Enhanced Layout Optimization and Wind Aerodynamic Models for Wind Farm Design

by

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A thesis submitted in conformity with the requirements for the degree of Doctor of Philosophy
Graduate Department of Mechanical & Industrial Engineering
University of Toronto

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Abstract

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The proposed project is motivated by the need to develop wake models and optimization algorithms that can accurately capture the wake losses in an array of wind turbines and optimize the turbine placements. In the past 4 years, we have developed capabilities to improve the layout design of wind farms located on complex terrains, as contributions from four major tasks.

The outcome of the first task was the creation of a wake interaction model capable of describing the effects of overlapping wakes that can be used in combination with existing mathematical optimization tools for wind farm layout design. Such a model was derived and evaluated against existing wake interaction methods. This wake interaction model enables a mechanistic approach to account for multiple overlapping wakes while remaining compatible with established mathematical optimization methods.

In the second task, this wake interaction model was used in conjunction with full-scale CFD simulations to design wind farm layouts. We developed an optimization algorithm that intelligently integrates a mathematical optimization approach to design wind farm layout on complex terrains with full-scale CFD simulations.

The two subsequent tasks were focused on developing a wake model capable of producing comparable accuracy as full-scale CFD simulations but at a significantly lower computational cost. The third task focused on studying the effects of turbine blade geometry and atmospheric turbulence on turbine wake development. The findings of this step contributed to the fourth task of developing a new wake model capable of simulating wakes on complex terrains. This model has been validated against full-scale CFD simulations of a turbine placed on the terrain of the Gros-Morne Wind Farm in Quebec. The proposed model allows for fast simulation of wakes, making it ideal for designing wind farm layouts on complex terrains.
To Charlie Matsubara
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# Contents

<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bibliography</td>
<td></td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>Introduction</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>Literature Review</td>
<td>3</td>
</tr>
<tr>
<td>2.1</td>
<td>Wind Farm Optimization Problem</td>
<td>3</td>
</tr>
<tr>
<td>2.2</td>
<td>Wake Models</td>
<td>4</td>
</tr>
<tr>
<td>2.2.1</td>
<td>Single Wake</td>
<td>4</td>
</tr>
<tr>
<td>2.3</td>
<td>Optimization</td>
<td>7</td>
</tr>
<tr>
<td>3</td>
<td>Multiple Turbine Wake Interactions</td>
<td>9</td>
</tr>
<tr>
<td>3.1</td>
<td>Introduction</td>
<td>9</td>
</tr>
<tr>
<td>3.2</td>
<td>Wake Modeling</td>
<td>10</td>
</tr>
<tr>
<td>3.2.1</td>
<td>Single Wake Model</td>
<td>10</td>
</tr>
<tr>
<td>3.2.2</td>
<td>Wake Interaction Models</td>
<td>10</td>
</tr>
<tr>
<td>3.3</td>
<td>Proposed Wake Model</td>
<td>11</td>
</tr>
<tr>
<td>3.3.1</td>
<td>Energy Balance</td>
<td>11</td>
</tr>
<tr>
<td>3.3.2</td>
<td>Model Fitting</td>
<td>14</td>
</tr>
<tr>
<td>3.4</td>
<td>Optimization</td>
<td>15</td>
</tr>
<tr>
<td>3.4.1</td>
<td>Model</td>
<td>15</td>
</tr>
<tr>
<td>3.5</td>
<td>Description of Tests</td>
<td>18</td>
</tr>
<tr>
<td>3.6</td>
<td>Results and Discussion</td>
<td>20</td>
</tr>
<tr>
<td>3.7</td>
<td>Conclusions</td>
<td>22</td>
</tr>
<tr>
<td>4</td>
<td>Layout Optimization on Complex Terrains</td>
<td>26</td>
</tr>
<tr>
<td>4.1</td>
<td>Introduction</td>
<td>26</td>
</tr>
<tr>
<td>4.2</td>
<td>Previous Work</td>
<td>27</td>
</tr>
</tbody>
</table>
List of Tables

3.1 Wake interaction models .................................................. 11
3.2 Wind turbine parameters .................................................. 20
3.3 Layout of 1 x 20 domain for a land strip of 2 km long. The x coordinates [m] for turbines T1–T5 and the resulting annual energy production (AEP) [GWh] are shown. .................. 20
3.4 Layout of 1 x 100 domain for a land strip of 2 km long. The x coordinates [m] for turbines T1–T5 and the resulting annual energy production (AEP) [GWh] are shown. .......... 21
3.5 Annual energy production [GWh] for WR6 10 x 10 on a 4 km x 4 km domain. The best solution found for each case is indicated in boldface type. ......................... 22
3.6 Forfeited annual revenue for different wake interaction methods with WR6 10 x 10 on a 4 km x 4 km domain, assuming an electricity price of $0.1/kWh [1, 2]. The best solutions found (Table 3.5) are used as reference values. .......................... 22
3.7 Annual energy production [GWh] for WR36 20 x 20 on a 4 km x 4 km domain. The best solution found for each case is indicated in boldface type. ......................... 25
3.8 Forfeited annual revenue for different wake interaction methods with WR36 20 x 20 on a 4 km x 4 km domain, assuming an electricity price of $0.1/kWh [1, 2]. The best solutions found (Table 3.7) are used as reference values. ......................... 25
4.1 Influence of relaxation parameter on solution quality and computational cost .......... 38
5.1 Mesh sensitivity analysis ................................................... 51
List of Figures

1.1 Global map of wind speeds at 80 m above the ground, courtesy of Vaisala [3].

2.1 Turbine wakes in Horns Rev Wind Farm in Denmark [4].

2.2 Wind turbine wake velocity recovery.

2.3 Wake regions of a wind turbine.

3.1 Two overlapping turbine wakes, inlet A of a streamtube is upstream in the free stream and outlet B is in the wake overlap.

3.2 Layout of Horns Rev wind farm, arranged in 8 rows and 10 columns. Arrows show wind directions and the corresponding spacing, expressed in rotor diameters [D].

3.3 Wind speeds experienced by a row of turbines separated by 7 diameter distances apart. Comparison between measurements (Horns Rev) and the proposed model (with Jensen and Frandsen) are shown. Error bars represent the standard deviation of the measurements.

3.4 Wind speeds experienced by a row of turbines separated by 9.4 diameter distances apart. Comparison between measurements (Horns Rev) and the proposed model (with Jensen and Frandsen) are shown. Error bars represent the standard deviation of the measurements. Note that the experimental data exhibits a non-monotonic behavior that cannot be captured by the single-wake models commonly used in the literature [5, 6].

3.5 Wind speeds experienced by a row of turbines separated by 10.4 diameter distances apart. Comparison between measurements (Horns Rev) and the proposed model (with Jensen and Frandsen) are shown. Error bars represent the standard deviation of the measurements.

3.6 Simple wind farm domain divided into 36 cells under a one-directional wind regime. The distance between cell center is five times the rotor diameter. The wake of turbine placed in cell \( j \) propagates downwind to affect cell \( i \). Consequently, the placement of turbine in cell \( j \) affects the decision whether to place a turbine in cell \( i \) or not.

3.7 A row of 5 turbines with a constant wind blowing from the west.
3.8 Layouts for the case of WR6 and 50 turbines with 10 x 10 grid and different interaction models, LS, SKED, present work (Horns Rev), and present work \( (\alpha = 1) \).

3.9 Layouts for the case of WR36 and 40 turbines with 20 x 20 grid and different interaction models, LS, SKED, present work (Horns Rev), and present work \( (\alpha = 1) \), from left to right.

4.1 Flowchart of the optimization algorithm process.

4.2 Turbine wake created by west wind. The wake from turbine at location \( i \) propagates downstream, affecting location \( j \).

4.3 Actuator disk model by El Kasmi and Masson [7].

4.4 2.8 km x 2.8 km wind farm domain in Carleton-sur-Mer.

4.5 Wind rose for Carleton-sur-Mer. [8]

4.6 Wind farm domain for CFD simulations.

4.7 Optimal layout found at the end of each iteration. The circles mark the turbines that were relocated in that iteration. Note that after only 3 iterations, the algorithm did not identify additional turbine locations that would lead to improvements in the optimization objective.

4.8 Progression of the objective values (varying the relaxation factor) based on wake effects known at each iteration.

4.9 Optimal layout found at the end of each iteration with relaxation parameter, \( C \), set to 0.7. The circles mark the turbines that were relocated in that iteration. Note that after only 3 iterations, the algorithm did not identify additional turbine locations that would lead to improvements in the optimization objective.

4.10 Optimal layout found at the end of each iteration with relaxation parameter, \( C \), set to 0.4. The circles mark the turbines that were relocated in that iteration. Note that after only 3 iterations, the algorithm did not identify additional turbine locations that would lead to improvements in the optimization objective.

4.11 Optimal layout found at the end of each iteration with relaxation parameter, \( C \), set to 0.2. The circles mark the turbines that were relocated in that iteration. Note that after only 3 iterations, the algorithm did not identify additional turbine locations that would lead to improvements in the optimization objective.

4.12 Optimal layout found at the end of each iteration with relaxation parameter, \( C \), set to 0. The circles mark the turbines that were relocated in that iteration. Note that after 8 iterations, the algorithm did not identify additional turbine locations that would lead to improvements in the optimization objective.
4.13 Effects of relaxation parameter on computational cost (fraction of maximum number of
CFD evaluations). ................................................................. 45
4.14 Effects of relaxation parameter on layout efficiency. ................................................ 46

5.1 An actuator disk model. ................................................................. 49
5.2 Axial induction factor along a LM8.2 turbine blade. .................................................. 50
5.3 Simulation domain ................................................................. 51
5.4 Velocity profile 1 rotor diameter downstream of the turbine with turbulence intensities of
5% and 10%. ............................................................................. 52
5.5 Velocity profile 2 rotor diameters downstream of the turbine with turbulence intensities
of 5% and 10%. ............................................................................. 53
5.6 Velocity profile 6 rotor diameters downstream of the turbine with turbulence intensities
of 5% and 10%. ............................................................................. 54
5.7 Velocity profile 1 rotor diameter downstream of the turbine with different force profiles
at a turbulence intensity of 5%. ................................................... 55
5.8 Velocity profile 2 rotor diameters downstream of the turbine with different force profiles
at a turbulence intensity of 5%. ................................................... 55
5.9 Velocity profile 6 rotor diameters downstream of the turbine with different force profiles
at a turbulence intensity of 5%. ................................................... 56
5.10 Velocity profile 1 rotor diameter downstream of the turbine with different force profiles
at a turbulence intensity of 10%. .................................................. 56
5.11 Velocity profile 2 rotor diameters downstream of the turbine with different force profiles
at a turbulence intensity of 10%. .................................................. 57
5.12 Velocity profile 6 rotor diameters downstream of the turbine with different force profiles
at a turbulence intensity of 10%. .................................................. 57

6.1 Near and far wake regions of a wind turbine ......................................................... 59
6.2 Actuator disk velocity and pressure distribution ...................................................... 59
6.3 Actuator disk theory, the wake experiences an initial wake expansion in the near wake. .... 61
6.4 Streamwise velocity contour for both CFD and proposed model. ................................ 62
6.5 Increase in turbulence viscosity inside a turbine wake. .............................................. 64
6.6 Streamwise velocity profiles on a flat terrain with new turbulent viscosity fitting. ........ 65
6.7 CFD domain with terrain based on Gros-Morne Wind Farm. ...................................... 66
6.8 Streamwise velocity at hub height. ........................................................................... 66
6.9 Streamwise velocity on a complex terrain using CFD and proposed model. ................. 67
Chapter 1

Introduction

Wind energy has experienced tremendous growth in the past two decades, driven by government policies and falling costs of wind energy [9]. In 2014, the European Union set a legally binding target that by the year 2030, at least 27% of the energy consumption must come from renewable sources [10]. According to European Wind Energy Association, of the electricity generated by renewables, nearly half will come from wind energy [10]. Similarly, Canada and the United States have both experienced and expect to continue to experience the trend of double-digit growths in their respective wind energy industries in the next decade [11, 12].

In fact, wind energy resource potential in North America is among the best in the world [13]. According to Lu et al. [13], the wind energy potential in the United States is more than 5 times that of its annual energy consumption. In terms of total wind energy potential by country, Russia, Canada, United States, Australia, and China are at the top of the list, with vast majority of that resource located inland. By observing the global map of wind energy resource, shown in Figure 1.1, it is clear that higher wind speeds are seen near coastal and mountainous regions.

The wind energy resource potential of a site is the single most important factor in determining the future performance of a wind farm. Consequently, much of the work available in the literature is focused on using scientific forecasting methods to find optimal locations for potential wind farm sites. Wind turbines produce power by extracting energy from the air and they can be arranged into wind farms. Commercial wind farms are typically made up of dozens of large wind turbines placed on areas that span over several kilometers. One of the key aspects of wind farm design is the placement of wind turbines, as aerodynamic losses, or wake losses, are one of the major sources of inefficiency in wind farms [14].

The wind farm layout optimization (WFLO) problem is concerned with the optimal placement of turbines in a geographical area to maximize energy production and minimize costs. In order to optimize turbine placements, aerodynamic wake models capable of describing these wake losses need to be used in
conjunction with optimization approaches to evaluate turbine layouts. Previously, much effort has been focused on developing aerodynamic models [5, 6, 15, 16, 17] and optimization approaches [18, 19, 20, 21, 22, 23] for flat and uniform terrains; however the wind energy development in the United States and Canada has been concentrated inland [24]. Therefore, there is a need to develop wake models capable of simulating wake effects and optimization approaches to optimize wind farm layouts on complex terrains.

