tr>
<td>Load 1</td>
<td>-</td>
<td>0.89 MW</td>
<td>-</td>
<td>0.89 MW</td>
<td>-</td>
<td>0.87 MW</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.18 MVAr</td>
<td>-</td>
<td>0.18 MVAr</td>
<td>-</td>
<td>0.17 MVAr</td>
</tr>
<tr>
<td>Load 2</td>
<td>-</td>
<td>0.59 MW</td>
<td>-</td>
<td>0.59 MW</td>
<td>-</td>
<td>0.57 MW</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.13 MVAr</td>
<td>-</td>
<td>0.13 MVAr</td>
<td>-</td>
<td>0.12 MVAr</td>
</tr>
<tr>
<td>Load 3</td>
<td>-</td>
<td>1.16 MW</td>
<td>-</td>
<td>1.16 MW</td>
<td>-</td>
<td>1.13 MW</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.54 MVAr</td>
<td>-</td>
<td>0.54 MVAr</td>
<td>-</td>
<td>0.53 MVAr</td>
</tr>
<tr>
<td>Load 4</td>
<td>-</td>
<td>1.11 MW</td>
<td>-</td>
<td>1.11 MW</td>
<td>-</td>
<td>1.08 MW</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.09 MVAr</td>
<td>-</td>
<td>0.09 MVAr</td>
<td>-</td>
<td>0.08 MVAr</td>
</tr>
<tr>
<td>Load 5</td>
<td>-</td>
<td>0.63 MW</td>
<td>-</td>
<td>0.63 MW</td>
<td>-</td>
<td>0.61 MW</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.07 MVAr</td>
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<td>0.07 MVAr</td>
<td>-</td>
<td>0.07 MVAr</td>
</tr>
<tr>
<td>Load 6</td>
<td>-</td>
<td>0.70 MW</td>
<td>-</td>
<td>0.70 MW</td>
<td>-</td>
<td>0.67 MW</td>
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<tr>
<td></td>
<td></td>
<td>0.38 MVAr</td>
<td>-</td>
<td>0.38 MVAr</td>
<td>-</td>
<td>0.36 MVAr</td>
</tr>
<tr>
<td>Load 7</td>
<td>-</td>
<td>0.86 MW</td>
<td>-</td>
<td>0.86 MW</td>
<td>-</td>
<td>0.41 MW</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.33 MVAr</td>
<td>-</td>
<td>0.33 MVAr</td>
<td>-</td>
<td>0.15 MVAr</td>
</tr>
<tr>
<td>Capacitor Cp</td>
<td>-</td>
<td>2 MVAr</td>
<td>-</td>
<td>2 MVAr</td>
<td>-</td>
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</tr>
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</table>
Appendix F

Extended Performance Evaluation of the Hierarchical Control Structure in VPP Mode

Robustness of the developed hierarchical control structure operating in the VPP-PQ and VPP-PV modes, against operating point changes, topological uncertainties, and three-phase and single-phase line-to-ground faults is investigated in Chapter 4. Now, in this appendix, performance of the developed hierarchical control structure during load disconnection and re-connection and nonlinear load energization is evaluated.

F.1 VPP Mode: Strong Utility Grid (ESCR=14.09)

In this series of studies, microgrid is operating in the VPP-PQ mode, transferring 1.02 MVA to the utility grid at a leading power factor of 0.98. In the VPP-PQ mode, the length of the line $TL_1$ is 0.8 km which results in a ESCR=14.09. The steady-state operating point is as of given in Table E.1, Appendix E.

F.1.1 Load change

To investigate the robustness of the developed hierarchical control structure to the local uncertainties in a microgrid, system response to the sudden disconnection and re-connection of $Load_1$ is simulated in this section. $Load_1$ is seen as the parallel combination of $L_1$ and $L_2$ in Figure 2.1. At $t = 0.2$ s, $Load_1$ disconnects from the grid, causing a decrease of 0.53 pu in the power demand, and then re-connects after 3 cycles at $t = 0.25$ s. Active and reactive power consumed by $Load_1$, its three-phase instantaneous input
Figure F.1: Sudden disconnection and connection of Load1: (a) and (b) real and reactive power consumed by Load1 ($S_{base}=3$ MVA), (c) and (d) Bus1 three-phase instantaneous voltage and input current ($V_{base}=11.2676$ kV and $I_{base}=0.17$ kA).

current, and the three-phase instantaneous voltage measured at Bus1 of Figure 2.1 are shown in Figure F.1.

Figure F.2 (a) and (b) depicts the real and reactive powers provided by the microgrid at the PCC and their specified set points. As it can be seen, the real and reactive powers transferred to the utility grid through the PCC settle down to their set points within 35 ms and 70 ms, respectively, with 1% criterion. This depicts the robustness of the developed hierarchical control structure in response to transients caused by considerable load changes. Real and reactive powers generated by the four DERs and their reference commands are also provided in Figure F.2 (e), (f), (i), and (j). Figure F.2 (c), (g), and (k) depicts that despite this considerable load change in the microgrid, voltage at the PCC and subsequently at the PC of DERs is regulated at 1 pu by the host utility grid. Transients in the three-phase instantaneous output current of the microgrid as well as the output current of DER1 and DER4 are also illustrated in plots (d), (h), and (l).

F.1.2 Nonlinear Load Energization

To investigate the robust performance of the developed hierarchical control structure when the microgrid is subjected to a nonlinear load, the case of induction machine energization is studied in this section. The motor is rated at 500 hp, 2.3 kV, and connected to Bus 5 of Figure 2.1 through a 13.8 kV/2.3 kV, $Y_g/\Delta$ transformer. The induction motor parameters are provided in Chapter 2 of [41].

