Abstract

Modern researchers and educators are increasingly noting the need to better prepare high school mathematics students for entry into a post-secondary environment (Gula et al., 2015; Lovric & Kajander, 2005; Sahmbi, 2014). This research paper, based on the results of a qualitative study, sought to collect and organize accompanying sets of strategies that could prove useful in easing the transition from high school to post-secondary mathematics. Semi-structured interviews were conducted with two mathematics educators – one a professor at an Ontarian university involved in a math and computer science outreach program and the other an associate professor in a teacher training program, who has had over 20 years of experience teaching high school math students. These interviews revealed support for combining strategies such as PBL and multi-step problems related to higher-level mathematics, while introducing experts from the post-secondary community to help facilitate them. In addition, the interviews suggested the use in pairing differentiated instruction with technological learning objects to extend the mediums and approaches for solving math problems. Lastly, results showed that using modified rote activities such as procedural variation (Lai & Murray, 2012) could act as a means of reflecting on the nature of math problems, while simultaneously providing the support and practice students would need for PBL and technology-based activities.

Keywords: mathematics transition, problem-based learning, procedural variation, learning objects, differentiated instruction.
Acknowledgements

This research study could not have been completed without the support and effort of the academic community at OISE. In particular, I would like to thank my research supervisors, Cristina Guerrero and Arlo Kempf, who spent a great deal of time and effort ensuring that the MTRPs they oversaw were more than up to par. Their support and detailed feedback have helped greatly in shaping this study. Of course, much of my work would have proven more difficult if not for my Master of Teaching colleagues, with whom I have embarked on a two-year process of collaborating, revising, and refining both my research and my pedagogical practices. Thanks as well to my participants, P₁ and P₂, who have dedicated their professional opinions and reserves of knowledge towards this project. Last, but not least of all, I would also like to thank my family, and in particular my mother and father, whose continual support throughout both my undergraduate and graduate career has helped motivate me towards my goals.
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Chapter 1: Introduction

1.0 Introduction to the Research Study

Since the early 2000s, the nature of the Ontario mathematics curriculum has been in a jarring state of unrest. A series of changes at the high school level—most notably, the removal of the Ontario Academic Credit in 2003 and overhauls to the Advanced Functions & Introductory Calculus and Geometry & Discrete Mathematics courses in 2007—have left mathematics students with less time and less content to prepare them for post-secondary education. As a result, more educators are witnessing students who are fundamentally unprepared for post-secondary mathematics (Gula et al., 2015; Lovric & Kajander, 2005; Sahmbi, 2014).

This research study seeks to empower individual educators to prepare students for the more complex and multifaceted mathematics they will inevitably encounter; it does not, however, aim to make overarching claims about how best to remedy the Ontario mathematics curriculum, which may prove both an interesting and beneficial pursuit for future research. In this MTRP, I draw on a combination of academic research and semi-structured interviews with educators to suggest that the answer to preparing students for transition to post-secondary mathematics courses is best achieved through a holistic combination of complementing pedagogical strategies.

1.1 Research Problem

In Ontario, secondary and post-secondary educators are finding that more and more students are unprepared for their first year in higher-level mathematics courses (Gula et al., 2015; Lovric & Kajander, 2005; Sahmbi, 2014). While a number of researchers concern themselves with this topic (Hmelo-Silver, 2004; Habash & Suurtamm, 2010; Abramovich, 2014; Gula et al.,
2015), a major drawback to such work is that often, it focuses on a single underlying contributor to the lack of mathematical preparedness. In this MTRP, I indicate that researchers concerned with math transition can generally be grouped into one of three ‘camps’ of thought: those who see the importance in having high-school opportunities for applied mathematics, those who see mathematics engagement as a prerequisite for post-secondary learning, and those who believe in a ‘practice-makes-perfect’ model of rote learning. The issue is that these proposed solutions, robust as they are, seldom comment on their position within the broader classroom context, where several strategies and resources often need to work together in order to facilitate preparation for post-secondary education. To this effect, my MTRP works to unify the three camps where possible and provide teachers with practical results that they can extend to their own teaching practice.

1.2 Research Purpose

In this research paper, I outline elements of practical and meaningful instruction that teachers of students bound for post-secondary education can use in their classrooms. I do this by first exploring the strategies that researchers recommend for reducing the gap between students’ mathematical knowledge at the end of high school and the knowledge expected of them at the start of their post-secondary careers. I then combine these ideas with suggestions from two Ontarian educators: a retired teacher and current associate professor for a teacher education program in the province, and a professor from the University of Waterloo who works with the Centre for Education in Mathematics and Computing (CEMC). As a result, I develop a set of pedagogical strategies that work hand-in-hand to improve student learning and better prepare high school students for post-secondary options.
1.3 Main & Subsidiary Research Questions

Throughout this MTRP, I explore the major question of which teaching strategies best complement each other in helping to ease student transition from high school to post-secondary mathematics; I also seek to explore whether educators use combinations of these strategies within their classes, and in the cases where they do not, ask how the strategies might be made to work in combination with each other. As subsidiary questions, my research study engages in a discussion of how educators are preparing students for post-secondary mathematics. Specifically, what teaching strategies do they use or recommend, and are there specific resources they rely on to ease the transition? Do educators’ instructional techniques share similarities with the research already being conducted on this topic? Lastly, do those educators favour one research model over the other?

1.4 Reflexive Positioning Statement

As someone who has lived his entire life in Ontario, I have had the benefit of studying mathematics under the province’s “discovery learning”-styled classes at Brebeuf College School and the “rote learning”-styled courses at the University of Toronto. Throughout my education, I have developed an interest in understanding the ‘how’ of teaching mathematics. How do mathematics instructors present their ideas? How do students demonstrate their understanding? How are core mathematical concepts best learned? This passion for understanding mathematics pedagogies and how they can be refined to produce the most efficient means of instruction has come to form the basis for my research study.
One of the challenges in writing this MTRP had initially been that because I was educated under the TCDSB during my high school career, I lacked direct experience in the difficulties that math students within the public school system encountered. Through the practicum placements in my teacher education program, however, I have since had the opportunity to work at public schools in a teacher role.

As an educator, I taught students at the secondary level in English and mathematics, while as a tutor, I have worked in one-on-one settings with students from elementary, secondary, and post-secondary school.

1.5 Overview

Chapter 1 of this research study introduced the topic of mathematics transition in an Ontarian context, presenting the three camps of thought concerned with mathematical pedagogy. It posed the question as to whether there existed a set of strategies grounded in ideas from each camp that best prepared students for higher-level mathematics. In addition, it described the questions I would be posing about mathematics transition, along with my relationship to the topic and background as a researcher. Chapter 2 reviews a wide array of literature in order to introduce academic thoughts about the topic to date and explain the foundational collection of ideas that fuel the investigations undertaken in Chapter 4. Chapter 3 elaborates on details of carrying out this MTRP, which serves as a qualitative study grounded in synthesized research and semi-structured interviews. Chapter 4 reports on my research findings and discusses them through the lens of the related academic literature. Lastly, Chapter 5 identifies the implications of this research study’s findings and explains the limitations of its results, along with potential academic avenues that could be pursued in the future.
Chapter 2: Literature Review

2.0 Deciphering the Research

To understand the issue of mathematics transition, one must first understand the overarching academic discussions about it. In Chapter 1 of this research study, I mentioned three ‘camps’ that researchers often fell into when discussing the topic of math transition. Here I define the word camp to represent a collection of thinkers connected by some core issue in math transition and post-secondary preparation that they attempt to solve. As such, this chapter includes a discussion of three main types of researchers: those who see applied mathematics as a way to prepare students at the high school level for post-secondary math, those who promote mathematics engagement as a way of maintaining student interest in pursuing the subject, and those who value rote learning as a fundamental means of giving students both the practice and foundations that will be required by post-secondary institutions. There is also a brief discussion of an unofficial fourth camp, into which I place researchers whose ideas, while they do not directly adhere to any of the major groups, do serve to extend the conversation in meaningful ways.

Historically, ideas about mathematics transition and preparation for it have not always developed in a linear fashion; scholarly thought about applied mathematics, mathematics engagement, and even rote learning instead exists on a spectrum of proponents, with each idea garnering both support and criticism to date. Yet the debate about math transition, if conducted well, carries the potential to function much like a symposium, where numerous voices come together to comment and build on each other’s ideas. In this literature review, I develop the academic groundwork that will later surround my analysis of interviews with educators in
Chapter 4. I do this by contextualizing key researchers related to math transition within their supporting camps. I conclude by discussing ideas that fall outside of the three major camps but nonetheless work to extend their ideas, and lastly summarize the collection of this chapter’s research.

2.1 The First Camp: Proponents of Application

The first camp of thought consists of researchers who seek to understand the benefits of the applied maths – the study of mathematics for use in real world and often interdisciplinary problem-solving. Researchers involved with applied mathematics often insist on the importance of making the subject pertinent to the jobs that students will eventually apply for. This means that while students either directly enter the workforce or remain in the education system for several more years, the end goal of applied mathematics is ultimately to prepare them for a job. To do this, researchers suggest that mathematics should be both interdisciplinary and comprehensible. The first of these two criteria is easily-enough understood within the transitional context; if post-secondary education exists to extend and specialize student knowledge, then high school mathematics should include material that allows the subject to be transferred over into those disciplines that make use of it.

2.1.0 Mathematics & problem-based learning. On a general level, high school math classes serve as frequent testing grounds for new models of applied mathematics. The most recent of these is problem-based learning (PBL), a technique that emphasizes complex questions meant to evoke deeper thinking. PBL exposes students to open-ended problems; it then requires them to use self-directed learning in combination with group work to generate various answers.
Hmelo-Silver (2004) describes PBL as a system of problem solving that promotes a “social construction of knowledge” (p. 243). According to her, PBL is particularly strong in preparing students for thinking in a dynamic way that can still be used after graduation. As opposed to only knowing the solutions to a fixed set of problems, students instead learn the process of problem-solving itself. Furthermore, the PBL model is cyclical in that students generate hypotheses, apply them to the given open-ended problems, and reflect on their knowledge in order to achieve two goals: locating the errors – if any – that exist in their ideas and abstracting their final solutions.

While reflecting on the process of problem-solving allows students to check their work, abstraction allows the students to use their solutions in other situations; PBL often if not always requires these situations to relate to the workforce. With respect to this, Hmelo-Silver notes that PBL works best as an interdisciplinary tool, citing several instances – assessing a chemical spill and planning a trip along the Appalachian Trail being two examples – where student solutions require a combination of mathematical and scientific thinking (p. 244). The last and arguably
most important component of PBL is that teachers take on a supervisory role instead of an instructional one, resulting in the decentralization of knowledge and reiterating the importance of teamwork amongst peers (p. 239). In particular, teamwork becomes a skill transferable to post-secondary education and the workforce simultaneously.

However, not all researchers in support of applied mathematics favour PBL. Those in opposition to it often cite the model’s lack of both structure and instruction as its most damning quality. Kirschner et al. (2006) voice concern over PBL’s approach to storing information, arguing from a psychological perspective that the model overlooks basic distinctions between long-term memory, in which old information is permanently stored, and working memory, in which new information is held temporarily while being processed. They argue that,

Learners must construct a mental representation or schema irrespective of whether they are given complete or partial information. Complete information will result in a more accurate representation that is also more easily acquired. [Problem-based learning] is based therefore, on an observation that, although descriptively accurate, does not lead to a prescriptive instructional design theory or to effective pedagogical techniques (Kirschner, 2004, p. 78).

The solution Kirschner and his colleagues propose is “direct guidance” (DG), something that most instructors who incorporate PBL are forced to utilize anyway when the model falls short (p. 79).

In the model of direct guidance, teachers specifically establish tools and techniques for solving problems, providing students with examples and evidence before allowing them to approach open-ended questions on their own. This prevents frustration and, more importantly, misconceptions from occurring. Citing Moreno (2004), along with other classroom studies,
Kirschner et al. claim that deep and meaningful knowledge occurs when students are given guidance to start and afterwards left to explore content rather than being left to explore content from the beginning (Kirschner, 2006, p. 79). If PBL paints the teacher as a kind of project manager who oversees student work from afar, then DG, in contrast, sees him as the foreman, working closely with students to produce a refined and well-informed result. In the high school context, this has the potential to prevent flawed knowledge in those seeking to either continue their study of mathematics or apply it to a job after graduation.

Ultimately, while Kirschner and Hmelo-Silver agree on the need for application and extension of mathematical ideas, they differ specifically on the process of learning them.