The focus on this work is to develop wake models capable of describing the wake effects and applying mathematical optimization tools to wind farm design. In Chapter 3, a model that accounts for multiple wake interactions is presented. This model allows for a mechanistic approach to describe multiple wake interactions that is compatible with existing mathematical optimization methods. In Chapter 4, the wake interaction model was used in conjunction with computational fluid dynamics (CFD) simulations to design wind farm layouts. We proposed an optimization algorithm that intelligently integrates CFD simulation data with a mathematical optimization approach to design wind farm layouts on complex terrains.

The following two Chapters are focused on developing a wake model with comparable accuracy with CFD simulations with significantly lower computational cost. Chapter 5 focuses on studying the effects of turbine blade geometry and atmospheric turbulence on turbine wake development, leading to a new wake model capable of simulating wakes on complex terrains, described in Chapter 6.
Chapter 2

Literature Review

2.1 Wind Farm Optimization Problem

The main objective of a WFLO problem is to maximize energy production while minimizing costs. The power production of a wind farm is dependent on incoming wind speeds, which are themselves dependent on terrain topography, atmospheric conditions, and upstream turbine wakes. In particular, production loss due to the wake interference of upstream turbines, called wake losses (Figure 2.1), can reduce the annual energy of a wind farm by as much as 10% to 20% [14].

Figure 2.1: Turbine wakes in Horns Rev Wind Farm in Denmark [4].
Wind turbines extract kinetic energy from the wind via interactions between the turbine blades and the wind. The aerodynamic forces produced by the wind turns the rotor to generate electricity. The air behind the turbine is slowed down with its turbulence intensity increased [16]. This region of decelerated air is called the wake [16]. The wake expands as it travels downstream, mixing with surrounding air and increasing its velocity back to undisturbed conditions after some distance. This distance is crucial as the performance of turbines downstream is dependent on the incoming wind conditions. If turbines are too close to each other, the wind cannot recover to its upstream state [25], causing losses in energy generation for turbines downstream [14, 17, 26]. Therefore, a better understanding of turbine wake recovery process is crucial for optimal wind farm design and planning [25].

The main purpose of this review is to present the relevant state-of-the-art tools for designing wind farms on complex terrains. Most of the development related to optimization and wake modelling have focused on flat and uniform terrains. However, some of these important advances can be adapted to design wind farms on complex terrains. This review examines key optimization approaches and wake models available in the literature.

This chapter is divided into two main sections, existing wake models and optimization approaches. The wake model section is divided into single wake and multiple wake interaction models. The single wake model section discusses a variety of models, ranging from low-fidelity analytical models to full-scale computational fluid dynamics models. While single wake models are well-developed, models that can describe the interactions of multiple overlapping wakes is limited in the literature. Overall, we found that the phenomena of wake interactions is not well understood, suggesting research gaps and opportunities to be pursued.

From the design optimization perspective, the field of optimizing turbine placement is relatively mature. In the literature, a wide selection of optimization approaches have been applied to solve WFLO problems. These established optimization approaches usually rely on analytical wake models for solution evaluation. However, analytical wake models do not account for effects on complex terrains; this makes their use limited. Based on these observations, we developed a set of tools to bridge the gap between wake models and optimization approaches to determine wind turbine placement on complex terrains.

### 2.2 Wake Models

#### 2.2.1 Single Wake

A turbine wake expands and propagates downstream, mixes with the mean flow, then its velocity recovers over large distances, shown in Figure 2.2. Wake models are used to quantify this wake recovery and propagation process.
Wind turbine wakes can be divided into two regions: near wake and far wake regions [27, 28]. The near wake region begins from the turbine to a specific location and then is followed by the far wake region as shown in Figure 2.3. The length of the near wake region varies, ranging from 1 to 4 diameters downstream of the rotor [29, 30]. Previous studies have shown that the velocity and turbulence profiles in the near wake region are strongly influenced by the rotor geometry [31, 32]. In the far wake, the influence of rotor geometry becomes less important while terrain and atmospheric effects become more dominant. In addition, the velocity profile becomes approximately Gaussian and the pressure gradient due to the turbine becomes negligible [27]. The far wake region is the region of interest for wind farm layout design, as turbines are typically placed far apart such that they are in the far wake regions of upstream turbines [33].

Far wake models are divided into two categories, kinematic and field models [28]. Kinematic wake models such as Jensen [5], Frandsen [6], Ishihara [34], and Larsen [35] are analytical models that can be solved very quickly, making them ideal for optimization. Field models such as Ainslie [15] use a reduced form of the Reynolds-averaged Navier-Stokes equations with added turbulence model.
In addition, with advances in computing power, numerical wake models have been introduced and implemented in commercial software packages. For instance, Ainslie [15] first developed the eddy viscosity wake model, which was implemented in the WindFarmer software [36]. Then, similar implementations of the model was done in OpenWind [37] and The Farm Layout Program (FLaP) [38]. The Energy Research Centre of the Netherlands introduced a wake model based on the parabolic Navier-Stokes equations using $k – \varepsilon$ turbulence model [39]. These models specifically simulate wakes on flat terrains and cannot be used to model wake effects on complex terrains.

Full-scale computational fluid dynamics (CFD) models have been applied to study wind turbine wakes [14, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50]. Full-scale CFD simulations involve solving the full Navier-Stokes equations. One of the main challenges in modelling turbine wakes is how the turbine rotor should be represented in CFD simulations. In order to reduce the computational cost, the actuator disk model [51] is used to describe the complex rotor geometries. The results of this approach has shown to produce good agreement with full rotor modelling approach [52]. Jafari et al. [53] proposed a novel approach in which actuator disk theory is used to describe the wake expansion and velocity and turbulence profiles are prescribed at the end of the near wake region.

The main challenge in developing wake models for complex terrains is reducing the computational cost. During the design optimization process, wake models are used to evaluate the wake effects of different layouts. This process is iterative and requires a large number of solution evaluations, thus low cost wake model is necessary to evaluate large pools of candidate solutions. In commercial software, wake models for complex terrains rely on variations of wake superposition approaches [54]. Recently, a virtual particle model [55] has also been developed to simulate wakes on complex terrains. This model utilizes the concept of solving for the velocity deficit instead of velocity directly by assuming velocity deficit is analogous to concentration of particles. However, due to the assumptions, this model requires parameter tuning that cannot be known a priori nor be generalized.

Multiple Wake Interactions

The interaction of multiple superimposed wakes is not fully understood, as it involves complex turbulence phenomena [56]. A number of descriptions exist in the literature to determine the wind speed due to the presence of multiple turbine wakes. In particular, four descriptions available in the literature [16, 57, 58].

These descriptions are recursive functions, which is dependent on conditions upstream which is not known a priori. However, these functions are not mechanistic in nature, lacking physical basis, which makes improvement through experimental results difficult. Furthermore, these descriptions are non-linear, leading to non-linear objective functions which makes it challenging to use with well-established mathematical programming approaches [18, 19, 59]. In Chapter 3, a physics-based wake interaction...
model that leads to linear objective functions is presented.

2.3 Optimization

The main objective of a wind farm layout optimization (WFLO) problem is to maximize energy production while minimizing costs such as installation and maintenance costs. Determining the optimal layout of a set of wind turbines in a given area is a complex problem. Wind farm layout problems can be modelled in two ways, namely (1) continuous and (2) discrete. In discrete models [20, 21, 60], the turbines can only be placed in a countable set of pre-determined locations inside the wind farm, while in continuous models [61, 62, 63, 64, 65, 66, 67], turbines can be placed anywhere in the farm, considering their coordinates as continuous variables. Metaheuristic algorithms such as evolutionary algorithms [60, 64, 65, 66, 67], particle swarm optimization [63, 68], and extended pattern search [62], have been the primary tools to solve continuous models. Although powerful in tackling non-linear problems, metaheuristics cannot guarantee global optimality.

A discrete model can be solved by using mathematical programming approaches, which are promising in solving WFLO problems [19, 59, 69, 70, 71, 72]. Donovan [59, 73] introduced a mixed-integer programming (MIP) model for solving the WFLO problem. Although commercial MIP solvers are widely available and are capable of proving optimality of solutions, they are limited to solving convex and linear or quadratic problems. Both Donovan and Fagerfjäll [70] attempted to address this problem by simplifying their wake model at the expense of losing accuracy. To address this issue, Archer et al. [74] improved the simplified wake model by introducing a wind interference coefficient, while Turner et al. [19] suggested more accurate linear and quadratic wake models that can be solved by MIP solvers. Furthermore, Zhang et al. [69] proposed a constraint programming model that incorporates the full non-linearity of the problem. Although these approaches allows to proof of optimality, they rely on the relatively coarse discretization of wind farm domains, because the solution time required to solve increase exponentially with finer discretizations.

On the other hand, continuous models are solved using evolutionary metaheuristic algorithms [60, 62, 63, 64, 65, 66, 67, 68, 75, 76] and nonlinear optimization methods [71, 77]. A significant portion of the literature has focused on improving the model by including realistic constraints and features. For instance, Yamani Douzi Sorhabi et al. [78] studied the impact of land use constraints on the impact of noise and energy in a multi-objective optimization. Furthermore, Serrano-González et al. [79, 80] included infrastructure costs and wind data uncertainty in their optimization model. While Saavedra-Moreno et al. [81] improved the model by considering multiple wind distributions for different locations in the wind farm domain. Although the WFLO problem refers specifically to the design optimization
of turbine placements [82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102] sometimes the number of turbines, turbine type [103], and turbine height [95, 104, 105] are considered in the optimization problem [106].

Most of the publications in the literature consider the terrains to be uniform and flat, ignoring the effects of terrain elevation. Terrain topography strongly influences the local energy potential of a farm as well as the wake propagation and recovery process. The work by Saavedra-Moreno et al. [81] considered the effects of terrain topography on local wind speed but did not account for them in the wake propagation and recovery process. The major hurdle is that the wake propagation and recovery process on complex terrains is difficult to model accurately and cost-effectively for optimization. Feng and Shen [54] assumed that the wind turbine wake propagates along a complex terrain at hub height. Then, Song et al. [55] introduced a wake model based on particle simulations and integrating it with various optimization algorithms [95, 105, 107, 108, 109] to design wind farms on complex terrains. Given the reciprocal relationship between optimization approaches and wake models, it is important to consider this dependency during their respective development, and this is the main objective of this thesis.
Chapter 3

Multiple Turbine Wake Interactions

3.1 Introduction

There are a number of publications on discrete modelling of wind turbine placement in wind farms [20, 21, 110, 111]. An example is the work by Mosetti et al. [20], where the wind farm is divided into 10 by 10 square cells and each turbine is placed in the centre of each cell. The cell sizes are chosen to enforce distance constraints between turbines, e.g., turbines cannot be placed closer than five turbine rotor diameters apart. Although layout solutions to discrete models may be of lower spatial resolution than of continuous models, discrete models can be solved using powerful mathematical programming approaches [19, 59, 69], which can guarantee optimality of the solutions for linear and quadratic functions and constraints. In a well-designed discrete model, knowing the optimality of solutions can save tremendous amount of time in the optimization process.

A mixed integer programming problem (MIP) consists of an objective function and a mix of integer and continuous constraints. The layout optimization problem can be modelled in this mathematical programming approach by discretizing the wind farm domain into possible turbine placement locations, with binary decision variables denoting if a turbine is placed at a specific location or not. These problems can be solved using algorithms such as branch and bound [112]. Fagerjäll [70], Donovan et al. [59], Zhang et al. [69], and Turner et al. [19] have studied the application of branch and bound methods in WFLO problems. Applying mathematical programming methods to solve the layout problem is promising due to the optimality of the solutions can be known, as opposed to metaheuristics that provide no guarantee of convergence. Furthermore, solver efficiency can be improved through alternative problem formulations and problem-specific branching strategies [69]. The objective of this chapter is to introduce a novel wake interaction model that leads to linear MIP formulations, thus guaranteeing the optimality of solutions to WFLO problems.
3.2 Wake Modeling

3.2.1 Single Wake Model

The Jensen model [5] is one of the most widely used wake models. It assumes a linearly expanding wake and uniform velocity profile inside the wake. As a result of momentum conservation, the decelerated wind behind the rotor recovers to free stream speed after travelling a certain distance downstream of the turbine [5]. The velocity downstream from the rotor is given by

$$u(x) = u_0 \left[ 1 - \frac{1 - \sqrt{1 - C_T}}{(1 + 2k \frac{x}{D})^2} \right],$$  \hspace{1cm} (3.1)

where $C_T$ is the thrust coefficient of the turbine, $D$ is the turbine rotor diameter, $u_0$ is the wind speed in the free stream, and $k$ is the Wake Decay Constant, which is generally taken to be 0.075 for onshore farms and 0.04 to 0.05 for offshore farms.

The power production of each turbine $i$ is based on the incoming wind speed that it experiences,

$$P_i = \frac{1}{2} \rho A u_i^3 (\eta_{gen} C_P),$$  \hspace{1cm} (3.2)

where $A$ as the rotor area, $\rho$ is the density of the air, $\eta_{gen}$ is the generator efficiency, and $C_P$ is the rotor power coefficient. The annual energy production (AEP) of a wind farm is defined as the integration of power production (kW) over time (hr),

$$AEP = 8766 \sum_{i=1}^{N} \sum_{d \in L} p_d P_i, d.$$  \hspace{1cm} (3.3)

where $p_d$ is the probability of wind state $d$, defined as a (speed, direction) pair, $L$ is the set of wind states with non-zero probability for the specific wind farm site, $N$ is the total number of turbines, and 8766 is the effective number of hours in a year.

3.2.2 Wake Interaction Models

The interaction of multiple superimposed turbine wakes is not fully understood, as it involves complex turbulence phenomena [56]. A number of descriptions exist in the literature to determine the wind speed due to the presence of multiple turbine wakes upstream. In particular, four descriptions available in the literature [16], listed in Table 3.1, will be introduced in this section. In these equations, $u_i$ is the wind speed at turbine $i$, $u_{ij}$ is the wind speed at turbine $i$ due to (the wake of) turbine $j$ and the summations and the products are taken over the $n$ turbines upstream of turbine $i$ [16, 57, 58].

