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Influence of End Effect on Rock Strength in True Triaxial Compression Test

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Abstract

The influence of the end effect on rock strength in true triaxial compression test was studied using a numerical approach. The influence of σ_2 on rock strength was isolated by using the 2D Mohr-Coulomb failure criterion that depends only on σ_1 and σ_3 . Thus, any enhancement to the rock strength with the increase of σ_2 can be attributed to the end effect. It was shown that the end effect can result in an apparent σ_2 effect, as long as the coefficient of friction (μ) at rock specimen-steel platen contacts is not zero and the specimen in the σ_2 loading direction is squat. When the strengthening due to the increase of σ_2 predicted by a theoretical failure criterion was

added to the strengthening due to the end effect, the results were in good agreement with the observed σ_2 effect from some previous laboratory tests, indicating that the observed σ_2 effect in true triaxial compression test could be partially influenced by the end effect, particularly when σ_3 was low. It is suggested to decrease the end effect to a level where the apparent σ_2 effect is very small so that the obtained test results are more meaningful to characterize the actual σ_2 effect.

Keywords: True triaxial compression test; intermediate principal stress; rock strength; end effect; 3D empirical failure criterion; numerical modeling

Draft

Introduction

Accurate estimation of rock strength is one the most important tasks in rock engineering design. Different approaches including laboratory, theoretical, empirical, and numerical approaches have been developed and used to investigate rock strength (Hudson et al., 1972; Read and Hegemier, 1984; Dai et al., 2010; Xu et al., 2016) and many failure criteria have been developed to model rock strength. Among them, the 2D Mohr-Coulomb (M-C) and Hoek-Brown (H-B) failure criteria, which are based on conventional triaxial ($\sigma_1 > \sigma_2 = \sigma_3$) compression test results and do not take the intermediate principal stress (σ_2) into account, have been widely used to estimate rock strength. Although a rock mass is generally under a true triaxial stress state ($\sigma_1 > \sigma_2 > \sigma_3$) in situ, it is commonly accepted that *"a simplification to ignore the influence of intermediate principal stress on rock material strength is justifiable"* (Brown, 2008). Moreover, there is no significant deviation of predicted rock strengths between the results using these 2D failure criteria and field measurements, suggesting that these simple failure criteria are reliable (Hoek et al., 2002; Eberhardt, 2012; Labuz and Zang, 2012). In addition, the well-known Griffith theory, developed from a theoretical approach, also ignores the effect of σ_2 in its original form.

True triaxial compression test apparatuses were developed to study rock strength in a true triaxial stress state and to investigate the σ_2 effect. A ground-breaking work was carried out by Mogi (1967), who designed a true triaxial compression test apparatus and for the first time observed that σ_2 had a large effect on rock strength. Similar results were later reported by other investigators (Wiebols and Cook, 1968; Michelis, 1987; He et al., 2010; Li et al., 2015). In particular, Haimson and Chang (2000) developed a new true triaxial compression test apparatus and observed that rock strength increased significantly with the increase of σ_2 . The observed σ_2 effect is normally characterized by an increase of rock strength with the increase of σ_2 , followed

by reaching a strength plateau and a strength drop soon after with the further increase of σ_2 (Fig. 1). It should be noted that while the σ_2 effect has been widely reported from most laboratory test results, the observed strengthening effect of σ_2 on some rocks is not significant (Takahashi and Koide, 1989; Chang and Haimson, 2005). Nevertheless, these laboratory observations provoked the attention of considering σ_2 in 3D empirical failure criteria (Mogi, 1967, 1971; Haimson and Chang, 2000; Al-Ajmi and Zimmerman, 2005; You, 2009).

In this study, a method to quantify the observed σ_2 effect is proposed and is schematically shown in Fig. 1. Blue point $(0, \sigma_c)$ represents the uniaxial compressive strength (UCS, or σ_c); blue points (σ_2, σ_1) represent conventional triaxial stress states at different σ_3 ; and red points (σ_2^*, σ_1^*) represent the stress state at the strength plateaus at different σ_3 . The slope between point (σ_2, σ_1) at the origin of the σ_1 – σ_2 curve and point (σ_2^*, σ_1^*) at the summit of the σ_1 – σ_2 curve gives the maximum percentage increase of rock strength caused by σ_2 . In this way, the strengthening effect of σ_2 on different rocks can be quantified.

Basing on the published data of true triaxial compression tests, some of the observed σ_2 effects are quantified and summarized in Table 1. Test results that do not show the strength plateau in the σ_1 – σ_2 curves are not included. It is seen that the σ_2 effect is not the same in different rocks. Moreover, it seems that the observed σ_2 effect on rock strength is more significant when the applied σ_3 is low, and it decreases with the increase of σ_3 . For instance, an unusually high percentage of strength increase of 114% for Westerly granite is observed at $\sigma_3/\sigma_c = 0$, and it drops to 49% at $\sigma_3/\sigma_c = 0.38$.

There are arguments on the adequacy of some of the test results that show significant influence of σ_2 on rock strength. Colmenares and Zoback (2002) concluded that σ_2 hardly

affected the strength of some rocks and in such cases the 2D M-C and H-B failure criteria fitted the test data equally well, or even better, than more complicated 3D failure criteria. Cai (2008b) clarified numerically that spalling and onion-skin formation in underground opening was mainly attributed to the existence of σ_2 , and σ_2 had a limited influence on rock strength when σ_3 was low. This statement tends to agree with field observation at the URL Mine-by tunnel (Cai and Kaiser, 2014) and theoretical prediction from Wiebols and Cook (1968). Accordingly, Cai (2008b) pointed out that a large percentage increase of rock strength due to σ_2 , as observed from some laboratory tests, was mainly attributed to the end effect.

End effect exists ubiquitously in rock laboratory tests (Fig. 2), and it is initiated under compression due to the elastic mismatch between the rock specimen and the metal platens and the friction between them (Hoskins and Horino, 1968; Choi et al., 1996). It is practically impossible to exclude the end effect completely from rock property testing, even if anti-friction measures are employed (Brady, 1971). For instance, the relation between end effect and slenderness effect is a well-studied subject from the 1960s to the 1990s (Babcock, 1969; Brady, 1971; Kotsovos, 1983; Tang et al., 2000). Laboratory studies on the slenderness effect show that the use of lubricants to minimize the end effect is sometimes not effective (Meikle and Holland, 1965; Pellegrino et al., 1997; Liang et al., 2015).

End effect plays different roles in affecting the rock strength in different laboratory tests. In uniaxial compression test, the end effect has a large influence on the rock strength when the specimen is squat (Fig. 2) (Bieniawski and Bernede, 1979). In conventional triaxial compression test, Mogi (2007) concluded that the end effect in triaxial compression test can be greatly reduced with the increase of confinement. There is still a lack of systematic investigation on how the end effect affects rock strength in true triaxial compression tests. Compared with the

conventional triaxial compression test, which employs hydraulic pressure to apply lateral confinement and hence has very little or no friction-induced constraint at the lateral surfaces of the specimen, friction exists at the lateral rock specimen-metal platen contacts in true triaxial compression test because metal platens are normally used to apply either σ_2 or both σ_2 and σ_3 . Thus, the end effect's influences on rock strength between these two types of triaxial compression tests are different. More importantly, the end effect usually becomes more significant when the specimen is squatter ($H/W < 2$). Fig. 3 illustrates that the ratio of the length on the contact where σ_2 is applied (l_1) to the length on the contact where σ_3 is applied (l_2) is 1 in most true triaxial compression tests. This is another reason that the end effect in true triaxial compression test cannot be completely omitted due to the shape of the specimens.

