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New and Improved Three and One-Third Parabolic Channel and Most Efficient Hydraulic Section

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Abstract. Several parabolic-shaped open channel sections are available in the literature, including quadratic and semi-cubic parabolic sections. This paper presents a three and one-third parabolic cross-section that has superior characteristics compared to those of previous parabolic-shaped sections. The section characteristics, including two approximate formulae for the wetted perimeter and a simple iterative formula for the normal water depth are presented. The exact solution for the most efficient hydraulic section is derived. The results show the width-depth ratio for the most efficient hydraulic section is 2.1273. Practical applications of the proposed most efficient hydraulic section are presented, including direct formulas for the discharge and explicit formulas of normal and critical depths. The results show that the proposed section improves the hydraulic characteristics with other parabolic sections and trapezoidal section.

Key Words: Three and one-third parabolic section; Open channel; Most efficient hydraulic section.

1. Introduction

Researchers have carried out numerous efforts to develop new channel sections that maximize the discharge under the same flow area or minimize construction cost under the same discharge (Das, 2007; Easa 2016). Existing parabolic channel sections can be divided into two categories: simple parabolic-shaped sections and complex parabolic-shaped sections. The simple parabolic-shaped sections, which have no horizontal bottom, include semi-cubic parabolic section (Han 2015), quadratic parabolic section (Loganathan 1991; Chahar 2005; Aksoy and Altan-Sakarya 2006) and cubic parabolic section (Han and Easa 2016). The complex parabolic-shaped sections, which have a horizontal bottom, include sections with semi-cubic parabolic sides (Han 2015), quadratic parabolic sides (Das 2007), and two-segment quadratic parabolic sides (Easa 2009).

The most efficient hydraulic (MEH) section has the largest flow capacity under the same flow area, and at the same time can make construction cost close to the minimum (Jain et al., 2004). It has been extensively used in channel design. For the simple sections, the semi-circular section has the largest capacity. However, the side slope of the semi-circular section at water surface is vertical and therefore the freeboard must be vertical which poses construction difficulties (Vatankhah 2014). After comparison with the quadratic parabolic, triangular, trapezoidal, and other sections, it was desirable to develop a new open channel cross-section that has the smallest deviation from the semi-circular section.

In this research, different combinations of the exponent parameters (m and n) of the general parabolic section ($y = cx^{m/n}$) were evaluated for $m = 1-5$ and $n = 1-20$. The results show that the discharge of the MEH section is maximum when $m = 10$ and $n = 3$. Therefore, a

new open channel section called three and one-third (10/3) parabolic section is proposed.

Clearly, this new section type can enrich the simple family of parabolic sections.

2. Characteristics of Proposed 10/3 Parabolic Section

2.1 Flow Area and Wetted Perimeter

The geometry of the 10/3 parabolic section is shown in Fig. 1. Let B be the water surface width and h be the water depth. Then, the function of the proposed section is given by

$$[1] \quad y = c |x|^{10/3}$$

where c = shape factor of the 10/3 parabolic section, y = vertical axis (m), and x = horizontal axis (m). Since $y = h$ at $x = B/2$, then the relationship between c , h , and B is given by

$$[2] \quad c = 8 \frac{\sqrt[3]{2}h}{B^{10/3}}$$

where B = water surface width (m) and h = water depth (m). The flow area A is obtained using integration as

$$[3] \quad \begin{aligned} A &= 2 \left(\frac{Bh}{2} - \int_0^{B/2} y dx \right) \\ &= hB - \frac{32^{2/3} B^{13/3} c}{208} \end{aligned}$$

Substituting Eqs. (2) into (3), A is expressed in terms of h and B as

$$[4] \quad A = \frac{10hB}{13}$$

The variable P can be derived by integration of the arc length as

$$\begin{aligned}
 [5] \quad P &= 2 \int_0^{\frac{B}{2}} ds = 2 \int_0^{\frac{B}{2}} \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx \\
 &= 2 \int_0^{\frac{B}{2}} \left(1 + 3\sqrt{100c^2 x^{14/3} + 9}\right) dx
 \end{aligned}$$

Clearly, it is difficult to compute the exact wetted perimeter using Eq. [5] and an approximation (presented later) is needed.