2.1.1 The applicable process & complicated problems. What can be agreed upon by most researchers is that mathematics does require a clear and reflexive process in order to yield solutions, though to cover the cases made by numerous models outside of PBL and DG would require too-extensive of a discussion. Fortunately, a long-standing cornerstone of the mathematical process exists in George Pólya, who suggested a four-step model of problem-solving in 1945 that is still used today. In his work How To Solve It, Pólya identifies his four steps as understanding the question posed, devising a plan to solve it, carrying out the plan, and reflecting on one’s work (Pólya, 1945, pp. 5-6). This notion of procedural mathematics fits into the schemes of both PBL and DG and furthermore goes hand-in-hand with ideas about concrete mathematical applications. Nonetheless, one might reasonably question the usefulness of such a simple and dated model; in truth, several researchers have gone great lengths to apply a critical lens to Pólya’s ideas and bring him into the 21st century.
Abramovich (2014) calls attention to the nearly-universal fact that with the rise of the computer, applied mathematics – particularly at the university level – is being conducted through digital programs in order to solve increasingly difficult problems. He suggests that teacher candidates learn to take advantage of procedure-based technologies such as geometry programs, graphing software, and electronic spreadsheets so that they can implement them in their classrooms for the benefit of their students (Abramovich, 2014, p. 1035). Still, he is careful to temper this ambitious goal by writing that the most meaningful learning always stems from combining digital aid with constructive feedback (p. 1041). With respect to applied mathematics, Abramovich points out that engines such as Wolfram Alpha and Maple can power through complex but routine calculations, saving time for students who wish to use complicated mathematics as groundwork for answering harder questions (p. 1044). This, interestingly enough, places the technology-savvy student at a unique advantage in fields such as finance, engineering, and computer science – though it speaks little to those who find using these programs difficult.

In this respect, colleges face a very different kind of procedural problem. Gula et al. (2015) examine the issue of poor mathematics grades in college students across Ontario, which, as she states, can be attributed to socio-economic circumstances (p. 1131). Colleges, which act as the entry-point to post-secondary education for many low-income families, inherently create the problem of how to educate a relatively unlearned population about the applied mathematical process in a short span of time. She and her colleagues answer the issue with JUMP, an introductory mathematics program used in elementary and secondary education – transposed to first-year college classes. Her research bases itself off of a previous study conducted with JUMP
on a smaller test group; in her work, Gula expands her sample size and adds a control group to measure results more effectively.

Unique to the modified JUMP program is the fact that it addresses the problem of transitional gaps in knowledge from the post-secondary side. Gula admits the fact that students entering college come from various backgrounds, and as a result, standardizing both their knowledge and her measurement of it becomes the lynchpin of her and her colleagues’ work (pp. 1132, 1140). To address the lack of mathematical knowledge in students, Gula suggests the use of “explicit instruction”, or the breakdown of problems into smaller, easy-to-solve steps (p. 1132). Through her work, she finds that college students perform better when they are taught through tightly-managed lessons. This apparent sacrifice of mathematical freedom for the sake of mathematical knowledge still relies in part on elements of DG though, and once students master basic concepts, some simple PBL situations can be introduced. Moreover, the structures of these lessons still adhere to Pólya’s four-step model and, as a result, give students the mathematical and procedural tools required for learning. The end result is a comprehensive lesson structure for students learning applied mathematics. Below is an example of a lesson for exponents, taken from Gula’s study:

First, the students’ relevant prior knowledge, in this case knowledge of multiplication, is recalled. Second, the new topic (e.g., the power of a number) is introduced in relation to this prior knowledge. Finally, terminology (e.g., base and exponent) relevant to the new topic is introduced. These first three steps are enacted succinctly and with as little cognitive overhead as possible: furthermore, the students are instantly given practice to familiarize them to this topic and the basic definitions. The practice is designed to cover multiple perspectives on the same topic, with the final question requiring a slight
extension of the topic by having the students evaluate expressions to further solidify the connection between this topic and relevant prior knowledge (Gula et al., 2015, p. 1134). Gula’s overall results over the 13-week implementation of JUMP show a modest increase in student success, though variances in instructors, students, and the types of jobs students are studying for do leave notable discrepancies. (pp. 1141-1143)

2.1.2 Engineering & the specifics of applicable mathematics. By this point, the need to make mathematics an interdisciplinary, process-oriented, and comprehensible subject is a clear goal of the first camp. However, the particulars of achieving this remain equally important, and so a final commentary must be briefly undertaken to discuss the specifics of making math practical. Habash and Suurtamm (2010) address applied mathematics with respect to the study of engineering, arguing that for an education system to produce better engineers, it must first provide engineering-related content in its math and science courses; moreover, educators must show students the interconnectedness of math, science, and engineering. They point to the multidisciplinary field of mechatronics as a current model of this, and in their research refer to a week-long course they had built around it, which ran each year from 2006 to 2008. The course introduced students from grades 7 to 12 to topics in calculus and science through the lens of group work and problem-solving (Habash & Suurtam, 2010, p. 138). In addition, the course included a presentation on vectors and system engineering specifically for Grade 11 and 12 students. The presentation was given by undergraduate engineers, and emphasized the potential connections between mathematics and jobs in mechanical, electrical, and computer engineering (p. 137).
A survey conducted by the researchers afterwards found that out of the 44 student attendees, all were interested in the presentations; 97% said that the presentations helped them understand the connection between math, science, and engineering, and 86% said that the presentation made engineering seem more interesting to them (p. 142). Important to note is the course’s emphasis on mathematics as an interdisciplinary tool, along with its introduction of undergraduate engineers – those most embedded in the process of using mathematics in a post-secondary setting – to act in a co-operative role with students. In this sense, learning about the usefulness of the subject in other fields becomes loosely similar to an apprenticeship, where students are guided towards using mathematics by those with experience over the course of several content-specific workshops (p. 137).

2.2 The Second Camp: Proponents of Engagement

The second camp of thought on mathematics transition is, to some extent, more unified than the first. Proponents of this camp emphasize the need for teachers to engage students in a classroom setting – some of the most common recommendations for going about this include utilizing digital technology, making mathematics a differentiated process, and targeting deterrents of engagement.

2.2.0 Teaching with technology. While technologies such as graphing programs and calculating engines have already been discussed in the context of mathematics procedure, research on their uses in engaging students has also produced meaningful results. Kay and Knaack (2008) evaluate the potency of “learning objects”, or web-based interactive programs, in the classroom (p. 447). They include studies from both secondary and post-secondary areas, and
with respect to the later, note that at the undergraduate level, approximately 50% to 60% of selected students enjoyed using learning objects. While overall approval was not particularly strong amongst the surveyed groups, students did respond well to learning objects that included animations, self-assessment, an attractive interface, control of learning pace, and ease of use (p. 448). Furthermore, the learning objects appeared to bring with them practical implications, which Kay and Knaack measure through large-scale statistical surveys. These surveys show ranging increases in pass rates, from 12% to 23%, after implementation of web-based programs (p. 448).

Fewer learning object studies have been carried out at the secondary level, though of those that have, Kay and Knaack discuss four, which argue that high school students, without teacher prompting, focus more on the aesthetic qualities of learning objects as opposed to their functionality (p. 449). They point out that while many secondary school teachers feel that learning objects are important for enhancing student understanding, little to no research has been conducted on their effects on student performance.

Kay and Knaack’s own study sampled 850 secondary students spanning three boards of education. In it, they instructed teachers to select learning objects from the Learning Object Research Development and Evaluation Collaboratory database (LORDEC) and apply them to their classrooms; all students used the learning objects, completed pre- and post-study tests, and were given an anonymous survey afterwards. While preparation times for lessons involving learning objects varied, the study found that students generally appreciated the usefulness of learning objects – Kay and Knaack indicate that student reception to the given technologies statistically averaged out to 5.2 on a scale of 7 (pp. 454-455).
In terms of student performance, Kay and Knaack found that the marks of those who used learning objects to study for tests increased on average by 29.3% from 40.5% to 69.9% (p. 457). The magnitude of this increase suggests that the learning objects used played a significant role in student engagement. Combining this with the ways in which these technologies were used by teachers, Kay and Knaack make the claim that learning objects, when used to accompany, extend, and review lessons find significant success in secondary classrooms.

2.2.1 Intelligences and variety: differentiated instruction in mathematics. The motivation behind differentiated instruction (often referred to as DI) is relatively simple: there is no one best way of teaching a subject – rather, the closest an instructor might come is creating a “heterogeneous” learning community (Tomlinson, 1999, p. 1). Simply put, researchers in favour of differentiated instruction point to classrooms where teachers successfully rely on combinations of instructional techniques and assessment tools. This adds up to a sort of educational juggling act, where instructors retain student interest with new and exciting activities while matching his or her assessments to the different types of learners in the class. As such, differentiated instruction goes hand-in-hand with the Theory of Multiple Intelligences, or MI Theory (Gardner & Hatch, 1999).

This theory establishes eight major intelligences, listed below, and Gardner and Hatch claim that each of these categories represents an area of performance for a student. If a student is strongest in one specific type of intelligence, then that student is, according to Gardner and Hatch, predisposed to learning best when instruction plays to that strength (p. 7).
**Verbal-Linguistic** | **Logical-Mathematical** | **Visual-Spatial** | **Bodily-Kinesthetic**
--- | --- | --- | ---
Musical | Interpersonal | Intrapersonal | Naturalistic
(Added after 1999)

*Figure 2.* The table above lists the eight major intelligences discussed by Gardner and Hatch. Adapted from “Multiple Intelligences Go to School: Educational Implications of the Theory of Multiple Intelligences” by H. Gardner and T. Hatch, 1989, Educational Researcher, 18 (8), pp. 4-10.

Ultimately, multiple intelligences can be measured through providing numerous learning opportunities that value the strengths of each type of learner, and though Gardner and Hatch focus their study on elementary students, other researchers have wasted little time in applying MI Theory to high school classes. Korpershoek et al. (2013) frame MI Theory in terms of how problems are presented in mathematics, using results from several Dutch secondary schools (See Appendix A, 1). Their focus rested largely on word problems that required students to combine their comprehensions of linguistics, reasoning, and mathematical facts. Resultantly, their study concluded that variances in students’ characteristics were in fact at least partially responsible for their mathematics test scores; for example, differences in gender saw boys outperforming girls in strict mathematics but faring poorer than them in reading ability, and differences in ethnicity saw native-Dutch students outperforming non-natives (Korpershoek et al., 2013, n.p.). In addition, Korpershoek’s results found that the courses outside of mathematics that still relied on mathematical procedures – such as chemistry and physics – were significantly affected by how well students could balance their knowledge of math and literacy (p. 1030).

Other studies, such as those of Sintanakul and Sanrach (2016) and Valanides (1997) use MI Theory and learning strengths to illuminate issues with streaming. Sintanakul and Sanrach,
writing about the Taiwanese education system, describe how students are given a choice of five high school learning plans: “[the] Science-Math Plan, Arts-Math Plan, Language-Arts Plan, Social-Arts Plan, and General Arts Plan” (p. 555). Each is centred on preparation for a different post-secondary outcome and is chosen by students after they have taken the Grade 9 Ordinary National Education Test (O-NET). The test, which measures aptitudes in several areas, is also taken in Grade 12, which Sintanakul and Sanrach use to create a model for determining which intelligences a student is more likely to develop over the course of their high school education. For them, the idea of potential intelligences developing over time works in combination with Taiwanese learning plans to determine which subjects each plan best emphasizes and which combination of potential intelligences fits best with it. For instance, both the Science-Math Plan and Arts-Math Plan include mathematics, but in the first, it is combined with science and in the latter, it is combined with English (p. 556). In this case, students exhibiting verbal-linguistic potential, and who are nonetheless interested in a career in mathematics, are better suited to elect the Arts-Math Plan; in contrast, those with the potential for logical-mathematical intelligence are better served by choosing the Science-Math Plan.

Valanides, who bases his research in Cyprus, speaks further to the idea of developing intelligences by focusing on levels of reasoning in relation to various types of mathematics – specifically, science-based math, economic math, and “unified” or common math. Using results from a standardized Test of Logical Thinking (TOLT) in combination with Piaget’s Stages of Cognitive Development (Piaget, 1932), Valanides identifies students as being at any of the concrete, transitional, or formal operational stages of development (For an elaboration of these terms, see Appendix A, 2). These developments play a crucial role in MI Theory and differentiated instruction, though Valanides’ results must be first be examined before the
connection becomes apparent. His survey of TOLT scores found that economics students were more significantly successful in the formal operational areas of the test than unified students; furthermore, unified students appeared to be largely concrete thinkers (p. 178). Valanides adds that Piaget’s notion of equilibrium – that one’s surrounding environmental factors can affect his or her learning – may very well play a decisive role in high school curricula. If the ways lessons are structured suggest that formal operational problem-solving is preferred, students who rely more heavily on concrete problem-solving will automatically show less engagement with material (p. 179).

2.2.2 Deterrents of engagement: self-concepts and the learning environment.

Valanides (1997) touches on a key issue towards the end of his article – that a school system itself might predispose students to certain post-secondary outcomes. We see this notion work in a positive way through PBL, co-operative workshops, and learning objects. In these cases, students become accustomed to thinking about workforce-related problems, see their possible careers through workshop presenters, and familiarize themselves with using technology to solve mathematical problems.

The guiding hand of education, however, can also have negative effects, and several researchers of mathematical pedagogy have chosen to address the topic of engagement by first solving the problem of disengagement.

Nardi and Steward (2003) analyze student perceptions of mathematics lessons through case studies in Norfolk, England in order to determine how the conditions of a classroom can affect student disaffection with the subject. Specifically, they focus on students who are least likely to openly comment about the quality of their education in compulsory mathematics
courses, and in doing so, accept teaching methods that leave them unhappy. Amongst the several problems Nardi and Steward pinpoint, “depersonalization” most vividly depicts the effects of teacher attitudes on their classes.