Accurate estimation of rock strength in a true triaxial stress state is driven by increased tunneling activities at depth. It is thus necessary to study the influence of end effect on rock strength in true triaxial compression test. In this study, we propose that the end effect exists in true triaxial compression test and it can influence true triaxial compression test results and thereby the interpretation of the actual σ_2 effect. It is hypothesized that the confined zones, resulted by the end constraint near the rock specimen-steel platen contacts where σ_2 is applied, can increase the actual σ_3 and hence the rock strength. Furthermore, enlightened by some indirect evidences, it is reckoned that different deformation behaviors of rocks can result in different levels of end constraint near the contacts where σ_2 is applied, thus the observed σ_2 effect can be different. Additionally, in true triaxial compression tests, the strengthening effect of end constraint on rock strength may be more significant at low confinement and so is the observed σ_2 effect. To verify our hypothesis, previous laboratory test results are used for model

119 calibration, and a numerical experiment using the ABAQUS/Explicit Finite Element Method
120 (FEM) tool is carried out.

121 Numerical simulation of true triaxial compression test

122 *Selection of published true triaxial compression test results*

123 True triaxial compression test has become popular in recent years and there are a number of true
124 triaxial compression test apparatuses for soil, concrete and rock testing (Shi et al., 2012). It is
125 necessary to select test results with precaution for this numerical study. Firstly, the reliability of a
126 true triaxial compression test apparatus should be well recognized and the tests should be
127 conducted with a high level of repeatability under carefully controlled test conditions. Secondly,
128 test results with various rocks should be considered to ensure the validity of this numerical
129 experiment.

130 Basing on above considerations, we have chosen two types of true triaxial compression
131 test apparatuses and their corresponding test results. The first one is the Mogi-type machine,
132 which had been used to test seven types of rocks. We choose the test results of Solnhofen
133 limestone (Mogi, 2007), which shows clearly the influence of σ_2 on rock strength. We also
134 choose the test data of Takahashi and Koide (1989), who designed a true triaxial compression
135 test apparatus (a new Mogi-type machine) slightly different from Mogi's design to accommodate
136 larger specimens. The second type of apparatus was developed by Chang and Haimson (2000),
137 and we call it the Haimson-type machine. The major difference between these two types of
138 apparatuses is that the pistons of the Mogi-type machine that apply two perpendicular principal
139 stresses σ_1 and σ_2 are positioned in the vertical direction, while a biaxial apparatus is positioned
140 horizontally in the Haimson-type machine to apply σ_1 and σ_2 . The test results of KTB

amphibolite and Westerly granite by Chang and Haimson (2000), which are well known in the field of true triaxial compression testing, are selected. Thus, representative test data of igneous (granite), sedimentary (limestone, shale), and metamorphic (amphibolite) rocks are all included in this study.

Mohr-Coulomb failure criterion and modeling strategy

The research focus is the influence of end effect on true triaxial compression test results. Because the end effect inevitably exists in a true triaxial compression test, 3D empirical failure criteria (Mogi, 1967; Al-Ajmi and Zimmerman, 2005) developed from true triaxial compression test results might also contain rock strength strengthening due to the end effect. Therefore, a rock model that excludes the influence of σ_2 effect on rock strength is preferable to study the contribution of end effect to the observed σ_2 effect. As a result, the M-C failure criterion, which does not consider σ_2 , is used to define the rock strength while the actual contact condition of rock specimen in a true triaxial compression test is modeled explicitly.

The 2D M-C failure criterion is one of the most commonly used failure criteria in rock engineering. As a failure criterion with mathematical simplicity, the physical meaning of the material parameters is clear. The rock strength of the M-C failure criterion is defined by the cohesive strength component and the frictional strength component (Martin, 1997; Hajiabdolmajid et al., 2002; Mogi, 2007). Despite one of its shortcomings, i.e., the linear nature of this failure criterion which may make it difficult to predict rock strength accurately when the applied confinement is high (Hoek and Brown, 1997; Jaeger et al., 2007), the strength parameters of the M-C failure criterion can be readily obtained or calibrated by performing simple rock property testing. More importantly, unlike other materials, rocks exhibit a confinement-dependent mechanical behavior, and rock failure under a moderate confinement is typically

characterized by an inclined shear failure plane relative to the σ_1 loading direction (Hudson and Harrison, 2000). The M-C failure criterion depicts the characteristic of rock strength and failure type under confinement conditions satisfactorily. The shortcoming of the M-C failure criterion for representing nonlinear failure envelopes of rocks in the σ_1 – σ_3 space can be overcome by restricting confinement within a limited range of interest (Pariseau, 2007; Mogi, 2007; Labuz and Zang, 2012; Brady and Brown, 2013).

The modeling strategy of using a simple failure criterion can isolate the end effect in the observed σ_2 effect. As it is hypothesized in this study, the end effect in a true triaxial compression test can result in confined zones similar to that in an uniaxial compression test and enhance the actual σ_3 of the rock and hence its strength. By increasing σ_2 in a true triaxial compression test, if it is verified that a material whose strength is only σ_3 -dependent experiences a strengthening due to the end effect, then a both σ_3 and σ_2 dependent material, which is more likely being the real material property of rocks, should be able to experience at least the same or even greater strengthening with the same strengthening effect due to end constraint. This is because that for rocks which are inherently confinement-dependent, the confinement they are subjected to might not be exclusively caused by σ_3 loading. Instead, the confinement might be influenced by σ_2 loading, leading to a combined influence of both σ_3 and σ_2 on rock strength. As a result, the M-C failure criterion is considered suitable for the proposed investigation.

Friction at rock specimen-steel platen contacts

The coefficient of friction (μ) at rock specimen-steel platen contacts is an important parameter that determines the end effect in rock laboratory tests (Gaffney, 1976). In this study, a set of tilt tests was performed to determine a reasonable range of μ values for rock specimen-steel platen

contacts. For a polished, smooth and dry rock specimen-steel platen contact without any lubricants, the average tilting angle is about 15° , which gives a static μ of 0.27.

In true triaxial compression tests, friction-reducing materials are routinely employed to reduce the end constraints. Mogi (2007) employed Teflon sheets and thin copper shims as lubricants. Takahashi and Koide (1989) used thin copper sheets and Teflon sheets with silicon grease. Haimson and Chang (2000) inserted thin copper shims and stearic lubricant at the rock specimen-steel platen contacts.

According to the aggregate state of the material zones involved, friction states can be classified as: solid friction, boundary friction, mixed friction, and fluid friction (Grote and Antonsson, 2009). Solid friction acts at direct contacts where material zones exhibit solid properties; boundary friction refers to the boundary layers on the contacts, each consists of a molecular film coming from a lubricant; fluid friction occurs when lubricating film is created between the friction body surfaces to reduce wear; mixed friction is a mixed form of boundary and fluid frictions. Apparently, the friction state at a lubricated specimen-platen contact is neither a solid friction nor a fluid friction state. According to the Handbook of Mechanical Engineering (Grote and Antonsson, 2009), the μ value ranges for boundary and mixed frictions are 0.1 – 0.2 and 0.01 – 0.1, respectively.

For lubricated rock specimen-steel platen contacts, it is reported that the μ values can be as low as 0.024 (He et al., 2014) and as high as 0.1 (Rashed and Peng, 2015), or even 0.39 in some cases (Hawkes and Mellor, 1970). A μ value of 0.05 is reported in Labuz and Bridell (1993) and a μ range of 0.04 ± 0.003 is given in the Handbook on Mechanical Properties of Rocks by Vutukuri et al. (1974). Accordingly, friction at lubricated specimen-platen contacts can be in a

208 mixed friction state. Hence, $\mu = 0.05$ to 0.1 may seem to be an appropriate range to represent
209 friction at lubricated rock specimen-steel platen contacts.

