Reciprocal Partnership: An intervention to enhance mathematics self-efficacy and achievement of first and second-semester college students

By

Kerry Kwan

A thesis submitted in conformity with the requirements for the degree of Doctor of Philosophy
Department of Curriculum, Teaching and Learning
Ontario Institute for Studies in Education of the
University of Toronto

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Doctor of Philosophy, 2016

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Abstract

Community colleges are calling to support students who are at risk of restricting their career options because they do not have the mathematical groundings to pursue math-related careers. In response, I conceptualize a mathematics intervention program namely, Reciprocal Partnership, which is defined as the collaboration of student dyads to engage in reciprocal learning and teaching under the influence of constructive and collaborative environments that are structured by the Three Learning Situations framework.

Reciprocal Partnership is proposed to enrich college students in their mathematical learning and to support their social development during their secondary-tertiary transition for mathematics knowledge and skills are strongly correlated with students’ college success and career aspiration. Therefrom, the purpose of this research is to investigate the effect of Reciprocal Partnership on the mathematics self-efficacy and achievement of first- and second-semester college students through a mixed methods approach. It also examines the structure of Reciprocal Partnership to inform the design of effective intervention programs for mathematical learning.
Data from final examination grade, and pre- and post-surveys were analyzed through descriptive and inferential statistics, and were used to cross-validate findings from semi-structured interviews. Quantitative results reveal significant effect of Reciprocal Partnership on the mathematics self-efficacy of only students in the first semester, and no significant effect on the mathematics achievement of both students in the first and second semester. However, qualitative results identify a number of benefits for both groups of students such as gains in mathematical knowledge and skill, confidence, motivation, social connection, and comfort. Results from this study suggest the emphasis of mathematics intervention programs on all three learning situations (exploratory, explanatory, and extensional) over only the explanatory situation to maximize learning outcomes.
Acknowledgements

With sincere gratitude, I want to thank my supervisor, Dr. Douglas McDougall, for his all supports from the brainstorming stage to the final oral examination. Without his inspirations and guidance, I would not have conceptualized my theoretical framework and formulated my research questions. I deeply appreciate his insights that stimulate my learning.

I also want to thank Dr. Cathy Marks Krpan for her suggestion to celebrate the positive results of my research. I am grateful for Dr. Jim Hewitt to advise me on my methodology and data analysis. In addition, I want to thank Dr. Qing Li for her advice to elaborate on my data triangulation. I cannot thank my supervisor, community members and external examiners enough for all their advices, feedback, and supports that have made my graduate experience more rewarding and meaningful.

My deepest appreciation goes to my beloved ones. First, I want to thank my mother and my late father for inspiring me to achieve a doctoral degree and a career in education. Second, I want to thank my parents-in-law for their unconditional care that helps me to perform my potentials. Third, I am very grateful to have a sister and a brother-in-law who encourage me to continue my pursuit. Finally, I want to thank my dearest husband for his patience and supports in every stage of the process, and his love that has made all this possible.

I hope this dissertation set an example for my son, my niece and my nephew to embrace their goals, and also services to remind them that they will always be braced wholeheartedly throughout their own pursuits by those who love them.
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Chapter One: Introduction

1.1 Introduction

The purpose of this research is to investigate the effects of an intervention, which I have named Reciprocal Partnership, on students’ mathematics self-efficacy and achievement. It also examines the structure of Reciprocal Partnership to inform the design of effective intervention programs for mathematical learning. Reciprocal Partnership is proposed to enrich college students in their mathematical learning and to support their social development during their transition to college. I have conceptualized the conceptual framework namely, Three Learning Situations, that underlies the tenets of Reciprocal Partnership. The Three Learning Situations framework is based on theories of collaborative learning and constructivism, and combines the structure of reciprocal peer tutoring and supplemental instruction to maximize academic and social outcomes on students’ learning.

Reciprocal Partnership is designed as a response to the increasing need for community colleges to support students who are at risk of restricting their career options because they do not have the mathematical groundings to pursue math-related careers. As a mathematics teacher who has taught at various educational settings (college, gifted after-school programs, special education program, tutorials, and etc.), I become very responsive to the needs of students who are struggling to learn mathematics for reasons ranging from insufficient fundamental knowledge to math-anxiety. I learn from my college students that many of them have difficulties adjusting to the academic expectations and social challenges of higher education. The inability to assimilate into college setting and meet curricular expectations of mathematics courses at the college
level significantly limits their career path and impedes their psychological growth. This motivates me to design an intervention, Reciprocal Partnership, and conduct this research to examine its effects on students’ mathematical learning and affective development during their secondary-tertiary transition.

1.2 Research Context

Community colleges are challenged with burgeoning difficulties to support students through their secondary-tertiary transition. Among these difficulties include increasing students’ diversity, managing students’ assessment, and producing educated workforce (Schuetz, 2002). However, there are growing concerns on the vast number of college students who are underachieving particularly in the subject of mathematics (Phelps & Evans, 2006).

Recently in the College Mathematics Project, Orpwood and colleagues (2012) collected data across major Ontario colleges to study students’ academic standings in mathematics. They found that many college freshmen were struggling with fundamental knowledge in mathematics and were incapable of undertaking basic numerical calculations. In particular, college students who earned a below average grade in preparatory mathematics courses were unable to perform elementary mathematics. Similar issues are found in the United States as majority of students who enter community college need to take remedial courses in mathematics (Schuetz, 2002).

Underachievement in mathematics is an evident obstacle for entry into many scientific and technical occupations (Robinson, Schofield & Steers-Wentzell, 2005). Not only is mathematics an important determiner of pursuit in scientific majors and other studies, success in mathematics is strongly correlated with students’ success in college
and career aspiration (National Mathematics Advisory Panel, 2008; Nite, Morgan, Capraro, Capraro & Peterson, 2014). According to the College Mathematics Project (Orpwood et al., 2012), students who experience great difficulties in preparatory mathematics courses have high chance of withdrawing from college programs that they may have otherwise pursued.

An imminent concern surfaces: When many college students withdraw from math-related majors, pre-mature closure of career options and loss in human potential will result (Cohen & Brawer, 2003). As such, the importance of mathematics in students’ career aspiration warrants the need to address the challenges that students face during their secondary-tertiary transition in mathematical learning.

While it is important to help students acquire mathematics groundings as a means to expand their career option, the social challenges that students face during their secondary-tertiary transition also require significant attentions. Transition to college is a long process of assimilation eminently because it imposes great academic demands along with radical personal and environmental changes (Gueudet, 2008). Tinto (2006) urges the need for students to integrate into both the academic and social communities of their college at the beginning of their program where abrupt withdrawal is most likely to occur. Features of successful assistance programs in mathematics should attend to both students’ social and cognitive domains (Edman & Brazil, 2009). It seems, thereupon, essential to increase students’ mathematics achievement along with supporting their affective development, thereby widening their career options and facilitating their transition to college.
1.3 Purpose of the Study

The purpose of this study is to investigate an intervention, Reciprocal Partnership that I have designed to enhance students’ success in assimilating into college setting, while helping them to meet mathematical expectations at the college level. It is a response to address the increasing need for community colleges to support students who are at risk of restricting their career options because they have gaps in their mathematical groundings to pursue math-related careers.

1.4 Statement of the Problem

This study examines the effects of Reciprocal Partnership on the mathematics self-efficacy and achievement of first and second-semester college students. It also investigates the structure of Reciprocal Partnership to inform the design of effective intervention programs for mathematical learning. The affective effects of Reciprocal Partnership on students’ mathematics self-efficacy and the insights that it will bring to inform mathematics intervention programs are determined through surveys and interviews. The impact of Reciprocal Partnership on students’ mathematics achievement are examined both quantitatively and qualitatively through final examination grades and interviews respectively. The overarching research questions are stated as follows: 1) what effects does Reciprocal Partnership have on the mathematics self-efficacy of first and second-semester college students? 2) What effects does Reciprocal Partnership have on the mathematics achievement of first and second-semester college students? 3) What insights does Reciprocal Partnership have to inform the design of effective intervention programs for mathematical learning?
1.5 **Significance of the Study**

Findings from this research will extend understanding of students’ social and academic needs in learning mathematics at the college level, while informing educators and administrators on the design of mathematics intervention programs that effectively integrates students into college life and improve learning. Additionally, this research will raise questions on how collaboration between first and second-semester college students can be optimized to improve students’ success and adjustment. It will provide experimental data for the use of cross-level collaboration and communication to mediate secondary-tertiary transition. It is important to note that the use of cross-level collaboration and communication is strongly supported in literature (e.g. Capstick & Fleming, 2002; Clark & Lovirc, 2009; Congos & Schoeps, 2003).

1.6 **Background of the Researcher**

I always enjoy working with mathematics and teach it with enthusiasm. My teaching profession has motivated me to constantly acquire pedagogical knowledge and challenged me with teaching obstacles to reflect on my own understanding of mathematics. I strive to become not only a better teacher but also a better learner in mathematics. This is the reason why I have decided to research on mathematics education from the perspectives of the learners.

I have taught mathematics at various settings. These include classrooms with majority of students who experience academic difficulty and gifted after-school programs with students who wish to excel further in mathematics. These settings allow me to identify the distinctive learning styles and thinking processes of the above two extreme groups of students in order to attend to their learning needs. I come to realize that the
main difference among these two groups of students in their mathematics success is their level of confidence in mathematics. Their self-efficacy plays a key role in shaping their aspiration and motivation. Consequently, the desire to help students who are struggling to learn mathematics has settled in me as I incline to believe that every student is capable of achieving mathematics excellence when they develop the confidence to do so.

The issue raised by the College Mathematics Project (Orpwood et al., 2012) has bought my attention to college students who are struggling in mathematics during their secondary-tertiary transition. After speaking to my college students, I learn that many of them do not have the confidence to enrol in mathematics courses or to pursue math-related careers because they believe they are incapable of meeting mathematics expectations at the college level. My concern, as a mathematics professor, is to improve students’ learning experiences so they can develop the academic competency and self-efficacy not to restrict their career paths by reason of self-inefficacy. My attention is particularly for incoming students who have insufficient mathematics groundings, while trying to overcome social changes and academic demands during their transition to college.

For the above reasons, I am proposing in this study a mathematics intervention namely, Reciprocal Partnership which I have designed to address both cognitive and affective developments of college students in their mathematical learning. Drawing from my research background, I decide to conduct a more comprehensive investigation using both qualitative and quantitative methods. As a college professor and a researcher in mathematics education, my inquisition is to focus on students’ learning needs through the lens of examining viable means to enhance their learning experiences in mathematics.
1.7 Conceptual Framework

I define Reciprocal Partnership as the collaboration among dyads to engage in reciprocal learning and teaching under the influence of constructive and collaborative environments that are structured by the Three Learning Situations framework. It is a collaborative and constructive approach in which students are paired to reciprocally help one another construct and consolidate knowledge, and reflect and articulate understanding, while developing the autonomy for mathematical learning.

In Reciprocal Partnership, students reciprocally draw from their course content to raise questions on areas where they need assistance, and on areas that they can confidently tutor their partner. By raising these two types of questions, three learning situations can be created: when both students have no knowledge of the question (exploratory); when only one of the students has knowledge of the question (explanatory); and when both students have knowledge of the question (extensional). In the three learning situations, the dyads engage in distinctive learning activities (problem solving, tutoring, or comparing solution methods). These forms of learning are proposed to capitalize on the social and cognitive benefits of collaborative learning, and adhere to the tenets of constructivism. Reciprocal Partnership is intended to address both the social and academic needs of students during their transition to post-secondary learning in mathematics. Therefore, it encompasses both social and cognitive components.

The social component of Reciprocal Partnership is drawn from research on collaborative learning. Collaborative learning is an effective pedagogy in creating motivation, enhancing self-efficacy, improving academic achievement, and reducing social isolation (e.g. Cabrera, Nora, Crissman, & Terenzini, 2002; La Rocca, Margottini,
Collaborative student-student and student-faculty interactions have shown to positively influence students’ academic and personal development (Terenzini, Cabrera, Colbeck, Parente & Bjorklund, 2001), and their satisfaction with college experiences (So & Brush, 2008). This is because social isolation and lack of social support are key contributors for students to develop attrition towards college life, discontinue their study and result in low academic achievement (Hogan, Parker, Wiener, Watters, Wood & Oke, 2010; Parker, Summerfeldt, Hogan & Majeski, 2004).

By incorporating elements of collaborative learning, Reciprocal Partnership aims to establish the supports for students’ social development. For example, it incorporates reciprocity, an essential factor of collaborative learning, by having students reciprocally teach and learn from one another. In other words, students alternate between the role of receiving and giving help. This social component of Reciprocal Partnership is examined through its impact on students’ affective development, specifically their mathematics self-efficacy. Mathematics self-efficacy is chosen because it is corroborated as a strong predictor of achievement, adjustment and perceived career options for college students (Chemers, & Gracia, 2001; Hall & Ponton, 2005; Zajacova, Lynch, & Espenshade, 2005).

The cognitive component of Reciprocal Partnership is based on the theory of constructivism. From the constructivists’ perspective, knowledge is actively constructed by learners when they relate new information with their prior knowledge into integrated cognitive constructs (network of knowledge) (Fosnot, 2013). According to social constructivism, this cognitive progress is mediated and transmitted by social interactions.
(Palincsar, 2005). These tenets of constructivism underlie the conceptual framework of Reciprocal Partnership to enhance students’ understanding in mathematics.

1.8 Limitations of Study

This study involves interviewing and surveying first and second-semester students at a specific community college of Ontario. Thereupon, it is subjected to a number of limitations. First, perspectives of participants are confined to a single community college: hence, interpretations are based on a specific setting. Second, participants are limited to only students of the courses that are targeted for this research, and hence findings cannot be generalized to all other subject areas. Third, interpretations to the findings are limited to male students as there is only one female participant. Fourth, other variables such as students’ prior academic achievement, learning style, and attitudes toward mathematics learning that may have affected their mathematics self-efficacy are not taken into account. Thereupon, there may be other valid interpretations to the findings than those mentioned in this research.

1.9 Plan of Thesis

This thesis is divided into five chapters. Chapter one is an overview of the research context and purpose, statement of problem, background of the researcher, conceptual framework, and limitations.

Chapter two provides a review of literatures on theories of constructivism and collaborative learning from which the conceptual framework of Reciprocal Partnership is based. Areas of focus include secondary-tertiary transition, supplemental instruction, reciprocal peer tutoring, and mathematical understanding.
Chapter three illustrates the methodological approach. It provides the epistemology and reasons for the use of a mixed methods approach. In details, it describes the research context, research design, participants, data collection, data analysis, and validity.

Chapter four reports the descriptive and inferential statistics of the quantitative data. It also annotates themes that have emerged from analysis of the qualitative data.

Chapter five answers the three research questions and summarizes findings from the quantitative and qualitative data. It also discusses convergence and divergence of the two forms of data, and includes discussions on limitations of the study and implications for future research.
Chapter Two: Literature Review

2.1 Introduction

Transition to college is a complex and life-changing process in which students go through various challenges that require academic and social changes in their ways of thinking and behaving (Gueudet, 2008). They need to acquire the ability to integrate into their new environment, while managing greater academic demands. The challenge becomes even greater for incoming students who do not have sufficient mathematical skills and knowledge to be successful in math-related courses. They will likely result in a shift out of mathematics programs (Orpwood et al., 2012). In order to maximize the successes of these students in college, their affective and cognitive growth must be supported during their secondary-tertiary transition (Edman & Brazil, 2009).

I have designed a mathematics intervention called Reciprocal Partnership to support incoming college students in their mathematical learning, while facilitating their assimilation into college life. It is defined as the collaboration of student dyads to engage in reciprocal learning and teaching under the influence of constructive and collaborative environments that are structured by the Three Learning Situations framework. In order to provide a holistic intervention for students who are trying to integrate into college communities, while meeting mathematics expectations at the college level, Reciprocal Partnership is designed with both social and cognitive components.

2.2 Social Component

The social component of Reciprocal Partnership is based on the tenets of collaborative learning. Thereupon, Reciprocal Partnership is the collaboration of student dyads in which they learn from each other and teach one another in collaborative learning
situations. It incorporates elements of collaborative learning to capitalize on students’ engagement and socialization as a mean to facilitate their transition to college.

2.2.1 Collaborative learning

Collaborative learning has been promoted on the grounds that it creates citizenship by encouraging collective responsibility and preparing students for workforce where productivity benefits from group efforts (Cabrera et al., 2002). Its theoretical stances lie on cognitive and motivational perspectives. According to the cognitive theory, “Zone of Proximal Development”, learners acquire effective learning strategies and higher order of thinking to accomplish more complex tasks when interacting, discussing, and processing materials with other individuals (Vygotsky, 1978). The exchange of dialogue and the negotiation of meanings among learners spur them to reorganize or re-examine their mental constructs which results in learning (Palincsar, 2005).

Based on motivational theory, collaborative work increases individual’s engagement (Johnson & Johnson, 2003). During group work, members are responsible for the learning of each other and must exert effort to help one another reach academic goals. They develop greater confidence and motivation to undertake educational endeavours when they receive assistances (Storm & Storm, 2002). In sum, by creating the social interactions that stimulate cognitive reorganizations and reinforce collective efforts, collaborative learning supports both conceptual and attitudinal growth of the learners.

2.2.2 Effects of Collaborative Learning

Effectiveness of collaborative learning has been well researched and documented. For example, in a study, achievement was significantly attained through the use of
students’ collaboration in higher education (Terenzini et al., 2001). When working collaboratively to solve mathematical problems, students verbalize their thinking, conceptualize questions, experiment with other’s ideas, and compare viewpoints, all of which enhance their application of knowledge and meta-cognition (Iiskala, Vauras, Lehtinen, & Salonen, 2011). Meta-cognition helps students to develop self-regulatory application of knowledge and skills which are needed for academic success in higher education (Cornford, 2002).

Beside improved cognitive achievement, collaborative learning allows students to develop positive attitudes toward education, and deeper appreciations for ethical diversity (Cabrera et al., 2002). For example, collaborative learning creates the setting where students negotiate and compromise positively, and acquire ownership of knowledge as a means to practice effective group work (Cockrell et al., 2000). In collaborative learning, there are more openness to different viewpoints, better negotiation in conflicts, and more empathy to help one another to develop understanding (Cabrera et al., 2002).

Collaborative learning has also been shown to assist students’ adjustment and integration. It promotes positive interactions which in turn, enhances students’ motivation and reduces examination anxiety (La Rocca, Margottini, & Capobianco, 2014). It is effective in reducing social isolation of individuals (Cabrera et al., 2002), and insufficient social support is the primary reason for college students to develop attrition towards college life, result in low academic performance, and discontinue their study (Hogan et al., 2010; Parker et al., 2004).

In collaborative group work, risk-taking behaviours are encouraged which subsequently decrease anxiety in learning, while increasing self-efficacy. In fact, group
learning is known to raise students’ self-efficacy more than individual learning (Johnson, Johnson & Smith, 2007). As a result, Reciprocal Partnership is proposed to promote positive social interactions through collaborative learning, then it follows that increased openness to diversity, positive attitudes and self-efficacy will occur.

2.2.3 Mathematics Self-efficacy

Reciprocal Partnership is assumed to inherit many learning benefits of collaborative learning; however, mathematics self-efficacy is the affective benefit to be focused on for this research because literature indicates its strong correlation with career aspiration and academic achievement (Chemers, & Gracia, 2001; Hall & Ponton, 2005; Zajacova et al., 2005). Self-efficacy is described by Bandura (1977) as one’s beliefs about one’s capabilities to attain specific goals and is a predictor of whether a person will undertake a course of action, how much effort the person will spend in pursuing goals, and whether the person will preserve through challenges and unexpected difficulties.

Self-efficacy has strong association with learners’ emotional and motivational states by way of determining whether they would take a more positive and calm approach in stressful environments (Zimmerman, 2000). Students with high self-efficacy see themselves more capable of meeting expectations and coping with stresses associated with college expectations (Chemers, Hu & Garcia, 2001). They become competent to handle environmental stress positively through analyzing risk factors, using metacognitive strategies, managing time, and solving problems.

Self-efficacy of undergraduates has been shown to influence their grade (Zajacova et al., 2005), their persistence in college courses (Chemers & Gracia, 2001) and their self-regulatory skills (Zimmerman & Cleary, 2006). For example, in a study on 300 college
students, self-efficacy was positively correlated with adjustment and was suggested to strongly affect academic performances, and anxiety level experienced by the students (Javed & Nizami, 2012). In another study, learners with greater confidence displayed calm and thoughtful manner, and resulted better in satisfaction and retention (Zimmerman, 2000).

Mathematics self-efficacy, in particular, is defined as a “more specific estimate of confidence in one’s ability to perform well with regard to particular mathematics tasks or in particular math and math-related courses” (Hackett, 1985, p. 48). It is a stronger indicator than prior achievement in determining students’ achievement in mathematics (Skaalvik & Skaalvik, 2006). It substantially influences students’ range of perceived options, expressed interests, and occupational preferences in math-related fields (Betz & Hackett, 2006). It is also negatively correlated with mathematics anxiety (Jain & Dowson, 2009; Ma & Xu, 2004) and is one of the most significant factors to diminish mathematics anxiety (Lee, 2009).

Mathematics anxiety is denoted as “the state of nervousness and discomfort brought upon by the presentation of mathematical problems [that] may impede mathematics performance irrespective for true ability” (Hoffman, 2010, p.276). It increases students’ attrition towards learning mathematics, which in turn weakens their mathematics competence and forecloses their career options in mathematics (Ashcraft, 2005). Mathematics anxiety also compromises learners’ cognitive resources used in task processing. The uneasiness and distress created by mathematics anxiety diverge learners’ working memory from their task focus, while impeding their performance in mathematics.
Based on negative correlation, a decrease in mathematics anxiety can potentially be resulted from an increase in mathematics self-efficacy.

According to the self-efficacy theory, one’s perceived competence in an endeavour is highly associated with one’s choice of behaviour (Bandura, 1977). When a learner has a sense of efficacy in mathematics, he or she would be more motivated to engage in mathematics related courses. Students’ perceptions on their math ability are negatively related to mathematical anxiety and positively related to achievement outcomes, and it is their mathematics self-efficacy rather than the achievement outcomes that have a stronger influence on their career choices in mathematics major (Betz & Hackett, 2006). Consequently, mathematics self-efficacy is measured in this study to determine effects of Reciprocal Partnership on students’ affective development.

2.3 Cognitive Component

The cognitive component of Reciprocal Partnership is underpinned by theories of constructivism. Higher education is pressed with the ever-increasing need to prepare students to be self-directed learners (Hussain & Sultan, 2010). It is important for students to critically think about information presented to them and to develop the autonomy to extend their knowledge. These become the compelling reasons for higher education to espouse approaches such as student-centered (Mokhtar, Tarmizi, Ayub & Tarmizi, 2010), inquiry-based learning (Chang, 2011), problem-based learning (Savery, 2015), and discovery-based learning (Hodge, 2006), all of which are derived from constructivism.

