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Influence of End Effect on Rock Strength in True Triaxial Compression Test

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Abstract

The influence of the end effect on rock strength in true triaxial compression test was studied using a numerical approach. The influence of $\sigma_2$ on rock strength was isolated by using the 2D Mohr-Coulomb failure criterion that depends only on $\sigma_1$ and $\sigma_3$. Thus, any enhancement to the rock strength with the increase of $\sigma_2$ can be attributed to the end effect. It was shown that the end effect can result in an apparent $\sigma_2$ effect, as long as the coefficient of friction ($\mu$) at rock specimen-steel platen contacts is not zero and the specimen in the $\sigma_2$ loading direction is squat. When the strengthening due to the increase of $\sigma_2$ predicted by a theoretical failure criterion was
added to the strengthening due to the end effect, the results were in good agreement with the observed $\sigma_2$ effect from some previous laboratory tests, indicating that the observed $\sigma_2$ effect in true triaxial compression test could be partially influenced by the end effect, particularly when $\sigma_3$ was low. It is suggested to decrease the end effect to a level where the apparent $\sigma_2$ effect is very small so that the obtained test results are more meaningful to characterize the actual $\sigma_2$ effect.

**Keywords:** True triaxial compression test; intermediate principal stress; rock strength; end effect; 3D empirical failure criterion; numerical modeling
Introduction

Accurate estimation of rock strength is one of the most important tasks in rock engineering design. Different approaches including laboratory, theoretical, empirical, and numerical approaches have been developed and used to investigate rock strength (Hudson et al., 1972; Read and Hegemier, 1984; Dai et al., 2010; Xu et al., 2016) and many failure criteria have been developed to model rock strength. Among them, the 2D Mohr-Coulomb (M-C) and Hoek-Brown (H-B) failure criteria, which are based on conventional triaxial ($\sigma_1 > \sigma_2 = \sigma_3$) compression test results and do not take the intermediate principal stress ($\sigma_2$) into account, have been widely used to estimate rock strength. Although a rock mass is generally under a true triaxial stress state ($\sigma_1 > \sigma_2 > \sigma_3$) in situ, it is commonly accepted that "a simplification to ignore the influence of intermediate principal stress on rock material strength is justifiable" (Brown, 2008). Moreover, there is no significant deviation of predicted rock strengths between the results using these 2D failure criteria and field measurements, suggesting that these simple failure criteria are reliable (Hoek et al., 2002; Eberhardt, 2012; Labuz and Zang, 2012). In addition, the well-known Griffith theory, developed from a theoretical approach, also ignores the effect of $\sigma_2$ in its original form.

True triaxial compression test apparatuses were developed to study rock strength in a true triaxial stress state and to investigate the $\sigma_2$ effect. A ground-breaking work was carried out by Mogi (1967), who designed a true triaxial compression test apparatus and for the first time observed that $\sigma_2$ had a large effect on rock strength. Similar results were later reported by other investigators (Wiebols and Cook, 1968; Michelis, 1987; He et al., 2010; Li et al., 2015). In particular, Haimson and Chang (2000) developed a new true triaxial compression test apparatus and observed that rock strength increased significantly with the increase of $\sigma_2$. The observed $\sigma_2$ effect is normally characterized by an increase of rock strength with the increase of $\sigma_2$, followed
by reaching a strength plateau and a strength drop soon after with the further increase of $\sigma_2$ (Fig. 1). It should be noted that while the $\sigma_2$ effect has been widely reported from most laboratory test results, the observed strengthening effect of $\sigma_2$ on some rocks is not significant (Takahashi and Koide, 1989; Chang and Haimson, 2005). Nevertheless, these laboratory observations provoked the attention of considering $\sigma_2$ in 3D empirical failure criteria (Mogi, 1967, 1971; Haimson and Chang, 2000; Al-Ajmi and Zimmerman, 2005; You, 2009).

In this study, a method to quantify the observed $\sigma_2$ effect is proposed and is schematically shown in Fig. 1. Blue point $(0, \sigma_c)$ represents the uniaxial compressive strength (UCS, or $\sigma_c$); blue points ($\sigma_2, \sigma_1$) represent conventional triaxial stress states at different $\sigma_3$; and red points ($\sigma_2^*, \sigma_1^*$) represent the stress state at the strength plateaus at different $\sigma_3$. The slope between point ($\sigma_2, \sigma_1$) at the origin of the $\sigma_1$–$\sigma_2$ curve and point ($\sigma_2^*, \sigma_1^*$) at the summit of the $\sigma_1$–$\sigma_2$ curve gives the maximum percentage increase of rock strength caused by $\sigma_2$. In this way, the strengthening effect of $\sigma_2$ on different rocks can be quantified.

Basing on the published data of true triaxial compression tests, some of the observed $\sigma_2$ effects are quantified and summarized in Table 1. Test results that do not show the strength plateau in the $\sigma_1$–$\sigma_2$ curves are not included. It is seen that the $\sigma_2$ effect is not the same in different rocks. Moreover, it seems that the observed $\sigma_2$ effect on rock strength is more significant when the applied $\sigma_3$ is low, and it decreases with the increase of $\sigma_3$. For instance, an unusually high percentage of strength increase of 114% for Westerly granite is observed at $\sigma_3/\sigma_c = 0$, and it drops to 49% at $\sigma_3/\sigma_c = 0.38$.

There are arguments on the adequacy of some of the test results that show significant influence of $\sigma_2$ on rock strength. Colmenares and Zoback (2002) concluded that $\sigma_2$ hardly
affected the strength of some rocks and in such cases the 2D M-C and H-B failure criteria fitted the test data equally well, or even better, than more complicated 3D failure criteria. Cai (2008b) clarified numerically that spalling and onion-skin formation in underground opening was mainly attributed to the existence of \( \sigma_2 \), and \( \sigma_2 \) had a limited influence on rock strength when \( \sigma_3 \) was low. This statement tends to agree with field observation at the URL Mine-by tunnel (Cai and Kaiser, 2014) and theoretical prediction from Wiebols and Cook (1968). Accordingly, Cai (2008b) pointed out that a large percentage increase of rock strength due to \( \sigma_2 \), as observed from some laboratory tests, was mainly attributed to the end effect.

End effect exists ubiquitously in rock laboratory tests (Fig. 2), and it is initiated under compression due to the elastic mismatch between the rock specimen and the metal platens and the friction between them (Hoskins and Horino, 1968; Choi et al., 1996). It is practically impossible to exclude the end effect completely from rock property testing, even if anti-friction measures are employed (Brady, 1971). For instance, the relation between end effect and slenderness effect is a well-studied subject from the 1960s to the 1990s (Babcock, 1969; Brady, 1971; Kotsovos, 1983; Tang et al., 2000). Laboratory studies on the slenderness effect show that the use of lubricants to minimize the end effect is sometimes not effective (Meikle and Holland, 1965; Pellegrino et al., 1997; Liang et al., 2015).

End effect plays different roles in affecting the rock strength in different laboratory tests. In uniaxial compression test, the end effect has a large influence on the rock strength when the specimen is squat (Fig. 2) (Bieniawski and Bernede, 1979). In conventional triaxial compression test, Mogi (2007) concluded that the end effect in triaxial compression test can be greatly reduced with the increase of confinement. There is still a lack of systematic investigation on how the end effect affects rock strength in true triaxial compression tests. Compared with the
conventional triaxial compression test, which employs hydraulic pressure to apply lateral
confinement and hence has very little or no friction-induced constraint at the lateral surfaces of
the specimen, friction exists at the lateral rock specimen-metal platen contacts in true triaxial
compression test because metal platens are normally used to apply either $\sigma_2$ or both $\sigma_2$ and $\sigma_3$. Thus, the end effect's influences on rock strength between these two types of triaxial
compression tests are different. More importantly, the end effect usually becomes more
significant when the specimen is squatter ($H/W < 2$). Fig. 3 illustrates that the ratio of the length
on the contact where $\sigma_2$ is applied ($l_1$) to the length on the contact where $\sigma_3$ is applied ($l_2$) is 1 in
most true triaxial compression tests. This is another reason that the end effect in true triaxial
compression test cannot be completely omitted due to the shape of the specimens.

Accurate estimation of rock strength in a true triaxial stress state is driven by increased
tunneling activities at depth. It is thus necessary to study the influence of end effect on rock
strength in true triaxial compression test. In this study, we propose that the end effect exists in
true triaxial compression test and it can influence true triaxial compression test results and
thereby the interpretation of the actual $\sigma_2$ effect. It is hypothesized that the confined zones,
resulted by the end constraint near the rock specimen-steel platen contacts where $\sigma_2$ is applied,
can increase the actual $\sigma_3$ and hence the rock strength. Furthermore, enlightened by some
indirect evidences, it is reckoned that different deformation behaviors of rocks can result in
different levels of end constraint near the contacts where $\sigma_2$ is applied, thus the observed $\sigma_2$
effect can be different. Additionally, in true triaxial compression tests, the strengthening effect of
end constraint on rock strength may be more significant at low confinement and so is the
observed $\sigma_2$ effect. To verify our hypothesis, previous laboratory test results are used for model
calibration, and a numerical experiment using the ABAQUS/Explicit Finite Element Method (FEM) tool is carried out.