2.2 Comparing Discharge Under the Same B and h

Comparing with Square Parabola ($y=cx^2$) Cross-Section

The classic parabola section is defined as $y = cx^2$, where A and P are given in Monadjemi

(1994). Assuming a dimensionless parameter $\omega_2 = \frac{Q_{10/3}}{Q_2}$, then

$$\begin{aligned}
 [6] \quad \omega_2 &= \frac{Q_{10/3}}{Q_2} = \frac{A_{10/3}^{5/3} / P_{10/3}^{2/3}}{A_2^{5/3} / P_2^{2/3}} \\
 &= \frac{\left(\frac{10hB}{13}\right)^{5/3} / \left[2 \int_0^{\frac{B}{2}} \left(1 + 3\sqrt{100c^2 x^{14/3} + 9}\right) dx\right]^{2/3}}{\left(\frac{2}{3}Bh\right)^{5/3} / \left[1/8 \frac{\ln\left(4h + \sqrt{B^2 + 16h^2}\right)B^2 - \ln(B)B^2 + 4\sqrt{B^2 + 16h^2}h}{h}\right]^{2/3}}
 \end{aligned}$$

where A_2 = flow sectional area (m^2), P_2 = wetted perimeter (m) of $y = cx^2$ section, Q_2 = discharge under the same B and h (m^3/s).

The variable ω_2 is verified numerically with $h = 0.01$ -20 m and $B = 0.01$ -100 m, with an increment of 0.01 m. The results show that the minimum $\omega_2 = 1.220$, which means that the discharge of the proposed section is 22% larger than that of the quadratic parabolic section discharge.

Comparing with Cubic Parabola ($y=cx^3$) Cross -Section

The cubic parabola cross-section is defined as $y = cx^3$ (Han and Said 2016). Assuming a dimensionless parameter which, $\omega_3 = \frac{Q_{10/3}}{Q_3}$, which can be used to compare the discharge under the same B and h . ω_3 can be written as

$$[7] \quad \omega_3 = \frac{Q_{10/3}}{Q_3} = \frac{A_{10/3}^{5/3} / P_{10/3}^{2/3}}{A_3^{5/3} / P_3^{2/3}} = \frac{\left(\frac{10hB}{13}\right)^{5/3} / \left[2 \int_0^B \left(1/3 \sqrt{100c^2 x^{14/3} + 9}\right) dx\right]^{2/3}}{\left(\frac{3}{4}hB\right)^{5/3} / \left[2 \int_0^{B/2} \sqrt{9c^2 x^4 + 1} dx\right]^{2/3}}$$

where, $Q_{10/3}$ = flow discharge of the proposed section (m^3/s) and Q_3 = discharge of cubic parabola section under the same B and h (m^3/s). A_3 = flow sectional area of the cubic parabola section (m^2), and P_3 = wetted perimeter for $y = cx^3$ section (m).

The variable ω_3 is verified numerically with $h = 0.01\text{--}20$ m and $B = 0.01\text{--}100$ m, with an increment of 0.01 m. The results show that the minimum ω_3 is also greater than one, which means that the discharge of the proposed section is larger than that of the quadratic parabola section discharge under the same B and h .

The general section in the form $y = cx^{m/n}$ was evaluated for different combinations of m and n , where $m = 1\text{--}5$ and $n = 1\text{--}20$. The results show the flow capacity of the proposed cross-section is larger than that of other parabola sections for given B and h .

3 Practical Aspects

3.1 Wetted Perimeter Approximation

There are several approximate methods for the integration of arc length. Here, the Gauss-Legendre integration method is used (Abramowitz and Stegun 1972). The three-point approximation is given by

$$[8] \quad P = \frac{h^{3/10} \left(0.18519\sqrt{9 + 0.0037642J} + 0.18519\sqrt{9 + 57.235J} + 0.074074\sqrt{144 + 62.996J} \right)}{c^{3/10}}$$

where $J = c^{3/5} h^{7/5}$

Similarly, the four-point approximation is given by

$$[9] \quad P = \frac{\left(0.11595\sqrt{9 + 0.00039259J} + 0.21738\sqrt{9 + 0.56640J} + 0.21738\sqrt{9 + 15.428J} + 0.11595\sqrt{9 + 71.476J} \right) h^{3/10}}{c^{3/10}}$$

For given h and B , c can be computed using Eq. [2].

To verify the accuracy of Eqs. (8) and (9), consider a 10/3 parabolic channel with shape factor $c = 0.29$. It is required to compare the results of the explicit approximate formulas of Eqs. [8] and [9] for the wetted perimeter and those of numerical integration for $h = 1.0$ - 3.0 m (Table 1).