210 It must be pointed out that the loading conditions in sliding tests (or direct shear test)
211 designed specifically to measure the coefficient of friction and that in rock laboratory strength
212 tests are different. Different testing arrangements can result in different test results. For instance,
213 instead of employing sliding tests to investigate the μ values for lubricated rock specimen-steel
214 platen contacts, Hawkes and Mellor (1970) first applied a torque to a cylinder steel block, which
215 was atop a hollow sandstone specimen, and then gradually relaxed the axial load concentrically
216 applied to the rock-steel system to measure the μ values under different contact conditions. The
217 obtained μ values for the lubricated specimen-platen contacts were noticeably higher than that
218 obtained by the sliding tests.

219 It is stated in the Handbook of Mechanical Engineering that “friction does not represent a
220 constant property of a material but rather depends on variables, e.g., on the load and the elements
221 involved in the friction process with their properties and interactions.” The applied normal stress
222 on the sliding plane in a sliding test is not high, usually equivalent or slightly higher than the
223 pressure caused by the weight of the sliding body. In laboratory rock strength tests, the applied
224 normal stress on lubricated contacts is normally much higher than that used in sliding tests.
225 Therefore, the μ values of the lubricated specimen-platen contacts under high normal stresses
226 could be different than that revealed from the sliding tests and the effectiveness of using
227 lubricants in mitigating the end effect needs further investigation.

228 In this numerical experiment, acknowledging that all the considered true triaxial
229 compression tests had employed anti-friction measures, base values of μ in the range of 0.05 to
230 0.1 at the rock specimen-steel platen contacts were used in the modeling. To isolate the actual

end effect as well as the potential σ_2 effect due to the end effect from true triaxial compression test results, a minimum μ value of 0 was used. Finally, to appreciate the fact that the actual μ value under high normal stresses for some rocks might be different from that obtained by sliding tests, a maximum μ value of 0.2 was used.

FEM models

The ABAQUS/Explicit FEM tool is considered suitable for this numerical experiment. Firstly, the potential increase of rock strength due to the end effect is the research focus and a precise modeling the fracture process of rocks using Discrete Element Method (DEM) is not necessary. Rock failure criteria like the M-C failure criterion that can be used to predict rock strength is available in the tool. Secondly, the tool can simulate complex contact behaviors for the rock specimen-steel platen contacts at a low computation cost. In addition, the explicit algorithm in the tool is preferred to solve nonlinear material and contact problems.

3D simulation models of cylinder specimens in uniaxial and conventional triaxial compression tests, and a square prism specimen in true triaxial compression test, as well as their corresponding loading conditions are shown in Fig. 4a, b, and c, respectively. Arrows in these figures show the direction of the principal stresses. Thick orange arrows indicate that the principal stress is applied by steel platens and thin blue arrows indicate that the principal stress is applied by hydraulic pressure. In the modeling, the applied σ_3 and σ_2 are pressure controlled, which means that once the applied principal stress reaches an assigned magnitude, the pressure will be hold constant on the applied surface. The applied σ_1 is displacement controlled, which means that σ_1 is applied at a constant strain rate.

The diameter of the cylinder and the width of the square prism specimens are 50 mm and the heights of the specimens are 100 mm. This gives a height to width ratio (H/W , or H/D for cylinder specimens) of 2, which is the same as that used in the laboratory tests. The top and bottom steel platens are 51 mm in width (or diameter) and 25 mm in height (Fig. 4a – c). To honor the laboratory test conditions, steel platen loading instead of hydraulic pressure loading is used to apply σ_2 in the true triaxial compression test simulations. Two lateral steel platens used for applying σ_2 are 50 mm in width, 98 mm in height, and 25 mm in thickness (Fig. 4c). Steel properties ($E = 200$ GPa, $\nu = 0.3$) are assigned to the platens.

The FEM models of a cylinder specimen in both uniaxial and triaxial loading tests ($\sigma_2 = \sigma_3 \geq 0$) and a square prism specimen in true triaxial compression test are presented in Fig. 5. The total numbers of hexagonal elements in the two specimens are 8844 and 9537, which is relatively fine for the stress modeling. The slight difference in element numbers between the two specimens has little influence on their peak strengths, and this is verified by examining the modeling results of rock strengths. Moreover, the FEM geometrical models for different rock types under different true triaxial stress states are all the same (Fig. 5b).

Calibration of material parameters

Rock properties need to be calibrated first to carry out the numerical experiment. The rocks calibrated in this study are hard rocks with UCS over 100 MPa, some are over 200 and 300 MPa (Section “Selection of published true triaxial compression test results”). The stress–strain curves of the hard rocks are relatively linear in the pre-peak deformation stage (Chang and Haimson, 2000; Haimson and Chang, 2000; Mogi, 2007), especially before reaching the crack damage threshold at a stress level of approximately 0.7 to 0.8 times of the peak strength where hard rocks

basically exhibit elastic behavior. Therefore, the pre-peak deformation behavior of the rocks in compression is simplified as linear elastic, as suggested by Fang and Harrison (2001) and Hoek (2007). Young's modulus and Poisson's ratio are important elastic parameters to characterize the deformation behavior of rocks, and their values can be readily obtained from the laboratory test results provided in the origin works. Table 2 presents the physical and deformation parameters of the rocks considered in the study.

The M-C failure criterion is used to determine the peak strengths of the rocks. The M-C strength parameters – cohesion (c) and friction angle (ϕ) – can be calibrated based on the relations between σ_1 and σ_3 obtained from conventional triaxial test results. The M-C failure criterion is a linear failure criterion that depicts a linear relation between σ_1 and σ_3 . However, the relation between σ_1 and σ_3 shown in conventional triaxial tests is nonlinear when the rock specimens are subjected to high confinements (Section “Mohr-Coulomb failure criterion and modeling strategy”). Thus, if only one set of M-C strength parameters are used to characterize the rock behavior, a mismatch between the numerical and experimental results may occur, especially when the range of confinement being simulated is large (e.g., σ_3 of 0 to 150 MPa for KTB amphibolite). Therefore, to characterize the rock behaviors well under different applied σ_3 in the true triaxial compression tests, specimens under each applied σ_3 was assigned with different sets of M-C strength parameters, calibrated based on the conventional triaxial test results in a range of confinements which are close to the applied range of σ_3 .

Table 3 to Table 6 present the M-C strength parameters of KTB amphibolite, Westerly granite, Solnhofen limestone, and Yuubari shale, respectively. The fitting equations based the test data ($\sigma_1, \sigma_2 = \sigma_3$) at different applied σ_3 give the linear form of the M-C failure criterion in

the σ_1 – σ_3 space. Thus, the M-C strength parameters, the cohesion and the friction angle, at each applied σ_3 can be calibrated accordingly.

The strain-softening behavior was used to model the post-peak deformation behavior of rocks. The evolution process of cracking in the post-peak deformation stage leads to a significant cohesion loss (Martin, 1997). The cohesion loss can be modeled by the M-C failure criterion by degrading the rock's cohesive strength as a function of plastic strain. Our numerical experiment and laboratory true triaxial compression test focus on the peak strength of rocks, not the post-peak deformation behavior. The strain-softening model is for the post-peak deformation behavior modeling and it has no impact on the peak strength of rocks (Kias and Ozbay, 2013; Hemami and Fakhimi, 2014; Xu and Cai, 2015). In addition, there are no solid laboratory test results of complete stress–strain curves of rocks in true triaxial compression test due to the limitation of test machines. Therefore, the post-peak deformation behavior of the rocks is not discussed in this study.

