Representative Stall Model of Regional Aircraft for Simulator Training
Using a Spline Shape Prescriptive Modeling Approach

by

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for the degree of Master of Applied Science
Graduate Department of Institute for Aerospace Studies
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Abstract

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Loss-of-control following aerodynamic stall remains the largest contributor to fatal civil aviation accidents. Aerodynamic models past stall are required to train pilots on stall recovery techniques using ground-based simulators, which are safe, inexpensive, and accessible. A methodology for creating representative stall models, which capture essential stall characteristics, is being developed for classes of twin-turboprop commuter and twin-engine regional jet aircraft. Despite having lower fidelity than type specific stall models generated from wind tunnel, flight test, and/or CFD studies data, these models are configuration adjustable and significantly cheaper to construct for high angle-of-attack regimes. Baseline specific stall models are modified to capture changes in aerodynamic coefficients due to configuration variations from a baseline to a target aircraft. A Shape Prescriptive Modeling approach combining existing theory and data using least-squares splines is used to make coefficient change predictions. Initial results are satisfactory and suggest that representative models are suitable for stall training.
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Symbols and Abbreviations

α  Angle-of-attack
β  Sideslip angle
\(x_E\)  x-position of C.G. in inertial frame
\(y_E\)  y-position of C.G. in inertial frame
\(z_E\)  z-position of C.G. in inertial frame
φ  Euler roll angle
θ  Euler pitch angle
ψ  Euler yaw angle
u  Airspeed along x-axis in body-fixed frame
v  Airspeed along y-axis in body-fixed frame
w  Airspeed along z-axis in body-fixed frame
p  Roll rate in body-fixed frame
q  Pitch rate in body-fixed frame
r  Yaw rate in body-fixed frame
X  Force along x-axis in body-axis frame
Y  Force along y-axis in body-axis frame
Z  Force along z-axis in body-axis frame
L  Roll Moment about x-axis in body-axis frame
M  Pitch Moment about y-axis in body-axis frame
N  Yaw Moment about z-axis in body-axis frame
\(L\)  Lift force along x-axis in wind-/stability-axis frame
\(Y\)  Side force along y-axis in wind-/stability-axis frame
\(D\)  Drag force along z-axis in wind-/stability-axis frame
\(l\)  Roll Moment about x-axis in wind-/stability-axis frame
\(m\)  Pitch Moment about y-axis in wind-/stability-axis frame
\(n\)  Yaw Moment about z-axis in wind-/stability-axis frame
\((\cdot)_B\)  Quantity in Body Axis
\((\cdot)_S\)  Quantity in Stability Axis
\(C_i\)  Non-dimensional aerodynamic coefficient for force or moment \(i\)
\(\eta_j\)  Stall efficiency factor for component \(j\) contribution
C.G.  Centre of gravity
LOC  Loss-of-control
FAA  U.S. Federal Aviation Administration
UTIAS  University of Toronto Institute for Aerospace Studies
Chapter 1

Introduction

1.1 Background & Motivation

Over the past few decades, the largest contributor to worldwide fatal aviation accidents involving commercial aircraft has been loss-of-control (LOC). In fact from 1999 to 2008 alone, LOC accounted for 22 accidents and 1,991 fatalities out of 91 accidents and 4,970 fatalities from all causes [66]. Of the accidents resulting from LOC, the leading cause is aerodynamic stall [37]. Memorable recent examples include Air France 447, Colgan Air 3407, and AirAsia 8501, which led to the death of all passengers and crew onboard [14]. In all cases, the pilots were unable to recover the aircraft from an aerodynamic stall that developed into LOC before crashing. LOC is defined as a flight condition where the aircraft motion cannot be predictably controlled by pilot inputs, where small state and control changes lead to lead to large responses, or where the aircraft experiences high angular rates, divergent, and/or oscillatory behaviour [31]. The result is that the pilot has difficulty maintaining heading, altitude, and wings-level flight, which may eventually lead to a deadly crash. It is therefore proposed that better pilot training in stall recovery would be a wise investment to significantly reduce these types of catastrophic accidents [19].

1.2 Flight Simulator Training

Since environmental and systems factors leading to LOC are difficult to predict, total avoidance is not possible. Research shows that few airline pilots ever experience aerodynamic stall or have been extensively trained in the special procedures needed to recover from stall in large transport aircraft [66]. Training them in full size transport aircraft in upset conditions is impractical due to the high costs and associated risks. As part of the overall training strategy that includes stall training in aerobatic aircraft and class room instructions, ground-based simulators which are safe, inexpensive, and accessible will be used to conduct Upset Prevention and Recovery Training (UPRT) [53]. Recently the US Federal Aviation Administration (FAA) issued a Notice of Proposed Rule Making (NPRM) mandating that ground-based flight simulator stall recovery training shall become a requirement for civil aviation pilots [20]. A key requirement for effective training is simulation fidelity which primarily depends on the accuracy of the aircraft physics models. Due to the current lack of adequate models however, most existing training simulators are inadequate for stall training. As part of a collaborative project led by the FAA, the Vehicle
Simulation group at UTIAS is developing a methodology to extend regional turbojet and turboprop aircraft flight models into the stall regime.

1.3 Representative Stall Modeling

Flight models with extended envelopes accurate in the post-stall regime (hereafter referred to simply as stall models) have been created in the past, but these were type specific [23, 39]. They were built using large sets of highly expensive wind-tunnel tests to generate high-fidelity aerodynamic databases covering the full range of flight conditions for one geometric configuration. It is desirable to create a less expensive representative stall model that captures at least the essential stall characteristics necessary for meaningful simulator training. The model should work for a range of aircraft within a category by adjusting their aerodynamic coefficients based on differences in parameters such as wing sweep, tail height, nacelle location, etc., so that given a specific baseline stall model as the input, a configuration adjusted target stall model within the category can be generated and returned as the output. This approach saves both time and money compared to re-creating individual specific stall models, although it is understood that they may not be as accurate since general trends are used. It is hypothesized that training with representative models, while perhaps not as good as type specific, can still lead to useful training.

Preliminary evidence supporting this hypothesis was presented in References [17, 24, 58]. Donaldson et al [17] built a representative model of a P-8A aircraft and found that some useful training could be accomplished in full-flight simulators using it. Gingras et al [24] developed a representative model of a B737-800, which along with another representative model were tested by Schroeder et al [58] in a simulator and then compared with a type specific model built by Boeing. Results from these studies showed that the test pilots evaluations of the models were not statistically different between the representative model and the high fidelity model. In addition, a quasi-transfer-of-training study using airline pilots found a wide range in the performance of the pilots trained using the two different models. Many of the results however, showed insignificant difference in the training benefit of the high fidelity model over the representative model. These findings suggest that representative stall models are comparable to specific models in terms of pilot training benefits, yet are much more cheaper to develop.

In this study, focus is placed on demonstrating a satisfactory methodology for creating representative stall models for the class of regional twin-engine jet aircraft with swept-wings and T-tails. Since the aircraft rigid-body equations of motion are well-understood and easily implemented, the majority of work centres around establishing important relationships between variations in aerodynamic coefficients in stall and aircraft geometry. The aerodynamic coefficients for stall will be estimated using data from a combination of analytical tools, CFD, wind-tunnel, flight test, and accident data found in literature. In case data is unavailable for some configuration of interest, similar type aircraft data and knowledge on aircraft stall characteristics is used to synthesize forces and moments. A comprehensive literature review is conducted to gather available knowledge of how aircraft parameters affect aerodynamic coefficients near and past stall. The model predictions will be validated against available post-stall data.
1.4 Shape Prescriptive Modeling

In the context of developing a representative stall model for varying aircraft configurations, a purely analytical approach is limited either by its applicability to only certain aircraft parts (such as lifting surfaces) or by its computational expense as it approaches CFD, while a purely empirical approach simply interpolating available data fails to accurately account for many effects and does not filter out data noise using more rigorous physics-based arguments. To mitigate these and other problems with existing stall modeling attempts as well as to meet current project requirements, an alternative modeling approach that combines experimental data and theoretical knowledge is explored in this study. Shape Prescriptive Modeling (SPM) or shape constrained spline regression modeling, using concepts borrowed from statistics, strikes a balance among robustness in modeling a range of aerodynamic coefficient terms, direct control over model flexibility, applicability to limited and noisy data, and justifying model assumptions with physical insight from either experienced pilots or engineers [16].

In general, regression or curve fitting reduce data to functional relationships, from which values or characteristics of the physical processes can be estimated or predicted. Various functional models can be used. Polynomial fitting can be suitable for simple relationships, but are tricky to use because of problems associated with under/over-fitting and Runge’s phenomenon [16]. Another commonly used tool is nonlinear regression using exponentials, gaussians, or trigonometric functions. While attractive, these models are often not chosen with valid physics based justifications, but rather because their basic shapes happen to somewhat match the data - e.g. fitting a gaussian curve to a roughly bell shaped dataset. If the fit is inadequate, another functional form is tried, often at the expense of neglecting accurate knowledge of the underlying process. For modeling aircraft stall behaviour, it is important to attribute and correlate major curve characteristics to aerodynamic phenomena, so shape guessing is not recommended.

The preferred approach is to start with an understanding of the underlying process to be modeled gathered from existing literature. The challenge then becomes building that knowledge in a usable mathematical form, without going into unnecessary details. For example, one may know that a relationship has an interval which must be monotone increasing, or that there is a specific data point the curve must pass through. To build in such behaviour, Shape Prescriptive Modeling is ideal. The concept of SPM is Bayesian in nature, as it involves quantifying a priori process knowledge in mathematical forms, followed by estimating a predictive model of the process with that information built-in. The process knowledge becomes a prescription for the function shape, which prevents over-fitting. The more that is known regarding the physical phenomenon, the more detailed the shape prescription, and the less data noise or random error detract from useful underlying trends.

In this study, SPM will be implemented using cubic (or lower order) splines and a combination of shape constraints such as monotonicity, knot placement, polynomial degree, slope, curvature, values at specific locations, max/min bounds etc., with the help of the publicly available “Shape Language Modeling” MATLAB Toolbox by John D’Errico [16]. These constraints, carefully chosen to reflect specific stall physics, first reduce the spline degrees-of-freedom (nDOF) or number of curve parameters until an appropriately reduced curve flexibility is reached; the remaining DOFs/parameters are then fitted to experimental data of various configurations using least-squares spline regression. Thus the constraints basically filter out noise in the data that are either random or caused by elusive secondary effects.
nonessential to the stall effect in question; the fitting and smoothing of datasets extract relationships between curve parameters/DOFs and aircraft parameters. The model accuracy depends on the quantity and quality of experimental data as well as quality of theoretical insight into the physical phenomenon. However, even with little training data and rudimentary physical understanding, many useful trends can be modeled to support stall training. Finally, making predictions becomes a straight-forward task of trend interpolation.

There are limitations of this method. Many of the spline shape constraints are based on heuristics gathered using past findings in literature, engineering judgement, and/or experience. The final fitted spline may give inaccurate results for certain coefficients or configurations, since the method minimizes global errors for all configurations instead of for individual coefficient versus geometry curves. In the post-stall regime, small changes in geometry (e.g. wing dressing components such as vortilons) can lead to quite large changes in the aerodynamic contributions of aircraft components due to flow separation and other interactions. Therefore the models developed using this method will need to be checked for the correct basic post-stall behaviour.

1.5 Document Structure

The next chapters are organized as Literature Review, Methodology, Model Synthesis, Results and Discussion, and Conclusions. Literature review summarizes relevant knowledge on flight dynamics, stall characteristics, configuration aerodynamic effects data, followed by examining existing stall modeling attempts, and a scope of study. The Methodology chapter explains the representative stall model architecture, how SPM is applied to stall modeling, and some LOC prediction metrics. The Model Synthesis chapter delves into details of how individual configuration variation effects are modeled. Lastly, the Results and Discussion chapter will show state variable limits checks, validation of individual configuration effect predictions, followed by a baseline to target aircraft validation, and an application of LOC prediction criteria to a representative stall model.

It is worth mentioning that there is a difference between a methodology and a method. Research methods are the tools, techniques or processes that are used to yield specific results. On the other had, a methodology is about the principles that guide research practices; it explains why certain methods or tools are used. This study presents a methodology for developing representative stall models; various methods are used within the study along with justifications for their selection.
Chapter 2

Literature Review

Fundamentals of flight dynamics are first reviewed in the context of flight modeling for simulator training followed by the basics of stall aerodynamics and characteristics. Public literature data sources on aircraft configuration effects on aerodynamic forces and moments at high angles-of-attack are then summarized using a matrix of possible terms, with relative importance highlighted. Finally, a summary of which coefficients and configuration variation combinations are addressed in this study is given.

2.1 Flight Dynamics Model

Assuming a flat non-rotating earth with no wind, a rigid body aircraft with no rotating inertia, a typical xz-plane of symmetry, we apply Newton’s second law in translation and rotation, and express quantities in body axes [36] to get,

\[
F_A + F_G + F_T = m\dot{V} + \omega \times mV \tag{2.1a}
\]

\[
M_A + M_T = I\dot{\omega} + \omega \times I\omega \tag{2.1b}
\]

Letting \( F_A = [X, Y, Z]^T \), \( M_A = [L, M, N]^T \), \( V = [u, v, w]^T \), \( \omega = [p, q, r]^T \), and for the time being zero thrust \( F_T = [0, 0, 0]^T \), the aircraft’s non-linear flight dynamics equations of motion in the body frame are Equations 2.2, which can be expressed in the 1st order ODE form \( \dot{x} = f(x, u) \) as in Equations 2.3,
where the C’s are the inertia ratios obtained after solving the coupled equations above,

\[ C_0 = (I_x I_z - I_{xz}^2)^{-1} \]
\[ C_1 = I_{xz}(I_x - I_y + I_z)C_0 \]
\[ C_2 = (I_y - I_z)I_z - I_{xz}^2)C_0 \]
\[ C_3 = I_x C_0 \]
\[ C_4 = I_{xz} C_0 \]
\[ C_5 = (I_z - I_x)/I_y \]
\[ C_6 = -I_{xz}/I_y \]
\[ C_7 = 1/I_y \]
\[ C_8 = I_{xz}(I_y - I_z - I_x)C_0 \]
\[ C_9 = ((I_x - I_y)I_x + I_{xz}^2)C_0 \]
\[ C_{10} = I_{xz} C_0 \]
\[ C_{11} = I_x C_0 \]

To complete the set of equations, the following 6 kinematic equations are also required:

\[ x_E = u(\cos \theta \cos \psi) + v(\sin \phi \sin \theta \cos \psi - \cos \phi \sin \psi) + w(\cos \phi \sin \theta \cos \psi + \sin \phi \sin \psi) \]  
(2.5a)
\[ y_E = u(\cos \theta \sin \psi) + v(\sin \phi \sin \theta \sin \psi + \cos \phi \cos \psi) + w(\cos \phi \sin \theta \sin \psi - \sin \phi \cos \psi) \]  
(2.5b)
\[ z_E = u(-\sin \theta) + v(\sin \phi \cos \theta) + w(\cos \phi \cos \theta) \]  
(2.5c)
\[ \dot{\phi} = p + q \sin \phi \tan \theta + r \cos \phi \tan \theta; \quad \dot{\theta} = q \cos \phi - r \sin \phi; \quad \dot{\psi} = (q \sin \phi + r \cos \phi) \sec \theta \]  
(2.5-d,e,f)

Now given the values of aerodynamic forces and moments \(X, Y, Z, L, M, N\) in body frame at each time step, the state of the aircraft \(\{u, v, w, p, q, r, x_E, y_E, z_E, \phi, \theta, \psi\}\) can be calculated through numerical integration of the dynamic and kinematic equations. The results can be run in the real time environment of a training simulator to produce the visual and motion cues in the cockpit as well as flight instruments and sounds [53]. Since it is often easier to express aerodynamic terms in the wind or stability axis (e.g. for wind-tunnel experiments), frame transformations are needed (see Etkin [18] for definitions)

\[
\begin{bmatrix}
X \\
Y \\
Z \end{bmatrix}_{\text{body}} = C_{bw} \begin{bmatrix}
L \\
C \\
D \end{bmatrix}_{\text{wind}} \quad (2.6)
\]
\[
\begin{bmatrix}
X \\
Y \\
Z \end{bmatrix}_{\text{body}} = C_{bs} \begin{bmatrix}
L \\
Y \\
D \end{bmatrix}_{\text{stability}} \quad (2.7)
\]
\[
\begin{bmatrix}
L \\
M \\
N \end{bmatrix}_{\text{body}} = C_{bw} \begin{bmatrix}
t \\
m \\
n \end{bmatrix}_{\text{wind}} \quad (2.8)
\]
\[
\begin{bmatrix}
L \\
M \\
N \end{bmatrix}_{\text{body}} = C_{bs} \begin{bmatrix}
t \\
m \\
n \end{bmatrix}_{\text{stability}} \quad (2.9)
\]

\[
C_{bw} = \begin{bmatrix}
\cos \alpha \cos \beta & -\cos \alpha \sin \beta & -\sin \alpha \\
\sin \beta & \cos \beta & 0 \\
\sin \alpha \cos \beta & -\sin \alpha \sin \beta & \cos \alpha 
\end{bmatrix} \quad (2.10)
\]
\[
C_{bs} = \begin{bmatrix}
\cos \alpha & 0 & -\sin \alpha \\
0 & 1 & 0 \\
\sin \alpha & 0 & \cos \alpha
\end{bmatrix} \quad (2.11)
\]
\[ V = \sqrt{u^2 + v^2 + w^2}; \quad \alpha = \tan^{-1}(w/u); \quad \beta = \sin^{-1}(v/V) \] (2.12)

Finally by expressing forces and moments in non-dimensional form, flight conditions such as air density, velocity, scale are separated from aerodynamics due to geometry and state/input. In stability axis,

\[ L = \frac{1}{2} \rho V^2 SC_L \] (2.13a)
\[ Y = \frac{1}{2} \rho V^2 SC_Y \] (2.13b)
\[ D = \frac{1}{2} \rho V^2 SC_D \] (2.13c)
\[ l = \frac{1}{2} \rho V^2 SbC_l \] (2.13d)
\[ m = \frac{1}{2} \rho V^2 SbC_m \] (2.13e)
\[ n = \frac{1}{2} \rho V^2 SbC_n \] (2.13f)

The aerodynamic coefficient will depend on various states and inputs, keeping in mind that everything also depends on aircraft configuration geometry, so that for each \( i = L, Y, D, l, m, n \)

\[ C_i = C_i(V; \alpha, \beta, p, q, r, \dot{\alpha}, \dot{\beta}, \dot{p}, \dot{q}, \dot{r}, \delta_{\text{controls}}, \ldots) = f(\text{configuration}) \] (2.14)

where \( V, \alpha, \beta \) are the static variables, \( p, q, r \) the dynamic variables, \( \dot{\alpha}, \dot{\beta}, \dot{p}, \dot{q}, \dot{r} \) the unsteady variables, and \( \delta_{\text{controls}} = [\delta_a, \delta_e, \delta_r, \delta_T, \delta_f] \) the control inputs. If a complete set of aerodynamic coefficients for all states can be generated, the model can be built. The mathematical structure of how the \( C_i \) terms are calculated defines the aerodynamic model. The following partitioning of \( C_i \) works reasonably well,

\[ C_i = C_i,\text{static}(V; \alpha, \beta) + \Delta C_i,\text{dynamic}(\alpha, p, q, r) + \Delta C_i,\text{unsteady}(\dot{\alpha}, \dot{\beta}) + \Delta C_i,\text{controls}(\alpha, \delta_a, \delta_e, \delta_r, \ldots) \] (2.15)

Hence the main difficulty in developing a simulation model is determining all these \( C_i,k \) coefficient terms due to aerodynamic effects which can be complicated. There are several ways to calculate or estimate these aerodynamic coefficients. For models in the normal flight envelope, some aerodynamic coefficients are given as functions converted into linear look-up tables, while others are expressed in terms of aerodynamic derivatives which are linear with respect to state variables but may themselves vary nonlinearly with flight condition or aircraft configuration [53]. For specific stall models such as in Reference [39], look-up tables extended to high \( \alpha, \beta \) ranges are used. Possible sources of the model data can be theoretical estimates, CFD, and/or wind tunnel data. Real data from flight tests or accidents may also be used, but would require the use of a parameter estimation algorithm to estimate the aerodynamic forces and moments. High fidelity models need to combine data from all these methods for best results.