The results indicate that the maximum relative error is 0.25% and 0.04% using the three and four-point approximate formulas, respectively. Thus, the approximate formulas are very accurate.

3.2 Normal water depth calculation

The discharge of the uniform flow of open channel can be written using Manning formula as (Chow 1959; Akgiray 2005; Easa 2011)

$$[10] \quad Q = \frac{1}{n} \frac{A^{5/3} i^{1/2}}{P^{2/3}}$$

where Q = flow discharge (m^3/s), n = roughness of channel bed, i = longitudinal grade of channel bottom.

Normal water depth can be computed by solving Eq. [10]. However this equation is a nonlinear equation and it is difficult to solving. Here, using the numerical fittings method, a simple iteration formula is proposed as

$$[11] \quad h_{k+1} = 0.65 \frac{Q^{0.6} n^{0.6} B^{0.4} \left(0.057976 \sqrt{9 + 0.000015457 a^2 B^{14/3}} + W \right)^{0.4} a^{0.3}}{h^{0.3} i^{0.3}}$$

where, h_{k+1} = normal depth of the $k+1$ (th) times of iteration, h = normal depth of the k (th) times of iteration, $B = 2 \frac{h^{3/10}}{a^{3/10}}$, and

$$[12] \quad W = 0.10869 \left(\sqrt{9 + 0.022301 a^2 B^{14/3}} + \sqrt{9 + 0.60745 a^2 B^{14/3}} \right) + 0.057976 \sqrt{9 + 2.8142 a^2 B^{14/3}}$$

As an example, for $Q = 15 \text{ m}^3/\text{s}$, $n = 0.014$, $c = 0.2$, $i = 1/11000$, using the numerical method $h = 4.31677 \text{ m}$. Using the simple iteration formula of Eq. [11] with an initial value for $h = 1.0 \text{ m}$, then $h = 4.317 \text{ m}$ only after three iterations.

4 Most Efficient Hydraulic Section

4.1 Model Structure of MEH Section

The **MEH** maximizes Q for given A or P or alternatively minimizes A or P for given Q . The case of given P is represented by the following optimization model

$$[13] \quad \text{Maximize} \quad Q = \frac{1}{n} \frac{A(h, B)^{5/3} i^{1/2}}{P(h, B)^{2/3}}$$

$$[14] \quad \text{Subject to} \quad A(h, B) = C$$

$$\text{where, } C = \text{constant flow area, } A = \frac{10hB}{13}$$

Substituting Eq. [14] into Eq. [13], the objective function will become,

$$[15] \quad \text{Maximize} \quad Q = \frac{1}{n} \frac{C^{5/3} i^{1/2}}{P(h, B)^{2/3}}$$

Because n , i and C are constant value. This optimization model will become into

$$[16] \quad \text{Minimize} \quad P(h, B)$$

$$[17] \quad \text{Subject to} \quad \frac{10hB}{13} = C$$

4.2 Solution of Model for MEH Section

Although Eqs. (8) and (9) provide a simple and practical method for P of the 10/3 parabolic section, they are approximate expressions. The partial derivatives of P can be obtained by expressing the wetted perimeter P of Eq. (5) using the super-geometric function as

$$[18] \quad P = \frac{7B}{10} H_y \left(\left[3/14, 1/2 \right], \left[\frac{17}{14} \right], -\frac{400h^2}{9B^2} \right) + 1/10 \sqrt{9B^2 + 400h^2}$$

where $H_y = \text{Hypergeom}([a, b], c, z)$, which is the super-geometric function

Assuming a dimensionless parameter, $\eta = \frac{B}{h}$ and after simplifying, Eq. [14] and Eq. [18]

become

$$[19] \quad h = 1/10 \frac{\sqrt{130} \sqrt{\eta C}}{\eta}$$

$$[20] \quad P = 1/10h \left(7 \text{Hypergeom} \left([3/14, 1/2], [\frac{17}{14}], -\frac{400}{9\eta^2} \right) \eta + \sqrt{9\eta^2 + 400} \right)$$

Substituting Eq. [19] into Eq. [20], the optimization model (Eq. [16] and Eq. [17]) becomes

$$[21] \text{Minimize} \quad P = \frac{\sqrt{130}\sqrt{\eta C}}{100\eta} \left(7 \text{Hypergeom} \left([3/14, 1/2], [\frac{17}{14}], -\frac{400}{9\eta^2} \right) \eta + \sqrt{9\eta^2 + 400} \right)$$