During the researchers’ interviews with high school students, students noted that teachers poorly paced their lessons, instructional approaches prevented conversations and questions that could have served individual needs, and that they had concerns that students who were in higher “sets” – the equivalent of Ontario’s streams – were given better teachers (Nardi & Steward, 2003, p. 360). In addition, the way in which mathematics was exposed to students tended to affect how comfortable they were with it. Nardi and Steward point out that,

Beneath this dissatisfaction with mathematics as dry proceedings lies, perhaps, a longing for deeper, more essential understanding and for engagement with mathematics that goes beyond… a following of the cues provided by the teacher (or the textbook). As execution of procedures and memorisation is perceived as an efficient route to a better performance in mathematics, this deeper understanding—and the implied enjoyment—is seen in juxtaposition to what students perceive as their professional obligation towards task completion. (Nardi & Steward, 2003, p. 356)

Most notably, Nardi and Steward claim that students develop strong negative feelings towards mathematics – and furthermore, a negative self-conception as a student of mathematics – when their attempts at understanding material fail (pp. 356-357)

The notion of self-conception is studied by other researchers such as Areepattamannil and Freeman (2008), who in conducting research within the Greater Toronto Area, assert that student notions about themselves within their school communities and mathematics classes contribute to actual academic performance. In particular, their survey results showed that positive math self-
concept was especially important for successful immigrant students (pp. 705, 724-726).

Areepattamannil and Freeman point out that this positive self-concept works in combination with families who actively support achievement and schools that can boast opportunities for improving one’s status in Western society (p. 727).

Negative self-concepts, meanwhile, are more popularly dissected and find themselves the subject of numerous studies (Richardson & Woolfolk, 1980; Hunsley and Flessati, 1988; Flessati and Jamieson, 1991; Brady and Bowd, 2005). Carrying the nomenclature of “math anxiety”, concerns about one’s own performance in mathematics often affect their engagement with lessons. If a student feels overwhelmed, he or she may disassociate from the material, a problem that Shields et al. (2005) discuss in depth and to which they suggest several solutions (pp. 326-327).

The lynchpin of these solutions is creating a classroom environment focused on “student inquisition, discovery, learning and the exploration of ideas” (p. 327). To accomplish this, Shields et al. consider an intersection of several factors: teacher attitude, curriculum, pedagogy, classroom culture, and assessment (pp. 327-329). The result is a teacher who can balance praise and criticism by building a positive, interactive learning climate through equitable and differentiated instruction; in addition, this teacher should offer fair and content-based critiques of student work (p. 329).

One major barrier to this goal, however, can come directly from the teacher’s own confidence in his or her mathematical capability. Brady and Bowd (2005) reflect on their study of 238 pre-service education students from a Canadian university, learning under conditions that had led to fewer math teachers holding neither a major nor a minor in the subject (Brady &
Bowd, 2005, pp. 40, 44). Their study centred on questions about what contributed to a lack of confidence in teaching mathematics.

Of those surveyed, several reported that a lack of formal education in the subject made it difficult for them to project a genuine enthusiasm for what they were teaching. In addition, one teacher answered that her continuing education failed to refresh what she had learned about mathematics in school; as a result, she had to reteach herself core content over the course of her practica (p. 43). Brady and Bowd’s suggestions for teacher programs in Ontario is to properly review mathematics material in a meaningful way, while for teachers, they make the suggestion of staying aware of the attitudes they project in the classroom, since these nuances are most likely to affect the classroom climate (p. 41).

Ultimately, researchers concerned with mathematics engagement argue for a combination of promoting positive self-concept in both students and teachers, along with working to create a meaningful and open learning environment.

2.3 The Third Camp: Proponents of Rote

The final camp is perhaps the most interesting of the three. Rote learning – also called stimulus-response learning – is defined by a streamlined process wherein an instructor provides students with relevant information, often in the form of concrete facts. Students then commit themselves to memorizing content by repeatedly practicing it with the end goal being that a student can replicate that content later on (Holmes & McGregor, 2007). Undeniably, there is a certain procedural charm to it; what students need to know is clearly laid out, and the process of learning relies heavily on simple repetition. Research-wise, however, recent years have seen academics abandoning this style of teaching for newer, and as is often argued, sounder
pedagogical theories. That said, rote learning does come with its proponents, whose research provides a relevant and reliable defence of the model. This section will serve to illustrate both the critiques and claims of what rote learning can provide.

2.3.0 Cards against humanity: the case against rote. Rote learning, both inside and outside of mathematics, is a common target of modern academic criticism – those in opposition to it claim rigor and inflexibility to be its largest weaknesses (Mayer, 2002; Stanford Center for Opportunity Policy in Education, 2013). However, an earlier study conducted by Hilgard et al. (1953) does more to map out the boundaries of rote learning in comparison to a teaching style that is focused on student understanding. Hilgard and his colleagues contextualize the limits of rote by claiming that a meaningful understanding of mathematics is the most powerful form of engagement over longer periods of time, though rote learning might ultimately be preferred when time allocated for instruction and learning is limited (p. 288).

Hilgard et al.’s discussion focuses on a two-day survey they conducted involving card tricks and the techniques associated with them. Of the students who participated, each was placed into either of two main groups – those who were to learn through rote memorization and those who would be guided in their understanding of the techniques. Building on the previous experiments of Katona (1940), Hilgard and his colleagues introduced a card trick called “13 Spades” and used it to measure student comprehension (For a detailed outline of the trick, see Appendix A, 3-1 and 3-2). In their study, an experimenter showed two subject groups the trick, then asked them to reproduce it. In both cases, the trick was introduced with time restraints and a rigorous schedule of learning, then applying.
For the group of students using rote learning, the experimenter provided complete information on how the trick worked, but did not offer a correction or explanation of the errors students made when attempting to replicate the trick – instead, students directly moved on to the next task. For those who learned through understanding, the experimenter offered both correction and explanations of errors. Both groups of S then attempted more difficult problems, which required them to adapt what they had learned to slightly different card tricks (For details, see Appendix A, 3-3).

Hilgard et al.’s results showed that while the rote group of students performed better at short-term replication, the understanding group was able to adapt their results more successfully. Despite this, the understanding group required more time to learn the tricks and was poorer at retaining knowledge of them overnight (pp. 291-292). In addition, when more complicated results were introduced, both groups showed difficulty in adapting their learned knowledge of the tricks. Hilgard et al. claim that these results begin to illustrate understanding as a building process requiring time whereas rote learning serves students well in terms of immediate or “original” learning (p. 292).

2.3.1 Philosopjies, eastern education, undergraduate expectations: the case for rote.

Other academics, such as Akin (2001), go further to speak to rote learning’s long-term effects. Akin points out that thinking about what one is doing can in fact inhibit the speed at which the given task is completed; in this, he draws parallels between rote learning for the purpose of memorizing educational content and rote learning for the purpose of performing household tasks (Akin, 2001, p. 2). Generally speaking, he asserts that rote learning is the foundation upon which more complex understanding is built.
All of these arts involve thinking, but the thought occurs at higher levels which are built upon a foundation of unthinking facility. You think about how to vary a sauce not how to crack an egg, about what is the appropriate emphasis for a musical passage not what note is flat in the key of F. (Akin, 2001, p. 3)

The notion of rote learning as a foundational form of instruction is mirrored in the philosophical arguments of Handa (2011), who suggests that rote learning works most efficiently when aimed towards the goal of understanding content; as such, what many mistake for mindless repetition is in fact a process based on the will to practice for the sake of learning (p. 266).

A common theme between these two writers is that rote learning, whether automatic or intentional, is something undergone by students as a means to an end. Concrete examples of this are found in Lai (2012), who questions why rote-grounded mathematics classes in China produce stronger student comprehension than Western models (pp. 3-4). Lai’s answer is that Chinese instructors see repetition and understanding as going hand-in-hand, and because of this, they ground their pedagogy in the theory of variation. This learning theory suggests that some portions of a lesson may carry over into other lessons, or remain “constant”, while others change; the constant content is continually re-learned while new content that works with it is introduced in new classes (p. 5). In other words, seeing mathematics procedures used in various ways and in combination with each other allow students to develop an understanding that is both flexible and comprehensive.

Lai’s observations can be extended to a Western context, as Peters and Higbea (2014) show through their study of student preference for “stimulus-response” learning in Michigan. Peters and Higbea survey undergraduate students to determine what it is they expect from their classes and instructors. Of the students surveyed, a theme of learning as the process necessary for
one’s future position in a globalized and capitalistic system emerged (p. 92). This played into students’ requests that teachers give them a clear outline of course expectations, discuss relevant knowledge, and match assessments properly to topics discussed in class (p. 104). Furthermore, students were shown to utilize learning strategies outside of classes that most efficiently used their time, even if professors tried to foster deeper understandings of material. Peters and Higbea note that this approach to the subject content often came at a cost of “deep learning and understanding” (p. 103). Even more interesting are their findings that instructors who attempted to promote thoughtful analysis of a subject were ultimately undermined by undergraduate students who preferred clear and uncomplicated methods of instruction (p. 104).

Placed within the context of the previous literature in this chapter, the American college mindset of knowledge for the sake of its use in society can quite clearly be extrapolated to Ontarian students. That said, if high schools in Ontario are to match ideas of meaningful education with universities’ promises of prosperous careers, then four key lessons from pro-rote researchers must first be acknowledged:

1. *Abandoning rote entirely is detrimental to students.*

2. *Yet admitting its usefulness as a starting block from which deeper understanding may be built is beneficial.*

3. *Developing deeper understanding in results-focused students will usually if not always be difficult.*

4. *However, this development may best be achieved through using a variety of approaches to solve what would otherwise be seen as standard rote problems.*

### 2.4 The (Unofficial) Fourth Camp: Extensions of Camp Ideas

In addition to the three major camps concerned with mathematics transition, a fourth and final collection of researchers, whose ideas extend those already mentioned, should be addressed.
From this collection, two articles stand out as emphasizing a broader discussion of improving mathematics instruction by focusing on the transitional period. In doing so, they provide a useful means of applying previously-discussed concepts specifically to the period of mathematical transition. This section offers discourse on transitionary learning environments and partnerships with technology.

2.4.0 Transition as a rite of passage. Clark and Lovric (2008) build a fascinating model that encompasses aspects of sociology and sees mathematics transition as a “rite of passage” – specifically, they describe this period as an instance where students make decisions that will most directly affect their futures. Because of this, they argue, students require learning environments that make those decisions easier (Clark & Lovric, p. 755). To do this, however, requires acknowledgement of the fact that high school and university are grounded in different cultural ideals.

Mathematics, mathematical thinking and applications of mathematics are integral parts of our daily life and our culture. However, ‘different groups based on professional, social, cultural characteristics, race, ethnicity, gender, etc. have different views of mathematics and its role in the society’, and thus the transition becomes a multi-layered phenomenon, relevant not only to the future university students, but also to their families, and broader community, as well as society as a whole. (Clark & Lovric, 2008, p. 757)

Clark and Lovric also draw on a report by several mathematical institutes, titled Tackling the Mathematics Problem (1995) in order to elaborate on what mathematics within the societal context means. Strictly speaking, lower mathematical competency may be attributed in part to the fact that there are currently more students moving on to post-secondary education now than
there have been in previous years (Clark & Lovric, 2008, p. 758). However, among them, “high-attaining students” are still lacking in fundamental concepts related to mathematics. Clark and Lovric take aim at educational mandates and reforms such as the No Child Left Behind Act of 2001, complex assessments, and a movement away from traditional mathematics courses (Clark & Lovric, 2008, p. 758). The primary motivator for their arguments is a fear that relaxing mathematics instruction at the high school level will lead to a necessary decline in university-level mathematics, which will need to adjust itself to compensate for less-competent students (p. 763).

They also add that students themselves have a broad definition of what it is they want from education – enjoyment of the subject, support for their learning, access to materials, and so forth (p. 759). Clark and Lovric suggest that, at the post-secondary level, students should be placed into groups as a form of coping with the new environment; students may also be placed with partners further along in the program than them, where the senior students take on a mentoring role (p. 760). On a general level, students should be able to feel that their communities are there to support them, though one of the most important aspects of their rite of passage is that they themselves must learn to work towards improving themselves through hardship. Clark and Lovric address this specifically by noting that high schools should refrain from practices such as inflating grades, passing students “at any cost”, and shying away from teaching formal logic (pp. 762-763).

Ultimately, Clark and Lovric admit the unavoidable “shock of the new” that students face when entering university, but they argue that this shock must be faced directly by students, and that professors should act as guides who, while still ensuring that students are all brought to a
standard level of learning during their courses, ensure as well that they are brought to a
*university* level of learning.

**2.4.1 In partnership with technology.** Clark and Lovric, in addressing educational
policy as a contextual issue, open up an avenue of exploration that falls outside the jurisdiction
of the three major research camps. Nonetheless, using it to frame discussions of the pedagogical
ideas discussed so far proves useful for an overall understanding of factors affecting the success
or failure of transition.