Simulation results

Deformation behavior

The deformation behavior of rocks obtained from numerical modeling fits the laboratory test data well, as long as the pre-peak deformation behavior of the rocks obtained in laboratory testing is relatively linear. For instance, Westerly granite is a fine-grained, uniform, nearly isotropic rock (Lockner, 1998). As illustrated in Fig. 6, the slope of the rock's pre-peak stress–strain curve is basically linear for $\sigma_2 = \sigma_3 = 60$ MPa. Using the same Young's modulus and Poisson's ratio obtained from the laboratory test (Table 2), the modeling results of both the deformation behavior and the peak strength agree well with the test results, indicating that the

numerical model is suitable for rock behavior modeling in uniaxial or triaxial stress states ($\sigma_2 = \sigma_3 \geq 0$).

Rock strength

Fig. 7 to Fig. 10 compare the modeling results with the laboratory test results for KTB amphibolite, Westerly granite, Solnhofen limestone, and Yuubari shale, respectively. As mentioned above in Section “Friction at rock specimen-steel platen contacts”, the exact μ value at the rock specimen-steel platen contacts that apply σ_2 are unknown in the laboratory tests and four values ($\mu = 0, 0.05, 0.1, 0.2$) have been used in the numerical simulation. The μ values beside the rock name indicate the coefficient of friction at the specimen-platen contacts in the modeling, for both σ_1 and σ_2 loadings.

The dash lines in Fig. 7 to Fig. 10 represent the modeling results of conventional triaxial compression tests. All modeling results fit the laboratory test results well. Hence, the modeling strategy of using the linear M-C failure criterion to predict the rock strength under given range of confinement ($\sigma_2 = \sigma_3 \geq 0$) is demonstrated. The applicability of the M-C failure criterion for predicting rock strength illustrated in this numerical experiment agrees with the conclusions drawn by other researchers (Pariseau, 2007; Labuz and Zang, 2012; Brady and Brown, 2013).

The solid lines in Fig. 7 to Fig. 10 represent the modeling results of true triaxial compression tests. All modeling results show that under each applied σ_3 , rock strength (σ_1) increases with the increase of σ_2 if the μ value is greater than 0. There is a measurable increase of rock strength for $\mu = 0.05$, a noteworthy increase of rock strength for $\mu = 0.1$, and a significant increase of rock strength for $\mu = 0.2$. It seems that the simulation results of all types of rocks fit the laboratory results well for $\mu = 0.2$, suggesting that the simulation captured one characteristic

of the observed σ_2 effect mentioned above, i.e., the observed σ_2 effect decreases with the increase of applied σ_3 .

If there is no end effect, i.e., the μ value is 0 at the specimen-platen contacts, the modeling results show that the specimen's strength is independent of σ_2 . This is because that the 2D M-C failure criterion was used and the increase of the specimen's strength is irrelevant of σ_2 . Note that there is a very small difference in the rock strengths given by the cylinder ($\sigma_2 = \sigma_3$) and square prism ($\sigma_2 > \sigma_3$) geometrical models. This is caused by the geometrical model difference, which is negligible as long as their loading conditions are the same ($\mu = 0$). For $\mu > 0$, the strengthening of the specimen is exclusively attributed to the end effect caused by σ_2 . As will be discussed in Section "Discussions", there is a legitimate σ_2 effect on rock strength. However, due to the way that loads were applied in laboratory true triaxial compression tests, the test results inevitably contain the influence of the end effect and a method to isolate the end effect from the test data is proposed in Section "Discussions".

Failure modes

Fig. 11 to Fig. 13 present typical failure modes by the numerical simulation ~~and the laboratory tests~~ of KTB amphibolite, Westerly granite, and Solnhofen limestone, respectively. The red zones in numerical 2D and 3D views represent the zones in which large plastic strains occur, which can be used to indicate strain localization. The viewing plane for all the three figures is the σ_1 - σ_3 plane (15° rotation of the σ_1 axis for the 3D view).

The simulation captures the characteristics of the failure modes of the true triaxial compression test results (Chang and Haimson, 2000; Haimson and Chang, 2000; Mogi, 2007).

With regard to the failure planes occurring on the σ_1 - σ_3 plane and striking in the σ_2 loading

direction, it is understood that σ_2 will constrain the failure planes in a way that the rock is easier to fail in a direction parallel to the σ_2 loading direction (Cai, 2008b). For the KTB amphibolite and the Westerly granite specimens, the numerical simulation results show conjugate fault planes but only one fault plane was observed in the laboratory test. This may be due to the reason that material heterogeneity was not considered in the numerical modeling. An approach had been adopted in the numerical work of Senent et al. (2013) by introducing a very small defect in the homogenous specimen to break the symmetry of the shear bands and a single fault plane was successfully captured.

Discussions

Relation between the σ_2 effect and the increase of actual σ_3

The increase of rock strength in the numerical modeling result is not directly related to the increase of σ_2 , because the actual σ_2 effect is deliberately excluded from the assigned material model. It is seen that friction mobilized at the specimen-platen contacts increases σ_3 in the specimen and as a result the specimen's strength is increased. Resultant displacements on planes cutting through the mid-height of the Westerly granite specimen in Fig. 14 show different modes of lateral expansion under three loading conditions. In the conventional triaxial compression test (Fig. 14a), the lateral expansions are relatively uniform and normal to the surface where confinement is applied ($\sigma_2 = \sigma_3 = 60$ MPa). In the true triaxial compression test, the expansions in the direction where σ_2 is applied are smaller than that where σ_3 is applied. This is because that it is easier for a specimen to expand in a direction in which it has less resistance. For the specimen under the same applied σ_3 (60 MPa), the lateral expansions in the direction where σ_3 is

applied increase with the increase of σ_2 , leading to a larger relative deformation and hence a greater end constraint at the contacts where σ_2 is applied (Fig. 14b and c).

Fig. 15 presents the contours of σ_3 on the side, top, and two vertical surfaces revealed from a quarter cut of the specimen at peak load. Specifically, red zones, which represent a range of σ_3 that is very close to the applied σ_3 (60 MPa), can be focused to evaluate the disturbance of the actual σ_3 distribution due to the end effect. For the specimen under conventional triaxial loading (Fig. 15a), because hydraulic pressure loading and pressure control are employed to apply confinement, $\sigma_3 = 60$ MPa is maintained around the lateral surface of the cylinder specimen. Therefore, the red zone is uniformly distributed and consists of the largest percentage of the specimen volume. In comparison, even though hydraulic pressure loading and pressure control are employed to apply σ_3 in true triaxial loadings, $\sigma_3 = 60$ MPa is maintained only on the contacts where σ_3 is applied. In fact, there are more non-uniform distributions of σ_3 observed on the contacts where σ_2 is applied and inside the specimen (Fig. 15b and c). Moreover, the transition of the contour zones in the true triaxial loadings indicates that the actual σ_3 is increased with the increase of σ_2 .

It is inferred that the end constraint is activated on the specimen-platen contacts where σ_2 is applied because of the relative deformations of the specimen and the platens. The end constraint acts on the contacts and inside the specimen in a direction coinciding with the σ_3 direction, the actual σ_3 is thus increased in the specimen, leading to an increase of the specimen's strength. End constraint acting on the specimen-platen contacts can be increased as an increase of either the contact roughness (μ value) or the normal stress (σ_2). As a result, the specimen's strength is increased with the increase of σ_2 , even though the assigned specimen strength is

independent of σ_2 . In this numerical experiment of true triaxial compression test, the influence of end effect on the increase of rock strength is studied. It is found that as long as the end effect exists (i.e., the μ value is greater than 0), rock strengthening due to the end effect exists as well. In the laboratory test results, this strength increase due to the end effect was not excluded and it constituted part of the observed σ_2 effect.