If a high fidelity model at and past stall can be built for any configuration within a class, it can be used as the input baseline model, upon which the representative stall model of the target model in the same class will be built. The fact that such an accurate baseline stall model will be available is a fundamental assumption to make this methodology feasible. Once this is available, two additional challenges must be overcome to build the configuration adjustable representative stall model without custom CFD calculations or wind tunnel tests. One is capturing essential aircraft stall characteristics by understanding how they relate to aerodynamic force and moment coefficients for the configurations of interest. The other is gathering enough usable data on how these coefficients change with respect to variations in configuration parameters such as wing sweep, tail height, nacelle location, etc. in the stall regime. The scarcity of stall data with configuration effects will require combinations of theoretical insight and available data. The next two sections review existing knowledge of these two areas.
2.2 Stall Aerodynamics and Characteristics

An aerodynamic stall is a condition where the angle-of-attack increases past a critical value leading to a reduction in lift. At this critical angle-of-attack, the attached flow starts to separate from the airfoil surface as seen in Figure 2.2, so that further increasing the angle-of-attack produces even less lift and more drag [18]. Besides loss of lift, the lift curve becomes negative; this makes the stall unstable because, as the lift is reduced, the vertical component of air velocity increases and in turn increases angle-of-attack further. There are many ways that an aircraft may behave upon stalling. For example, the sudden loss of lift can lead to rapid altitude loss. One wing may stall before the other causing the aircraft to lose lateral stability and roll suddenly to one side. A spin may occur if an aircraft is stalled and there is an asymmetric yawing moment applied to it. For a T-tailed aircraft, a deep stall in which the turbulent wake of a stalled wing flows over the horizontal stabilizer and elevators is especially dangerous as the pilot loses elevator control authority needed to adjust pitch and recover [15]. Stall behaviour is heavily dependent on the aircraft configuration. While some stall characteristics can act as benign stall warning signs, others can lead to loss-of-control.

There are significant difficulties associated with modeling stall dynamics for a complete aircraft. The aerodynamic forces and moments become more strongly coupled; for example, an airplane with roll-off tendencies will have a lateral rolling moment exacerbated by an increases in the longitudinal angle-of-attack variable. Component contributions and interference become more nonlinear, so that aerodynamic forces and moments become sensitive to even small changes in aircraft configuration. The quasi-steady assumption that the aerodynamic coefficients don’t depend on time histories or derivatives of state variables becomes invalid. Finally, because the aircraft often becomes unstable and there is often a lack of comprehensive flight test data, it becomes difficult to perform a thorough validation of the post-stall aircraft model. Errors in the time histories between a flight test in the real aircraft and a flight model with the same control inputs can become arbitrarily large due to instabilities that lead to chaotic motions, even when the aerodynamic model is reasonably accurate.
For stall prevention and recovery training, some of these difficulties can be alleviated. According to Reference [20], it should be enough that models developed for ground simulator training have stall characteristics comparable to the real aircraft within a certain envelope past stall. Beyond the envelope, the training simulation could stop and reset until the pilot learns to not exceed it, and/or warn the pilot that the dynamics are no longer representative. This would at least ensure that the conditions and procedures pilots need to learn can be simulated; actual training benefits can be tested later with pilot groups in a controlled study. Of course with relaxed accuracy requirements, efforts will still be made to achieve adequate model accuracy, especially for aerodynamic coefficients with a large impact on stall behaviour.

A summary of stall characteristics essential for simulator training flight models, identified from literature and experience, is presented below. They are described along with typical aerodynamic coefficient curves and the impact of key configuration parameters. While priority is placed on static aerodynamic phenomena, discussions on dynamic, unsteady, controls, and secondary effects are included for completeness.

### 2.2.1 Reduced Longitudinal Static Stability

A **g-break** is the sudden change in vertical acceleration due to loss of lift at stall. It is primarily due to wing flow separation that is large and fast enough to cause a negative jump in the g-force and sudden sinking felt by the pilot. This occurs when the static $C_L(\alpha)$ curve decreases past $C_{L,max}$, and the effect is more pronounced with a sharper peak/drop in the curve. Because the wings generate the vast majority of total aircraft lift, their shape parameters (sweep, airfoil, etc.) affect the sharpness of $C_L(\alpha)$ and hence the g-break characteristic the most as seen in Figure 2.3.

![Figure 2.3: $C_L(\alpha)$ curve relation to wing sweep affecting g-break at stall.](Reprinted from Reference [3])

A **pitch-break** is a sudden change in pitch attitude near stall. Depending on design, the pitch break can be stable ($C_{m_{\alpha}} < 0$) if the static $C_m(\alpha)$ curve steepens before stall so that the nose pitches back down, or unstable if the $C_m(\alpha)$ curve slope starts to become more positive so that the aircraft experiences a pitch up aggravating the stall. The configuration parameters of the wing (sweep, aspect ratio) and the
horizontal tail (height) have the largest effect on pitch break. For example with swept wings the tips tend to stall before the root leading to a nose-up moment increment see in Figures 2.4 and 2.5.

![Figure 2.4: Planform vs. stall. (Reprinted from Reference [69])](image1)

![Figure 2.5: Sweep and AR effect on $C_m(\alpha)$ pitch break tendency. (Reprinted from Polhamus [48])](image2)

The stall location on the wing can affect the up-/down-wash at the tail which can have a large impact on the pitch moment. Inboard stall tends to cause upwash which causes a nose down moment, while outboard stall tends to increase downwash leading to nose up moment. The position of the horizontal tail affects the effect of wing downwash on the tail contribution to pitching moment as well as the the angle-of-attack at which the stalled wing wake hits the tail. For low-tailed aircraft, the tail quickly moves below the wing wake. As a result, the $C_m(\alpha)$ curve is relatively unaffected and stays negative, or there is a further stabilizing nose-down pitch break due to a steepening of the $C_m(\alpha)$ curve at stall. However for high tails such as T-tails, the wing wake is often just starting to blanket the horizontal tail near stall. This causes the $C_m(\alpha)$ curve to bend up sharply near stall as seen in Figure 2.6(b).

![Figure 2.6: Tail height effect on $C_m$ vs. $C_L$ pitch break tendency. (Reprinted from Chambers [12])](image3)
2.2.2 Reduced Lateral-Directional Static Stability

A degradation or loss in directional and lateral stability \((C_{n\beta} > 0 \text{ and } C_{l\beta} < 0 \text{ in stability axis})\) at stall results from altered forebody aerodynamics and/or turbulent flow around the aircraft decreasing the dynamic pressure on surfaces that provide stabilizing moments. The vertical tail, the fuselage, and the wing are the main contributors to lateral-directional stability. For example at stall, the stabilizing effects of the vertical tail is reduced due to blanketing from the wake of the wing-body. As a result the contributing factors to \(C_{n\beta}\), which determines directional static stability, are diminished during stall shown in Figure 2.7. Configuration parameters that affect how much of the vertical tail is blanketed by the wing-fuselage such as fuselage size, wing location, and tail size are the biggest factors affecting \(C_{n\beta}\). Directional instability leads to nose-slice/yaw-off or even spins.

![Figure 2.7: \(C_{n\beta,V}\) reduction at high \(\alpha\).
(Reprinted from Chambers [12])](image)

During stall the lateral stabilizing effects of the wing due to dihedral and sweep can be diminished or even reversed when the flow separates on the wing asymmetrically. This can affect the wing’s contribution to the coefficients \(C_{l\beta}\), and is largely sensitive to configurations of the wing planform parameters and slats/flaps as seen in Figure 2.8. The compromised roll stability during stall when combined with asymmetric wing stall often leads to wing-drop/roll-off.

![Figure 2.8: Example lateral characteristics of wing configuration effecting \(C_{l\beta}\).
(Reprinted from Chambers [12])](image)
2.2.3 Dynamic & Unsteady Characteristics

The previous stall characteristics correspond to static aerodynamic coefficients $C_{i,\text{static}}(\alpha)$ for $i = L, m, n, l$. Although they cover the basic behaviours to model for training, other behaviours associated with dynamic $C_{m_\alpha}, C_{n_r}, C_{l_\beta}, C_{n_\beta}$ and unsteady $C_{m_a}, C_{n_a}, C_{l_\beta}$ terms should be mentioned as well as they are linked to dynamic stability and damping (e.g. $C_{n_r} < 0$ to counter spin and $C_{l_\beta} < 0$ to counter wing rock) [41]. Unfortunately data on these terms, extracted from rotary-balance or forced-oscillation wind tunnel tests, are lacking for most configurations, depend highly on rotation rates, have coupled dynamic and unsteady terms, and can be difficult to use as derivatives due to strong nonlinearities. They are important however, as trends show significant deviations near stall as shown in the figures below. The configuration parameters that affect these coefficients are mostly similar to their static counterparts in the same axis. The following figures show typical variations with angle-of-attack:

![Figure 2.9: Pitch damping changes w.r.t. rotation rates $k$.](image1)

![Figure 2.10: Roll damping curve for one configuration.](image2)

![Figure 2.11: Yaw damping changes w.r.t. sweep angles.](image3)

In the pitch axis, $C_{m_\alpha}$ and $C_{m_a}$ depend on horizontal tail and wing planform specifications. At stall conditions, these contributions are altered by flow separation, greatly reducing the stabilizing damping terms as seen in Figure 2.9; it is also seen that the amount of damping lost is highly dependent on rotation rate $k$. This can lead to unstable pitching behaviour that can combine with a pitch-up leading to pitch bucking oscillations, or to falling-leaf motions when combined with lateral-direction instability.

In the roll axis, $C_{l_\beta}, C_{l_\alpha}, C_{l_\beta}$ depend mostly on wing planform. At stall conditions, these contributions are altered by flow separation, greatly reducing the stabilizing damping terms as seen in Figure 2.10. With this loss of damping in roll, an asymmetrical roll moment can cause an oscillation known as wing rock. The location of the roll damping sign reversal corresponds to the location of the wings’ $C_{L,max}$, so highly swept wings would result in earlier positive $C_{l_\beta}$ (negative damping) [8].

In the yaw axis, $C_{n_r}, C_{n_\beta}, C_{n_\beta}$ depend mostly on the vertical tail and fuselage. At stall, these contributions are altered by flow separation, reducing the stabilizing damping terms as seen in Figure 2.11. Under these conditions, $C_{n_r}$ is less negative and a spin may occur if an aircraft is stalled and there is an asymmetric yawing moment applied to it. The spin is a stall-induced autorotation where one wing stalls more deeply first causing it to drop; combined with a yawing moment, the stalled wing’s higher drag and lower lift leads to a sustained yaw towards the stalled wing. Altitude can be lost very rapidly.
2.2.4 Reduced Control Effectiveness

In the stall regime, control surfaces may be engulfed in the separated flow of the wing or another surface. This significantly reduces the forces and moments (represented by $\Delta C_{i,\text{controls}}(\alpha, \delta_a, \delta_e, \delta_r)$ terms) produced by them. To model representative control effects in stall, the control derivatives need to scaled accordingly to reduce their authority in the stall regime. Figures 2.12 and 2.13 from Reference [45] both suggest that the control moments are reduced in a roughly linear fashion after a certain point around stall. Reduction of rudder control authority in yaw, $\Delta C_{n \delta_r}(\alpha)$, is caused by similar effects as reduced directional stability of the vertical tail, i.e. lower dynamic pressure due to disturbed flow by the wing-body. Roll control reduction for the ailerons would be affected by the wing-planform-dependent stall progression already discussed for wing-related pitch break, reducing $\Delta C_{l \delta_a}(\alpha)$. Pitch control reduction for the elevators would be similarly affected as the horizontal tail shadowing by the wing wake as discussed for tail-related pitch break, reducing $\Delta C_{m \delta_e}(\alpha)$.

![Figure 2.12: $\Delta C_n$ due to aileron $\delta_a$ and differential stabilizers $\delta_d$ vs. Angle-of-Attack.](Reprinted from Nguyen [45])

![Figure 2.13: $\Delta C_l$ due to aileron $\delta_a$ and differential stabilizers $\delta_d$ vs. Angle-of-Attack.](Reprinted from Nguyen [45])

2.2.5 Secondary characteristics

Aerodynamic buffeting

Buffeting is a high frequency vibration felt by the pilot that occurs typically at low speeds and high angles-of-attack. The separated boundary layer is unsteady which leads to vibrations either on the wing itself or when the varying wake impinges on the tail. Because buffeting is a cue to the pilot that a stall is imminent, it should be simulated to train stall prevention. Without working out the buffeting forces and moments, the effect can be simply added to the simulation motion base as a high-frequency special effect [39]. Buffeting characteristics depend on the relative locations of the wing with the tail assembly, and the flow separation behaviour near stall. Buffeting will not be included using the present study’s methodology.

Deep stall

For T-tailed aircraft a dangerous phenomenon called deep stall and associated stall lock-in can occur, where the aircraft has a stable trim point at a very high angle-of-attack but the elevator authority is insufficient to create a nose-down restoring pitching moment. As seen in Figures 2.6 and 2.14, the relative
height between the wing and horizontal tail has the largest effect on deep stall characteristic. Since the representative model range requirement is up to 10 degrees past stall only, modeling reduced stability before deep stall associated with T-tail aircraft even occurs should suffice.

![Figure 2.14: Tail height affecting $C_m(\alpha)$ and deep stall.](Reprinted from Ray [52])

In summary, the essential stall characteristics for a representative stall training model are pitch break, g-break, reduced lateral-directional stability potentially leading to roll-off or yaw-off. These stall characteristics manifest themselves in the static coefficients $C_m, C_L, C_{\ell\beta}, C_{n\beta}$ respectively. Some dynamic and unsteady characteristics, manifested in the $C_{m_a}, C_{n_r}, C_{l_p}, C_{l_q}, C_{m_{\theta}}, C_{n_{\phi}}, C_{l_{\beta}}$ terms, that can lead to spins and wing-rock, will be incorporated when their impact is large and relevant data is available. Controls terms that model reduced pilot input authority, represented by $\Delta C_{m_{\delta e}}, \Delta C_{l_{\delta a}}, \Delta C_{n_{\delta r}}$, are not considered in the current work. Lastly, the secondary effects of deep stall and buffeting are outside the scope of this current study. It is assumed that $C_i = C_i(\alpha)$ for all $i$; coming sections on scope of study and state variable boundaries will justify this sole dependence on $\alpha$ as a starting point.

### 2.3 Configuration Effects Data

To predict configuration effects on stall characteristics, experimental data on how post-stall aerodynamic coefficients vary with respect to design parameters are needed. The data will be either used as training datasets in spline regression or as validation data. Although some basic configuration effects have already been discussed in the previous section on stall characteristics, detailed studies of individual configuration parameters done in wind tunnel tests for configurations similar to T-tailed regional turbojet aircraft are required to yield accurate results. Tables 2.1 and 2.2 summarize the availability and sources of existing wind tunnel studies found in the open literature. The coefficients are split into static, dynamic, unsteady, and controls contributions. Note that red cells represent strong effects, yellow cells medium effects and unhighlighted cells represent relatively weak effects. It can be seen that not all important contributions are readily available, and also that not all published effects are necessarily important for stall modeling. A comprehensive post-stall model would include every term in these tables, but emphasis is placed on effects with the biggest impact on the final aerodynamic model. The Scope of Study section will list which cells within these tables this study will address.
### Longitudinal Aero Coefficients References

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Table 2.1: References to public post-stall wind tunnel data with configuration variations (longitudinal). Red cells indicate high importance for post-stall modeling; yellow indicates medium importance.

### Lateral Aero Coefficients References

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Table 2.2: References to public post-stall wind tunnel data with configuration variations (lateral). Red cells indicate high importance for post-stall modeling; yellow indicates medium importance.

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1 Static lateral effects of the wing are due to asymmetrical lift during stall, where the effect is altered by wing configuration.

2 Important effect but calculated directly using tail arm and $(C_Y)_v$; little need for literature data.
2.4 Existing Configuration/Stall Modeling Techniques

Similar efforts found in literature, either only build configuration adjustable linear models or only model high angle-of-attack flight for one configuration. Most existing methods are therefore insufficient for achieving a model with the desired stall characteristics as well as configuration variation effects. For example, the USAF Data Compendium (DATCOM) \[29\] methods can predict a complete set of stability and control derivatives without resorting to outside information for many given configurations and flight conditions. Although very helpful for most configuration variation effects, the DATCOM methods are not applicable in the post-stall regime. This is true for most other analytical estimation techniques including the classic methods presented in the works of Roskam \[54\] and Etkin \[18\].

In terms of stall aerodynamic modeling methods, Paul et al in Reference \[46\] offers a post-stall modeling method using iterative decambering that can quickly calculate basic aerodynamic coefficients for moderate-fidelity level simulations. However the results are both not very accurate and currently limited to straight wings and low tails not accounting for fuselage and other effects. The post-stall models developed by Murch and Foster \[42\], while incorporating high fidelity wind tunnel data, only apply to one configuration. The representative modeling method used by Donaldson et al \[17\] and Gingras \[24\] while similar to the present philosophy of representative modeling for simulator training, also only applies to one full configuration. Considering these and more examples such as the SUPRA model in Reference \[2\], it is concluded that the existing methods either only model one specific configuration in the stall regime, or account for configuration variation effects only in pre-stall. Thus the desired representative model poses a unique problem, which can only be solved by combining knowledge of stall aerodynamics with data of configuration variation effects.

