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Bianchi type- V modified holographic Ricci dark energy model in self creation theory of gravitation

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Abstract

Anisotropic and spatially homogeneous Bianchi type- V space time is considered in the presence of matter and anisotropic dark energy in Barber's (Gen. Relativ. Gravit. 14, 117, 1982) second self creation theory of gravitation. Anisotropic modified holographic Ricci dark energy model is presented using linearly varying deceleration parameter (Akarsu and Dereli: Int. J. Theor. Phys. 51, 612, 2012), energy density of modified holographic Ricci dark energy and relation between the metric potentials. Some physical and kinematical properties of the model are discussed through graphically by choosing suitable values to the parameters and which are found to be in good agreement with modern cosmological observational data.

Key words: Bianchi type- V metric, modified Ricci dark energy, holographic dark energy, self creation theory, deceleration parameter.

1 Introduction

It is well known that in recent years there has been a considerable interest in modified theories of gravitation which are viable alternatives to general relativity. Brans and Dicke [1] formulated a scalar tensor theory of gravitation which incorporates Machs-principle in a relativistic frame work. Barber [2] produced two continuous creation theories. The first is a modification of Brans Dicke theory and the second is an adoption of general relativity to include continuous creation of matter and is within the limits of observation. These modified theories create the universe out of self contained gravitational and matter fields. Several authors have investigated various cosmological models in Barbers second self creation theory, some of them are Singh [3], Reddy [4], Reddy and Venkateswarlu [5], Shanti and Rao [6], Pradhan and Vishwakarma [7], Rao et al. [8], Reddy and Naidu [9], Rao and Vinutha [10], Naidu et al. [11].

The evidences for the acceleration of the universe have come from the direct recent cosmological observations, e.g., Supernovae Ia ([12]; [13]), cosmic microwave background radiation ([14]), large scale structure ([15]), baryon acoustic oscillations ([16]) and weak lensing ([17]). One approach to explain the acceleration of the universe is to put some new physical field of matter such as k-essence, quintessence and phantom etc.

Holographic dark energy is one of the potential candidate to the solution of dark energy problem, which is based on the holographic principle. According to this principle the entropy of a system scales not with its volume, but its surface area ([18]; [19]). A cosmological version of this principle was proposed by Fischer and Susskind [20] and Cohen et al. [21]. Recently, Grand and Oliveros [22] suggested a new holographic Ricci dark energy model with density $\rho_\Lambda = 3M_{pl}^2(\eta H^2 + \zeta \dot{H})$. Later, Chen and Jing [23] modified this model by assuming the density of dark energy contains the Hubble parameter H , the first order and the second order derivatives. The expression of the energy density of modified holographic Ricci dark energy (MHRDE) is given by $\rho_\Lambda = 3M_{pl}^2(\beta_1 H^2 + \beta_2 \dot{H} + \beta_3 \ddot{H} H^{-1})$. Thus, if we take the whole universe into account, then the vacuum energy related to the holographic principle may be viewed as dark energy, usually called holographic dark energy. Hence several authors, in recent years, have investigated holographic dark energy models in anisotropic space-times. This is because of the fact that observational data suggests that the anomalies found in the cosmic microwave background (CMB) simulated increasing interest in anisotropic models. Also, these models will certainly help for a better understanding of the early stages of evolution of the universe.

Kiran et al. [24] have discussed minimally interacting holographic Bianchi type-V dark energy models in Saez-Ballester scalar tensor theory of gravitation. Sarkar [25] explored holographic dark energy model with linearly varying deceleration parameter, cosmic coincidence and the future singularity of the Kantowski-Sachs universe. Recently, Rao and Prasanthi [26, 27] have investigated some dark energy models with anisotropic background. Reddy et al. [28] have discussed Kantowski-Sachs minimally interacting holographic dark energy model with linearly varying deceleration parameter in Saez-Ballester theory. Ramesh and Umadevi [29] have obtained minimally interacting holographic dark energy model in Saez-Ballester theory using linearly varying deceleration parameter. Das and Sultana [30] have studied Bianchi type-V I_0 anisotropic MHRDE cosmological model. Santhi et al. [31] have investigated LRS Bianchi type-V dark energy model in a scalar tensor theory of gravitation. Very recently, and Santhi et al. [32]-[34] have discussed anisotropic MHRDE cosmological models in scalar tensor theories of gravitation. Rao et al. [35] have studied Bianchi type-III MHRDE model in a self-creation cosmology.