**Numerical simulation of true triaxial compression test**

**Selection of published true triaxial compression test results**

True triaxial compression test has become popular in recent years and there are a number of true triaxial compression test apparatuses for soil, concrete and rock testing (Shi et al., 2012). It is necessary to select test results with precaution for this numerical study. Firstly, the reliability of a true triaxial compression test apparatus should be well recognized and the tests should be conducted with a high level of repeatability under carefully controlled test conditions. Secondly, test results with various rocks should be considered to ensure the validity of this numerical experiment.

Basing on above considerations, we have chosen two types of true triaxial compression test apparatuses and their corresponding test results. The first one is the Mogi-type machine, which had been used to test seven types of rocks. We choose the test results of Solnhofen limestone (Mogi, 2007), which shows clearly the influence of $\sigma_2$ on rock strength. We also choose the test data of Takahashi and Koide (1989), who designed a true triaxial compression test apparatus (a new Mogi-type machine) slightly different from Mogi’s design to accommodate larger specimens. The second type of apparatus was developed by Chang and Haimson (2000), and we call it the Haimson-type machine. The major difference between these two types of apparatuses is that the pistons of the Mogi-type machine that apply two perpendicular principal stresses $\sigma_1$ and $\sigma_2$ are positioned in the vertical direction, while a biaxial apparatus is positioned horizontally in the Haimson-type machine to apply $\sigma_1$ and $\sigma_2$. The test results of KTB
amphibolite and Westerly granite by Chang and Haimson (2000), which are well known in the field of true triaxial compression testing, are selected. Thus, representative test data of igneous (granite), sedimentary (limestone, shale), and metamorphic (amphibolite) rocks are all included in this study.

**Mohr-Coulomb failure criterion and modeling strategy**

The research focus is the influence of end effect on true triaxial compression test results. Because the end effect inevitably exists in a true triaxial compression test, 3D empirical failure criteria (Mogi, 1967; Al-Ajmi and Zimmerman, 2005) developed from true triaxial compression test results might also contain rock strength strengthening due to the end effect. Therefore, a rock model that excludes the influence of \( \sigma_2 \) effect on rock strength is preferable to study the contribution of end effect to the observed \( \sigma_2 \) effect. As a result, the M-C failure criterion, which does not consider \( \sigma_2 \), is used to define the rock strength while the actual contact condition of rock specimen in a true triaxial compression test is modeled explicitly.

The 2D M-C failure criterion is one of the most commonly used failure criteria in rock engineering. As a failure criterion with mathematical simplicity, the physical meaning of the material parameters is clear. The rock strength of the M-C failure criterion is defined by the cohesive strength component and the frictional strength component (Martin, 1997; Hajiabdolmajid et al., 2002; Mogi, 2007). Despite one of its shortcomings, i.e., the linear nature of this failure criterion which may make it difficult to predict rock strength accurately when the applied confinement is high (Hoek and Brown, 1997; Jaeger et al., 2007), the strength parameters of the M-C failure criterion can be readily obtained or calibrated by performing simple rock property testing. More importantly, unlike other materials, rocks exhibit a confinement-dependent mechanical behavior, and rock failure under a moderate confinement is typically
characterized by an inclined shear failure plane relative to the $\sigma_1$ loading direction (Hudson and Harrison, 2000). The M-C failure criterion depicts the characteristic of rock strength and failure type under confinement conditions satisfactorily. The shortcoming of the M-C failure criterion for representing nonlinear failure envelopes of rocks in the $\sigma_1-\sigma_3$ space can be overcome by restricting confinement within a limited range of interest (Pariseau, 2007; Mogi, 2007; Labuz and Zang, 2012; Brady and Brown, 2013).

The modeling strategy of using a simple failure criterion can isolate the end effect in the observed $\sigma_2$ effect. As it is hypothesized in this study, the end effect in a true triaxial compression test can result in confined zones similar to that in an uniaxial compression test and enhance the actual $\sigma_3$ of the rock and hence its strength. By increasing $\sigma_2$ in a true triaxial compression test, if it is verified that a material whose strength is only $\sigma_3$-dependent experiences a strengthening due to the end effect, then a both $\sigma_3$ and $\sigma_2$ dependent material, which is more likely being the real material property of rocks, should be able to experience at least the same or even greater strengthening with the same strengthening effect due to end constraint. This is because that for rocks which are inherently confinement-dependent, the confinement they are subjected to might not be exclusively caused by $\sigma_3$ loading. Instead, the confinement might be influenced by $\sigma_2$ loading, leading to a combined influence of both $\sigma_3$ and $\sigma_2$ on rock strength. As a result, the M-C failure criterion is considered suitable for the proposed investigation.

**Friction at rock specimen-steel platen contacts**

The coefficient of friction ($\mu$) at rock specimen-steel platen contacts is an important parameter that determines the end effect in rock laboratory tests (Gaffney, 1976). In this study, a set of tilt tests was performed to determine a reasonable range of $\mu$ values for rock specimen-steel platen contacts.
contacts. For a polished, smooth and dry rock specimen-steel platen contact without any lubricants, the average tilting angle is about 15°, which gives a static \( \mu \) of 0.27.


According to the aggregate state of the material zones involved, friction states can be classified as: solid friction, boundary friction, mixed friction, and fluid friction (Grote and Antonsson, 2009). Solid friction acts at direct contacts where material zones exhibit solid properties; boundary friction refers to the boundary layers on the contacts, each consists of a molecular film coming from a lubricant; fluid friction occurs when lubricating film is created between the friction body surfaces to reduce wear; mixed friction is a mixed form of boundary and fluid frictions. Apparently, the friction state at a lubricated specimen-platen contact is neither a solid friction nor a fluid friction state. According to the Handbook of Mechanical Engineering (Grote and Antonsson, 2009), the \( \mu \) value ranges for boundary and mixed frictions are 0.1 – 0.2 and 0.01 – 0.1, respectively.

For lubricated rock specimen-steel platen contacts, it is reported that the \( \mu \) values can be as low as 0.024 (He et al., 2014) and as high as 0.1 (Rashed and Peng, 2015), or even 0.39 in some cases (Hawkes and Mellor, 1970). A \( \mu \) value of 0.05 is reported in Labuz and Bridell (1993) and a \( \mu \) range of 0.04 ± 0.003 is given in the Handbook on Mechanical Properties of Rocks by Vutukuri et al. (1974). Accordingly, friction at lubricated specimen-platen contacts can be in a
mixed friction state. Hence, \( \mu = 0.05 \) to 0.1 may seem to be an appropriate range to represent friction at lubricated rock specimen-steel platen contacts.

It must be pointed out that the loading conditions in sliding tests (or direct shear test) designed specifically to measure the coefficient of friction and that in rock laboratory strength tests are different. Different testing arrangements can result in different test results. For instance, instead of employing sliding tests to investigate the \( \mu \) values for lubricated rock specimen-steel platen contacts, Hawkes and Mellor (1970) first applied a torque to a cylinder steel block, which was atop a hollow sandstone specimen, and then gradually relaxed the axial load concentrically applied to the rock-steel system to measure the \( \mu \) values under different contact conditions. The obtained \( \mu \) values for the lubricated specimen-platen contacts were noticeably higher than that obtained by the sliding tests.

It is stated in the Handbook of Mechanical Engineering that “friction does not represent a constant property of a material but rather depends on variables, e.g., on the load and the elements involved in the friction process with their properties and interactions.” The applied normal stress on the sliding plane in a sliding test is not high, usually equivalent or slightly higher than the pressure caused by the weight of the sliding body. In laboratory rock strength tests, the applied normal stress on lubricated contacts is normally much higher than that used in sliding tests. Therefore, the \( \mu \) values of the lubricated specimen-platen contacts under high normal stresses could be different than that revealed from the sliding tests and the effectiveness of using lubricants in mitigating the end effect needs further investigation.

In this numerical experiment, acknowledging that all the considered true triaxial compression tests had employed anti-friction measures, base values of \( \mu \) in the range of 0.05 to 0.1 at the rock specimen-steel platen contacts were used in the modeling. To isolate the actual
end effect as well as the potential $\sigma_2$ effect due to the end effect from true triaxial compression
test results, a minimum $\mu$ value of 0 was used. Finally, to appreciate the fact that the actual $\mu$
value under high normal stresses for some rocks might be different from that obtained by sliding
tests, a maximum $\mu$ value of 0.2 was used.

**FEM models**

The ABAQUS/Explicit FEM tool is considered suitable for this numerical experiment. Firstly,
the potential increase of rock strength due to the end effect is the research focus and a precise
modeling the fracture process of rocks using Discrete Element Method (DEM) is not necessary.
Rock failure criteria like the M-C failure criterion that can be used to predict rock strength is
available in the tool. Secondly, the tool can simulate complex contact behaviors for the rock
specimen-steel platen contacts at a low computation cost. In addition, the explicit algorithm in
the tool is preferred to solve nonlinear material and contact problems.