The Geometric Superposition (GS) assumes the ratio of the wind speed at a location relative to the
free stream speed is a product of velocity ratios caused by upstream turbines. The Linear Superposition of Velocity Deficits (LSVD) considers that the velocity deficit at a given turbine is equal to the sum of the velocity deficits caused by all turbines upstream from it. The Sum of Energy Deficits (SED) assumes the kinetic energy deficit in the wakes is additive. The Sum of Squares (SS) sums up the squares of the velocity deficits of the upstream wakes.

### Table 3.1: Wake interaction models

<table>
<thead>
<tr>
<th>Name</th>
<th>Formula</th>
</tr>
</thead>
<tbody>
<tr>
<td>Geometric Superposition (GS)</td>
<td>$\frac{u_i}{U_\infty} = \prod_{j=1}^{n} \frac{u_{ij}}{u_j}$</td>
</tr>
<tr>
<td>Linear Superposition of Velocity Deficits (LSVD)</td>
<td>$(1 - \frac{u_i}{U_\infty}) = \sum_{j=1}^{n} (1 - \frac{u_{ij}}{u_j})$</td>
</tr>
<tr>
<td>Sum of Energy Deficits (SED)</td>
<td>$(U_\infty^2 - u_i^2) = \sum_{j=1}^{n} (u_j^2 - u_{ij}^2)$</td>
</tr>
<tr>
<td>Sum of Squares (SS)</td>
<td>$(1 - \frac{u_i}{u_\infty})^2 = \sum_{j=1}^{n} (1 - \frac{u_{ij}}{u_j})^2$</td>
</tr>
</tbody>
</table>

Two main issues exist that hinder the use of these wake interaction models. Firstly, with the exception of SED, the physical basis of these descriptions is unclear, which makes improvement through experimental data difficult. Secondly, using all of the mentioned wake interaction models in deterministic optimization methods remains a challenge, for several reasons. If the objective function of the WFLO problem is to maximize power or energy production, all of the above wake interaction models (except SED) would lead to non-linear objective functions and linear approximations will be required to improve solvability. In addition, all of the models mentioned are recursive functions. Specifically, the term $u_j$, i.e. the incoming wind speed that a turbine $j$ experiences, is dependent on the conditions upstream, which are unknown a priori, thus precluding the use of well-established mathematical programming methods for its solution [19, 59, 69].

In the literature, comparisons of the different wake interaction models with experimental measurements have demonstrated that the sum of squares model, despite lacking physical meaning, is the most accurate [16]. However, as mentioned previously, it is difficult to implement sum of squares into a MIP formulation. In this work, a wake interaction model based on the principle of energy balance is presented as a physics-based, linear alternative to the sum of squares model, leading to linear objective functions.

### 3.3 Proposed Wake Model

#### 3.3.1 Energy Balance

In the proposed wake interaction model, energy balance is used to describe the wind speed recovery in the far wake, where the flow is fully developed [25]. The speed recovery in the wake is due to mixing with the surrounding air, causing changes in kinetic energy, this change is represented by a mixing head,
Figure 3.1: Two overlapping turbine wakes, inlet A of a streamtube is upstream in the free stream and outlet B is in the wake overlap.

Without loss of generality, consider a simple case of two wakes overlapping, shown in Figure 3.1. Energy analysis is done along a streamtube from A to B, ignoring the presence of the bottom turbine. The mixing head $h_{AB}$ becomes

$$h_{AB(1)} = P_{A1} + \alpha_{A1} \frac{u_0^2}{2g} - P_{B1} - \alpha_{B1} \frac{u_{B1}^2}{2g} ,$$

(3.4)

where $\alpha_{A1}$ and $\alpha_{B1}$ are kinetic energy correction factors, accounting effects of nonuniform speed profiles in the streamtube. The other terms $P$, $u$, and $g$ represent local pressure, wind speed, and gravity, respectively. In a flat terrain, the pressure in the wake is assumed to have recovered to mean flow pressure, thus $P_{A1} = P_{B1}$. However, if pressure changes need to be captured, the pressure terms could be kept easily in the analysis. The mixing head $h_{AB}$ can be simplified to

$$h_{AB(1)} = \alpha_{A1} \frac{u_0^2}{2g} - \alpha_{B1} \frac{u_{B1}^2}{2g} ,$$

(3.5)

Similarly, the same analysis can be done again from A to B, ignoring the top turbine, leading to

$$h_{AB(2)} = \alpha_{A2} \frac{u_0^2}{2g} - \alpha_{B2} \frac{u_{B2}^2}{2g} .$$

(3.6)
When the wakes of two turbines overlap, we assume in this work that the mixing gains in the combined wake is equal to the energy loss in the free stream, thus the analysis from A to B becomes

$$\alpha_\infty \frac{u_0^2}{2g} = \alpha_B \frac{u_B^2}{2g} + \sum h,$$

(3.7)

or

$$u_B^2 = \frac{\alpha_\infty}{\alpha_B} u_0^2 + \sum_j \left( \frac{\alpha_{Bj}}{\alpha_B} u_{Bj}^2 - \frac{\alpha_{Aj}}{\alpha_B} u_0^2 \right),$$

(3.8)

where $n$ is the total number of overlapping wakes at point B. The assumption that $u_A \approx u_0$ may not hold if an upstream turbine is too close to downstream turbines, e.g., for wake interactions when turbines are densely placed together. Specifically, our computer experiments showed that the proposed model outperformed benchmarks for wind farms in which the inter-turbine spacing was larger than seven turbine rotor diameters (7D), with relative performance degrading (but still comparable) in the 5–7D range, presumably because the experimental data used to calibrate the proposed model corresponds to a wind farm in which the closest turbines are a distance of 7D apart. In any case, however, we note that our assumption of $u_A \approx u_0$ is not expected to be a limitation because an inter-turbine spacing constraint is typically enforced during wind farm design and typical wind turbine densities (turbines per square kilometre) found in existing wind farms such as Horns Rev and Nysted [113].

The kinetic energy correction factors can be determined experimentally, and we considered them in this work as model fitting parameters to be estimated based on available data from real wind farms [14]. Consequently, experimental data can be used to improve the accuracy of the model. Based on turbulent pipe flows, these values of these coefficients should be close to 1 [114]. To simplify the analysis, the ratios $\frac{\alpha_\infty}{\alpha_B}$, $\frac{\alpha_{Bj}}{\alpha_B}$, and $\frac{\alpha_{Aj}}{\alpha_B}$ are denoted as $\alpha_{r,1}$, $\alpha_{r,2}$, and $\alpha_{r,3}$, respectively, assuming these coefficients are constant in the far wake region for all $j$. Equation (3.8) becomes

$$u_i^2 = \alpha_{r,1} u_0^2 + \sum_j (\alpha_{r,2} u_{ij}^2 - \alpha_{r,3} u_0^2).$$

(3.9)

If no experimental or detailed CFD (computational fluid dynamics) data is available, the model can be used as a surrogate model, or metamodel [115, 116, 117] for the SS model, in which the model is a linear approximation of SS. The coefficients can be obtained with synthetic data generated from direct evaluation of the SS model, an approach that will be explored in future work. In addition, the coefficients could be set to 1 based on considerations that are typically valid in turbulent pipe flows. In this work, we will compare the performance of wind farm layouts that are determined with the proposed model both when the coefficients are regressed to data from the Horns Rev wind farm [118] and also when they
are considered as constants set to 1.

3.3.2 Model Fitting

The coefficients of the proposed model are determined using experimental data from Horns Rev wind farm in Denmark. Horns Rev wind farm is made up of 80 2 MW turbines in a structured layout of 8 rows and 10 columns, as shown in Figure 3.2. Publicly available data from wake measurement at wind directions indicated in Figure 3.2, are used for parameter fitting. The wind speeds along a row of turbines are presented as a function of distance between upwind and downwind turbines. Figures 3.3, 3.4, and 3.5 show the wind speeds in a row of 5 turbines separated by constant distances of 7, 9.4, and 10.4 diameters at 6 m/s, respectively. The error bars are the standard deviations of the measurements of the available rows [118].

\[ \alpha_{r,1} = 0.936, \quad \alpha_{r,2} = 0.9375, \quad \alpha_{r,3} = 0.8885, \]

Based on the Horns Rev data shown in Figures 3.3–3.5, the model coefficients are found to be \( \alpha_{r,1} = 0.936, \ \alpha_{r,2} = 0.9375, \ \text{and} \ \alpha_{r,3} = 0.8885, \) and the fitted wake interaction model, Eq. (3.9), is also shown in the figures. These coefficients minimize the root mean square error when comparing with experimental data. The proposed model performs within the error bars for 7 and 10.4 diameter distances, while the wind speeds predicted by the model do not fall within the error bars for 9.4 diameter distances. A closer examination of the experimental data showed that the wind speed recovered faster for the 9.4D case than for the 10.4D case, a behavior that does not correspond to what the single-wake Jensen model would predict. Consequently, this non-monotonic and faster-than-expected recovery for the 9.4D case
is not an artifact of the proposed wake interaction model, but it is instead attributable to the Jensen’s model inability to reproduce the observed wake recovery.

It is important to demonstrate that the proposed wake interaction model is suitable for other wake models other than the Jensen model. As a comparison, the Frandsen wake model [6] is used with the proposed wake interaction model, and the results of model fitting are also shown in Figures 3.3–3.5. Note that the proposed wake interaction model fits the experimental data within its error bars, indicating that it can be used to model wake interactions regardless of the approach used to model the single-wake speed recovery.

Figure 3.3: Wind speeds experienced by a row of turbines separated by 7 diameter distances apart. Comparison between measurements (Horns Rev) and the proposed model (with Jensen and Frandsen) are shown. Error bars represent the standard deviation of the measurements.

3.4 Optimization

3.4.1 Model

The wind farm layout problem can be formulated as a mixed integer programming (MIP) problem. The wind farm domain is partitioned into a set of cells, with the restriction that each cell can have at most one turbine placed in its geometric centre. A simple wind farm under a simple one-directional wind regime shown in Figure 3.6 is discretized such that inter-turbine distance constraint of 5 rotor diameters
Chapter 3. Multiple Turbine Wake Interactions

Figure 3.4: Wind speeds experienced by a row of turbines separated by 9.4 diameter distances apart. Comparison between measurements (Horns Rev) and the proposed model (with Jensen and Frandsen) are shown. Error bars represent the standard deviation of the measurements. Note that the experimental data exhibits a non-monotonic behavior that cannot be captured by the single-wake models commonly used in the literature [5, 6].

is enforced. The proposed MIP formulation, similar to that proposed by Donovan [59, 119] and Zhang et al. [69], will be in the form of kinetic energy deficit (equivalent to the summation portion of Eq. (3.9)), meaning that the objective of the formulation would be to minimize the kinetic energy loss (mixing head). Let $x_i$ be a binary decision variable, indicating whether a turbine is placed ($x_i = 1$) or not ($x_i = 0$) in cell $i$, $i = 1, .., n$. Let $p_d$ be the probability of wind state $d$, where a wind state is defined as a (speed, direction) pair, and let $L$ be the the total number of wind states with non-zero probability. Then, the optimization formulation can be written as
where the first constraint describes the total number of wind turbines $k$ to be placed in the wind farm domain, which is held constant. In Eq. (3.10c), $Q$ is a binary matrix that is calculated prior to the optimization as an aid to enforce the inter-turbine distance constraints. For example, if cells $i$ and $j$ are closer than 5 rotor diameters apart, a row would be generated in the matrix $Q$ with 1’s in the $i$-th and $j$-th columns and 0’s everywhere else.

Also, in Eq. (3.10a), the $h_{ij}$ term denotes the kinetic energy deficit (mixing head) at cell $i$ caused by a turbine at cell $j$. This is found by using the summation term in Eq. (3.9), $h_{ij} = -\alpha_{r,2}u_{ij}^2 + \alpha_{r,3}u_0^2$. 
As previously described, the coefficients $\alpha_{r,2}$ and $\alpha_{r,3}$ are used to fit the wake interaction model to experimental measurements. The Jensen wake model is used to determine the wind speed for each individual wake.

![Figure 3.6: Simple wind farm domain divided into 36 cells under a one-directional wind regime. The distance between cell center is five times the rotor diameter. The wake of turbine placed in cell $j$ propagates downwind to affect cell $i$. Consequently, the placement of turbine in cell $j$ affects the decision whether to place a turbine in cell $i$ or not.](image)

### 3.5 Description of Tests

Three sets of tests were conducted to evaluate different wake interaction models used for MIP in the literature. The MIP formulation was chosen since the optimality of the solutions can be determined, allowing for a fair and accurate comparison between the underlying wake interaction models. The first set of tests is aimed at illustrating the importance of wake interaction models for layout optimization. The second set of tests aims to study how the current proposed model performs against existing wake interaction models, namely the Linear Superposition (LS) method used by Zhang [69] and the Sum of Kinetic Energy Deficits (SKED) approximation by Turner et al. [19]. The last set of tests is intended to assess the quality of the near-optimal solutions produced from each model if optimal solutions cannot be found in the time allotted for the optimization run.