2.5 Departure Prediction Criteria

LOC accidents are not caused by stall per se, but by departure from controlled flight that often follow a stall. It is therefore important to understand aircraft departure tendencies and factors that influence it. In the context of aircraft stability and control, a “departure” refers to when an aircraft goes from controlled flight to uncontrolled flight, departing from the intended flight path; here it is used synonymously with LOC. Departure may be characterized by sustained divergent, large amplitude, uncommanded aircraft motions, such as nose slice or pitch-up, spin or deep stall; an momentary angle-of-attack excursion alone is not considered a departure if it does not result in sustained uncommanded aircraft motion \[57\]. It is usually associated with directional instability, aerodynamic asymmetry, and unstable roll and yaw rate damping \[15\]. It should be noted that departure resistance, though directly related to stability, is not always the same as static or dynamic stability; it can be a combination of those concepts. The idea of departure prediction criteria or departure susceptibility metrics became increasingly popular from studies on fighter aircraft LOC in the post world war era, which aimed to cheaply predict departure tendencies of new fighter configurations early in the design process. Seltzer et al \[60\] has produced significant work summarizing existing and proposing new departure prediction metrics. A few of these metrics are reviewed here for qualitative stall model analysis.

Once a representative model is developed, the post-stall flight dynamics can be validated by experienced
test pilots in the simulator. Using departure prediction metrics, this study explores an additional method for assessing the model representativeness, as well as the sensitivity of departure tendencies to certain configuration changes. A model’s predicted departure characteristics can be checked against results from existing studies of a similar configuration at various angles-of-attack. Since the representative models establish relationships between configuration and aerodynamic coefficients, it will be possible to work backwards to find critical configuration thresholds at the boundary of certain metrics in stall; this is facilitated by the SPM method’s ability to algebraically relate configuration parameter(s) and aerodynamic coefficients. Although these metrics are based on linearized rigid body equations of motion, they have value when used as additional validation tools prior to actual pilot training studies [41].

**Metrics based on Static Stability**

From basic aerodynamics, the most obvious parameters used to predict departure tendencies are the linear static stability derivatives. $C_{m_\alpha} < 0$ indicates static pitch stability preventing pitch-up and deep stall; $C_{n_\beta} > 0$ indicates static directional/weathercock stability which helps prevent yaw-off; $C_{l_\beta} < 0$ indicates static lateral stability (effective dihedral) preventing potential roll-off. An aircraft that was designed to be statically stable in all three axes in normal flight, may see one of these criteria violated near or at stall, increasing the likelihood for departure from controlled flight.

**Metrics based on Dynamic Stability**

Even if all static stability terms are satisfied, a lack of dynamic stability could lead to diverging behaviour and/or unstable oscillations. Thus the open-loop linear dynamic stability derivatives are useful as departure prediction criteria. $C_{m_q} < 0$ ensures pitch damping to prevent prolonged longitudinal oscillations, $C_{n_r} < 0$ gives positive yaw damping which helps prevent spins, and $C_{l_p} < 0$ indicates roll damping which helps prevent wing-rock. An aircraft that was designed to be statically and dynamically stable in all three axes in normal flight, may see one of these dynamic criteria violated near or at stall, leading to one of the unstable high angle-of-attack phenomena and departure described earlier.

**Open-Loop Departure Susceptibility Metric**

$C_{n_\beta,dyn}$ is an open-loop stability metric also known as the lateral divergent parameter, derived from the general aircraft rigid body equations-of-motion by considering the stability of the linear uncoupled lateral-directional dynamics. More specifically, it is derived by applying Routh’s stability criteria to the state-space representation, $\dot{x} = Ax$, of the linearized equations of motion described in Section 2.1. $C_{n_\beta,dyn} > 0$ is a necessary (but not sufficient) condition for all of the eigenvalues of $A$ to have negative real parts, and has proven useful for determining departure in yaw despite many simplifying approximations for many configurations [40]. It is therefore a quick indication of dynamic stability without calculating the aircraft eigenvalues by only using static aerodynamic derivatives.

$$C_{n_\beta,dyn} = (C_{n_\beta})_B \cos \alpha - \left( \frac{I_z}{I_x}_B \right) (C_{l_\beta})_B \sin \alpha > 0 \quad (2.16)$$

**Lateral Control Departure Parameter**

LCDP is a closed-loop metric that can indicate lateral-directional stability with respect to roll control. It indicates conditions where roll reversal is likely to occur and departure is likely to follow. The typical
condition for LCDP having a negative value arises when yaw due to aileron control becomes negative, usually the results of the second term in Equation 3.24b over powering the positive values of $C_{n,3}$

$$LCDP = (C_{n,3})_{S} - (C_{l,3})_{S} \left( \frac{C_{n,4}}{C_{l,4}} \right)_{S} > 0 \quad (2.17)$$

**Integrated Bihrle-Weissman Chart**

Effort has been made to use both LCDP and $C_{n,3, dyn}$ to define regions of (in)stability as shown in Figure 2.5. A study done by Weissman [70] has shown that this tool for predicting departure characteristics and spin susceptibility provides strong correlation with test results for a variety of aircraft types. Stall models can thus be graphed on the Bihrle-Weissman chart to gain qualitative understanding of how certain configuration parameters affect overall stability. As the angle-of-attack is increased, the values of LCDP and $C_{n,3, dyn}$ for the stall model trace out a curve, which varies (shifted or stretched) with configuration parameter(s). The regions on the chart generally relate to the following behaviour:

- **A** - Highly departure and spin resistant
- **B** - Spin resistant, objectionable roll reversals can induce departure and post stall gyrations
- **C** - Weak spin tendency, strong roll reversal results in control induced departure
- **D** - Strong departure, roll reversal, spin tendencies
- **E** - Weak spin tendency, moderate departure and roll reversals, affected by secondary factors
- **F** - Weak departure and spin resistance, no roll reversals, heavily influenced by secondary factors
- **U** - High directional instability.

It is proposed that a representative stall model with correct departure tendencies, would give more confidence for its applicability in simulator training.

### 2.6 Scope of Study

This study focuses on lateral-directional coefficients with the exception of longitudinal contributions of the horizontal tail. Wing contributions, though obviously important, are excluded as they are covered in a parallel UTIAS study using a semi-analytical technique. The project requires that stall behaviour be modelled up to 10 degrees past stall, or roughly up to 25°. Furthermore only coefficients that are – linked to the essential stall characteristics, referenced in the public data matrix, and have high impact on the stability metrics – are modeled. Configuration parameters varied are those typical within a family of transport aircraft, where fuselage length and wing-tail distances differ while local design parameters (i.e. wing planform, tail planform) remain relatively similar. The application of the modeling methodology to a variety of aerodynamic terms should illustrate how it may apply to other coefficients and/or configurations. To summarize, the following are developed in detail using the SPM method:
• Horizontal tail (H) contributions to longitudinal static coefficient – $C_m(\alpha)$

• Vertical tail (V) contributions to lateral-directional static coefficients – $C_{Y_\beta}(\alpha), C_{l_\beta}(\alpha), C_{n_\beta}(\alpha)$

• Fuselage (F) and forebody (Fb) contributions to directional static coefficient – $C_{n_\beta}(\alpha)$

• Horizontal tail (H) and Vertical tail (V) contributions to pitch damping and yaw damping coefficients - $C_{m_q}(\alpha)$ and $C_{n_r}(\alpha)$ respectively

Static coefficient estimates are validated against wind tunnel results from Reference [51], which contains post-stall data for lateral-directional coefficients for a twin engine T-tail jet. Qualitative departure tendency analysis for a baseline and target aircraft using the Bihrlle-Weissman chart is briefly carried out in Section 5.3.
Chapter 3

Methodology

The high-level aerodynamic model is first defined and modified to show how it will accommodate configuration variation effects to predict the target model given a baseline model. Then, the procedure for estimating individual configuration change effects and application of the SPM method are explained. Lastly, basic departure prediction metrics as applied to target and baseline stall models are described.

3.1 Aerodynamic Model Synthesis

It has been shown earlier that the full aerodynamic model can be partitioned into,

\[ C_i = C_{i,\text{static}}(\alpha, \beta, V) + \Delta C_{i,\text{dynamic}}(\alpha, p, q, r) + \Delta C_{i,\text{unsteady}}(\dot{\alpha}, \dot{\beta}) + \Delta C_{i,\text{controls}}(\alpha, \delta_a, \delta_e, \delta_r, \ldots) \]  (3.1a)

or more compactly,

\[ C_i = C_{i,\text{ST}} + C_{i,DYN} + C_{i,UNS} + C_{i,CTL} = \sum_k C_{i,k} \]  (3.1b)

for \( i = L, D, Y, l, m, n \). Here \( C_{i,DYN}, C_{i,UNS}, C_{i,CTL} \) shall always imply \( \Delta C_{i,DYN}, \Delta C_{i,UNS}, \Delta C_{i,CTL} \) respectively. This suppression of the \( \Delta \) notation simplifies the incorporation of configuration change terms, by eliminating the need to explicitly write out double delta’s, for example \((\Delta \Delta C_{i,DYN})_{\text{tail}}\). Each partition coefficient \( k \) can be written as a sum of aircraft component contributions from vertical tail \((.)_V\), fuselage \((.)_F\), wing \((.)_W\), etc. plus any additional component contributions. So for all \( i \) and \( k \),

\[ C_{i,k} = \sum_j (C_{i,k})_j = (C_{i,k})_F + (C_{i,k})_W + (C_{i,k})_V + (C_{i,k})_H + (C_{i,k})_{I_{NH}} + \ldots \]  (3.2)

Now instead of directly predicting each of the \((C_{i,k})_j\) terms of the target aircraft, the stall aerodynamic differences between two aircraft due to configuration variations are captured by adding \( M \) contribution change terms \((\Delta C_{i,k})_j\) to the baseline \( C_{i,k} \), each corresponding to the change in component \( j \)'s contribution due to all configuration changes. Note that each of the \( M \) contribution change terms is still a function of many configuration parameters, not only those describing \( j \) itself. This approach is preferred for simplicity and accuracy since the direct predictions in pre-stall will be slightly off-target but transformed to match regardless. For consistency and compactness in the upcoming discussion, a notation system for aerodynamic coefficients \( C_{i,k}^a \) and configuration change effects on \( j \)-contribution \((\Delta C_{i,k})_j\) is defined:
The representative stall model for the target aircraft $C_{TS}^T$ is generated by adjusting an existing stall model of the baseline aircraft within the same class $C_{BS}^B$, using the sum of component-$j$ contribution changes $(\Delta C_{i,k})_j$ due to relative differences in configuration parameters. This is followed by matching and blending to an existing pre-stall model of the target aircraft $C_{TP}^T$, ensuring pre-stall fit and curve continuity. By only extending models within a class, the aircraft parameter range for the extension would be relatively small. In addition the extension only requires the near post-stall region, as stall recovery training stays within 10 degrees past stick-shaker angle-of-attack. In short, the modeling problem statement is: Given $C_{BS}^B$ and $C_{TP}^T$, find $C_{TS}^T$ using $(\Delta C_{i,k})_j$ terms for all $i, k, j$ when available.

**Step 1**

The baseline aircraft stall model $C_{BS}^B$ is separated into its $k$ partition coefficients. Breaking the model down this way ensures clarity, since each $k$-coefficient in the stall model can be separately validated.

$$C_{BS}^B = \sum_{k} C_{i,k}^{BS} = C_{i,ST}^{BS}(\alpha,...) + C_{i,DYN}^{BS}(\alpha,...) + ... \quad (3.3)$$

Each of the $k$-partition terms is appended by $M$ available $(\Delta C_{i,k})_j$ terms to capture configuration change effects on $j$-component contributions, going from the baseline to the target configuration. This gives the estimated target stall model coefficients $C_{i,k}^{TE}$.

$$C_{i,k}^{TE} = C_{i,k}^{BS} + M \sum_{j} (\Delta C_{i,k})_j \quad (3.4)$$

In Equation 3.4, the $(\Delta C_{i,k})_j$ terms can be added on to any baseline model structure. The modified coefficients (static, dynamic, etc.) terms are then recombinined to get the full estimated target stall model,

$$C_{i}^{TE} = \sum_{k} C_{i,k}^{TE} \quad (3.5)$$

**Step 2**

The estimated target stall model $C_{i}^{TE}$ is transformed (shifted and scaled using constants $a, b, c, d$) such that it matches the given target aircraft pre-stall model $C_{i}^{TP}$ before $\alpha_{stall}$. The result is called $C_{i}^{TE}$.

$$C_{i}^{TE} = T(C_{i}^{TE}) = a[C_{i}^{TE}(b\alpha - c)] + d \quad \text{such that,} \quad (3.6a)$$
\[(C_{T_i}^{P} - C_{T_i}^{E})^2 \text{ is minimized on } [0^\circ, \alpha_{\text{stall}}] \quad (3.6b)\]

**Step 3**

The given pre-stall model of the target aircraft \(C_{T_i}^{P}\) and the transformed estimate of the target aircraft stall model \(C_{T_i}^{E}\) are then blended linearly over a certain range \([\alpha_1, \alpha_2]\) to ensure smoothness near \(\alpha_{\text{stall}}\) to arrive at the desired target aircraft stall model \(C_{T_i}^{S}\).

\[
C_{T_i}^{S} = \begin{cases} 
C_{T_i}^{P} & \text{for } \alpha_{\text{min}} < \alpha < \alpha_1 \\
(1 - \frac{\alpha - \alpha_1}{\alpha_2 - \alpha_1})C_{T_i}^{P} + \frac{\alpha - \alpha_1}{\alpha_2 - \alpha_1}C_{T_i}^{E} & \text{for } \alpha_1 < \alpha < \alpha_2 \\
C_{T_i}^{E} & \text{for } \alpha_2 < \alpha < \alpha_{\text{max}}
\end{cases} 
\quad (3.7)
\]

The application of these steps to real data will be shown in the Results and Discussions chapter.

### 3.2 Configuration Change Effects

The \((\Delta C_{i,k})_j\) terms in Step-1 are obtained by making SPM predictions (') for the target \((C_{T_i}^{i,k})_j\) and baseline \((C_{B_i}^{i,k})_j\) configurations, and then finding the difference between the two as shown in Equation 3.8. Using this approach, minor inaccuracies in the individual predictions are cancelled out, whereas the impacts of configuration changes are extracted.

\[
(\Delta C_{i,k})_j = (C_{T_i}^{i,k})_j - (C_{B_i}^{i,k})_j 
\quad (3.8)
\]

This works provided that the SPM method can estimate both target and baseline \(j\)-component contributions. Oftentimes available data only contains total aircraft force/moment coefficients or multi-component (cross-term) contributions rather than single component contributions. For simplicity, this study will strive to only use datasets where \(i, k, j\) are all equal (same axis, partitioned coefficient, component contribution) between the target and baseline configuration datasets. Finally, in case an accurate baseline stall model \(C_{B_i}^{S_i,k}\) cannot be obtained, the SPM predicted \(C_{T_i}^{S_i,k}\) can be used directly as the target extended model \(C_{T_i}^{S_i,k}\).

### 3.3 Shape Prescriptive Modeling Procedure

The Shape Prescriptive Modeling (SPM) method provides a framework for generating \((C_{B_i}^{i,k})_j, (C_{T_i}^{i,k})_j\) or simply \((C_{i,k})_j\) predictions by combining a priori knowledge of stall aerodynamics with least-squares spline curve fitting on usable configuration effects data. SPM first cuts down on the number of parameters/DOF in a generic cubic spline using theoretical arguments to prescribe function shape constraints; the constrained spline is then fitted to experimental data to capture configuration variation effects and to correlate remaining spline curve parameters to geometry parameters. The SPM fit is a balance between meeting the shape prescription constraints and minimizing errors to data. The resulting spline model can be expressed in closed form and is used to predict stall aerodynamic coefficients for new configurations. The accuracy of this method depends on the amount and quality of experimental data as well as established theory on stall phenomena, both of which were summarized in the Literature Review chapter. To apply SPM to stall modeling, the following 5 steps are followed:
1. Define \((C_i'_{i,k})_j\) in terms of aerodynamic effects, modeled by spline functions of configuration and \(\alpha\).

2. Set shape constraints based on physical arguments and/or known trends to limit spline DOFs.

3. Fit constrained spline to data for several sets where only one configuration parameter is varied.

4. Relate geometric parameter changes with curve parameter changes using low order polynomials.

5. Estimate new spline curve parameters for new configuration based on the relationship found in Step-4; these new curve parameters are then substituted in the expression for \((C_i'_{i,k})_j\).

Each of the five steps will be discussed in detail for component contribution coefficients considered in the next chapter on Model Synthesis. In the Results and Discussion chapter, the SPM estimate is applied twice to get both \((C_T'_{i,k})_j\) and \((C_B'_{i,k})_j\) to calculate \((\Delta C_{i,k})_j\), which is then summed to get the target extended model using \(C_{i,k}^E = C_{i,k}^S + \sum_j N_j (\Delta C_{i,k})_j\) as described earlier.

### 3.3.1 Stall Efficiency Factors

In Step-1 above, the coefficients \((C_i'_{i,k})_j\) are defined to give acceptable pre-stall estimates and incorporate stall effects using \(\alpha\)-dependent spline functions. Stall efficiency factors \(\eta_j^s(\alpha)\) will be used as an aid to highlight and/or model the onset and magnitude of changes in aerodynamic coefficients with respect to angle-of-attack, as literature results commonly measure efficiency in stall. Efficiency factors are suitable for interpreting stall because changes in aerodynamic force/momentum contribution are magnified with increased angle-of-attack. Additionally for representative models, the efficiency factor is an appropriate level of abstraction to capture general trends for training purposes with sufficient accuracy without unnecessary details associated with lower level often unknown effects. The term \(\eta_j^s(\alpha)\), which is required to be 1 at \(\alpha = 0\) and close to 1 in pre-stall, is obtained by dividing the the estimated high-\(\alpha\) SPM estimate \((C_i'_{i,k})_j\) by the classical linear coefficient estimate equations for very low angles-of-attack (e.g. Etkin, Roskam, DATCOM) for the \(j\)-component contribution \((C_i)_{j}\). The efficiency factors are thus functions of aerodynamic effects such as local dynamic pressure ratio \(q_j/q_\infty\), sidewash \(\sigma\), downwash \(\epsilon\), etc., which have been represented by spline functions dependent on angle-of-attack and configuration parameters \(p_1, p_2\), etc. In cases where aerodynamic effects data and physical understanding are lacking, the efficiency factor can be modelled directly as a blackbox using spline functions. Hence,

\[
(C_i'_{i,k})_j = (C_i)_{j} \cdot \eta_j^s(\alpha)
\]

\[
\eta_j^s(\alpha) = f(\epsilon, \sigma, q_j/q_\infty, \ldots)
\]

where \(S_y(\alpha, \ldots)\) is the spline function used to model effect \(y\). Note that while related, in this study the stall efficiency factor should be distinguished from the commonly used (horizontal or vertical) tail efficiency factor which represents only the ratio of local dynamic pressures, so that generally \(\eta_j^s \neq \eta_t = q_t/q_\infty\).