Motivated by above discussion, in this work we have investigated anisotropic and spatially homogeneous Bianchi type-V MHRDE cosmological model in Barber second self creation theory using linearly varying deceleration parameter proposed by Akarsu and Dereli [36]. The plan of our paper is follows: Section 2 is devoted to metric and field equations. In section 3 we have obtained solution of the field equations and the anisotropic MHRDE model in self creation theory. In section 4, we have discussed some properties of the model. We concluded our paper in last section.

2 Metric and Field equations

We consider a spatially homogeneous Bianchi type- V metric of the form

$$ds^2 = dt^2 - A^2(t)dx^2 - B^2(t)e^{2x}dy^2 - C^2(t)e^{2x}dz^2, \quad (1)$$

where A, B, C are functions of cosmic time t only.

The field equations in Barber's second self creation theory of gravitation are given by

$$\begin{aligned} R_{ij} - \frac{1}{2}Rg_{ij} &= -8\pi\phi^{-1}(T_{ij} + \bar{T}_{ij}) \\ \square\phi &= \phi_{;\nu}^{\nu} = \frac{8\pi}{3}\mu(T + \bar{T}) \end{aligned} \quad (2)$$

where R_{ij} is the Ricci tensor, R is the Ricci scalar, T_{ij} is energy-momentum tensor of matter, $\bar{T}_{ij}$ is the energy-momentum tensor of MHRDE and μ is the coupling constant. The energy momentum tensors for pressure less matter and anisotropic DE are given by

$$\begin{aligned} T_{ij} &= \text{diag}[\rho_m, 0, 0, 0] \\ \bar{T}_{ij} &= \text{diag}[\rho_\Lambda, -\omega_\Lambda\rho_\Lambda, -(\omega_\Lambda + \delta)\rho_\Lambda, -(\omega_\Lambda + \gamma)\rho_\Lambda] \end{aligned} \quad (3)$$

where $\omega_\Lambda = \frac{p_\Lambda}{\rho_\Lambda}$ is equation of state (EoS) parameter of DE, p_Λ and ρ_Λ are pressure and energy density of DE respectively. ρ_m is the energy density of the matter and δ and γ are deviations (skewness parameters) from ω_Λ on both y and z directions.

Now in comoving coordinate system, the field equations (2) for the metric (1) using (3) can be written as

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} - \frac{1}{A^2} = -\frac{8\pi}{\phi}\omega_\Lambda\rho_\Lambda \quad (4)$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\dot{A}\dot{C}}{AC} - \frac{1}{A^2} = -\frac{8\pi}{\phi}(\omega_\Lambda + \delta)\rho_\Lambda \quad (5)$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} - \frac{1}{A^2} = -\frac{8\pi}{\phi}(\omega_\Lambda + \gamma)\rho_\Lambda \quad (6)$$

$$\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{A}\dot{C}}{AC} - \frac{3}{A^2} = \frac{8\pi}{\phi}(\rho_m + \rho_\Lambda) \quad (7)$$

$$2\frac{\dot{A}}{A} - \frac{\dot{B}}{B} - \frac{\dot{C}}{C} = 0 \quad (8)$$

$$\ddot{\phi} + \dot{\phi}\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right) = \frac{8\pi}{3}\mu(T + \bar{T}) \quad (9)$$

where an overhead dot stands for ordinary differentiation with respect to cosmic time t .