The first set of tests determines the effects of wake interaction models in WFLO problems by studying the placement of 5 turbines on a horizontal land strip that is 2 km long and 100 m wide, under a constant wind of 6 m/s blowing from west to east. Due to the land’s narrow width, the turbines can only be placed downstream of each other, as shown in Figure 3.7. To test the effect of the discretization resolution on the resulting wind farm layouts, two discretization resolutions are tested, dividing the 2 km x 0.1 km wind farm domain into 20 cells and 100 cells. The turbine parameters for all tests are shown in Table
3.2. The second set of tests is conducted to evaluate the performances of different wake interaction models (LS and SKED) and against the proposed physics-based, linear model with coefficients determined from Horns Rev data. In addition, to assess the applicability of the proposed model when no measurement data is available, we also studied the performance of the proposed model with all coefficients set to 1, i.e., $\alpha_{r,1} = \alpha_{r,2} = \alpha_{r,3} = 1$. These test cases involved a simple wind regime and a varying number of turbines, using a 4 km x 4 km square wind farm domain, discretized into 10 x 10 cells, each cell is a square of size 400 m x 400 m. The wind regime includes six equally probable wind directions at 60° increments starting from the north, with a constant wind speed of 6 m/s. This wind regime is denoted as WR6. The turbine details are the same as in the previous test set, as shown in Table 3.2. Note that the size of these problems is sufficiently small such that they can be solved to optimality in a reasonable amount of time, so that any observed differences in performances can only be attributed to the wake interaction models, rather than lack of convergence to their respective optimal solutions [120].

The final test is performed to evaluate the ability for the models to produce good near-optimal solutions. One of the advantages that mathematical programming optimization methods provide is the known optimality of solutions. However, in large complex problems, it may not be possible to solve them to optimality in sufficient time. Thus, the ease to solve a particular model becomes very important. In this test, turbines are placed in a 4 km x 4 km square domain, divided into 20 x 20 cells with WR36 wind regime, where the wind blows at a constant wind speed of 6 m/s from 36 directions.

The optimization model was implemented using MATLAB and Gurobi 5.6 on a Dell Poweredge T420 Tower Server, 8 Intel Xeon processor E5-2400, and 164 GB RAM. The Gurobi linearization function to convert quadratic integer problems to mixed-integer linear problems was used. Then, the problems are solved through a branch-and-bound process. The total simulation time limit was set to 384 hr of CPU time. In order to ensure a fair comparison with other results reported in the literature, all the optimization results are re-evaluated with the sum of squares model, regardless of the model that was used to obtain the results.
Table 3.2: Wind turbine parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rotor Diameter</td>
<td>80 m</td>
</tr>
<tr>
<td>Thrust Coefficient</td>
<td>0.805</td>
</tr>
<tr>
<td>Power Curve</td>
<td>$1.68u^3$ kW</td>
</tr>
<tr>
<td>Wake Decay Constant</td>
<td>0.05</td>
</tr>
</tbody>
</table>

Table 3.3: Layout of 1 x 20 domain for a land strip of 2 km long. The x coordinates [m] for turbines T1–T5 and the resulting annual energy production (AEP) [GWh] are shown.

<table>
<thead>
<tr>
<th>Model</th>
<th>T1</th>
<th>T2</th>
<th>T3</th>
<th>T4</th>
<th>T5</th>
<th>AEP</th>
</tr>
</thead>
<tbody>
<tr>
<td>Present Work</td>
<td>50</td>
<td>450</td>
<td>1050</td>
<td>1550</td>
<td>1950</td>
<td>8.17</td>
</tr>
<tr>
<td>LS</td>
<td>50</td>
<td>450</td>
<td>950</td>
<td>1450</td>
<td>1950</td>
<td>8.15</td>
</tr>
<tr>
<td>SKED</td>
<td>50</td>
<td>450</td>
<td>950</td>
<td>1550</td>
<td>1950</td>
<td>8.12</td>
</tr>
</tbody>
</table>

3.6 Results and Discussion

The first test case illustrates the importance of wake interaction models. Based on intuition about problem behavior, it is clear that first and last turbines would be placed in the first and last cells, leaving the remaining 3 turbines to be placed in the rest of the domain. This simple case also served as a validation check for the problem formulation.

The layouts found highlight the influence and importance of wake interaction models. Table 3.3 shows the optimal turbine positions for 1 x 20 domain discretization. The influence of the wake interaction models on turbine positions is apparent. For example, the proposed model places the third turbine 100 m further downstream than SKED and LS, while the fourth turbine is placed in the same location as SKED. In this simple case, the differences in AEP found using different models are noticeable, with the proposed model outperformed the others.

The wind farm domain is further discretized into 1 x 100 cells to test the influence of discretization resolution. The results are shown in Table 3.4. The differences in turbine layout positions for the three wake interaction models have narrowed but are still noticeable. In terms of AEP, all three layouts have similar performance as the problem is too restrictive, leaving turbines little room to move to improve performance. For this simple case, it is clear that the solutions are grid dependent.

Typically, smaller problems require lower computation cost, thus optimal solutions can be found very quickly with MIP formulations. For example, the optimal layouts for the first set of tests (1 x 20 domain) were found in less than 0.05 seconds (wall clock time) for each case. However, as the number of cells increases, the difficulty of the problem increases exponentially [121]. Thus, it may not be possible to solve the problem to optimality in a timely fashion for finer grids. For example, in the 1 x 100 domain, optimal solutions were reached in less than 5 seconds, a 100-fold increase even though the problem size increased only 5-fold.
Table 3.4: Layout of 1 x 100 domain for a land strip of 2 km long. The x coordinates [m] for turbines T1–T5 and the resulting annual energy production (AEP) [GWh] are shown.

<table>
<thead>
<tr>
<th>Model</th>
<th>T1</th>
<th>T2</th>
<th>T3</th>
<th>T4</th>
<th>T5</th>
<th>AEP</th>
</tr>
</thead>
<tbody>
<tr>
<td>Present Work</td>
<td>10</td>
<td>410</td>
<td>1010</td>
<td>1590</td>
<td>1990</td>
<td>8.34</td>
</tr>
<tr>
<td>LS</td>
<td>10</td>
<td>410</td>
<td>990</td>
<td>1530</td>
<td>1990</td>
<td>8.33</td>
</tr>
<tr>
<td>SKED</td>
<td>10</td>
<td>450</td>
<td>990</td>
<td>1530</td>
<td>1990</td>
<td>8.34</td>
</tr>
</tbody>
</table>

In the second set of tests, the performance of proposed model, LS model, and SKED model are evaluated under WR6 wind regime, for the problem of placing 40 to 70 turbines in the 4 km x 4 km wind farm domain; note that Turner et al. [19] showed that cases with these numbers of turbines are particularly challenging to solve. Table 3.5 shows the resulting AEP for this case, comparing the LS, the SKED, and the proposed wake interaction model with two different sets of model coefficients, namely (a) coefficients fitted to data and (b) coefficients set to 1. The Gurobi solver was able to solve all the problem instances to optimality within a wall-clock time of 200 seconds.

In Table 3.5, the best solutions found are indicated in boldface type, note that the proposed model outperforms both the LS and SKED wake interaction models. Moreover, even when the proposed model coefficients are set to 1, i.e., assuming that there is no experimental data available to calibrate the model, the proposed model outperforms the others.

In order to relate the AEP values shown in Table 3.5 with economic gains/losses, Table 3.6 shows the annual revenues that would be forfeited if the proposed model was not used. The layout found using proposed model with Horns Rev coefficients produces additional revenue of $26,000–458,000 USD compared with other models, assuming a wind energy price of approximately $0.10 USD/kWh [1, 2]. Even when the proposed model is used with nominal coefficients, not fitted to any experimental data, larger AEP values, and corresponding financial gains, can still be realized. It is important to note that the AEP values were calculated using a wind speed of only 6 m/s, which is considered to be a low wind speed when compared with the rated wind speed of large wind turbines. Of course, the absolute differences in AEP will be higher at higher speeds. For example, the amount of revenue forfeited by using the SKED model (instead of our proposed model with nominal coefficients of $\alpha = 1$) in the case with 70 turbines would increase from $26K at a 6 m/s wind speed to $62K at 8 m/s and $120K at 10 m/s.

The optimized layouts found with different wake interaction models for 50 turbines under the WR6 wind regime are shown in Figure 3.8. The LS layout (Figure 3.8a) is the worst performing, with the two top rows and bottom occupied while layouts from SKED (Figure 3.8b) and proposed model (Figures 3.8c and 3.8d) occupied the top and bottom rows and spreading the turbines out in the remaining domain. Overall, the results for the second test case show that the proposed model produces better results than existing wake interaction models and that choosing an appropriate wake interaction model
Table 3.5: Annual energy production [GWh] for WR6 10 x 10 on a 4 km x 4 km domain. The best solution found for each case is indicated in boldface type.

<table>
<thead>
<tr>
<th>Number of Turbines</th>
<th>LS</th>
<th>SKED</th>
<th>Present Work (Horns Rev)</th>
<th>Present Work ($\alpha \equiv 1$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>40</td>
<td>114.97</td>
<td>114.97</td>
<td>114.97</td>
<td>114.97</td>
</tr>
<tr>
<td>50</td>
<td>125.82</td>
<td>128.75</td>
<td>129.51</td>
<td>129.03</td>
</tr>
<tr>
<td>60</td>
<td>137.13</td>
<td><strong>141.71</strong></td>
<td>141.71</td>
<td><strong>141.71</strong></td>
</tr>
<tr>
<td>70</td>
<td>148.45</td>
<td>149.25</td>
<td><strong>149.51</strong></td>
<td>149.25</td>
</tr>
</tbody>
</table>

Table 3.6: Forfeited annual revenue for different wake interaction methods with WR6 10 x 10 on a 4 km x 4 km domain, assuming an electricity price of $0.1/kWh [1, 2]. The best solutions found (Table 3.5) are used as reference values.

<table>
<thead>
<tr>
<th>Number of Turbines</th>
<th>LS</th>
<th>SKED</th>
<th>Present Work (Horns Rev)</th>
<th>Present Work ($\alpha \equiv 1$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>40</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>50</td>
<td>$370K$</td>
<td>$75K$</td>
<td>-</td>
<td>$48K$</td>
</tr>
<tr>
<td>60</td>
<td>$458K$</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>70</td>
<td>$106K$</td>
<td>$26K$</td>
<td>-</td>
<td>$26K$</td>
</tr>
</tbody>
</table>

is very important for layout optimization.

The third set of tests was aimed to determine how the models would perform when they’re not solved to optimality, within the allowed simulation time limit of 384 CPU hours. Under the WR36 wind regime, 20 to 70 turbines were placed in the wind farm domain. The layouts found for 40 turbines are shown in Figure 3.9. In these layouts, the turbines arranged themselves along the perimeter of the domain and spread out in the interior. The AEP values and the annual revenue forfeited due to using different layouts produced from LS, SKED, and proposed model are shown in Table 3.7 and Table 3.8, respectively. In the WR36 wind regime, as the number of turbines increase, the annual revenue forfeited increases due to the increased wake interactions. The present model with coefficients of 1’s outperformed that of LS and SKED when the number of turbines range from 20 to 50, but not for 60 and 70 turbines. On the other hand, the present model with coefficients from Horns Rev outperformed all other models. The solutions found in this case are not globally optimal but the results demonstrated that under resource constraints, the proposed model can produce better solutions compared with existing models.

3.7 Conclusions

In the present work, a new physics-based wake interaction model that leads to linear MIP formulations was introduced. The proposed linear wake interaction model was compared with existing wake interaction
Chapter 3. Multiple Turbine Wake Interactions

Figure 3.8: Layouts for the case of WR6 and 50 turbines with 10 x 10 grid and different interaction models, LS, SKED, present work (Horns Rev), and present work (α = 1).
Figure 3.9: Layouts for the case of WR36 and 40 turbines with 20 x 20 grid and different interaction models, LS, SKED, present work (Horns Rev), and present work ($\alpha = 1$), from left to right.
Table 3.7: Annual energy production [GWh] for WR36 20 x 20 on a 4 km x 4 km domain. The best solution found for each case is indicated in boldface type.

<table>
<thead>
<tr>
<th>Number of Turbines</th>
<th>LS</th>
<th>SKED</th>
<th>Present Work (Horns Rev)</th>
<th>Present Work ($\alpha \equiv 1$)</th>
</tr>
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<tbody>
<tr>
<td>20</td>
<td>59.61</td>
<td>59.76</td>
<td>59.92</td>
<td>59.80</td>
</tr>
<tr>
<td>30</td>
<td>86.05</td>
<td>85.86</td>
<td>86.15</td>
<td>85.94</td>
</tr>
<tr>
<td>40</td>
<td>109.96</td>
<td>109.73</td>
<td>110.63</td>
<td>110.13</td>
</tr>
<tr>
<td>50</td>
<td>131.57</td>
<td>131.36</td>
<td>132.66</td>
<td>131.97</td>
</tr>
<tr>
<td>60</td>
<td>149.75</td>
<td>150.67</td>
<td>151.46</td>
<td>150.23</td>
</tr>
<tr>
<td>70</td>
<td>164.66</td>
<td>165.14</td>
<td>165.50</td>
<td>165.12</td>
</tr>
</tbody>
</table>

Table 3.8: Forfeited annual revenue for different wake interaction methods with WR36 20 x 20 on a 4 km x 4 km domain, assuming an electricity price of $0.1/kWh [1, 2]. The best solutions found (Table 3.7) are used as reference values.

<table>
<thead>
<tr>
<th>Number of Turbines</th>
<th>LS</th>
<th>SKED</th>
<th>Present Work (Horns Rev)</th>
<th>Present Work ($\alpha \equiv 1$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>$31K$</td>
<td>$16K$</td>
<td>-</td>
<td>$12K</td>
</tr>
<tr>
<td>30</td>
<td>$10K$</td>
<td>$29K$</td>
<td>-</td>
<td>$21K</td>
</tr>
<tr>
<td>40</td>
<td>$67K$</td>
<td>$90K$</td>
<td>-</td>
<td>$50K</td>
</tr>
<tr>
<td>50</td>
<td>$110K$</td>
<td>$131K$</td>
<td>-</td>
<td>$69K</td>
</tr>
<tr>
<td>60</td>
<td>$170K$</td>
<td>$79K$</td>
<td>-</td>
<td>$122K</td>
</tr>
<tr>
<td>70</td>
<td>$84K$</td>
<td>$36K$</td>
<td>-</td>
<td>$38K</td>
</tr>
</tbody>
</table>

models in the literature that are suitable for MIP formulations. While the interaction of multiple wakes is not fully understood, the physics-based model can be fitted with experimental data or can be used as a MIP-friendly surrogate model for non-linear wake interaction models, e.g., sum of squares.