### 3.3.2 Spline Function Definition

A spline is a function defined by piecewise polynomials, with continuity and smoothness conditions set at the knots \(\alpha_i\), where the polynomials connect. Splines are versatile as they can take on a range of function
shapes while ensuring smoothness. Interpolation using splines is preferred over using polynomials because similar results can be achieved while Runge’s phenomenon \cite{16}, where high order polynomials become highly oscillatory/unstable, is avoided. Splines also make for easier physical interpretation than high-order polynomials, since the degree is kept low. Spline functions are used here to model trends in the post-stall regime as per the SPM method, where the only independent (x-axis) variable will be $\alpha$, and configuration parameters will be related to the curve coefficients $c_{ik}$. To represent a general spline $S(\alpha)$ of order $n$ with $m$ intervals, piece-wise polynomials $P_i$ of degrees no higher than $n$ with coefficients $c_{ik}$, are chosen such that

$$S(\alpha, \alpha_i, c_{ik}) = \begin{cases} P_1(\alpha) = \sum_{k=0}^{n} c_{1k} \alpha^k & \alpha_0 \leq \alpha < \alpha_1 \\ P_2(\alpha) = \sum_{k=0}^{n} c_{2k} \alpha^k & \alpha_1 \leq \alpha < \alpha_2 \\ \vdots & \vdots \\ P_m(\alpha) = \sum_{k=0}^{n} c_{mk} \alpha^k & \alpha_{m-1} \leq \alpha \leq \alpha_m \end{cases} \quad (3.11a)$$

$$P_i^{(j)}(\alpha_i) = P_{i+1}^{(j)}(\alpha_i) \text{ for } j = 0, 1, ..., n - 1 \quad (3.11b)$$

Note that the continuity requirements are relaxed to $C_1$ from hereon, so that only values and slopes but not curvature need be the same at the knots. The polynomial basis $\{1, \alpha, \alpha^2, ..., \alpha^n\}$ spans a space of possible curve shapes on each interval. The coefficients are to be related to aircraft configuration parameters. An alternative compact representation of splines using truncated polynomials can also be used, but the first piecewise notation will be used mostly for clarity.

$$S(\alpha) = \sum_{k=0}^{n} b_k \alpha^k + \sum_{i=1}^{m-1} c_i (\alpha - \alpha_i)^n \quad (3.12)$$

$$\begin{cases} 0 & x < \alpha_i \\ (\alpha - \alpha_i)^n & x \geq \alpha_i \end{cases} \quad (3.13)$$

The previously mentioned SPM steps are applied here now for a simple example going from a generic spline to a configuration dependent aerodynamic coefficient. In general multiple intervals are used to model curve detailed, so choosing knot locations $\alpha_i$ can become a difficult problem. However in this initial study, only two (sometimes three) intervals on $[0^\circ, \alpha_{crit}, 25^\circ]$ are used to capture general trends so that,

$$S(\alpha, \alpha_{crit}, c_{ik}) = \begin{cases} P_1(\alpha) = \sum_{k=0}^{n} c_{1k} \alpha^k & 0^\circ \leq \alpha < \alpha_{crit} \\ P_2(\alpha) = \sum_{k=0}^{n} c_{2k} \alpha^k & \alpha_{crit} \leq \alpha \leq 25^\circ \end{cases} \quad (3.14)$$

where $n \leq 3$ so the highest order would be cubic splines. This expression will serve as a starting point for all the Step-2’s of the SPM method seen in the next chapter, where shape constraints are set before fitting the spline function to data. With each physics based spline shape constraint, the general expression takes on a more defined mathematical form with fewer unknown coefficients $c_{ik}$. In this simple example, take the stall efficiency factor for component-j as the spline $\eta_j^s = S$. If known (and cited) or derived physical phenomena, call them, “Stall Effects A, B, C” impose the following shape prescriptions:

- Stall Effect A [Ref. x] $\Rightarrow$ Knot placement: Efficiency function trend changes sharply at $\alpha_{crit}$.
- Stall Effect B [Ref. y] $\Rightarrow$ Values: Efficiency is 1 when angle-of-attack is less than or equal to $\alpha_{crit}$. 
• Stall Effect C [Ref. z] ⇒ Polynomial order and slope: The efficiency decreases linearly after $\alpha_{crit}$.

Then many of the coefficients $c_{ik}$ become zeroes or fixed values, greatly simplifying the general spline expression from Equation 3.14. The total number of remaining free polynomial coefficients $c_{ik}$ plus the knot location $\alpha_{crit}$ of the spline equal the number of parameters/DOFs still left, which are to be fitted to data (Step-3) and correlated to configuration parameter(s) (Step-4). The simple example spline for the $j$-contribution efficiency factor $\eta_j^s$ now becomes Equation 3.15,

$$
\eta_j^s(\alpha, \alpha_{crit}, c_1) = \begin{cases} 
1 & 0^\circ \leq \alpha < \alpha_{crit} \\
1 + c_1(\alpha - \alpha_{crit}) & \alpha_{crit} \leq \alpha \leq 25^\circ
\end{cases}
$$

(3.15)

leaving just two free curve parameters $c_1, \alpha_{crit}$ to be fitted against data via spline regression techniques. Each fit to a particular configuration’s $\eta_j^{s, data}$ returns these curve parameters calculated to minimize residuals in the least-squares sense within constraints; this is then repeated for various values of one configuration parameter to return a set of curve parameters. So in this simple generic example, the constrained spline $\eta_j^s(\alpha, \alpha_{crit}, c_1)$ is fitted to a set of $\eta_j^{s, data}(\alpha)$ data each with, say, a different wing sweep $\Lambda_w$. In Step-4 the resulting parameters $c_1, \alpha_{crit}$ are each plotted against the configuration parameter varied $\Lambda_w$, and fitted with low-order polynomials (i.e. linear or quadratic) which would give relationships $c_1 = f(\Lambda_w)$ and $\alpha_{crit} = g(\Lambda_w)$. If these were substituted into the spline to express the function in configuration-to-coefficient form, they could be described by Equations 3.16 and Figure 3.1.

$$
(C'_{i,k})_j(\alpha, \Lambda_w) = (C_{i,k})_j \cdot \eta_j^s(\alpha, \Lambda_w)
$$

(3.16a)

$$
\eta_j^s(\alpha, \Lambda_w) = \begin{cases} 
1 & 0^\circ \leq \alpha < g(\Lambda_w) \\
1 + f(\Lambda_w)(\alpha - g(\Lambda_w)) & g(\Lambda_w) \leq \alpha \leq 25^\circ
\end{cases}
$$

(3.16b)

Figure 3.1: Simple example of spline function with two DOF.

This method would however only work if all other configuration parameters are the exact same for the dataset and the SPM prediction aircraft, or if the current parameter had a completely independent
effect. Because there are almost always multiple configuration parameters affecting each coefficient, the effects of varying one geometric parameter while holding all others constant are combined using Taylor approximations at an initial configuration (subscript 0) from available data closest to the new configuration (superscript ' for SPM prediction). This is done for each curve parameter as part of Step-5. Using the current example and a 1st-order expansion, if both the wing sweep \( \Lambda_w \) and the fuselage length \( l_F/b \) were varied such that the Step-4 yields \( c_1 = f(\Lambda_w, l_F/b) \) and \( \alpha_{crit} = g(\Lambda_w, l_F/b) \), and \( \Delta \Lambda_w = \Lambda'_w - \Lambda_{w,0} \) etc., then

\[
e'_1 = (c_1)_0 + \left( \frac{\partial c_1}{\partial \Lambda_w} \right)_0 \Delta \Lambda_w + \left( \frac{\partial c_1}{\partial (l_F/b)} \right)_0 \Delta (l_F/b) + \ldots \\
\alpha'_{crit} = (\alpha_{crit})_0 + \left( \frac{\partial \alpha_{crit}}{\partial \Lambda_w} \right)_0 \Delta \Lambda_w + \left( \frac{\partial \alpha_{crit}}{\partial (l_F/b)} \right)_0 \Delta (l_F/b) + \ldots
\]

Note that while the curve parameter \((c_1, \alpha_{crit})\) changes due to configuration changes are estimated using this linear technique, the resulting spline curve can still be cubic (or of lower order). This is similar for example, when a quadratic function \( ax^2 + bx + c \) is used to model a physical phenomenon, but the curve parameter \((a, b, c)\) variations due to physical changes are estimated using linear approximations.

### 3.3.3 Data Compatibility

Before carrying out Step-3 where the least-squares spline fitting is carried out, compatibility between the stall effect being modeled by the spline and the available data is required. In the case of multi-component contributions, if data is available, the cross-term interference effects can be found using,

\[
(C_{i,k})_{AB} = (C_{i,k})_A + (C_{i,k})_B + (C_{i,k})_{IAB} \tag{3.19a}
\]

\[
(C_{i,k})_{IAB} = (C_{i,k})_{AB} - (C_{i,k})_A - (C_{i,k})_B \tag{3.19b}
\]

where the subscript ()_{IAB} indicates cross-term interference effects for components A, B. Other times, data and the spline do not represent the same quantity. If for example, the dynamic pressure at component-\( j \), \( \frac{q_j}{q_\infty} \) is the effect to be splined by \( S_{q_j} \), the ideal dataset would contain data measuring \( \frac{q_j}{q_\infty} \) vs. \( \alpha \) for a few different values of a certain configuration parameter \( p_1 \) as in,

\[
\frac{q_j}{q_\infty}(\alpha) = S_{q_j}(\alpha, p_1) \sim \left( \frac{q_j}{q_\infty} \right)^{data} \tag{3.20}
\]

If the data does not measure the same quantity, it can be converted. For example, if the effect to be modeled was the overall stall efficiency \( \eta^*_j \), but the data contains the aerodynamic coefficient \( C_{i,k}(\alpha) \), the data would be transformed using,

\[
\eta^*_j(\alpha) = S_{\eta_j}(\alpha, p_1) \sim \left( \eta^*_j \right)^{data}(\alpha) = \left[ \frac{C_{i,k}^{data}(\alpha) \left( \frac{\partial C_{i,k}^{data}(\alpha)}{\partial (\alpha = 0)} \right)_{j}}{C_{i,k}(\alpha = 0)} \right] \tag{3.21}
\]

which would be then fit to the spline function for \( \eta^*_j \). Lastly in some cases, there is only data for the total coefficient without any data of the contributing factors or lower level effects represented by splines. For example the efficiency is modeled as the product of two effects \( e_1, e_2 \), each represented by a spline,

\[
\eta^*_j(\alpha) = f(S_{e_1}(\alpha, p_1), S_{e_2}(\alpha, p_2)) = S_{e_1}(\alpha) \cdot S_{e_2}(\alpha) \tag{3.22}
\]
but data is only available for the overall coefficient for $j$-contribution $C_{i,k}^{data}$. In this study, these cases are not considered. Instead, only datasets with easily compatible data will be used.

### 3.3.4 Spline Regression

Following data conditioning, the best fit curve is determined by minimizing the squares of residuals between the spline and data points on each interval, subject to previously selected shape constraints. From basic statistics, the sum of the squared residuals is given by,

$$SS = \sum_{i=1}^{n} (y_i^{data} - S_y(\alpha_i, c_{ik}))^2$$  \hspace{1cm} (3.23)

where $y_i^{data}$ is the dependent variable from training data and $S_y(\alpha_i, c_{ik})$ is the predicted value of $y_i$ from the spline fit based on angle-of-attack and spline parameters. The goal is to find parameters $c_{ik}$ within constraints that minimize $SS$. In practice, the minimization is carried out using the MATLAB Optimization Toolbox and built-in spline functions.

### 3.4 Application of Departure Prediction Criterion

The Bihrle-Weissman chart, based on the lateral-directional stability metrics $C_{n\beta, dyn}$ and LCDP, can be used as a simple qualitatively assessment of the impact of and sensitivity to certain configurations on departure tendencies. Once a representative stall model is created, the two metrics are calculated by substituting the total aircraft lateral-directional moment coefficients into the expressions for $C_{n\beta, dyn}$ and LCDP below. The baseline aircraft metrics will be calculated using,

$$C_{n\beta, dyn}^{B}(\alpha) = (C_{n\beta})_B \cos \alpha - \left(\frac{I_z}{I_x}\right)_B (C_{l\alpha})_B \sin \alpha$$  \hspace{1cm} (3.24a)

$$LCDP^{B}(\alpha) = (C_{n\beta})_S - (C_{l\alpha})_S \left(\frac{C_{n\delta\alpha}^{B}}{C_{l\delta\alpha}^{B}}\right)_S$$  \hspace{1cm} (3.24b)

Similarly the target aircraft metrics will be calculated using the same equations by substituting the $B$ superscripts with $T$. Note that LCDP is defined using stability-axis derivatives, whereas $C_{n\beta, dyn}$ is defined using body-axis derivatives; these are indicated using the subscripts $(\_)_B, (\_)_S$. When necessary, the moment coefficients are converted between these references frames using,

$$\begin{bmatrix} C_{l\alpha} \\ C_{n\beta}^{stability} \end{bmatrix} = \begin{bmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{bmatrix} \begin{bmatrix} C_{l\alpha} \\ C_{n\beta}^{body} \end{bmatrix}$$  \hspace{1cm} (3.25)

Qualitative results of the Bihrle-Weissman chart analysis can then be compared against results found in the literature on the LOC tendencies of similar configuration. In addition the sensitivity of departure tendencies can be tested by varying geometric parameters to gauge their impact.

#### 3.4.1 Aileron Effectiveness

In calculating LCDP above, aileron contributions to roll and yaw moments are needed. For a swept wing where the tips usually stall first, the ailerons are engulfed in separated flow early on and lose a
significant amount of control authority. For a straight tapered wing on the other hand, the wing roots typically stall first leaving the ailerons unaffected at first even after the wing has lost part of its lift. The adverse yaw due to differential lift and hence differential induced drag between the two wings due to deflected ailerons, is altered when the aileron loses lift upon entering stall. In general, control derivatives are difficult to predict accurately using simple analyses, so wind-tunnel testing or CFD results usually are needed. A simplistic way to estimate the needed ratio of aileron derivatives, is to use post-stall data of high angle-of-attack aileron moment contributions data collected on an aircraft with swept wings from Reference [45], so that

$$\frac{C_{\text{nsA}}}{C_{l_{A}}}(\alpha) = \eta_{l_{A}}(\alpha)$$

(3.26)

can be used to estimate LCDP using Equation 3.24b
Chapter 4

Model Synthesis

The five steps of the SPM method are applied to estimate the body-axis moment coefficient contributions of the horizontal tail, vertical tail, and fuselage forebody in the stall regime. At least one literature dataset for configuration effect extraction is used for each component contribution; only configuration parameters with the most important effects on coefficient curves are addressed. Rough estimates for damping coefficients in pitch and yaw are also included.

4.1 Horizontal Tail Contributions

The pitching moment contributions of various parts of a swept-wing transport category aircraft up to high angles-of-attack are shown in Figure 4.1. The horizontal tail provides almost all of the stabilizing negative pitch moment that offset the positive moment of the wing-body alone. It is therefore imperative for stall modeling to capture the longitudinal moment contributions of the horizontal tail. At high angles-of-attack the downwash due to wing experienced by the horizontal tail can change dramatically, depending on stall progression pattern of the wing and related wingtip vortex behaviour. In addition, the relative positions of the horizontal tail and the wing define the angle-of-attack at which the stalled turbulent wake shadows the horizontal tail, decreasing the local dynamic pressure reducing the tail lift. Additionally for the fuselage mounted nacelles seen in T-tail transport jets, the location of the nacelle can also significantly blanket the tail leading to further reduced dynamic pressure. Lastly while very unlikely, the horizontal tail itself can stall when the local angle-of-attack becomes large enough. All four effects mentioned diminish the ability of the tail to provide the pitching moment needed to maintain longitudinal stability. The SPM procedure is applied to predict static pitch moment $(C_m)_H = f(\alpha, H, W, N)$.