Martinovic et al. (2013) speak to this in their commentary on how technology is used in
post-secondary settings. They test a computer program known as Maple, which students can use
to model mathematical problems at Brock University, asking how the current mathematics
curriculum in Ontario prepares high school students for technology-integrated mathematics
courses and how, at the university level, mathematics instruction can be designed to help aid in
partnerships between students and math-based applications (Martinovic et al, 2013, p. 77).

Their model for using computer-technologies in university courses requires students to be
able to identify the correct digital technology for the task at hand, have access to digital
resources, and use these resources effectively (p. 81). Martinovic et al. point out that new
generations, including the “Net Generation”, provide a useful opportunity where new university
students are already versed in using technology – hence, only training for the specific
mathematics applications is required (p. 81). However, the Ontario curriculum, which only
provides sporadic support for introducing digital math-based technologies during high school
leaves students lacking in knowledge of these math-based applications, which could otherwise be
present by the beginning of university (pp. 85-86).
The result is that students are learning about using mathematics in combination with technology, but are not being taught how to specifically apply this knowledge in “intelligent partnerships” with math-based applications. Martinovic et al. suggest that bridge this gap, university professors may take on a more active role in offering intelligent partnerships programs such as Mathematics Integrated with Computers (MICA), which has been linked to a tripling of Brock mathematics majors between 2001 and 2006 (p. 97).

Generally, the approach Martinovic et al. take to enhancing technology in high school classes is a top-down one.

By making more explicit the partnership role that technology may have in learning mathematics, it is our hope that more new and innovative strategies will trickle down to even younger students than undergraduate or secondary school. Both doing and learning school mathematics could be enhanced this way, making exciting the prospect of what could be done in the mathematics class of the future (Martinovic et al., 2013, p. 97).

2.5 A Summary of Ideas

The extent and variety of research conducted on improving mathematics transition offers a wide array of options for assembling a model that best prepares students for post-secondary education.

The first camp proposes making mathematics an applicable subject, and suggested ideas include PBL high school models, amendments to first-year post-secondary courses – as has been the case with JUMP – and creating cooperative learning environments through mentorship programs, such as Habash and Suurtamm’s 2-week mechatronics initiative.
The second camp argues for the creation of engaging learning environments through teaching with interactive and thought-provoking technologies, varying teaching styles by combining differentiated instruction with MI Theory, and targeting deterrents of engagement, such as negative self-concepts and learning environments.

The third camp argues for the necessity of rote as a foundational layer upon which deeper understandings of content may be built.

Lastly, the fourth camp extends ideas about learning environments and technologies as connected to the context of students’ transitional periods – or rites of passage – and the Ontario curriculum’s overarching goals for high school students.
Chapter 3: Methodology

3.0 Procedure

This study sought a pedagogical model suited to preparing high school math students for post-secondary education. To accomplish this, it took on the form of a qualitative case study, collecting and analyzing firsthand experiences from Ontarian educators. I conducted face-to-face interviews, partially through a “phenomenological” lens, which focuses on evaluating individuals’ experiences as centred on a common experience (Creswell, 2013, p. 42) – in this case, the experience was perception of mathematical preparedness/unpreparedness in the educators’ classrooms. The interviews focused on three key areas:

1) *How teachers and professors interpreted mathematical preparedness/unpreparedness in their classes.*

2) *Which of the three camps of thought they most associated with, and what suggestions or thoughts they had with respect to each camp.*

3) *The actions they took and ideas they suggested to help students prepare for post-secondary education.*

The interviews directly followed this MTRP’s literature review, which introduced three main camps, along with a fourth unofficial camp that extended ideas from the previous three. The first camp consisted of researchers concerned with making mathematics applicable to the workforce and the world. To do this, they put forth solutions involving problem-based learning, direct guidance, digital aid, and co-opt programs.

The second saw engagement as a tool for laying the foundations of mathematical preparedness. They also pursued technology, though as a medium that connected students to lesson content through specific learning objects. In addition, they discussed multiple intelligences, differentiated instruction, self-concept, and learning environments.
The third camp believed rote learning to be the teacher’s most powerful tool in preparing students for post-secondary education, and as such, suggested that unaided rote learning was useful for short-term practical memorization, though rote learning aimed at some meaningful purpose was much more powerful. In addition, researchers from this camp discovered that students preferred rote memorization as a style of learning and that abandoning rote was impossible. Resultantly, this camp suggested that rote learning could and should be used in combination with meaningful teaching in order to enhance it, as opposed to trying to remove rote entirely.

The fourth camp briefly covered transitionary environments, where students experienced post-secondary education as a milestone in their lives and as such, needed to face changes in their social and educational environments. It also discussed partnerships with technology, and in doing so, addressed an apparent incompatibility between the Ontario Curriculum and teaching technologies.

3.1 Instruments

Following a review of relevant literature, I conducted face-to-face interviews with three in-service secondary teachers and one professor, who all commented on mathematics transition. I used semi-structured interviews, which were guided by a list of interview questions (see Appendix C). Each interview was recorded and transcribed, then analyzed for shared themes in the participants’ experiences (Creswell, 2013, pp. 242). I grouped the questions into the following sections:

- General Questions
- Questions About Application
• Questions About Engagement
• Questions About Rote
• Reflection (for any time remaining)

I used these sections to separate and classify interviewees’ answers based on specific themes and ideas, following a framework that mirrored that of this MTRP’s literature review.

3.2 Participants

Two participants, referred to as P₁ and P₂ in this MTRP, contributed their knowledge and opinions through the interview portion of my study. P₁ is a professor at the University of Waterloo, involved with the Center for Education in Mathematics and Computing (CEMC), which promotes math and computer science by providing resources and activities to secondary school students and teachers. P₂ is a current associate professor for a teacher education program in Ontario. He leads courses on mathematics education and has over 20 years of experience in mathematics instruction at the secondary school level.

Both participants have had ample experience in gauging the effectiveness of different teaching strategies; in particular, they have had practice in implementing a range of pedagogical styles during their classes. As such, the participants were able to comment on the contributors and detriments to mathematical preparedness for high school students based on unique vantage points granted by their respective roles in the education system.

3.3 Data Collection and Analysis

After the interviews took place, I transcribed them, then coded and analyzed them. According to Creswell, researchers carrying out phenomenological studies look for common experiences of the participants surrounding a particular phenomenon (Creswell, 2013, p. 42).
Therefore, I analyzed the data to find common experiences these teachers had surrounding levels of mathematics preparedness, particularly in Grade 12 and first-year post-secondary students. I also looked for commonalities amongst \( P_1 \) and \( P_2 \)’s answers. This aided me in analyzing their pedagogical approaches to teaching math as they happened to align with one or a more of the three camps; through this, I assembled a collection of successful practices that could be used in Grade 12 mathematics classrooms to ease transition to the post-secondary environment.

### 3.4 Ethical Review Procedures

All voluntary participants in this study were given all relevant information about it and its content before they were asked to commit to participation. Prior to the interview, participants received a consent letter (see Appendix B), which gave them time to make an informed decision as to whether or not they wished to participate. In addition, participants were given time and opportunity to clarify points of information; they were also free to withdraw from the study at any time, including during the interview.

After informing participants of the topic and purpose of this study, each participant was given a pseudonym to guarantee his or her anonymity; transcripts and recordings were kept on my computer until after my research for this project was complete, at which point, they were deleted. Participants had full knowledge of the interview process and were aware that it would be recorded. Interviews took place in a location of the participants’ choice in order to ensure their comfort. After the transcribing of the interviews took place, participants received a copy of their signed consent form, along with a transcribed copy of the interview. They were also allowed the opportunity to review the paper prior to submission in order to correct/clarify any conclusions.
drawn from the information they gave me. Lastly, they were given my contact information and were permitted to review the final draft of the research paper.

3.5 Limitations

The most pressing limitations of this study came from its timeframe and range. To speak to the first issue, more time to analyze results from interviewees would have proven useful for deepening the understandings and connections produced from the work.

In terms of range, the number of teachers interviewed, along with the durations of the interviews limited possible avenues of conversation that could have been pursued. As a Master of Teaching student at OISE, I most readily had access to educators in the immediate area; however, this study did not include the perspectives of rural Ontarian teachers. This served to limit the generalizability of the study. In addition, I would have appreciated the opportunity to interview students about their thoughts on mathematics transition, which would have provided crucial perspectives for analyzing my research findings. Due to restrictions and preserving student privacy, I was unable to interview them.

3.6 Conclusion

This chapter has explained my own research methodology, which aims to collect various views from interviewees about how best to prepare students for math transition. In the following chapter, I develop my research into an analysis of each interview, drawing on ideas from the camps of thought outlined in the Chapter 2 literature review to support and emphasize certain aspects of P₁ and P₂’s responses. Through this two-pronged analysis, I define a collection of pedagogical strategies and resources that, when used in Grade 12 mathematics classrooms, carry
the potential to complement each other and guide students towards better preparedness for post-secondary education.
Chapter 4: Findings

4.0 Introduction

Contained in this chapter are the findings of this MTRP’s qualitative study. The study includes excerpts from two participants, referred to here as P₁ and P₂ in order to preserve their anonymity. P₁ is a professor at the University of Waterloo, involved with the Center for Education in Mathematics and Computer Science (CEMC). The CEMC is an outreach initiative aimed at providing resources and activities to secondary school students in order to promote math and computer science. P₂ is a current associate professor within an Ontarian teacher education program. He leads courses on mathematics education and has over 20 years of experience in teaching math at the secondary school level.

Each participant was recruited in person based on their experience in mathematics education and educational initiatives, as laid out by the requirements of this research project (see Appendix B for the letter of consent outlining the project’s requirements). The findings presented in this chapter were drawn from two audio interviews, consisting of 52 and 45-minute sections of dialogue for P₁ and P₂ respectively. Each of the interviews was initially coded into pieces of data, which were then rearranged and grouped into larger themes in accordance with Creswell (2013).

Each theme includes excerpts or observations from the interviews, along with accompanying discourse that shows how these educators’ pedagogical strategies branch from one of the three camps of mathematics research – applicable mathematics, mathematics engagement, and rote learning – that were outlined in Chapter 2. Specifically, the study’s findings aim to provide insight into this MTRP’s research question, which asks what combinations of strategies best complement each other in easing the transition from secondary to
post-secondary mathematics courses, and whether or not educators use these strategies in their math classrooms.

The first theme illuminates a topic of conversation that I found to be the most prevalent in both the research and interview data: using problem-based learning over extended periods of time and in combination with complementing teaching strategies like introducing experts and promoting student leadership. I open the theme with a discussion of what I define to be “problem-based learning (PBL) projects”, or structured inquiries into real-world and interdisciplinary problems. Based on interview data, I illuminate the prerequisites for implementing this kind of project, then explore how different resources and techniques enhance PBL projects to produce lessons that prompt higher-order thinking and mirror real-world and often multi-step challenges. In doing this, I offer an answer to my MTRP’s subsidiary research question: What teaching strategies do educators use or recommend, and are there specific resources they have relied on to ease mathematics transition?

In the second theme I discuss how differentiated instruction and technology both fill key gaps in the PBL project model. Interview data from my participants suggested that differentiated instruction worked best in allowing the teacher to guide as opposed to strictly instruct students, hence improving students’ self-reliance and perseverance towards long-term goals. This instructional strategy however does have its drawbacks, as $P_2$ in particular found it to be time-consuming and requiring a great deal of forethought. Here, technology can help to streamline differentiation by giving teachers an easy medium over which to present content and create collaborative online communities of shared work. Through my analysis of these two sub-themes, I respond to the second part of my research question: how combining teaching strategies has worked to improve student learning in their classes.
My third theme emphasizes the concept of higher-order rote learning as an underpinning to PBL projects; discussed by P₁ and P₂ as practicing mathematics but in a variety of ways, my interview findings showed specific similarities to ideas such as “procedural variation”, discussed by Lai and Murray (2009). Previously in research, rote was lauded as a way of building up a basic knowledge of static facts and materials (Holmes & McGregor, 2007), however, in this theme I discuss its use as seen by P₁ and P₂ as a gateway to more deeply understanding the kinds of complex problems found in post-secondary mathematics. I begin by describing the benefits my interviewees have seen in using rote to approach math problems from various angles, then follow by elaborating on how rote works in partnership with PBL projects as a whole by giving students the opportunity to pursue challenges and experience failure in, as P₁ describes, a controlled and constructive way.

The chapter concludes by offering commentary on how the holistic combination of these strategies and resources helps to ease student transition from secondary to post-secondary mathematics classrooms.

4.1 Applicable Mathematics Strategies

In the literature review of Chapter 2, several proponents of applicable mathematics offered suggestions to improve learning and knowledge for math transition through pedagogies that enforce higher-order (or more complex) and open-ended problems (Hmelo-Silver, 2004; Habash & Suurtamm, 2010). In discussing these kinds of strategies for teaching applicable math, I found that P₁ and P₂ both had valuable insight to offer, especially with respect to problem-based learning (PBL). PBL helps to promote higher-order skills in students by challenging them with open-ended questions that require integrative thinking to solve, and both educators saw
value in this kind of pedagogy for easing math transition. While P₁ pointed to resources such as *Task Maths* (which deals with themed math questions relating to the real world), P₂ indicated that there were very specific prerequisites that needed to be fulfilled before PBL could begin to be used for improving student skillsets. My discussion of this content begins in the following sub-theme.