Three test cases were conducted to evaluate the performance of the proposed model. The first major finding of this study is that under the wind regime tested, better optimal layouts were found with the proposed model compared to optimal layouts found with SKED and LS models. This result illustrated the importance of selecting appropriate wake interaction models for WFLO problems, as they affect energy production and consequently, wind farm economics. The second major finding is that even when optimal solutions cannot be obtained due to resource constraints, the proposed model still outperformed that of SKED and LS models. This has strong implications when globally optimal solutions cannot be easily found, and near-optimal solutions are used as inputs for local search procedures to further improve layout solutions.

The current research was not specifically designed to evaluate factors related to the capabilities of the optimization solver. Considerable more work will need to be done to determine the performance of these models for large complex problems and the MIP solver’s ability to find good solutions efficiently.
Chapter 4

Layout Optimization on Complex Terrains

4.1 Introduction

Most studies on wind farm layout optimization have focused on optimizing layouts on flat and uniform topography [18, 19, 20, 21, 22, 23, 102]. However, wind speeds over complex terrains are very different than they are over flat terrains, since complex flow structures can form as wind flows over various land features. Consequently, energy production is strongly influenced by local topography. Furthermore, the lack of analytical, closed-form mathematical models for wakes over complex terrains makes it difficult to evaluate and optimize wind farm layouts. As a result, Feng and Shen [54] modified an adapted Jensen wake model to estimate the wake effects of a wind farm on a two-dimensional Gaussian hill. Taking a different approach, the virtual particle model developed by Song et al. [55] modeled the turbine wake as concentration of non-reactive particles undergoing a convection-diffusion process in a relatively low-cost model that describes the wake more accurately than a modified flat terrain wake model. Despite these efforts, reducing the computational cost of wake evaluations while maintaining accuracy during the optimization process remains a challenge. Hence, subsequent work [107, 108, 109] has focused on better integration of wake modeling and optimization algorithms.

Computational fluid dynamics (CFD) models (e.g. actuator disk and actuator line) have been developed to simulate complex wake phenomena and their interactions with terrains [40, 46, 47, 48, 49, 50, 122]. However, these simulations are expensive and must be used sparingly during the optimization process.

Deterministic optimization approaches such as mixed-integer programming (MIP) [18, 19, 69, 123, 124] have been shown to be promising in solving WFLO problems. These models can provide global
4.2 Previous Work

4.2.1 Optimization Models

A number of approaches to tackle the WFLO problem have been developed in the literature. The WFLO problem can be modeled by two approaches, discrete and continuous. In discrete models [20, 21, 60], the wind farm domain is divided into a number of possible turbine locations, while for continuous models [61, 62, 63, 64, 65, 66, 67], the turbine location is represented by two-dimensional continuous coordinates. Continuous models are typically solved using evolutionary metaheuristic algorithms [65, 66, 75, 81, 125, 126, 127, 128, 129] and nonlinear optimization methods [71, 77]. A discrete model can be solved by using mathematical programming approaches, which have the significant advantage of providing optimality bounds [19, 22, 59, 69, 70].

4.2.2 CFD Models

Computational fluid dynamics models have been applied to simulate wind turbine wakes, using Reynolds-averaged Navier-Stokes (RANS) [40, 46] and Large Eddy Simulation (LES) [26, 47, 130, 131, 132] turbulence models to simulate the turbulent wake phenomena. In addition to turbulence modeling, there are two main approaches to model rotor geometry: actuator disk/line and direct blade modeling. In an actuator disk [40, 46, 48, 133, 134, 135] or actuator line [136, 137, 138] approach, the turbine is modeled by imposing aerodynamic forces through a disk representing the rotor or lines representing the turbine blades, respectively. In a direct blade modeling approach [52, 130, 139], the turbine geometries
are inserted into the computational domain, allowing a more accurate representation of the aerodynamic
effects than the actuator disk/line approach at the expense of higher computational cost. The actuator
disk approach is less computationally expensive and less accurate. Despite the introduction of these
models in turbine wake modeling, it remains difficult to apply these models in optimization algorithms
to solve the WFLO problem due to the computational expense of CFD models.

## 4.3 Proposed WFLO Optimization Algorithm

While optimization and wake modeling have been applied individually to WFLO, there is a significant
challenge in combining them. An optimization algorithm typically must evaluate a very large number
of solutions and partial solutions. However, a single CFD simulation is so computationally expensive
that very few can be conducted in a reasonable run-time. In our approach, the optimization model
is first used with less accurate, less expensive data to identify promising turbine locations. The wake
effects of turbines placed at those locations are updated using CFD simulations. The CFD data is then
used iteratively by the optimization model to identify newly promising locations. Figure 4.1 shows a
schematic of our approach.

The principal idea behind the proposed algorithm is that on a complex terrain, the wind energy
potential of a location is influenced by the local terrain topography, thus different turbine locations will
have different “turbine placement potentials”. The proposed algorithm utilizes a MIP model to search
through promising locations through a combination of estimated wake effects and CFD simulation data.

Looking at the flowchart of the proposed algorithm in Figure 4.1, firstly, a flow field over the complex
terrain without turbines is generated using CFD. The initial wake effects can be calculated by superim-
posing a flat terrain wake onto the complex terrain as described in Section 4.3.2. This initial problem
is then solved to determine where the turbines should be placed. However, due to inaccuracies in the
initial wake estimate, placing turbines at these locations may not produce the optimal layout. Hence
CFD simulations are conducted at these locations to improve the accuracy of the initial estimated wake
effects. This process is repeated until no new improving turbines locations are found. In other words,
the wake effects described in the optimization model becomes more accurate with each iteration. Hence,
the optimal solution of the current iteration is more accurate than those found in previous iterations.
If the problem cannot be solved to optimality due to run-time limits, then it becomes necessary to
compare the near-optimal solutions from previous iterations. Conceivably, other optimization methods
such as metaheuristics are also compatible with this algorithm; however, without proof of optimality,
the termination criteria for the optimization problem would need to be defined appropriately.
4.3.1 MIP Optimization Model

A number of mixed-integer programming formulations have been developed to tackle the WFLO problem [19, 69, 123, 124]. A MIP model consists of an objective function, a set of constraints, and a mix of integer and continuous variables. To describe the WFLO problem, the wind farm is discretized into possible turbine locations with corresponding binary decision variables denoting if a turbine is located at each location or not. The formulation used in this work, similar to that of the work of Kuo et al. [123, 124], has an objective function of maximizing the sum of the kinetic energy experienced by each turbine, as follows. Let the wind farm domain be divided into a total of $N$ cells, let $K$ be the number of turbines to be placed (considered a constant in the formulation), and let $x_i$ be a binary variable denoting whether a turbine is placed in the $i$-th cell. The optimization problem is

\[
\begin{align*}
\text{max} & \quad \sum_{i=1}^{N} \sum_{s \in S} p_s x_i \left( u_{0,s,i}^2 - \sum_{j \in J} (u_{0,s,j}^2 - u_{s,ij}^2) x_j \right) \\
\text{s.t} & \quad \sum_{i=1}^{N} x_i = K \\
& \quad d_{ij} x_i + d_{ji} x_j \leq 1 \quad \forall i, j \\
& \quad x_i \in \{0, 1\} \quad \forall i = 1, ..., N
\end{align*}
\] (4.1a)

(4.1b)

(4.1c)

(4.1d)
where the binary terms $d_{ij}$ and $d_{ji}$ indicate the violation of the distance constraint between $i$-th and $j$-th cells, which need to be calculated in advance. Namely, $d_{ij} = d_{ji} = 1$ if the distance constraint is violated when turbines are placed both in the $i$-th and $j$-th locations, and $d_{ij} = d_{ji} = 0$ otherwise. In Eq.(4.1a), $p_s$ is the probability of wind state $s$, and $S$ is the total number of wind states, where a wind state is defined as a (wind speed, wind direction) pair. Most importantly, $u^2_{0,s,i,j} - u^2_{s,ij}$ denotes the kinetic energy deficit at cell $j$ caused by a turbine at cell $i$, which is dependent on the wind state, $s$. Figure 4.2 shows a wake from turbine located in cell $i$, propagating downstream to affect cell $j$.

![Figure 4.2: Turbine wake created by west wind. The wake from turbine at location $i$ propagates downstream, affecting location $j$.](image)

In this formulation, all the single wake effects caused by a turbine must be calculated in advance for all possible locations. That is, when a turbine is placed in cell $i$, its single wake effects on all remaining cells must be known for all possible turbine locations and wind states. Hence, the number of potential turbine locations (i.e., the number of cells) multiplied by the number of wind states determines the number of wake calculations required (i.e., $N\vert S\vert$) to define the MIP formulation. In the proposed algorithm, the promising turbine locations are identified from the optimal MIP layout solutions using less accurate data and CFD simulations are only conducted at these locations. In this way, we seek to achieve the same wake accuracy as running $N\vert S\vert$ CFD simulations with a fraction of the computational cost.

When multiple turbines wakes are present, their combined effect on wind speed recovery is approximated by using an energy balance approach by Kuo et al. [124]. This form is suitable for MIP formulation due to its linearity and sound physical basis. Energy balance is done along a streamtube from the free stream mixing into the wake, assuming the wake losses are additive for overlapping wakes. The MIP model can be solved using mathematical programming approaches to compute the optimal turbine layout for each set of inputs.
4.3.2 Wake Modeling

In order to identify a promising turbine placement to evaluate with a CFD simulation, we must first solve the MIP model with approximate wake effects. These wake effects are calculated using an approximate wake model by superimposing CFD simulation data of a flat terrain wake onto the complex terrain, using Eq.(4.2) and Eq.(4.3). The assumptions made here are that the wake propagates downstream along the terrain surface at hub height and that the wake will experience a speed-up factor due to terrain effects, i.e.

\[
uc_{t,s,j} = S_{s,j} u_{ft,s,j},
\]

\[
u_{w}^{ct,s,ij} = S_{s,j} u_{w}^{ft,s,ij},
\]

where \(uc_{t,s,j}\) and \(u_{ft,s,j}\) are the free stream wind speeds on complex and flat terrains in cell \(j\) in wind state \(s\), and \(u_{w}^{ct,s,ij}\) and \(u_{w}^{ft,s,ij}\) describe the wind speeds in the wake on complex and flat terrains in cell \(j\) due to a turbine in cell \(i\), respectively. In other words, \(S_{s,j}\), the speed-up factor due to terrain effects experienced in cell \(j\) (in comparison with flat terrain flow field) in wind state \(s\), is calculated without the presence of turbines, and then used to “carry” the wakes downstream, similar to the implementation used by Feng and Shen [54] and in several commercial software packages [54]. In this work, whenever CFD simulation data is available, the speed-up factor \(S_{s,j}\) is corrected using simulation results. It should be noted that while superimposing wakes onto terrains is not an accurate representation of the actual wake effects, this work also addresses the effects of accuracy of initial wake approximation on solution quality and computational cost (see following section).

When promising turbine locations are available, CFD simulations are conducted to simulate wake effects of turbines at those locations. The actuator disk model and the extended \(k-\varepsilon\) turbulence model by El Kasmi and Masson [7] are used in this study. Specifically, an actuator disk is inserted into the computational domain and the turbulent dissipation zones are prescribed upstream and downstream of the disk, shown in Figure 4.3. Appropriate boundary conditions (e.g. inlet, outlet, surface roughness) must be prescribed to accurately simulate the atmospheric boundary layer.

To summarize how MIP and CFD are combined, the proposed algorithm is as follows:

1. Generate flow field over the complex terrain without turbines using CFD.
2. Construct the initial wake effects using the approximated method described in wake modelling section.
3. Solve the optimization problem to identify the most promising locations.
4. Run single turbine CFD simulations at locations found in the previous step.
(5) Update the wake effects from CFD results \((u_{s,ij} \text{ term})\) in optimization (Eq.(4.1a)).

(6) Repeat steps (3–5) until the solution converges.

---

4.3.3 Impact of the Initial Wake Approximation

In this algorithm, the final layout is dependent on the initial wake approximation. The assumption that wakes propagate in a straight line at the hub height may not hold for complex terrains, thus resulting in a vast overestimate of the velocity deficit in certain cells and an underestimate in others. If the velocity deficit is overestimated in some cells in the initial approximation, those cells might never be considered in future layout solutions. Thus a relaxation parameter, \(C\), is introduced to reduce the velocity deficit in the initial wake approximation. Specifically, the velocity deficit is multiplied by the relaxation parameter, \(C\), to force an underestimate of the velocity deficit and mitigate the effect of poor approximations of wake behavior on complex terrains.