3 Solution of the field equations

By integrating equation (8), we get $A^2 = bBC$, where b is an integrating constant. Without loss of generality we take b as unity, so that

$$A^2 = BC \quad (10)$$

The set of field equations (4)-(9) is a system of five independent equations with eight unknowns A , B , ϕ , ω_Λ , ρ_Λ , ρ_m , γ and δ . In order to solve the above system completely we need three more equations relating these parameters, so we consider the following physically plausible conditions:

The shear scalar is proportional to expansion scalar of the model, which leads to the relationship between the metric potentials ([37])

$$B = A^l \quad (11)$$

The varying deceleration parameter is given by ([36])

$$q = -kt + n - 1 \quad (12)$$

where $k \geq 0$ and $n \geq 0$ are constants and for $k = 0$, equation (12) reduces to the law of Berman which yields model with constant deceleration parameter. Here we are concerned with linearly varying deceleration parameter given by equation (12) when $k > 0$ and $n \geq 0$ in Bianchi type- V metric in the framework of Barber's second self creation theory. In this particular case, we get the average scale factor as

$$a(t) = (ABC)^{\frac{1}{3}} = a_1 \exp \left[\frac{2}{\sqrt{n^2 - 2c_1k}} \operatorname{arc tanh} \left(\frac{kt}{\sqrt{n^2 - 2c_1k}} - 1 \right) \right] \quad (13)$$

where a_1 , c_1 are constants of integration. For convenience, in the following we consider the solution for $k > 0$ and $n > 0$ choose the integration constant $a_1 = 1$, $c_1 = 0$. The reason for considering the solution only for $k > 0$, $n > 0$ is not only for simplicity but also for compatibility with the observed universe. The condition $k > 0$ means that we are dealing with increasing acceleration ($\dot{q} = -k < 0$). With this, the equation (13) reduces to

$$a(t) = \exp \left[\frac{2}{n} \operatorname{arc tanh} \left(\frac{kt}{n} - 1 \right) \right] \quad (14)$$

The energy density of MHRDE in Barber's second self creation theory as

$$\rho_\Lambda = \frac{3\phi}{8\pi} (\beta_1 H^2 + \beta_2 \dot{H} + \beta_3 \ddot{H} H^{-1}), \quad (15)$$

here H is the Hubble's parameter, β_1 , β_2 and β_3 are constants.

Also, we use the extra physically valid condition because the field equations are highly nonlinear

$$T + \bar{T} = 0 \quad (16)$$

Now using equations (10), (11), (14) and (16), we obtain the expressions for metric potentials as

$$\begin{aligned} A &= \exp \left[\frac{2}{n} \operatorname{arc tanh} \left(\frac{kt}{n} - 1 \right) \right] \\ B &= \exp \left[\frac{2l}{n} \operatorname{arc tanh} \left(\frac{kt}{n} - 1 \right) \right] \\ C &= \exp \left[\frac{2(2-l)}{n} \operatorname{arc tanh} \left(\frac{kt}{n} - 1 \right) \right]. \end{aligned} \quad (17)$$

and the scalar field ϕ as

$$\phi = \frac{\phi_0 \exp \left[\frac{-6}{n} \operatorname{arc tanh} \left(\frac{kt}{n} - 1 \right) \right]}{6k^2[kt^2 - 2nt]} \quad (18)$$

where ϕ_0 is an integrating constant.