2.3.1 Effects of Constructivism

Pedagogies based on constructivism have been reported to produce significant gains in achievement and promote positive attitude towards mathematical learning in
higher education (Abdulwahed, Jaworski & Crawford, 2012). For example, problem-based learning which adopts student-centered and active learning approaches has shown to increase students’ participation, interests and achievement in learning of calculus (Mokhtar et al., 2010). Students develop greater appreciation for the use of mathematics in everyday activities and obtain significant gains in achievement through participating in courses designed to incorporate exploration of mathematics applications and inquiries (Ward, Campbell, Goodloe, Miller, Kleja, Kombe & Torres, 2010).

Constructivist pedagogies suggest to enhance learning outcomes through the use of cooperative and collaborative group work (Hussain, 2012). Through creating discourses in collaborative learning, students demonstrate more attentiveness, while displaying less frustration (Fielding-Wells & Makar, 2008), and cooperative learning has shown to improve students’ affective development (Johnson & Johnson, 2003). Studies have reported strong correlation between students’ affective factors such as self-efficacy, motivation, and beliefs in mathematical learning with students’ achievement in mathematics (Betz & Hackett, 2006; Chemers & Gracia, 2001). It is evident that constructivist pedagogies also attend to students’ affective development in their mathematical learning.

2.3.2 Tenets of Constructivism

Constructivism is a theoretical stance relating to human knowledge and learning (Fosnot, 2013). It regards learning as an active process in which learners construct knowledge by relating new information with their prior understanding. It is diverged into two main branches: radical and social constructivism.
Radical constructivism derives mainly from Piaget (1950) and focuses on studying the cognitive mechanism by which meanings are constructed. To an extent, it studies how learners make sense of their social settings. Social constructivism, on the other hand, derives from Vygotsky (1978) and focuses on how learners’ social and cultural setting influences their construction of knowledge. Both branches address the learning of individual in relation with their physical and social world (Abdulwahed et al., 2012).

Radical constructivism upholds two general doctrines. First, knowledge is not passively received since it is constructed by the learner through active cognitive activities, and learning occurs when learners construct meaning out of new information by relating it to what they already know (Fosnot, 2013). This involves the cognitive process of accommodating and integrating new knowledge into existing web of cognitive constructs (Faulkenberry & Faulkenberry, 2006). Understanding mathematics becomes the act of generating relationships between ideas, concepts, images, or information into a fabric of knowledge (Thompson, 2013). It is often described as the “state of knowledge” when relations are formed between new ideas and the learner’s existing knowledge (Carpenter & Lehrer, 1999).

Second, knowledge is closely associated with the learner’s view of the world (Thompson, 2013). Learning is an evolving process in which learners settle cognitive conflicts between new information and their perception of the world. They assume their cognitive constructs are true unless these constructs contradict with their experience of the world. Learners will re-examine their mental constructs and modify or replace them when contradictions arise (Fosnot, 2013). In other words, “knowledge cannot be thought
of as a copy of an external reality, and claims of truth cannot be grounded in claims about reality” (Thompson, 2013, p. 1).

Social constructivism begins with the above mentioned tenets and initiates “the idea of mathematical objectivity as a social construct” (Thompson, 2013, p. 4). Specifically, construction of meanings is mediated or transmitted by social interactions (Vygotsky, 1978). During social exchanges, learners offer ideas and test their ideas against one another’s viewpoints giving rise to conceptual discrepancies that prompt the learners’ internal cognitive conflicts (Kim, 2001). In resolving their cognitive conflicts, learners undergo further negotiation of one another’s reasoning, in order to reach joint meaning (Palincsar, 2005).

In group setting, the exchange of ideas allows learners to compare new information offered by others with their existing knowledge then modify or replace the existing knowledge, if necessary. Similarly, conceptual controversy emerged from social exchange evokes feelings of cognitive uneasiness in the learners, which in turn, spurs them to seek additional information and recognize existing cognitive constructs (Palinscar, 2005). In the end, new meanings are constructed. Therefore, social interactions are suggested to generate the cognitive conflicts necessary for knowledge construction that might not otherwise occur during individual work.

The above tenets of constructivism and collaborative learning form the basis from which I posit the theoretical perspectives of the Three Learning Situations framework. I design this conceptual framework of Reciprocal Partnership to create the three collaborative and constructive learning situations (exploratory, explanatory, and extensional) that support mathematical understanding.
2.4 Three Learning Situations Framework

The Three Learning Situations framework, as I have developed, points to the different states of understanding that a dyad of learners possesses when encountering a mathematical problem. The different states (one possesses understanding, both possess understanding or both do not possess understanding) give rise to the three learning situations. First, it is possible for both learners of a dyad to have little or no understanding of the mathematical problems raised by one of the learners. This leads to the exploratory situation in which the learning activity is problem solving. Second, it is possible for only one of the learners to have knowledge of the raised problems creating the explanatory situation. In this situation, the learner who possesses understanding tutor the other learner through providing explanations. Finally, it is possible for both learners of the dyad to possess sufficient knowledge of the raised problems. In such circumstance namely, extensional situation, the dyad collaboratively compares each other’s solution methods. Therefore, the different states of understanding among a student dyad lend to three learning situations: exploratory, explanatory and extensional.

2.4.1 Exploratory situation

The exploratory situation occurs when both learners of a dyad possess little or no understanding of the mathematical problems raised by one of the learners. In this situation, problem solving is the learning activity and is used as a vehicle for student dyads to collaboratively construct mathematical understanding. Problem solving is “a process by which an individual uses previously acquired knowledge, skills, and understanding to satisfy the demands of an unfamiliar situation” (Norwood, 1995, p.
It is contended by reformers as consistent with the way how mathematics should be learned.

The processes of problem solving require mathematical knowledge to be constructed through reasoning and proving (Ball & Bass, 2000). In other words, mathematical knowledge is not attained by formal deductions but rather through progressive problem solving in which learners continuously rethink and redefine their tasks (Carpenter & Lehrer, 1999). Therefore, in the exploratory situation, problem solving is not only viewed as the effect of mathematical instruction but also as the mechanism through which mathematical knowledge is constructed.

Formerly, problem solving was presented after the acquisition of knowledge to help learners practice their newly developed knowledge (Lesh & Zawojewski, 2007). This is prevalent in traditional teaching where mathematical concepts were instructed with full procedures and explanations, and problem solving was used afterward as a practice. This form of teacher-oriented instructions suppresses students’ creative thinking skills and deprives them of the opportunity to conceptualize meaning because students tend to imitate or memorize procedures presented to them by the teacher (Roh, 2003).

In one study, a student, who was not directly introduced to standard algorithms, had the opportunity to discover her own algorithms and solve the mathematical problems with straightforward computation (Ebby, 2005). Mathematics is contended as a social activity in which learners are continuously engaging in sense-making through solving problems (Carpenter & Lehrer, 1999). Thus, in the exploratory situation, the focus is having learners explore mathematical ideas through problem-solving.
Learning through problem-solving in the exploratory situation is not proposed to occur completely devoid of guidance. Although learners are expected to unconditionally explore mathematical concepts, a facilitator is needed to provide guidance during problem solving when difficulty arises and to negotiate mathematical meanings with the learners (Hmelo-Silver & Barrows, 2006). According to Hmelo-Silver and Barrows (2006), when learners are diverging away from key concepts the facilitator can redirect them back to the learning objectives by legitimizing certain assumptions. During group discussions, the facilitator may use probing questions to elicit ideas or provide constructive feedbacks on the learning progress. In doing so, the facilitator is modeling, questioning, and scaffolding to guide learners through open dialogue.

The exploratory situation is also organized around having learners solve problems in pairs to afford them the opportunities to take risk in their mathematical endeavours, to be exposed with different perspectives, and to communicate mathematically with each other. Learners are partnered to solve the mathematical problems that neither one can do through individual effort. By forming dyads, learners are placed in the context of collaborative learning to overcome the challenges of solving mathematical problems because according to Johnson, Johnson and Smith (2007) working in pairs is more effective than working alone.

When working with a partner, learners are more willing to experiment with new and untested ideas (Storm & Storm, 2002). This is because collaborative learning encourages open communication, peer support and respect of individual’s perspectives (Cabrera et al., 2002). Its focus on process over end result lessens intimidation and encourages risk-taking behaviours (Johnson & Johnson, 2003). Moreover, social
exchange between dyads can spur learners to critically re-examine their own assumptions, and to undergo cognitive reorganization for the integration of new conceptual relations. This form of reflection and metacognition is necessary for solving mathematical problems (Iiskala, Vauras, Lehtinen, & Salonen, 2011). Damon and Phelps (1989) proclaim that “in the context of joint exploration with a partner, learner feels more like a fellow explorer than an isolated incompetent” (p. 14).

In general, the exploratory situation is centered on learning through facilitated problem solving. Learners are mediated by a facilitator to discover their own solution and to become continual learners. Presumably, they will collaborate to identify the conditions of the problem, evaluate options, compare approaches, reflect on their understanding, and engage in negotiations of mathematical meanings with the attempt to apply prior knowledge or conceptualize new ones. As such, in the exploratory situation, learners are expected to undertake collaborative learning and autonomous acquisition of mathematical knowledge, and by this means, make the learning a social activity and their own endeavour.

2.4.2 Explanatory situation

In the explanatory situation, a learner of the dyad possesses understanding of the mathematical problems raised by one of the learners. The learner with the knowledge articulates or explains his or her understanding to the other learner. According to Carpenter and Lehrer (1999), “the classrooms are discourse communities in which all students discuss about different ways of viewing important mathematical ideas, explain why their conjectures and conclusions make sense, why a procedure they have used is valid for the given problem” (p. 26). Mathematics should be “a language for thought”
rather than a prescribed set of procedures to find the correct answer (Carpenter & Lehrer, 1999).

In the process of providing elaborated answers or explanations, learners meaningfully relate new information with prior knowledge, which results in better retention of information (Larsen, Butler & Roediger III, 2013). Reflective verbalization produces specific way of thinking for solving complex problems (Wetzstein & Hacker, 2004). By verbalizing the strategic steps of solving a problem, learners can arrive at the solution with greater ease (Montague, Warger & Morgan, 2000). This process stimulates more attentive construction of relations between ideas that might not otherwise occur. Webb (1989) said it most richly (cited in Pressley, Wood, Woloshyn, Martin, King, & Menke, 1992):

In explaining to someone else, the helper must clarify, organize, and possibly reorganize the material (see Bargh & Schul, 1980). [The helper] may discover gaps in his or her own understanding or discrepancies with other’s work or previous work [and] may search for new information and subsequently resolve those inconsistencies, thereby learning the material better than before…When an explanation given to a teammate is not successful (the teammate does not understand it or does not use it to solve the problem correctly), the helper is forced to try to formulate the explanation in new or different ways. This may include using different language …generating new or different examples, linking examples to target the student’s prior knowledge or work completed previously, using alternative symbolic representations of the same material... and translating . . . the same material. All of these activities will likely expand and solidify the helper’s understanding of the material. (p. 29)

Given the cognitive effects of explanation, the explanatory situation focuses on the learner who possesses knowledge of the mathematical problem to assume the role of a tutor. Since tutors need to apply their knowledge and verbalize it in creative ways to help their tutees understand, “the very factors that make tutoring cognitively demanding may also contribute to tutor learning” (Roscoe & Chi, 2007, p. 540). For example, in order to
explain the subject matter as a whole, the tutor needs to form elaborated representations of the subject concepts by arranging the bits of ideas into organized cognitive structures (O'Donnell & King, 2014). This cognitive elaboration facilitates the tutor’s understanding of the concepts with its underlying principles (O'Donnell & King, 2014).

Researchers posit that tutors benefit most from the tutoring process (Allen, 2013; Galbraith & Winterbottom, 2011; Robinson, Schofield & Steers-Wentzell, 2005). Since tutors need to spend time preparing for the tutelage, such required preparation may be one of the reasons why tutors benefit more than tutees (Bargh & Schul, 1980). The time spent on studying the teaching materials allows cognitive representations of the materials to be processed more often, and hence creating more accessible retrievals of the information. When people learn in order to teach, they build greater content-specific knowledge structure and attend to more process-learning (they learn how to learn) (O'Donnell & King, 2014).

In addition, there are greater intrinsic motivations for learning with the intention to teach someone (Sobral, 2004). Learning in order to teach is learning information that can be used to act on one’s environment which is a source of motivation. The increased motivation leads to more diligent learning of the material which in turn facilitates memory (Deci, Koestner & Ryan, 2001). The role theory states: “Enactment of a role produces changes in self-concept, attitudes, cognitions, and behaviour consistent with the role expectations” (Allen & Feldman, 1973, p. 1). When learners enact the role of a teacher by tutoring, they inherit the attributes of that role: attentive to learning; accountable for another’s learning; effective in applying verbal reinforcements; higher self-esteem; more positive attitudes toward school; responsive to questions; active in
listening; and high in academic competence (Allen & Feldman, 1973). Expectation on the role of the teacher to demonstrate expertise on subject matter may motivate the tutor to exert more effort to learn the teaching materials. All these intrinsic factors of a teacher are suggested to be displayed in learners who assume the role of a tutor.

When tutors shape their explanations for the purpose of tutoring someone, they are forced to present concepts in accordance with what the tutees understand and think (O'Donnell & King, 2014). This allows tutors to see new relations among ideas that they have not previously thought of. Correspondingly, when tutors encounter difficulty in providing explanations to a question, they may re-examine and reflect on their own knowledge (Roscoe & Chi, 2007). This metacognitive examination is a form of reflecting and regulating one’s own learning progress, and contributes to knowledge-building (Roscoe & Chi, 2004). In sum, the act of explaining facilitates organization of cognitive structures, generation of elaborated relations with prior knowledge, and reflection of one’s knowledge.

Although the emphasis is on the learning outcomes of providing explanations, the explanatory situation also focuses on the social effects of collaborative learning to create mutuality and equity. In collaborative learning, learners work together rather than compete with each other in their learning (Cabrera et al., 2002). This promotes balance discourses, equal status and joint efforts (Damon & Phelps, 1989). To capitalize on these features, the explanatory situation incorporates reciprocity. By means of reciprocity, the explanatory situation affords learners the opportunity to take turns tutoring one another.

According to the equity theory, when there is unequal exchange of help, the development of social skills necessary for maintaining close relationships will be
hindered (Bakker et al., 2000). Likewise, unequal status among members of group learning could adversely affect interaction and learning (Brock, Rovegno & Oliver, 2009). Learners who perceived themselves as relatively higher in status tend to dominate in decision-making and voicing their opinions. Mutual contingency or reciprocity, on the other hand, increases the chances of formulating and testing shared ideas even among diverse or heterogeneous groups (Hmelo-Silver, 2013). Therefore, reciprocity is argued to play an important role in the development of effective group work and social relationships, and it is this feature through which the explanatory situation is attempted to enlarge the mutuality and equality effects of collaborative learning.

Nevertheless, tutoring has a mutual effect on the academic achievement and social growth of both the tutor and tutee (Topping, 2005). While tutors improve the learning of themselves and their tutees by providing explanations, tutees can to some extent influence the learning of the tutor (Roscoe & Chi, 2004). They may direct the learning objectives by choosing the topics of discussion during tutelage. This way, tutees can determine the depth of the topics and manipulate the opportunities to study underlying ideas. Questions asked by tutees can prompt tutors’ awareness of their weakness or misconceptions in the teaching materials, and tutors’ reflection on their own learning (King, 2008). This mutual effect between the tutor and tutee may create the social support necessary for the learners to become more accountable and responsible for their own and others’ learning.

Overall, the explanatory situation allows the learner who possesses knowledge of the mathematical problem to assume the role of a tutor. This role will be reciprocally enacted by both learners to accentuate the essential elements of collaborative learning -
mutuality and equity. That is, opportunities are created to capitalize on the cognitive benefits of articulation and social effect of reciprocity.

### 2.4.3 Extensional Situation

In the extensional situation, both learners of the dyad possess understanding of the mathematical problems raised by one of the learners. Subsequently, they share different viewpoints and compare each other’s thinking through discussing one another’s solution methods, explaining and justifying their own approaches, and reflecting on the underlying mathematical concepts. To be specific, learners are making sense of one another’s approaches as a collaborative work centering on acquiring alternative methods and challenging each other’s thinking. This is based on an aphorism: “You can learn more from solving one problem in many different ways than you can from solving many different problems, each in only one way” (cited in Silver, Ghousseini, Gosen, Charalambous, & Font Strawhun, 2005, p. 288). When learners are exposed to a variety of mathematical approaches or a range of solution strategies, they can build stronger repertoires in solving future problems (Carpenter & Lehrer, 1999). Multiple solution methods can promote connections of different ideas in the learners’ existing knowledge networks which in turn consolidate understanding (Silver et. al., 2005).

The extensional situation creates the opportunity for learners to compare, contrast and discuss multiple solutions through collaboration. In a study, students who compared alternative solution methods were twice more likely than students who reflected the same method to use shortcuts and justify these uses in solving algebraic equations (Rittle-Johnson & Star, 2007). These students were presented with multiple methods through worked examples, and they were asked to collaborate with a partner in generating
explanations. Consequently, they gained more in conceptual and procedural knowledge by comparing multiple solutions, and became more flexible in using multiple methods and less adhered to a single solution method.

When learners compare two example cases with common fundamental principles, their analogical reasoning is activated (Gentner, Loewenstein & Thompson, 2003). Analogical reasoning allows learners to acquire new concept by raising analogy of a concept that they understand. They can better recall and transfer knowledge through comparing two example cases because it allows them to identify the common principles rather than the surface details (Gentner et al., 2003). On the other hand, comparing or analyzing contrasting cases is argued to cultivate learners’ differentiated knowledge. This knowledge allows them to differentiate cases accordingly to their precise qualities, “much as a botanist can distinguish subspecies of a given flower” (Schwartz & Bransford, 1998, p. 475). It helps the learners to be attentive or reflective to information and develop better conceptual knowledge (Guo & Pang, 2011). Seemingly in the extensional situation, learners compare similarities or differences among solution methods to develop conceptual understanding and transfer acquired knowledge.

In the extensional situation when both learners possess prior knowledge of the mathematical problem, their solution methods become the worked examples for comparison. Worked examples help to reduce cognitive load because with solution examples learners can devote their working memory capacity towards studying the conditions of the problem and its underlying principles rather than defining the task parameters to solve the problem (Atkinson, Derry, Renkl, & Wortham, 2000). According to the cognitive load theory, there is limited working memory to process distinct item at
any given time, and inappropriate learning activities or materials can overload the
capacity of the working memory which can result in reduced learning (Paas, Renkl &
Sweller, 2003).

Conventional problem solving strategies such as the means-ends process requires
heavy cognitive load because the learner must “simultaneously consider the current
problem state, the goal state, the relation between the current problem state and the goal
state, and the relations between problem-solving operators” (Sweller, 1988, p. 261). This
approach will lead to the problem goal but in exchange for schema acquisition (Paas,
Renkl & Sweller, 2003). By definition, schema is “cognitive construct that allows
problem solvers to group problems into categories in which the problems in each
category require similar solutions” (Cooper & Sweller, 1987, p. 348). Without possessing
schemas, learners will increase demands on their working-memory because schemas
allows grouping of certain problem states into a single element that produces automation
of the operators in the problem (Paas, Renkl & Sweller, 2003).

The means-ends search as argued by Cooper and Sweller (1987) can result in
inferior learning because cognitive resources are devoted to searching for legitimate
strategies over studying the underlying concepts associated with the mathematical
problem. Learners who study worked-out examples by explaining and justifying each
step have better performance, and less investment of time and mental effort (Van Gog,
Paas & van Merriënboer, 2006). Under certain circumstances, it is more effective for the
learner to observe the use of a mathematical operator than to apply it (Paas, Renkl &
Sweller, 2003). In the extensional situation, learners are expected to refine or expand the
parameters of a step in their solution method, relate the outcome of a step with another, and explain why a procedural step is taken to infer underlying principles.

There is a general shift in mathematical learning from one standpoint that views mathematical knowledge as the ability to find the correct answer with a single standard method to another standpoint that values multiple approaches to solving a mathematical problem (Silver et al., 2005). Grounded in this shift, the extensional situation encourages learners to compare each other’s reasoning and thinking modes to examine alternative solutions and identify underlying concepts. Therefore, in the extensional situation, learners become one another’s resource to build a repertoire of different solution methods by comparing and constructing relationships among these methods.

The three learning situations are unlike stages of learning - there is no hierarchy or progression. They are not lined in a successive sequence in which learners succeed from a pre-mature to a more sophisticated level of understanding. The dyad does not need to traverse from one situation to another in an orderly fashion. This implies that not any one of the three learning situations is more important than the others in terms of supporting the mental activities introduced by Carpenter and Lehrer (1999) as conducive to mathematical understanding. These mental activities will be discussed in the following sections. In essence, the three learning situations have their own unique collaborative and constructive activity (solving problems, tutoring, and comparing solution methods) that mediates social interactions for mathematical learning to occur.

2.4.4 The Five Mental Activities

The three learning situations engage some form or degree of the five mental activities introduced by Carpenter and Lehrer (1999). Seeing mathematical understanding
as an emerging social phenomenon which can change in accordance with association and re-association of ideas, Carpenter and Lehrer (1999) introduce five types of interrelated mental activities from which mathematical understanding emerges: construct relationships, extend and apply mathematical knowledge, reflect about experiences, articulate what one knows, and make mathematical knowledge one’s own.

**Constructing Relationships:** Learners make sense of new ideas by connecting and relating these ideas to their prior knowledge for learning is a process of forming conceptual relations (Carpenter & Lehrer, 1999). Mathematical understanding, thereupon, is developed when ideas are related with other ideas through some means. The means of connection varies depending on how the relations are formed cognitively by the individual learner.

**Extending and Applying Mathematical Knowledge:** In addition to simply connecting ideas together, mathematical understanding requires ideas to be related in integrated cognitive constructs (Carpenter & Lehrer, 1999). When ideas are connected in such a way rather than in isolated pieces, there are multiple paths for retention and retrieval of the ideas. In parallel, when new ideas are organized in multiple connections, they become liable for incorporation or integration into existing knowledge structures, thereby extending the knowledge base of the learners (Carpenter & Lehrer, 1999).

**Reflecting About Experiences:** Reflection as stated by Carpenter and Lehrer (1999) involves learners consciously examining new or prior ideas on how the ideas are related to each other. This process prompts reorganization of cognitive structures to fit new revelations. Carpenter and Lehrer (1999) attribute learners’ ability to reflect on what they know and how they know to their sophistication of understanding. Reflection is,
therefore, argued to be crucial for the development of mathematical understanding and is necessary for both teaching and learning (Elbers, 2003).

**Articulating what one knows:** Articulation is a “public form of reflection” (Carpenter & Lehrer, 1999). It is communicating one’s knowledge through various means such as verbalizing, writing, and drawing, while identifying the main ideas so the essence can be conveyed. By articulating thoughts to others, learners’ exiting cognitive constructs are re-examined for more elaborate relations and may undergo reorganization to incorporate new connections (O’Donnell & King, 2014). Simply stated, the act of articulation requires learners to reflect on the relations among ideas and rearrange cognitive constructs in ways that are communicative and comprehensible to others.