When the wake effects are underestimated, more turbine locations or cells will be explored so more CFD simulations are required. Hence, the relaxation parameter \(C\) controls how aggressively the optimization space is explored, balancing the need for better accuracy in wake modeling with the total computational cost of the optimization. Specifically, the \(u_{s,ij}\) term from Eq.(4.1a) is re-written as \(U_{0,s,j} - CD_{s,ij}\), where \(D_{s,ij}\) is the velocity deficit at cell \(j\) caused by turbine at cell \(i\) in wind state \(s\). The \(U_{0,s,j} - CD_{s,ij}\) term is only used when CFD data is not available (these cells are defined as set \(N_2\)). If CFD data is available (defined as set \(N_1\)), then the simulation data is used directly for \(u_{s,ij}\) and the
relaxation parameter is not used. The new MIP formulation is written as,

\[
\begin{align*}
\max & \sum_{i \in \mathcal{N}} \sum_{s \in \mathcal{S}} p_{s,i} \left[ U_{0,s,i}^2 - \sum_{j \in J} (U_{0,s,j}^2 - u_{s,ij}^2)x_j \right] \quad (4.4a) \\
& + \sum_{i \in \mathcal{N}} \sum_{s \in \mathcal{S}} p_{s,i} \left[ U_{0,s,i}^2 - \sum_{j \in J} (2U_{0,s,j} - CD_{s,ij})CD_{s,ij}x_j \right] \quad (4.4b) \\
\text{s.t} & \sum_{i=1}^{N} x_i = K \quad (4.4c) \\
d_{ij}x_i + d_{ji}x_j \leq 1 \quad \forall i, j \quad (4.4d) \\
x_i \in \{0, 1\} \quad \forall i = 1, \ldots, N. \quad (4.4e)
\end{align*}
\]

### 4.4 Case Study: The Carleton-sur-Mer Wind Farm

The proposed algorithm is tested on a 2.8 km x 2.8 km wind farm domain in Carleton-sur-Mer, Quebec, Canada. The topography was extracted from Google Maps\textsuperscript{TM} (https://goo.gl/maps/XTpzd), with a roughness length assumed to be 0.1 m. The terrain elevation in meters above sea level is shown in Figure 4.4. The optimization domain is discretized into a uniform grid of 20 x 20 cells, separated at cell center by a distance of 140 m. A wind farm layout of 20 turbines is optimized for this terrain. These turbines are assumed to have a constant thrust coefficient of 0.8, hub height of 80 m, and rotor diameter of 80 m. These turbine parameters were obtained from the Carleton Wind Farm. The proximity constraint between turbines is set as 5 rotor diameters apart.

For this wind farm domain, information regarding the wind speed and directions are available from the Canadian Wind Energy Atlas [8]. A power law velocity profile is used to describe the wind speed at varying heights

\[
u(z) = 6\left(\frac{z-139}{50}\right)^{0.16},
\]

where \(z\) is the height above sea level. This velocity profile is used to define inlet boundary conditions for CFD simulations. The wind rose used for this domain is shown in Figure 4.5, noting that the dominant wind direction is from the west. The turbulent kinetic energy and dissipation rate at the inlet are prescribed as \(k = \frac{(u^*)^2}{\sqrt{C^*_\mu}}\) and \(\varepsilon(z) = \frac{(u^*)^3}{\kappa(z-139)}\), where \(C^*_\mu = 0.033\) and \(\kappa = 0.4\). With the assumptions for ground roughness and the height (1000 m) of the boundary layer, the friction velocity \(u^* = 0.4m/s\). The velocity and turbulence quantities are fixed at the top boundary, as other types of boundary conditions such as symmetry or slip wall could cause undesirable streamwise gradients [48, 140]. In case the wind
is not aligned with the x-direction, the velocity inlet takes the form of \( u_x(z) = 6 \left( \frac{z-139}{50} \right)^{0.16} \cos(\theta) \) and \( u_y(z) = 6 \left( \frac{z-139}{50} \right)^{0.16} \sin(\theta) \), where \( \theta \) is the wind direction relative to the x-axis [141]. The ground is taken as wall boundary and the outlet face is considered as pressured outlet boundary.

4.4.1 Initial Results

To summarize the WFLO problem, 20 turbines are placed in a domain (Figure 4.4) that is discretized into uniformly sized 20 x 20 cells. Based on the wind rose, Figure 4.5, there are 12 wind directions with a power law wind velocity profile as given in Eq.(4.5). The proximity constraint between turbines was set to be 5 diameters distance apart. In the initial test, the relaxation parameter has been set to \( C = 1 \).

The MIP model can be solved under 30 seconds using Gurobi 5.6, so that the bulk of the computational expense is dedicated to CFD simulations. For each cell, a CFD simulation needs to be conducted for every wind direction, or in this case, 12 CFD simulations per cell. With 400 possible locations, and 12 wind directions, the maximum number of single turbine CFD simulations is 400 x 12 = 4800.

Each CFD simulation is performed for a domain of 2.8 km x 2.8 km in length and width, with a height up to 1000 m above sea level, shown in Figure 4.6a. Initially, the CFD simulations are conducted
without the presence of turbines for all 12 wind directions, with the domain discretized into 1.2 million cells in the domain. When a turbine is placed in the domain, the number of cells is increased to 1.6 million cells to better capture the wake effects downstream of the turbine, shown in Figure 4.6b.

In the first iteration, the flow field in the absence of turbines is obtained from CFD simulations. The turbine wake from flat terrain is modified using Eq.(4.3) to approximate the wake effects without conducting any CFD wake simulations. The layout found in this first iteration is shown in Figure 4.7a.

In the second iteration, the wakes for wind turbines placed at these 20 locations are simulated using CFD and the initial wake effects are updated. The new layout that was found is shown in Figure 4.7b. In this new layout, three turbines are relocated compared to the first iteration. The turbine wakes from
these three locations (indicated by circles) are simulated and updated. In the third and final iteration, the layout found in Figure 4.7c is identical to that of the second layout, indicating that the algorithm has converged.

![Figure 4.7: Optimal layout found at the end of each iteration. The circles mark the turbines that were relocated in that iteration. Note that after only 3 iterations, the algorithm did not identify additional turbine locations that would lead to improvements in the optimization objective.](image)

### 4.4.2 Manipulating the Relaxation Parameter

A parametric study on the relaxation parameter, $C$, was conducted, considering the values $C = \{1, 0.7, 0.4, 0.2, 0\}$, to study the effects of the initial wake approximation on the solution quality and computational cost. In a WFLO problem for complex terrains, the solution upper bound in terms of energy production is one where the turbines are placed at locations where the wind speeds are the highest, ignoring the wake effects. This upper bound for this test case is found to be 2177.48. Normalizing all the objective values
found in this study with this upper bound provides a relative comparison of the solutions found using different values of $C$. This normalized value is defined as the layout efficiency. The influence of the values of $C$ on the progression on the objective values (based on wake effects known at each iteration) is shown in Figure 4.8.

The solutions found for different values of $C$ are shown in Figures 4.9–4.12. The influence of $C$ on the number of iterations, number of CFD simulations, final objective value, layout efficiency, and run-time is shown in Table 4.1. It can be seen that as $C$ decreases in value, better layouts are produced. It is notable that for the cases where $C \geq 0.2$, only three iterations and a small fraction of the total number of CFD simulations are needed for convergence. For the case of $C = 0$, eight iterations are required for convergence and more CFD simulations are needed (compared with higher values of $C$) as the algorithm searched through 52 turbine locations in the domain. In other words, when the wake deficits are not accounted for, the algorithm will “blindly” search through the most promising cells in terms of wind resource until the optimal solution is found. This behavior can be seen in Figure 4.12, where large number of turbines are relocated to neighbouring locations from one iteration to the next, until all the promising cells are exhausted. While computational cost is not a major concern when the size of the problem is relatively small, and can be solved to optimality relatively quickly, this can be a significant downside when the problem increases in size, e.g. larger number of possible turbine locations and more complex wind regimes. For the test cases where $C \geq 0.2$, the total run-time is approximately 300 hours on a Dell Precision T1700 PC, but the run-time more than doubled when $C = 0$, demonstrating the importance of the relaxation parameter in controlling the computational cost. It is important to note that the solution found in the $C = 0$ case is the globally optimal solution. That is, if CFD simulation data is available for all cell locations ($N|S| = 4800$ CFD simulations, approximately 5300 hours), the optimal solution would be identical to that of $C = 0$, unless the presence of turbine wakes can locally improve the energy potentials of some locations.

For all the different values of $C$ tested, the final layout solutions are within 2% of the upper bound. The difference in performance between the best ($C = 0$) and worst ($C = 1$) solutions is less than 1.5%, demonstrating the algorithm’s capability to find good solutions even with poor initial estimation of wake effects. Figures 4.13 and 4.14 show the effects of the relaxation parameter on computational cost and layout efficiency. In terms of solution quality, underestimating the wake deficit (e.g. $C = 0.2$) is desirable as the path of wake propagation is difficult to predict prior to CFD simulations. When higher values of $C$ are used, velocity deficits experienced by downstream turbines may be overestimated in some cells. The consequence is that certain promising locations may be ignored during the search. However, when wake deficits are underestimated with lower values of $C$, the computational cost increases. In this particular problem, a low $C$ value of 0.2 did not dramatically increase the computational cost relative to
Table 4.1: Influence of relaxation parameter on solution quality and computational cost

<table>
<thead>
<tr>
<th>$C$</th>
<th># of Iterations</th>
<th># of CFD Evaluations</th>
<th>Final Objective Value</th>
<th>Layout Efficiency (%)</th>
<th>Run-time (hr)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3</td>
<td>$23 \times 12 = 276$</td>
<td>2133.25</td>
<td>97.97</td>
<td>303.63</td>
</tr>
<tr>
<td>0.7</td>
<td>3</td>
<td>$22 \times 12 = 264$</td>
<td>2146.66</td>
<td>98.58</td>
<td>290.43</td>
</tr>
<tr>
<td>0.4</td>
<td>3</td>
<td>$21 \times 12 = 252$</td>
<td>2150.83</td>
<td>98.78</td>
<td>277.23</td>
</tr>
<tr>
<td>0.2</td>
<td>3</td>
<td>$24 \times 12 = 288$</td>
<td>2165.16</td>
<td>99.43</td>
<td>316.83</td>
</tr>
<tr>
<td>0</td>
<td>8</td>
<td>$52 \times 12 = 624$</td>
<td>2165.80</td>
<td>99.46</td>
<td>686.47</td>
</tr>
</tbody>
</table>

larger values, but did improve solution quality significantly. Note that this algorithm is not a globally seeking algorithm, hence the final solution is dependent on the initial layout. Based on the finding, the relaxation factor has the effect of forcing the algorithm to converge into locally optimal solutions.

Choosing the “right” $C$ to produce good layout will depend on the terrain topography. If the terrain is too rugged and the flow experiences rapid changes where the streamlines can deviate significantly from the terrain profile, a smaller $C$ would be ideal in finding good layouts. As the local changes in the topography is less pronounced, a larger value of $C$ would be preferred. An intuitive and adaptive scheme of varying values of $C$ for every iteration can be developed, borrowing the idea from simulated annealing [142], e.g. starting with initial low $C$ and adjusts as the algorithm progresses. In addition, better prediction of the initial wake effect is needed for improving solution quality and computational cost. These two areas of improvement will be the focus in future studies.
Figure 4.8: Progression of the objective values (varying the relaxation factor) based on wake effects known at each iteration.
Figure 4.9: Optimal layout found at the end of each iteration with relaxation parameter, $C$, set to 0.7. The circles mark the turbines that were relocated in that iteration. Note that after only 3 iterations, the algorithm did not identify additional turbine locations that would lead to improvements in the optimization objective.
Chapter 4. Layout Optimization on Complex Terrains

Figure 4.10: Optimal layout found at the end of each iteration with relaxation parameter, $C$, set to 0.4. The circles mark the turbines that were relocated in that iteration. Note that after only 3 iterations, the algorithm did not identify additional turbine locations that would lead to improvements in the optimization objective.
Figure 4.11: Optimal layout found at the end of each iteration with relaxation parameter, $C$, set to 0.2. The circles mark the turbines that were relocated in that iteration. Note that after only 3 iterations, the algorithm did not identify additional turbine locations that would lead to improvements in the optimization objective.
Chapter 4. Layout Optimization on Complex Terrains
Figure 4.12: Optimal layout found at the end of each iteration with relaxation parameter, $C$, set to 0. The circles mark the turbines that were relocated in that iteration. Note that after 8 iterations, the algorithm did not identify additional turbine locations that would lead to improvements in the optimization objective.

4.5 Conclusion

In this work, an algorithm that optimizes wind farm layouts on complex terrains was introduced. This algorithm combines CFD simulations with mathematical programming methods for layout optimization. To the best of the authors’ knowledge, this is the first WFLO study that makes use of mathematical programming methods with CFD wake simulations. The proposed iterative approach identifies promising turbine locations to minimize the number of CFD simulations required in optimization while finding good layouts, even when the optimization relies on inaccurate wake models during the first iterations. The proposed approach starts with an approximate wake model that superimposes a flat terrain wake model on the topography, and this model is adaptively refined based on CFD simulations that are conducted only at promising turbine locations. This chapter presents a better and more efficient optimization of wind turbine layouts on complex terrain, because of better modeling accuracy and the theoretical convergence bounds of MIP models.

In order to study the effects of initial wake approximation on solution quality and computational cost, we introduced a relaxation parameter to control how the optimization space is explored. It was found that regardless of the parameter value, the difference in performance for best and worst layouts found is less than 1.5%, indicating that the algorithm is capable of finding good layouts even under poor initial wake approximations. Finding a suitable value for the relaxation parameter will depend on the balance between computational cost and solution quality as low values of the relaxation parameter may
Figure 4.13: Effects of relaxation parameter on computational cost (fraction of maximum number of CFD evaluations).
Figure 4.14: Effects of relaxation parameter on layout efficiency.
improve solution quality at the expense of computational cost while the reverse may hold true for high values.

Further work in developing the proposed novel approach for WFLO combining CFD simulations of wake behavior with mathematical programming is needed to study the scalability of the algorithm to larger problem instances, i.e., to wind farms with more potential turbine locations. The implications of this work are that CFD can be a valuable tool in WFLO problems and that good potential turbine locations can be identified in advance to significantly reduce the number of expensive simulations.
Chapter 5

Influence of Rotor Geometry and Atmospheric Turbulence

5.1 Introduction

One of the main challenges in modelling turbine wakes is how the turbine rotor should be represented in CFD simulations. To reduce the computational cost, the actuator disk approach [51] is used to model the complex rotor geometries. The results from this modelling approach have been shown to produce good agreement with the full rotor modelling approach [52].