Step 1 - Model structure and stall effect spline functions

Starting with the standard equation for the tail pitching moment, the following approximation is made since $-\frac{l_H}{c} \gg (h - h_{nwb})$ and because the horizontal tail volume $V_H = \frac{S_H l_H}{S_c}$,

$$
(C_m)_H = \frac{S_H}{S} \left[ -\frac{l_H}{c} + (h - h_{nwb}) \right] \frac{q_H}{q_\infty} C_{L_{\alpha.h}} (\alpha_{wb} - i_t - \epsilon) \\
\approx -C_{L_{\alpha.h}} V_H \frac{q_H}{q_\infty} (\alpha_{wb} - i_t - \epsilon_{\infty})
$$

(4.1)
Chapter 4. Model Synthesis

Figure 4.1: Pitching moment contribution breakdown. (Reprinted from Shevell [61])

Where the downwash angle is commonly approximated using $\epsilon_\infty = \frac{2C_{L,w}(\alpha)}{\pi AR_w}$. In stall however, both the dynamic pressure and downwash angle at the tail vary nonlinearly; the model for the stall modified contribution of the horizontal tail to the pitching moment should be,

$$(C'_m)_{H}(\alpha) = -C_{L,h} V_H (\alpha - i_t - \epsilon(\alpha)) \frac{q_h}{q_\infty}(\alpha)$$ (4.2)

Where $\alpha - i_t - \epsilon(\alpha) = \alpha_t$ is the local angle-of-attack at the horizontal tail. It has been shown that for most configurations, the horizontal tail will not stall before $\alpha_{wb} = 25^\circ$ because the downwash keeps the tail's local angle-of-attack relatively low. Therefore the tail surface lift curve slope can be approximated using basic linear estimates from DATCOM [29]. At much higher angles-of-attack however, the eventual nonlinearity of the horizontal tail lift curve can be captured by substituting $C_{L,h}$ with nonlinear lift curves, but for the present simple model,

$$C_{L,h} = \frac{2\pi AR}{2 + \sqrt{4 + \frac{2R^2B^2}{K^2}(1 + \tan^2 \Lambda_c/2)}}$$ (4.3)

where $c_{l\alpha}$ below is estimated using the closest NACA airfoil data available,

$$B = \sqrt{1 - M^2} \quad \text{and} \quad K = \frac{c_{l\alpha}}{2\pi/B} \quad \text{with} \ c_{l\alpha} \ \text{being the section lift curve slope} \ (4.4)$$

**Spline Functions**

The two effects that change significantly with angle-of-attack are modeled using spline functions. Both the downwash $\epsilon(\alpha)$ and the dynamic pressure ratio $\frac{q_h}{q_\infty}(\alpha)$ depend on wing (W) planform, location of the horizontal tail (H) relative to the wing, and nacelle (N) location.

$$\frac{q_h}{q_\infty}(\alpha) = S(\alpha, W, H, N) \quad (4.5a)$$

$$\epsilon(\alpha) = S(\alpha, W, H, N) \quad (4.5b)$$

According to studies in Reference [10], the key configuration parameters affecting dynamic pressure and downwash changes are the relative position of the horizontal tail to the wings and fuselage-mounted nacelles. Downwash at the tail is affected by wing sweep and aspect ratio; however only swept wings are considered first. These can be seen in Figures 4.2 and 4.3.
The component dependencies $W, H, N$ are thus replaced by more specific configuration parameters,

$$\frac{q_h}{q_\infty}(\alpha) = S(\alpha, \theta_{WH}, \theta_{NH})$$ (4.6a)
$$\epsilon(\alpha) = S(\alpha, \theta_{WH}, \theta_{NH}, AR_w)$$ (4.6b)

where the wing-tail and the nacelle-tail shadowing angles are denoted $\theta_{WH}$ and $\theta_{NH}$ respectively. It is expected that as $\alpha$ gets closer to either of these angles, the tail efficiency diminishes for that component. The stall efficiency factor $\eta^*_H(\alpha)$ can be used to illustrate effect of stall on $(C_m)_H$ where $\eta^*_H(0) = 1$.

$$\eta^*_H(\alpha) = \frac{(C'_m)_H(\alpha)}{(C_m)_H(\alpha = 0)} = \frac{\alpha - i_t - \epsilon(\alpha)}{-i_t - \epsilon(0)} \cdot \frac{q_h}{q_\infty}(\alpha)$$ (4.7)

**Model Simplifications - Nacelle Interactions**

For T-tailed regional jets, the fuselage-mounted nacelles can have a significant impact on the pitch stability past stall via dynamic pressure shadowing effects on the horizontal tail. It is necessary to determine the extend of this impact for the angle-of-attack region of interest (10 degrees past stall). Using data from References [35] and [6], the interference effect in pitching moment between nacelle and horizontal tail is extracted using Equation 3.19b,

$$\frac{(C_m)_{I_{NH}}(\alpha)}{(C_m)_{NH} - (C_m)_N - (C_m)_H} = \frac{(C_m)_{FWNH} - (C_m)_{FWN} - (C_m)_H}{[(C_m)_{FWNH} - (C_m)_{FWN}] - [(C_m)_{FWH} - (C_m)_{FW}]}$$ (4.8)

This is applied to the three nacelle locations show in Figure 4.4, where location 2 is most forward,
The resulting nacelle interaction effects on $C_m$ in Figure 4.8 show that they actually become significant past $\alpha = 25^\circ$. If the pitching moment is needed past this range, the nacelle effects can be added on to the pitching moment curves, but for the on-going discussion on downwash and dynamic pressure spline functions, they are put aside to focus on wing-tail interactions.

$$q_h(\alpha) = S(\alpha, \theta_{WH}, \Lambda_w) \quad (4.9a)$$

$$\epsilon(\alpha) = S(\alpha, \theta_{WH}, AR_w) \quad (4.9b)$$

**Step 2 - Stall physics as shape prescriptions**

Starting with the general spline defined in Equation 3.14 for representative stall modeling, known and/or derived physical characteristics of stall converted into SPM shape constraints, are applied to arrive at
a lower-order piecewise equation. This is done to get rid of as many spline DOFs as possible, until an appropriate nDOF remain for parameter correlation.

\[
S(\alpha, c_{ik}) = \begin{cases} 
P_1(\alpha) &= \sum_{k=0}^{n} c_{1k}\alpha^k \quad 0^\circ \leq \alpha < \alpha_{crit} \\
P_2(\alpha) &= \sum_{k=0}^{n} c_{2k}\alpha^k \quad \alpha_{crit} \leq \alpha < 25^\circ 
\end{cases} \quad (4.10)
\]

**Experimental Insights**

Using data from extensive wind tunnel studies done by Kettle [35] at the Royal Aircraft Establishment, the downwash and dynamic pressure changes at the horizontal tail are analyzed. Measurements of pitching moment \( C_m(\alpha) \) were made on a swept wing aircraft with high tails and fuselage-mounted nacelles, by varying positions of the wings and horizontal tail, including tail incidence angles, in order to examine high angle-of-attack stall problems.

![Figure 4.9: Horizontal tail placement locations. (Reprinted from Kettle [35])](image)

The following technique is used to extract the average downwash and dynamic pressure at the horizontal tail using pitching moment data alone (nacelles off). Recall that,

\[
\alpha_t = \alpha_{wb} - i_t - \epsilon_t \quad (4.11)
\]

Using Figures 4.10 and 4.11 showing total pitching moment for the horizontal tail at various incident angles \( i_t \) and with the tail off, it can be deduced that the intersecting points between the tail-off curve and the various pitch moment curves are incident angles at which the tail angle-of-attack \( \alpha_t \) is virtually zero.

![Figure 4.10: Tail-off \( C_m \) with tail incidence angles for low wing. (Adapted from Kettle [35])](image)  
![Figure 4.11: Tail-off \( C_m \) with tail incidence angles for high wing. (Adapted from Kettle [35])](image)
This is because at the intersections, having the horizontal tail mounted produces no additional pitching moment compared to the case without a tail at all. In other words, at the highlighted intersecting points,

$$\epsilon_{t}(\alpha) = \alpha_{wb} - i_{t}$$  \hspace{1cm} (4.12)

So the relationship between the wing-body angle-of-attack and downwash at the horizontal tail can be plotted as \(\epsilon_{t}(\alpha)\) as shown in Figure 4.12. As for the average dynamic pressure at the horizontal tail \(\frac{q_{h}}{q_{\infty}}\), differentiating the pitching moment Equation 4.2 with respect to tail incidence angles \(i_{t}\) at locations where the local angle-of-attack \(\alpha_{t} = 0\), gives Figure 4.13

$$\frac{dC_{m}}{di_{t}} \bigg|_{\alpha_{t}=0} = C_{L,h} V_{n} \frac{q_{h}}{q_{\infty}}(\alpha)$$  \hspace{1cm} (4.13)

It is seen that for high wings (ie. low tail), the downwash passes over the tails past some \(\alpha\).

![Figure 4.12: Downwash vs. relative tail height.](image)

![Figure 4.13: Dynamic pressure vs. tail height.](image)

Additional supporting evidence of these trends are seen in the downwash and dynamic pressure profiles measured along various horizontal tail span locations shown in Figure 4.14. Note that these observations confirm the previous finding that nacelle effects are not significant until around \(\alpha = 30^\circ\). With these, the following shape constraints for downwash and dynamic pressure at the horizontal tail can be set.

**Shape primitives for \(\frac{q_{h}}{q_{\infty}}(\alpha)\)**

As the angle-of-attack increases, the wake from the wing, and the fuselage-mounted nacelles in the case of T-tailed turbojet configurations, blanket the horizontal tail. From the profile seen in Figure 4.14, below the critical angle-of-attack the dynamic pressure is constantly at 1. Past \(\alpha_{crit}\) the curve bends down sharply towards about 50% at 25\(^\circ\). The following will define the shape constraints with reference to aerodynamic phenomena gathered in the literature:

- Disregarding power effects, before stall neither the wings nor the nacelles (if applicable) generate perturbed airflow that affect the horizontal tails \([6]\) \Rightarrow Prior to \(\alpha_{crit}\) the spline is constantly 1.

- After the angle-of-attack increases past stall, the separated wing wake impinges the horizontal tail; this effect is fairly linear \([10]\) \Rightarrow Past \(\alpha_{crit}\) the curve should be linearly monotone decreasing.
Applying these shape constraints, only two curve DOFs \((c_1, \alpha_{\text{crit}})\) are left in the spline expression,

\[
\frac{q_h}{q_\infty}(\alpha, c) = \begin{cases} 
1 & 0^\circ \leq \alpha < \alpha_{\text{crit}} \\
1 + c_1(\alpha - \alpha_{\text{crit}}) & \alpha_{\text{crit}} \leq \alpha < 25^\circ 
\end{cases} 
\]

(4.14)

where \(c_1 \leq 0\) for non-positive slope and continuity conditions are implemented in MATLAB code.

**Shape primitives for \(\epsilon(\alpha)\)**

The wing downwash \(\epsilon\) significantly alters the effective angle-of-attack at the horizontal tail. As the wing-body angle-of-attack increases past stall, the downwash from the wings increases at higher rate as seen in Figure 4.14. This is because for swept wings, as the tips stall first, the vortices move inboard and closer to the tail assembly, enhancing the local downwash. The following will define the shape prescriptions with reference to specific aerodynamic phenomena:

- Downwash is proportional to the wing’s lift before stall according to the common used formula
  \[
  \epsilon_\infty = \frac{2C_L}{\pi AR_w} \]  
  [10] ⇒ Prior to stall the downwash curve should be monotone increasing linear.

- After stall depending on wing sweep and aspect ratio, the wing vortices will create additional downwash (swept configurations) [10] ⇒ After stall the downwash curve will change to slope, which is more/less positive depending on the tail height.

Note that while all \(\alpha_{\text{crit}}\) parameter should technically be fitted to data, in certain cases (such as the current spline) all data would be fitted to one value anyway. Therefore, that known value is used directly to simplify the spline fitting process. Applying these shape constrains and taking the critical angle-of-attack to be 15 degrees for this dataset, only two curve parameters remain:

\[
\epsilon(\alpha, c) = \begin{cases} 
    c_1\alpha & 0^\circ \leq \alpha < 15^\circ \\
    15c_1 + c_2(\alpha - 15^\circ) & 15^\circ \leq \alpha < 25^\circ 
\end{cases} 
\]

(4.15)

Requirements of monotonicity etc. are implemented in MATLAB code. Note that the value of the
starting constant $c_1$ is expected to be related to the downwash estimate for $\epsilon_\infty = \frac{2C_L}{\pi AR_\infty}$ which is linear prior to wing stall.

**Step 3 - Least squares spline fitting to data**

Although results in Figures 4.12 and 4.12 confirm previously stated assumptions about the downwash and dynamic pressure changes past stall, they are not enough to provide meaningful predictions.

**Fitting $\frac{q_h}{q_\infty}(\alpha)$**

To further understand the effect on dynamic pressures, data of a regional turbojet aircraft from Taylor [65] are used to fit the constrained splines. The wing-tail angle $\theta_{WH}$ values are tabulated in Step-4.

**Figure 4.16: Fitted spline for dynamic pressure with tail positions**

**Figure 4.17: Fitted splines with various tail positions**

The effect of wing sweep on the dynamic pressure, though lacking in data, is fitted as well and shown in Figures 4.18 and 4.19. The wing sweep values are shown in Step-4.

**Figure 4.18: Fitted spline for dynamic pressure with/without wing sweep**

**Figure 4.19: Fitted splines with/without wing sweep**
Fitting $\epsilon(\alpha)$

Studies by Taylor et al analyze configurations of a T-tailed swept wing aircraft are used to study the effect of $\theta_{WH}$, which are shown in Figures 4.20 and 4.21. The $\theta_{WH}$ values are tabulated in Step-4.

Figure 4.20: Fitted spline for downwash for various tail heights

Figure 4.21: Fitted spline for downwash for various tail heights

Additional measurements of downwash at the horizontal tail from References [22, 55, 56] all with highly swept (45°) wings, are fitted to the the same constrained spline function to find the effect of wing aspect ratio and tail height on downwash.

Figure 4.22: Downwash vs. relative tail height and $AR_w$ using data from three sources

Figure 4.23: Downwash vs. relative tail height and $AR_w$ using data from three sources

Step 4 - Configuration parameter fitting to spline parameters

Parameters for $\frac{\epsilon_h}{\epsilon_\infty}(\alpha)$

The spline coefficients $c$ and $\alpha_{crit}$ that define the curves shown in Figures 4.17 and 4.19 are plotted with respect to aircraft configuration parameters. First the tail shadowing angle is calculated for the three tail heights and presented in Table 4.3
### Table 4.1: Wing-tail shadow angles for various tail heights.

<table>
<thead>
<tr>
<th>$z_h/c$</th>
<th>High</th>
<th>Mid</th>
<th>Low</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta_{W\theta}$</td>
<td>23°</td>
<td>14°</td>
<td>0°</td>
</tr>
</tbody>
</table>

Figures 4.24 and 4.25 show that with higher tails (i.e., higher shadow angles), the average dynamic pressure at the tail is much lower when the wing stalls, which is seen in the more negative slope of the second interval. These trends show that when the wing sweep is increased the critical angle-of-attack is higher, likely because the wing tips stall more than the root, which is closer to the tail assembly.

Figure 4.24: Variation of dynamic pressure curve parameters with wing-tail shadowing angle

Figure 4.25: Variation of dynamic pressure curve parameters with wing sweep

**Parameters for $\epsilon(\alpha)$**

The downwash curve parameters versus tail height are shown in Figures 4.26 and 4.27

Figure 4.26: Variation of downwash curve parameters with wing-tail shadow angle.

Figure 4.27: Variation of downwash curve parameters with wing aspect ratio.

Note that the seemingly large errors in Figure 4.27 are not as important as a consistent trend followed
by all sets of points, since only the slopes will be used to estimate new configuration curve parameters.
The trends here show that lower horizontal tails will lead to a more negative slope of the downwash angle at the horizontal tail, because the tail would have dipped below the wing level.

Step 5 - Parametric relationship combination

The estimate for \( (C^I_m)'_H(\alpha, \theta_{WH}, \Lambda_w, AR_w) \) can be calculated from \( \frac{q_w}{q_{\infty}}(\alpha, \theta_{WH}, \Lambda_w) \) and \( \epsilon(\alpha, \theta_{WH}, AR_w) \), which are determined by finding their curve parameters for new configurations. For \( \frac{q_w}{q_{\infty}}(\alpha) \), using the results from the previous step.

\[
\alpha'_{\text{crit}} = (\alpha_{\text{crit}})_0 + \frac{\partial \alpha_{\text{crit}}}{\partial \theta_{WH}} \Delta \theta_{WH} + \frac{\partial \alpha_{\text{crit}}}{\partial \Lambda_w} \Delta \Lambda_w
\]

\[
c'_1 = (c_1)_0 + \frac{\partial c_1}{\partial \theta_{WH}} \Delta \theta_{WH} + \frac{\partial c_1}{\partial \Lambda_w} \Delta \Lambda_w
\]

where ()_0 indicate the coefficients from available data closest to the new configuration and (') indicate SPM prediction values for the new configuration. Similarly, for \( \epsilon(\alpha) \)

\[
c'_1 = (c_1)_0 + \frac{\partial c_1}{\partial \theta_{WH}} \Delta \theta_{WH} + \frac{\partial c_1}{\partial AR_w} \Delta AR_w
\]

\[
c'_2 = (c_2)_0 + \frac{\partial c_2}{\partial \theta_{WH}} \Delta \theta_{WH} + \frac{\partial c_2}{\partial AR_w} \Delta AR_w
\]

These will be applied twice - for \( (C^I_m)'_H \) and \( (C^B_m)'_H \) for calculating \( \Delta (C_m)_H \), shown in Section 5.2.

4.2 Vertical Tail Contributions

Perturbed flow around the aircraft fuselage past stall significantly decreases the dynamic pressure at the vertical tail [44]. In addition in sideslip, the wing-tip vortices which shift depending on the wing stall pattern, alter the sidewash field at the vertical tail. Both effects reduce the sideforce generated by the vertical tail due to sideslip. Since the vertical tail is the main contributor to stabilizing yaw (Figure 4.28) and roll moments due to sideslip angle, these effects lead to greatly reduced lateral-directional stability that can lead to LOC. Configuration parameters that affect how much of the vertical tail flow is disturbed by the fuselage wake and/or wing-induced sidewash in stall are the relative locations and sizes of the wing (W), fuselage (F), and vertical tail (V). The SPM procedure is applied to estimate the stall-regime vertical tail contribution to sideforce due to sideslip \( (C_{Y_{\beta}})_V = f(\alpha, V, F, W) \), which is then used to calculate the roll moment due to sideslip \( (C_{l_{\beta}})_V \), and yaw moment due to sideslip \( (C_{n_{\beta}})_V \).

![Figure 4.28: Importance of vertical tail for directional stability (Reprinted from Chambers [12])]
Step 1 - Model structure and stall effect splines

For vertical tail contributions to static lateral-directional coefficients, both \((C'_l)_v\) and \((C'_n)_v\) can be expressed to depend on the sideforce due to vertical tail \((C'_Y)_v\), which is assumed to act through the centre of pressure (cp) of the vertical tail. Recall that the prime (‘) indicates the stall modified SPM predicted version of any coefficient. In the body-axis, \((C'_l)_v(\alpha) = (C'_Y)_v z_v \frac{b}{l_v} \) \hspace{1cm} (4.18a)

\((C'_n)_v(\alpha) = -(C'_Y)_v l_v \frac{b}{l_v} \) \hspace{1cm} (4.18b)

\((C'_Y)_v(\alpha) = -C_{L_{\beta,v}} \frac{S_v}{S} (1 - \frac{\partial \sigma}{\partial \beta}(\alpha)) \frac{q_v}{q_{\infty}}(\alpha) \) \hspace{1cm} (4.18c)

where \(z_v = z_{cg} - z_{cp,v}\) and \(l_v = x_{cg} - x_{cp,v}\). The vertical tail sideforce \((C'_Y)_v\) derived in [64] is calculated using the vertical tail lift curve slope \(C_{L_{\beta,v}}\) and the area ratio \(\frac{S_v}{S}\), both of which do not change with respect to angle-of-attack. Since the representative model requires a large \(\alpha\) but only moderate \(\beta\), the lift curve slope of the vertical tail \(C_{L_{\beta,v}}\) used in both the static and damping terms, is estimated using the linear approximation from DATCOM [29] in Equation 4.19, which could be replaced with a 2D lift curve slope of the vertical tail if needed for higher sideslip angles.

\[ C_{L_{\beta,v}} = \frac{2\pi AR}{2 + \sqrt{4 + \frac{AR^2B^2}{K^2}(1 + \frac{\tan^2 A/2}{B^2})}} \] \hspace{1cm} (4.19)

where \(c_{l_{\alpha}}\) below is estimated using the closest NACA airfoil data available,

\[ B = \sqrt{1 - M^2} \quad \text{and} \quad K = \frac{c_{l_{\alpha}}}{2\pi/B} \quad \text{with} \quad c_{l_{\alpha}} \quad \text{being the section lift curve slope} \] \hspace{1cm} (4.20)

The estimate also depends the sidewash gradient \((1 - \frac{\partial \sigma}{\partial \beta})\) and the dynamic pressure ratio \(\frac{q_v}{q_{\infty}}\) (often represented by \(\eta_v\) in other texts), which do change significantly with respect to angle-of-attack, especially past stall. The sidewash gradient term depends largely on relative wing-tail (this will considered first) and fuselage-tail positioning, whereas the dynamic pressure ratio depends most heavily on the relative size of fuselage to the vertical tail, for low sideslip [48]. Both of these effects change with \(\alpha\) and are modeled by splines.