Complementary evidences related to end effect in true triaxial compression tests

Some rocks show a strength increase due to σ_2 while others do not, and different end constraints initiated by different deformation behaviors of rocks could be the culprit. Using the same true triaxial compression test apparatus and testing method, Chang and Haimson (2005) found that the Long Valley Caldera rocks did not exhibit any meaningful σ_2 effect. After examining the deviatoric stress ($\sigma_1 - \sigma_3$)–volumetric strain ($\Delta V/V$) curves and the scanning electron microscope results, we notice that the strength independency on σ_2 of the Long Valley Caldera rocks is due to the non-dilatant deformation behavior of the rocks.

Fig. 16 show the deviatoric stress–volumetric strain curves for KTB amphibolite and Long Valley Caldera metapelite in true triaxial compression tests (Chang and Haimson, 2000; Chang and Haimson, 2005). It is seen that even though these two metamorphic rocks were tested by the same true triaxial compression test apparatus, their deformation behaviors are different. The deviatoric stress–volumetric strain curve of the amphibolite is nonlinear, indicating that dilation develops gradually when the deviatoric stress is above 700 MPa, while that of the metapelite is almost linear to the peak stress, indicating that there is an absence of dilation before peak load. The reversal point in the amphibolite’s deviatoric stress–volumetric strain curve

marks the onset of unstable crack growth (i.e. crack damage threshold) that usually corresponds to a stress level of approximately 0.7 to 0.8 times of the rock strength (Zhao and Cai, 2010). From this point, the plastic volumetric strain rate outnumbers the elastic volumetric strain rate, leading to a substantial growth of microcracks and dilation. Hence, for an amphibolite specimen that experiences dilatancy in compression, the plastic volumetric strain resulting from the dilation contributes to a large lateral expansion of the specimen before the peak load is reached. This large lateral deformation of the rock led to high constraint from the loading platens and thus the strength increase due to σ_2 was large.

Enlightened by our modeling results – the lateral expansion in the mid-height of specimens shown in Fig. 14 and the disturbance of actual σ_3 distribution in specimens in Fig. 15 – it becomes clear that if there is little lateral expansion (e.g., those two Long Valley Caldera rocks), there will be very small relative deformation at the rock specimen-steel platen contacts and the end effect is thus small.

Using ABAQUS, we calibrated some true triaxial compression test results and our modeling results suggest that the end effect due to σ_2 can result in a strength increase even if the rock strength used in the numerical modeling is independent of σ_2 . Shi et al. (2012) used FLAC3D to simulate the loading boundary effects in true triaxial compression test and revealed that the end effect can result in an apparent σ_2 effect. Gerstle et al. (1978) and Gerstle et al. (1980) carried out true triaxial compression tests on concrete specimens made from the same material composition and concluded that the σ_2 effect varied considerably due to the different loading methods and anti-friction measures. Therefore, the point highlighted in our study, i.e., the end effect can result in a different interpretation of the σ_2 effect on rock strength, is supported by the experimental results of Gerstle et al. (1978) and Gerstle et al. (1980). Uniaxial

compression tests using well-polished, high strength Beishan granite (Zhao et al., 2012; Zhao et al., 2013) revealed that even anti-friction measures were employed, rock strength was noticeably increased if the H/D ratio of the cylinder specimen was decreased from 2 to 1. Therefore, it is practically impossible to neglect the end effect in true triaxial compression test results because the H/W ratio in the σ_2 direction is normally less than 1 (0.5 in most tests).

Contribution of end effect to observed σ_2 effect

It is numerically proved that when testing a rock-like material whose strength is independent of σ_2 , there is an apparent σ_2 effect caused exclusively by the end effect. The next step is to confirm that the apparent σ_2 effect constitutes part of the observed σ_2 effect in previous laboratory test results. Although the actual μ values in previous true triaxial compression tests are unknown, the μ values can never be 0 and a range of 0.05 to 0.1 seems to be appropriate for lubricated rock specimen-steel platen contacts (Section “Friction at rock specimen-steel platen contacts”). Accordingly, laboratory test results that show a significant σ_2 effect might contain an apparent σ_2 effect resulted from the end effect in a way similar to that revealed in our modeling results. It is worth noting that, as justified in Section “Mohr-Coulomb failure criterion and modeling strategy”, although a σ_3 -only-dependent material property might not be precise enough to characterize the actual rock strength property, it is sufficient to emphasize the contribution of the end effect to the observed σ_2 effect. The increase of rock strength due to the end effect in true triaxial compression test simulations based on a both- σ_3 -and- σ_2 -dependent material model should not be less than that based on a σ_3 -only-dependent material model.

In order to further verify the influence of the end effect on true triaxial compression test results, the modeling results of the apparent σ_2 effect needs to be added to the actual σ_2 effect

and compared with the observed σ_2 effect in previous laboratory test results. Approaches that can consider the actual σ_2 effect while excluding the end effect include theoretical approach (Wiebols and Cook, 1968) and numerical investigation using the DEM (Fjær and Ruistuen, 2002; Cai, 2008a; Schöpfer et al., 2013). DEM modeling employs micromechanical properties to capture the failure process of rocks and thereby avoids the discretion of selecting an empirical failure criterion that only reflects the macro-mechanical behavior of rocks. However, the micromechanical parameters used in DEM modeling are hypothetical and cannot be measured in laboratory tests. These parameters can only be obtained by trials-and-errors in the calibration process while they are changed systematically until a set of parameters yield a macro behavior that best fit the actual one (Potyondy and Cundall, 2004).

In this study, we choose the theoretically-based effective strain energy criterion developed by Wiebols and Cook (1968) instead of the DEM modeling approach to characterize the actual σ_2 effect. The effective strain energy criterion (Wiebols and Cook, 1968) was derived based on a hypothesis that the effective shear strain energy stores around Griffith cracks due to the sliding of crack surfaces, and rock failure occurs when the total effective shear strain energy, which depends on the coefficient of sliding friction (μ_s) of the crack surfaces, reaches a critical value. It predicts a strengthening of rock from increasing σ_2 followed by a strength plateau and a weakening of rock once σ_2 exceeds a certain magnitude, which was verified experimentally by Wiebols and Cook (1968) and later confirmed by other researchers (Mogi, 1971; Chang and Haimson, 2000). Because the σ_2 effect predicted by the effective strain energy criterion excludes the end effect, it can be considered as the “true” or the “actual” σ_2 effect.

All failure criteria have their limitations in representing the true physics of rock deformation under complex loading, and the effective strain energy criterion has with no

exception. Laboratory test results show that the failure envelopes of rocks in conventional triaxial compression tests are normally nonlinear, especially near the brittle-ductile transition zones (Mogi, 1974), but the failure envelopes of conventional triaxial compression tests predicted by the effective strain energy criterion are linear. A precise prediction of a rock's failure envelope in the conventional triaxial stress state is the benchmark to ensure the accuracy of the predicted σ_1 – σ_2 curve in the true triaxial stress state, because the conventional triaxial stress state is the origin of the σ_1 – σ_2 curve. Acknowledging the shortcoming of the effective strain energy criterion, only the failure envelopes in the low confinement range will be considered. In addition, it is preferable to choose rocks whose σ_1 – σ_3 relations in conventional triaxial compression tests are relatively linear. Another shortcoming of the effective strain energy criterion is that the μ_s value is difficult to be determined. Wiebols and Cook (1968) provided failure envelopes for five μ_s values (0.25, 0.5, 0.667, 0.85, and 1) and pointed out that there is a relation between μ_s and $\tan\phi$ (where μ_s and $\tan\phi$ are close to each other but $\mu_s < \tan\phi$). Accordingly, we choose the μ_s value based on the $\tan\phi$ value of a rock.