Now the metric (1) can be written as

$$\begin{aligned} ds^2 = dt^2 - \exp \left[\frac{4}{n} \operatorname{arc tanh} \left(\frac{kt}{n} - 1 \right) \right] dx^2 - \exp \left[\frac{4l}{n} \operatorname{arc tanh} \left(\frac{kt}{n} - 1 \right) \right] e^{2x} dy^2 \\ - \exp \left[\frac{4(2-l)}{n} \operatorname{arc tanh} \left(\frac{kt}{n} - 1 \right) \right] e^{2x} dz^2 \end{aligned} \quad (19)$$

From equations (15) and (17), we get the energy density of MHRDE as

$$\rho_\Lambda = \frac{\phi_0 \exp \left[\frac{-6}{n} \operatorname{arc tanh} \left(\frac{kt}{n} - 1 \right) \right]}{4\pi k^2(kt^2 - 2nt)^3} \left[\beta_1 + \beta_2(kt - n) + \beta_3 \left(\frac{k}{2}(2nk - kt^2) + (kt - n)^2 \right) \right]. \quad (20)$$

Now from the field equations (4)-(7), (17), (18) and (20), we obtain the following:
the EoS parameter of MHRDE as

$$\omega_\Lambda = \frac{(2nk - kt^2)^2 \exp \left[\frac{-4}{n} \operatorname{arc tanh} \left(\frac{kt}{n} - 1 \right) \right] - 4[(l^2 - 2l + 4) + 2kt - 2n]}{12[\beta_1 + \beta_2(kt - n) + \beta_3 \left(\frac{k}{2}(2nk - kt^2) + 2(kt - n)^2 \right)]} \quad (21)$$

the skewness parameters as

$$\delta = \frac{16\pi k^2(kt^2 - 2nt)(l - 1)(kt - n + 3)}{\phi_0 \exp \left[\frac{-6}{n} \operatorname{arc tanh} \left(\frac{kt}{n} - 1 \right) \right] [\beta_1 + \beta_2(kt - n) + \beta_3 \left(\frac{k}{2}(2nk - kt^2) + 2(kt - n)^2 \right)]} \quad (22)$$

$$\gamma = \frac{16\pi k^2(kt^2 - 2nt)(1 - l)(kt - n + 3)}{\phi_0 \exp \left[\frac{-6}{n} \operatorname{arc tanh} \left(\frac{kt}{n} - 1 \right) \right] [\beta_1 + \beta_2(kt - n) + \beta_3 \left(\frac{k}{2}(2nk - kt^2) + 2(kt - n)^2 \right)]}. \quad (23)$$

and the energy density of matter as

$$\begin{aligned} \rho_m = \frac{\phi_0 \exp \left[\frac{-6}{n} \operatorname{arc tanh} \left(\frac{kt}{n} - 1 \right) \right]}{12\pi k^2(kt^2 - 2nt)^3} \left\{ l(2 - l) + 2 - \beta_1 - \beta_2(kt - n) \right. \\ \left. - \beta_3 \left(\frac{k}{2}(2nk - kt^2) + 2(kt - n)^2 \right) \right\} - \frac{\phi_0 \exp \left[\frac{-10}{n} \operatorname{arc tanh} \left(\frac{kt}{n} - 1 \right) \right]}{16\pi k^2(kt^2 - 2nt)} \end{aligned} \quad (24)$$

Figure 1: Plot of EoS parameter of MHRDE versus time t for $l = 1.5$, $k = 0.097$, $n = 1.6$, $\beta_1 = 3.1$, $\beta_2 = -2.95$ and $\beta_3 = 3.5$.

Thus the metric (19) together with (20)-(24) and (18) constitutes a Bianchi type- V MHRDE cosmological model in Barber's second self creation theory of gravitation.

Behavior of EoS parameter for MHRDE (ω_Λ) in terms of cosmic time t is shown in Fig. 1. It can be observe that EoS parameter is decreasing function of time and crosses the phantom divide line (PDL) ($\omega_\Lambda = -1$). Later on EoS parameter varies in phantom region only, which characterizes the behavior of dark energy.