Clearly, Eq. [21] is equivalent to $\frac{\partial P}{\partial \eta} = 0$. Then finding derivative of P with respect to the

variables η and simplifying results in

$$[22] \quad 357\sqrt{9\eta^2 + 400}\eta^2 H_{y1} + 459\eta^3 + 5600\sqrt{9\eta^2 + 400}H_{y2} - 20400\eta = 0$$

where $H_{y1} = \text{Hypergeom} \left([3/14, 1/2], [17/14], -\frac{400}{9\eta^2} \right)$, $H_{y2} = \text{Hypergeom} \left([17/14, 3/2], [31/14], -\frac{400}{9\eta^2} \right)$

Since $h > 0$, solving Eq. [22] numerically yields the only and feasible solution as

$$[23] \quad \eta = 2.1273028$$

This means that the exact ratio of water surface width to depth for the most efficient hydraulic

section is 2.1273. It is a important parameter. Substituting $\eta = \frac{B}{h} = 2.1273$ into Eq. [2], the

most efficient hydraulic shape factor c is obtained as

$$[24] \quad c = 0.8141h^{-7/3}$$

It should be noted that the same results can been obtained using Lagrange multipliers optimization method (Monadjemi 1994).

4.3 Application Aspects of MEH Section

Substituting for $\eta = \frac{B}{h} = 2.1273$ into Eqs. [3] and [18], A and P are obtained as

$$[25] \quad A = 1.6364h^2$$

$$[26] \quad P = 3.2298h$$

Substituting Eqs. [25] and [26] into Eq. [10] yields the following explicit formula for the discharge,

$$[27] \quad Q = 1.0400 \frac{h^{8/3} \sqrt{i}}{n}$$

which can be applied to compute the discharge of the MEH section given the water depth.

From Eq. [27], the explicit formula for water depth is given by

$$[28] \quad h = 0.9854Q^{3/8}$$

where $Q = \frac{Qn}{\sqrt{i}}$ (m³/s).

The water depth can be computed using Eq. (24) given the discharge. Combining Eqs. [28] and [24], the explicit formula of the factor of the MEH section is obtained as

$$[29] \quad c = 0.8425Q^{7/8}$$

which can be used to directly design the best hydraulic section using flow discharge.

4.4 Explicit Formulas for Normal and Critical Water Depths

The normal water depth is determined by Manning formula and is used in channel design, operation, flood control, and flow measurement and maintenance in open channels. Clearly, Eq. [28] can also be used for computing the normal water depth,

$$[30] \quad h_0 = 0.9854Q^{3/8}$$

The critical water depth signifies the state of flow at which the specific energy is minimum for given Q . It represents the gradually varied flow into subcritical and supercritical flows, and plays an important role in analyzing important aspects such as water surface profile, hydraulic jump, and energy dissipation. The general formula of the critical water depth is given by

$$[31] \quad \frac{\beta Q^2}{g} = \frac{A^3}{B}$$

where β = kinetic energy correction factor and g = acceleration of gravity (m/s^2).

Substituting for $B=2.1273h$ and for A from Eq. [25] into Eq. [31], the critical water depth is derives as following

$$[32] \quad h_c = 0.8654 \frac{\sqrt[5]{\beta Q^{2/5}}}{\sqrt[5]{g}}$$

Example 2: Consider an open channel with $Q = 26 \text{ m}^3/\text{s}$, $n = 0.014$, $i = 1/9000$, and $\beta = 1$. Design a 10/3 parabolic-shaped channel having maximum discharge. Substituting the known data into Eqs. [29] and [30], one obtains $c = 0.038$ and $h = 3.719 \text{ m}$. Then, using Eqs. [23] and [33], one obtains $B = \eta h = 7.912 \text{ m}$ and $h_c = 2.018 \text{ m}$. From Eqs. [25] and [26], one gets $A = 22.635 \text{ m}^2$ and $P = 12.012 \text{ m}$.

4.5 Comparison with Other Parabolic Sections (Under Optimal Conditions)

For the 10/3 parabolic section, substituting Eqs. [28] into [25] and [26], B , P , and A can be expressed in terms of Q as

$$[33] \quad B = 2.0963\delta^{3/8}$$

$$[34] \quad P = 3.1826\delta^{3/8}$$

$$[35] \quad A = 1.5890\delta^{3/4}$$

$$\text{where } \delta = \frac{Qn}{\sqrt{i}}.$$

Similarly, for the quadratic parabolic section $y = cx^2$, the optimal width-depth ratio can be obtained as $\eta = B/h = 2.0555$ (Han 2015). The characteristics of the quadratic, cubic, semi-cubic parabolic, and semi-circle section are presented in Table 2.