### 4.1.0 Prerequisites for problem-based learning (PBL)

A good starting point for the conversation about complex and open-ended math problems is that of problem-based learning. A way of working through situational and often multi-step word problems, PBL was in fact referenced by both P₁ and P₂ in their interview responses. This pedagogy emphasizes a student-driven instructional strategy, where the learners themselves are challenged to lead and generate conversations. In my interview with P₂, I was offered his thoughts on the circumstances under which proper PBL could take place.

> If you take teachers and put them in a school system where… maybe they teach six classes a day, not a lot of time off during the day, high pressure, lots of students in the classroom, you know, they’re, they’re quite busy. You don’t provide them with much P.D. It’s really hard to help teachers change, you know, because they’re just trying to stay above water. So definitely you need some time, plenty for teachers to work together on these things, and… develop skills to help them teach this way. You definitely need that, and you know, you need teachers who are open-minded, willing to try things.

In this excerpt, P₂ addresses the need for what I believe are three crucial precursors to PBL: time to implement it, opportunity to practice and refine it, and professional development.
Let us begin by considering the first of these things. In order to best implement new forms of problem-based learning in meaningful ways within classrooms, teachers may ultimately benefit from setting aside blocks of time in-class to specifically create problem-based learning situations. In most Ontarian school boards, the length of a class is roughly 70 minutes. An instructor who dedicates 20 minutes at the start of each lesson to an extended PBL project may create a situation where an exercise in critical thought introduces students to a better understanding of the lesson content, while also doubling as a “minds on” activity.

Consider the practical example: a teacher might choose to merge the fields of architecture and mathematics through a semester-long open-ended project, whereby teams of students use AutoCAD (a computer-aided design program relied on by architects) to create floor plans for a school. This alone would already require several math concepts in combination—trigonometry, volumes and surface areas, and optimization, depending on what restraints the teacher chose to place on the project.

Business and finance could also play a role in a second phase of this PBL example, where the student teams must determine the costs of building and maintaining their school. Research on materials, lighting and heating prices, and other factors could be distilled into variables and formulas (for example, by using \( n \) to represent the number of hours a school needs to be heated for during the 194 school days in which it is operational every year, then multiplying that by the cost of lighting per hour).

A teacher might even want to take this a step further in a third phase and integrate the arts by having each student team create a small-scale model of their school. These models could be set up around the school, along with the accompanying mathematics, as a demonstration of the students’ interdisciplinary proficiency to the school community. To put it all shortly, such an
exercise in problem-based learning could potentially achieve three key qualities discussed through the research: intrinsic motivation, the ability to collaborate, and interdisciplinary proficiency (Hmelo-Silver, 2004, pp. 244, 246, 260).

To be clear, this, like many PBL projects, is a daunting task for both students and teachers, however, breaking it up into manageable and consistent chunks throughout the year can help teachers streamline its introduction and promote a genuine application of mathematics with respect to essential workforce skills. This topic, however, is left to a discussion in 4.3.1.

The two other prerequisites for PBL discussed by P2 can be more easily and directly met since, one being opportunity to practice and the other being professional development, they already go hand-in-hand. Professional development already establishes an environment apt to providing opportunities for the practicing and refinement of PBL. Hence, the obvious suggestion would be to dedicate more professional development time to learning, practicing, and most importantly sharing these strategies. Outside of professional development, teachers might consider sharing PBL ideas with each other, partnering to use the same PBL projects if they both teach the same kinds of classes, and keeping collections of PBL projects for future teachers to refer to. This last suggestion hints at the possibility of a functional and school-run PBL repository, something that, based on my own knowledge of Ontarian schools, is not often enough a priority. Teachers might also rely on sources of projects – and moreover knowledge – from outside of their schools; here, field experts become key.

4.1.1 The Student-Expert Relationship. Noticeably, problem-based learning fits well into the scheme of mentorship activities, such as those discussed in Habash and Suurtamm (2010), where engineering students from the University of Ottawa connected with secondary
school students to tackle open-ended and interdisciplinary problems from the field. In this section I define an expert as a professor, worker, or post-secondary student with higher-than-secondary knowledge of a work-related field in mathematics. When it came to the topic of experts in Ontarian classrooms, P₁ showed enthusiasm for such initiatives. As a member of the CEMC, last year he visited over 300 secondary schools both in and outside of Canada in order to host workshops on the application of math to fields such as computer engineering. In describing the initiative, P₁ stated, “the workshops are designed to spread some enthusiasm and love for mathematics and computer science as well as to talk about the careers and applications in those areas, which is a question teachers often get asked.” In particular, it was apparent that he and his colleagues were able to more aptly answer field-specific questions that students had and could do it in a more efficient and up-to-date manner than teachers who were only knowledgeable about the school subject itself.

While the power of such a strategy varies depending on the resources and personnel available for school visits, P₁ did mention that in addition to visiting secondary schools, his outreach program also hosts workshops on the university campus and provides online courseware and resources. These same strategies for outreach are referenced by Habash and Suurtamm (2014, p. 138), and importantly suggest that postsecondary institutions capable of taking on the roles of preparing resources, speakers, and venues in cases where such things may be too difficult or time-consuming for a secondary school to organize, meaning that secondary schools with this difficulty could focus their attention on promoting the programs in partnership with universities. Of course ultimately the responsibility falls to school boards and universities to first make contact with each other, though there is nothing to stop secondary school teachers from suggesting this to their school administrators.
By asking what role the introduction of experts plays within a pedagogical model involving PBL, one will quickly see an obvious tie-in, as experts are a direct and efficient way of informing the structure that a PBL project takes. In particular, an expert could potentially provide contextual information to students, offer them suggestions for improving their project-related ideas, help coordinate the projects’ details with the teacher, or oversee certain phases. This could benefit all parties by providing students and teachers with access to genuine field experts, but also rewarding the experts with recognition in the form of volunteer certificates (or for the University of Toronto, with an indication on their Co-Curricular Record).

Of course in analysing the data from my interviews, I also considered the question of how PBL projects, being longer and multi-step tasks, could function in a peer community when experts were not available to help guide learning. One solution is to create a spirit of student leadership through perseverance and the internalization of teacher strategies in a way that also conveniently helps prepare students for more active roles in post-secondary school. In the following sub-theme, I expand on this notion.

**4.1.2 Perseverance as a method for improving student leadership.** Under the applicable math research camp in Chapter 2, the discussion of Pólya’s 4-step model – understanding the question posed, devising a plan to solve it, carrying out the plan, and reflecting on one’s work (Abramovich, 2014) – led to an appreciation that not every problem in mathematics has a clear and easy-to-see solution. Creating students who understand and reflect this in their work comes as a result of a gradual transition towards having students learn to pace themselves and think through problems in an integrative manner. My interview with P₁ highlighted this fact, with P₁ stating:
We find that... students are good at solving questions that they know how to do—that they think they know how to do, but when they come to a question that they cannot immediately see a solution to... it’s a challenge for them to, to learn to persist, to know that, okay I didn’t get this... I didn’t get the answer now, but if I tried again tomorrow, or if I tried again in three days, if I work at it a little bit every day, even if I don’t seem to be getting somewhere, I will eventually get somewhere.

Having students who can complete procedural problems does not necessarily help those same students when they encounter situations, often in post-secondary education, where more intensive and extended thinking is required. The question then becomes, “How does an educator train students to anticipate and persevere in a situation where there is no easy or immediate solution to a problem?” Recalling the work of Gula et al. (2015), a study group of Ontarian college students, who had difficulty in completing math work, required problems that were broken down into smaller steps with consistent support given by the instructor. P1’s interview responses suggested a similar approach, though with the added need to push students towards internalizing the strategies used originally by their instructors. To accomplish this, P1 pointed to the importance of the teacher as a model for perseverance.

It’s not me pushing down onto the students; it’s me working with the students... to be enthusiastic, to be engaged, to know what you’re talking about, to be willing to admit when you don’t know what you’re talking about; you know, students asks a question, and instead of making up an answer, you say “Well you know what? I don’t know. I’ll find out and I’ll tell you tomorrow.”

While Gula’s initial suggestion of explicit instruction was and is useful for padding students who need additional coaching, an important takeaway is that honest and open dialogue can pave the
way to success in longer-spanning problems (such as PBL projects) by providing the opportunity to teach students in a manner that guides them towards self-sufficiency.

The Euclid Mathematics Contest, a test-based event run by Waterloo and directed at students in their final year of secondary school, incorporates this kind of guiding framework into the way it poses its questions. In describing the contest’s layout to me, P₁ pointed out, “The Euclid Contest consists of a mixture of short answer and full solution questions. The earliest questions on the paper are much more curriculum based, are closer to things that they [the Grade 12 students] will have learned in their classrooms.” P₁ also described how the more complicated questions would at points require students to draw on several areas of their expertise. This meant that successful students not only generated pieces of knowledge, but used a degree of intuition to assemble those pieces to solve multi-step questions. On the part of the secondary math teacher, a similar approach might be used in math projects and drawn out over a longer period of time. Specifically, this entails easing into more complicated questions over each class, with an eventual emphasis on using teacher questions and assessments in ways that empower students to think for themselves instead of simply being given answers.

For PBL projects in particular, a teacher might choose to have project goals build on each other, with the smaller initial goals becoming prerequisites for later ones. To this effect (and similar to the framework of the Euclid Mathematics Contest), teachers could use small milestones at the start of the project as prerequisites for a larger coalition of student work that would be presented at the end of the project through a showcase event. This idea of having students display their work and process to each other and to the school community was touched on by P₂ during my interview with him:
One idea is where you start off with a problem, and the students work on it, and they figure out what they need to know… there’s intervention from the teacher as well, but they solve the problem and, what also… generally comes with the problem-based learning, is where in the end students present their solutions.

The benefit in sharing and, more importantly, explaining one’s work at the end of the PBL project is that students are given the chance to realize and understand the process that has led to that final achievement. Similar to P₁’s notion of working at a problem “a little bit every day”, P₂’s suggestion of a culminating presentation of student work provides a holistic way for those students to reflect on the work they previously did. Furthermore it shows the students the eventual value in perseverance, prompting them in addition to actively reflect on and discuss the strategies they used throughout the project.

This said, there are still questions left to answer about the PBL project framework – mainly, how can teachers ensure that any of the strategies discussed thus far reach all students in a meaningful and individualized way that aids them in their path to post-secondary education? As monumental as this question may appear, in the following theme, I discuss how teachers can tailor PBL projects to different types of learners through differentiated instruction and how classroom resources such as online programs can help students benefit from their peer communities in a way that decentralizes the role of the teacher.

4.2 Improving PBL Projects Through Differentiated Instruction and Technology

Two large themes from the research camp of mathematics engagement – differentiated instruction (Tomlinson, 1999) and technology in the classroom (Kay & Knaack, 2008) re-emerged throughout my interviews with P₁ and P₂. In particular, P₁ referenced the importance
and manner of differentiated instruction occurring at the post-secondary level, while P₂ commented on the importance of using technology to foster a deeper and more conceptual understanding of mathematics. The following sub-themes will analyse the interview data with respect to differentiated instruction and digital technologies as means of reaching all students within a classroom.

**4.2.0 Differentiating instruction to reach more students.** In my interview with P₁, the topic of differentiating classroom instruction as a way to improve all students’ understanding of content (Tomlinson, 1999) arose when P₁ commented on the strategies he used in his classes with post-secondary students:

> [As professors] we do find ourselves lecturing more, but we’ve been trying over the last few years to incorporate more time for the students to try things in class, so rather than me talking for 50 minutes, you know, I’ll talk for 15 or 20 and then I’ll give them a question to try for 5 minutes and to work with a partner on. Then we’ll talk about it, then… I will teach a bit more, and then give them another chance to work on something… I certainly am a strong believer in interactions in my classrooms, whether it’s from a quote on quote small class of 60 here at the university to a large class of 200 or 300… I think interaction is really important—it helps engage students and it helps me gauge where things are in the classroom.

This break from traditional lecture-style instruction to instead facilitate critical, student-driven reasoning about math problems is a quality that can be easily carried over into secondary school classes. The benefit in differentiated instruction is that it gives a teacher the opportunity to briefly present the lesson material, then move on to host an activity if students appear to be
disengaging with or failing to understand concepts in that particular way. The teacher may then perhaps change pace again to allow for an individual work period if some students still do not understand the content. Now, however, the teacher is able to move around and assist various students specifically, keeping them focused and answering any questions they might have in a more direct and thorough way. At points, if a teacher chooses to, he or she can even return full-circle with consolidations of tasks as P\textsubscript{1} suggests by inserting class discussions throughout the lesson, or by pointing out important rules and concepts. Conveniently, this kind of strategy also answers the problem of how to reach diverse student learners discussed by Gardner and Hatch (1989), who purported the existence of multiple intelligences – or ways of learning content – that varied from student to student.