3D simulation models of cylinder specimens in uniaxial and conventional triaxial
compression tests, and a square prism specimen in true triaxial compression test, as well as their
corresponding loading conditions are shown in Fig. 4a, b, and c, respectively. Arrows in these
figures show the direction of the principal stresses. Thick orange arrows indicate that the
principal stress is applied by steel platens and thin blue arrows indicate that the principal stress is
applied by hydraulic pressure. In the modeling, the applied $\sigma_3$ and $\sigma_2$ are pressure controlled,
which means that once the applied principal stress reaches an assigned magnitude, the pressure
will be hold constant on the applied surface. The applied $\sigma_1$ is displacement controlled, which
means that $\sigma_1$ is applied at a constant strain rate.
The diameter of the cylinder and the width of the square prism specimens are 50 mm and the heights of the specimens are 100 mm. This gives a height to width ratio (H/W, or H/D for cylinder specimens) of 2, which is the same as that used in the laboratory tests. The top and bottom steel platens are 51 mm in width (or diameter) and 25 mm in height (Fig. 4a–c). To honor the laboratory test conditions, steel platen loading instead of hydraulic pressure loading is used to apply $\sigma_2$ in the true triaxial compression test simulations. Two lateral steel platens used for applying $\sigma_2$ are 50 mm in width, 98 mm in height, and 25 mm in thickness (Fig. 4c). Steel properties ($E = 200$ GPa, $\nu = 0.3$) are assigned to the platens.

The FEM models of a cylinder specimen in both uniaxial and triaxial loading tests ($\sigma_2 = \sigma_3 \geq 0$) and a square prism specimen in true triaxial compression test are presented in Fig. 5. The total numbers of hexagonal elements in the two specimens are 8844 and 9537, which is relatively fine for the stress modeling. The slight difference in element numbers between the two specimens has little influence on their peak strengths, and this is verified by examining the modeling results of rock strengths. Moreover, the FEM geometrical models for different rock types under different true triaxial stress states are all the same (Fig. 5b).

**Calibration of material parameters**

Rock properties need to be calibrated first to carry out the numerical experiment. The rocks calibrated in this study are hard rocks with UCS over 100 MPa, some are over 200 and 300 MPa (Section “Selection of published true triaxial compression test results”). The stress–strain curves of the hard rocks are relatively linear in the pre-peak deformation stage (Chang and Haimson, 2000; Haimson and Chang, 2000; Mogi, 2007), especially before reaching the crack damage threshold at a stress level of approximately 0.7 to 0.8 times of the peak strength where hard rocks
basically exhibit elastic behavior. Therefore, the pre-peak deformation behavior of the rocks in compression is simplified as linear elastic, as suggested by Fang and Harrison (2001) and Hoek (2007). Young’s modulus and Poisson’s ratio are important elastic parameters to characterize the deformation behavior of rocks, and their values can be readily obtained from the laboratory test results provided in the origin works. Table 2 presents the physical and deformation parameters of the rocks considered in the study.

The M-C failure criterion is used to determine the peak strengths of the rocks. The M-C strength parameters – cohesion \((c)\) and friction angle \((\phi)\) – can be calibrated based on the relations between \(\sigma_1\) and \(\sigma_3\) obtained from conventional triaxial test results. The M-C failure criterion is a linear failure criterion that depicts a linear relation between \(\sigma_1\) and \(\sigma_3\). However, the relation between \(\sigma_1\) and \(\sigma_3\) shown in conventional triaxial tests is nonlinear when the rock specimens are subjected to high confinements (Section “Mohr-Coulomb failure criterion and modeling strategy”). Thus, if only one set of M-C strength parameters are used to characterize the rock behavior, a mismatch between the numerical and experimental results may occur, especially when the range of confinement being simulated is large (e.g., \(\sigma_3\) of 0 to 150 MPa for KTB amphibolite). Therefore, to characterize the rock behaviors well under different applied \(\sigma_3\) in the true triaxial compression tests, specimens under each applied \(\sigma_3\) was assigned with different sets of M-C strength parameters, calibrated based on the conventional triaxial test results in a range of confinements which are close to the applied range of \(\sigma_3\).

Table 3 to Table 6 present the M-C strength parameters of KTB amphibolite, Westerly granite, Solnhofen limestone, and Yuubari shale, respectively. The fitting equations based the test data \((\sigma_1, \sigma_2 = \sigma_3)\) at different applied \(\sigma_3\) give the linear form of the M-C failure criterion in
the \( \sigma_1 - \sigma_3 \) space. Thus, the M-C strength parameters, the cohesion and the friction angle, at each applied \( \sigma_3 \) can be calibrated accordingly.

The strain-softening behavior was used to model the post-peak deformation behavior of rocks. The evolution process of cracking in the post-peak deformation stage leads to a significant cohesion loss (Martin, 1997). The cohesion loss can be modeled by the M-C failure criterion by degrading the rock’s cohesive strength as a function of plastic strain. Our numerical experiment and laboratory true triaxial compression test focus on the peak strength of rocks, not the post-peak deformation behavior. The strain-softening model is for the post-peak deformation behavior modeling and it has no impact on the peak strength of rocks (Kias and Ozbay, 2013; Hemami and Fakhimi, 2014; Xu and Cai, 2015). In addition, there are no solid laboratory test results of complete stress–strain curves of rocks in true triaxial compression test due to the limitation of test machines. Therefore, the post-peak deformation behavior of the rocks is not discussed in this study.

Simulation results

**Deformation behavior**

The deformation behavior of rocks obtained from numerical modeling fits the laboratory test data well, as long as the pre-peak deformation behavior of the rocks obtained in laboratory testing is relatively linear. For instance, Westerly granite is a fine-grained, uniform, nearly isotropic rock (Lockner, 1998). As illustrated in Fig. 6, the slope of the rock’s pre-peak stress–strain curve is basically linear for \( \sigma_2 = \sigma_3 = 60 \) MPa. Using the same Young’s modulus and Poisson’s ratio obtained from the laboratory test (Table 2), the modeling results of both the deformation behavior and the peak strength agree well with the test results, indicating that the
numerical model is suitable for rock behavior modeling in uniaxial or triaxial stress states ($\sigma_2 = \sigma_3 \geq 0$).

**Rock strength**

Fig. 7 to Fig. 10 compare the modeling results with the laboratory test results for KTB amphibolite, Westerly granite, Solnhofen limestone, and Yuubari shale, respectively. As mentioned above in Section “Friction at rock specimen-steel platen contacts”, the exact $\mu$ value at the rock specimen-steel platen contacts that apply $\sigma_2$ are unknown in the laboratory tests and four values ($\mu = 0, 0.05, 0.1, 0.2$) have been used in the numerical simulation. The $\mu$ values beside the rock name indicate the coefficient of friction at the specimen-platen contacts in the modeling, for both $\sigma_1$ and $\sigma_2$ loadings.

The dash lines in Fig. 7 to Fig. 10 represent the modeling results of conventional triaxial compression tests. All modeling results fit the laboratory test results well. Hence, the modeling strategy of using the linear M-C failure criterion to predict the rock strength under given range of confinement ($\sigma_2 = \sigma_3 \geq 0$) is demonstrated. The applicability of the M-C failure criterion for predicting rock strength illustrated in this numerical experiment agrees with the conclusions drawn by other researchers (Pariseau, 2007; Labuz and Zang, 2012; Brady and Brown, 2013).

The solid lines in Fig. 7 to Fig. 10 represent the modeling results of true triaxial compression tests. All modeling results show that under each applied $\sigma_3$, rock strength ($\sigma_1$) increases with the increase of $\sigma_2$ if the $\mu$ value is greater than 0. There is a measurable increase of rock strength for $\mu = 0.05$, a noteworthy increase of rock strength for $\mu = 0.1$, and a significant increase of rock strength for $\mu = 0.2$. It seems that the simulation results of all types of rocks fit the laboratory results well for $\mu = 0.2$, suggesting that the simulation captured one characteristic.
of the observed $\sigma_2$ effect mentioned above, i.e., the observed $\sigma_2$ effect decreases with the increase of applied $\sigma_3$.

If there is no end effect, i.e., the $\mu$ value is 0 at the specimen-platen contacts, the modeling results show that the specimen’s strength is independent of $\sigma_2$. This is because that the 2D M-C failure criterion was used and the increase of the specimen’s strength is irrelevant of $\sigma_2$. Note that there is a very small difference in the rock strengths given by the cylinder ($\sigma_2 = \sigma_3$) and square prism ($\sigma_2 > \sigma_3$) geometrical models. This is caused by the geometrical model difference, which is negligible as long as their loading conditions are the same ($\mu = 0$). For $\mu > 0$, the strengthening of the specimen is exclusively attributed to the end effect caused by $\sigma_2$. As will be discussed in Section “Discussions”, there is a legitimate $\sigma_2$ effect on rock strength. However, due to the way that loads were applied in laboratory true triaxial compression tests, the test results inevitably contain the influence of the end effect and a method to isolate the end effect from the test data is proposed in Section “Discussions”.

**Failure modes**

Fig. 11 to Fig. 13 present typical failure modes by the numerical simulation and the laboratory tests of KTB amphibolite, Westerly granite, and Solnhofen limestone, respectively. The red zones in numerical 2D and 3D views represent the zones in which large plastic strains occur, which can be used to indicate strain localization. The viewing plane for all the three figures is the $\sigma_1-\sigma_3$ plane (15° rotation of the $\sigma_1$ axis for the 3D view).

The simulation captures the characteristics of the failure modes of the true triaxial compression test results (Chang and Haimson, 2000; Haimson and Chang, 2000; Mogi, 2007).