**Spline Functions**

The stall effects of decreased dynamic pressure and sidewash gradient due to wing and/or fuselage interference, are modeled using spline functions \(S(\alpha, p_1, p_2, ... )\) such that,

\[ \frac{q_v}{q_{\infty}} = S(\alpha, V, F) \] \hspace{1cm} (4.21a)

\[ \frac{\partial \sigma}{\partial \beta} = S(\alpha, V, W) \] \hspace{1cm} (4.21b)

These choices of component dependencies on \(F, W, V\) and specific configuration parameters therein are based on literature results. According to a study by Polhamus [48], two configuration changes in particular affect the vertical tail’s sideforce contribution the most, and hence its contribution to directional stability. These two were the forebody length and the wing height as shown in Figures 4.29 and 4.30.
At high angles-of-attack the forebody alters the dynamic pressure where downwind vertical tail operates. The height and sweep of the wing changes the sidewash field which alters the vertical tail’s flow field, also decreasing its efficiency. Thus the local dynamic pressure and sidewash will be parametrized using dimensionless ratios related to the forebody length and relative wing height/sweep respectively, so that the simple stall effect spline functions are,

\[
\frac{q_v}{q_\infty} = S(\alpha, \frac{b_v}{L_F}) \quad (4.22a)
\]

\[
\frac{\partial \sigma}{\partial \beta} = S(\alpha, \frac{z_w}{b_v}, \Lambda_w) \quad (4.22b)
\]

The stall efficiency factor \( \eta^*_V \) can be used to highlight and compare trends at high angles-of-attack. Unlike the dynamic pressure ratio, the sidewash gradient is not 1 at low angles-of-attack, so its value at \( \alpha = 0 \) is factored out as \( (1 - \frac{\partial \sigma}{\partial \beta})_{\alpha = 0} \) so that \( \eta^*_V(0) = 1 \), making stall effect plots easier to interpret so that,

\[
\eta^*_V(\alpha) = \frac{(C'_{Y\beta})_V(\alpha)}{(C'_{Y\beta})_V(\alpha = 0)} = \frac{q_v}{q_\infty} \frac{(1 - \frac{\partial \sigma}{\partial \beta})_{\alpha = 0}}{(1 - \frac{\partial \sigma}{\partial \beta})_{\alpha = 0}} \quad (4.23)
\]

Note that this efficiency is only required to be unity at \( \alpha = 1 \), with minor deviations even prior to stall.

**Step 2 - Stall physics as spline shape prescriptions**

Again, starting with the general spline defined in Equation 3.14 for representative stall modeling, known and/or derived physical characteristics of stall converted into SPM shape constraints, are applied to arrive at a lower-order piecewise equation.

\[
S(\alpha, c_{ik}) = \begin{cases} 
P_1(\alpha) = \sum_{k=0}^{n} c_{1k}\alpha^k & 0^\circ \leq \alpha < \alpha_{crit} \\
P_2(\alpha) = \sum_{k=0}^{n} c_{2k}\alpha^k & \alpha_{crit} \leq \alpha < 25^\circ
\end{cases} \quad (4.24)
\]

**Shape primitives for \( \frac{q_v}{q_\infty}(\alpha) \)**

As the angle-of-attack increases past stall, the vertical tail becomes increasingly immersed in the separated wake of the fuselage-forebody. From experimental results seen in literature, this effect has an onset close to the wing stall point and typically reduces the vertical tail’s dynamic pressure linearly thereafter.
The dynamic pressure ratio at the vertical tail aerodynamic centre, has the characteristic curve shown in Figure 4.31. Using this as a reference, it is seen that the efficiency is equal to 1, until the approximate expected stall angle-of-attack, and then decreases linearly to approximately 50% at around 25°, which is the maximum range this study requires. This curve trend is also in agreement with many flight models used in industry for simulator needs.

![Figure 4.31: Dynamic pressure ratio trend measured at vertical tail.](Reprinted from Chambers [12])

The following will define the shape prescriptions with reference to specific aerodynamic phenomena as well as the trend seen in Figure 4.31,

- Disregarding power effects, prior to some stall angle-of-attack, the flow velocity at the vertical tail equals that of the free-stream \[ \frac{q_v}{q_\infty} \] should be 1 prior to some \( \alpha_{crit} \)

- Once the fuselage starts generating a turbulent flow upon stall, the wake will increasingly impinge upon the vertical tail linearly with increased angle-of-attack \[ \frac{q_v}{q_\infty} \Rightarrow \text{After } \alpha_{crit}, \text{ the efficiency is monotone decreasing in a linear fashion.} \]

Applying these shape constrains and taking \( \alpha_{crit} = 12^\circ \) since the current dataset will always fit to this value, only 1 DOF is left in the spline expression 4.24 so the above becomes,

\[
\frac{q_v}{q_\infty}(\alpha, c) = \begin{cases} 
1 & 0^\circ \leq \alpha < 12^\circ \\
1 + c_1 (\alpha - 12^\circ) & 12^\circ \leq \alpha < 25^\circ 
\end{cases} 
\] (4.25)

Various values for the remaining constants (e.g. the slope of the linear portion) are to be determined for various geometric configurations in Step-3.

**Shape primitives for \( \frac{\partial \sigma}{\partial \beta} (\alpha) \)**

As the angle-of-attack increases past stall, the sidewash gradient factor \( (1 - \frac{\partial \sigma}{\partial \beta}) \) at the vertical tail increases. It typically has a characteristic curve shown in Figure 4.32, which is converted to give the shape of only \( \frac{\partial \sigma}{\partial \beta} \) as shown in red in Figure 4.33. The sidewash gradient does not necessarily start from zero prior to the stall angle-of-attack. The sidewash gradient becomes positive and destabilizing when the angle-of-attack is large enough to put the vertical stabilizer below the wingtip vortices [47].

The following will define the shape prescriptions with reference to specific aerodynamic phenomena:

- Prior to stall, the sidewash field is not altered by the wings or fuselage \( \Rightarrow \) The curve should be constant before some \( \alpha_{crit} \)
Figure 4.32: Dynamic pressure ratio and sidewash gradient trends measured at vertical tail. (Reprinted from Chambers [12])

Figure 4.33: Sidewash gradient at Vertical Tail

- The value and rate of change of the sidewash gradient increase with higher angle-of-attack \[\alpha_{\text{crit}}\] ⇒ Past \(\alpha_{\text{crit}}\) the curve should be monotone increasing (linear or quadratic)

Applying these shape constrains and taking \(\alpha_{\text{crit}} = 10^\circ\) since the existing dataset will always fit to this value, the above becomes,

\[
\frac{\partial \sigma}{\partial \beta}(\alpha, c) = \begin{cases} 
  c_0 & 0^\circ \leq \alpha < 10^\circ \\
  c_0 + c_1(\alpha - 12) & 10^\circ \leq \alpha < 25^\circ
\end{cases}
\]  

(4.26a)

Having now limited the spline expressions for \(\frac{q_v}{q_\infty}\) and \(\frac{\partial \sigma}{\partial \beta}\) to only a few curve parameters, the prescribed curves are fit to configuration datasets. This will allow correlation between remaining free curve parameters to geometric parameters.

**Step 3 - Least squares spline fitting to data**

**Fitting \(\frac{q_v}{q_\infty}\)(\(\alpha\))**

The dataset used is taken from NASA Langley wind tunnel studies by Queijo [50], who measured vertical tail efficiency and interference effects, for \(\alpha = [0^\circ, 25^\circ]\), \(\beta = [-5^\circ, +5^\circ]\). The study contains experiments done for various sizes of vertical tails and fuselages with and without wings, as seen in Figure 4.34. Note that \(F1\) is the shortest fuselage, and \(V1\) is the smallest vertical tail.

Figure 4.34: Vertical tail and fuselage variations. (Reprinted from Queijo [50])
Compatibility between the spline model and experimental data already exists, as the measured $\eta_{\text{data}}^*$ represents the same quantity as in the model’s. The current dataset pertains to vertical tail and fuselage combinations without the wing, so it is assumed that the measured efficiency mostly reflects the dynamic pressure ratio due the fuselage wake as $(1 - \frac{\partial \sigma}{\partial \beta})_{\alpha=0} \approx 1$, so that

$$\eta_{\text{data}}^* (\alpha) \sim \frac{q_v}{q_{\infty}} (\alpha) \quad (4.27)$$

Having set shape constraints, the spline function for $\frac{q_v}{q_{\infty}}$ is fitted to data $\eta_{\text{data}}^*$ for the three different fuselage and vertical tail sizes for a total of 9 different configurations. This leaves the unconstrained parameters to contain configuration trend information. The results of this least squares spline fit are shown in Figures 4.35 and 4.36. The numerical values of the geometry are shown in Step-4.

Fitting $\frac{\partial \sigma}{\partial \beta} (\alpha)$

The same procedure is applied to another dataset, now with wings. The fitted results indicate that unlike the fuselage tail alone setup, the addition of the wing shifts the angle-of-attack at which the middle knot should be placed. This hints that the wing configuration parameter(s) chosen should be correlate to this. The dataset is taken from Fisher [21], which measures vertical tail sidewash gradient for various wing positions and sweep for $\alpha = [0^\circ, 28^\circ]$. The configurations are shown in Figure 4.37.
Again, this dataset is already compatible because the plots display sidewash gradient directly,

\[ \frac{\partial \sigma}{\partial \beta} \mid_{\text{data}} (\alpha) \sim \frac{\partial \sigma}{\partial \beta} (\alpha) \]  

(4.28)

Data for three wing heights for both straight and swept wings are fitted to the constrained spline. Results of this procedure are shown in Figures 4.38 and 4.39, where the wing sweep and height values are shown in Step-4.

It can be seen that both the wing height and sweep have significant influence on the sidewash gradient. Step-4 will correlate geometry parameters to curve parameters.

**Step 4 - Configuration parameter fitting to spline parameters**

**Parameters for \( \frac{\partial \sigma}{\partial \beta} (\alpha) \)**

The variations in fuselage length and the vertical tail span can be combined into one parameter \( b_v/L_F \) calculated in Table 4.2, as they both essentially affect the amount of blanketing by the forebody on the vertical tail.

<table>
<thead>
<tr>
<th>( L_F )</th>
<th>( b_v )</th>
<th>( b_v/L_F )</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.5 ft</td>
<td>0.1632</td>
<td>0.2332</td>
</tr>
<tr>
<td>3.3 ft</td>
<td>0.1224</td>
<td>0.1749</td>
</tr>
<tr>
<td>5.0 ft</td>
<td>0.0816</td>
<td>0.1166</td>
</tr>
</tbody>
</table>

The variation of the second interval slope of the dynamic pressure ratio is plotted against this blanketing parameter in Figure 4.40.
Chapter 4. Model Synthesis

Figure 4.40: Variation of vertical tail dynamic pressure curve slope with relative vertical tail size.

The trends show that when the vertical tail is small compared to the fuselage, the dynamic pressure at the tail drops more rapidly past stall.

Parameters for $\frac{\partial \sigma}{\partial \beta} (\alpha)$

Similarly to the dynamic pressure, the spline parameters are fit to the variations in wing height and sweep. The results are show in Figures 4.41 and 4.42.

It can be seen that higher wings, as they’re closer to the vertical tail, lead to higher additional sidewash and thus lower efficiency. According to a study by Phillips et al in [47], the sidewash gradient become positive when the vertical tail dips below the wing vortices. This confirms that higher wings will be above the vertical tail C.P. earlier as the angle-of-attack is increased.
Step 5 - Parametric relationship combination

The estimate for \( (C'_{Y,\beta})_V (\alpha, b_{VF}, \Lambda_w, \bar{z}_w) \) can be calculated from \( \frac{\partial}{\partial \beta} (\alpha) \) and \( \frac{\partial}{\partial \beta} \left( \frac{b_V}{L_F} \right) \), which are determined by finding their curve parameters for new configurations. For \( \frac{\partial}{\partial \beta} (\alpha) \), using the results from the previous step.

\[
c'_1 = (c_1)_0 + \frac{\partial c_1}{\partial \left( \frac{b_V}{L_F} \right)} \Delta b_V \tag{4.29}
\]

where \( ()_0 \) indicate the coefficients from available data closest to the new configuration and \( (') \) indicate SPM prediction values for the new configuration. Similarly, for \( \frac{\partial}{\partial \beta} \left( \frac{b_V}{L_F} \right) \)

\[
c'_0 = (c_0)_0 + \frac{\partial c_0}{\partial \left( \frac{b_V}{L_F} \right)} \Delta \bar{z}_w + \frac{\partial c_0}{\partial \Lambda_w} \Delta \Lambda_w \tag{4.30a}
\]

\[
c'_1 = (c_1)_0 + \frac{\partial c_1}{\partial \left( \frac{b_V}{L_F} \right)} \Delta \bar{z}_w + \frac{\partial c_1}{\partial \Lambda_w} \Delta \Lambda_w \tag{4.30b}
\]

These will be applied twice - for \( (C'_{Y,\beta})_V \) and \( (C'_{b,Y})_V \) for calculating \( (\Delta C_{Y,\beta})_V \), shown later in Section 5.2.

4.3 Fuselage-Forebody Contributions

Studies show that at high angles-of-attack, forebody contributions have significant, sometimes dominant impact on the directional stability for most fixed-wing aircraft types [9]. Figure 4.43 shows how the forebody alone can negatively affect yaw stability. In the current context of modeling regional transports, a common and often the only major configuration change within a class is actually the fuselage length, which changes the moment arm between the forebody and aircraft C.G., affecting the directional characteristics at stall. Assuming a cylindrical fuselage, the yaw moment due to the fuselage without forebody is linear with respect to sideslip and almost constant with respect to angle-of-attack [64]. However, in wind tunnel tests the fuselage and forebody contributions are measured together, so their effects must first be separated. SPM is applied to predict yaw moment due to sideslip \( (C'_{n,\beta})_F = f(\alpha, F, f b) \) for the fuselage-forebody combination. Focus is placed on the effects of forebody ellipticity, fineness ratio, and fuselage length. Surface dressing components such as forebody strakes and vortex generators are known to have notable contributions at stall, but are not included in this study.

Step 1 - Model structure and stall effect spline functions

The pre-stall model for fuselage moment contributions \( (C_{n,\beta})_F \) is based on linear estimates for the fuselage moment contribution from Stengel [64], with a scaling factor of \( (1 - \frac{d_{max}}{L_F})^{1.3} \) that accounts for deviations from the basic slender-body theory assumptions. \( V_F \) is the volume of the fuselage approximated using, \( V_F \approx S_{base} L_F \) and \( d_{max} \) is the maximum diameter, and \( L_F \) is the fuselage length, \( S_{base} = \pi d^2 / 4 \) is the cross-section area assuming an approximately circular cross-section,

\[
(C_{n,\beta})_F = -\frac{2V_F}{S_w b_w} (1 - \frac{d_{max}}{L_F})^{1.3} \tag{4.31}
\]
To model stall contributions, this linear estimate is modified by adding the yawing moment contribution of the forebody at high angles-of-attack ($C_{n\beta fb}$). Note that typically fuselages provide a destabilizing moment as indicated by the negative yawing moment coefficient. Depending on forebody cross-sectional shape however, this may be reversed at stall. The combined stall regime coefficient estimated is,

$$(C_{n\beta}')_F(\alpha) = (C_{n\beta})_F + (C_{n\beta fb})(\alpha) \quad (4.32)$$

**Spline Function**

The yawing moment due to forebody configuration ($C_{n\beta fb}$) will be modeled as a spline function of $\alpha$ and various fuselage-forebody configuration parameters,

$$(C_{n\beta fb})(\alpha) = S(\alpha, F, fb) \quad (4.33)$$

The actual dependencies are specified in terms of the following parameters below.
Thus the spline function takes the form,

\[
(C_{n\beta})_{fb}(\alpha) = S(\alpha, FR_{fb}, E_{fb}, \frac{l_{fb}}{c}) = \frac{l_{fb}}{c} S(\alpha, FR_{fb}, E_{fb})
\]

where the forebody length \(\frac{l_{fb}}{c}\) is factored out, so the quantity to be splined basically represents the sideforce generated by the forebody due to sideslip,

\[
(C_Y)_{fb}(\alpha) = S(\alpha, FR_{fb}, E_{fb})
\]

where the parameters are fineness ratio \(l_{fb} = x_{fb} - x_{CG}\), forebody arm \(FR_{fb} = \frac{l_{fb}}{d_{fb}}\), and ellipticity \(E_{fb} = \frac{h_{fb}}{w_{fb}}\). Here \(h_{fb}\) and \(w_{fb}\) are the forebody’s average height and width. The stall effect efficiency factor \(\eta_s\) can be used to illustrate the deviations from linear estimates with respect to \(\alpha\). Here the efficiency factor is a slight misnomer in the typical sense, because it may also amplify rather than only diminish the aerodynamic contribution of a component.

\[
\eta_s(\alpha) = \frac{(C'_{n\beta})_{F}(\alpha)}{(C_{n\beta})_{F}(\alpha = 0)} = 1 + \frac{(C_{n\beta})_{fb}(\alpha)}{(C_{n\beta})_{F}}
\]

**Step 2 - Stall physics as shape prescriptions**

Again to reduce spline DOFs, known and/or derived physical characteristics of stall converted into SPM shape constraints are applied to arrive at a lower-order piecewise equation.

\[
S(\alpha, c_{ik}) = \begin{cases} 
P_1(\alpha) = \sum_{k=0}^{n} c_{1k} \alpha^k & 0^\circ \leq \alpha < \alpha_{crit} \\
P_2(\alpha) = \sum_{k=0}^{n} c_{2k} \alpha^k & \alpha_{crit} \leq \alpha < 25^\circ
\end{cases}
\]

As the angle-of-attack increases past stall, the side force increases linearly, as seen in Figure 4.45. It is also seen that for a horizontal ellipse with sideslip, separated vortex pattern creates a restoring sideforce, whereas that of a vertical ellipse creates a destabilizing sideforce; the circular cross section shows relatively negligible sideforce.