The failure envelopes of KTB amphibolite with applied σ_3 ranging from 0 to 60 MPa are used for validation. The calibrated ϕ values of KTB amphibolite in this applied σ_3 range (Table 3) are relatively constant compared with other rocks (Table 4 and Table 5), denoting relatively linear σ_1 – σ_3 relations. Furthermore, the calibrated $\tan\phi$ in this σ_3 range is slightly higher than $\mu_s = 1$. Therefore, the theoretical σ_2 effect predicted by the effective strain energy criterion for KTB amphibolite in the true triaxial compression tests can be obtained by assuming $\mu_s = 1$. Then, the influence of end effect on the observed σ_2 effect can be analyzed, by adding the modeling results, which quantify the end effect in the true triaxial compression tests, to the theoretical results that reflect the actual σ_2 effect only.

The combination of the numerical end effect and the theoretical σ_2 effect on rock strength is shown in Fig. 17 and compared with the experimental test results, where σ_1 , σ_2 , and σ_3 are normalized to σ_c . The lower bound (dash line) corresponds to the theoretical result of Wiebols and Cook (1968), which includes only the influence of σ_2 on rock strength for $\mu_s = 1$. The upper bound (solid line) corresponds to the results of strength increase due to both the actual σ_2 effect for $\mu_s = 1$ (according to Wiebols and Cook (1968)) and the end effect for $\mu = 0.1$ (obtained from our numerical modeling). Thus, the yellow highlighted zones between the upper bound and lower bound of the σ_1 – σ_2 curves represent the apparent σ_2 effect due to the end effect for a μ value ranging from 0 to 0.1.

Although there was strength variability in the test results of KTB amphibolite, most test data fell within the yellow highlighted zones, indicating that the end effect caused by σ_2 loading somehow constituted part of the observed σ_2 effect. Because a laboratory test result inevitably contains an apparent σ_2 effect if the friction at the rock specimen-steel platen contacts is not zero, 3D empirical failure criteria developed from such a true triaxial compression test result may overestimate the rock strength. Although the actual μ values in the previous laboratory tests were unknown, our study shows that the end effect exists in true triaxial compression test for $\mu > 0$ and the μ values in those tests could range from 0.05 to 0.1. Therefore, it is suggested that when using previous and future true triaxial compression test results, the influence of the end effect on the test results needs to be investigated. As illustrated in this study, with knowledge of the actual μ value at the rock specimen-steel platen contacts, the apparent σ_2 effect may be separated from the observed σ_2 effect. In this way, the actual σ_2 effect could be characterized for the development of 3D empirical failure criteria for rocks.

Recent laboratory work to minimize end effect and to characterize actual σ_2 effect

It was proposed in Section “Contribution of end effect to observed σ_2 effect” that with knowledge of the apparent σ_2 effect resulted by end constraint, it is possible to use previous true triaxial compression test data for the development of 3D empirical failure criteria of rocks. However, this approach should be applied individually to each laboratory test because the coefficients of friction at the rock specimen-steel platen contacts were not the same in the tests.

An alternative approach is to decrease the friction at the rock specimen-steel platen contacts to a level where the contribution of the end effect to the observed σ_2 effect is very small such that the obtained test data can be used directly to develop 3D empirical failure criteria. To achieve this goal, we must ensure that the end effect has been greatly reduced. Based on our modeling results, we conclude that the strengthening of rock strength due to the end effect for a μ value ranging from 0.05 to 0.1 cannot be neglected. Hence, the coefficient of friction at the rock specimen-steel platen contacts must be reduced to a value much less than 0.05. In addition, the characteristics of the obtained test results should be somehow in accordance with that predicted by theoretical results (e.g., Wiebols and Cook, 1968).

A series of laboratory tests were carried out at Northeastern University (China) to study how friction at the rock specimen-steel platen contacts can be reduced. Test specimens were prepared using Linghai granite samples collected from a quarry site in Liaoning Province, China. Linghai granite is homogeneous and isotropic with a high strength. Using a polishing machine with a very high machining accuracy, the specimen ends were ground flat and polished down to ± 0.01 mm. To prevent the anti-friction material from flowing into the rock specimen, a copper sheet of 0.02 mm thickness was put between the specimen and the platen. Then, friction tests

with increasing normal forces were conducted to study the effectiveness of two anti-friction measures, one by using Teflon and the other by using a mixture of stearic acid and Vaseline at a 1:1 ratio, recommended by Labuz and Bridell (1993). The black line in Fig. 18 shows the μ values of the specimens' polished surfaces without lubricant. Because the specimens' end and the platen's end surfaces had been polished smoothly, the obtained average μ value of 0.15 is low, which is lower than that of some contacts smeared with lubricants (Section "Friction at rock specimen-steel platen contacts"). The results shown in Fig. 18 clearly indicate that the mixture of stearic acid and Vaseline at a 1:1 ratio in combination with a thin copper sheet can reduce the μ value of the lubricated rock specimen-steel platen contact to about 0.02. Referring to the numerical simulation results in Section "Rock strength", it can be seen that the end effect caused by $\mu = 0.02$ is small and can be neglected.

True triaxial compression tests were conducted using a newly developed true triaxial compression test apparatus (Feng et al., 2016). This novel test apparatus was designed and fabricated on the basis of the Mogi-type apparatus with many improvements implemented to address key issues such as off-center suppression, loading gap removal, and more importantly, end effect reduction (Feng et al., 2016). Linghai granite specimens, with a rectangular prismatic shape of $50 \times 50 \times 100 \text{ mm}^3$, were used in the test. The effect of σ_2 on rock strength at low σ_3 (up to 50 MPa) was focused in this laboratory test. The mixture of stearic acid and Vaseline at a 1:1 ratio and thin copper sheets were used to reduce friction at the contacts. Additionally, compared with some previous true triaxial compression tests that barely reached the strength plateau in the σ_1 - σ_2 curves, the full range of σ_2 loading ($\sigma_3 \leq \sigma_2 \leq \sigma_1$) was attempted to obtain the complete σ_2 effect curves. The laboratory test results are shown in Fig. 19.

The overall characteristics of the observed σ_2 effect on the strength of Linghai granite agree with that of previous laboratory test results, showing an increase of rock strength with the increase of σ_2 before reaching the strength plateau and then a decrease of rock strength as σ_2 further increases. The complete strength drop from the strength plateau to the maximum loading range of σ_2 ($\sigma_2 = \sigma_1$), a characteristic predicted by Wiebols and Cook (1968) but normally missed in some previous laboratory test results, was captured in this test. Additionally, we noticed a characteristic of the observed σ_2 effect on Linghai granite that differs from previous laboratory results. The σ_2 effect on rock strength as observed in previous test results is normally the greatest at low σ_3 and drastically decreased with the increase of σ_3 (Table 1); however, in the Linghai granite test results the rates of rock strength increase at the peak due to σ_2 are similar at different σ_3 (Table 7), which are about 51% to 56% in all cases. It is numerically proved in Sections “Rock strength” and “Contribution of end effect to observed σ_2 effect” that the apparent σ_2 effect resulted by the end constraint can lead to an overestimation of the actual σ_2 effect. On the other hand, the theoretical results (Wiebols and Cook, 1968) that exclude the end effect show a very small decrease of the peak strength increase due to σ_2 with the increase of σ_3 (Table 8, with different μ_s). Therefore, these indirect evidences, supported by numerical and theoretical results, suggest that the anti-friction measure adopted in our tests is effective. In other words, the σ_2 effect shown in the test results is dominantly the actual σ_2 effect. The apparent σ_2 effect due to the end effect is small in our test results.