With regard to the failure planes occurring on the $\sigma_1-\sigma_3$ plane and striking in the $\sigma_2$ loading
direction, it is understood that $\sigma_2$ will constrain the failure planes in a way that the rock is easier to fail in a direction parallel to the $\sigma_2$ loading direction (Cai, 2008b). For the KTB amphibolite and the Westerly granite specimens, the numerical simulation results show conjugate fault planes but only one fault plane was observed in the laboratory test. This may be due to the reason that material heterogeneity was not considered in the numerical modeling. An approach had been adopted in the numerical work of Senent et al. (2013) by introducing a very small defect in the homogenous specimen to break the symmetry of the shear bands and a single fault plane was successfully captured.

**Discussions**

*Relation between the $\sigma_2$ effect and the increase of actual $\sigma_3$*

The increase of rock strength in the numerical modeling result is not directly related to the increase of $\sigma_2$, because the actual $\sigma_2$ effect is deliberately excluded from the assigned material model. It is seen that friction mobilized at the specimen-platen contacts increases $\sigma_3$ in the specimen and as a result the specimen’s strength is increased. Resultant displacements on planes cutting through the mid-height of the Westerly granite specimen in Fig. 14 show different modes of lateral expansion under three loading conditions. In the conventional triaxial compression test (Fig. 14a), the lateral expansions are relatively uniform and normal to the surface where confinement is applied ($\sigma_2 = \sigma_3 = 60$ MPa). In the true triaxial compression test, the expansions in the direction where $\sigma_2$ is applied are smaller than that where $\sigma_3$ is applied. This is because that it is easier for a specimen to expand in a direction in which it has less resistance. For the specimen under the same applied $\sigma_3$ (60 MPa), the lateral expansions in the direction where $\sigma_3$ is
applied increase with the increase of $\sigma_2$, leading to a larger relative deformation and hence a 
greater end constraint at the contacts where $\sigma_2$ is applied (Fig. 14b and c).

Fig. 15 presents the contours of $\sigma_3$ on the side, top, and two vertical surfaces revealed 
from a quarter cut of the specimen at peak load. Specifically, red zones, which represent a range 
of $\sigma_3$ that is very close to the applied $\sigma_3$ (60 MPa), can be focused to evaluate the disturbance of 
the actual $\sigma_3$ distribution due to the end effect. For the specimen under conventional triaxial 
loading (Fig. 15a), because hydraulic pressure loading and pressure control are employed to 
apply confinement, $\sigma_3 = 60$ MPa is maintained around the lateral surface of the cylinder 
specimen. Therefore, the red zone is uniformly distributed and consists of the largest percentage 
of the specimen volume. In comparison, even though hydraulic pressure loading and pressure 
control are employed to apply $\sigma_3$ in true triaxial loadings, $\sigma_3 = 60$ MPa is maintained only on the 
contacts where $\sigma_3$ is applied. In fact, there are more non-uniform distributions of $\sigma_3$ observed on 
the contacts where $\sigma_2$ is applied and inside the specimen (Fig. 15b and c). Moreover, the 
transition of the contour zones in the true triaxial loadings indicates that the actual $\sigma_3$ is increased 
with the increase of $\sigma_2$.

It is inferred that the end constraint is activated on the specimen-platen contacts where $\sigma_2$ 
is applied because of the relative deformations of the specimen and the platens. The end 
constraint acts on the contacts and inside the specimen in a direction coinciding with the $\sigma_3$ 
direction, the actual $\sigma_3$ is thus increased in the specimen, leading to an increase of the specimen’s 
strength. End constraint acting on the specimen-platen contacts can be increased as an increase 
of either the contact roughness ($\mu$ value) or the normal stress ($\sigma_2$). As a result, the specimen’s 
strength is increased with the increase of $\sigma_2$, even though the assigned specimen strength is
independent of $\sigma_2$. In this numerical experiment of true triaxial compression test, the influence of end effect on the increase of rock strength is studied. It is found that as long as the end effect exists (i.e., the $\mu$ value is greater than 0), rock strengthening due to the end effect exists as well. In the laboratory test results, this strength increase due to the end effect was not excluded and it constituted part of the observed $\sigma_2$ effect.

**Complementary evidences related to end effect in true triaxial compression tests**

Some rocks show a strength increase due to $\sigma_2$ while others do not, and different end constraints initiated by different deformation behaviors of rocks could be the culprit. Using the same true triaxial compression test apparatus and testing method, Chang and Haimson (2005) found that the Long Valley Caldera rocks did not exhibit any meaningful $\sigma_2$ effect. After examining the deviatoric stress ($\sigma_1 - \sigma_3$)–volumetric strain ($\Delta V/V$) curves and the scanning electron microscope results, we notice that the strength independency on $\sigma_2$ of the Long Valley Caldera rocks is due to the non-dilatant deformation behavior of the rocks.

Fig. 16 show the deviatoric stress–volumetric strain curves for KTB amphibolite and Long Valley Caldera metapelite in true triaxial compression tests (Chang and Haimson, 2000; Chang and Haimson, 2005). It is seen that even though these two metamorphic rocks were tested by the same true triaxial compression test apparatus, their deformation behaviors are different. The deviatoric stress–volumetric strain curve of the amphibolite is nonlinear, indicating that dilation develops gradually when the deviatoric stress is above 700 MPa, while that of the metapelite is almost linear to the peak stress, indicating that there is an absence of dilation before peak load. The reversal point in the amphibolite’s deviatoric stress–volumetric strain curve
marks the onset of unstable crack growth (i.e. crack damage threshold) that usually corresponds
to a stress level of approximately 0.7 to 0.8 times of the rock strength (Zhao and Cai, 2010).

From this point, the plastic volumetric strain rate outnumbers the elastic volumetric strain rate,
leading to a substantial growth of microcracks and dilation. Hence, for an amphibolite specimen
that experiences dilatancy in compression, the plastic volumetric strain resulting from the
dilation contributes to a large lateral expansion of the specimen before the peak load is reached.
This large lateral deformation of the rock led to high constraint from the loading platens and thus
the strength increase due to $\sigma_2$ was large.

Enlightened by our modeling results – the lateral expansion in the mid-height of
specimens shown in Fig. 14 and the disturbance of actual $\sigma_3$ distribution in specimens in Fig. 15
– it becomes clear that if there is little lateral expansion (e.g., those two Long Valley Caldera
rocks), there will be very small relative deformation at the rock specimen-steel platen contacts
and the end effect is thus small.

Using ABAQUS, we calibrated some true triaxial compression test results and our
modeling results suggest that the end effect due to $\sigma_2$ can result in a strength increase even if the
rock strength used in the numerical modeling is independent of $\sigma_2$. Shi et al. (2012) used
FLAC3D to simulate the loading boundary effects in true triaxial compression test and revealed
that the end effect can result in an apparent $\sigma_2$ effect. Gerstle et al. (1978) and Gerstle et al.
(1980) carried out true triaxial compression tests on concrete specimens made from the same
material composition and concluded that the $\sigma_2$ effect varied considerably due to the different
loading methods and anti-friction measures. Therefore, the point highlighted in our study, i.e.,
the end effect can result in a different interpretation of the $\sigma_2$ effect on rock strength, is supported
by the experimental results of Gerstle et al. (1978) and Gerstle et al. (1980). Uniaxial
compression tests using well-polished, high strength Beishan granite (Zhao et al., 2012; Zhao et al., 2013) revealed that even anti-friction measures were employed, rock strength was noticeably increased if the H/D ratio of the cylinder specimen was decreased from 2 to 1. Therefore, it is practically impossible to neglect the end effect in true triaxial compression test results because the H/W ratio in the $\sigma_2$ direction is normally less than 1 (0.5 in most tests).

**Contribution of end effect to observed $\sigma_2$ effect**

It is numerically proved that when testing a rock-like material whose strength is independent of $\sigma_2$, there is an apparent $\sigma_2$ effect caused exclusively by the end effect. The next step is to confirm that the apparent $\sigma_2$ effect constitutes part of the observed $\sigma_2$ effect in previous laboratory test results. Although the actual $\mu$ values in previous true triaxial compression tests are unknown, the $\mu$ values can never be 0 and a range of 0.05 to 0.1 seems to be appropriate for lubricated rock specimen-steel platen contacts (Section “Friction at rock specimen-steel platen contacts”). Accordingly, laboratory test results that show a significant $\sigma_2$ effect might contain an apparent $\sigma_2$ effect resulted from the end effect in a way similar to that revealed in our modeling results. It is worth noting that, as justified in Section “Mohr-Coulomb failure criterion and modeling strategy”, although a $\sigma_3$-only-dependent material property might not be precise enough to characterize the actual rock strength property, it is sufficient to emphasize the contribution of the end effect to the observed $\sigma_2$ effect. The increase of rock strength due to the end effect in true triaxial compression test simulations based on a both-$\sigma_3$-and-$\sigma_2$-dependent material model should not be less than that based on a $\sigma_3$-only-dependent material model.