**Making mathematical knowledge one’s own:** Mathematical learning is guided by the learner’s experiences of the world, and hence the process of understanding becomes personally owned by the learner (Carpenter & Lehrer, 1999). According to Carpenter and Lehrer (1999), making mathematical knowledge one’s own does not developed from having the learner merely absorbs information without any “cognitive inputs” and “personal investments”. Learners need to think critically rather than mindlessly accepting information presented to them. This means, they must have the predisposition to learn mathematics for personal goals and through personal means in order to develop the autonomy to extend their knowledge (Carpenter & Lehrer, 1999).

The above mental activities are engaged to some degree or extent in all three learning situations. To be specific, all five mental activities are presumably engaged through the learning activity of problem solving in the exploratory situation, tutoring in
the explanatory situation and comparing one another’s solution method in the extensional situation.

Problem solving engages the five mental activities for the following reasons. It allows learners to articulate their thinking through describing their approaches to the questions, explaining why certain strategies are used, and justifying why their conjectures make sense. Articulation, in turn, involves learners consciously reflecting on the relation between their existing knowledge and conditions of the problem (Carpenter & Lehrer, 1999). In order to solve a mathematical problem, learners need to draw on their knowledge to evaluate, compare, and analyze various mathematical approaches. By this, they are applying and extending their mathematical knowledge. Moreover, when learners are experimenting with parameters of the mathematical problem, they are constructing relations among ideas, reviewing the relations and assessing how they operate upon each other. Thus, the learners “develop their own stances about different forms and practices of mathematics” (Carpenter & Lehrer, 1999, p. 23) and mathematical learning becomes their personal goals.

Likewise, providing explanations in the form of tutoring supports the five mental activities when learners articulate their knowledge coherently for others to understand. Having learners provide explicit links between their ideas is providing them the “opportunities to relate what they are learning to their existing knowledge in ways that support the extension and application of that knowledge” (Carpenter & Lehrer, 1999, p. 26). The act of articulating one’s thinking is the primary stimulus for one to reflect. When learners are expected to provide explanations, they tend to reflect on it. In turn, “reflection is inherently personal, and encouraging reflection is critical in helping
students develop a sense of ownership of their knowledge” (Carpenter & Lehrer, 1999, p. 30).

Comparing solution methods is also contended by Carpenter and Lehrer (1999) to support the five mental activities. Through discussing alternative strategies, learners can acquire a repertoire of strategies and construct multiple connections among ideas thus extending their understanding (Silver et. al., 2005). This process involves relating new ideas to the analogies of what the learners understand (Gentner et al., 2003). Moreover, in comparing one another’s solution methods, learners must articulate the rationale or justification for their solution methods, and by having them regularly “negotiating how solutions are alike and different”, they reflect more on their own solutions (Carpenter, Fennema, Fuson, Hiebert, Human, Murray, Olivier & Wearne., 1999, p. 58). When learners serve as one another’s source of information, they are more likely to take responsibility of their own learning (Cockrell et al., 2000). Carpenter and colleagues (1999) explained it most precisely:

The discussion of alternative strategies not only provides an opportunity for children to reflect on and articulate their thinking, it also provides opportunity to construct relationships among different strategies by juxtaposing alternative strategies and discussing commonalities and differences among them. It provides an opportunity for [students] to discuss how their strategies can be applied to different problems in different ways, and it communicates to [students] that their own strategies are valued. Thus the discussion of alternative strategies not only serves the goal of providing opportunity for students to articulate their thinking, it provides a bases for developing the other forms of mental activity from which mathematical understanding emerge. (p. 59)

In particular, the five mental activities are highly related and this assertion was elaborated further by Fennema, Sowder, and Carpenter (1999):

Although each mental activity is described separately, in reality they are not isolated but integrated. Constructing relationships without thinking about or reflecting on those relationships is difficult. Articulation facilitate reflection
because describing (or articulating) how a problem is solved require thinking about (or reflecting on) what has been done. Because these mental activities are personal, the understanding that develops becomes the learner’s own. So we come full circle: Understanding is constructed, reflected on, and articulated by the learner and the knowledge that results is his or her own. (p. 186)

It is extrapolated that engagement of one mental activity can intricately activate the others. By providing learners the opportunity to construct knowledge through problem solving, tutoring or comparing solution methods, learners are concurrently extending and applying knowledge, reflecting about their experiences, articulating what they know and making knowledge their own. Therefore, the five mental activities introduced by Carpenter and Lehrer (1999) are engaged to some degree or extent in each learning situation. For this reason, the three learning situations are suggested to complement each other. Complementary, in this perspective, means that each situation contributes to a more complete support of mathematical understanding by each adding to the whole understanding with the mental activities that it aims to focus.

The degree of focus for one situation to support more or less of any one of the five mental activities can be complemented by the focus of the other two situations on another activity. For example, the explanatory situation provides more opportunities for articulation and reflection when the learner who assumes the role of a tutor regularly explains his or her understanding to a partner. Comparably, in the exploratory situation, problem solving is the prominent learning activity allowing learners to “apply existing knowledge in the generation of new knowledge” (Carpenter & Lehrer, 1999, p. 29). Since learners are creating their own solution methods to solve the problem, this freedom to construct their own procedures empowers learners to assume ownership of their learning (Carpenter et al., 1999). On the other hand, the extensional situation takes place in large
measure by having learners extend their knowledge through making connections between alternative solutions, thereby constructing relationships.

Furthermore, form of the mental activities may differ in each learning situation. For example, reflection can be distinguished into two forms (Carpenter & Lehrer, 1999). The first form occurs while the learning activity such as problem solving is being carried out. This form requires the learners to make connections between their existing knowledge and the parameters of the problem in order to experiment with various mathematical strategies (Carpenter & Lehrer, 1999). The second form of reflection is carried out after problem solving has been completed such as comparing solution methods and tutoring. This form of reflection involves the re-examination of existing cognitive constructs in order to justify conjectures or provide explanations (Carpenter & Lehrer, 1999). Although, learners undergo reflection in all three situations, the form of this mental activity differs depending on whether the learning activity is problem solving, tutoring or comparing solution methods.

It is argued that any one of the three learning situations alone is insufficient to support all five mental activities. There is not one learning situation that can fully accommodate all learning process, needs or styles. According to Carpenter and Lehrer (1999), the development of understanding is closely associated with the learner’s personal experience. Such variable will determine how knowledge is constructed and becomes a dynamic element of learning. Because of this dynamic variable, it is an over simplification for any one of the three learning situations to fully accommodate all learning needs and completely engage all five mental activities. Therefore, together the three learning situations provide a more complete support of mathematical understanding.
For this reason, mathematical understanding is best resulted from all three learning situations since they complementarily support the five mental activities introduced by Carpenter and Lehrer (1999). Figure 1 is a visual representation of the five mental activities as they fit into the Three Learning Situations framework.

Note. This figure illustrates the five mental activities as they fit into the Three Learning Situations framework. It displays “mathematical understanding” at the peak of the pyramid where it is supported by the three learning situations: extensional, exploratory, and extensional. The five mental activities are listed in all three learning situations and the focused mental activities of each situation are bolded.

Figure 1. The Three Learning Situations Framework.
2.5 Structure of Reciprocal Partnership

It is argued that mathematical understanding is best supported by all three learning situations, and hence Reciprocal Partnership is designed with a structure that makes possible for all of them to occur. For that reason, the structure of Reciprocal Partnership combines features of supplemental instruction and reciprocal peer tutoring for the purpose of creating all three learning situations and harnessing on the learning benefits of these two forms of tutoring.

2.5.1 Supplemental Instruction

Supplemental instruction is a form of group tutoring that support students who are enrolled in “high risk” courses (Phelps & Evans, 2006). High risk courses refer to college credit courses with high number of students who are under-performing (Arendale, 1998). Supplemental instruction is extensively shown to improve achievement of at-risk students (Malm, Bryngfors, & Morner, 2011), lessen factors identified to hinder students’ retention (Congos & Schoeps, 2003), and increase passing rate of college courses that are historically recognized as difficult (Phelps & Evans, 2006). It broadens classroom experience by incorporating study skills (time management, note-taking, metacognition, etc.) with review of course contents (Congos & Schoeps, 2003). In general, supplemental instruction is an intervention program designed to maximize college success.

Supplemental instruction is organized in workshops that occur outside of the mainstream curriculum and welcomes any student who wishes to participate (Phelps & Evans, 2006). Arendale (1998) described the details of the workshops as follows: The workshops are arranged in small groups and each group is facilitated by a leader who is usually an upper level student and has already taken the targeted high risk course with an
honourable grade. During the workshops, the leader would instigate discussion, elicit responses, pose critical and probing questions, encourage participation, demonstrate study skills, and provide feedbacks. There is a supervisor, or a trained professional who oversees the workshops and trains the leaders in applying instructional strategies during the workshops.

The structure of supplemental instruction involves college freshman or those who are enrolled in high risk courses to assume the role of a tutee in choosing or raising questions on areas that they need assistance (Malm et al., 2011). These questions are addressed to the upper level student who will then facilitate the learning of a group of students in the session (Arendale, 1998). When students have the opportunity to raise their questions on areas that they wish to improved, they are empowered to direct the learning objectives of the tutelage to some extend (King, 2008). Raising this type of questions leads to two learning situations: explanatory and exploratory.

**Explanatory Situation:** When students choose or raise questions that they need assistance on and addressed them to their partner who has knowledge of these questions then the explanatory situation is created. Their partner will then act as a tutor to facilitate their learning through providing explanations, while clarifying misconceptions, breaking down problems, reviewing concepts and offering alternative perspectives or examples. In supplemental instruction, the emphasis is for the students to benefit from the role of a tutee in receiving explanations and clarifications that facilitate their learning. On the other hand, in Reciprocal Partnership the learning emphasis is on the tutor who engages in the cognitive processes of providing explanations.
**Exploratory Situation:** When students generate or pose questions on which they need further assistance and addressed them to their partner who does not have knowledge of these questions then the exploratory situation is created. However, in supplemental instruction, this situation is usually overlooked because when the tutor (student leader) cannot provide explanation to the questions posed by the tutee, the supervisor will legitimately take over the role of the tutor. On the other hand, in Reciprocal Partnership, when both students have no solutions to the posed question, they are expected to explore and experiment with the problem. The supervisor becomes the facilitator of the whole learning process by probing questions, encouraging discussions and importantly, providing feedbacks that concern behaviours for collaborative and constructive learning.

**2.5.2 Reciprocal Peer Tutoring**

Literatures argue that the learning benefits of being a tutor surpass that of being a tutee (e.g. Allen, 2013; Galbraith & Winterbottom, 2011; Robinson, Schofield & Steers-Wentzell, 2005). With respect to this, Pigott, Fantuzzo, and Clement (1986) developed an instructional strategy called reciprocal peer tutoring. It is a type of collaborative learning that allows students to reciprocally fulfill the role of the tutor and tutee (De Backer, Van Keer, & Valcke, 2012). Students benefit from enacting as a tutor to provide explanations and as a tutee to receive assistance. Students who experience in both roles learn more than students who only take on either one role (Dioso-Henson, 2012). For this reason, reciprocal peer tutoring allows students to reciprocally tutor one another for the goal of maximizing learning, while creating reciprocal and equitable relationships.

Equity and reciprocity are some of the main factors for collaborative learning to be effective (Barron, 2000). Unequal status among partnership suppresses ideas, and
devalues the need for cognitive conflicts that simulate knowledge integration and construction (Brock et al., 2009). In parallel, reciprocity enhances students’ perceived responsibility for one another when they take turn enacting the role of a tutor (Dioso-Henson, 2012), and accentuates mutuality where both students alternatively learn from each other (Duran & Monereo, 2005). Students who assume the responsibility to teach another student, while receiving help are more positive in their own learning experiences and have more comfort in their learning (Cheng & Ku, 2009). It is “difficult to determine whether gains are due to tutoring or being tutored” (Roscoe & Chi, 2007, p. 538).

Reciprocal peer tutoring has been shown to improve understanding and communication of students (Krych, March, Bryan, Peake, Pawlina, & Carmichael, 2005), and elevate students’ social or self-perceived competency (Van Keer & Verhaeghe, 2005). It involves each student of a dyad to prepare and generate test questions for his or her partner and then provide explanations when the questions are answered incorrectly (Cheng & Ku, 2009).

The student who generates questions assumes the role of a tutor. On the contrary, in supplement instruction, the generator acts as the tutee. Therefrom, in reciprocal peer tutoring, the generator must have knowledge of the questions that he or she poses. Most distinctively, the students generate the questions in a reciprocal fashion. The purpose is to afford students the opportunity to harness the learning benefits from tutoring someone or providing explanations (Pigott et al., 1986). Reciprocal Partnership adapts this structure by having students take turns generating or choosing questions of which they have knowledge. This lends to the explanatory and extensional situations.
**Explanatory Situation:** When students generate questions of which they have knowledge and pose them to their partner who does not have understanding of these questions then the explanatory situation is created. This explanatory situation is slightly different from the one created in supplementary instruction because the tutor is the generator or the chooser of the questions rather than the tutee.

**Extensional Situation:** When students choose or raise questions of which they have knowledge and addressed them to their partner who has understanding of these questions then the extensional situation is created. However, in reciprocal peer tutoring, this situation is dismissed since students only need to address questions answered incorrectly by their partners. When questions are answered correctly, the students do not need to provide further explanations because they would be unable to enact the role of a tutor. Whereas in Reciprocal Partnership, learning is encouraged beyond the point where students arrive at the correct answer. For example, in the extensional situation, students may examine each other’s interpretation of the questions, see alternative perspectives, study similarities and differences in the approaches, discover any misconceptions or errors, and understand reasons for those errors— all of which help to extend the students’ knowledge base.

Therefore, the structure of Reciprocal Partnership is to have student dyads choosing questions in areas on which they need assistance and generating questions to which they have full solutions as required in supplemental instruction and reciprocal peer tutoring respectively. By allowing each member of the dyad to administer these two types of questions (without and with understanding) to his or her partner, all three learning situations can be created. In sum, this structure maximizes the occurrences of all three
learning situations to capitalize on the learning activities (problem solving, tutoring, or comparing solution methods) that support the five mental activities of mathematical understanding. Figure 2 is a visual representation of the Reciprocal Partnership structure that combines both supplemental instruction and reciprocal peer tutoring to maximize the occurrences of all three learning situations.

*Note.* This figure illustrates the structure of Reciprocal Partnership that combines both supplemental instruction and reciprocal peer tutoring to maximize the occurrences of all three learning situations. As displayed in the figure, supplemental instruction creates only the exploratory situation and explanatory situation, and reciprocal peer tutoring creates only the explanatory situation and extensional situation. Reciprocal Partnership is, therefore, a combination of both tutoring methods to create all three learning situations.

*Figure 2.* Structure of Reciprocal Partnership.

**2.5.3 Reciprocal Partnership for College Students**

The context of this research is to enhance mathematical learning of first and second-semester college students, while supporting first-semester students through their transition to college. Specifically, it involves first-semester students who are taking the
first mathematics course targeted in this research and second-semester students who are enrolled in the subsequent course. The purpose is to address the growing concern for college students who are more likely to withdraw from math-related courses and become at risk of restricting career options due to their insufficient mathematical groundings (Orpwood et al., 2012). Therefore, Reciprocal Partnership is subjected to the following specifications of the context in which this research is based.

First, Reciprocal Partnership encompasses cross-grade dyads to mitigate factors that are known to impede learning of college mathematics for both first and second-semester students. For cross-age tutoring program, the gap between ages or grades should be kept relatively modest in order to capitalize on the benefits of tutoring (Robinson et al., 2005). Accordingly, student dyads in this research are formed by partnering first and second-semester students. Having completed the course themselves, second-semester students become the best resource and support for first-semester students to overcome difficult subject matter of the course (Capstick & Fleming, 2002). Second-semester students can informally depict conditions of learning at the college level and offer survival strategies (i.e. manoeuvring around campus buildings, purchasing useful textbooks, etc.). They can teach first-semester students the skills to effectively manage their time and approach course material (Arendale, 1998). Clark and Lovirc (2009) state it best:

In providing assistance to first-year students there is a growing trend in the use of second- and third- year students in a mentoring role...pass on their experiences and help others with problems. Having gone through these problems themselves recently, they are in the position to understand and emphasize with struggling first-year students. (p. 760)
In return, first-semester students can also share their study skills, while going through the course materials with second-semester students. Through modeling or sharing their study skills, both first and second-semester students are presumably to have a better grasp of the learning strategies. This is because without learning the underlying study strategies attached to the course subject matter, students will experience academic difficulty in the succeeding course, and will disengage with the learning (Adrendale, 1998).

By partnering with first-semester students, second-semester students can also re-learn materials or concepts that they have not mastered or have once perceived as difficult during their first semester. This is intended for second-semester students to gain an appreciation of what is required of them in upper-level courses. When they assume the role of a tutor and engage in the process of providing explanations and revisiting teaching materials, they can consolidate their own understanding. Although, second-semester students have completed the course, they may not have learned the course content entirely and may have gaps in their understanding where these gaps can possibly be filled through tutoring first-semester students. This is in parallel with Roscoe and Chi’s (2007) account on how tutors can strengthen their understanding through tutoring:

[Tutor] may still possess knowledge gaps and misconceptions, and their knowledge of the material and procedures may be largely implicit, fragmented, and poorly organized. This may be true even though the tutor has shown good grades and test achievement (Chi, 2000). The tension between the demands of effective explaining and peer tutors’ less than perfect knowledge may push tutors to engage in reflective knowledge-building. For example, to generate relevant explanations that address key topics in a meaningful way, tutors may have to think carefully about conceptual relationships and prioritize information…Similarly, striving to producing complete explanations that integrate key concepts and principles could push peer tutors to reassess the depth and breadth of their own prior knowledge…To the extent that peer tutors attempt to repair these
problems through elaboration and inferences, their understanding should be enhanced. (p. 545)

Moreover, by partnering with a second-semester student, first-semester students may benefit from the role of a tutee. By being tutees, students’ learning can be enriched by the explanations of their tutor. In such situation, concepts that first-semester students have not fully mastered may be explained to them by their partner who may have developed a better understanding. On the other hand, given the opportunity to enact the role of a tutor, it is intended for first-semester students to develop a sense of self-regulation, empowerment, and control over their own learning. The experience of tutoring another student is an apt context for students who are “at-risk” of dropping out of school to develop perceptions of success in school, and sense of belonging to the school community (Nazzal, 2002).

The tutoring experience can also be a vehicle to prepare students for workforce and help them foster an interest in teaching. For example, in one study, upper-level students developed confidence and gained greater understanding of course material through holding the responsibility to tutor a peer (Tien et al., 2002). These students viewed their mentoring experience as honourable and beneficial. Likewise, students who experience the role of a mathematics tutor develop appreciation for the subject, and demonstrate better interpersonal skills and greater responsibility for their own learning (Carmody & Wood, 2009). Therefore, Reciprocal Partnership is proposed to create opportunity for students to better themselves through participating in collaborative learning, taking leadership in reaching out to their colleagues, and providing support for one another in their mathematical learning by partnering first-and second-semester students.
Second, Reciprocal Partnership adapts the pro-active approach of supplemental instruction to target mathematics courses that are considered essential or fundamental for developing mathematics groundings rather than students who are at risk. Targeting the courses rather than the students allows high-achievers to participate and thus, the intervention will no longer be regarded as remedial and any associated stigma will be removed (Arendale, 1998).

Third, Reciprocal Partnership requires the second-level students to be former students of the same course that the first-semester students are undertaking but does not require them to have an honourable grade. This is to encourage students of varying abilities to participate. The use of groups or dyads with students of mixed abilities is most effective to capitalize on the benefits of collaborative learning (Johnson & Johnson, 2003). Moreover, having second-semester students with mixed abilities removes any attached stigma for taking part in Reciprocal Partnership. Therefore, underperforming students who might avoid seeking assistance may participate in what is now perceived as an opportunity to engage in collaborative learning.

One negative perception may emerge when the second-semester student is not a high-achiever: the “blind leading the blind”. However, below-average students were found to be effective in tutoring each other with significant gain in achievement (Menesses & Gresham, 2009). Having a higher-performing partner does not necessarily leads to better results and less-knowledgeable tutor can has positive and significant influence on the learning outcomes of the tutelage (Hughes & Fredrick, 2006).

In Reciprocal Partnership, the emphasis is not only on the learning of the tutee but also on the tutor. Having low-achieving students to enact the role of a tutor is an effective
means for increasing academic and social gains of these students (White, 2000). Tutors do not need to be high-achieving students and it is beneficial for low-achieving students to assume the role of a tutor (Robinson et al., 2005). Robinson and colleagues (2005) explained:

[S]erving as a cross-age tutor provides a context where low-achieving students can receive additional instruction and practice with subject matter that is below their grade level and where there is less threat to their self-esteem than in other circumstances. (p. 357)

On a similar note, using cross-grade dyads raises an issue in the explanatory situation where the lower grade student, who is typically less knowledgeable on the subject materials, may enact the role of the tutor and be perceived as incapable of teaching the upper semester student. However, achievement does not result only in pairing with a partner who is more competent but can be obtained simply from having a partner with whom one can interact, discuss and exchange viewpoints (Falchikov, 2001). In a study, both tutor and tutee were reported to result in positive gains when students who were low-performing or at-risk assumed the role of the tutor (Nazzal, 2002). For this, in the explanatory situation where the less knowledgeable student can possibly be tutoring a more knowledgeable tutee, the emphasis is to allow the low performing student to benefit from the role of the tutor and to afford both students the opportunity for successive exchanges of ideas to refine thinking and consolidate understanding.

2.6 Summary

The social component of Reciprocal Partnership is based on the theory of collaborative learning and its academic component draws on the tenets of constructivism. These theories underlie theoretical perspectives of the Three Learning Situations framework, which in turn frames the structure of Reciprocal Partnership. The Three
Learning Situations framework posits that different states of understanding among a student dyad lends to three learning situations: exploratory, explanatory and extensional. The learning activity (problem solving, tutoring, and comparing solution methods) of each learning situation engages the five mental activities introduced by Carpenter and Lehrer (1999) as conducive to mathematical understanding. It is suggested that mathematical understanding is best resulted from all three learning situations. Therefore, the structure of Reciprocal Partnership combines both the mechanism of reciprocal peer tutoring and supplementary instruction to maximize the occurrences of all three learning situations.
Chapter Three: Methodology

3.1 Introduction

The purpose of this study is to investigate the effects of Reciprocal Partnership on students’ mathematical learning and affective development during their secondary-tertiary transition. It also examines the structure of Reciprocal Partnership to inform the design of effective intervention programs for mathematical learning. I employ a mixed methods approach to examine whether first and second-semester students at a community college would benefit academically and psychologically through attending the Reciprocal Partnership workshops. This method is chosen because it allows me to use different methods to cross-validate findings.