As previously mentioned, to obtain the MEH section, the general section in the form $y = cx^{m/n}$ was evaluated for different combinations of m and n , where $m = 1-5$ and $n = 1-20$. The results show that the discharge of the MEH section is maximum when $m = 10$ and $n = 3$. This means that the parabola section $y = cx^{10/3}$ is the MEH section.

The results show that the water depth of the 10/3 parabola section is smaller than that of the square parabola, cubic parabola and semi-cubic parabola section for given Q . In addition, comparison (Table 1) shows that B , P , and A of the proposed section are smaller than those of other parabola section and trapezoidal sections for given Q . In other words the discharge of the 10/3 parabola section is larger than these of other parabola and trapezoidal sections too.

Comparison with the semi-circle section shows that the difference in the flow area between the 10/3 parabolic and semi-circular sections is very small. For example, for $Q = 10 \text{ m}^3/\text{s}$, $n = 0.014$, and $i = 1/10000$, $A = 11.5003 \text{ m}^2$ for the 10/3 parabolic section and $A = 11.4590 \text{ m}^2$ for the semi-circular section (difference = 0.0413 m^2). However, the slope of the semi-circular section at the water surface is vertical which generally makes the construction of the freeboard rather difficult and exposes it to potential damaged.

5 Conclusions

In this paper, a new so-called three and one-third parabolic channel section has been proposed.

For practical purposes, the section characteristics, including an explicit approximation for the wetted perimeter are presented, and a simple iteration algorithm to calculate the normal water depth is proposed. A model of the best hydraulic section along with its solution is developed. The results show that the width-depth ratio for the MEH section is 2.1273. Using this ratio, explicit formulas for the discharge, shape factor, normal depth, and critical depth are derived. The comparison results show that the proposed section has a larger discharge than the quadratic, cubic, and semi parabolic sections as well as the trapezoidal section. The hydraulic characteristics of the new section were found to be superior to those of other parabolic-shaped sections under optimal and non-optimal conditions.

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Table 2 The hydraulic characteristic of different shaped channel sections

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Table 1 Results of the wetted perimeter using approximate formulas

h (m)	Theoretical P_t (m)	Three-Point Formula P_3 (m)	Absolute Error ($P_t - P_3$)	Four-Point Formula P_4 (m)	Absolute Error ($P_t - P_3$)
1.0	3.840	3.838	0.002	3.840	0.000
1.5	4.908	4.901	0.007	4.909	0.001
2.0	5.951	5.939	0.012	5.953	0.002
2.5	6.981	6.965	0.017	6.984	0.003
3.0	8.004	7.984	0.020	8.007	0.003

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Table 2 The hydraulic characteristic of different shaped channel sections

Section Type	B/h	Water Surface Width , B (m)	Flow Area, A (m^2)	Wetted Perimeter, P (m)	Shape Factor, a
Semi-circle	2.0	$2.0078\delta^{3/8}$	$1.58323\delta^{3/4}$	$3.1539\delta^{3/8}$	—
10/3 Parabolic	2.127	$2.0963\delta^{3/8}$	$1.5890\delta^{3/4}$	$3.1826\delta^{3/8}$	$c = 0.8425\epsilon^{-\frac{7}{8}}$
3.0 Parabolic	2.114	$2.1165\epsilon^{3/8}$	$1.5900\epsilon^{3/4}$	$3.1847\epsilon^{3/8}$	$c = 0.8470h^{-2}$
2.0 Parabolic	2.056	$2.2213\delta^{3/8}$	$1.6004\delta^{3/4}$	$3.2401\delta^{3/8}$	$c = 0.9467h^{-1}$
3/2 Parabolic	2.019	$2.3355\delta^{3/8}$	$1.6213\delta^{3/4}$	$3.3469\delta^{3/8}$	$c = 0.9862\frac{1}{\sqrt{h}}$
Trapezoidal	2.309^b	$2.2351\delta^{3/8}$	$1.6224\delta^{3/4}$	$3.3527\delta^{3/8}$	$z=\sqrt{3} / 3$

^a $\delta = Qn / \sqrt{i}$. ^b the ration of bottom width to depth, $b / h = 1.155$.

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Fig. 1 Geometry of the 10/3 parabolic section

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