Such a style of engagement-based and differentiated teaching rejects the archetype of teachers as givers of knowledge and replaces it with teachers who can afford to be mobile in their classes by handing agency over to students and fostering a spirit of self-reliance. In such a case, differentiated instruction makes for an easy gateway to applicable mathematics, as it provides both the time and opportunity to practice tailoring instruction to different students’ needs.

One drawback to note though is that because differentiation requires a teacher to have a combination of instructional strategies (and potentially assessments) prepared, dealing with a class of 20 to 30 students can quickly become a great deal of work. In my interview with him, P\textsubscript{2} mentioned an experience of his in practicing PBL tasks, noting:

One thing I never really got good at was the assessment part of it. The marking. You know, I would go to the English teachers in the school and ask them about marking the
[differentiated PBL] word problems, and they would say, ‘Well, did you ask the students about so and so ahead of time?

It is clear from $P_2$’s example that careful planning and time is required in easing the move towards differentiated PBL projects. To lessen the pressure on any individual teacher, in 4.1.1 and 4.1.2 I mentioned two potential solutions – to rely on a community of teachers and / or repositories of PBL projects, and to introduce experts to the classroom. In the next theme, I comment on a third, digital possibility: the introduction of technology.

4.2.1 Technology as a means of easing differentiation. Despite allowing teachers to tailor instruction to different types of learners, differentiated instruction inevitably causes more work. A technological solution to this not only reduces the workload on the part of the teacher, but has the added benefit of enhancing the student community and improving interactivity within PBL projects. Chapter 2’s literature review referenced the notion of “learning objects”, interactive mathematics programs discussed by Kay and Knaack (2008). In my second interview, $P_2$ provided an example of how these interactive technologies could be implemented in secondary school to easily create rich and multifaceted tasks that would have otherwise been more difficult and time-consuming for teachers to assemble:

Say you have a system of two equations and two unknowns. So you solve it—you get your answer. Here’s a really cool thing you can do. You can go back to the original problem, look at the two equations that were given, and you can take one of the coefficients and change it, and change it again and again… and you sit back and you watch how the solution changes when you do that.
P₂, who has used this kind of exploratory math in his teaching, introduced me to computer applications designed to help facilitate precisely this. “Desmos”, for example, is a free online graphing tool that lets users build functions (piecewise, polynomial, exponential, etc…) through equations, then graphs a visual and colour-coded representation of the student’s work. “Geometer’s Sketchpad” is another resource that is available to school boards for a fee and allows for the creation and manipulation of shapes, graphs, equations, and measurements. These two programs serve as useful tools for adding visual and interactive elements to math, but they also provide potential platforms for student-driven work centred on real-world problems. P₂’s discussion of Desmos led me to an activity from the site referred to as “Water Line” (see Appendix D, 1-1 and 1-2 for further descriptions of the online project), where students can draw empty cups that the program then fills with water. As the cups fill, students must model the relationship between the height of the water in the cup and the time that has passed by sketching a graph. Students can also reverse-engineer the problem by using graphs to anticipate curvature or chokepoints in the cup they are dealing with. In this case, the Water Line activity could help to facilitate key discussions about fluid mechanics – a branch of physics used in engineering – in a way that can be understood at the secondary level.

Yet another benefit to Water Line is the inclusion of a “class cupboard”, onto which students can place cups they themselves draw; from there, the cups can be shared with the class, and students can attempt to sketch graphs for each other’s cups. By this point, technology has already provided a platform for creating and sharing ideas, leaving the teacher and his or her students with more time to discuss the mathematics and post-secondary connections behind the activity.
Still, while P₂ was enthusiastic about having technology help facilitate engagement and math application, he did also warn against redundancy in its use, something that few researchers pointed out. “Why do stuff you could have done without tech? In that case, students don’t learn anything new or extend their ideas, they just rely on tech to do things they would have normally learnt to do by hand.” P₂’s preference for technology that facilitates exploration instead of becoming a kind of learning crutch leads to an important notion – that the richness of computer programs in mathematics stems from when technology itself is used in a fashion that accommodates extended learning. In this sense, it fits hand-in-glove with PBL projects, but nevertheless, one should always consider the context in which technology is being used. To this effect, P₂ also emphasized the importance of preventing a classroom learning object from becoming “just a toy or a distraction” and of preventing teachers from relying on technology “just for the sake of it”. In addition, he mentioned that a degree of preparation and willingness to work with the teacher was necessary on the students’ part.

P₂’s comments illuminated the need for rigor within instruction, whether that instruction was in the form of a common curriculum-based lesson or a more expansive, student-driven, differentiated, and potentially technological PBL project. In the following theme, I discuss my findings related to how the rote camp of mathematics research satisfies this need and can contribute to laying the groundwork for PBL projects.

4.3 Rote as a Supporting Framework for PBL Projects

In Chapter 2, I outlined rote learning as a pedagogical approach whereby students aim to memorize facts through repetition and practice (Holmes & McGregor, 2007). The results of an experiment by Hilgard et al. (1953), pointed to a strength in rote learning, but only when it was
used for short-term memorization of a math process as opposed to long-term and meaningful understanding. Yet other researchers claim that rote itself lays the very groundwork for meaningful thinking, considering it a lexicon of mathematical concepts (Akin, 2001; Handa, 2011; Lai & Murray, 2012). Others yet point to the fact that rote learning is a way for students – particularly those bound for post-secondary education – to manage their time, choosing the most efficient means of memorizing information fast, despite being given opportunities to engage in deeper understanding (Peters & Higbea, 2014). In the following sub-themes, I consider the implications of these ideas for attuning students to post-secondary course demands and providing opportunities for challenge and practice within PBL projects, focusing this discussion through the lens of evidence given by each of my participants.

4.3.0 Aiding math transition by linking repetition with understanding. Despite the benefits of possessing an engaging and diverse framework for PBL projects, one must also consider the broader context of mathematics transition – a Western education system embedded in a marks-driven society. In my interview with him, P₁ spoke briefly to the level of competitiveness in Ontarian post-secondary institutions. In particular, he referenced the University of Waterloo’s admission requirements: “To get into our first year programs, you need an average of, at least in the high 80s if not the 90s to be admitted.” While on some level marking is subjective depending on the school and teacher conducting it, other factors, particularly a student’s ability to reproduce and expand on the math concepts taught, more directly contribute to the mark that student receives. In consideration of this, P₁ pointed out what he believed were weaknesses in current secondary school classes and how rote carried the potential to fix them:
The challenges we see are often challenges of practice, that students… can handle the content that we’re teaching and they can handle the content that they’ve learned through Grade 12, but often then struggle with some of the… more basic things. They struggle with some of the algebraic manipulations, they struggle with some of the fraction manipulations, things like that—things that, you know, for better off for worse, our curriculum in Ontario as it is does not, in my opinion, provide sufficient time to practice.

P₁’s stance on giving students the time and resources to practice more rigorous and fundamental concepts is mirrored by Peters and Higbea (2014), who show similar pre-existing attitudes in post-secondary students; specifically, the undergraduate students they surveyed displayed a strong predisposition towards wanting efficiency in instruction and class content, along with the opportunity for memorizing processes and gaining marks, an attitude that Peters and Higbea claim came at the expense of “promoting deep learning and understanding” (Peters & Higbea, 2014, p. 103).

The problem of balancing the opportunity for rote (practice of problems in an efficient way) and deeper understanding (through engaging, student-driven work) has one potential solution in the form of “procedural variation”. Lai and Murray (2012) discussed the benefit in what they referred to as procedural variation, a style of rote learning used by students in China (p. 8), illustrating how rote could be used as a means of giving students a variation of math problems to which they would then transfer strategies they had learned. For Lai, the link between rote and meaningful learning came from the application of strategies or concepts – the variation ensuring that students understood how the strategy or concept functioned. In the context of the secondary school classroom, P₁ briefly offered his opinion, that the benefit in this kind of technique was “to try a variety of different approaches, to have some time to practice, to have
some time to problem solve, to have some time for students to work and… you circulate, and… help them when they get stuck.” In this sense, while students used rote to practice approaching problems from different angles, a teacher could provide support in a decentralized way. P₂, in his teaching practice, has relied on similar techniques, and provided me with examples of assignments he used in his classes (see Appendix D, 2-1 through 2-3 for this problem and its iterations). For these assignments, a single problem based on laying cable to provide electricity for a house was given, but P₂ changed the problem in several ways for different student groups, and in each case, different trigonometric and function-based techniques were required to find solutions. This movement from practicing repetitive and concrete math concepts to the more fluid practice of approaches to problems is, I think, a valuable aspect of rote. P₁ added that,

To a certain point… in a Grade 12 class like Calculus and Vectors for example where you’ve got students who are almost certainly going onto university and more than likely going into a STEM discipline of some kind, I think it is important that teachers give—start to do a bit of abstraction with students, even if it’s not something that is explicitly encouraged upon.

Hence modified rote activities, through procedural variation, carry the potential to help students gear themselves towards the more abstract “understanding” of concepts. The value in this lies in comparing an originally procedural repetition with the questions of “Why and how does this procedure work?”

In the following sub-theme, I link this idea back to the PBL project framework, and discuss exactly how P₁ and P₂ saw the two concepts working together.
4.3.1 Rote as a means of facilitating higher-order thinking. A key element of rote lies in its use for memorizing, internalizing, and then recognizing certain patterns. Akin (2001) references this in his description of math as cumulative, and of its memorization as like a household attic from which students “dust off” certain information (p. 10). My interview with P1 also underscored the importance of this kind of memorization and subsequent use of memories:

Just by playing with numbers, just by practicing this, you begin to see when you look at… a set of information, you begin to find and see patterns in that. Even if you’re not doing it quite actively, your brain will manage all of these things and will find things for you, that… if you haven’t had that fluency – you don’t have practice – your brain isn’t going to do.

According to him, there is then an integrative element to rote mathematics, one that builds the foundation for being able to recognize patterns and concepts that have later extensions to the kinds of higher-order problems used in post-secondary education. Meanwhile, P2 brought up the fact that, in addition to this, rote could be made to work in partnership with other strategies – particularly PBL. “PBL was certainly a centrepiece of my teaching, but it was usually mixed in with traditional [rote] elements.” P2 referred to rote as a way of not only practicing basic concepts, but of achieving fluency in higher-order skills; throughout the conversation, he also emphasized the importance of striking a balance between PBL and rote. “It’s like this giant pendulum swinging around… [but] you need balance. You know, all of these things are important… I would stop trying to swing… over to one [pedagogical strategy], and then, you know… we then swing over to another one.”

In this sense, P1 perceived rote as a way of practicing the more complex skills required by PBL projects, the importance of which is doubly-appealing when one considers the cost in terms
of time, resources, and capital that post-secondary courses (where fewer chances are generally given) carry. A teacher considering this might still wonder how exactly rote facilitates higher-order skills (such as independent or team work, perseverance, making connections between different areas of lesson content, and so on) especially given their complexity. Yet even higher-order skills ground themselves in procedures, whether those procedures make up the prerequisite knowledge for applying the skills or act as part of the skills. For example, using perseverance to solve a multi-step problem requires the will to keep attempting that problem and the creativity to try new methods.

Practicing these things, P₂ pointed out, has in his experience helped students improve in persevering when it came to solving longer problems. P₁ meanwhile saw rote learning as an opportunity for challenging students and allowing them to fail in a constructive way. “I think it is important that… in a controlled way, teachers find a way to challenge students… with questions they’re not initially going to be able to do, but that they, um, but they then can get with a little bit of work.” P₁ also added. “I think it is very important that all students have a chance to learn how to fail. That’s not necessarily to learn to fail a course, but to learn to fail to do a specific task, and then to succeed at doing that task.” In other words, the more opportunities are presented to students, the more chances they have to reflect on and understand their processes – where they succeeded and now as well where they failed – leading to them producing better work in the future. This aligns well with the PBL project framework both practically and in terms of the framework’s aim: to prepare secondary students for math transition by allowing them to practice higher-order skills that have applicable elements in their classrooms.
4.4 Conclusion

In seeking a combination of practices that could help ease the transition to post-secondary math for students, the study uncovered data from participants that exposed a potential solution in the form of PBL projects. Through these projects, delivered in roughly 20-minute periods over a series of classes, student-driven explorations of concepts tied to post-secondary fields of study could be made. In particular, P₁ pointed out that PBL projects could directly connect to post-secondary fields of study by relying on guidance from experts (students or professors) engaged in the areas the projects focused on, though if no experts were available, teachers could also have students self-guide and engage in problem-solving through internalizing strategies like perseverance.

To ensure teachers could reach all students (through instruction and assessment) via the PBL projects, teachers could also make use of differentiated instruction, though P₂ pointed out a weakness in that it often tended to create more work and require thinking ahead on the teacher’s part. As a solution to this, learning object technology was found to be useful for reducing the time and effort of making and presenting content while also creating collaborative communities for students.

Lastly, by using an underpinning of rote to view the same problem from different angles and practice higher-order skills – both emphasized in post-secondary classrooms – teachers could give students both the time and opportunity to practice the understanding and application of concepts, leading to a finalized framework where students had full access to the guidance, expertise, platforms, and strategies that eased math transition.