Mixed methods research is defined as “the class of research where the researcher mixes or combines quantitative and qualitative research techniques, methods, approaches, concepts or language into a single study” (Johnson & Onwuegbuzie, 2004, p. 17). A well accepted paradigmatic view of mixed methods research is pragmatism, which promotes the use of any philosophical and/or methodological approach that is appropriate to the research under study (Johnson & Onwuegbuzie, 2004). Tashakkori and Teddlie (1998) made the point clear:

[S]tudy what interests and is of value to you, study it in the different ways that you deem appropriate, and use the results in ways that can bring about positive consequences within your value system. (p. 30)

Different approaches provide alternative viewpoints and combined methods provide more evidence (Creswell & Clark, 2007). Disadvantage of the mixed methods approach includes the requirement of the researchers’ expertise on more than one methodology, the complexity of integrating different type of data, and the difficulty of
resolving discrepancies resulted from separate methods (Creswell, 2013). On the other hand, its advantage includes minimizing the weakness of one method with the strengths of the other by confirming, cross-validating, or corroborating findings (Creswell & Clark, 2007). Integration of both qualitative and quantitative data is also prominently considered as a means to create comprehensive analysis and interpretation (Creswell, 2013).

3.2 Research Approach and Design

My purpose of using the mixed-methods approach centers on data triangulation. “The value of triangulation is not as a technological solution to a data collection and analysis problem, it is a technique which provides more and better evidence from which researchers can construct meaningful propositions about the social world” (Mathison, 1988, p. 15). It enhances validity of a research by assuring variances in the findings reflect the variable being studied rather than the method(s) being used (Morse, Barrett, Mayan, Olson & Spiers, 2002).

By employing a mixed methods approach, I hoped to bring together the non-overlapping strength of the quantitative methods (generalization) and the qualitative methods (small group of participants, in-depth). The mixed methods approach used in my research was a one-phase design in which the quantitative and qualitative data were collected within the same timeframe. Both methods were not equally weighted where the qualitative data were more emphasized and the quantitative data were used only to cross-validate the interview responses through descriptive and inferential statistics. The intent was to produce valid and well-substantiated conclusions for the research questions.

I examined the affective impact of Reciprocal Partnership on the participants by using the pre-and post-surveys on mathematics self-efficacy, and the academic impact by
comparing participants’ final examination grades with previous class averages of the final examination. Survey ratings on mathematics self-efficacy and final examination grades of the participants were collected for quantitative analysis. Interview transcripts were analyzed qualitatively with regards to the structure of the workshops and the impacts of Reciprocal Partnership on participants’ mathematics self-efficacy and achievement. Although both qualitative and quantitative data were analyzed separately, they were converged for the purpose of cross-validation at the discussion stage.

3.3 Research Context

This study has taken place at a community college of Ontario. The college is one of the participating institutes in the College Mathematics Project (Orpwood et al., 2012) that reveals the need to support college students in their mathematical learning before they pre-maturely restrict themselves from math-related pursuits. The college has more than 80,000 students with 30% Chinese, 20% Indian and 10% South Korean and offers about 160 programs which include certificates, degrees, and diplomas. Every program requires a minimum of one mathematics credit and each credit course usually spans across a semester (fourteen weeks). This participating college has served as the context in which the structure of Reciprocal Partnership and its effects on students’ mathematics self-efficacy and achievement have been examined.

3.4 Participants

The participants of this research were first and second-semester students of the Civil Engineering department at the participating community college. The criterion for selecting the first-semester participants required them to be enrolled in the first-semester mathematics course (Math 1) targeted for this study. On the other hand, the criterion for
selecting the second-semester participants required them to be enrolled in the succeeding mathematics course (Math 2). Participants were recruited under a voluntary basis. Recruitment letters (see Appendix A) were distributed during class time with a brief introduction of the research purpose. Both quantitative (numeric) and qualitative (text) data were collected from the first and second-semester participants.

3.5 Data Collection

I was the primary researcher to conduct the data collection. This study began with an initial workshop at the beginning of the Reciprocal Partnership intervention. The initial workshop was designed to describe the structure of Reciprocal Partnership with particular attention to the three learning situations and their specifications. For example, the learning activity (solving problem, tutoring or comparing solution methods) of each situation was described and the task of each participant to prepare two sets of questions (with knowledge and without knowledge) was explained. Other focuses of the initial workshop were to introduce the participants to one another, and to establish a safe learning environment in which goals should be achieved through collaborative effort, open communication, and mutual participation.

During the initial workshop, consent forms (see Appendix B) were distributed for the participants to signify their participation in the study and their disclosure of personal information for the purpose of this research. Confidentiality and rights of participants in a research study were iterated. Information forms were also distributed for participants to complete, and this information (e.g. gender, ethnical background, program of study etc.) was used to identify any discernible patterns that might affect data analysis (see
Appendix C). The pre-survey was administered to all participants and the data were used to establish a baseline measure of their mathematics self-efficacy.

In addition, participants were given a package with information on what they needed to prepare for the Reciprocal Partnership workshops (see Appendix E). The package also included blank pages for participants to write their questions and record all worked steps. Dyads of first and second-semester participants were formed and pairing of the participants was based on their matching time schedule and availabilities. Participants were paired with the same partner throughout the intervention and were asked to attend two Reciprocal Partnership workshops per week. In the final week of the intervention, participants were invited to complete the post-survey and their final examination grades were collected. After the intervention, semi-structured interviews of about forty minutes were conducted with all participants. Confidentiality and the rights of participants in a research study were reiterated again during the interviews.

3.5.1 Final Examination Grade

Final examination grades of the first and second-semester participants were collected to measure the effects of Reciprocal Partnership on their academic achievement. Final examination grades were chosen over the final course grades because the final examination was standardized among the different instructors who were teaching the same course. Testing items on the final examination were ensured by the instructors to correspond with their teaching objectives. This should warrant content validity of the final examination measures.
3.5.2 Mathematics Self-Efficacy Survey

A survey based on the Mathematics self-efficacy scale (MSES) was used to gather quantitative data on students’ mathematics self-efficacy for first and second-semester participants, and was administered before and after they underwent the Reciprocal Partnership intervention. These participants were in the treatment groups. The pre- and post-surveys were also administered to students who were enrolled in Math 1 and Math 2 but did not partake in the Reciprocal Partnership workshops and other intervention programs during their enrollment in these courses. These participants became the control groups. The survey ratings of the control groups were used to compare with the survey ratings of the treatment groups as a means to detect any significant change in students’ mathematics self-efficacy before and after the intervention.

MSES is created by Betz and Hackett (1983) after they have reviewed existing measures of mathematics anxiety and math confidence. MSES is reported to have moderate item-total rating correlations for its subscales and a high level of internal consistency (coefficient alpha = 0.96). Accordingly, this instrument is accounted for strong evidence of reliability and validity.

There are 52 items in MSES which are divided into three subscales identified as relevant to the study of math-related self-efficacy expectations. The first subscale represents the essential skills for solving problems found on standardized mathematics tests. It is consisted of 18 items drawn from the Mathematics Confidence Scale (Dowling, 1978). The second subscale includes mathematics behaviours used in “everyday” math tasks such as balancing a check book. It consists of 18 items resembling the Math Anxiety Rating Scale (Richardson & Suinn, 1972). The final subscale represents self-
assessed perceived ability to perform the mathematics knowledge and skills with a grade of B or better in 16 math-related college courses.

All three subscales require the rating of one’s confidence in their ability to successfully perform the math problems, math tasks and math courses. Rating is based on a 10-point continuum with 0 indicating no confidence and 9 signifying complete confidence. The median value of the ratings was calculated across the three subscales to measure participants’ mathematics self-efficacy.

3.5.3 Semi-structured Interview

Post-experimental interviews were conducted with the participants to gather their reflective self-reports regarding the structure of the Reciprocal Partnership workshops and the impacts of the workshops on their mathematics self-efficacy and academic achievement. The primarily purpose was to elicit valuable perceptions of students who participated in the workshops with the intention to find incidental information that were relevant to the study but could not be detected from using the quantitative method alone. Both first and second-semester students in the treatment group were interviewed individually for about 40 minutes. The interviews were semi-structured with open-ended questions (see Appendix D).

3.6 Reciprocal Partnership Workshop

The Reciprocal Partnership intervention was organized in workshops pairing first and second-semester college students. In the workshops, students undertook the three learning situations to solve problems, tutor one another, or compare solution methods. In specific, they discussed and solved mathematics programs, explained and justified solution methods, reviewed course content, answered probable examination questions,
and evaluated each other’s worked examples. The workshops were designed for these main purposes: 1) To help first-semester students construct deeper understanding of course content, while gaining confidence to socialize at the college level; 2) To support second-semester students in their effort to facilitate the integration of first-semester students, and 3) to re-learn fundamental mathematics concepts that would sustain them through their upper-level courses.

The workshops began in the first week of the academic term. They were held twice per week throughout the semester and took place outside of the mainstream curriculum. This was due to the following reasons: Students’ attrition is highest during the initial period, and adjustment to the college must be established during students’ first week of their program (Tinto, 2006); Longer tutoring programs are not necessary more effective than shorter ones in fostering better academic achievement (Robinson, Schofield, & Steers-Wentzell, 2005).

Participants were expected to prepare a set of 2 to 3 questions of which they had knowledge and another set on which they needed assistance. Participants drew their questions from text, lecture notes, supplementary readings, previous tests or exams of Math 1. Their questions were either in a form of reviewing course lectures, studying difficult concepts, going through test questions or predicting future exam questions. This is because reviewing former tests and exams helps prioritize information and helps students understand the type of questions perceived as important (Adrendale, 1998).

Questions generated by the participants became the learning objectives of the Reciprocal Partnership workshops. Question generation is an effective cognitive strategy to foster comprehension and self-regulation (King, 2008). For this reason, participants
were given the opportunity to generate the two different sets of questions that directed the learning objectives:

When posing questions of which they had knowledge, students attempted to use questions that mirrored the course content or presented the questions in alternative perspectives. Having participants generated this type of questions afforded them the opportunity to prepare the teaching material and be held accountable for one another’s learning. Students who anticipate teaching someone and actually perform the teaching are reported to have higher intrinsic motivation and more positive engagement in their learning (Sobral, 2004).

When generating questions on which they needed assistance, students directed the Reciprocal Partnership workshops on areas that they wished to focus or improve. This was to encourage students to self-regulate their own learning, identify areas of strength and weakness, and ultimately take the initiatives to address their own learning needs.

Time was designated at the beginning of each workshop for casual discussions. The purpose was to address any potential problems from lack of interactions between partners and to provide a platform for discussions and sharing of learning strategies. Another purpose was for the second-semester students to share their experience and knowledge about the expectations or demands of Math 1 since they were the former students of this course. According to Adrendale (1998), students must understand the nature of the courses or the expectations of the instructor so they can demonstrate the expected thinking processes or achievements.

During the brief discussion period before the Reciprocal Partnership workshops, second-semester participants might have provided the following assistances: Directed
their partner to set objectives and goals for each week of the course or each Reciprocal Partnership workshops; Helped their partner to manage their time for exams and assignments by referring back to the course syllabus; Helped their partner to connect prior knowledge with current course material, and predict future test questions.

In order to provide ongoing supports during the workshops, I monitored the participants’ implementation of the Reciprocal Partnership structure, answered their questions, and responded to any of their learning needs. Particularly, in the exploratory situation where the participants were solving the mathematical problems under my guidance, I provided probing-questions, clarified misconceptions, and steered them to their understanding. I also took the administrative role to conduct the initial workshop. In general, I ensured that the participants were implementing the Reciprocal Partnership procedures properly, and communicating respectfully.

3.7 Data Analysis

Personal information of the participants including gender, program of study, and ethnic background were gathered and used to determine any discernible patterns that might affect analysis of the data. Final examination grades and the pre-and post-surveys were analyzed quantitatively through descriptive and inferential statistical methods. Responses from the post-experimental interviews were analyzed qualitatively through thematic analysis. The purpose was to conduct a parallel analysis of these two types of data and combine them at the discussion stage for cross-validation purposes.

3.7.1 Quantitative Analysis

Quantitative data were analyzed by descriptive and inferential statistics. SPSS statistics software version 14.0 was used to calculate the inferential results. The median
of the treatment group’s final examination grades was calculated and was measured against previous class averages of the final examination for the two mathematics courses (Math 1 and Math 2). The median of the pre- and post-survey ratings and the difference between the medians were also calculated. Analyses of these data were descriptive statistics using graphs and tables, and inferential statistics using non-parametric tests.

Nonparametric tests are used in situations where the population is not presumed to follow a specific probability distribution and where little is known about the parameters of the testing variable in the population (Gibbons & Chakraborti, 2011). They do not assume normal distribution of data and do not estimate key parameters of the population distribution such as the mean and standard deviation. They are less powerful than their parametric counterparts because they may be less likely to reject the null hypothesis when the alternative hypothesis is true (Gibbons & Chakraborti, 2011). They also require modification of the hypothesis since they examine the population center using the median instead of the mean.

The main reason to use the nonparametric tests in this research was the small sample size as it was too small to assume that the sampling distribution and the distribution of the variable (mathematics self-efficacy and academic) in the population were normal. For small sample size, the asymptotic statistical significant level (exact p-value) (Dineen & Blakesley, 1973) is presented in SPSS for nonparametric test. Asymptotic statistical significant level is only an approximation to the true p-value and this approximation will improve as the sample size increases. Since the sample size for this research was less than 20, the asymptotic statistical significant level was provided and the significant level was set at .05.
3.7.2 Qualitative Analysis

Qualitative data collected from post-experimental interviews were analyzed using thematic analyses. This approach involves classifying recurring responses into common themes (Braun & Clarke, 2012). Qualitative analysis software, Nvivo 10, was used to code the actual texts of the interviews. The coded transcripts were categorized to identify emerging themes that provided meanings to the research questions.

Braun and Clarke (2012) described the process of thematic analysis in six phases: The first phase involves familiarizing with the data by transcribing, reading, and making notes on the data. Second phase requires the generation of initial codes based on interesting features. The third phase is to search for themes by collating initial codes. The fourth phase is to review the potential themes by checking the relation of the themes with the coded data. The fifth phase involves defining and naming the themes by refining the specifics of each theme. The final phase is to produce the report by relating the analysis back to the research question. According to Braun and Clarke (2012), thematic analysis involves providing meanings to recognised and organized patterns (themes) across the entire data set.

3.8 Internal Validity

The use of more than one method allows the weakness of one approach to be complemented by the strength of another, and hence strengthens the validity of the research (Morse et al., 2002). Validity of this research was intended to increase through the use of cross-validation. For example, descriptive statistics from the quantitative data (pre- and post-survey ratings, and the final examination grades) were used to cross-validate findings from qualitative data (interview responses). Member checks were also
conducted which involved having the participants to confirm interpretations on their responses. This procedure should establish validity by providing opportunity for the participants to assess and confirm sufficiency of the data interpretations (Lincoln & Guba, 1985).

3.9 Ethical Considerations

This study was reviewed by the Office of Research Ethics (ORE) of the University of Toronto, and by the ORE of the community college where data were collected.

Participants were recruited and asked verbally to take part in this study. They were reminded of their right to decline answering any uncomfortable questions and withdraw from the research at any time. They were invited to sign the consent form to signify their participation and disclosure of personal information for the purpose of this research. Pseudonyms were used to ensure confidentiality of the participants, and all documents of this study were stored in locked cabinets for at least five years after the research had been completed.
Chapter Four: Findings

4.1 Introduction

This chapter begins by describing the background of the participants. It then explores the qualitative and quantitative data according to the three research questions: 1) what effects, if any, does Reciprocal Partnership have on the mathematics self-efficacy of first and second-semester college students? 2) What effects, if any, does Reciprocal Partnership have on the achievement of first and second-year semester students? 3) What insights does Reciprocal Partnership have to inform the design of effective intervention programs for mathematical learning?

To examine the research questions, pre and post surveys (Betz & Hackett, 1983) on mathematics self-efficacy were conducted on all of the participants (control and treatment groups), final examination grades of the participants were collected, and semi-structured interviews were conducted only with the treatment groups. Quantitative data collected from the surveys were analyzed through descriptive and inferential statistics using SPSS statistics software version 14.0. On the other hand, qualitative data gathered from the interviews were coded using Nvivo 10 and analyzed through thematic analysis (Braun & Clarke, 2012). Descriptive results are displayed in the following graphs and charts, and inferential results are reported in APA format. Finally, themes that emerged from the qualitative analysis are conveyed in narratives.

4.2 Background Information

Participants in the first-semester were taking the first mathematics course (Math 1). Math 1 was offered in a compressed schedule, which spanned across seven weeks.
instead of the normal fourteen weeks at the time when the research took place. There was only one section for Math 1 with a total of 36 students.

Students who were in the second-semester completed Math 1 and were currently taking the second mathematics course (Math 2). Math 2 was offered through fourteen weeks and in four sections with a total of 105 students. Three different professors taught the course. Math 1 was a perquisite for Math 2 and both courses were mandatory for the Civil Engineering Program.

Gender was not equally distributed. There was only one female (student S5) and she was in the second semester. This was because the Civil Engineering Program had always been male dominated. Out of the twelve participants, student S5 was the only English Language Learner. Ethnical background of the participants was quite diverse which included two Chinese, two Indian, two Caucasian, one Brazilian, one Ecuadorian, one Albania, one Filipino, one Grenada, and one Persian student.

There were more students from Math 2 than from Math 1 who had volunteered. The criterion for selecting these students was based on their matching availability or timetable with the students in Math 1. Six pairs of students were formed to participate in the Reciprocal Partnership workshops – a pair consisted of a student in Math 1 and a student in Math 2. Other students who enrolled in Math 1 and Math 2 were invited to complete the pre and post-surveys. Their survey ratings were included in the quantitative analysis as a control group based on the criterion that they had not participated in any mathematics tutorial or intervention program, while enrolling in the courses (Math 1 and Math 2).
There were two treatment groups, where one group was in the first semester and the second group was in the second semester. As a result, there were two corresponding control groups (students who did not participated in the workshops or other intervention programs but were enrolled in either Math 1 or Math 2). Participants in the treatment groups were numbered from 1 to 12 and participants who were assigned to the control group were numbered from 13-34. The letter “S” was used to indicate students who were in the second semester and “F” was used to indicate students who were in the first semester. The numbers of participants in each group are provided in Table 1.

Table 1

*Number of participants in each group*

<table>
<thead>
<tr>
<th>Groups</th>
<th>Treatment</th>
<th>Control</th>
</tr>
</thead>
<tbody>
<tr>
<td>First Semester Students</td>
<td>6</td>
<td>8</td>
</tr>
<tr>
<td>Second Semester Students</td>
<td>6</td>
<td>14</td>
</tr>
</tbody>
</table>

4.3 Descriptive Statistics

Descriptive statistics of the pre and post-survey ratings and final examination grades are displayed in the following graphs and tables. As shown in Figure 3, all of the participants in the treatment group who were in the first semester rated their mathematics self-efficacy higher in the post-survey. The pre-survey had a range of 4.62 between the maximum (Md = 8.48) and minimum (Md = 3.86) ratings. Comparatively, the post-surveys had a smaller range of 3.72 between the maximum (Md = 8.88) and minimum (Md = 5.16) ratings.
Student F3 had the smallest increase in his mathematics self-efficacy when comparing the pre-survey (Md = 8.48) with the post-survey (Md = 8.88) ratings. On the other hand, student F2 had the largest increase when comparing the pre-survey (Md = 3.86) with the post-survey (Md = 6.68) ratings. Students (F2, F8, and F9) who initially had lower median (Md = 3.86, 4.54, and 4.29 respectively) on their pre-survey had the larger increases in mathematics self-efficacy (Md = 2.82, 1.85 and 1.12 respectively).

![Figure 3](image)

*Figure 3.* Pre and post-survey ratings of students in first semester who were in the treatment group.

According to Figure 4, all of the students in the treatment group who were in the second semester rated their mathematics self-efficacy higher in the post-survey. The pre-survey had a range of 2.86 between the maximum (Md = 8.49) and minimum (Md =
5.63) ratings. Comparatively, the post-surveys had a smaller range of 1.47 between the maximum (Md = 8.82) and minimum (Md = 7.35) ratings.

Student S5 had the smallest increase in his mathematics self-efficacy when comparing the pre-survey (Md = 8.49) with the post-survey (Md = 8.82) ratings. On the other hand, student S7 had the largest increase when comparing the pre-survey (Md = 5.63) with the post-survey (Md = 7.35) ratings. Students (S1 and S7) who initially had lower median on their pre-survey (Md = 7.12 and 5.63 respectively) had the larger increases in mathematics self-efficacy (Md = 0.76 and 1.72 respectively).

Figure 4. Pre and post-survey ratings of students in second semester who were in the treatment group.

Furthermore, first-semester students who were in the treatment group had lower median in both the pre- and post-survey compared to students in the second semester. This was the same case between the first and second-semester students in the control
The median difference in the table refers to the difference between the medians of the pre-survey and the post-survey ratings. Students in the treatment group who were in the first semester had higher median difference than students in the second semester. In other words, first-semester students had larger increases in their mathematics self-efficacy than second-semester students. Table 2 shows the median of pre and post-survey ratings for first and second-semester students in the treatment and control groups.

Table 2

<table>
<thead>
<tr>
<th>Mathematics Self-efficacy</th>
<th>Pre-Survey Median</th>
<th>Post-Survey Median</th>
<th>Median Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Groups</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Treatment 1</td>
<td>5.32</td>
<td>6.61</td>
<td>0.99</td>
</tr>
<tr>
<td>Control 1</td>
<td>6.57</td>
<td>6.67</td>
<td>0.19</td>
</tr>
<tr>
<td>Treatment 2</td>
<td>7.74</td>
<td>8.35</td>
<td>0.74</td>
</tr>
<tr>
<td>Control 2</td>
<td>6.60</td>
<td>7.25</td>
<td>0.31</td>
</tr>
</tbody>
</table>

Note. Treatment 1 refers to first-semester students who participated in the Reciprocal Partnership workshops and enrolled in Math 1. Control 1 refers to first-semester students who did not participate in any intervention program but enrolled in Math 1. Treatment 2 refers to second-semester students who participated in the Reciprocal Partnership workshops and enrolled in Math 2. Control 2 includes second-semester students who did not participate in any intervention program but enrolled in Math 2.

First and second-semester students also had higher median difference than their respective control group. Comparing the median difference between both treatment and control groups was an initial analysis to detect effect of Reciprocal Partnership on students’ mathematics self-efficacy. As a result, the above descriptive statistics provided
preliminary evidence of positive effect of Reciprocal Partnership on mathematics self-efficacy. This apparent effect had been calculated to have a median difference of 0.99 for students in the first semester and 0.74 for students in the second semester.

Figure 5. Final examination grade of students in first semester compared to median of previous class averages for the final examination of Math 1.