Figure 4.45: Yaw moment due to forebody aerodynamics at high-\(\alpha\) for \(E = 0.625, 1, 1.6\).
The following items define the spline shape prescriptions.

- The forebody sideforce is minimal and does not vary prior to $\alpha_{crit}$ $\Rightarrow$ The first interval should be constant.

- Past stall, the asymmetric vortex buildup leads to increased side force, which increases linearly. $\Rightarrow$ Past $\alpha_{crit}$ the curve is constrained to be monotone linearly increasing or decreasing.

Applying these and taking $\alpha_{crit} \approx 15^\circ$ for the current dataset as it is already the best fit, two DOFs are left,

$$
(C_{n_{beta}})_{fb}(\alpha, c) = \begin{cases} 
     c_1 & 0^\circ \leq \alpha < 15^\circ \\
     c_1 + c_2(\alpha - 15) & 15^\circ \leq \alpha < 25^\circ
\end{cases}
$$

(4.38)

**Step 3 - Least squares spline fitting to data**

The dataset is chosen from a NASA Langley wind tunnel study on forebody aerodynamics at high angles-of-attack by Brandon et al [9]. In the study, forebody fineness ratio was varied for three forebody shapes (circular, horizontal ellipse, vertical ellipse) and for two wing positions.

![Forebody shapes tested in wind tunnel study](Reprinted from Brandon [9])

The data is collected on a wing-fuselage combination, but because a hemispherical cap is used in place of a forebody to measure moments without the forebody, it is possible to extract the contribution of the forebody alone, making it compatible with the chosen stall model using,

$$
(C_{n_{beta}})_{fb} = (C_{n_{beta}})_{FW+fb} - (C_{n_{beta}})_{FW} \\
(C_{Y_{beta}})_{fb}(\alpha) = \frac{(C_{n_{beta}})_{fb}}{I_{fb}/c} \sim S(\alpha, FR_{fb}, E_{fb})
$$

(4.39a, 4.39b)

The constrained spline is fitted to data for 3 fineness ratios for horizontal and vertical elliptical forebodies. Results are shown in Figures 4.47 and 4.48.
Step 4 - Configuration parameter fitting to spline parameters

The effects of forebody fineness ratio and ellipticity on the curve parameters are shown in Figures 4.49 and 4.50, respectively. Note that for \(c_2 = f(FR_{fb})\) the absolute value is used to gather a trend, since depending on the ellipticity, \(c_w\) increases or decreases with fineness ratio.

The trends confirm that if the ellipticity is less than 1 in the case of a horizontal ellipse, the forebody sideforce is restoring and so has a positive slope, and vice versa. When the fineness ratio is increased, the magnitude of either restoring or destabilizing sideforce is increased.

Step 5 - Parametric relationship combination

The estimate for \((C_{n,\beta})_{fb}(\alpha)\) can be calculated if both spline parameters can be determined to first find \((C_{n,\beta})_{fb}(\alpha, FR_{fb}, E_{fb})\) for the new configuration. To get the combined effects for arbitrary new
configurations,
\[ c'_1 = (c_1)_0 + \frac{\partial c_1}{\partial E_{fb}} \Delta E_{fb} + \frac{\partial c_1}{\partial FR_{fb}} \Delta FR_{fb} \]  
(4.40a)
\[ c'_2 = (c_2)_0 + \frac{\partial c_2}{\partial E_{fb}} \Delta E_{fb} + \frac{\partial c_2}{\partial FR_{fb}} \Delta FR_{fb} \]  
(4.40b)
where \((\cdot)_0\) indicate the coefficients from available data closest to the new configuration and \((\cdot')\) indicate SPM prediction values for the new configuration. These will be applied twice - for \((C_{n\beta}^{T'})_F\) and \((C_{n\beta}^{B'})_F\) for calculating \((\Delta C_{n\beta})_F\), shown later in Section 5.2.

### 4.4 Damping Coefficients

The next few estimates for dynamic coefficients for pitch and yaw damping are done using a more crude approach due to limited data. The terms to be modelled by spline functions are not individual effects but the overall efficiency factors, scaling the low angle-of-attack estimates into the stall region. The SPM approach steps are shortened, shape prescriptions are based on observed trends, and results are only valid estimates for general direction and order of magnitude. Steady-state conditions are assumed, so unsteady \(\dot{\alpha}, \dot{\beta}\) derivative terms are not considered here.

#### 4.4.1 Horizontal Tail

The horizontal tail provides the most significant contribution to damping in pitch, more so than the wing-body combination [38]. Changes to flow conditions at the horizontal tail therefore alter the pitch damping parameter significantly as seen in Figures 4.51 and 4.52. The derivative with respect to \(q\) accounts for the effect of increased tail angle-of-attack when the aircraft is pitching; the derivative with respect to \(\dot{\alpha}\) arise from time lag of the wing downwash affecting the horizontal tail, which is not considered in this study. Similar to the static case, during a stall these coefficients change due to decreased dynamic pressure and increased wing-induced downwash at the tail. The key configuration parameters here are those related to the wing, nacelles, and horizontal tail. Thus, SPM is used to estimate the dynamic pitch moment due to pitch rate \((C'_{m\alpha})_H = S(\alpha, F, W, H)\) term.

![Figure 4.51: Effect of tail area on pitch damping.](Reprinted from Wiley [72])
![Figure 4.52: Effect of nacelle on pitch damping.](Reprinted from Wiley [72])
Step 1 - Model definition

The linear estimate derived by Stengel [64] for pitch moment due to pitch rate from the horizontal tail, is extended to high angles-of-attack ranges using the $\alpha$-dependent efficiency,

$$ (C'_{m_q})_H(\alpha) = -2C_{L_{\alpha,h}} V_H \frac{I_H}{c} \eta_H^q(\alpha) \quad (4.41) $$

where the factor $\eta_H^q$ captures the effects of changing local dynamic pressure $\frac{q_h}{q_\infty}$ and the local downwash lag $(1 - \frac{\partial \epsilon}{\partial \tilde{q}})$. However, since little data exist for these effects separately, the efficiency factor is approximated as a black box using a spline function.

$$ \eta_H^q(\alpha) = S(\alpha, F, W, H) \quad (4.42) $$

The specific geometric parameters will be based on available public data.

Step 2 - Spline shape prescriptions

Judging from available experimental studies, the damping coefficient efficiency $\eta_H^q(\alpha)$ has a sharp peak just past the primary stall point, after which it falls down to below pre-stall values. So the efficiency will be modeled using three intervals, and spline shape prescriptions will be,

- Prior to the onset of stall flow, the pitch damping remains very close to the linear estimate at low angles-of-attack $\Rightarrow$ The pitch damping efficiency starts at $(0, 1)$ and remains 1 until some $\alpha_{\text{crit},1}$

- Approaching the stall point, the horizontal tail provides an increased amount of pitch damping [38] $\Rightarrow$ The second interval should be monotone linear increasing until some $\alpha_{\text{crit},2}$

- After the peak, the pitch damping efficiency decreases sharply and reaches values below pre-stall levels due to the altered flow conditions at the horizontal tail [38] $\Rightarrow$ The third interval should be monotone decreasing linear.

The efficiency factor spline would take the following form, which has 4 parameters

$$ \eta_H^q(\alpha) = \begin{cases} 
1 & 0^\circ \leq \alpha \leq \alpha_{\text{crit},1} \\
1 + c_2(\alpha - \alpha_{\text{crit},1}) & \alpha_{\text{crit},1} \leq \alpha \leq \alpha_{\text{crit},2} \\
1 + c_2(\alpha_{\text{crit},2} - \alpha_{\text{crit},1}) + c_3(\alpha - \alpha_{\text{crit},2}) & \alpha_{\text{crit},2} \leq \alpha < 25^\circ 
\end{cases} \quad (4.43) $$

subject to the constraints above and continuity conditions.

Step 3 - Spline regression

The dataset used is taken from NACA wind tunnel studies by Lichtenstein [38], in which the aircraft pitch damping coefficient $C_{m_q}$ is measured for nine horizontal tail locations. The same study also includes effects of fuselage length and horizontal tail area.
The non-dimensional parameters considered will be the relative wing-tail angle, fuselage length, and horizontal tail area,

\[ \eta_q^H(\alpha) = S(\alpha, \theta_{WH}, \frac{L_F}{b}, \frac{S_H}{S}) \]  

(4.44)

For data-model compatibility, the measured data for the total aircraft \( C_{m_q} \) will be converted to the horizontal tail contribution efficiency,

\[ (C_{m_q})_H = (C_{m_q})_{FWVH}^{data} - (C_{m_q})_{FW}^{data} \]  

(4.45a)

\[ \eta_q^H(\alpha)^{data} = \frac{(C_{m_q})_H(\alpha)}{(C_{m_q})_H(\alpha = 0)} \sim \eta_q^H(\alpha) \]  

(4.45b)

The constrained spline is fitted to three tail sizes and three fuselage lengths, shown in Figures 4.56 and 4.57, where the tail position labels from Figure 4.53 are used, and the actual \( \theta_{WH} \) values are shown in Step-4.
Step 4 - Configuration parameter fitting to spline parameters

At high angles-of-attack, varying either the tail height or tail arm has the effect of changing the wing-tail wake shadow angle. Thus both variations can be collapsed into one, that is $\theta_{WH}$ using

$$\theta_{WH} = \tan^{-1} \frac{z_h - z_w}{x_h - x_w}$$

<table>
<thead>
<tr>
<th>$\theta_{WH}$°</th>
<th>Forward</th>
<th>Middle</th>
<th>Rearward</th>
</tr>
</thead>
<tbody>
<tr>
<td>Upper</td>
<td>17.5</td>
<td>15.3</td>
<td>14.4</td>
</tr>
<tr>
<td>Center</td>
<td>10.4</td>
<td>9.6</td>
<td>9.0</td>
</tr>
<tr>
<td>Lower</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

The effect of the tail shadow angle $\theta_{WH}$ on the 4 curve DOFs that define pitch damping efficiency are plotted in Figure 4.58. The fitted trends show that as the shadowing angle $\theta_{WH}$ increases, the critical angles-of-attack increase because the tail will be affected by the wing at higher angles-of-attack, the slope of the last interval slope also becomes steeper.
The effect of horizontal tail area and fuselage length on the tail contribution to pitch damping efficiency curve parameters are plotted in Figures 4.59 and 4.60 respectively. The trends show that with increased fuselage length, the post-stall slope is reduced.

**Step 5 - New Configuration Prediction**

The estimate for \( (C_T')_H (\alpha) \) can be calculated if all 4 spline curve parameters can be determined to first find \( \eta_H'' (\alpha, \theta_{WH}, L_F/b_w, S_h/S_w) \) for the new configuration.

\[
\begin{align*}
\alpha'_{\text{crit},1} &= (\alpha_{\text{crit},1})_0 + \frac{\partial \alpha_{\text{crit},1}}{\partial \theta_{WH}} \Delta \theta_{WH} + \frac{\partial \alpha_{\text{crit},1}}{\partial (L_F/b_w)} \Delta (L_F/b_w) + \frac{\partial \alpha_{\text{crit},1}}{\partial (S_h/S_w)} \Delta (S_h/S_w) \\
\alpha'_{\text{crit},2} &= (\alpha_{\text{crit},2})_0 + \frac{\partial \alpha_{\text{crit},2}}{\partial \theta_{WH}} \Delta \theta_{WH} + \frac{\partial \alpha_{\text{crit},2}}{\partial (L_F/b_w)} \Delta (L_F/b_w) + \frac{\partial \alpha_{\text{crit},2}}{\partial (S_h/S_w)} \Delta (S_h/S_w) \\
c'_2 &= (c_2)_0 + \frac{\partial c_2}{\partial \theta_{WH}} \Delta \theta_{WH} + \frac{\partial c_2}{\partial (L_F/b_w)} \Delta (L_F/b_w) + \frac{\partial c_2}{\partial (S_h/S_w)} \Delta (S_h/S_w) \\
c'_3 &= (c_3)_0 + \frac{\partial c_3}{\partial \theta_{WH}} \Delta \theta_{WH} + \frac{\partial c_3}{\partial (L_F/b_w)} \Delta (L_F/b_w) + \frac{\partial c_3}{\partial (S_h/S_w)} \Delta (S_h/S_w)
\end{align*}
\]

where ()_0 indicate the coefficients from available data closest to the new configuration and ('') indicate SPM prediction values for the new configuration. These can then be applied twice - for \( (C_{m_q})_H \) and \( (C_{m_q}^B)'_H \) for calculating \( \Delta C_{m_q} \).

**4.4.2 Vertical Tail**

The vertical tail is the most important contributor to yaw damping, followed by the wings. The derivative terms with respect to yaw rate \( r \) arise from the increased instantaneous angle-of-attack at the vertical tail during directional rotation; the yaw damping derivative term with respect to sideslip rate \( \dot{\beta} \) arises mainly from time lag of the wing sidewash affecting the vertical tail, but will not be considered here. Similar to the static moments, during stall the diminished dynamic pressure and sidewash gradient significantly reduce the damping effects, as seen in Figure 4.61. The factors that affect these effects are again the configuration parameters of the fuselage (F) and wings (W) in addition to the vertical tail (V) itself. SPM will predict the yaw moment coefficient due to yaw rate \( (C_{n_y}')_V = f(\alpha, V, W, F) \)

![Figure 4.61: Example yaw damping effects for thin fuselage. (Reprinted from Queijo [49])](image-url)
Chapter 4. Model Synthesis

Step 1 - Model definition

The derivation of the expression for \((C'_{n_r})_V\) from Stengel [64] are based on the assumption that sideslip angles are small. Similar to the case for pitch damping, the linear estimate is extended using the efficiency factor to high angles-of-attack,

\[
(C'_{n_r})_V(\alpha) = -2C_{L_{\text{\beta},V}} \frac{l_v}{b} \eta'_V(\alpha)
\]  

(4.47)

where the efficiency factor \(\eta'_V\) depends on the local dynamic pressure \(\frac{q_v}{\infty}\) and the local sidewash gradient \((1 - \frac{\partial \sigma}{\partial \hat{r}})\). However, little conclusive theory or data exist for these effects. Therefore, the crude approach directly approximating the efficiency using the spline function is used,

\[
\eta'_V(\alpha) = S(\alpha, V, F, W)
\]  

(4.48)

The specific geometric parameters will be whichever ones are found in publicly available data.

Step 2 - Spline shape prescriptions

Similar to damping in pitch, the typical yaw damping coefficient experiences a sharp increase approaching stall. However it does not return to pre-stall levels after the stall.

- Prior to stall the airflow over the vertical tail has not been perturbed \(\Rightarrow\) Before some \(\alpha_{\text{crit}}\) the efficiency is 1.

- Past stall the yaw damping efficiency increases and decreases. \(\Rightarrow\) A cubic function initially increasing is used on second spline interval.

Taking \(\alpha_{\text{crit}} \approx 15^\circ\) for current dataset which is the best fit, the efficiency factor would take the following form, with 2 curve parameters

\[
\eta'_V(\alpha) = \begin{cases} 
1 & 0^\circ \leq \alpha < 15^\circ \\
1 + c_2(\alpha - 15)^2 + c_3(\alpha - 15)^3 & 15^\circ \leq \alpha < 25^\circ 
\end{cases}
\]  

(4.49)

Step 3 - Spline regression

Dataset used is taken from two NACA wind tunnel studies by Queijo and Jaquet [32, 49], in which the yaw damping coefficient \(C_{n_r}\) is measured for several configurations of a swept wing model. The parameters considered will be fuselage diameter, wing height, and vertical tail area,

\[
\eta''_V(\alpha) = S(\alpha, \frac{D_F}{b_w}, \frac{z_w}{D_F}, \frac{S_v}{S_w})
\]  

(4.50)

For data-model compatibility, the measured data for the total aircraft \(C_{n_r}\) will be transformed to the vertical tail contribution’s efficiency,

\[
(C_{n_r})_V = (C_{n_r})_{\text{FWV}} - (C_{n_r})_{\text{FW}}
\]  

(4.51a)

\[
\eta''_V(\alpha)^{\text{data}} = \frac{(C_{n_r})_V(\alpha)}{(C_{n_r})_V(\alpha = 0)} \sim \eta''_V(\alpha)
\]  

(4.51b)
The constrained spline is fitted to the dataset above, which examines the yaw damping for four different tail sizes. Results are shown in Figures 4.62 and 4.63, where $V_1$ represents the smallest vertical tail area with the actual values shown in Step-4.

Similarly, the yaw damping due to vertical tail for various fuselage diameters and wing heights are fitted to the same spline and shown in Figures 4.64 and 4.65. In these plots, $F_1$ is the thinnest fuselage and $W_1$ is the lowest wing; actual values are seen in Step-4.

**Step 4 - Configuration parameter fitting to spline parameters**

The effect of the vertical tail area on the 3 curve DOFs that define yaw damping efficiency are plotted in Figures 4.66, 4.67, and 4.68.
Step 5 - Parametric relationship combination

The estimate for \( (C'_{n_r})_V (\alpha) \) can be calculated if both spline curve parameters can be determined to first find \( \eta'_r (\alpha, \frac{D_F}{b_w}, \frac{z_w}{D_F}, \frac{S_v}{S_w}) \) for the new configuration.

\[
c'_2 = (c_2)_0 + \frac{\partial c_2}{\partial (\frac{z_w}{D_F})} \Delta \left( \frac{z_w}{D_F} \right) + \frac{\partial c_2}{\partial (\frac{D_F}{b_w})} \Delta \left( \frac{D_F}{b_w} \right) + \frac{\partial c_2}{\partial (\frac{S_v}{S_w})} \Delta \left( \frac{S_v}{S_w} \right) \tag{4.52a}
\]

\[
c'_3 = (c_3)_0 + \frac{\partial c_3}{\partial (\frac{z_w}{D_F})} \Delta \left( \frac{z_w}{D_F} \right) + \frac{\partial c_3}{\partial (\frac{D_F}{b_w})} \Delta \left( \frac{D_F}{b_w} \right) + \frac{\partial c_3}{\partial (\frac{S_v}{S_w})} \Delta \left( \frac{S_v}{S_w} \right) \tag{4.52b}
\]

where \(()_0\) indicate the coefficients from available data closest to the new configuration and \( (')\) indicate SPM prediction values for the new configuration. These can then be applied twice - for \( (C'^{T'}_{n_r})_V \) and \( (C'^{B'}_{n_r})_V \) for calculating \( (\Delta C_{n_r})_V \).
Chapter 5

Results and Discussion

State variables limits typical of LOC scenarios are presented to justify earlier modeling assumptions. Results of applying the representative stall model methodology from a baseline to a target aircraft are validated using experimental data. LOC prediction criteria are briefly discussed.