In order to further verify the influence of the end effect on true triaxial compression test results, the modeling results of the apparent $\sigma_2$ effect needs to be added to the actual $\sigma_2$ effect
and compared with the observed $\sigma_2$ effect in previous laboratory test results. Approaches that can consider the actual $\sigma_2$ effect while excluding the end effect include theoretical approach (Wiebols and Cook, 1968) and numerical investigation using the DEM (Fjær and Ruistuen, 2002; Cai, 2008a; Schöpfer et al., 2013). DEM modeling employs micromechanical properties to capture the failure process of rocks and thereby avoids the discretion of selecting an empirical failure criterion that only reflects the macro-mechanical behavior of rocks. However, the micromechanical parameters used in DEM modeling are hypothetical and cannot be measured in laboratory tests. These parameters can only be obtained by trials-and-errors in the calibration process while they are changed systematically until a set of parameters yield a macro behavior that best fit the actual one (Potyondy and Cundall, 2004).

In this study, we choose the theoretically-based effective strain energy criterion developed by Wiebols and Cook (1968) instead of the DEM modeling approach to characterize the actual $\sigma_2$ effect. The effective strain energy criterion (Wiebols and Cook, 1968) was derived based on a hypothesis that the effective shear strain energy stores around Griffith cracks due to the sliding of crack surfaces, and rock failure occurs when the total effective shear strain energy, which depends on the coefficient of sliding friction ($\mu_s$) of the crack surfaces, reaches a critical value. It predicts a strengthening of rock from increasing $\sigma_2$ followed by a strength plateau and a weakening of rock once $\sigma_2$ exceeds a certain magnitude, which was verified experimentally by Wiebols and Cook (1968) and later confirmed by other researchers (Mogi, 1971; Chang and Haimson, 2000). Because the $\sigma_2$ effect predicted by the effective strain energy criterion excludes the end effect, it can be considered as the “true” or the “actual” $\sigma_2$ effect.

All failure criteria have their limitations in representing the true physics of rock deformation under complex loading, and the effective strain energy criterion has with no
exception. Laboratory test results show that the failure envelopes of rocks in conventional triaxial compression tests are normally nonlinear, especially near the brittle-ductile transition zones (Mogi, 1974), but the failure envelopes of conventional triaxial compression tests predicted by the effective strain energy criterion are linear. A precise prediction of a rock’s failure envelope in the conventional triaxial stress state is the benchmark to ensure the accuracy of the predicted $\sigma_1-\sigma_2$ curve in the true triaxial stress state, because the conventional triaxial stress state is the origin of the $\sigma_1-\sigma_2$ curve. Acknowledging the shortcoming of the effective strain energy criterion, only the failure envelopes in the low confinement range will be considered. In addition, it is preferable to choose rocks whose $\sigma_1-\sigma_3$ relations in conventional triaxial compression tests are relatively linear. Another shortcoming of the effective strain energy criterion is that the $\mu_s$ value is difficult to be determined. Wiebols and Cook (1968) provided failure envelopes for five $\mu_s$ values (0.25, 0.5, 0.667, 0.85, and 1) and pointed out that there is a relation between $\mu_s$ and tan$\phi$ (where $\mu_s$ and tan$\phi$ are close to each other but $\mu_s < \tan \phi$). Accordingly, we choose the $\mu_s$ value based on the tan$\phi$ value of a rock.

The failure envelopes of KTB amphibolite with applied $\sigma_3$ ranging from 0 to 60 MPa are used for validation. The calibrated $\phi$ values of KTB amphibolite in this applied $\sigma_3$ range (Table 3) are relatively constant compared with other rocks (Table 4 and Table 5), denoting relatively linear $\sigma_1-\sigma_3$ relations. Furthermore, the calibrated tan$\phi$ in this $\sigma_3$ range is slightly higher than $\mu_s = 1$. Therefore, the theoretical $\sigma_2$ effect predicted by the effective strain energy criterion for KTB amphibolite in the true triaxial compression tests can be obtained by assuming $\mu_s = 1$. Then, the influence of end effect on the observed $\sigma_2$ effect can be analyzed, by adding the modeling results, which quantify the end effect in the true triaxial compression tests, to the theoretical results that reflect the actual $\sigma_2$ effect only.
The combination of the numerical end effect and the theoretical $\sigma_2$ effect on rock strength is shown in Fig. 17 and compared with the experimental test results, where $\sigma_1$, $\sigma_2$, and $\sigma_3$ are normalized to $\sigma_c$. The lower bound (dash line) corresponds to the theoretical result of Wiebols and Cook (1968), which includes only the influence of $\sigma_2$ on rock strength for $\mu_s = 1$. The upper bound (solid line) corresponds to the results of strength increase due to both the actual $\sigma_2$ effect for $\mu_s = 1$ (according to Wiebols and Cook (1968)) and the end effect for $\mu = 0.1$ (obtained from our numerical modeling). Thus, the yellow highlighted zones between the upper bound and lower bound of the $\sigma_1$–$\sigma_2$ curves represent the apparent $\sigma_2$ effect due to the end effect for a $\mu$ value ranging from 0 to 0.1.

Although there was strength variability in the test results of KTB amphibolite, most test data fell within the yellow highlighted zones, indicating that the end effect caused by $\sigma_2$ loading somehow constituted part of the observed $\sigma_2$ effect. Because a laboratory test result inevitably contains an apparent $\sigma_2$ effect if the friction at the rock specimen-steel platen contacts is not zero, 3D empirical failure criteria developed from such a true triaxial compression test result may overestimate the rock strength. Although the actual $\mu$ values in the previous laboratory tests were unknown, our study shows that the end effect exists in true triaxial compression test for $\mu > 0$ and the $\mu$ values in those tests could range from 0.05 to 0.1. Therefore, it is suggested that when using previous and future true triaxial compression test results, the influence of the end effect on the test results needs to be investigated. As illustrated in this study, with knowledge of the actual $\mu$ value at the rock specimen-steel platen contacts, the apparent $\sigma_2$ effect may be separated from the observed $\sigma_2$ effect. In this way, the actual $\sigma_2$ effect could be characterized for the development of 3D empirical failure criteria for rocks.
Recent laboratory work to minimize end effect and to characterize actual $\sigma_2$ effect

It was proposed in Section “Contribution of end effect to observed $\sigma_2$ effect” that with knowledge of the apparent $\sigma_2$ effect resulted by end constraint, it is possible to use previous true triaxial compression test data for the development of 3D empirical failure criteria of rocks. However, this approach should be applied individually to each laboratory test because the coefficients of friction at the rock specimen-steel platen contacts were not the same in the tests.

An alternative approach is to decrease the friction at the rock specimen-steel platen contacts to a level where the contribution of the end effect to the observed $\sigma_2$ effect is very small such that the obtained test data can be used directly to develop 3D empirical failure criteria. To achieve this goal, we must ensure that the end effect has been greatly reduced. Based on our modeling results, we conclude that the strengthening of rock strength due to the end effect for a $\mu$ value ranging from 0.05 to 0.1 cannot be neglected. Hence, the coefficient of friction at the rock specimen-steel platen contacts must be reduced to a value much less than 0.05. In addition, the characteristics of the obtained test results should be somehow in accordance with that predicted by theoretical results (e.g., Wiebols and Cook, 1968).

A series of laboratory tests were carried out at Northeastern University (China) to study how friction at the rock specimen-steel platen contacts can be reduced. Test specimens were prepared using Linghai granite samples collected from a quarry site in Liaoning Province, China. Linghai granite is homogeneous and isotropic with a high strength. Using a polishing machine with a very high machining accuracy, the specimen ends were ground flat and polished down to $\pm$ 0.01 mm. To prevent the anti-friction material from flowing into the rock specimen, a copper sheet of 0.02 mm thickness was put between the specimen and the platen. Then, friction tests
with increasing normal forces were conducted to study the effectiveness of two anti-friction measures, one by using Teflon and the other by using a mixture of stearic acid and Vaseline at a 1:1 ratio, recommended by Labuz and Bridell (1993). The black line in Fig. 18 shows the $\mu$ values of the specimens’ polished surfaces without lubricant. Because the specimens’ end and the platen’s end surfaces had been polished smoothly, the obtained average $\mu$ value of 0.15 is low, which is lower than that of some contacts smeared with lubricants (Section “Friction at rock specimen-steel platen contacts”). The results shown in Fig. 18 clearly indicate that the mixture of stearic acid and Vaseline at a 1:1 ratio in combination with a thin copper sheet can reduce the $\mu$ value of the lubricated rock specimen-steel platen contact to about 0.02. Referring to the numerical simulation results in Section “Rock strength”, it can be seen that the end effect caused by $\mu = 0.02$ is small and can be neglected.