Moreover, the previous class averages of the final examination grades were used to estimate the hypothesized median of the population. For students in the first semester, their population referred to all of the students who took Math 1 from at least five previous semesters. For students in the second semester, their population referred to all students who took Math 2 from at least five previous semesters. As displayed in Figure 5, the final examination grades of first-semester students had a range of 35% between the maximum (M = 79%) and minimum (M = 44%) grades. Only student F3 performed better than the median of previous class averages (Md = 61%).
The final examination grades of students in the second semester had the range of 45% between the maximum (M = 95%) and minimum (M = 50%) grades. Student S10 was the only student who performed below the median of previous class averages (M = 61%). However, student S4 did not take the final examination and his result was not accounted for in the analysis. Figure 6 displays the final examination grade of students in second semester compared to median of previous class averages for the final examination of Math 2.

![Figure 6](image)

*Figure 6.* Final examination grade of students in second semester compared to median of previous class averages for final examination of Math 2.

First-semester students had a lower median in their final examination grades compared to the median of previous class averages for the final examination of Math 1. However, the second-semester students had a much higher median in their final
examination grades compared to the median of previous class averages for the final examination of Math 2. The median of the final examination grades for the second-semester students was higher than the students in the first semester. Table 3 displays the median of final examination grade for first and second-semester students and median of previous class averages for the final examination.

Table 3

*Median of final examination grade for first and second-semester students and median of previous class averages for the final examination*

<table>
<thead>
<tr>
<th>Groups</th>
<th>Final Examination Grades Median</th>
<th>Previous Class Averages Median</th>
</tr>
</thead>
<tbody>
<tr>
<td>First Semester Students</td>
<td>51.00</td>
<td>61.00</td>
</tr>
<tr>
<td>Second Semester Students</td>
<td>80.50</td>
<td>61.00</td>
</tr>
</tbody>
</table>

When asked which learning situation students would prefer over the other two situations, they made their choice based on the amount of learning that they had gained from engaging in the learning activity (problem solving, tutoring or comparing solutions) of each situation. 5 students preferred the exploratory situation because they felt that they learned most from deriving their own solution methods when solving the mathematical problems with their partner. Only one student preferred the explanatory situation because he enjoyed being tutored by his partner and gained knowledge from the experience. There were 3 students who preferred the extensional situation because they found value in learning alternative approaches, and in the process detected their own mistakes and worked at their own pace. There were 3 students who found all three learning situations equally helpful. These students insisted that not any one of the learning situation was
more effective than the others in terms of enhancing their mathematical learning as each situation benefited them in one or the other way. Figure 7 displays preference of the students for the three learning situations based on their learning gains.

![Figure 7](image)

*Figure 7. Preference of students for the three learning situations based on their learning gains.*

When comparing changes in students’ mathematics self-efficacy with their final examination grades, the top six students with the largest difference between their pre- and post-survey ratings were analyzed. Five out of the six students with the largest increase in their mathematics self-efficacy performed below the median of previous class averages for the final examination. The hypothesized population median of the final examination was 61% for both Math 1 and Math 2. Only student S7 with a large difference (Md = 1.72) between his pre-survey (Md = 5.63) and post-survey (Md = 7.35) ratings had a final
examination grade (M = 90%) that was higher than the hypothesized median (Md = 61%). Table 4 shows the median difference between the pre- and post-survey ratings, and the final examination grade of the six students with the largest increase in their mathematics self-efficacy.

Table 4

*Comparing changes in students’ mathematics self-efficacy with their final examination grades*

<table>
<thead>
<tr>
<th>Student</th>
<th>Median Difference between Pre- and Post-survey ratings</th>
<th>Final Examination Grade (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>F2</td>
<td>2.82</td>
<td>51.00</td>
</tr>
<tr>
<td>F8</td>
<td>1.85</td>
<td>51.00</td>
</tr>
<tr>
<td>S7</td>
<td>1.72</td>
<td>92.00</td>
</tr>
<tr>
<td>F9</td>
<td>1.12</td>
<td>53.00</td>
</tr>
<tr>
<td>F11</td>
<td>0.86</td>
<td>44.00</td>
</tr>
<tr>
<td>S1</td>
<td>0.76</td>
<td>60.00</td>
</tr>
</tbody>
</table>

4.4 **Inferential Statistics**

The Mann-Whitney U test was used to compare between the effect of the Reciprocal Partnership workshops and the mathematics courses (Math 1 and Math 2) on students’ mathematics self-efficacy. Specifically, it determined if there was a significant difference between the mathematics self-efficacy of students who participated in the Reciprocal Partnership workshops (treatment group) and students who did not (control group). Students in both the treatment and control groups were enrolled in the same mathematical course (Math 1 or Math 2) but students in the latter group did not
participated in any tutoring or supportive program including the Reciprocal Partnership workshops.

A Mann-Whitney U test is the nonparametric resemblance of independent sample t-test. It examines whether there is a difference in the median or distribution between two independent groups on a dependent variable (Gibbons & Chakraborti, 2011). The effect size for the Mann-Whitney U test can be determined by the $r$ value (Fritz, Morris & Richler, 2011). The formula to calculate $r$ is to divide the $z$ value by the square root of $N$ where $N$ is the total number of participants in the two independent groups. Magnitude of the effect size is based on Cohen’s (1988) criteria of $.1 = \text{small effect}, .3 = \text{medium effect}, .5 = \text{large effect}$. Hypotheses of the Mann-Whitney U test were specified as follows: Null hypothesis ($H_0$) stated that the median difference of survey ratings between the control and treatment groups was equal; Alternate hypothesis ($H_A$) stated that the median difference of survey ratings between the control and treatment groups was not equal.

The data (difference between the pre- and post-survey ratings) were examined to have met the three assumptions of the Mann-Whitney U test: First the dependent variable (mathematics self-efficacy) was measured at the ordinal scale. Second, the independent variable consisted of two categorical, independent groups (control and treatment). The pre and post survey ratings in each group were independent from each other such that there was no relationship between the observations in each group. Third, visual examination of histograms revealed that distribution of data between the two independent groups had similar shape.
Mann-Whitney U test results for first-semester students: A Mann-Whitney U test was employed to determine if there was significant difference between the mathematics self-efficacy ratings of the treatment and control groups in the first semester. Distributions of the difference between the pre and post survey ratings for both groups were similar, as assessed by visual inspection. Median of the mathematics self-efficacy ratings between treatment (Md = .99) and control (Md = .19) groups were statistically significant, \( U = 8.00, z = -2.07, p = .04 \), with a medium effect size \( (r = .31) \), using an exact sampling distribution for \( U \) (Dineen & Blakesley, 1973).

Mann-Whitney U test results for second semester: A Mann-Whitney U test was run to determine if there was significant difference between the mathematics self-efficacy ratings of the treatment and control groups in the second semester. Distributions of the difference between the pre and post survey ratings for both groups were similar, as assessed by visual inspection. Median of the mathematics self-efficacy ratings between treatment (Md = .74) and control (Md = .31) groups were not statistically significant, \( U = 22.00, z = -1.65, p = .10 \), with a small effect size \( (r = .14) \), using an exact sampling distribution for \( U \) (Dineen & Blakesley, 1973).

Furthermore, a Wilcoxon signed-rank test (one sample) was used in this research to compare the median of the final examination grades of the students in the treatment groups with the hypothesized median of the population. This test is known to measure the difference between the median of the sample and the hypothesized value for the population median, and is a common alternative to the parametric one sample t-test (Gibbons & Chakraborti, 2011). The quantitative data (final examination grades) were examined to have met the assumptions of the Wilcoxon signed-rank test (one sample):
First, the data were ordinal and second they were mutually independent. Second, the
distribution of the data was quite symmetric, as assessed by visual inspection of
histograms. Hypotheses of the Wilcoxon signed-rank test (one sample) were specified
as follows: Null hypothesis ($H_0$) stated that the sample median was equal to the
hypothesized population median; Alternate hypothesis ($H_A$) stated that the sample median
was not equal to the hypothesized population median.

**Wilcoxon signed-rank test (one sample) results for first-semester students:** Six
male students in the first semester of a community college were recruited to understand
the impact of Reciprocal Partnership workshops on their mathematics achievement. A
Wilcoxon signed-rank test (one sample) determined that there was no significance
difference between the median of the participants’ final examination grades ($Md = 51$)
and the hypothesized median ($Md = 61$), $p = .25$.

**Wilcoxon signed-rank test (one sample) results for second-semester students:**
Five male students and one female student in the second semester of a community college
were recruited to understand the impact of Reciprocal Partnership workshops on their
mathematics achievement. A Wilcoxon signed-rank test (one sample) determined that
there was no statistical significance difference between the median of the participants’
final examination grades ($Md = 80.50$) and the hypothesized median ($Md = 61$), $p = .12$.

### 4.5 Themes of Mathematics Self-efficacy

Students in the treatment groups were interviewed with open-ended semi-
structured questions about the affective and academic effects of Reciprocal Partnership
on their mathematical learning. Their responses were transcribed and coded using the
software, Nvivo 10 and the transcripts were analyzed according to the phrases of thematic
analysis (Braun & Clarke, 2012). Themes emerged from the analysis of the interview responses on mathematics self-efficacy.

Mathematics self-efficacy was described interchangeably as confidence in the students’ responses. All of the students replied that, after participating in the Reciprocal Partnership workshops, their confidence increased and the levels of increase were different among individuals. Analysis of the interview responses revealed three main avenues through which the students experienced the proliferated self-efficacy in mathematics: confidence from extra practice, confidence from solving difficult questions, and happiness and confidence from teaching another student.

4.5.1 Confidence from Extra Practice

The opportunity to spend extra practice on the mathematical problems improved students’ mathematical knowledge which in turn, bolstered their confidence. For example, student F2 was unable to solve trigonometry problems back in high school but “the amount of practice gave [him] more confidence” in solving those questions (Interview, June 22, 2015). He continued to describe it in details:

At first, I was struggling. Now, I am pretty confident and better than before…because by just taking the time and effort to work on the questions, it builds up my self-esteem and my confidence in school. (F2 Interview, June 22, 2015)

Likewise, his partner contended that he gained confidence because the workshops provided him the opportunity to work with the mathematical questions and receive feedback:

When I’m actually physically here, it gives me the chance to work on the questions and people can also give me inputs. Then I was like, “Okay, I understand how to do it. It makes sense”… I feel more confident talking about or working with the questions now. (S1 Interview, June 26, 2015)
To the students, extra practice of the mathematical questions involved solving and working through the questions. In the end, student S1 developed the persistency to solve the mathematical problems and illustrated how it allowed him to gain understanding and confidence: “It gave me confidence because once I kept doing it, I understood it. It was not as hard as it may seem. People make it seems like it’s a big deal but it’s not. It definitely gives me more confidence” (S1 Interview, June 26, 2015).

Student F6 described himself as a student who was never confident in mathematics but remarked, “After working on the questions, doing it, and practicing it, I understood more than when I first came to the workshop. I feel confident now” (Interview, June 22, 2015). On the same note, student F8 identified the positive correlation between practice and his increased confidence:

> It makes me feel smarter. I have not done that before with this much confidence. Every time when I’m doing math, I rely on someone to teach me…Since I take the opportunity and time to do the questions, I feel smarter and great inside. (F8 Interview, June 25, 2015)

### 4.5.2 Confidence from Solving Difficult Questions

According to the students, they also gained confidence by solving difficult questions. Student S1 asserted that he was “more confident to tackle one of the harder questions” and was “not afraid to see big problems anymore”. He then illustrated how he approached the difficult questions: “You have to try to grab some parts, and see if you can make an equation from what’s in the word problem” (S1 Interview, June 26, 2015).

Student F11 highlighted the growth of his confidence after solving difficult questions: “When we were able to get the right answer and then tried another question and got it right, confidence was growing. Soon enough, I felt more confident doing the type of problems that I had previously struggled with” (Interview, June 22, 2015).
Student S10 had greater confidence in solving questions that he struggled with when he was taking Math 1: “Now, I’m sure I’m taking the right steps and doing the questions right. I’m confident in solving those questions” (Interview, June 19, 2015). His partner echoed with similar response about solving questions on which he previously had difficulties: “[The workshops] give me more confidence. I can actually solve some of the equations that I didn’t know how to solve before” (F9 Interview, June 22, 2015).

4.5.3 Happiness and Confidence from Teaching another Student

Students felt more confident about their own mathematical knowledge and skills by teaching another student. Student F3 explained, “If you have taught it to somebody, you know how it works. You have better confidence in what you do” (Interview, June 22, 2015). His partner felt the same, “When I was teaching him, I felt like I know everything in Math 1 now. I gained more confidence in my own mathematics.” (S4 Interview, June 26, 2015). Student F8 felt confident when he had the opportunity to demonstrate his knowledge to his partner: “Teaching my partner means I know something. When I know “I know it”, I have more confidence in doing it” (Interview, June 25, 2015). His partner also felt confident remembering the materials from Math 1 and being able to teach him on those materials: “Most of the time I helped him and I feel confident that I still know some of the materials” (S7 Interview, June 19, 2015).

Furthermore, students in the first semester described the relationship with their partner as “one-sided” because they believed they did not have enough mathematical knowledge as their partner who was in the second semester. Student F6 said, “To be honest, I didn’t helped her much. She mostly helped me. I didn’t know much compared to her” and student F11 said the same, “I really didn’t know much and it was mostly him
helping me because I was really struggling with most of the materials”. However, when they were able to help their partner occasionally, student F6 exclaimed, “I felt smart in knowing how to do that kind of questions” (Interview, June 22, 2015) and student F11 celebrated, “It feels good. It makes me feel like I’m not the stupid one here because I actually know something that he doesn’t know” (Interview, June 22, 2015). Student S12 pointed out how his partner, student F11, was instantly more confident by correcting him on a question:

We were actually working on a problem…When he was able to tell me what to do, it boosted up his confidence and he was like “yeah, I can do bigger things in Math 1”. Also, there was actually one time when his solution was right and mine was wrong. Once he saw that he actually got a question over me that was correct, it really boosted up his confidence…I think what really motivated him to want to do well was when he saw that he could do better than what he was doing before. (S12 Interview, June 26, 2015)

Student S12 continued and talked about how observing the change in his partner’s confidence affected his own learning:

Sometimes, people underestimated themselves. So when I saw a student like my partner who feared [trigonometry] and then was able to understand it and helped me with it, it made me think that if I see derivatives in Calculus 2, it may seems big for me now but I’m going down the same path that he’s going. I’m going to take some classes, do some questions, and understand it. Maybe I will be so good at Calculus 2 one day that I can help somebody else…Seeing how he developed and how he elevated, it changed my confidence in how I can go through the same kind of learning path. (S12 Interview, June 26, 2015)

Although students in the first semester seldom tutored their partner, it was in the occasional moments when they were able to offer help that vastly increased their confidence. Student F8 boasted, “I taught him a couple of little tips on algebra. I felt so much more confident in myself” (Interview, June 25, 2015), and student F2 exclaimed, “Although, my partner helped me with the stuffs that I didn’t know but I also helped him with stuffs that he didn’t know. It was just a huge confidence boost” (Interview, June 22,
2015). Student F9 said that he felt pride when he was able to teach his partner: “It is pride that I know what he doesn’t know and be able to teach him. That way, I don’t look dumb and feel very confident” (Interview, June 22, 2015).

Student F2 had the opportunity to teach his partner on a question from Math 1 that his partner had never learned before. He reasoned, “When you are able to teach someone that feeling changes and uplifts your self-esteem” (F2 Interview, June 22, 2015). While he was teaching his partner on the whiteboard, other students gathered around to learn from him. He proclaimed: “It makes me feel like I have gained more leadership. I actually feel like I’m a leader for once” (Interview, June 22, 2015). His partner also mentioned about increased confidence after tutoring and helping him: “Because I actually learned stuff and I was able to apply it to help someone in that aspect, it gave me confidence and added strength in my math” (S1 Interview, June 26, 2015).

In addition, the opportunity to teach another student not only increased confidence, it also created happiness. “It’s like passing on the knowledge that you know on to the others” (F8 Interview, June 25, 2015). Student S5 remarked “During this workshop, I know something that can make me happy: it’s to help people. Because I know math, and I am so happy to help people especially when they really appreciate me” (Interview, June 25, 2015). Her partner agreed that helping others made him feel good about himself: “To me helping anyone, at the end of the day, feels good. When you leave this room with the knowledge that you give to someone and knowing that they can go home and do it by themselves, it feels good because I’m the one who helped them” (F6 Interview, June 22, 2015). Student S4 described the feeling of helping others as a sense of confidence, goodness, and self-realization:
When you teach someone else, it gives you a pretty good idea of where you stand. Helping someone else always gives a better feeling. It gives you confidence in your own ability. It gives you the ability to understand or analyze yourself and where you stand compared to your partner. (S4 Interview, June 26, 2015)

Moreover, some students gained confidence not only in their mathematical knowledge but also in their ability to teach mathematics. Student S5 said she developed an interest in tutoring and if she could not find a job in her field, she would “want to return to college or university to learn more math and become a math teacher”. She explained, “I feel like I know more about Math 1 and I can teach it now” (Interview, June 25, 2015). After coming to the workshops, student S12 felt confident to “accomplish more things at the college, getting involved in other programs like tutoring and signing up to be a tutor” (Interview, June 26, 2015). Finally, Student S4 praised about his ability to teach another student in mathematics:

I could make the person sitting with me understand and it gives me a lot of confidence that I can teach people. I can teach mathematics now. I feel like whatever I know I can teach and transfer what’s in my mind to someone else. (S4 Interview, June 26, 2015).

4.6 Themes of Other Affective Impacts

Although, there were no quantitative measures on other affective impacts of Reciprocal Partnership, the semi-structured interviews were able to elicit valuable perceptions of the students on how Reciprocal Partnership supported their affective growth. This included increased motivation to exert effort, closer connection with peers, and a sense of comfort in learning from the positive environment.

4.6.1 Motivated to Exert Effort

Beyond increased confidence, students exerted greater effort in solving the mathematical problems. Student S1 reflected, “[The workshops] definitely open my eyes
that nothing is impossible. I just have to put in more effort and at the end of the day it is paying off” (Interview, June 26, 2015). He mentioned further about the change in his attitude when he approached difficult questions:

[The workshops] open my ways of thinking because before when I couldn’t do a questions, I would just skip to the next question. But now I actually take the time to dissect it and then try to think about the ways that I can solve it. Once I actually solve it, it feels good. (S1 Interview, June 26, 2015)

The sense of responsibility to teach their partner encouraged the students to exert effort to study or to prepare questions for the workshops. Student S5 provided an example: “Before, I just leave out the finance questions. Now, I have to solve the question so that I can teach someone” (Interview, June 25, 2015). His partner also “put in more effort and initiatives towards math” than last semester in which he failed the course (F6 Interview, June 22, 2015).

Another pair of students also talked about their motivation. For example, student F8 generated the questions for the workshops diligently and treated it as an assignment because he knew that his partner would be prepared for the workshops. He said, “I would write one question down that I need help on and one question that I can help others on. I did it for each day so that I was ready, my partner was ready and we can be on the same page” (Interview, June 25, 2015). His partner also mentioned about spending extra time studying:

I don’t normally study… [The workshops] make me spend extra couple of hours on math...This is what I took from this program that for math you actually have to do the questions and spend time on it. (S7 Interview, June 19, 2015)

In addition, student S4 said he was motivated to work with the mathematical problems because he was having fun with his partner, “We tease each other like “you can’t do this or I can do that”. It made it more enjoyable. We were having fun, while
doing math” (Interview, June 26, 2015). Student S1 also stated, “Like you have to help your partner so you don’t want to look bad. It gives you an extra push to actually do some of the questions where when a teacher gives it to you as homework you probably wouldn’t do it” (S1 Interview, June 26, 2015). His partner was also motivated to solve the mathematical questions because learning was like playing a game:

>We were checking who’s right and who’s wrong. I was doing it like it was a game…Every time when you are doing a test, it’s like life and death. But here, it just feels like I’m having fun with my partner. (F2 Interview, June 22, 2015)

Student F11 described how it became more interesting and fun when he worked with a group of students:

>We all work together. We compare and figure out the problem. It makes math more fun when there are more people…It makes the situation more interesting in a group-oriented environment. You don’t get feedbacks from just one person. You get feedback from a whole group of people. I think that is very helpful for a person to become a better learner in what they are doing…This was fun for me. (F11 Interview, June 22, 2015)

His partner mentioned further about how the workshops “got students more interested in actually coming and doing more questions” and remarked on his own motivation:

>Personally, once I go home I can’t open my books. My bags stayed at the front of the door until the next morning so I have to study here in this workshop… I feel like this program really got you to push your fence a little harder…Now when I’m alone at home, I am more motivated to open my books and do some math questions. This workshop helped me a lot to learn the little pieces that I have missed in Math 1. It got me more involved in doing math so I was more active in my Math 2 course solving questions. (S12 Interview, June 26, 2015)

4.6.2 Closer Connection with Peers

All of the students described the relationships with their partner positively and acknowledged their partner’s kindness and friendliness. Student S5 praised her partner as “kind and nice”. She continued, “When I was tutoring him, he was very patient. That’s really important. He helped me a lot with my speaking and he accepted my accents”
Likewise, her partner labelled her as “a great, smart, very helpful person” and remarked, “We get along. She knows what she’s doing and what she is saying. Interacting with her comes easily. She has been very helpful in this workshop. I’m glad she is my partner. We are friends” (F6 Interview, June 22, 2015).

Student F9 described his partner as follows: “He is a friendly guy. He is a good guy to interact with” (F9 Interview, June 22, 2015). He also mentioned about how they mutually respected one another:

When we interacted, we both have respects for each other. He knew stuffs that I didn’t know but he wasn’t all like “oh, I know more than you”. That really helps when somebody is humble like that. (F9 Interview, June 25, 2015)

Student F2 insisted, “It is important to teach one another patiently and sensitively like it’s a favour for a favour” (Interview, June 22, 2015). He continued, “If I know something by heart, I would dumb it down for a person and my partner took the time and opportunity to dumb it down for me too” (F2 Interview, June 22, 2015). His partner described their learning as “teamwork” especially with other students in the workshops:

I also enjoy the aspect where it is like a collective group. When we solved the question together, everyone had their own takes to it. Everyone was putting in their own ideas and I saw teamwork. It was definitely a good thing to have. (S1 Interview, June 26, 2015).

Students illustrated the means through which they built their partnerships. Student F8 commented about his partner, “I like my partner. He is very good. He shows up on time. When he couldn’t make it, he notifies me and you” (Interview, June 25, 2015). He asserted, “My partner accepted my knowledge and I accepted his knowledge. That’s how I think the partnership should works” (F8 Interview, June 25, 2015). His partner described him as “a nice guy and very easy going” (S7 Interview, June 19, 2015).
Furthermore, friendships were established not only between the partners but also among other students in the workshops. Many students acknowledged the opportunities to meet other people, work with one another outside of the workshops, and talk with each other in the hallways. Student S12 said, “I got to meet a lot of students. They understand my character” (S12 Interview, June 26, 2015). Student S4 gave similar example: “I meet other people. They are very friendly. When I borrow a textbook from them, they are very humble and kind. Maybe next time, I will make them my partner” (Interview, June 26, 2015). He talked about the friendship between him and his partner, “We were friends before but in this workshop we can build up our knowledge together. I guess our friendship grew and we get to know each other more” (S4 Interview, June 22, 2015).