In Chapter 5, I will work to contextualize these findings within my own teaching practice and in the broader educational community as a whole. I will elaborate on the significance of my
project’s findings with respect to their implications for educators and researchers and will also provide suggestions for implementing them in classrooms. Finally, I will address areas for further research into topics referenced this MTRP.
Chapter 5: Discussion

5.0 Introduction

The purpose of this research study was to explore the set of strategies associated with preparing students for mathematics transition and to suggest a practical way of combining them in the classroom. To this effect, I directed the study towards answering two overarching questions: “Which teaching strategies best complement each other in helping to ease student transition from high school to post-secondary mathematics?” and “Do educators use combinations of these strategies within their classes; in the cases where they do not, how might the strategies be made to work in combination with each other?” In addressing these questions, I explored the teaching strategies and resources employed by two skilled educators, P₁ and P₂, and drew connections between their comments and academic research.

In this chapter, I review my findings in the form of PBL projects and discuss their significance. I then elaborate on the implications this study has for teachers, administrators, and other educators. I continue on to explore the study’s effects on the educational community as a whole and on my own teaching pedagogy, and offer recommendations for in-service and pre-service teachers using PBL projects. Lastly, I point to areas for further research that may help to build on the idea of PBL projects and aid teachers in preparing their students for mathematics transition.

5.1 Overview of Key Findings and their Significance

Chapter 4 discussed three main themes. The first of these revolved around the idea of PBL projects: complex, real-world problems extended over periods of time in Grade 12 math
classes. My interview findings pointed to the idea that PBL projects were well-poised to prepare students for post-secondary mathematics when used in combination with the introduction of experts (post-secondary students or professors) and enhanced student leadership. For example, while Habash and Suurtaam (2010) used undergraduate engineers to speak to students about jobs involving mathematics (p. 137), P₁ mentioned his own work with the Center for Education in Mathematics and Computing (CEMC), where he used similar workshops and online resources for both teachers and students. These were used to improve student knowledge about professions and academic streams involving math and computer science, and on a general level, are useful for alerting students to what the math they learn in high school can be used for.

The second theme examined ways of enhancing PBL projects in the classroom via differentiated instruction and learning object technology, which were used as cultivators of engagement and student interest in post-secondary math. While P₁’s pedagogical strategies for teaching university students reflected many of the qualities of differentiated instruction, P₂ pointed to the benefit in using learning object technology to ease the teacher workload associated with having to make instruction differentiated. In this way, differentiated instruction and learning object technology can be seen to work hand-in-hand to engage all students in a way that places less stress on the teacher.

The third and last theme measured the uses of rote learning as a framework for post-secondary mathematics preparation. It offered that rote learning could become the practical basis for viewing and working with PBL projects by returning to and reflecting on math questions in non-redundant ways. P₂ in particular provided examples of his work to demonstrate how this could be done by asking different questions about the same situation – providing electricity to a house – given different initial information (see Appendix D, 2).
Overall, the significance of these findings is that they provide a relevant set of strategies that can work together to streamline the transition to post-secondary mathematics, in which a deeper and more patient approach to thinking about problems is preferred over quick and easy solutions. In the following section, I discuss how these strategies affect networks of educators and students and how PBL projects and the accompanying strategies can reshape how we see education aimed at math transition.

5.2 Implications

5.2.0 The educational community. Consisting of teachers, administrators, and a range of other professionals, the educational community is both large and diverse. The findings in this MTRP carry value specifically for those networks of educators working closely with upper-year high school students, along with those planning on working with them in the future. PBL projects and the strategies and resources that supplement them (such as differentiation in combination with learning objects such as Waterline and procedural variation) have far-reaching implications for school communities in terms of both teacher collegiality and student leadership. Differentiation and procedural variation, for example, automatically enforce and emphasize different ways of approaching similar problems; technology meanwhile provides a new and potentially faster medium for problem-solving, and all of these methods can work as part of the larger concept of the PBL project, which emphasizes complex problem with a longer duration. Meanwhile, for pre-service teachers, PBL projects and the strategies that work in partnership with them, provide ways to begin framing their instruction in terms of educational policy documents such as the Ministry of Education’s Growing Success (2010). Outside of these areas,
PBL projects also act as a first step towards better connecting secondary and post-secondary educators through cooperative projects.

5.2.0 Implications for the school community and in-service teachers. In Chapter 4, P₁ pointed to the cooperative nature of PBL projects, stating the need for teachers who were both “open-minded” and had time to develop PBL skillsets. On one end, this requires a willingness of teachers to participate constructively with colleagues in order to design more complex tasks for students; on the other, it points to the need for a supportive school administration in terms of giving teachers time to plan and co-construct lesson material. Collegiality amongst teachers, especially when promoted by school administration through professional development, can open the door to the interdisciplinary proficiency mentioned in Chapter 4 by Hmelo-Silver (2004, pp. 244, 246, 260). In this sense, PBL projects and inter-departmental cooperation fit well together; for example, a math and physics teacher might choose to work on the Waterline activity from Desmos as part of a larger PBL project. In such a case, each teacher would contribute their own expertise to the activity, and both the math and physics class could then go on to use Waterline in their classes. In this way, inter-departmental cooperation becomes mutually beneficial. With the permission of principals, PBL projects could also take students outside of the classroom, for example, to measure and survey a field in order to plan the construction of a mall.

5.2.0.1 Implications for pre-service teachers. For pre-service teachers, PBL projects and how they are constructed take a different route. Lacking access to school departments, technology resources, and static classes of students, it may be difficult for them to see the immediate benefit in assembling such complex and time-consuming projects. Nonetheless, few are better-poised to take advantage of the PBL project design than these teachers. PBL projects play to a certain strength of pre-service teachers who are usually recent graduates of post-
secondary institutions. These educators, after all, have learned under and have the contact information of a range of experts in the form of their undergraduate and program-specific professors and, at times, colleagues, whose expertise they can draw from when working on PBL projects. Hence, retaining a network of associates from post-secondary institutions can in this sense help pre-service teachers build PBL projects that draw on field experts, as suggested by Habash and Suurtaam (2010). In addition to this, pre-service teachers have the time to think through meaningful ways of implementing PBL projects in their classrooms and fitting them with elements of the Ontario curriculum, as opposed to in-service teachers who are limited by constraints of preparation time and the lessons and content they are already required to teach within their periods. In terms of educational policy documents, this also means that pre-service teachers have more time to adjust their PBL projects to adhere to ministry requirements. For example, Growing Success (2010) places an emphasis on means of assessment; in particular, it references Learning Skills and Work Habits, which appear in secondary school report cards and are used to gauge how students display skills like collaboration, initiative, and self-regulation (p. 11). A pre-service teacher might think of ways to use group work, student-driven activities, and multi-step problems respectively to have students demonstrate each of the aforementioned qualities throughout the PBL project. Of course on a practical level as well, being able to tie larger premeditated projects into curriculum and policy documents is a strength that pre-service teachers can play to in their résumé, and the PBL project as a whole can become a strong addition to any teacher portfolio.

5.2.0.2 Implications for the secondary and post-secondary relationship. Lastly, PBL projects as a whole have implications for a more connected relationship between secondary and post-secondary institutions. In terms of these institutions’ administrations, PBL projects are a call
to an open dialogue when it comes to teaching upper-year students and discussing ways to best prepare them for the higher-order thinking required by post-secondary mathematics classes. For example, workshop and online-driven initiatives put on by universities – such as those that P₁ and the CEMC run – can help high school math teachers better understand the needs of their students and pave the way for success in the subject. Meanwhile, high school teachers on their end can work towards building initiatives that foster collaboration with post-secondary educators and create a culture of shared ideas; this culture in turn, acts as a space where the opinions of educators from the different tiers of education can be heard and incorporated. Gardner and Hatch (1989) mention various types of student learners, and P₁’s suggestions reflected an accompanying use in differentiating teaching styles to deliver content in ways that these types of learners could benefit from.

5.2.1 My professional identity and practice. The research contained in my MTRP also carries implications for my own professional practice. As a pre-service teacher, it is fairly easy for me to think back on my high school education and consider the kinds of questions I had in mathematics classes versus what I encountered in my first year of university. The first step in my mind to easing transition is perhaps to admit that the blanket idea of ‘first-year university mathematics’ is not quite so simple. One must consider types of mathematics in the forms of calculus, linear algebra, analysis, probability, and so forth. Each area demands a different kind of mathematical knowledge, but certain skillsets, like those that the PBL project model and related strategies aim to enhance – inquiry, leaderships, perseverance, technological literacy – all help to lessen the learning curve significantly by preparing students for a more comprehensive
workload. In this sense, I align with P₁ in that I believe in the value of asking students to take on the kinds of challenging multi-step math questions that do not always yield simple answers. At the high school level, this can better prepare them for what will be asked of them in the future.

PBL projects can also double as ways to train students in skills that the education system often skims over – thinking about long-term budgeting goals, using banking systems, developing fiscal responsibility and reducing debt, and even starting a small business. Many of my undergraduate colleagues often looked back on their high school careers and regretted that they missed the opportunity to learn these skills. To me, this has served as a wakeup call for the province and a chance for PBL projects to lead into not only academic, but the kind of career-relevant mathematics content that not only permeates higher education, but the workforce and real-life issues as well.

5.3 Recommendations

The collection of strategies I have assembled within this MTRP can be used by in-service and pre-service teachers in different ways; however, for students to benefit fully from them, they require the support of school administrators, post-secondary administrators and professors, and teachers’ own colleagues. In short, the implications of using PBL projects and the strategies that work in partnership with them are best described by their requirements – that a variety of educators and experts actively collaborate to produce diverse, engaging, and most importantly post-secondary-relevant lessons that encourage leadership, the use of technology, perseverance in problem-solving, and reflective thought about approaches to math problems.
In-service teachers are best-poised to begin implementing PBL projects immediately and as such, should take advantage of their situation. Embedded within their high school, they often have direct access to key elements of the PBL project – in particular, photocopiers, materials from their school’s art department, and learning object technology. In addition, they are often better able to gauge which teaching techniques work best with their classes, and so differentiated instruction should, in theory, come more easily to them. Meanwhile, in-service teachers should rely on collaboration in a way that specifically encourages teachers from different disciplines to work together and plan out lesson content that draws on the strengths of each contributor. This reduces the burden for teachers wishing to create new PBL projects and activities; it also coincidentally makes managing and scheduling for sought-after resources such as computer labs and projectors much easier.

Pre-service teachers, while lacking familiar classes of students, can – and should – still engage with PBL projects and associated strategies, though in a slightly different way. The benefits of being a pre-service teacher concerned with math transition can be summed up by the time and resources pre-service teachers are able to allocate to their work. Unburdened by classes of students, marking, and unit planning – and furthermore, having available resources in teacher education programs – pre-service teachers are well-suited to thinking ahead through lesson plans. In addition, though they may not have a class to test it on, they do have the time to create and explore variations of PBL projects and digital activities, strategies for differentiated instruction, and problems that encourage both practice and reflection. Moreover, programs such as the Master of Teaching at OISE often emphasize the creation of larger unit plans, and so discussions about strategies for math transition can be streamlined into class projects of making lesson and unit plans. In this way, thinking ahead makes it easier for pre-service teachers to conceptualize,
test, and tweak their particular projects; the effort as well can double as padding for a teaching portfolio.

5.4 Areas for Further Research

The limitations of this MTRP in both its direction and scope leave room for a variety of extending research. One potential area of focus for future researchers is that of the specific differences between college and university. This MTRP generalized post-secondary education in its discussion of math transition. Yet differences in the kinds of approaches that these institutions take towards mathematics education may affect how a secondary educator chooses to tailor a PBL project.

Secondly, because this MTRP was teacher focused, its scope covered what teachers themselves could do from within the Grade 12 classroom to prepare students for post-secondary math. It did not examine any of the paid-for or extra-curricular transitional programs (private services) within Ontario nor the effects these had on students who made use of their services. In extension of this MTRP’s ideas for classroom strategies, researchers might choose to engage in an analysis of how the strategies surrounding PBL projects compare to those used in private math transition courses. In particular, do these paid-for courses attempt to include strategies related to applied and engaging mathematics, and do they offer the opportunity to practice and reflect on math problems in a variety of ways?

5.5 Concluding Comments
When considering the topic of mathematics transition, educators often aim to address specific issues in their classes that may hinder students’ pursuit of postsecondary options. Interviews with P₁ and P₂, however, have pointed towards the value in a holistic approach to postsecondary preparation in the form of PBL projects. Grounded in problem-based learning and extended over longer periods of time, these projects can be used as ways to introduce students to more complex mathematical tasks. The projects are diversified through differentiated instruction, and this kind of instruction is itself eased through the use of learning object technology. Overall, the project is then supported by a framework of reflective rote learning, where students practice similar problems, but from a variety of different ways. As a multifaceted set of approaches to math education, PBL projects and the resources and strategies that support them allow for a way of designing engaging math tasks that improve student collaboration and knowledge of the subject. Importantly, these projects also open the door to the introduction of classroom experts, who can speak directly to student inquiries about the field of mathematics and careers involved in it while as a peer-to-peer exercise, they act as a vehicle for improving student leadership.