True triaxial compression tests were conducted using a newly developed true triaxial compression test apparatus (Feng et al., 2016). This novel test apparatus was designed and fabricated on the basis of the Mogi-type apparatus with many improvements implemented to address key issues such as off-center suppression, loading gap removal, and more importantly, end effect reduction (Feng et al., 2016). Linghai granite specimens, with a rectangular prismatic shape of $50 \times 50 \times 100$ mm$^3$, were used in the test. The effect of $\sigma_2$ on rock strength at low $\sigma_3$ (up to 50 MPa) was focused in this laboratory test. The mixture of stearic acid and Vaseline at a 1:1 ratio and thin copper sheets were used to reduce friction at the contacts. Additionally, compared with some previous true triaxial compression tests that barely reached the strength plateau in the $\sigma_1-\sigma_2$ curves, the full range of $\sigma_2$ loading ($\sigma_3 \leq \sigma_2 \leq \sigma_1$) was attempted to obtain the complete $\sigma_2$ effect curves. The laboratory test results are shown in Fig. 19.
The overall characteristics of the observed $\sigma_2$ effect on the strength of Linghai granite agree with that of previous laboratory test results, showing an increase of rock strength with the increase of $\sigma_2$ before reaching the strength plateau and then a decrease of rock strength as $\sigma_2$ further increases. The complete strength drop from the strength plateau to the maximum loading range of $\sigma_2$ ($\sigma_2 = \sigma_1$), a characteristic predicted by Wiebols and Cook (1968) but normally missed in some previous laboratory test results, was captured in this test. Additionally, we noticed a characteristic of the observed $\sigma_2$ effect on Linghai granite that differs from previous laboratory results. The $\sigma_2$ effect on rock strength as observed in previous test results is normally the greatest at low $\sigma_3$ and drastically decreased with the increase of $\sigma_3$ (Table 1); however, in the Linghai granite test results the rates of rock strength increase at the peak due to $\sigma_2$ are similar at different $\sigma_3$ (Table 7), which are about 51% to 56% in all cases. It is numerically proved in Sections “Rock strength” and “Contribution of end effect to observed $\sigma_2$ effect” that the apparent $\sigma_2$ effect resulted by the end constraint can lead to an overestimation of the actual $\sigma_2$ effect. On the other hand, the theoretical results (Wiebols and Cook, 1968) that exclude the end effect show a very small decrease of the peak strength increase due to $\sigma_2$ with the increase of $\sigma_3$ (Table 8, with different $\mu_s$). Therefore, these indirect evidences, supported by numerical and theoretical results, suggest that the anti-friction measure adopted in our tests is effective. In other words, the $\sigma_2$ effect shown in the test results is dominantly the actual $\sigma_2$ effect. The apparent $\sigma_2$ effect due to the end effect is small in our test results.

**End-effect-induced $\sigma_2$ effect at different $\sigma_3$ levels**

Based on previous discussions, the phenomenon of the drastic decrease of the $\sigma_2$ effect with the increase of $\sigma_3$ in the test results found in literature (Table 1) can be examined. The numerical
experiment demonstrates that the end-effect-induced $\sigma_2$ effect decreases with the increase of $\sigma_3$ (Fig. 7 to Fig. 10). Fig. 20 below illustrates how the end-effect-induced $\sigma_2$ effect is related to $\sigma_3$. As explained in Section “Relation between the $\sigma_2$ effect and the increase of actual $\sigma_3$”, the end effect can increase the overall $\sigma_3$ of the specimen, making it higher than the applied $\sigma_3 (\Delta\sigma_3$). In other words, the end-effect-induced $\sigma_2$ effect (pink coordinate systems) is manifested by the increase of $\Delta\sigma_3$ with the increase of $\sigma_2$, and that in turn leads to the increase of rock strength ($\Delta\sigma_1$). In the numerical modeling, the end-effect-induced $\sigma_2$ effect depends on the friction angle when the M-C failure criterion is used. The higher the friction angle is, the greater the end-effect-induced $\sigma_2$ effect will be.

The friction angle calibrated (Table 3 to Table 6) decreases with the increase of $\sigma_3$ (blue and green dash lines) and this matches well the nonlinearity of the $\sigma_1$–$\sigma_3$ failure envelopes (grey curve). Therefore, all the modeling results with various $\mu$ values reflect that the end-effect-induced $\sigma_2$ effect decreases with the increase of $\sigma_3$. Note that the $\sigma_1$–$\sigma_3$ failure envelope representing conventional triaxial compression tests (grey coordinate system) is plotted in Fig. 20. This is because that true triaxial compression tests ($\sigma_2 \geq \sigma_3$) normally begin with a stress state of $\sigma_2 = \sigma_3$; true triaxial compression test results are obtained by increasing $\sigma_2$ from the applied $\sigma_3$.

The calibrated friction angle (blue dash lines) cannot always reflect precisely the slopes of the true $\sigma_1$–$\sigma_3$ failure envelope at different applied $\sigma_3$ levels. For instance, at very low $\sigma_3$ levels especially at $\sigma_3 = 0$, the calibrated friction angle can underestimate the slope of the failure envelope. On the other hand, at very high $\sigma_3$ levels, the calibrated friction angle can overestimate the slope of the failure envelope. Hence, the simulation results (Fig. 7d) at intermediate $\sigma_3$ levels
match laboratory test results well (green dash lines), while at very low and very high $\sigma_3$ levels the modeling results are not equally good.

We consider that as long as the trend of the end-effect-induced $\sigma_2$ effect revealed from the numerical modeling confirms our research hypothesis (the end-effect-induced $\sigma_2$ effect is large), there is no need to require that the modeling results match the test results perfectly. In fact, a perfect match between the modeling results and the test results can only suggest that the $\sigma_2$ effect observed in the laboratory test results are completely caused by the end effect. This implication is not correct because both theoretical and laboratory studies have shown that there is a legit $\sigma_2$ effect to rock strength. More importantly, the differences between the modeling and the test results at very low and very high $\sigma_1$ levels support the viewpoint: the end effect affects true triaxial compression test results; the $\sigma_2$ effect decrease with the increase of $\sigma_3$.

It was experimentally confirmed by Mogi (2007) that rock strength increase due to the end effect increases with the decrease of $\sigma_3$. Hence, it can be interpreted that in the previous laboratory test results, the end-effect-induced rock strength increase was high when the applied $\sigma_3$ was low, which resulted in a greater apparent $\sigma_2$ effect. The end effect became smaller at high applied $\sigma_3$. Fig. 21 presents some conventional triaxial compression test results of rocks and their H-B fitting curves. Nonlinearity of the failure envelopes is observed ubiquitously in these rocks, illustrating that the same end-effect-induced $\Delta\sigma_3$ at a low $\sigma_3$ can result in a greater increase of rock strength $\Delta\sigma_1$ than that at a high $\sigma_3$. Therefore, the end effect can result in a large apparent $\sigma_2$ effect when $\sigma_3$ is low and the observed $\sigma_2$ effect at low $\sigma_3$ in some previous test results might be exaggerated. It is suggested that if true triaxial compression test results contain an unknown influence of end effect on rock strength, the test results at high $\sigma_3$, which are less influenced by the end effect, might be more appropriate for developing 3D empirical failure criteria for rocks.
Conclusions

Laboratory test results show that the use of lubricants cannot eliminate the end effect completely, especially when the rock specimen is squat in the $\sigma_2$ loading direction. The influence of end effect on rock strength in true triaxial compression test is studied using a numerical approach. The influence of $\sigma_2$ on rock strength is purposely excluded in the material model in the numerical study so that any increase of rock strength with the increase of $\sigma_2$ while the applied $\sigma_3$ is kept constant can be attributed to the end effect. It is seen from the simulation results that friction at the rock specimen-steel platen contacts has a large influence on rock strength, because the $\sigma_3$ in the specimen is increased due to the end constraint at the $\sigma_2$ loading contacts. Thus, the end effect can result in an apparent $\sigma_2$ effect that contributes to the observed strength increase due to $\sigma_2$ in true triaxial compression tests. This point can explain why some rocks show the $\sigma_2$ effect while others do not because different end constraints can be initiated by different rock deformation behaviors. This study made a contribution to better understanding true triaxial compression test results by decomposing the observed $\sigma_2$ effect on rock strength in laboratory tests into two parts: one from the end effect and the other from the actual $\sigma_2$ effect. The actual $\sigma_2$ effect can be explained using the theory of Wiebols and Cook (1968).

Considering that the end effect can result in an apparent $\sigma_2$ effect that may mislead the interpretation of the true triaxial compression test results, some suggestions are made for future study of the $\sigma_2$ effect. Firstly, previous laboratory test results are still useful in developing 3D empirical failure criteria for rocks. An improvement can be made by employing a reverse strategy to subtract the apparent $\sigma_2$ effect from the observed $\sigma_2$ effect, provided that the actual end effect in the true triaxial compression test results can be quantified. Otherwise, one can refer
to the test results obtained at high $\sigma_3$, where the relative influence by the end effect is small. According to our recent laboratory work and supported by the numerical and theoretical results, it is seen that previous test results that showed a significant $\sigma_2$ effect at low $\sigma_3$ might include a large end effect. Hence, future study should focus on quantifying the actual $\sigma_2$ effect under low confinement conditions because it is important for rock engineering practices. Moreover, when conducting true triaxial compression test to study the $\sigma_2$ effect, attention should be paid to minimizing the end effect. Our recent laboratory work shows that using a novel test apparatus and effective anti-friction measures, the end effect can be decreased to minimize the apparent $\sigma_2$ effect such that the characteristics of the obtained $\sigma_2$ effect are very close to the true $\sigma_2$ effect.

Note that the conclusions drawn here are based on the numerical simulation results using a homogeneous model. Material heterogeneity of rocks may influence the failure modes. Using heterogeneous models, researchers have found that $\sigma_2$ has a small influence on rock strength (Cai, 2008b) when $\sigma_3 = 0$ and $\sigma_2$ is not high. Thus, future work needs to consider the heterogeneity of rock properties to quantify the end effect on rock strength in true triaxial compression tests.

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**References**


Figure captions

Fig. 1. Schematic of the proposed method to quantify \( \sigma_2 \) effect.