Student S7 made many friends and was greeted by them in the hallways. He said, “I made new friends. Wherever I go around the college, everyone was “Hey!”” (Interview, June 26, 2015). His partner had the same social connection with other students in the workshops: “You don’t just get help in math, you get time to hangout. We would do math problems and we would talk. It was like a study session that you get to hang out with people. You get to mingle and do work at the same time” (F8 Interview, June 25, 2015).

Student S5 compared the friendly environment of the workshops with the unsociable setting of the classroom. She proclaimed, “I can make so many friends. If you are in class, you just go to class and after that everyone is like strangers and just leaves. But in here, you can make friends” (Interview, June 25, 2015). His partner mentioned about gathering with her and other students to work on the mathematical questions after the workshops: “We get together. We were doing questions as a group. Like on Friday,
we did questions together. It was a big achievement for me to interact with other people and do the questions” (F6 Interview, June 22, 2015). Student F11 commented on the enthusiasm of his partner to teach him:

He is really motivated and enthusiastic about teaching and helping people. That was very encouraging for me to see. He makes sure that he explains the problems in every little details for me to understand. (F11 Interview, June 22, 2015)

His partner annotated the close conversation with him after they saw each other in the hallway:

You actually have someone that you can become friends with. I had a really friendly relationship with my partner. Even if he sees me in the hallway, he can just ask me a quick question and I’m always there to help him. I have seen him around the campus a few times. We actually got a drink together. I told him about how it is hard coming from high school to the college environment where you are more independent and you have to take care of yourself. I explained all that to him and he really liked it… I also told him about other courses and programs, recommending to him which ones he should take and which ones he shouldn’t take. (S12 Interview, June 26, 2015)

4.6.3 Sense of Comfort from Positive Learning Environment

Students felt comfortable to learn in the workshops because of the friendly and positive learning environment. A student simply said, “It is very welcoming. I feel more comfortable when I come to the math workshops” (S5 Interview, June 25, 2015). Her partner made the same contention:

I learned not just from my partner but from other partners as well. We actually shared ideas on how to solve problems. It was a great friendly environment for learning and working with people… Even when you and your partner had a problem there were other people you could go to get help. People were there to help me. People were good to me and I was good to them… It was just very comfortable. (F6 Interview, June 22, 2015)

Learning was less intense in the workshops because the environment made the students less afraid to take risks and make mistakes. For example, Student S10 remarked, “My partner and the other people are here to learn so we don’t mind making mistakes.
We feel comfortable with each other. People here don’t mind if you make mistakes. It is a good learning environment” (Interview, June 19, 2015). In addition, student F11 felt comfortable making mistakes in the workshops because students were accepting one another’s errors:

It is a fun environment even if you mess up, everyone laughs and they are like, “don’t worry, we will figure it out together”. Again, it goes back to the whole positive environment. Everyone is on the same page here. Everyone here is trying to help each other out. That’s what I like about it. (F11 Interview, June 22, 2015)

Similarly, his partner described the learning environment as less intense because students in the workshops were friendly:

It’s like the environment is different here. There is less pressure, it’s less intense, you are calmer, your brain is able to comprehend everything in a different manner, and you are not trying to rush everything in your head…You are able to ask your questions as much as you want, and you are able to work with other students. The environment is like math friendly because when you come in here, everyone is just doing math. (S12 Interview, June 26, 2015)

Furthermore, student S7 said, “The workshop allows you to partner with someone who you can develop a bond or become acquainted with so I feel comfortable to make mistakes”, and “there are lots of students who are willing to help and to make you feel comfortable” (Interview, June 19, 2015). His partner felt comfortable learning collectively with the other people in the workshops:

It was comfortable and the setting was good. Everybody did their own thing and when more than 2 people needed help, we got to a big group and we work together on the board. It was very supportive and it had a very good vibe. You get the support of your partner and the rest of the people in the program. (F8 Interview, June 25, 2015)

Student F8 continued to mention about feeling less “dumb” in the workshops because he was comfortable with everyone around him: “Coming here, feeling comfortable with everybody and getting the support from them, I don’t feel so stupid because I find that
other people are also not getting the stuff that I’m not getting” (Interview, June 25, 2015). Similarly, student F2 remarked, “If you are working alone, you feel dumb. If you are working with others, you feel dumb together but happy” (Interview, June 22, 2015).

4.7 Themes of Mathematics Achievements

When asked about the mathematical achievements that they gained from participating in the workshops, only two students mentioned about higher grades. However, all of the students described their achievement in terms of increased mathematical knowledge and improved mathematical skills. They provided details of their learning and analysis of these details revealed four emerging themes on how they gained mathematical knowledge and skills: solve problems with partner, see different mathematical approaches, and reciprocally tutor one another.

4.7.1 Solve Problems with Partner

By solving the questions raised by the students themselves during the workshops, the students developed mathematical understanding. One student pointed out, “I actually understand what is happening and I learn more things through this workshop than I ever have before” (S1 Interview, June 26, 2015). Another student emphasized on the mathematical knowledge that he would not have otherwise gained if he had not participated in the workshops: “I got a better understanding of the problems than I would’ve just going to class. The biggest achievement was those linear and algebra problems. In that it was worth more to me than anything else- the understanding of those questions” (F8 Interview, June 25, 2015).

Student F6 remarked on the importance of problem solving, “Everyone has to learn by solving by themselves to get the practice, and to get to know how to do it. You
just practice until you get it right. If it’s wrong, it’s wrong until you get the right answer” (Interview, June 22, 2015). One student pointed out how he developed understanding by persistently solving the mathematical problems:

I’m getting it. I’m finally getting it. I’m not perfect with it but I understand a lot more than I have previously…After solving it, this constant way of never giving up,…I feel more comfortable answering the problems instead of just sitting there starring at the question for 5 minutes and not giving you an answer. (F11 Interview, June 22, 2015)

Similarly, student S4 described how problem solving allowed him to practice the questions as a journey to learn new things:

We tried 5 different things and 5 different methods to solve the question. All of them were wrong and then the sixth one was right but all those five attempts helped us practice the other things that we learned before. It’s like a journey on your way to the answer…You learned many new things along the way. (S4 Interview, June 26, 2015)

Student F2 also depicted how he developed understanding from solving mathematical problems through perseverance: “After the first try, then the second, and on the third try, you can finally understand. It’s practicing to solve it” (Interview, June 22, 2015). In the words of student F9, “once you solve it step by step, you learn how to do it” (Interview, June 22, 2015), and his partner said the same, “once you practice more often, you get more natural at it” (S10 Interview, June 19, 2015).

Furthermore, students insisted on the opportunity to solve the questions first before having the solutions presented to them. Student F6 explained, “When everyone is trying to dissect the question, I find it more interesting than just the teacher giving the answer. I like seeing how people think. I think it is cool that everyone is trying different methods and eventually one works up” (Interview, June 22, 2015). Student S1 also
detailed on how he and his partner learned from solving the problem when the solution was not given to them:

When we solve a question, we try to formulate an answer and then see what we did wrong before going back to the actual [solution]. I like this process more than just having a solution given to me…because giving us the answer doesn’t really teach us. It just shows us the patterns. We just memorize it but we have never understood it and we can’t really solve it because we never really know how to break down the questions...But in this workshop, we actually see how to break down the question, and start from scratch on how to make our own equations. I understand it more from solving the question than just having a solution given to me... It helps us because it gives us the independence to tackle the question ourselves first. (S1 Interview, June 26, 2015)

His partner agreed with him saying, “I like the part where you let us solve the questions first. It gives us the time to think and when we can’t figure out the questions ourselves, we can ask you as our last resort” (F2 Interview, June 22, 2015).

Student F9 stressed the importance of finding the solution himself by solving the problem: “If you give us the answers, it might not teach us how to do it or how to get the final answer. It is a negative impact because we don’t really learn anything” (Interview, June 22, 2015). His partner elaborated on this viewpoint: “Sometimes when you do the question again you get something different because you didn’t understand how to solve it in the first place. Solving the problem is important because you will learn to work with the question” (S10 Interview, June 19, 2015).

Student F8 pointed out the need for active participation in his own learning, “Because when you do it yourself, you feel like you have accomplished more than just sitting there and being shown how to do it” (Interview, June 25, 2015). He also appreciated his partner for allowing him to solve the problems before offering help. “The way my partner did it was really helpful. He let me solve those questions first. In that way, it helps me because I can practice applying it” (F8 Interview, June 25, 2015). On the
same note, his partner enjoyed solving problems with him and described the process as “solving a puzzle that [they] can work together trying to think of the possible answers, and looking for hints and glues” (S7 Interview, June 19, 2015).

Moreover, during problem solving, students emphasized the need to work with another student. According to them, solving problems with another student produced better understanding of the questions because there were more exchange of perspectives, constant interactions and immediate encouragement. Student F2 described it as “collaborating our mind sets together. When one of us doesn’t know, then we help each other” (Interview, June 22, 2015). Similarly, his partner said, “As teamwork, we were able to solve questions that might not be possible if it was just me” (S1 Interview, June 26, 2015).

Student S4 preferred not to study mathematics alone: “I used to think that I need to practice alone with math. I always study alone but now I think that if I study with someone else, it’s surely going to help me. Sometimes you just don’t seem to get it even if the answer is right in front of you. When there is someone to interact with you, it kind of helps out” (Interview, June 26, 2015). He commented on how working with his partner allowed him to stay focused: “When I try the questions by myself and don’t get the answer, it becomes difficult for me to concentrate because there are thousands of different things to do. But when someone who is also doing it with me, he brings me back to focus” (S4 Interview, June 26, 2015). His partner felt bonded with him when they solve the problems together, “Each person comes up with an idea. You are bonded to each other” (F3 Interview, June 22, 2015).
Student S1 found it more interesting to work with his partner because they “have to use both of [their] minds trying to understand what’s happening. It’s interesting to have that student-student interaction instead of having a professor or tutor trying to give it to you” (Interview, June 26, 2015). He continued to explain, “The interaction was back and forth. We were able to help each other out. It helps me understand because when you actually talk to someone, it slips in the mind better than just doing the question” (S1 Interview, June 26, 2015). However, “when you are doing it by yourself and you can’t do a question, you have to wait until the next day to see a teacher or tutor. But when you are working with a group of friends and everyone has their own ideas, you can see how they think and get help from them” (S1 Interview, June 26, 2015). His partner agreed with his point of view: “If I encountered some difficult questions on my own, of course I feel intimidated. When I work with others, it does help me to understand more. It’s getting that encouragement” (F2, Interview, June 22, 2015).

Student F9 thought it was important to solve problems with a partner because you can learn from one another and get immediate feedbacks. He commented, “You get a partner to interact with and someone who can help you with the questions. We go through the equations together. It helped me improved my overall math skills” (Interview, June 22, 2015). He also said when he solved an equation by working together with his partner, they “actually get more out of it because [they] can immediately correct each other’s mistakes” (F9 Interview, June 22, 2015). Likewise, student F11 argued, “Even if you know things, you can learn more from another person” (Interview, June 22, 2015).
4.7.2 See Different Mathematical Approaches

Students contended that seeing different mathematical approaches increased their mathematical knowledge. Comparing solution with a partner is “an opportunity to help you understand different ways of approaching the question from your usual ways of thinking” (S1 Interview, June 26, 2015). For example, student S1 found it interesting to learn alternative methods from his partner: “I like the aspects that he was solving them in a sense that I have never seen it before. It shows that there are different ways of going at a question. I find that interesting” (S1 Interview, June 26, 2015).

Students commented further on how they learned simpler approaches to solve the mathematical problems: “There are many ways to do the questions but there are also easy ways. I learned from my partner to find easy ways” (F6 Interview, June 22, 2015).

Student F11 elaborated on his experience:

There were shorter routes that I could take to complete the problems than the ways I was taught previously when I was in high school and elementary school. He showed me those ways and I was very grateful for him to show them to me because it will help me in the long term and helped me finish the problem quicker now than previously. (F11 Interview, June 22, 2015)

Student F3 and his partner (student S4) described how they taught each other an alternative method. Student F4 expressed, “My partner showed me a different way that I have never seen it before. It was kind of interesting to see that he took a different approach and yet we ended up with the same answer” (Interview, June 26, 2015).

Similarly, student F3 commented, “If you have two solutions to one problem, you will gain an extra way to solve the problem by learning one another’s way” (F3 Interview, June 22, 2015). He continued with an example on how he learned an alternative method from student S4:
He taught me something that I couldn’t have learned in class. There are things that if you don’t think about it you couldn’t solve the question. Like, there is one question, he showed me how to simplify an equation by factoring out the GCF [greatest common factor]. I never did this and this is one of thing that I never think about. (F3 Interview, June 22, 2015)

Furthermore, students learned to find their own mistakes by comparing approaches with their partners. Student 10 stated, “Comparing solutions is like a way to check your answers. If your answer is different from your partner, you can check who made the mistake” (Interview, June 19, 2015). He was dumbfounded by the vast amount of possible approaches to one question: “I always thought there is only one way to do math but when someone is doing a way that is different from what you are doing, you can see that there are many other ways to do the same question…Yes, much more than I thought” (S10 Interview, June 19, 2015).

Student S5 also appreciated the variety of solution methods to one question and commented, “We can see different ways to solve problems. I think that’s good because there are so many ways to do one question” (Interview, June 25, 2015). His partner, student F6, stressed, “The fact that at the beginning I didn’t know anything but now I learn how to do the questions in different ways, I feel that was an achievement”, and continued with the following remark:

There are different ways of doing the questions depending on the questions that you are dealing with. There’s different ways to solve it. She has her own way of solving it and I have my own ways of solving it. That’s where comparing solutions comes in…Comparing solutions helps because we go step-by-step. We ask each other how to do the question in each step of the way. (F6 Interview, June 22, 2015)

Student S7 viewed comparing solution methods as a means to detect his own mistakes and said, “I like comparing solutions because we could look for the mistakes that we made. It’s an important aspect of mathematics to find out where you made the
mistakes. Especially, when you are doing the equation, you need to know where you went wrong” (Interview, June 19, 2015). His partner also noticed the difference between finding his own mistakes compared to someone pointing it out to him:

In comparing solutions, you get your answer checked. It makes me feel a lot better about myself because in tutoring if I don’t get the right answer then the tutor will describe to me how to get that answer…In comparing solutions, you find your own mistakes. (F8 Interview, June 25, 2015)

4.7.3 Tutor One Another Reciprocally

Students were gaining knowledge, while tutoring their partner. In the words of student F8, “When I teach it to somebody else, it makes me remember what I know. So when I’m teaching something to the others, it just brings back memories of doing it before.” (Interview, June 25, 2015).

The teaching experience was viewed by students in the second semester as reinforcement of their own understanding. Student S5 said it precisely, “It is like a review of Math 1 at the same time I’m learning math 2” (Interview, June 25, 2015). The same remark was found in another student’s response: “Although, I helped him most of the time but I don’t mind because it is a good review for me” (S10 Interview, June 19, 2015).

Student S1 explained, “You can demonstrate what you know. It helps you understand what you are doing because you are trying to explain it. It sticks in your mind” (S1 Interview, June 26, 2015). By helping his partner on the mathematical questions, student S7 consolidated his understanding of Math 1:

Most of the time I helped him but by helping him, it is like reviewing the materials… Doing those questions again helps me to retain them. It’s a good practice to work with the questions for the second time. When he helped me on that one question, I got to learn what I didn’t learn before. (S7 Interview, June 19, 2015)
Although students in the second year predominately enacted the role as the tutor, there were opportunities when they learned from students who were in the first semester. These opportunities were created because students in the second semester did not fully master or comprehend all of the materials in Math 1. For example, student S1 explained, “Because every teacher teaches differently, I didn’t know some of the materials that he knows. He taught me degrees, minutes and seconds. I learned that from him instead” (Interview, June 26, 2015). Student S10 gave a similar example: “In the first semester, I learned a lot but there were a lot that I didn’t understand very well like graphing and algebra…I am confident in trigonometry so I can teach him trigonometry and he teaches me graphing” (Interview, June 19, 2015). He stressed, “Now I can solve the question in class with what I have learned from my partner. It’s good because when I’m helping him, I’m studying at the same time” (Interview, June 19, 2015). Student S7 also remarked, “Sometimes, as a Math 2 student, I don’t remember everything or learn everything in Math 1. It’s good to be able to review those concepts again. (Interview, June 19, 2015).

According to student S12, he had fully mastered Math 1 by tutoring his partner on questions that he did not learn before:

I feel like I know everything about Math 1 more. Even though I passed it with a 90%, there were times during the workshops that I was like, “Wait! Did I really learn this in Math 1?”… For example, we were doing factoring and I never did long division. So coming here and helping him out with long division, I actually learned something…Right now, I’m at the point where I feel like I know everything about Math 1 and have accomplished Math 1 to the fullest extent to be a tutor. That’s a math achievement. (S12 Interview, June 26, 2015)

Similar remark was found in the response of student S1:

Helping someone who is in a lower grade, it’s going to be more of you tutoring them. I guess it is more of me being able to help someone. But, there were many times when he actually showed me things that I didn’t know. I liked that because it helped me understood even more of Math 1 that I missed before. It gave me
more knowledge of Math 1. It showed that just because you are at higher math
doesn’t mean you know everything. (S1 Interview, June 26, 2015)

Students in the first semester remembered or understood the mathematical
problems better when it was explained to them by their partner. For example, student F2
said, “Every time when he explained something to me, it would just click in my head and
I would not forget it. It was a good experience and I enjoyed learning from him”
(Interview, June 22, 2015). Student F6 commented further on how he understood his
partner who was very patient with him: “When she is teaching me, I understand a lot.
When I do that question again, the same question that she have explained to me, I know
what to expect from it and know what to do with it. It is a good feeling” (F6 Interview,
June 22, 2015).

Student F11 learned to solve the mixture problems by providing his partners
explanations to the questions, and said, “It helps me to understand the mixture problems
better because I actually did it and explained it to someone instead of someone showing it
to me. That helped me a lot” (Interview, June 22, 2015). On the other hand, his partner
noticed how they understood each other’s teaching better compared to listening to a
lecture: “When I explained to him how to get to a solution from my way and his
professor’s way, he tended to understand my way better. Not that it was better, it was the
same thing just that it was more clear” (S12 Interview, June 26, 2015).

Moreover, students found the opportunity to reciprocate the help from their
partner added value to their learning. Student F3 believed that reciprocally tutoring one
another increased retention: “When you teach someone and they teach you back, it goes
in your head and it stays” (F3 Interview, June 22, 2015). Student S10 thought it prevented
problems between him and his partner: “We didn’t have much problem interacting with
each other. I guess it’s because I could help him and he could help me when I’m not quite sure of how to do the questions” (Interview, June 19, 2015).

Other students voiced their preference for the opportunity to reciprocally enact the role of a tutor and tutee. Student F8 enjoyed the opportunity to also teach his partner:

“When you are getting tutored, they are showing you how to do it but I like it when I get to show how to do it too” (F8 Interview, June 25, 2015). On the other hand, student S1 found it beneficial when his partner was able to return the teaching: “It’s also good when your partner also teaches you because it shows that you don’t know everything. That’s beneficial because he shows me stuffs that I have never learned before” (S1 Interview, June 26, 2015). Student S10 described reciprocal teaching as “more interesting”:

I prefer the back and forth teaching because you both get some benefits from this program. It’s not like only one person is teaching. It can get boring sometimes because if the other person is not interested than it is like trying to teach someone who doesn’t want to learn. The reciprocal teaching and learning is better because there are more interactions and it’s more interesting. (S10 Interview, June 19, 2015)

Likewise, student S12 viewed the opportunity to reciprocate the tutoring as motivating:

There’s a need for a balance of sharing the learning objectives. If I’m constantly coming here, and I’m just tutoring him and unable to ask any question for myself, I might be like “what am I doing here?” Since it was reciprocal for both sides, it was great because I felt like when I came here to tutor him, I was actually learning. It was motivating. (S12 Interview, June 26, 2015)

His partner detailed on why he also preferred the reciprocal relationship:

What I like about the reciprocal relationship is that you don’t feel insignificant. What I mean insignificant, I mean like when you go to a tutorial, this person is the absolute teacher. You listen to everything he says. He knows what he is talking about and he is never wrong. You can never question his authority over you. He is the tutor. This workshop was different because as smart as he was, I knew my partner made mistakes. Even when he was showing me the questions, I questioned certain things that he did. I let him knows that he made mistakes and he was like “oh, you are right, I didn’t know”. Even being in Math 2 and he knew a little bit more than me, he respected when I helped him and corrected his mistakes. It was
like mutual understanding. There was respect from both sides. We could help each other out. It didn’t matter if one knew a little more. “I make mistakes, I need help” – that was the feeling that I got from him. (F11 Interview, June 22, 2015)

4.8 Themes of Structure of Workshop

Asides from mathematics achievement and self-efficacy, students commented on the structure of the workshops. The structure of the workshops required the students to prepare two sets of questions on Math 1: questions on which they need help and on which they can help their partner. They took turns asking the two sets of questions in a reciprocal fashion to engage in the three learning activities: tutoring, comparing solutions and problem solving.

4.8.1 Preference for One of the Learning Situations

When asked which learning situation they preferred more in terms of their learning gains, the students provided the following reasons for their choice. Student S4 chose problem solving (exploratory situation), saying “I think problem solving is more helpful because when two minds come to the same problem it takes seconds to get the answer” (Interview, June 26, 2015). Other students preferred the exploratory situation for the same reasons why they think problem solving helped them to improve their mathematical knowledge and skills. For example, student F6 thought problem solving was more helpful because “problem solving is like practicing” (Interview, June 22, 2015). Student S10 explained, “Solving the problem is important because you will learn to work with the question” (Interview, June 19, 2015). Student F2 preferred problem solving because it allowed him to try the question first. This is the same reason why student S1 preferred the exploratory situation:

Definitely problem solving was the most helpful because when someone else tutors you, you may not understand it. When you do it, you will understand it.
much better… Because when you tried yourselves, you understood which way to go with the question. (S1 Interview, June 26, 2015)

Only one student preferred tutoring (explanatory situation) over the other learning activities. He gave the following reason:

I rarely need help on something. Those ones when I needed help on are more helpful so I like tutoring more…For my learning, I prefer the situation when I need help on like being tutored. (F3 Interview, June 22, 2015)

Student S5 preferred comparing solutions (extensional situation) because she liked to learn different approaches to a question; “I prefer comparing solutions because we can learn different ways to solve problems…I think comparing solution is helpful for me to know and understand the question” (Interview, June 25, 2015). In addition, student F9 found analyzing the solution methods step-by-step was very helping: “Comparing the solutions was the best. We actually sat down and we actually compared answers…When you compare steps to see how you both get the same answer, it helps you learn better and helps you understand the equations better because you can always ask your partner how to do it step-by-step” (Interview, June 22, 2015). Student F8 preferred comparing solution for a different reason. He liked to work through the question at his own pace:

I really like comparing solution more because you can do it yourself and get the answer checked…I like to work at my own pace. When we did the problem solving together, my partner will be working faster than I am and I work slower because I want to do the question right. When I’m rush, I don’t get the question right that’s why I like comparing solutions. With comparing solution, I felt like I did it myself even if I got the wrong answers. (F8 Interview, June 25, 2015)

Student F11, S7 and S12 had no preference for any one of the learning situations over the other two. Student F11 claimed, “All of them are equally helpful to me. I don’t think one is better than the other. They all equally helped me to improve my learning in math…I don’t see any situation better than the other. I see them all equal” (Interview,
Student S7 simply said, “I don’t find any one of them more helpful than the other” (Interview, June 19, 2015), and student S12 exclaimed, “I think it was perfect the way it was. I was able to solve problems, compare solutions as well as tutoring someone at the same time” (Interview, June 26, 2015).