As educators look for new ways to advance their students’ academic performance and readiness for post-secondary math, a shift in how we think about the subject is slowly starting to occur. PBL projects open the way for an introduction of content beyond the secondary classroom, and for the inclined educator, they can even be used as tools for exploration into interdisciplinary studies. Yet ultimately, on a general level, seeing the value in how different approaches to mathematical instruction fit together to enhance math transition can help teachers give students the well-rounded skillsets they require to succeed.
References


Table 1 Postsecondary enrolments by province of study and for the territories. (2016, November


Appendices

Appendix A: Additional Information About the Research Literature

1. Korpershoek and his colleagues tested 1,446 Dutch secondary school students taking advanced mathematics, physics, and chemistry. The students came from a combination of senior general secondary school and pre-university education – two tracks which are specifically meant to prepare students for higher education. Results from these groups were based on statistics for each of the three subjects, which Korpershoek’s team categories under the labels below:

Fixed effects
- Math ability
- Reading ability
- Sex
- Ethnicity
- Reading ability × sex
- Reading ability × ethnicity
- Math ability × sex
- Math ability × ethnicity
- Sex × ethnicity
- Random effects

Random Effects
- Student level variance
- School level variance
- Intraclass correlation coefficient

Explained variance
- Student level
2. In terms of Piaget’s model of developmental stages in a student’s life, concrete students are able to apply logic, though only when a concrete or physical example can be provided or thought of. Formal operational students build on this and are able to generalize and abstract their thinking. Transitional students fall between the two categories. Generally, a progression is expected to occur between the developmental stages, where students move from concrete thinking to formal operational thinking.

3-1. The “13 Spades” card trick demonstrates number patterns in mathematics. It calls for all members of the spades suit to be used together as a small deck. The cards must be placed in the order: A, Q, 2, 8, 3, J, 4, 9, 5, K, 6, 1, 7. A is drawn from the deck first and placed face-up on a table. The next card, 2, is placed at the bottom of the deck. The card after that, 3, is placed face-up on the table, next to A, and the card after it, 4, is placed on the bottom of the deck. This continues until all the cards are exhausted.

3-2. Generally speaking, the cards in “13 Spades” adhere to the order 1, n-1, 2, n-2, 3, n-3, … The i\textsuperscript{th} card in this trick can be defined as any odd term in the above sequence (regardless of its value). Resultantly, the (i+1)\textsuperscript{th} card carries a value of n − i. So for example, if the third term in a deck of 6 pre-arranged cards has a value of 2, the card after 2 will be 6 − i = 6 − (3) = 3.

3-3. One of the final tasks in the study required students to create an order of cards that would produce a similar sequence as the “13 Spades” trick, but by placing two cards at the bottom of the deck this time instead of one. This required students to be able to generalize and adapt what they had learned, and in this respect, both test groups showed difficulty, suggesting that as opposed to the simple dichotomy of memorization vs. understanding, there are, as Hilgard points out, “degrees of understanding” (Hilgard et al., 1953, p. 291)
Appendix B: Letter of Consent for Interview

Date:

Dear ______________________________,

My Name is Matthew Colquhoun and I am a student in the Master of Teaching program at the Ontario Institute for Studies in Education at the University of Toronto (OISE/UT). A component of this degree program involves conducting a small-scale qualitative research study. My research will focus on math transition and creating a pedagogical model to help students ease the transition from high school to post-secondary mathematics. I am interested in interviewing teachers who have experience working with mathematics students and addressing their educational needs through a variety of teaching approaches. I think that your knowledge and experience will provide insights into this topic.

Your participation in this research will involve one 45-60 minute interview, which will be transcribed and audio-recorded. I would be grateful if you would allow me to interview you at a place and time convenient for you, outside of school time. The contents of this interview will be used for my research project, which will include a final paper, as well as informal presentations to my classmates. I may also present my research findings via conference presentations and/or through publication. You will be assigned a pseudonym to maintain your anonymity and I will not use your name or any other content that might identify you in my written work, oral presentations, or publications. This information will remain confidential. Any information that identifies your school or students will also be excluded. The interview data will be stored on my password-protected computer and the only person who will have access to the research data will be my course instructor for CTL7015. You are free to change your mind about your participation at any time, and to withdraw even after you have consented to participate. You may also choose
to decline to answer any specific question during the interview. I will destroy the audio recording after the paper has been presented and/or published, which may take up to a maximum of five years after the data has been collected. There are no known risks to participation, and I will share a copy of the transcript with you shortly after the interview to ensure accuracy.

Please sign this consent form, if you agree to be interviewed. The second copy is for your records. I am very grateful for your participation.

Sincerely,

Matthew Colquhoun

xxx-xxx-xxxx

OISE Faculty Contact: Dr. Angela Macdonald
Contact Information: angela.macdonald@utoronto.ca

Consent Form
I acknowledge that the topic of this interview has been explained to me and that any questions that I have asked have been answered to my satisfaction. I understand that I can withdraw from this research study at any time without penalty.
I have read the letter provided to me by Matthew Colquhoun and agree to participate in an interview for the purposes described. I agree to have the interview audio-recorded.
Signature: ______________________________________
Name: (printed) ______________________________________
Date: ______________________________________
Appendix C: Interview Protocol

Introductory Statement

The purpose of this research project is to outline a model of teaching that appropriately prepares graduating students for post-secondary education. It seeks to learn if the best model of teaching comes from a combination of using applied mathematics, engaging students in the subject, and including repetition (or rote learning) in lessons.

During the interview, the researcher may ask for elaboration on your response to any question, or, depending on the time available, may skip over certain questions. Remember that throughout this interview, you have the right to withdraw yourself or refrain from answering any question that the researcher asks. In addition, you will be granted a pseudonym in the MTRP document in order to preserve anonymity. This interview will be preserved through an audio recording device for the purpose of transcribing the conversation. Once this MTRP is published, all audio recordings of interviewees will be promptly destroyed.

Do you have any questions before we begin?

Please state your name and the grade level (or year of post-secondary school) that you teach.

General Questions

1. Generally, what issues do you as an educator see as being challenges in math classrooms to graduating students? Why?

2. The three main models for learning covered in this MTRP are teaching math as applicable to the workforce, making math engaging to students, and using rote learning / repetition. Of these three, is there one that you as a teacher prefer to use in your
classroom instruction? Alternatively, do you feel you use a combination of two or more of the models, or do you feel you use none of them?

a) If you only prefer one, would you consider combining it with others? Why or why not?

b) If you prefer a combination, how have these models interacted in your classroom? Alternatively, have there been times where they conflicted?

c) If none of them suit you, what are your suggestions for how students could best be taught?

3. Which teaching strategies do you employ in the classroom? For example, do you give out many paper assignments, do you lecture to students, do you use group work often, do you use technology in your classroom, etc.?

4. Do you think it is important to include technology in classroom lessons, or does it get in the way of student learning? (Note: technology can refer to smart boards, calculators, graphing applications, etc.)

Questions About Application

1. Habash and Suurtamm (two researchers discussed in this MTRP) discuss a program where engineers visit high school mathematics classrooms and explain the subject’s uses in the workforce.

   a) Do you think these types of programs might be useful to students?

   b) Have you ever considered using them in your classes? If so, why?

   c) If not, can you see any challenges to these programs being implemented?

2. Problem-based Learning is a style of instruction that relies on situations often encountered in “the real world”. It is usually accompanied by group work, student-guided learning, and a decentralized classroom model.

   a) Have you seen PLB used at your school / have you yourself used it?

   b) Do you think that it works? Why or why not?

   c) If you have seen/used PBL at your school, in what ways has it either helped or hindered students’ learning?

Questions About Engagement
1. Based on your experience as a teacher, what do you think causes student disengagement? (If the teacher requires prompting or additional information: Some ideas that have been suggested within this MTRP have been poor teacher-student communication and students’ lack of confidence in their own mathematical abilities.)

2. What do you believe teachers can do to increase student engagement?

3. Differentiated instruction uses various teaching methods, projects, and assignments that favour different styles of learning (ex. Visual learning, tactile learning, auditory learning, etc…). This MTRP discusses differentiated instruction as a popular and modern way of keeping different kinds of students engaged.

   a) Have you ever actively used differentiated instruction in your classes? What was your experience with it?

   b) Do you think that differentiated instruction will work well in every classroom? In your opinion, what kind of teacher is best suited to apply differentiated instruction?

Questions About Rote

1. Rote learning is a process through which students use memorization of facts, along with repetitive drills to remember procedures.

   a) Do you think that rote learning is useful for all classroom instruction, or only for certain types of instruction?

   b) Can you think of instances where you used rote learning in your classroom? How was it used and what were the outcomes?

2. Hilgard et. Al (researchers discussed in this MTRP who measured the effectiveness of rote through teaching card tricks to students) suggest that rote is useful for giving students a basis of knowledge, but afterwards, students need to apply that knowledge in more dynamic ways. In other words, they suggest that rote learning provides factual knowledge, while using these facts provides deeper understanding of material.

   a) Based on your experience, do you think that Hilgard’s statements about rote learning are true?

   b) How do you think teachers can best use rote learning to teach students in their math classes?

Reflection (for any time remaining)

1. Do you know of any extra-curricular courses or programs used to prepare students for post-secondary mathematics?
a) If not, do you think that more could/should be done to spread awareness of these programs?

b) If so, do you think that aspects of these programs could be included in the Grade 12 math course? Why or why not?

2. Do you think that time is an issue for teachers who wish to provide more opportunities for students to learn and practice mathematical concepts? Do you have any suggestions for what can be done about this by either teachers or the school board?

3. Are there any additional thoughts or comments you have about bridging the gap in mathematical knowledge between high school and post-secondary education?

Concluding Statement

Thank you for taking the time to meet with me and discuss this MTRP. Your recorded answers will be kept until the end of my writing this MTRP, at which point, they will be deleted from my records. You are welcome to follow up with me about the project at any time. In addition, if you have any questions or concerns, you can reach me at

matthew.colquhoun@mail.utoronto.ca.
Appendix D: Examples of Waterline and Procedural Variation

Figure 1. A description of how students will use the Water Line project. Reproduced from “Water Line” (n.d.). [https://teacher.desmos.com/waterline](https://teacher.desmos.com/waterline).
Figure 1. The class cupboard, onto which students can place the cups they create. Reproduced from “Water Line” (n.d.), [https://teacher.desmos.com/waterline](https://teacher.desmos.com/waterline).
2-1.

**Providing Electricity to a Remote Area (Version 1)**

The owners of a home on an island (point B in Figure 1) would like to have a cable installed that will provide them with electricity. Let the distance between B and C be the shortest distance from the island to the shore. The distance from C to B is 10 km. There is a source of electricity 20 km due west at point A. The cost of laying a cable underwater is five times as much as using a land line.

Question 1

The owners of the home would like to find the most economical way of being provided the electricity. Without doing any analysis, what does your mathematical intuition tell you about how the electricity should be provided to the home owners?

Question 2

Select at least 10 different points P on AC (Figure 2) and use a strategy of your own choice to determine the cost of providing power to the home for each point. From the list of points you selected, which one has the lowest cost? Could there be a better point to use? Why or why not?
Providing Electricity to a Remote Area (Version 2)

The owners of a home on an island (point B in Figure 1) would like to have a cable installed that will provide them with electricity. Let the distance between B and C be the shortest distance from the island to the shore. The distance from C to B is 10 km. There is a source of electricity 20 km due west at point A. The cost of laying a cable underwater is five times as much as using a land line.

Question 1

The owners of the home would like to find the most economical way of being provided the electricity. Without doing any analysis, what does your mathematical intuition tell you about how the electricity should be provided to the home owners?

Question 2

Let P be a point on AC (Figure 2), let $x$ represent the length of PC and let $y$ represent the total cost of connecting a cable from A to P on land and from P to B underwater.

(a) What is the domain of $x$?

(b) Express $y$ as a function of $x$. Use technology to graph $y$ versus $x$. Determine the value of $x$ for which $y$ is a minimum.

Question 3

What do your answers to Question 2 tell you about the real-life situation of obtaining the most economical method of obtaining electricity?
Providing Electricity to a Remote Area (Version 3)

The owners of a home on an island (point B in Figure 1) would like to have a cable installed that will provide them with electricity. Let the distance between B and C be the shortest distance from the island to the shore. The distance from C to B is 10 km. There is a source of electricity 20 km due west at point A. The cost of laying a cable underwater is five times as much as using a land line.

Question 1

The owners of the home would like to find the most economical way of being provided the electricity. Without doing any analysis, what does your mathematical intuition tell you about how the electricity should be provided to the home owners?

Question 2

Let P be a point on AC (Figure 2), let $x$ represent the length of PB and let $y$ represent the total cost of connecting a cable from A to P on land and from P to B underwater.

(a) What is the domain of $x$?

(b) Express $y$ as a function of $x$. Use technology to graph $y$ versus $x$. Determine the value of $x$ for which $y$ is a minimum.

Question 3

What do your answers to Question 2 tell you about the real-life situation of obtaining the most economical method of obtaining electricity?