End-effect-induced σ_2 effect at different σ_3 levels

Based on previous discussions, the phenomenon of the drastic decrease of the σ_2 effect with the increase of σ_3 in the test results found in literature (Table 1) can be examined. The numerical

experiment demonstrates that the end-effect-induced σ_2 effect decreases with the increase of σ_3 (Fig. 7 to Fig. 10). Fig. 20 below illustrates how the end-effect-induced σ_2 effect is related to σ_3 . As explained in Section “Relation between the σ_2 effect and the increase of actual σ_3 ”, the end effect can increase the overall σ_3 of the specimen, making it higher than the applied σ_3 ($\Delta\sigma_3$). In other words, the end-effect-induced σ_2 effect (pink coordinate systems) is manifested by the increase of $\Delta\sigma_3$ with the increase of σ_2 , and that in turn leads to the increase of rock strength ($\Delta\sigma_1$). In the numerical modeling, the end-effect-induced σ_2 effect depends on the friction angle when the M-C failure criterion is used. The higher the friction angle is, the greater the end-effect-induced σ_2 effect will be.

The friction angle calibrated (Table 3 to Table 6) decreases with the increase of σ_3 (blue and green dash lines) and this matches well the nonlinearity of the σ_1 – σ_3 failure envelopes (grey curve). Therefore, all the modeling results with various μ values reflect that the end-effect-induced σ_2 effect decreases with the increase of σ_3 . Note that the σ_1 – σ_3 failure envelope representing conventional triaxial compression tests (grey coordinate system) is plotted in Fig. 20. This is because that true triaxial compression tests ($\sigma_2 \geq \sigma_3$) normally begin with a stress state of $\sigma_2 = \sigma_3$; true triaxial compression test results are obtained by increasing σ_2 from the applied σ_3 .

The calibrated friction angle (blue dash lines) cannot always reflect precisely the slopes of the true σ_1 – σ_3 failure envelope at different applied σ_3 levels. For instance, at very low σ_3 levels especially at $\sigma_3 = 0$, the calibrated friction angle can underestimate the slope of the failure envelope. On the other hand, at very high σ_3 levels, the calibrated friction angle can overestimate the slope of the failure envelope. Hence, the simulation results (Fig. 7d) at intermediate σ_3 levels

match laboratory test results well (green dash lines), while at very low and very high σ_3 levels the modeling results are not equally good.

We consider that as long as the trend of the end-effect-induced σ_2 effect revealed from the numerical modeling confirms our research hypothesis (the end-effect-induced σ_2 effect is large), there is no need to require that the modeling results match the test results perfectly. In fact, a perfect match between the modeling results and the test results can only suggest that the σ_2 effect observed in the laboratory test results are completely caused by the end effect. This implication is not correct because both theoretical and laboratory studies have shown that there is a legit σ_2 effect to rock strength. More importantly, the differences between the modeling and the test results at very low and very high σ_3 levels support the viewpoint: the end effect affects true triaxial compression test results; the σ_2 effect decrease with the increase of σ_3 .

It was experimentally confirmed by Mogi (2007) that rock strength increase due to the end effect increases with the decrease of σ_3 . Hence, it can be interpreted that in the previous laboratory test results, the end-effect-induced rock strength increase was high when the applied σ_3 was low, which resulted in a greater apparent σ_2 effect. The end effect became smaller at high applied σ_3 . Fig. 21 presents some conventional triaxial compression test results of rocks and their H-B fitting curves. Nonlinearity of the failure envelopes is observed ubiquitously in these rocks, illustrating that the same end-effect-induced $\Delta\sigma_3$ at a low σ_3 can result in a greater increase of rock strength $\Delta\sigma_1$ than that at a high σ_3 . Therefore, the end effect can result in a large apparent σ_2 effect when σ_3 is low and the observed σ_2 effect at low σ_3 in some previous test results might be exaggerated. It is suggested that if true triaxial compression test results contain an unknown influence of end effect on rock strength, the test results at high σ_3 , which are less influenced by the end effect, might be more appropriate for developing 3D empirical failure criteria for rocks.

Conclusions

Laboratory test results show that the use of lubricants cannot eliminate the end effect completely, especially when the rock specimen is squat in the σ_2 loading direction. The influence of end effect on rock strength in true triaxial compression test is studied using a numerical approach. The influence of σ_2 on rock strength is purposely excluded in the material model in the numerical study so that any increase of rock strength with the increase of σ_2 while the applied σ_3 is kept constant can be attributed to the end effect. It is seen from the simulation results that friction at the rock specimen-steel platen contacts has a large influence on rock strength, because the σ_3 in the specimen is increased due to the end constraint at the σ_2 loading contacts. Thus, the end effect can result in an apparent σ_2 effect that contributes to the observed strength increase due to σ_2 in true triaxial compression tests. This point can explain why some rocks show the σ_2 effect while others do not because different end constraints can be initiated by different rock deformation behaviors. This study made a contribution to better understanding true triaxial compression test results by decomposing the observed σ_2 effect on rock strength in laboratory tests into two parts: one from the end effect and the other from the actual σ_2 effect. The actual σ_2 effect can be explained using the theory of Wiebols and Cook (1968).

Considering that the end effect can result in an apparent σ_2 effect that may mislead the interpretation of the true triaxial compression test results, some suggestions are made for future study of the σ_2 effect. Firstly, previous laboratory test results are still useful in developing 3D empirical failure criteria for rocks. An improvement can be made by employing a reverse strategy to subtract the apparent σ_2 effect from the observed σ_2 effect, provided that the actual end effect in the true triaxial compression test results can be quantified. Otherwise, one can refer

to the test results obtained at high σ_3 , where the relative influence by the end effect is small. According to our recent laboratory work and supported by the numerical and theoretical results, it is seen that previous test results that showed a significant σ_2 effect at low σ_3 might include a large end effect. Hence, future study should focus on quantifying the actual σ_2 effect under low confinement conditions because it is important for rock engineering practices. Moreover, when conducting true triaxial compression test to study the σ_2 effect, attention should be paid to minimizing the end effect. Our recent laboratory work shows that using a novel test apparatus and effective anti-friction measures, the end effect can be decreased to minimize the apparent σ_2 effect such that the characteristics of the obtained σ_2 effect are very close to the true σ_2 effect.

Note that the conclusions drawn here are based on the numerical simulation results using a homogeneous model. Material heterogeneity of rocks may influence the failure modes. Using heterogeneous models, researchers have found that σ_2 has a small influence on rock strength (Cai, 2008b) when $\sigma_3 = 0$ and σ_2 is not high. Thus, future work needs to consider the heterogeneity of rock properties to quantify the end effect on rock strength in true triaxial compression tests.

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Figure captions

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Fig. 10. Simulation of true triaxial compression tests of Yuubari shale: (a) $\mu = 0$, (b) $\mu = 0.05$, (c) $\mu = 0.1$, (d) $\mu = 0.2$.

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907 different applied σ_3 levels (refer to the grey coordinate system).

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909 curves: (a) KTB amphibolite, (b) Westerly granite, (c) Solnhofen limestone, (d) Yuubari
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911 Mogi, 2007).

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Table 1. Quantified σ_2 effect in different rocks (data from Chang and Haimson, 2000; Haimson and Chang, 2000; Mogi, 2007).