At $t=0.2$ s, motor energizes from the stall under full load condition. Figure F.3 (a) to (c) shows the real power, reactive power, and the inrush current consumed by the induction motor. The per-unit values of the electric torque, mechanical torque,
Figure F.2: System response to sudden load disconnection and connection: (a) to (d) microgrid’s output real and reactive power, and three-phase instantaneous voltage and output current measured at the PCC, (e) to (l) DER$_1$ to DER$_4$ output real and reactive powers, three-phase instantaneous output voltage and output current.

and mechanical speed of the motor are demonstrated in Figure F.3 plots (d) to (f), respectively. The per-unitization is done based on the motor ratings.

Figure F.4 (a) to (f) show transients in the real and reactive power provided by the microgrid at the PCC and generated by the four DERs and their reference commands. As depicted, the developed hierarchical control structure is able to maintain the stability of the microgrid during the start-up period of the motor and to keep the tracking error within ±7% range. Within 300 ms after motor energization, the control structure provides the required power at the PCC with zero tracking error, with 1% criterion.

The instantaneous magnitude waveform of the voltage and the output current of the microgrid and DERs, measured at the PCC and PC$_1$ to PC$_4$, are also shown in Figure F.4 (g) to (k) and (m) to (q). The successful implementation of the generalized SPWM along with the dual internal current controllers to balance the output current of the DERs is illustrated in plots (n) to (q). Microgrid frequency measured at the PCC and PC$_1$ to PC$_4$ are also provided in Figure F.4 (l) and (r). As depicted, frequency is maintained at 60 Hz by the utility grid.
Figure F.3: System response to induction motor energization: (a) and (d) real and reactive power consumed by the motor, (c) inrush motor current, (d) to (f) electrical torque, mechanical torque, and mechanical speed of the motor.

Figure F.4: System response to induction motor energization: (a) to (f) output real and reactive power at the PCC and generated by the four DERs, (g) to (k) the instantaneous magnitude waveform of the voltage at the PCC and PCC's frequency, (l) and (r) the instantaneous magnitude waveform of the output current of the microgrid and the four DERs, (m) to (q) the instantaneous magnitude waveform of the output current of the microgrid and the four DERs, (l) and (r) microgrid frequency measured at the PCC and PC1 to PC4.

F.2 VPP Mode: Weak Utility Grid (ESCR=2.16)

In this series of studies, microgrid is operating in the VPP-PV mode, transferring 1.00 MW to the utility grid while regulating PCC’s voltage at 1 pu. In the VPP-PV mode,
the length of the line TL₁ is 20 km which results in a ESCR=2.16. The steady-state operating point is as of given in Table E.1 Appendix E.

F.2.1 Load change

In this scenario, Load₁ disconnects from the grid, at t=0.35 s, causing a decrease of 0.51 pu in the power demand, and then connects back after 3 cycles at t=0.4 s.

Figure F.5 (a) to (g) depicts the real and reactive power provided by the microgrid at the PCC and generated by the four DERs as well as real and reactive powers consumed by Load₁. Instantaneous magnitude and three-phase waveform of the PCC voltage are also provided in Figure F.5 (h) and (i). As it can be seen, despite this severe load change, the real power transferred to the utility grid and the voltage at the PCC are regulated to their specified values, which depicts the robustness of the developed hierarchical control structure. Three-phase instantaneous output current of the microgrid and the four DERs as well as load current are provided in Figure F.5 (o) to (u).

F.2.2 Nonlinear Load Energization

At t=0.35 s, motor energizes from the stall under full load condition. Figure F.6 (a) to (c) shows the real power, reactive power, and the inrush current consumed by the induction motor. The per-unit values of the electric torque, mechanical torque, and mechanical speed of the motor are demonstrated in Figure F.6 (d) to (f), respectively. The per-unitization is done based on the motor ratings.

Figure F.7 (a) to (f) show the transients in the real and reactive powers provided by the microgrid and the four DERs. The instantaneous magnitude waveform of voltage measured at the PCC and PC₁ to PC₄ are also shown in plots (g) to (k). As it can be seen in Figure F.7, the developed hierarchical control structure tracks the specified real power and voltage setpoints within 310 and 250 ms, respectively, despite the severe transients caused by the nonlinearity of the motor energization process. The instantaneous magnitude waveform of the output currents of the microgrid and the four DERs are also provided in Figure F.7 (m) to (q).
Figure F.5: System response to sudden load disconnection and connection: (a) and (b) output real and reactive powers provided by the microgrid at the PCC, (c) to (f) real and reactive powers generated by DER1 to DER4, (h) and (i) instantaneous magnitude and three-phase waveform of the PCC voltage, (j) to (m) instantaneous three-phase output voltage of DER1 to DER4, (o) to (s) instantaneous three-phase output current of the microgrid and DER1 to DER4, (t) and (u) instantaneous three-phase consumed by loads L1 and L2.

Figure F.6: System response to induction motor energization: (a) and (b) real and reactive power consumed by the motor, (c) inrush motor current, (d) to (f) electrical torque, mechanical torque, and mechanical speed of the motor.
Figure F.7: System response to induction motor energization: (a) to (f) output real and reactive power at the PCC and generated by the four DERs, (g) to (k) the instantaneous magnitude waveform of the voltage at the PCC and PC₁ to PC₄, (m) to (q) the instantaneous magnitude waveform of the output current of the microgrid and the four DERs, (l) and (r) microgrid frequency measured at the PCC and PC₁ to PC₄.
Bibliography