4.8.2 Challenges for Following the Structure of Workshops

Students revealed that they had difficulties scheduling time to come to the workshops and creating questions to ask their partners. Some students could not attend few workshops due to their tight schedule and heavy workload but managed to make up for most missed workshops. Student F2 said it concisely, “It is just time management. We have other courses that we have to focus on” (Interview, June 22, 2015). His partner was faced with the same obstacle, saying “It is kind of hard because of the tests and assignments that were coming up” (S1 Interview, June 26, 2015).

One student described the awkward situation when his partner could not attend the workshops: “If your partner doesn’t show up, you are practically alone or practically solving questions by yourself. I guess it is very partner-dependent” (F9 Interview, June 22, 2015). Student F8 explained, “It is every morning. We wake up and struggle to get up. There is traffic in the morning and it does affect one of us” (Interview, June 25, 2015). He continued to elaborate on the situation when his partner was occupied with priorities from other courses:

I think one problem is when your partner can’t make it here and has other priorities. My partner was popping in and out of the room. It was not the same as him sitting down and having the time to do this. I understand that things get in the way especially with college, you are trying to do your assignments on time and passing the grade. (F8 Interview, June 25, 2015)
Student 12 provided a suggestion to solve the conflict between the class timetables and the schedule of the workshops:

For the first few sections, I was here and I was always on-time but then I slacked off a bit and the reason was because I got really busy. Also, I had four hours of class before the workshops and after the fifth hour I was starving. It would be nice if the workshops were a little bit more flexibility on the scheduled time where you could just come here, sign in and start. (S12 Interview, June 26, 2015)

His partner suggested to pair different partners when needed.

My only issue is when you don’t have a partner and your partner is busy. He was sometimes here and he was sometimes not. There was some inconsistency but I can understand because he was really busy and he had a lot of things going on…If there are people who don’t have partners coming in and if there are not enough partners to go around, I think we should group people with other partner. (F11 Interview, June 22, 2015)

Students also faced the challenge of creating questions for the workshops because their course schedule is different from their partner. Due to the discordant course schedules, students in the second semester did not know which questions were related to the topics that their partner was on in Math 1. Student S1 commented, “You just don’t know which questions you should do. Should you do all the questions in one chapter or should it be different? Also, I can’t determine which questions are related to what my partner is doing in Math 1” (Interview, June 26, 2015). Student F9 had similar experiences:

My partner gave me some questions but he mostly took them from the textbook. I guess he was in a different course, so he was trying to go down to my level. It was hard for him to ask questions related to other topics like geometry because he didn’t learned it so he did mostly mixture problems...We needed to find questions that are related to the math course. If they teach us a topic in the course, we should be doing it in the workshops so that way we can cover what I learned from the class. (F9 Interview, June 22, 2015)

Other students had difficulties creating the questions because they simply did not have a question to ask and suggested to have some questions provided for them. Student
F3 remarked, “Finding questions that I need help on is a challenge. Most of the time, I don’t have any questions. I think the questions should be pre-set and divided by types…If you have all types of questions, you can orientate yourself to what you know and what you don’t know” (Interview, June 22, 2015).

Student F8 agreed with this viewpoint: “Sometimes I didn’t know what to do or what to ask or what to put down. Only if the questions are prepared for us because sometimes we can’t come up with all six questions” (Interview, June 25, 2015). His partner elaborated on the difficulty to coordinate with each other when both courses were on different topics: “It was hard to coordinate with your partner if the courses were not on the same topic. It was like when we were learning the geometry unit in 3D and he was learning it in 2D or when we hadn’t started the trigonometry and he was already finished with the unit” (S7 Interview, June 19, 2015).

Another reason why the partners found it difficult to coordinate with each other was because Math 1 was a compressed course. Student S12 gave the precise reason, “When I took my math course, it was a whole semester long but his was only two months. It was hard for me to see where he was at in his class and what materials he was learning” (Interview, June 26, 2015). Student S7 echoed, “My partner was in a compressed program and sometimes I felt like he was going very fast. We were trying to do one unit while the course already covered few units” (Interview, June 19, 2015).

For students who were in the first semester, the fast-track schedule of Math 1 made it difficult for them to coordinate the pace of the workshops with the course. Student F6 said, “The challenge was that this semester was fast pace and fast track. It was hard for me to catch up and go with the pace of the course in the workshops” (Interview,
June 22, 2015). Student F9 contended: “It was a compressed course and went through all topics quickly. It was hard to understand and hard to look back on how well you did. Even if you did, you always have to go forward because the class kept moving forward. So it made it hard studying in the workshops” (Interview, June 22, 2015).

Although some students had difficulties scheduling time for the workshops, they arranged with their partner to make up for their missed attendances. Similarly, despite the difficulty that few students had in preparing their questions before the workshops, they were able to create the questions while they were working with their partner during the workshops. Nevertheless, students made valuable suggestions to overcome the above challenges for the intention to effectively implement the workshops.
Chapter 5: Discussion and Interpretation of Findings

5.1 Introduction

In this study, Reciprocal Partnership holds promises for meeting the complex academic and social needs of college students in their mathematical learning. It creates a positive learning environment enriched with social interactions and constructive learning to support the learning needs of students. It also extends the classroom learning experience, and maximizes the learning benefits of collaborative learning and peer tutoring to help students overcome difficult course materials. Much of the students’ reasoning for gains in mathematics achievement and self-efficacy resemble many reported benefits of peer tutoring and collaborative learning.

Qualitative data suggest a number of positive effects of the Reciprocal Partnership workshops. These effects include the focuses of this study: mathematics self-efficacy and achievement. On the other hand, inferential statistics reveal insignificant difference between the final examination grades of students who participate in the workshops and the previous class averages. There is also insignificant gain in mathematics self-efficacy of students in the second semester compared to their control group. Explanations for the discrepancies between the qualitative and quantitative are embedded in the following discussions.

5.2 Research Question One

What effects, if any, does Reciprocal Partnership have on the mathematics self-efficacy of first and second-semester college students? Quantitative data gathered from this question reveal only significant increase in mathematics self-efficacy for students in the first semester compared to their control group. However, increased confidence is
reported in all students’ interview responses. This qualitative finding is consistent with those of earlier studies investigating on the use of collaborative learning and peer tutoring. Collaborative learning is reported to promote greater self-efficacy (Poellhuber et al., 2008), and peer tutoring is corroborated to increase the comparative personal efficacy (Van Keer & Verhaeghe, 2005). Consequently, opportunities created for collaborative learning and peer tutoring in the Reciprocal Partnership workshops have fuelled the gains in students’ mathematics self-efficacy. As reported in students' responses, the sources for increased mathematics self-efficacy are from extra practice, solving difficult questions, and teaching another student.

When students have extra practice on the questions, they are more likely to have higher self-efficacy in that area. Reciprocal Partnership workshops afford students the opportunity to practice the questions on which they need help or can help others. For students in the first semester, they are practicing their newly acquired knowledge from Math 1, and for students in the second semester, they are revisiting Math 1 as a reinforcement of their understanding. These extra practices are speculated to help students develop greater control in solving mathematical problems that ultimately lead to higher self-efficacy. This is supported by students’ responses and is revealed in descriptive statistics where all of the students have rated a higher if not the same level of mathematics self-efficacy in their post-survey.

When students mutually help one another to solve mathematical problems in the Reciprocal Partnership workshops, they are more likely to take risks and tackle the harder questions as in collaborative learning. Many of the students find themselves capable of solving questions that they cannot solve before. By solving the difficult questions,
students develop a sense of self-efficacy to meet higher expectations which is parallel with descriptive statistics. According to descriptive statistics, students who have the most increases in confidence are those who initially have the lowest self-efficacy. In other words, students, who originally perceive themselves as incapable of performing the mathematical tasks, have the highest increase in their confidence when they overcome difficulties of the tasks.

When students are afforded the tutoring experience, their self-perceived competency is elevated (Van Keer & Verhaeghe, 2005), and the experience of tutoring another student is an apt context to empower students (Menesses & Gresham, 2009). This may explain why students in the first semester have the greatest boast in confidence in the few occasions when they tutor their partner who is in the second semester. This is also in line with descriptive statistics which show that first-semester students have a higher median of gain in mathematics self-efficacy than second-semester students.

The tutelage opportunities empower not only students in the first semester to take ownership of their learning but also students in the second semester to find a job as a tutor. According to Tien and colleagues (2002), tutoring experience instill in students an interest to teach. For example, student S12 and S5 feel so confident about their knowledge in Math 1 after tutoring another student that they have applied for a tutoring position.

5.2.1 Additional Findings

It is not surprising that additional affective effects of Reciprocal Partnership are identified in students’ interview responses. Collaborative learning has been shown to create appreciation for ethical diversity (Cabrera et al., 2002), increase motivation and
improve performance (Johnson et al., 2007), and peer tutoring is supported to reduce attrition (Godfrey, 2008), and improve learning (Carmody & Wood, 2009). The post-experimental interviews draw out valuable students’ perspectives on affective impacts of Reciprocal Partnership that resemble benefits of collaborative learning and peer tutoring, such as increased motivation to exert effort, closer connection with peers, and a sense of comfort in their learning from the positive environment.

During the workshops, students are motivated to complete their assignments and to solve the mathematical problems. This is because students feel obliged to prepare for the tutelage. They go through textbooks to search for appropriate questions and refer to previous class notes to answer their partner’s questions. According to research, collaborative work increases individual’s engagement because members are responsible for the learning of one another (Johnson & Johnson, 2003). Moreover, learners are persistent in their task, have greater sense of competence, and display better personal control and intrinsic motivation when tutoring another learner (Carmody & Wood, 2009; Falchikov, 2001; Godfrey, 2008). Therefore, when the students learn for the intention to teach, they are more motivated to act on their attempts.

The students are motivated and engaged in solving the mathematical problems also because they are having fun collaborating with their partner and enjoy helping one another. For example, students tease each other, while comparing to see who gets the right answer, and they believe that at the end of the day, helping others brings good feelings. When group members receive help from one another, they are more confident and motivated to undertake educational endeavours (Storm & Storm, 2002).
In addition, the positive social effects of Reciprocal Partnership can be attributed to the social congruence between the partners. According to Schmidt and Moust (1995), social congruence occurs when tutors “seek an informal relationship with the tutee and display an attitude of personal interest and caring” (p. 709). This is evident among the partnerships as all students have described their partner positively. Social congruence is created because Reciprocal Partnership is structured to have students reciprocally tutor one another, thereby creating mutual benefit, equal status, and social support among the students. Students asserted that they bonded with their partner through mutual respect, encouragement, and acknowledgment of one another’s viewpoints.

In addition to social congruence, social connections are established not only among the partners but also among other students in the workshops. According to the students, the learning environment of the workshops is positive with everyone respecting and helping one another. In essence, they feel welcomed. Students also have a sense of belonging when they are in the workshops working with other students who are also struggling with mathematics. This is echoed in literature, “in the context of joint exploration with a partner, learner feels more like a fellow explorer than an isolated incompetent” (Damon & Phelps, 1989, p. 14).

Furthermore, under the mutual and social learning environment of the Reciprocal Partnership workshops, students feel comfortable about learning mathematics. They are less intimidated to make mistakes. Johnson, Johnson and Smith (2007) explain that collaborative group work encourages risk-taking behaviours and subsequently lead to reduced anxiety. When mathematics is learnt in a way where understanding supersedes getting the right answer, anxiety is reduced among the learners (Stuart, 2000). The safe
learning environment of the workshops reduces anxiety by encouraging exchange of ideas, acceptance of mistakes, and support of understanding.

In situation where mathematics anxiety is diminished, learners are empowered to construct their own solution methods and are encouraged to voice their mathematical ideas and perspectives (Finlayson, 2014). Mathematics anxiety can make learning of mathematics inaccessible to students, but the learning becomes plausible if students work under the positive influence of cooperative learning and engage in activities that develop understanding as in the three learning situations. Therefore, students have comfort in their learning when solving questions with their partner because they are less anxious to make mistakes and take risks.

Moreover, when comparing the changes in students’ mathematics self-efficacy with their achievement, five out of the six students who have the largest increase in their mathematics self-efficacy perform below the median of previous class averages on their final examination. This descriptive statistics only compare students’ grades with the control group but not with the students’ previous mathematics performance. It is equally important to question whether the increase in confidence has allowed students to achieve better than their usual mathematics performance rather than just how they perform compared to the class average. However, this question cannot be determined as data are not collected on students’ previous mathematics achievement to make the comparison. Perhaps, future research can examine students’ achievement in terms of comparison with their previous performance as well as with the norm.

Future research can also examine whether Reciprocal Partnership has greater effects on mathematics self-efficacy of low-achievers as descriptive statistics indicate that
students who perform lower than the previous class averages on the final examination have the most increase in mathematics self-efficacy. Nevertheless, no conclusion can be made in this research with regards to the correlation between mathematics self-efficacy and achievement. Literature, on the other hand, indicates positive correlation between self-efficacy and academic achievement of undergraduates (Chemers, & Gracia, 2001; Hall & Ponton, 2005; Zajacova et al., 2005). For example, Skaalvik and Skaalvik (2006) contend that self-efficacy is a stronger indicator of mathematics achievement than students’ prior performance in mathematics. Likewise, Betz and Hackett (2006) support that students’ mathematics self-efficacy is positively related to their achievement outcomes.

5.3  **Research Question Two**

What effects, if any, does Reciprocal Partnership have on the mathematics achievement of first and second-semester college students? As Reciprocal Partnership has positive impacts on students’ mathematics self-efficacy, academic achievement should follow. This is because students’ perceptions on their math ability have strong effects on performance (Pietsch, Walker & Chapman, 2003), and students’ self-efficacy, motivation, and beliefs in mathematical learning are directly correlated with their achievement in mathematics (Chemers & Gracia, 2001).

According to the qualitative analysis, students perceive their mathematics achievement not in terms of higher grade on assignments or tests but rather gains in their mathematical understanding and skills. Only two students have mentioned about obtaining higher grades after participating in the Reciprocal Partnership workshops. However, all of them have reported gains in mathematical knowledge and skills. Their
interview responses reveal three avenues through which they achieve the mathematical gains: opportunities to solve problems with partner, reciprocally tutoring one another and seeing different approaches to a question. These three avenues are discussed in the context of the three learning situations (exploratory, explanatory, and extensional), and the five mental activities described by Carpenter and Lehrer (1999).

**Exploratory Situation:** In the exploratory situation, where both students of a dyad do not have knowledge of the posed questions, opportunities are presented for students to solve the problems together under the guidance of a facilitator (the primary investigator of this research). Problem solving is used as the learning activity through which mathematical knowledge is constructed. The students contend that problem solving is important for them to develop understanding because they have the opportunity to work through the problems. Similar argument is made by Carpenter and Lehrer (1999): mathematical knowledge should be constructed through progressive problem solving.

Specifically, problem solving provides the context through which students engage the five mental activities: extend and apply their knowledge, construct relationship, reflect about experiences, articulate what one knows, and make mathematical knowledge one’s own. Students point out that through solving problems they can break down the questions into steps, formulate equations, and work through various approaches which ultimately allow them to attain mathematical understanding. They find most of their learning resulted from devising procedures to solve the mathematical problems. This is because when students are constructing their own solution methods they are actively relating new concepts and procedures to their prior knowledge, thereby extending and applying knowledge (Carpenter & Lehrer, 1999).
In the same way, students appreciate the opportunity to work through the mathematical problems before solutions are given. Student F9 stresses the importance of finding the solution himself by solving the problem, and student S8 points out the need for active participation in his own learning. When the solution is given beforehand, students would simply memorize the procedures presented to them (Roh, 2003). In other words, “when [students] are not provided with algorithms to learn: they construct them” (Carpenter et al, p.57). By constructing their own solution methods, students recognize the reasons for the use of the methods and connect the procedures to the underlying concepts. As a result, students develop understanding and assume ownership of the mathematical knowledge that they have constructed.

Students also illustrate their needs to solve problems with a partner because there are frequent exchanges of perspectives, constant interactions and immediate encouragements. Solving problems with a partner provides the median through which the students articulate their assumptions, ideas, and conjectures to one another. In doing so, they engage in personal reflection which involves consciously examining relations between parameters of the mathematical problems and their existing knowledge. Both reflection and articulation of what one knows are critical in supporting mathematical understanding. Simply stated, “The development of students’ personal involvement in learning with understanding is tied to classroom practices in which communication and negotiation of meanings are important facets” (Carpenter & Lehrer, 1999, p. 23).

**Explanatory Situation:** In the explanatory situation, opportunities are created in which students who have knowledge of the posed question enact the role of the tutor to provide explanations. The processes of generating and verbalizing explanations are
proved to enhance learning (Wetzstein & Hacker, 2004; Montague et al., 2000), and elaborate explanation is contended to support retention of information (Larsen, Butler & Roediger III, 2013). Learning should involve not only connecting new concepts with existing knowledge but also re-examining or reorganizing relations as in reflection and articulating what one knows (Carpenter & Lehrer, 1999).

Students contend that the opportunity to tutor their partner allows them to consolidate their understanding. This is because the process of providing explanations requires learners to form elaborated cognitive representations of ideas in order to convey meanings (O'Donnell & King, 2014). In other words, learners relate new information with existing knowledge in organized constructs that results in better retention of information (Larsen, Butler & Roediger III, 2013). Through articulating their thoughts, learners are also prompted to reflect on their own understanding which in turn, allows them to recognize new relations that they may have not realized before (Montague, Warger & Morgan, 2000). Both articulation of thoughts and reflection on one’s own knowledge are conducive to mathematical understanding (Carpenter & Lehrer, 1999).

Students in the second semester perceive the tutoring experience as a learning opportunity to gain better understanding of the course materials through reviewing and explaining the concepts. Articulating what one knows requires students to consciously examining relations to identify and describe key idea in ways that make sense to others (Carpenter & Lehrer, 1999). By having students describe their solution methods, explain the purpose of the procedures and clarify misconceptions, students develop not only the ability to articulate what they know but also to reflect on their experiences. In doing so,
they assume ownership of their own learning and the learning of their partner (Carpenter & Lehrer, 1999).

Students in the second semester also acknowledge that, as Math 2 students, they may not know everything in Math 1. The processes of providing explanations and revisiting teaching materials are supported by Roscoe and Chi (2007) to help students in the higher level consolidate understanding because they may have gaps in their learning, and these gaps can be filled through tutoring another student. On the other hand, students in the first semester not only increase their confidence when tutoring their partner, they also develop better understanding when they are being tutored. Literature suggest that there are reduced attrition (Godfrey, 2008), and improved learning (Carmody & Wood, 2009) when students are tutored by another student.

In addition, the reciprocal feature of the explanatory situation allows students to alternatively enact the role of the tutor and tutee. Students think it is more interesting and motivating to reciprocate the teaching because there are more interactions and better sharing of learning objectives. This can be explained as follows: where there are greater mutual contingency or reciprocity, there are greater interactions for formulating and testing shared ideas (Hmelo-Silver, 2013). Studies show that students gain more understanding when they enact both roles as a tutor and tutee (Dioso-Henson, 2012), and experience more comfort in their learning when they assume the responsibility to teach another student (Cheng & Ku, 2009).

**Extensional Situation:** In the extensional situation, where both students have knowledge of the posed questions, students compare one another’s solution methods. Exposing students to a variety of mathematical approaches helps students to build the
repertoires of solution methods to solve future problems (Carpenter & Lehrer, 1999) and multiple solution methods allow students to connect different ideas in their existing web of knowledge, thereby consolidating understanding (Silver et al., 2005).

As pointed out by the students in this study, they learn more efficient methods to solve problems by comparing solution methods with their partner. Likewise, in Rittle-Johnson and Star’s (2007) study, learners who compared alternative solution methods had the tendency to reflect the same method to use shortcuts. Another reason for students to develop better understanding of the mathematical questions when comparing one another’s solution methods is because students’ solution methods serve as the worked examples. Worked examples can be used to reduce cognitive loads because the learners’ working memory can be devoted towards studying the conditions and the underlying principles rather than occupied completely to solve the problem (Atkinson, Derry, Renkl, & Wortham, 2000). Another reason is “when students compare and contrast alternative strategies, they are identifying similarities and disparities among the strategies by means of constructing relationships” (Carpenter & Lehrer, 1999, p. 29). Thereby, they are making the relationships explicit through reflection and articulation, which as mentioned before leads to taking ownership of one’s learning.

Furthermore, students find comparing solutions allows them to identify their own mistakes. By comparing or analyzing contrasting cases, students develop differentiated knowledge which allows them to be attentive to information that they may otherwise missed (Schwartz, Oppezzo & Chin, 2011) and to develop conceptual knowledge (Guo & Pang, 2011). Therefore, students illustrate the benefits of seeing different mathematical approaches as learning the more effective method to arrive at the solution, examining the
questions step-by-step and identifying one’s own mistakes. These benefits have been created in the extensional situation when both members of a dyad extend their knowledge by comparing one another’s solution methods.

The overarching purpose of creating all three learning situations (exploratory, explanatory and extensional) is to maximize engagement of the five mental activities introduced by Carpenter and Lehrer (1999) as conducive to mathematical understanding. Each situation supports all five mental activities in somewhat degree and form. As the mental activities are interrelated, the three learning situations are also complementarily supporting the mental activities, such that not one learning situation can fully or completely engage all five mental activities. For examples, all three learning situations create discourses in which students articulate their thinking about mathematical ideas and perspectives. These discourses encourage students to reflect on constructing relationships that give meanings to new ideas, thereby applying or extending their existing knowledge. Ultimately, students adapt the mathematical learning as a personal involvement and make the mathematical knowledge one’s own. Therefore, the five mental activities are engaged complementarily by the three learning situations either through problem solving in the exploratory situation, tutoring another student in the explanatory situation and comparing one another’s solution methods in the extensional situation.

5.4 Result Discrepancies

Qualitative data reveal relevant gains in students’ mathematical knowledge and skills after they participate in the Reciprocal Partnership workshops but inferential statistics provide contrasting results. For example, some students perform below the median of previous class averages for the final examination. The median of their final
examination grades is also insignificantly different from the median of the previous class averages. This discrepancy between the qualitative and quantitative data can be explained by Rittschof and Griffin’s (2001) notion that standardized test may not be a viable tool to reveal the learning of relevant incidental information. As asserted by the students, even with unsatisfactory performance on their final examination, they gain extra knowledge of the course materials by solving more difficult questions, learning different approaches to arrive at solutions and reciprocally tutoring one another. It is reasonable to say that these proclaimed gains of mathematical knowledge and skills may have been “left untested”.