KTB amphibolite	σ_3/σ_c	0.00	0.19	0.38	0.63	0.95	
	$(\sigma_1^* - \sigma_1) / (\sigma_2^* - \sigma_2)$	69%	70%	39%	45%	29%	
Westerly granite	σ_3/σ_c	0.00	0.10	0.19	0.30	0.38	0.50
	$(\sigma_1^* - \sigma_1) / (\sigma_2^* - \sigma_2)$	114%	103%	88%	59%	49%	58%
Dunham dolomite	σ_3/σ_c	0.09	0.17	0.25	0.32	0.40	0.47
	$(\sigma_1^* - \sigma_1) / (\sigma_2^* - \sigma_2)$	78%	63%	43%	42%	38%	32%
Solnhofen limestone	σ_3/σ_c	0.06	0.13	0.26			
	$(\sigma_1^* - \sigma_1) / (\sigma_2^* - \sigma_2)$	34%	33%	36%			
Yamaguchi marble	σ_3/σ_c	0.15	0.30	0.49			
	$(\sigma_1^* - \sigma_1) / (\sigma_2^* - \sigma_2)$	104%	79%	53%			
Mizuho trachyte	σ_3/σ_c	0.45	0.60	0.75	1.00		
	$(\sigma_1^* - \sigma_1) / (\sigma_2^* - \sigma_2)$	34%	30%	26%	25%		
Manazuru andesite	σ_3/σ_c	0.14	0.29	0.50			
	$(\sigma_1^* - \sigma_1) / (\sigma_2^* - \sigma_2)$	82%	54%	52%			
Inada granite	σ_3/σ_c	0.09	0.18	0.44	0.66	0.88	
	$(\sigma_1^* - \sigma_1) / (\sigma_2^* - \sigma_2)$	104%	71%	57%	65%	69%	
Orikabe monzonite	σ_3/σ_c	0.17	0.34	0.59	0.85		
	$(\sigma_1^* - \sigma_1) / (\sigma_2^* - \sigma_2)$	115%	80%	66%	67%		

Table 2. Physical and deformation parameters of the rocks.

Rock types	Density (kg/m ³)	Young's modulus (GPa)	Poisson's ratio	References
KTB amphibolite	2920	65	0.29	Haimson and Chang (2002)
Westerly granite	2700	60	0.28	Bhat et al. (2011)
Solnhofen limestone	2500	80	0.20	Renner and Rummel (1996)
Yuubari shale	2200	40	0.20	Gercek (2007)

Table 3. Calculation of the M-C strength parameters of KTB amphibolite (data from Colmenares and Zoback, 2002, original data from Chang and Haimson, 2000).

σ_3 level (MPa)	Data ($\sigma_1, \sigma_2 = \sigma_3$) (MPa)	Fitting equation	Cohesion (MPa)	Friction angle (°)
0	(158, 0) (176, 0) (410, 30)	$\sigma_1 = 8.4\sigma_3 + 176$	31	52
30	(158, 0) (410, 30) (702, 60)	$\sigma_1 = 9.1\sigma_3 + 151$	26	53
60	(410, 30) (702, 60) (868, 100)	$\sigma_1 = 6.4\sigma_3 + 318$	62	47
100	(410, 30) (702, 60) (868, 100) (1147, 150)	$\sigma_1 = 6.2\sigma_3 + 232$	48	46.5
150	(868, 100) (1147, 150)	$\sigma_1 = 5.6\sigma_3 + 310$	66	44

Table 4. Calculation of the M-C strength parameters of Westerly granite (data from Al-Ajmi and Zimmerman, 2005, digitized from Haimson and Chang, 2000).

σ_3 level (MPa)	Data ($\sigma_1, \sigma_2 = \sigma_3$) (MPa)	Fitting equation	Cohesion (MPa)	Friction angle (°)
0	(201, 0) (231, 2)	$\sigma_1 = 15.0\sigma_3 + 201$	26	61
20	(201, 0) (430, 20) (605, 38)	$\sigma_1 = 10.7\sigma_3 + 206$	33	56
38	(430, 20) (605, 38) (747, 60)	$\sigma_1 = 7.9\sigma_3 + 306$	54	51
60	(605, 38) (747, 60) (889, 77)	$\sigma_1 = 7.3\sigma_3 + 325$	60	49
77	(747, 60) (889, 77) (1012, 100)	$\sigma_1 = 6.6\sigma_3 + 385$	76	47
100	(889, 77) (1012, 100)	$\sigma_1 = 5.4\sigma_3 + 477$	101	44

Table 5. Calculation of the M-C strength parameters of Solnhofen limestone (data from Mogi, 2007).

σ_3 level (MPa)	Data ($\sigma_1, \sigma_2 = \sigma_3$) (MPa)	Fitting equation	Cohesion (MPa)	Friction angle (°)
20	(310, 0) (397, 20) (449, 40)	$\sigma_1 = 3.5\sigma_3 + 328$	88	34
40	(310, 0) (397, 20) (449, 40)	$\sigma_1 = 3.5\sigma_3 + 316$	84	34
60 *	(310, 0) (397, 20) (449, 40) (473, 60)	$\sigma_1 = 2.7\sigma_3 + 326$	98	28
80	(473, 60) (528, 80)	$\sigma_1 = 2.8\sigma_3 + 308$	93	28

* Conventional triaxial compression test data at $\sigma_3 = 60$ MPa diverges somewhat from the failure envelop; hence, calibration of the friction angle was adjusted accordingly.

Table 6. Calculation of the M-C strength parameters of Yuubari shale (data from Colmenares and Zoback, 2002, digitized from Takahashi and Koide, 1989).

σ_3 level (MPa)	Data ($\sigma_1, \sigma_2 = \sigma_3$) (MPa)	Fitting equation	Cohesion (MPa)	Friction angle (°)
25	(161, 25) (228, 50)	$\sigma_1 = 2.7\sigma_3 + 94$	29	27
50	(161, 25) (228, 50)	$\sigma_1 = 2.7\sigma_3 + 94$	29	27

Table 7. Quantified σ_2 effect on Linghai granite.

σ_3/σ_c	0.00	0.06	0.12	0.18	0.31
$(\sigma_1^* - \sigma_1)/(\sigma_2^* - \sigma_2)$	56%	52%	52%	53%	51%

Table 8. Quantified σ_2 effect predicted by the theory of Wiebols and Cook (1968) for different μ_s .

$\mu_s = 0.25$	σ_3/σ_c	0.00	0.20	0.40	0.60	0.80
	$(\sigma_1^* - \sigma_1)/(\sigma_2^* - \sigma_2)$	44%	43%	43%	43%	43%
$\mu_s = 0.50$	σ_3/σ_c	0.00	0.10	0.20	0.30	0.40
	$(\sigma_1^* - \sigma_1)/(\sigma_2^* - \sigma_2)$	67%	66%	65%	64%	64%
$\mu_s = 0.67$	σ_3/σ_c	0.00	0.05	0.10	0.15	0.20
	$(\sigma_1^* - \sigma_1)/(\sigma_2^* - \sigma_2)$	79%	78%	77%	75%	74%
$\mu_s = 0.80$	σ_3/σ_c	0.00	0.05	0.10	0.15	0.20
	$(\sigma_1^* - \sigma_1)/(\sigma_2^* - \sigma_2)$	87%	83%	80%	79%	78%
$\mu_s = 1.00$	σ_3/σ_c	0.00	0.05	0.10	0.15	0.20
	$(\sigma_1^* - \sigma_1)/(\sigma_2^* - \sigma_2)$	101%	88%	83%	81%	79%

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● Laboratory results - - Modeling results ($\sigma_2 = \sigma_3$) — Modeling results ($\sigma_2 > \sigma_3$)

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