5.1 State Variable Limitations

Before verifying the configuration effect terms synthesized for the stall model, some simplifications can be made by setting numerical limits for the aircraft state variables such as \( \alpha, \beta, p, q, r \), representative of typical stall flight conditions. This defines an envelope within which the stall model should be reasonably accurate or representative. These boundaries also provide further justification for the choice of configuration effects and coefficients selected in the previous scope of study.

5.1.1 Literature Findings

Flight data from LOC motions of fighters and commercial transports show that both can reach high body rates, but transports reach lower maximum wind angles [43], as shown in Table 5.1. One example that supports this is a post-stall modeling study done by Foster et al [23] showing \( \alpha - \beta \) envelopes in Figures 5.1 and 5.2 where a transport aircraft does not exceed \( \alpha_{\text{max}} \approx 45^\circ, \beta_{\text{max}} \approx 20^\circ \).

Table 5.1: State variable limits in LOC motions (Adapted from Murch [43])

<table>
<thead>
<tr>
<th>Configuration</th>
<th>( \alpha_{\text{max}} )</th>
<th>( \beta_{\text{max}} )</th>
<th>( \dot{p}_{\text{max}} )</th>
<th>( q_{\text{max}} )</th>
<th>( r_{\text{max}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fighter</td>
<td>70</td>
<td>45</td>
<td>0.129</td>
<td>0.0062</td>
<td>0.043</td>
</tr>
<tr>
<td>Transport</td>
<td>47</td>
<td>20</td>
<td>0.126</td>
<td>0.0045</td>
<td>0.047</td>
</tr>
</tbody>
</table>

The FAA NPRM only mandates that training simulator models are representative up until 10 degrees past stall, which for typical transport class would mean \( \alpha_{\text{max}} \approx 15^\circ + 10^\circ \approx 25^\circ \). As seen in Figure 5.2, before crossing \( \alpha = 25^\circ \), sideslip stays within \( \beta < \pm 10^\circ \) and stability problems as shown in Figure 5.1 are primary static not dynamic. This is one example among similar studies [11,51] that successfully used linearity in \( \beta \) to assess high-\( \alpha \) aerodynamics. This suggests that for \( \alpha < 25^\circ \), linearity in sideslip \( \beta \) can be assumed without implying linearity in \( \alpha \), so that \( \beta \) derivative terms as functions of \( \alpha \) can be
used in early stages of stall modeling, i.e. $C_{n, ST}(\alpha, \beta) \approx C_{n_s}(\alpha)\beta$. This simplified approach is used in this study for lateral-directional coefficients.

5.1.2 Accident Data Analysis

To test these claims against an example, accident data of the Colgan Air Flight 3407 crash involving a T-tail turboprop [33], known for highly violent rolling oscillations upon stall induced LOC, are analyzed with the help of results from Crider [14]. Given the Euler angles, body acceleration, angle-of-attack shown in Figures 5.3, 5.4, and 5.5 in addition to altitude and airspeed data (not shown), the values of $p, q, r, \alpha, \beta$ with respect to time are estimated by differentiation of Euler angles transformed by Equation 5.1 and integration of Equations 5.2. Data was conditioned by evenly time-spaced linear interpolation, smoothing of signal noise after differentiation, and de-trending of drift after integration. Without using sophisticated parameter estimation, this yields approximate boundaries of the flight state variables.
Figure 5.5: Angle-of-attack data in last 50s.

\[
\begin{bmatrix}
    p \\
    q \\
    r
\end{bmatrix}
= 
\begin{bmatrix}
    1 & 0 & -\sin\theta \\
    0 & \cos\phi & \sin\phi\cos\theta \\
    0 & -\sin\phi & \cos\phi\cos\theta
\end{bmatrix}
\begin{bmatrix}
    \dot{\phi} \\
    \dot{\theta} \\
    \dot{\psi}
\end{bmatrix}
\quad (5.1)
\]

\[
\begin{align*}
    \dot{u} &= a_x - g \sin(\theta) - qw + rv 
    \quad \text{(5.2a)} \\
    \dot{v} &= a_y + g \cos(\theta) \sin(\phi) - ru + pw 
    \quad \text{(5.2b)} \\
    \dot{w} &= -a_z + g \cos(\theta) \cos(\phi) - pv + qu 
    \quad \text{(5.2c)}
\end{align*}
\]

Figure 5.6: Estimated $p, q, r$ rates in last 50s.

Figure 5.7: Estimated $\alpha$ vs. $\beta$ ranges in last 50s.

Figure 5.6 shows that estimated maximum body rates match those in literature, with the exception of the slightly larger roll rates as expected for this particular accident. Figure 5.7 shows that before $\alpha$ crosses $20^\circ$ for the first time into a fully developed LOC, sideslip stays in a range of $\beta < \pm 5^\circ$. However, at $\alpha \approx 20^\circ$ when the plane departs from controlled flight, the sideslip rapidly exceeds $10^\circ$. Therefore it would be questionable to assume linearity in $\beta$ and only static instability near or past just $\alpha \approx 20^\circ$.

Considering this finding with previous scope of study and availability of data, it should be advised that the total representative stall model may likely be only valid up to stall and until $5^\circ$ past stall, especially for configurations and conditions prone to very high sideslip and rotation rates. However this particular accident could be an outlier, atypical of transport configuration LOC cases as summarized in Table 5.1. Ideally this method of checking relevant accidents for their state variable limits should be repeated to as many datasets as possible, for a more complete summary of these boundaries.
5.2 Baseline to Target Aircraft Model

The original problem statement for this study was: Given $C_{i}^{B}$ and $C_{i}^{T}$, find $C_{i}^{S}$ using $(\Delta C_{i,k})_{j}$ terms for all $i, k, j$ where available. To validate the methodology’s predictive power, two known stall aerodynamic models are used. The baseline aircraft model $C_{i}^{B}$ is the T-tailed twin-engine turbojet configuration tested in Aoyagi [6], whereas the target aircraft model $C_{i}^{T}$ is taken from the pre-stall portion of a similar aircraft tested in Ray [51]. The model synthesis steps from the previous chapter are applied to static pitch and yaw moment coefficients for verification. The results are compared to the post-stall portion of the target dataset. The two aircraft geometries are show in Figures 5.8 and 5.9,

![Figure 5.8: Baseline aircraft geometry](Reprinted from Aoyagi [6])  
![Figure 5.9: Target aircraft geometry](Reprinted from Ray [51])

and the specific configuration parameter changes are listed in Table 5.2

<table>
<thead>
<tr>
<th>Parameter</th>
<th>$\theta_{WH}$</th>
<th>$\theta_{NH}$</th>
<th>$AR_{w}$</th>
<th>$\Lambda_{w}$</th>
<th>$b_{w}/L_{F}$</th>
<th>$z_{w}/b_{w}$</th>
<th>$FR_{fb}$</th>
<th>$E_{fb}$</th>
<th>$\ell_{fb}/r$</th>
<th>Re</th>
<th>Mach</th>
</tr>
</thead>
<tbody>
<tr>
<td>Baseline</td>
<td>20.6°</td>
<td>32.8°</td>
<td>5.38</td>
<td>35°</td>
<td>0.25</td>
<td>-0.11</td>
<td>3.6</td>
<td>1</td>
<td>2.85</td>
<td>6.5e6</td>
<td>0.12</td>
</tr>
<tr>
<td>Target</td>
<td>23.1°</td>
<td>34.5°</td>
<td>7.8</td>
<td>28°</td>
<td>0.22</td>
<td>-0.36</td>
<td>3.1</td>
<td>1</td>
<td>3.86</td>
<td>1.2e6</td>
<td>0.3</td>
</tr>
<tr>
<td>$\Delta_{B\rightarrow T}$</td>
<td>+2.5°</td>
<td>+1.7°</td>
<td>+2.42</td>
<td>-7°</td>
<td>-0.03</td>
<td>-0.25</td>
<td>0</td>
<td>+1.01</td>
<td>-5.3e6</td>
<td>+0.18</td>
<td></td>
</tr>
</tbody>
</table>

The baseline model study shows that the effect of varying the Reynolds number within the same order of magnitude is a slight vertical shift in the pitching moment coefficient, which will be automatically corrected in the offsets in Step-2. Both datasets are in the incompressible regime and no data for Mach variation effects is available, so this is assumed to be negligible. Before applying any configuration change effects, the stall angles-of-attack at $C_{L,max}$ between the two aircraft are matched by horizontally shifting one curve, which is equivalent to virtually setting the wing incidence angles until both configurations have the same $\alpha_{crit,wb}$. The two datasets after this shift are shown in Figures 5.10 and 5.11, where the common stall angle is indicated by the vertical dotted line. The goal now is to recreate the post-stall
region of the target configuration for $15^\circ \leq \alpha \leq 25^\circ$ using the steps outlined previously in the section on Model Synthesis.

Step 1

The baseline stall model $C_i^{B_{ST}}$ is the sum of partitions $k = ST, DYN, UNS, CTL$

$$C_i^{B_{ST}} = \sum_k C_i^{B_{ST,k}}$$

(5.3)

but since only the $C_{n,ST}^{B_{ST}}$ and $C_{m,ST}^{B_{ST}}$ terms are considered here, the $k$-subscript can be dropped. Linearity with respect to sideslip $\beta$ is assumed as per previous section, so that sideslip derivatives are used as a low order approximation. Each term will be appended with corresponding $(\Delta C_{i,k})_j$ terms to get the estimated target stall model $C_{T,E}^{i,k}$, so that

$$C_{T,E}^{m} = C_m^{B_{ST}} + \sum_j (\Delta C_m)_j = C_m^{B_{ST}} + (\Delta C_m)_{FW} + (\Delta C_m)_H + (\Delta C_m)_I_{NH}$$

(5.4a)

$$C_{T,E}^{n,\beta} = C_{n,\beta}^{B_{ST}} + \sum_j (\Delta C_{n,\beta})_j = C_{n,\beta}^{B_{ST}} + (\Delta C_{n,\beta})_{W} + (\Delta C_{n,\beta})_V + (\Delta C_{n,\beta})_F$$

(5.4b)

where the last two terms in both coefficients are to be estimated using the SPM methods, and the wing-fuselage contributions are obtained from data. Whether from available data or SPM predictions, each delta term is calculated via the subtraction,

$$(\Delta C_i)_j = (C_i^{T})_j - (C_i^{B})_j$$

(5.5)

For the SPM predictions, the curve parameter versus geometry variation graphs from the previous Chapter are used to estimate the component contributions for both the baseline and target configurations, before finding their difference. Note that $(C_{n,\beta}^{B})_{W}$ is actually not available, so it will be estimated from similar sources [68]. The values for $C_{T,E}^{i,k}$ using this approach are shown in Figures 5.12 and 5.13,
Figure 5.12: Baseline and Estimated Target pitch moment coefficients.

Figure 5.13: Baseline and Estimated Target yaw moment due to sideslip coefficients.

Note that it is expected that the initial estimate can be highly offset, because many effects from the original trend datasets are compounded. However, this will be corrected for in the next steps.

Step 2

The estimated target stall model $C_{T_E}^T$ terms above are then each transformed (scaled and offset using $a, b$) such that it matches the target aircraft pre-stall model $C_{T_E}^{P_i}$ before $\alpha_{stall}$. The result is $C_{T_E}^T$

$$C_{T_E}^E = T(C_{T_E}^E) = a(C_{T_E}^T) + b$$

such that,

$$\sum_{\alpha}(C_{T_E}^E - C_{T_E}^{P_i})^2$$

is minimized on $[0^\circ, \alpha_{stall}]$ (5.6b)

This is applied to both static coefficients $C_{n_{\beta}}^E(\alpha)$ and $C_m^E(\alpha)$.
Step 3

Finally, the given pre-stall model of the target aircraft $C_T^P$ and the transformed estimate of the target aircraft stall model $C_T^E$ are then blended linearly over a certain range $[\alpha_1, \alpha_2]$, large enough to ensure smoothness, around $\alpha_{stall}$ to arrive at the desired target aircraft stall model $C_T^S$.

$$C_T^S = \begin{cases} 
    C_T^P & 0^\circ \leq \alpha < \alpha_1 \\
    (1 - \frac{\alpha - \alpha_1}{\alpha_2 - \alpha_1})C_T^P + \frac{\alpha - \alpha_1}{\alpha_2 - \alpha_1}C_T^E & \alpha_1 \leq \alpha < \alpha_2 \\
    C_T^E & \alpha_2 \leq \alpha \leq 25^\circ 
\end{cases} \quad (5.7)$$

where $i = m, n_\beta$. This is applied to both static coefficients $C_{n_\beta}^E(\alpha)$ and $C_m^E(\alpha)$, and compared to the post-stall regions of the target configuration.

Figure 5.16: Baseline and final Blended Target pitch moment coefficients, compared to reference.

Figure 5.17: Baseline and final Blended Target yaw moment coefficients, compared to reference.

These final results show that the predictions for $C_T^S$ and $C_{n_\beta}^S$, although not perfect, fall within reasonable margins of error from the true post-stall values of the target aircraft configuration. It is therefore also reasonable to assume that the representative stall model can be applied to simulator training for stall prevention and recovery.

### 5.3 LOC Prediction Criteria

Now that full aircraft coefficients have been estimated, they can be substituted into some basic departure prediction metrics for qualitative analysis. As discussed in Section 2.5, the metric $C_{n_\beta, dyn}$ and LCDP are well-cited and relatively simple to calculate giving preliminary insight into departure tendencies. These two metrics are calculated for the baseline and target models above, using

$$C_{n_\beta, dyn} = C_{n_\beta} \cos \alpha - \left( \frac{I_z}{I_x} \right)_B C_{l_\beta} \sin \alpha \quad (5.8a)$$

$$LCDP = (C_{n_\beta})_S - (C_{l_\beta})_S \left( \frac{C_{n_\beta}}{C_{l_\beta}} \right)_S \quad (5.8b)$$
where the value of $C_{n \beta, \text{dyn}}(\alpha) = \eta \delta_a$ is estimated using data from Reference [45] as described in Section 3.4.1, and mass inertia terms are estimated using design geometries.

![Figure 5.18: $C_{n \beta, \text{dyn}}$ for Baseline and Target configurations.](image)

![Figure 5.19: LCDP for Baseline and Target configurations.](image)

### 5.3.1 Stability Trajectory

Both departure tendency metrics of the baseline and target aircraft are plotted on the Integrated Bihrle-Weissman Chart in Figure 5.20. The overall qualitative LOC tendencies of the chart regions shown in Section 2.5, are also labelled in Figure 5.20 shown below.

![Figure 5.20: Integrated Bihrle-Weissman Chart for Baseline vs. Target configurations.](image)

As expected, both categories of aircraft are quite stable. However, the larger vertical tail of the baseline configuration, provides more stabilizing moments well into the post-stall regime. It can be seen that at higher angles-of-attack, both models start to veer into less stable regions. Since the model build-up method has been shown to provide decent estimates, future studies can combine the SPM predictions with these LOC criteria for early stage post-stall design considerations.
Chapter 6

Conclusions

This study is part of a first attempt methodology for creating a configuration adjustable representative stall model for the class of swept-wing T-tail regional transport aircraft. This effort has potential for improving UPRT in near and post stall flight regimes, and meeting the newly introduced FAA requirements for future flight simulator stall training, with the goal of reducing fatal accidents involving stall induced LOC. Initial results show that for the static coefficients considered, the predictions generally fall within reasonable margins of error. The dynamic/unsteady coefficients responsible for damping considered should only be trusted for order of magnitude, and need to be verified in future studies. This shows potential for the stall models to generate representative aircraft motions for the purpose of pilot training.

Summary of key trends and heuristics for representative stall modelling:

- Nacelle effects on the pitch moment past stall is heavily dependent on their relative position to the horizontal tail. However for many configurations, their effects below $\alpha \approx 25^\circ$ are minimal.

- The average dynamic pressure at the horizontal tail drops more significantly for higher tail shadow angles, because the tail shadow angle is closer to $\alpha_{stall}$.

- The average downwash at the horizontal tail will decrease past stall if the tail is placed low enough relative to the wings, such that tip vortices pass over the tail.

- Lower wing aspect ratio leads to higher downwash at the tail in general until a critical angle.

- The average dynamic pressure at the vertical tail is heavily dependent on the relative sizes of the fuselage and tail. Larger fuselage to vertical tail size ratio means more body blanketing.

- The sidewash gradient at the vertical tail is larger for higher wings due to the proximity of the stalled wing tip vortices.

- The forebody generated yawing moment will be stabilizing for horizontal elliptical cross-sections but destabilizing for vertical elliptical cross sections, scaled by fineness ratio and moment arm.

- The pitch/yaw damping due to horizontal/vertical tail is improved with larger tail surface areas.
6.1 Research Significance

Apart from the models, important contributions of this study include firstly the comprehensive literature review conducted summarizing essential stall characteristics and their relationships with model aerodynamic coefficients. A reference matrix of publicly available data on stall aerodynamics organizes literature with respect to aerodynamic coefficient and aircraft component. Second, a model extension and blending method is introduced, whereby the representative model is guaranteed to be correct prior to stall, make a smooth transition into post-stall, and at the very least give correct trends extending the given pre-stall model of the target aircraft using change effects added to the post-stall model of the baseline. Thirdly, to account for individual component contributions, change terms \(\Delta C_{i,k}^j\) are defined and subsequently calculated using the Shape Prescriptive Modeling approach, which while simple can offer fast engineering estimations using physics-based constraints and limited data. Lastly, LOC prediction criteria are used as a rough analytical approximation to give insight into sensitivity of overall aircraft stability to configuration parameter changes.

6.2 Recommendations for Future Work

The methodology presented here combined with a parallel study focusing on wing contributions can be applied to two baseline simulator models to produce target representative stall models. They are to be implemented in the UTIAS research simulator which generates visual and motion cues based on the flight model dynamics. This will enable transfer of training studies to be conducted on groups of pilots, to help refine the model and test stall training benefits. Before the implementation however, important items for future studies have been identified throughout the course of this study.

- Effects of Mach number and Reynolds number in stall regions should be be studied in depth.
- Extend the stall model’s range to higher sideslip angles where nonlinear effects cannot be ignored.
- Aerodynamic contributions of various wing dressing and lower level components should be included.
- Validation for same class aircraft with greater configuration deviations should be tested.
- Model the reduced effectiveness of control surfaces including ailerons, elevators, rudder.
- Research applicability of modern machine learning techniques to large experimental datasets.
Bibliography


[57] USAF Test Pilot School. Flying qualities flight testing phase - chapter 10 high angle of attack.