Fig. 2. Illustration of confined zones due to end effect in specimens with different slenderness ratios in rock uniaxial compression tests, after Van Vliet and Van Mier (1996).

Fig. 3. Illustration of the shape and the corresponding end effect in true triaxial compression test.

Fig. 4. 3D models of three types of compression tests in numerical experiment: (a) uniaxial compression test, (b) conventional triaxial compression test, (c) true triaxial compression test.

Fig. 5 FEM models of rock specimens and loading platens for different compression test simulations: (a) uniaxial and triaxial, and (b) true triaxial.

Fig. 6. Simulation of the deformation behavior of Westerly granite under confinement of \( \sigma_2 = \sigma_3 = 60 \) MPa.

Fig. 7. Simulation of true triaxial compression tests of KTB amphibolite: (a) \( \mu = 0 \), (b) \( \mu = 0.05 \), (c) \( \mu = 0.1 \), (d) \( \mu = 0.2 \).

Fig. 8. Simulation of true triaxial compression tests of Westerly granite: (a) \( \mu = 0 \), (b) \( \mu = 0.05 \), (c) \( \mu = 0.1 \), (d) \( \mu = 0.2 \).

Fig. 9. Simulation of true triaxial compression tests of Solnhofen limestone: (a) \( \mu = 0 \), (b) \( \mu = 0.05 \), (c) \( \mu = 0.1 \), (d) \( \mu = 0.2 \).

Fig. 10. Simulation of true triaxial compression tests of Yuubari shale: (a) \( \mu = 0 \), (b) \( \mu = 0.05 \), (c) \( \mu = 0.1 \), (d) \( \mu = 0.2 \).

Fig. 11. Numerical and laboratory (Chang and Haimson, 2000) results of typical failure modes of a KTB amphibolite specimen.

Fig. 12. Numerical and laboratory (Haimson and Chang, 2000) results of typical failure modes of a Westerly granite specimen.

Fig. 13. Numerical and laboratory (Mogi, 2007) results of typical failure modes of a Solnhofen limestone specimen.

Fig. 14. Resultant displacements at peak load on the plane cutting through the mid-height of the Westerly granite specimens.

Fig. 15. \( \sigma_3 \) contours in the Westerly granite specimens at peak load.

Fig. 16. Deviatoric stress–volumetric strain curves for: (a) KTB amphibolite and (b) Long Valley Caldera metapelite in true triaxial compression tests (digitized and modified from Chang and Haimson, 2000 and Chang and Haimson, 2005, respectively).

Fig. 17. Combination of numerical end effect and theoretical \( \sigma_2 \) effect on rock strength, compared with the experimental data of KTB amphibolite.
Fig. 18. μ values of rock specimen-steel platen contacts measured by shear tests with different normal forces.

Fig. 19. Laboratory true triaxial compression test results showing the $\sigma_2$ effect on the strength of Linghai granite.

Fig. 20. Comparison of end-effect-induced $\sigma_2$ effect (refer to the pink coordinate system) at different applied $\sigma_3$ levels (refer to the grey coordinate system).

Fig. 21. Conventional triaxial compression test results of some rocks and their H-B fitting curves: (a) KTB amphibolite, (b) Westerly granite, (c) Solnhofen limestone, (d) Yuubari shale (Takahashi and Koide, 1989; Chang and Haimson, 2000; Haimson and Chang, 2000; Mogi, 2007).
Table 1. Quantified $\sigma_3$ effect in different rocks (data from Chang and Haimson, 2000; Haimson and Chang, 2000; Mogi, 2007).

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<td></td>
</tr>
<tr>
<td>Yamaguchi marble</td>
<td>0.15</td>
<td>0.30</td>
<td>0.49</td>
<td>0.45</td>
</tr>
<tr>
<td></td>
<td>104%</td>
<td>79%</td>
<td>53%</td>
<td></td>
</tr>
<tr>
<td>Mizuho trachyte</td>
<td>0.45</td>
<td>0.60</td>
<td>0.75</td>
<td>34%</td>
</tr>
<tr>
<td></td>
<td>104%</td>
<td>79%</td>
<td>53%</td>
<td></td>
</tr>
<tr>
<td>Manazuru andesite</td>
<td>0.14</td>
<td>0.29</td>
<td>0.50</td>
<td>0.14</td>
</tr>
<tr>
<td></td>
<td>82%</td>
<td>54%</td>
<td>52%</td>
<td></td>
</tr>
<tr>
<td>Inada granite</td>
<td>0.09</td>
<td>0.18</td>
<td>0.44</td>
<td>0.66</td>
</tr>
<tr>
<td></td>
<td>104%</td>
<td>71%</td>
<td>57%</td>
<td>65%</td>
</tr>
<tr>
<td>Orikabe monzonite</td>
<td>0.17</td>
<td>0.34</td>
<td>0.59</td>
<td>0.85</td>
</tr>
<tr>
<td></td>
<td>115%</td>
<td>80%</td>
<td>66%</td>
<td>67%</td>
</tr>
</tbody>
</table>

Table 2. Physical and deformation parameters of the rocks.

<table>
<thead>
<tr>
<th>Rock types</th>
<th>Density (kg/m$^3$)</th>
<th>Young’s modulus (GPa)</th>
<th>Poisson’s ratio</th>
<th>References</th>
</tr>
</thead>
<tbody>
<tr>
<td>KTB amphibolite</td>
<td>2920</td>
<td>65</td>
<td>0.29</td>
<td>Haimson and Chang (2002)</td>
</tr>
<tr>
<td>Westerly granite</td>
<td>2700</td>
<td>60</td>
<td>0.28</td>
<td>Bhat et al. (2011)</td>
</tr>
<tr>
<td>Solnhofen limestone</td>
<td>2500</td>
<td>80</td>
<td>0.20</td>
<td>Renner and Rummel (1996)</td>
</tr>
<tr>
<td>Yuubari shale</td>
<td>2200</td>
<td>40</td>
<td>0.20</td>
<td>Gercek (2007)</td>
</tr>
</tbody>
</table>

https://mc06.manuscriptcentral.com/cgj-pubs
Table 3. Calculation of the M-C strength parameters of KTB amphibolite (data from Colmenares and Zoback, 2002, original data from Chang and Haimson, 2000).

<table>
<thead>
<tr>
<th>σ3 level (MPa)</th>
<th>Data (σ1, σ2 = σ3) (MPa)</th>
<th>Fitting equation</th>
<th>Cohesion (MPa)</th>
<th>Friction angle (°)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>(158, 0) (176, 0) (410, 30)</td>
<td>σ1 = 8.4σ3 + 176</td>
<td>31</td>
<td>52</td>
</tr>
<tr>
<td>30</td>
<td>(158, 0) (410, 30) (702, 60)</td>
<td>σ1 = 9.1σ3 + 151</td>
<td>26</td>
<td>53</td>
</tr>
<tr>
<td>60</td>
<td>(410, 30) (702, 60) (868, 100)</td>
<td>σ1 = 6.4σ3 + 318</td>
<td>62</td>
<td>47</td>
</tr>
<tr>
<td>100</td>
<td>(410, 30) (702, 60) (868, 100) (1147, 150)</td>
<td>σ1 = 6.2σ3 + 232</td>
<td>48</td>
<td>46.5</td>
</tr>
<tr>
<td>150</td>
<td>(868, 100) (1147, 150)</td>
<td>σ1 = 5.6σ3 + 310</td>
<td>66</td>
<td>44</td>
</tr>
</tbody>
</table>


<table>
<thead>
<tr>
<th>σ3 level (MPa)</th>
<th>Data (σ1, σ2 = σ3) (MPa)</th>
<th>Fitting equation</th>
<th>Cohesion (MPa)</th>
<th>Friction angle (°)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>(201, 0) (231, 2)</td>
<td>σ1 = 15.0σ3 + 201</td>
<td>26</td>
<td>61</td>
</tr>
<tr>
<td>20</td>
<td>(201, 0) (430, 20) (605, 38)</td>
<td>σ1 = 10.7σ3 + 206</td>
<td>33</td>
<td>56</td>
</tr>
<tr>
<td>38</td>
<td>(430, 20) (605, 38) (747, 60)</td>
<td>σ1 = 7.9σ3 + 306</td>
<td>54</td>
<td>51</td>
</tr>
<tr>
<td>60</td>
<td>(605, 38) (747, 60) (889, 77)</td>
<td>σ1 = 7.3σ3 + 325</td>
<td>60</td>
<td>49</td>
</tr>
<tr>
<td>77</td>
<td>(747, 60) (889, 77) (1012, 100)</td>
<td>σ1 = 6.6σ3 + 385</td>
<td>76</td>
<td>47</td>
</tr>
<tr>
<td>100</td>
<td>(889, 77) (1012, 100)</td>
<td>σ1 = 5.4σ3 + 477</td>
<td>101</td>
<td>44</td>
</tr>
</tbody>
</table>
Table 5. Calculation of the M-C strength parameters of Solnhofen limestone (data from Mogi, 2007).