4 Some properties of the model

Spatial volume of the model as

$$V = \sqrt{-g} = \exp \left[\frac{6}{n} \operatorname{arc tanh} \left(\frac{kt}{n} - 1 \right) \right] \quad (25)$$

Directional Hubble's parameters are given by

$$H_1 = \frac{\dot{A}}{A} = \frac{2}{2nt - kt^2}; \quad H_2 = \frac{\dot{B}}{B} = \frac{2l}{2nt - kt^2}; \quad H_3 = \frac{\dot{C}}{C} = \frac{2(2-l)}{2nt - kt^2} \quad (26)$$

Mean Hubble's parameter

$$H = \frac{\dot{a}}{a} = \frac{2}{2nt - kt^2} \quad (27)$$

Expansion scalar is

$$\theta = 3H = \frac{6}{2nt - kt^2} \quad (28)$$

Shear scalar is

$$\sigma^2 = \frac{1}{2} \sum_{i=1}^3 H_i^2 - \frac{1}{6} \theta^2 = \frac{2(1-l)^2}{(2nt - kt^2)^2} \quad (29)$$

Anisotropic parameter is

$$A_h = \frac{1}{3} \sum_{i=1}^3 \left(\frac{H_i - H}{H} \right)^2 = \frac{2(l-1)^2}{3} \quad (30)$$

Lot of dark energy models have been proposed in order to explain the accelerating expansion of the universe. However a sensitive test is required, which can differentiate between these models. For example, $(r, s) = (1, 0)$, $(1, 1)$ show the Λ CDM and CDM limits, while the regions $(s > 0$ and $r < 1)$ correspond to the phantom and quintessence DE. For this purpose two new parameters $\{r, s\}$ called statefinder are defined by Sahni et al. [38] as

$$\begin{aligned} r &= \frac{\ddot{a}}{H^3 a} = \frac{n(n-1) - kt + 2}{2} \\ s &= \frac{r-1}{3(q-\frac{1}{2})} = \frac{n(n-1) - kt}{6(n-kt+1)} \end{aligned} \quad (31)$$

Figure 2: Plot of deceleration parameter versus time t for $k = 0.097$ and $n = 1.6$.

In Fig. 2 we describe the behavior of deceleration parameter q versus cosmic time t . It can be observe that the deceleration parameter is initially $q = 0.6$ and the present (i.e. at $t = 13.7$) value of deceleration parameter $q \approx -0.73$. Also, the model enters into the present accelerating phase of the universe at $t \approx 6.2$. These values are consistent with the recent observational results ([12];[39]). In Fig. 3 we obtain the plane of statefinders by plotting r versus s . It can be observe that the $r-s$ plane corresponds to Λ CDM model. Also, it provides the phantom and quintessence regions of DE, and Chaplygin gas model.

Figure 3: Plot of (r, s) plane.

5 Conclusions

We have presented anisotropic Bianchi type- V modified holographic Ricci dark energy cosmological model in Barber's second self creation theory of gravitation using linearly varying deceleration parameter proposed by Akarsu and Dereli [36]. We have obtained some important physical and kinematical parameters of the model and discussed their cosmological significance.

Spatial volume of the universe increases with time showing the spatial expansion and it is zero at $t = 0$ giving a initial singularity at $t = 0$. It is also observed that the parameters $H, \theta, \sigma^2, \phi$ and all other parameters diverge at $t = 0$ and decrease as time increases, which shows inflationary scenario of the universe. We have observed that there is a smooth transition of the model from deceleration to the accelerated phase which agrees with the present scenario of modern cosmology. The EoS parameter exhibits quintom behavior, i.e., initially varies in quintessence region ($\omega_\Lambda > -1$), crosses the PDL and remains in the phantom region ($\omega_\Lambda < -1$). Initial value of deceleration parameter is $q = 0.6$ and the model enters into the accelerating expansion phase at $t \approx 6.2$. The present (i.e. at $t = 13.7$ Gyr) value of the deceleration parameter is $q = -0.73$. Thus, the deceleration parameter is consistent with the recent observational results. The $r-s$ plane corresponding to model meets the Λ CDM limit. It is also pointed out that the $r-s$ plane possess the regions of Chaplygin gas, quintessence and phantom models.

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