In addition, the compressed schedule of the first mathematics course (Math 1) may be a potential reason for the contrast between the weak examination performance of some students and the positive responses of all students on their gains in mathematical knowledge and skills. For example, students in the second semester have difficulty creating questions aligned with the topics of Math 1, and students in the first semester find it difficult to coordinate the pace of Math 1 with the pace of the workshops.

The compressed schedule of Math 1 also spans across only seven weeks so there may be insufficient time for both the participants and researcher to adjust to the structure of the Reciprocal Partnership workshops. Falchikov (2001) argues that positive mathematics achievement on examination may not result if intervention is implemented in courses that do not afford students more time to engage in the learning activities. Perhaps, sufficient time is needed for students to experience the Reciprocal Partnership workshops before treatment effect on achievement can be detected.

In addition, heightened confidence reported by the students contradict inferential statistics which reveal insignificant increase of mathematics self-efficacy for second-
semester students compared to their control group. However, type II errors may be in place. First, descriptive statistics show a large difference between the survey ratings of the treatment (Md = .72) and the control (Md = .31) group. This indicates the likelihood of the non-parametric test to accept the null hypothesis when the alternative hypothesis is true. Second, the high median of the pre-survey ratings (Md = 7.74) has little room for significant increase making ceiling effects likely. In order words, the insignificant increase in confidence of second-semester students may be due to the fact that they are highly confident to begin with.

Another reason for the result discrepancies is connected with “self-selection bias” (Van de Mortel, 2008), where more high achieving students in the second semester have participated in the study compared to more low achieving students in the first semester. A reason is because students have not realized at the time of volunteering for the research that Reciprocal Partnership allows for equal status and mutual benefits among the partners. As such, students may have misperceived that, like any tutoring program, Reciprocal Partnership requires the higher level students to enact to the role of a tutor, and hence more knowledgeable students in the second semester have volunteered.

Likewise, fewer high-achieving students in the first semester have volunteered because they may have anticipated being only the tutee. This may have distorted the research data creating a median lower than previous class averages on the final examination for students in the first semester but higher for students in the second semester.

Result discrepancies may also be caused by the small sample size. There are only six participants in the first semester and six in the second semester. The small sample size
may increase the probability of both type I and type II error because there are insufficient data to substantiate the statistics tests.

It is important to note that social desirability effects, the participants’ reactivity to being studied (Van de Mortel, 2008), may underlie the subjective responses of the students on the positive outcomes. However, the interview responses are consistent across all students with explanations and examples on how they gain mathematics self-efficacy, knowledge and skills. Details of their undertaking to spend extra practice on the mathematical questions, overcome previous challenges on difficult questions, tutor another student, solve mathematical problems, see different mathematical approaches, and reciprocally tutor one another lend supports on the positive effects of Reciprocal Partnership on students’ mathematical self-efficacy and achievement.

5.5 Research Question Three

What insights does Reciprocal Partnership have to inform the design of effective intervention programs for mathematical learning? To answer this question, descriptive statistics reveal that there are five students who prefer the exploratory situation (problem solving), three students who prefer the extensional situation (comparing solutions), and three students who find all three situations equally significant in supporting their mathematical learning. However, there is only one student who prefers the explanatory situation (tutoring) over the other two learning situations. Where most intervention programs emphasize mainly if not only on the explanatory situation, this research reveals that students find most of their learning gains from the other two learning situations. Examples of prevalent intervention programs that focus mainly on the explanatory situation include supplemental instruction and reciprocal peer tutoring.
Supplemental instruction is one of the widely used intervention program in the United States (Malm, et al, 2011). It is a form of group tutoring where the tutee raises questions on areas of which he or she has little or no knowledge. This typical form of tutoring creates only the explanatory and exploratory situations. For example, when the tutee raises questions to the tutor who then provides explanations to these questions, the explanatory situation is created. Whereas, when the tutor cannot provides explanations to the questions raised by the tutee, both the tutor and tutee have no knowledge of the poised questions, and this epitomizes the exploratory situation. However, since the explanatory situation is the focus of the program, the exploratory situation is often neglected.

On the other hand, reciprocal peer tutoring creates opportunities where both members of a dyad assume the role of a tutor by having them take turns posing questions to their partner, and then provide explanations to the questions that their partner answers incorrectly. For this reason, the tutor must have knowledge of the questions that he or she poses and this form of tutoring creates only the explanatory and extensional situations. For example, when the tutee cannot answer the questions posed by the tutor, the tutor provides explanations to the posed questions thus, the explanatory situation is created. Whereas, when tutee can answer the questions posed by tutor, both the tutor and tutee have knowledge of the posed questions: hence, the extensional situation is created. However, the explanatory is also the emphasis of reciprocal peer tutoring and so the extensional situation is often disused.

Therefore, the structure of Reciprocal Partnership combines both feature of supplemental instruction and reciprocal peer tutoring to maximize the occurrences of all
three learning situations. For example, it adapts the approach of reciprocal peer tutoring to create opportunity for students to assume both the role of a tutor and tutee. The purpose is to reap the learning benefits derived from teaching (as a tutor) and receiving assistances (as a tutee). However, in reciprocal peer tutoring, students do not have the opportunity to pose questions on areas that they need assistances. Consequently, they will lose the influence of directing the tutelage to improve their weakness.

By generating questions on topics that they wish to improve, students can influence the depth of the enquiry, and to some extent determine the learning objectives of the tutelage (Roscoe & Chi, 2007). This is one of the reasons why Reciprocal Partnership mirrors supplemental instruction in having students generate questions that they have little or no knowledge of so they can influence the learning objectives of the tutelage. Another main reason to adapt the structure of supplemental instruction is to create the extensional situation which cannot be created in the structure of reciprocal peer tutoring. Simply by having students generate questions of which they have and do not have knowledge, Reciprocal Partnership combines the mechanism of supplemental instruction and reciprocal peer tutoring to inherit the learning benefits attributed to these two forms of tutoring, and to create the three learning situations that are projected to complementarily enhance students’ mathematical understanding.

Occurrences of any one of the three learning situations are already embedded in the structure of most mathematics intervention programs. As mentioned above, supplemental instruction can create the explanatory and exploratory situations whereas, the explanatory and extensional situations can be created by reciprocal peer tutoring. However, the exploratory and extensional situations are often misused, neglected, or
undermined. By emphasizing all three learning situations, Reciprocal Partnership can fully support students’ mathematical understanding through problem solving, tutoring, and comparing solution methods.

Students agree that all three learning situations support their mathematical learning. They provide various reasons for their preferred learning situation based on their learning needs. For example, students prefer problem solving (exploratory situation) because they value the opportunity to work through the problem, practice the question, and collaborate with their partner. In contrast, students prefer comparing solution methods (extensional situation) because they like to learn various mathematical approaches, analyze the questions step-by-step, and work at their own pace. Distinctively, student F3 prefers being tutored (extensional situation) because he rarely needs help on mathematical problems and when he finally has the opportunity to be tutored, it was very helpful.

The various reasons provided by the students indicate that each learning situation differs from one another in supporting their learning needs. As speculated by the Three Learning Situations framework, not one learning situation can fully accommodate all learning process, needs or styles. Therefore, mathematical understanding is best resulted from all three learning situations as they complementarily help students develop mathematical understanding.

Results of this study reveal that students gain mathematical knowledge and skills from each of the learning situations, and most students prefer the exploratory or the extensional situation over the explanatory situation in terms of their learning outcomes. On the contrary, the emphasis of most intervention program is the explanatory situation.
This suggests the potential benefit of redesigning intervention programs from those that emphasize only on the explanatory situation to those that include all three learning situations.

5.5.1 Suggestions for Improvement

Students face a couple of challenges following the structure of the workshops. This include managing their time. Due to their tight schedule, some students have difficulties allocating time for the workshops especially when they have upcoming tests and assignments. However, in most cases, they make up for the missed workshops. Some students find it awkward to attend the workshops when their partner is absent. Another challenge includes creating the two set of questions. Few students simply do not have any question to ask but some students find it difficult to create questions in accordance with the schedule and syllabus of Math 1. Due to the compressed schedule of Math 1, students have difficulty aligning the pace of the workshops with the pace of the course. As their final remark, some students provide suggestions to resolve these two challenges.

Students suggest having drop-in schedules in which they can come whenever they are available and be partnered with any student who is present at the time when their partner cannot attend the workshops. Students also suggest to have pre-set questions to facilitate the workshops when they cannot generate the questions themselves. These suggestions shed light on how to maximize the benefits of Reciprocal Partnership and can be considered in future research to improve implementation of the workshops.

5.6 Major Findings

Qualitative analysis reveal that all students gain confidence by participating in the Reciprocal Partnership workshops. A simple reason for the increase is extra practice on
the mathematical questions. The time and effort spent to work through the questions create a sense of contentment for the students. Another reason is the opportunity to teach another student. Students feel rewarding and happy to pass their knowledge to another student. In particular, students in the first semester have higher self-efficacy after occasionally teaching their partner who is in the second semester. These occasional moments greatly change the perspectives of first-semester students in their mathematical ability. Few students gain confidence not only in their capability to work with mathematics but also the ability to teach it. A final reason for the increased confidence includes overcoming difficult questions. With the help and encouragement from a partner, students are confident to render challenging questions and in turn, experience greater self-efficacy.

Students mention further about other affective effects of Reciprocal Partnership: increased motivation, social connection and comfort in learning. They exert more efforts to solve mathematical problems under the positive influence of other students and given the opportunity to teach another student. Students also value the opportunities created in the workshops to meet people, to establish social connection with one another and to be accepted and respected by other students. This allows them to have comfort in their learning and makes learning mathematics more enjoyable.

The above affective impacts of Reciprocal Partnership have powerful implication in diminishing adversities in mathematical learning such as mathematics anxiety, high failure rates, premature foreclosures of career path, and large academic gaps in performance. It is evident in the qualitative data that students develop more positive attitudes towards learning mathematics. They find enjoyment in a subject that is
notoriously known to create anxiety. Their respects and encouragements for one another have become a norm that foster the positive learning environments of the workshops. Nevertheless, the positive effects of Reciprocal Partnership on students’ affective development can potentially address many negative emotions and perspectives associated with the learning of mathematics.

Quantitative data, on the other hand, reveal significant increase of mathematics self-efficacy for students in the first semester compared to their control group, and insignificant difference in the increase of mathematics self-efficacy between second-semester students and their control group. In other words, the gain in mathematics self-efficacy for students in the second semester cannot be attributed to their participation in the Reciprocal Partnership workshops without the influence of the mathematics course. However, this result is likely to be affected by a type II error because of ceiling effects on the high pre-survey ratings of second-semester students.

Despite insignificant gains in mathematics achievement for both first and second semester students, qualitative data reveals positive effects on students’ mathematical achievement in terms of gains in mathematical knowledge and skills. Two possible interpretations are suggested to explain this discrepancy. First, the final examination may be ineffective for detecting learning gains of relevant incidental information from the workshops. Second, the duration of the intervention may be too short for the students to improve their mathematical skills or knowledge. It is inferred from statistics results that Reciprocal Partnership has insignificant effect on mathematics achievement of college students in the first semester and second semester; however, qualitative data reveals rather positive effects on students’ mathematical knowledge and skills.
Mathematics achievement is described by the students as gains in mathematical knowledge and skills as opposed to improved academic performance on tests and exams. The reasons for the gains include solving problems with a partner. They value the opportunity to work through the questions first before having the solution presented to them. Another reason for the gain is seeing different approaches to a question by comparing one another’s solution methods. Some students learn more effective or simpler routes to arrive at the solution, and others detect their own mistakes when comparing one another’s solution methods.

The final reason for the gains in mathematical skills and knowledge is the opportunity to reciprocally tutor one another. For students in the second semester, they consolidate their own understanding by retrieving and retaining information when tutoring their partner. Although they predominately enact the role of the tutor, there are occasions when they learn from their partner on materials of Math 1 that they have never learned before or have forgotten. For students in the first semester, they gain understanding when tutoring their partner and being tutored by them. Both groups of students appreciate the opportunity to reciprocate the help from their partner which creates more interesting, motivating, and mutual interactions.

Students also make two suggestions for improving the structure of the workshops: A drop-in schedule along with the flexibly to form their own partner will better meet their tight schedule; Pre-set questions from which they can choose will facilitate circumstances when they cannot generate their own questions. Future study on Reciprocal Partnership may consider implementing these suggestions to improve the structure of the workshops.
It is important to note that students report gains in mathematical knowledge and skills through problem solving (exploratory situation), tutoring one another (explanatory situation), and comparing solution methods (extensional situation). Results reveal that most students prefer the exploratory and extensional situations over the explanatory situation in terms of gains in their learning. Where most intervention programs emphasize mainly if not only on the explanatory situation, this study suggests the emphasis on all three learning situations as a feature to be adapted in mathematics intervention programs to maximize learning outcomes.

5.7 Limitations of the Study

While this research reveals promising results, it has several limitations. First, variables that may have affected results such as students’ prior academic achievement, learning style and attitudes toward mathematics learning have not been taken into account. Second, limitations are related but not restricted to the highly homogeneous nature of the participants. In particular, the gender of the students is unevenly distributed where there is only one female participant. Perspectives of participants are also confined to a single community college and to those who have taken the mathematics courses (Math 1 and Math 2) so interpretations are based on a specific setting and subject. Accordingly, findings of this study cannot be generalized to other subject areas and settings, and across genders.

Although this study is limited in its generalization, the study of particular events such as the effects of Reciprocal Partnership on a specific group of students in a specific setting plays an important role in the development of a general explanatory construct. Cobb, Yackel, and Wood (1992) say it precisely:
It is assumed that the goals of making sense of particular events in one classroom and of developing a more general orienting framework are interdependent in that the analysis of particulars constitutes occasions to reconsider what needs to be explained and to revise general explanatory constructs. Conversely, the segmentation of classroom life into significant potentially understandable events reflects one’s theoretical orientation. Thus, particular events empirically ground theoretical constructs, and theoretical constructs inform the interpretation of particular events (Erickson, 1986; Mishter, 1986). (p. 100)

5.8 Implications for Future Research

Future researchers may want to use a more comprehensive control group and widen the research to include more participants. Since Reciprocal Partnership has demonstrated to have many affective and academic benefits for both students in the first and second semester, further studies can also focus more on the effect of Reciprocal Partnership as a cross-level intervention. Literature supports the used of cross-level collaboration and communication to mediate secondary-tertiary transition (e.g. Capstick & Fleming, 2002; Clark & Lovirc, 2009; Congos & Schoeps, 2003). Further studies on the use of Reciprocal Partnership as a systematic intervention can inform the effective design to not only improve achievement but also to establish a system of social support and intellect exchange across all grades.

5.9 Conclusion

The increasing diversity of college students and the need for mathematical preparation of incoming college students impose a challenge for today’s higher education to effectively enhance mathematical learning of the students, and mediate their secondary-tertiary transition. An effective intervention should provide the necessary social contacts for students to combat isolation and overcome difficulties assimilating into the college setting. Further, it should afford opportunities for problem solving, articulating one’s understanding, and acquiring alternative solution methods as a means
to deliver a holistic support of mathematical understanding. These features are realized in Reciprocal Partnership.

Reciprocal Partnership encourages students to collaborate and actively involved in their own learning by integrating the strategies of supplemental instruction and reciprocal peer tutoring to create the three learning situations. Qualitative data reveals the effectiveness of Reciprocal Partnership in helping students gain mathematical knowledge and skills, while sustaining their social development. It appears to be a promising and effective intervention. Therefore, the demonstrated effects of Reciprocal Partnership to enhance mathematical understanding, bolster mathematics self-efficacy, build social relationship, increase motivation, and establish positive learning environment warrant further study and greater consideration. It is hoped that future studies can further reveal the positive effects of Reciprocal Partnership on students’ mathematical learning during their secondary-tertiary transition.
References


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Appendix A

Recruitment Form

Volunteers needed for research on mathematical learning

Dear students:

In light of the research project called Reciprocal Partnership, we are recruiting first and second-semester students to collaborate in learning and reviewing course materials of Math 1. If you are interested, please contact Kerry Kwan to register for the information session__________________________

You have the right to withdraw from this study at any time, as well as not to answer any questions and disclose any information. There are no known risks or benefits to you for participating in this research. All information that may identify you in any way will be kept confidential.

Sincerely,
Kerry Kwan
OISE/University of Toronto
kerry.kwan@mail.utoronto.ca
Appendix B

Consent Form for Participant Group

Dear student,

My name is Kerry Kwan and I am a PhD candidate at the Ontario Institute for Studies in Education, University of Toronto (OISE/UT). I am conducting a research to examine a program, namely Reciprocal Partnership, on students’ mathematics achievement and self-efficacy. The purpose of this study is to inform educators and researchers on the use of Reciprocal Partnership to enhance students’ mathematical learning at the college level.

I like to invite you to participate in this study. Upon your consent, you will collaborate with another student to reciprocally teach and learn from one another in workshops that are structured according to the framework of Reciprocal Partnership. The workshops will be held twice per week throughout a semester. There will be a pre- and post-survey for you to rate your confidence in mathematics. Personal information and written work will be collected only with your consent. I will conduct observations/notes-taking during the workshops and an interview with you at the end of this study. It will take about 45 minutes and will be tape-recorded. Your final examination grade will be collected. You will also be given the transcript of the interview to confirm its validity and will be able to search for the final report of this study through the database/library of the University of Toronto.

This study has been approved by the University of Toronto Ethics Office and the Research Ethics Board of your college. If you wish to have more information or ask any questions about your rights as a research participant, you may contact U of T Office of Research Ethics, 12 Queens’ Park Crescent West, McMurrich Building, 2nd floor, M5S 1S8, Toronto, Phone: 416-946-3273.

Any information that might identify you in written work, oral presentations or publications will remain confidential. It is important that you are aware of your right to ask questions about this research and to have those questions answered by me before, during, or after the research. Most importantly, you have the right to withdraw from this study at any time, as well as decline to answer any questions and disclose any information. Data pertaining to this study will be stored in locked cabinets for up to five years after the research has been presented and/or published. There are no known risks or benefits to you for participating in this study.

Please sign at the bottom of the page if you agree to participate and a second copy will be given to you for your records. Thank you very much for your participation.

I acknowledge that the topic of this study has been explained to me and that any questions that I have asked have been answered to my satisfaction. I understand that I can withdraw at any time through any means of contact or communication with Kerry Kwan. I have read the letter provided to me by Kerry Kwan, and agree to participate in this study, disclose personal information, and provide written work for the purpose described.

Students’ Signature: _____________________________________________________________________

Students’ Name (printed):  ______________________________________________________________

Date:  _____________________________________________________

Yours sincerely,

Kerry Kwan
OISE/University of Toronto
kerry.kwan@mail.utoronto.ca

Thesis supervisor: Dr. Douglas E. McDougall
Dear student,

My name is Kerry Kwan and I am a PhD candidate at the Ontario Institute for Studies in Education, University of Toronto (OISE/UT). I am conducting a research to examine a program, namely Reciprocal Partnership, on students’ mathematics achievement and self-efficacy. The purpose of this study is to inform educators and researchers on the use of Reciprocal Partnership to enhance students’ mathematical learning at the college level.

I like to invite you to participate in this study by completing a survey at the beginning and end of the semester. The survey is for you to rate your confidence in math-related questions.

This study has been approved by the University of Toronto Ethics Office and the Research Ethics Board of your college. If you wish to have more information or ask any questions about your rights as a research participant, you may contact U of T Office of Research Ethics, 12 Queens’ Park Crescent West, McMurrich Building, 2nd floor, M5S 1S8, Toronto, Phone: 416-946-3273.

Any information that might identify you in written work, oral presentations or publications will remain confidential. It is important that you are aware of your right to ask questions about this research and to have those questions answered by me before, during, or after the research. Most importantly, you have the right to withdraw from this study at any time, as well as decline to answer any questions and disclose any information. Data pertaining to this study will be stored in locked cabinets for up to five years after the research has been presented and/or published. There are no known risks or benefits to you for participating in this study.

Please sign at the bottom of this page if you agree to complete the surveys and a second copy will be given to you for your records. Thank you very much for your participation.

I acknowledge that the topic of this study has been explained to me and that any questions that I have asked have been answered to my satisfaction. I understand that I can withdraw at any time through any means of contact or communication with Kerry Kwan. I have read the letter provided to me by Kerry Kwan, and agree to participate in this study by completing the surveys and disclose personal information for the purpose described.

Students’ Signature: ________________________________________________________________

Students’ Name (printed): ________________________________________________________________

Date: ________________________________

Yours sincerely,

Kerry Kwan
OISE/University of Toronto
kerry.kwan@mail.utoronto.ca

Thesis supervisor: Dr. Douglas E. McDougall
Appendix C

Information Form

Name:________________________________________________________

Phone number:_________________________________________Gender:______

Email:____________________________________________________________________________

Ethnical Background:________________________________________________________________

Program/Major:______________________________________________________________________

Course:______________________________  Semester of program:____

Professor of the course:______________________________________________________________

What is your opinions/thoughts/perspectives on mathematics?

____________________________________________________________________________________

____________________________________________________________________________________

____________________________________________________________________________________

____________________________________________________________________________________

____________________________________________________________________________________

____________________________________________________________________________________
Appendix D

Interview Questions

1. What are your career aspirations? Have they changed since participating in these workshops? If so, why and how?

2. What are your views on learning mathematics? Have they changed since participating in these workshops? If so, why and how?

3. Has your confidence to succeed in college changed since the beginning of these workshops? If so, why and how?

4. Describe your interactions and relationship with your partner throughout these workshops.

5. Describe your feelings and thoughts when you helped your partner and also when you received help from him or her on the mathematical questions.

6. Describe your feelings and thoughts working with a student who is in a different grade than you.

7. Describe any challenges that you may have encountered with regards to the implementation of the workshops and collaborating with your partner.

8. What sense of achievement in terms of working with mathematics, if any, do you feel you have gained through these workshops?

9. How would you compare these workshops to other tutoring or support program that you have experienced before?

10. Do you find any one of three learning situations (tutoring, problem solving, and comparing solutions) more helpful? If yes, which one and how?

11. What suggestions would you provide to improve these workshops?
Appendix E

Workshop Package

Reciprocal Partnership

Prepare 2 to 3 questions per set

Set 1
Questions you need help on
When your partner has knowledge of your question
  Tutoring
  Your partner helps you on your question
When your partner has no knowledge of your question
  Problem solving
  Both of you try to solve the question

Set 2
Questions you can help others on
When your partner has knowledge of your question
  Comparing
  Both of you compare each other’s solution methods
When your partner has no knowledge of your question
  Tutoring
  You help your partner on your question

Collaboration among you and your partner will be sustained as both of you develop mathematical understanding through respecting one another’s thinking, and mutually helping each other.
Question you need help on

Tutoring □

Problem Solving □

Question you can help others on

Tutoring □

Comparing Solutions □