<table>
<thead>
<tr>
<th>σ level (MPa)</th>
<th>Data (σ₁, σ₂ = σ₃) (MPa)</th>
<th>Fitting equation</th>
<th>Cohesion (MPa)</th>
<th>Friction angle (°)</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>(310, 0) (397, 20) (449, 40)</td>
<td>σ₁ = 3.5σ₃ + 328</td>
<td>88</td>
<td>34</td>
</tr>
<tr>
<td>40</td>
<td>(310, 0) (397, 20) (449, 40)</td>
<td>σ₁ = 3.5σ₃ + 316</td>
<td>84</td>
<td>34</td>
</tr>
<tr>
<td>60</td>
<td>(310, 0) (397, 20) (449, 40) (473, 60)</td>
<td>σ₁ = 2.7σ₃ + 326</td>
<td>98</td>
<td>28</td>
</tr>
<tr>
<td>80</td>
<td>(473, 60) (528, 80)</td>
<td>σ₁ = 2.8σ₃ + 308</td>
<td>93</td>
<td>28</td>
</tr>
</tbody>
</table>

* Conventional triaxial compression test data at σ₃ = 60 MPa diverges somewhat from the failure envelop; hence, calibration of the friction angle was adjusted accordingly.


<table>
<thead>
<tr>
<th>σ level (MPa)</th>
<th>Data (σ₁, σ₂ = σ₃) (MPa)</th>
<th>Fitting equation</th>
<th>Cohesion (MPa)</th>
<th>Friction angle (°)</th>
</tr>
</thead>
<tbody>
<tr>
<td>25</td>
<td>(161, 25) (228, 50)</td>
<td>σ₁ = 2.7σ₃ + 94</td>
<td>29</td>
<td>27</td>
</tr>
<tr>
<td>50</td>
<td>(161, 25) (228, 50)</td>
<td>σ₁ = 2.7σ₃ + 94</td>
<td>29</td>
<td>27</td>
</tr>
</tbody>
</table>

Table 7. Quantified σ₂ effect on Linghai granite.

<table>
<thead>
<tr>
<th>σ₁/σ₂</th>
<th>0.00</th>
<th>0.06</th>
<th>0.12</th>
<th>0.18</th>
<th>0.31</th>
</tr>
</thead>
<tbody>
<tr>
<td>(σ₁⁺ - σ₁)/ (σ₂⁺ - σ₂)</td>
<td>56%</td>
<td>52%</td>
<td>52%</td>
<td>53%</td>
<td>51%</td>
</tr>
</tbody>
</table>
Table 8. Quantified $\sigma_2$ effect predicted by the theory of Wiebols and Cook (1968) for different $\mu_s$.

<table>
<thead>
<tr>
<th>$\mu_s$</th>
<th>$\sigma_3/\sigma_c$</th>
<th>0.00</th>
<th>0.20</th>
<th>0.40</th>
<th>0.60</th>
<th>0.80</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$(\sigma_1^* - \sigma_1)/(\sigma_2^* - \sigma_2)$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\mu_s = 0.25$</td>
<td>44%</td>
<td>43%</td>
<td>43%</td>
<td>43%</td>
<td>43%</td>
<td></td>
</tr>
<tr>
<td>$\mu_s = 0.50$</td>
<td>67%</td>
<td>66%</td>
<td>65%</td>
<td>64%</td>
<td>64%</td>
<td></td>
</tr>
<tr>
<td>$\mu_s = 0.67$</td>
<td>79%</td>
<td>78%</td>
<td>77%</td>
<td>75%</td>
<td>74%</td>
<td></td>
</tr>
<tr>
<td>$\mu_s = 0.80$</td>
<td>87%</td>
<td>83%</td>
<td>80%</td>
<td>79%</td>
<td>78%</td>
<td></td>
</tr>
<tr>
<td>$\mu_s = 1.00$</td>
<td>101%</td>
<td>88%</td>
<td>83%</td>
<td>81%</td>
<td>79%</td>
<td></td>
</tr>
</tbody>
</table>
Figure 1. Schematic of the proposed method to quantify $\sigma_2$ effect.

Figure 2. Illustration of confined zones due to end effect in specimens with different slenderness ratios in rock uniaxial compression tests, after Van Vliet and Van Mier (1996).
Figure 3. Illustration of the shape and the corresponding end effect in true triaxial compression test.
Figure 4. 3D models of three types of compression tests in numerical experiment: (a) uniaxial compression test, (b) conventional triaxial compression test, (c) true triaxial compression test.
Figure 5 FEM models of rock specimens and loading platens for different compression test simulations: (a) uniaxial and triaxial, and (b) true triaxial.

Figure 6. Simulation of the deformation behavior of Westerly granite under confinement of $\sigma_2 = \sigma_3 = 60$ MPa.
Figure 7. Simulation of true triaxial compression tests of KTB amphibolite: (a) $\mu = 0$, (b) $\mu = 0.05$, (c) $\mu = 0.1$, (d) $\mu = 0.2$. 

- Laboratory results
- Modeling results ($\sigma_2 = \sigma_3$)
- Modeling results ($\sigma_2 > \sigma_3$)
Laboratory results  — Modeling results ($\sigma_2 = \sigma_3$)  — Modeling results ($\sigma_2 > \sigma_3$)

Figure 8. Simulation of true triaxial compression tests of Westerly granite: (a) $\mu = 0$, (b) $\mu = 0.05$, (c) $\mu = 0.1$, (d) $\mu = 0.2$. 

Westerly Granite ($\mu = 0$) 

Westerly Granite ($\mu = 0.05$) 

Westerly Granite ($\mu = 0.1$) 

Westerly Granite ($\mu = 0.2$)
Figure 9. Simulation of true triaxial compression tests of Solnhofen limestone: (a) $\mu = 0$, (b) $\mu = 0.05$, (c) $\mu = 0.1$, (d) $\mu = 0.2$. 

Laboratory results  
- - Modeling results ($\sigma_2 = \sigma_3$)  
- - Modeling results ($\sigma_2 > \sigma_3$)
Figure 10. Simulation of true triaxial compression tests of Yuubari shale: (a) \( \mu = 0 \), (b) \( \mu = 0.05 \), (c) \( \mu = 0.1 \), (d) \( \mu = 0.2 \).
<table>
<thead>
<tr>
<th>Plastic strains</th>
<th>Numerical 3D view</th>
<th>Numerical 2D view</th>
</tr>
</thead>
<tbody>
<tr>
<td>+4.0e-02</td>
<td></td>
<td></td>
</tr>
<tr>
<td>+2.0e-02</td>
<td></td>
<td></td>
</tr>
<tr>
<td>+0.0e+00</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(numerical parameters: \(\sigma_3 = 0 \text{ MPa}, \sigma_2 = 30 \text{ MPa} \quad \mu = 0.05\))

Figure 11. Numerical and laboratory (Chang and Haimson, 2000) results of typical failure modes of a KTB amphibolite specimen.

<table>
<thead>
<tr>
<th>Plastic strains</th>
<th>Numerical 3D view</th>
<th>Numerical 2D view</th>
</tr>
</thead>
<tbody>
<tr>
<td>+8.0e-02</td>
<td></td>
<td></td>
</tr>
<tr>
<td>+4.0e-02</td>
<td></td>
<td></td>
</tr>
<tr>
<td>+0.0e+00</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(numerical parameters: \(\sigma_3 = 100 \text{ MPa}, \sigma_2 = 312 \text{ MPa} \quad \mu = 0.2\))

Figure 12. Numerical and laboratory (Haimson and Chang, 2000) results of typical failure modes of a Westerly granite specimen.
(numerical parameters: \(\sigma_3 = 40 \text{ MPa}, \quad \sigma_2 = 80 \text{ MPa}, \quad \mu = 0.05\))

<table>
<thead>
<tr>
<th>Plastic strains</th>
<th>Numerical 3D view</th>
<th>Numerical 2D view</th>
</tr>
</thead>
<tbody>
<tr>
<td>+4.0e-02</td>
<td></td>
<td></td>
</tr>
<tr>
<td>+2.0e-02</td>
<td></td>
<td></td>
</tr>
<tr>
<td>+0.0e+00</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Figure 13. Numerical and laboratory (Mogi, 2007) results of typical failure modes of a Solnhofen limestone specimen.
Figure 14. Resultant displacements at peak load on the plane cutting through the mid-height of the Westerly granite specimens.
Figure 15. $\sigma_3$ contours in the Westerly granite specimens at peak load.
Figure 16. Deviatoric stress–volumetric strain curves for: (a) KTB amphibolite and (b) Long Valley Caldera metapelite in true triaxial compression tests (digitized and modified from Chang and Haimson, 2000 and Chang and Haimson, 2005, respectively).

Figure 17. Combination of numerical end effect and theoretical $\sigma_2$ effect on rock strength, compared with the experimental data of KTB amphibolite.
Figure 18. µ values of rock specimen-steel platen contacts measured by shear tests with different normal forces.

Figure 19. Laboratory true triaxial compression test results showing the σ2 effect on the strength of Linghai granite.
Figure 20. Comparison of end-effect-induced $\sigma_2$ effect (refer to pink coordinate system) at different applied $\sigma_3$ levels (refer to grey coordinate system).
Figure 21. Conventional triaxial compression test results of some rocks and their H-B fitting curves: (a) KTB amphibolite, (b) Westerly granite, (c) Solnhofen limestone, (d) Yuubari shale (Takahashi and Koide, 1989; Chang and Haimson, 2000; Haimson and Chang, 2000; Mogi, 2007).