When the details of turbine rotor geometry or its aerodynamic forces are not available, it is common to model a turbine rotor as a disk or lines exerting a uniform force on the flow [133, 143, 144, 145]. This approach assumes that the variable of interest is independent of the rotor geometry far downstream of the turbine. This assumption was also made by Jafari et al. [53] and Ainslie [146] in their respective modelling approaches. In these numerical models, the far wake is the region of interest, and the near wake region is modelled analytically. These modelling approaches rely on two underlying assumptions: 1) the wake recovery process in the far wake region, where the atmospheric conditions are dominant, can be determined without knowing the blade profile, and 2) the location where the far wake region begins is relatively insensitive to blade geometry and atmospheric conditions. However, the validity of these assumptions has not been studied in the literature. Thus, understanding the influence of turbine geometry and atmospheric conditions on the wake recovery process is the primary objective of this study.
5.2 Wind Turbine Model

In an actuator disk model, the turbine is modelled as a porous disk with an infinite number of blades that extract momentum from the flow [51, 147]. A visual representation of the model is shown in Figure 5.1. The actuator disk is a momentum sink that exerts aerodynamic forces on the incoming flow. This thrust force \( T \) can be calculated as:

\[
T = \int_A (p_1 - p_2) dA, \tag{5.1}
\]

where \( p_1 \) and \( p_2 \) represent the pressure acting on the upstream and downstream faces of the disk, respectively, and \( A \) is the rotor swept area. This thrust force can be normalized, and the thrust coefficient \( (C_T) \) is introduced:

\[
C_T = \frac{T}{0.5 \rho A u_0^2} = 4a(1-a), \tag{5.2}
\]

where \( a = 1 - \frac{u_{wake}}{u_0} \) is the axial induction factor, \( u_0 \) and \( u_{wake} \) are the wind speed in the free stream and the wake, respectively. As previously mentioned, the aerodynamic forces that turbine blades exert on the flow are not uniform along the radial direction. However, in the absence of detailed turbine rotor information, these forces are assumed to be uniform.

![Figure 5.1: An actuator disk model.](image)

In this work, an actuator disk model is used in conjunction with the extended \( k - \varepsilon \) turbulence model developed by El Kasmi et al. [41]. This \( k - \varepsilon \) model is a modification of the standard \( k - \varepsilon \) model in which an additional term is added to more appropriately model the turbulence energy transfer rate from large-scale to small-scale turbulence. The constants used by this model, corresponding to neutral atmospheric conditions, are: \( \sigma_k = 1.0, \ \sigma_\varepsilon = 1.3, \ C_{\varepsilon 1} = 1.176, \ C_{\varepsilon 2} = 1.92, \ C_{\varepsilon 4} = 0.37, \) and \( C_{\mu} = 0.033. \)
5.3 Model Validation

The wake model by El Kasmi et al. [41] has been validated in the literature, producing good agreement with experimental data. The focus of this section is to validate the accuracy of the aerodynamic forces prescribed on the actuator disk with that of wind tunnel experimental data from a test turbine. The turbine used in this study has three LM8.2 blades with a rotor diameter of 19.06 m. The axial induction factor, shown in Figure 5.2 [148], is used to calculate the forces acting on the incoming flow.

![Axial Induction Factor](image)

Figure 5.2: Axial induction factor along a LM8.2 turbine blade.

Through the use of Eq. (5.2), aerodynamic forces can be prescribed on the actuator disk, and then the mechanical power can be calculated as:

\[ P = \int_{A} (p_1 - p_2)u_{disk} dA, \]  

where \( u_{disk} \) is the wind speed at the disk.

A mesh sensitivity analysis was performed to ensure mesh independence. Three different mesh sizes were used in the validation process and their corresponding mechanical powers (calculated using Eq. (5.3)) are shown in Table 5.1. For all three mesh sizes tested, they yielded similar mechanical power when compared with experimental data. The error from Mesh 1 is approximately 9% with respect to the experimental data and is the mesh used in this study.
### Table 5.1: Mesh sensitivity analysis

<table>
<thead>
<tr>
<th>Number of Elements</th>
<th>Mesh 1</th>
<th>Mesh 2</th>
<th>Mesh 3</th>
<th>Measurement</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Elements</td>
<td>1.2M</td>
<td>4.0M</td>
<td>8.1M</td>
<td>–</td>
</tr>
</tbody>
</table>

### 5.4 Numerical and Experimental Setup

This main focus on this work is to study the effects of turbine blade geometry and atmospheric turbulence on wind turbine wake development. To study the influence of blade geometry, the wake development of an actuator disk with an aerodynamic force profile of an LM8.2 blade and that of a uniform force profile are compared. In other words, to examine the effect of rotor geometry on wake development, two sets of forces corresponding to an LM8.2 blade and uniform profiles that extract identical mechanical power are prescribed for comparison. Then, the effects of atmospheric turbulence are studied under two different conditions, turbulent intensities of 5% and 10%.

The CFD simulation domain is 24 rotor diameters in the streamwise direction and 10 rotor diameters in both the spanwise and vertical directions. The turbine is placed 4 rotor diameters downstream of the inlet at a hub height of 20 m. The geometry of the simulation domain is shown in Figure 5.3.

![Simulation domain](image)

Figure 5.3: Simulation domain

A log law velocity profile is prescribed at the inlet:

\[
U(y) = \frac{u^*}{\kappa} \ln \left( \frac{y}{y_0} \right),
\]

where \( \kappa = 0.4 \), \( y \) is the height above ground in the vertical direction, \( y_0 \) is the surface roughness, and \( u^* \) is the friction velocity. The turbulent kinetic energy and dissipation rate at the inlet are prescribed as \( k = \frac{(u^*)^2}{\sqrt{C_\mu}} \) and \( \varepsilon(y) = \frac{(u^*)^3}{\kappa y} \), where \( C_\mu = 0.033 \), and the friction velocity \( u^* \) is assumed to be 0.16
The roughness length $y_0$ is prescribed as $4.036 \times 10^{-6} \text{ m}$ and $8.984 \times 10^{-3} \text{ m}$, corresponding to turbulence intensities of 5% and 10%, respectively [141]. The velocity and turbulence quantities are fixed at the top boundary, and the bottom boundary is taken as a wall boundary, while the outlet is considered a pressure outlet, and the symmetry boundary conditions are applied on the two boundaries in the spanwise direction [48].

### 5.5 Results and Discussion

The effects of turbulence on the wake recovery process have been previously studied [16, 42, 149]. In general, higher atmospheric turbulence improves the mixing process and thus accelerates the recovery process. Figures 5.4, 5.5, and 5.6 show the velocity profiles in the spanwise direction for the LM8.2 blade profile with different inlet turbulence intensities (TI) at 1, 2, and 6 rotor diameters downstream. It is seen that the recovery process for turbulence intensity at 10% is significantly faster than at 5%. Thus, it can be expected that the length of the near wake region shortens as turbulence intensity increases.

To study the influence of turbine blade geometry, a uniform thrust coefficient is used to compare with the force profile from that of the LM8.2 blade. Figures 5.7, 5.8, and 5.9 show the velocity profiles for turbulence intensity at 5%, for 1, 2, and 6 rotor diameters downstream of the turbine. At 1 and 2 rotor diameters downstream, the velocity profiles from uniform thrust coefficient do not match well with results generated from the LM8.2 blade profile. Initially, the profile generated with the LM8.2 blade is
bimodal, and then by 2.5 diameters downstream the velocity profile becomes Gaussian, marking the end of the near wake region. By 6 rotor diameters downstream, the velocity profiles from both force profiles become nearly identical. This has strong implications, as it shows that the wake recovery process is nearly independent of the blade profile in the far wake region. However, if the focus is modelling the near wake region, which ends at approximately 2.5 rotor diameters downstream of the turbine in this scenario, then the uniform force approach is not sufficient.

A similar comparison is done to study the effects of increasing atmospheric turbulence on the wake recovery process under different distribution of forces. Figures 5.10, 5.11, and 5.12 show the velocity profiles under a turbulence intensity of 10%. At approximately 1.5 rotor diameters downstream, the velocity profile becomes Gaussian, marking the end of the near wake region. This has implications for wake models that analytically model or bypass the near wake region, such as the one developed by Jafari et al. [53], where the near wake region is assumed to be 1 rotor diameter downstream compared to 2.5 and 1.5 rotor diameters for turbulence intensities of 5% and 10%, respectively. Looking at Figures 5.11 and 5.12, the velocity profiles of both LM8.2 and the uniform forces are nearly identical, echoing the previous results with a 5% turbulence intensity.
5.6 Conclusion

This work set out to answer two main questions: 1) Does the wake recovery depend on the blade profiles? 2) How does turbulence affect the near and far wake regions? Our results show that the near wake region is strongly affected by the turbine geometry, as evidenced by the shape of the velocity profiles. In addition, the length of the near wake region is dependent on atmospheric turbulence conditions, where the near wake region ends 2.5 and 1.5 rotor diameters downstream of the turbine for the turbulence intensities of 5% and 10%, respectively. Increased turbulence improves the mixing process, shortening the length of the near wake region. Typically, the effects of the blade geometry become less significant with higher turbulence intensity.

In the far wake region, increased turbulence allows accelerated recovery as atmospheric turbulence becomes the dominant factor, which has been reported in previous studies. The results show that in the far wake region, this recovery is nearly independent of the blade geometry. This suggests that it is not necessary to model the aerodynamic force distribution and that a uniform force is sufficient to model the wake in the far wake region. In other words, given the bulk aerodynamic forces that a turbine exerts on the flow, a uniform force actuator disk is sufficient to model the rotor if the far wake is the region of interest. For wake models that bypass or analytically model the near wake region, it is necessary to identify where the near wake region ends and such a transition must be appropriately modelled.
Figure 5.7: Velocity profile 1 rotor diameter downstream of the turbine with different force profiles at a turbulence intensity of 5%.

Figure 5.8: Velocity profile 2 rotor diameters downstream of the turbine with different force profiles at a turbulence intensity of 5%.
Figure 5.9: Velocity profile 6 rotor diameters downstream of the turbine with different force profiles at a turbulence intensity of 5%.

Figure 5.10: Velocity profile 1 rotor diameter downstream of the turbine with different force profiles at a turbulence intensity of 10%.
Figure 5.11: Velocity profile 2 rotor diameters downstream of the turbine with different force profiles at a turbulence intensity of 10%.

Figure 5.12: Velocity profile 6 rotor diameters downstream of the turbine with different force profiles at a turbulence intensity of 10%.
Chapter 6

Wake Model for Complex Terrains

6.1 Introduction

Due to cost-competitiveness of onshore wind farms, most of the wind energy development in North America have been focused on onshore farms [24]. One of the key aspects to wind farm design is the placement of wind turbines, as wake losses is one of the major losses in wind farms [14]. In order to optimize turbine placements, wake models are required to evaluate the layout solutions. However, most of the wake model development have been focused on flat and uniform terrains [18, 19, 20, 21, 22, 23]. Therefore, there is a need to develop wake models capable of simulating wake effects on complex terrains.

During the design optimization process, wake models are used to evaluate the wake effects of different layout solutions. This process is iterative and requires a large number of solution evaluations, thus low-cost wake model is necessary to evaluate large pools of candidate solutions. In commercial software, wake models for complex terrains rely on variations of wake superposition approaches [54]. In addition to wake superposition, only one wake model capable of simulating wakes on complex terrains has been developed. This model, virtual particle model [55], utilizes the concept of solving for velocity deficit instead of velocity by assuming that velocity deficit is analogous to concentration of particles.

The main challenge in developing wake models for complex terrains is reducing the computational cost. Currently, a number of full-scale computational fluid dynamics (CFD) models [40, 46, 47, 48, 49, 50, 122] have been developed to simulate wake effects. However, these high-fidelity simulations are costly and difficult to integrate with existing optimization approaches as these models rely on solving the full Navier-Stokes equations in large computational domains. In this work, the wake development and propagation process on complex terrains is simulated approximately as an advection-diffusion problem, reducing its computational cost while maintaining accuracy comparable with full-scale CFD models, and compatible with the requirements of the wind farm layout optimization problem.
In the following section and subsections, background information on wake turbine wakes is described, followed by a detailed description of the proposed model and numerical discretization. Finally, the performance of the proposed model is discussed.

### 6.1.1 Wind Turbine Wake

There are two main regions in a wind turbine wake, near and far wake regions, shown in Figure 6.1. In the near wake region, the blade geometry has a strong influence on the velocity and pressure profiles, while in the far wake region, the velocity profile becomes Gaussian and the blade influence on pressure gradient is negligible [27, 31, 32].

![Figure 6.1: Near and far wake regions of a wind turbine](image)

### 6.1.2 Actuator Disk Model

The actuator disk model is one of the common methods of modelling wind turbine rotor, owing to its simplicity. This model assumes an initial wake expansion behind the turbine and its velocity and pressure profiles are known [51]. The velocity and pressure profiles described by the actuator disk model are illustrated in Figure 6.2.

![Figure 6.2: Actuator disk velocity and pressure distribution](image)
6.2 Proposed Wake Model

A low-cost numerical wake model that can be used to optimize layouts on complex terrains while maintaining high accuracy is proposed. It utilizes ideas from the virtual particle model [55] and the turbulent viscosity development from the Ainslie wake model [15]. The aim of this approach is to make use of previous development to simplify the problem in order to reduce computational cost.

The model starts with Reynolds-Average Navier-Stokes equations, shown in Eq.(6.1). Given any flow over a complex terrain, it must satisfy the Reynolds-averaged Navier-Stoke equations, where \( u_0 \) is an unique solution that describes the flow field in a given domain. If a turbine is placed in the same domain, then the original flow field changes due to the presence of the turbine. The flow field solution that satisfies the momentum equations can then be defined as \( u_{\text{wake}} \), described in Eq.(6.2), where \( f \) is the body force exerted by the turbine. The notation \( u_{\text{wake}} = u_0 - \Delta u \) is used, where \( u_{\text{wake}} \) is the wind velocity due to the presence of a wake, \( u_0 \) is the wind velocity in the free stream, and \( \Delta u \) is the velocity deficit experienced in the flow due to the wake.

\[
\begin{align*}
\frac{u_{0,j}}{\partial x_j} - \frac{1}{\rho} \frac{\partial p_0}{\partial x_i} + \nu_{t,0} \frac{\partial^2 u_{0,i}}{\partial x_j^2} = \frac{\partial^2 u_{\text{wake},i}}{\partial x_j^2} + \frac{\partial p_{\text{wake}}}{\partial x_i} + \nu_{t,\text{wake}} \frac{\partial u_{\text{wake},i}}{\partial x_j} + f_i, \\
\frac{u_{\text{wake},j}}{\partial x_j} = \frac{1}{\rho} \frac{\partial p_{\text{wake}}}{\partial x_i} + \nu_{t,\text{wake}} \frac{\partial^2 u_{\text{wake},i}}{\partial x_j^2} + f_i.
\end{align*}
\] (6.1, 6.2)

In order to obtain the difference (velocity deficit) between the two flow fields, Eq.(6.1) is subtracted by Eq.(6.2),

\[
\frac{u_{0,j}}{\partial x_j} - \frac{u_{\text{wake},j}}{\partial x_j} = -\frac{1}{\rho} \left( \frac{\partial p_0}{\partial x_i} - \frac{\partial p_{\text{wake}}}{\partial x_i} \right) + \nu_{t,0} \frac{\partial^2 u_{0,i}}{\partial x_j^2} - \nu_{t,\text{wake}} \frac{\partial^2 u_{\text{wake},i}}{\partial x_j^2} - f_i. 
\] (6.3)

If Eq.(6.3) can be solved, then the influence of a turbine on the flow field can be quantified. Before this Eq.(6.3) can be solved, a number of simplifications and assumptions can be applied to reduce the complexity.

Firstly, for the purpose of wind farm layout design, turbines are placed in the far wake region of upstream turbines, where pressure gradient caused by the turbine is considered to be negligible, and the turbulent viscosity is assumed to be approximately the same (\( \nu_{t,\text{wake}} \approx \nu_{t,0} \)). Furthermore, to avoid directly simulating the near wake region, where the flow field is dependent on turbine blade profiles, the body force term \( f \) is then described using the actuator disk model, by prescribing the wake expansion and velocity profile at the end of the near wake. Then, \( u_{\text{wake}} = u_0 - \Delta u \) can be substituted into Eq.(6.3), which simplifies to
\[(u_{0,j} - \Delta u_j) \frac{\partial \Delta u_i}{\partial x_j} + \Delta u_j \frac{\partial u_{0,i}}{\partial x_j} = \nu_{t,wake} \frac{\partial^2 \Delta u_i}{\partial x_j^2}. \]  

(6.4)

The first term of the equation represents the advection term, describing how the velocity deficit is adveceted. The second term accounts for the effects of velocity gradients while the third term describes the diffusion of velocity deficit. Furthermore, the advection term can be assumed to be more dominant than the diffusion term in the streamwise direction.

Figure 6.3: Actuator disk theory, the wake experiences an initial wake expansion in the near wake.

Equation (6.4) describes the influence of a turbine on the flow field. The variable of interest is velocity deficit, \( \Delta u \). The free stream velocity \( u_0 \) and viscosity \( \nu_{t,wake} \) are obtained from full-scale CFD or experimental measurements. As discussed previously, the actuator disk model is used to describe the momentum extracted due to the presence of a turbine. It states that the wake experiences an initial wake expansion (Fig. 6.3) [51, 53] in the near wake and this expansion at the end of the near wake (\( \approx 2 \) rotor diameters downstream of the turbine) is described as,

\[ r_{wake,2D} = \frac{r_{turbine}(1 - a)}{1 - 2a} \]  

(6.5)

and

\[ u_{wake,2D} = u_0 - 2a, \]  

(6.6)

respectively. In these equations, \( r_{turbine} \) is the rotor radius of the turbine, \( a = \frac{1 - \sqrt{1 - C_T^2}}{2} \) is the induction factor and \( C_T \) is the thrust coefficient of the turbine. Due to the presence of the turbine, a modification to the turbulent viscosity is required to describe the non-equilibrium between the velocity field and the turbulence field [15], up to about 5 diameters downstream of the rotor. Ainslie described this non-equilibrium in the form of,

\[ \nu_{t,wake}(x) = \nu_{t,e} + \nu_{t,0} = F [2k_1r_{wake,2D}(u_0(x) - u_e(x))] + \nu_{t,0}(x) \]  

(6.7)
Chapter 6. Wake Model for Complex Terrains

\[ F = \begin{cases} 
0.65 + \left( \frac{x-4.5}{25.32} \right)^{2/3} & 2 \leq x \leq 5.5 \\
1 & x > 5.5 
\end{cases} \tag{6.8} \]

where \( \nu_{t,c} \) and \( u_c \) are the turbulent viscosity and velocity at the wake centerline, respectively, and \( x \) is the distance downstream of the turbine in rotor diameter. The constant \( k_1 \) is found experimentally. The term \( u_0(x) - u_c(x) \), defines the velocity deficit in the wake centerline. The filter function \( F(x) \) accounts for the non-equilibrium based on fitting with experimental data. Ainslie’s \( \nu_{t,c} \) function produced inadequate fit when compared our full-scale CFD simulation, thus we redefine \( \nu_{t,c} \) to ensure better fit with full-scale CFD simulation data, discussed in the next section.

6.3 Results and Discussion

In this section, the turbulent viscosity term, \( \nu_{t,c} \), is redefined to fit better with full-scale CFD simulation results on a flat terrain. Then, the model is compared with full-scale CFD simulations of a turbine placed on a complex terrain found at the Gros-Morne Wind Farm in Quebec.

![Velocity contour for CFD simulation.](image1)

![Velocity contour for proposed model.](image2)

Figure 6.4: Streamwise velocity contour for both CFD and proposed model.
6.3.1 Turbulent Viscosity Model Fitting

A turbine of 80 m in both rotor diameter and hub height is placed in a 3000 m x 1000 m x 1000 m domain. The turbine is placed 1000 m downstream (x-direction) of the inlet, 80 m for hub height (y-direction), and 500 m in the spanwise direction (z-direction). At the inlet, a logarithmic velocity profile is prescribed, \( u(y) = \frac{u^*}{\kappa} \ln\left( \frac{y}{y_0} \right) \), where \( u(y) \) represents the streamwise wind speed with varying height \( y \) and \( u^* \) is the friction velocity which is assumed to be 0.4 m/s [150]. Turbulent kinetic energy and dissipation rate at the inlet are prescribed as \( k = \left( \frac{u^*}{\sqrt{C_{\mu}}} \right)^2 \) and \( \varepsilon(y) = \left( \frac{u^*}{\kappa y} \right)^3 \), where \( C_{\mu} = 0.033 \) and \( \kappa = 0.4 \). At the top boundary, velocity and turbulence quantities are correspondingly prescribed at 1000 m. The side boundaries are set as symmetry boundary conditions and a pressure outlet is prescribed at the outlet boundary. The turbulence model used for full-scale CFD simulations is Reynolds stress model (RSM), with \( C_{\mu} = 0.033 \), identical to that used by Makridis and Chick [48].

As previously mentioned, the \( \nu_{t.c}(x) \) from Eq.(6.7) is redefined as \( \nu_{t'}(x,y,z) \) and fitted to full-scale CFD simulations to ensure closer comparison between the proposed model and full-scale CFD simulations. As observed by Ainslie, \( \nu_{t'} \) is a function of velocity deficit, and that this relationship is approximately parabolic, shown in Figure 6.5. The fitted function is shown in Eq.(6.9),

\[
\nu_{t'}(x,y,z) = -7.28916 \times 10^{-5} (2r_{wake,2D}[u_0(x,y,z) - u_{wake}(x,y,z)])^2 \\
+ 3.64535 \times 10^{-2} (2r_{wake,2D}[u_0(x,y,z) - u_{wake}(x,y,z)]) + 1.6457. 
\]

In the proposed model, the domain is discretized evenly into 10 m x 10 m x 10 m cells, or 3 million cells in this domain. Comparatively, the full-scale CFD simulation uses 0.78 million cells. This is due to the advanced meshing capabilities available in commercial CFD packages, while only uniform-sized cells can be assigned in the in-house code for the proposed model. Then the velocity profiles of the full-scale CFD and the proposed models at 5 and 10 rotor diameters downstream of the rotor are compared, shown in Figures 6.6a and 6.6b. Looking at these profiles, there is a very good agreement between the two models in terms of accuracy with error less than 5% and 3% for 5 and 10 rotor diameters, respectively; however, the full-scale CFD simulation took approximately 3 hours to complete with 3 processors while the proposed model converged in 2 minutes with a single processor. These wake effects are apparent up to 20 rotor diameters, and the error is less than 4%. Based on these results, the proposed model is able to describe the wake recovery process accurately while keeping the computational cost orders of magnitude lower.
6.3.2 Complex Terrains

In this test case, the same turbine at the flat terrain scenario is placed on a complex terrain based on the Gros-Morne Wind Farm in Quebec, Canada, shown in Figure 6.7. The inlet profile is prescribed using the log law previously described, with a surface roughness length, $y_0$, of 0.0002 m, corresponding to open sea terrains, as the wind blows inland from the sea. As with the flat terrain simulation, the friction velocity is assumed to be 0.4 m/s [150]. The roughness length of domain is prescribed to be 0.04 m, corresponding to roughness of crop terrain [51].

The simulation domain is 4500 m in the streamwise direction, 1790 m in the spanwise direction, and 1500 m in height. The full-scale CFD domain is discretized into 2.3 million cells, with higher resolution closer to the ground. In the proposed model, the domain is discretized into 10 m x 10 m x 10 m cells, with a total of approximately 12 million cells in the domain.

Initially, a full-scale CFD simulation without the turbine is conducted to obtain the natural flow field over the complex terrain. Then, using the actuator disk model, the velocity profile is prescribed 2 rotor diameters downstream of the turbine. Due to the terrain topography, the velocity has multiple components, thus the components of the velocity deficit prescribed at the end of the near wake should correspond to the velocity components at that location. In other words, the velocity deficit is prescribed in the direction of the velocity vector two diameters downstream.
6.4 Conclusions

This work focuses on developing a low cost and accurate wake model capable of simulating wake effects on complex terrains. The model solves a simplified variation of the Navier-Stokes equations with three major simplifications to reduce computational cost while maintaining accuracy:

1. Near wake is modelled using the actuator disk theory
2. Far wake is the region of interest
3. Velocity deficit is the variable of interest

This model was validated against CFD simulation of a turbine placed on the terrain of Gros-Morne Wind Farm in Quebec. The proposed model allows for fast simulation of wake effects on complex terrains, making it ideal for designing wind farm layouts on complex terrains.
Figure 6.7: CFD domain with terrain based on Gros-Morne Wind Farm.

Figure 6.8: Streamwise velocity at hub height.
Figure 6.9: Streamwise velocity on a complex terrain using CFD and proposed model.
Chapter 7

Future Work

This thesis presents a new optimization algorithm, a wake model, and a wake interaction model to tackle the challenges of designing the layout of wind farms on complex terrains, as well as a study for quantifying the influence of atmospheric turbulence and terrain on wake propagation and recovery process. While the methodologies used to develop these models and algorithm are fundamentally sound, the predicted improvements cannot be verified through real wind farm data. In addition, validating CFD simulations with field data remains difficult due to the experimental uncertainties. One of the major assumptions made in this work is the condition of atmospheric stability, which is considered to be constant and neutral. The effects of atmospheric stability and other physical phenomena such as wake meandering on wind farm design need to be captured in future work. In terms of future research, there are several possible directions.

Wake Modelling and Optimization Integration

In this thesis, we proposed an optimization algorithm that integrates full-scale CFD simulations into the optimization process that reduces computational cost, as well as a fast wake model for complex terrains with similar accuracy as full-scale CFD simulations. This wake model can be used to replace full-scale CFD simulations to cut optimization times by orders of magnitude. This is a very promising method for wind farm layout design.

Robust Wind Farm Design

Uncertainty is present in all stages of engineering design. The operations research community is very familiar with optimization under uncertainty. In terms of wind farm layout optimization, Zhang et al. [151] included wind direction uncertainty into the layout optimization process. The concept of robust design can also be applied to uncertainty in wake modelling. A robust wind farm design will help reduce
fluctuations in power generation due to wake losses [152].

**Real-Time Optimization of Wind Farm Production**

After a farm has been designed and installed in place. It is likely that the energy production does not match that of forecast found through simulations. We observed in the literature that many wake models underestimate the wake effects [28] experienced in the field. Wind farm control is an active research area that improves farm performance by controlling individual turbines. A recent study by Fleming et al. [153] shows that pitch and yaw angles can induce deflections in the wake, reducing the wake losses experienced by downstream turbines. Real-time optimization is possible with the tools that we have developed, and as such, is a very promising research direction.

**Hybrid Renewable Energy Systems**

Wind energy is typically not a stand-alone source of energy, due to its intermittent nature. We have experienced proliferation in energy storage research in recent years. Particularly for wind energy, pumped hydro [154], flywheels [155], and smart battery technologies [156] are promising storage technologies that can stabilize the intermittency. Furthermore, wind energy can be used in conjunction with other energy sources such as solar and gas turbines. The integration of these systems to provide cost-effective and stable power will become increasingly important.
Bibliography


